2015-DSE-MATH-EP(M2)-Q06

6(a)

$M^T = -M$
$\Rightarrow |M^T| = |-M|$
$\Rightarrow |M| = -|M|$
$\Rightarrow 2|M| = 0$
$\Rightarrow |M| = 0$

6(b)(i)

$A + I$
$= \begin{pmatrix} -1 & a & b \\ -a & -1 & -8 \\ -b & 8 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & a & b \\ -a & 0 & -8 \\ -b & 8 & 0 \end{pmatrix}$

$\Rightarrow (A + I)^T = \begin{pmatrix} 0 & a & b \\ -a & 0 & -8 \\ -b & 8 & 0 \end{pmatrix}^T$

$\Rightarrow (A + I)^T = \begin{pmatrix} 0 & -a & -b \\ a & 0 & 8 \\ b & -8 & 0 \end{pmatrix}$

$\Rightarrow (A + I)^T = -(A + I)$
$\Rightarrow |A + I| = 0$

6(b)(ii)

$|A^3 + I|$
$= |(A + I)(A^2 - A + I)|$
$= |A + I|\,|A^2 - A + I|$
$= (0)\,|A^2 - A + I|$
$= 0$

Therefore $A^3 + I$ is singular.