Simulating Ranked Choice Voting in the 2021 Canadian Federal Election

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Abstract

First-past-the-post voting is currently used for Canadian federal elections and is criticized for not being representative of actual voting preferences. This study simulates alternative outcomes of different ranked choice voting (RCV) systems, including Instant Runoff Voting (IVR), Borda Count, and Defeat-Dropping Condorcet (DDC) and how confident we are in our predictions through variance among simulations. In addition, the effect on variance when simulating using stratification will be observed as well. Voting preference rankings were simulated using a Dirichlet distribution based on voting data of the 2021 federal election and Ipsos polling data on second choice preferences. Our results of our simulations suggest that IVR, Borda Count, and DDC favour the Liberal party. In addition, using an F-test it was found that simulations using stratification had the variance decrease when compared to not using stratification. These findings highlight how the choice of an RCV method influences the federal election results and strategic considerations for parties in advocating an electoral reform.

1. Introduction

First-past-the-post (FPTP) is the voting system Canada currently uses for federal elections. This system entails each voter selecting a single party and seats are distributed by the popular vote of each riding. Although FPTP seems fair and simple to implement it has been heavily criticised for not being representative of actual voting preferences. One of the criticisms is the spoiler effect, which is when the inclusion of an additional candidate in an election affects the results of the election. A paper on the spoiler effect illustrates one of the famous examples of it in the year 2000 during the United States presidential election where Ralph Nader ran in the green party, and is highly speculated that he stole many votes from Al Gore who ran in the Democratic party, causing Gore to lose (Atkinson et al.) It is also believed that FPTP voting does not accurately represent the voting preferences of the population. For example in the 2019 federal election in the 4 western provinces (British Columbia, Alberta, Saskatchewan and Manitoba) the conservative won 53% of the votes and Liberals won 21% of the votes, yet they were rewarded with 68% and 14% member of parliaments from those provinces respectively (Fair Vote Canada) .

An alternative to FPTP is ranked choice voting (RCV), which hypothetically allows voting preferences to be more proportionally represented in parliament. One potential alternative to FPTP voting is Instant Runoff Voting (IVR). This is where the least popular candidate is eliminated and their votes transfer to the voters next choice, this process repeats recursively until one candidate receives a majority of the votes. Another alternative is Defeat Dropping Condorcet (DDC) where one candidate beats all other candidates in a one-on-one vote, based on the rankings. If no candidate is a Condorcet winner then FPTP is implemented. Borda Count is another alternative method where for each ballot every candidate is given points based on how highly they ranked on the ballot. The winning candidate has the highest cumulative points across all ballots.

Although all simulated methods are designed to result in more representative elections none can perfectly represent the interests of the voting public. Each voting method comes with benefits and tradeoffs that have to be weighed accordingly. Our intention with this report is not to definitively define the best voting system but provide information to help decision makers make an informed decision.

IRV is perhaps the simplest of the alternative voting systems we are simulating, but its simplicity comes at the cost of suboptimality. Consider a scenario where candidate A is the second choice of the entire population, but receives no first choice votes. Obviously, candidate A having the second choice backing of the entire population should be a contender for the role but under IRV rules they are dropped from consideration first. This flaw arises because IRV only takes into account the current state of votes and fails to consider the impact of future states.

Borda count attempts to fix some of the flaws of IRV but introduces other complexities in the process. Unlike instant runoff voting, Borda count accounts for future states by assigning points to each positional ranking and rewarding each candidate appropriately. However, in doing so it introduces ways for the vote to be manipulated by candidates in the pre-election period. Consider an election with exactly three candidates: a candidate with one first place vote and a candidate with three third place votes would both receive three points putting them at a tie. However in a more crowded election space with 100 candidates the candidate with one first place vote would have 100 points while the third place candidate would receive 294 points putting the second candidate ahead. Candidates with significant information advantage can manipulate the point system to skew elections in their favor without changing the underlying preferences of the population by simply registering additional candidates.

DDC fixes the problems introduced by Borda count but comes with tradeoffs. Unlike Borda count, DCC attempts to find a candidate that would defeat every other candidate in one-on-one matchups. If a Condorcet winner cannot be found another ranked voting system can be used. DCC is overall the most equitable of the electoral systems we are implementing. In the event a Condorcet winner cannot be found the fallback system with its inherent flaws is introduced. If a Condorcet can be found, determining a winner can be extremely complicated with the algorithm having a high time complexity. In addition such a drastic change to the electoral system from something as simple as FPTP to DDC will take significant time and effort on the part of the government to educate and inform the public.

