Endogenous War Aims and State Resolve

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Abstract

How do rulers formulate their war aims and how are those war aims shaped by the necessity of bargaining with their subjects over wartime resource allocation? To answer these questions, I study a model in which a ruler must decide to make peace or pursue war and how much of the resources of the state to demand for the war effort. In equilibrium, the relationship between the effect of military victory on the future course of the war and the effect of military victory on diplomacy emerges as the central factor influencing both subject resolve and ruler war aims. I find (1) *subject resolve declines as military status quo improves;* (2) *rulers limit their war aims;* (3) *when more domestic support is needed for policy implementation limitations on war aims increase;* (4) *democracies extract more resources during capital intensive wars while autocracies extract more resources during labor intensive wars;* (5) *material inequality limits war aims.*

1 Introduction

How do rulers formulate their war aims? How are the war aims chosen by rulers shaped by the necessity of bargaining with their subjects over wartime resource allocation? The purpose of this essay is to further our understanding of the relationship between war termination and the means available to the state executive in charge of conducting the war effort. To answer these questions, I study an infinite-horizon model of a state at war in which a ruler must decide, in each period, whether to make peace or pursue war and how much of the resources generated by his subjects to demand for the war effort. The state subjects then have the opportunity to veto the demands made by their ruler and conclude peace on their own terms.

The questions above that I address in this simple stylization are fundamental to the study of war for two reasons. First, most conflicts do not end in the military collapse of one party.[1] This stands in stark contrast to the fact that a military victory is *the* central objective of military planners.[2] If, as this implies, the decision to terminate war is a political one, then the question for political scientists becomes *why* rulers pursue limited policy goals by formulating limited war aims, rather than fighting it out.[3] Moreover, without knowledge of how rulers choose the conditions that they expect to end their wars, it is difficult to understand how they will form expectations about how long war is likely to last, how costly it is likely to be, and so difficult to understand what will influence rulers to begin a conflict in the first place (Wagner 2000). Similarly, it will be difficult to explain how subjects determine what costs in war they are willing to bear, how resolved the state will be as a whole, and so difficult to analyze the political process by which war becomes costly.

The second reason that the questions addressed by this paper are fundamental is that they suggest that existing approaches to explaining war termination are incomplete. The two primary rationalist explanations for war initiation are informational deficiencies and commitment problems (Fearon, 1995). Given that war rarely results in a military victory for one side, an explanation for the outbreak and settlement of war based upon information must explain how the informational deficiency is resolved by the fighting. Similarly, an explanation based on commitment must explain how the commitment problem is alleviated by war.

Schelling (1960) described war as a bargaining process, a sometimes nasty, always costly attempt to convince/compel an opponent to accept terms (demands for particular war aims) by achieving an advantageous military status quo. Under the analogy of war as a bargaining process the delay of an adversary to accept the war aims demanded by their opponent is dependent upon the current military status quo and expectations

[1]Pillar (1983). See Table 2, p. 25. Only 20% of interstate wars end with capitulation of one side.

[2]e.g. von Moltke the Younger's conscious effort to inflict a decisive defeat upon the French.

[3]Political, as opposed to the operational questions of tactics, strategy, and logistics.

about how the war will progress in the future that are endogenous through the beliefs of rulers about relative military advantage (Blainey 1988). Once enough information has been communicated by battlefield events or diplomatic signaling to allow expectations to converge, war will end. This explanation for war termination will be incomplete to the extent that a significant portion of the costs in war are related to mobilization of resources to be "invested" into the war effort. *Expectations about the future military status quo will be endogenous not only through beliefs held about relative military advantage, but also through the knowledge that at future military status quos, state subjects will only be willing to part with so much to continue the war effort.* This will in turn affect what each side will be willing to concede to the other, and so affect war termination.

While most of the work on war termination has concentrated on the information transmission potential of war, recently Leventoğlu and Slantchev (2007) have constructed an explanation for the termination of wars caused by commitment problems:

> ...the very desirability of peace creates a commitment problem that undermines its likelihood. Because players have incentives to settle as soon as possible, they cannot credibly threaten to fight long enough if an opponent launches a surprise attack. This decreases the expected duration and costs of war and causes mutual deterrence to fail. Fightings destructiveness improves the credibility of these threats by decreasing the benefits from continuing the war and can eventually lead to peace.

However, state foreign policy elites do not choose to suffer the costs of war in a vacuum, and may not be able to credibly threaten to suffer the costs of war under the pressure of their subjects to keep wartime costs down. In effect, this could result in a failure of deterrence which would close the windows available to make peace.

In this paper I will analyze the bargaining between a ruler and his subjects over the extraction of state resources during war using a formal model. I argue that under certain circumstances the ruler will be unable to leverage his political control over the state into resource extraction during war; this will induce him to pursue limited war aims. In doing so, I will make three contributions to the study of war.

First, I will endogenize the *resolve* of the state as a bargain struck between ruler and subjects over wartime resource allocation. I will offer a dynamically rich characterization of how the willingness of subjects to bear the costs of war changes with military victories and defeats. The reaction of subject resolve to battlefield conditions is not obvious *a priori*; in 1940 the defeats suffered by the allies on the Western Front induced French will to collapse and British will to strengthen. Moreover, it cannot be derived from existing work which uniformly assumes exogenous resolve (Smith 1998; Slantchev 2003b; Powell 2004; Smith and Stam 2004).

I distinguish between two sources affecting the probability of prevailing in war - *mobilization* of resources and *momentum* in war. We may think of the latter as derived from geography (e.g. difficult terrain), problems of supply, or the technology of war (e.g. tanks versus infantry). The manner in which the subjects are willing to bear the costs of war depends on the relationship between momentum in war and the marginal effect of one more military victory at the negotiating table. In equilibrium, subject willingness to allocate resources to the war effort will decline as military status quo improves.

The second contribution I make will be to endogenize war aims and analyze the logic by which the ruler will formulate them. In the model, I equate each military status quo with a value representing the best available settlement for the ruler and his subjects that their opponent state would be compelled (willing under the threat of continued fighting) to accept. The military status quos where the ruler chooses to make peace will represent his war aims. In equilibrium, the ruler will pursue limited war aims by making peace at a least sufficiently favorable military status quo, and at all more favorable military status quos than this. The logic by which the ruler formulates these war aims depends crucially on the manner in which the subjects are willing to bear the costs of war. Thus, like the characterization of subject resolve, the form taken by the war aims of the ruler will be determined by the relationship between momentum and the marginal effect of military victory at the negotiating table.

War aims are difficult or impossible to observe; to date, Goemans's *War and Punishment* represents one of the few attempts to measure and operationalize the war aims of state rulers. The third contribution is a set of comparative statics results that links war aims and resource extraction to observable characteristics and provides insight into systematic relationships between aims in war, resolve, and structural aspects of the state. First, I find that when more domestic support is needed for policy implementation limitations on war aims are greater. Democratic states with more inclusive voting rules will muster less effort during wars than purely majoritarian states and therefore pursue less ambitious war aims. As a consequence of this, if wealthier subjects are likely to be politically relevant, then democracies ought to be able to expend more effort when the technology of war is capital intensive, while oligarchic autocracies expend more effort when the technology of war is labor intensive. Second, I find that unequal states will, on average, mobilize fewer resources in war.

Related Literature

The willingness of subjects to pay the costs of war is linked to the war aims chosen by the ruler. Once the ruler chooses war aims, his subjects will know the value associated with continuing the war, and will be able to determine whether a particular level of mobilization is warranted. Once they determine what levels of mobilization they will approve, the ruler can then adjust the expanse of his war aims to bring them in line with what can be realistically achieved under the resource constraints imposed by his subjects.

No existing study addresses the full scope of the interaction between war aims and the willingness of subjects to bear the costs of war via an attempt to endogenize the probability of victory, war aims, resolve, and consider the incentives of both rulers and ruled. Thus, no existing study provides a complete theoretical picture of the logic by which rulers formulate their war aims. In almost all models of unitary rational actors deciding upon war, the resolve of the state appears as an exogenous parameter of the model (e.g. Bueno de Mesquita and Lalman 1992; Powell 1993, 1999; Fearon 1994; Schultz 2001). Few models endogenize the probability of victory (Brito and Intriligator 1985; Powell 1993, 2006; Bueno de Mesquita et al. 1999; Slantchev 2005). Fewer still attempt to model the formulation of war aims (e.g. Slantchev 2003a, 2003b; Smith and Stam 2004; Powell

2004).[4,5]

Since Wagner (2000, 2008) expressed the desirability of relaxing the costly lottery assumption in formal models of war, recent work has extended the formal analysis of war to costly processes: Smith (1998); Filson and Werner (2002, 2004); Slantchev (2003a, 2003b); Powell (2004); Smith and Stam (2004); Leventoğlu and Slantchev (2007). The model I will present below is perhaps closest in spirit (though not in formulation) to that of Slantchev (2003a). He derives the valuable insight that endogenous settlement depends upon the ability of states to impose and bear the costs of war, but does not analyze the extraction of state resources.

Much of the existing work analyzing war as a costly process has been concerned with how war can transmit information (e.g. Filson and Werner 2002, 2004; Slantchev 2003b; Smith and Stam 2004; and Powell 2004). In order to focus on the transmission of information, these efforts have abstracted away from the specifics of wartime resource mobilization. To explicate the logic by which rulers formulate war aims, I have opted for a richer treatment of the dynamics of resource extraction in war.[6]

Because of my interest in the dynamics of subject willingness to bear the costs of war, like Slantchev (2003b), I will make use of Smith's (1998) model of warfare as a costly stochastic process. Smith finds "monotone" Markov Perfect Equilibria, in which states only choose to stop fighting once the military status quo becomes sufficiently unfavor-

[4] Yet one can hardly wage war without some idea of the value of the prize or without paying for it, as rulers found during the gunpowder revolution and subsequent five centuries, often to their consternation (and bankruptcy/military humiliation). See Parker (1988).

[5] Selectorate theory (Bueno de Mesquita et al. 1999) analyzes three of these four concepts but does not model war aims. Also, see Jackson and Morelli (2007) for the ruler/population interaction in war initiation.

[6] For simplicity, I do not model international bargaining. Despite this limitation, my results require only the weak assumption that military victory is perceived to be valuable in diplomacy.

able. This contrasts sharply with my results, which suggest that the ruler chooses to limit his war aims, and is willing to make peace once sufficiently close to, but often considerably short of, a military victory.

2 The Model

Consider a model of a state at war. The state consists of a *ruler* R and a *set of subjects* $S = \{1, 2, \ldots, s\}$. The ruler and subjects interact in discrete time for a possibly infinite number of periods $t \in \{0, 1, 2, 3, \ldots\}$. The task of the ruler and his subjects is to bargain over the level of mobilization adopted by the state in period t.

Denote the *current military status quo* by the state variable $k \in K = \{0, 1, 2, 3, \ldots, N\}$. At the beginning of period t, if the ruler and his subjects remain at war with an (unmodeled) international opponent, then the ruler must either make peace based upon the current military status quo or continue the war. If the ruler chooses to make peace in period t, then redistribution of a settlement based upon the current military status quo k occurs, payoffs are allocated, and the game ends. If the ruler does not make peace, then he must bargain with his subjects over the current level of military mobilization.

We model this by allowing the ruler and his subjects to divide a feasible set of state resources which belongs to the subjects; each subject $i \in S$ receives a constant flow of benefits in each period of size λ_i, such that $\sum_{i \in S} \lambda_i$ is normalized to 1. In any period, the bargaining takes the form of a take-it-or-leave-it offer from the ruler to the subjects of a fraction of resources to extract $x \in [0, 1]$ from each subject. Since the total resources available to the subjects sum to 1, x is the total mobilization available to the ruler. The subjects observe the mobilization demand x from the ruler and simultaneously choose whether or not to accept it. If the subjects choose to reject x, then we may interpret this as choosing to replace the ruler with somebody more amenable to making peace at the current military status quo; the game ends with the ruler dispossessed of his office (and possibly his life) and the subjects in possession of a peace settlement based upon current military status quo k.[7]

Successful extraction of a mobilization demand x requires that the ruler gain the approval of a *winning coalition* of subjects. The available winning coalitions are determined by the relative political power of subjects in society, which I will call the *voting rule*. Let $\mathcal{D}$, a non-empty subset of all possible subsets of S, denote the collection of winning coalitions. I will make the minimal assumption that $\mathcal{D}$ is monotonic so that a coalition that contains within it a winning coalition is itself winning. The mobilization demand x will be accepted if and only if some $C \subseteq S$ choose to accept x such that $C \in \mathcal{D}$. Examples of voting rules permitted by these assumptions include unanimity (i.e. $\mathcal{D} = \{S\}$), q-rule, (i.e. $\mathcal{D} = \{C \subseteq S | q \leq |C|, q \geq \frac{s+1}{2}\}$), of which majority rule is a special case, and dictatorship. In addition, weighted q-rule, bicameralism, bicameralism with an executive, and numerous configurations of veto players are also permitted as representations of the political power within society. These examples, many of which have been studied elsewhere, represent only a small number of the possibilities allowed by $\mathcal{D}$.[8]

If the subjects choose to accept the mobilization demand of the ruler, then a battle is fought and nature draws a new military status quo, k', from K. Given a level of mobilization x at military status quo k, denote the probability that the next period military status quo is k' as $p(k' \mid x, k)$; x is consumed by the fighting. Obviously, $\sum_{k'=0}^{N} p(k' \mid x, k) = 1$. If $k' = 0$, then the game ends with the ruler and his subjects conquered by their international opponent. If $k' = N$, then the ruler and his subjects are the conquerors; payoffs

[7]Interpreting the rejection of a mobilization demand as replacement of the ruler by someone more amenable to making peace requires that within the pool of elites from which potential replacement rulers come there is a sufficient amount of heterogeneity in preferences. When elites are homogeneous, the potential replacement would make the same demand as the incumbent ruler, and the subjects would be indifferent - this would allow the ruler to extract enormous amounts of resources without fear of political instability.

