ME 57200 Aerodynamic Design

Lecture #12: Elemental Flows

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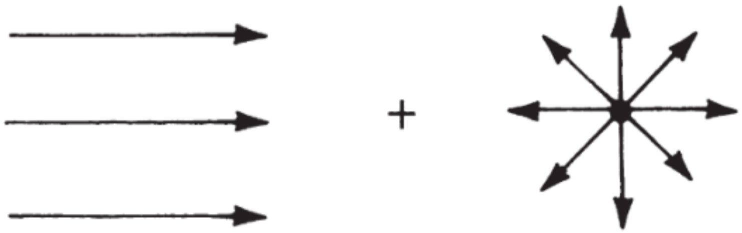
Elementary Flows

- **Combination of a uniform flow with a source flow**

Consider a polar coordinate system with a source of strength Λ located at the origin. Superimpose on this flow a uniform stream with velocity V_∞ moving from left to right.



Uniform stream

$$\psi = V_\infty r \sin\theta$$

Source

$$\psi = \frac{\Lambda}{2\pi}\theta$$

$$\boxed{\nabla^2 \phi = 0}$$

$\varphi_1, \varphi_2 \ldots, \varphi_n$ represent *n* separate solutions

$\varphi = \varphi_1 + \varphi_2 \ldots + \varphi_n$ is also a solution

Elementary Flows

- **Combination of a uniform flow with a source flow**

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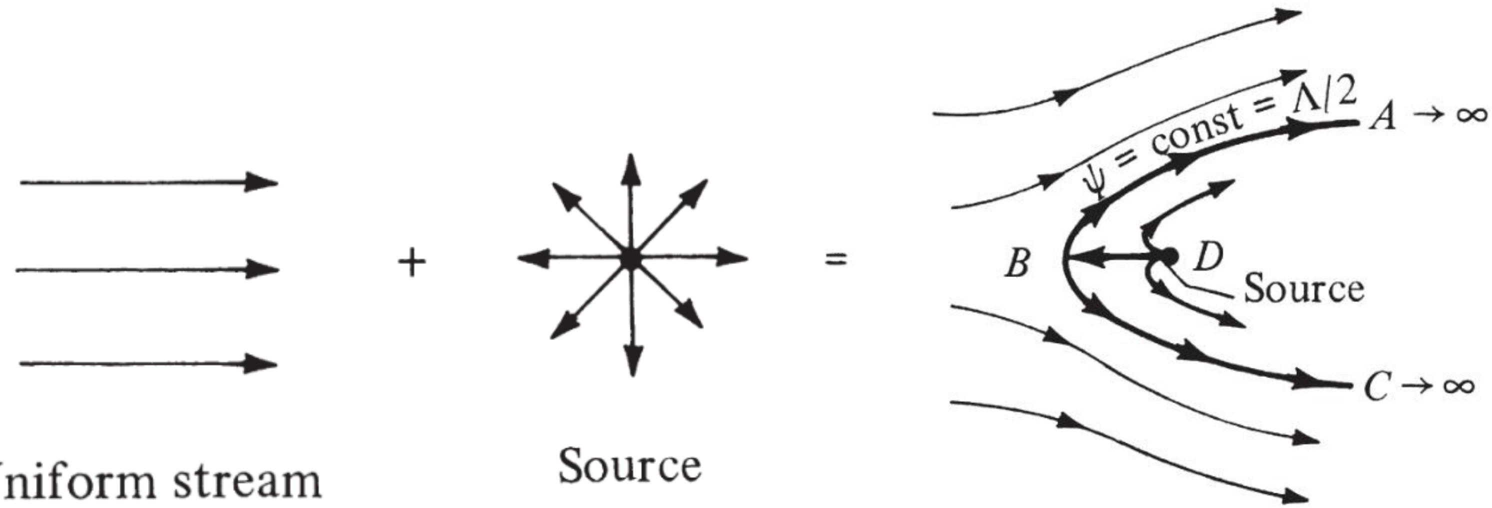