There are many considerations when implementing a new voting system. For example, which parties benefit from the new voting system and by how much. This paper aims to simulate the previously defined RCV methods and investigate the results among the simulations, to provide information on what parties benefit from the various RCV systems. Variance among the simulations will be taken into consideration as simulations with lower variance will mean we are more confident in our results than ones with higher variance. This paper will also investigate if simulating ranked choice voting using stratification reduces variance among simulations, over not using stratification.

2. Methodology

To simulate ranked choice voting for the Canadian federal 2021 election we used data from the election, which is provided by the government and can be publicly accessed (Elections Canada). The data consist of the amount of votes each party received in each riding. Because not every riding had a candidate from every party, missing data was present. The missing data was not extrapolated because if a riding did not have a candidate from a particular party, then it was believed that voters in that riding would not be interested in seeing that party win. In addition we used data on voter's second choice from a poll done by Ipsos, which is a global market research company, to simulate the rankings.Table 1 displays the probability of a voter's second choice based on their first choice.

	Second	**Choice**					
First Choice		Cons	Libs	NDP	Bloc	Green	PPC
	Cons	/	0.1186	0.1779	0.6541	0.0247	0.0247
	Libs	0.2051	/	0.5550	0.1192	0.0965	0.0241
	NDP	0.1465	0.4762	/	0.1208	0.2198	0.0366
	Bloc	0.4181	0.3636	0.1818	/	0.0182	0.0182
	Green	0.08	0.3	0.5	0.15	/	0.02
	PPC	0.7	0.05	0.04	0.2	0.01	/

Table 1: Second Choice voting Preference from Ipsos

Rankings were simulated using a Dirichlet distribution, which is a multi-dimensional case of the beta distribution. The parameter for the Dirichlet distribution is a vector of size k called alpha and it returns a probability vector of size k where all elements sum to 1, where the elements are determined by randomness and from the size of the element in the alpha vector at the corresponding index. This is ideal for simulations, because when simulating rankings we can get different k-dimensional discrete distributions that are relatively similar to voting preference for the next choice each time we sample from the Dirichlet distribution. When the magnitude of the alpha values is higher, there is more certainty meaning the "tails" are smaller, and when the magnitude of the alpha values are lower, there is less certainty resulting in larger tails.

$$f(x_1,\ldots,x_K;\alpha_1,\ldots,\alpha_K) = \frac{1}{\mathrm{B}(\boldsymbol{\alpha})}\prod_{i=1}^{K} x_i^{\alpha_i - 1} \quad \text{for } x_i \geq 0, \sum_{i=1}^{K} x_i = 1$$

$$\text{Where } \mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^{K}\Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{K}\alpha_i\right)}$$

Equation 1: Dirichlet distribution

Because different areas within Canada tend to vote in different directions, each simulation should represent each different area proportionally to Canada's population to lower bias and variance of our simulations. To ensure this, voters can be stratified into their predetermined ridings and each riding will be sampled equally in every simulation. Not using stratification voters will be chosen randomly, which will lead to oversampling and undersampling of different areas and voting preferences in every simulation, hypothetically increasing bias and variance in the results of the simulations.

Although we strived to make our simulations as accurate and dependable as possible our methodology was not without flaw. The first choice ranking for all our simulations was enlicit assumption that the choices electors made in the 2021 federal elections were genuine and voters did not engage in strategic voting. However, IPSOS data from 2021 suggest that up to one third of voters voted defensively rather than vote for their favourite party (Ipsos). Taking into account the political climate of left wing parties in Canada it's not unreasonable to assume that the liberal party benefited greatly in our simulations due to this effect. This suggests that the staggering losses seen by the liberals in our simulations to third parties might be greater than predicted and that our simulations actually define a lower bound for possible losses. In addition this methodology also fails to account for any reversals to the spoiler effect. In the current FPTP system the domineeringly based on the voting outcomes of the 2021 federal election. This carries with it the importance of the conservative party in right wing Canadian politics deters smaller players from entering the race significantly skewing simulation outcomes by a lack of third parties siphoning away from conservatives. The asymmetry of party distribution significantly benefited the conservatives in our simulations. Under a more equitable election system smaller players may choose to enter the race balancing the siphoning effect we saw with left leaning parties.

