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Brief paper

Persistent awareness coverage control for mobile sensor networks☆

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ABSTRACT

In this paper the persistent awareness coverage problem for mobile sensors with awareness loss is considered, where persistent coverage and awareness coverage are addressed simultaneously. The goal is to cover the mission domain periodically with a finite period and guarantee full awareness coverage of a finite set of points of interest. A closed path is designed such that it is possible to develop periodic speed controllers for mobile sensors. When there is no constraint on the period, the least number of mobile sensors that are needed for the persistent awareness coverage task is derived. Given a network of mobile sensors and a finite period, it is shown that the persistent awareness coverage task can be accomplished if there exists a solution to a set of linear inequalities. Finally, if there is no awareness loss, the proposed approach guarantees full awareness coverage of the whole mission domain even if only one sensor is deployed.

1. Introduction

Recently, mobile sensor networks have attracted much attention due to its versatility in many applications (Anisi, Ogren, & Hu, 2010; Casbeer et al., 2006; Fan, Feng, Wang, & Song, 2013; Kingston, Beard, & Holt, 2008; Leonard et al., 2007; Tang & Özgüner, 2005). In this paper we focus on the persistent awareness coverage problem for mobile sensors with awareness loss, where the goal is to guarantee full awareness coverage of a finite set of points of interest and persistent coverage of the mission domain simultaneously. This problem emerges in many practical applications, where some regions are of much more importance with respect to the other regions in the mission domain. Consequently, one needs to discriminate these two classes of regions. For example, in an application where a mobile sensor network is employed to survey a nuclear power plant persistently, the nuclear reactor must be covered with full awareness while the other regions of less importance only need to be revisited frequently.

Coverage control can be generally classified into static coverage control and dynamic coverage control. In static coverage control, the goal is to optimize the locations of sensors to improve the quality of service provided by mobile sensor networks (Cortés, Martínez, & Bullo, 2005; Cortés, Martínez, Karatus, & Bullo, 2004; Kwok & Martínez, 2010; Li & Cassandras, 2005; Schwager, Rus, & Slotine, 2009; Zhong & Cassandras, 2011). In Li and Cassandras (2005), given a probabilistic sensing model and a density function representing the occurring frequency of random events, a distributed gradient-based algorithm is proposed to maximize the joint detection probabilities of these events. In Zhong and Cassandras (2011), coverage control combined with data collection for mobile sensor networks is addressed by solving an optimization problem trading off these two objectives. A Voronoi partition is employed in Cortés et al. (2004) to develop a distributed coverage control law and the resulting optimal network configuration is that each sensor is located at the centroid of its corresponding Voronoi partition. In Cortés et al. (2005), this work is extended to a more realistic scenario where agents have limited sensing and/or communication ranges. In Kwok and Martínez (2010), Voronoi partition based coverage is addressed for a group of nonholonomic agents. A decentralized adaptive controller is developed in Schwager et al. (2009) to accomplish the static coverage task without *a priori* knowledge of the mission domain.

When the mission domain cannot be fully covered by any static configuration of a sensor network, the problem of dynamic coverage (Hokayem, Stipanovic, & Spong, 2007; Hussein & Stipanovic, 2007; Song, Feng, Fan, & Wang, 2011; Wang & Hussein, 2010) arises in which each point in the mission domain is sampled by the mobile sensors until a prescribed coverage level is achieved. In Hussein and Stipanovic (2007), coverage control laws for connected

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networks are developed to achieve satisfactory coverage of the mission domain with guaranteed collision avoidance. An awareness coverage model is proposed in Wang and Hussein (2010), which characterizes how "aware" a network of vehicles is of events occurring over a large-scale domain. A decentralized switching control strategy is proposed to drive all points' awareness coverage to a neighborhood of full awareness when awareness loss is not considered. The persistent coverage problem is addressed in Hokayem et al. (2007), where the goal is to cover all points in the mission domain periodically. An open path is first designed for the agents. Then, the path is divided into several parts and each agent moves along its corresponding part at a constant speed back and forth to cover the mission domain repeatedly.

Problems that are closely related to persistent awareness coverage include persistent surveillance and monitoring, where repetitive motion of the agents is required (Elmaliach, Agmon, & Kaminka, 2007; Nigam, Bieniawski, Kroo, & Vian, 2012; Nigam & Kroo, 2008; Smith, Schwager, & Rus, 2012). In Smith et al. (2012), the changing environment is modeled as an accumulation function defined over a finite set of locations. The function increases at locations that are not covered by any agent and decreases at locations that are covered by an agent. Given several closed paths, agents' speed controllers are computed to prevent the accumulation function from growing unbounded at any location. The persistent surveillance problem is addressed in Nigam and Kroo (2008), where the goal is to minimize the time between visitations to the same region. In Nigam et al. (2012), the authors further take into account endurance constraints of the aerial vehicles in the persistent surveillance problem. In Elmaliach et al. (2007), area patrolling under frequency constraints is addressed, where a given region is required to be continually sampled such that each point in this region is revisited with equal frequency.

