ME 57200 Aerodynamic Design

Lecture #2: Aerodynamic Forces

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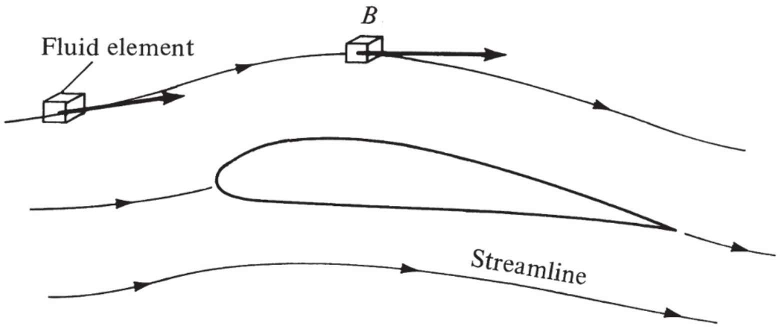
Fundamental Aerodynamic Variables

- Flow velocity:

An extremely important consideration in aerodynamics.

- Flow in motion
- A vector: has both magnitude and direction



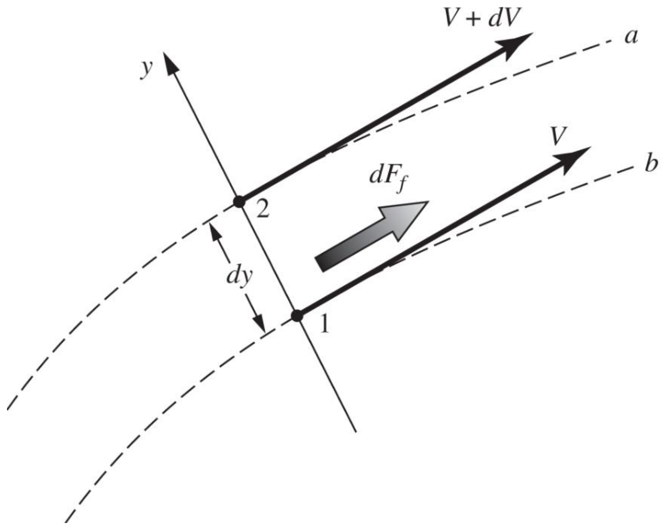
Fundamental Aerodynamic Variables

- Friction, shear stress, viscous flow:

Act tangentially along the flow direction.

- Play a role internally in a flow
- Related to velocity gradients

$$\tau = \mu \frac{dV}{dy}$$



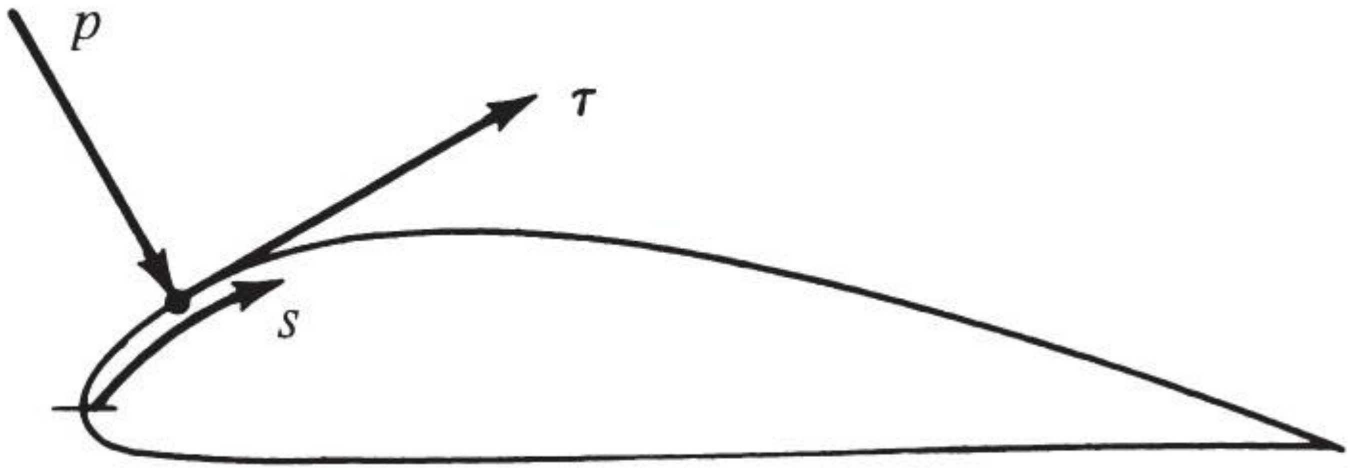
Aerodynamic Forces

- Types of forces acting on aircraft?

Aerodynamic Forces

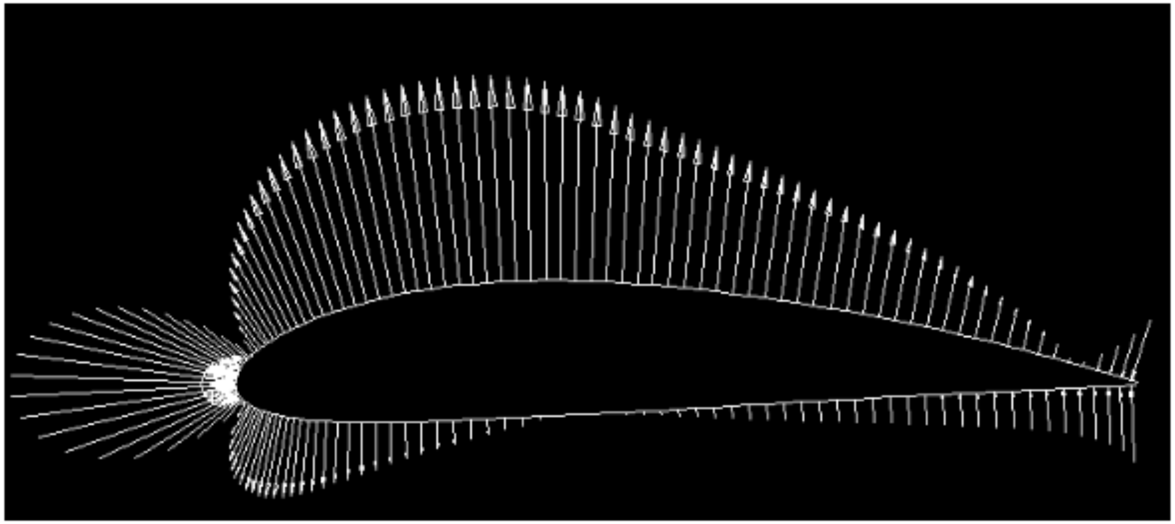
- Types of forces acting on aircraft?
  - Pressure distribution over the surface
    - Acts normal to the surface
  - Shear stress distribution over the surface
    - Acts tangential to the surface
  - No matter how complex the body shape may be, the aerodynamic forces and moments on the body are due to the above two basic sources.

