ME 51500/I5800 L
75 min.

Exam 2
Closed Book, Open Notes*

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12/05/2023

*** Only original handwritten notes and homeworks are allowed. Photocopied notes and homework solution sheets are not permitted. Except for a hand calculator, no cell phone or electronic equipment of any kind is allowed.**

Show all work and give units in final answers.

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[40] 1. A reconnaissance satellite is in a 300 km × 500 km orbit about the earth. It is desired to circularize the orbit to the apogee height and change the inclination by 10° using a single burn.

a) Draw the velocity diagram for the burn and label all relevant velocities and angles.
b) Calculate the magnitude of the required ΔV.
c) Calculate the angle between the required ΔV and the direction of travel just before the burn. Be sure to show this angle on the velocity diagram you drew for part (a).
d) If the satellite's mass (including the mass of the fuel) is 1000 kg and it is equipped with a constant thrust engine having a specific impulse of 270 sec, calculate the mass of fuel required for the maneuver.

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[30] 2. Consider the vertical ascent of a rocket having initial mass m_0 in a constant gravity field g, powered by an engine with specific impulse I_{sp} burning fuel at a constant rate $\dot{m}_e$. The density variation in the atmosphere is given by $\rho = \rho_0 e^{-\frac{h}{H}}$ where h is the altitude and the surface density ρ_0 and scale height H are constant and assumed known. The drag coefficient C_D and cross sectional area A of the vehicle in the flight direction may also be assumed constant and known. Write the system of first-order ordinary differential equations, with initial conditions, which need to be solved to determine the velocity and altitude of the vehicle as a function of time. DO NOT SOLVE THE EQUATIONS.

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[30] 3. A four-foot diameter spherical satellite weighing 2000 lb orbits the earth at an inclination of 30° with a perigee altitude of 800 nautical miles and an eccentricity of 0.1.

a) Estimate the orbital lifetime of the satellite in years.
b) Determine the time-averaged rate of node regression in degrees per day (including direction) due to
   i) The earth's oblateness.
   ii) Atmospheric drag.

