ME 51500/I5800 L 75 min.	Exam 1 Closed Book, Open Notes*	P. Ganatos 10/27/2022

*** Only original handwritten notes and homeworks are allowed. Photocopied notes and homework solution sheets are <u>not</u> permitted. Except for a hand calculator, no cell phone or electronic equipment of any kind is allowed.**

Show all work and give <u>units</u> in final answers.

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[50] 1. A satellite in earth orbit has a specific angular momentum of 70,000 km^2/sec and specific energy of $-15\ km^2/sec^2$.

a) Calculate the parameter p, the semi-major axis, and the eccentricity of the orbit. [15 points]
b) Calculate the period of the orbit. [5 points]
c) Calculate the apogee and perigee <u>altitudes</u>. [10 points]
d) Calculate the maximum and minimum velocities. [10 points]
e) Calculate the value(s) of the true anomaly where the altitude is midway between the apogee and perigee altitudes. [10 points]

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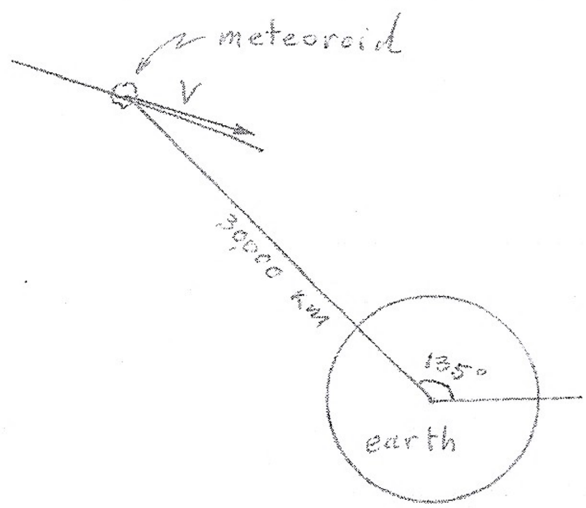
[50] 2. A meteoroid is first observed approaching the earth when it is 30,000 km from the center of the earth with a true anomaly of 135°, as shown in the figure below. If the geocentric speed of the meteoroid V is 5.154932912592882 km/sec when it is first observed, determine:

a) Whether the trajectory is elliptic, parabolic or hyperbolic. [10 points]
b) Whether the meteoroid flies by or strikes the earth. [10 points]
c) The time until the meteoroid reaches its closest approach to the earth or strikes the earth. [20 points]
d) Draw the trajectory of the meteoroid on the hodographic plane and label the point where the meteoroid is first observed and the point referred to in part (c). [10 points]



<u>Physical constants</u>

<u>The Earth</u>

Mean Radius = 6368 km $\mu_{earth} = 3.986 \times 10^5\ km^3/sec^2$

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1) $h = 70{,}000\ km^2/sec \qquad \mathcal{E} = -15\ km^2/sec^2$

a) $p = \frac{h^2}{\mu} = \frac{(70{,}000)^2}{3.986\times10^5} = \underline{\underline{12{,}293\ km}}$ [5]

$\mathcal{E} = -\frac{\mu}{2a} \Rightarrow a = -\frac{\mu}{2\mathcal{E}} = -\frac{3.986\times10^5}{2(-15)} = \underline{\underline{13{,}287\ km}}$ [5]

$p = a(1-e^2) \Rightarrow e = \sqrt{1-\frac{p}{a}} = \sqrt{1-\frac{12{,}293}{13{,}287}} = \underline{\underline{0.273514}}$ [5]

b) $T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(13{,}287)^3}{3.986\times10^5}} = \underline{\underline{15{,}242 sec}} = \underline{\underline{4.23\ hrs}}$

[5]

c) $r = \frac{p}{1+e\cos\theta}$

$r_p = r\Big|_{\theta=0} = \frac{p}{1+e} = \frac{12{,}293}{1+0.273514} = 9653\ km$

$H_p = r_p - r_E = 9653 - 6368 = \underline{\underline{3285\ km}}$ [5]

$r_a = r\Big|_{\theta=\pi} = \frac{p}{1-e} = \frac{12{,}293}{1-0.273514} = 16{,}921\ km$

$H_a = r_a - r_E = 16{,}921 - 6368 = \underline{\underline{10{,}553\ km}}$ [5]

d) $h = r_p V_p = r_a V_A$

$V_p = \frac{h}{r_p} = \frac{70{,}000}{9653} = \underline{\underline{7.25163\ km/sec}}$ [5]

$V_a = \frac{h}{r_a} = \frac{70{,}000}{16{,}921} = \underline{\underline{4.13687\ km/sec}}$ [5]

e) For $H = \frac{H_p + H_a}{2} = \frac{3285 + 10{,}553}{2} = 6919$ km

$r = H + r_e = 6919 + 6368 = 13{,}287$ km [4]

$$r = \frac{p}{1 + e\cos\theta} \Rightarrow \cos\theta = \frac{\frac{p}{r} - 1}{e} = \frac{\frac{12{,}293}{13{,}287} - 1}{0.273514} = -0.273514$$

$$\boxed{\theta = 106^\circ, 254^\circ}$$

[3] [3]

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2) a) $\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r} = \frac{(5.1549329125928 82)^2}{2} - \frac{3.986 \times 10^5}{30{,}000} = \underline{\underline{0}}$ [5]

$\therefore$ Trajectory is <u>parabolic</u> [5]

b) For parabolic trajectory

$$r = \frac{p}{1 + \cos\theta} \Rightarrow p = r(1 + \cos\theta) = 30{,}000(1 + \cos 135^\circ)$$
$$= 8786.8 \text{ km}$$

At perigee $\theta = 0$

$r_p = \frac{p}{2} = \frac{8786.8}{2} = 4393.4$ km $< r_e = 6368$ km [5]

$\therefore$ <u>Meteoroid will collide</u> [5]

c) $r = \dfrac{p}{1+\cos\theta}$

At impact

$$\cos\theta_2 = \frac{p}{r_2} - 1 = \frac{8786.8}{6368} - 1 = 0.379837$$

$$\theta_2 = 1.18118 \text{ rad} = 67.6764° \quad [5]$$

$$D_1 = \sqrt{p}\tan\frac{\theta_1}{2} = \sqrt{8786.8}\tan\left(\frac{135°}{2}\right) = 226.303 \text{ km}^{1/2} \quad [3]$$

$$D_2 = \sqrt{p}\tan\frac{\theta_2}{2} = \sqrt{8786.8}\tan\left(\frac{67.6764°}{2}\right) = 62.8429 \text{ km}^{1/2} \quad [3]$$

$$t_1 = \frac{1}{2\sqrt{\mu}}\left(pD_1 + \frac{D_1^3}{3}\right) = \frac{1}{2\sqrt{3.986\times10^5}}\left[(8786.8)(226.303) + \frac{(226.303)^3}{3}\right]$$

$$= 4634.29 \text{ sec} \quad [3]$$

$$t_2 = \frac{1}{2\sqrt{\mu}}\left(pD_2 + \frac{D_2^3}{3}\right) = \frac{1}{2\sqrt{3.986\times10^5}}\left[(8786.8)(62.8429) + \frac{(62.8429)^3}{3}\right]$$

$$= 502.79 \text{ sec} \quad [3]$$

$$t_1 - t_2 = 4634 - 503 = \underline{\underline{4131 \text{ sec}}} = \underline{\underline{1.148 \text{ hr}}} \quad [3]$$

d)