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Course: Math 672
Assignment: Homework #9 problem 3
Due: 5/9/17

3. Classify the units, irreducibles, and zerodivisors of the ring $\mathbb{Z}_{(p)}$. Is this ring a UFD?

Solution: Note that the multiplicatively closed set for $\mathbb{Z}_{(p)}$ is $S = \mathbb{Z} - \{p\}$. Thus, the units of $\mathbb{Z}_{(p)}$ are the elements in the set $\mathbb{Z}_{(p)} - (\frac{p}{1})$, the set of irreducibles is $\{\frac{a}{b} \in \mathbb{Z}_{(p)} | a = p\}$, and there are no zerodivisors.

Claim: $\mathbb{Z}_{(p)}$ is a UFD.

Proof of Claim: Recall that in Homework #8, we proved that $\mathbb{Z}_{(p)}$ has precisely 3 ideals: (0), $(\frac{p}{1})$, and (1). Thus as each of these ideals is principal, $\mathbb{Z}_{(p)}$ is a PID. Therefore $\mathbb{Z}_{(p)}$ is a UFD.