A Bargaining Model with Partially Observed Covariates

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April 15, 2010

Abstract

We derive a statistical estimator that can be used to model equilibrium behavior in the popular ultimatum bargaining game when the determinants of player reservation utility are partially observed. This procedure gives the analyst the ability to estimate the effect of substantively interesting covariates on equilibrium behavior by (1) incorporating their influence directly into the equilibrium conditions that generate the data and (2) explicitly accounting for the fact that the proposing player is unable to observe characteristics of the responding player captured by the covariates. We apply the model to analyze the experimental bargaining data of Henrich (2000), and Henrich et al (2005). Our analysis suggests that, in contrast to the results of these studies, economic and demographic characteristics determine bargaining behavior.

1 Introduction

The theoretical and empirical study of bargaining is crucially important to the investigation of many political phenomena in a variety of substantive settings (e.g. Banks 1990, Bennett 1996, Baron 1989, Fearon 1995, Huth and Allee 2002, Laver and Schofield 1990, London 2002, Morrow 1989, Wagner 2000, Powell 1987, Powell 1996). Theories of bargaining underlie explanations of the initiation and termination of wars (Goemans 2000), legislative appropriational politics (Baron and Ferejohn 1989), democratization (Acemoglu and Robinson 2006), and the formation of the state (Tilly 1992). Bargaining also lies at the heart of a large literature in experimental economics, political science, and anthropology (e.g. Roth et. al. 1991, Davis and Holt 1993, Kagel and Roth 1995, Slonim and Roth 1998, Henrich 2000, Henrich et. al. 2005, Tingley 2010).

Despite the importance of bargaining in the study of politics, economics, and society, it remains difficult to specify the link between substantive variables, theories, and outcomes. Underlying most theories of bargaining are strategic interactions between the participants. Without somehow incorporating this strategic structure when pursuing empirical investigation of theories the inferences drawn by the researcher are likely to be biased and incorrect (Signorino 1999, Signorino and Yilmaz 2003). Thus, without an explicit model of the process that generates the data, it is particularly difficult for the researcher to ascertain the effects of substantive variables, e.g. congressional district demographics, inequality, military balance of power, etc., on the bargaining process. Unfortunately, there is no "off the shelf" statistical estimator readily available that can account for strategic interaction. As a result, attempting to bring a theoretical model involving bargaining to the data remains problematic. This makes many hypotheses hard to operationalize and even harder to test.

To address these problems, we require an integration of theoretical bargaining models and statistical methods. In particular, researchers need a statistical model that permits them to make theoretically consistent inferences about the relationship between substantive variables, the bargain struck, and the probability of bargaining failure. In other words, we need estimators that explicitly model the strategic interaction in data generating process. Some scholarship has begun to incorporate strategic structure directly into statistical models (e.g. McKelvey and Palfrey 1995, McKelvey and Palfrey 1996, McKelvey and Palfrey 1998, Signorino 1999) with more recent work incorporating bargaining directly (e.g. Diermeier, Eraslan, and Merlo 2003).

In this paper, we build on work by Ramsay and Signorino (2010) and derive a generalized statistical model for ultimatum bargaining games. The ultimatum bargaining game is both simple and also extensively studied theoretically and experimentally. In this game there are two players. The first player, often called the "proposer," is conditionally allotted a divisible resource. The proposer then offers a portion of the allotted resource to the second player, the responder. The responder, aware of both the offer made and the total amount of resources allotted, can then either accept or reject the offer made by the proposer. If the responder accepts the offer, then he receives the offer and the proposer gets the allotted resources less the offer paid out to the proposer. If the responder rejects the offer then neither receives anything. In the event of either an acceptance or rejection, the game ends.

We extend the model derived by Ramsay and Signorino (2010) to allow for situations in which the proposing player does not observe all of the characteristics of the responding player. For example, in experimental bargaining situations, experiments are frequently conducted under conditions of anonymity. Proposing players do not observe demographic characteristics of the responding player. In international national crises, the proposing state may not observe the military capacity of the responding state. The model derived in this paper explicitly accounts for this partial observability and the effect that it has on the optimal offer of the proposer.

We first characterize equilibrium play in the ultimatum game. The equilibrium conditions explicitly capture the relationship between player utilities and outcomes. We then use these conditions to construct a statistical estimator that models this relationship under the conditions of partial observability discussed above. Next, we conduct a Monte Carlo experiment by generating strategic bargaining data and then estimating the relationships between the regressors and the dependent variable(s). We compare these results to the inferences that would be made by employing traditional OLS and Tobit models, as well as a structural bargaining estimator that does not account for partial observability. Finally, we apply our model to investigate demographic effects on offers and acceptance/rejection choices in experimental ultimatum game data gathered in 15 small scale societies.

2 The Model

[Figure 1 about here.]

Among the simplest of bargaining situations that incorporates explicit strategic interaction is the ultimatum bargaining game. In the ultimatum game, two players must divide a contested prize. We will model the prize as a compact interval on the real line, $\mathbb{R}$. Without loss of generality, normalize this interval to $[0, Q]$. At the beginning of the game, player 1 makes a proposal for a division of the contested prize. Player 1 offers an *agreement*, a pair $(Q - y, y)$, where $Q - y$ is the allocation for player 1 and y is the allocation for player 2. Player 2 then observes the offered agreement and decides whether to *accept* or *reject* it. If player 2 accepts the agreement, then the contested prize is divided according to $(Q - y, y)$. If player 2 rejects the agreement, then each receives some reservation value, R_i.

In order to derive a statistical estimator for the ultimatum game, we will assume that the total reservation utility to players in the event of a rejection consists of two components - one part which is publicly observable to both players and the analyst, and a second part which is private information. In particular, we will assume that the players and the analyst have complete information about the size of the resource to be divided, Q, and the reservation value R_i. In addition, endow each player with a type, $\varepsilon_i \in \mathbb{R}$, which is assumed to be private information to i. We assume that player i's type is an independently distributed random variable following a cumulative distribution function F_{ε_i} with density f_{ε_i} which is well defined on $\mathbb{R}$. Let this random variable have mean 0 and finite variance σ_i^2. Let the prior beliefs of the opposing player (and the analyst) about player i's type be F_{ε_i}.

