Distributed Receding Horizon Coverage Control for Multiple Non-holonomic Mobile Robots

Fateme Mohseni*, Ali Doustmohammadi, Mohammad Bagher Menhaj*****

Electrical Engineering Department, Amirkabir University of Technology, Tehran, Iran
E-mails: {(fatememohseni@aut.ac.ir),**(dad@aut.ac.ir),***(menhaj@aut.ac.ir)}*

Abstract: This paper presents a distributed receding horizon coverage control algorithm for controlling a group of non-holonomic mobile robots with the assumption that the robots dynamics are decoupled from each other. The objective of the coverage algorithm considered here is to maximize the detection of the occurrence of the events. In order to solve each receding horizon coverage control problem, each robot needs to know the trajectories of its neighbors during the optimization time interval. Since this information is not available, an algorithm is presented to assume the trajectory of the neighboring robots. By using the proposed algorithm, the resulting closed-loop behavior is near optimal, adaptive, distributed, scalable, robust, and stable. Simulation results validate the proposed algorithm.

Keywords: Distributed control, Coverage control, Non-holonomic mobile robots, Optimization.

1. INTRODUCTION

During recent years, formulating a cooperative control design among distributed agents, assigned to a specific task that can navigate autonomously without collision has received significant attention. Researchers have shown that multiple robots could potentially accomplish a task more efficiently than a single robot. Furthermore, a major factor for consideration in developing reliable distributed control algorithms is location of nodes for the robot network in the mission space. This is referred to as the coverage control as explored in Cortes et al. (2004) and Meguerdichian and his associates (2001). It has been determined that the level of sensitivity and the domain of coverage of mobile robots in their deployment location is essential to the overall efficiency of the system. Researchers have used Voronoi partitioning of the region model to reduce difficulties of the locational optimization as done in Okabe et al. (1992). Dunbara and his colleagues (2005) have suggested an algorithm for formation control in a multi-agent system based on receding horizon control. Franco and others (2005) proposed a stable receding horizon cooperative control for a class of distributed agents and Defoort (2010), suggested another distributed receding horizon planning for multi-robot systems. Mohseni et al. (2012) proposed a centralized receding horizon coverage control approach for controlling linear sensory networks and then in Mohseni et al. (2012), this centralized approach was extended to a distributed receding horizon coverage control (DRHCC) algorithm for controlling linear sensory networks. All of these authors supposed that the dynamic of system is holonomic. This paper presents a distributed receding horizon coverage control algorithm for controlling a group of non-holonomic mobile robots with the assumption that the robots' dynamics are decoupled from each other. The algorithm will provide for maximum event detection through confluence of robots position to a centroidal Voronoi configuration.

2. BACKGROUNDS

2.1 Non-holonomic mobile robot dynamic model

As was explored in Fierro and Lewise (1996) and Bloch et al. (1992) a large class of non-holonomic mobile robots is described by the following dynamic equations

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = w \end{cases} \qquad (1)$$

where v and w are the linear and angular velocity respectively.

2.2 Locational optimization

This section presents some facts regarding the method used to describe *coverage control* for mobile sensing network as done in Cortes and his colleagues (2004), Okabe and Boot and Sugihara (1992). Assume that S be a convex space in $\Re^2$ and $P = (p_1, ..., p_n)$ is augmented vector whose elements are the positions of the n mobile robots, i.e. $p_i \in S$ denotes i^{th} robot position. Furthermore, assume that movement of each robot is confined in S and $W = \{W_1, ..., W_n\}$ is a tessellation of S such that $I(W_i) \bigcap I(W_j) = \varnothing$. $I(.)$ denotes interior space of each W_i and $\bigcup_{i=1}^{n} I(W_i) = S$. To obtain the probability of an event occurring at a point in S, the mapping $\phi : S \to \Re^+$ is defined. Note that in this sense, ϕ is the distribution density function. Let's assume that d describes the distance between any given point s inside the mission space S and location of a given robot. As robot i moves further away from point s, its sensing performance deteriorates. The deterioration is defined by function

$g:\Re^{+}\to\Re^{+}$ that indicates the probability that a sensor will not detect an event. As a measurement for system's performance, coverage cost function is described as

$$H(P,W)=\sum_{i=1}^{n}H(p_i,W_i)=\sum_{i=1}^{n}\int_{W_i}g(d(s,p_i))\phi(s)ds$$

where H is a differentiable function. Notice that the cost function H must be minimized in regards to location of robots and partition of the space.

