2014-DSE-MATH-EP(M2)-Q07

7(a)

Let P(n) be the statement that $A^{n+1} = 2^n A$ for all positive integer n.

When n=1, A^2

$$= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

$$= 2 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} = 2A$$

Therefore P(1) is true.

Assume that P(k) is true for some positive integer $k \geq 1$. Then

A^{k+1}

$= A^k A$

$= 2^k AA$

$= 2^k A^2$

$= 2^k \cdot 2A$

$= 2^{k+1} A$

P(k+1) is true. Therefore by mathematical induction, P(n) is true.

7(b)

Consider

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2 - 2 = 0$$

$\Rightarrow A^{-1}$ does NOT exist.

$\Rightarrow A^2 A^{-1} = 2AA^{-1}$ is invalid.

$\Rightarrow A = 2I$ is invalid.