In this game, we can denote the pure strategy of player 1, the proposer, as $y : \mathbb{R} \longrightarrow [0, Q]$, a mapping from the type of player 1 into the portion of the contested resource offered to player 2. Similarly, denote the pure strategy of player 2, the responding player, as $a : [0, Q] \times \mathbb{R} \longrightarrow \{0, 1\}$, a mapping from the offer made to player 2 and his type into a decision to accept (1) or reject (0). Assume utilities are given by:

$$u_1(y, a) = \begin{cases} Q - y & \text{if } a(y, \varepsilon_2) = 1 \\ R_1 + \varepsilon_1 & \text{if } a(y, \varepsilon_2) = 0 \end{cases}$$

$$u_2(y, a) = \begin{cases} y & \text{if } a(y, \varepsilon_2) = 1 \\ R_2 + \varepsilon_2 & \text{if } a(y, \varepsilon_2) = 0 \end{cases}$$

Existence and uniqueness of a Perfect Bayesian Equilibrium of the ultimatum game with private components in the reservation utilities are assured by the arguments of Ramsay and Signorino (2010) under certain circumstances. We restate their result here for clarity:

Proposition 2.1. *(Ramsay and Signorino 2010) If F_{ε_2} is log-concave then there exists a unique Perfect Bayesian Equilibrium to the ultimatum bargaining game.*

Given existence and uniqueness, we briefly characterize the equilibrium. After observing any offered agreement $(Q-y,y)$, player 2 faces the choice between accepting the agreement and obtaining y or rejecting the agreement and obtaining his reservation utility $R_2+\varepsilon_2$. It follows immediately that in equilibrium:[1]

$$a^*(y,\varepsilon_2)=\begin{cases}1 & \text{if } y\geq R_2+\varepsilon_2\\ 0 & \text{if } y<R_2+\varepsilon_2.\end{cases}$$

Player 1 can reason that player 2 will respond in this way and plan his offered agreement accordingly. The complicating factor is that player 1 does not observe the private component of the reservation utility for player 2, ε_2. In equilibrium, player 1 must balance the rewards associated with a favorable agreement against the risk associated with being rejected. Given this, from the perspective of player 1, the expected utility associated with offering an agreement $(Q-y,y)$ will be:

$$E(u_1(y,a))=p(a(y,\varepsilon_2)=1)(Q-y)+p(a(y,\varepsilon_2)=0)(R_1+\varepsilon_1).$$

Note that the probability that player 2 chooses to accept the proffered agreement, i.e. that $a(y,\varepsilon_2)=1$, is:

$$\begin{aligned}p(a(y,\varepsilon_2)=1)&=p(y\geq R_2+\varepsilon_2)\\&=p(y-R_2\geq\varepsilon_2)\\&=F_{\varepsilon_2}(y-R_2).\end{aligned}$$

[1] Note that we break indifference between acceptance and rejection in favor of acceptance. Since the goal is to derive a statistical estimator for the ultimatum game, this will matter only on a measure zero set of model parameters.

Since in equilibrium, it must be that the optimal offer $y^*(\varepsilon_1)$ satisfies:

$$y^*(\varepsilon_1) \in \underset{[0,Q]}{\text{argmax}} \{E(u_1(y, a))\}$$

any interior solution to the optimization problem facing player 1 must therefore satisfy the first-order condition on his expected utility:

$$y^* = Q - R_1 - \varepsilon_1 - \frac{F_{\varepsilon_2}(y^* - R_2)}{f_{\varepsilon_2}(y^* - R_2)} \tag{1}$$

The optimal strategy for player 1 is therefore:

$$y^*(\varepsilon_1) = \begin{cases} Q & \text{if } Q < Q - R_1 - \varepsilon_1 - \frac{F_{\varepsilon_2}(Q-R_2)}{f_{\varepsilon_2}(Q-R_2)} \\ y^* & \text{if there exists } y^* \text{ such that (1) is satisfied} \\ 0 & \text{if } 0 > -R_1 - \varepsilon_1 - \frac{F_{\varepsilon_2}(-R_2)}{f_{\varepsilon_2}(-R_2)}. \end{cases}$$

These equilibrium conditions are a complete characterization of the optimal behavior of both the proposing and the responding player. In the next section, we will use these equilibrium conditions to derive a statistical estimator for the ultimatum game. Before we proceed to this step we establish that the optimal offer $y^*(\varepsilon_1)$ is monotonic in model parameters.

Proposition 2.2. *If F_{ε_2} is log-concave then the optimal offer $y^*(\varepsilon_1)$ is monotonic in ε_1, R_1, and R_2.*

Proof. Let: $G(y, \varepsilon_1) = F_{\varepsilon_2}(y - R_2)(Q - y) + (1 - F_{\varepsilon_2}(y - R_2))(R_1 + \varepsilon_1)$ and define $\rho \equiv -\varepsilon_1$. Then:

$$\frac{\partial G(y, \varepsilon_1)}{\partial y} = Q + \rho - y - R_1 - \frac{F_{\varepsilon_2}(y - R_2)}{f_{\varepsilon_2}(y - R_2)}$$

$$\frac{\partial^2 G(y, \varepsilon_1)}{\partial \rho \partial y} = 1 > 0.$$

Thus $G(y, \varepsilon_1)$ satisfies increasing differences and so single-crossing. By Theorem 4 of Milgrom and Shannon (1994) $y^*(\varepsilon_1)$ is monotonic in ρ and so monotonic in ε_1. An analogous argument establishes that $y^*(\varepsilon_1)$ is monotonic in R_1. To see that $y(\varepsilon_1)$ is monotonic in R_2 note that by log-concavity of F_{ε_2} it follows that $\frac{F_{\varepsilon_2}(y-R_2)}{f_{\varepsilon_2}(y-R_2)}$ is non-increasing in its argument and so: $\frac{\partial^2 G(y,\varepsilon)}{\partial R_2 \partial y} \geq 0$. The function $G(y, \varepsilon_1)$ again satisfies increasing differences and so single-crossing. Monotonicity of $y^*(\varepsilon_1)$ in R_2 follows from Theorem 4 of Milgrom and Shannon (1994). $\square$

3 A Statistical Estimator

For the remainder of this paper, we will study a particular class of distribution functions for the type of the players. Our focus will be on analyzing strategic situations in which there are characteristics of the responding player that contribute systematically to his reservation utility yet are unobserved by the proposing player. To fix ideas, assume that we have data on both players actions, i.e. a proposed division of the contested resource by player 1 and a decision to accept that division or reject it by player 2. Assume that reservation values are a linear combination of covariates (x, z, w) and type $(\varepsilon_1, \varepsilon_2)$ for each player. Thus let:

$$\begin{aligned} R_1 &= \beta_0 + \sum_{i=1}^{m} x_i \beta_i = x\beta \\ R_2 &= \underbrace{\gamma_0 + \sum_{i=1}^{k} z_i \gamma_i}_{\text{Observed by 1}} + \underbrace{\sum_{i=1}^{l} w_i \delta_i}_{\text{Unobserved by 1}} = z\gamma + w\delta \end{aligned}$$