2.3 Centroidal Voronoi partition

A collection of points $P=\{p_1,...,p_n\}$ commonly referred to as Voronoi domain or Voronoi cell connected to point p_i are defined by

$$V_i=\{s\in S:d(s,p_i)\le d(s,p_j),\forall j\ne i\} \quad (2)$$

Definition 1 [It was defined in Aurenhammer and Klein]**:** For robot i all neighboring Voronoi robots (meaning N_i) are described as collection of robots with shared Voronoi cell border.

Based on definition of Voronoi partitioning, we have $\min_{i\in 1,...,n} g(d(s,p_i))=g(d(s,p_i))$ for each $s\in V_j$. Accordingly,

$$H(P,V(P))=\int_S \min_{i\in 1,...,n} g(d(s,p_i))\phi(s)ds \quad (3)$$

Proposition 1 [it was explained in Cortes et al. (2004)]**:** Following is a sufficient condition for minimizing $H(P)$

$$\frac{\partial H_V(P)}{\partial p_i}=\frac{\partial H(p_i,V_i)}{\partial p_i}=\int_{V_i}\frac{\partial}{\partial p_i}g(d(s,p_i))\phi(s)ds=0 \quad (4)$$

The (generalized) mass and first moment (not normalized) and centre of i^{th} Voronoi cell are defined as

$$M_{V_i}=\int_{V_i}\phi(s)ds,\quad L_{V_i}=\int_{V_i}s\phi(s)ds,\quad C_{V_i}=\frac{L_{V_i}}{M_{V_i}} \quad (5)$$

respectively. Using the above definition and proposition and letting $g=\frac{1}{2}\|s-p_i\|$, (4) can be written as

$$\frac{\partial H_V(P)}{\partial p_i}=M_{V_i}(p_i-C_{V_i}) \quad (6)$$

Thus, to minimize H_V , each robot must not only be a generator point of its own Voronoi cell but also must be at the centre of the cell. We will refer to a robots' configuration as a centroidal Voronoi configuration, if it gives rise to a centroidal Voronoi partition, as was explored in Cortes and his associates (2004).

2.4 Receding horizon control (RHC)

As been stated in Dunbar, (2004) RHC is an optimization approach that can be used for systems, even if some constraints on states and inputs exist. In RHC, the current control law is obtained by solving a finite horizon optimal problem at each sampling instant. Each optimization generates an open-loop optimal control trajectory, and the first portion of this optimal control trajectory is applied to the system until next sampling time. In the sections that follow, the RHC approach is used to drive a group of n non-holonomic mobile robots to the centroidal Voronoi configuration.

3. CENTRALIZED RECEDING HORIZON COVERAGE CONTROL (CRHCC)

The CRHCC objective is to asymptotically force a group of n non-holonomic mobile robots toward

$$q_d=\left(C_{V_x},C_{V_y},\theta_d\right) \quad (7)$$

Let $q(t)=(q_1,...,q_n),\ q(t)=(P(t),\theta(t))=(x(t),y(t),\theta(t))$, be a n-vector whose elements are robots' states, i.e. $q_i(t)=(p_i(t),\theta_i(t))$, and $C_V=(C_{V_1},...,C_{V_n})$ be a vector of Voronoi cells' centroid. The overall system dynamic can be described as

$$\dot{q}(t)=f(q(t),u(t))\qquad t\ge 0,\qquad q(0)\qquad given \quad (8)$$

where $f=[v\cos\theta\quad v\sin\theta\quad w]^T$, $u(t)=[v(t)\quad w(t)]^T$, $q(0)$ is known, and $q(t),u(t)$ are state and input vectors respectively. It is assumed that there exist some constraints on state and input, i.e. $q(t)\in\aleph^n$ and $u(t)\in\mathcal{U}^n$ where $\aleph^n$ and $\mathcal{U}^n$ are the state and input constraints sets respectively. Now the position error and the desired orientation angle for each robot are defined as

$$p_{e_i}=(x_{e_i},y_{e_i})=\left(x_i-C_{V_{x_i}},y_i-C_{V_{y_i}}\right) \quad (9)$$

$$\theta_{d_i}=\begin{cases}\arctan(\frac{y_{e_i}}{x_{e_i}})=\arctan\left(\frac{y_i-C_{V_{y_i}}}{x_i-C_{V_{x_i}}}\right); & p_i\ne C_{V_i}\\ 0; & p_i=C_{V_i}\end{cases} \quad (10)$$

respectively. Notice that using only (10), we can't get a unique θ_{d_i} for each robot, so an extra constraint must be chosen for θ_{d_i} , to be unique at every instant. This constraint is defined as follow:

$$\left(\cos\theta_{d_i},\sin\theta_{d_i}\right).\left(x_i-C_{V_{x_i}},y_i-C_{V_{y_i}}\right)\le 0 \quad (11)$$

Lemma 1 [Lemma 2.4 in Aurenhammer and Klein]**:** Two points of P in Voronoi diagram are connected with a Delaunay edge, iff their corresponding Voronoi cells are adjacent.

