ME 57200 Aerodynamic Design

Lecture #19: Compressible Flow

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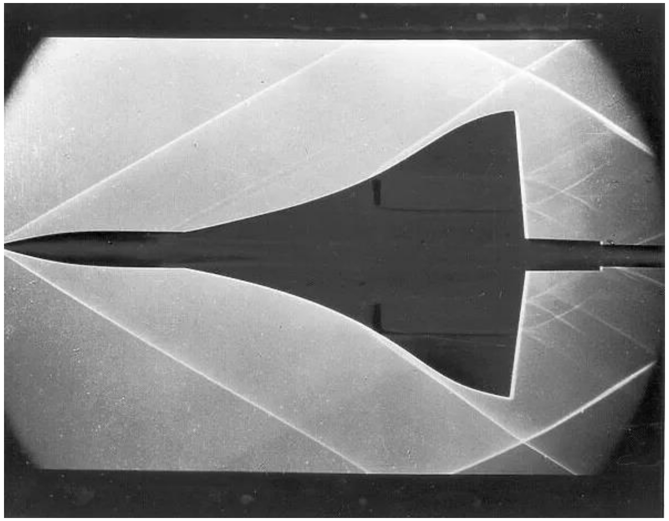
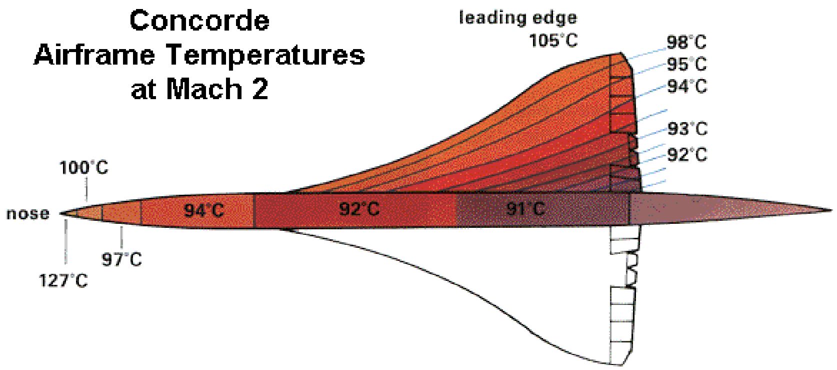
Compressible Flow

<u>High-speed flow</u>





- Energy transformations and temperature changes are important considerations
  - Science of Thermodynamics

Compressible Flow

Review of Thermodynamics

- Perfect Gas

$$\boxed{p = \rho RT}$$

where R is the specific gas constant, which is a different value for different gases.

For air at standard conditions, $R = 287\ \mathrm{J/(kg \cdot K)}$

$$\boxed{pv = RT}$$

where v is the specific volume, that is, the volume per unit mass; $v = 1/\rho$.

Compressible Flow

Review of Thermodynamics

- Internal Energy and Enthalpy $h = e + pv$

For a perfect gas, both e and h are functions of temperature only:

$$e = e(T)$$

$$h = h(T)$$

$$de = c_v dT \qquad dh = c_p dT$$

where c_v and c_p are the specific heats at constant volume and constant pressure

- For moderate temperatures (for air, T < 1000 K), the specific heats are reasonably constant.
- A perfect gas where c_v and c_p are constants is defined as calorically perfect gas

$$\boxed{e = c_v T} \qquad \boxed{h = c_p T}$$

Compressible Flow

Review of Thermodynamics

For a specific gas, c_p and c_v are related through the equation

$$c_p - c_v = R$$

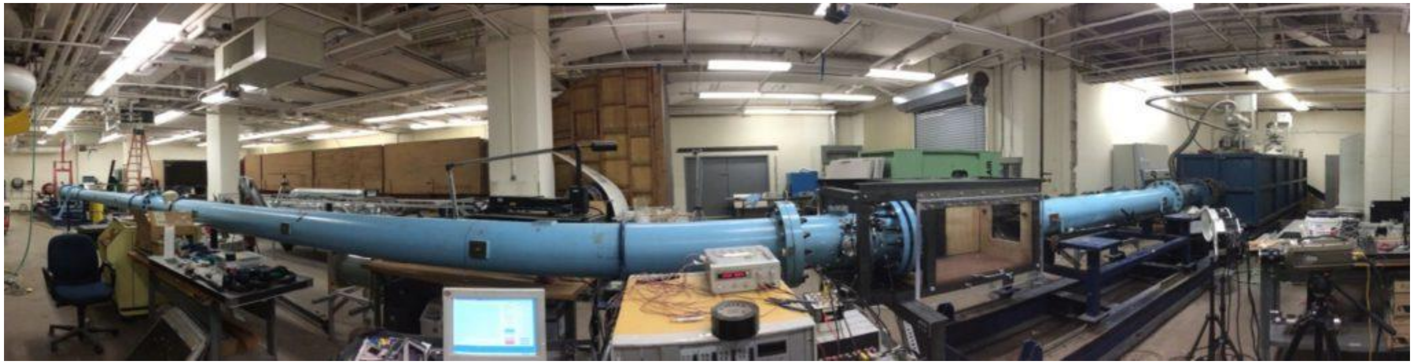
Define $\gamma \equiv c_p/c_v$. For air at standard conditions, $\gamma = 1.4$.