The methodology and implementation for the IRV simulations are slightly different than the other voting systems due to limitations in time and memory when running individual simulations. Starting with the stratified or riding specific simulations, we were capable of distributing rankings to each voter in the riding, which meant no random sample of voters needed to take place. In the case, where no party receives greater than 50% of the vote and votes to another party are to be redistributed, we used the dirichlet distribution and used the preference matrix (Table 1) for the alpha values. The outputted proportion vectors would be used to assign a certain percentage of the redistributed votes to other parties. This process would occur for each round until a party receives over 50% of the vote. After all ridings completed the same process, each party will tally all their winning ridings just like FPTP and a winner is determined. Moving on to the methods utilized in the unstratified simulations, we incorporated similar processes to which the United States of America uses for their federal elections. That would be the electoral college which means that we treat each province or territory as a single riding and the party that wins will accumulate all the seats that are present in that jurisdiction, therefore, Ontario is treated as a single riding however, only one party will capture all 121 parliamentary seats.

DDC

Errors in Implementation

For the sake of faster computing times, the first version of the DDC code used some time saving optimizations which led to incorrect results. In the voter simulation component, multiple pre-computation strategies were used, the most notable few being: cumulative probability distributions for certainty levels were calculated once during initialization rather than repeatedly during each voter generation and all random numbers for a batch of voters were generated using NumPy's 'np.random.random()' instead of individual 'random.random()' calls. The first-choice party selection process was optimized by pre-computing vote share weights and utilizing Python's 'random.choices()' function with weight parameters, replacing the manual cumulative probability calculation loop.

Taking a closer look, the most significant optimization involved the pre-computation of cumulative probability distributions for certainty levels, where the optimized version calculated these probabilities once during initialization and stored them in a pre-computed array. The non-optimized version recalculates the cumulative probabilities for each individual voter.

The second optimization involved random number generation. The random number generation strategy differs fundamentally between the two versions: the optimized implementation generates all random numbers for a batch of voters at once using NumPy's vectorized 'np.random.random()' function, while the non-optimized version generates individual random numbers on-demand using Python's 'random.random()' function. This difference is particularly problematic because NumPy's random number generator and Python's built-in random module use different underlying algorithms and state management systems, producing different random number sequences, especially as no seed was set. This in turn cascaded through the entire simulation and affected all subsequent probabilistic calculations including party selection and preference ordering.

The cumulative effect of these differences became particularly significant when considering the scale of the simulation, with 500 Monte Carlo iterations, thousands of voters per riding, and hundreds of ridings, even small systematic biases can, and did, compound to produce substantially different overall results. All of these ‘optimizations’ were removed for subsequent testing and more accurate results were obtained. A comparison between the results of the two versions of the DDC code is exhibited next.

Comparison - ‘Optimized’ Code vs Unoptimized

A test run of 500 iterations for both implementations was conducted. The results are as follows:

Conservative Party Results:

Striking differences appeared in the Conservative Party results, where the optimized version produced an average of 168.9 seats with a standard deviation of 4.41, while the unoptimized version yielded only 121.5 seats with a standard deviation of 2.12. This represents a massive systematic bias of approximately 47 seats in favor of the Conservative Party in the optimized version, which is statistically significant and cannot be explained by random variation. The optimized version also shows higher variance (coefficient of variation of 2.6% vs 1.7%), suggesting that the optimization changes have greater variability in Conservative Party performance.

Similar differences to these were present for all parties, summarized in the table below.

Party	Optimized Seats	Unoptimized Seats	Optimized CV (%)	Unoptimized CV (%)
Conservative	168.9	121.5	2.6	1.7
Liberal	92.6	156.0	5.1	1.4
NDP	20.7	26.5	9.6	5.0
Bloc	53.6	31.9	4.0	3.3
Green	2.2	2.1	18.0	14.3
PPC	0.0	0.0	-	-

Table 2: DDC Seat Distribution Based on Code Optimality

It is important to highlight that the unoptimized code yielded a Liberal win, different to the erroneously ‘optimized’ code’s Conservative win. For the purposes of the report, the unoptimised, better code will be used.

Methodology - Stratified

The implementation consists of several interconnected components that work together to simulate realistic elections using real Canadian electoral data and voter preference modeling. 500 simulations were run with a voter sample size of 1000 within each riding.

