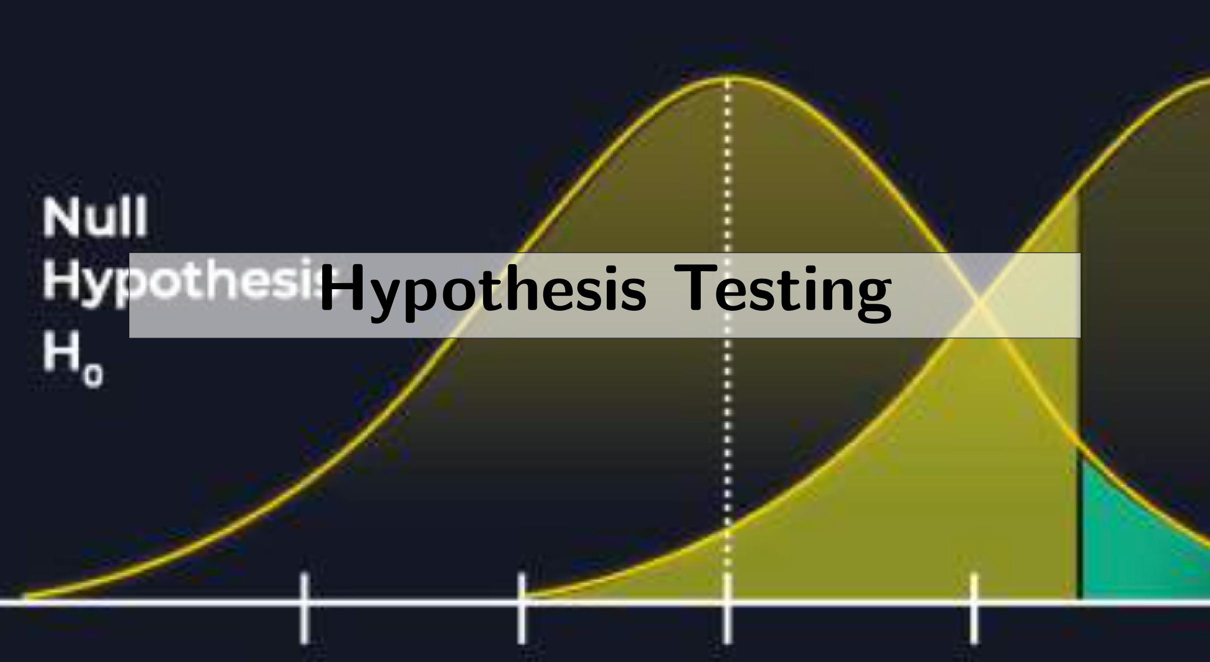
Hypothesis Testing



Today:

- Define the parts of a hypothesis test;
- Define p-values, understand what they mean, and relate them to type I errors;
- Work through an example of a hypothesis test in a political context.

Hypothesis testing is a model that helps us decide between different hypotheses using **falsification**.

Parts of a hypothesis test

1. A **Null hypothesis $\mathbf{H_0}$**;
   - Usually a claim that there is no effect or nothing of interest;
   - We will be looking to decide if the data falsifies the null hypothesis;

2. An **Alternative hypothesis $\mathbf{H_A}$**;
   - Usually a claim that there IS an effect;
   - We want to create support for this by falsifying its converse;

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3. A **test statistic**;
   - A point estimate summary of the data that we can use to decide between H_0 and H_A;
   - Observed test statistic: the actual value of the test stat we observed IRL;
   - Null distribution: sampling distribution of the test statistic assuming H_0 is true;

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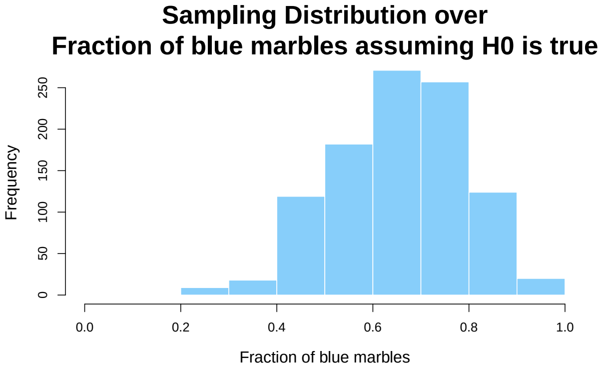
4. A **rejection criterion** (also called the significance level);
   - If we see this happen to the test statistic then we will decide we have falsified or rejected H_0;
   - Largely an arbitrary choice.

