ME 51500 L - Orbital Mechanics

ME I5800 L - Trajectories and Orbits

Instructor: Peter Ganatos

Office: ST-252

Office Hours: Tu, Th 12:00 - 2:00 PM

Tel: (212) 650-5215

E-mail: ganatos@ccny.cuny.edu

Prereq: ME 24700 (Dynamics)

ME 51500: Orbital Mechanics

Course (Catalog) Description

The two-body problem. Lagrangian dynamics. Hamiltonian equations. Perturbations. Satellite orbits and ballistic trajectories. Effects of drag on satellite orbits. The general three-body problem. Coordinate transformations. Computational methods. Design project. Prereq.: ME 24700. 3 class hrs./wk.; 3 credits.

Instructor: Peter Ganatos, Professor of M.E.

Course Materials

Textbooks: *Orbital Mechanics for Engineering Students*, H. Curtis, Elsevier, fourth edition, 2021 (ISBN 978-0128240250) (required).

Orbital Mechanics, J. E. Prussing and B. A. Conway, Oxford University Press, second edition, 2012 (ISBN 978-0199837700) (optional).

Fundamentals of Astrodynamics, R. R. Bate, D. D. Mueller, J. E. White and W. W. Saylor, Dover Publications, second edition, 2020 (ISBN 978-0486497044) (optional).

References: NASA General Mission Analysis Tool (GMAT) Documentation, available online at: https://documentation.help/GMAT/

Systems Tool Kit (STK) Tutorial, Analytical Graphics, Inc., available online at: http://help.agi.com/stk/11.0/index.htm#training/TutorialOverview.htm

Course Structure / Grading

This course is conducted using the web-enhanced course format. This means that you are required to attend all lectures in person and participate on Blackboard which may be accessed through the CUNY website http://www.cuny.edu and clicking on the login link on the upper right hand corner of the page. Here you will find weekly assignments which are due on the indicated date. Certain assignments will be collected and graded or there may be a quiz based on the homework on the date due. No makeups will be given for missed quizzes and no additional time will be given if you are late. There will also be two 75-minute in-class exams and a final exam which will also be in person. All exams are closed book, but original handwritten notes and homeworks will be allowed. Your final grade for the course will be determined as follows:

Exam 1	20%
Exam 2	20%

Final Exam	30%
Collected Homeworks and Quizzes	20%
Design Project	10%

The work you submit is expected to be your own. Anyone caught cheating during an exam or submitting an assignment bearing any resemblance to another student's work will receive a grade of negative 100%. Six absences in lectures will result in a letter grade reduction. Three late arrivals or leaving before the end of class are equivalent to one absence. To preserve grading fairness for all students, the course grade breakdown and grade assignment as specified in the course syllabus will be strictly followed. No individual exception to the grading policy will be allowed.

Course Outcomes

a. **Specific Outcomes of Instruction**

1) Knowledge of Newton's law of gravitation and formulation of the N-body problem.
2) Knowledge of the two-body problem, satellite orbits and ballistic trajectories.
3) Knowledge of orbital maneuvers, interception and rendezvous.
4) Knowledge of the effect of atmospheric drag on satellite orbits.
5) Knowledge of the three-body problem and libration (Lagrange) points.
6) Knowledge of interplanetary trajectories, sphere of influence and the patched conic method.

b. **Student Outcomes in ABET Criterion 3 or Other Outcomes Addressed by the Course**

The course contributes to the following Student Outcomes:

1. An ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics.
2. An ability to apply engineering design to produce solutions that meet specified needs with consideration of public health, safety, and welfare, as well as global, cultural, social, environmental, and economic factors.
7. An ability to acquire and apply new knowledge as needed, using appropriate learning strategies

Course Outline

Topics:

1) Introduction, the history of spaceflight, projectile motion, Newton's law of gravitation, formulation of the N-body problem, the ten known integrals and their meaning. (1 week)
2) The two-body problem. (1.5 weeks)
3) Ballistic trajectories, minimum energy trajectory. (1 week)