These two points (or corresponding robots) are called neighbors. By drawing robots' Voronoi diagram and its corresponding Delaunay graph, the set of robots' positions can be shown with a graph where its vertexes are robots positions and its edges are connecting segment between any two neighboring robots. We denote the coverage graph topology by $G=(V,E),V=\{1,...,n\},E\subset V\times V$. Each edge in graph is illustrated with an ordered pair $(i,j)\in E$, where i,j are any two neighboring robots. The coverage graph is assumed to be undirected. Hence, if $(i,j)\in E$, then $(j,i)\in E$. Robots i,j are called neighbors if in the coverage graph $(i,j)\in E$. The set of neighbors of i^{th} robot is denoted by $\mathrm{N}_i\subset V$. E_0 is defined as a sub-set of E where it contains all elements of E that only one of the pairs between $(i,j),(j,i)\in E$ is included in E_0. Each element of E_0 is denoted by e_i. Accordingly, $E_0=\{e_1,...,e_M\}$, where M is the number of Delaunay edges.

The desired connecting vector between any two neighbors in a coverage graph, denoted by D_{ij} is defined as

$$D_{ij}=q_{d_j}-q_{d_i} \tag{12}$$

Definition 2: "*Coverage vector" and "coverage matrix"*

The *"coverage vector*" denoted by COV is defined as

$$COV=(cov_1,...,cov_l,...,cov_M,cov_{M+1},...,cov_{M+n}),\ where$$

$$cov_l=q_i-q_j+D_{ij},\quad l=1,...,M,\quad \forall(i,j)\in E_0 \tag{13}$$

$$cov_{M+k}=q_k-q_{d_k},\quad k=1,...,n \tag{14}$$

It is clear that the robots will be in the desired configuration, namely $q=q_d$, when $COV\equiv 0$. Hence, we can write the linear mapping from q to COV as

$$COV=Tq+\bar{D} \tag{15}$$

Where $\bar{D}=(...,D_{ij},...,-q_{d_k},...)$. We call T as *"coverage matrix"*.

From definition of the coverage vector, we know that

$$COV=Tq+\bar{D}\rightarrow if\quad q=q_d\quad then\quad COV\equiv 0\Rightarrow$$

$$Tq_d+\bar{D}=0\Rightarrow \bar{D}=-Tq_d \tag{16}$$

Substitution of (16) into (15) yields:

$$COV=Tq-Tq_d=T(q-q_d) \tag{17}$$

Lemma 2: The coverage matrix T in (15) has full rank.

Definition 3: The centralized receding horizon coverage control cost function is defined as

$$\bar{H}(q(t),u(.),h_p)=$$

$$\int_t^{t+h_p}[\sum_{(i,j)\in E_0}\xi\left\|q_i(\tau)-q_j(\tau)+D_{ij}\right\|^2+\xi\left\|q(\tau)-q_d\right\|^2$$

$$\eta\left\|\begin{bmatrix}v(t)\\w(t)\end{bmatrix}\right\|^2]d\tau+\sigma\left\|q(t+h_p,q(t))-q_d\right\|^2 \tag{18}$$

where ξ,η,σ are positive weighting constants.

Proposition 2: The CRHCC cost function (18) designed to drives a group of n non-holonomic mobile robots to a centroidal Voronoi configuration can be written as

$$\bar{H}(q(t),u(.),h_p)=\int_t^{t+h_p}\left\|q(\tau;q(t))-q_d\right\|_Q^2+\left\|u(\tau)\right\|_R^2 d\tau$$

$$+\left\|q(t+h_p;q(t))-q_d\right\|_G^2 \tag{19}$$

Proof: It can be proven using Definitions 1, 2, 3 and Lemmas 1, 2, and by defining the weighting matrixes $Q=\xi T^T T$, $G=\sigma I$ and $R=\eta I$ (where I is identity matrix).■

Problem 1: *"CRHCC problem"*

Find

$$\bar{H}^*(q(t),h_p)=\min_{u(.)}\bar{H}(q(t),u(.),h_p),\ \text{with}$$

$$\bar{H}(q(t),u(.),h_p)=\int_t^{t+h_p}\left\|q(\tau;q(t))-q_d\right\|_Q^2+\left\|u\right\|_R^2 d\tau+$$

$$\left\|q(t+h_p;q(t))-q_d\right\|_G^2$$

subject to:

$$\left.\begin{array}{l}\dot{q}(\beta)=f(q(\beta),u(\beta))\\ u(\beta)\in\mathcal{U}\\ q(\beta;q(t))\in\aleph\end{array}\right\}\beta\in\left[t,t+h_p\right],$$

$$q(t+h_p;q(t))\in\Psi(\omega):=\left\{q:\left\|q-q_d\right\|_G^2\le\omega,\omega\ge 0\right\} \tag{20}$$