Aerodynamic Forces

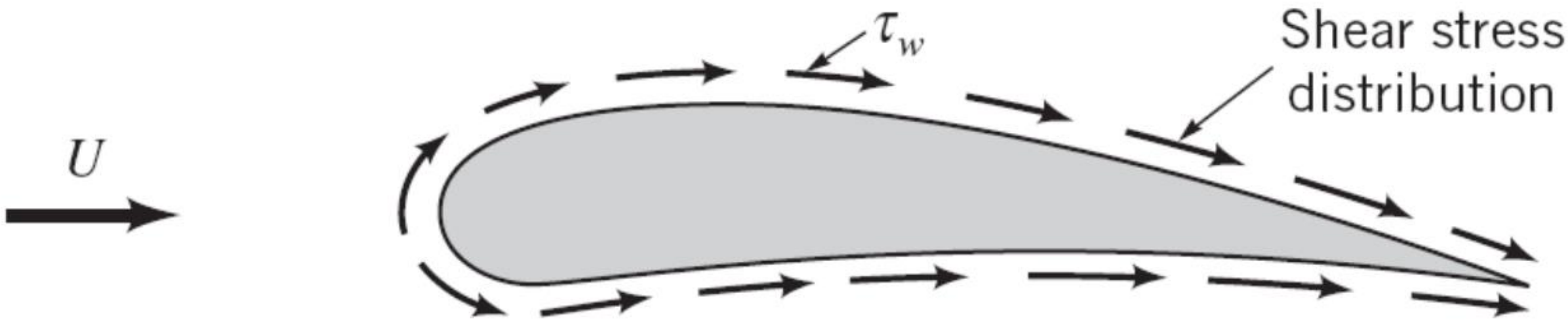


$p = p(s)$ = surface pressure distribution
$\tau = \tau(s)$ = surface shear stress distribution

