1997-AL-P-MATH-1-Q03

3(a)

(*) has nontrivial solutions

$\Rightarrow \Delta = 0$

$\Rightarrow \begin{vmatrix} \lambda & k & 0 \\ 0 & -\lambda & 1 \\ 1 & k & 1 \end{vmatrix} = 0$

$\Rightarrow \lambda(-\lambda - k) - k(-1) = 0$

$\Rightarrow -\lambda^2 - k\lambda + k = 0$

$\Rightarrow \lambda^2 + k\lambda - k = 0$

3(b)

Quadratic equations in λ in (a) has equal roots

$\Rightarrow \sqrt{k^2 + 4k} = -\sqrt{k^2 + 4k}$

$\Rightarrow 2\sqrt{k^2 + 4k} = 0$

$\Rightarrow k^2 + 4k = 0$

$\Rightarrow k(k + 4) = 0$

$\Rightarrow k = 0$ or $k = -4$

When k=0,

$\Rightarrow \lambda^2 = 0$

$\Rightarrow \lambda = 0$

$\Rightarrow (*) \begin{cases} z = 0 \\ x + z = 0 \end{cases}$

$\Rightarrow$ Solution of (*) is : $\begin{cases} x = 0 \\ y = t \ \in R \\ z = 0 \end{cases}$

When k=-4,

$\Rightarrow \lambda^2 - 4\lambda + 4 = 0$

$\Rightarrow (\lambda - 2)^2 = 0$

$\Rightarrow \lambda = 2$

$\Rightarrow (*) \begin{cases} 2x - 4y = 0 \\ -2y + z = 0 \\ x - 4y + z = 0 \end{cases}$

$$\Rightarrow (*) \begin{cases} x - 4y + z = 0 \\ 2x - 4y = 0 \\ -2y + z = 0 \end{cases}$$

Consider the augmented matrix

$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 2 & -4 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

$\Rightarrow$ Solution of (*) is $x = t$, $y = \dfrac{1}{2}t$, $z = t \in R$