[8]For additional examples, see Austen-Smith and Banks (2000) and Banks and Duggan (2000).

associated with total victory are allocated, and the game ends. If $k' \in \{1, 2, \ldots, N-1\}$, then payoffs are allocated, period t ends, and period $t+1$ begins. Ruler and subjects discount the future by $\delta \in (0,1)$ between periods.

Strategies and Preferences

Define the feasible set of *actions* for the ruler as $[0,1] \bigcup \{P\}$, where P is the decision to make peace. A strategy for the ruler will consist of a (possibly degenerate) probability distribution over the set of actions, i.e. the mobilization demand and the decision to make peace, at each military status quo. A strategy for the subjects consists of a decision rule for what levels of mobilization to accept from the ruler at each military status quo.

Conceivably, the strategies employed by the ruler and his subjects could be extremely complex, depending on the past histories of play in a variety of ways. However, rather than tax the cognitive abilities of the players, I will restrict attention to stationary strategies which condition only on the payoff relevant information summarized by the current military status quo.[9] These are the simplest strategies in which we can examine the dynamics of the willingness of subjects to pay the costs of war.

Formally, let $\Delta([0,1] \bigcup \{P\})$ denote the set of probability distributions on $[0,1] \bigcup \{P\}$. Define a mixed strategy for the ruler as a function mapping from the current military status quo to probability distributions over the set of feasible actions for the ruler, $\pi : K \longrightarrow \Delta([0,1] \bigcup \{P\})$. Finally, denote by $\pi(k)(P)$ the probability that $\pi(k)$ places on P, and note that this implies that $\int_0^1 \pi(k)(dx) = 1 - \pi(k)(P)$. A strategy for subject $i \in S$ is a function mapping from the mobilization demanded by the ruler x and the current military status quo k to the probability that i accepts x at military status quo k; let $a_i(x,k)$ denote this probability. Denote a strategy profile by $\sigma = (\pi, a_1, \ldots, a_s)$.

In order to complete the specification of the model, I will need to specify the payoffs for peace at each military status quo, as well as the stage payoffs to the ruler and subjects while the war is on-going. To specify the payoffs to the ruler and his subjects for peace,

[9] Maskin and Tirole (2001).

let $V_i^P(k)$ be the value of the settlement to $i \in S \bigcup \{R\}$ after peace has been made where:

$$V_i^P(k) = \begin{cases} \lambda_i + \lambda_i \gamma_S(k) & \text{if } k > 0 \text{ and the ruler makes peace} \\ 0 & \text{if } k = 0 \end{cases}$$

$$V_R^P(k) = \begin{cases} b + \gamma_R(k) & \text{if } k > 0 \text{ and the ruler makes peace} \\ 0 & \text{if } k = 0. \end{cases}$$

That is, in the event that any peace better than total defeat breaks out, the subjects secure their resources (of size λ_i). In addition, each subject will obtain a benefit from the portion of the settlement from war available to the subjects, $\gamma_S(k)$, proportional to his allocation of societal resources, $\lambda_i \gamma_S(k)$.[10] The ruler maintains his control over the state, obtaining a benefit $b > 0$ for doing so. In addition, the ruler also obtains some benefit from the negotiated settlement of the war, $\gamma_R(k)$. I will assume that military success can be used to extract diplomatic concessions from the international opponent so that the benefit to making peace at a particular military status quo improves as military status quo k improves, i.e. for all $k, k' \in K, \gamma_S(k') > \gamma_S(k)$ if $k' > k$, with an analogous assumption for the ruler. In the event of a military defeat, the ruler loses his office and the subjects have their resources expropriated; both get nothing. These payoffs capture Schelling's (1960) observation that military victory is valuable at the negotiating table.

Finally, if the peace has been reached by rejecting the mobilization demand made by the ruler, then each subject pays a one period cost; in an autocratic regime, successful resistance to the state may entail large personal costs (e.g. death) while in a democracy, it might be simply the cost of organizing a substantial group of voters to go to the polls. Denote the value for peace made at military status quo k after a rejection of the mobilization demand by $V_i^{P,r}(k) = ((1-\delta)\eta + \delta)V_i^P(k)$ where $\eta \in [0,1]$; $1-\eta$ represents the cost of opposing the ruler.[11] The ruler loses his office and his benefits from the spoils of

[10] War is often observed to have redistributional effects. I model the redistributive effects of war through the distribution of costs, rather than the distribution of rewards.

[11] i.e. the closed form of: $\eta(1-\delta)V_i^P(k) + \delta \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta)V_i^P(k)$.

war. This reflects that there may be very real costs associated with attempting to deny a ruler his control over a particular policy process, in this case the execution of the war; at the same time, holding office is valuable for the ruler, while losing control over the policy process may be quite costly.

I will assume that the ruler and his subjects must wait to benefit from the spoils of war until the conflict has been settled and a peace agreement worked out. Thus, in each period that the war continues, the ruler obtains a stage payoff equivalent to only the utility for holding onto his office, $u_R(x) = b$. In each period that the war continues subject i obtains a stage payoff that consists of whatever resources he has generated in that period less his share of whatever society has agreed to mobilize for the war effort: $u_i(x) = \lambda_i - c(x, \lambda_i)\lambda_i$. The term $c(x, \lambda_i)$ represents that share of the cost of mobilizing x born by subject i; $c(x, \lambda_i)$ is continuous, strictly increasing, and weakly convex in x, with $c(0, \lambda_i) = 0$ and $c(1, \lambda_i) = 1$ for any λ_i.[12] I will assume that the share of the mobilization demand x born by a particular subject is correlated with the portion of total societal resources controlled by that subject.

Assumption 1. *(Correlated Share of Costs) For all $x \in (0, 1)$, either:*

1. *Those with more wealth bear a larger share of any particular fixed cost of war relative to those with less, i.e. $\lambda_i < \lambda_j$ implies that $c(x, \lambda_i) < c(x, \lambda_j)$, or*
2. *Those with more wealth bear a smaller share of any particular fixed cost of war relative to those with less, i.e. $\lambda_i < \lambda_j$ implies that $c(x, \lambda_i) > c(x, \lambda_j)$.* [13]

[12]This implies that $c(x, \lambda_i)$ is invertible over x.

[13]A tax on capital like that enacted by the Imperial German government prior to the outbreak of the Great War would expropriate the societal resources of the rich to a greater degree than the poor. Alternatively, conscription disproportionately affects the poor, especially where a buy-out clause exists, as in France during the Revolutionary and Napoleonic Wars and the 19th century, McNeill (1982). In the first case, war would redistribute from wealthy to poor; in the second case, from the poor to the wealthy.

The ruler and his subjects care about the same thing in the long term - the expected normalized sum of their discounted stage payoffs: $E_t\left((1-\delta)\sum_{t=0}^{\infty}\delta^t u_i(x)\right)$. Given a stationary strategy profile σ, for $i \in S\bigcup\{R\}$, we write the expected utility induced by σ recursively in the following way:

$$V_i(k \mid \sigma) = \pi(k)(P)V_i^P(k) + \int_0^1 W_i^1(x,k \mid \sigma)\pi(k)(dx).$$

In this first expression, $V_i(k|\sigma)$ is the value to player i for continuing the game at the beginning of the period, before any decisions have been made, while $V_i^P(k)$ is the value for making peace, as specified above. Next let:

$$W_i^1(x,k \mid \sigma) = \begin{cases} (1-a(x,k))V_i^{P,r}(k) + a(x,k)W_i^2(x,k \mid \sigma) & i \in S \\ \\ a(x,k)W_i^2(x,k \mid \sigma) & i = R \end{cases}$$

be the value to i for continuing the game once the ruler has chosen to fight by mobilizing x where:

$$a(x,k) = \sum_{C\in\mathcal{D}}\left(\prod_{i\in C} a_i(x,k)\right)\left(\prod_{i\in S\setminus C}(1-a_i(x,k))\right)$$

is the probability that x is accepted. Finally let the expected payoff to i for fighting the war once a mobilization demand x has been accepted be:

$$W_i^2(x,k \mid \sigma) = (1-\delta)u_i(x) + \delta\sum_{k'\in K} p(k' \mid x,k)V_i(k' \mid \sigma).$$

The first term in the expression for $W_i^2(x,k|\sigma)$ is the stage payoff allocated at the end of the period, while the second term is the expected value for continuing to fight the war.

We will seek a Markov Perfect Equilibrium given by the following conditions:

Definition 1. *A Markov Perfect Equilibrium is a strategy profile* $\sigma^* = (\pi^*, a_1^*, \ldots, a_s^*)$ *so that:*

$$\pi^*(k)(P) = 1 \quad \textit{if } V_R^P(k) > W_R^1(x, k \mid \sigma^*) \tag{1}$$

$$\pi^*(k)\left(\underset{x\in[0,1]}{\operatorname{argmax}}\left\{W_R^1(x,k \mid \sigma)\right\}\right) = 1 \quad \textit{if } V_R^P(k) < W_R^1(x,k \mid \sigma^*) \tag{2}$$

$$\textit{where } x \in \operatorname{argmax}_{x\in[0,1]}\left\{W_R^1(x,k \mid \sigma)\right\}$$

$$a_i^*(x,k) = \begin{cases} 1 & \textit{if } V_i^{P,r}(k) < W_S^2(x,k \mid \sigma^*) \\ 0 & \textit{if } V_i^{P,r}(k) > W_S^2(x,k \mid \sigma^*) \end{cases} \quad \textit{for all } i \in S. \tag{3}$$

for all military status quos $k \in K\backslash\{0, N\}$.

The first two conditions require that the ruler mobilizes/makes peace optimally while the second requires optimal voting on mobilization demands; I will require of my subjects that they vote as though they were pivotal, even in circumstances in which they are not.[14]

It may also be convenient to work with a set of acceptable mobilization levels that contains all the mobilization demands where subject i would be at least as well off if they were accepted: $A_i(k,\sigma) = \{x \in [0,1] | V_i^{P,r}(k) \leq W_i^2(x,k|\sigma)\}$. Given these individual acceptance sets, define the social acceptance set as $A(k,\sigma) = \bigcup_{C\in\mathcal{D}}\left(\bigcap_{i\in C} A_i(k,\sigma)\right)$. This will denote all the mobilization demands that the ruler could propose that would be accepted by at least one winning coalition.

Finally, I will define state resolve and war aims. I identify state resolve as the largest socially acceptable mobilization demand at each military status quo, since this will be the largest cost society is willing to bear before the subjects challenge the ruler for control over the state to avoid the burdens of further war.[15] To reinforce this, I restate it formally:

[14]This amounts to refining away weakly dominated strategies, eliminating implausible equilibria where everybody votes to accept a mobilization demand even though everyone would be better off if the demand were rejected and vice-versa.

[15]In addition, in the equilibrium that I will analyze, the *ex ante* expectation of total wartime costs corresponds with the expected number of times a particular military status quo is realized.

Definition 2. *The resolve of the state at military status quo k is the largest socially acceptable mobilization demand. Denote this by* $\overline{x}(k, \sigma) = \max\{A(k, \sigma)\}$.

I identify war aims with the set of military status quos at which the ruler chooses to make peace. Thus, given a strategy π, denote by $K(\pi)$ the set of military status quos at which the ruler is unwilling to continue the fight. That is $K(\pi) = \{k \in K \backslash \{0, N\} \mid \pi(k)(P) = 1\} \bigcup \{N\}$. Then:

Definition 3. *The war aims of the ruler are* $K(\pi)$ *.*

I have associated each military status quo with a value that represents the concessions that the international opponent would be compelled to accept under the threat of continued fighting from that military status quo. Given this, the war aims of the ruler consist of those military status quos that the ruler aims to achieve in order to compel his enemy to make diplomatic concessions that are sufficiently favorable as to make continuing the war relatively unattractive. They are the conditions that lead to peace once they are realized.

3 Equilibrium

The model specified in the last section is a stochastic game with an uncountably infinite strategy space.[16] General equilibrium existence results for such games have proved elusive and thus, before I derive substantive results, first I will argue that at least one equilibrium exists, possibly in mixed strategies.[17]

In the Appendix, I establish that correlation of the share of costs of mobilization with the pre-war distribution of societal resources (Assumption 1) means that if a particular subject is willing to accept a particular mobilization demand x then that demand will also be acceptable to either all wealthier subjects or all poorer subjects (Lemma 6.2).[18]

[16]See Neyman and Sorin (2003) for a technical presentation of characteristics of stochastic games.

[17]Dutta and Sundaram (1998).

[18]All proofs are either in the Appendix or available upon request.

With this result in mind, for the remainder of the paper I will relabel S so that for any $k \in K \backslash \{0, N\}$ $A_1(k, \sigma) \subseteq A_2(k, \sigma) \subseteq \ldots \subseteq A_s(k, \sigma)$. With this sorting of the subjects, the set of mobilization demands that are socially acceptable will be convex. We can then argue for the existence of a Markov Perfect Equilibrium in which the ruler offers only what is acceptable to his subjects.

Theorem 3.1. *If Correlated Share of Costs, A5, and A6 hold, then there exists a Markov Perfect Equilibrium. In any Markov Perfect Equilibrium: (i) the ruler only makes proposals that will be accepted with positive probability, i.e. if $\pi(k)(x) > 0$ then $x \in A(k, \sigma^*)$; (ii) the subjects will not mix on the equilibrium path unless there is exactly one acceptable level of mobilization.*

It is worth noting that we have shown that in any equilibrium in simple (i.e. stationary) strategies the ruler will choose his wartime policy to avoid the use of the collective rejection power by his subjects. This reinforces the relevance of the link between limited war aims and subject preferences reflected historically in the fears of rulers of dissent and coordination against them.