Pressure Distribution around Airfoil

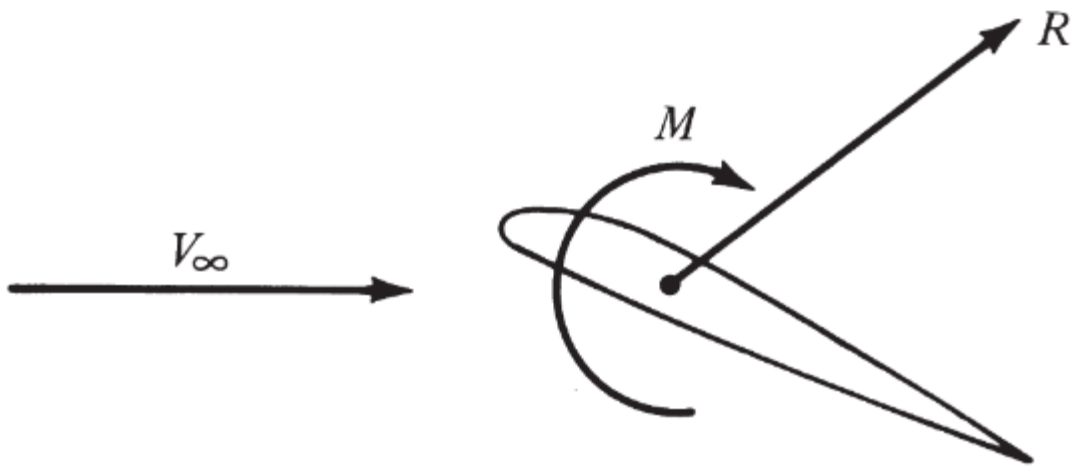


Shear Stress around Airfoil

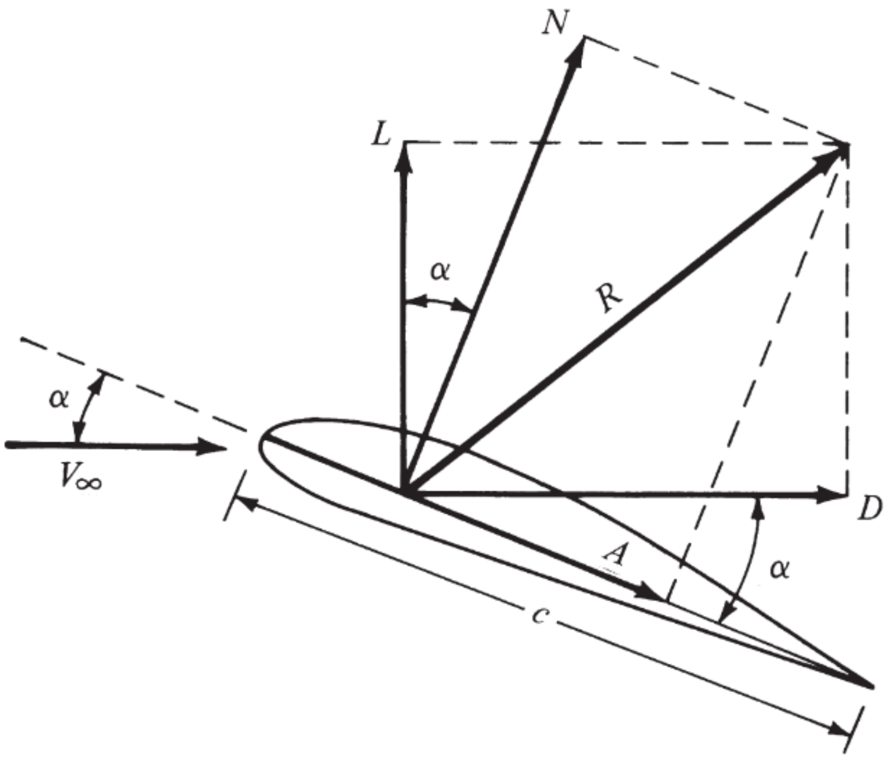


Aerodynamic Forces

- The net effect of the pressure and shear stress integrated over the body surface is a resultant aerodynamic force R and moment M on the body



Aerodynamic Forces



$L \equiv$ lift $\equiv$ component of R perpendicular to V_∞

$D \equiv$ drag $\equiv$ component of R parallel to V_∞

$N \equiv$ normal force $\equiv$ component of R perpendicular to c

$A \equiv$ axial force $\equiv$ component of R parallel to c

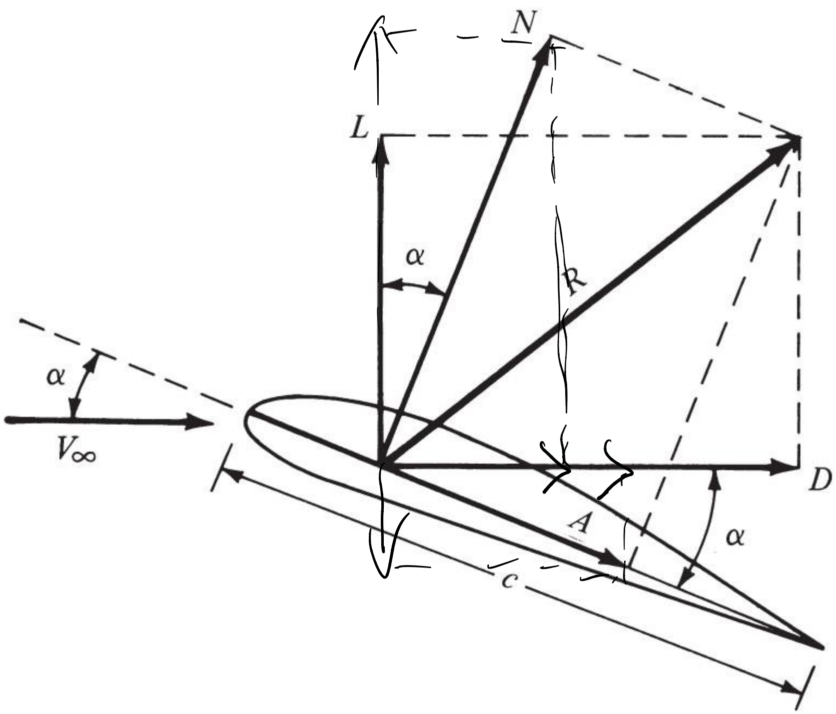
Vectored Thrust

Vectored Thrust

It can give an advantage of low-speed, plus high angle-of-attack maneuverability, compared to conventional-thrust aircraft.

Aerodynamic Forces

- Relations between N & A and L & D?