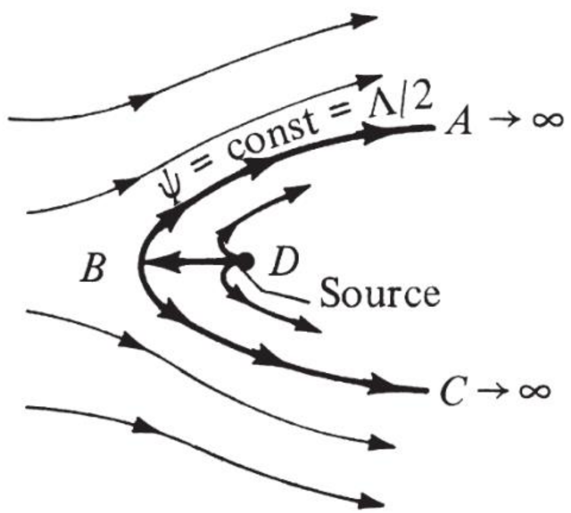
$$\psi = V_\infty r \sin\theta$$

$$\psi = \frac{\Lambda}{2\pi}\theta$$

$$\boxed{\psi = V_\infty r \sin\theta + \frac{\Lambda}{2\pi}\theta}$$

Elementary Flows

- **Combination of a uniform flow with a source flow**



$$\boxed{\psi = V_\infty r \sin\theta + \frac{\Lambda}{2\pi}\theta}$$

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = V_\infty \cos\theta + \frac{\Lambda}{2\pi r}$$

$$V_\theta = -\frac{\partial \psi}{\partial r} = -V_\infty \sin\theta$$

To find the stagnation point

$$V_\infty \cos\theta + \frac{\Lambda}{2\pi r} = 0$$

$$\boxed{(r, \theta) = (\Lambda/2\pi V_\infty, \pi)}$$

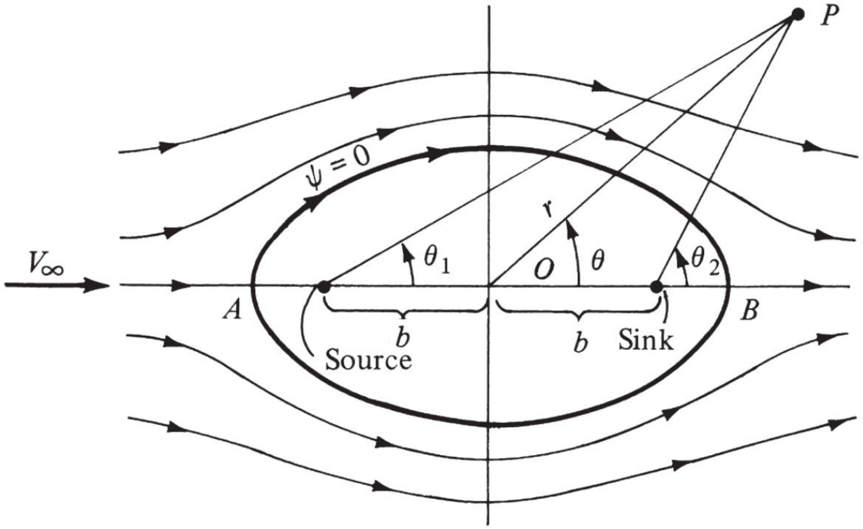
$$V_\infty \sin\theta = 0$$

The streamline that goes through the stagnation point

$$\boxed{\psi = V_\infty \frac{\Lambda}{2\pi V_\infty} \sin\pi + \frac{\Lambda}{2\pi}\pi = \frac{\Lambda}{2}}$$

