1994-CE-A-MATH-1-Q04

$$egin{aligned} y &= tan(rac{1}{x}) \ &\Rightarrow rac{dy}{dx} = sec^2(rac{1}{x}) \cdot rac{d}{dx}(rac{1}{x}) \ &\Rightarrow rac{dy}{dx} = -rac{1}{x^2} \cdot sec^2(rac{1}{x}) \end{aligned}$$

Therefore,

$$egin{aligned} x^2 rac{dy}{dx} + y^2 + 1 \ &= -rac{1}{x^2} \cdot sec^2(rac{1}{x}) \cdot x^2 + tan^2(rac{1}{x}) + 1 \ &= -sec^2(rac{1}{x}) + sec^2(rac{1}{x}) - 1 + 1 \ &= 0 \end{aligned}$$

$$x^{2} \frac{dy}{dx} + y^{2} + 1 = 0$$

$$\Rightarrow \frac{d}{dx} \left[x^{2} \frac{dy}{dx} + y^{2} + 1 \right] = 0$$

$$\Rightarrow \frac{d}{dx} \left(x^{2} \frac{dy}{dx} \right) + \frac{d}{dx} (y^{2}) + \frac{d}{dx} (1) = 0$$

$$\Rightarrow 2x \frac{dy}{dx} + x^{2} \frac{d^{2}y}{dx^{2}} + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow x^{2} \frac{d^{2}y}{dx^{2}} + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow x^{2} \frac{d^{2}y}{dx^{2}} + 2(x + y) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d^{2}y}{dx^{2}} + \frac{2(x + y)}{x^{2}} \frac{dy}{dx} = 0$$