Equilibrium War Aims and Resource Extraction

The next task is to use the model to generate insight into the logic underlying the relationship between war aims and the bargaining between ruler and subjects over wartime resource allocation, and so provide a theory of how rulers formulate their war aims. In order to do this, I will need to be able to describe one distribution over the future military status quos as unambiguously "better" than another. First, I assume that investing more resources into the war effort will increase the probability of winning battles, i.e. of a military status quo that is more valuable at the negotiating table. Formally:

Assumption 2. *(Correlated Victory) For all $x, x' \in [0,1]$ and $k \in K \backslash \{0, N\}$, for any non-constant weakly increasing function $h(k)$, we have $\sum_{k' \in K} p(k' \mid x, k) h(k') > \sum_{k' \in K} p(k' \mid x', k) h(k')$ if and only if $x > x'$.*[19]

[19]Bawa (1975). This assumption is also known as first-order stochastic dominance.

As a consequence of this assumption, the expected future military status quo will be more valuable at the negotiating table with more mobilization.

The nature of the dependence of the probability distribution over the military status quos in the next period, $p(\cdot)$, on the current military status quo k is not as straightforward. As I noted in the introduction, we may think of this dependence as *momentum* in war. On the one hand, one could imagine that once the armed forces of a state have taken a particularly advantageous position, for example conquering one side of a river, taking a ridge, etc., it becomes much harder to dislodge them. In addition, "fast" technology that allows for the use of blitzkrieg style warfare which favors quick penetration and the bypass of fortified positions lends itself to increasing momentum in warfare, as the Battle for France of 1940 attests. This would imply that more favorable military status quos have a beneficial effect on the future progress of the war, in the sense of Assumption 2 above. Alternatively, as Van Creveld (1977) has argued persuasively, the history of warfare is a history of logistical and supply difficulties which have only been exacerbated by the advent of the modern age; upon winning a victory, armies reach further from their supply depots and their momentum inevitably slows as problems of supply set in, making it difficult to move against their enemies and bringing defeat and retreat.[20] This suggests that dependence of momentum in war on current military status quo k ought to work in exactly the reverse fashion. I formalize both possibilities:

Definition 4. *For all $x \in [0,1]$ and any non-decreasing function $h(\cdot)$ say that military status quo k has increasing momentum if $\sum_{k' \in K} p(k' \mid x, k+1)h(k') > \sum_{k' \in K} p(k' \mid x, k)h(k')$. Say that k has declining momentum if $\sum_{k' \in K} p(k' \mid x, k+1)h(k') \leq \sum_{k' \in K} p(k' \mid x, k)h(k')$.*

[20]This was as true in the 1991 liberation of Kuwait as it was in 1940, 1914, 1871, 1812, and even of warfare in the 17^{th} century. In fact, Van Creveld argues that problems of supply are exacerbated in modern warfare, since modern military equipment makes it extremely difficult/impossible to obtain supplies from the area through which the armies pass while fighting. Such difficulties hindered Operation Barbarossa in $1941 - 1942$ and sapped the Wehrmacht of the initial momentum gained in the invasion.

Since uncontested ground is always taken with little cost, mobilization of resources must be a fundamentally important component of performance in war. Therefore, I will concentrate on analyzing behavior when the impact of momentum on battlefield success is small relative to the impact of mobilization on battlefield success, i.e. the dependence of $p(\cdot)$ on k is small relative to the dependence of $p(\cdot)$ on x.

Assumption 3. *(Bounded Momentum) For all $k \in K \backslash \{0, N\}$ $p(0|0,k)$ is sufficiently large.*

In practice, this will mean that expending effort is a necessary condition for victory in war - the war will not fight itself entirely through momentum.

In the analysis, I want to focus on the case in which the bargaining between the ruler and subjects is relevant. For this reason, I will focus attention on the case in which subject willingness to bear the costs of war is constrained:

Assumption 4. *(Bounded Investment in War) Relabel S so that $A_1(k,\sigma) \subseteq A_2(k,\sigma) \subseteq \ldots \subseteq A_s(k,\sigma)$. Then for all $k \in K \backslash \{0, N\}$ $\max\{i \in S | \{i, i+1, \ldots, s\} \in \mathcal{D}\}$ satisfies:*

$$1 > c_{\lambda_i}^{-1}\left(1 + \frac{\delta \sum_{k' \in K} p(k'|1,k) V_i^P(k') - V_i^{P,r}(k)}{\lambda_i(1-\delta)}\right)$$

where $c_{\lambda_i}^{-1}(\cdot)$ is the inverse of $c(\cdot, \lambda_i)$ over x.

In the context of the equilibrium this will imply that at each military status quo, there is at least one *politically relevant* individual to whom mobilizing all the resources of the state is unacceptable.[21] Assumptions 1 – 4 will be maintained for the remainder of the paper.

Under these assumptions, I turn to describing the equilibrium war aims formulated by the ruler, the extraction of state resources, and the logic that underlies them. I will focus on constructing an equilibrium in which (1) at any military status quo, almost every subject $i \in S$ accepts any level of mobilization that leaves him *at least as well off as voting to reject mobilization at the current military status quo*, (2) the ruler quits the war by *making peace once he is sufficiently close to total victory*, and (3) the ruler *mobilizes as much as he*

[21] This will make certain results cleaner but will not substantively change their meaning.

can where he chooses to continue the war. This will be an equilibrium in which the ruler formulates and pursues limited war aims.

The characterization of the equilibrium proceeds in several steps. The first is to analyze the decision of the subjects to accept or reject a mobilization demand given an arbitrary strategy for the ruler. The second step involves fixing a particular strategy for the ruler and deriving the dynamic relationship satisfied by subject resolve (willingness to bear the costs of war) as battles are won and lost, i.e. across military status quos. Finally, I will derive the relationship between war aims and subject resolve and find an equilibrium.

I begin by considering the decision facing the subjects. For any strategy π for the ruler, I can write down exactly what any given subject $i \in S$ will be willing to accept by noting that i will be at least as well off accepting a mobilization demand x so long as:

$$V_i^{P,r}(k) \leq (1-\delta)(\lambda_i - c(x,\lambda_i)\lambda_i) + \delta \sum_{k' \in K} p(k' \mid x, k) V_i(k'|\sigma)$$

Manipulating this yields the following implicit relationship that must be true for any mobilization demand that could be accepted, $x \in A_i(k,\sigma)$:

$$x \leq c_{\lambda_i}^{-1}\left(1 + \frac{1}{\lambda_i(1-\delta)}\left\{\delta \sum_{k' \in K} p(k'|x,k)V_i(k'|\sigma) - V_i^{P,r}(k)\right\}\right) \equiv C_i(k,\sigma,x).$$

Given this, identify the largest mobilization demand that subject i will be willing to accept wherever $A_i(k,\sigma) \neq \emptyset$ by defining the function:

$$\overline{x}_i(k,\sigma) = \begin{cases} \max\{x|x = C_i(k,\sigma,x)\} & \text{if } 1 > C_i(k,\sigma,1) \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

With this in mind it is possible to identify the largest mobilization demand that would be acceptable to all the members of a coalition $C \subseteq S$. If there is at least one mobilization demand that would simultaneously satisfy all members of C, then the largest mobilization demand acceptable to C will be that which leaves the "most expensive" member of C

exactly indifferent between accepting and rejecting. This most expensive member will be the individual with the smallest tolerance for paying the costs of war; therefore, the largest mobilization demand acceptable to C is $\min_{i \in C}\{\overline{x}_i(k,\sigma)\}$ so long as $\bigcap_{i \in C} A_i(k,\sigma)$ is non-empty. Once I know the largest acceptable mobilization demand for an arbitrary coalition, I can identify the largest *socially* acceptable mobilization demand, the resolve of the state subjects, as:

$$\overline{x}(k,\sigma) = \max\{A(k,\sigma)\} = \max_{C \in \mathcal{D}} \left\{ \min_{i \in C} \{\overline{x}_i(k,\sigma)\} \right\}$$

This will be the largest mobilization demand that all the members of at least one winning coalition will vote to accept; if the voting rule is majority rule, then the ruler will need to satisfy more than half of his subjects. If the voting rule is oligarchic (in a way we shall make precise later), then the ruler will at least need to satisfy all of the oligarchs in his society.

Finally, by Lemma 6.2, the subjects can be labelled so that those with a higher label are willing to accept anything those with a lower label are, at any military status quo k, i.e. $A_1(k,\sigma) \subseteq A_2(k,\sigma) \subseteq \ldots \subseteq A_s(k,\sigma)$. I can exploit the fact that this ordering of the subjects will not change across different military status quos to argue that the same subject determines the largest acceptable level of mobilization across all military status quos (Lemma 6.5 in the Appendix). This means that the constraints imposed upon the ruler by the preferences of his subjects are summarized across all military status quos by a single "least resolved" politically relevant individual that the ruler must make concessions to in order to have his mobilization demand accepted - a *pivotal subject*.[22] Given the labeling of the individuals, identify the pivotal subject as $\underline{i}(\mathcal{D}) = \max\{i \in S | \{i, \ldots, s\} \in \mathcal{D}\}$.

It follows from this that the largest socially acceptable level of mobilization is given

[22]For almost all distributions of wealth, the pivotal subject will be a unique individual - when several individuals have identical allotments of the societal resources generated in each period, there may be a group of pivotal subjects, all of whom will be simultaneously satisfied by a concession from the ruler.

by $\overline{x}(k, \sigma) = \overline{x}_{\underline{i}(\mathcal{D})}(k, \sigma)$ at any military status quo k. By doing this, we have identified the largest amount of resources that the ruler can extract from his subjects at any military status quo k without risking certain rejection of his demand and internal challenge to his rule.

The next goal is to characterize (1) the dynamics of subject resolve as battles are won and lost, i.e. as military status quo changes and (2) the relationship *between* the war aims of the ruler and the resolve of the state subjects. Consider a strategy for the ruler π^* such that for some military status quo $k^* \in K$:

$$\pi^*(k)(P) = 1 \text{ if } k \geq k^* \tag{5}$$

$$\pi^*(k)(\overline{x}(k, \sigma)) = 1 \text{ if } k < k^* \tag{6}$$

The military status quo k^* will be the worst military status quo at which the ruler makes peace. Thus, π^* will be a strategy profile in which the ruler adopts limited aims in war; he chooses to quit once he gets sufficiently close to victory, and extracts as much as is politically feasible from his subjects as long as the war continues.

Given the ruler's strategy π^* I can now derive a result that will demonstrate how subject resolve changes as battles are won and lost:

Lemma 3.2. *Fix strategy π^*. If at military status quo k the momentum of war is declining or increasing but sufficiently small then the largest level of mobilization the subjects are willing to accept will be declining as military status quo improves, i.e. if $A(k, \sigma) \neq \emptyset$ then either $\overline{x}(k, \sigma) > \overline{x}(k+1, \sigma)$ or $A(k+1, \sigma) = \emptyset$.*

The largest acceptable mobilization demand, the resolve of the state subjects, declines as military status quo improves. The intuition that underlies this result is that military victory has two simultaneous effects: it increases the value to the subjects for opting out of the war because a more favorable military status quo compels the international opponent to make more valuable concessions; it also affects the expected future course of the war through momentum. When the effect of military victory on diplomacy outweighs the effect of military victory on the expected future course of the war through momentum, *independent of the mobilization of resources for the war*, then the cost of forgoing peace at more favorable military status quos is not compensated for by any increased expected military gains from being in a more favorable military status quo. Winning a battle and improving the military status quo implies that war will become relatively less valuable than peace; the subjects will be less willing to invest resources into it.[23]

Finally, fixing the strategy π^*, the largest acceptable level of mobilization, $\overline{x}(k, \sigma)$, and hence the resolve of the state, is completely determined by the worst military status quo at which the ruler makes peace, k^*. Thus, I can summarize the dependence of the resolve of the state subjects on the war aims of the ruler by $\overline{x}(k, k^*)$ and derive how their resolve changes, and hence how ability of the ruler to extract resources for the war effort changes, as the expanse of the ruler's war aims shift.

Lemma 3.3. *Fix strategy π^*. Then more ambitious war aims imply a smaller largest acceptable mobilization demand, i.e. $k^* > k^{*\prime}$ implies that for any military status quo k we have $\overline{x}(k, k^*) < \overline{x}(k, k^{*\prime})$.*

If the ruler has more ambitious aims in war, then he will have fewer resources with which to pursue them. When the ruler chooses to expand his war aims by contesting an extra military status quo, he effectively asking his subjects to pay larger costs for war in expectation; as a result the value for continuing to fight for the subjects will decline, and so the subjects will be less willing to invest resources into the war effort. This implies that the war aims formulated by the ruler represent the optimal tradeoff between ambition for a favorable settlement and the means available to achieve it.

Using the intuition from Lemmas 3.2 and 3.3, I can construct an equilibrium. To find this equilibrium, we can imagine the ruler choosing war aims (a worst military status quo at which to make peace, k^*). This set of war aims will allow the subjects to decide on the largest level of mobilization they are willing to accept at each military status quo, and will result in subject resolve that is declining as military status quo improves so that

[23] In fact, with no dependence of $p(\cdot)$ on k, this will be true for an arbitrary strategy π.

$\overline{x}(1,\sigma^*) > \overline{x}(2,\sigma^*) > \overline{x}(3,\sigma^*) > \ldots$ (Lemma 3.2). If the ruler finds that the amount of resources he can extract from the subjects is large enough to make fighting profitable for him at the worst military status quo at which he is presently making peace, k^*, then he will expand the ambition of his war aims. This will result (by Lemma 3.3) in the subjects being less willing to bear the costs of war: their resolve will decline from its initial level. Alternatively, if the ruler finds that the amount of resources he can extract from his subjects is so small that making peace is profitable at the best military status quo at which he is presently fighting, $k^* - 1$, then he will further limit his war aims. This will result in the subjects being more willing to bear the costs of war: their resolve will increase from its initial level. When neither expansion nor reduction of war aims is profitable, then the ruler and his subjects will have reached an equilibrium that determines war aims and resolve. In doing so, the ruler takes into account not only the present but also all possible future realizations of the military status quo, and the resources available to him in those futures. The subjects take into account not only the present mobilization demand, but also the total expected cost of the war and its likely outcome.

