ME 57200 Aerodynamic Design

Lecture #23: Oblique Waves and Expansion Waves

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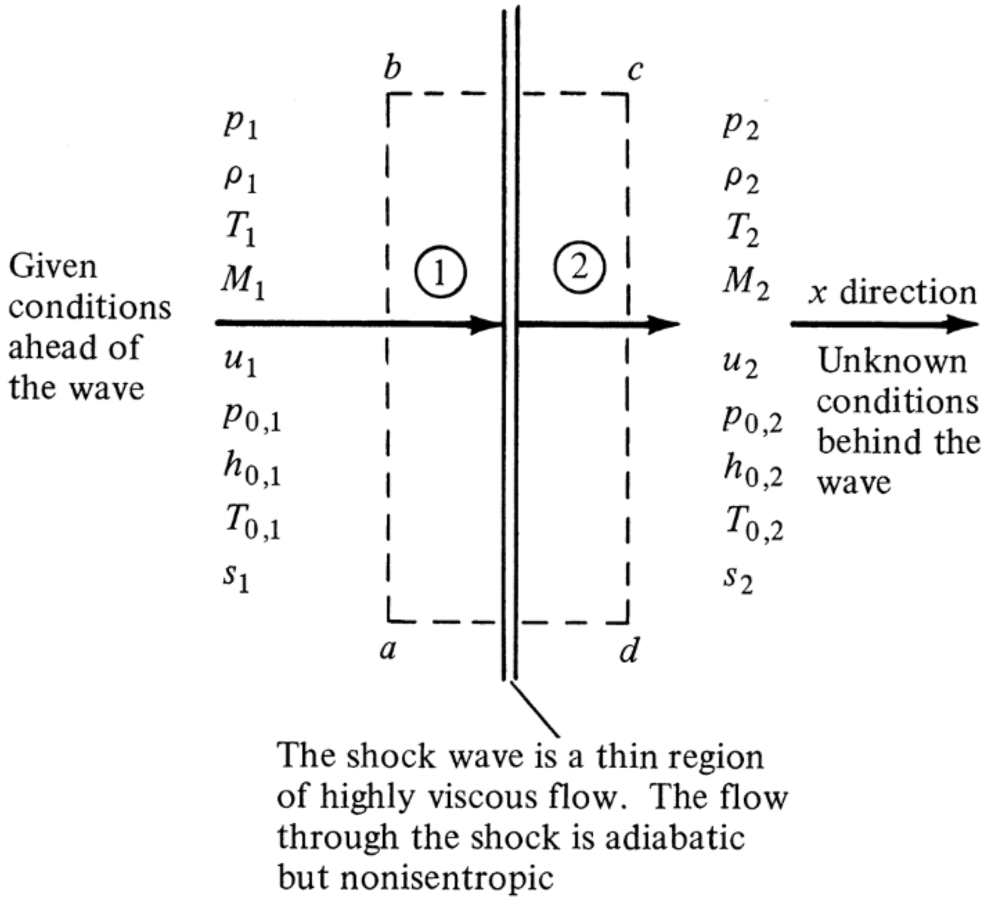
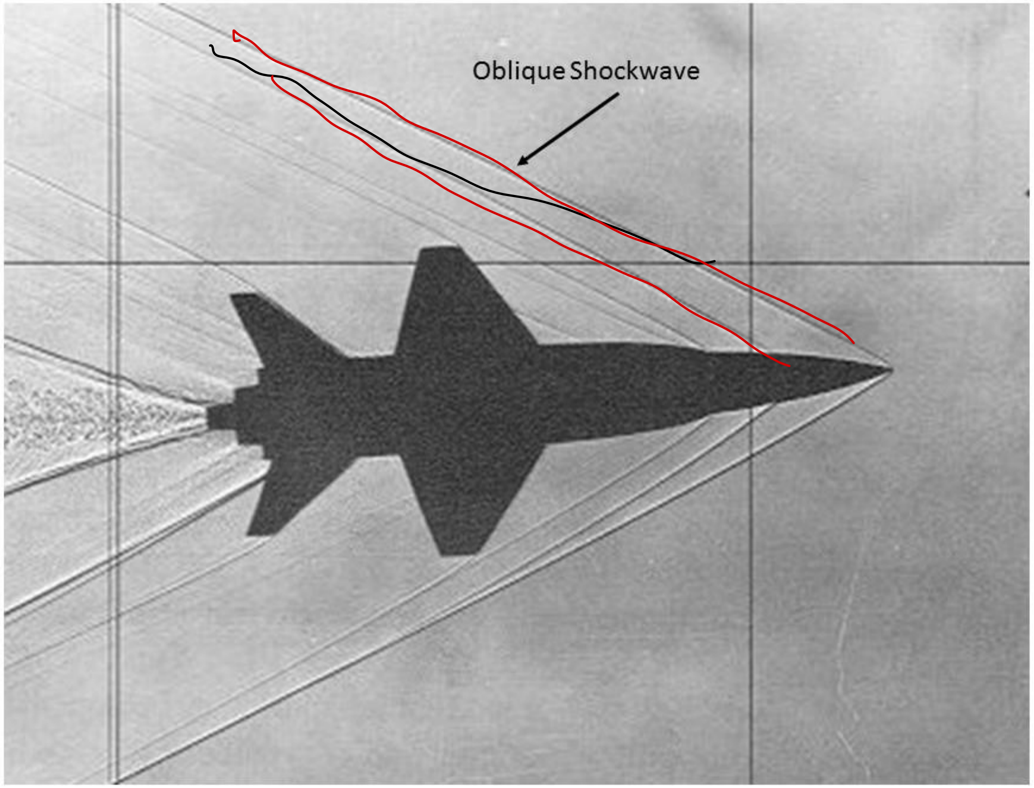
Shock Waves