Trajectory planning for mobile sensors has been studied extensively in literature (Choi & How, 2010; Kant & Zucker, 1986; Tang & Özgüner, 2005). In Tang and Özgüner (2005), trajectory planning for multiple target surveillance is addressed by solving an optimization problem whose objective is the minimization of the average time between two consecutive observations of each target. The continuous trajectory of mobile sensors is planned in Choi and How (2010) to reduce the uncertainty in quantities of interest. In Kant and Zucker (1986), it is established that decoupling path planning from speed control is an efficient approach to the complex trajectory planning problem. Following this idea, we first construct a closed path for mobile sensors based on the pioneering work in Hokayem et al. (2007) such that when a sensor completes one cycle of the path all points in the mission domain can be covered by this sensor. Then, the agents' speed controllers are designed to accomplish the persistent awareness coverage task. The main contributions of this work are threefold. Firstly, based on a necessary and sufficient condition for achievement of full awareness coverage of an arbitrary point in the mission domain, it is shown that the least number of sensors that are needed for persistent awareness coverage can be derived by solving a mixed-integer nonlinear programming problem (MINLP). Secondly, a formal formulation of persistent coverage of an arbitrary point in the mission domain is provided such that it is possible for us to consider the case of persistent coverage with a given period. When a mobile sensor network and a finite period are given, it is shown that the persistent coverage task can be accomplished if a set of linear inequalities has a solution. Finally, if there is no awareness loss, the proposed approach can drive all points' awareness coverage to full awareness exactly rather than to a neighborhood of full awareness even if only one mobile sensor is deployed.

The remainder of this paper is organized as follows. In Section 2, the problem of persistent awareness coverage is formulated. In Section 3, trajectory planning of mobile sensors is presented to accomplish the persistent awareness coverage task with and without a given period. A simulation example is given in Section 4 to illustrate the main results. Finally, Section 5 concludes the paper.

2. Problem formulation

Consider a mobile sensor network operating in workspace $\mathbb{R}^2$. The position of each sensor $A_i,\ i \in S = \{1, 2, \ldots, n\}$ is denoted by q_i. The mission domain D is a convex polygonal region and an arbitrary point in D is denoted by q. $\mathcal{P} = \{p_1, \ldots, p_m\}$ is a finite set of points of interest in D.

In this paper, we use the sensor model proposed in Hussein and Stipanovic (2007) and Wang and Hussein (2010)

$$M_i(q_i, q) = \begin{cases} \dfrac{G_i}{{r_i}^4}(\|q_i - q\|^2 - r_i^2)^2 & \text{if } \|q_i - q\| \le r_i, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $r_i > 0$ is the limited sensing range. We assume the sensors have uniform sensing function, that is, r_i and G_i are identical for all sensors.

In this work, we consider a mobile sensor network's awareness coverage (Wang & Hussein, 2010) of the mission domain which is a distribution $x(q, t)$ describing how "aware" the mobile sensor network is of events occurring at a specific location q at time t. Without any loss of generality, we assume that $x(q, t) \in (-\infty, 0]$. When $x(q, t) = 0$ it indicates that the point q attains full awareness coverage. For each point $q \in D$, the awareness coverage $x(q, t)$ is assumed to satisfy the following differential equation

$$\begin{aligned} &\dot{x}(q, t) = -\left(\sum_{i=1}^{n} M_i(q_i, q) - \alpha\right) x(q, t), \\ &x(q, 0) = x_0(q), \end{aligned} \tag{2}$$

where $\alpha \ge 0$ is a constant awareness loss.

In practice, full awareness coverage of the whole mission domain may not be necessary because not all points in the mission domain are of the same importance. Moreover, in the case that agents' resource and capability are limited it is often unaffordable to be fully aware of all points in the mission space. Thus, in this work we focus on guaranteeing full awareness coverage of a set of points of interest while covering the mission domain periodically with a finite period T^*. Note that a point q is covered by the mobile sensor network in time interval $[t_s, t_f]$ if and only if

$$\int_{t_s}^{t_f} \sum_{i=1}^{n} M_i(q_i, q) d\tau > 0. \tag{3}$$

Then, the goal of persistent awareness coverage is to plan a network of mobile sensors' trajectory such that $x(p_j, t) \to 0,\ \forall p_j \in \mathcal{P}$ as $t \to \infty$ and there exists a finite period T^* satisfying $\int_{t_0}^{t_0+T^*} \sum_{i=1}^{n} M_i(q_i, q) d\tau > 0,\ \forall t_0 \ge 0,\ \forall q \in D$. Furthermore, we consider the persistent awareness coverage problem with a given period of time, where all points in the mission domain are needed to be covered by the mobile sensor network in the time period.

3. Trajectory planning for persistent awareness coverage

In this section, we plan the mobile sensor network's trajectory to cover the mission domain periodically and guarantee full awareness coverage of all points of interest simultaneously. Path planning is decoupled from speed control to solve this trajectory planning problem as suggested in Kant and Zucker (1986).