Physical constants

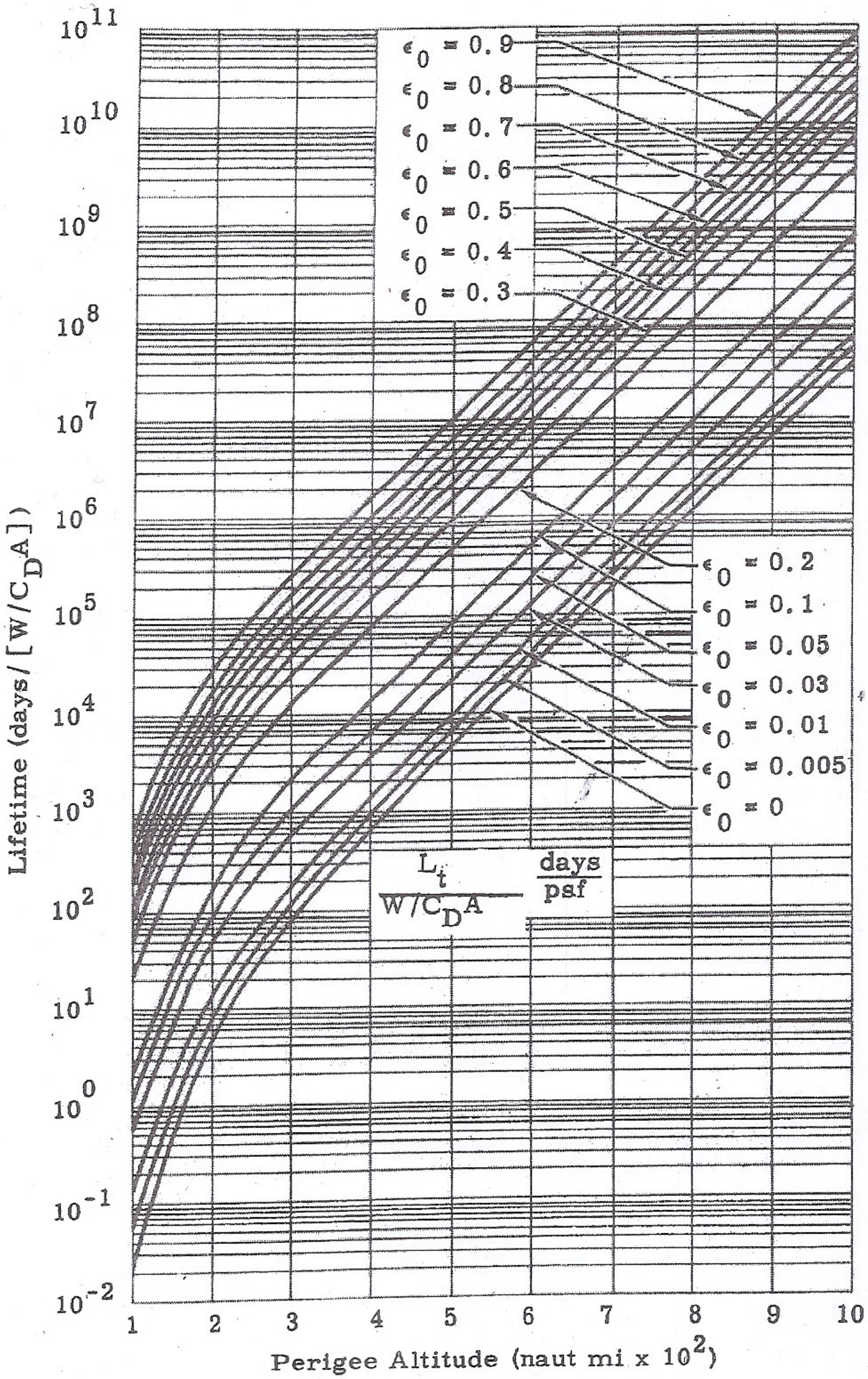
The Earth

Mean radius = 6368 km

Equatorial radius = 6378 km

$\mu = 3.986 \times 10^5$ km³/sec²

$J_2 = 1.08263 \times 10^{-3}$



$\epsilon_0 = 0.9$
$\epsilon_0 = 0.8$
$\epsilon_0 = 0.7$
$\epsilon_0 = 0.6$
$\epsilon_0 = 0.5$
$\epsilon_0 = 0.4$
$\epsilon_0 = 0.3$
$\epsilon_0 = 0.2$
$\epsilon_0 = 0.1$
$\epsilon_0 = 0.05$
$\epsilon_0 = 0.03$
$\epsilon_0 = 0.01$
$\epsilon_0 = 0.005$
$\epsilon_0 = 0$
$\frac{L_t}{W/C_D A} \; \frac{\text{days}}{\text{psf}}$
Lifetime (days/[W/C_DA])
10^{11} 10^{10} 10^9 10^8 10^7 10^6 10^5 10^4 10^3 10^2 10^1 10^0 10^{-1} 10^{-2}
1 2 3 4 5 6 7 8 9 10
Perigee Altitude (naut mi x 10^2)

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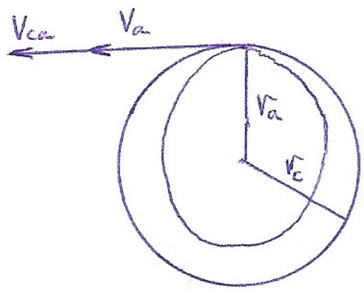
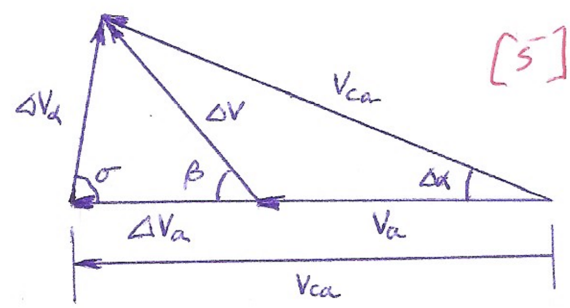
1) a)





[5]

b) $r_c = r_a = h_a + r_e = 500 + 6368 = 6868 \text{ km}$ [4]