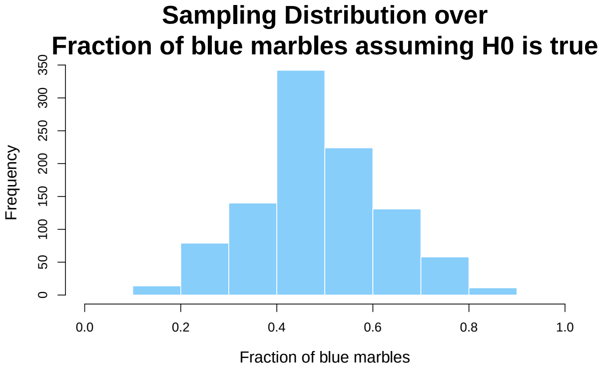
Null distribution

- **Remember the marbles in the bag?** We didn't talk about it but our test statistic was the fraction of blue marbles in the sample;
- Imagine when we did that our null hypothesis was that the fraction of blue marbles $= 66\%$. Then the sampling (null) distribution ought to look like:



Null distribution

- **Remember the marbles in the bag?** We didn't talk about it but our test statistic was the fraction of blue marbles in the sample;
- Imagine when we did that our null hypothesis was that the fraction of blue marbles $= 50\%$. Then the sampling (null) distribution ought to look like:



Null distribution

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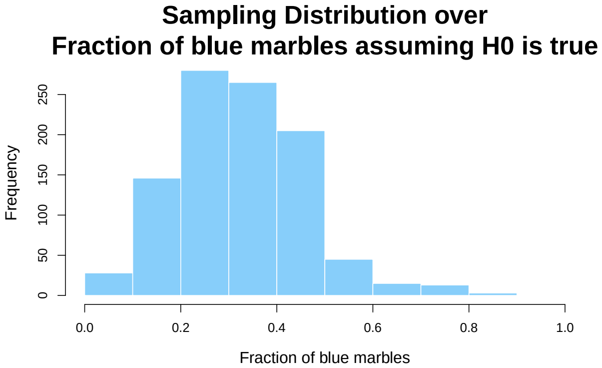
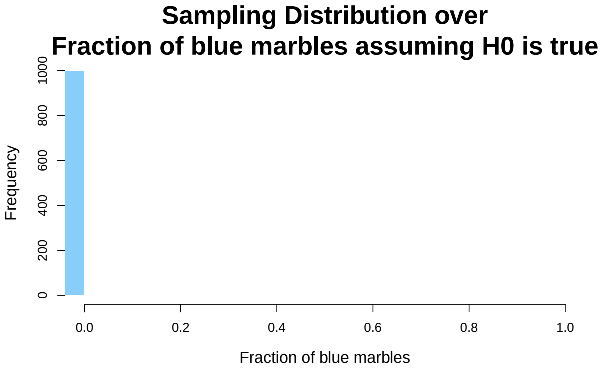


Null distribution

- **Remember the marbles in the bag?** We didn't talk about it but our test statistic was the fraction of blue marbles in the sample;
- Imagine when we did that our null hypothesis was that the fraction of blue marbles $= 0\%$. Then the sampling (null) distribution ought to look like:



Choosing the rejection criterion: Type I errors and p-values

- A **type I error** is when we reject H_0 when it is true;
  - This would mean mistakenly endorsing H_A;
  - A similar idea is that of a false positive;
  - The probability of a type I error is usually called α;

- A **p-value** is defined as the probability of getting a more extreme value than the observed test statistic given that H_0 is true;
  - Measures surprise;
  - Comes from the null distribution;
  - The lower the p-value the lower the risk of a type I error;

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  - Comes from the null distribution;
  - The lower the p-value the lower the risk of a type I error;
- Choosing a rejection criterion or significance level is equivalent to choosing a cutoff for α – by convention this is usually 0.05:
  - We reject H_0 when we are sufficiently unlikely to be making a type I error.
  - We reject H_0 when the p-value is less than our α cutoff of 0.05.

Applying this to reason about voters...

