

1991-AL-P-MATH-1-Q01

1991-AL-P MATH 1 #01

$$\begin{aligned} & \begin{vmatrix} a^3 & b^3 & c^3 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^3 & b^3 & c^3 \\ a & b & c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & b^3 - a^3 & c^3 - a^3 \\ 0 & b - a & c - a \end{vmatrix} \\ &= \begin{vmatrix} b^3 - a^3 & c^3 - a^3 \\ b - a & c - a \end{vmatrix} \\ &= (b^3 - a^3)(c - a) - (c^3 - a^3)(b - a) \\ &= (b - a)(b^2 + ab + a^2)(c - a) - (c - a)(c^2 + ac + a^2)(b - a) \\ &= (b - a)(c - a)(b^2 + ab + a^2 - c^2 - ac - a^2) \\ &= (b - a)(c - a)(b^2 + ab - c^2 - ac) \\ &= (b - a)(c - a)(b^2 - c^2 + ab - ac) \\ &= (b - a)(c - a)[(b - c)(b + c) + a(b - c)] \\ &= (b - a)(c - a)(b - c)(a + b + c) \end{aligned}$$