1998-AL-P-MATH-1-Q01

1(a)

(*) has infinitely many solutions
$\Rightarrow \Delta = 0$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 2 \\ 1 & 0 & k+1 \\ k & -1 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 4 - k(k+1) - 2(k+1) + 2 = 0$$

$$\Rightarrow -k^2 - k - 2k - 2 + 6 = 0$$

$$\Rightarrow k^2 + 3k - 4 = 0$$

$$\Rightarrow (k+4)(k-1) = 0$$

$\Rightarrow k = -4$ or $k = 1$

1(b)

When k=-4,
(*) $\begin{cases} 2x + y + 2z = 0 \\ x - 3z = 0 \\ -4x - y + 4z = 0 \end{cases}$

Consider augmented matrix,

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 1 & 0 & -3 & 0 \\ -4 & -1 & 4 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 2 & 1 & 2 & 0 \\ -4 & -1 & 4 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & -1 & -8 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 8 & 0 \end{array}\right]$$

$\Rightarrow$ Solutions are $x = 3t$, $y = -8t$, $z = t \in R$

When k=1,
(*) $\begin{cases} 2x + y + 2z = 0 \\ x + 2z = 0 \\ x - y + 4z = 0 \end{cases}$

Consider augmented matrix,

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & -1 & 4 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 1 & 2 & 0 \\ 1 & -1 & 4 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right]$$

$\Rightarrow$ Solutions are $x = -2t$, $y = 2t$, $z = t \in R$