3.1. Path planning

In Hokayem et al. (2007), an open path has been designed for the persistent coverage problem. In this subsection, a closed path is constructed based on the work in Hokayem et al. (2007) such

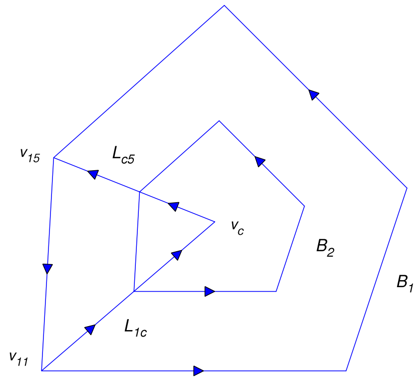

Fig. 1. Closed path Ω for mobile sensor network.

that when sensors move along the path perpetually all points in the mission domain are covered periodically. The boundary of the mission domain D is supposed to be a convex polygon denoted by B_1 with vertices $v_{1k} = (x_{1k}, y_{1k}),\ k = 1, \ldots, \gamma$. This assumption is not restrictive because in dynamic coverage agents are not required to stay inside the mission domain. If the boundary of the mission domain is non-convex, one can consider persistent awareness coverage of a convex domain that covers the non-convex mission domain.

Given the vertices of the polygon B_1, the geometric center $v_c = (x_c, y_c)$ can be calculated. Then, one can construct the inner polygons using the approach proposed in Hokayem et al. (2007). Define $d_{\max} = \max_{k\in\{1,\ldots,\gamma\}} \|v_{1k} - v_c\|$ as the largest distance from the vertices of B_1 to the geometric center v_c. Let λ be the least integer satisfying $2\lambda r \geq d_{\max}$, where $0 < r < r_i$ is a positive constant. Then, the ratios $\rho_j = 2(j-1)r_i/d_{\max},\ j = 2, \ldots, \lambda$ can be calculated. Given these definitions, one can define the inner polygons B_j for $j = 2, \ldots, \lambda$ as $B_j \triangleq (1-\rho_j)B_1$ with vertices being given by $v_{jk} = \rho_j v_c + (1-\rho_j)v_{1k},\ k = 1, \ldots, \gamma$.

Since the polygons $B_j,\ j = 1, 2, \ldots \lambda$ are similar, when we connect v_c and $v_{1\gamma}$ with a line denoted by $L_{c\gamma}$ the vertices $v_{j\gamma},\ j = 2, \ldots, \lambda$ are surely on this line. The line connecting v_c and v_{11} is denoted by L_{1c}. Without loss of generality, we assume that the sensors move along the path counterclockwise. The following sequential points provide a closed path for the mobile sensors,

$$\Omega : \underbrace{v_{11}, \ldots, v_{1\gamma}, v_{11}}_{B_1}, \underbrace{v_{21}, \ldots, v_{2\gamma}, v_{21}}_{B_2}, \ldots, \underbrace{v_{\lambda 1}, \ldots, v_{\lambda\gamma}, v_{\lambda 1}}_{B_\lambda}, \underbrace{v_c, v_{\lambda\gamma}, \ldots, v_{1\gamma}}_{L_{c\gamma}}, v_{11}. \tag{4}$$

Let Q be the set of points on Ω, that is, $Q = B \bigcup L_{c\gamma} \bigcup L_{1c}$, where $B = \bigcup_{j=1,\ldots,\lambda} B_j$. Fig. 1 illustrates an example of the closed path Ω.

Lemma 1. *When a sensor completes one full cycle of the closed path Ω, all points in the mission domain D can be covered.*

Proof. It has been proved in Hokayem et al. (2007) that if the geometric center v_c and every point on the polygons $B_j,\ j = 1, 2, \ldots, \lambda$ are visited by a sensor with sensing range r, then all points in the mission domain can be within the sensing range of this sensor. Note also that $r_i > r$ and $M_i(q_i, q) > 0$ if $\|q_i - q\| < r_i$. Thus, one has $\int_0^T M_i(q_i, q)d\tau > 0,\ \forall q \in D$. □

3.2. Speed controller

Let $\mathbb{R}_+$ be the set of strictly positive real numbers. We derive the speed controller $v : Q \to \mathbb{R}_+$ along the path Ω to accomplish the persistent awareness coverage task. For simplicity, the speed controller only relies on each agent's current position, that is, all agents' speed is identical whenever they arrive at a same point $\tilde{q} \in Q$. Since $v(\tilde{q}) > 0$ for $\tilde{q} \in Q$, each agent completes one cycle of the closed path in a period

$$T = \int_\Omega \frac{1}{v(\tilde{q})} d\tilde{q}. \tag{5}$$

Given these definitions, the following theorem provides a necessary and sufficient condition under which an arbitrary point in the mission domain attains full awareness coverage.