Elementary Flows

- **Combination of a uniform flow with a source flow and a sink flow**



Rankine oval

$$\psi = V_\infty r \sin\theta + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

$$OA = OB = \sqrt{b^2 + \frac{\Lambda b}{\pi V_\infty}}$$

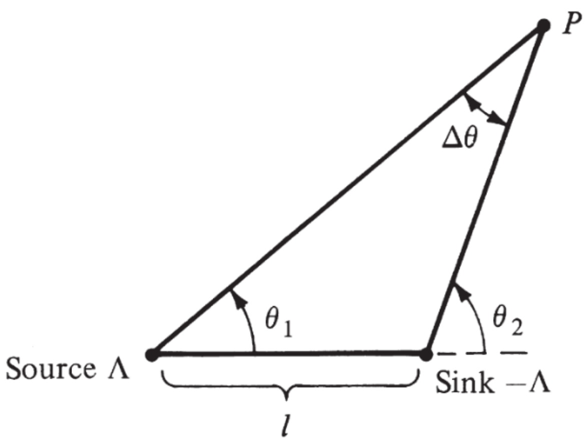
The stagnation streamline is given by

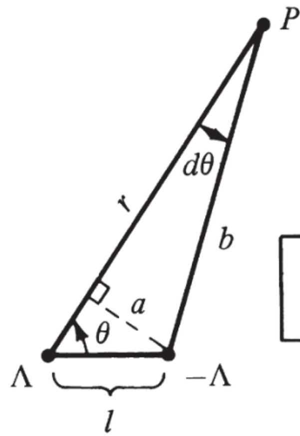
$$\psi = 0,$$

$$V_\infty r \sin\theta + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2) = 0$$

Elementary Flows

- **Doublet Flow**





$$\boxed{\begin{array}{l} l \to 0 \\ l\Lambda = \text{const} \end{array}}$$

