Unit 2 Related Rates

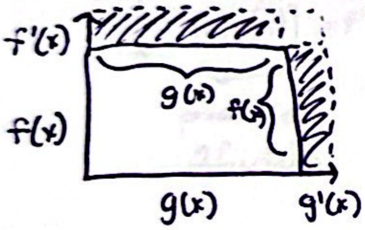
26	27	28	29	30
	Notes: Product Rule, Quotient Rule, Chain Rule	**Unit 1 Test**	Implicit Differentiation, Related Rates	
September 2	3	4	5	6
STUDENT/TEACHER HOLIDAY	Notes cont.	ICA1	ICA2	**Quiz** Pep rally
9	10	11	12	13
Review	**Unit 2 Test**			

Derivatives: Product Rule, Quotient Rule, Chain Rule / Transcendental Derivatives

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x)+f(x)g'(x)$$

Think Area



* Change in the product of the functions

Quotient Rule

$$\frac{d}{dx}\underbrace{\left[\frac{f(x)}{g(x)}\right]}_{\text{Quotient of two functions}} = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$$

Chain Rule

$$\frac{d}{dx}\underbrace{\left[f(g(x))\right]}_{\text{composition of two functions}} = f'(g(x))g'(x)$$

Transcendental Function Derivatives

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

Co-functions

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Exponentials

$$\frac{d}{dx}(e^x) = e^x$$

* The slope at a point is the same as the output

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Composition

$$\frac{d}{dx}(\ln u) = \frac{u'}{u}$$

u is a function!

↳ expression in terms of x!

Review of Log Properties: Name of property identifies operation in a single log arg.

Product Rule

$$\ln(xy) = \ln x + \ln y$$

Quotient Rule

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

Power Rule

$$\ln(x^n) = n\ln x$$

n = constant

Differentiate each function.

1. $y = e^{3x} + \sin(5x) + \ln(9x+1)$

$f(g(x)) \to f(x) = e^x,\ g(x) = 3x$

$f'(g)\cdot g' = e^{3x}\cdot 3$

$$\frac{dy}{dx} = e^{3x}\cdot 3 + \cos(5x)\cdot 5 + \frac{9}{9x+1}$$

2. $y = \tan^8 x$

$y = (\tan x)^8$

$$\frac{dy}{dx} = \underbrace{8(\tan x)^7}_{f'g}\cdot\underbrace{\sec^2 x}_{g'}$$

3. $y = \dfrac{x}{\cos x}$ ($x \leftarrow f$, $\cos x \leftarrow g$) $\qquad \dfrac{f'g - fg'}{g^2}$

$$\frac{dy}{dx} = \frac{\overset{f'}{1}\cdot\overset{g}{\cos x} - \overset{f}{x}\cdot\overset{g'}{(-\sin x)}}{\cos^2 x \leftarrow g^2}$$

$$\frac{dy}{dx} = \frac{\cos x + x\sin x}{\cos^2 x}$$

4. $f(x) = \sqrt[3]{5x^2+6x}$

$f(x) = (5x^2+6x)^{1/3}$

$f'(x) = \frac{1}{3}(5x^2+6x)^{-2/3}(10x+6)$

$$f'(x) = \frac{10x+6}{3\sqrt[3]{(5x^2+6x)^2}}$$

5. $y = \ln\left(\dfrac{3x+1}{5x-1}\right)$

$y = \ln(3x+1) - \ln(5x-1)$

$$\frac{dy}{dx} = \frac{3}{3x+1} - \frac{5}{5x-1}$$

6. $A(t) = \pi(r(t))^2$

$$\frac{dA}{dt} = \underbrace{2\pi(r(t))'}_{f'g}\cdot\underbrace{\frac{dr}{dt}}_{g'}$$

7. $g(x) = \dfrac{x^5+x^4+x^3}{x}$

* single term in the denominator

$g(x) = x^4 + x^3 + x^2$

$g'(x) = 4x^3 + 3x^2 + 2x$

$x \neq 0$

8. $y = \sec(x^2+x)$

$$\frac{dy}{dx} = \underbrace{\sec(x^2+x)\tan(x^2+x)}_{f'g}\cdot\underbrace{(2x+1)}_{g'}$$

