Mean Value Theorem Proof

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Mean value theorem stated that if $f(x)$ in continuous on an interval $[a, b]$ and differentiable on (a, b), the there exists at least one c in (a, b) where $f'(c) = \frac{f(b)-f(a)}{b-a}$. Let $g(x) = f(x) - \left(\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right)$, notice that $g(a) = g(b) = 0$. By using Rolle's theorem, there exists a value $c \in (a, b)$ where $g'(c) = 0$. Also notice that $g'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$. Combining both statements, we can get the equation
$0 = g'(c) = f'(c) - \frac{f(b)-f(a)}{b-a}$
$0 = f'(c) - \frac{f(b)-f(a)}{b-a}$
$\frac{f(b)-f(a)}{b-a} = f'(c)$
$f'(c) = \frac{f(b)-f(a)}{b-a}$
QED