The reservation value for player 1, the proposer, is a function of characteristics, for example, gender, age, education, etc. These characteristics are observable to both player 2 and the analyst. The reservation value for player 2, the responder, is also a function of characteristics. Some of these, the covariates in matrix z, are observable to player 1 and the analyst. Others, those in matrix w, are not observed by player 1, but are observed by the analyst. If the covariates that are unobserved by player 1 structure the choice made by player 2 in a systematic manner, and if player 1 knows how they *should* matter if he could observe them, then this partial observability should be modeled explicitly.[2] Our interest in this section is in developing a model that takes this partial observability into account to produce estimates of β, γ, and δ, the covariate effects on the choices made by the players.

For the remainder of the paper, we will assume that each ε_i is independent of the other and distributed normally, with mean 0 and variance σ_i^2. This will greatly simplify derivation of the estimator; because the outcome of the bargaining model consists of two dependent variables, the agreement proposed by player 1 and the decision to accept it or reject it by player 2, our probability model is a joint density over those variables. Independence reduces this joint density to the product of two univariate densities. Finally, since the normal distribution satisfies log-concavity, the Perfect Bayesian Equilibrium characterized above exists and is unique.

Given the functional form assumptions on the distribution over ε_i, we can derive the estimator for the statistical bargaining game. Making use of the equilibrium characteriza-

[2]For example, one could imagine that the proposing player knows that an individual with more education is likely to respect standards of fairness (a hypothetical example) and so is more likely to reject unfavorable agreements (because his reservation utility is larger). The proposing player knows that education influences the choice of the responding player systematically, and moreover knows how education would influence the responding player, but simply does not observe the level of education of the responding player. We will later demonstrate the consequences of ignoring this partial observability via a Monte Carlo experiment.

tion above:

$$
\begin{aligned}
p(a(y,\varepsilon_2)=1) &= p(y \geq R_2 + \varepsilon_2) \\
&= p(y \geq z\gamma + w\delta + \varepsilon_2) \\
&= p(y - z\gamma - w\delta \geq \varepsilon_2) \\
&= \Phi^{0}_{\sigma_2^2}(y - z\gamma - w\delta)
\end{aligned}
$$

where $\Phi^{\mu}_{\sigma^2}(\cdot)$ is the normal CDF with mean μ and variance σ^2.

For player 1, the distribution of $y^*(\varepsilon_1)$ is more complicated. We need to derive the expression for the optimal offer made by player 1 in anticipation of player 2's acceptance probability. This is complicated by the fact that only some of the characteristics of the responding player are observable to the proposing player. Assume that from the point of view of player 1, for $i = 1, \ldots, l$ the covariate w_i is independently distributed according to F_{w_i} with mean μ_{w_i} and variance $\sigma^2_{w_i}$. This implies that for any particular agreement offered by player 1, the probability of an acceptance from the perspective of the analyst diverges from the probability of an acceptance from the perspective of player 1:

$$
\begin{aligned}
p(a(y,\varepsilon_2)=1) &= p(y \geq R_2 + \varepsilon_2) \\
&= p\left(y \geq z\gamma + w\delta + \varepsilon_2\right) \\
&= p\left(y - z\gamma \geq w\delta + \varepsilon_2\right) \\
&= F_{\varepsilon_2 + w\delta}\left(y - z\gamma\right).
\end{aligned}
$$

This implies that we require the joint cumulative distribution function for the random variable $\varepsilon_2 + w\delta$. Call this variable u. We will assume that player 1 has prior beliefs over the values taken on by w, namely that w_i is normally distributed. Following a well-known moment generating function argument, the cumulative distribution function for a linear combination of normally distributed random variables is itself normal. Thus the distribution for $u = \varepsilon_2 + w\delta$ is normal so that:

$$p(a(y, \varepsilon_2) = 1) = \Phi^{\mu_u}_{\sigma_u^2}(y - z\gamma)$$

where:

$$\mu_u = \sum_{i=1}^{l} \mu_{w_i}\delta_i$$
$$\sigma_u^2 = \sigma_2^2 + \sum_{i=1}^{l} \delta_i^2 \sigma_{w_i}^2.$$

Having derived the appropriate distribution for the probability of acceptance from player 1's point of view, making use of equation (1), any interior optimal offer must satisfy:

$$y^* = Q - x\beta - \varepsilon_1 - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}$$

where $\phi^{\mu}_{\sigma^2}(\cdot)$ is the normal probability density function with mean μ and variance σ^2.

By Proposition 2.2 the previous implicit expression for y^* is monotonic in ε_1. Therefore we can simply apply the method of monotonic transformations (Casella and Berger 2002, Theorem 2.1.5). Under this transformation: $f_{y^*}(y^*) = f_{\varepsilon_1}(h^{-1}(y^*))\text{abs}\left(\frac{d(h^{-1}(y^*))}{dy^*}\right)$. Thus:

$$y^* = h(\varepsilon_1)$$
$$\varepsilon_1 = h^{-1}(y^*) = Q - R_1 - y^* - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}.$$

Since ε_1 is also assume to be distributed normal with mean 0 and variance σ_1^2, we can write:

$$f_{y^*}(y^*) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left\{ -\frac{\left(Q - x\beta - y^* - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}\right)^2}{2\sigma_1^2} \right\} \text{abs}\left(\frac{d(h^{-1}(y^*))}{dy^*}\right)$$

$$\begin{aligned}
\frac{d(h^{-1}(y^*))}{dy^*} &= \frac{d\left(Q - R_1 - y^* - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}\right)}{dy^*} \\
&= -2 - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)} \left(\frac{y^* - z\gamma - \sum_{i=1}^{l} \mu_{w_i}\delta_i}{\sigma_2^2 + \sum_{i=1}^{l} \sigma_{w_i}^2 \delta_i^2}\right)
\end{aligned}$$

The full expression for the distribution on the optimal offer thus becomes:

$$\begin{aligned}
f_{y^*}(y^*) = &\frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left\{ -\frac{\left(Q - x\beta - y^* - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^*-z\gamma)}\right)^2}{2\sigma_1^2} \right\} \\
&\times \text{abs}\left(-2 - \frac{\Phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)}{\phi^{\mu_u}_{\sigma_u^2}(y^* - z\gamma)} \left(\frac{y^* - z\gamma - \sum_{i=1}^{l} \mu_{w_i}\delta_i}{\sigma_2^2 + \sum_{i=1}^{l} \sigma_{w_i}^2 \delta_i^2}\right)\right). \qquad (2)
\end{aligned}$$