4) Orbit determination, orbital elements, equatorial and ecliptic systems, the hodographic plane, Lagrangian coefficients. (1.5 weeks)
5) Orbital maneuvers, interception and rendezvous, impulsive maneuvers, relative motion between vehicles, linearized orbit theory, the Clohessy-Wiltshire equations (2 weeks)
6) Orbit perturbations, Lagrange's planetary equations. (1 week)
7) Effect of atmospheric drag on satellite orbits. (1 week)
8) The three-body problem, specialized solutions, libration points. (1.5 weeks)
9) Interplanetary trajectories, sphere of influence, patched conic method, direct and gravity assisted interplanetary trajectories. (1.5 weeks)
10) Mechanics of powered flight, specific impulse and exhaust velocity, energy considerations, spacecraft motion under continuous thrust. (1 week)
11) Tests (1 week)

Design Project: Students are assigned a problem to design a trajectory or sequence of maneuvers needed for a specific space mission under a given set of constraints. Students are required to perform all necessary calculations, and create a computer simulation. A written report is required.

Availability of Matlab R2023a for Mechanical Engineering Students

CUNY has a university-wide license of Matlab and Simulink (including 50 toolboxes) which permits currently enrolled students to install a copy of the software on their own computer. Follow the steps outlined below to register and download the software:

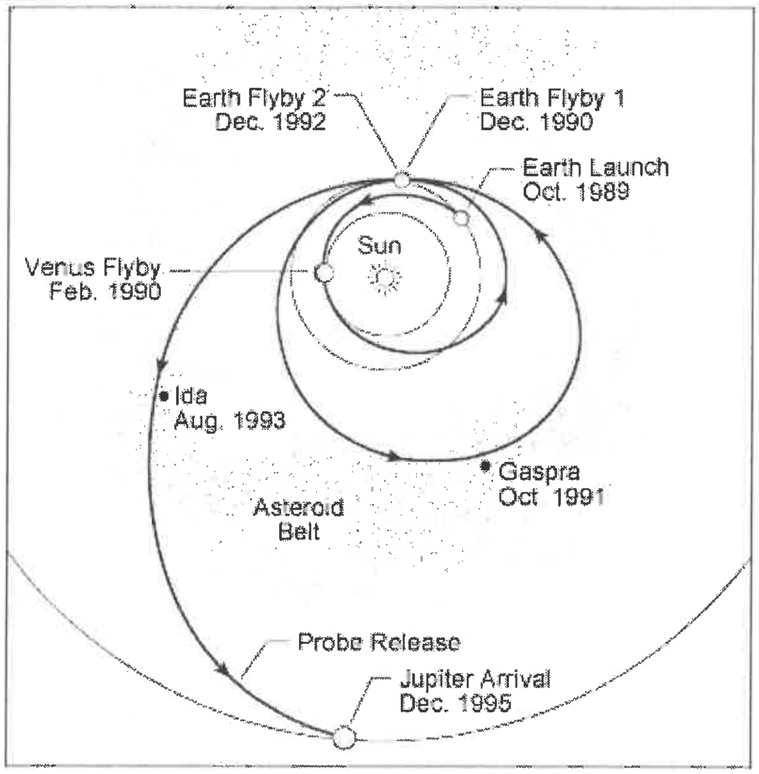
1) Go to https://www.mathworks.com/mwaccount/register and Create MathWorks Account.
2) You **MUST** use your **citymail** e-mail account (i.e. username@citymail.cuny.edu)
3) **Make sure you select "United States"** for the **Country/Region**
4) **Make sure you select "Student"** for **Which best describes you?**
5) Make sure you select **"Yes"** for **Are you at least 13 years or older?**
6) Click the **Create** button.
7) After your account is created, be sure you can log in to the site.
8) Send an e-mail to sitelicenserequest@ccny.cuny.edu from your CCNY citymail e-mail address and request to be added to the Matlab license. Please note, it may take between two and three business days to get a reply.
9) **After you are added to the license, log in and click "My Account" on the top right. You should see a box saying "Account services" and you should be able to download the** software from there.

Please contact Peter Ganatos, ST-252, (212) 650-5215, ganatos@ccny.cuny.edu if you encounter any difficulties.

