Limited Dependent Variables

November 8, 2022

Objectives

- Distinguish between different dependent variable structures;
- Revisit the assumptions of OLS;
- Learn (more of) the limitations of OLS;

So far you've dealt with:

1. Model a linear relationship between dependent variable y and one or more independent variables $x_1, x_2, x_3, \ldots$:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \varepsilon;$$

2. Model a NON-linear relationship between dependent variable y and many independent variables x:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \ldots + \varepsilon;$$

3. Model relationships between independent variables:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon;$$

4. Model pseudo-experimental data using difference in differences.

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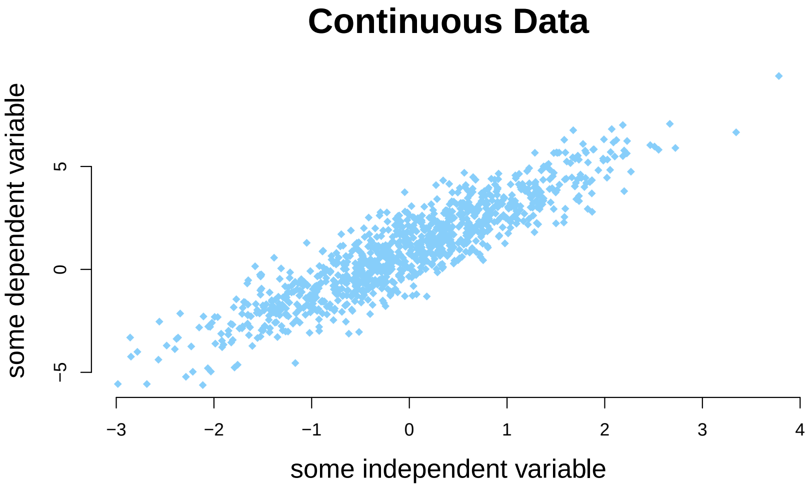

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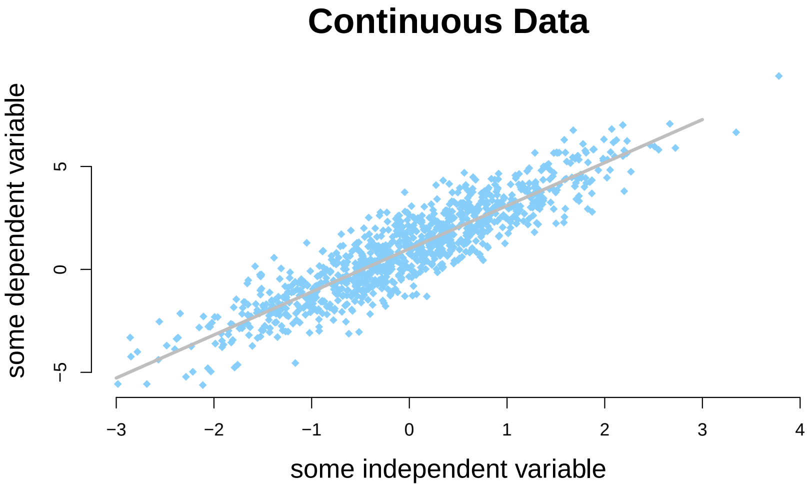
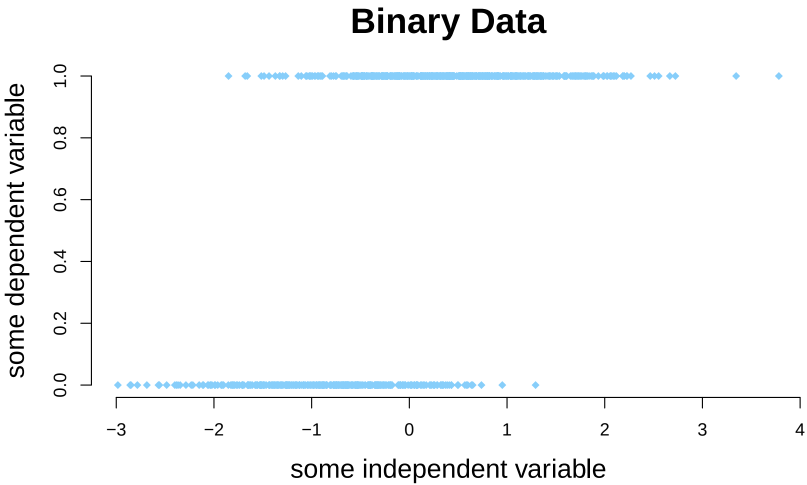


So far you've dealt with:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.9689	0.0305	31.76	0.0000
x	1.9734	0.0301	65.57	0.0000

actual relationship: $y = 1 + 2x + \varepsilon$

Ok, but what about...