- Cavalier Johnson: a political candidate running a campaign for Mayor of Milwaukee;
- Johnson's campaign claims that he will win the election – in this case that means getting more than 50% of votes from Milwaukee voters;
- How to assess the Johnson campaign claim? Conduct an experiment:
  - **Sample** n voters from Milwaukee;
  - Record the number of voters who will vote for Johnson.

Applying this to reason about voters – let's build a hypothesis test

- Suppose we collect a sample of 15 voters and we see that 6 of them plan to vote for Johnson. Do we believe the Johnson campaign claim?

- H_0:

- H_A:

- Test statistic:

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- Suppose we collect a sample of 15 voters and we see that 6 of them plan to vote for Johnson. Do we believe the Johnson campaign claim?
- H_0: the Johnson campaign claim is true, i.e. the fraction of voters who will vote for Johnson is $\pi = 0.5$;
- H_A: the Johnson campaign claim is false, i.e. the fraction of voters who will vote for Johnson is $\pi < 0.5$;
- Test statistic:

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- H_A: the Johnson campaign claim is false, i.e. the fraction of voters who will vote for Johnson is $\pi < 0.5$;
- Test statistic: the number of Johnson voters in our sample, in this case 6;

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- Suppose we collect a sample of 15 voters and we see that 6 of them plan to vote for Johnson. Do we believe the Johnson campaign claim?

- Our hypothesis test model says we reject H_0 if:
  - The probability of a more extreme result than our observed test statistic assuming H_0 is true is less than 0.05;

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  - The probability of a more extreme result than our observed test statistic assuming H_0 is true is less than 0.05;
  - The probability of a more extreme result than our observed test statistic according to the null distribution is less than 0.05;

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  - The p-value is less than 0.05;

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  - The probability of 6 or fewer Johnson voters in our sample according to the null distribution is less than 0.05;
  - The p-value is less than 0.05;

- So what is the probability of 6 or fewer Johnson voters in our sample? It would be:

$$p(0 \text{ Johnson voters}) + p(1 \text{ Johnson voter}) + p(2 \text{ Johnson voters})$$
$$p(3 \text{ Johnson voters}) + p(4 \text{ Johnson voters}) + p(5 \text{ Johnson voters})$$
$$+ p(6 \text{ Johnson voters}).$$

Applying this to reason about voters – let's build a hypothesis test

- Where do these probabilities come from?

$$p(0 \text{ Johnson voters}) = ???$$
$$p(1 \text{ Johnson voter}) = ???$$
$$p(2 \text{ Johnson voters}) = ???$$
$$p(3 \text{ Johnson voters}) = ???$$
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Applying this to reason about voters – let's build a hypothesis test

- Where do these probabilities come from? Well, from the null distribution.

$$
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- Where do these probabilities come from? Well, from the null distribution.
- Remember the **binomial distribution** that models the probability of k successes in n trials given that each trial has probability of success π?

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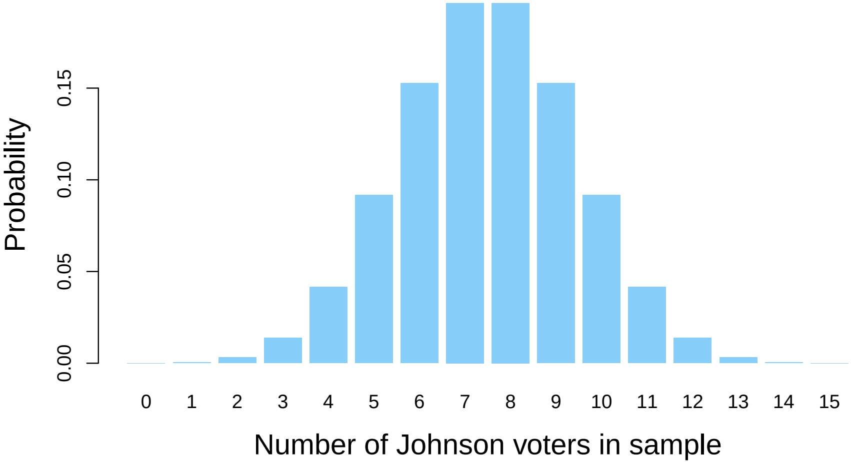
Applying this to reason about voters – let's build a hypothesis test

- Where do these probabilities come from? Well, from the null distribution.