$r_p = h_p + r_e = 300 + 6368 = 6668 \text{ km}$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{6868 - 6668}{6868 + 6668} = 0.014775 \quad [3]$$

$$V_{ca} = \sqrt{\frac{\mu}{r_c}} = \sqrt{\frac{3.986 \times 10^5}{6868}} = 7.61822 \frac{\text{km}}{\text{sec}} \quad [3]$$

$$V_a = V_{ca}\sqrt{1-e} = 7.61822\sqrt{1 - 0.014775} = 7.56173 \frac{\text{km}}{\text{sec}} \quad [3]$$

$$\Delta V_a = V_{ca} - V_a = 7.61822 - 7.56173 = 0.05649 \frac{\text{km}}{\text{sec}} \quad [3]$$

$$\Delta V_\alpha = 2 V_{ca} \sin\frac{\Delta\alpha}{2} = 2(7.61822)\sin\frac{10^\circ}{2} = 1.32794 \frac{\text{km}}{\text{sec}} \quad [3]$$

$$\sigma = 90^\circ - \frac{\Delta\alpha}{2} = 90^\circ - \frac{10^\circ}{2} = 85^\circ \quad [3]$$

$$\Delta V = \sqrt{(\Delta V_a)^2 + (\Delta V_\alpha)^2 - 2(\Delta V_a)(\Delta V_\alpha)\cos\sigma}$$

$$= \sqrt{(0.05649)^2 + (1.32794)^2 - 2(0.05649)(1.32794)\cos 85^\circ}$$

$$\boxed{\Delta V = 1.32422 \frac{km}{sec}}$$ [3]

c) $$\frac{\Delta V_\alpha}{\sin\beta} = \frac{\Delta V}{\sin\sigma}$$

$$\sin\beta = \frac{\Delta V_\alpha}{\Delta V}\sin\sigma = \frac{1.32794}{1.32422}\sin 85^\circ = 0.998999$$

$$\boxed{\beta = 1.52605\ rad = 87.4362^\circ}$$ [5]

d) $$c = I_{sp}\, g = (270)(0.00981) = 2.6487 \frac{km}{sec}$$

$$\Delta m = m_0\left(1 - e^{-\frac{\Delta V}{c}}\right) = 1000\left(1 - e^{-\frac{1.32422}{2.6487}}\right)$$

$$\boxed{\Delta m = 393.44\ kg}$$ [5]

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2) $$\boxed{m\frac{dV}{dt} = \dot{m}_e c - mg - \frac{1}{2} C_D \rho V^2 A}$$ [12] $$\boxed{V(0) = 0}$$ [3]

where $\boxed{m = m_0 - \dot{m}_e t}$ [3] $\boxed{c = I_{sp}\, g}$ [3] $\boxed{\rho = \rho_0 e^{-\frac{h}{H}}}$ [3]

$\boxed{\frac{dh}{dt} = V}$ [3] $\boxed{h(0) = 0}$ [3]

3) $D = 4\,ft$ $\quad W = 2000\,lb$ $\quad i = 30°$ $\quad h_p = 800\,n\text{-}mi$ $\quad e = 0.1$

a) $A = \frac{\pi D^2}{4} = \frac{\pi (4)^2}{4} = 12.5664\,ft^2$ [5]

$$\frac{W}{C_D A} = \frac{2000}{(2)(12.5664)} = 79.5773\,psf \quad [5]$$

For $h_p = 800\,n\text{-}mi$, $e = 0.1$

Lifetime chart gives $\frac{L_t}{W/C_D A} = 2 \times 10^7\,\frac{days}{psf}$ [5]

$$L_t = 2 \times 10^7 \left(\frac{W}{C_D A}\right) = (2 \times 10^7)(79.5773)$$

$$L_t = 1.59155 \times 10^9\,days \left(\frac{1\,year}{365.26\,days}\right)$$

$$\boxed{L_t = 4.3573 \times 10^6\,years} \quad [5]$$

b) i) Oblateness

$$r_p = a(1-e) \Rightarrow a = \frac{r_p}{1-e} = \frac{6368 + 800(1.852)}{1-0.1} = 8722\,km$$

$$\dot{\Omega}_{avg} = -\frac{3}{2}\frac{\sqrt{\mu}\,J_2 R^2}{(1-e^2)^2 a^{7/2}} \cos i$$

$$= -\frac{3}{2}\frac{\sqrt{3.986 \times 10^5}\,(1.08263 \times 10^{-3})(6378)^2}{[1-(0.1)^2]^2 (8722)^{7/2}} \cos 30°$$

$$\dot{\Omega}_{avg} = -6.86724 \times 10^{-7}\,rad/sec$$

or

$$\boxed{\dot{\Omega}_{avg} = 3.399^\circ \text{ per day to the west}}$$

[5]

ii) Atmospheric Drag

From eq (10.60) in the notes, $\frac{d\Omega}{dt}$ depends only on T_h.

For atmospheric drag, $T_h = 0$ $\therefore$ $\boxed{\frac{d\Omega}{dt} = 0}$ [5]