PHY045 Problem Set 2

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Problem PS2-1. (a)

The output for x=3.0 and dx=0.1 is 6.1. This makes sense in terms of equation 1 because the computer does the calculation $\frac{3.1^2 - 3^2}{0.1}$, which is 6.1. The output for x=3.0, dx=0.01 is 6.01, which makes sense because when you plug the numbers into equation 1 you get 6.01.

Problem PS2-2. (a)

The output for x=0.0 and dx=0.01 is -0.005000. This is near the correct value of -sin(0), which is 0. The output for x=1.57 and dx=0.01 is -0.999987. This is the correct output because $-sin(\frac{\pi}{2})$ is -1.

Problem PS2-3. (a)

The output for x=0.9,dx=0.001 is 8.857759. This is correct because when I plugged in the same values into equation 1 I got 8.857759448, which is about the same thing as the computer got.

Problem PS2-4. (a)

The line $x = a + j * dx$ serves the purpose of increasing x until it gets to the the same value as b. This allows for the function to plug in the different values of x to get the values of the function that are necessary for knowing the height of the boxes. The line $integral = integral + f * dx$ adds the areas of the boxes to the overall value of the integral.

Problem PS2-6. (a)

The answer for a=2, b=5, and N=10 for my trapezoid rule code is 39.04500. Using the same values, the answer using the rectangular rule is

35.89500. Considering the theoretical answer is 39, the trapezoid method is more accurate for N=10. For N=100, the trapezoid rule code outputted 39.000450. The output for the rectangular rule is 38.685450. Thus, the trapezoid method is more accurate once again. The reason the trapezoid rule is more accurate is because it follows the shape of the function better than the rectangular rule. The rectangular rule only uses one value of the function at a time to calculate the areas of the rectangles, while the trapezoidal rule uses two points. Using a trapezoid instead of a rectangle makes it so that each area is more accurate, so the sum of the areas results in a more accurate integral.

Problem PS2-7. (a)

The output for the values a=0,b=5, and N=10 is 6.98517. This is very close the the theoretical value of 7.06858. The output for N=100 is 7.06594, which is closer to the theoretical value. The output for N=1000 is 7.06850, which is even closer to the theoretical value.

Problem PS2-8. (a)

A method to get a rough estimate for the value of the integral would be to notice that the numerator is less than 1, and the denominator can be greater than one. This would suggest that the value of the integral is probably between 0 and 1.

Problem PS2-9. (a)

The first few lines of code output make sense because the initial position starts at 5 and not much time has passed, so the position should not drastically change over this time period.

(b)

Farther down in the code we see that the position has changed from positive to negative. This makes sense because the code is for a mass on a spring, so the mass should cross x=0 after an amount of time, which so happens to be near t=0.8 seconds.

Problem PS2-10. (a)

The behavior of the velocity makes sense because it increases as the mass gets closer to the natural resting position. The velocity equals -10 around t-0.80 seconds because this is the time where the distance from the center is near zero, so all of the potential energy given to the mass by pulling the spring has gone into the kinetic energy of the mass.

Lab 2 Derivation

Wednesday, January 17, 2024 6:03 PM

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$A: \int_0^3 \sqrt{9-x^2}\, dx = ?$$

Let $x = 3\sin\theta \Rightarrow dx = 3\cos\theta\, d\theta$

$$\Rightarrow A \Longleftrightarrow \int_0^{\frac{\pi}{2}} \left(\sqrt{9-9\sin^2\theta}\right) 3\cos\theta\, d\theta$$

$$= 9\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta}\, \cos\theta\, d\theta$$

$$= 9\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

Note: $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$$= \frac{9}{2}\int_0^{\frac{\pi}{2}} \left(1+\cos(2\theta)\right) d\theta$$

$$= \frac{9}{2}\left[\theta + \frac{1}{2}\sin(2\theta)\right|_0^{\frac{\pi}{2}}$$

$$= \frac{9}{2}\left[\frac{\pi}{2} + \frac{1}{2}\overset{0}{\cancel{\sin(\pi)}} - 0 - 0\right]$$

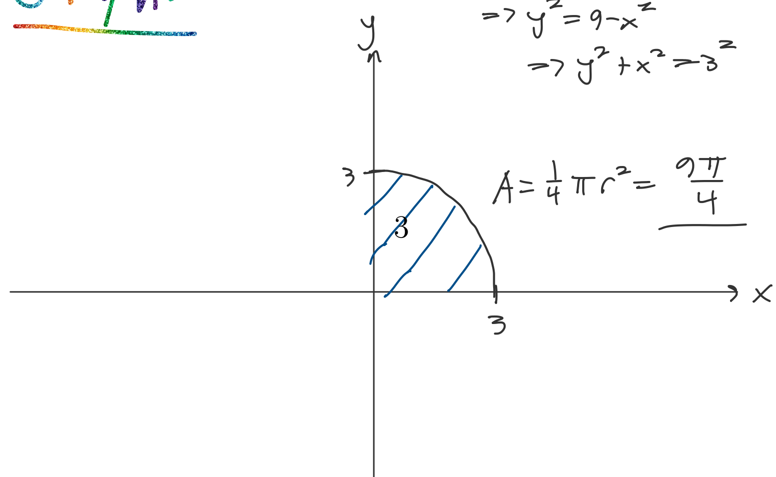
$$= \frac{9\pi}{4} \approx 7.068583471$$

Graph:

$y = \sqrt{9-x^2}$

$\Rightarrow y^2 = 9 - x^2$

$\Rightarrow y^2 + x^2 = 3^2$



$A = \frac{1}{4}\pi r^2 = \frac{9\pi}{4}$