Derivative Lazy Proofs

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Here are some proofs or rules useful in taking derivatives

Addition Rule

$y = f(x) + g(x)$

$y' = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)+g(x+\Delta x)-f(x)-g(x)}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \left(\frac{f(x+\Delta x)-f(x)}{\Delta x} + \frac{g(x+\Delta x)-g(x)}{\Delta x}\right)$

$y' = f'(x) + g'(x)$

Constant Multiplier

$y = kf(x)$

$y' = \lim\limits_{\Delta x \to 0} \frac{kf(x+\Delta x)-kf(x)}{\Delta x}$

$y' = k \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$y' = kf'(x)$

Power Rule

$y = x^n$

$y' = \lim\limits_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \frac{nx^{n-1}\Delta x + C_2^n x^{n-2}\Delta x^2 + \cdots + \Delta x^n}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \left(nx^{n-1} + \Delta x(C_2^n x^{n-2} + \cdots + \Delta x^{n-2})\right)$

$y' = nx^{n-1}$

Quotient Rule

$y = \frac{f(x)}{g(x)}$

$y' = \lim\limits_{\Delta x \to 0} \frac{\frac{f(x+\Delta x)}{g(x+\Delta x)} - \frac{f(x)}{g(x)}}{\Delta x}$

$y' = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)g(x)-f(x)g(x+\Delta x)}{\Delta x g(x)g(x+\Delta x)}$

$y' = \lim\limits_{\Delta x \to 0} \left(\frac{1}{g(x)g(x+\Delta x)} * \frac{f(x+\Delta x)g(x)-g(x)f(x)+g(x)f(x)-f(x)g(x+\Delta x)}{\Delta x}\right)$

$y' = \lim_{\Delta x \to 0} \left(\frac{1}{g(x)g(x+\Delta x)} \right) * \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x)g(x) - g(x)f(x) + g(x)f(x) - f(x)g(x+\Delta x)}{\Delta x} \right)$
$y' = \frac{1}{g(x)^2} \lim_{\Delta x \to 0} \left(\frac{g(x)(f(x+\Delta x) - f(x))}{\Delta x} - \frac{f(x)(g(x+\Delta x) - g(x))}{\Delta x} \right)$
$y' = \frac{1}{g(x)^2} \left(g(x) \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} - f(x) \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} \right)$
$y' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$
According to the Quotient Rule, $(\frac{1}{g(x)})' = \frac{-g'(x)}{g(x)^2}$
Product Rule
$y = f(x)g(x))$
$y = \frac{f(x)}{\frac{1}{g(x)}}$
$y' = \frac{\frac{f'(x)}{g(x)} - f(x)\frac{-g'(x)}{g(x)^2}}{\frac{1}{g(x)^2}}$
$y' = g(x)f'(x) + f(x)g'(x)$
Chain Rule
$y = f(g(x))$
$y' = \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{x - a}$
$y' = \lim_{x \to a} \left(\frac{f(g(x)) - f(g(a))}{g(x) - g(a)} * \frac{g(x) - g(a)}{x - a} \right)$
Let $u = g(x)$, and $a' = g(a)$
$y' = \lim_{x \to a} \frac{g(x) - g(a)}{x - a} * \lim_{u \to a'} \frac{f(u) - f(a')}{u - a}$
$y' = f'(u) * g'(x)$
$y' = f'(g(x)) * g'(x)$