Theorem 1. *For an arbitrary point $q \in D$, $\lim_{t\to\infty} x(q, t) = 0$ if and only if*

$$\int_0^T \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau > 0. \tag{6}$$

Proof. From the definition (2), one has

$$x(q, t) = x_0(q) e^{-\int_0^t (\sum_{i=1}^n M_i(q_i,q) - \alpha) d\tau}. \tag{7}$$

Then, $\lim_{t\to\infty} x(q, t) = 0$ if and only if $\lim_{t\to\infty} \int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau$ goes to infinity.

Let k be the largest nonnegative integer such that $kT \leq t$. Note that $q_i(t) = q_i(t+T),\ \forall t \geq 0$. Then, one has

$$\int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau = k \int_0^T \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau + \int_{kT}^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau.$$

We first prove the necessity by contradiction. Let $\varepsilon = \int_0^T \left(\sum_{i=1}^n M_i (q_i, q) - \alpha \right) d\tau$. Assume $\varepsilon \leq 0$. Note that $M_i(q_i, q) \leq G_i, \forall i \in S$ always holds and $t - kT < T$. Then, one has

$$\int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau \leq \int_{kT}^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau \leq \int_{kT}^t nG_i d\tau < nTG_i.$$

This contradicts the fact that $\int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau$ goes to infinity as $t \to \infty$.

Next, we prove that the condition is also sufficient. Note that $M_i(q_i, q) \geq 0$ always holds. Then, one has

$$\int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau = k\varepsilon + \int_{kT}^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau \geq k\varepsilon - \int_{kT}^t \alpha d\tau > k\varepsilon - \alpha T.$$

When time goes to infinity, one also has $k \to \infty$. Then, if $\varepsilon > 0$, $\int_0^t \left(\sum_{i=1}^n M_i(q_i, q) - \alpha \right) d\tau \to \infty$ and $\lim_{t\to\infty} x(q, t) = 0$. This completes the proof of sufficiency. □

Theorem 1 indicates that periodic motion of networked sensors guarantees full awareness coverage of point q if and only if awareness coverage $x(q, t)$ increases strictly when each sensor completes one cycle of the path Ω. The following corollary is a direct result of Lemma 1 and Theorem 1.

Corollary 1. *When $\alpha = 0$, a sensor's periodic movement along the path Ω with any speed controller $v(\tilde{q})$ can guarantee $\lim_{t\to\infty} x(q, t) = 0,\ \forall q \in D$.*

In fact, Corollary 1 implies that even if only one sensor is deployed full awareness of the whole mission domain can be guaranteed when awareness loss is not considered. However, one sensor often cannot drive awareness coverage of all points of interest to full awareness when awareness loss is taken into account. Then, it is natural to ask how many mobile sensors are needed for the persistent awareness coverage task.

To transform the condition for full awareness coverage of points of interest into constraints on the speed controller, we make the following assumption on the inverse of the speed controller.

Assumption 1. $v^{-1}(\tilde{q})$ can be represented by

$$v^{-1}(\tilde{q}) = \sum_{l=1}^{L} a_l \mathcal{K}_l(\tilde{q}), \tag{8}$$

where $\mathcal{K}_l(\tilde{q}) : Q \to \mathbb{R}_+,\ l = 1, \dots, L$ are bounded continuous basis functions and $a_l,\ l = 1, \dots, L$ are free parameters satisfying $0 < a_{\min} \le a_l \le a_{\max},\ l = 1, \dots, L$ with $a_{\max}$ and $a_{\min}$ being upper bound and lower bound respectively.

This assumption implies that the inverse of the speed controller can be described by a linear combination of a set of basis functions. Note that $\int_0^T M_i(q_i, p_j)d\tau = \int_\Omega M_i(\tilde{q}, p_j)v^{-1}(\tilde{q})d\tilde{q}$. Since the sensing function (1) is identical for all sensors, the necessary and sufficient condition (6) can be rewritten as

$$n \int_\Omega M_i(\tilde{q}, p_j)v^{-1}(\tilde{q})d\tilde{q} > \alpha T. \tag{9}$$

Substituting Eqs. (5) and (8) into the above inequality, one has

$$n \sum_{l=1}^{L} a_l \int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q} > \alpha \sum_{l=1}^{L} a_l \int_\Omega \mathcal{K}_l(\tilde{q})d\tilde{q}.$$

The following theorem provides a lower bound on the number of mobile sensors that are needed for persistent awareness coverage.

Theorem 2. *The least number of mobile sensors that are needed to accomplish the persistent awareness coverage task can be obtained by solving the following MINLP*

$$\begin{aligned}
&\textit{min } n \\
&\textit{s.t. } n \sum_{l=1}^{L} a_l \int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q} - \alpha \sum_{l=1}^{L} a_l \int_\Omega \mathcal{K}_l(\tilde{q})d\tilde{q} > 0, \\
&\quad \forall j \in \{1, \dots, m\}, \\
&a_{min} \le a_l \le a_{max}, \quad \forall l \in \{1, \dots, L\}, \\
&n \ge 1,\ n \in \mathbb{N},
\end{aligned} \tag{10}$$

where $a_1, \dots, a_L$ and n are the optimization variables.