This distribution is unconstrained. However, the possible agreements available to player 1 as potential proposals are bounded by the fact that y can be no larger than Q and no smaller than 0. This implies that the observed y^* is censored from both above and below. This censored distribution leads to the following likelihood. Denote the i^{th} observed optimal offer by y_i^* and define a set of dummy variables $d_i(\cdot)$ such that:

$$d_i(k) = \begin{cases} 1 & \text{if } k = 0 \text{ and } y_i^* = 0 \\ 1 & \text{if } k = y^* \text{ and } 0 < y_i^* < Q_i \\ 1 & \text{if } k = Q \text{ and } y_i^* = Q_i \\ 1 & \text{if } k = a \text{ and player 2 accepts in the } i^{th} \text{ observation} \\ 0 & \text{otherwise.} \end{cases}$$

Assuming that we have data on the decisions made by each player (i.e. y^* and acceptance/rejection) the likelihood would be:

$$\begin{aligned} \mathcal{L} = \prod_{i=1}^{N} & \left(\int_{-\infty}^{0} f_{y^*}(y^*) dy^* \right)^{d_i(0)} \left(\int_{Q}^{\infty} f_{y^*}(y^*) dy^* \right)^{d_i(Q)} \\ & \times \left(f_{y^*}(y_i^*) \right)^{d_i(y^*)} \left(\Phi^0_{\sigma_2^2}(y_i^* - z_i\gamma - w_i\delta) \right)^{d_i(a)} \left(1 - \Phi^0_{\sigma_2^2}(y_i^* - z_i\gamma - w_i\delta) \right)^{1-d_i(a)}. \end{aligned} \quad (3)$$

This is much like a censored normal model; we can think of there being a latent best offer which we observe when its realization is feasible, i.e. between 0 and Q. When the latent best offer is not feasible, we observe a boundary point. With this likelihood, we may obtain estimates of β, γ, δ, and the variance parameters via either maximum likelihood estimation or Bayesian simulation methods.

4 Monte Carlo

In this section, we present the results of a Monte Carlo experiment. The purpose of the experiment is two-fold. First, we demonstrate that the distribution we have derived recovers the correct parameter estimates. Second, we demonstrate the bias that occurs when we compare this technique with more typical regression techniques, for example OLS, Tobit, and when we ignore the partial observability in the covariates.

For the analysis in this section, the Ultimatum Bargaining Game pictured in Figure 1 was used as the data generating process. For each observation, the total value of the contested resource, Q, was drawn from the uniform distribution on $[0, 10]$. The public reservation values consisted of two variables for each player; these were drawn from the standard normal distribution, as was the private systematic component of player 2's reservation value, w. Private reservation values, ε_1 and ε_2 were drawn normal with mean 0 and variances σ_1^2 and σ_2^2. We then computed the optimal offer y^* by numerically solving equation (1) for a fixed-point and then applying the constraint that the observed optimal offer lie within the feasible set $[0, Q]$ if necessary. Finally, given the optimal offer and the covariates, we sampled the decision of the responding player to accept that optimal offer or not by comparing y^* with $z_1\gamma_1 + z_2\gamma_2 + \varepsilon_2$. The data for a given observation consist of the size of the contested resource, Q, the covariates x_1, x_2, z_1, z_2, and w, the optimal offer y^*, and the decision to accept or reject. In each iteration of the Monte Carlo algorithm, we drew data drawn according to the scheme above, and then applied Maximum Likelihood Estimation to attempt to recover the parameters.

Parameter Recovery

Our first Monte Carlo experiment was devoted to ascertaining whether the distribution we have derived in (2) and the likelihood based upon it in (3) can be used to recover the model parameters correctly. The true values were set at $\beta = (3.5, -0.5)$, $\gamma = (1, -0.5)$, $\delta = 1.25$, $\sigma_1^2 = 0.5$, $\sigma_2^2 = 0.75$, and $\sigma_w^2 = 1.2$. We drew 1000 datasets of 1000 observations each. For each dataset, the data generating process produced data in which roughly 20% of observations were censored, mostly at 0.

In Figure 2, we demonstrate the results of the first Monte Carlo analysis in which we simply try to recover the true parameters of the model. Each graph in the figure plots the density over the parameter estimates obtained from the 1000 Monte Carlo experiments.

In each case, the modal value of the density is quite close to the true value (the heavy dotted line in the plots). Moreover, in no case does the empirical 95% region include 0, suggesting that in each iteration of the Monte Carlo experiment, the statistical model extracted the correct inference from the data; with a larger number of iterations, these 95% regions will cluster more tightly around the true value.

[Figure 2 about here.]

[Table 1 about here.]

Bias in Alternative Approaches

When analyzing bargaining data it is common practice to employ standard techniques, such as OLS, FGLS, Tobit, Logit, and Probit. One sensible and theoretically motivated justification for this is that, under certain circumstances, a structural statistical model need not be derived directly from a formal model. As the argument goes, if one can demonstrate monotonicity of the solution to the maximization problems facing the players in the model parameters then parametric models with a linear link function should be perfectly appropriate for data analysis. Since the optimal offer in the ultimatum bargaining game is monotonic in the systematic component of the reservation utility by Proposition 2.2 it will also be monotonic in the included regressors (so long as no regressor appears in the reservation value of both players).

Despite the monotonicity of the optimal offer $y^*(\varepsilon_1)$ in the model parameters, comparison of the inferences extracted from the data by the bargaining estimator with partial observability to the inferences that would be gotten from OLS and Tobit reveals that the traditional methods lead to faulty conclusions when modeling the offer data. Table 1 provides the reader with a summary of the bias detected in the use of traditional methods to model the offer data. In the first column of Table 1 we list the true parameter value. The second column lists the mean of the density of the Monte Carlo experiment and provides a summary of the variance of the parameter estimates obtained across all the iterations of the Monte Carlo experiment. In the third column of Table 1, we compare the Monte Carlo results from the bargaining estimator with partially observable covariates to traditional OLS. The linear model gets the sign incorrect on β_1, β_2, and γ_2; it also fails to capture the correct magnitude on δ, the effect of the partially observable covariate. Tobit leads to similarly poor results. More worrisome, in a large percentage of the Monte Carlo iterations these techniques assigned statistical significance to these estimates, leading to faulty inferences. We summarize these comparisons graphically in Figure 3.