To state this result formally, fix $\sigma^* = (\pi^*, a_1^*, \ldots, a_s^*)$ where the ruler's strategy π^* is as described by equations (5) and (6). The strategy for the subjects is more complicated:

$$a_i^*(x,k) = \begin{cases} 1 & \text{if } x \leq C_i(k,\sigma^*,x) \text{ and either } i \neq \underline{i}(\mathcal{D}) \text{ or } |A_i(k,\sigma)| > 1 \\ \min\left\{1, \frac{V_R^P(k)}{W_R^2(x,k|\sigma^*)}\right\} & \text{if } x = C_i(k,\sigma^*,x), i = \underline{i}(\mathcal{D}), \text{ and } |A_i(k,\sigma)| = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

That is, subjects accept any mobilization demand that leaves them at least as well off as voting for rejection - the pivotal subject mixes when he is willing to accept only one level of mobilization with probability sufficient to make the ruler exactly indifferent between making peace and mobilizing. Finally, identify k^*, the worst military status quo at which the ruler makes peace, as:

$$k^* = \begin{cases} 1 & \text{if } V_R^P(k) > W_R^1(\overline{x}(k,1), k \mid \sigma^*) \text{ for all } k \in K \backslash \{0, N\} \\ k & \text{if } V_R^P(k-1) < W_R^1(\overline{x}(k-1,k), k-1 \mid \sigma^*) \\ & \text{and } V_R^P(k) > W_R^1(\overline{x}(k,k), k \mid \sigma^*) \text{ for some } k \in K \backslash \{0, N\} \\ N & \text{if } V_R^P(k) < W_R^1(\overline{x}(k,N), k \mid \sigma^*) \text{ for all } k \in K. \end{cases} \tag{8}$$

Then the behavior of the ruler and subjects discussed above is self-reinforcing in that it forms a Markov Perfect Equilibrium of the model:

Proposition 3.4. *(The Culminating Point of Victory) Let* $\widehat{k} = \max\{k \in K | W_R^2(\overline{x}(k,k), k|\sigma^*) \geq V_R^P(k)\}$. *For any voting rule* $\mathcal{D}$, *if at each military status quo k the momentum of war is either declining or increasing but sufficiently small and* $W_R^2(\overline{x}(\widehat{k}, \widehat{k}+1), \widehat{k}|\sigma^*) \geq V_R^P(\widehat{k})$ *then there exists a Markov Perfect Equilibrium in which:*

(i) *The ruler pursues limited aims in war by making peace once the military status quo is sufficiently favorable. His equilibrium war aims are* $K(\pi^*) = \{k^*, k^*+1, \ldots, N\}$.

(ii) *The ruler proposes the largest socially acceptable level of mobilization: for all* $k \in K \backslash K(\pi^*)$, *the equilibrium proposal strategy for the ruler is* $\pi^*(k)(\overline{x}(k, \sigma^*)) = 1$;

(iii) *For all subjects* $i \in S$, $a_i^*(x,k)$ *is as described above in equation* (7). *In equilibrium, subject resolve is declining as military status quo improves, i.e.* $\overline{x}(1,\sigma^*) > \overline{x}(2,\sigma^*) > \overline{x}(3,\sigma^*) > \ldots$.

There is only one equilibrium of this form.

For convenience, I refer to this equilibrium as a *monotonic* Markov Perfect Equilibrium. In Proposition 3.4 I argue that, despite the considerable freedom the ruler has in choosing his war aims, those war aims take a simple form. The war aims formulated by the ruler in equilibrium are completely summarized by military status quo k^*, the worst military status quo at which the ruler seeks peace. At all better military status quos than k^* the ruler makes peace, and at all worse military status quos than k^* he mobilizes the largest acceptable level of mobilization. The critical military status quo beyond which the ruler will seek peace, k^*, is a measure of precisely how limited his war aims will be.

In addition, Proposition 3.4 yields a sharp characterization of the equilibrium dynamics of the resolve of the state subjects - their willingness to bear the costs of war - as battles are won and lost. Via Lemma 3.2, an improved military status quo results in a lower tolerance for the costs of war and, as a result, the ruler will be able to extract fewer of his total societal resources as the military status quo improves. This suggests that in wars where complete military defeat is catastrophic for the losing side, an initial defeat would actually act to strengthen the resolve of the state to pay the costs necessary to resist their opponent.

The condition upon which both Lemma 3.2 and Proposition 3.4 rely is the relationship discussed above between the marginal diplomatic advantages one more military victory provides at the negotiating table and the marginal military advantages one more victory provides on the battlefield through momentum. This underlines the importance of distinguishing between strength in war derived from the costs that a society is willing to bear and strength in war derived purely from the momentum of war. Indeed, the analysis in this section suggests that the first form of strength in war is explicitly a function of the second. When the effect of military victory on international diplomacy outweighs the effect of military victory on the expected future course of the war independent of the effort invested by the state into the fighting, this will have a profound impact on the ability of the ruler to mobilize the resources of his state - subject resolve will decline as military status quo improves. *In equilibrium, the relationship between the marginal effect of one more military victory on the expected future course of the war and the marginal effect of one more military victory on international diplomacy emerges as the central factor influencing both subject resolve in war and the war aims of the ruler.*

4 War Aims, Resource Extraction, and the Distribution of Power and Wealth within the State

The general form of the war aims that will be pursued by the ruler and the dynamics of subject resolve in the monotonic Markov Perfect Equilibrium of Proposition 3.4 do not depend upon the distribution of political power within the state, or on the model parameters. However, both the expanse of the war aims of the ruler, summarized by k^*, and the extent to which the subjects are willing to bear the costs of war will depend on these factors. Comparative statics analysis of the equilibrium yields theoretical insights into how two variables affect the political process of bargaining over wartime resource allocation and the formulation of the war aims of a ruler. I consider in turn the impact of shifting the distribution of political power and the distribution of wealth within the state. Throughout this section, all the results will compare behavior conditional on the ruler and subjects playing the monotonic Markov Perfect Equilibrium of Proposition 3.4.

Political Inclusiveness

In the first comparative static result, I argue that within the class of democratic states, the institutional effect of broadening the distribution of power within the state hampers the ability of the ruler to extract state resources and pursue ambitious war aims.

Proposition 4.1. *(Effect of Inclusiveness) Fix $\mathcal{D}$ and $\mathcal{D}'$ as quota rules with $q > q'$. Then in the two states with distributions of political power defined by these voting rules the state with greater quota is less resolved and has less ambitious war aims. That is, for all military status quos k we have $\overline{x}(k, \sigma) < \overline{x}(k, \sigma')$ and $k^*(\mathcal{D}) \leq k^*(\mathcal{D}')$.*

The distribution of political power within the state follows a quota rule when the winning coalitions available to the ruler consist of all those coalitions of a size larger than the quota, q. Under a quota rule, political inclusiveness is a function of the quota of "aye" votes needed for successful approval of a mobilization demand. As the quota increases, the size of the winning coalition the ruler is required to construct becomes larger; to bring these extra subjects on board with his plans for resource extraction during war, the ruler is required to moderate the severity of the demands he makes of them. As a result, the state will be less resolved during war. Because the ruler has fewer resources at his disposal, he may find it advantageous to make peace where it is relatively valuable to do so. Since military victory allows the ruler and his subjects to compel their international opponent to make concessions, the temptation for the ruler to make peace is largest at the most advantageous military status quos where he is currently fighting, $k^* - 1, k^* - 2, \ldots$. Therefore, when war becomes less valuable to the ruler because he has fewer resources with which to pursue it, he will reduce the ambition of his war aims by choosing to decrease k^*.

Autocracy and Democracy

In the second comparative static result, I argue that whether a democratic state can extract more resources than an autocratic state depends largely on who bears the costs of war in the autocracy. This result provides a theoretical underpinning to the empirical observation made by Kugler and Domke (1986) that the extractive capacity of a state at war does not have a strong unconditional relationship with regime type.

In this paper, the essence of an institutionally democratic state is that the distribution of political power is "anonymous" in that it does not recognize *a priori* that an individual subject is special in possessing a disproportionate amount of political power. Thus, when trying to gain support for mobilization, the ruler of a majoritarian democratic state can construct any majority coalition he wishes, and does not need to satisfy "special interests."[24] By contrast, the rulers of autocratic states often lack this flexibility in that their pillars of support in society are fixed and cannot be readily changed. For example, in feudal monarchies, the king required the support of his barons to rule; when he did not have it, as when Edward IV lost the support of the powerful Earl of Warwick Richard

[24] This could be modified to allow for the existence of any number of veto players, i.e. collegial voting rules in which $\bigcap_{C \in \mathcal{D}} C \neq \emptyset$. It could also be modified to allow for several houses, each of which must satisfy a quota rule amongst its members.

Neville (the "Kingmaker"), internal strife was often the result.[25] These autocracies are fundamentally *oligarchic* in nature in that the ruler requires the support of a few special individuals to extract resources during war or govern more generally. To capture this intution formally, let $O = \bigcap_{C \in \mathcal{D}} C$ denote the set of veto players and say that the voting rule $\mathcal{D}$ is oligarchic if those veto players form a winning coalition so that $O \in \mathcal{D}$.

Proposition 4.2. *(Autocracies and Democracies) Suppose that $\mathcal{D}$ is oligarchic and $\mathcal{D}'$ is quota rule. Then in the two states with distributions of political power defined by these voting rules the oligarchic state is more resolved and has more ambitious war aims (i.e. for all k we have $\overline{x}(k, \sigma') < \overline{x}(k, \sigma)$ and $k^*(\mathcal{D}') \leq k^*(\mathcal{D})$) than the quota-rule state if and only if* $\min\{i \in O\} > N - q + 1$.

The autocratic state with an oligarchic distribution of political power will be able to extract more resources during war than a democratic state governed by quota rule depending on the share of costs of mobilization born by the oligarchs.

When the oligarchs bear a small share of the costs of mobilization the autocratic ruler will be able to take advantage of the disproportionate amount of political power held by these subjects to appeal to a winning coalition whose members are willing to approve large amounts of extraction. By contrast, when the oligarchs bear a large share of the costs of mobilization the autocratic ruler will still require their support and so will have to make a much smaller mobilization demand in order to get it. The democratic ruler can always take advantage of the anonymity in the distribution of political power imposed by democratic institutions to appeal to the winning coalition whose members bear the smallest share of the cost of mobilization.

Proposition 4.2 highlights the importance of the relationship between the distribution of political power within society and the subjects most importuned by the costs of mobilization in determining resource extraction and war aims. This immediately suggests that

[25] Similarly, Hitler could not rule without the support of prominent Nazi party members whose authority and positions were relatively fixed and could not be changed without considerable effort, e.g. Himmler, Goering, Speer, and his Gauleiters. See Speer (1970) for an account.

an important but currently under-appreciated determinant of wartime state effectiveness is the interaction between regime type and the technology of war.

Proposition 4.3. *(The Technology of War) Suppose that $\mathcal{D}$ is oligarchic and $\mathcal{D}'$ is quota rule. Assume that the wealthy are politically relevant in the oligarchic autocracy (i.e. $O = \{i \in S | \lambda_i \geq \lambda_l\}$ for $l > N - q + 1$). Then in the two states with distributions of political power defined by these voting rules: (1) if war is capital intensive (i.e. Assumption 1 part 1 holds), then majoritarian democracies will be more resolved in war and have more ambitious war aims than autocratic oligarchies. (2) If war is labor intensive (i.e. Assumption 1 part 2 holds), then majoritarian democracies will be less resolved in war and have less ambitious war aims than autocratic oligarchies.*

I defined the technology of war as capital intensive if there are redistributive effects from the wealthy to the poor - the wealthy bear a larger share of the cost of mobilization. War is labor intensive when the redistributive effects work in the opposite direction, from the poor to the wealthy - the poor bear a larger share of the cost of mobilization. If the oligarchs that the autocratic ruler must appeal to are wealthy subjects, then a capital intensive war that redistributes from the wealthy to the poor will be one in which the wealthy bear a larger share of the cost of mobilization. The wealthy will be unwilling to approve large levels of mobilization, and the autocratic ruler will be unable to extract large amounts of resources. A labor intensive war that redistributes from the poor to the wealthy will be one in which the wealthy bear a smaller share of the cost of mobilization and so will be more willing to approve large levels of mobilization. The autocratic ruler will be able to take advantage of their political power to extract large amounts of resources.

Inequality

Finally, I argue that material inequality within the state will hamper the ability of the ruler to extract resources from his subjects and will decrease the ambition of the ruler's goals during war.

Proposition 4.4. *(Effect of Inequality) Fix a voting rule $\mathcal{D}$. Assume that war is labor intensive (so that Assumption 1 part 2 holds) and consider two distributions of wealth $\lambda = \{\lambda_1, \ldots, \lambda_{\underline{i}(\mathcal{D})}, \ldots \lambda_s\}$*

and $\lambda' = \{\lambda'_1, \ldots, \lambda'_{\underline{i}(\mathcal{D})}, \ldots \lambda'_s\}$ *such that for all* $i \leq \underline{i}(\mathcal{D})$, $\lambda'_i < \lambda_i$ *and for all* $i > \underline{i}(\mathcal{D})$, $\lambda'_i > \lambda_i$. *Then in the two states with these distributions of wealth, the state with greater material inequality is less resolved and has less ambitious war aims. That is, for all k we have* $\overline{x}(k, \sigma') < \overline{x}(k, \sigma)$ *and* $k^*(\lambda') \leq k^*(\lambda)$.[26]

In the model, each subject generates resources in each period. The distribution of these resources reflects the degree of inequality within the state.

