

1991-CE-A-MATH-1-Q02

For any $\Delta \in R$,

$$f(x + \Delta) - f(x)$$

$$= \frac{1}{1 + x + \Delta} - \frac{1}{1 + x}$$

$$= \frac{(1 + x) - (1 + x + \Delta)}{(1 + x + \Delta)(1 + x)}$$

$$= \frac{-\Delta}{(1 + x + \Delta)(1 + x)}$$

Therefore,

$$f'(x) = \lim_{\Delta \rightarrow 0} \frac{f(x + \Delta) - f(x)}{\Delta}$$

$$= \lim_{\Delta \rightarrow 0} \frac{-\Delta}{(1 + x + \Delta)(1 + x)} \cdot \frac{1}{\Delta}$$

$$= \lim_{\Delta \rightarrow 0} \frac{-1}{(1 + x + \Delta)(1 + x)}$$

$$= \frac{-1}{(1 + x)(1 + x)}$$

$$= \frac{-1}{(1 + x)^2}$$