- Remember the **binomial distribution** that models the probability of k successes in 15 trials given that each trial has probability of success $\pi = 0.5$?

$$p(0 \text{ Johnson voters}) = p(k = 0, n = 15, \pi = 0.5) \approx 0.00003$$
$$p(1 \text{ Johnson voter}) = p(k = 1, n = 15, \pi = 0.5) \approx 0.00046$$
$$p(2 \text{ Johnson voters}) = p(k = 2, n = 15, \pi = 0.5) \approx 0.00320$$
$$p(3 \text{ Johnson voters}) = p(k = 3, n = 15, \pi = 0.5) \approx 0.01389$$
$$p(4 \text{ Johnson voters}) = p(k = 4, n = 15, \pi = 0.5) \approx 0.04166$$
$$p(5 \text{ Johnson voters}) = p(k = 5, n = 15, \pi = 0.5) \approx 0.09164$$
$$p(6 \text{ Johnson voters}) = p(k = 6, n = 15, \pi = 0.5) \approx 0.15274$$

The null distribution in our hypothesis test (comes from the Binomial)



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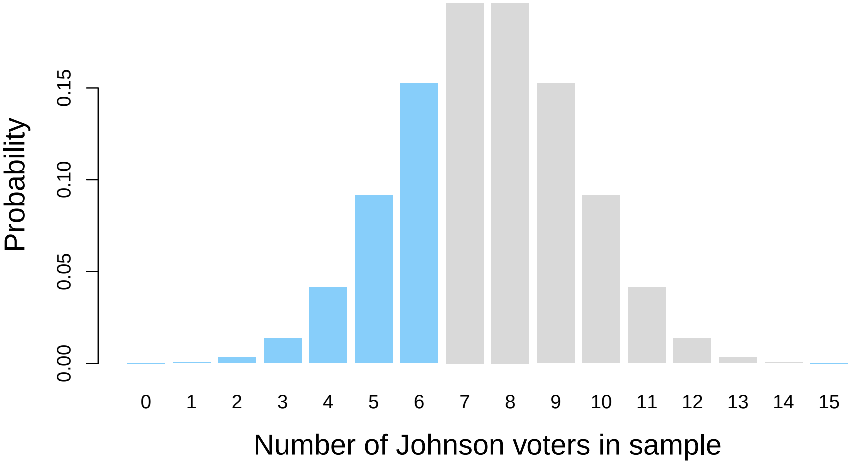
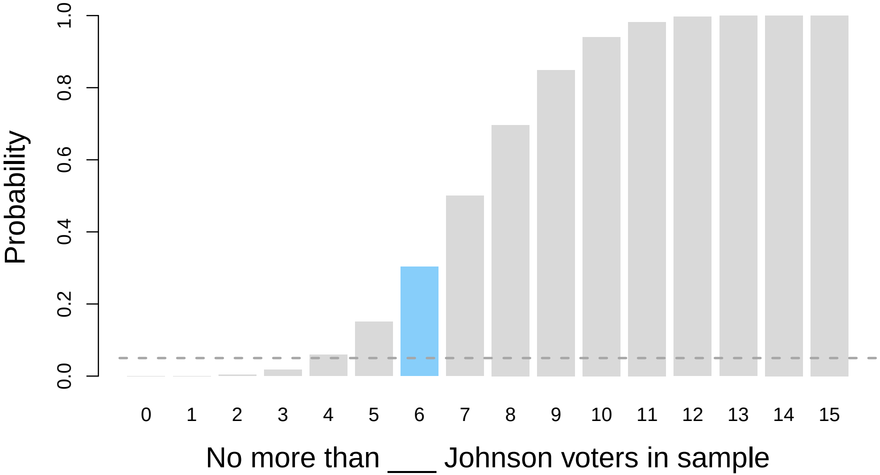


The possible *p*-values in our hypothesis test



So...

- In our Johnson voter example the p-value is $\approx 0.3 > 0.05$ meaning we fail to reject H_0 that Johnson will win;

- If we had seen $4, 5, \ldots, 15$ Johnson voters in our sample of 15 the p-value would have been greater than 0.05 and so we would have failed to reject H_0 that Johnson will win;

- If we had seen 0, 1, 2, or 3 Johnson voters in our sample of 15 the p-value would have been less than 0.05 and so we would have rejected H_0 that Johnson will win;

- Whenever the p-value < 0.05 we would reject H_0.

Why should we care?

Hypothesis testing is the key technique used to falsify theories in modern science.