$$L = N \cdot \cos\alpha - A \cdot \sin\alpha$$

$$D = N \cdot \sin\alpha + A \cdot \cos\alpha$$

Type of Forces Acting on Airfoil

- How to calculate the aerodynamic forces

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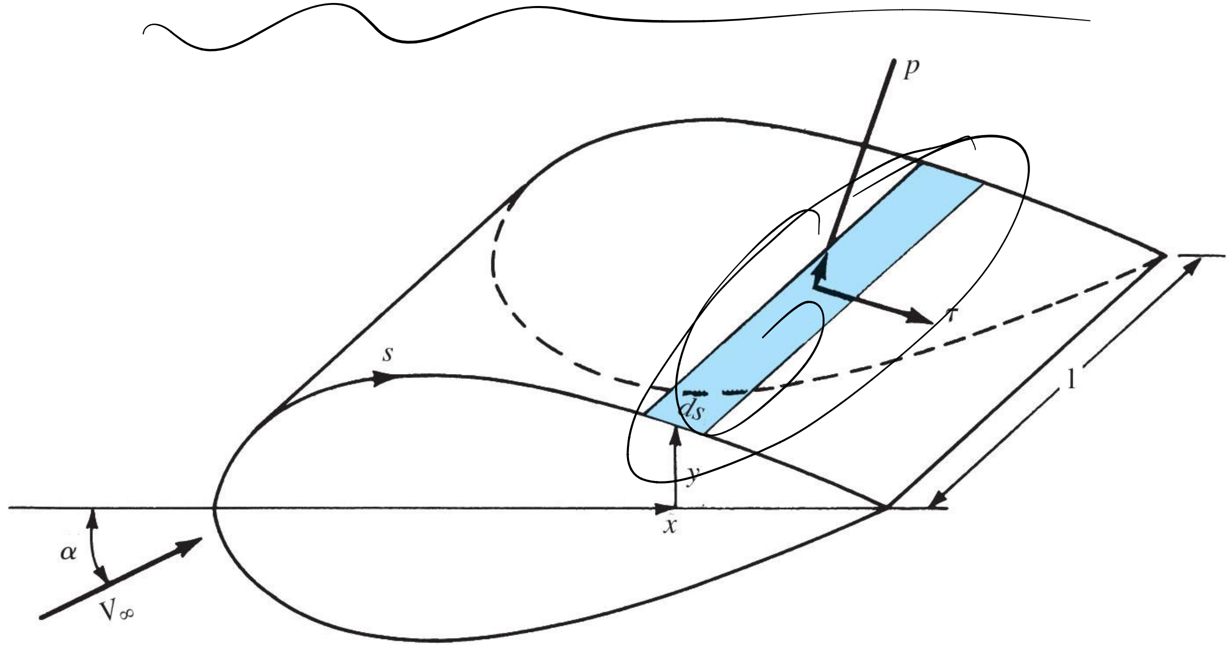
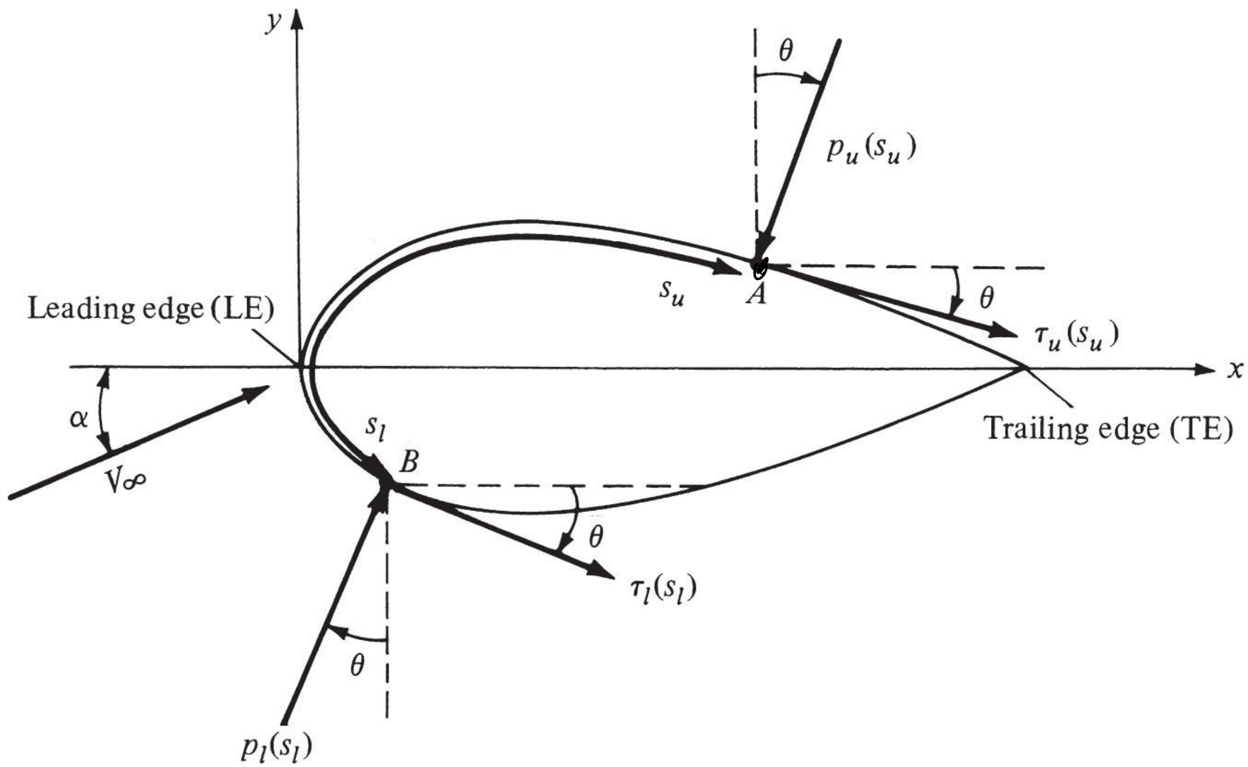


