PP-DSE-MATH-EP(M2)-Q05

5(a)

$cos(x+1) + cos(x-1) = k\ cos\ x$

$\Rightarrow 2\ cos(\frac{(x+1)+(x-1)}{2})\ cos(\frac{(x+1)-(x-1)}{2}) = k\ cos\ x$
$\Rightarrow 2\ cos\ x\ cos(1) = k\ cos\ x$
$\Rightarrow k = 2\ cos(1)$

5(b)

Consider following determinant for any real numbers $a_1,\ a_2,\ b_1,\ b_2,\ c_1,\ c_2$

$$\begin{vmatrix} a_1 & a_1+a_2 & a_2 \\ b_1 & b_1+b_2 & b_2 \\ c_1 & c_1+c_2 & c_2 \end{vmatrix}$$

$= a_1(b_1+b_2)c_2 + (a_1+a_2)b_2c_1 + a_2b_1(c_1+c_2) - a_1b_2(c_1+c_2) - (a_1+a_2)b_1c_2 - a_2(b_1+b_2)c_1$

$= a_1b_1c_2 + a_1b_2c_2 + a_1b_2c_1 + a_2b_2c_1 + a_2b_1c_1 + a_2b_1c_2 - a_1b_2c_1 - a_1b_2c_2 - a_1b_1c_2 - a_2b_1c_2 - a_2b_1c_1 - a_2b_2c_1$

$= a_1b_1c_2 - a_1b_1c_2 + a_1b_2c_2 - a_1b_2c_2 + a_1b_2c_1 - a_1b_2c_1 + a_2b_2c_1 - a_2b_2c_1 + a_2b_1c_1 - a_2b_1c_1 + a_2b_1c_2 - a_2b_1c_2$

$= 0$

Also, from 5(a), we have

$cos\ 2 = \frac{cos\ 3 + cos\ 1}{k}$, $cos\ 5 = \frac{cos\ 6 + cos\ 4}{k}$ and $cos\ 8 = \frac{cos\ 9 + cos\ 7}{k}$

Therefore,

$$\begin{vmatrix} cos\ 1 & cos\ 2 & cos\ 3 \\ cos\ 4 & cos\ 5 & cos\ 6 \\ cos\ 7 & cos\ 8 & cos\ 9 \end{vmatrix} = \frac{1}{k} \begin{vmatrix} cos\ 1 & cos\ 1 + cos\ 3 & cos\ 3 \\ cos\ 4 & cos\ 4 + cos\ 6 & cos\ 6 \\ cos\ 7 & cos\ 7 + cos\ 9 & cos\ 9 \end{vmatrix} = 0$$