5(e)

(i)

$$\frac{2}{x^2-1} \equiv \frac{a}{x-1} + \frac{b}{x+1}$$

$\Rightarrow 2 = a(x+1) + b(x-1)$
$\Rightarrow (a+b)x + a - b = 2$

$$\Rightarrow \begin{cases} a+b=0 \\ a-b=2 \end{cases}$$

$\Rightarrow a = 1,\, b = -1$

(ii)

A = (2, $\frac{7}{3}$) and B = (3, $\frac{3}{2}$)

Equation of line joining AB is:

$$\frac{y-\frac{3}{2}}{x-3} = \frac{\frac{3}{2}-\frac{7}{3}}{3-2}$$

$$\Rightarrow \frac{y-\frac{3}{2}}{x-3} = -\frac{5}{6}$$

$$\Rightarrow y = -\frac{5}{6}(x-3) + \frac{3}{2}$$

$$\Rightarrow y = -\frac{5}{6}x + 4$$

Note that:

$$f(x) = \frac{x^2+3}{x^2-1} = 1 + \frac{4}{x^2-1} = 1 + \frac{2}{x-1} - \frac{2}{x+1}$$

Area = $\int_2^3 [\,-\frac{5}{6}x + 4 - f(x)\,]dx$

$$= \int_2^3 [\,-\frac{5}{6}x + 3 - \frac{2}{x-1} + \frac{2}{x+1}]dx$$

$$= [\,-\frac{5}{12}x^2 + 3x - 2\,ln(x-1) + 2\,ln(x+1)\,]_2^3$$

$$= [\,-\frac{5}{12}x^2 + 3x + 2\,ln(\frac{x+1}{x-1})\,]_2^3$$

$$= -\frac{25}{12} + 3 + 2\ ln(\frac{2}{3})$$

$$= \frac{11}{12} - 2\ ln(\frac{3}{2})$$

$$\approx 0.1057$$