# 1993-AL-P-MATH-1-Q06

## 6(a)

$Let\ A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{13} \end{pmatrix}$

$A^T = -A$
$\Rightarrow A^T + A = 0$

$\Rightarrow \begin{pmatrix} 2a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \\ a_{12}+a_{21} & 2a_{22} & a_{23}+a_{32} \\ a_{13}+a_{31} & a_{23}+a_{32} & 2a_{13} \end{pmatrix} = 0$

$\Rightarrow a_{11} = 0,\ a_{22} = 0,\ a_{33} = 0,\ a_{12} = -a_{21},\ a_{13} = -a_{31},\ a_{23} = -a_{32}$

$\Rightarrow A = \begin{pmatrix} 0 & a_{12} & a_{13} \\ -a_{12} & 0 & a_{23} \\ -a_{13} & -a_{23} & 0 \end{pmatrix}$

$\Rightarrow det\ A = 0 \begin{vmatrix} 0 & a_{23} \\ -a_{23} & 0 \end{vmatrix} - a_{12} \begin{vmatrix} -a_{12} & a_{23} \\ -a_{13} & 0 \end{vmatrix} + a_{13} \begin{vmatrix} -a_{12} & 0 \\ -a_{13} & -a_{23} \end{vmatrix}$

$\Rightarrow det\ A = 0\ \cdot 0 - a_{12} \cdot a_{13} \cdot a_{23} + a_{12} \cdot a_{13} \cdot a_{23}$

$\Rightarrow det\ A = 0$

## 06(b)

$I - B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -2 & 74 \\ 2 & 1 & -67 \\ -74 & 67 & 1 \end{pmatrix}$

$\Rightarrow I - B = \begin{pmatrix} 0 & -2 & 74 \\ 2 & 0 & -67 \\ -74 & 67 & 0 \end{pmatrix}$

$\Rightarrow (I-B)^T = \begin{pmatrix} 0 & 2 & -74 \\ -2 & 0 & 67 \\ 74 & -67 & 0 \end{pmatrix} = -(I-B)$

$\Rightarrow det\ (I-B) = 0$

Also $I - B^4 = (I+B^2)(I-B^2) = (I+B^2)(I+B)(I-B)$
$\Rightarrow det\ (I-B^4) = det\ (I+B^2) \cdot det\ (I+B) \cdot det\ (I-B)$
$\Rightarrow det\ (I-B^4) = det\ (I+B^2) \cdot det\ (I+B) \cdot 0$
$\Rightarrow det\ (I-B^4) = 0$