1992-AL-P-MATH-1-Q03

1992-AL-P MATH 1 #03

1. $B^{-1}\ exists \Rightarrow det\ B \neq 0 \Rightarrow \lambda \neq 0$
2. $B^{-1}AB = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$$\Rightarrow AB = B\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 1 & \lambda \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 1 & \lambda \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2 & 0 \\ 1 & 3\lambda \end{pmatrix} = \begin{pmatrix} -2a & 0 \\ a & b\lambda \end{pmatrix}$$

$\Rightarrow a = 1$ and $b = 3$

3. $(B^{-1}AB)^{100} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}^{100}$

$$\Rightarrow B^{-1}A^{100}B = \begin{pmatrix} 1^{100} & 0 \\ 0 & 3^{100} \end{pmatrix}$$

$$\Rightarrow A^{100} = B\begin{pmatrix} 1 & 0 \\ 0 & 3^{100} \end{pmatrix}B^{-1}$$

$$\Rightarrow A^{100} = \begin{pmatrix} -2 & 0 \\ 1 & \lambda \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3^{100} \end{pmatrix}B^{-1}$$

$$\Rightarrow A^{100} = \begin{pmatrix} -2 & 0 \\ 1 & 3^{100}\,\lambda \end{pmatrix}B^{-1}$$

$$\Rightarrow A^{100} = \begin{pmatrix} -2 & 0 \\ 1 & 3^{100}\,\lambda \end{pmatrix}\frac{1}{-2\lambda}\begin{pmatrix} \lambda & 0 \\ -1 & -2 \end{pmatrix}$$

$$\Rightarrow A^{100} = \frac{1}{-2\lambda}\begin{pmatrix} -2 & 0 \\ 1 & 3^{100}\,\lambda \end{pmatrix}\begin{pmatrix} \lambda & 0 \\ -1 & -2 \end{pmatrix}$$

$$\Rightarrow A^{100} = \frac{1}{-2\lambda}\begin{pmatrix} -2\lambda & 0 \\ \lambda - \lambda\, 3^{100} & -2\,\lambda\, 3^{100} \end{pmatrix}$$

$$\Rightarrow A^{100} = \begin{pmatrix} 1 & 0 \\ \frac{3^{100}-1}{2} & 3^{100} \end{pmatrix}$$