The heart of the system is the 'DefeatDroppingCondorcet' class, which implements the DDC voting method through a series of steps. The algorithm begins by creating a pairwise comparison matrix from ranked ballots, where each cell 'matrix[i][j]' represents the number of voters who prefer party i over party j. This matrix processes all possible pairs of parties for each ballot, using manual list searches to find party indices. The algorithm then enters an iterative loop where it first checks for the existence of a Condorcet winner (a party that defeats all others in pairwise comparisons), and if none exists, it identifies and removes the weakest defeat (the pairwise comparison with the smallest margin of victory) from the matrix. This process continues until a Condorcet winner emerges, ensuring that the final winner is the most broadly acceptable choice among the electorate.

The 'VoterSimulator' class implements a voter preference modeling system based on real IPSOS polling data that captures the complexity of human voting behavior. The system models four distinct certainty levels among voters: "absolutely_certain" (46% of voters who rank only one party), "fairly_certain" (39% who rank two parties), "not_very_certain" (11% who rank three parties), and "not_at_all_certain" (4% who rank four parties). This distribution is used to determine how many parties each simulated voter will rank, reflecting the reality that many voters have strong preferences for their first choice but varying levels of certainty about their subsequent preferences.

The system employs a Dirichlet distribution-based approach to model uncertainty in voter preferences beyond their first choice. For each voter, the system uses their first-choice party to determine an ideological preference order for the remaining parties, then applies Dirichlet distribution sampling with parameters that vary based on the voter's certainty level. Higher alpha values (10.0 for absolutely certain voters, 5.0 for fairly certain, 2.0 for not very certain, and 1.0 for not at all certain) create more concentrated probability distributions, while lower values create more uniform distributions. The Dirichlet parameters are further refined by considering each party's position in the ideological preference order, with parties closer to the first choice receiving higher alpha values, ensuring that the simulated preferences reflect realistic ideological clustering.

The output includes seat distributions across all simulations, calculated variance and standard deviations for each party's performance, and changes from the First-Past-The-Post (FPTP) system to the DDC system. It also includes histograms like those attached in the result section.

<u>Methodology - Unstratified</u>

This implementation treats the entire country as a single electoral district while maintaining the integrity of regional representation, particularly for Quebec and the Bloc Québécois party. The methodology begins by aggregating voting data from all 338 federal electoral districts into 13 "super-ridings" based on the first two digits of riding numbers, creating massive districts each containing millions of voters.

The core innovation lies in the voter preference simulation system, which uses a two-stage approach to create realistic ranked ballots. First, the system samples a voter's first choice based on actual 2021 election vote shares, ensuring the overall distribution matches real-world voting patterns. Then, for subsequent preferences, it employs a Dirichlet distribution-based model incorporating empirical data about how voters of different parties tend to rank other parties. Rather than generating individual ballots for millions of voters (which would be extremely computationally prohibitive), the implementation uses weighted sampling to generate representative samples of 10,000 ballots that maintain the same statistical properties as the full population.

3. Analysis

The stratified simulations were quite similar to the outcome seen in the 2021 federal election. The main difference between the IRV results and the FPTP results, was the liberals on average losing a few seats and the NDP gaining a few extra seats. This outcome may not be too surprising considering that both of these parties are left leaning on the political spectrum therefore, they tend to split votes in various ridings. Now looking at the distribution and variance of the simulations, we can see that the spread looks awfully close to a normal distribution with a coefficient of variance between 0.03 and 0.09 for the parties' seat counts.

