Squeeze Theorem Lazy Proof

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The squeeze theorem stated that $a_n \leq b_n \leq c_n$, $(a_n) \to k$ and $(c_n) \to k$, then $(b_n) \to k$. In order to prove this theorem, we need to show that for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $n > N$ implies $|b_n - k| < \varepsilon$. Given by the situation, $|a_n - k| < \varepsilon$ for N_1 and $|c_n - k| < \varepsilon$ for N_2, then let $n > \max\{N_1, N_2\}$ so both statements holds true. The lowest that a_n will be is given by $k - a_n < \varepsilon$, $k - \varepsilon < a_n$. The highest that c_n will be is given by $c_n - k < \varepsilon$, $c_n < k + \varepsilon$. So we have $k - \varepsilon < a_n \leq b_n \leq c_n < k + \varepsilon$, $k - \varepsilon < b_n < k + \varepsilon$, which can be rewritten as $|b_n - k| < \varepsilon$. Hence, by the $N - \varepsilon$ definition of sequence convergence, $(b_n) \to k$ if $a_n \leq b_n \leq c_n$, $(a_n) \to k$ and $(c_n) \to k$. QED