Derivative Lazy Proofs

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Here are some proofs or rules useful in taking derivatives

Addition Rule

$y = f(x) + g(x)$

$y' = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)+g(x+\Delta x)-f(x)-g(x)}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \left(\frac{f(x+\Delta x)-f(x)}{\Delta x} + \frac{g(x+\Delta x)-g(x)}{\Delta x} \right)$

$y' = f'(x) + g'(x)$

Constant Multiplier

$y = kf(x)$

$y' = \lim\limits_{\Delta x \to 0} \frac{kf(x+\Delta x)-kf(x)}{\Delta x}$

$y' = k \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$y' = kf'(x)$

Power Rule

$y = x^n$

$y' = \lim\limits_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \frac{nx^{n-1}\Delta x + C_2^n x^{n-2}\Delta x^2 + \cdots + \Delta x^n}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \left(nx^{n-1} + \Delta x(C_2^n x^{n-2} + \cdots + \Delta x^{n-2}) \right)$

$y' = nx^{n-1}$

Chain Rule

$y = f(g(x))$

$y' = \lim\limits_{x \to a} \frac{f(g(x))-f(g(a))}{x-a}$

$y' = \lim\limits_{x \to a} \left(\frac{f(g(x))-f(g(a))}{g(x)-g(a)} * \frac{g(x)-g(a)}{x-a} \right)$

Let $u = g(x)$, and $a' = g(a)$

$y' = \lim\limits_{x \to a} \frac{g(x)-g(a)}{x-a} * \lim\limits_{u \to a'} \frac{f(u)-f(a')}{u-a}$

$y' = f'(u) * g'(x)$
$y' = f'(g(x)) * g'(x)$