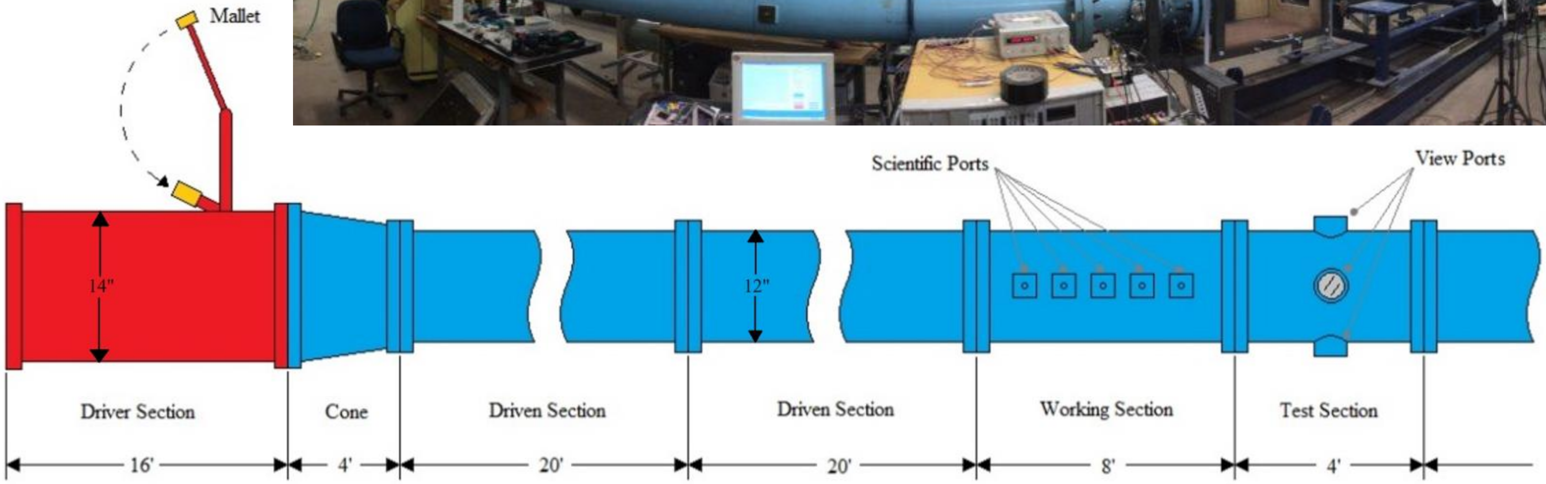
$$\boxed{c_p = \frac{\gamma R}{\gamma - 1}}$$

$$\boxed{c_v = \frac{R}{\gamma - 1}}$$

Shock Tube

- One type of supersonic wind tunnel is a blow-down tunnel, where air is stored in a high-pressure reservoir, and then, upon the opening of a valve (or diaphragm), exhausted through the tunnel into a vacuum tank or simply into the open atmosphere at the downstream end of the tunnel.





CCNY Shock Tube Facility

Practice Example

- Consider a reservoir with an internal volume of 30 m^3. As air is pumped into the reservoir, the air pressure inside the reservoir continually increases with time. Consider the instant during the charging process when the reservoir pressure is 10 atm. Assume the air temperature inside the reservoir is held constant at 300 K by means of a heat exchanger. Air is pumped into the reservoir at the rate of 1 kg/s. Calculate the time rate of increase of pressure in the reservoir at this instant.

Practice Example

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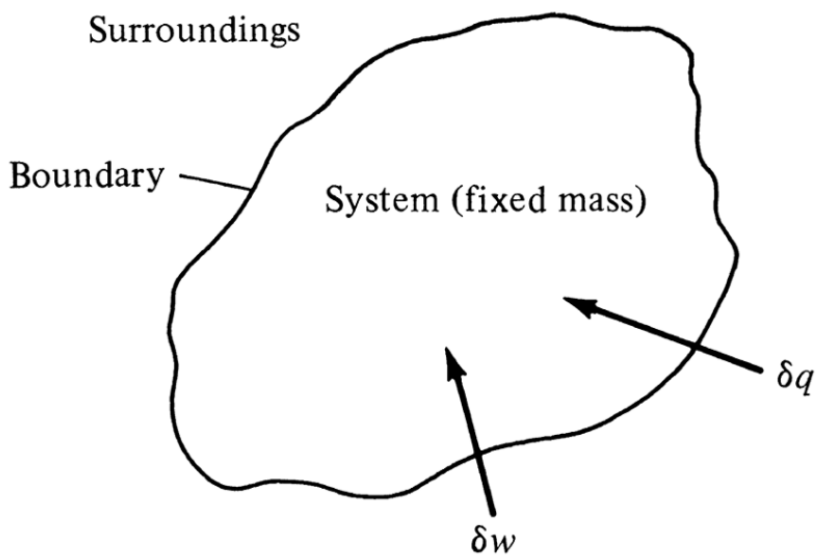
$$\rho = \frac{M}{V} \qquad \frac{d\rho}{dt} = \frac{1}{V}\frac{dM}{dt} = \frac{1\,\mathrm{kg/s}}{30\,\mathrm{m}^3} = 0.0333$$