9. $h(x) = \overset{f}{x^8}\ \overset{g}{\cos x}$ $\qquad f'g + fg'$

$h'(x) = 8x^7\cdot\cos x + x^8(-\sin x)$

$h'(x) = 8x^7\cos x - x^8\sin x$

10. $y = \dfrac{\cot x \to f}{x^2 \to g}$ $\qquad \dfrac{f'g - fg'}{g^2}$

$$\frac{dy}{dx} = \frac{(-\csc^2 x)\cdot x^2 - \cot x\cdot 2x}{(x^2)^2}$$

$$\frac{dy}{dx} = \frac{-x^2\csc^2 x - 2x\cot x}{x^4}$$

$$\frac{dy}{dx} = \frac{-x\csc^2 x - 2\cot x}{x^3}$$

11. $f(x) = \cos^5(7x) = (\cos(7x))^5$

*composition of 3 functions

$\frac{d}{dx}\left[f(g(h(x)))\right] =$

$f'(g(h(x))\cdot g'(h(x))\cdot h'(x)$

$f'(x) = 5\cos^4(7x)\cdot(-\sin(7x))\cdot 7$

$f'(x) = -35\cos^4(7x)\sin(7x)$

12. $y = \sin(\sqrt{x})$

$y = \sin(x^{1/2})$

$$\frac{dy}{dx} = \cos(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{\cos(\sqrt{x})}{2\sqrt{x}}$$

13. $V(t)=\frac{4\pi}{3}(r(t))^3$

$$\frac{dV}{dt}=4\pi(r(t))^2\cdot\frac{dr}{dt}$$

14. $y=\sin x\cos x$ $f \cdot g$ $f'g+fg'$

$$\frac{dy}{dx}=\cos x\cdot\cos x+\sin x\cdot(-\sin x)$$

$$\frac{dy}{dx}=\cos^2 x-\sin^2 x=\cos(2x)$$

$$y=\frac{1}{2}(2\sin x\cos x)=\frac{1}{2}\sin(2x)$$

$$\frac{dy}{dx}=\frac{1}{2}\cos(2x)\cdot 2=\cos(2x)$$

15. $y=e^x\csc x$ $f \cdot g$ $f'g+fg'$

$$\frac{dy}{dx}=e^x\csc x+e^x(-\csc x\cot x)$$

$$\frac{dy}{dx}=e^x\csc x(1-\cot x)$$

16. $g(x)=e^{(\tan x)}+\sqrt{7x}$

$$g(x)=e^{(\tan x)}+\sqrt{7}\cdot x^{1/2}$$

$$g'(x)=e^{(\tan x)}\cdot\sec^2 x+\frac{\sqrt{7}}{2}x^{-\frac{1}{2}}$$

$$g'(x)=e^{(\tan x)}\sec^2 x+\frac{\sqrt{7}}{2\sqrt{x}}$$

17. $y=\ln(x^2+3)$

$$\frac{dy}{dx}=\frac{2x}{x^2+3}$$

18. $y=\frac{e^x}{x-2}$ $e^x \to f$, $x-2 \to g$ $\frac{f'g-fg'}{g^2}$

$$\frac{dy}{dx}=\frac{e^x\cdot(x-2)-e^x\cdot 1}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{e^x(x-2-1)}{(x-2)^2}=\frac{e^x(x-3)}{(x-2)^2}$$

