

Differentiability Implies Continuity Proof

Jeremy Chow

12/18/18

The statement stated that if $f'(c)$ exists at $x = c$, then $f(x)$ is continuous at $x = c$. In order to proof so, we realize that

$$\begin{aligned} 0 &= 0 * f'(c) \\ &= 0 * \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} \\ &= \lim_{x \rightarrow c} (x - c) * \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} \\ &= \lim_{x \rightarrow c} \frac{(f(x) - f(c))(x - c)}{x - c} \\ &= \lim_{x \rightarrow c} (f(x) - f(c)) \\ &= -f(c) + \lim_{x \rightarrow c} f(x) \end{aligned}$$

Therefore

$\lim_{x \rightarrow c} f(x) = f(c)$ (which is the limit definition of continuity)

QED