Proof. Recall that T is the time period in which agent A_i completes one full cycle of the closed path Ω. Thus, T is a finite period during which all points in the mission domain are covered by the mobile sensor network. It follows from Theorem 1 that the persistent awareness coverage task can be accomplished if and only if the MINLP is feasible.

Next, we prove the feasibility of the MINLP. The first constraint in the MINLP (10) can be rewritten as

$$n > \frac{\alpha \sum_{l=1}^{L} a_l \int_\Omega \mathcal{K}_l(\tilde{q})d\tilde{q}}{\sum_{l=1}^{L} a_l \int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q}}, \quad \forall j \in \{1, \dots, m\}. \tag{11}$$

When agent A_i completes one cycle of Ω, all points in the mission domain are covered. Thus, $\int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q} > 0,\ \forall j \in \{1, \dots, m\}$. Note also that $0 < a_{\min} \le a_l \le a_{\max}$. Therefore, the left term in inequality (11) is finite. Let

$$N = \max_{j \in \{1,\dots,m\}} \frac{a_{\max}\alpha \sum_{l=1}^{L} \int_\Omega \mathcal{K}_l(\tilde{q})d\tilde{q}}{a_{\min} \sum_{l=1}^{L} \int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q}}. \tag{12}$$

Denote $\lceil x \rceil$ as the least integer that is larger than or equal to x. When $n \ge \lceil N \rceil + 1$, all constraints in the MINLP (10) are satisfied. Therefore, the MINLP is solvable. □

Remark 1. For the first constraint in the MINLP, the computational complexity of the first term and the second term is $O(mL)$ and $O(L)$, respectively. Hence, one can compute the first constraint with complexity $O(mL)$. Note also that the computation complexity of the second constraint is $O(L)$ and the third constraint can be computed at one time. Thus, the computational complexity of the MINLP is $O(mL)$.

Remark 2. Recall that $0 < r < r_i$. If the constant r is chosen to be close to r_i and there exist points of interest $p_j,\ j = 1, \dots, l$ located at the middle of two parallel edges on neighboring polygons, the sensor functions $M_i(\tilde{q}, p_j),\ \tilde{q} \in Q,\ j = 1, \dots, l$ will be close to zero. Then, the denominator in Eq. (12) will be close to zero, which leads to a requirement of a large number of mobile sensors. To avoid this problem, one can choose the constant r sufficiently away from r_i such that if the point of interest p_j is within the sensing range of a sensor located at $\tilde{q} \in Q$, one has $M_i(\tilde{q}, p_j) \ge \tau_j,\ \forall i \in S,\ \forall \tilde{q} \in Q,\ \forall p_j \in \mathcal{P}$ with τ_j being a positive constant. Note also that a lower value of r leads to more inner polygons to be constructed, which is undesirable. Thus, one needs to trade off these two aspects when choosing the constant r.

For a network of heterogeneous sensors with capability $M_\phi,\ \phi \in \{1, \dots, \Phi\}$, let n_ϕ be the number of sensors with sensing function M_ϕ. The necessary and sufficient condition (6) can be rewritten as $\sum_{\phi=1}^{\Phi} n_\phi \int_\Omega M_i(\tilde{q}, p_j)v^{-1}(\tilde{q})d\tilde{q} > \alpha T$. Then, one has the following corollary.

Corollary 2. *Given a network of heterogeneous sensors with capability $M_\phi,\ \phi \in \{1, \dots, \Phi\}$, the least number of sensors required for persistent awareness coverage can be obtained by solving the following MINLP*

$$\begin{aligned}
&\textit{min } \sum_{\phi=1}^{\Phi} n_\phi \\
&\textit{s.t. } \sum_{\phi=1}^{\Phi} \sum_{l=1}^{L} n_\phi a_l \int_\Omega M_\phi(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q} \\
&\qquad - \alpha \sum_{l=1}^{L} a_l \int_\Omega \mathcal{K}_l(\tilde{q}) \\
&d\tilde{q} > 0, \quad \forall j \in \{1, \dots, m\}, \\
&a_{min} \le a_l \le a_{max}, \quad \forall l \in \{1, \dots, L\}, \\
&n \ge 1,\ n \in \mathbb{N},
\end{aligned} \tag{13}$$

where $a_l,\ l = 1, \dots, L$ and $n_\phi,\ \phi = 1, \dots, \Phi$ are the optimization variables.

Next, we consider the persistent coverage problem when a network of mobile sensors and a time period are given. Initially, all sensors are supposed to be located on the closed path Ω. Without any loss of generality, we also assume that when agent A_i completes one cycle of the path Ω, it arrives at $q_{i+1}(0), \dots, q_n(0), q_1(0), \dots, q_{i-1}(0)$ sequentially and finally returns back to its initial location $q_i(0)$. Denote $q_{n+1}(0)$ as $q_1(0)$. One can define $\Omega(q_i(0), q_{i+1}(0)), i \in S$ as the part of the path Ω with starting point $q_i(0)$ and ending point $q_{i+1}(0)$. Then, $\bigcup_{i\in S} \Omega(q_i(0), q_{i+1}(0)) = \Omega$. Note that all points in the mission domain can be covered every period of time T when an agent moves along Ω perpetually. When a network of mobile sensors is deployed, a finite period during which every point in D is covered by the sensor network can be given by the following theorem.