19. $h(x)=x^2\csc(3x)$ $f'g+fg'$

$$h'(x)=2x\cdot\csc(3x)+x^2(-\csc(3x)\cot(3x)\cdot 3)$$

$$h'(x)=x\csc(3x)[2-3x\cot(3x)]$$

20. $y=\tan(e^x+1)$

$$\frac{dy}{dx}=\sec^2(e^x+1)\cdot e^x$$

$$\frac{dy}{dx}=e^x\sec^2(e^x+1)$$

21. $y=\ln\left[(4x+3)^5\right]=5\ln(4x+3)$

$$\frac{dy}{dx}=5\cdot\frac{4}{4x+3}$$

$$\frac{dy}{dx}=\frac{20}{4x+3}$$

22. $h(x)=x^8\ln x$ $f'g+fg'$

$$h'(x)=8x^7\cdot\ln x+x^8\cdot\frac{1}{x}$$

$$h'(x)=8x^7\ln x+x^7$$

$$h'(x)=x^7(8\ln x+1)$$

23. $y=(\ln x)^{10}$

$$\frac{dy}{dx}=10(\ln x)^9\cdot\frac{1}{x}$$

$$\frac{dy}{dx}=\frac{10(\ln x)^9}{x}$$

24. $y=\ln\left(\frac{3x+1}{5x-1}\right)$

$$y=\ln(3x+1)-\ln(5x-1)$$

$$\frac{dy}{dx}=\frac{1}{3x+1}-\frac{1}{5x-1}$$

Implicit Differentiation

1. Which of the following functions would the Chain Rule have to be applied in finding the derivative of y?

$\boxed{\text{a. } y=\sqrt{f(x)}}$ b. $y=f(x)$ $\boxed{\text{c. } y=(f(x))^{1/5}}$ d. $y=5\cdot f(x)$

$y=(f(x))^{1/2}$

$\frac{dy}{dx}=\frac{1}{2}(f(x))^{-\frac{1}{2}}\cdot f'(x)$

$\frac{dy}{dx}=f'(x)$

$\frac{dy}{dx}=\frac{1}{5}(f(x))^{-\frac{4}{5}}\cdot f'(x)$

$\frac{dy}{dx}=5\cdot f'(x)$

2. Differentiate the explicit form of the equation $y=\frac{1}{x}$ and then also differentiate the implicit form $xy=1$.

$y=\frac{1}{x}=x^{-1}$

$\frac{dy}{dx}=-x^{-2}=\boxed{-\frac{1}{x^2}}$

$f'g+fg'$

$xy=1 \Leftrightarrow \overset{f}{x}(\overset{g}{y(x)})=1$

$1\cdot y+x\cdot\frac{dy}{dx}=0$

Isolate $\frac{dy}{dx}$

$\frac{dy}{dx}=-\frac{y}{x}=\frac{-1/x}{x}=-\frac{1}{x}\cdot\frac{1}{x}=\boxed{-\frac{1}{x^2}}$

3. Find $G'(x)$ for each function.

a. $G(x)=(f(x))^3$

$G'(x)=3(f(x))^2\cdot f'(x)$

b. $G(x)=y^3 \rightarrow (y(x))^3$

$G'(x)=\underbrace{3y^2}_{\text{Outer}}\cdot\underbrace{\frac{dy}{dx}}_{\text{Inner}}$

4. Find $\frac{dy}{dx}$ in terms of x and y for each equation

a. $5y=3y^2+x^4$ $\rightarrow 3(y(x))^2$

$5\cdot\frac{dy}{dx}=6y\cdot\frac{dy}{dx}+4x^3$

$5\cdot\frac{dy}{dx}-6y\frac{dy}{dx}=4x^3$

$\frac{dy}{dx}(5-6y)=4x^3$

$\boxed{\frac{dy}{dx}=\frac{4x^3}{5-6y}}$

b. $3xy-1=x-\sin y$

$\overset{f}{(3x)}\overset{g}{y}$ $f'g+fg'$

$-\sin(y(x))$ outer inner

$3y+3x\cdot\frac{dy}{dx}-0=1-\cos y\cdot\frac{dy}{dx}$

$3x\frac{dy}{dx}+\cos y\frac{dy}{dx}=1-3y$

$\frac{dy}{dx}(3x+\cos y)=1-3y$

$\boxed{\frac{dy}{dx}=\frac{1-3y}{3x+\cos y}}$

5. Find the equations of line tangent and also normal to the circle at the given point.

$\frac{dy}{dx}=?$

$\frac{dy}{dx}$ ↑ slope!

$x^2+y^2=169$ (5,12)

$x^2+y^2=13^2$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

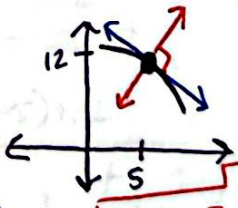
$\frac{dy}{dx}=-\frac{x}{y}$

slope at a point?