Chronological Highlights of spaceflight

1957 - Sputnik I
1958 - Explorer I
1959 - Luna II, III
1961 - Vostok I, MR-3
1962 - MA-6, Telstar I, Mariner 2, Vostok VI
1964 - Voskhod I, Mariner 4
1965 - Voskhod II, GT-4, 7, 6
1966 - GT-8, Luna 9, Surveyor 1
1967 - Apollo 1, Soyuz 1 accidents
1968 - Apollo 7, 8
1969-1972 - 6 Apollo lunar landings, Luna rovers & sample return, Salyut-Soyuz missions.
1973 - Skylab
1975 - ASTP
1976 - Viking landings on Mars
1977 - Voyager launchings
1981 - STS-1
1986 - Challenger accident, launch of Mir Space Station
1989 - Launch of Galileo to Jupiter

Galileo trajectory to Jupiter



1990 - Launch of HST

1997 - Pathfinder (Sojourner) landing on Mars

1998 - Launch of first components of ISS (Zarya/Unity)

2003 - Columbia accident

2004 - Spirit, Opportunity rover landings on Mars

2005 - Cassini enters Saturn orbit, Huygens probe landing on Titan.

2006 - Launch of New Horizons flyby probe to Pluto (arrived July 14, 2015, Ultima Thule flyby Jan. 1, 2019)

2008 - Phoenix landing near northern polar cap of Mars

2011 - Completion of construction of ISS -
end of shuttle program (135 flights)

2012 - Landing of Curiosity nuclear powered rover
on Mars.

2014 - First unmanned test flight of Orion capsule
on Delta 4 Heavy. (2 earth orbits, 20,000 mph re-entry)

2018 - Launch of Falcon Heavy by SpaceX.
Landing of Insight to study interior of Mars

2020 - Landing of Chang-e 4 lunar rover on back side of moon by China
Launch of Crew Dragon to ISS by SpaceX

2021 - Landing of Perseverance rover & Ingenuity
helecopter on Mars.

2022 - JWST Operational at L2, Artemis I test flight

Future Plans

Until at least 2030 - continuation of flights to the ISS

unmanned cargo flights

- Progress (Russia)
- Dragon (SpaceX) (return capability)
- Cygnus (Northrup-Grumman)
- HTV (Japan)

manned flights

- Soyuz (Russia)
- Crew Dragon (SpaceX)
- CST-100 Starliner (Boeing)
- Dream Chaser (Sierra Nevada)

US Future Plans

2024 - Artemis II (manned circumlunar flight)

2025 - Artemis III (next man and first woman on lunar surface using SpaceX Starship lunar landing vehicle)

2026 - NASA/ESA Mars Sample Return Mission (Sample return 2033)

2028 - Artemis IV (Begin assembly of Lunar Gateway) (2nd Artemis landing using Starship)

2029 - Artemis V (Further assembly of Lunar Gateway) (3rd Artemis landing using Blue Origin Landing vehicle)

2030 - 2040 Manned Mars mission ?

China Future Plans

2020's - Continued flights to Tiangong "Heavenly Palace" space station in earth orbit.

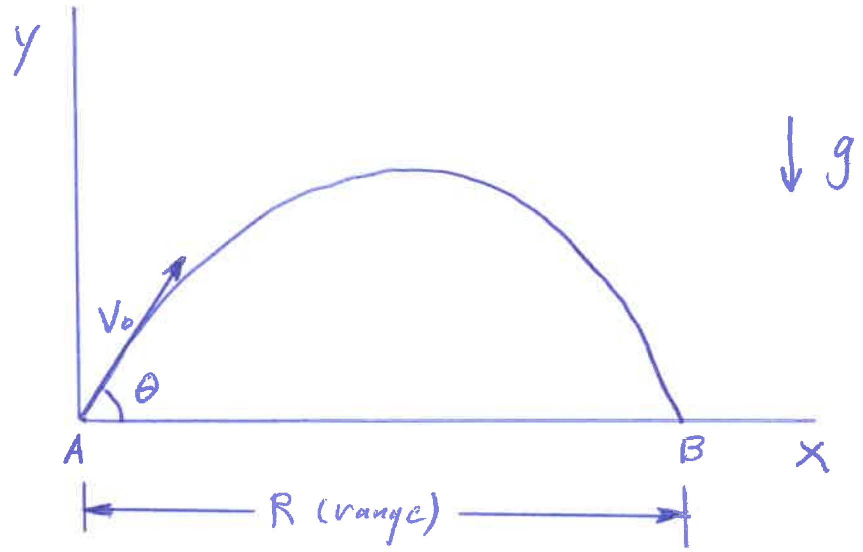
2030's - Manned lunar missions

Russia Future Plans

Not well defined

- New space station
- Unmanned lunar & planetary science missions
- Manned lunar missions?