Theorem 3. *When a network of mobile sensors moves along the path Ω perpetually, all points in the mission domain can be covered in a period T_{max} which is given by*

$$T_{max} = \max_{i\in S} T_i, \tag{14}$$

where $T_i = \int_{\Omega(q_i(0),q_{i+1}(0))} 1/v(\tilde{q})d\tilde{q}$ is the time period during which an agent moves from $q_i(0)$ to $q_{i+1}(0)$ along Ω.

Proof. For simplicity, denote $T_0 = T_n$ as the time that agent A_n moves from $q_n(0)$ to $q_1(0)$ along Ω. Let $\sigma = arg\max_{i\in S} T_i$. One can define T_σ^j as the traveling time from $q_\sigma(0)$ to $q_j(0)$, $j \in \{1, \dots, n\}$,

$$T_\sigma^j = \begin{cases} \displaystyle\sum_{i=\sigma}^{j-1} T_i & j > \sigma, \\ \displaystyle\sum_{i=\sigma}^{n-1} T_i + \sum_{i=0}^{j-1} T_i & j \le \sigma. \end{cases} \tag{15}$$

Then, one has $\int_{t_0}^{t_0+T} M_\sigma(q_\sigma, q)d\tau = \int_{t_0}^{t_0+T_\sigma^{\sigma+1}} M_\sigma(q_\sigma, q)d\tau + \sum_{j=0,j\neq\sigma}^{n-1} \int_{t_0+T_\sigma^j}^{t_0+T_\sigma^{j+1}} M_\sigma(q_\sigma, q)d\tau$.

Note that all agents' traveling speed is identical whenever they arrive at the same point $\tilde{q} \in Q$. Thus, if two agents start from the same location then they will move the same distance along the path during the same time period. That is, if $q_i(t_0^i) = q_j(t_0^j)$, one has $q_i(t + t_0^i) = q_j(t + t_0^j)$, $\forall t \ge 0$. Recall that T_σ^j is the traveling time from $q_\sigma(0)$ to $q_j(0)$, $j \in \{1, \dots, n\}$, that is, $q_\sigma(T_\sigma^j) = q_j(0)$. Then, $q_\sigma(t + T_\sigma^j) = q_j(t)$, $\forall t \ge 0$, $\forall j \in \{1, \dots, n\}$. Therefore, for any $j \in \{0, \dots, n-1\}$ and $j \neq \sigma$, one has $\int_{t_0+T_\sigma^j}^{t_0+T_\sigma^{j+1}} M_\sigma(q_\sigma, q)d\tau = \int_{t_0}^{t_0+T_j} M_j(q_j, q)d\tau$, $\forall t_0 \ge 0$. Then, $\int_{t_0}^{t_0+T} M_\sigma(q_\sigma, q)d\tau = \sum_{j=1}^{n} \int_{t_0}^{t_0+T_j} M_j(q_j, q)d\tau$. Note that $T_\sigma \ge T_j$, $\forall j \in S$. Thus, $\int_{t_0}^{t_0+T_\sigma} \sum_{j=1}^{n} M_j(q_j, q)d\tau \ge \int_{t_0}^{t_0+T} M_\sigma(q_\sigma, q)d\tau > 0$, $\forall t_0 \ge 0$, $\forall q \in D$ which implies that all points in the mission domain can be covered by the mobile sensor network every period $T_{\max}$. $\square$

Given Theorems 2 and 3, one can obtain the following result.

Theorem 4. *Given a network of n mobile sensors and a time period T^*, all points in the mission domain D are covered by the mobile sensor network every T^* and $\lim_{t\to\infty} x(p_j, t) = 0$, $\forall j \in \{1, \dots, m\}$ if the following linear inequalities in terms of the parameters a_l, $l = 1, \dots, L$*

$$\begin{cases} n\displaystyle\sum_{l=1}^{L} a_l \int_\Omega M_i(\tilde{q}, p_j)\mathcal{K}_l(\tilde{q})d\tilde{q} - \alpha \sum_{l=1}^{L} a_l \int_\Omega \mathcal{K}_l(\tilde{q})d\tilde{q} > 0, \\ \quad \forall j \in \{1, \dots, m\} \\ \displaystyle\sum_{l=1}^{L} a_l \int_{\Omega(q_i(0),q_{i+1}(0))} \mathcal{K}_l(\tilde{q})d\tilde{q} \le T^*, \quad \forall i \in \{1, \dots, n\} \\ a_{min} \le a_l \le a_{max}, \quad \forall l \in \{1, \dots, L\} \end{cases} \tag{16}$$

have a solution.