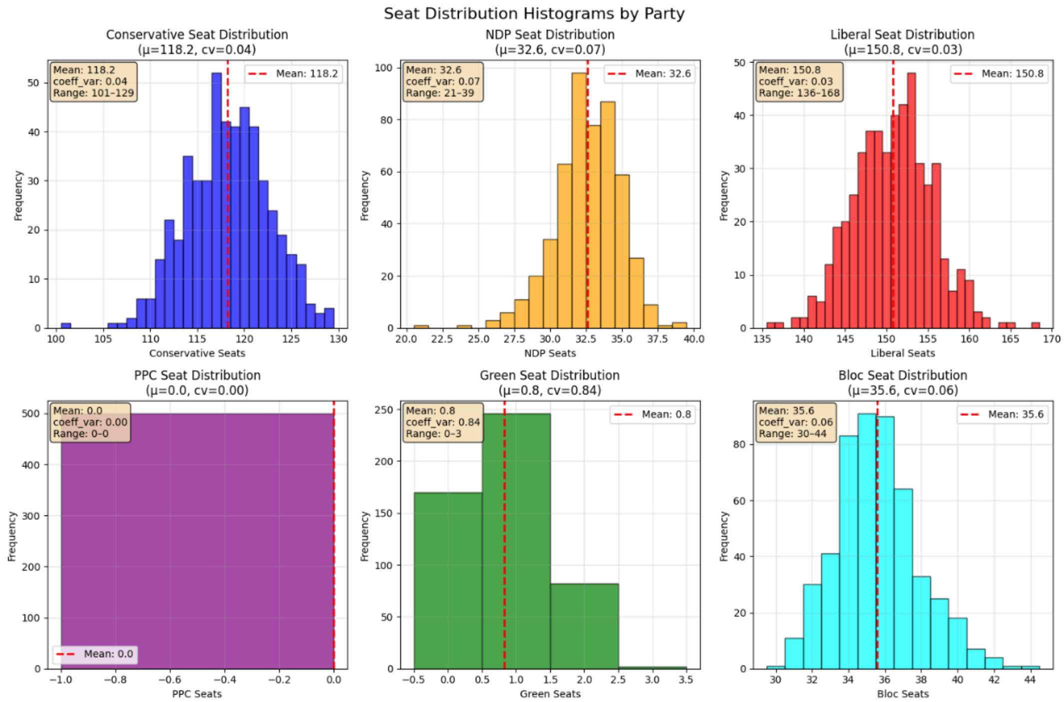

Figure 1: IRV Stratified Seat Distribution

For the unstratified simulation, meaning we treat each province or territory as its own riding but with proportional seat count, the simulations give us a wide range of outcomes. The Bloc can vary from 78 to 0 seats from Quebec, which shows the high variance that occurs with unstratified voting. In general, we see bimodal or even trimodal distribution with these parties due to enormous seat counts in Ontario, Quebec, and British Columbia.

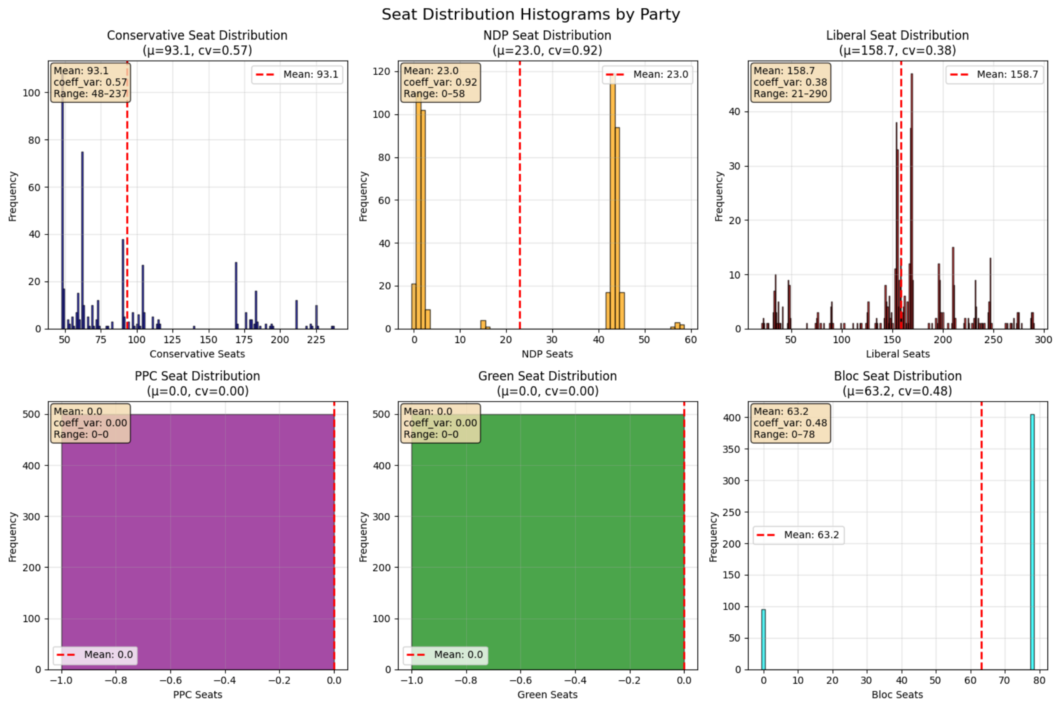

Figure 2: Unstratified IRV Seat Distributtion

- DDC results (stratified, unstratified and plots)