Note that (20) represents the terminal constraint, similar to Dunbar (2005). Assume that the first segment of the optimal control problem is solved at the time instant $t_0\in\Re$, h_c is the receding horizon update period, and the closed-loop system that we wish to stabilize at q_d is

$$\dot{q}(\tau)=f(q(\tau),u^*(\tau)),\tau\ge t_0 \tag{21}$$

where $u^*(\beta;q(t))$, $\beta\in\left[t,t+h_p\right]$, is the open-loop optimal solution of Problem 1. This optimal control solution is applied to the system until $t+h_c$, i.e. the applied control to the system in the time interval $\tau\in\left[t,t+h_c\right)$, $0<h_c\le h_p$ is $u^*(\tau)=u^*(\tau;q(t)),\tau\in\left[t,t+h_c\right)$. The open-loop optimal state trajectory is denoted as $q^*(\tau;q(t))$.

4. DISTRIBUTED RECEDING HORIZON COVERAGE CONTROL (DRHCC)

In CRHCC approach, the control law requires centralized information and computations. The distributed approach proposed in this section, avoids the disadvantages associated with centralized approaches. Let q_i and u_i be state and control input of the i^{th} robot respectively, where $i=1,...,n$. It is assumed that robots' dynamics are decoupled from each other and hence their dynamics can be written as

$$\dot{q}_i(t)=f_i(q_i(t),u_i(t)),\quad q_i(0)\quad given \tag{22}$$

Where, $q_i(t)=\begin{bmatrix}x_i(t)\\y_i(t)\\\theta_i(t)\end{bmatrix},f=\begin{bmatrix}v_i\cos\theta_i\\v_i\sin\theta_i\\w_i\end{bmatrix},u_i(t)=\begin{bmatrix}v_i(t)\\w_i(t)\end{bmatrix}$

To achieve the desired cost function, the coupling that is inherent with the centralized approach is eliminated by defining n different costs, one for each robot, and only the couplings between any given robot and its neighbors are present. To facilitate the results, the terminal constraint and the terminal cost are assumed to be decoupled i.e. $G=diag(G_1,...,G_n)$. It is also assumed that h_p,h_c are identical for all robots. Considering (8) and defining $q(t)=(q_1,...,q_n),u=(u_1,...,u_n)$, the overall system can be decomposed into n sub-systems having the dynamics given by (22). Accordingly, the objective is to design a DRHCC for each robot that drives that to the desired state, i.e. $q_{d_i}=\left(C_{V_i},\theta_{d_i}\right)$, while cooperating with its neighbors.

Definition 4: The DRHCC cost function for each robot with the objective of reaching to its desired state in a cooperative way with its neighbors, is defined as

$$\bar{H}_i(q_i(t),q_j(t),u_i(.),h_p)=$$
$$\int_t^{t+h_p}(\sum_{j\in N_i}\frac{\xi}{2}\left\|q_i(\tau)-q_j(\tau)+D_{ij}\right\|^2+\xi\left\|q_i(\tau)-q_{d_i}\right\|^2+ \tag{23}$$
$$\eta\left\|\begin{bmatrix}v_i(t)\\w_i(t)\end{bmatrix}\right\|^2]d\tau+\sigma\left\|q_i(t+h_p,q_i(t))-q_{d_i}\right\|^2$$

In the newly considered system, the state and the control constraints are separated for each robot, i.e. $q_i(t)\in\aleph$ and $u_i(t)\in\mathcal{U}$. Given $R=diag(R_1,...,R_n)$, control cost can be rewritten as $\|u\|_R^2=\sum_{i=1}^{n}\|u_i\|_{R_i}^2$, where each $R_i=\sigma I$ is a positive definite matrix. To proceed further, the concepts of "*distributed coverage vector*" and "*distributed coverage matrix*" are needed. Before that, some notations must be introduced.