$\left.\frac{dy}{dx}\right|_{(5,12)}=-\frac{5}{12}$

$\boxed{y_{tan}=-\frac{5}{12}(x-5)+12}$

Normal? opposite reciprocal



$\boxed{y_{nor}=\frac{12}{5}(x-5)+12}$

6. Differentiate with respect to t. (Treat x and y as functions of t)

a. $x^3-x-y^3=12$

$(x(t))^3-x(t)-(y(t))^3=12$

$\boxed{3x^2\cdot\frac{dx}{dt}-\frac{dx}{dt}-3y^2\cdot\frac{dy}{dt}=0}$

b. $5x^3-2xy^3-y^6=3$ $f'g+fg'$

$5(x(t))^3-\underbrace{2(x(t))}_{f}\underbrace{(y(t))^3}_{g}-(y(t))^6=3$

$\boxed{15x^2\cdot\frac{dx}{dt}-\left(2\cdot\frac{dx}{dt}\cdot y^3+2x\cdot 3y^2\cdot\frac{dy}{dt}\right)-6y^5\cdot\frac{dy}{dt}=0}$

Application Problems ← **Related Rates** → Implicit Differentiation

A spherical balloon is inflated with gas at a rate of 500 cubic centimeters per minute. How fast is the radius of the balloon (increasing) increasing at the instant the radius is 45 cm?

① instantaneous rate of change
② derivative

$$\left[V=\frac{4}{3}\pi r^3\right]\frac{d}{dt} \quad r^3 \to (r(t))^3$$

$$\frac{dV}{dt}=\frac{4}{3}\pi\cdot 3r^2\cdot\frac{dr}{dt}$$

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

$$500=4\pi(45)^2\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt}=\frac{5}{81\pi}\ \text{cm/min}}$$

Rate the volume is increasing

$$\frac{dV}{dt}=500\ \text{cm}^3/\text{min}$$

Radius

$r=45$ cm

Rate the radius is changing

$$\frac{dr}{dt}=?$$

Given the formula for volume of a cylinder $V=\pi r^2h$:

a) differentiate with respect to time t. $V(t)=\pi\underbrace{(r(t))^2}_{f}\underbrace{h(t)}_{g}$ $\quad f'g+fg'$

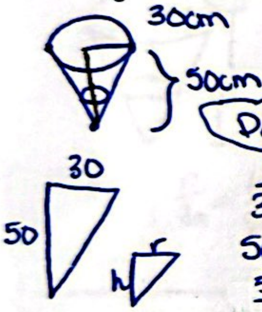
$$\frac{dV}{dt}=\underbrace{2\pi r\cdot\frac{dr}{dt}}_{f'}\cdot\underbrace{h}_{g}+\underbrace{\pi r^2}_{f}\cdot\underbrace{\frac{dh}{dt}}_{g'}$$

b) differentiate with respect to time t, knowing that the height is 3 times the radius.

$$h=3r \qquad V=\pi r^2h=\pi r^2(3r)=3\pi r^3$$

$$\frac{dV}{dt}=9\pi r^2\cdot\frac{dr}{dt}$$

A conical shaped bin is filling up with rain water at a rate of 4 cubic cm/sec. The bin has a radius of 30 cm and a height of 50 cm. Determine the rate at which the radius is changing when the height of water in the bin is 10 cm.



$$V=\frac{\pi}{3}r^2h$$

Do not know $\frac{dh}{dt}$, so eliminate h

$$\frac{r}{30}=\frac{h}{50}$$

$$50r=30h$$

$$\frac{50r}{30}=\frac{5r}{3}=h$$

$$\frac{5r}{3}=10$$

$$r=6$$

$$\frac{dr}{dt}=?\qquad V=\frac{\pi}{3}r^2\left(\frac{5r}{3}\right)$$

$$V=\frac{5\pi}{9}r^3$$

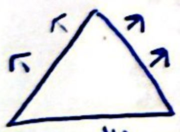
$$\frac{dV}{dt}=\frac{5\pi}{3}r^2\cdot\frac{dr}{dt}$$

$$4=\frac{5\pi}{3}(6)^2\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{\cancel{4}\cdot 3}{5\pi\cdot\cancel{36}}=\frac{\cancel{3}}{5\pi\cdot\cancel{9}}=\frac{1}{5\pi\cdot 3}$$

$$\boxed{\frac{dr}{dt}=\frac{1}{15\pi}\ \text{cm/sec}}$$

The base of a triangle is increasing at a rate of 4 cm/min while the area is increasing at a rate of 7 cm²/min. At what rate is the altitude of the triangle changing when the base is 10 cm and the area is 100 cm²?



