Mean Value Theorem Proof

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Fermat's theorem stated that if $f(c)$ is a local extrema of a differentiable function $f(x)$, then $f'(c) = 0$. WLOG, assume $f(c)$ is the local maximum, then for δ being positive value approaching 0, $\lim\limits_{\delta \to 0^+} f(c + \delta) \leq \lim\limits_{\delta \to 0^+} f(c)$,
$\lim\limits_{\delta \to 0^+} (f(c + \delta) - f(c)) \leq 0$
$\lim\limits_{\delta \to 0^+} \frac{f(c+\delta)-f(c)}{\delta} \leq 0$
$f'(c) \leq 0$
For δ being negative value approaching 0, let δ' be absolute value of δ, so
$\lim\limits_{\delta' \to 0^+} f(c - \delta') \leq \lim\limits_{\delta' \to 0^+} f(c)$
$\lim\limits_{\delta' \to 0^+} (f(c) - f(c - \delta')) \geq 0$
$\lim\limits_{\delta' \to 0^+} \frac{f(c)-f(c-\delta')}{\delta'} \geq 0$
$f'(c) \geq 0$
For both statements to hold true, $0 \leq f'(c) \leq 0$, $f'(c) = 0$. QED