1. Motion of Projectile on Flat Earth



R depends on V_0 & θ

Newton's 2nd law:

$$m\frac{d^2x}{dt^2} = 0 \qquad m\frac{d^2y}{dt^2} = -mg \qquad (1.1)$$

$$\frac{dx}{dt} = C_1 \qquad \frac{dy}{dt} = -gt + C_3$$

$$x = C_1 t + C_2 \qquad y = -\frac{1}{2}gt^2 + C_3 t + C_4$$

@ $t=0$ $\quad x=0,\ y=0 \quad \Rightarrow \quad C_2 = 0,\ C_4 = 0$

@ $t=0$ $\quad \frac{dx}{dt} = V_0\cos\theta,\ \ \frac{dy}{dt} = V_0 \sin\theta \quad \Rightarrow$

$$C_1 = V_0\cos\theta, \qquad C_3 = V_0 \sin\theta$$

$$\left.\begin{aligned} x(t) &= (V_0 \cos\theta)\, t \\ y(t) &= -\tfrac{1}{2} g t^2 + (V_0 \sin\theta)\, t \end{aligned}\right\} \quad (1.2)$$

When $y = 0$

$$(V_0 \sin\theta)\, t = \tfrac{1}{2} g t^2$$

$$T = \frac{2 V_0 \sin\theta}{g} \qquad \text{(time of flight)} \qquad (1.3)$$

$$R = \frac{2 V_0^2 \sin\theta \cos\theta}{g} = \frac{V_0^2 \sin 2\theta}{g} \qquad \text{(range)} \qquad (1.4)$$

θ for maximum range

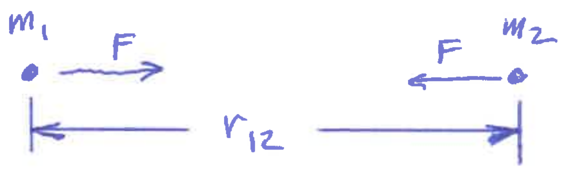
$$\frac{dR}{d\theta} = \frac{2 V_0^2 \cos 2\theta}{g} = 0 \implies \boxed{\theta = 45^\circ}$$

$$\boxed{R = \frac{V_0^2}{g}} \qquad \text{maximum range}$$

$$\boxed{V_0 = \sqrt{gR}} \qquad \text{minimum velocity} \qquad (1.5)$$

Valid close to earth's surface where g = constant
for small ranges where earth can be considered flat.

