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AMS 131: Quiz 8

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Someone offers you the possibility to play a gambling game with the following rules. First, you decide how much money you're willing to put at risk in this game: this amount — let's call it A — is referred to as your *stake* (all the monetary quantities are in dollars in this problem). Having chosen your stake, you're allowed to bet any amount $0 \leq B \leq A$ (thus, as a decision problem, for any fixed value of A, your possible actions in this situation correspond to values of B). If you win the bet, which occurs with probability $0 < p < 1$, your stake becomes $(A + B)$; if you lose, it becomes $(A - B)$, and this (of course) occurs with probability $(1 - p)$; and (crucially) p is known to you. Let X denote the value of your stake after the gamble has occurred, and suppose that you agree with Daniel Bernoulli that a reasonable utility function is $U(x) = 1 + \log(x)$.

(a) Write out the probability mass function (PMF) for X.

$$P(X = x) = \left\{ \begin{array}{cc} p, & \text{if } x = A + B \\ 1 - p, & \text{if } x = A - B \\ 0, & \text{otherwise} \end{array} \right\}$$

(b) Work out your expected utility $E[U(X)]$ in this game, as a function of A, B and p.

$$U(X) = 1 + log(x)$$

$$E[U(X)] = 1 + p * log(A + B) + (1 - p) log(A - B) = 1 + log(A - B) + p * log(\frac{A + B}{A - B})$$

(c) Intuitively, what should you do (i.e., what value of B should you choose) if $p < \frac{1}{2}$? Explain briefly.

If $p < \frac{1}{2} 1 - p > p$, the probability of losing is higher than the probability of winning the value of B. Therefore, I would want to choose a small value of B so that there is nothing to lose. In some cases 0, especially if p is very small.

(d) Show that when $p \geq \frac{1}{2}$ your expected utility is maximized with the choice $B = (2\,p - 1)\,A$. Is this answer intuitively reasonable? Explain briefly.

$$\frac{dE[U(X)]}{dB} = \frac{p}{A+B} - \frac{1-p}{A-B} = 0 \Rightarrow$$
$$\Rightarrow p(A-B) - (1-p)(A+B) = 0$$
$$\Rightarrow 2pA - (A-B) = 0 \Rightarrow B = (2p-1)A$$

also

$$\frac{d^2E[U(X)]}{dB^2} = -p(A+B)^2 = -\frac{p}{(A+B)^2} - \frac{(1-p)}{(A-B)^2} < 0$$

Therefore, $B = (2p-1)A$ maximizes utility.

Since $2p - 1 \geq 0$, $p \geq \frac{1}{2}$, and B increases with any increase in p; the chance of winning the amount B is greater than losing it.