As stated in Section 2, N_i is the set that contains the neighbors of the i^{th} robot. Therefore, there exists a Delaunay edge between each robot and its neighbors. Let $q_{-i}=(q_{i,1},...,q_{i,|N_i|})$, $C_{V_{-i}}=(C_{V_{i,1}},...,C_{V_{i,|N_i|}})$ and $q_{d_{-i}}$, denote the state and centroid and desired state vectors of the neighbors of robot i respectively, where $|N_i|$ is the number of elements in N_i. Now for each i^{th} robot, define the following vector:

$$\overline{cov}^i=(....,\overline{cov}_l^i,...,\overline{cov}_{|N_i|+1}^i),\quad where$$
$$\overline{cov}_l^i=q_i-q_j+D_{ij}\qquad l=1,...,|N_i|,\forall j\in N_i \tag{24}$$
$$\overline{cov}_{|N_i|+1}^i=q_i-q_{d_i} \tag{25}$$

Let the linear mapping from $q^i=(q_i,q_{-i})$ to $\overline{cov}^i$ be written as

$$\overline{cov}^i=\bar{T}^iq^i+\bar{D}^i, \tag{26}$$
$$\bar{D}^i=\left(...,D_{ij},...,-q_{d_i}\right),j=1,...,|N_i| \tag{27}$$

We can now state the following definition:

Definition 5: *"Distributed coverage vector" and "distributed coverage matrix"*

The "*distributed coverage vector*" is defined as

$$cov^i=(...,cov_l^i,...,cov_{|N_i|+1}^i),\quad where$$
$$cov_l^i=\frac{1}{2}\overline{cov}_l^i,l=1,...,|N_i| \tag{28}$$
$$cov_{|N_i|+1}^i=\overline{cov}_{|N_i|+1}^i \tag{29}$$

The "*distributed coverage matrix*" is defined as matrix T^i in the following equation

$$cov^i=T^iq^i+D^i \tag{30}$$

where $D^i=\left(...,\frac{1}{2}D_{ij},...,q_{d_i}\right)\forall j\in N_i$ and $q^i=\left(q_i,q_{-i}\right)$.

Since $Q\neq diag(Q_1,...,Q_n)$, the term $\frac{1}{2}$ is added in (28) in order to satisfy the following equation:

$$\|q-q_d\|_Q^2=\sum_{i=1}^{n}\left\|\begin{matrix}q_i-q_{d_i}\\q_{-i}-q_{d_{-i}}\end{matrix}\right\|_{Q_i}^2=\sum_{i=1}^{n}\left\|q^i-q_d^i\right\|_{Q_i}^2,\quad q_d^i=(q_{d_i},q_{d_{-i}})$$

Note that if robot i and its neighbors are located at their desired state, i.e. $q^i=q_d^i$, then (24), (25) and therefore (28), (29) will be equal to zero. Hence

$$T^iq_d^i+D^i=0\Rightarrow D^i=-T^iq_d^i\Rightarrow cov^i=T^iq^i-T^iq_d^i\Rightarrow$$
$$cov^i=T^i(q^i-q_d^i) \tag{31}$$

Proposition 3: The cost function given by (23) can be rewritten as

$$\bar{H}_i(q^i(t),u(.),h_p)=\int_t^{t+h_p}(\left\|q^i(\tau)-q_d^i\right\|_{Q_i}^2+\|u_i(\tau)\|_{R_i}^2)d\tau$$
$$+\left\|q_i(t+h_p;q_i(t))-q_{d_i}\right\|_{G_i}^2 \tag{32}$$

and $\sum_{i=1}^{n}\bar{H}^i(q^i(t),u_i(.),h_p)=\bar{H}(q(t),u(.).h_p)$, where $\bar{H}$ is the CRHCC cost function.

Proof: It can be proven using Definitions 2, 3, 4, 5, Proposition 2 and Lemmas 1, 2.■

Based on pertinent literature this form of DRHCC cost function is very useful for stability analysis.

Now suppose that n DRHCC optimal problems, one corresponding to each robot, are all solved at a common time instant called "update time", denoted by $t_k=t_0+h_ck$, $k\in\{0,1,...\}$. As stated in (23), (32), for each cost function, there is a term that contains coupling between the corresponding robot and its neighbors. So, in every update time, when the local optimal problems are solved, each robot requires to know the state trajectories of all its neighbors over time interval $\left[t_k,t_k+h_p\right],k\in N$. But, such information doesn't exist at instant t_k. Therefore each robot must assume some state trajectories for its neighbors at $[t_k,t_k+h_p]$ and then solves its optimal control problem. The trajectories that each robot assumes for its neighbors are called assumed trajectories similar to Dunbar and Murray (2006). To ensure agreement between the actual and the assumed trajectories, an additional constraint called "*agreement constraint*" is added to DRHCC problem of each robot.