<u>Stratified (Results mentioned in ‘Errors of Implementation’ section above)</u>

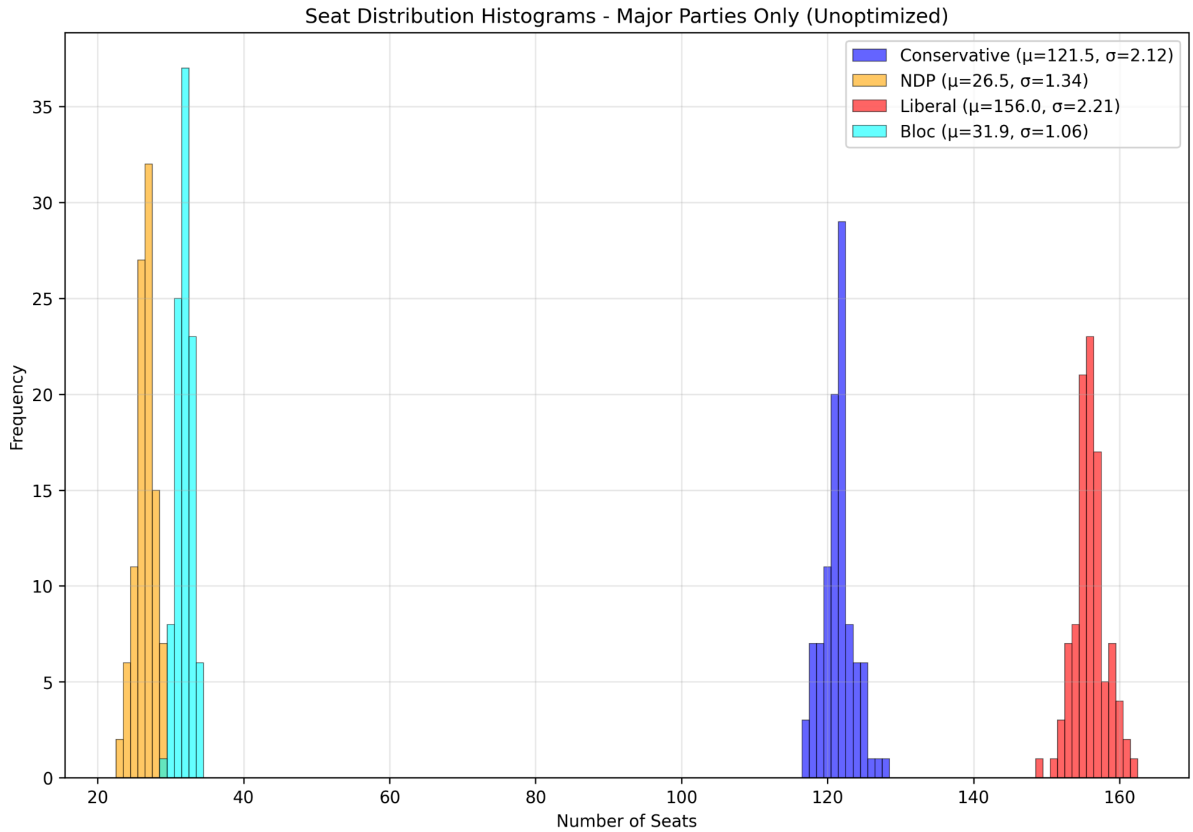

Figure 3: DDC Stratified Seat Distribution

Unstratified
Sample simulation results of 100 iterations show the Liberal Party winning 155 seats (45.9%), Conservative Party 104 seats (30.8%), Bloc Québécois 78 seats (23.1%), and NDP just 1 seat (0.3%), with Green and PPC winning zero seats on average. This outcome suggests DDC heavily favors centrist parties with broad appeal, as the Liberal Party's moderate positioning likely makes it an acceptable second choice for many voters, while smaller parties are nearly eliminated from representation.

- Borda Count

In the Borda Count stratified simulations it appears that the Liberal party lost many seats and the NDP gained many seats, in comparison to the actual 2021 election. This could be because often left leaning voters would rank Liberal and NDP in the top two, hence the seats are more evenly distributed between the two because of the weighted ranking. When simulating Borda Count not using stratification it appears the Bloc party loses many seats. This is because only Quebec voters prioritize the Bloc party, therefore when sampling from Canada as a whole and not dividing it by ridings, Quebec voting preferences get overruled by the rest of Canada, hence typically winning 0 seats.