Normal Shock Waves

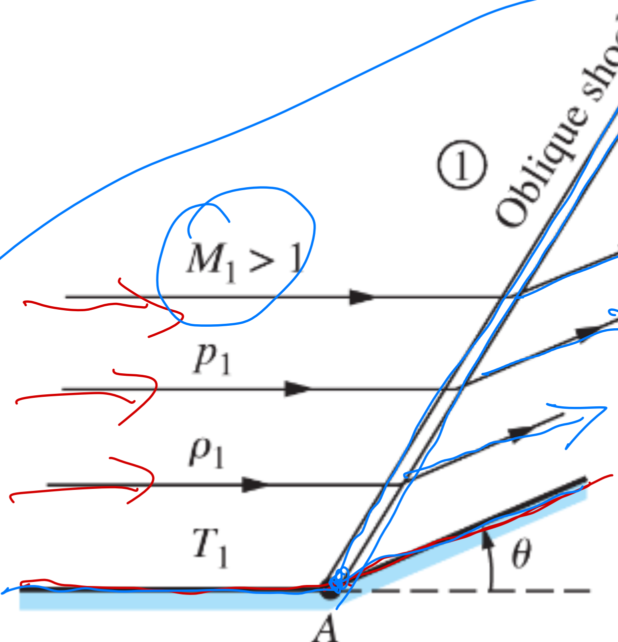


Shock Waves

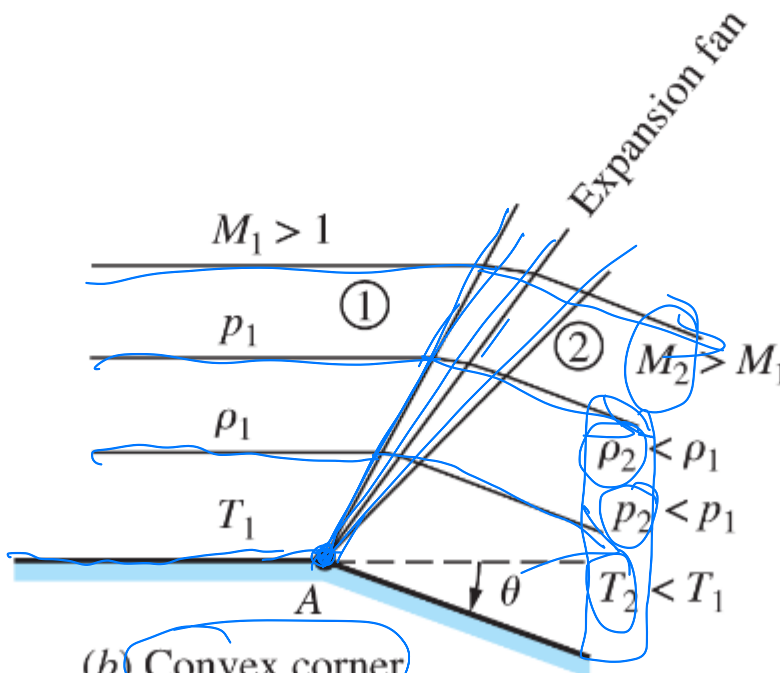
Oblique Shock Waves



Oblique Shock VS. Expansion Wave



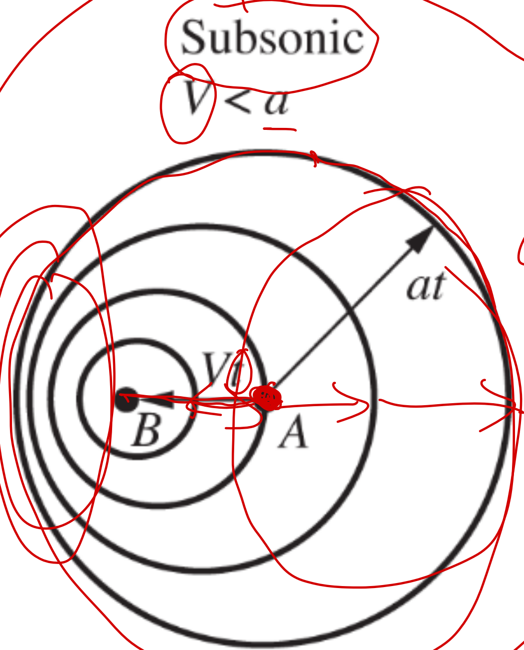
(a) Concave corner

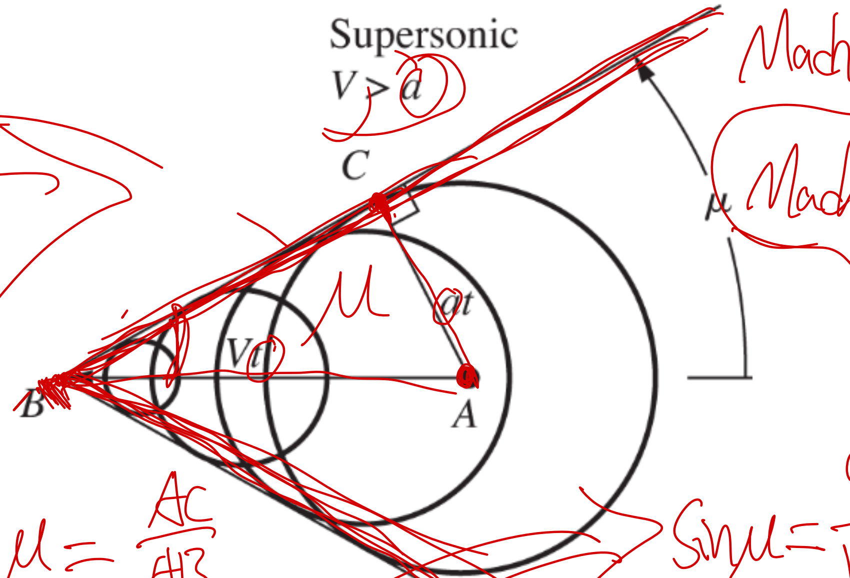


(b) Convex corner

Shock Waves

<u>Source of Oblique Waves</u>





$\sin \mu = \frac{AC}{AB}$

$AC = a \cdot t$

$AB = V \cdot t$

$\sin \mu = \frac{at}{Vt}$

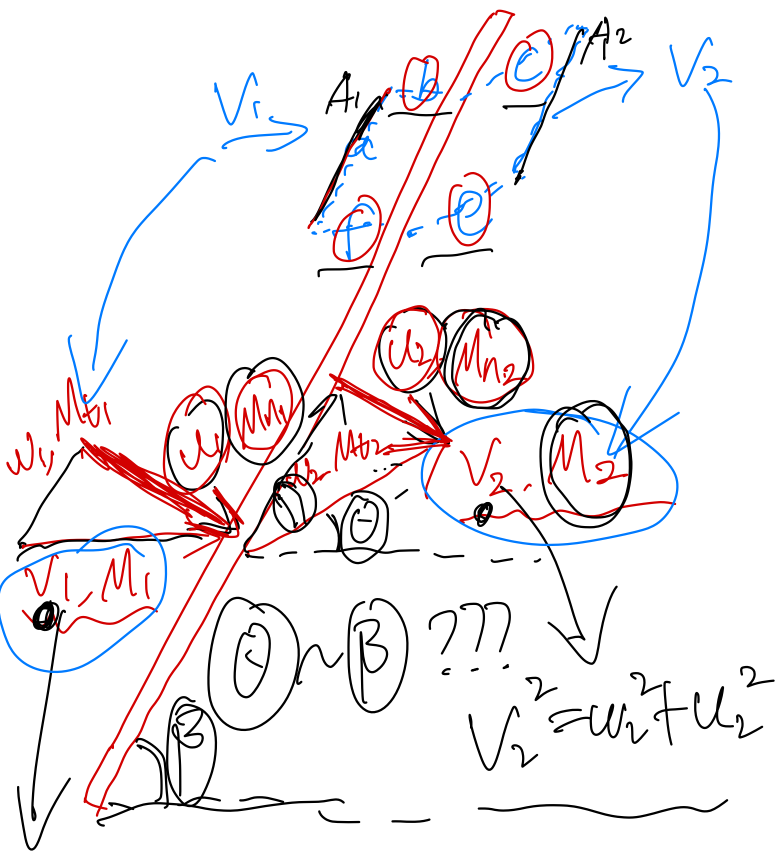
