2018-DSE-MATH-EP(M2)-Q7

7(a)

$MX = XM$

$\Rightarrow \begin{pmatrix} 7 & 3 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} a & 6a \\ b & c \end{pmatrix} = \begin{pmatrix} a & 6a \\ b & c \end{pmatrix} \begin{pmatrix} 7 & 3 \\ -1 & 5 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 7a+3b & 42a+3c \\ -a+5b & -6a+5c \end{pmatrix} = \begin{pmatrix} a & 33a \\ 7b-c & 3b+5c \end{pmatrix}$

$\Rightarrow 7a+3b = a$ and $42a+3c = 33a$
$\Rightarrow b = -2a$ and $c = -3a$

7(b)

$X = \begin{pmatrix} a & 6a \\ b & c \end{pmatrix} = \begin{pmatrix} a & 6a \\ -2a & -3a \end{pmatrix} = a \begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix}$

$|X| = \left| a \begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix} \right| = a^2 \begin{vmatrix} 1 & 6 \\ -2 & -3 \end{vmatrix} = 9a^2 \neq 0$

Therefore X is non-singular.

7(c)

$(X^T)^{-1}$
$= (X^{-1})^T$

$= (\frac{1}{9a^2} \cdot a \begin{pmatrix} -3 & -6 \\ 2 & 1 \end{pmatrix})^T$

$= (\frac{1}{9a} \begin{pmatrix} -3 & -6 \\ 2 & 1 \end{pmatrix})^T$

$= \frac{1}{9a} \begin{pmatrix} -3 & 2 \\ -6 & 1 \end{pmatrix}$