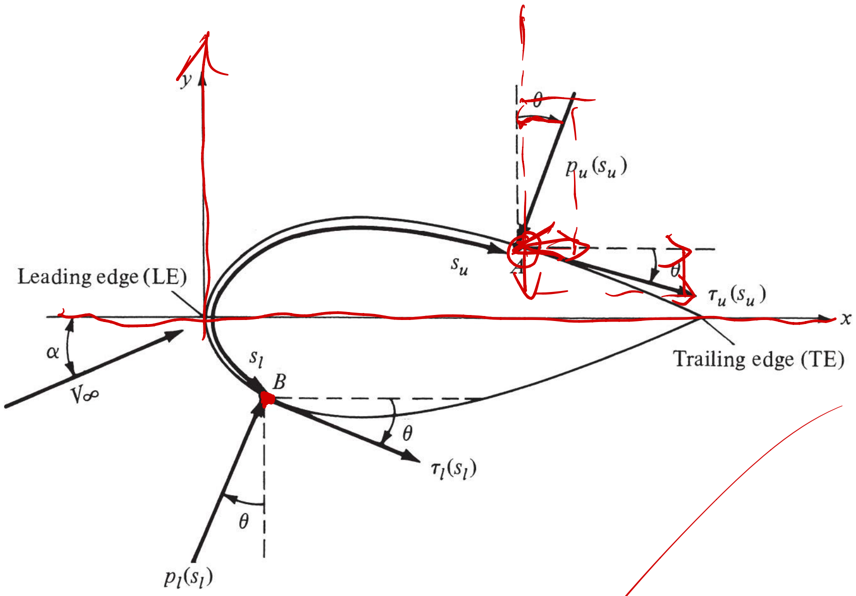
Type of Forces Acting on Airfoil

- How to calculate the aerodynamic forces



Type of Forces Acting on Airfoil

- How to calculate the aerodynamic forces

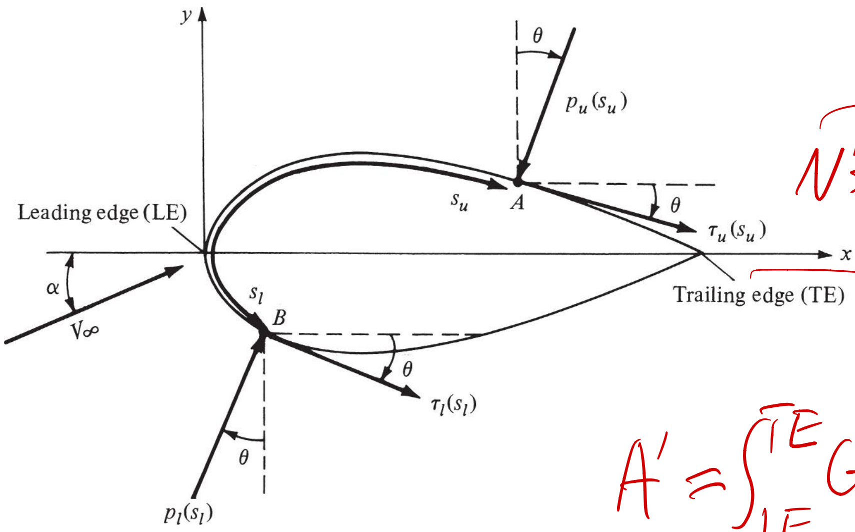


$$\begin{cases} dN_u' = -P_u ds_u \cdot \cos\theta - \tau_u ds_u \cdot \sin\theta \\ dA_u' = -P_u ds_u \cdot \sin\theta + \tau_u ds_u \cdot \cos\theta \end{cases}$$

$$\begin{cases} dN_l' = P_l ds_l \cos\theta - \tau_l ds_l \sin\theta \\ dA_l' = P_l ds_l \cdot \sin\theta + \tau_l ds_l \cdot \cos\theta \end{cases}$$

Type of Forces Acting on Airfoil

- How to calculate the aerodynamic forces



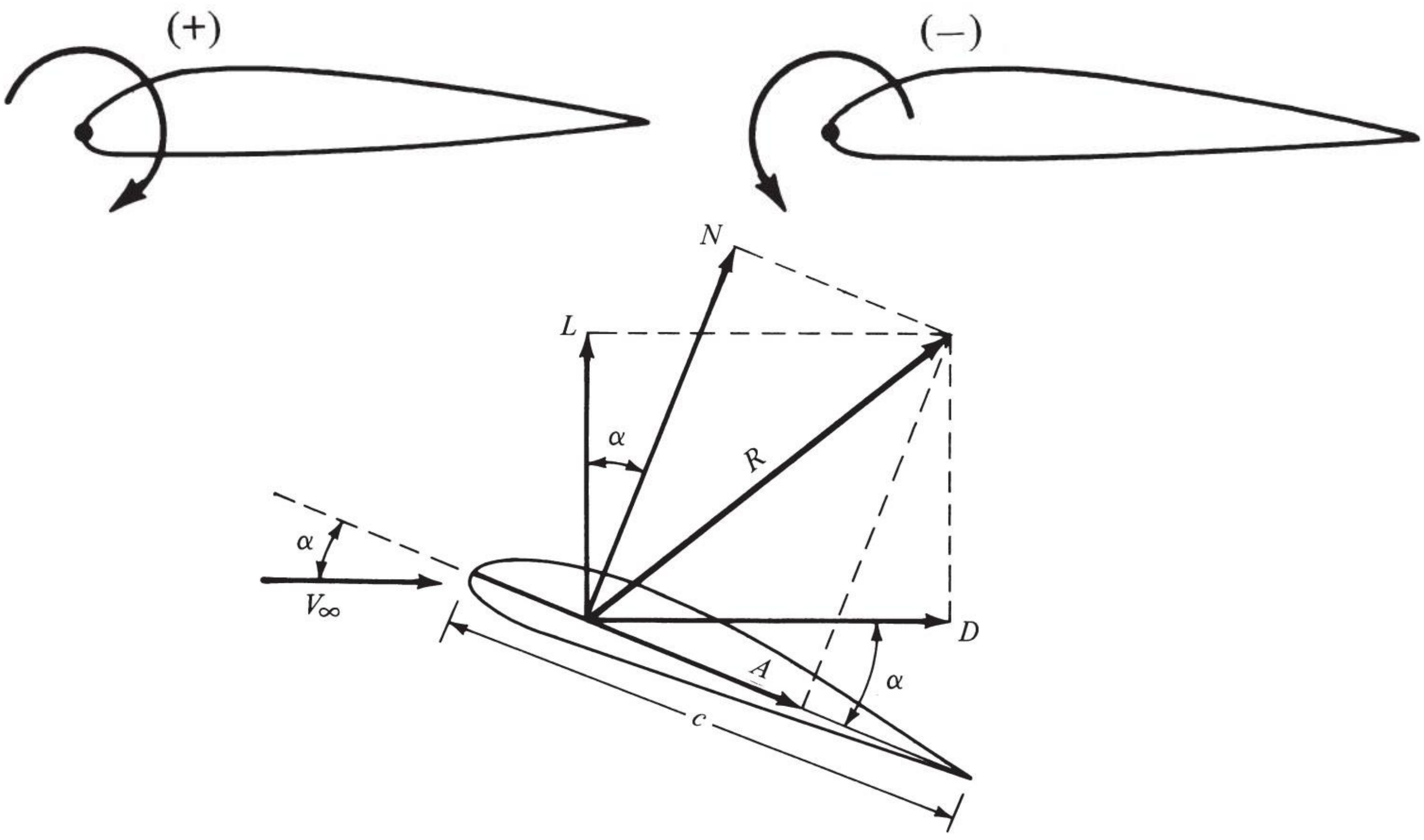
$$N' = \int_{LE}^{TE} dN_u' + \int_{LE}^{TE} dN_l'$$

$$N' = -\int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int_{LE}^{TE} (p_l \cos\theta - \tau_l \sin\theta) \cdot ds_l$$

$$A' = \int_{LE}^{TE} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{LE}^{TE} (p_l \sin\theta + \tau_l \cos\theta)\, ds_l$$

