# 1986-CE-A MATH-2-Q11

## 11(a)(i)

Let $x = 2\ sin\theta$
$\Rightarrow \frac{dx}{d\theta} = 2\ cos\theta$

Also,

- When x=1, $sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$
- When x=2, $sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$
- $\sqrt{4-x^2} = \sqrt{4-4\ sin^2\theta} = 2cos\theta$

Then $\int_1^2 \sqrt{4-x^2}\ dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\ cos\theta)(2\ cos\theta)\ d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4\ cos^2\theta\ d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\ (1 + cos\ 2\theta)\ d\theta$

$= \int_{\frac{\pi}{3}}^{\pi} (1 + cos\ \beta)\ d(\beta)$ where $\beta = 2\theta$

$= \pi + sin\pi - \frac{\pi}{3} - sin\frac{\pi}{3}$

$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

## 11(a)(ii)

$3 + 2x - x^2$
$= 4 - 1 + 2x - x^2$
$= 4 - (1 - 2x + x^2)$
$= 2^2 - (x-1)^2$

Let $x - 1 = 2\ sin\theta$

- $\frac{dx}{d\theta} = 2\ cos\theta$
- $\sqrt{3+2x-x^2} = \sqrt{2^2-(x-1)^2} = \sqrt{2^2 - 2^2\ sin^2\theta} = 2cos\theta$
- when x=0, $sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$
- when x=1, $sin\theta = 0 \Rightarrow \theta = 0$

Then $\int_0^1 \sqrt{3+2x-x^2}\,dx$

$= \int_{-\frac{\pi}{6}}^{0} 4\,cos^2\theta\,d\theta$

$= \int_{-\frac{\pi}{3}}^{0} (1+cos\,\beta)\,d(\beta)$

$= 0 + sin0 - (-\frac{\pi}{3}) - sin(-\frac{\pi}{3})$

$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

# 11(b)(i)

P(x, y) is on $C_2$

$\Rightarrow (x-1)^2 + (y-\sqrt{3})^2 = 4$

$\Rightarrow (y-\sqrt{3})^2 = 4-(x-1)^2$

$\Rightarrow y-\sqrt{3} = -\sqrt{4-(x-1)^2}$ ($\because$ P is below the centre, therefore, $y-\sqrt{3}<0$)

$\Rightarrow y = \sqrt{3} - \sqrt{4-(x-1)^2}$

$\Rightarrow y = \sqrt{3} - \sqrt{3+2x-x^2}$

# 11(b)(ii)

Area of the shaded region

$= \int_0^1 [\,\sqrt{3}x - (\sqrt{3} - \sqrt{3+2x-x^2})\,]dx + \int_1^2 [\,\sqrt{4-x^2} - (\sqrt{3} - \sqrt{3+2x-x^2})\,]\,dx$

$= \sqrt{3}\int_0^1 xdx - \sqrt{3}\int_0^1 dx + \int_0^1 \sqrt{3+2x-x^2}dx + \int_1^2 \sqrt{4-x^2}dx - \sqrt{3}\int_1^2 dx + \int_1^2 \sqrt{3+2x-x^2}\,dx$

$= \sqrt{3}\int_0^1 xdx - \sqrt{3}\int_0^2 dx + \int_0^2 \sqrt{3+2x-x^2}dx + \int_1^2 \sqrt{4-x^2}dx$

$= \frac{\sqrt{3}}{2} - 2\sqrt{3} + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1+cos\,\beta)\,d(\beta) + \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} - 2\sqrt{3} + \frac{2\pi}{3} + \sqrt{3} + \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

$= \frac{4\pi}{3} - \sqrt{3}$