ME 55600/I0200
Homework #8a: solutions

1. $\Lambda = 20 ft^3/s$
$a = 10\ ft$
$U = 10\ ft/s$

Combine the source, sink, and uniform flow:

$$\phi = \frac{\Lambda}{2\pi}\left[\frac{1}{2}\ln(r^2 + a^2 - 2ra\cos\theta) - \frac{1}{2}\ln(r^2 + a^2 + 2ra\cos\theta)\right] - Ux$$

Since $r^2 = x^2 + y^2$ and $r\cos\theta = x$, then

$$\phi = \frac{\Lambda}{4\pi}[\ln(x^2 + y^2 + a^2 - 2ax) - \ln(x^2 + y^2 + a^2 + 2ax)] - Ux$$

Differentiate to determine velocity components:

$$u = -\frac{\partial\phi}{\partial x} = -\frac{\Lambda}{4\pi}\left[\frac{2(x-a)}{x^2 + y^2 + a^2 - 2ax} - \frac{2(x+a)}{x^2 + y^2 + a^2 + 2ax}\right] - U$$

$$v = -\frac{\partial\phi}{\partial y} = -\frac{\Lambda}{4\pi}\left[\frac{2y}{x^2 + y^2 + a^2 - 2ax} - \frac{2y}{x^2 + y^2 + a^2 + 2ax}\right]$$

Substitute point (15,15) and calculate the results:

$$\left.\begin{array}{l} u = 10.029\ ft/s \\ \\ v = 0.134\ ft/s \end{array}\right\} \quad |V| = \sqrt{u^2 + v^2} = 10.03\ ft/s$$

2. (a) Combining the two vortices, the stream function is:

$$\psi = -\frac{\Lambda}{2\pi}\ln r_1 + \frac{\Lambda}{2\pi}\ln r_2$$

or

$$\psi == \frac{\Lambda}{4\pi}[\ln(x^2 + y^2 + a^2 - 2ax) - \ln(x^2 + y^2 + a^2 + 2ax)]$$

Differentiate to get velocity components:

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\Lambda}{4\pi}\left[\frac{2y}{x^2 + y^2 + a^2 - 2ax} - \frac{2y}{x^2 + y^2 + a^2 + 2ax}\right]$$

$$v = \frac{\partial \psi}{\partial y} = \frac{\Lambda}{4\pi}\left[\frac{2(x-a)}{x^2+y^2+a^2-2ax} - \frac{2(x+a)}{x^2+y^2+a^2+2ax}\right]$$

(b) At the plane of symmetry $x = 0$, the two velocity components are

$$u = 0, \qquad v = -\frac{\Lambda a}{\pi(y^2+a^2)}$$

To determine the pressure distribution, use Bernoulli's equation $\frac{p}{\rho} + U^2 = const$
The constant is obtained from the conditions far from the vortices:

At $y \to \infty,\ p \to p_\infty \quad U \to 0,$ therefore $\frac{p}{\rho} + U^2 = \frac{p_\infty}{\rho}$

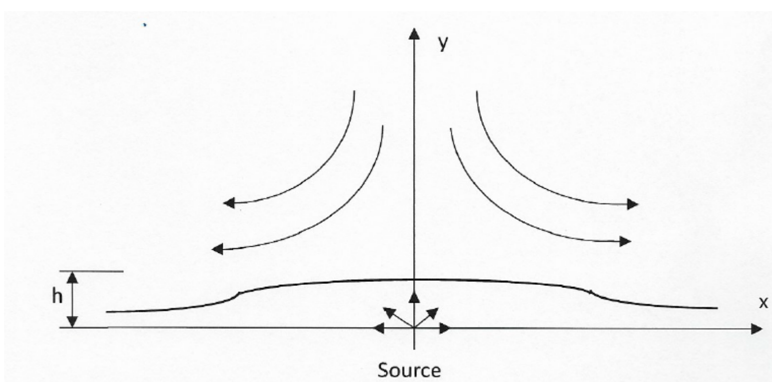
Since $U = v$ at $x = 0$, the pressure distribution is:

$$\frac{p_\infty - p}{\rho} = \left[\frac{\Lambda a}{\pi(y^2+a^2)}\right]^2$$

The pressure decreases toward the center of the vortex.

3. The stream function is a combination of the stagnation flow and the source

$$\psi = Axy + \frac{\Lambda}{2\pi}\theta = \frac{A}{2}r^2 \sin 2\theta + \frac{\Lambda}{2\pi}\theta$$



The stagnation point at the bump will be at $x = 0, y = h$ or $\theta = \frac{\pi}{2}, r = h$.

In cylindrical coordinates, the velocity components are:

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial \theta} = -\left(Ar\cos 2\theta + \frac{\Lambda}{2\pi r}\right)$$

$$v = \frac{\partial \psi}{\partial r} = Ar \sin 2\theta$$

At the stagnation point, $\theta = \frac{\pi}{2}, r = h$ both velocity components are zero. Since $v = 0$, the radial velocity components is zero when:

$$-Ah + \frac{\Lambda}{2\pi h} = 0 \qquad\qquad h = \sqrt{\frac{\Lambda}{2\pi A}}$$