Substantively, consider a war in which the ruler is extracting resources from his subjects that take the form of conscription with the option to pay to avoid personal service in the front lines.[27] A middle income individual might be able to pay to avoid personal service at the front. However, if we exacerbate inequality, making him poorer and the wealthy richer at his expense, then attempting to pay to avoid service would consume a larger portion of his income. Because of this, he would have been less willing to approve having that mobilization demand imposed upon him. So long as he is politically relevant, society as a whole would have been less willing approve that mobilization demand. An unequal state will be a state in which a ruler will be relatively less able to extract resources for the war effort, and so may reduce the ambition of his war aims.

5 Discussion

Not every war can be pursued to annihilation of the enemy. In the analysis of the model, this emerges not from the realities of military defeat by the enemy, but from voluntary limitations on aims in war. These voluntary limitations have the essential features of what von Clausewitz (1832) termed the *Culminating Point of Victory*. Arguing from his experiences in the Napoleonic Wars, von Clausewitz noted that:

[26]The case where war is capital intensive is analogous.

[27]During the American Civil War, the North instituted such a policy in 1863, offering conscripted men the option to pay \$300 to avoid service in the Union army. Similarly, France maintained such a policy (except for the Levée en Masse of mid-1793, see Forrest 1989) through much of the 19th century.

> Before his [Bonaparte's] time, every campaign had ended with the winning side attempting to reach a state of balance in which it could maintain itself. At that point, the progress of victory stopped, and a retreat might even be called for. This culminating point in victory is bound to recur in every future war in which the destruction of the enemy cannot be the military aim... The natural goal of all campaign plans, therefore, is the turning-point at which attack becomes defense.[28]

This culminating point of victory is the best achievable military status quo available and so defines the war aims of the attacking side. For von Clausewitz, culminating points of victory are primarily functions of military concerns - the necessity of depleting manpower on enemy territory to besiege and garrison fortresses, interaction with a hostile population, and movement away from sources of supply. However, these concerns are not the entire story.

In the model, the culminating point of victory is captured by k^*, the worst military status quo at which the ruler seeks peace. Thus, culminating points of victory are also driven by the political interaction between ruler and subjects. When the ruler and his subjects are threatened by their mutual opponent with an unfavorable military status quo in which their enemy can only be induced to cease fighting by making large concessions at the negotiating table, vetoing the mobilization demands of the ruler and making peace is not valuable. In such threatening circumstances, the subjects will tolerate large levels of mobilization to push back the frontlines and improve the military status quo, thereby reducing the threat from their mutual enemy. The reason why the ruler may not be able to carry the war to total victory is that once the threat has been reduced by improving military status quo, vetoing the continued demands of the ruler is more valuable. If this increase in the value of immediate peace is not canceled by a concurrent improvement in the expected future course of the war, then subject resolve to tolerate costs will decline resulting in a culminating point of victory short of the military overthrow of the enemy. Because of the reduced means at his disposal, the ruler will not be able to commit to carry on the war to victory and will limit his sought-for gains instead.

Thus, when the effect of military victory on international diplomacy outweighs the effect of military victory on the expected future course of the war independent of the

[28]von Clausewitz pp. 684-690.

effort invested by the state into the war, the ruler will be increasingly unable to leverage his control over the state to extract resources from his subjects as victories are won on the battlefield. This will induce a culminating point of victory which results in the ruler adopting limited war aims. Limited wars arise when rulers are cognizant not only of the links they make between their political goals and the results of the battlefield, but also take note of the culminating points of victory generated by the resolve of their subjects.

This argument yields comparative statics that have implications for the relationship between democracy and war outcome and the empirical analysis of public opinion during war. I will now discuss these in turn.

Democracy and War

Democratic states have been observed to win more of the wars they fight (Lake 1992; Siverson 1995; Bennett and Stam 1996; Reiter and Stam 1998a, 1998b). Some have argued that democracies possess a greater war-fighting ability than autocracies and that this is due to the greater ability of democracies to extract resources from the public (e.g. Lake 1992; Bueno de Mesquita et al. 1999). In the model analyzed here, the military effectiveness of the state is determined by the capacity of the ruler to extract resources by proposing acceptable mobilization demands to his subjects. It follows from this analysis that if democratic states do possess a greater war-fighting ability then they do not derive this advantage from systematically greater resource extraction.

First, "more democratic" electoral rules within a democratic state hinder the ability of the ruler to extract state resources. By Duverger's hypotheses, plurality rule tends to favor a two-party state, while proportional representation tends to favor multiparty systems (Duverger 1954; Duverger 1986; Cox 1997). Since differing electoral rules induce differing numbers of effective political parties (Powell 2000, Lijpart 1994), if party discipline is strong and no party maintains a majority, these differing electoral rules also induce differing *de facto* policy-making quota requirements, even if the institution is *de jure* governed by majority rule. By Proposition 4.1, increases in quota imply that the ruler of the state will be less able to extract resources during war. This implies that states governed by proportional representation will mobilize fewer of the assets of the state during wartime. Increasing the effective representation of subject preferences is often taken to be one of the crucial components of democracy. If this is so, then the implication of Proposition 4.1 is that increasing "democracy" within the state hinders resource extraction and hence decreases the effectiveness of the state during war.

Second, from Proposition 4.2, anonymity in the distribution of political power within the state conferred by institutional democracy may or may not be advantageous relative to autocratic states - it largely depends on whether the oligarchs bear large costs for mobilization of state resources, and so on whether resource extraction is capital or labor intensive (Proposition 4.3). Taken together, these comparative statics suggest that the effect of democracy on resource extraction during war is highly conditional on (1) the particular democratic institutions of the state and (2) the sort of resources being extracted from the population.

Democratic states may have one systematic advantage over autocrats that does not appear in this model: the ability to commit to uphold agreements. Prior to or during war it may be difficult for the ruler to commit not to take advantage of the power he has vis-á-vis his subjects in the post-war world. This may undermine pre-war agreements struck between ruler and subjects over division of the expected spoils. Thus an advantageous peace will be less valuable to the subjects, who will be less willing to pay the costs of war to achieve one. The result will be reduced resource extraction, and so reduced ability in war.

This sort of commitment problem limited the ability of Charles I of England, and the House of Stuart more generally, to raise funds for war, and even maintain financial solvency; Charles I was compelled by resistance in Parliament to employ nearly forgotten laws like the 1279 Distraint of Knighthood in situations of dubious legality, as well as forced "loans," to raise funds. These measures were enforced by the royal courts and the Star Chamber, bodies beyond Parliamentary control.[29] The conflicts surrounding the use of these extra-Parliamentary institutions imposed drastic limits on the means of war for

[29]See Kishlansky (1997) and Miller (2000).

the Stuart Dynasty. By contrast, after the Glorious Revolution, the crown "gained access to an unprecedented level of funds."[30] North and Weingast argue that this was due to changes in the ability of the government to commit to not unilaterally alter the terms of promised agreements that it entered into.

Since democratic states that have embraced the rule of law are more likely to be able to make credible commitments to either split the spoils of war equitably or at least pay back debts incurred during fighting, these states may be able to secure more resources during the fighting and pursue more ambitious war aims than autocrats.

These arguments suggest that the net effect of democracy on resource extraction during war will be ambiguous. If democracies have a systematic advantage during war it is derived from something other than resource extraction, perhaps leadership (Reiter and Stam 1998b).

Pre-War Uncertainty

The commitment problem described above has a second implication regarding war initiation. Since Fearon (1995), incomplete information regarding either state resolve or the probability of victory has received attention as a cause of war initiation. Since wars flare up and die out it follows that if informational deficiencies cause wars, then those deficiencies must themselves wax and wane. My results link resolve in war and probability of overall victory strongly with the underlying societal arrangements of the state. However, the underlying parameters that influence society e.g. inequality, democratic tendencies, economic development, resource distribution, etc., *are observable and simply do not shift that much over time*, suggesting that they are not a source of uncertainty.

To the extent that the commitment problem described above shapes the mobilization demands that subjects will be willing to accept, it also shapes wartime resolve and the distribution of power between the international belligerents. It follows that the ability of rulers to commit to a distribution of the spoils of war, which may result from private negotiations and deals brokered behind closed doors, acts as a source of private informa-

[30]North and Weingast (1989).

tion. Indeed, in this model, it is precisely when the ability to commit is unobservable to outside actors that pre-war uncertainty will exist over the extent of wartime state resolve and the ability of the ruler and his subjects to win the war.

Public Opinion During War

The logic by which rulers formulate their war aims in the model also has implications for the dynamics of public opinion during war. Shifts in wartime public opinion are said to affect U.S. congressional and presidential elections (Carson et al. 2001; Gartner et al. 2004; Karol and Miguel 2007; Voeten and Brewer 2006), the tenure of rulers beyond America (Bueno de Mesquita et al. 2003), war outcomes, and public discourse more broadly (Zaller 1992; Berinsky 2007). Despite the importance of public opinion in the politics of warfighting, our knowledge of its determinants remains inadequate (Feaver and Gelpi 2004; Gartner 2008).

The standard argument (e.g. Mueller 1973) is that increases in casualties (i.e. costs) will decrease approval for the executive of the state, a proxy for approval for the war. Other scholars have argued that success during war, elite behavior, and trajectory of costs matter more than the absolute number of casualties (Gelpi, Feaver, and Reitler 2005; Berinsky 2007; Gartner 2008).

My analysis suggests another important variable shapes wartime public approval for the war effort but has been unexplored in existing studies: the value of the outside of option of an immediate peace. In the model, as military victories accrue, the ruler and subjects can use this leverage to extract more valuable concessions from their international opponent. The increasing value of peace with military status quo induces the subjects to be less willing to support large amounts of resource extraction as the military status quo improves; subject support for the war effort will decline.

The value of the outside option of peace has the potential to be at least as important in determining support for resource mobilization and hence the war effort as the previously mentioned factors. For example, in late 1965, President Johnson dispatched Secretary of Defense Robert McNamara to analyze the situation in Vietnam. McNamara, who had previously delivered glowing reports on the progress of the Strategic Hamlet program, compiled a much less optimistic assessment, suggesting that 500,000 American troops would be needed in the next eighteen months. In response, Johnson unilaterally halted the bombing of North Vietnam for a month and initiated a peace overture to Hanoi that was, at least from the perspective of the American public, genuine. Privately, Secretary of State Dean Rusk cynically told South Vietnamese Ambassador Lodge that the attempted peace talks were "an exercise [by Johnson] in public relations:"

> The prospect of large-scale reinforcements in men and defense budget increases for the next eighteen-month period requires solid preparation of the American public. A crucial element will be a clear demonstration that we have fully explored every alternative but that the aggressor has left us no choice.[31]

As it turned out, the peace initiative failed: an attempt to deliver a note to the North Vietnamese Embassy in Moscow in early 1966 was rejected out of hand. However, it did give Johnson the demonstration that he needed to show that the military status quo was not favorable enough to extract political concessions from Hanoi. As a result, Congress approved the administration's request to deploy the requisite half-million troops to Vietnam.

The effect of the value of the outside option of peace on the willingness of subjects to bear the costs of war will likely compete with two countervailing effects. First, the subjects may be uninformed about the likelihood of winning the war; battlefield defeats may lead them to believe that a favorable military conclusion is unlikely, while battlefield victories may convince them of the opposite. Second, the subjects may infer from battlefield defeats that their ruler is incompetent and incapable of running the war.

Whether these countervailing effects will change the relationship between subject resolve and military status quo depends on the same logic as the results I have analyzed here. If the effect of military victory on the beliefs of the subjects about competence or the probability of victory is small relative to the effect of victory on international diplomacy, then the same dynamics are likely to obtain as in the results I have derived here. If the

[31]See Karnow's *Vietnam: A History*.

effect is large, then the effect of suffering a defeat may dominate the effect of battlefield outcome on international diplomacy, while a large victory may do the opposite. The subjects will be more willing to bear the costs of war as military status quo improves until the status quo is sufficiently good, at which point the effect of victory on international diplomacy begins to dominate the effect of victory on beliefs, and the subjects become less willing to bear the costs of war as military status quo further improves.

Empirical Dynamics of Extraction

Perhaps the most striking and counter-intuitive finding of the monotonic equilibrium characterized in Proposition 3.4 is that the willingness of subjects to tolerate the costs of war declines as victories accrue. This suggests that when military status quo worsens analysts ought to observe relatively more mobilization and when it improves they ought to observe relatively less mobilization. To demonstrate support for this result I consider the French Revolutionary War of the First Coalition.

The new French rulers who seized control of France at the collapse of the *ancien régime* were inexperienced, and engaged in inflammatory attempts to promote unwelcome revolution abroad largely for domestic political reasons. This helped to move Austro-French relations on the road to war in 1792. By the time the Girondins took the assembly in France, swept prudence aside, and voted an ultimatum to the Holy Roman Emperor Leopold II, the Austrian position had hardened, and war between France on one side, and allied Austria and Prussia on the other, was the result.

The initial Allied invasion of France was stymied on the fields of Valmy and Jemappes in 1792; the French chased the Prussians from northern France and gained a series of costless conquests in Belgium. As a result, the military status quo was relatively favorable for France in late 1792. The army used by the French in late 1792 was a professional army that looked much like it had prior to the Revolution - its maintenance did not involve large extractive demands imposed upon society by the state. These military successes were not to last; the decision to execute the king and invade Belgium turned much of

Europe against the Revolutionaries.[32] Thus, by early 1793, France found herself at war with Britain, Spain, Austria, Prussia, the United Provinces, and Piedmont, invaded on three fronts, her early conquests jeopardized, and the military status quo shifted against her. The French state responded by rallying the entire nation against the invaders by imposing enormous extractive demands. In August of 1793, the Revolutionary Convention instituted the Lévee en Masse with the famous declaration that:

> ...all Frenchmen are permanently requisitioned for service into the armies. Young men will go forth to battle; married men will forge weapons and transport munitions; women will make tents and clothing and serve in hospitals; children will make lint from old linen; and old men will be brought to the public squares to arouse the courage of the soldiers, while preaching the unity of the Republic and hatred against Kings.

