ME 572 Aerodynamic Design
HW #4 (Due at 11:59 pm on Friday, Mar 08)

Problem 1 [10 pt]
A Pitot tube on an airplane flying at standard sea level reads 1.09×10^5 N/m^2. What is the velocity of the airplane? (Round the final answer to two decimal places.)

Solution:

$$p_0 = p_\infty + \frac{1}{2}\rho V_\infty^2$$

4 Points

$$V_\infty = \sqrt{\frac{2(p_0 - p_\infty)}{\rho}}$$

4 Points

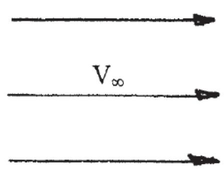
$$= \sqrt{\frac{2(1.09\ \mathrm{N/m^2} - 1.01\ \mathrm{N/m^2}) \times 10^5}{1.23\ \mathrm{kg/m^3}}}$$

$$= 114.05\ \ \mathrm{m/s}$$

2 Points

Problem 2 [10 pt]
Show that a uniform flow with velocity V_∞ is a physically possible incompressible flow and that it is irrotational.

Solution:



$$\vec{V} = V_\infty \vec{i} \qquad V_\infty = u = \text{constant}$$

$$\nabla \cdot \vec{V} = \cancelto{0}{\frac{\partial u}{\partial x}} + \cancelto{0}{\frac{\partial v}{\partial y}} + \cancelto{0}{\frac{\partial w}{\partial z}} = \boxed{0}$$

4 Points

The flow is a physically possible incompressible flow.

$$\nabla \times V = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & \cancelto{0}{v} & \cancelto{0}{w} \end{vmatrix} = \vec{i}\,(0\text{-}0) - \vec{j}\,\left(0 - \cancelto{0}{\frac{\partial u}{\partial x}}\right) + \vec{k}\,\left(0 - \cancelto{0}{\frac{\partial u}{\partial y}}\right)$$

6 Points

$$\boxed{\nabla \times \vec{V} = 0}$$ The flow is irrotational.

Problem 3 [10 pt]
Show that a source flow is a physically possible incompressible flow everywhere except at the origin. Also show that it is irrotational.

Solution:

For a source flow,

$$\vec{V} = V_r\, \vec{e}_r = \frac{\Lambda}{2\pi r}\, \vec{e}_r$$

2 Points

In polar coordinates:

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(r\, V_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\, \frac{\Lambda}{2\pi r}\right] + \frac{1}{r}\frac{\partial (0)}{\partial \theta}$$

2 Points

$$\nabla \cdot \vec{V} = \frac{1}{r}\cancelto{0}{\frac{\partial}{\partial r}\left(\frac{\Lambda}{2\pi}\right)} + 0 = \boxed{0}$$

2 Points

The flow is a physically possible incompressible flow except the origin where r = 0

To show that the flow is irrotational, calculate $\nabla \times \vec{V}$.

$$\nabla \times \vec{V} = \frac{1}{r}\begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \frac{\Lambda}{2\pi r} & 0 & 0 \end{vmatrix}$$

2 Points

$$\nabla \times \vec{V} = -r\, \vec{e}_\theta \left(\cancelto{0}{\frac{\partial 0}{\partial r}} - \cancelto{0}{\frac{\partial \Lambda / 2\pi r}{\partial z}}\right) + \vec{e}_z\left(\cancelto{0}{\frac{\partial 0}{\partial r}} - \cancelto{0}{\frac{\partial \Lambda / 2\pi r}{\partial \theta}}\right) = \boxed{0}$$

2 Points

The flow is irrotational everywhere.