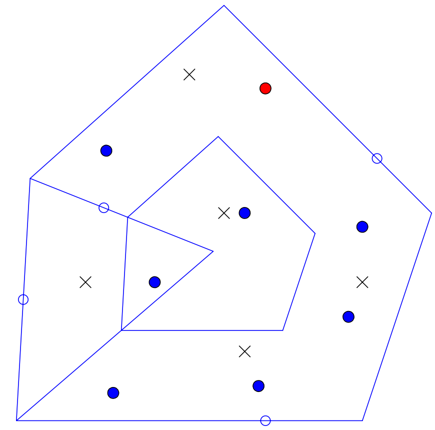

Fig. 2. A mission domain includes 8 points of interest.

Proof. When $T_{\max} \le T^*$, $\int_{t_0}^{t_0+T^*} \sum_{i=1}^{n} M_i(q_i, q)d\tau \ge \int_{t_0}^{t_0+T_{\max}} \sum_{i=1}^{n} M_i(q_i, q)d\tau > 0$, $\forall t_0 \ge 0$, $\forall q \in D$ which indicates all points in the mission domain can be covered every T^*. Recall that $T_{\max} = \max_{i\in S} T_i$. Thus, $T_{\max} \le T^*$ if $T_i \le T^*$, $\forall i \in S$, that is, $\sum_{l=1}^{L} a_l \int_{\Omega(q_i(0),q_{i+1}(0))} \mathcal{K}_l(\tilde{q})d\tilde{q} \le T^*$, $\forall i \in \{1, \dots, n\}$. It has been proved that $\lim_{t\to\infty} x(p_j, t) = 0$, $\forall j \in \{1, \dots, m\}$ if and only if the first inequality is satisfied. Thus, the persistent awareness coverage task with the given period can be accomplished if the linear inequalities have a solution. $\square$

Remark 3. In practice, it is often desirable to cover the mission domain as frequently as possible, that is, to minimize the period $T_{\max}$. To obtain the optimal speed controllers that minimize $T_{\max}$, one can solve a linear program whose objective is $T_{\max}$ and constraints are inequalities in (16).

Remark 4. To implement the developed speed controller in real-time, we firstly select an arbitrary sensor as the leader. Then, let the leader move along the path with a constant speed while the other sensors remain static on the closed path Ω. Since when a sensor completes one cycle of the planned path all points in the mission domain can be covered by this sensor, when the leader completes a cycle of Ω it can obtain all sensors' initial location information $q_i(0)$, $i \in 1, \dots, n$ and the location information of all points of interest p_j, $j \in 1, \dots, m$. Given a time period T^*, the leader can compute the speed controller efficiently by solving the linear inequalities (16). Then, the leader moves along the path with the computed speed and communicates the speed controller to the other sensors when it moves to the sensors' communication range. Note that all sensors remain static on the closed path except the leader. Therefore, when the leader completes another cycle of the path, all sensors acquire the information of the speed controller. Then, they move along Ω with the designed speed and guarantee the accomplishment of the persistent awareness coverage task.

4. Simulation results

In this section we give a simulation example to illustrate the main results. The mission domain D is a convex polygonal region and there are eight points of interest denoted by solid circles in D. All sensors have uniform sensing capability, that is, $r_i = 1$, $G_i = 1$, for $i = 1, \dots, 4$. Their initial locations are on the closed path and denoted by hollow circles in Fig. 2. We parameterize the inverse of the speed controller as a linear combination of five Gaussian functions $\mathcal{K}_l = \exp\{-(\tilde{q} - \mu_l)^2/2\}/2\pi$, $l = 1, \dots, 5$, where the locations of μ_l are marked with crosses in Fig. 2. The lower bound and upper bound on the variable a_l are $a_{\min} = 0.01$ and

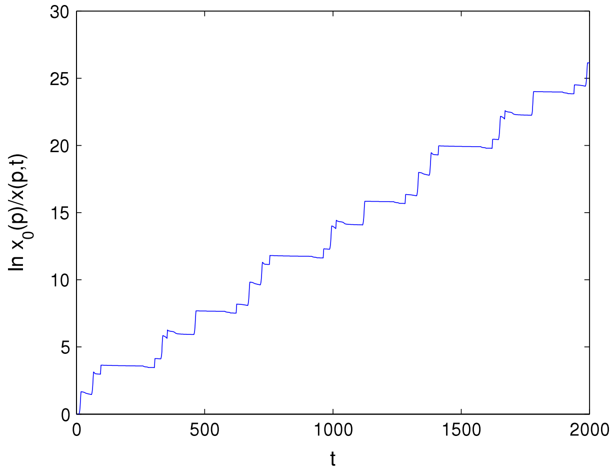

Fig. 3. Time evolution of function $\ln x_0(p)/x(p, t)$ at the point of interest denoted by red solid circle in Fig. 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

