The City College of New York

AERO-THERMAL FLUIDS LABORATORY – ME43600

THE CITY COLLEGE OF NEW YORK
DEPARTMENT OF MECHANICAL ENGINEERING
STEINMAN HALL 233
160 CONVENT AVENUE
NEW YORK CITY, NY 10031

Sep 1, 2023

Course Outline

- Introduction – safety, lab reports, plotting, error analysis, signal processing, lab-specific hardware/software data acquisition, measurement devices, engineering principles
- Laboratory Experiments
  1. Wind Tunnel
  2. Viscous Pipe Flow
  3. Heat Exchanger
  4. Cross-Section Fin
  5. Pelton Wheel
- Design Project Presentation and Report
- Final Exam

Tell them what they will be doing

Then introduce them to the underlying physics

Tell them the instrumentation they will be using

Error Analysis

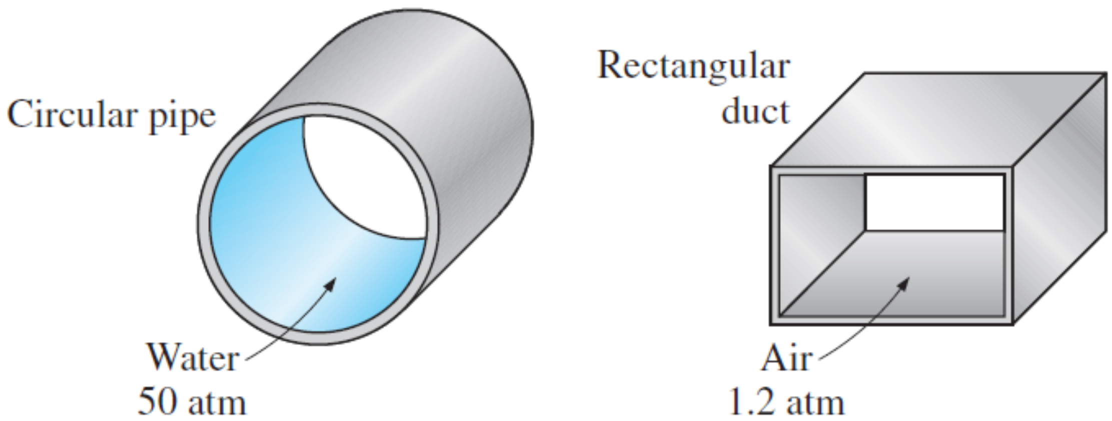
Background- *Internal Flow*

Here we will study the difference between laminar and turbulent flow in pipes, their major differences and analyze fully developed pipe flow.

Calculate head loss in pipe flows and determine pumping power requirements

Background- *Internal Flow*

- Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to friction, which is directly related to the pressure drop and head loss during flow through pipes and ducts.
- The pressure drop is then used to determine the pumping power requirement.



Background- *Internal Flow*

Flow inside pipes are complicated and analytical solutions are only available for simplified flows.

Flow does not vary with x, it only varies in r direction i.e, radially. This type of flow is called as **uniform flow**.

x momentum:

$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{Assumption 2}} + \underbrace{\cancel{u_r \frac{\partial u}{\partial r}}}_{\text{Assumption 3}} + \underbrace{\cancel{\frac{u_\theta}{r}\frac{\partial u}{\partial \theta}}}_{\text{Assumption 6}} + \underbrace{\cancel{u\frac{\partial u}{\partial x}}}_{\text{Continuity}}\right)$$

$$= -\underbrace{\cancel{\frac{\partial P}{\partial x}}}_{\text{Assumption 5}} + \underbrace{\rho g_x}_{\rho g \sin\alpha} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}}}_{\text{Assumption 6}} + \underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{\text{Continuity}}\right)$$

or

Result of x momentum: $$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{-\rho g \sin\alpha}{\mu} \qquad (3)$$

Background- *Internal Flow*

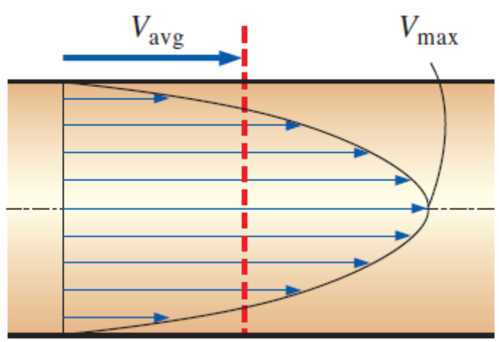
Here, we will rely on experimental and empirical methods to study pipe flows

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

Mass conservation should still be satisfied

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

Average velocity

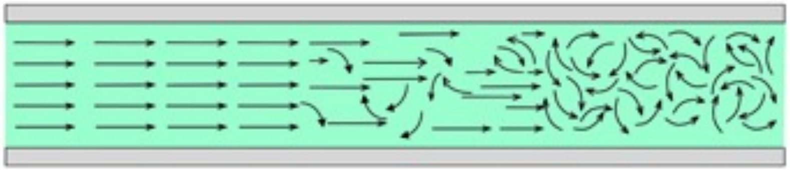


Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

Background- *Internal Flow*

Laminar flow think about **sheets**

Turbulent flows think about **eddies**



smooth streamlined flow influenced by viscosity

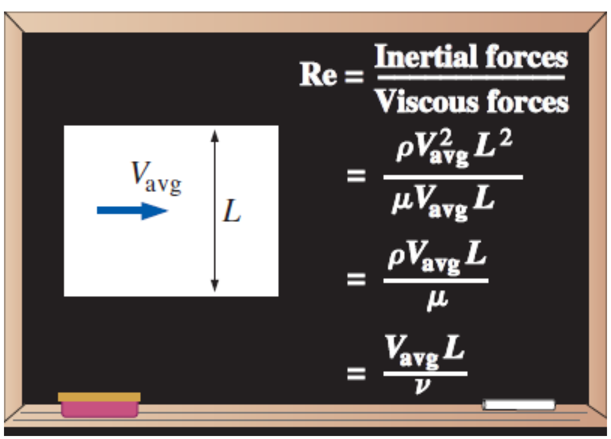
enhanced mixing and chaotic 3 dimensional flow field.

Background- *Internal Flow*

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$



At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**).
At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (**laminar**).

Critical Reynolds number, Re_{cr}:
The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

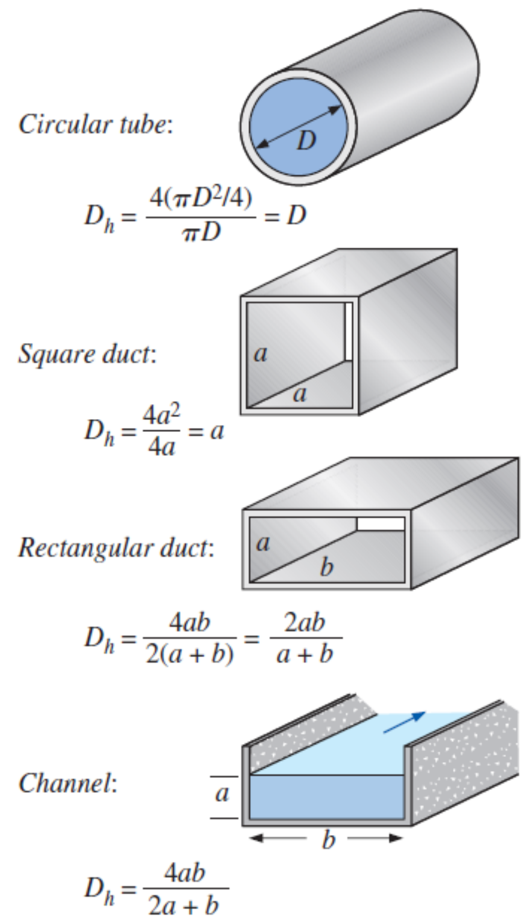
Background- *Internal Flow*

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter**

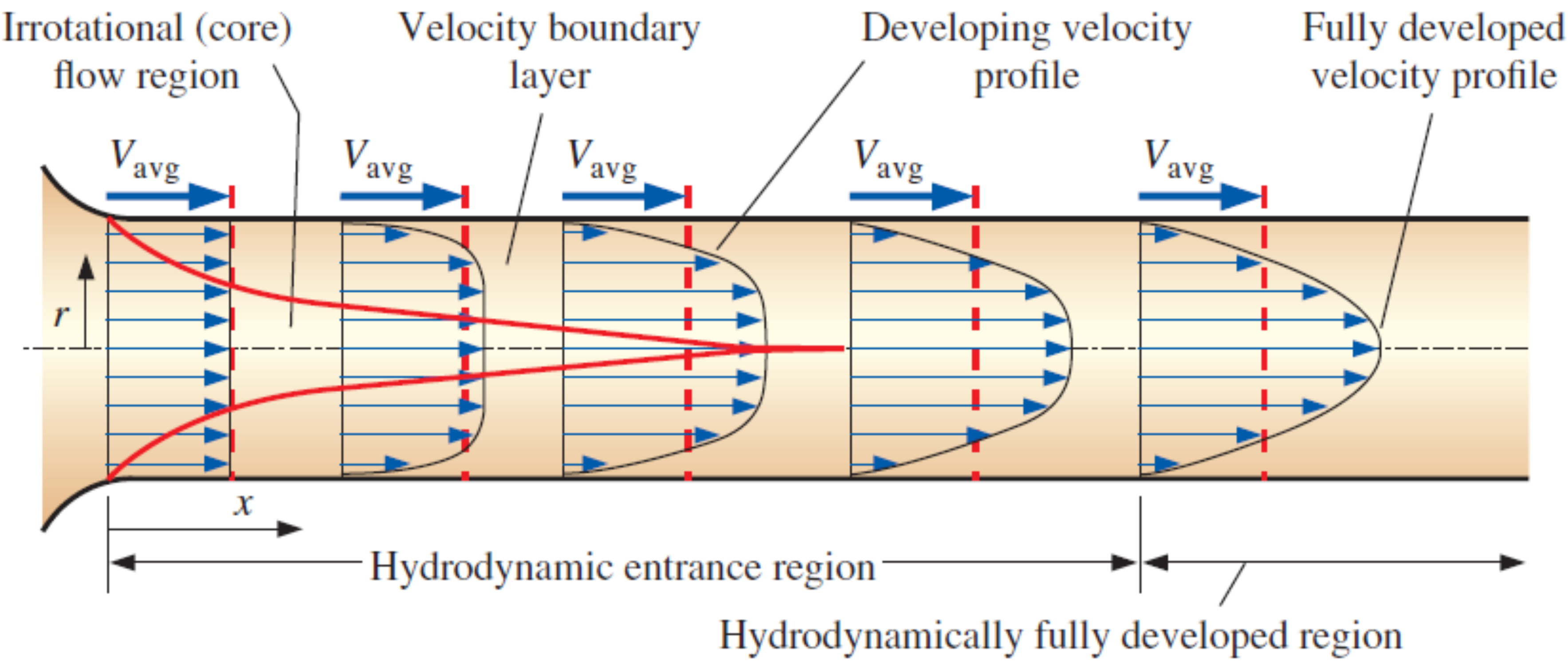
$$D_h = \frac{4A_c}{p}$$

	$\mathrm{Re} \lesssim 2300$	laminar flow
$2300 \lesssim$	$\mathrm{Re} \lesssim 10{,}000$	transitional flow
	$\mathrm{Re} \gtrsim 10{,}000$	turbulent flow

The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.



Entrance Region



Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.
Boundary layer region: The viscous effects and the velocity changes are significant.
Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.

Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

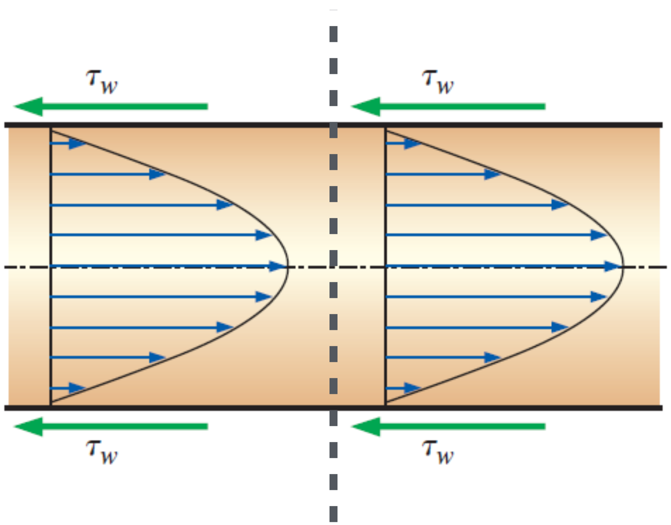
Hydrodynamic entry length L_h: The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile the normalized temperature profile remain unchanged.

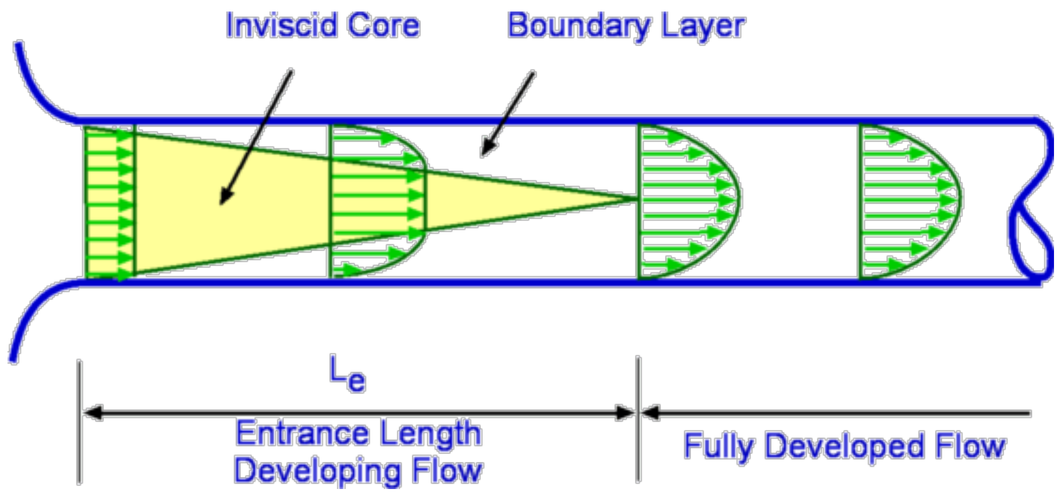
$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$



No Change

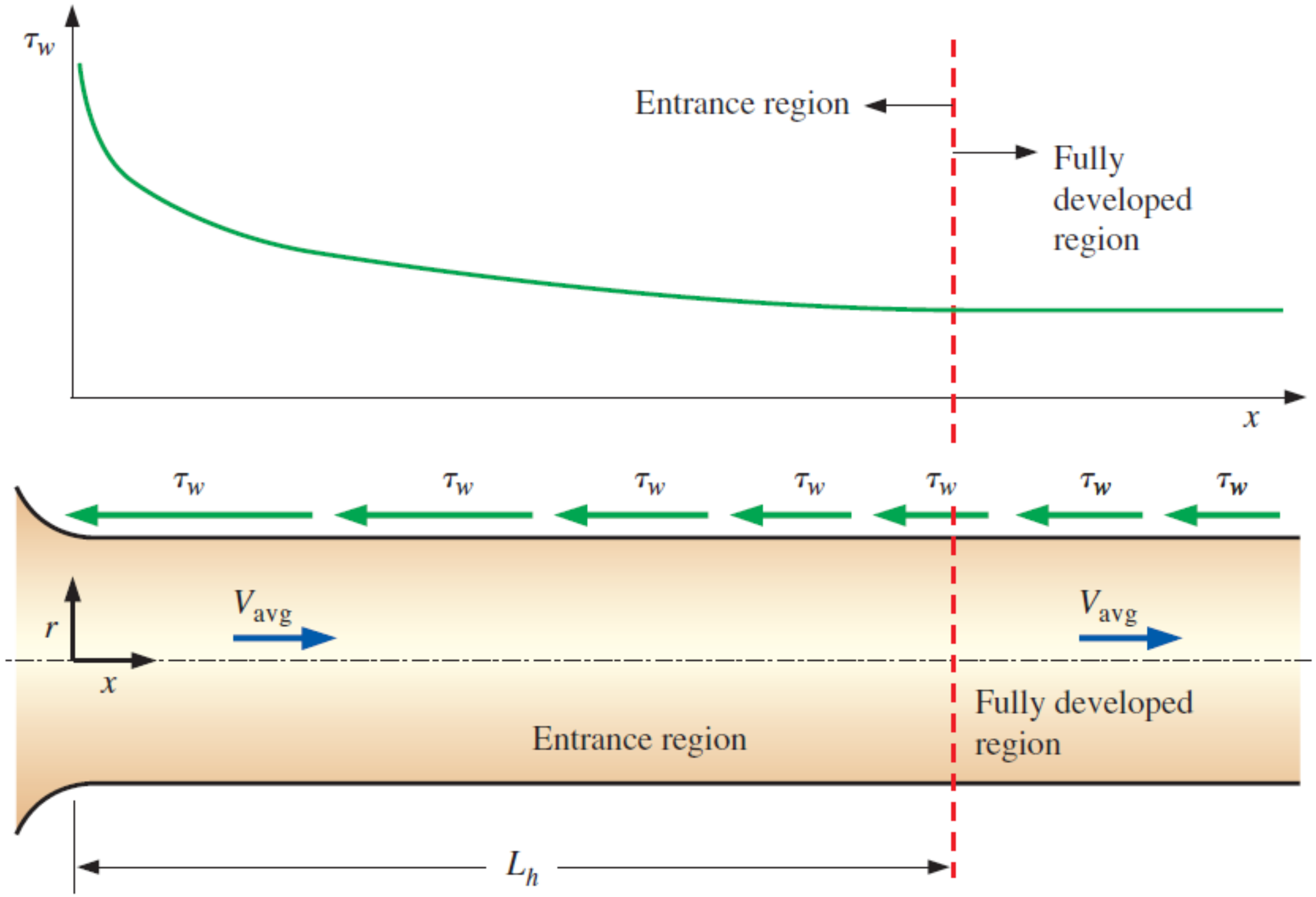
In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