[Figure 3 about here.]

[Table 2 about here.]

Table 2 reports a comparison on the inferences derived from data generated according to the bargaining process with partial observability. As in the previous Monte Carlo experiment, we drew 1000 datasets of 1000 observations each. We set the true values at $\beta = (1.0, -0.5)$, $\gamma = (1, -0.5)$, $\delta = 1.25$, $\sigma_1^2 = 0.5$, $\sigma_2^2 = 1$, and $\sigma_w^2 = 1.25$. Column 1 of Table 2 reports the mean of the density over estimates produced by the 1000 iterations of the Monte Carlo experiment. The model correctly captures the effects of the parameters. In column 2 of Table 2, we report the inferences drawn by implementing a bargaining model that ignores partial observability in the covariates of the responding player. The estimates of covariate effect differ wildly across different iterations of the Monte Carlo experiment. Simply ignoring partial observability and modeling only part of the bargaining data generating process does not, in general, seem to improve inference much.

These results suggest that, even in the best case scenario for the use of traditional statistical techniques, in a model in which all relationships between regressors and dependent variables are monotonic, neglecting the bargaining structure leads to faulty inferences.

5 Bargaining Experiments in Small Scale Societies

Researchers from across the social sciences have conducted bargaining experiments based upon the ultimatum game in a variety of settings in both the developed and developing world. Common across these experiments are deviations from the predictions of the canonical theoretical ultimatum game model based purely on self-interest. Given that pure self-interest does not appear to determine behavior in the ultimatum bargaining game, this research begs the question of what factors do determine behavior. Are there tastes for or inclinations towards fairness and or punishment for the lack? If so are these tastes explained by individual attributes, e.g. gender, age, education, wealth, etc., or by the cultural or economic aspects of the group to which an individual belongs?

We demonstrate our approach to modeling bargaining data generated by an ultimatum game by addressing these questions. In doing so, we apply our statistical estimator to ultimatum game data gathered under a cross-cultural study conducted by Henrich (2000), Henrich et. al. (2005), Henrich et. al. (2006). Field researchers conducted ultimatum bargaining game experiments in 12 different countries, on four different continents, involving subjects from 14 different societies. These societies ranged from students at UCLA to Peruvian aboriginal peoples. We provide a summary of these societies taken from Table 1 of Henrich et. al. (2005).

[Table 3 about here.]

In each of these societies, the ultimatum game as described above was introduced. Players were generally paid in cash and were anonymous to one another, but not to the experimenters. The players were incentivized in the games with substantial sums of money (in the appropriate currency). For this game, the canonical complete information ultimatum game model restricts payoffs to concern with material rewards and hence predicts that responders, faced with a choice between nothing and a positive payoff, should accept any positive offer. Proposers should thus offer the smallest positive amount available. We summarize the data from these experiments in Figure 4. The main lesson of Figure 4 is that in almost every case, proposers deviated from offering the expected smallest positive amount. Instead most proposals were strictly higher, concentrating between 0 and 50% of the total stakes. Moreover, a significant number of proposals offers strictly more than 50% of the total stakes to the responder.

[Figure 4 about here.]

Henrich et. al. summarize the results of analyzing this data as follows. First, in each experiment proposers violated this prediction. Second, large variability in behavior across groups. Third, high levels of market integration and payoffs to cooperation promote greater levels of cooperation in the ultimatum game and so greater deviation from the predictions of the canonical model. Finally, individual-level economic and demographic characteristics do not explain behavior.

[Table 4 about here.]

We apply the statistical bargaining estimator derived in this paper to this data. In each csae, the model parameters were estimated via Maximum Likelihood Estimation; the standard errors were bootstrapped. The estimation results appear in Table 4. The bargaining estimator models both the proposal stage and the acceptance/rejections stage - the dependent variable in Table 4 is therefore an outcome consisting of a proposal (coded as a percentage of the total stakes) and a decision to accept or reject that proposal. The top part of the table are parameter estimates for the effects of covariates that influence the systematic component of the reservation value for the proposer, x. The middle part of the table contains parameter estimates for the effects of covariates that influence the observable systematic component of the reservation value for the responder, z. The bottom of the table lists parameter estimates for the systematic component of the responder reservation value that is not observable by the proposer. We boldface the parameter estimates whose 95% bootstrapped confidence intervals do not include zero.

In the first two columns of Table 4, we concentrate on model specifications that mirror those originally examined in Henrich et. al. (2005). These model specifications rely on two independent variables that summarize the economic environment of the proposing and responding player. Payoffs to cooperation measures the degree to which economic life depends on cooperation with cooperative institutions that are not kin-related. Market integration measures the frequency of market exchange. Summaries of these variables are available in the last two columns of Table 3.

In the first column, we parameterize the reservation value to the proposer with payoffs to cooperation and market integration. We model the responder reservation value with a constant. The results indicate that payoffs to cooperation and market integration do not systematically shape the reservation value of the proposer; the 95% bootstrapped confidence intervals both include zero. This contrasts starkly with the analysis of Henrich et. al. (2005); using a linear model they find that payoffs to cooperation and market integration shape the mean group offers.

In the second column of Table 4 we parameterize the observable responder reservation value with payoffs to cooperation and market integration. The proposer reservation value is held to a constant. The results indicate that, while payoffs to cooperation and market integration do not systematically affect the proposer reservation value, they do affect the reservation value of the responder. Though the probability model is complex and so interpretation of the covariate effects is not straightforward, an argument analogous to that for Lemma 2.2 implies that the proposal will be increasing in size as payoffs to cooperation and market integration increases. This is broadly consistent with the findings of Henrich et. al. (2005). Interestingly, however, these economic variables affect the proposal through the expectations of the proposer as to whether or not a particular proposal will be accepted - a notion rejected by Henrich et. al. (2005). The columns 3-5 of Table 4 contain the results of the bargaining model estimated with fixed effects. The importance of payoffs for cooperation and market integration vanishes after introducing these fixed effects.

6 Conclusion

In this paper, we derive a statistical estimator that can be used when the data generating process is best described as an equilibrium to an ultimatum bargaining game in which the proposer is only partially able to observe some of the characteristics of the responder. This statistical model allows that researcher to estimate the effects of covariates on bargaining processes and outcomes in a theoretically consistent way. Monte Carlo experiments suggest that substantive inferences regarding the effect of variables will be different and faulty if the strategic structure and partial observability in the data generating process is ignored in the process of estimation.

