Gentrification index measure

There is no general accepted definition of "gentrification". The term was coined by British sociologist Ruth Glass in 1964. She was describing the displacement of lower-class worker residents in urban neighborhoods by middle-class people in Islington, London[1]. This vague and abstract definition makes it even harder to measure the extent of gentrification of a whole city quantitatively.

The idea behind getting a number for the "amount" of gentrification happening in a city is to merely focus on the change of the household income with respect to time. All other census data, which claim to measure gentrification, can be viewed as its effects according to [2]. Hence, we decided to divide a city into districts by virtue of the town subdivision (???), and compared the median household income per month (MHI) of each district from data 10 years apart. To assure that the increase is free from inflation and other economic growth factors we calculate the increase of MHI in each district $x_i = x(i = District)$ relative to the increase of the mean MHI of the whole city $\tilde{x}$. More precisely,

$$x_i = \frac{\mathrm{MHI}(i,2016) - \mathrm{MHI}(i,2007)}{\mathrm{MHI}(i,2016)}, \quad \tilde{x} = \frac{\widehat{\mathrm{MHI}}(2016) - \widehat{\mathrm{MHI}}(2007)}{\widehat{\mathrm{MHI}}(2016)}, \qquad (1)$$

where $\widehat{\ }$ denotes the average of MHI in a particular year over all districts i. Then we use $\tilde{x}$ as a reference for the increase x_i. Tab. 1 shows this "relative" increase of the MHI, $r_i = (x_i - \tilde{x})/\tilde{x}$, in case of Berlin. Furthermore, we are also interested in the change of the number of household $H_i = \mathrm{N}(i,2016)/\mathrm{N}(i,2007)$ in a certain district. This also influences gentrification. If, for instance, rental fees increase but the number of household decrease in a certain area, then we can assume that several cheaper households were displaced by a few expensive ones.

District	$\frac{\mathrm{N}(i,2007)}{1000}$	$\frac{\mathrm{N}(i,2016)}{1000}$	H_i	$\mathrm{MHI}(i,2007)$	$\mathrm{MHI}(i,2016)$	r_i
1	161.4	152.0	0.94	1825	2325	−0.03
2	119.3	124.4	1.04	1550	1775	−0.43
3	199.7	182.5	0.91	1575	2125	+0.16
4	125.9	133.0	1.06	1675	2025	−0.22
5	187.8	212.3	1.13	1425	1825	−0.01
6	189.4	183.5	0.97	1575	1975	−0.09
7	173.5	164.7	0.95	1175	1800	+0.56
8	218.8	214.9	0.98	1525	2150	+0.31
9	166.4	169.8	1.02	1350	1700	−0.07
10	141.9	159.5	1.12	1450	1825	−0.08
11	127.9	132.8	1.04	1525	1950	−0.07
12	128.9	134.6	1.04	1625	2050	−0.07

Table 1: Median household income per month in EUR in Berlin (city divided into districts according to Fig. 1) in years 2007 & 2016.

Figure 1: Map of Berlin's districts with random numbering. Image: © Increa

The next step is to define a measure for the distance of a district with respect to each other. A generic solution is to use the underling structure of an city, the districts itself as distance units. Therefore, we introduce the number of minimal district-border-crossings, which are necessary to get from district i to district j, as our distant measure. For example, the distance, denoted as $\langle \cdot, \cdot \rangle$, of district 1 to district 8 in Fig. 1 is equal to 3, i.e. $\langle 1, 8 \rangle = 3$. Hence, differences in the values r_i of district further apart contribute less to the gentrification index g_i of an particular district i.

To compute the distances of the districts from a map in this measure, we applied ... ??? (a technique from graph theory / machine learning) ...

The final step consists of putting the quantities $\langle \cdot, \cdot \rangle$ and r_i together in a way which results in a meaningful measure of gentrification. We propose that the following formula is a comprehensive quantity to measure the gentrification of a city,

$$G = \sum_{i=1}^{N} \frac{G_i}{H_i}, \quad G_i = \sum_{j=1,\, j\neq i}^{N} \frac{|r_i - r_j|^2}{\langle i, j \rangle^2}. \tag{2}$$

We call G and G_i the gentrification index of a city and the gentrification index of the city district i respectively. Due to the fact that gentrification is a rather subjective notion, and the fact that in the perspective of citizens it manifests itself the most when adjacent neighborhoods are compared, we take the square of our distance measure $\langle \cdot, \cdot \rangle$. This has the effect to soften the differences between two districts when they are far apart. The meaning of the numerator in Eq. (2) is clear when we write out the squares, $\left(x_i^2 - x_j^2\right)/\tilde{x}^2$. The numerator is smaller, if the mean MHI of the city $\tilde{x}$ is high. Then, two district with a big difference in the MHI $x_i - x_j$ are going to make G_i small, whereas on those where the

difference is small $\tilde{x}$ has no effect at all. With this feature we can take into account the overall increase of the MHI in a city. The gentrification index of an city with a low $\tilde{x}$ but a distinctive shift of citizens of different working-classes shall be especially big, as we can then assume that wealthier people haven't moved in from beyond the city at an above average rate.

Although, the above argumentation is sensible, we can't be sure if (2) is the best formula for the index. Thus, we also present some alternatives to G_i, and compare these to Eq. (2). Namely,

$$\begin{array}{rl} G_i' = & \sum_{j=1,\, j\neq i}^{N} \frac{|r_i - r_j|^2}{\langle i,j\rangle} \\ G_i'' = & \sum_{j=1,\, j\neq i}^{N} \frac{|r_i - r_j|}{\langle i,j\rangle^2} \\ G_i''' = & \sum_{j=1,\, j\neq i}^{N} \frac{|r_i - r_j|}{\langle i,j\rangle} \end{array} \quad . \tag{3}$$

Tab. 2 shows the correlation between the gentrification indexes in each district and the quantities r_i, H_i for the various types of G_i, and the gentrification index G as well. Based on this result we propose the formula in Eq. (2) as our best measure for the gentrification of a city.

G	8.85	11.20	18.52	23.54
corr $(\cdot,\cdot)$	G_i	G_i'	G_i''	G_i'''
r_i	0.39	0.35	0.33	0.28
H_i	0.77	0.78	0.85	0.86

Table 2: Correlation

References

[1] Ruth Glass (1964). London: aspects of change. London: MacGibbon & Kee.

[2] Steve Holland, Gentrification: Causes and Consequences, Journal of Lutheran Ethics, Vol. 16, Issue 1, Jan 2016