Evolution of Boundary Layer



"The boundary layer is not a static phenomenon; it is dynamic. it grows meaning that its thickness increases as we move downstream. From Fig. it is seen that the boundary layer from the walls grows to such an extent that they all merge on the centerline of the pipe. Once this takes place, inviscid core terminates and the flow is all viscous. The flow is now called a Fully Developed Flow. The velocity profile becomes parabolic. Once the flow is fully developed the velocity profile does not vary in the flow direction. In fact in this region the pressure gradient and the shear stress in the flow are in balance. The length of the pipe between the start and the point where the fully developed flow begins is called the Entrance Length."

pressure drop is high at the entrance region of the pipe



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Entry Length

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

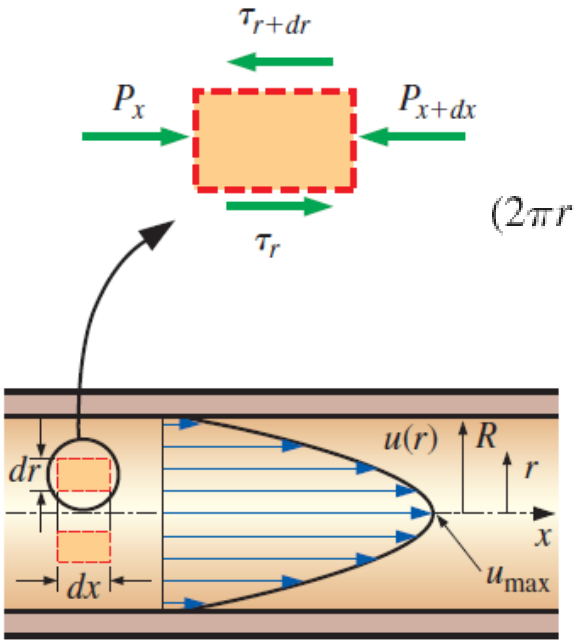
$$\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05\text{Re}$$

$$\frac{L_{h,\,\text{turbulent}}}{D} = 1.359\text{Re}^{1/4}$$

The pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe.

This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it underpredicts the wall shear stress and thus the friction factor.

Laminar Flow



Method 1

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx}-P_x}{dx} + \frac{(r\tau)_{r+dr}-(r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \tau = -\mu(du/dr)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Method 2

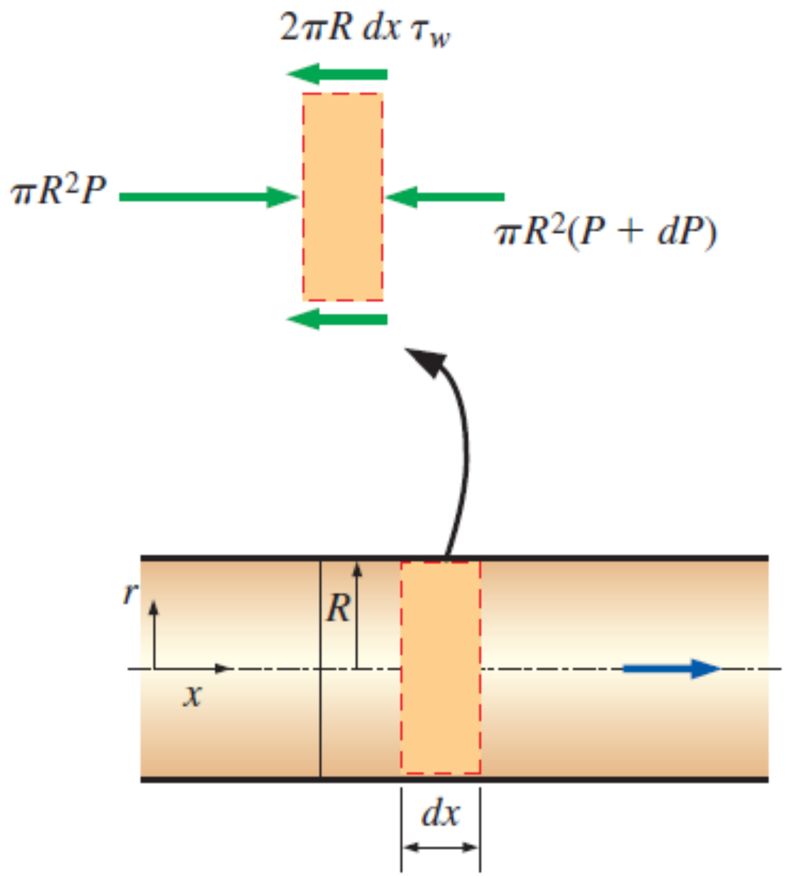
$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{Assumption 2}} + \underbrace{\cancel{u_r\frac{\partial u}{\partial r}}}_{\text{Assumption 3}} + \underbrace{\cancel{\frac{u_\theta}{r}\frac{\partial u}{\partial \theta}}}_{\text{Assumption 6}} + \underbrace{\cancel{u\frac{\partial u}{\partial x}}}_{\text{Continuity}}\right)$$

$$= -\boxed{\underbrace{\frac{\partial P}{\partial x}}_{\text{Assumption 5}}} + \underbrace{\cancel{\rho g_x}}_{\cancel{\rho} g\sin\alpha} + \mu\left(\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}}}_{\text{Assumption 6}} + \underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{\text{Continuity}}\right)$$

Laminar Flow

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Pressure Force

Shear Force

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

Fully Developed $\partial u/\partial r = 0$ at $r = 0$

No Slip $u = 0$ at $r = R$

Boundary conditions

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Average velocity

$$\boxed{V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)}$$

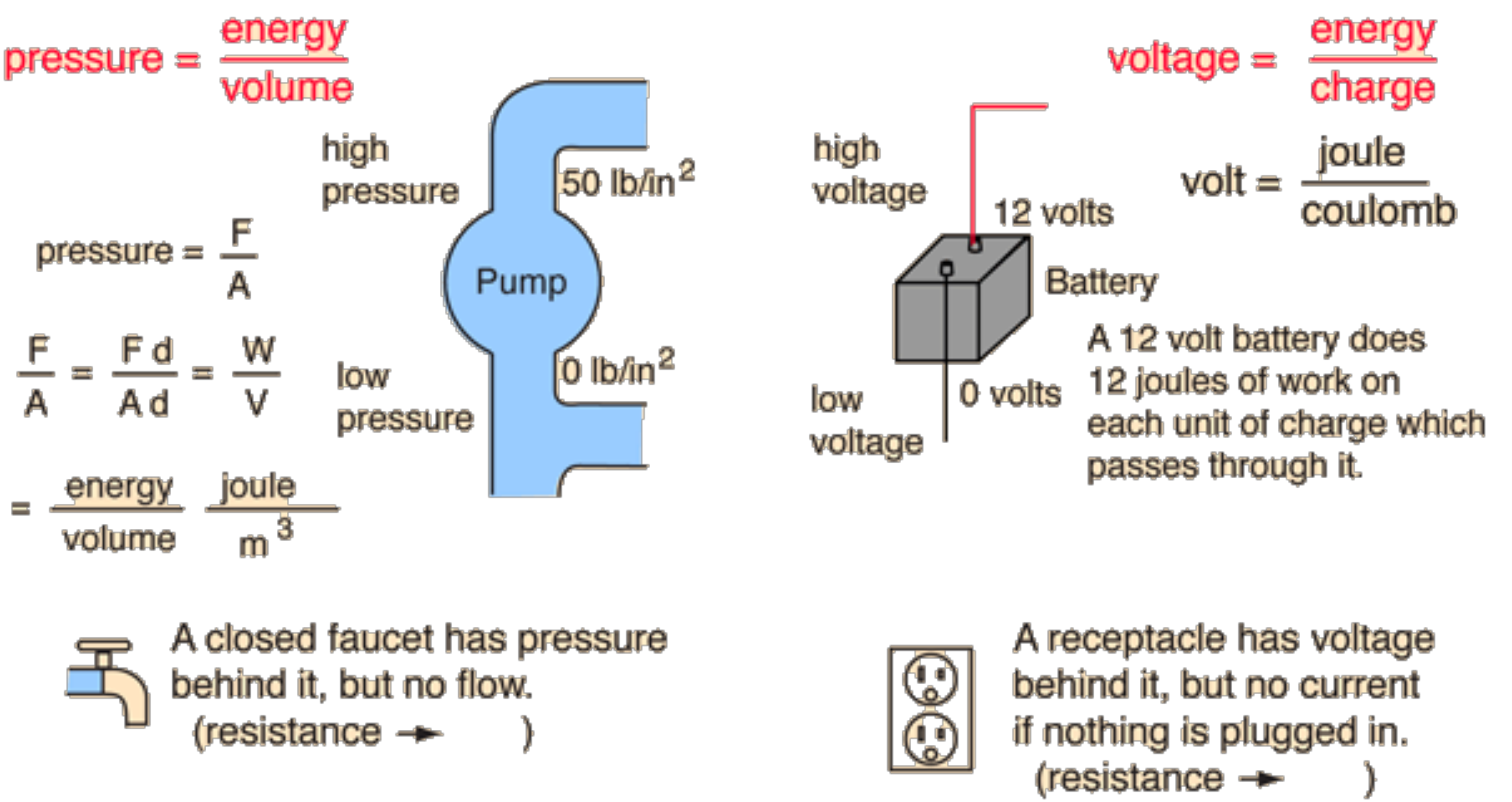
$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

Velocity profile

$$u_{max} = 2V_{avg}$$

Maximim velocity at centerline

Pressure Drop & Head Loss



The Pressure loss is an important parameter in pipe flows. The pressure drop is directly related to the power requirements of a pump

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

Laminar flow:

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

Pressure Loss

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$

$$h_L = \frac{\Delta P_L}{\rho g}$$

Head Loss

Friction factor

Dynamic Pressure

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

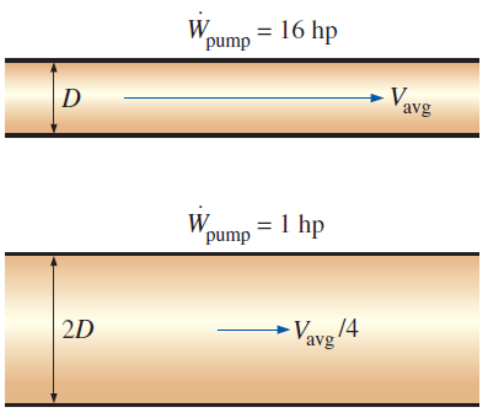
For Laminar flow in pipes $f = \dfrac{64}{\text{Re}}$

In laminar flow, the friction factor is only a function of Reynolds Number and is independent of pipe roughness

$$\dot{W}_{pumpL} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

$$\dot{V} = V_{avg}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$



The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

For laminar flow in horizontal pipe, the pressure drop is directly proportional to pressure loss.

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

energy equation for steady, incompressible one-dimensional flow

$$P_1 - P_2 = \rho(\cancel{\alpha_2 V_2^2 - \alpha_1 V_1^2})/2 + \rho g[(\cancel{z_2 - z_1}) + \cancel{h_{\text{turbine},\, e} - h_{\text{pump},\, u}} + h_L]$$

If the z2 and z1 are equal and there is no external work and V2 and V1 are equal, the pressure drop is equal to the pressure loss $\rho g h_L$

Turbulent Boundary Layer

Wall Layer

$u(y) = f\,(\mu, \tau_w, \rho, y)$

$$u^+ = \frac{u}{u^*} = F\left(\frac{yu^*}{\nu}\right) \qquad u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$$

Outer Layer

$U - u(y) = f\,(\delta, \tau_w, \rho, y)$

$$\frac{U - u}{u^*} = G\left(\frac{y}{\delta}\right)$$

Overlap Layer Layer

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \frac{yu^*}{\nu} + B$$

Viscous sublayer

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$

Viscous sublayer: $$\frac{u}{u_*} = \frac{yu_*}{\nu}$$

Thickness of viscous sublayer: $$y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

thickness of viscous sublayer is proportional to viscosity and inversely proportional to velocity

ν/u_* Viscous length; it is used to nondimensionalize the distance *y* from the surface.

Nondimensionalized variables: $y^+ = \frac{yu_*}{\nu}$ and $u^+ = \frac{u}{u_*}$

Normalized law of the wall: $u^+ = y^+$

Outer layer

Outer turbulent layer:

$$\frac{u_{\max} - u}{u_*} = 2.5 \ln \frac{R}{R - r}$$

Power-law velocity profile:

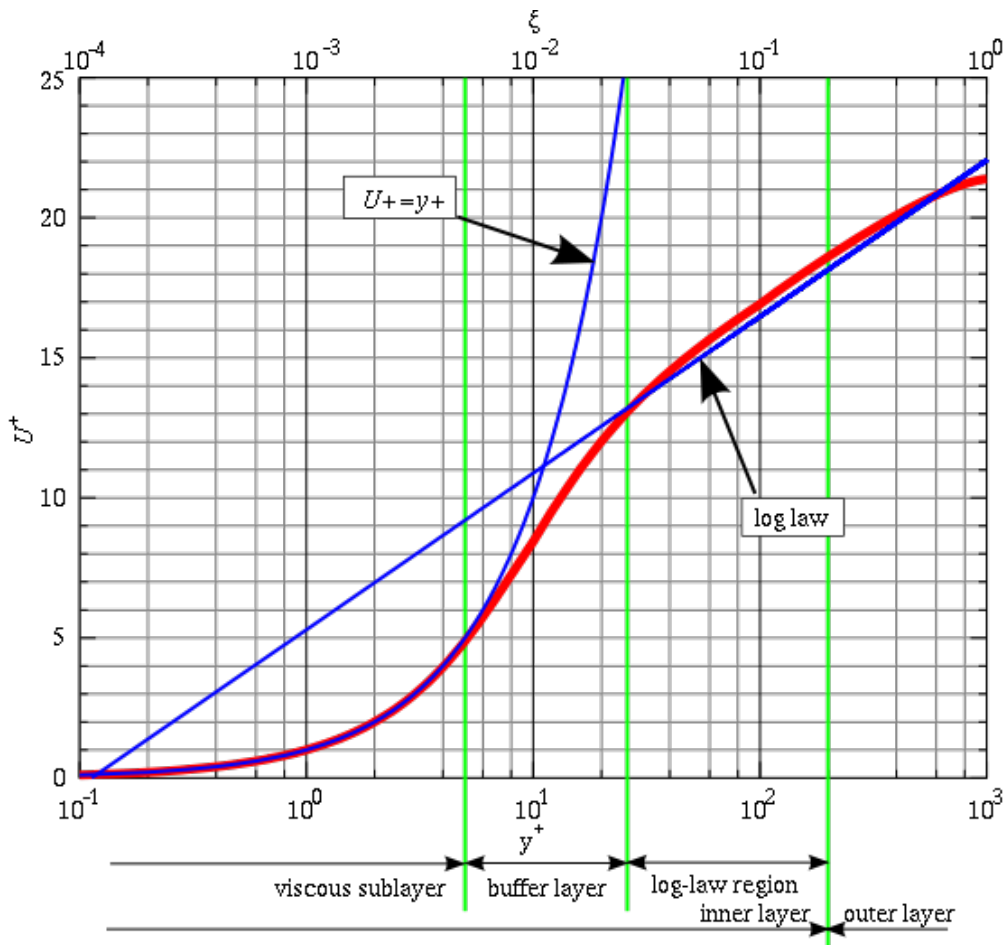
$$\frac{u}{u_{\max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{\max}} = \left(1 - \frac{r}{R}\right)^{1/n}$$