$$\psi = \frac{\Lambda}{2\pi}(\theta_1 - \theta_2) = -\frac{\Lambda}{2\pi}\Delta\theta$$

$$\psi = \lim_{\substack{l \to 0 \\ \kappa = l\Lambda = \text{const}}} \left(-\frac{\Lambda}{2\pi} d\theta\right)$$

$$a = l \sin\theta$$

$$b = r - l\cos\theta$$

$$d\theta = \frac{a}{b}$$

$$d\theta = \frac{a}{b} = \frac{l\sin\theta}{r - l\cos\theta}$$

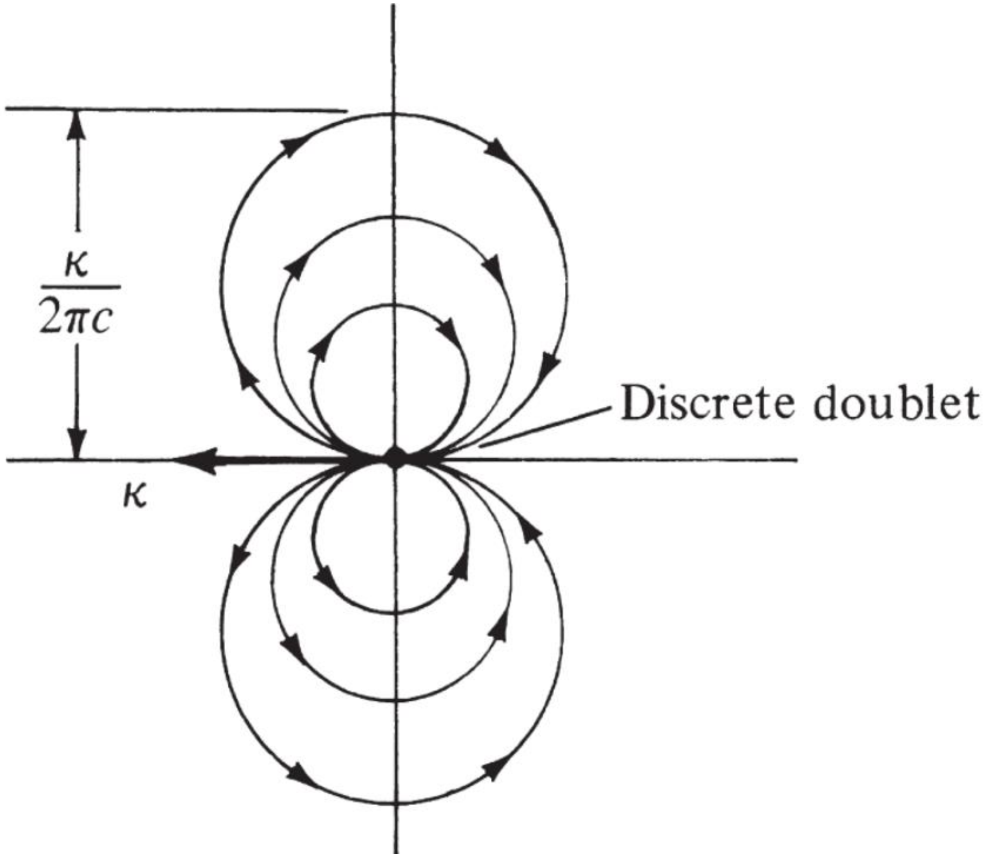
$$\psi = \lim_{\substack{l \to 0 \\ \kappa = \text{const}}} \left(-\frac{\Lambda}{2\pi}\frac{l\sin\theta}{r - l\cos\theta}\right)$$