$A=100$

$\frac{dA}{dt}=7$

$b=10,\ \frac{db}{dt}=4$

$$100=\tfrac{1}{2}(10)(h)$$

$$100=5h$$

$$h=20$$

$$A=\tfrac{1}{2}bh \qquad b'h+bh'$$

$$\frac{dA}{dt}=\frac{1}{2}\left(\frac{db}{dt}h+b\frac{dh}{dt}\right)$$

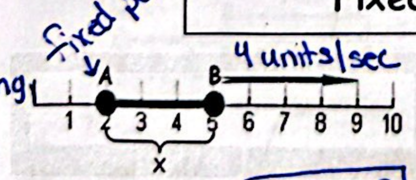
$$7=\frac{1}{2}\left(4\cdot 20+10\cdot\frac{dh}{dt}\right)$$

$$7=\frac{1}{2}\left(80+10\frac{dh}{dt}\right)$$

$$7=40+5\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt}=-6.6\ \text{or}\ -\frac{33}{5}\ \text{cm/min}}$$

Fixed/moving pts

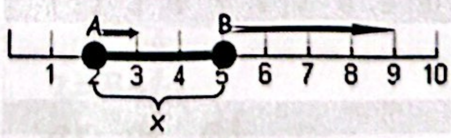


A is not moving, so $\frac{dA}{dt}=0$

$$X=B-A$$

$$\frac{dx}{dt}=\frac{dB}{dt}-\frac{dA}{dt}$$

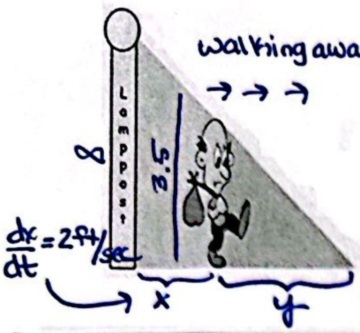
$$\boxed{\frac{dx}{dt}=\frac{dB}{dt}}$$

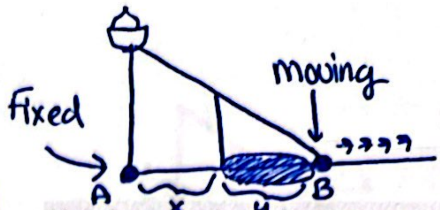


$$X=B-A$$

$$\boxed{\frac{dx}{dt}=\frac{dB}{dt}-\frac{dA}{dt}}$$

A 3.5-foot hobbit walks at a rate of 2 ft/sec away from a lamppost that has a height of 8 feet. When he is 12 feet from the base of the light.





a) at what rate is the length of his shadow changing?

$$\frac{y}{x+y} = \frac{3.5}{8}$$

$$8y = 3.5(x+y)$$

$$8y = 3.5x + 3.5y$$

$$4.5y = 3.5x$$

$$y = \frac{3.5}{4.5}x \qquad y = \frac{7/2}{9/2}x \qquad y = \frac{7}{9}x$$

$$dy/dt = \frac{7}{9}\frac{dx}{dt} = \frac{7}{9}\cdot 2 = \boxed{\frac{14}{9} \text{ ft/sec}}$$

b) at what rate is the tip of his shadow moving? (entire base)

$$\frac{d}{dt}(x+y) = (B-A)\frac{d}{dt}$$

$$\frac{dx}{dt} + \frac{dy}{dt} = \frac{dB}{dt} - \frac{dA}{dt} \to 0$$

$$2 + \frac{14}{9} = \frac{dB}{dt}$$

$$\frac{dB}{dt} = \frac{18}{9} + \frac{14}{9}$$

$$\boxed{\frac{dB}{dt} = \frac{32}{9} \text{ ft/sec}}$$

You shine a flashlight, making a circular spot of light on a wall where the area of the circle and the radius are both changing with respect to time as you move ~~away from~~ towards the wall.

a) At what rate is the radius of the light on the wall changing when the area is ~~increasing~~ decreasing at a rate of 10π cm²/sec and the circle has a radius of 2?