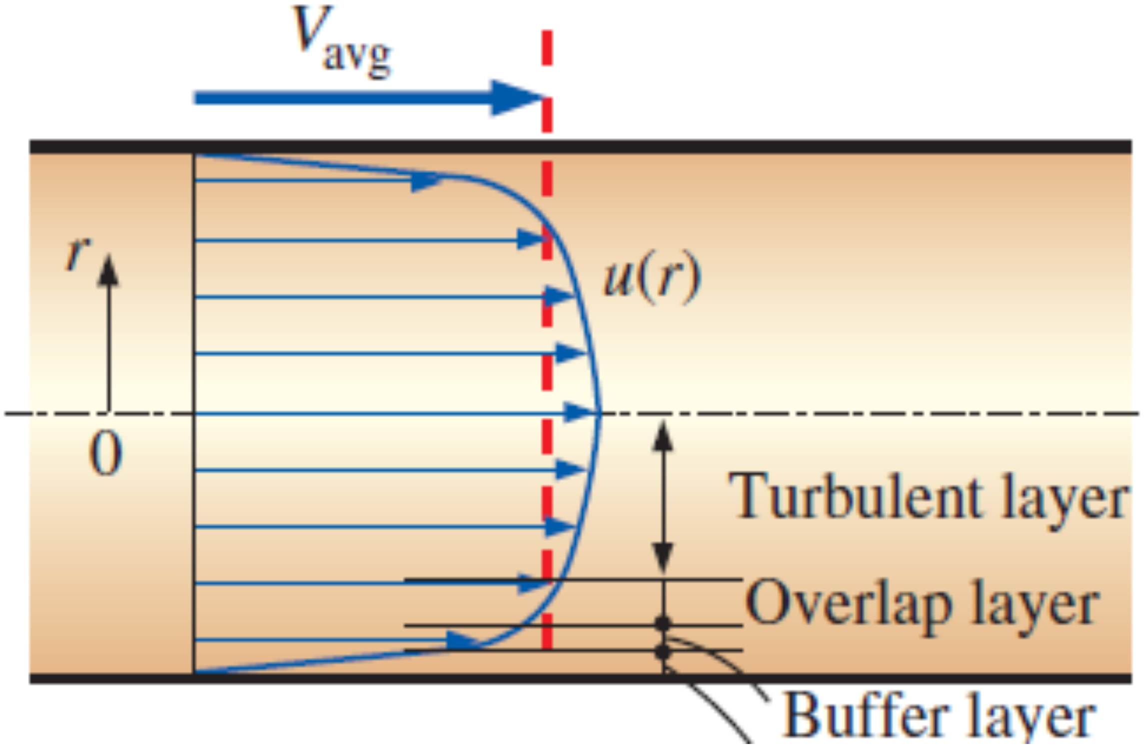
Deviation of velocity from the centerline velocity is called as velocity defect and these relationships are commonly known as velocity defect law

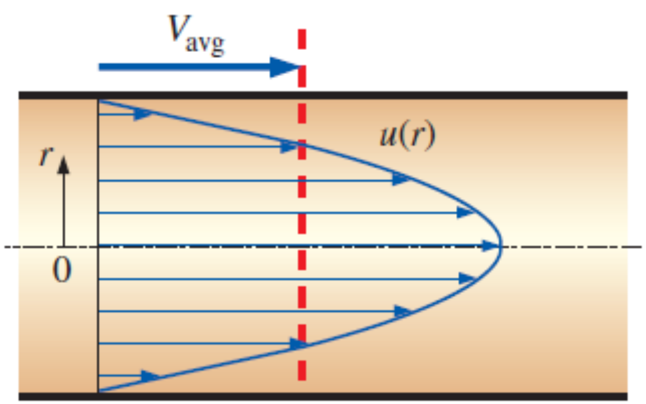
Overlap layer

Overlap layer: $$\frac{u}{u_*} = 2.5 \ln \frac{yu_*}{\nu} + 5.0 \quad \text{or} \quad u^+ = 2.5 \ln y^+ + 5.0$$

The logarithmic law: $$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{yu_*}{\nu} + B$$







Laminar flow

The very thin layer next to the wall where viscous effects are dominant is the **viscous** (or **laminar** or **linear** or **wall**) sublayer.

The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

Next to the viscous sublayer is the **buffer layer**, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the **overlap** (or **transition**) **layer**, also called the **inertial sublayer**, in which the turbulent effects are much more significant, but still not dominant.

Above that is the **outer** (or **turbulent**) **layer** in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.

Turbulent Flow Solutions

Start from log law

$$\frac{u(r)}{u^*} \approx \frac{1}{\kappa} \ln \frac{(R - r)u^*}{\nu} + B$$

$$V = \frac{Q}{A} = \frac{1}{\pi R^2} \int_0^R u^* \left[\frac{1}{\kappa} \ln \frac{(R - r)u^*}{\nu} + B \right] 2\pi r \, dr$$

$$= \frac{1}{2} u^* \left(\frac{2}{\kappa} \ln \frac{Ru^*}{\nu} + 2B - \frac{3}{\kappa} \right)$$

$$\frac{V}{u^*} = \left(\frac{\rho V^2}{\tau_w} \right)^{1/2} = \left(\frac{8}{f} \right)^{1/2}$$

$$\frac{1}{f^{1/2}} = 2.0 \log \left(\mathrm{Re}_d f^{1/2} \right) - 0.8$$

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6 and are calculated from the Colebrook equation.

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

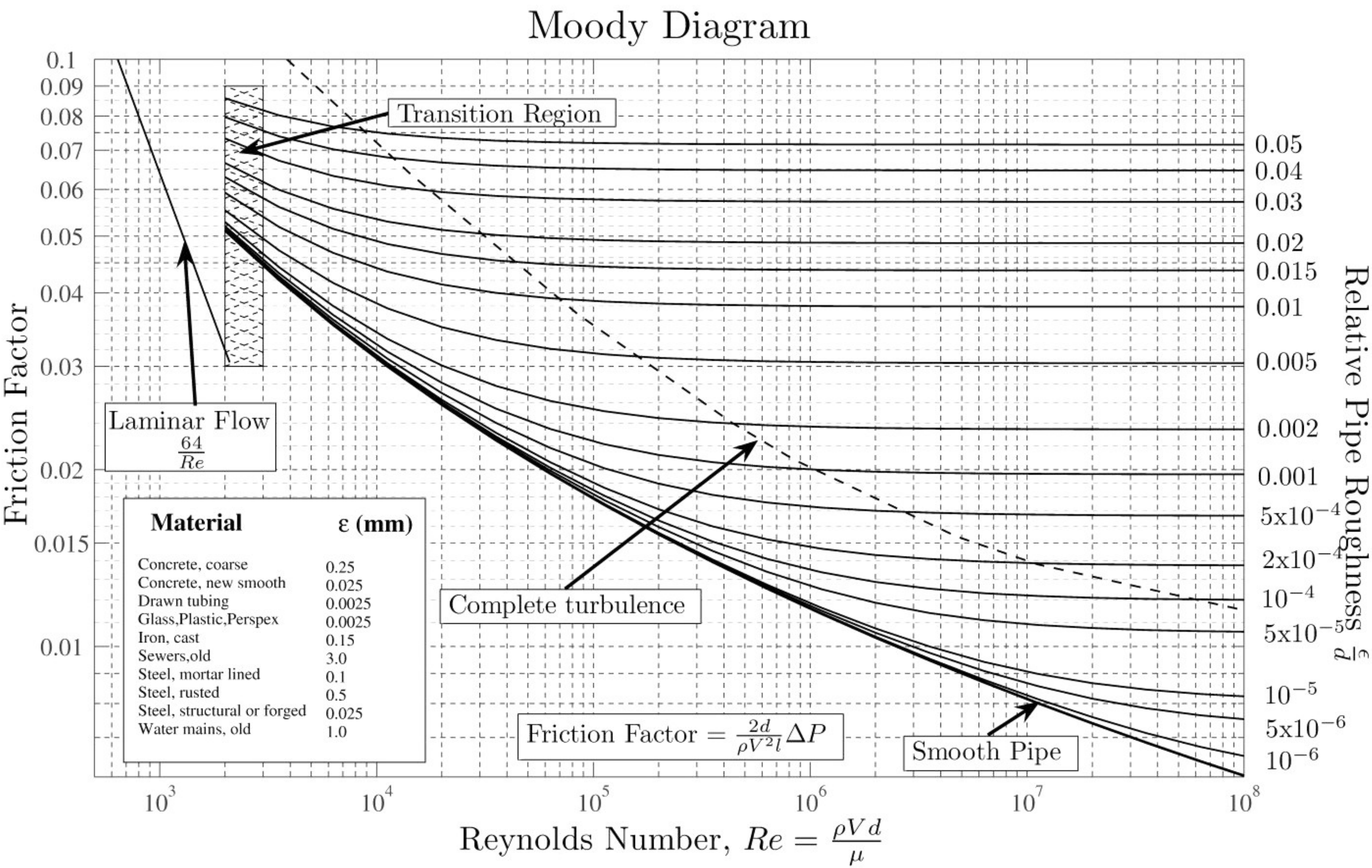
The friction factor in fully developed turbulent pipe flows is dependent on Reynolds number and the relative roughness factor

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

implicit Colebrook equation

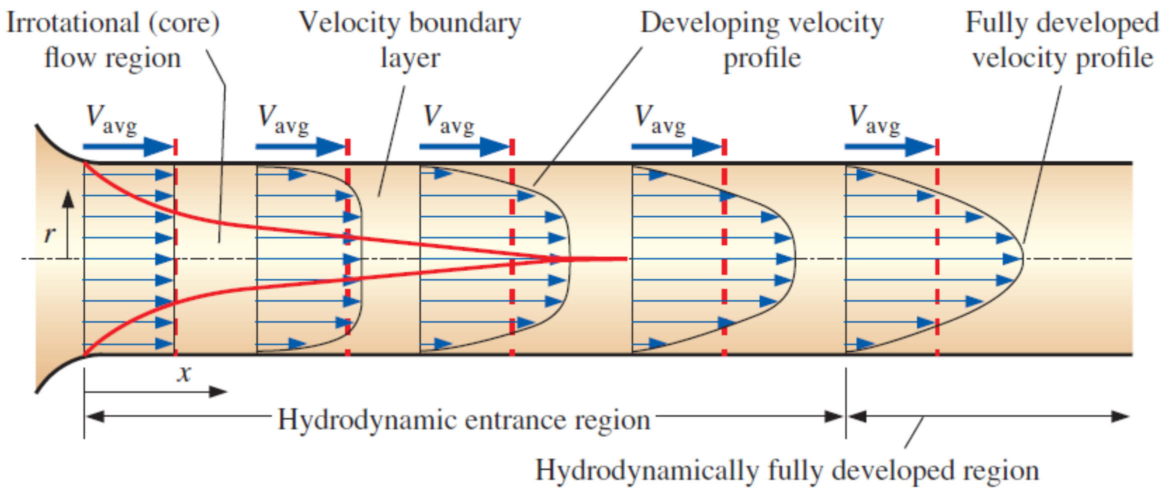
$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

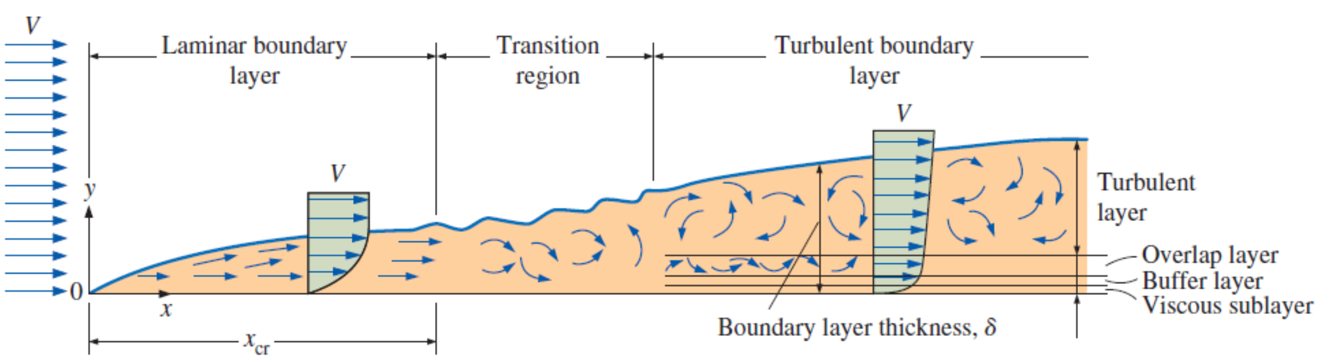
explicit Haaland equation

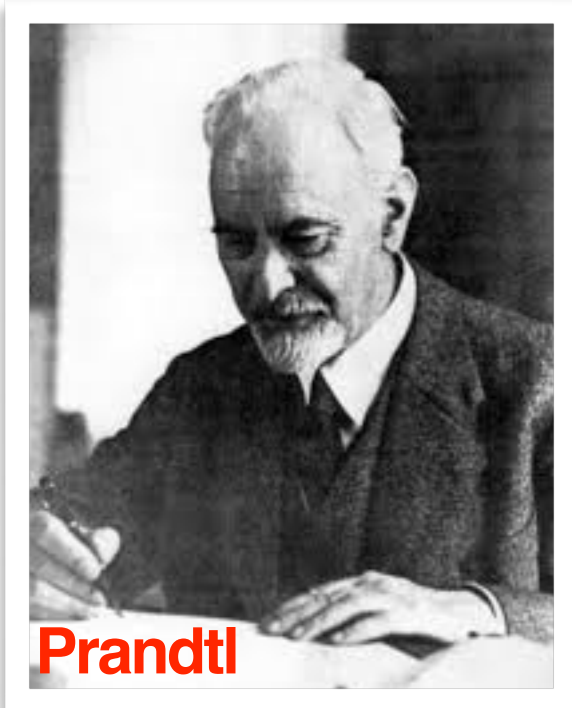

Moody Diagram
Transition Region
Laminar Flow
64/Re
Complete turbulence
Smooth Pipe
Friction Factor = $\frac{2d}{\rho V^2 l}\Delta P$
Friction Factor
Reynolds Number, $Re = \frac{\rho V d}{\mu}$
Relative Pipe Roughness $\frac{\epsilon}{d}$
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.01
10^3
10^4
10^5
10^6
10^7
10^8
0.05
0.04
0.03
0.02
0.015
0.01
0.005
0.002
0.001
5x10^{-4}
2x10^{-4}
10^{-4}
5x10^{-5}
10^{-5}
5x10^{-6}
10^{-6}
Material ε (mm)
Concrete, coarse 0.25
Concrete, new smooth 0.025
Drawn tubing 0.0025
Glass,Plastic,Perspex 0.0025
Iron, cast 0.15
Sewers,old 3.0
Steel, mortar lined 0.1
Steel, rusted 0.5
Steel, structural or forged 0.025
Water mains, old 1.0

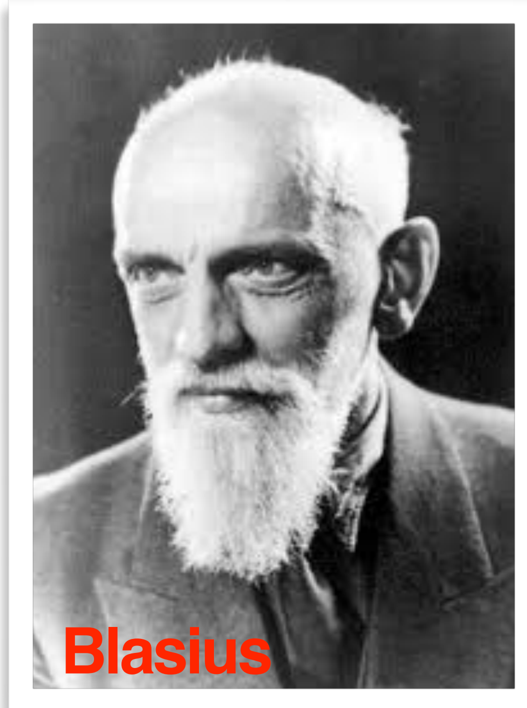
Background- *External Flow*

Internal Vs External









$$u(y) = f(Ue, x, y, \rho, \mu)$$

$$\frac{u}{U_e} = f\left(\frac{U_e x}{\upsilon}, \frac{y}{x}\right)$$

$$u(\partial u/\partial x) + v(\partial u/\partial y) = \nu\left(\partial^2 u/\partial y^2\right)$$

$$\partial u/\partial x + \partial v/\partial y = 0$$

subject to the boundary conditions $u(y{=}0){=}v(y{=}0){=}0$

$$\xi = (y/2)(U/\nu x)^{1/2} \quad \psi = (Ux/\nu)^{1/2}\zeta$$

$$\zeta\zeta'' + \zeta''' = 0$$

Blasius velocity profile

$y(U_e/\nu x)^{1/2}$	u/U_e	$y(U_e/\nu x)^{1/2}$	u/U_e
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

Analytical Approximation

$$\frac{F_v}{\rho W U_e^2} = \int_0^\delta \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy = \frac{2}{15}\delta$$

Assuming a parabolic velocity profile

$$F_v = \int_0^L \tau_w W dx$$

wall shear stress

$$\frac{1}{W}\frac{dF_v}{dx} = \tau_w = \mu\frac{du}{dy} = \frac{2\mu U_e}{\delta}$$

$$F_v = \sqrt{\frac{8}{15}\mu\rho U_e^3 W^2 L}$$

$$C_F = \frac{1.46}{\sqrt{Re_L}}$$

$$C_F = \frac{F_v}{\frac{1}{2}\rho U_e^2 L W} \qquad \text{and} \qquad Re_L = \frac{\rho U_e L}{\mu}$$

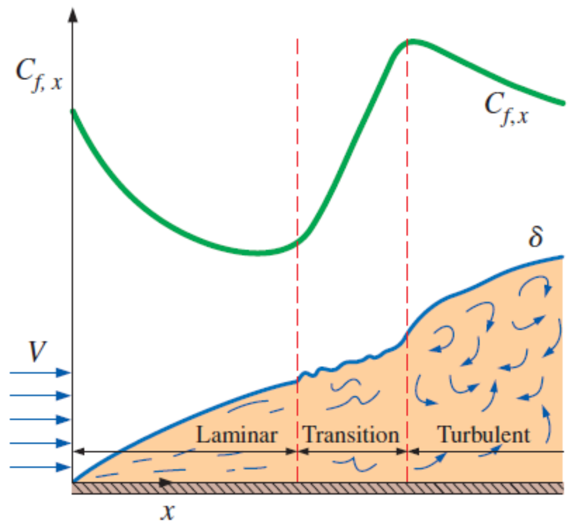
Cf is the total skin friction coefficient as it measures the total viscous drag on the plate. It can also be defined locally at every x location.

for a parabolic velocity profile

$$C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Coefficient of Friction

- The friction coefficient for laminar flow over a flat plate can be determined theoretically by solving the conservation of mass and momentum equations numerically.
- For turbulent flow, it must be determined experimentally and expressed by empirical correlations.