$$\boxed{\psi = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r}}$$

$$\boxed{\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r}}$$

Elementary Flows

- **Doublet Flow**

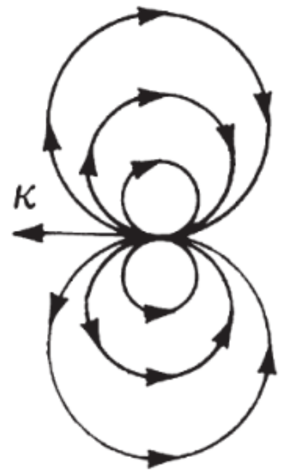


Elementary Flows

- **Nonlifting Flow over a Circular Cylinder**

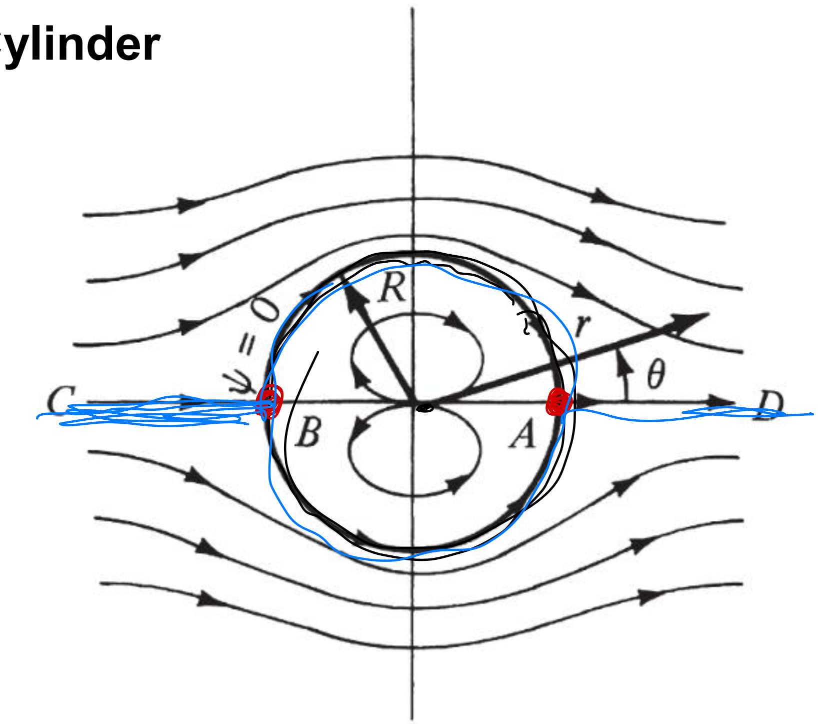
Uniform flow

$$\psi = V_\infty r \sin\theta$$



Doublet

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}$$



Flow over a cylinder

$$\psi = V_\infty r \sin\theta - \frac{\kappa}{2\pi}\frac{\sin\theta}{r}$$

Elementary Flows

$$\psi = V_\infty r \sin\theta - \frac{K}{2\pi}\frac{\sin\theta}{r}$$

$$\psi = V_\infty r \sin\theta \left(1 - \frac{K}{2\pi V_\infty r^2}\right)$$

$$R^2 = \frac{K}{2\pi V_\infty} \Rightarrow \psi = V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$\Rightarrow \begin{cases} V_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} = \dfrac{1}{\cancel{r}}\left(V_\infty \cancel{r} \cos\theta\right)\left(1 - \dfrac{R^2}{r^2}\right) = \underbrace{\left(1 - \dfrac{R^2}{r^2}\right) \cdot V_\infty \cos\theta}_{=\,0} \\ V_\theta = -\dfrac{\partial \psi}{\partial r} = -\left(1 + \dfrac{R^2}{r^2}\right) V_\infty \sin\theta \end{cases}$$

$$\begin{cases} V_r = 0 \\ V_\theta = 0 \end{cases} \Rightarrow \quad r = R, \quad \theta = 0, \pi$$

$$\Rightarrow (R, 0), \quad (R, \pi)$$

Elementary Flows

$\Rightarrow$ The equation of the stagnation streamline

$$\psi = (V_\infty r \sin\theta)\cdot\left(1 - \frac{R^2}{r^2}\right) = 0$$

$\Rightarrow$ ① $r = R$ for all values of θ, $R^2 = \frac{\kappa}{2\pi V_\infty}$

$\Rightarrow$ ② $\sin\theta = 0 \Rightarrow \theta = 0, \pi,$ for all values of r

$\Rightarrow$ the entire horizontal axis

Elementary Flows

$\Rightarrow$ { All the flow inside $\psi = 0$ comes from doublet
All the flow outside $\psi = 0$ comes from uniform flow.

※ The inviscid irrotational incompressible flow over a circular cylinder of radius R can be synthesized by adding a uniform flow with velocity V_∞ and a doublet of strength K, where R is related to V_∞ and K

$$R = \sqrt{\frac{K}{2\pi V_\infty}}$$

Elementary Flows

$$\begin{cases} V_r = \left(1 - \dfrac{R^2}{r^2}\right) V_\infty \cos\theta \\ V_\theta = -\left(1 + \dfrac{R^2}{r^2}\right) V_\infty \sin\theta \end{cases}$$

at $r = R$

$$\Rightarrow \begin{cases} V_r = 0 \\ V_\theta = -2 V_\infty \sin\theta \end{cases}$$