$= \frac{a}{V}$

$$\sin \mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$$

$$\boxed{\mu = \sin^{-1} \frac{1}{M}}$$

$M = \frac{V}{a}$

Oblique Shock Waves



$V_1^2 = w_1^2 + u_1^2$

① Continuity Equation:

$$\oiint_S \rho \vec{V} \cdot d\vec{S} = 0$$

$$-\rho_1 u_1 A_1 + \rho_2 u_2 A_2 = 0$$

$$\Rightarrow \boxed{\rho_1 u_1 = \rho_2 u_2}$$

② Momentum Equation:

$$\oiint_S (\rho \vec{V} \cdot d\vec{S}) w = -\oiint_S (p\, dS)_{tangential}$$

$$-(\rho_1 u_1 A_1) w_1 + (\rho_2 u_2 A_2) \cdot w_2 = 0$$

$$\Rightarrow \boxed{w_1 = w_2}$$

Oblique Shock Waves

$$\oiint_S (\rho \vec{V} \cdot d\vec{S})\, \underline{u} = -\oiint_S (p\, dS)_{normal}$$

$$-(\rho_1 u_1 A_1)\cdot u_1 + (\rho_2 u_2 A_2)\cdot u_2 = -(p_1 A_2 + p_2 A_2)$$

$$\Rightarrow \boxed{p_1 + \rho_1 u_1^2 = \underline{p_2} + \underline{\rho_2 u_2^2}}$$

Energy Equation: $\oiint_S \rho\left(e + \frac{V^2}{2}\right)\cdot \vec{V} \cdot d\vec{S} = -\oiint_S p \vec{V} \cdot d\vec{S}$