2. Newton's Law of Gravitation for 2 Bodies

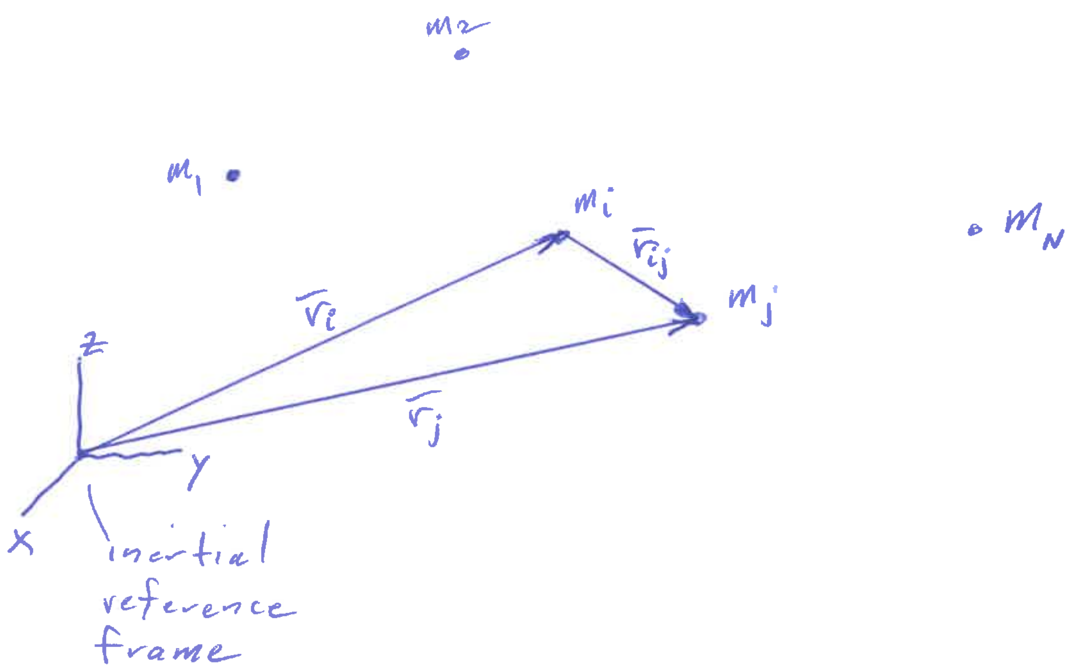


$$\boxed{F = \frac{G\, m_1\, m_2}{r_{12}^{\,2}}} \qquad (2.1)$$

G = Universal gravitational constant

$= 6.67259 \times 10^{-11} \dfrac{m^3}{kg - sec^2}$

3. N-body Problem



$$\bar{r}_i = x_i\,\hat{i} + y_i\,\hat{j} + z_i\,\hat{k}$$

$$\bar{r}_{ij} = \bar{r}_j - \bar{r}_i$$

$$r_{ij} = |\bar{r}_j - \bar{r}_i| = \left[(\bar{r}_j - \bar{r}_i)\cdot(\bar{r}_j - \bar{r}_i)\right]^{1/2}$$

$$= \left[(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2\right]^{1/2}$$

The force of attraction $\bar{F}_i$ on m_i is

$$\boxed{\bar{F}_i = G m_i \sum_{j=1}^{N}{}' \frac{m_j}{r_{ij}^3} (\bar{r}_j - \bar{r}_i)} \qquad (3.1)$$

Prime (′) means omit term $j = i$

The equation of motion of m_i is

$$\boxed{m_i \frac{d^2 \bar{r}_i}{dt^2} = G m_i \sum_{j=1}^{N}{}' \frac{m_j}{r_{ij}^3} (\bar{r}_j - \bar{r}_i)} \qquad (3.2)$$

or can write

$$\boxed{m_i \frac{d^2 \bar{r}_i}{dt^2} = \nabla_i U} \qquad (3.3)$$

where $\nabla_i = \frac{\partial}{\partial x_i} \hat{i} + \frac{\partial}{\partial y_i} \hat{j} + \frac{\partial}{\partial z_i} \hat{k}$

$$\boxed{U = \frac{G}{2} \sum_{i=1}^{N} \sum_{j=1}^{N}{}' \frac{m_i m_j}{r_{ij}}} \qquad (3.4) \quad \text{force function}$$

The force function U is equal to the total work done by the gravitational forces in assembling the system of N point masses from a state of infinite dispersion to a given configuration.

If the potential energy V of the system is taken to be zero when the particles are infinitely far apart, the P.E. of the assembled system is $-U$ $(V = -U)$.

To solve (3.2) requires $6N$ integrations..
Only 10 can be performed analytically.

Add all equations of (3.2)

$$\sum_{i=1}^{N} m_i \frac{d^2\bar{r}_i}{dt^2} = G \sum_{i=1}^{N} \sum_{j=1}^{N}{}' \frac{m_i m_j}{r_{ij}^{3}} (\bar{r}_j - \bar{r}_i) = 0$$

(since $\bar{r}_{ij} = -\bar{r}_{ji}$)

Integrate

$$\sum_{i=1}^{N} m_i \frac{d\bar{r}_i}{dt} = \bar{C}_1 \qquad (3.5)$$

Total linear momentum is conserved.

(3 integrals)

Integrate again

$$\sum_{i=1}^{N} m_i \bar{r}_i = \bar{C}_1 t + \bar{C}_2 \qquad (3.6)$$

(3 more integrals, total 6 integrals)

Since the center of mass of the system $\bar{r}_{cm}$ is

$$\bar{r}_{cm} = \frac{\sum\limits_{i=1}^{N} m_i \bar{r}_i}{\sum\limits_{i=1}^{N} m_i} \qquad (3.7)$$

Eq. (3.6) shows that the center of mass of the system has constant velocity.