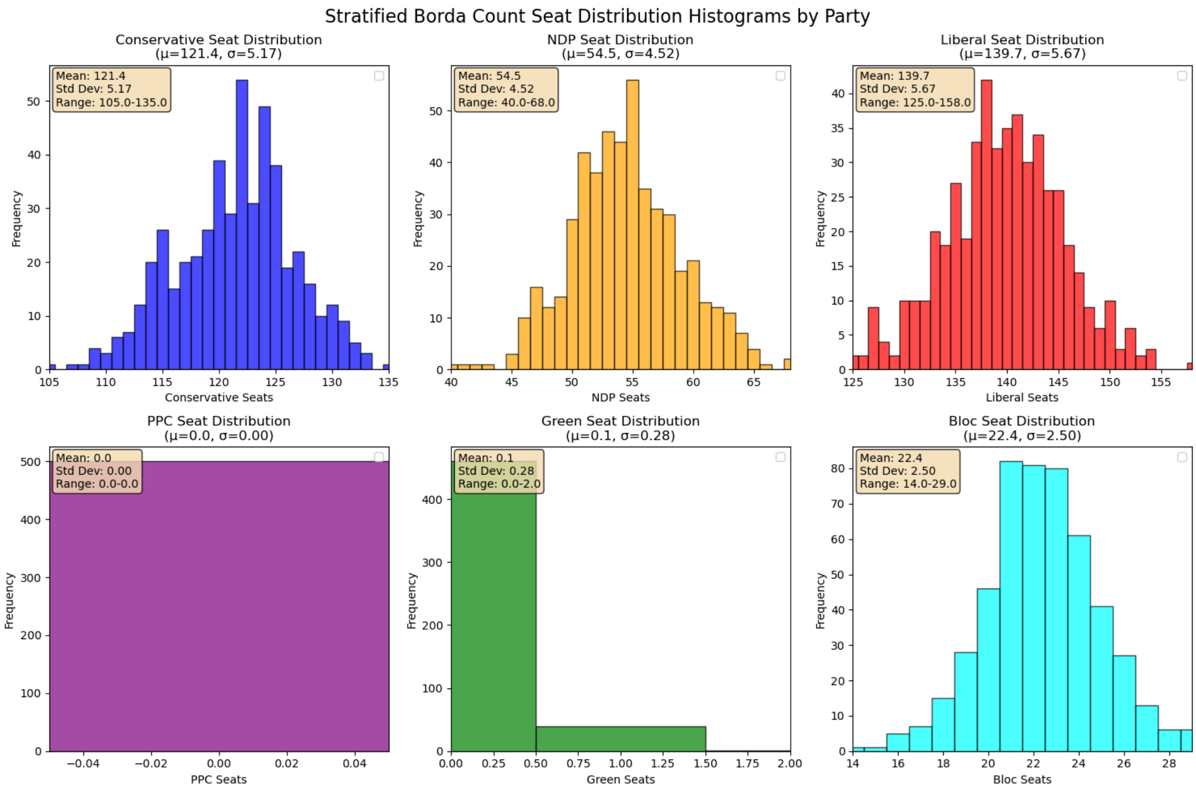

Figure 4: Stratified Borda Count Seat Distribution

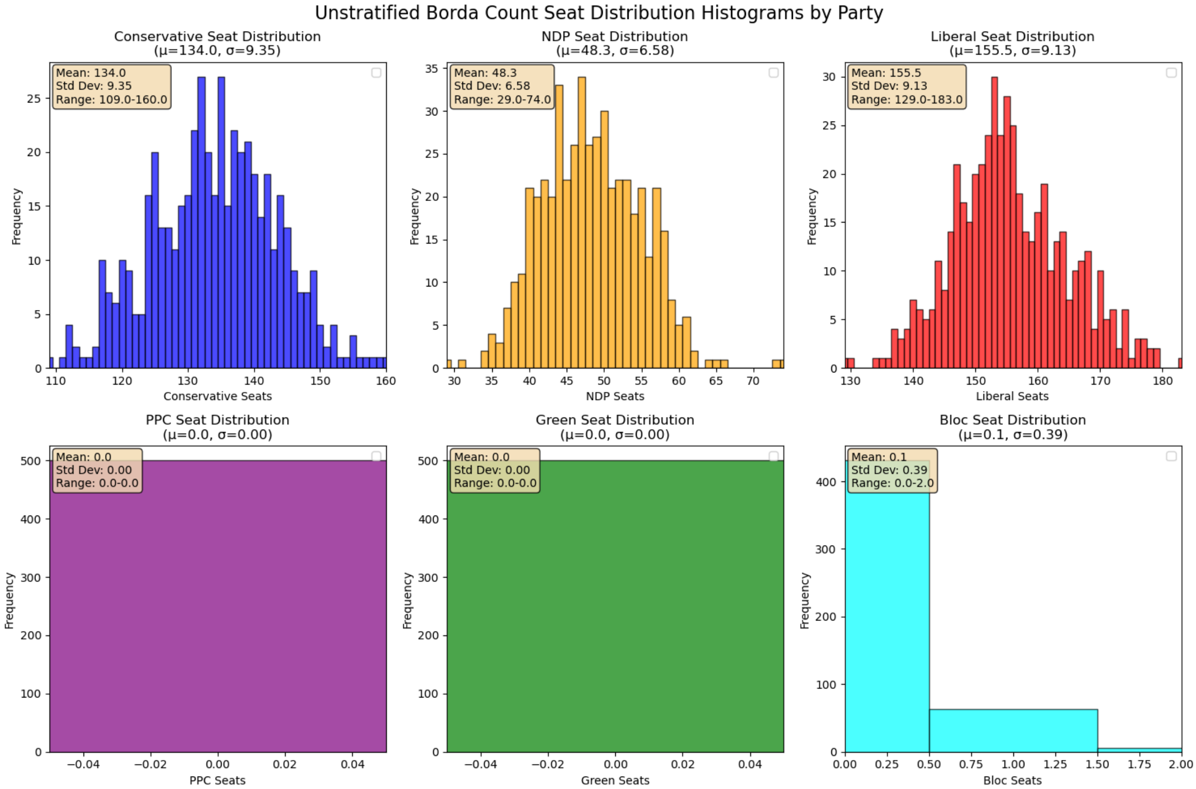

Figure 5: Unstratified Borda Count Seat Distribution

Comparing the coefficient of variances between using stratification and not using stratification in Borda Count, it appears that when applicable the p-value is below 0.05 for all parties. This means that using stratification results in a lower coefficient of variation than not using stratification for Borda count

	Stratified	**Unstratified**	**DF Stratified**	**DF Unstratified**	**F-Statistic**	**p-value**
Conservatives	4.2634	6.9751	499	499	1.636	2.272712e-08
NDP	8.3030	13.6114	499	499	1.639	2.005681e-08
Liberal	4.0611	5.8736	499	499	1.446	1.996226e-05
PPC	NA	NA	499	499	NA	NA
Green	343.3658	NA	499	499	NA	NA
Bloc	11.1784	259.4867	499	499	21.213	~0

Table 3: Borda Count Variance Comparison

4. Conclusion

This study explored the potential outcomes of implementing ranked choice voting (RCV) systems: Instant Runoff Voting (IRV), Borda Count, and Defeat-Dropping Condorcet (DDC) in the 2021 Canadian federal election through Monte Carlo simulations. Our findings demonstrate that the choice of RCV method significantly influences electoral outcomes, with each system favoring different political parties. IRV and Borda Count tended to benefit the Liberal Party, while DDC simulations initially showed a Conservative advantage; however, Liberals won upon discovery of major errors in the implementation. Additionally, stratification techniques effectively reduced variance in simulations, increasing confidence in the predicted outcomes.

The results highlight key trade-offs between fairness, complexity, and representativeness in electoral systems. While IRV offers simplicity, it may overlook broadly acceptable candidates eliminated early in runoff rounds. Borda Count provides a more nuanced ranking system but risks strategic manipulation. DDC, though computationally intensive, delivers more equitable results by prioritizing Condorcet winners. The stark differences in seat distributions under each system underscore the importance of electoral reform debates in Canada, as no single method perfectly captures voter intent without inherent biases.

Furthermore, the reduction in variance through stratification confirms that regional voting patterns must be accounted for in simulations to avoid skewed or unstable predictions. Future research could expand on these findings by incorporating dynamic voter behavior, third-party influences, and alternative preference modeling techniques. Ultimately, this study provides policymakers with empirical insights into how RCV systems could reshape Canadian elections, emphasizing that electoral reform requires careful consideration of both mathematical fairness and practical implementation.

5. References

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