The variation of the local friction coefficient for flow over a flat plate. Note that the vertical scale of the boundary layer is greatly exaggerated in this sketch.

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$$

$$\mathrm{Re}_x = Vx/\nu$$

$$\textit{Laminar:}\quad \delta = \frac{4.91x}{\mathrm{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5\times 10^5$$

$$\textit{Turbulent:}\ \delta = \frac{0.38x}{\mathrm{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}}, \quad 5\times 10^5 \le \mathrm{Re}_x \le 10^7$$

Average friction coefficient over the entire plate

Laminar: $$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$$

$$\mathrm{Re}_{cr} = 5 \times 10^5 = Vx_{cr}/\nu$$

$$\begin{aligned}
C_f &= \frac{1}{L}\int_0^L C_{f,x}\, dx \\
&= \frac{1}{L}\int_0^L \frac{0.664}{\mathrm{Re}_x^{1/2}}\, dx \\
&= \frac{0.664}{L}\int_0^L \left(\frac{Vx}{\nu}\right)^{-1/2} dx \\
&= \frac{0.664}{L}\left(\frac{V}{\nu}\right)^{-1/2} \frac{x^{1/2}}{\frac{1}{2}}\Bigg|_0^L \\
&= \frac{2\times 0.664}{L}\left(\frac{V}{\nu L}\right)^{-1/2} \\
&= \frac{1.328}{\mathrm{Re}_L^{1/2}}
\end{aligned}$$

When the laminar flow region is not disregarded

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,\,x,\,\mathrm{laminar}}\, dx + \int_{x_{cr}}^{L} C_{f,\,x,\,\mathrm{turbulent}}\, dx\right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \leq \mathrm{Re}_L \leq 10^7$$

The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface. The values shown here are for a laminar flat plate boundary layer.

For **Laminar** flow **Cf only depends on Reynolds number**.

For **Turbulent** flows, **surface roughness causes Cf to increase**. For very high Reynolds number in Turbulent flows, Cf is almost just a function of surface roughness

Fully rough turbulent regime: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

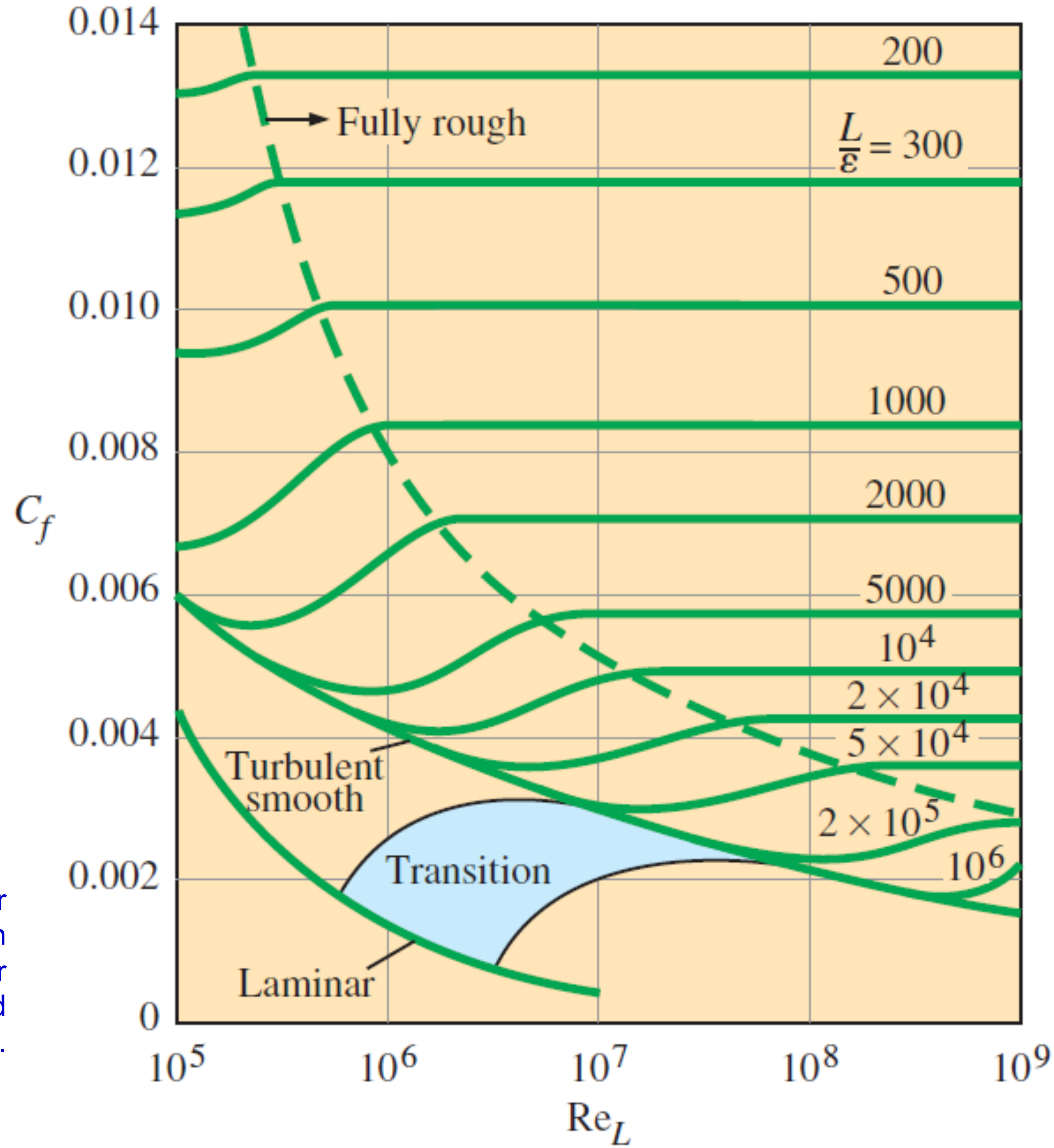
Relative Roughness, ε/L	Friction Coefficient, C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

* Smooth surface for Re = 10^7. Others calculated from Eq. 11–23 for fully rough flow.

C_f increases severalfold with roughness in turbulent flow.

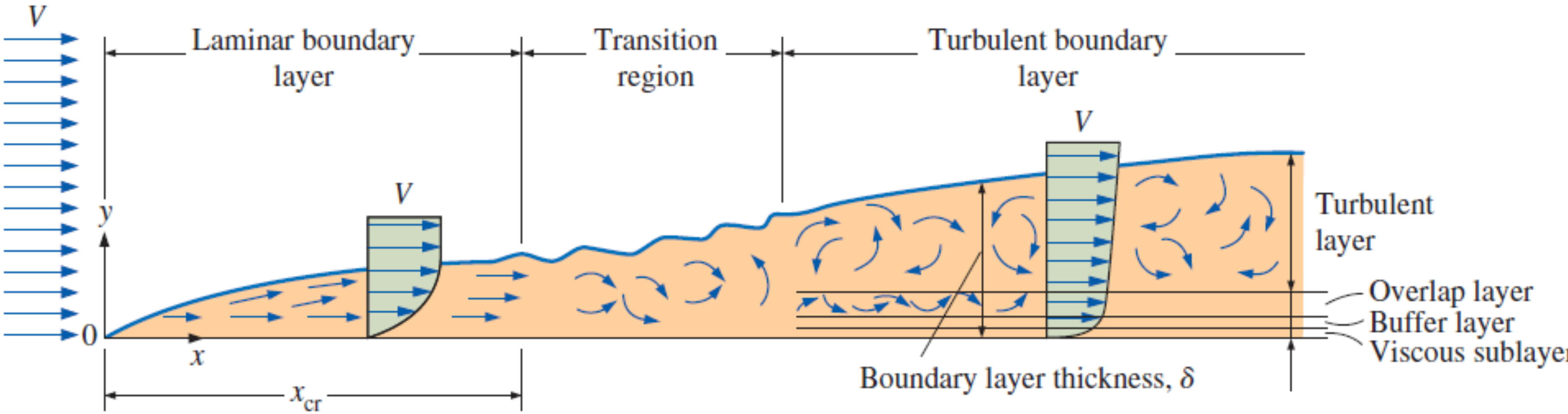
C_f is independent of the Reynolds number in the fully rough region.

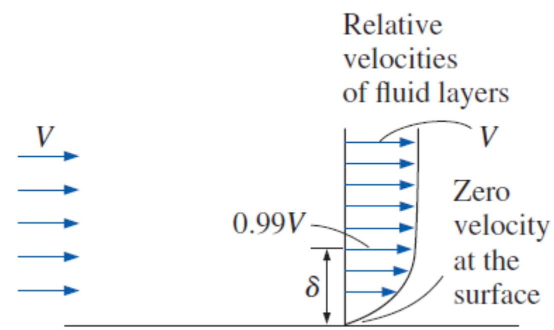
This chart is the flat-plate analog of the Moody chart for pipe flows.



Friction coefficient for parallel flow over smooth and rough flat plates for both laminar and turbulent flows.

Flow over Flat Plate Summary





The turbulent boundary layer can be considered to consist of four regions, characterized by the distance from the wall:

- viscous sublayer
- buffer layer
- overlap layer
- turbulent layer

Friction coefficient on a flat plate

$$C_D = C_{D,\text{ friction}} = C_f$$

$$F_D = F_f = \tfrac{1}{2} C_f A \rho V^2$$

Flow over a flat plate

$$C_{D,\text{ pressure}} = 0$$

$$C_D = C_{D,\text{ friction}} = C_f$$

$$F_{D,\text{ pressure}} = 0$$

$$F_D = F_{D,\text{ friction}} = F_f = C_f A \frac{\rho V^2}{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y}$$

$$\tau_{lam} = \mu\frac{\partial u}{\partial y}$$

$$\tau_{turb} = \mu\frac{\partial u}{\partial y} - \overline{\rho u' v'}$$

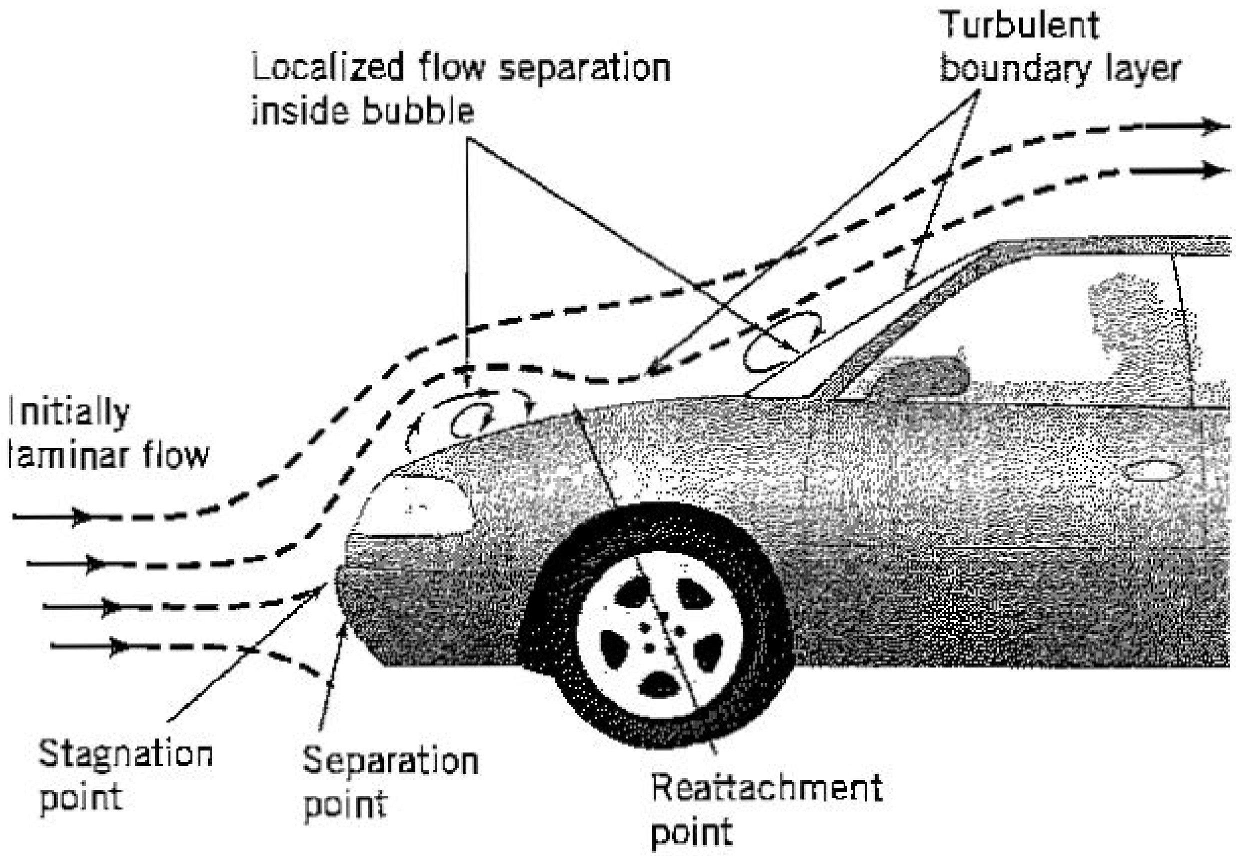
Cf accounts for viscous drag acting on the surface due to the presence of boundary layer

Are there other effects that cause drag?