The demand for societal resources was further intensified in September with the passage of the Law of the Maximum.[33] Given the threat of the revolution in danger, the public largely accepted these new burdens; the Lévee en Masse did not provoke widespread rioting or unrest, and desertion rates from the army during 1793 were low.[34]

The French were able to use these extractive demands to increase the size of their military forces to nearly 1.2 million men; with these new forces they defeated the allied invasions of France and by mid-1794, their armies were everywhere in the ascendent. The exertions of 1793 could not be sustained after the invaders were ejected from French soil and the military status quo improved; once the immediate danger was past with the victories of late 1793 and early 1794, extracting resources met with continually mounting resistance and unrest, and as a result, no further conscription was required until the end of the War of the First Coalition. Moreover, the Law of the Maximum was repealed.[35,36] These shifts in French mobilization are consistent with the equilibrium behavior of the

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[32] Schroeder (1994).

[33] Price controls intended to allow the government to buy war material at below-market value.

[34] Forrest (1989).

[35] McNeill (1982) pp. 195-197.

[36] There is evidence to suggest that the Nazi rulers of Germany during World War II exploited the defeat at Stalingrad similarly. Early post-World War II historical analysis

ruler and subjects derived in the model.

Conclusion

Plans for military victory formulated by war planners and military professionals inevitably meet the reality of finite resources and political constraints. To analyze how rulers meet the challenge posed by these constraints, I advanced a theory that explains the logic by which rulers formulate their war aims. At the center of this theory lies a formal model in which rulers choose the conditions under which they will end the war and seek to achieve their war aims by extracting resources from their subjects. Analysis of the model reveals that when the effect of military victory on international diplomacy outweighs the effect of military victory on the expected future course of the war, independent of mobilization of state resources, the willingness of subjects to bear the costs of war will decline as military status quo improves. The ruler will be unable to leverage his political control over the state into resource extraction during war, and will face a tradeoff between resources and the expanse of his war aims. This defines a culminating point of victory - the war aims of the ruler.

Not all states are be able to credibly threaten a fight to the finish. Certain states will be unable to commit to suffer large costs in war. This implies that these states may find

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argued that, because of Nazi sensitivity to internal unrest, the pre-1942 wartime economy of the Reich was optimized for a cheap blitzkrieg strategy in warfare. Although they may have overstated their case (Overy 1994, especially chapters 6-9), it is true that the Nazi régime was exceedingly aware of the potential for domestic instability prior to 1942. See Abelshauser (1998) pp. 142,150-151. In the wake of the disastrous shift in military status quo after the defeat at Stalingrad in 1942, there was a massive increase in mobilization of resources for the war effort. Two factors account for the expansion. First, by interfering in traditional methods of German industrial organization, Speer was able to greatly increase efficiency. Second, the expectations of the Nazi leadership about what the German people were willing to accept shifted. As a result much greater demands were placed on the population.

it particularly difficult to end wars brought on by commitment problems (Leventoğlu and Slantchev 2007). Comparative statics analysis suggests that: (1) increasing political inclusiveness hampers the ability of rulers to extract resources from the state during war and limits their war aims; (2) democratic states extract more resources during capital intensive wars while autocratic states extract more resources during labor intensive wars; (3) material inequality imposes greater constraints on the extraction of state resources from society. This implies that democratic states are not systematically able to extract more resources during war and derive any wartime advantages from another source.

These results also imply that there may be powerful forces of selection at work that shape the participation of states in the wars that analysts observe - for example, if autocracies are forced to fight wars that are heavily capital intensive, then what they can extract will not enable them to pursue ambitious war aims. If they are sufficiently unable to extract resources, then it will not be worth fighting a war at all, and we will never observe that war happen. Inequality will have similar effects, especially in democratic states. This will complicate empirical testing of these comparative statics.

In addition, the interpretation of limited war as the result of a commitment problem between rulers and ruled in the post-war world leads naturally to a potential source of uncertainty prior to and during war. To the extent that the commitment device worked out between rulers and ruled to guarantee a post-war division of the spoils is unobservable by the outside, resolve in war and the probability of victory will be a significant source of uncertainty prior to and during the fighting.

Finally, future work could extend the model in a number of ways. It would be useful to consider the strategic transmission of information about the current military status quo, since it may frequently be only indirectly observable by subjects, as well as uncertainty over the likelihood of victory in war. It would also be useful to consider the effect of history dependence on past mobilization, either by direct introduction into the strategies of the ruler and subjects, or by allowing the ruler to draw extraction from a pool of resources owned by the subjects that decreases over time.

6 Proofs

I will need to impose some structure on the probability distribution over the possible locations of the front, k. In particular, I will make the following assumptions:

Assumption A5. For all $k' \in K, k \in K \backslash \{0, N\}$, $p(k' \mid x, k)$ is continuous in x.

Assumption A6. For all $k, k' \in K \backslash \{0\}$, $p(k' \mid x, k)$ is strictly concave in x.

These conditions are *sufficiently* strong to produce strict quasi-concavity in $W_S^2(x, k \mid \sigma)$, but not necessary. The proof of existence, which relies on strict-quasiconcavity, is therefore more general than the assumption that guaranteed us concavity.

Lemma 6.1. *For all $k \in \{1, \ldots, N-1\}$ and all $i \in S$, $W_i^2(x, k \mid \sigma)$ is continuous and strictly concave in x. For any $C \subseteq S$ and strategy profile σ, $A_C(k, \sigma)$ is a convex set.*

Proof. Continuity follows directly from Assumption A5 and the definition of $W_i^2(x, k \mid \sigma)$. Concavity follows directly from Assumption A6 and that $W_i^2(x, 0 \mid \sigma) = 0$. $\square$

Fixing a strategy profile σ, we can represent the path of play for i by a transition probability, $P(\sigma)$.

$$P(\sigma) = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \begin{array}{c} \int_0^1 r(x,1) p(0|x,1) \\ \times \pi(1)(dx) \end{array} & \begin{array}{c} \int_0^1 r(x,1) p(1|x,1) \\ \times \pi(1)(dx) \end{array} & \begin{array}{c} \pi(1)(P) \\ + \int_0^1 (1 - r(x,1)) \pi(1)(dx) \end{array} & \begin{array}{c} \int_0^1 r(x,1) p(2|x,1) \\ \times \pi(1)(dx) \end{array} & \cdots & \begin{array}{c} \int_0^1 r(x,1) p(N|x,1) \\ \times \pi(1)(dx) \end{array} \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \begin{array}{c} \int_0^1 r(x,2) p(0|x,2) \\ \times \pi(2)(dx) \end{array} & \begin{array}{c} \int_0^1 r(x,2) p(1|x,2) \\ \times \pi(2)(dx) \end{array} & 0 & \begin{array}{c} \pi(2)(P) \\ + \int_0^1 (1 - r(x,2)) \pi(2)(dx) \end{array} & \cdots & \begin{array}{c} \int_0^1 r(x,1) p(N|x,1) \\ \times \pi(1)(dx) \end{array} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

The first row represents the distribution over the military status quo in the next period once the ruler and his subjects have been defeated. The second row represents the distribution over the military status quo in the next period so long as the war is on-going. The third row represents the distribution over the military status quo in the next period once peace has broken out at military status quo 1. This pattern repeats. Given σ and this transition probability, we can write the continuation values in closed form as vectors in the following way: $V_i(\sigma) = (1 - \delta) u_i(\sigma) + \delta P(\sigma) V_i(\sigma) = (1 - \delta)(I - \delta P(\sigma))^{-1} u_i(\sigma)$ where $u_i(\sigma)$

is the vector of expected stage utilities: $u_i(\sigma) = \Big(0, \int_0^1 \lambda_i(1-c(x,\lambda_i))\pi(1)(dx), \lambda_i + \lambda_i\gamma_S(1), \ \ldots, \ \int_0^1 \lambda_i($
$\lambda_i \Big(0, \int_0^1 (1-c(x,\lambda_i))\pi(1)(dx), 1+\gamma_S(1), \ldots, \int_0^1 (1-c(x,\lambda_i))\pi(N-1)(dx), 1+\gamma_S(N-1), 1+\gamma_S(N)\Big)$
$\lambda_i u(\sigma,\lambda_i)$. It is straightforward to show that the inverse in the expression for the continuation values exists and so I omit the proof. Thus for profile σ, we can write the continuation value $V_i(k|\sigma)$ to the subjects as the k^{th} entry of: $\lambda_i(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_i)$.

Lemma 6.2. *Fix a strategy profile σ and assume Correlated Share of Costs holds. Then for any $k \in K\backslash\{0,N\}$ $\lambda_i = \lambda_j$ implies that $A_i(k,\sigma) = A_j(k,\sigma)$ and for all $i,j \in S$ either:*

1. *$\lambda_i < \lambda_j$ implies that $A_j(k,\sigma) \subseteq A_i(k,\sigma)$; $1 \notin A_j(k,\sigma)$ implies that $\overline{x}_i(k,\pi) > \overline{x}_j(k,\pi)$ or*

2. *$\lambda_i < \lambda_j$ implies that $A_i(k,\sigma) \subseteq A_j(k,\sigma)$; $1 \notin A_i(k,\sigma)$ implies that $\overline{x}_i(k,\pi) < \overline{x}_j(k,\pi)$.*

Proof. Take any $i,j \in S$ such that $\lambda_i \leq \lambda_j$ and any $k \in K\backslash\{0,N\}$, and suppose that Assumption 1, Part (1) holds. Then I claim that part (1) above holds, i.e. that $A_j(k,\sigma) \subseteq A_i(k,\sigma)$ and $1 \notin A_j(k,\sigma)$ implies that $\overline{x}_j(k,\pi) > \overline{x}_i(k,\pi)$. If $A_j(k,\sigma) = \emptyset$ then we are done, so suppose that $A_j(k,\sigma) \neq \emptyset$. Then there exists $x \in [0,1]$ such that $V_j^{P,r}(k) \leq W_j^2(x,k|\sigma)$. We want to show that this implies that $V_i^{P,r}(k) \leq W_i^2(x,k|\sigma)$. At this x we have: $((1-\delta)\eta + \delta)\{\lambda_j(1+\gamma_S(k))\} \leq (1-\delta)\lambda_j(1-c(x,\lambda_j)) + \delta\sum_{k'} p(k'|x,k)V_j(k'|\sigma)$. Since we have fixed a stationary profile σ we can compute $V_j(k|\sigma)$ as:

$$\begin{aligned}
V_j(0|\sigma) &= \lambda_j(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_1 = 0\\
V_j(1|\sigma) &= \lambda_j(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_2\\
&\vdots\\
V_j(N|\sigma) &= \lambda_j(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_{2N-2} = \lambda_j + \lambda_j\gamma_S(N)
\end{aligned}$$

where $\lambda_j(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_k$ is the k^{th} entry of the vector of continuation values, computed as above. Thus we have: $((1-\delta)\eta+\delta)\{\lambda_j + \lambda_j\gamma_S(k)\} \leq (1-\delta)\lambda_j(1-c(x,\lambda_j)) + \delta\sum_{k'} p(k'|x,k)\lambda_j(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_{k'}$ which implies that: $((1-\delta)\eta + \delta)\{1+\gamma_S(k)\} \leq (1-\delta)(1-c(x,\lambda_j)) + \delta\sum_{k'} p(k'|x,k)(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_{k'}$. Given this, if $\lambda_i = \lambda_j$ then obviously $A_i(k,\sigma) = A_j(k,\sigma)$. Assume that $\lambda_i < \lambda_j$. If assumption

1, part (1) holds, then $\lambda_i < \lambda_j$ implies that $c(x,\lambda_i) < c(x,\lambda_j)$, and so $1-c(x,\lambda_i) > 1-c(x,\lambda_j)$. This also implies that $(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_i)_k \geq (1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_j)_k$ for all k. Thus: $\lambda_i((1-\delta)\eta+\delta)\{1+\gamma_S(k)\} < (1-\delta)\lambda_i(1-c(x,\lambda_i))+\delta\sum_{k'} p(k'|x,k)\lambda_i(1-\delta)(I-\delta P(\sigma))^{-1}u(\sigma,\lambda_i)_{k'}$ and so: $V_i^{P,r}(k) < (1-\delta)\lambda_i(1-c(x,\lambda_i))+\delta\sum_{k'} p(k'|x,k)V_i(k'|\sigma) = W_i^2(x,k|\sigma)$. But then $x \in A_i(k,\sigma)$, and so $A_j(k,\sigma) \subseteq A_i(k,\sigma)$ as required. Next suppose that $1 \notin A_j(k,\sigma)$. Then by continuity of $W_j^2(x,k|\sigma)$, $\overline{x}_i(k,\pi) > \overline{x}_j(k,\pi)$. By exactly analogous argument, we can establish that if Assumption 1, Part (2) holds, then Part (2) of the Lemma holds.

□

Lemma 6.3. *For all $k \in K\backslash\{0,K\}$, $A(k,\sigma)$ is convex valued.*

Proof. Take any $k \in K\backslash\{0,N\}$. If $A(k,\sigma)=\emptyset$ then the claim is vacuously satisfied, so suppose that $A(k,\sigma) \neq \emptyset$. If $|A(k,\sigma)|=1$, then we are done, so suppose that $|A(k,\sigma)| > 1$. Take any x_1 and $x_2 \in A(k,\sigma)$ such that $x_1 < x_2$. Then for $x \in (x_1,x_2)$ we want to show that $x \in A(k,\sigma)$. By Lemma 6.2, we can relabel S such that $A_1(x,k) \subseteq A_2(k,\sigma) \subseteq \ldots \subseteq A_s(k,\sigma)$. Let $i(x_2)=\min\{i \in S | x_2 \in A_i(k,\sigma)\}$. Similarly, let $i(x_1)=\min\{i \in S | x_1 \in A_i(k,\sigma)\}$. Without loss of generality, suppose that $i(x_2) > i(x_1)$. Then by Lemma 6.2 and our ordering, it follows that for all $i > i(x_2)$, $x_1 \in A_i(k,\sigma)$. Let $C_2=\{i(x_2),i(x_2)+1,\ldots,s\}$. By Lemma 6.1 $A_{C_2}(k,\sigma)$ is convex. Therefore, $x_2 \in A_{C_2}(k,\sigma)$ and $x_1 \in A_{C_2}(k,\sigma)$ imply that $x \in A_{C_2}(k,\sigma)$. Finally, monotonicity implies that $C_2 \in \mathcal{D}$, and so $x \in A(k,\sigma)$.