For the flow tangent to faces $\vec{V} \cdot d\vec{S} = 0$

$$\Rightarrow -\rho_1\left(e_1 + \frac{V_1^2}{2}\right) u_1 A_1 + \rho_2\left(e_2 + \frac{V_2^2}{2}\right)\cdot u_2 A_2 = -(-p_1 u_1 A_1 + p_2 u_2 A_2)$$

$$\Rightarrow -\rho_1 u_1\left(\boxed{e_1 + \frac{p_1}{\rho_1}} + \frac{V_1^2}{2}\right) + \rho_2 u_2\left(\boxed{e_2 + \frac{p_2}{\rho_2}} + \frac{V_2^2}{2}\right) = 0$$

$e + pv = h$

Oblique Shock Waves

$$\Rightarrow \rho_1 u_1 \left(h_1 + \frac{V_1^2}{2}\right) = \rho_2 u_2 \left(h_2 + \frac{V_2^2}{2}\right)$$

$$\Rightarrow h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_0$$

※: Total enthalpy is constant across the shock wave

$h_0 = C_p \cdot T_0 \Rightarrow$ the Total temperature is constant across the shock wave.

$$\Rightarrow h_1 + \frac{1}{2}\left(\cancel{w_1^2} + u_1^2\right) = h_2 + \frac{1}{2}\left(\cancel{w_2^2} + u_2^2\right)$$

$$\Rightarrow \boxed{h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}}$$

Oblique Shock Waves

* The changes across an oblique shock wave are governed only by the component of velocity normal to the wave.

$$M_{n1} = M_1 \cdot \sin\beta$$

$$M_{n2}^2 = \frac{1 + [(\gamma-1)/2] M_{n,1}^2}{\gamma M_{n,1}^2 - (\gamma-1)/2}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_{n,1}^2}{2 + (\gamma-1) M_{n,1}^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n,1}^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_2}\frac{\rho_1}{\rho_2}$$

$$\boxed{M_2 = \frac{M_{n,2}}{\sin(\beta-\theta)}}$$

Oblique Shock Waves

※: M_{n1} depends on both M_1 and β

⇒ The changes across an oblique shock wave depend on two parameters — M_1, and β.

※ normal shock is the condition when $\beta = \pi/2$

$\tan\beta = \frac{u_1}{w_1}$, $\tan(\beta-\theta) = \frac{u_2}{w_2}$, $w_1 = w_2$

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{u_2}{u_1} = \frac{\rho_1}{\rho_2}$$