Definition 6: *"Assumed Control"*

At every time interval $\tau\in\left[t_k,t_k+h_p\right]$, the assumed control for each robot is defined as follows

$\hat{u}_i(\tau;q_i(t_k))=0;$ *if* $q(t_k)=q_d$, otherwise

$$\hat{u}_i(\tau;q_i(t_k))=\begin{cases} u_i^*(\tau;q_i(t_{k-1})), & \tau\in\left[t_k,t_{k-1}+h_p\right) \\ \begin{bmatrix} -k_{v_i}\sqrt{e_{x_i}{}^2+e_{y_i}{}^2}\cos(e_{\theta_i}) \\ -k_{w_i}e_{\theta_i}\end{bmatrix}, & \tau\in\left[t_{k-1}+h_p,t_k+h_p\right]\end{cases}$$

where k_{v_i}, k_{w_i} are positive scalar gains and

$$e_{\theta_i}=\theta_i-\theta_{d_i}, \begin{bmatrix} e_{x_i} \\ e_{y_i}\end{bmatrix}=\begin{bmatrix}\cos(\theta_i) & \sin(\theta_i) \\ -\sin(\theta_i) & \cos(\theta_i)\end{bmatrix}\begin{bmatrix} x_i-C_{V_{xi}} \\ y_i-C_{V_{yi}}\end{bmatrix}$$

The actual and the assumed state trajectories are denoted by $q_i(.;q_i(t_k))$ and $\hat{q}_i(.;q_i(t_k))$ respectively.

The DRHCC problem can now be stated as follows:

Problem 2: *"DRHCC problem"*

With a given fixed update period time h_c and an optimization period time h_p, for every $i=1,...,n$ and at any sampling time $t_k, k\in N$, for $\beta\in\left[t_k,t_k+h_p\right]$ and with given $\hat{u}_{-i}(\beta,q_{-i}(t_k)), q_i(t_k), q_{-i}(t_k), \hat{u}_i(\beta;q_i(t_k))$ at $\beta\in\left[t_k,t_k+h_p\right]$ find $\bar{H}_i^*(q_i(t_k),q_{-i}(t_k),h_p)=\min\limits_{u(.)}\bar{H}_i(q^i(t_k),u_i(.;q_i(t_k),h_p)$

where $\bar{H}_i$ is defined as (23) and this optimization is subject to

$$\dot{q}_i(\beta)=f(q_i(\beta),u_i(\beta))$$

$$\dot{\hat{q}}_j(\beta)=f(\hat{q}_i(\beta),\hat{u}_i(\beta)), j\in N_i$$

$$u_i(\beta;q_i(t_k))\in\mathcal{U}, q_i(\beta;q_i(t_k))\in\aleph$$

$$\left\|q_i(\beta;q_i(t_k))-\hat{q}_i(\beta;q_i(t_k))\right\|\le h_c{}^2\kappa \quad ,\kappa\in(1,\infty) \tag{33}$$

$$q_i(t_k+h_p;q_i(t_k))\in\Psi_i(\omega_i)=:\left\{q:\sigma\left\|q_i-q_{d_i}\right\|^2\le\omega_i,\omega_i\ge 0\right\} \tag{34}$$

Equation (33) is called agreement constraint and (34) is target or terminal constraint. The optimal solution for each DRHCC problem is denoted by $u_i^*(\tau;q_i(t_k)),\tau\in\left[t_k,t_k+h_p\right]$ and the closed-loop system where we wish to stabilize it, is

$$\dot{q}(\tau)=f(q(\tau),u^*(\tau)),\tau\ge 0 \tag{35}$$

where $u^*(\tau;q(t_k))=(u_1^*(\tau;q_1(t_k)),...,u_n^*(\tau;q_n(t_k))),\ \tau\in\left[t_k,t_k+h_c\right)$.

The optimal state trajectory for i^{th} robot is denoted by $q_i^*(\tau;q_i(t_k)),\ \tau\in\left[t_k,t_k+h_p\right]$.

In DRHCC problem, initialization is more difficult. As stated before, each robot has an assumed trajectory. To solve the optimal problem corresponding to a robot at t_0, the assumed control information of its neighbors is needed. Since a portion of the assumed control of each step is assumed to be the optimal control that is obtained in the previous step, and since prior to t_0 no optimal problem has been solved, one must define an initialization method to obtain the assumed control. The time instant that this initialization occurs is denoted by t_0-h_c.

Algorithm 1: *"Initial setting method"*

At time instant t_0-h_c, the Problem 2 with initial state $q^i(t_0-h_c)$ and with $\hat{u}_i(\tau;q_i(t_0-h_c))=0$ for all $\tau\in\left[t_0-h_c,t_0-h_c+h_p\right]$ and $\kappa=+\infty$ must be solved.