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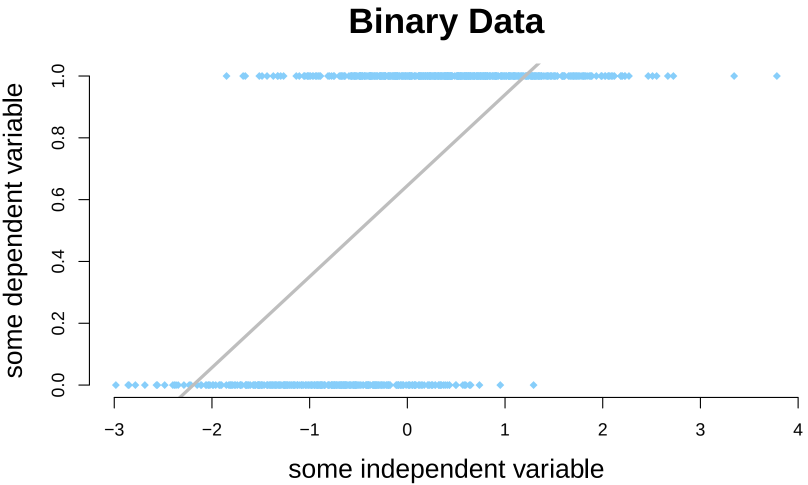

Ok, but what about...

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.6656	0.0123	54.21	0.0000
x	0.2630	0.0121	21.71	0.0000

actual relationship: $y = f(1 + 2x)$... uh oh!

And maybe even worse...

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	18.0014	1.8726	9.61	0.0000
x	32.1421	1.8476	17.40	0.0000

actual relationship: $y = f(1 + 2x)$... yikes!

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1. Dependent variable is a **linear** function of independent variables plus noise;

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_m x_m + \varepsilon$$

2. Independent variables are not related to each other – **no multicollinearity**;
3. Independent variables have **no measurement error**;
4. Noise term is a random variable following the **normal distribution**;

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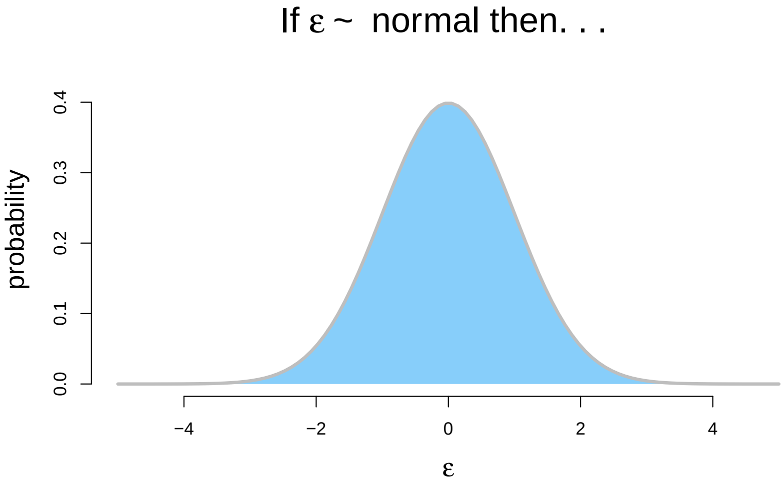

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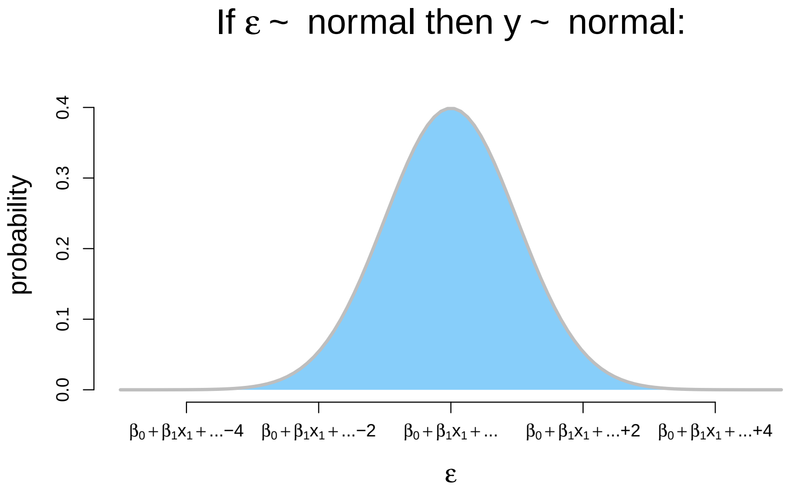
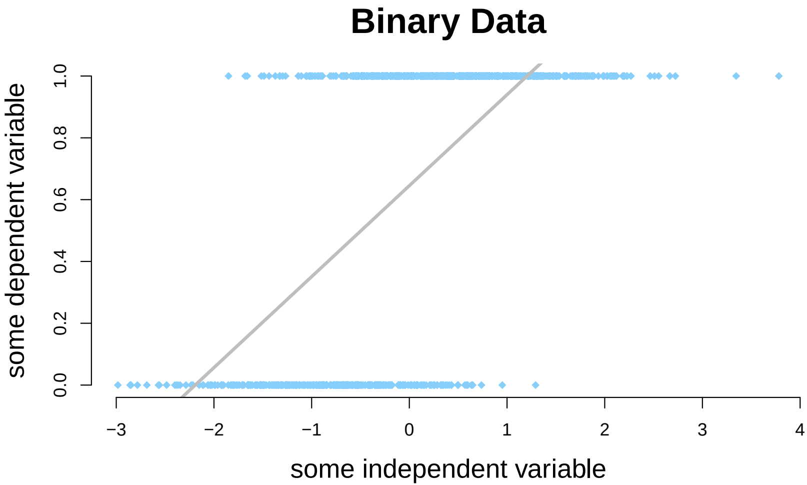


