Squeeze Theorem $\delta\varepsilon$ Lazy Proof

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The squeeze theorem stated that $a(x) \leq b(x) \leq c(x)$, $\lim_{x \to k} a(x) = L$ and $\lim_{x \to k} c(x) = L$, then $\lim_{x \to k} b(x) = L$. In order to prove this theorem, we need to show that for any $\varepsilon > 0$, there exists $\delta > 0$ such that $0 < |x - k| < \delta$ implies $|b(x) - L| < \varepsilon$.
Given by the situation,
$|a(x) - L| < \varepsilon$ for δ_1 and
$|c(x) - L| < \varepsilon$ for δ_2.
Let $\delta^* > \max\{\delta_1, \delta_2\}$ so both statements holds true.
The lowest that $a(x)$ will be is given by $L - a(x) < \varepsilon$, $L - \varepsilon < a(x)$.
The highest that $c(x)$ will be is given by $c(x) - L < \varepsilon$, $c(x) < L + \varepsilon$.
So we have
$L - \varepsilon < a(x) \leq b(x) \leq c(x) < L + \varepsilon$,
$L - \varepsilon < b(x) < L + \varepsilon$,
which can be rewritten as
$|b(x) - L| < \varepsilon$.
Hence, by the $\delta\varepsilon$ definition of sequence convergence, $\lim_{x \to k} b(x) \to L$ if $a(x) \leq b(x) \leq c(x)$, $\lim_{x \to k} a(x) \to L$ and $\lim_{x \to k} c(x) \to L$. QED