$r = 2$

$\frac{dA}{dt} = -10\pi$

$$\left[A = \pi r^2\right]\frac{d}{dt}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$-10\pi = 2\pi(2)\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{-10\pi}{4\pi} = -\frac{5}{2} \text{ cm/sec}}$$

b) Find the rate of change for the area ~~of the~~ when the circle has an area of 25π cm² and the radius is ~~changing~~ decreasing at rate of 3 cm/sec.

$\frac{dr}{dt} = -3$

$A = 25\pi$

$25\pi = \pi r^2$

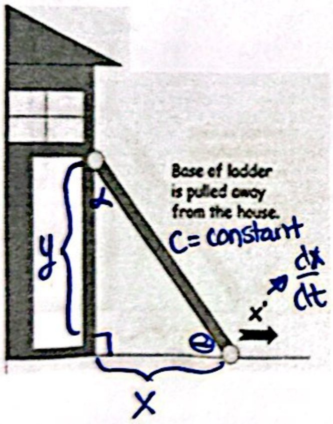
$r = 5$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(5)(-3)$$

$$\boxed{\frac{dA}{dt} = -30\pi \text{ cm}^2/\text{sec}}$$

Moving Ladder Problem



A rate of change will be given:

$\frac{dx}{dt} \to$ positive value

or

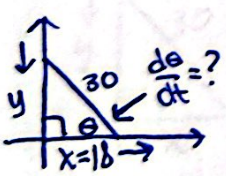
$\frac{dy}{dt} \to$ negative value

We will be solving for:

1. The rate of change of the top/bottom of the ladder corresponds to the rate of change of a side length. Pythagorean
2. Rate of change of an angle. SOHCAHTOA

A ladder 30 feet long rests against a vertical wall. If the top of the ladder slides down the wall at a speed of 3 ft/sec, then dy/dt

a) How fast is the angle between the ladder and the ground changing when the bottom of the ladder is 18 feet from wall? $\sin(\theta(t))$



$\frac{d\theta}{dt} = ?$

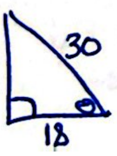
$$\sin\theta = \frac{y}{30}$$

$$\left[\sin\theta = \frac{1}{30}y\right]\frac{d}{dt}$$

$$\cos\theta\cdot\frac{d\theta}{dt} = \frac{1}{30}\cdot\frac{dy}{dt}$$

$$\frac{dy}{dt} = -3$$

$$\frac{3}{5}\cdot\frac{d\theta}{dt} = \frac{1}{30}(-3)$$

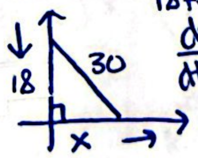


$$\cos\theta = \frac{18}{30} \qquad \cos\theta = \frac{3}{5}$$

$$\frac{d\theta}{dt} = -\frac{1}{10}\cdot\frac{5}{3}$$

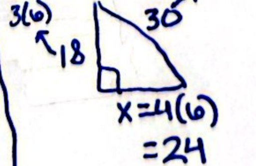
$$\boxed{\frac{d\theta}{dt} = -\frac{1}{6} \text{ radians/sec}}$$

b) How fast is the base of the ladder moving away from the wall when the top of the ladder is ~~20 feet~~ 18 feet from the ground?



$\frac{dx}{dt} = ?$

$$\frac{dy}{dt} = -3$$



$$x^2 + y^2 = 30$$

$$\left[(x(t))^2 + (y(t))^2 = 30\right]\frac{d}{dt}$$

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$2(24)\frac{dx}{dt} + 2(18)\cdot(-3) = 0$$

$$48\frac{dx}{dt} - 108 = 0$$

$$\boxed{\frac{dx}{dt} = \frac{108}{48} = \frac{9}{4} \text{ ft/sec}}$$

Notes: More Related Rates Problems

1) If $y^2 = y + 2\sqrt{x}$, and if $\frac{dx}{dt} = 4$, find $\frac{dy}{dt}$ when $y = 3$.