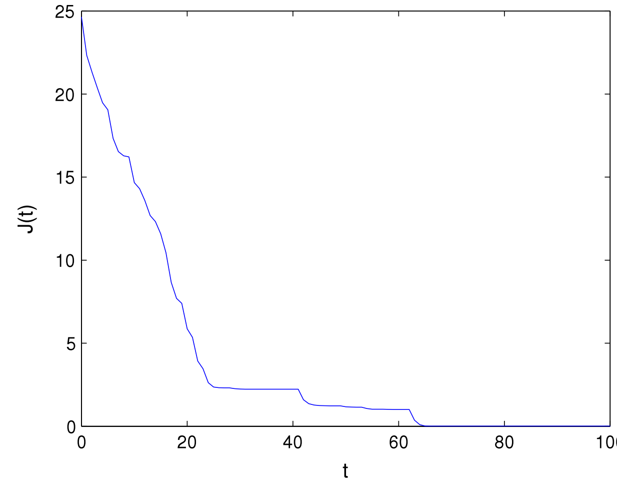

Fig. 5. Awareness coverage cost function $J(t)$ when $\alpha = 0$.

$a_{\max} = 2000$. Initial awareness coverage of the mission domain is assumed to be $x_0(q) = -1,\ \forall q \in D$.

When awareness loss is taken into account, that is, $\alpha = 0.039$, the least number of mobile sensors for the persistent awareness coverage task is derived by solving the MINLP (10) and the result is $n = 2$. Given four agents, we consider the persistent awareness coverage with given period $T^* = 50$. A feasible speed controller with parameters $[1.23, 12.15, 0.56, 1.44, 0.72]$ is obtained by solving the linear inequalities (16). Fig. 3 shows time evolution of the function $\ln x_0(p)/x(p, t) = \int_0^t \left(\sum_{i=1}^n M_i(q_i, p) - \alpha\right) d\tau$, where p is the location of the point of interest denoted by a red solid circle in Fig. 2. It can be seen that the function increases periodically and approaches infinity as time goes to infinity, which implies that the point of interest attains full awareness coverage. Fig. 4 provides a snapshot of awareness coverage of eight points of interest. Note that awareness coverage of all points of interest is driven to a neighborhood of full awareness at $t = 3600$. When awareness loss is zero, that is, $\alpha = 0$, the awareness coverage cost function $J(t) = \int_D x^2(q, t)dq$ is considered. Note that this function is equal to zero if and only if $x(q, t) = 0,\ \forall q \in D$. In Fig. 5, it is shown that the cost function is driven to zero, which indicates that all points in the mission domain attain full awareness coverage.

5. Conclusions

In this paper we have formulated the persistent awareness coverage control problem and planned the mobile sensors' trajectory to accomplish the persistent awareness coverage task. The least number of sensors required for this task has also been derived. It is also shown that the persistent awareness coverage task for a given network of mobile sensors with the given period can be accomplished if there exists a solution to a set of linear inequalities. Since sensor failure most likely leads to invalidation

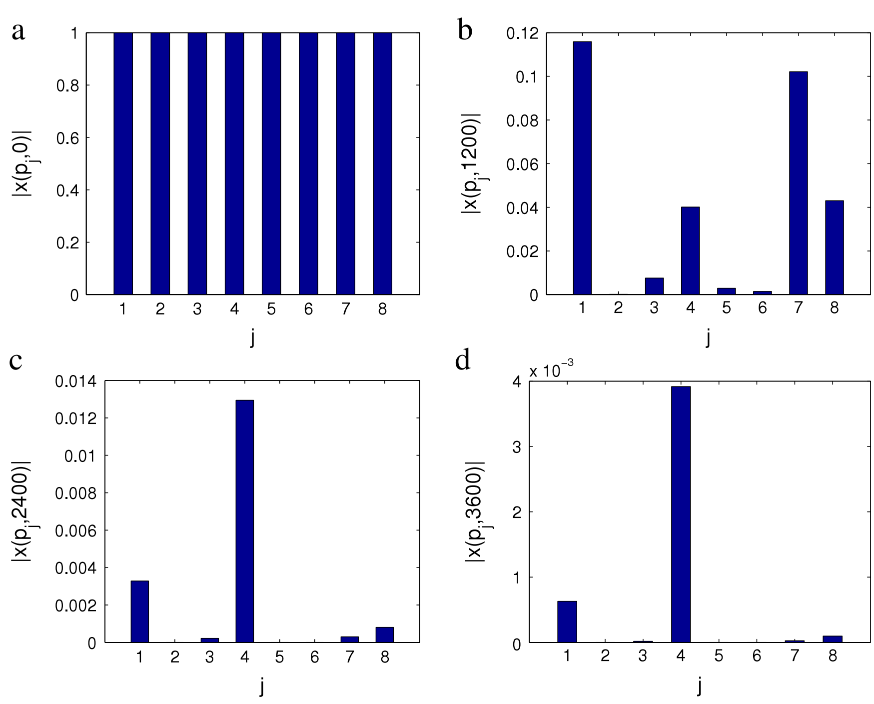

Fig. 4. Snapshot of absolute value of awareness coverage.

of the proposed approach, our on-going work is focused on developing speed controllers that are robust to sensor failure.

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