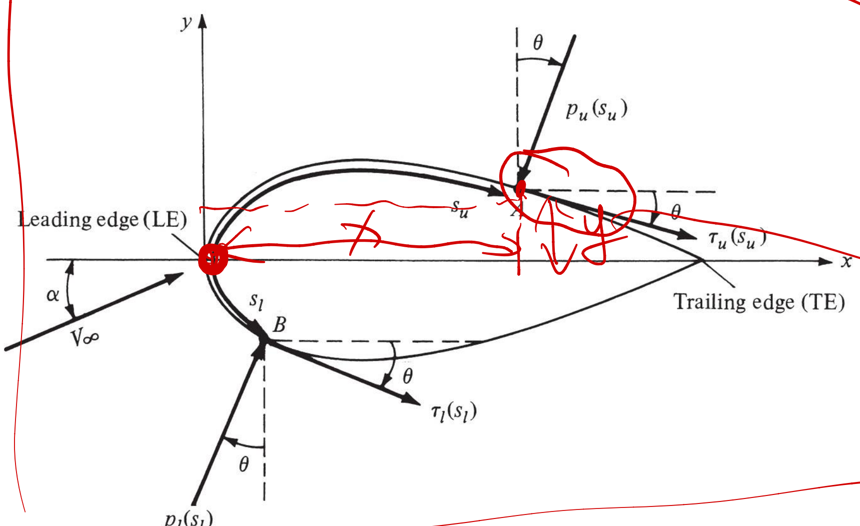
Type of Forces Acting on Airfoil

- Aerodynamic Moments

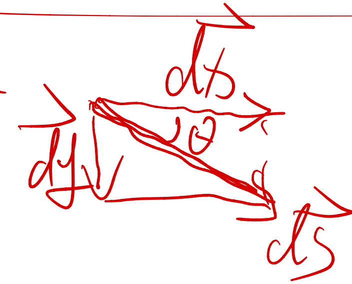


Type of Forces Acting on Airfoil

- Aerodynamic moments



$$M'_{LE} = \int_{LE}^{TE} dM'_u + \int_{LE}^{TE} dM'_l$$



$$dx = ds \cdot \cos\theta$$

$$dy = ds \cdot \sin\theta$$

$$dM'_u = (P_u \cos\theta + \tau_u \sin\theta) \cdot x \cdot ds_u + (-P_u \sin\theta + \tau_u \cos\theta)\, y \cdot ds_u$$

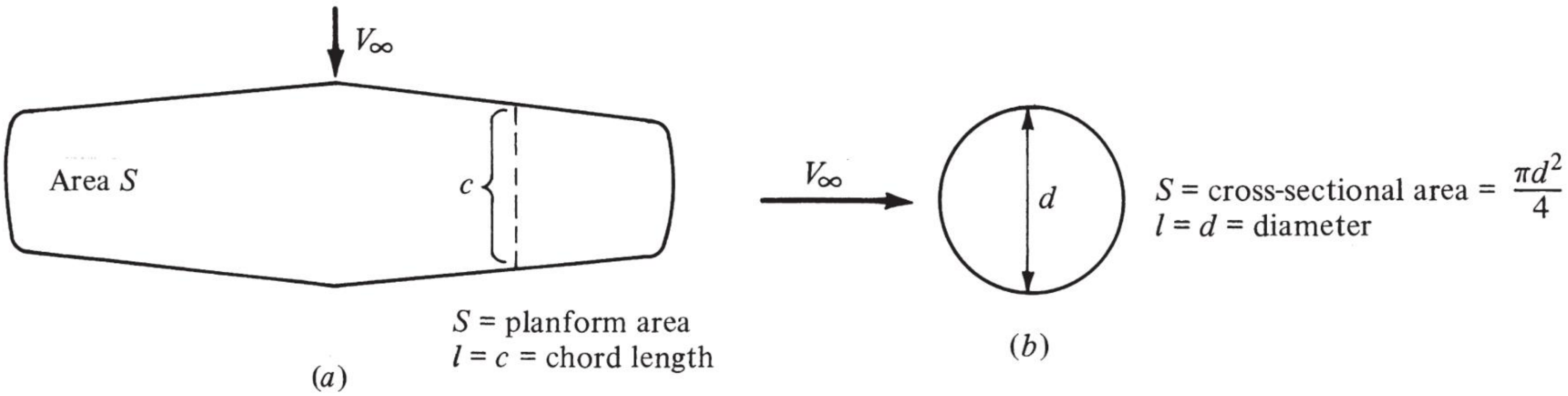
$$dM'_l = (-P_l \cos\theta + \tau_l \sin\theta)\, x \cdot ds_l + (P_l \sin\theta + \tau_l \cdot \cos\theta)\, y\, ds_l$$

Aerodynamic Forces

- Is it good enough to just know the aerodynamic forces moments?
- Why do we need dimensionless force and moment coefficients?

Aerodynamic Coefficients

- Dynamic pressure $q_\infty \equiv \frac{1}{2}\rho_\infty V_\infty^2$



Aerodynamic Coefficients

Lift coefficient: $C_L \equiv \dfrac{L}{q_\infty S}$

$$\frac{N}{\frac{N}{m^2} \cdot m^2} = 1$$

Drag coefficient: $C_D \equiv \dfrac{D}{q_\infty S}$

Normal force coefficient: $C_N \equiv \dfrac{N}{q_\infty S}$

Axial force coefficient: $C_A \equiv \dfrac{A}{q_\infty S}$

Aerodynamic Coefficients

M : N·m

Moment coefficient: $$C_M \equiv \frac{M}{q_\infty S l}$$

Aerodynamic Coefficients

- The symbols in capital letters (i.e., C_L, C_D, C_A , C_N , C_M) denote the force and moment coefficients for a complete three-dimensional body such as an airplane or a finite wing.

- For a two-dimensional body, the forces and moments are per unit span. The aerodynamic coefficients are denoted by lowercase letters:

$$c_l \equiv \frac{L'}{q_\infty c} \quad c_d \equiv \frac{D'}{q_\infty c} \quad c_m \equiv \frac{M'}{q_\infty c^2}$$

where the reference area $S = c(1) = c$.

