L'Hôpital's rule Proof (with $\frac{0}{0}$ only)

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L'Hôpital's rule stated that if $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ is indeterminate, and both functions can be differentiated at $x = a$, then $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$. In this proof we are only going to explore the case where the limit becomes $\frac{0}{0}$. Given that the limit will turn into $\frac{0}{0}$, it can be implied that $\lim\limits_{x\to a} f(x) = 0$ and $\lim\limits_{x\to a} g(x) = 0$, and since differentiability implied continuity, $f(a) = g(a) = 0$. Therefore
$\lim\limits_{x\to a} \frac{f(x)}{g(x)}$
$= \lim\limits_{x\to a} \frac{f(x)-0}{g(x)-0}$
$= \lim\limits_{x\to a} \frac{f(x)-f(a)}{g(x)-g(a)}$
$= \lim\limits_{x\to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}$
$= \frac{f'(a)}{g'(a)}$
$= \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$
QED