Harmonic Series Divergent Lazy Proof

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To proof that the sum of the harmonic series diverges, we need to proof that $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$. Indeed, $\sum_{n=1}^{\infty} \frac{1}{n}$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$$

$$> \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \cdots$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$$

$$= \infty$$

$$\because \sum_{n=1}^{\infty} \frac{1}{n} > \infty$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

QED