$$p + \frac{1}{2}\rho V^2 = p_\infty$$

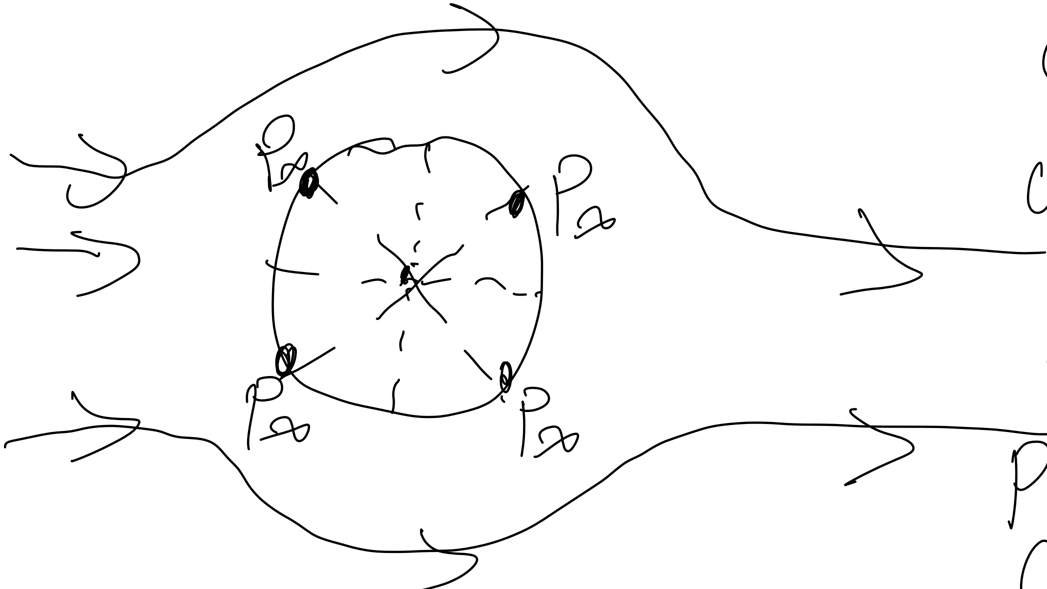
Elementary Flows

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = \frac{(\cancel{p_0} - \frac{1}{2}\rho V^2) - (\cancel{p_0} - \frac{1}{2}\rho V_\infty^2)}{\frac{1}{2}\rho V_\infty^2}$$

$$C_p = 1 - \frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

$$\Rightarrow C_p = 1 - \left(\frac{-2\cancel{V_\infty}\sin\theta}{\cancel{V_\infty}}\right)^2 = 1 - 4\sin^2\theta$$

Elementary Flows



Calculate the locations on the surface of the cylinder where the surface pressure equals the freestream pressure p_∞

$$\Rightarrow C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 0 \Rightarrow 1 - 4\sin^2\theta = 0$$

$$\sin^2\theta = \frac{1}{4} \Rightarrow \theta = 30^\circ, 330^\circ$$

$$\sin\theta = \pm\frac{1}{2} \qquad 150^\circ, 210^\circ$$

Elementary Flows

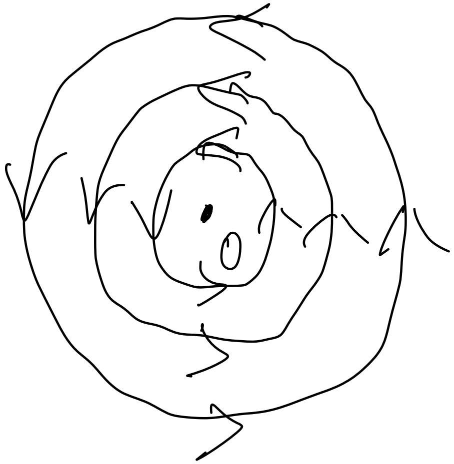
Aerodynamic forces: $\tau = 0$, $C_p = 1 - 4\sin^2\theta$

$$\Rightarrow \begin{cases} C_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\,dx = 0 \\ C_a = \frac{1}{c}\int_0^c (C_{p,u} - C_{p,l})\,dy = 0 \end{cases}$$

$\Rightarrow$ no lift, no drag, Not existing in real life.

Elementary Flows

Vortex Flow: a flow where all the streamlines are concentric circles about a given point. The velocity along any given circular streamline is constant but vary from one to another inversely with distance from the common center.



$$\begin{cases} V_r = 0 \\ V_\theta = \dfrac{Const}{r} \end{cases} \Rightarrow \begin{cases} \underbrace{\nabla \cdot \vec{V} = 0}_{} \text{ "Continuity equation"} \\ \nabla \times \vec{V} = 0 \text{ "Irrotational" except "0"} \end{cases}$$

In-Class Quiz