The optimal control that is obtained by solving this problem with the above conditions is the assumed control for the time interval $\left[t_0,t_0+h_p\right]$. $\kappa=+\infty$ implies that the agreement constraint is not important prior to t_0. State and control trajectories that are obtained at t_0-h_c over interval $\tau\in\left[t_0-h_c,t_0-h_c+h_p\right]$ are denoted by $q_i^*(\tau;q_i(t_0-h_c))$ and $u_i^*(\tau;q_i(t_0-h_c))$ respectively. This optimal control is applied to i^{th} robot over $\left[t_0-h_c,t_0\right)$. Now, the proposed DRHCC algorithm is given in Table 1.

TABLE 1: DRHCC ALGORITHM

Goal: Asymptotically drive a group of *n* non-holonomic mobile robots toward centroidal Voronoi configuration.
At instant t_0-h_c every robot:
A1- senses its state and transmits the information about its state to the neighbors and receives its neighbors' state
A2- computes its Voronoi region $V_i(t_0-h_c)$ and q_{d_i} according to (2), (5), (7), (10) and (11)
A3- follows the initial setting method given in Algorithm 1
At every update instant $t_k, k\in N$ each robot:
B1- senses its own and its neighbors' state (or receives neighbors state).
B2- computes its Voronoi region $V_i(t_k)$.
B3- computes q_{d_i} according to (5) and (7), (10) and (11)
B4- transmits the information about its q_{d_i} to each of its neighbors in the system and retrieves the same information from its neighbors
B5- computes its own and its neighbors assumed trajectory using (22)
B6- computes distributed optimal control trajectory $u_i^*(\tau;q_i(t_k))$, over interval $\tau\in[t_k,t_k+h_p]$ using Problem 2
Over every interval $[t_{k-1},t_k)$, each robot:
C1- applies the distributed optimal control trajectory that has been obtained at t_{k-1}
C2- computes its assumed control for $[t_k,t_k+h_p]$ according to Definition 6
C3- transmits its assumed control that was computed in C2 to every neighbors and receives their assumed control

5. STABILITY ANALYSIS

As stated before, the overall cost function for the system (8) is given by (18) and

$$V(t_k)=\bar{H}^*(q(t_k),h_p)=\sum_{i=1}^{n}\bar{H}_i^*(q_i^*(t_k),q_{-i}(t_k),h_p) \tag{36}$$

Proposition 4: The desired state given by (7), i.e. q_d, is an equilibrium point of the closed-loop system (35).

Proof: It can be proven similar to Mohseni (2012) and by using Definitions 1-6 and Algorithm 1.■

Theorem 1: Based on DRHCC algorithm given in Table 1, the closed-loop system (35) converges to centroidal Voronoi configuration and q_d is an asymptotically stable equilibrium point for that, with $\aleph$ as its region of attraction (ROA).

Proof: According to (23) and Proposition 3, it is clear that (36) is a non-negative, piecewise and continuous function and it is equal to zero if and only if $q(t_k) = q_d$. So $\bar{H}^*$ can be selected as a Lyapunov function. Hence, by using all above definitions, lemmas and propositions, and since $Q > 0$, according to Khalil (1996) and all above definitions, lemmas and propositions and using Barbalat's lemma it can be proven that $\|q(t) - q_d\| \to 0$ as $t \to \infty$. Thus, q_d is an asymptotically stable equilibrium point for closed-loop system (35) with region of attraction $\aleph$, as explored in Mohseni (2012).

6. SIMULATION RESULTS

The proposed DRHCC algorithm has been numerically simulated for 20 mobile robots. The event distribution density function is assumed to be a Gaussian density function equal to $e^{-[(x-0.1)^2+(y-0.9)^2]}$. It is also assumed that the robots are initially distributed randomly in the mission space as shown in Figure 1-(a). After 0.4 second, the robots converge to a centroidal Voronoi configuration shown in Figure 1-(b). The Robots' paths are shown in Figure 1-(c) and Figure 1-(d) shows the gradual reduction of $\bar{H}^*(q(t_k), h_p)$ towards zero.