□

It follows immediately from this result and strict concavity in the transition probabilities that the ruler R will have a *unique* optimal level of mobilization x at any $k \in K\backslash\{0,K\}$. Next I will redefine the acceptance sets of the subjects as $A(k,\omega)$ in the following way. Let $\pi=\{\pi(1),\pi(2),\ldots,\pi(n-1)\} \in \Delta\left([0,1]\bigcup\{P\}\right)^{N-1}$ denote the vector of mixtures, one for each military status quo in which decisions are made. Denote by V_i the vector $V_i=\{V_i(0),V_i(1),\ldots,V_i(N)\}$, the vector of continuation values for i, one for each state, $V_i(k) \in [0,\gamma_i(N)], k \in K$. Next, let $\omega=(V_R,V_1,\ldots,V_s,\pi) \in [0,\gamma_R(N)]^{N+1} \times [0,\gamma_1(N)]^{N+1} \times \ldots \times [0,\gamma_s(N)]^{N+1} \times (\Delta(X\bigcup\{P\}))^{N-1} = \Omega$. Define $A_i(k,\omega)=\{x \in$

$X|V_i^{P,r}(k) \leq W_i^2(x,k \mid \omega)\}$, a correspondence mapping from the state and strategy profile to the set of mobilization offers that will be acceptable to subject i at k given the remaining choices for peace and mobilization, $A_i : \Omega \times K \rightrightarrows X$, not necessarily non-empty. Finally, define $A(k,\omega) = \bigcup_{C\in\mathcal{D}} \left(\bigcap_{i\in C} A_i(k,\omega)\right)$. I will prove first that it is upper and lower hemi-continuous (UHC and LHC respectively) as a correspondence at most points of the strategy space and second that a Markov Perfect Equilibrium exists.

Lemma 6.4. *$A(k,\omega)$ is continuous as a correspondence at all ω such that there exists $C \in \mathcal{D}$ such that $|A_C(k,\sigma)| > 1$.*

It follows immediately from Lemma 6.4 that $A(k,\omega)$ is continuous at all ω where $|A(k,\omega)| > 1$.

Theorem 3.1.

Proof. We will use Glicksberg's fixed point theorem to establish that there must exist a fixed point ω^*. Let: $W_i^2(x,k \mid \omega) = (1-\delta)u_i(x) + \delta \sum_{\widehat{k}\in K} p(\widehat{k} \mid x,k)V_i(\widehat{k})$, a continuous function of ω, $i \in \{R\} \bigcap S$. Define a function $a(k,x \mid \omega)$ so that:

$$a(k,x \mid \omega) = \begin{cases} 1 & \text{if } x \in A(k,\omega) \text{ and } \mid A(k,\omega) \mid > 1 \\ 0 & \text{if } x \notin A(k,\omega) \\ a^* & \text{if } x \in A(k,\omega) \text{ and } \mid A(k,\omega) \mid = 1 \end{cases}$$

where $a^* = \min\left\{1, \frac{V_R^P(k)}{W_R^2(x,k|\omega)}\right\}$. This a^* will be the smaller of the probability of acceptance that leaves the ruler exactly indifferent between mobilizing and making peace, or 1.[37] Let $G(k,\omega) = \text{argmax}_{x\in A(k,\omega)}\{W_R^1(x,k \mid \omega)\}$ where $W_R^1(x,k \mid \omega) = a(k,x \mid \omega)W_R^2(x,k \mid \omega)$, and denote $x \in G(k,\omega)$ by x_g. Next, consider the correspondence $H : K \times \Omega \rightrightarrows X \bigcup \{P\}$,

[37] We may assign such an a^* because the fact that $|A(k,\omega)| = 1$ by assumption implies that there exists a winning coalition with some members who are all indifferent between accepting or rejecting. Therefore, allow those who strictly favor accepting to vote yes with certainty, and fix the acceptance probabilities of the indifferent subjects to achieve the required a^*.

where:

$$H(k,\omega) = \begin{cases} \{P\} & \text{if } V_R^P(k) > W_R^1(x_g, k \mid \omega) \\ \{G(k,\omega), P\} & \text{if } V_R^P(k) = W_R^1(x_g, k \mid \omega) \\ \{G(k,\omega)\} & \text{if } V_R^P(k) < W_R^1(x_g, k \mid \omega) \end{cases}$$

Note that as a correspondence, $H(k,\omega)$ will be non-empty and compact though not necessarily convex-valued, and $G(k,\omega)$ will be single-valued wherever $A(k,\omega) \neq \emptyset$.

We want to see that $H(k,\omega)$ is UHC. There are two cases to consider. 1) Suppose that $|A(k,\omega)| = 1$ and denote an arbitrary point where $|A(k,\omega)| = 1$ as ω_d, and $x \in A(k,\omega_d)$ as x_d. Consider any sequences $\omega^n \to \omega_d$ and $y^n \to y$. Then if for all n, $y^n \in H(k,\omega^n)$, we want to see that $y \in H(k,\omega_d)$. Suppose not. Then there exists $\widehat{y}$ so that $\widehat{y}$ is strictly better than y. We have two subcases to deal with. *i*) It could be that $\min\left\{1, \frac{V_R^P(k)}{W_R^2(x,k|\omega)}\right\} = \frac{V_R^P(k)}{W_R^2(x,k|\omega)}$ which implies that $W_R^1(x_d, k \mid \omega_d) = V_R^P(k)$. Thus $H(k,\omega_d) = \{x_d, P\}$. We then have a further two cases to consider: *a*) $\widehat{y} = P$. But then this implies that P is strictly better than P, a contradiction. *b*) $\widehat{y} = x \neq x_d$. But then it must be that $x, x_d \in A(k,\omega_d)$, which implies that $|A(k,\omega_d)| > 1$, a contradiction. *ii*) It could be that $\frac{V_R^P(k)}{W_R^2(x,k|\omega)} > 1$, which implies that $H(k,\omega) = \{P\}$, and UHC is trivial. Thus, $H(k,\omega)$ is UHC at ω_d.

2) Consider ω so that $|A(k,\omega)| \neq 1$. Then it could be that $A(k,\omega) = \emptyset$, in which case $H(k,\omega) = \{P\}$ and UHC is trivial, as before. Alternatively, it could be that $|A(k,\omega)| > 1$. By Lemma **??**, $A(k,\omega)$ is continuous as a correspondence at ω. It then follows from the Theorem of the Maximum that $G(k,\omega)$ is UHC at ω, which implies that $H(k,\omega)$ is UHC at ω.

Let $\Delta H(\omega) = \Delta H(1,\omega) \times, \ldots, \times \Delta H(N-1,\omega)$, where $\Delta H(k,\omega)$ is the set of all Borel probability measures on $H(k,\omega)$, $k \in K \backslash \{0, N\}$, and define a function $V_i(\omega) = V_i(0,\omega) \times \ldots \times V_i(N,\omega)$. For $\pi \in \Delta H(\omega)$, compute $V_i(k,\omega)$ according to: $V_i(k,\omega) = \pi(k)(P)V_i^P(k) + \int_X W_i^1(x,k \mid \omega)\pi(k)(dx)$ and note that $V_i(\omega)$ will be convex valued, since $G(k,\omega)$ is single-valued.

Finally, define $\mathcal{B} : \Omega \rightrightarrows \Omega$ as $\mathcal{B}(\omega) = \{V_R(\omega)\} \times \{V_i(\omega)\}_{i \in S} \times \Delta H(\omega)$. By Theorem 17.13 in Aliprantis and Border (2006), it follows that $\Delta H(\omega)$ is UHC. Thus, $\mathcal{B}(\omega)$ is a non-empty, compact, and convex valued UHC correspondence. Let $A(\omega) = A(1,\omega)\times,\ldots,\times A(N-1,\omega)$. By Glicksberg (1952) Theorem 1, $\mathcal{B}(\omega)$ has a fixed point, ω^*, so that $\omega^* = \mathcal{B}(\omega^*)$. This, along with $A(\omega^*)$, constitutes the continuation values and strategies of a Markov perfect equilibrium.

To see the second claim, suppose not. Then there exists some $k \in K\backslash\{0,N\}$ and some set of equilibrium strategies where, at k, $\pi^*(k)(P) < 1$ and $a^*(x,k) = 0$. The given set of strategies, σ^*, implies that there exists $x \in G(k,\sigma^*)$ so that $V_R(k \mid \sigma^*) = 0$. Then it must be that, since R is willing to mix across $x \in G(k,\sigma^*)$, for all such x, $V_R(k \mid \sigma) = 0$. Consider the deviation to $\widehat{\pi}(k)(P) = 1$. Then: $V_R(k \mid \sigma^*, \widehat{\pi}(k)(P)) \geq b > 0 = V_R(k \mid \sigma^*)$ which implies that σ^* is not an equilibrium. In a pure-strategy MPE, $a^*(x,k)$ cannot be 0, so it must be 1.

Finally, to see that there cannot be mixing on the equilibrium path if there is more than one alternative that would be accepted with positive probability, fix a Markov Perfect Equilibrium σ^*. Take any $k \in K$ such that for some x $\pi(k)(x) = 1$ (WLOG by Lemma 6.1). There are three possibilities. (a) it could be that $x \in \text{int}\{A(k,\sigma^*)\}$. Then there exists $C \in \mathcal{D}$ such that $W_i^2(x,k|\sigma^*) > V_i^{P,r}(k)$ for all $i \in C$, which implies that $a(k,\sigma^*) = 1$. (b) it could be that $x = \max\{A(k,\sigma^*)\}$. Then for any $a(k,\sigma^*) < 1$, by Lemma 6.1 there exists $x' < x$ such that $W_R^2(x',k|\sigma^*) > W_R^2(x,k|\sigma^*)$, implying that σ^* is not an equilibrium. (c) it could be that $x = \min\{A(k,\sigma^*)\}$. Then for any $a(k,\sigma^*) < 1$, by Lemma 6.1 there exists $x' > x$ such that $W_R^2(x',k|\sigma^*) > W_R^2(x,k|\sigma^*)$, again implying that σ^* is not an equilibrium. □

Lemma 6.5. *For any $k \in K\backslash\{0,N\}$ such that $A(k,\sigma) \neq \emptyset$, let $M_k = \{i \in S|\overline{x}(k,\pi) = \overline{x}_i(k,\pi)\}$. Then for any $k,k' \in K\backslash\{0,N\}$ such that $A(k,\sigma) \neq \emptyset$ and $A(k',\sigma) \neq \emptyset$, $M_k = M_{k'}$.*

Proof. The proof is straightforward and so is omitted. □

Lemma 6.6. *Fix π^*. Then if (1) Correlated Share of Costs and (2) Bounded Investment hold, then for all $k \in K\backslash K(\pi^*)$, $A_i(k,\sigma) \neq \emptyset$ implies that $V_{\underline{i}}(k|\sigma^*) = V_{\underline{i}}^{P,r}(k)$.*

Proof. This follows from the assumptions of the lemma and the definitions of π^* and $\overline{x}(k,\sigma)$. □

Lemma 3.2.

Proof. Fix the strategy π^* and take any $k \in K\backslash\{0, N-1, N\}$. I claim that for $\underline{i}(\mathcal{D})$, it must be that $A_{\underline{i}(\mathcal{D})}(k+1,\sigma) \subseteq A_{\underline{i}(\mathcal{D})}(k,\sigma)$. If $A_{\underline{i}(\mathcal{D})}(k+1,\sigma) = \emptyset$, then the claim is clearly true, so suppose not. Take any $x \in A_{\underline{i}(\mathcal{D})}(k+1,\sigma)$. Then by definition, it must be that $x \leq C_{\underline{i}(\mathcal{D})}(k+1,\sigma,x)$. Therefore, a sufficient condition for the claim to be true would be that $C_{\underline{i}(\mathcal{D})}(k+1,\sigma,x) < C_{\underline{i}(\mathcal{D})}(k,\sigma,x)$. Under strategy π^*, by Lemma 6.6, we require:
$c^{-1}_{\lambda_{\underline{i}(\mathcal{D})}}\left(1+\frac{1}{\lambda_{\underline{i}(\mathcal{D})}(1-\delta)}\left\{\delta\sum_{k'\in K}p(k'|x,k+1)V_{\underline{i}(\mathcal{D})}(k'|\sigma)-V^{P,r}_{\underline{i}(\mathcal{D})}(k+1)\right\}\right)$
$< c^{-1}_{\lambda_{\underline{i}(\mathcal{D})}}\left(1+\frac{1}{\lambda_{\underline{i}(\mathcal{D})}(1-\delta)}\left\{\delta\sum_{k'\in K}p(k'|x,k)V_{\underline{i}(\mathcal{D})}(k'|\sigma)-V^{P,r}_{\underline{i}(\mathcal{D})}(k)\right\}\right)$ or:

$$\gamma_S(k+1)-\gamma_S(k) > \delta\sum_{k'\in K}\frac{p(k'|x,k+1)-p(k'|x,k)}{\lambda_{\underline{i}(\mathcal{D})}((1-\delta)\eta+\delta)} \times \begin{cases} V^{P,r}_{\underline{i}(\mathcal{D})}(k') & \text{if } k'<k^* \\ V^{P}_{\underline{i}(\mathcal{D})}(k') & \text{if } k'\geq k^* \end{cases}.$$

There are two possibilities. (i) k has declining momentum. Then given the strategy π^*, $V_{\underline{i}(\mathcal{D})}(k|\sigma)$ is strictly increasing in k. By declining momentum, it follows that the right hand side is strictly less than 0. By strict monotonicity of $\gamma_S(k)$, the left hand side is strictly positive. Therefore, $C_{\underline{i}(\mathcal{D})}(k+1,\sigma,x) < C_{\underline{i}(\mathcal{D})}(k,\sigma,x)$, and so $A_{\underline{i}(\mathcal{D})}(k+1,\sigma) \subseteq A_{\underline{i}(\mathcal{D})}(k,\sigma)$. (ii) k has increasing momentum. Then, as above, the left hand side is strictly positive. By increasing momentum and π^*, the right hand side is also at least 0. Since $V_{\underline{i}(\mathcal{D})}(\cdot)$ is necessarily finite, for any positive value $\gamma_S(k+1)-\gamma_S(k)$, we can always pick $p(\cdot)$ so that the right hand side is closer to 0 than this. Therefore, if momentum at k is sufficiently small, $C_{\underline{i}(\mathcal{D})}(k+1,\sigma,x) < C_{\underline{i}(\mathcal{D})}(k,\sigma,x)$, and so $A_{\underline{i}(\mathcal{D})}(k+1,\sigma) \subseteq A_{\underline{i}(\mathcal{D})}(k,\sigma)$.

