Problem Set 5

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Problem PS5-1. (a)

The value of the sum of all the boxes is 1.00000. This is because the initial value is 1.00000, so the sum of the diffused numbers remains the same. In real life, this would be because of conservation of particles. It is good that the sum remains the same over each iteration because it would not make sense for particles to not be conserved.

Problem PS5-2. (a)

The total time elapsed during the simulation would be 10 seconds because Nt*dt = 100000*0.0001 = 10 seconds, which is mentioned in the lab handout. The plot of rho[x] vs. x is a curve that looks like it could be a combination of sines and cosines, which matches the solution presented in class (the solution is a Fourier series).

Problem PS5-3. (a)

For N=20 and seed=12345, the output of the code in the document is 0.17839530. This output was the same when I ran it a second time. For seed=54321, the output was 0.02803721. It seems to be that for each seed and N combination there is only 1 output. The output does not follow an obvious logical formula, so it is random in this sense. It is not completely random because you always get the same "random" number for each N and seed combination.

Problem PS5-4. (a)

The calculation for the first six moments for N = 1000000 is: Moment 1: 0.499877 Moment 2: 0.333117 Moment 3: 0.249764 Moment 4: 0.199780 Moment 5: 0.166474 Moment 6: 0.142693. This pattern implies that the first six moments are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ respectively.

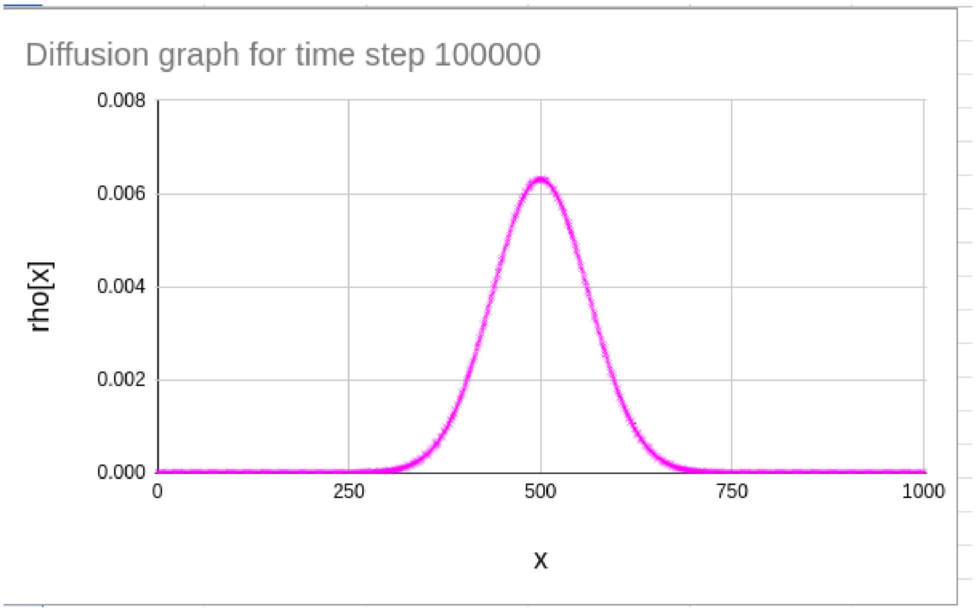

Figure 1: A graph of the diffusion curve for the 1000000th time step of the diffusion equation code.

Problem PS5-5. (a)

For the random walks that have an equal probability of stepping left or right after each step, the most likely final position would be back at/near zero. Considering that the farthest the final positions strayed from zero was 38 steps, some of the distances strayed farther than expected. The final positions would have been most likely to stay within plus or minus twenty, but only one of the five trials did so. This being said, two of the trials finished at a positive position and three finished at a negative position, so there was some balance with positive and negative.

For the random walks with unequal probabilities, the expected final position would follow the formula $xfinal = N(1-2p)$, which suggests the final position should have been -40 steps. Four of the five trials fell within the plus or minus twenty margin. All of the trials ended at negative positions, which also matched what was expected.

Problem PS5-6. (a)

Problem PS5-7. (a)

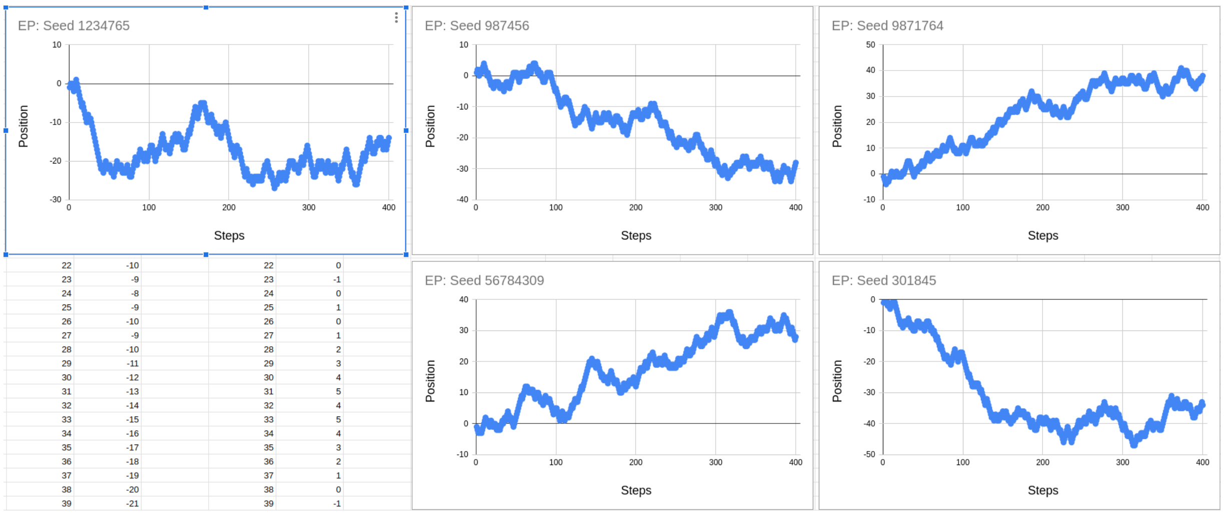

Figure 2: A graph of the five random walks for equal probabilities.