Seperation, Reattachment & Wakes

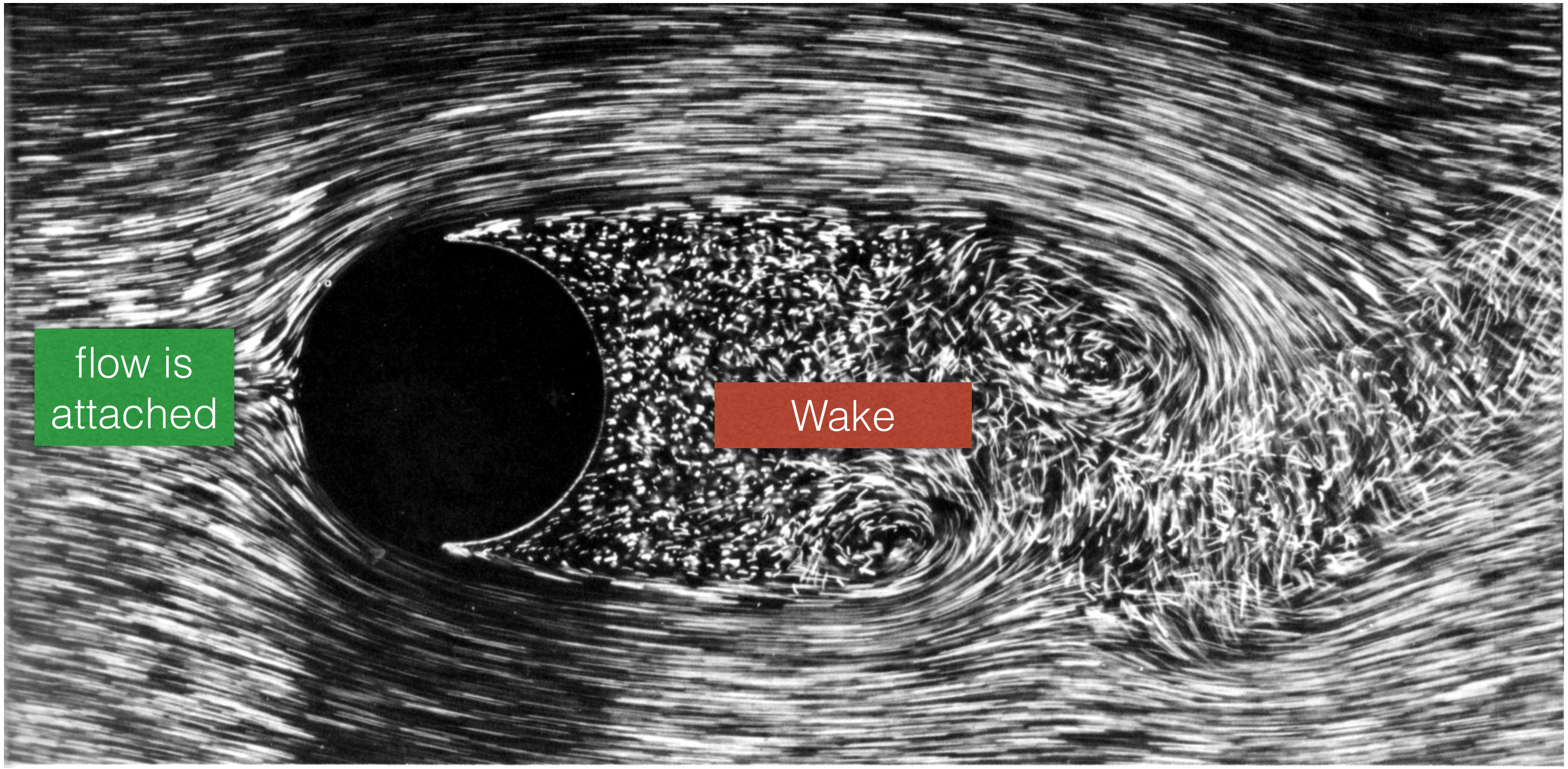
$$\frac{dP}{dx} > 0$$ **Adverse**

$$\frac{dP}{dx} < 0$$ **Favourable**

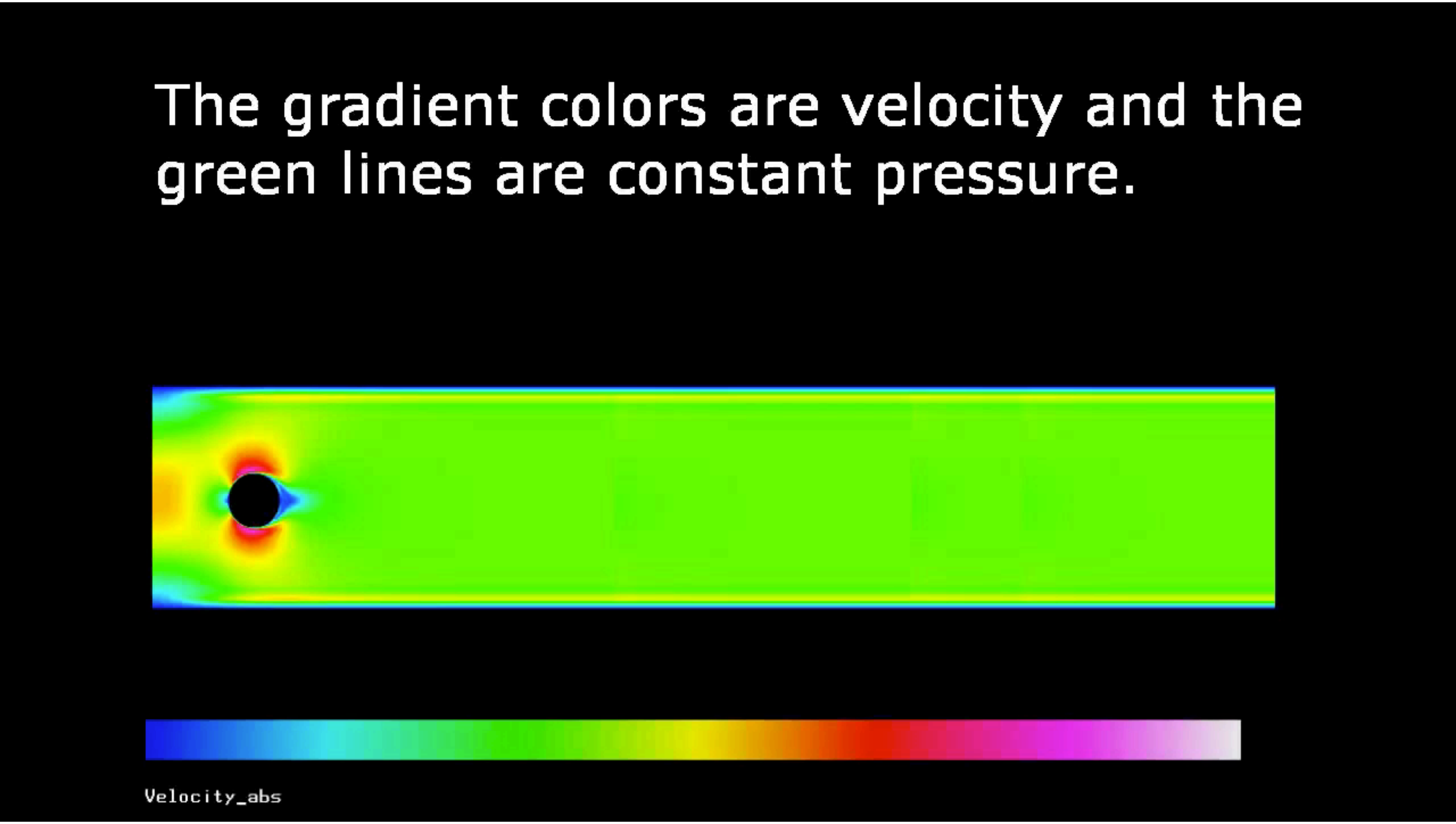


Sharp corners & Flow reversal give rise to adverse pressure gradient and causes flow to separate.

Re ~ 2000



The flow is separated in the wake zone which acts as a **momentum sink**, draining the kinetic energy in the flow.



The eddying motion behind the flow produces **pressure disturbance** in the flow field.
(the whistling you hear from a tree dancing in the wind)

So the drag force experienced by bodies can be split in to two components; **Viscous** drag & **Pressure** drag

Viscous drag

Due to boundary layers

Important for attached flows

Scales with Reynolds number

Related to surface area exposed

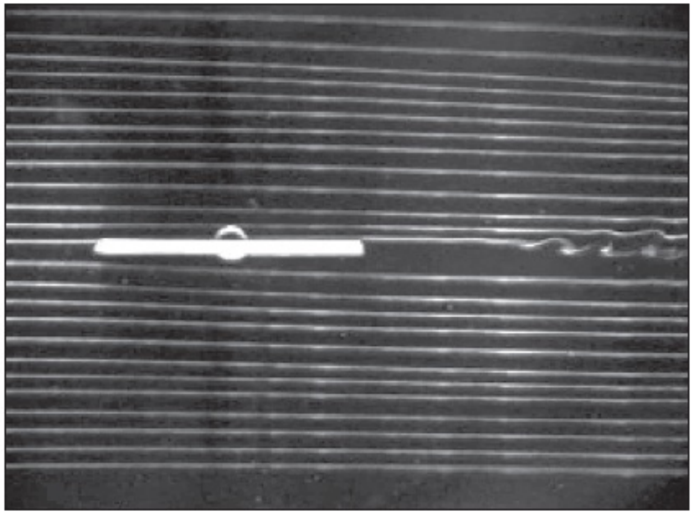
Pressure drag

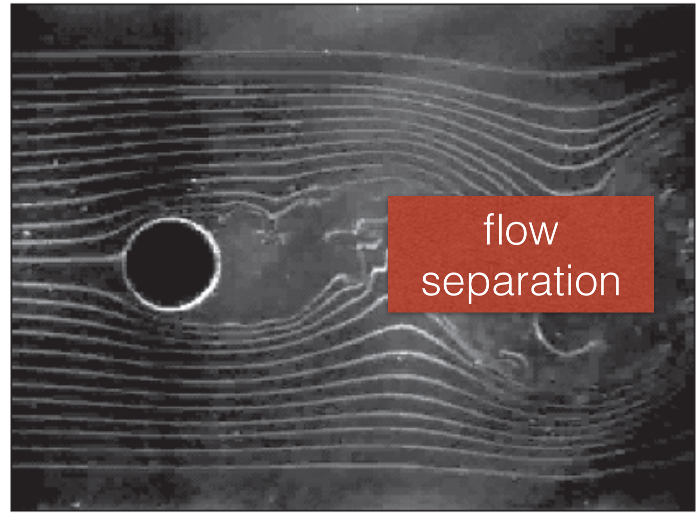
vortex shedding flow separation

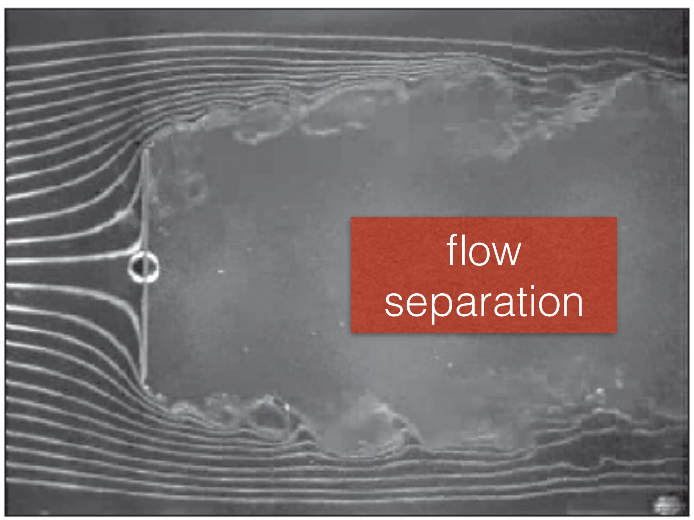
Separated flows

Cross section area & shape

Viscous drag





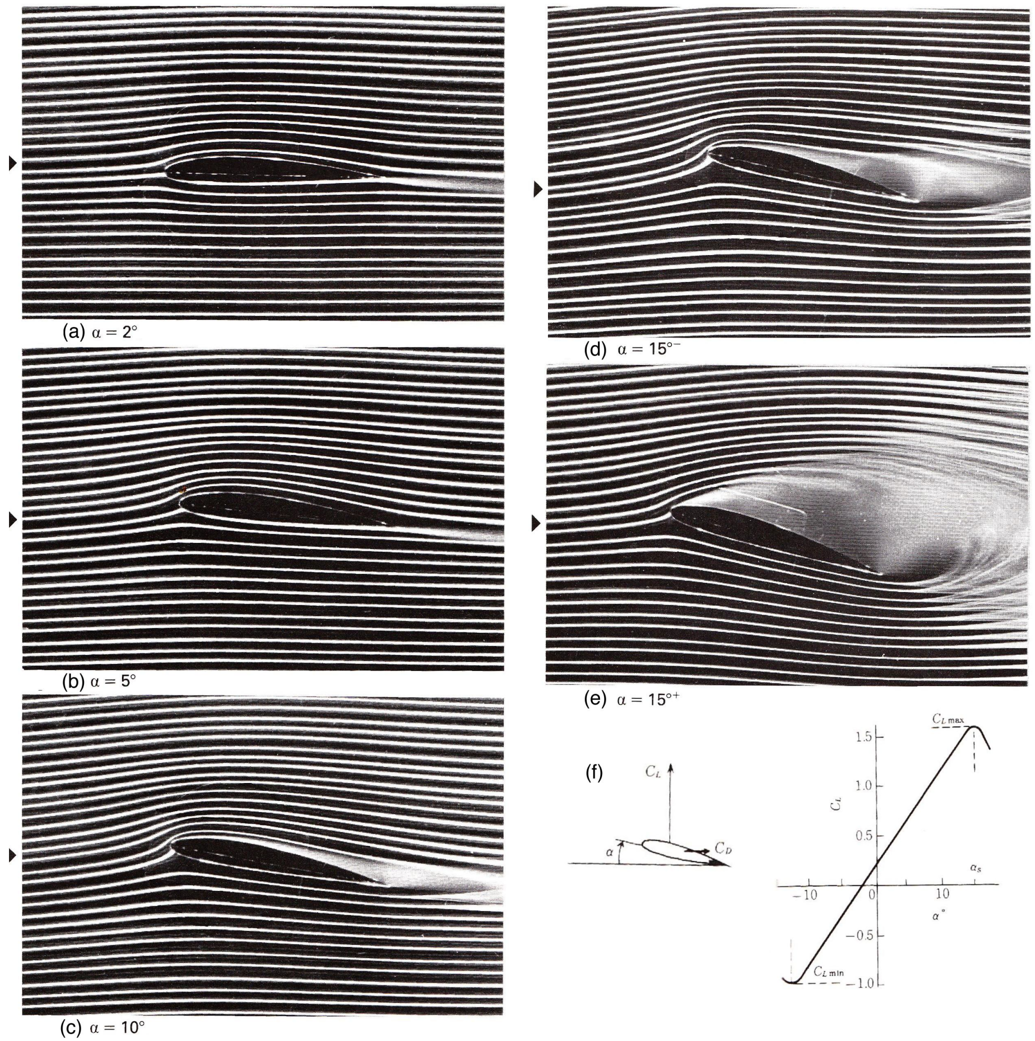


Pressure drag

Small pressure drag $\longrightarrow$ Streamlined

Large pressure drag $\longrightarrow$ Bluff body

Flow over Airfoil



As angle of attack increases, the pressure gradient along the airfoil becomes less favorable, leading to separation of the flow

