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AMS 131: Quiz 2

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(You can use another page if necessary in any or all parts of the problems.)

1. Each year starts on one of the seven days (Sunday through Saturday). Each year is either a leap year (i.e., it includes February 29) or not. How many different calendars are possible for a year? Explain briefly.

Each day of the week a calendar year may start on represents a unique calendar because that calendar year starts on a unique day. However, this does not take into account leap years in the situation where March 1st would normally fall on a particular day it is instead February 29th. Therefore, there are 7 unique non-leap year calendars and 7 unique leap-year calendars for a total of **14 unique possible calendars for a year.**

2. A box contains 100 balls, of which r are red. Suppose that the balls are drawn from the box one at a time, at random **without replacement**. Determine

(a) the probability that the first ball drawn will be red;

Recall: $\mathrm{P(A)} = \frac{\text{number of outcomes favorable to A}}{\text{total number of possible outcomes}}$ or in this case: $\mathbf{\frac{r}{100}}$

(b) the probability that the 50th ball drawn will be red; and

The above logic applies even if one considers it is the 50th draw as we only know that there are r red balls.

Suppose A is the draw of a red ball. There are $\binom{100}{r}$ sequences of length 100 consisting of r red balls and 100-r non-red balls.

The number of sequences in which any ball at any point is red is: $\binom{99}{r-1}$

This then shows that: $P(A) = \frac{\binom{99}{r-1}}{\binom{100}{r}} = \frac{r}{100}$

therefore the probability that the 50th ball will be red is still and will always be:

$\mathbf{\frac{r}{100}}$ given the fact that we don't know r and the draws are without replacement.

(c) the probability that the last ball drawn will be red.

For the same reasons above, drawing the last ball has the same odds of being red: $\mathbf{\frac{r}{100}}$

Explain briefly in each case. *Hint:* Imagine that the 100 balls are randomly ordered in a list, and then drawn in that order, which is equivalent to sampling at random without replacement. *Further Hint:* When choosing a sample (Y_1, Y_2) of size 2 at random without replacement from the population $\{1, 2, 7\}$ in class on Monday, what was the relationship between $P(Y_1 = 7)$ and $P(Y_2 = 7)$?