Oblique Shock Waves

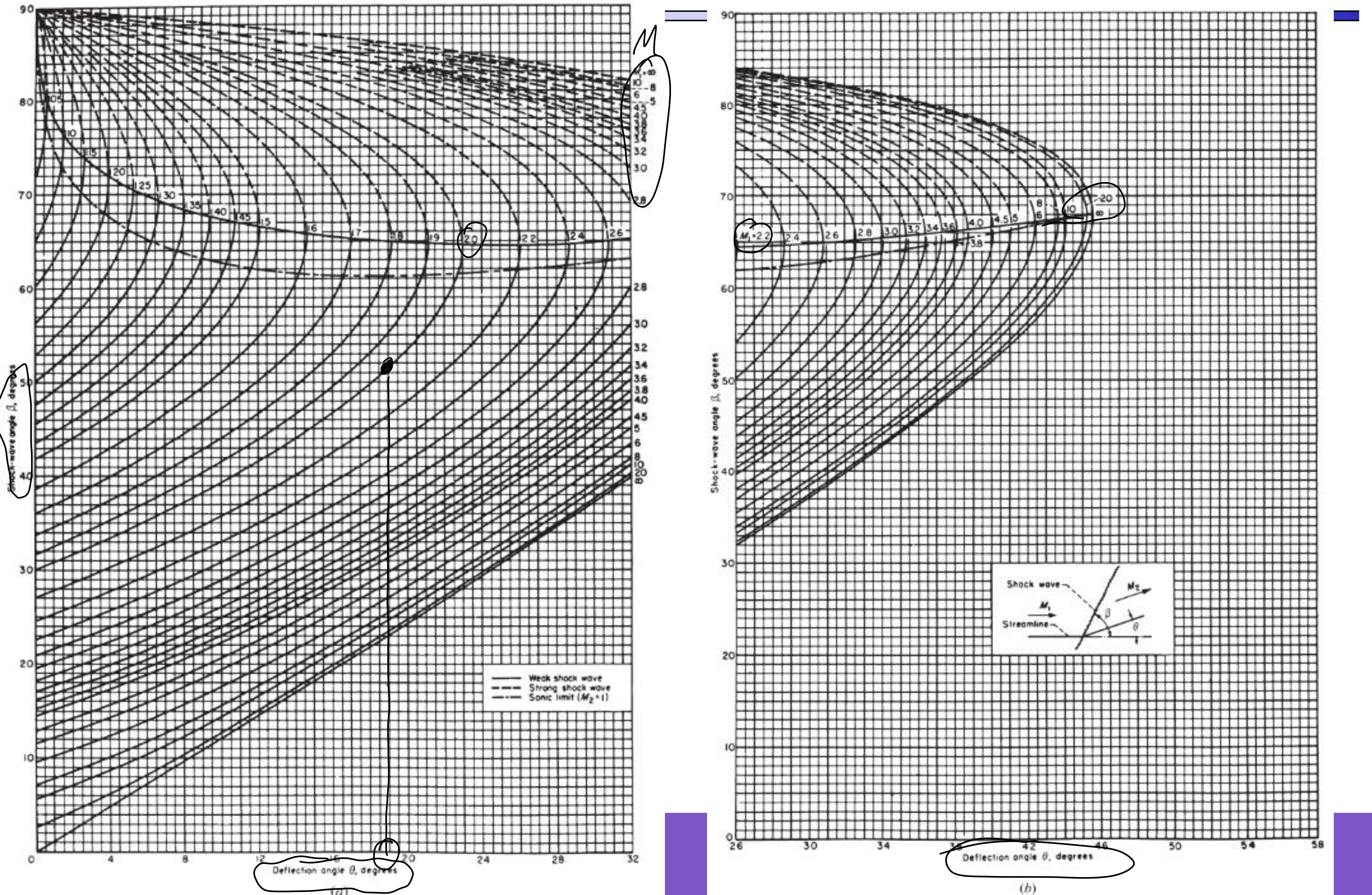
$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{2+(\gamma-1)M_1^2\sin^2\beta}{(\gamma+1)M_1^2\sin^2\beta}$$

$$\Rightarrow \boxed{\tan\theta = 2\cdot\cot\beta\,\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}}$$

M_1 and β

$\beta = 90^\circ$ for normal shock

Oblique Shock Waves



Shock Waves

APPENDIX B

Normal Shock Properties

M	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{0_2}}{p_{0_1}}$	$\frac{p_{0_2}}{p_1}$	M_2
0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1893 + 01	0.1000 + 01
0.1020 + 01	0.1047 + 01	0.1033 + 01	0.1013 + 01	0.1000 + 01	0.1938 + 01	0.9805 + 00
0.1040 + 01	0.1095 + 01	0.1067 + 01	0.1026 + 01	0.9999 + 00	0.1984 + 01	0.9620 + 00
0.1060 + 01	0.1144 + 01	0.1101 + 01	0.1039 + 01	0.9998 + 00	0.2032 + 01	0.9444 + 00
0.1080 + 01	0.1194 + 01	0.1135 + 01	0.1052 + 01	0.9994 + 01	0.2082 + 01	0.9277 + 00
0.1100 + 01	0.1245 + 01	0.1169 + 01	0.1065 + 01	0.9989 + 00	0.2133 + 01	0.9118 + 00
0.1120 + 01	0.1297 + 01	0.1203 + 01	0.1078 + 01	0.9982 + 00	0.2185 + 01	0.8966 + 00
0.1140 + 01	0.1350 + 01	0.1238 + 01	0.1090 + 01	0.9973 + 00	0.2239 + 01	0.8820 + 00