We than applied this model to experimental bargaining data gathered in a cross-cultural ultimatum bargaining experiment conducted in 14 small scale societies in the developing world. Our preliminary findings differ from previous work, suggesting that key independent variables affect the bargaining process through a fundamentally different pathway than revealed by the original analysis.

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List of Figures

Figure 1: The Ultimatum Bargaining Game.

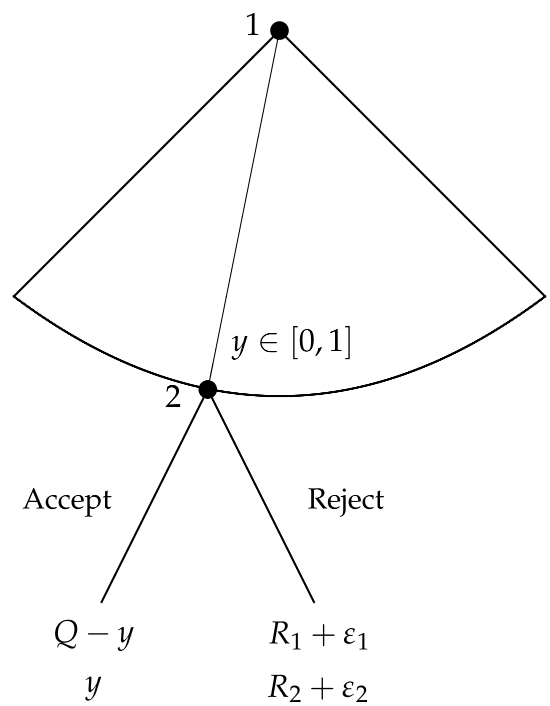
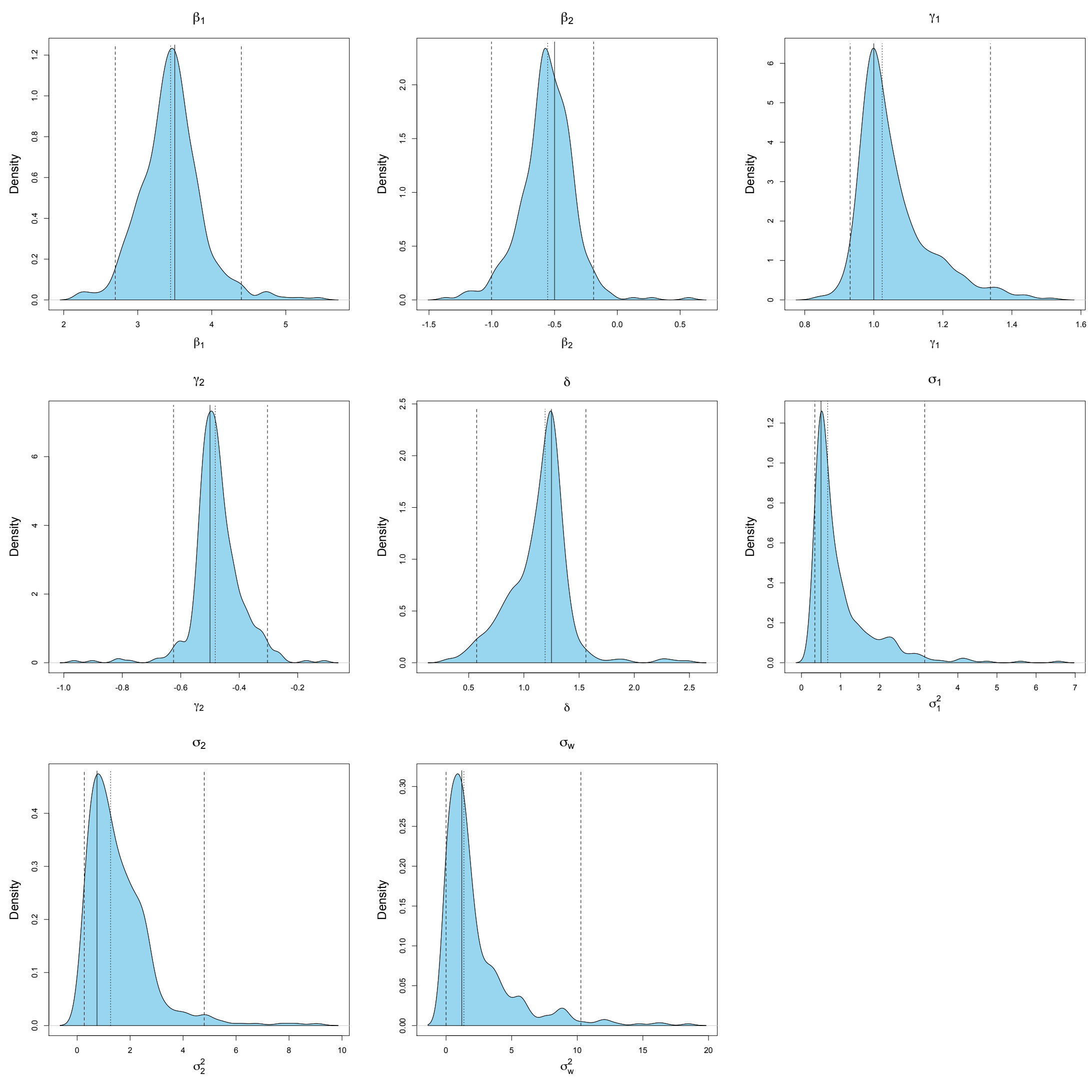
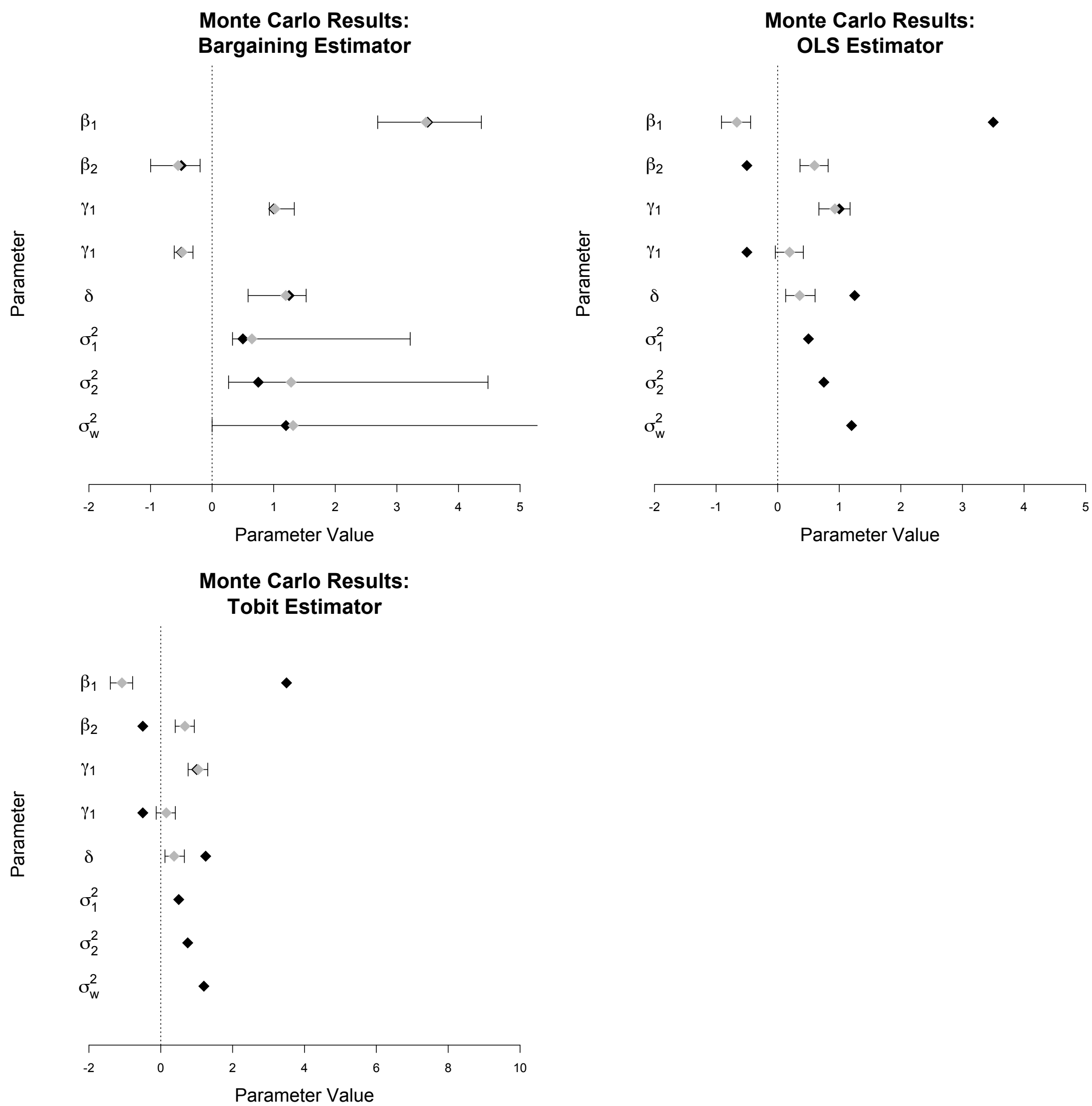