Flow over Cylinder

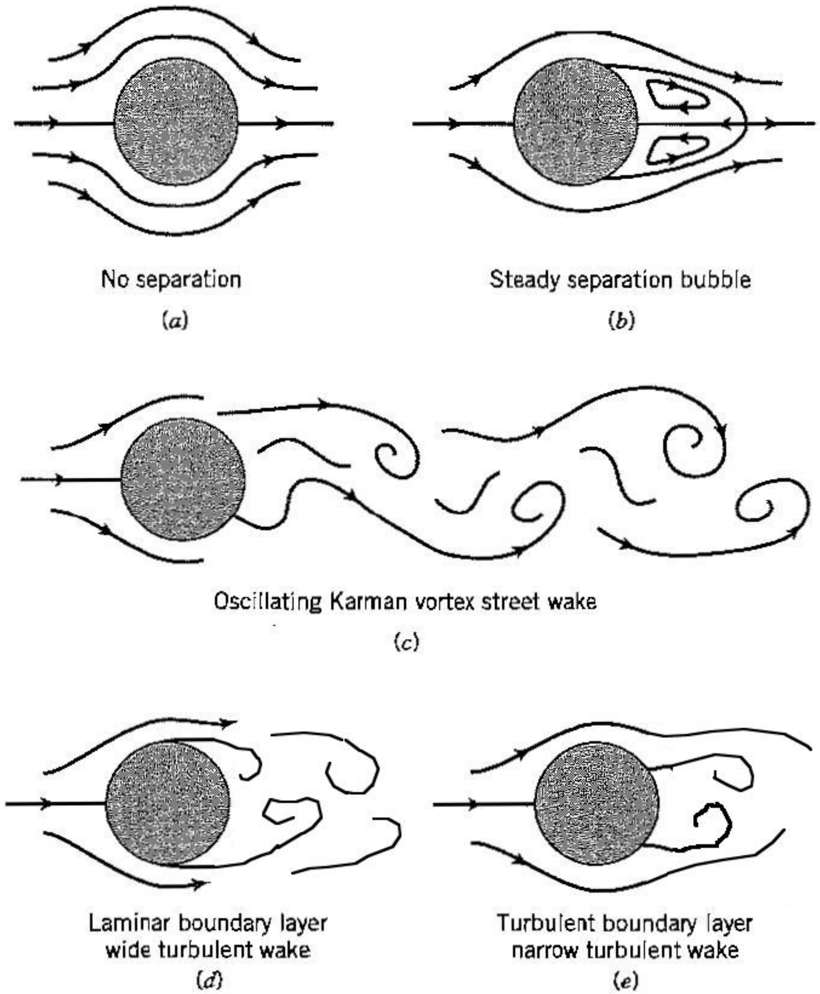
RE # 0.2

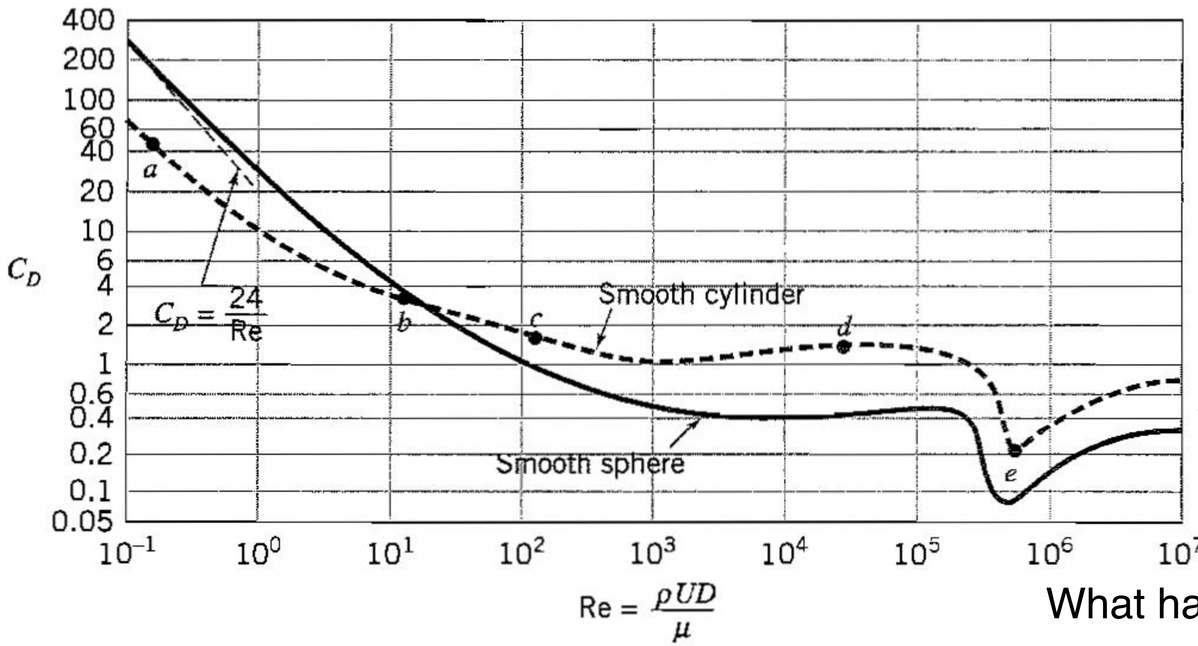
RE # 12

RE # 120

RE # 30000

RE # 500000

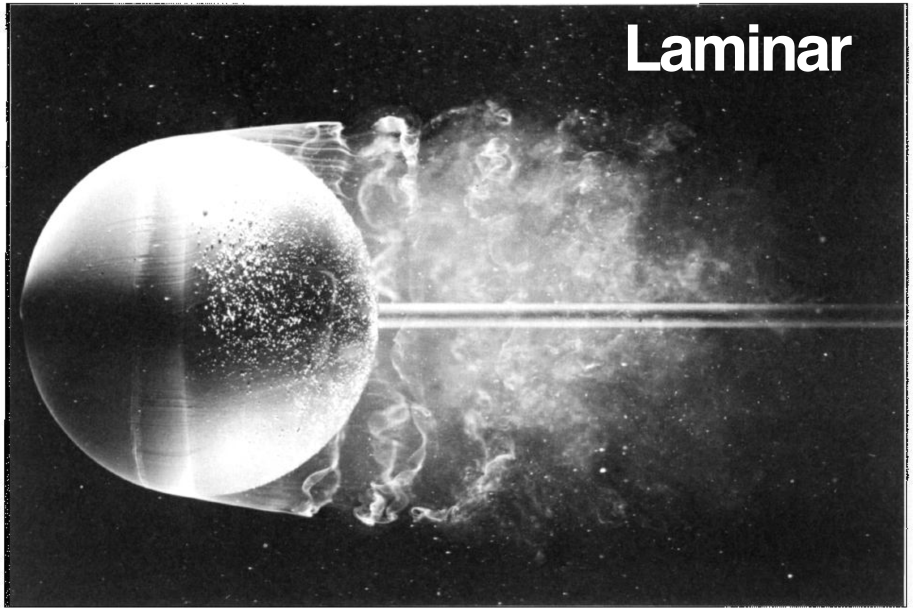


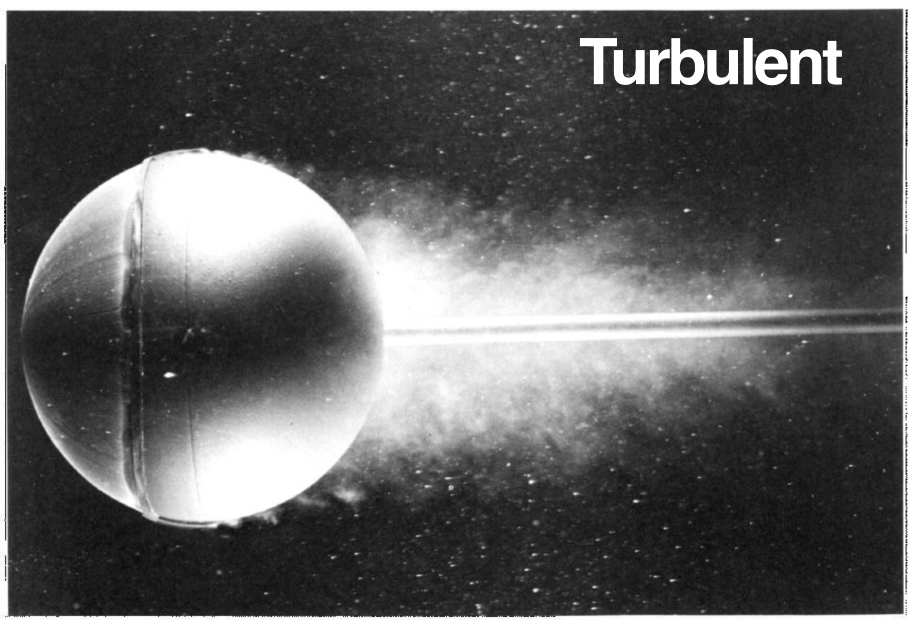


What happens for high Re

What causes Cd to decrease with Re#

What causes Cd to decrease with Re# ?





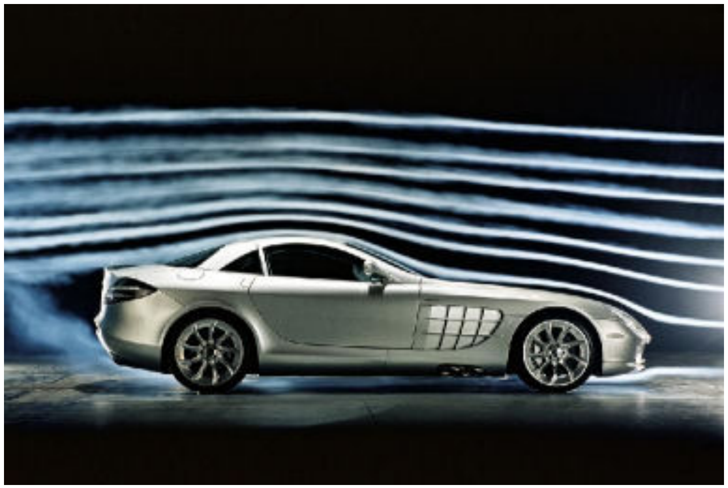
Turbulent BL has **more momentum near wall than a laminar BL** as turbulence is very effective in **mixing momentum**. This mixing replenishes lost momentum near wall due to viscous drag. So when the pressure gradient becomes adverse, the **turbulent boundary layer can persist longer** without separating.

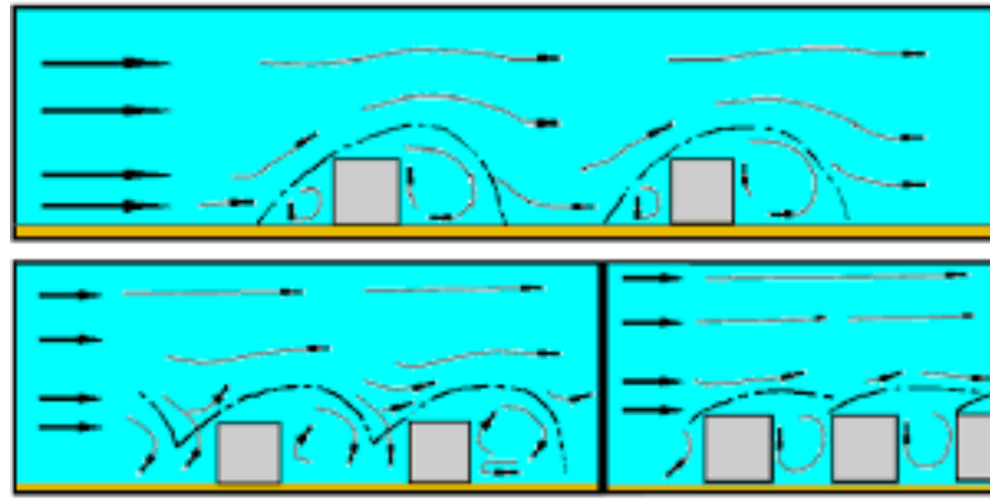
How to estimate drag on bluff bodies?

Important in aviation

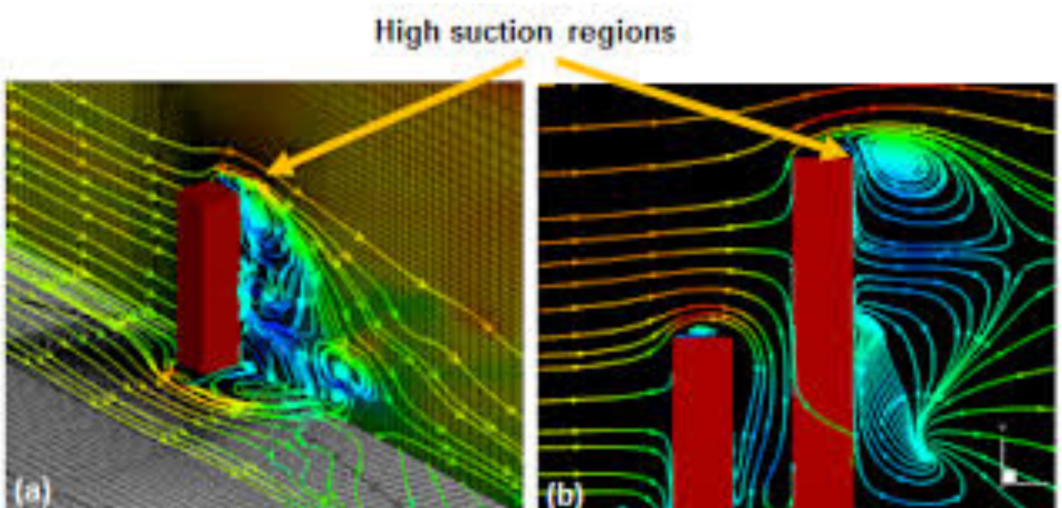
Common engineering use

Common engineering use





Urban Wind Flow Patterns With Various Simple Building Shapes and Spacings



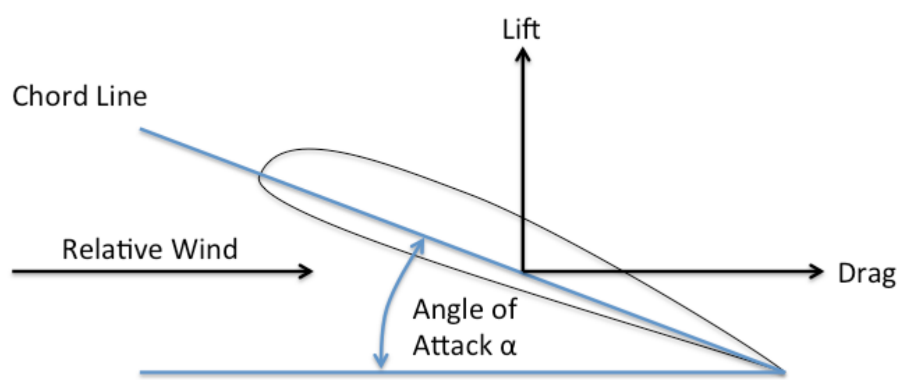
Important in sports

With dimples 250 yards,
without dimples 100 yards

In Cricket and Baseball, the uneven separation of the boundary layer is used to dip the ball sharply.

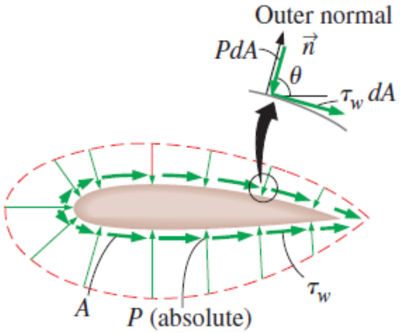
Aerodynamic forces are a result of surface forces, pressure force and shear stresses or viscous stresses.

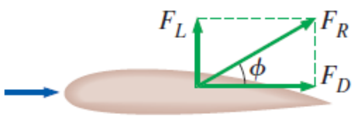
$$\vec{F} = \int_{S_{wet}} \vec{\tau}\, dS - \int_{S_{wet}} p\vec{n}\, dS,$$



Both pressure and viscous stresses affect lift and drag.

Force acting on the body in the direction of flow is called drag and normal to the flow is called lift.



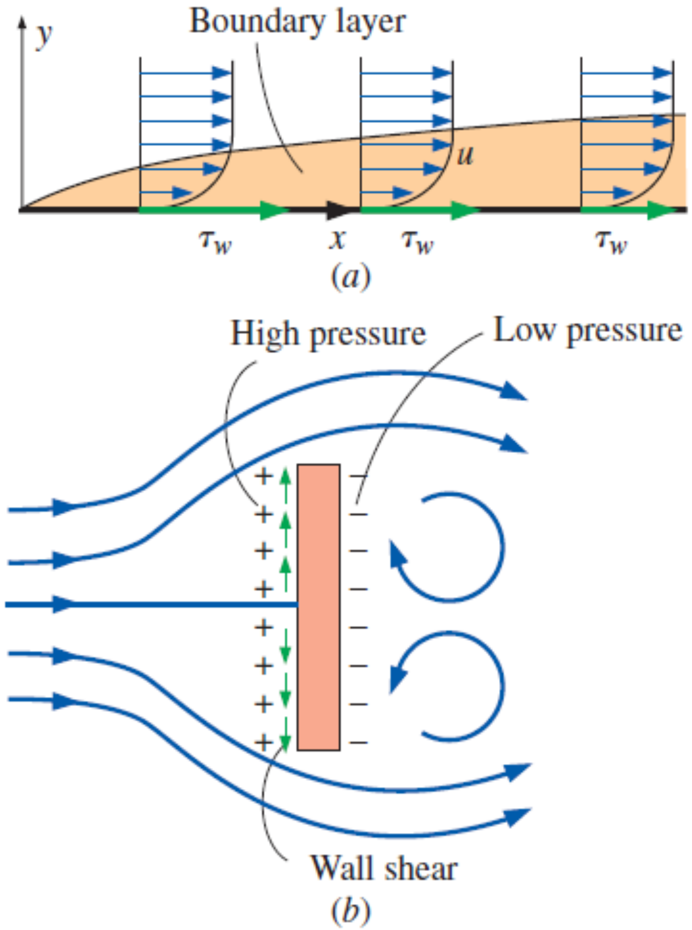


$$dF_D = -P\, dA \cos\theta + \tau_w\, dA \sin\theta$$

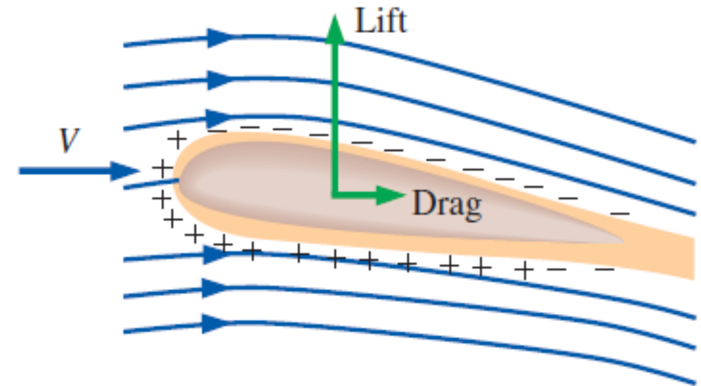
$$dF_L = -P\, dA \sin\theta - \tau_w\, dA \cos\theta$$

Drag force: $$F_D = \int_A dF_D = \int_A (-P\cos\theta + \tau_w \sin\theta)\, dA$$

Lift force: $$F_L = \int_A dF_L = -\int_A (P\sin\theta + \tau_w \cos\theta)\, dA$$



y
Boundary layer
u
τ_w
x
τ_w
τ_w
(a)
High pressure
Low pressure
Wall shear
(b)



Lift
V
Drag

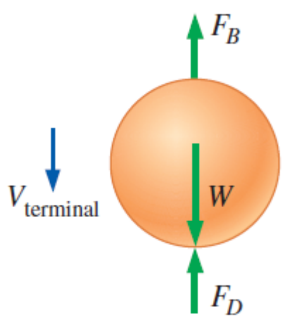
The drag and lift forces depend on size, shape and orientation of the object and on density of the fluid and the incident velocity.

To simplify the calculation, we simply used non-dimensional variables based on PI groups.

Drag coefficient: $$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Lift coefficient: $$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

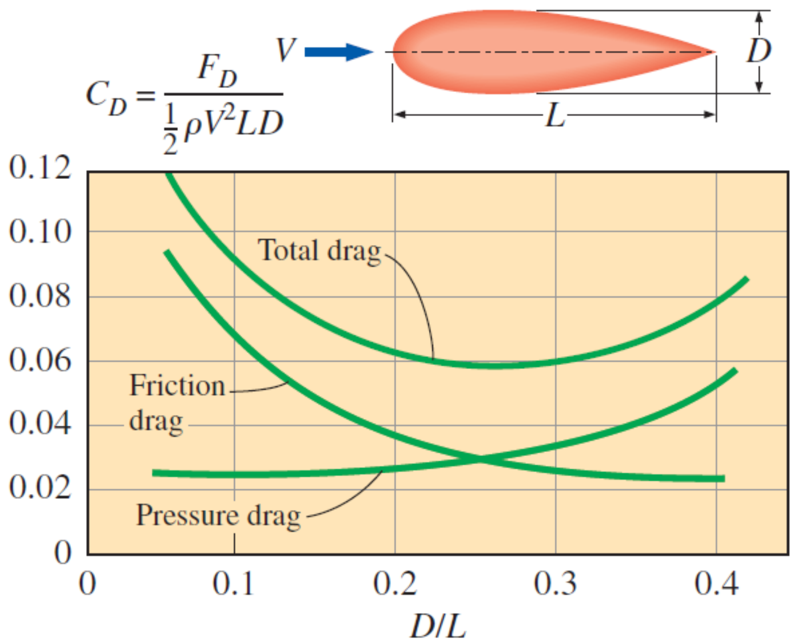
$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \qquad C_L = \frac{1}{L}\int_0^L C_{L,x}\,dx$$



$F_D = W - F_B$
(No acceleration)

Reducing Drag by Streamlining

Streamlining decreases pressure drag by delaying boundary layer separation and thus reducing the pressure difference between the front and back of the body but increases the friction drag by increasing the surface area. The end result depends on which effect dominates.