What does this mean? On the theory side:

- The **linear function** and **normal errors** assumptions require that y be able to take on any value!
- If the dependent variable is binary, i.e. always either 0 or 1 then...
  1. either **linear function** or **normal errors** are **wrong**, or...
  2. something exceedingly unlikely happened.

What does this mean? On the practical side:



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Let's use this to make predictions...

If $x =$	$y = 0.6656 + 0.2630x$	then $y = \ldots$
0	$y = 0.6656 + 0.2630 * 0$	0.6656

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Leads to nonsense!

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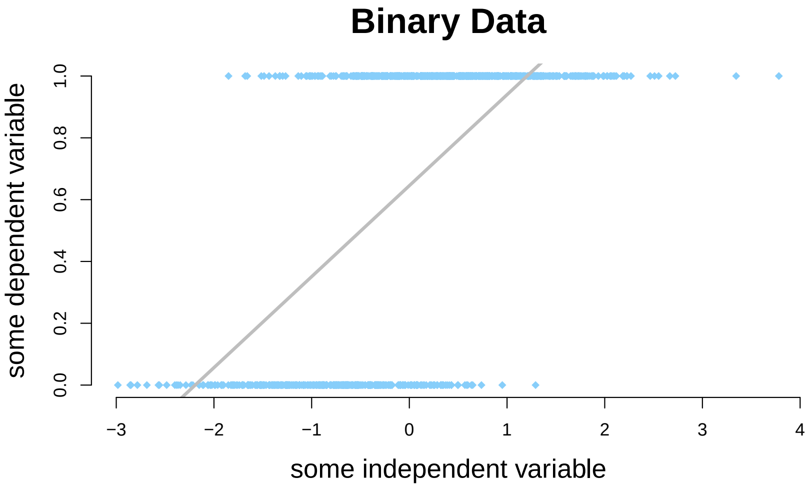

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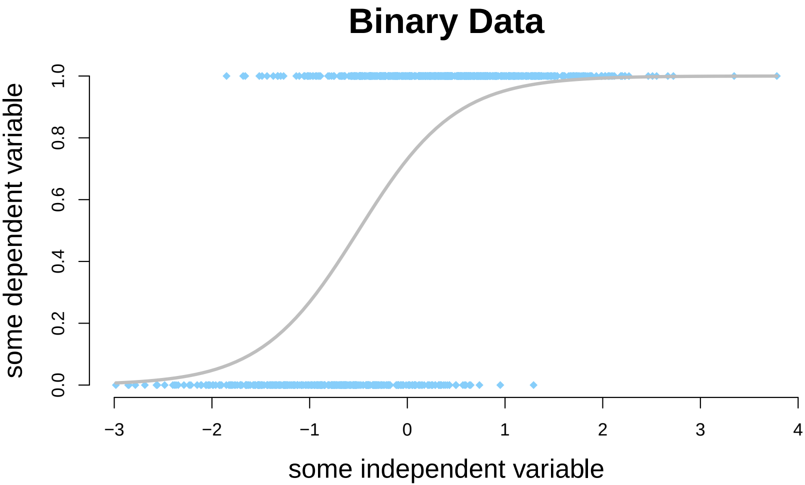

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GLMs have three required components:

1. A probability distribution that describes the dependent variable;
2. A linear model $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots$;
3. A link function that relates the linear model to the dependent variable distribution.

Binary data: GLM = Logistic regression;

Why should we care?

Limited dependent variables require different modeling strategies – we'll explore one of them next week.