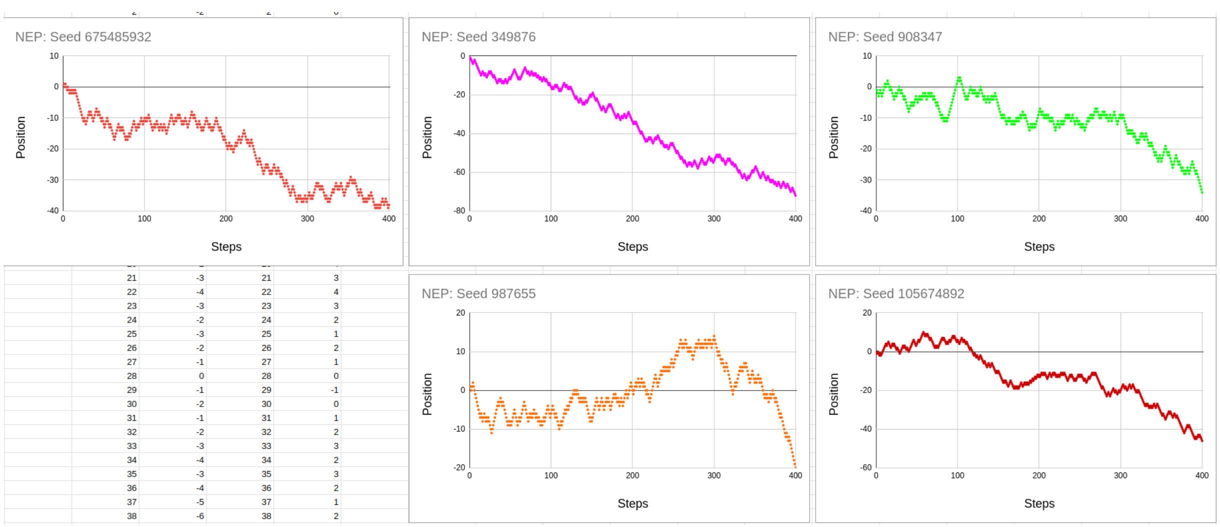

Figure 3: A graph of the five random walks for non-equal probabilities.

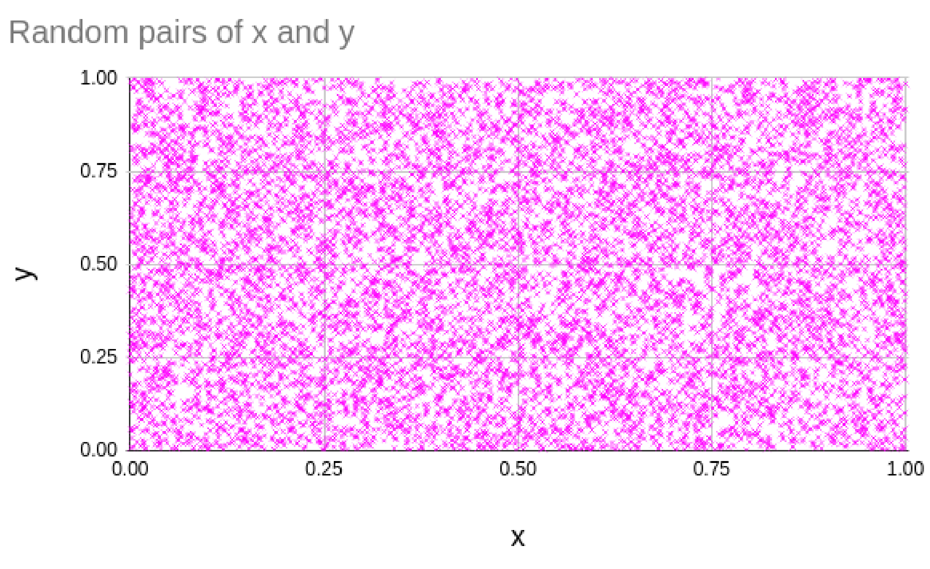

Figure 4: A graph of the set of random pairs. As expected, they are evenly distributed within the given range of x and y values.

Considering that x and y are two unrelated random numbers between zero and 1, the plot should be a random distribution of points where $0 < x < 1$ and $0 < y < 1$.

Problem PS5-8. (a)

The value of M/N for N=100 is 0.8, and N=10000 is 0.7902, and N=1000000 is 0.786009, and for N = 100000000 is 0.785417. Considering that the area of one fourth of the unit circle is pi/4, The outputted value is very close to what the area of one fourth of the unit circle should be, which is close to 0.7853981634.

Problem PS5-9. (a)

See my code to see which functions I tested.

Problem PS5-11. (a)

The main advantage of the Newton method is that it converges very quickly when compared to the Bisection method. If the chosen function is one that works with Newton Method, then it makes sense to use it. You also only have to input a single point, which is nice because you do not need to have a range where you know the root is between. The disadvantages of the Newton method are that it does not work for every function and you need to know the derivative of the function. If the function has a local minimum that does not cross the x-axis, then the Newton Method will not find a root.

The bisection method's advantages are that it can find the root of any function, so it works for functions that the Newton method does not work for. The disadvantages are that you need to have a range where you know the root is, and it converges slowly when compared to the Newton method.