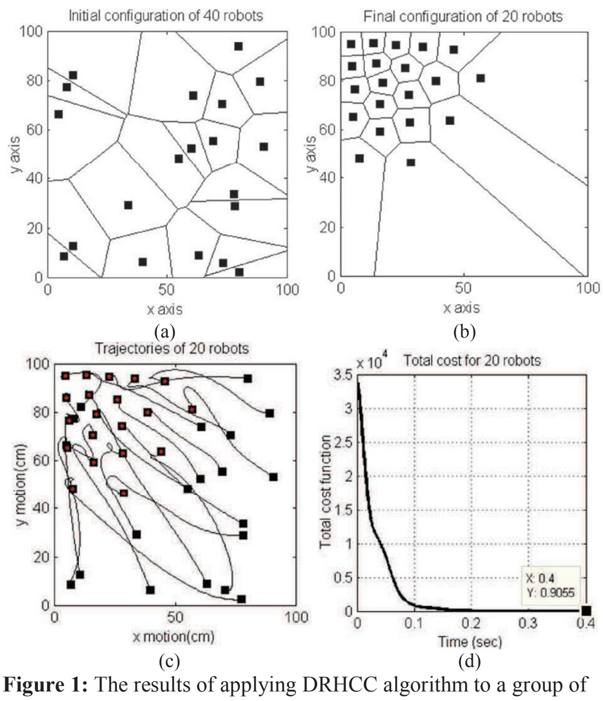

Figure 1: The results of applying DRHCC algorithm to a group of 20 mobile robots.

7. CONCLUSION

In this paper, the authors proposed a distributed receding horizon coverage control algorithm for controlling a group of non-holonomic mobile robots. In the proposed algorithm, the dynamics of every mobile robot was assumed decoupled from each other. The objective of the coverage algorithm considered here was to maximize the detection of the occurrence of the events. Simulation results validated the algorithm and convergence of the robots to the desired configuration with an admissible orientation. The proposed approach can be extended to large scale networks, systems with various constraints on their dynamics and inputs. Future research directions include extending the control laws to time-varying environments (e.g., consider a time-varying distribution density function), multi-agents formation control, non-isotropic sensors, etc.

REFERENCES

Aurenhammer, F., Klein, R. Voronoi Diagrams. Partially supported by the Deutsche Forschungsgemeinschaft, grant Kl 655 2-2.

Bloch, A.M., Reyhanoglu, M. and McClamorch, H. (1992). Control and stabilization of nonholonomic dynamic system. *IEEE Transaction on Automatic Control*, 37, 11, pp.1746-1757.

Khalil, H.K. (1996). *Nonlinear Systems*, Second Edition. Prentice Hall.

Cortes, J., Martinez, S., Karatas, T., and Bullo, F. (2004). Coverage control for mobile sensing networks, *IEEE Trans*. on Robotics and Automation, vol. 20, no. 2.

Defoort, M. (2010). Distributed receding horizon planning for multi-robot systems, *IEEE International Conference on Control Applications Part of 2010 IEEE Multi-Conference on Systems and Control*. Yokohama, Japan, September 8-10.

Dunbar, W. B. (2004). *Distributed Receding Horizon Control of Multi-agent Systems*, In Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy, California Institute of Technology Pasadena, California.

Dunbar, W. B., Murray, R. M. (2005). Distributed receding horizon control for multi-vehicle formation stabilization, *Automatica* 42 (2006) 549 – 558.

Fierro, R., Lewis, F. L. (1996). Control of a Non-holonomic mobile robot: backstepping Kinematics into dynamics, *Automation and robotics research institute the University of Texas* at Arlington 7300 Jack Newell BLvd. South forth worth,Tx76118-7115.

Franco, E., Parisini, T., and Polycarpou, M, (2005). Stable Receding-Horizon Cooperative Control of a Class of Distributed Agents, *American Control Conference*, June 8-10, Portland, OR, USA.

Khalil, H.K. (1996). *Nonlinear Systems, Second Edition*. Prentice

Meguerdichian, S., Koushanfar F., Potkonjak.M, and Srivastava, M. B, (2001). Coverage problems in wireless ad-hoc sensor networks, in Proc. of *IEEE INFOCOM* , pp. 1380-1387.

Mohseni, F. (2012). *Distributed Receding Horizon Coverage Control by Multiple Mobile Robots In unknown Environment*, M.Sc thesis, Amirkabir University of Technology, Tehran, Iran.

Mohseni, F., Doustmohammadi, A., and Menhaj, M. B. (2012). Centralized Receding Horizon Coverage Control for Mobile Sensory Networks, *IEEE Third International Conference on Intelligent Systems, Modeling and Simulation,* ISMS, Kota Kinabalo, Malasya, ISBN. 978-0-7695-4668-1, pp. 588-593.

Mohseni, F., Doustmohammadi, A., and Menhaj, M. B. (2012), Distributed Receding Horizon Coverage Control for Mobile Sensory Networks, *IEEE Third International Conference on Intelligent Systems, Modeling and Simulation, ,* ISMS, Kota Kinabalo, Malasya ISBN. 978-1-4673-0886-1, pp. 594-599.

Okabe, A., Boot, B., and Sugihara,K. (1992), Spatial Tessellations: Concepts and Applications of Voronoi Diagrams, *John Wiley & Sons Ltd.*