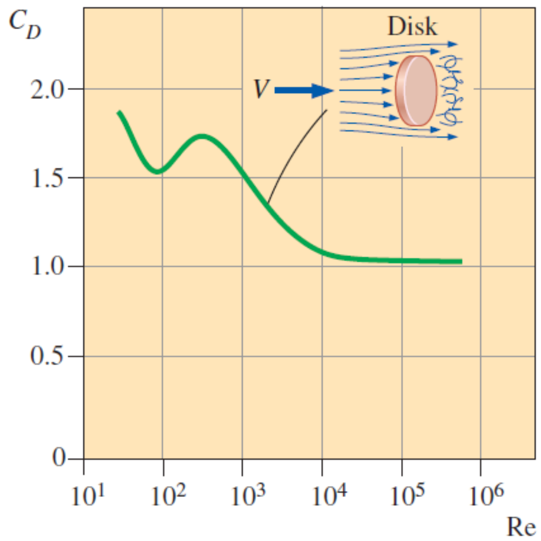


The variation of friction, pressure, and total drag coefficients of a streamlined strut with thickness-to-chord length ratio for Re = 4×10^4. Note that C_D for airfoils and other thin bodies is based on *planform* area rather than frontal area.

DRAG COEFFICIENTS OF COMMON GEOMETRIES

The drag behavior of various natural and human-made bodies is characterized by their drag coefficients measured under typical operating conditions.

Usually the *total* (friction+pressure) drag coefficient is reported.



The drag coefficient for many (but not all) geometries remains essentially constant at Reynolds numbers above about 10^4.

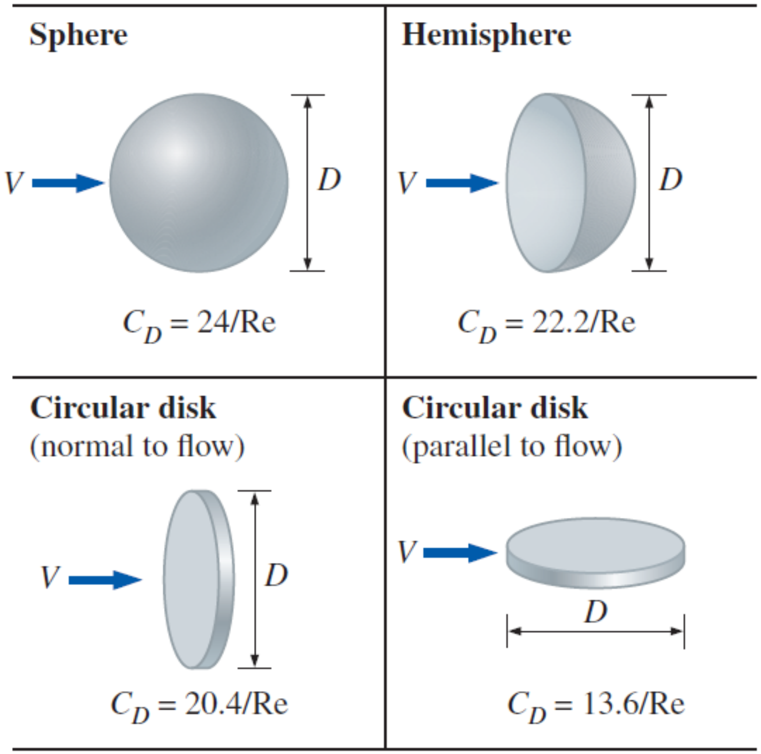
The drag coefficient exhibits different behavior in the low (creeping), moderate (laminar), and high (turbulent) regions of the Reynolds number.

The inertia effects are negligible in low Reynolds number flows (Re < 1), called *creeping flows*, and the fluid wraps around the body smoothly.

$$C_D = \frac{24}{\mathrm{Re}} \qquad (\mathrm{Re} \lesssim 1)$$

$$F_D = C_D A \frac{\rho V^2}{2} = \frac{24}{\mathrm{Re}} A \frac{\rho V^2}{2} = \frac{24}{\rho V D/\mu} \frac{\pi D^2}{4} \frac{\rho V^2}{2} = 3\pi\mu V D$$

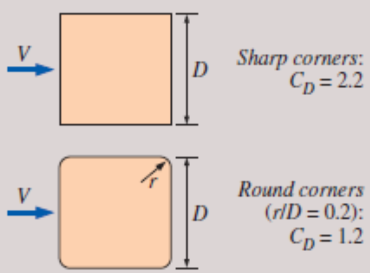
DRAG COEFFICIENTS OF COMMON GEOMETRIES



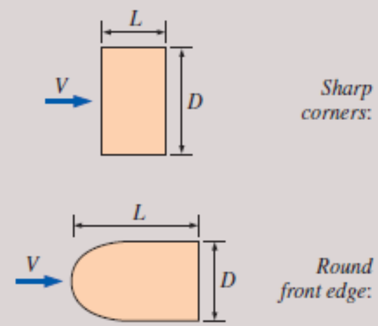
DRAG COEFFICIENTS OF COMMON GEOMETRIES

Drag coefficients C_D of various two-dimensional bodies for Re > 10^4 based on the frontal area $A = bD$, where b is the length in direction normal to the page (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)

Square rod



Rectangular rod

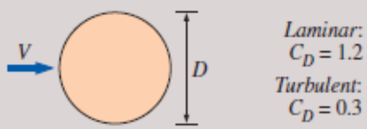


L/D	C_D
0.0*	1.9
0.1	1.9
0.5	2.5
1.0	2.2
2.0	1.7
3.0	1.3

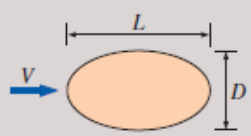
* Corresponds to thin plate

L/D	C_D
0.5	1.2
1.0	0.9
2.0	0.7
4.0	0.7

Circular rod (cylinder)

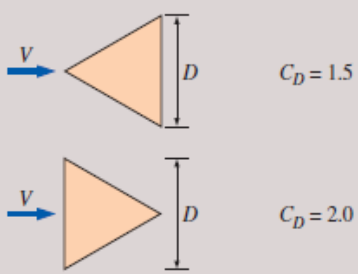


Elliptical rod

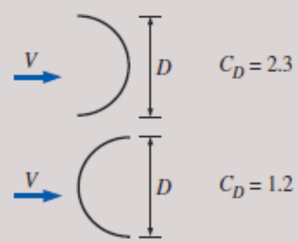


	C_D	
L/D	Laminar	Turbulent
2	0.60	0.20
4	0.35	0.15
8	0.25	0.10

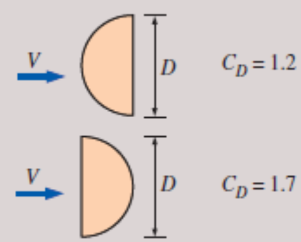
Equilateral triangular rod



Semicircular shell



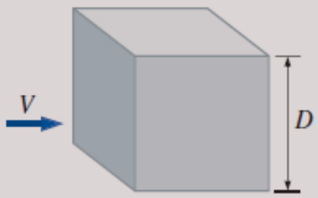
Semicircular rod



DRAG COEFFICIENTS OF COMMON GEOMETRIES

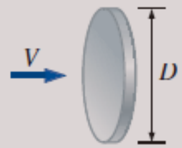
Representative drag coefficients C_D for various three-dimensional bodies based on the frontal area for Re > 10^4 unless stated otherwise (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)

Cube, $A = D^2$



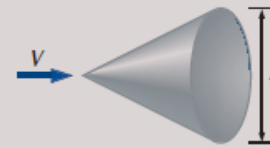
$C_D = 1.05$

Thin circular disk, $A = \pi D^2/4$



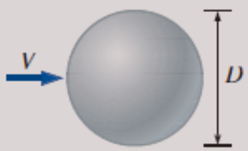
$C_D = 1.1$

Cone (for $\theta = 30°$), $A = \pi D^2/4$



$C_D = 0.5$

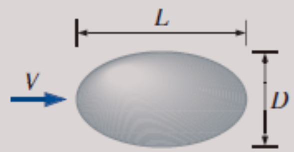
Sphere, $A = \pi D^2/4$



Laminar:
Re $\lesssim 2 \times 10^5$
$C_D = 0.5$
Turbulent:
Re $\gtrsim 2 \times 10^6$
$C_D = 0.2$

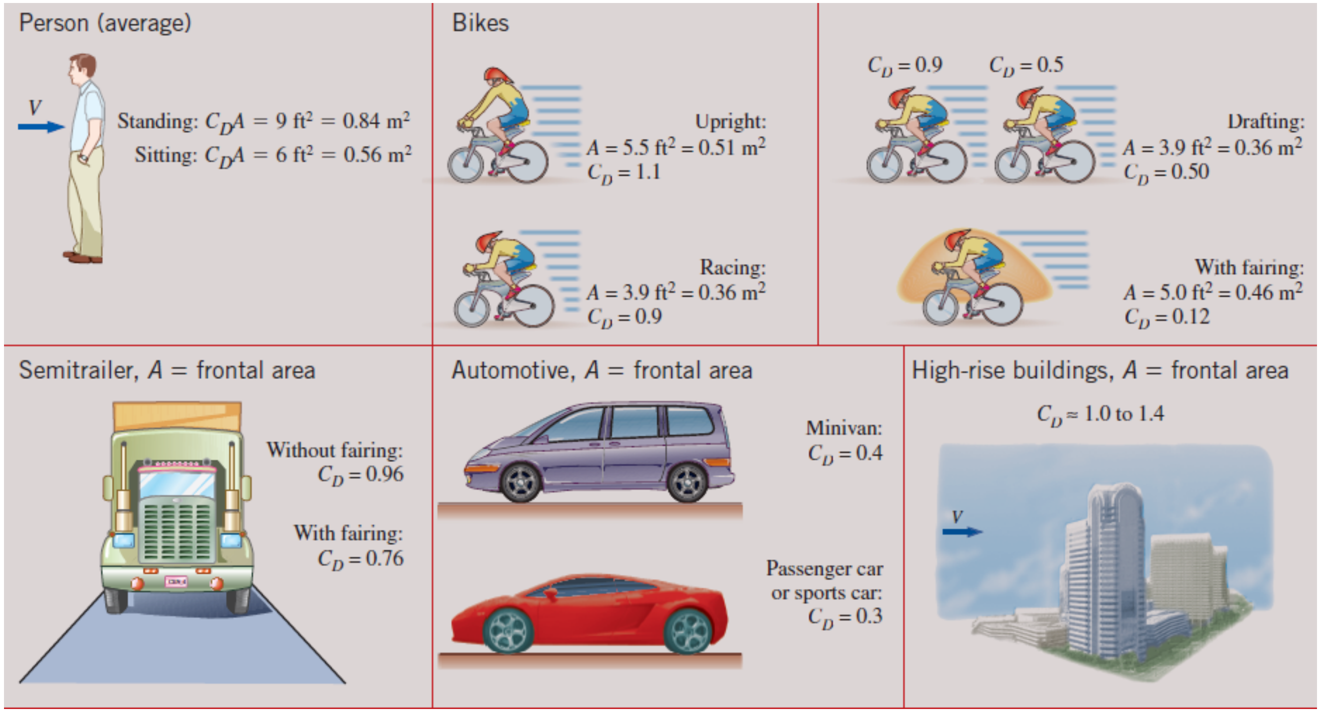
See Fig. 11–36 for C_D vs. Re for smooth and rough spheres.

Ellipsoid, $A = \pi D^2/4$



	C_D	
L/D	Laminar Re $\lesssim 2 \times 10^5$	Turbulent Re $\gtrsim 2 \times 10^6$
0.75	0.5	0.2
1	0.5	0.2
2	0.3	0.1
4	0.3	0.1
8	0.2	0.1

DRAG COEFFICIENTS OF COMMON GEOMETRIES



Turbines

A turbine extracts energy from a fluid which possesses high head. There are two types, reaction and impulse, the difference lying in the manner of head conversion. In the reaction turbine, the fluid fills the blade passages, and the head change or pressure drop occurs within the impeller. Reaction designs are of the radial-flow, mixed-flow, and axial-flow types and are essentially dynamic devices designed to admit the high-energy fluid and extract its momentum. *An **impulse turbine** first converts the high head through a nozzle into a high- velocity jet, which then strikes the blades at one position as they pass by*. The impeller passages are not fluid-filled, and the jet flow past the blades is essentially at constant pressure. Reaction turbines are smaller because fluid fills all the blades at one time.

Turbines

Turbine parameters are dependent on the output brake horsepower, which depends upon the inlet flow rate Q, available head H, im- peller speed n, and diameter D. The efficiency is the output brake horsepower divided by the available water horsepower $\varrho\boxed{?}gQH$. The dimensionless forms are C_Q, C_H, and C_P, defined just as for a pump, Eqs. (11.23). If we neglect Reynolds-number and roughness effects, the functional relationships are written with C_P as the independent variable:

$$C_H = C_H(C_P) \qquad C_Q = C_Q(C_P) \qquad \eta = \frac{\text{bhp}}{\rho g Q H} = \eta(C_P)$$

Turbines

Power Specific Speed

Rigorous form: $$N'_{sp} = \frac{C_P^{*1/2}}{C_H^{*5/4}} = \frac{n(\text{bhp})^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

Lazy but common: $$N_{sp} = \frac{(\text{r/min})(\text{bhp})^{1/2}}{[H\ (\text{ft})]^{5/4}}$$

Turbines- Pelton Wheel

Power Specific Speed

$$F = \rho Q(V_j - u)(1 - \cos\ \beta)$$

$$P = Fu = \rho Qu(V_j - u)(1 - \cos\ \beta)$$

$$u^* = 2\pi n^* r = \tfrac{1}{2} V_j$$

$$V_j = C_v (2gH)^{1/2} \qquad 0.92 \le C_v \le 0.98$$

Turbine efficiency $\eta = 2(1 - \cos\ \beta)\phi(C_v - \phi)$

$$\phi = \frac{u}{(2gH)^{1/2}} = \text{peripheral-velocity factor}$$

Course Outline

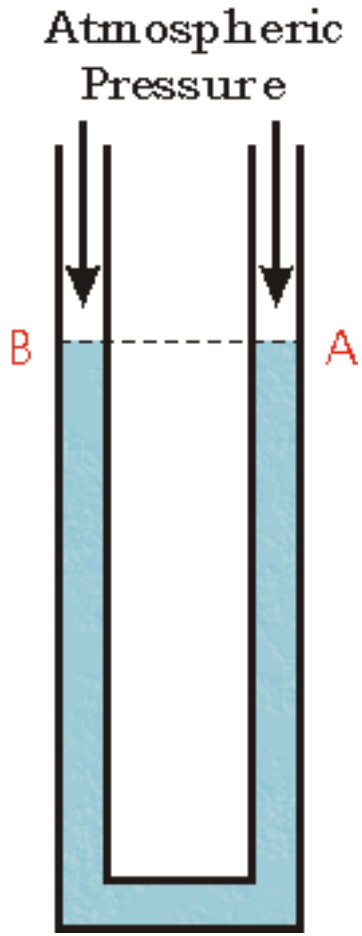
- Introduction – safety, lab reports, plotting, error analysis, signal processing, lab-specific hardware/software data acquisition, measurement devices, engineering principles
- Laboratory Experiments
  1. Wind Tunnel
  2. Viscous Pipe Flow
  3. Heat Exchanger
  4. Cross-Section Fin
  5. Pelton Wheel
- Design Project Presentation and Report
- Final Exam