Next, I claim that $\overline{x}(k,\sigma) > \overline{x}(k+1,\sigma)$. Suppose not. Then $\overline{x}(k,\sigma) \leq \overline{x}(k+1,\sigma)$. Note that by definition $\overline{x}(k,\sigma) = \overline{x}_{\underline{i}(\mathcal{D})}(k,\sigma)$. Thus, by the previous argument, it cannot be that $\overline{x}(k,\sigma) < \overline{x}(k+1,\sigma)$, since if it were $A_{\underline{i}(\mathcal{D})}(k+1,\sigma) \nsubseteq A_{\underline{i}(\mathcal{D})}(k,\sigma)$. Could it be that $\overline{x}(k,\sigma) = \overline{x}(k+1,\sigma)$? Observe that $\overline{x}(k,\sigma) = \overline{x}(k+1,\sigma)$ implies that:

$$\gamma_S(k+1)-\gamma_S(k) = \delta\sum_{k'\in K}\frac{\{p(k'|\overline{x}(k,\sigma),k+1)-p(k'|\overline{x}(k,\sigma),k)\}}{\lambda_{\underline{i}(\mathcal{D})}((1-\delta)\eta+\delta)} \times \begin{cases} V^{P,r}_{\underline{i}(\mathcal{D})}(k') & \text{if } k'<k^* \\ V^{P}_{\underline{i}(\mathcal{D})}(k') & \text{if } k'\geq k^* \end{cases}.$$

But this is impossible by the argument above. Therefore, $\overline{x}(k,\sigma) > \overline{x}(k+1,\sigma)$, as required. $\square$

Lemma 3.3.

Proof. This follows immediately from the definition of $c(x,\lambda_i)$ and the fact that for any $k \in K \backslash \{0,N\}$ and $i \in S$, $V_i^{P,r}(k) \leq V_i^P(k)$. $\square$

Next, as in the body of the paper, we may define the profile π^* as: $\pi^*(k)(P) = 1$ if $k \geq k^*$ and $\pi^*(k)(\overline{x}(k,\pi^*)) = 1$ if $k < k^*$. This will be a pure-strategy profile. Under this profile and given Theorem 3.1, we may rewrite the transition probability $P(\sigma)$ as an $N+1 \times N+1$ matrix where each row k less than k^* (except row 1) has the column k' entry as $p(k'|\overline{x}(k,\sigma),k)$ and each row k larger or equal k^* (and also row 1) has 1 on the diagonal and 0 off the diagonal. Given this transition probability, we may write the continuation values for the ruler in closed form, as above: $V_R(k|\sigma^*) = (1-\delta)(I-\delta P(\sigma^*))^{-1}u_R(\sigma^*)_k$, where $u_R(\sigma^*)$ is the vector of stage payoffs to the ruler. The matrix $(1-\delta)(I-\delta P(\sigma^*))^{-1}$ is itself a transition probability whose rows satisfy first order stochastic dominance in x.

Proposition 3.4.

Proof. Let σ^* denote the strategy profile in which (i) $a_i^*(x,k)$ is as specified in (7); (ii) $K(\pi^*) = \{k^*, k^*+1, \ldots, N\}$ for $k^* \in K$; (iii) $\pi^*(k)(\overline{x}(k,\sigma^*)) = 1$ for all $k \in K \backslash K(\pi^*)$. Assume that the hypotheses of the proposition are satisfied. By Definition 1, the formulation of $C_i(k,\sigma^*,x)$, and the strategy profile σ^*, (i) is clearly a best response for the subjects to σ^*. Observe that given strategy profile σ^* and the hyptheses of the proposition, the hypotheses of Lemma 3.2 are satisfied. The proof will proceed in three steps, and conclude with an argument that the equilibrium is unique.

Claim (1). Under strategy profile σ^*, for any k $x' > x$ implies that $W_R^2(x',k|\sigma^*) > W_R^2(x,k|\sigma)$. This follows immediately from the discussion above and that under strategy profile σ^*, $u_R(\sigma^*) = \{0, b, \ldots, b, V_R^P(k^*, \ldots, V_R^P(N))\}$ which is non-constant and weakly increasing in k. By the one stage deviation principle (iii) is a best response to σ^*.

Claim (**2**). Under σ^*, for all k such that $A(k,\sigma^*) \neq \emptyset$ we have $W_R^1(\overline{x}(k,\sigma^*),k|\sigma^*) > W_R^1(\overline{x}(k+1,\sigma^*),k+1|\sigma^*)$. For all k such that $A(k,\sigma^*) = \emptyset$ we have $W_R^1(x,k|\sigma^*) = W_R^1(x',k+1|\sigma^*)$ for any $x,x' \in [0,1]$. By the hypotheses of the proposition, the conditions of Lemma 3.2 are satisfied. There are two cases to consider. (i) suppose that $A(k,\sigma^*) = \emptyset$. Then by Lemma 3.2, $A(k+1,\sigma^*) = \emptyset$. It follows that for all $x,x' \in [0,1]$, $W_R^1(x,k|\sigma^*) = W_R^1(x',k+1|\sigma^*) = 0$. (ii) suppose that $A(k,\sigma^*) \neq \emptyset$. Then we have two subcases. (a) $A(k+1,\sigma^*) = \emptyset$. Then as above, $W_R^1(\overline{x}(k,\sigma^*),k|\sigma^*) > 0 = W_R^1(x,k+1|\sigma^*)$ for any $x \in [0,1]$. (b) suppose that $A(k+1,\sigma^*) \neq \emptyset$. Then by Lemma 3.2, $\overline{x}(k,\sigma^*) > \overline{x}(k+1,\sigma^*)$. If $W_R^2(\overline{x}(k,\sigma^*),k|\sigma^*) > W_R^2(\overline{x}(k+1,\sigma^*),k+1|\sigma^*)$ then it follows that $W_R^1(\overline{x}(k,\sigma^*),k|\sigma^*) > W_R^1(\overline{x}(k+1,\sigma^*),k+1|\sigma^*)$ and we are done, so suppose not. Then $W_R^2(\overline{x}(k,\sigma^*),k|\sigma^*) \leq W_R^2(\overline{x}(k+1,\sigma^*),k+1|\sigma^*)$. Then by Bounded Momentum, continuity, and the Intermediate Value Theorem there exists $x < \overline{x}(k,\sigma^*)$ such that $W_R^2(\overline{x}(k,\sigma^*),k|\sigma^*) = W_R^2(x,k+1|\sigma^*)$. Observe that for any $\varepsilon > 0$ we can choose $p(\cdot)$ to have sufficiently little dependence on k, i.e. sufficiently small increasing momentum in war, for all k' so that $\overline{x}(k,k^*) - x < \varepsilon$. Therefore, by strict monotonicity of $\gamma_S(\cdot)$ and an argument similar to that for Lemma 3.2 we can choose $p(\cdot)$ so that $\overline{x}(k,k^*) - x < \overline{x}(k,k^*) - \overline{x}(k+1,k^*)$. But then we have $x > \overline{x}(k+1,k^*)$. By claim (1) we have $W_R^1(\overline{x}(k,k^*),k|\sigma^*) = W_R^1(x,k+1|\sigma^*) > W_R^1(\overline{x}(k+1,k^*),k+1|\sigma^*)$, as required.

Claim (**3**). If there exists k such that for all $x \in A(k,\sigma^*)$ $V_R^P(k) \geq W_R^2(x,k|\sigma^*)$ then for all $k' > k$, for all $x \in A(k',\sigma^*)$ $V_R^P(k') > W_R^2(x,k'|\sigma^*)$. If there exists k such that there exists $x \in A(k,\sigma^*)$ with $V_R^P(k) \leq W_R^2(x,k|\sigma^*)$, then for all $k' < k$, there exists $x \in A(k',\sigma)$ with $V_R^P(k') < W_R^2(x,k'|\sigma^*)$. This follows immediately from claim (2) and strict monotonicity of $\gamma_R(\cdot)$ in k.

We now establish the existence of $k^* \in K$ that satisfies (8). For any $k \in K\backslash\{0\}$ and any $k' \in K\backslash\{0,N\}$ we may compute a collection of mobilization levels that would be accepted with positive probability, $\{A(1,\sigma^*),\ldots,A(N-1,\sigma^*)\}$. For any k' such that $A(k',\sigma^*) \neq \emptyset$ we may compute $\overline{x}(k',k)$ according to (6). By Lemma 3.2, this yields a vector of largest acceptable levels of mobilization: $\{\overline{x}(1,k),\ldots,\overline{x}(\overline{k}(k),k),\emptyset,\ldots,\emptyset\}$ where $\overline{k}(k) = \max\{k \in K|A(k,\sigma^*) \neq \emptyset\}$. Now fix $k-1 = \max\{k' \in K\backslash\{0\}|W_R^2(\overline{x}(k',k'),k'|\sigma^*) \geq V_R^P(k')\}$;

clearly $k-1 \leq \bar{k}(k-1)-1$. There are two cases. (i) it could be that $|A(k-1,\sigma^*)| = 1$. Then by the definition of $a^*_{\underline{i}(\mathcal{D})}$ it must be that $W^2_R(\overline{x}(k-1,k-1),k-1|\sigma^*) = V^P_R(k-1)$. Then by claim (3) for all $k' > k-1$ $V^P_R(k') < W^2_R(x,k|\sigma^*)$ for all $x \in [0,1]$ and for all $k' < k-1$ $V^P_R(k') < W^2_R(\overline{x}(k',k-1),k-1|\sigma^*)$. But then $k-1$ satisfies (8) and so $k-1 = k^*$. (ii) it could be that $|A(k-1,\sigma^*)| > 1$. Then if $W^2_R(\overline{x}(k-1,k-1),k-1|\sigma^*) = V^P_R(k-1)$ we are done by the same argument, so assume that $W^2_R(\overline{x}(k-1,k-1),k-1|\sigma^*) > V^P_R(k-1)$. Then by the hypotheses of the proposition and the definition of $k-1$ $W^2_R(\overline{x}(k-1,k),k-1|\sigma^*) \geq V^P_R(k-1)$ while $W^2_R(\overline{x}(k,k),k|\sigma^*) < V^P_R(k)$. By claim (3) for all $k' > k$ $W^2_R(x,k'|\sigma^*) < V^P_R(k')$ and for all $k' > k-1$ $W^2_R(\overline{x}(k',k),k|\sigma^*) > V^P_R(k')$. Then k satisfies (8) and so $k = k^*$. By the one stage deviation principle, this, in conjunction with claim (1), establishes that π^* is a best response to a^*_i. Therefore σ^* is a Markov Perfect Equilibrium.

To see that there is only one equilibrium of this form, suppose not. Then there exist at least two equilibria satisfying the conditions of the proposition, call them σ_1 and σ_2, with $\sigma_1 \neq \sigma_2$. Observe that the equilibrium strategies are uniquely determined by the choice of k^*. Thus, if $\sigma_1 \neq \sigma_2$, then it must be that $k^*_1 \neq k^*_2$. Assume without loss of generality that $k^*_1 < k^*_2$. Now take any $k \in \{k^*_1,\ldots,k^*_2-1\}$ and note that by Definition 1 it must be that $W^2_R(\overline{x}(k,\sigma_2),k|\sigma_2) \geq V^P_R(k) \geq W^2_R(\overline{x}(k,\sigma_1),k|\sigma_1)$. By claim (1), it must be that $\overline{x}(k,\sigma_2) > \overline{x}(k,\sigma_1)$. Then by Lemma 6.6 $\overline{x}(k,\sigma_1)$ implicitly satisfies: $\overline{x}(k,\sigma_1) \geq c^{-1}_{\lambda_{\underline{i}(\mathcal{D})}}\left(1+\frac{1}{\lambda_{\underline{i}(\mathcal{D})}(1-\delta)}\left\{\delta\sum_{k'<k^*_2} p(k'|\overline{x}(k,\sigma_1),k)V^{P,r}_{\underline{i}(\mathcal{D})}(k') \ +\sum_{k'\geq k^*_2} p(k'|\overline{x}(k,\sigma_1),k)V^{P}_{\underline{i}(\mathcal{D})}(k') - V^{P,r}_{\underline{i}(\mathcal{D})}(k)\right\}\right)$. Since $\overline{x}(k,\sigma_2) > \overline{x}(k,\sigma_1)$ by assumption, $\overline{x}(k,\sigma_2) \notin A(k,\sigma^*_2)$, a contradiction by Theorem 3.1. □

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