Take the cross-product of $\bar{r}_i$ with (3.2) and add

$$\sum_{i=1}^{N} m_i \bar{r}_i \times \frac{d^2 \bar{r}_i}{dt^2} = G \sum_{i=1}^{N} \sum_{j=1}^{N}{}' \frac{m_i m_j}{r_{ij}^3} \bar{r}_i \times (\bar{r}_j - \bar{r}_i) = 0$$

(since $\bar{r}_i \times \bar{r}_i = 0$, $\bar{r}_i \times \bar{r}_j = -\bar{r}_j \times \bar{r}_i$)

Integrate using $\frac{d}{dt}\left(\bar{r}_i \times \frac{d\bar{r}_i}{dt}\right) = \cancel{\frac{d\bar{r}_i}{dt} \times \frac{d\bar{r}_i}{dt}}^{\,0} + \bar{r}_i \times \frac{d^2\bar{r}_i}{dt^2}$

$$\sum_{i=1}^{N} m_i \left(\bar{r}_i \times \frac{d\bar{r}_i}{dt} \right) = \bar{C}_3 \qquad (3.8)$$

Total angular momentum is conserved.

(3 more integrals, total 9 integrals)

Finally take the scalar product of (3.3) with $\frac{d\bar{r}_i}{dt}$ and add

$$\sum_{i=1}^{N} m_i \frac{d^2\bar{r}_i}{dt^2} \cdot \frac{d\bar{r}_i}{dt} = \sum_{i=1}^{N} \nabla_i U \cdot \frac{d\bar{r}_i}{dt} = \frac{dU}{dt}$$

$$\left(\frac{\partial U}{\partial x_i} \hat{i} + \frac{\partial U}{\partial y_i} \hat{j} + \frac{\partial U}{\partial z_i} \hat{k} \right) \cdot \left(\frac{dx_i}{dt} \hat{i} + \frac{dy_i}{dt} \hat{j} + \frac{dz_i}{dt} \hat{k} \right)$$

Integrate using $\frac{d}{dt}\left(\frac{d\bar{r}_i}{dt} \cdot \frac{d\bar{r}_i}{dt} \right) = \frac{d\bar{r}_i}{dt} \cdot \frac{d^2\bar{r}_i}{dt^2} + \frac{d^2\bar{r}_i}{dt^2} \cdot \frac{d\bar{r}_i}{dt}$

$$= 2 \frac{d^2\bar{r}_i}{dt^2} \cdot \frac{d\bar{r}_i}{dt}$$

$$\frac{1}{2} \sum_{i=1}^{N} m_i \frac{d\bar{r}_i}{dt} \cdot \frac{d\bar{r}_i}{dt} - U = c$$

(1 more integral, total 10 integrals)

or $T + V = c \qquad (3.9)$

where $$T = \frac{1}{2}\sum_{i=1}^{N} m_i \frac{d\bar{r}_i}{dt}\cdot\frac{d\bar{r}_i}{dt} \qquad (3.10)$$ kinetic energy

$V = -U$ potential energy

Eq. (3.9) shows total of kinetic energy and potential energy of the system remain constant.

The N-body problem cannot be solved analytically. Only the relative motion of two bodies which requires 6 integrals can be solved analytically.

4. The Effect of Other Bodies on the Relative Motion of Two Bodies.

From (3.2) with $i = 1, 2$ write

$$(i=1) \qquad \frac{d^2\bar{r}_1}{dt^2} = \underbrace{G\frac{m_2}{r_{12}^3}(\bar{r}_2-\bar{r}_1)}_{j=2} + G\sum_{j=3}^{N}\frac{m_j}{r_{1j}^3}(\bar{r}_j - r_1) \qquad (4.1a)$$