$(3)^2 = 3 + 2\sqrt{x}$

$6 = 2\sqrt{x}$

$3 = \sqrt{x}$

$9 = x$

$\frac{d}{dt}\left[y^2 = y + 2(x)^{1/2}\right]$

$2y\frac{dy}{dt} = \frac{dy}{dt} + x^{-\frac{1}{2}} \cdot \frac{dx}{dt}$

$2(3)\frac{dy}{dt} = \frac{dy}{dt} + \frac{1}{\sqrt{9}}(4)$

$6\frac{dy}{dt} = \frac{dy}{dt} + \frac{4}{3}$

$6\frac{dy}{dt} - \frac{dy}{dt} = \frac{4}{3}$

$\frac{dy}{dt}(6-1) = \frac{4}{3}$

$\frac{dy}{dt} = \frac{4}{3} \cdot \frac{1}{5}$

$\boxed{\frac{dy}{dt} = \frac{4}{15}}$

2) Each side of a rectangle is increasing at a rate of 5 cm/sec. At what rate is the area of the rectangle increasing when the area of the rectangle is 36 cm^2 and the length is 9 cm? Hint: Start by differentiating the formula $A = lw$.

$A = l \cdot w = l(t) \cdot w(t)$

$\frac{dw}{dt} = \frac{dl}{dt} = 5 \text{ cm/sec}$

$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$

$A = l \cdot w$

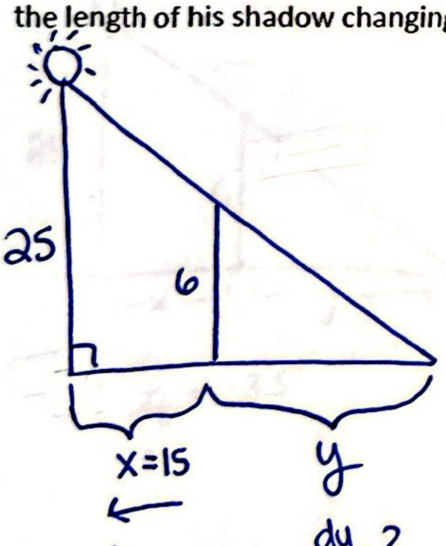
$36 = 9 \cdot w$

$4 = w$

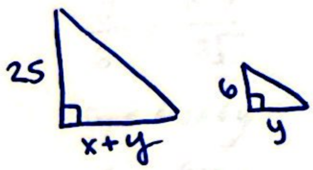
$\frac{dA}{dt} = 5(4) + 9(5)$

$\boxed{\frac{dA}{dt} = 65 \text{ cm}^2/\text{sec}}$

3) A man 6 ft tall walks at the rate of 3.5 ft/sec toward a street light that is 25 ft above the ground. At what rate is the length of his shadow changing when he is 15 ft from the base of the light?



$\frac{dx}{dt} = -3.5$ $\frac{dy}{dt} = ?$



$\frac{y}{6} = \frac{x+y}{25}$

$25y = 6x + 6y$

$19y = 6x$

$y = \frac{6}{19}x$

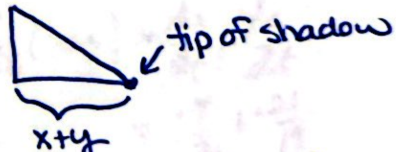
$\frac{d}{dt}\left[y = \frac{6}{19}x\right]$

$\frac{dy}{dt} = \frac{6}{19} \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{\cancel{6}^{3}}{19} \cdot \left(-\frac{7}{\cancel{2}_{1}}\right)$

$\boxed{\frac{dy}{dt} = -\frac{21}{19} \text{ ft/sec}}$

Rate of change of the tip of his shadow?



$\frac{d}{dt}[x+y] = \frac{dx}{dt} + \frac{dy}{dt}$

$= -\frac{7}{2} + \left(-\frac{21}{19}\right)$

$= -\frac{133}{19} - \frac{21}{19}$

$= \boxed{-\frac{154}{19} \text{ ft/sec}}$

$f'g+fg'$

4) Given $14x^2 + xy^3 = 41$.

a) Differentiate with respect to x.

