

1996-AL-P-MATH-1-Q05

5(a)

Consider augmented matrix of the system:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & c \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & -1 & 1 & b-a \\ 0 & 1 & 1 & c \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & -1 & 1 & b-a \\ 0 & 0 & 2 & b+c-a \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 1 & -1 & a-b \\ 0 & 0 & 1 & \frac{b+c-a}{2} \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{b+c-a}{2} \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{a+b-c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{b+c-a}{2} \end{array} \right]$$

$$\Rightarrow X = \frac{-a+b+c}{2}, Y = \frac{a-b+c}{2}, Z = \frac{a+b-c}{2}$$

5(b)

Let $Z = xy$, $Y = xz$, $X = yz$

By result in 5(a),

$$X = \frac{-a+b+c}{2}, Y = \frac{a-b+c}{2}, Z = \frac{a+b-c}{2}$$

$$\Rightarrow yz = \frac{-a+b+c}{2}, xz = \frac{a-b+c}{2}, xy = \frac{a+b-c}{2}$$

$$\Rightarrow yz = \frac{-a+b+c}{2}, xz = \frac{a-b+c}{2}, xy = \frac{a+b-c}{2}, \frac{y}{x} = \frac{-a+b+c}{a-b+c}, \frac{z}{y} = \frac{a-b+c}{a+b-c}, \frac{x}{z} = \frac{a+b-c}{-a+b+c}$$

$$\Rightarrow y \cdot \frac{a-b+c}{a+b-c} \cdot y = \frac{-a+b+c}{2}, z \cdot \frac{a+b-c}{-a+b+c} \cdot z = \frac{a-b+c}{2}, x \cdot \frac{-a+b+c}{a-b+c} \cdot x = \frac{a+b-c}{2}$$

$$\Rightarrow y^2 = \frac{(a+b-c)(-a+b+c)}{2(a-b+c)}, z^2 = \frac{(a-b+c)(-a+b+c)}{2(a+b-c)}, x^2 = \frac{(a+b-c)(a-b+c)}{2(-a+b+c)}$$

$$\Rightarrow x^2 = \frac{(a+b-c)(a-b+c)}{2(-a+b+c)}, y^2 = \frac{(a+b-c)(-a+b+c)}{2(a-b+c)}, z^2 = \frac{(a-b+c)(-a+b+c)}{2(a+b-c)}$$

$$\Rightarrow x^2 = \frac{(a+b-c)(a-b+c)}{2(-a+b+c)}, y^2 = \frac{(a+b-c)(-a+b+c)}{2(a-b+c)}, z^2 = \frac{(a-b+c)(-a+b+c)}{2(a+b-c)}$$

$$\Rightarrow x = \sqrt{\frac{(a+b-c)(a-b+c)}{2(-a+b+c)}}, y = \sqrt{\frac{(a+b-c)(-a+b+c)}{2(a-b+c)}}, z = \sqrt{\frac{(a-b+c)(-a+b+c)}{2(a+b-c)}}$$

or

$$x = -\sqrt{\frac{(a+b-c)(a-b+c)}{2(-a+b+c)}}, y = -\sqrt{\frac{(a+b-c)(-a+b+c)}{2(a-b+c)}}, z = -\sqrt{\frac{(a-b+c)(-a+b+c)}{2(a+b-c)}}$$