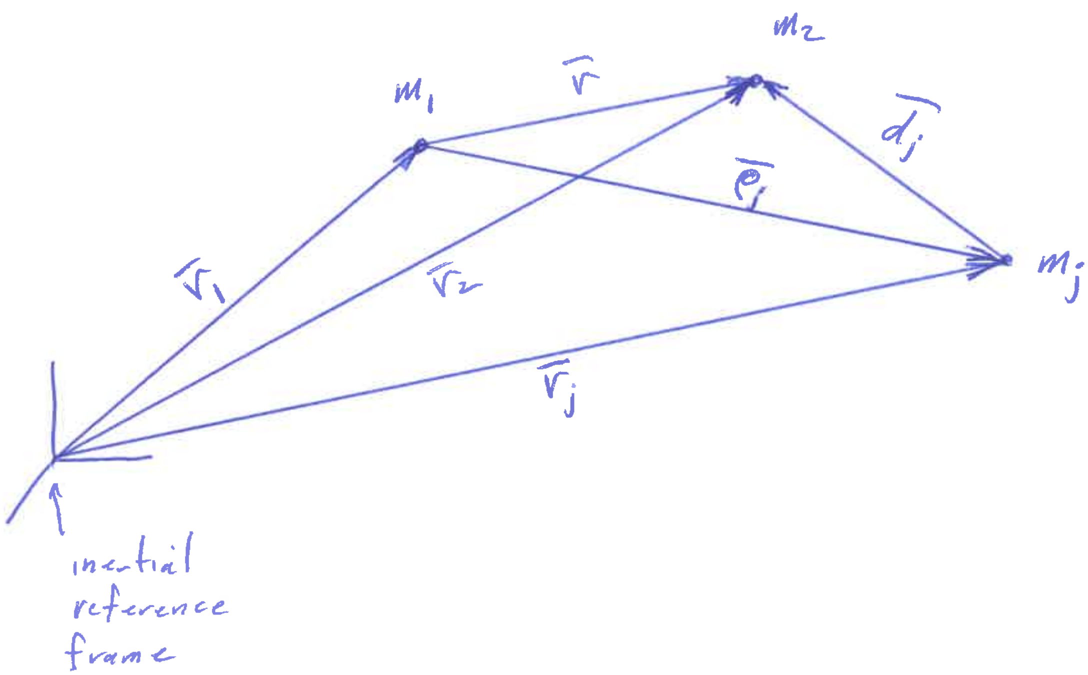
$$(i=2) \qquad \frac{d^2\bar{r}_2}{dt^2} = \underbrace{G\frac{m_1}{r_{21}^3}(\bar{r}_1-\bar{r}_2)}_{j=1} + G\sum_{j=3}^{N}\frac{m_j}{r_{2j}^3}(\bar{r}_j - \bar{r}_2) \qquad (4.1b)$$

Define

$$\bar{r} = \bar{r}_2 - \bar{r}_1 \qquad (4.2)$$

$$\bar{\rho}_j = \bar{r}_j - \bar{r}_1 \qquad (4.3)$$

$$\bar{d}_j = \bar{r} - \bar{\rho}_j = \bar{r}_2 - \bar{r}_j \qquad (4.4)$$



Subtract (4.1a) from (4.1b)

$$\frac{d^2\bar{r}}{dt^2} + \frac{\mu}{r^3}\bar{r} = -G\sum_{j=3}^{N} m_j\left(\frac{\bar{d}_j}{d_j^3} + \frac{\bar{\rho}_j}{\rho_j^3}\right) \qquad (4.5)$$

where $\mu = G(m_1 + m_2)$ (4.6)

is called the <u>gravitational parameter</u>,

The right hand side of (4.5) can be written in alternate form. It can be shown that

$$\frac{\bar{d}_j}{d_j^3} + \frac{\bar{\rho}_j}{\rho_j^3} = -\nabla\left(\frac{1}{d_j} - \frac{1}{\rho_j^3}\,\bar{r}\cdot\bar{\rho}_j\right) \qquad (4.7)$$

where ∇ is with respect to the components of $\bar{r}$.

Define

$$R_j = G\,m_j\left(\frac{1}{d_j} - \frac{1}{\rho_j^3}\,\bar{r}\cdot\bar{\rho}_j\right) \qquad (4.8)$$

called the <u>disturbing function</u> associated with the disturbing body m_j (scalar)

Therefore

$$\boxed{\frac{d^2\bar{r}}{dt^2} + \frac{\mu}{r^3}\,\bar{r} = \nabla\sum_{j=3}^{N} R_j} \qquad (4.9)$$

The right hand side of (4.9) accounts for the effect of other particles on the relative motion of two particles.