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2(a)

Let P(n) be the statement that $1^3+2^3+3^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$ for any positive integer n.

When n=1,
L.H.S. $=1^3=1$
R.H.S. $=\frac{1}{4}1^2(1+1)^2=\frac{1}{4}(4)=1$
L.H.S. = R.H.S., therefore, P(1) is true.

Assume that P(k) is true for an positive integer $k\geq 1$.
When n = k+1,
$1^3+2^3+3^3+\cdots+k^3+(k+1)^3$
$=\frac{1}{4}k^2(k+1)^2+(k+1)^3$
$=(k+1)^2[\ \frac{1}{4}k^2+(k+1)\ ]$
$=\frac{1}{4}(k+1)^2[\ k^2+4k+4)\ ]$
$=\frac{1}{4}(k+1)^2(k+2)^2$
$\Rightarrow$ P(k+1) is true.

Therefore, by mathematical induction, P(n) is true.