1999-AL-P-MATH-1-Q01

1(a)

(*) has non-trivial solutions.
$\Rightarrow \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -\lambda \\ 1 & \lambda & -1 \\ \lambda & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -\lambda + 1 + 1 - \lambda + \lambda(\lambda^2 - 1) = 0$$

$$\Rightarrow -2\lambda + 2 + \lambda^3 - \lambda = 0$$

$$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$$

$$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda - 2) = 0$$

$$\Rightarrow (\lambda - 1)^2(\lambda + 2) = 0$$

$\Rightarrow \lambda = -2$ or $\lambda = 1$

1(b)

When $\lambda = -2$,
(*) $\begin{cases} x + y + 2z = 0 \\ x - 2y - z = 0 \end{cases}$

Consider the augmented matrix.

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & -2 & -1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$\Rightarrow$ Solutions are $x = -t$, $y = -t$, $z = t \in R$

When $\lambda = 1$,
(*) x + y - z = 0

Solutions are $x = t - s$, $y = s \in R$, $z = t \in R$