$$28x + \left(1\cdot y^3 + x\cdot 3y^2\cdot \frac{dy}{dt}\right) = 0$$

$$28x + y^3 + 3xy^2\frac{dy}{dt} = 0$$

$$3xy^2\cdot\frac{dy}{dt} = -28x - y^3$$

$$\boxed{\frac{dy}{dt} = \frac{-28x-y^3}{3xy^2}}$$

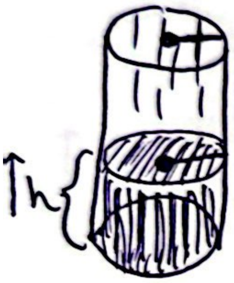
b) Differentiate with respect to t. Solve for $\frac{dx}{dt}$.

$$28x\cdot\frac{dx}{dt} + \frac{dx}{dt}y^3 + x\cdot 3y^2\cdot\frac{dy}{dt} = 0$$

$$\frac{dx}{dt}(28x+y^3) + 3xy^2\frac{dy}{dt} = 0$$

$$\boxed{\frac{dx}{dt} = \frac{-3xy^2\frac{dy}{dt}}{28x+y^3}}$$

5) A cylindrical can fills up with rain water at rate of 1.5 cm^3/sec. The can has a radius of 15 cm. Determine the rate of change of the rain water level in the can when the water level is at a height of 12 cm.



$\frac{dV}{dt} = 1.5 = \frac{3}{2}$

$r = 15$

$\frac{dh}{dt} = ?$

$h = 12$

$V = \pi r^2 h$

$V = \pi(15)^2 h$ → do not eliminate h $\left(\frac{dh}{dt}\right)$

$V = 225\pi h$

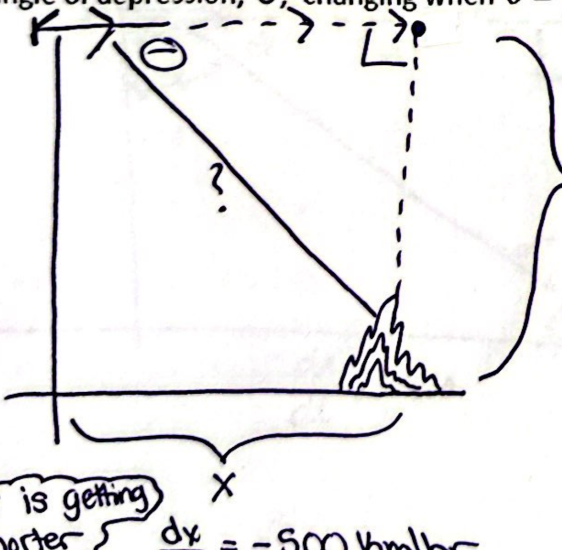
$\frac{dV}{dt} = 225\pi\cdot\frac{dh}{dt}$

$\frac{3}{2} = 225\pi\cdot\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{2\cdot 225\cdot\pi}$

$$\boxed{\frac{dh}{dt} = \frac{1}{150\pi}\ cm/sec}$$

6) An airplane, flying at 500 km/hr at a constant altitude of 6 km, is approaching a fire on the ground. A camera within the plane rotates such that it is pointed directly at the fire. How fast is the angle of depression, θ, changing when $\theta = \frac{\pi}{6}$?



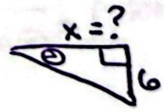
$\frac{dx}{dt} = -500$ km/hr

TOA $\tan\theta = \frac{6}{x}$

$\frac{d}{d\theta}\left[\tan\theta = 6x^{-1}\right]$

$\sec^2\theta\cdot\frac{d\theta}{dt} = -6x^{-2}\cdot\frac{dx}{dt}$

$\theta = \frac{\pi}{6}$



$\tan\frac{\pi}{6} = \frac{6}{x}$

$\frac{1}{\sqrt{3}} = \frac{6}{x}$

$x = 6\sqrt{3}$

$\sec^2\left(\frac{\pi}{6}\right)\frac{d\theta}{dt} = -\frac{6}{(6\sqrt{3})^2}\cdot(-500)$

$\left(\frac{2}{\sqrt{3}}\right)^2\cdot\frac{d\theta}{dt} = \frac{500}{18}$

$\frac{4}{3}\cdot\frac{d\theta}{dt} = \frac{500}{18}$ → $\boxed{\frac{d\theta}{dt} = \frac{\cancel{250}^{125}}{\cancel{9}_{3}}\cdot\frac{\cancel{3}^{1}}{\cancel{4}_{2}} = \frac{125}{6}\ \frac{rad}{hr}}$