

Derivative Lazy Proofs

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Here are some proofs or rules useful in taking derivatives

Addition Rule

$$y = f(x) + g(x)$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) + g(x+\Delta x) - f(x) - g(x)}{\Delta x}$$

$$y' = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} + \frac{g(x+\Delta x) - g(x)}{\Delta x} \right)$$

$$y' = f'(x) + g'(x)$$

Constant Multiplier

$$y = kf(x)$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{kf(x+\Delta x) - kf(x)}{\Delta x}$$

$$y' = k \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$y' = kf'(x)$$

Power Rule

$$y = x^n$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{nx^{n-1}\Delta x + C_2^n x^{n-2}\Delta x^2 + \dots + \Delta x^n}{\Delta x}$$

$$y' = \lim_{\Delta x \rightarrow 0} \left(nx^{n-1} + \Delta x(C_2^n x^{n-2} + \dots + \Delta x^{n-2}) \right)$$

$$y' = nx^{n-1}$$

Quotient Rule

$$y = \frac{f(x)}{g(x)}$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{\frac{f(x+\Delta x)}{g(x+\Delta x)} - \frac{f(x)}{g(x)}}{\Delta x}$$

$$y' = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x)g(x) - f(x)g(x+\Delta x)}{\Delta x g(x)g(x+\Delta x)}$$

$$y' = \lim_{\Delta x \rightarrow 0} \left(\frac{1}{g(x)g(x+\Delta x)} * \frac{f(x+\Delta x)g(x) - g(x)f(x) + g(x)f(x) - f(x)g(x+\Delta x)}{\Delta x} \right)$$

$$y' = \lim_{\Delta x \rightarrow 0} \left(\frac{1}{g(x)g(x+\Delta x)} \right) * \lim_{\Delta x \rightarrow 0} \left(\frac{f(x+\Delta x)g(x) - g(x)f(x) + g(x)f(x) - f(x)g(x+\Delta x)}{\Delta x} \right)$$

$$y' = \frac{1}{g(x)^2} \lim_{\Delta x \rightarrow 0} \left(\frac{g(x)(f(x+\Delta x) - f(x))}{\Delta x} - \frac{f(x)(g(x+\Delta x) - g(x))}{\Delta x} \right)$$

$$y' = \frac{1}{g(x)^2} \left(g(x) \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} - f(x) \lim_{\Delta x \rightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} \right)$$

$$y' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Chain Rule

$$y = f(g(x))$$

$$y' = \lim_{x \rightarrow a} \frac{f(g(x)) - f(g(a))}{x - a}$$

$$y' = \lim_{x \rightarrow a} \left(\frac{f(g(x)) - f(g(a))}{g(x) - g(a)} * \frac{g(x) - g(a)}{x - a} \right)$$

Let $u = g(x)$, and $a' = g(a)$

$$y' = \lim_{x \rightarrow a} \frac{g(x) - g(a)}{x - a} * \lim_{u \rightarrow a'} \frac{f(u) - f(a')}{u - a'}$$

$$y' = f'(u) * g'(x)$$

$$y' = f'(g(x)) * g'(x)$$