1996-AL-P-MATH-1-Q01

1996-AL-P MATH 1 #01

$A^3 - 5A^2 + 8A - 4I = (A - I)(A - 2I)^2$

$\Rightarrow A^3 - 5A^2 + 8A - 4I = (A - I)(A - 2I)^2 = \begin{pmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$\Rightarrow A^3 - 5A^2 + 8A - 4I = \begin{pmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$\Rightarrow A^3 - 5A^2 + 8A - 4I = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$\Rightarrow A^3 - 5A^2 + 8A - 4I = 0$

Hence,

$4I = A^3 - 5A^2 + 8A$

$\Rightarrow 4A^{-1} = A^2 - 5A + 8I$

$\Rightarrow 4A^{-1} = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix} - 5 \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

$\Rightarrow 4A^{-1} = \begin{pmatrix} -2 & 0 & -6 \\ 3 & 4 & 3 \\ 3 & 0 & 7 \end{pmatrix} - \begin{pmatrix} 0 & 0 & -10 \\ 5 & 10 & 5 \\ 5 & 0 & 15 \end{pmatrix} + \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

$\Rightarrow 4A^{-1} = \begin{pmatrix} 6 & 0 & 4 \\ -2 & 2 & -2 \\ -2 & 0 & 0 \end{pmatrix}$

$\Rightarrow A^{-1} = \frac{1}{2} \begin{pmatrix} 3 & 0 & 2 \\ -1 & 1 & -1 \\ -1 & 0 & 0 \end{pmatrix}$