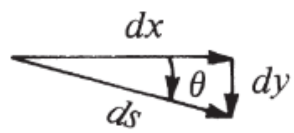
Aerodynamic Coefficients

- Two additional dimensionless quantities:

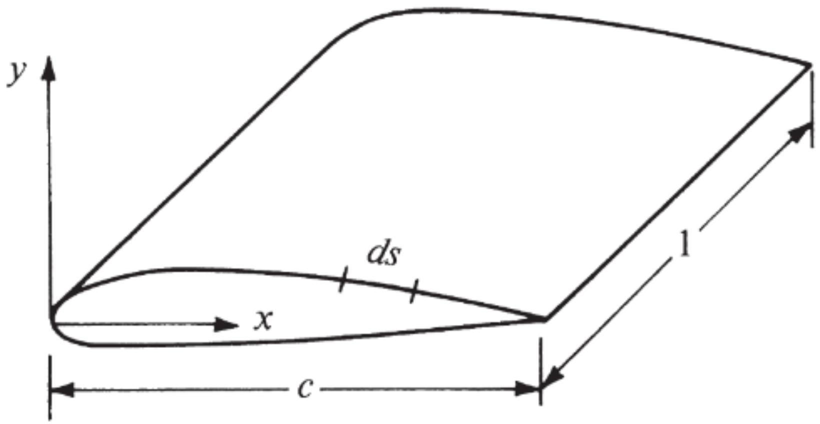
Pressure coefficient: $$C_p \equiv \frac{p - p_\infty}{q_\infty}$$

Skin friction coefficient: $$c_f \equiv \frac{\tau}{q_\infty}$$

Aerodynamic Coefficients



$$dx = ds\cos\theta$$



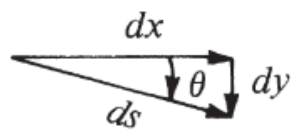
$$dy = -(ds\sin\theta)$$

$$S = c\,(1)$$

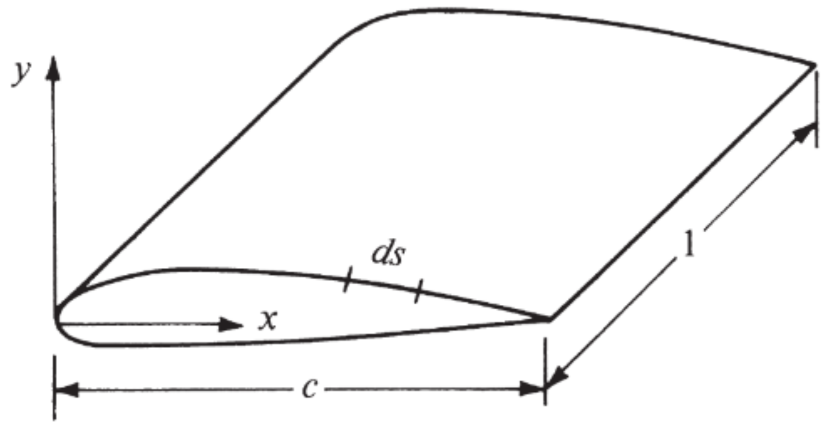
$$c_n = \frac{1}{c}\left[\int_0^c \left(C_{p,l} - C_{p,u}\right)\,dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right)\,dx\right]$$

$$c_a = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,l}\frac{dy_l}{dx}\right)\,dx + \int_0^c \left(c_{f,u} + c_{f,l}\right)\,dx\right]$$

Aerodynamic Coefficients



$$dx = ds \cos\theta$$

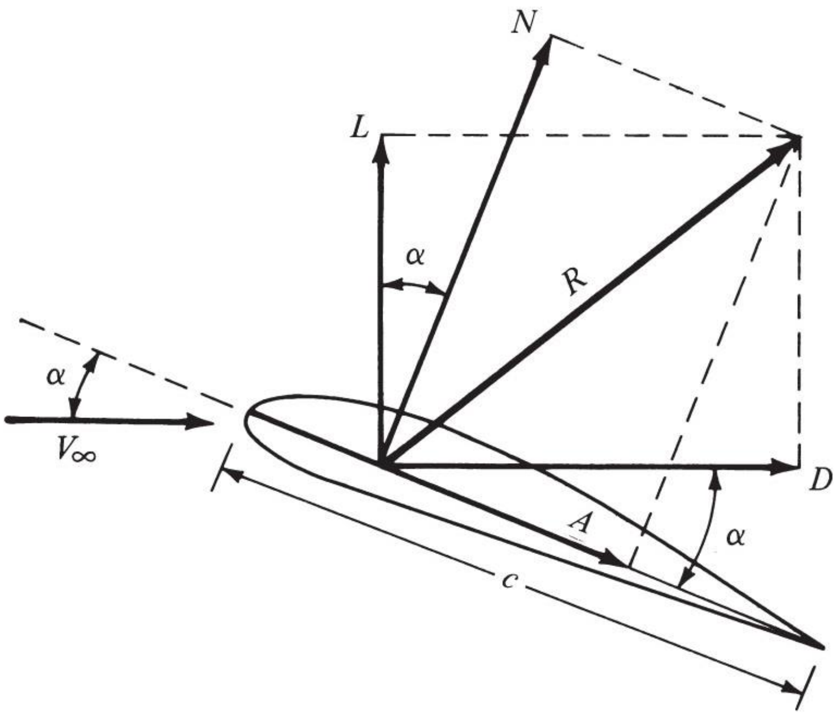


$$dy = -(ds \sin\theta)$$

$$S = c\,(1)$$

$$c_{m_{\mathrm{LE}}} = \frac{1}{c^2}\left[\int_0^c (C_{p,u} - C_{p,l})\; x\,dx - \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) x\,dx\right. \\ \left. + \int_0^c \left(C_{p,u}\frac{dy_u}{dx} + c_{f,u}\right) y_u\,dx + \int_0^c \left(-C_{p,l}\frac{dy_l}{dx} + c_{f,l}\right) y_l\,dx\right]$$