Figure 2: Empirical Distributions of Monte Carlo Analysis of the Bargaining Estimator with Partial Observability.



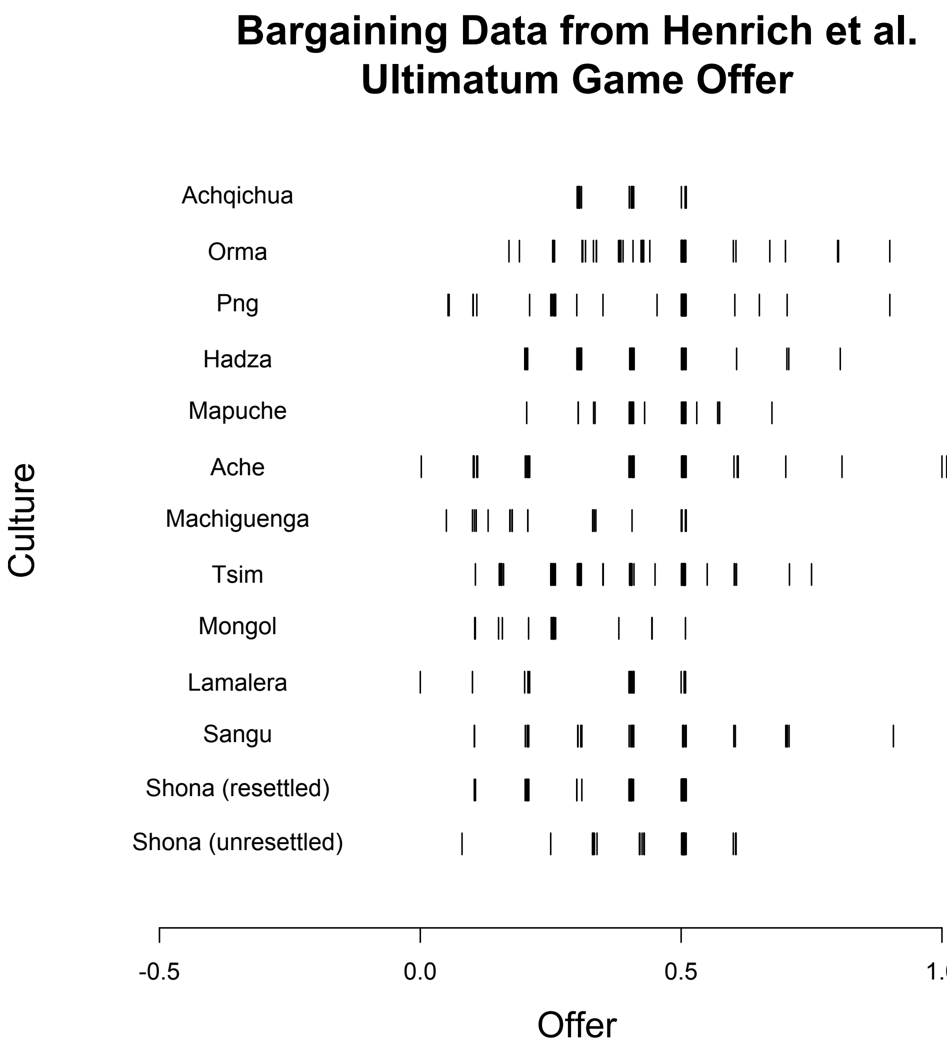
We plot the density over the estimates of the model parameters. The solid vertical line indicates the true value, while the light dotted vertical line indicates the empirical median value; the dashed vertical lines indicate the empirical 95% region.

Figure 3: Monte Carlo Estimation Results.



