1994-CE-A-MATH-1-Q04

$y = tan(\frac{1}{x})$

$\Rightarrow \frac{dy}{dx} = sec^2(\frac{1}{x}) \cdot \frac{d}{dx}(\frac{1}{x})$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} \cdot sec^2(\frac{1}{x})$

Therefore,
$x^2 \frac{dy}{dx} + y^2 + 1$

$= -\frac{1}{x^2} \cdot sec^2(\frac{1}{x}) \cdot x^2 + tan^2(\frac{1}{x}) + 1$

$= -sec^2(\frac{1}{x}) + sec^2(\frac{1}{x}) - 1 + 1$
$= 0$

$x^2 \frac{dy}{dx} + y^2 + 1 = 0$

$\Rightarrow \frac{d}{dx}[\, x^2 \frac{dy}{dx} + y^2 + 1 \,] = 0$

$\Rightarrow \frac{d}{dx}(x^2 \frac{dy}{dx}) + \frac{d}{dx}(y^2) + \frac{d}{dx}(1) = 0$

$\Rightarrow 2x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} + 2y \frac{dy}{dx} = 0$

$\Rightarrow x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$

$\Rightarrow x^2 \frac{d^2y}{dx^2} + 2(x + y) \frac{dy}{dx} = 0$

$\Rightarrow \frac{d^2y}{dx^2} + \frac{2(x + y)}{x^2} \frac{dy}{dx} = 0$