1 Line Integral of scalar-valued functions

- The idea is similar to the one of an Integral over a two dimensioal Surface. As analogy you can imagine to calculate the mass of a wire from its density.
- Assume you have a wire whose density is not constant over its length. We have a function $C(t)$ which describes a certain point on the wire. How can you calcuate it's mass from its density?
- We can segment the wire into lots of smaller segments and calculate their mass according to the specific densities. The length of the i-th segment is defined as: $||c(t_i) - c(t_i - 1)||$.
  The densitiy for a specific point is defined as: $f(c(t_i))$
- The mass of a segment is just the Linesegment $\cdot$ its density:
  $f(c(t_i)) \cdot ||c(t_i) - c(t_i - 1)||$
  To calculate the total mass of the whole wire you just have to sum up all the masses of the segments: $\sum_i^n f(c(t_i)) \cdot ||c(t_i) - c(t_i - 1)||$
- To turn this into an integral we define: $\Delta t_i = t_i - t_{i-1}$. Now multiply and divide each term by Δt_i and obtain the more complicated looking expression:
  $\sum_i^n f(c(t_i)) \cdot ||c(t_i) - c(t_{i-1})|| = \sum_i^n ||\frac{c(t_{i-1}+\Delta t_i)-c(t_{i-1})}{\Delta t_i}|| \cdot \Delta t_{i-1}$
- You realize, that the term in the expression $||.||$ is actually the definition of the derivate off $c(t)$. The counter is nothing else than a function c(t) applied to an interval and the denominator is that exact interval. So if you apply it to a two-dimensional function it's nothing else than $\frac{\Delta y}{Deltax}$ which defines the secant.

  For comparison here is the definition of the definition of a Secant: $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$
- If we let $\Delta t_i \to 0$ and $n \to \infty$ the above Riemann sum converges to the integral: $\int_a^b f(c(t)) \cdot ||c'(t)||dt$ which is often denoted as $\int_c f ds$.

2 Introduction to a line integral of a vector field

- The only change to the chapter above is that now we want to integrate a vector valued curve function along a curve. They are usually represented by a vector field.

A very easy interpretation is to imagine the amount of work that a foce field does on a par

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