$$\frac{dp}{dt} = RT\frac{d\rho}{dt} \qquad \frac{dp}{dt} = (287)(300)(0.0333) = \boxed{2867.13\,\frac{\mathrm{N}}{\mathrm{m}^2\,\mathrm{s}}}$$

Compressible Flow

Review of Thermodynamics

- First Law of Thermodynamics



$$\boxed{\delta q + \delta w = de}$$

Review of Thermodynamics

- First Law of Thermodynamics

For a given *de*, we are primarily concerned with three types of processes

- Adiabatic process: no heat is added to or taken away from the system
- Reversible process: no dissipative phenomena occur, that is, where the effects of viscosity, thermal conductivity, and mass diffusion are absent
- Isentropic process: both adiabatic and reversible

Compressible Flow

Review of Thermodynamics

- Second Law of Thermodynamics

Definition of entropy $$ds = \frac{\delta q_{\text{rev}}}{T}$$

where s is the entropy of the system, δq_{rev} is an incremental amount of heat added reversibly to the system, and T is the system temperature.

$$\boxed{ds = \frac{\delta q}{T} + ds_{\text{irrev}}}$$

where δq is the actual amount of heat added to the system during an actual irreversible process, and ds_{irrev} is the generation of entropy due to the irreversible, dissipative phenomena.

$$\boxed{ds_{\text{irrev}} \geq 0}$$

Compressible Flow

Review of Thermodynamics

- Second Law of Thermodynamics

$$\boxed{ds \geq \frac{\delta q}{T}}$$

If the process is adiabatic, $\delta q = 0$

$$\boxed{ds \geq 0}$$

Review of Thermodynamics

- Second Law of Thermodynamics

$$\boxed{Tds = de + pdv}$$

$$\boxed{Tds = dh - vdp}$$

$de = c_v\, dT$ and $dh = c_p\, dT$.

$$\boxed{s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}}$$

$$\boxed{s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}}$$

Review of Thermodynamics

- Second Law of Thermodynamics

Isentropic Relations $\delta q = 0.$ $ds_{\text{irrev}} = 0.$

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}}$$

Practice Example

- Consider the gas in the reservoir of the supersonic wind tunnel in the previous example. The pressure and temperature of the air in the reservoir are 20 atm and 300 K, respectively. The air in the reservoir expands through the wind tunnel duct. At a certain location in the duct, the pressure is 1 atm. Calculate the air temperature at this location if: (a) the expansion is isentropic and (b) the expansion is non-isentropic with an entropy increase through the duct to this location of 320 J/(kg K).

Practice Example

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(a) the expansion is isentropic

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300\left(\frac{1}{20}\right)^{\frac{0.4}{1.4}} = 300(0.05)^{0.2857}$$

$$= 300(0.4249) = \boxed{127.5\text{K}}$$

Practice Example

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(b) the expansion is non-isentropic with an entropy increase through the duct to this location of 320 J/(kg K)

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(287)}{0.4} = 1004.5\frac{\text{J}}{\text{kg}\cdot\text{K}}$$

$$320 = 1004.5\ln\left(\frac{T_2}{300}\right) - (287)\ln\left(\frac{1}{20}\right)$$

$$= 1004.5\ln\left(\frac{T_2}{300}\right) - (-859.78)$$

$$\ln\left(\frac{T_2}{300}\right) = \frac{320 - 859.78}{1004.5} = -0.5374$$

$$\frac{T_2}{300} = e^{-0.5374} = 0.5843$$

$$T_2 = (0.5843)(300) = \boxed{175.3\text{K}}$$

Practice Example

- Consider a Boeing 747 flying at a standard altitude of 36,000 ft. The pressure at a point on the wing is 400 lb/ft^2. Assuming isentropic flow over the wing, calculate the temperature at tis point.

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Solution

From **Appendix E**, at a standard altitude of 36,000 ft, p_∞ = 476 lb/ft^2 and T_∞ = 391 °R.

$$\frac{p}{p_\infty} = \left(\frac{T}{T_\infty}\right)^{\gamma/(\gamma-1)}$$

$$T = T_\infty\left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 391\left(\frac{400}{476}\right)^{0.4/1.4} = \boxed{372°\text{R}}$$