Aerodynamic Forces



$$L = N\cos\alpha - A\sin\alpha$$

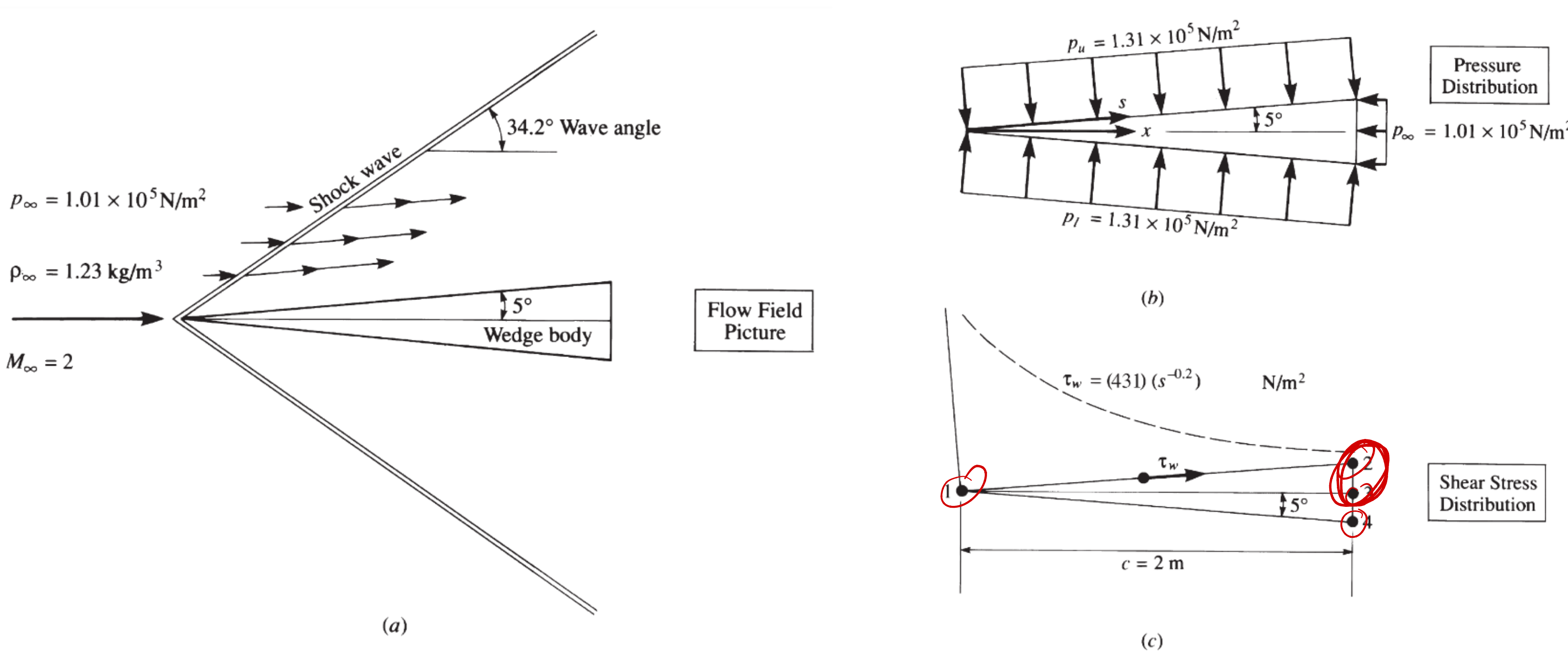
$$D = N\sin\alpha + A\cos\alpha$$

$$c_l = c_n \cos\alpha - c_a \sin\alpha$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha$$

In-class Practice

Consider the supersonic flow over a 5° half-angle wedge at zero angle of attack, as sketched in **Figure 1.23*a***. The freestream Mach number ahead of the wedge is 2.0, and the freestream pressure and density are 1.01×10^5 N/m^2 and 1.23 kg/m^3, respectively (this corresponds to standard sea level conditions). The pressures on the upper and lower surfaces of the wedge are constant with distance s and equal to each other, namely, $p_u = p_l = 1.31 \times 10^5$ N/m^2, as shown in **Figure 1.23*b***. The pressure exerted on the base of the wedge is equal to p_∞. As seen in **Figure 1.23*c***, the shear stress varies over both the upper and lower surfaces as $\tau_w = 431 s^{-0.2}$. The chord length, c, of the wedge is 2 m. Calculate the drag coefficient for the wedge.



In-class Practice

$$D' = \int_{LE}^{TE} (-P_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{LE}^{TE} (P_l \sin\theta + \tau_l \cos\theta)\, ds_l$$

$$\int_{LE}^{TE} (-P_u \sin\theta)\, ds_u = \int_{S_1}^{S_2} -(1.31 \times 10^5) \sin(-5^\circ)\, ds_u$$

$$+ \int_{S_2}^{S_3} -(1.01 \times 10^5) \sin 90^\circ)\, ds_u$$

$$= 5260\ N$$

$$\int_{LE}^{TE} P_l \sin\theta\, ds_l = 5260\ N$$

In-class Practice

$$\int_{LE}^{TE} \tau_u \cos\theta \, dS_u = \int_{s_1}^{s_2} 431 s^{-0.2} \cos(-5^\circ) \, dS_u$$

$$= 936.5N$$

$$\int_{LE}^{TE} \tau_l \cos\theta \, dS_l = \cdots = 936.5N$$

$$D' = 2 \times 5260N + 936.5N \times 2 = \boxed{1.24 \times 10^4 N}$$

$$\frac{D'}{\frac{1}{2}\rho V^2 \cdot S} = C_d$$

In-class Practice

An alternate solution?

1. Calculate the pressure coefficients and skin friction coefficients.
2. Then integrate the pressure coefficients and skin friction coefficients to obtain the drag coefficient.

In-Class Quiz