We plot the inferences from the estimates of the model parameters. The solid diamond indicates the true value, while the gray diamond indicates the empirical median value; the horizontal lines indicate the empirical 95% region.

Figure 4: Summary of Proposal Data.



We the proposal data from the cross-cultural experiment. The x-axis plots the offer as a percentage of the total stake. The thickness of the segment indicates how many offers took on that value. The y-axis lists culture.

List of Tables

Table 1: Recovering the Parameters.

True	Bargaining Model	OLS	Tobit
	Mean Estimate [Empirical 95% Region]	Mean Estimate [Empirical 95% Region]	Mean Estimate [Empirical 95% Region]
$\beta_1 = 3.5$	3.4734	-0.6628	-1.0807
	$[2.6889, 4.3718]$	$[-0.91223, -0.4392]$	$[-1.4041, -0.7835]$
$\beta_2 = -0.5$	-0.5518	0.5983	0.6724
	$[-0.9974, -0.1944]$	$[0.3629, 0.8194]$	$[0.4017, 0.9344]$
$\gamma_1 = 1$	1.0209	0.9297	1.0424
	$[0.9298, 1.3341]$	$[0.6698, 1.1758]$	$[0.7618, 1.3070]$
$\gamma_2 = -0.5$	-0.4846	0.1935	0.1538
	$[-0.6149, -0.3105]$	$[-0.0377, 0.4165]$	$[-0.1271, 0.4057]$
$\delta = 1.25$	1.1966	0.3576	0.3702
	$[0.5848, 1.5271]$	$[0.1301, 0.6078]$	$[0.1162, 0.6562]$
$\sigma_1^2 = 0.5$	0.6465	–	–
	$[0.3323, 3.2169]$	–	–
$\sigma_2^2 = 0.75$	1.2829	–	–
	$[0.2679, 4.4790]$	–	–
$\sigma_w^2 = 1.2$	1.3151	–	–
	$[0.0003, 9.6121]$	–	–
σ^2	–	–	1.7957
	–	–	$[1.2876, 2.4349]$

Table 2: Ignoring Partial Observability.

	True Value	Model w/ Partial Observability Mean Estimate	Model w/o Partial Observability Mean Estimate
β_1	1	0.9520	1.4495
		$[0.1439, 2.1125]$	$[-24.4219, 66.3152]$
β_2	-0.5	-0.5342	1.4143
		$[-1.2933, -0.2176]$	$[-41.6854, 37.7139]$
γ_1	1	1.0162	1.0836
		$[0.8764, 1.2840]$	$[0.0531, 2.9889]$
γ_2	-0.5	-0.4780	1.3515
		$[-0.6230, -0.2611]$	$[-0.0876, 7.9111]$
δ	1.25	1.2310	0.6415
		$[0.3244, 1.5228]$	$[-2.1333, 4.4914]$
σ_1^2	0.5	0.7805	0.0028
		$[0.3680, 7.5039]$	$[0.001, 2.2845]$
σ_2^2	1	1.0950	1.5485
		$[0.3120, 3.9095]$	$[0.2262, 8.6177]$
σ_w^2	1.25	1.3808	–
		$[0.0001, 37.5130]$	

Table 3: Ethnographic Summary of Societies

Group	Environment	Economic Base	Residence	Complexity	PC	AMI
Machiguenga	Tropical Forest	Horticulture	Semi-nomad	Family	1	4.5
Achqichua	Tropical Forest	Horticulture	Semi-nomadic	Family	1	2
Hadza	Savanna	Foraging	Semi-nomadic	Band	4	1.25
Aché	Semi-Tropical	Horticulture	Nomadic	Band	6	5
Tsimané	Tropical Forest	Horticulture	Semi-nomadic	Family	1	2.75
Png	Mountainous	Horticulture	Sedentary	Village	3	4.75
Mapuche	Plains	Farming	Sedentary	Family	2	4
Mongol	Desert	Pastorial	Transhumance	Clan	2	9
Sangu	Savanna	Agro-Pastorial	Sedentary	Clan-Chiefdom	5	6.5
Orma	Savanna	Pastorial	Sedentary	Chiefdom	2	9.25
Lamalera	Tropical Coast	Foraging	Sedentary	Village	7	9
Shona	Savanna	Farming	Sedentary	Village	1	10

Table 4: Bargaining Model Results

Parameter (x)	Estimate	Estimate	Estimate	Estimate	Estimate	
Constant	**−29.7650**	**−29.2243**	**−2.2088**	**−2.2551**	**−1.9606**	**−2.4486**
Payoffs to Coop.	−1.0371	–	–	–	−0.2252	–
Market Integration	−0.9080	–	–	–	−0.9775	–
Age	–	–	–	–	–	0.3177
Female	–	–	–	–	–	−0.0147
$\ln(\sigma_1)$	0.2755	0.6662	−1.8579	−2.1148	−2.1913	−1.7982
Parameter (z)	**Estimate**	**Estimate**	**Estimate**	**Estimate**	**Estimate**	
Constant	**−7.5407**	**−6.7181**	–	–	–	–
Unresettled Shona	–	–	**0.6478**	**1.2086**	0.4961	**0.7211**
Resettled Shona	–	–	**0.5700**	**1.2056**	0.4687	**0.6476**
Sangu	–	–	**0.5838**	0.2935	**0.6303**	**0.6810**
Lamalera	–	–	**1.2917**	0.8143	**0.9497**	**1.3193**
Mongol	–	–	**0.7447**	**0.9947**	0.5620	**0.8429**
Tsimané	–	–	**0.4382**	**0.6056**	**1.0341**	**0.4912**
Machiguenga	–	–	0.5274	**0.6536**	0.8016	0.6073
Aché	–	–	**0.6888**	0.1525	**0.8991**	**0.7389**
Mapuche	–	–	**0.5330**	0.5749	**0.8952**	**0.6067**
Hadza	–	–	**0.4256**	−0.0224	**1.1246**	**0.5229**
Png	–	–	**0.5789**	0.5138	**0.8771**	**0.6323**
Orma	–	–	**0.5744**	**0.9680**	0.4676	**0.6611**
Achqichua	–	–	**0.5638**	0.6491	**1.1786**	**0.6280**
Payoffs to Coop.	–	**0.1415**	–	1.7613	–	–
Market Integration	–	**0.1120**	–	−0.6416	–	–
Age	–	–	–	–	–	
Female	–	–	–	–	–	
$\ln(\sigma_2)$	3.6943	3.2801	1.4794	1.4497	1.7881	1.6456