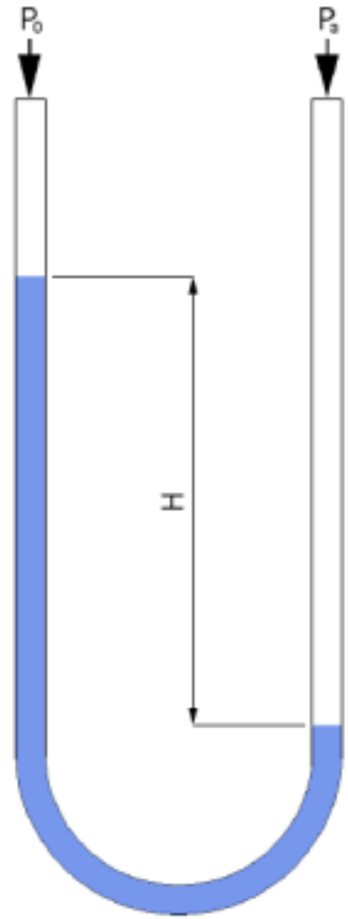
Measurement Devices

Physical Instruments

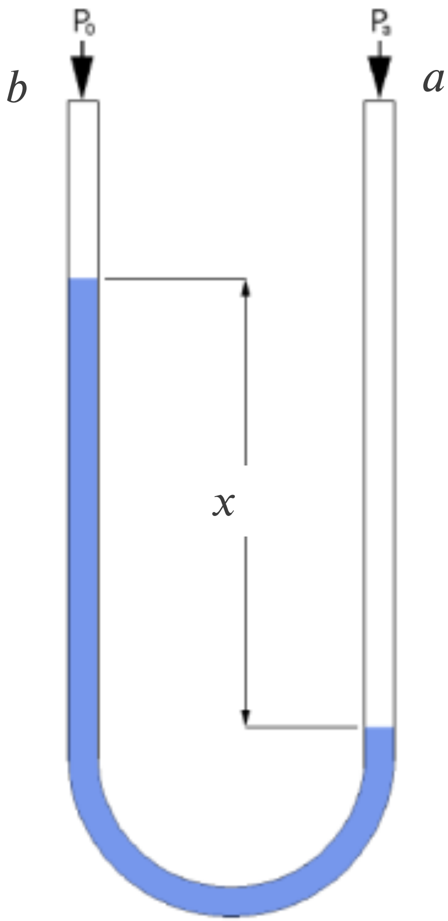
- Manometers
- Pitot Tubes
- Venturi Meters
- Thermocouples

Manometer





Manometer



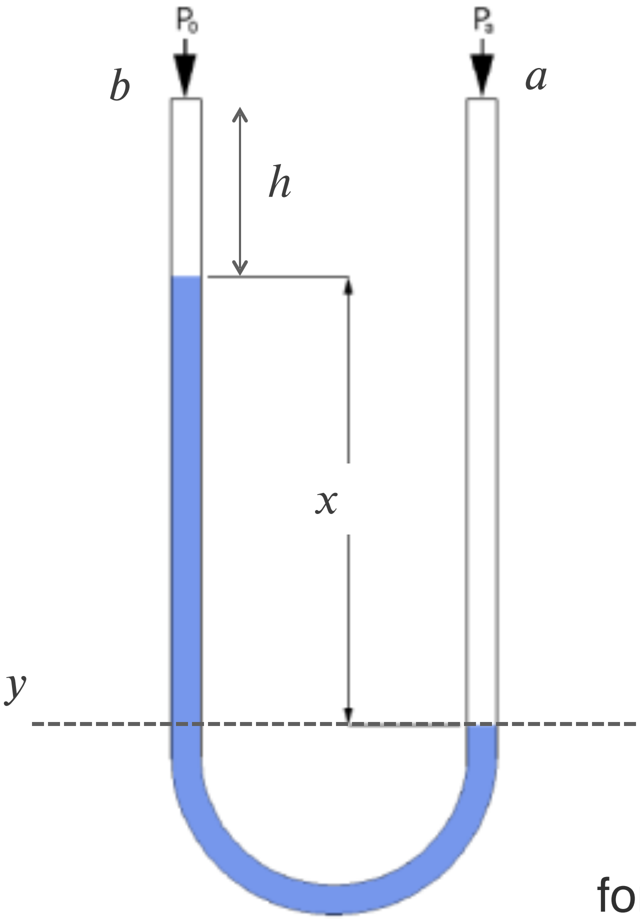
$$P_{a,tot} + \frac{1}{2}\rho_a v_a^2 + \rho_a g h_a = \text{ const}$$

Keep in mind dynamic pressure and total pressure

$$P_{a,tot} = P_0 + P_a$$

$$P_b - P_a = \ ?$$

Manometer



$$P_{a,y} = P_{b,y}$$

$$P_{a,y} = P_0 + P_a + \rho_a g\left(h + x\right)$$

$$P_{b,y} = P_0 + P_b + \rho_a gh + \rho_b gx$$

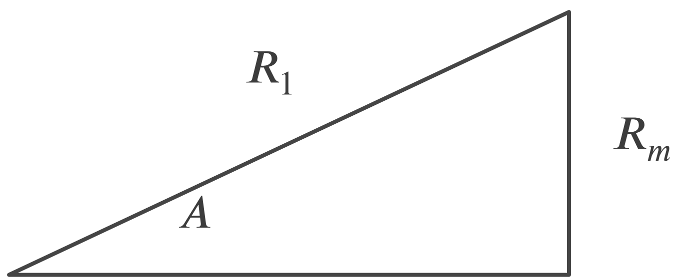
$$P_b - P_a = \rho_a gx - \rho_b gx = gx(\rho_a - \rho_b)$$

$$P_{a,y} = P_{b,y}$$

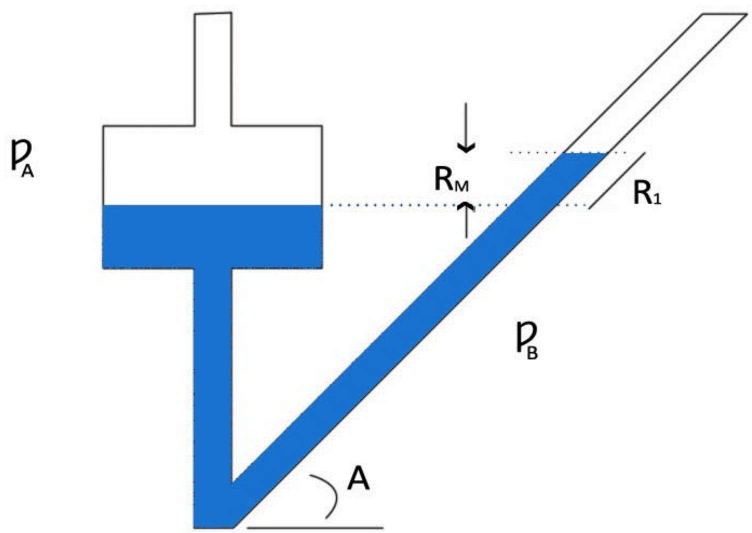
for continuous fluid

$$\boxed{P_b - P_a = gx(\rho_a - \rho_b)}$$

Manometer

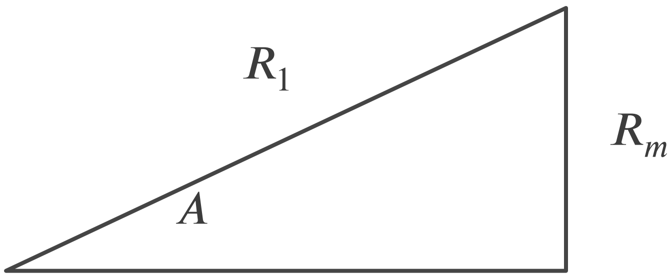


$$R_M = R_1 \cdot \sin A$$

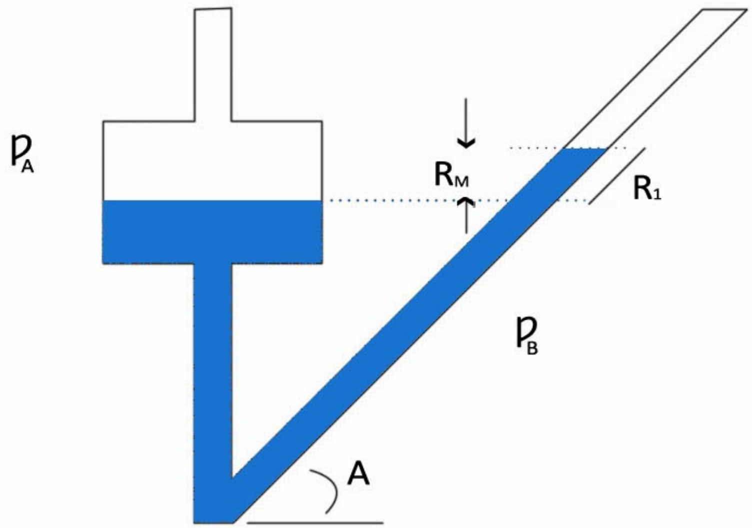


$$P_A - P_B = \ ?$$

Manometer



$$R_M = R_1 \cdot \sin A$$

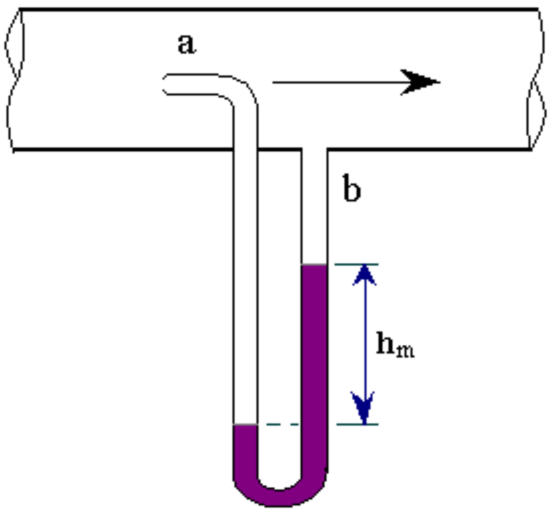


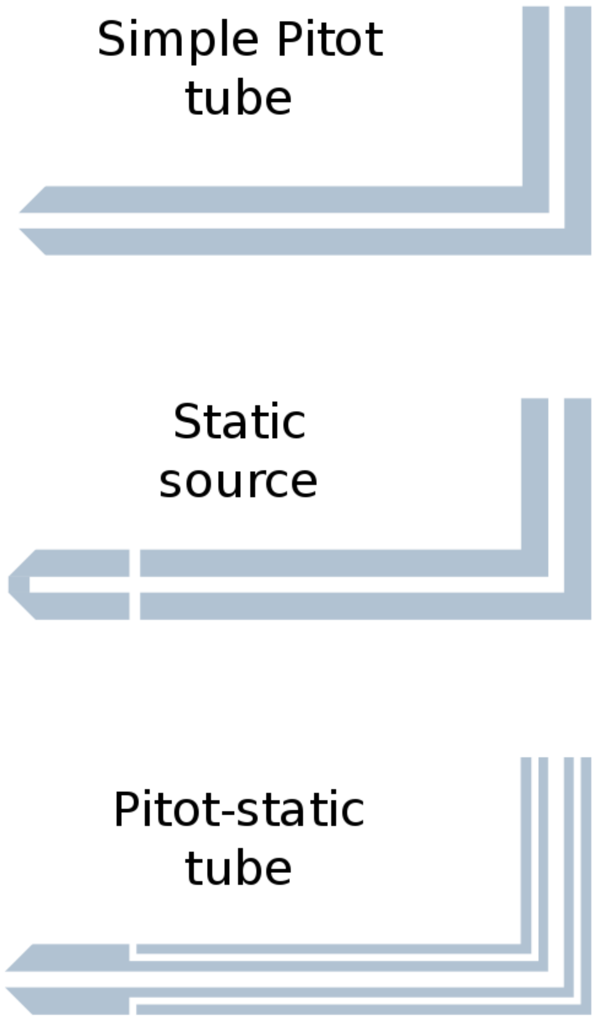
$$P_A - P_B = R_1.\mathrm{Sin}(A).g.(\rho_B - \rho_A)$$

ρ_A DENSITY OF FLUID A

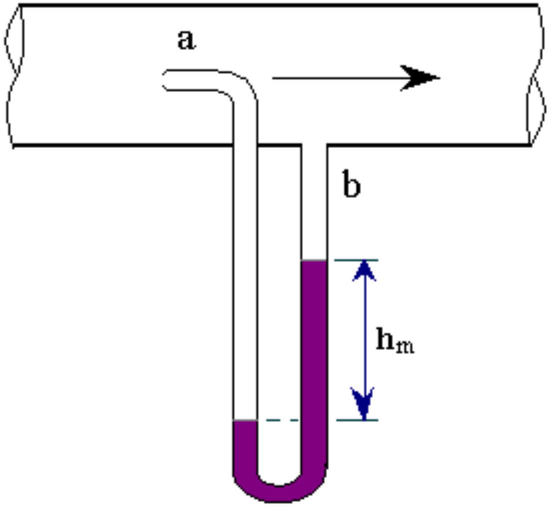
ρ_B DENSITY OF MANOMETRIC FLUID B

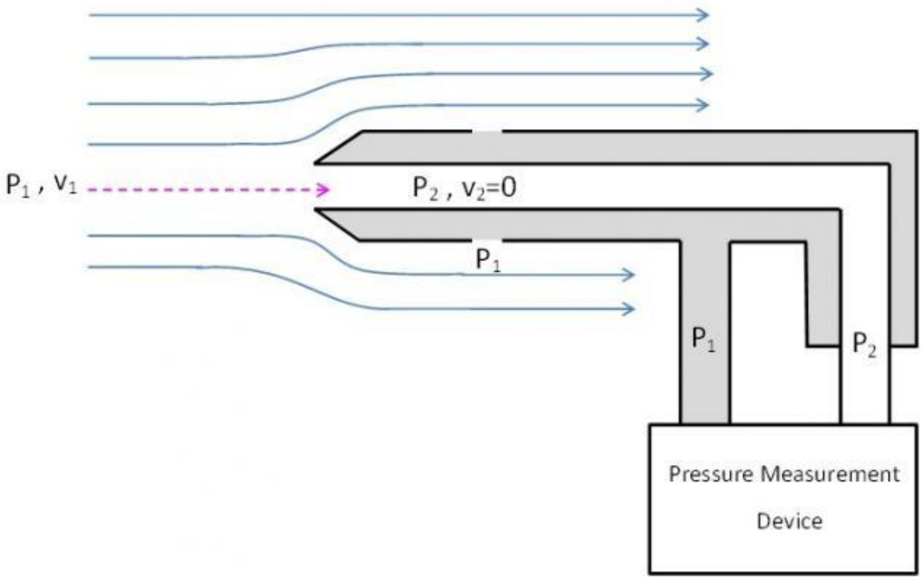
Pitot Tube



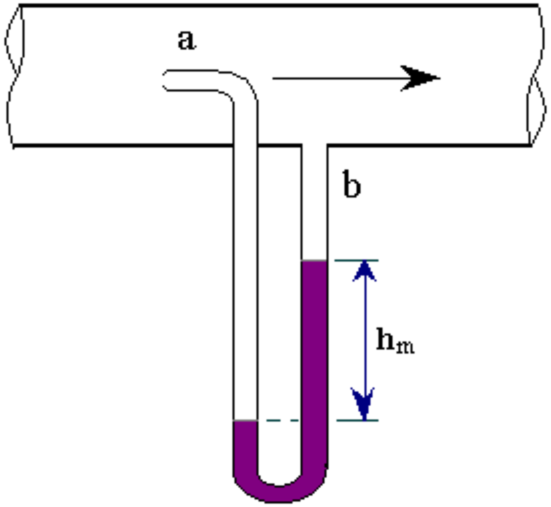


Pitot Tube



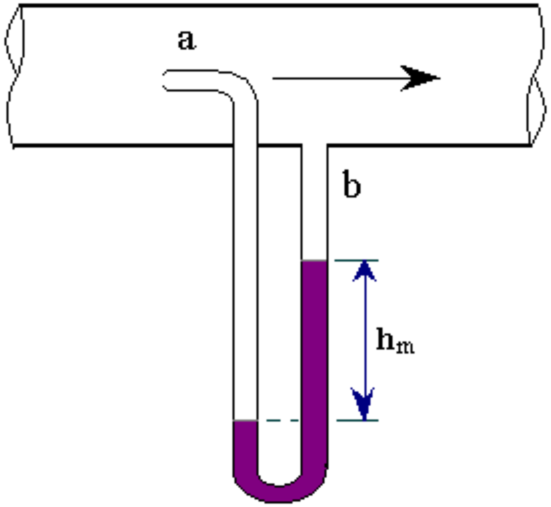


Pitot Tube



Using Bernoulli → Calculate velocity

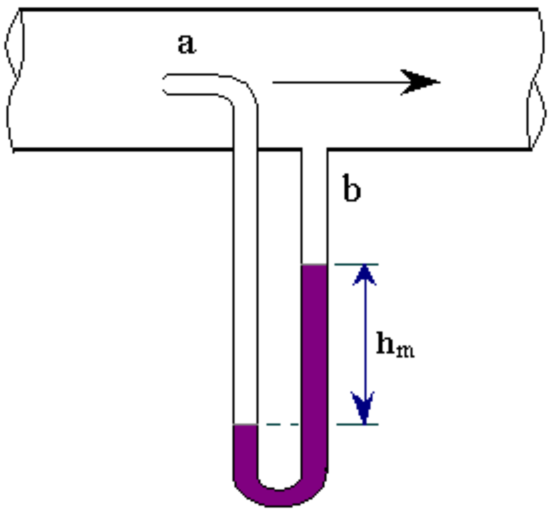
Pitot Tube



For measurement at static point in bend:

$$P_a = P_{static} + \frac{1}{2}\rho_a v_a^2$$

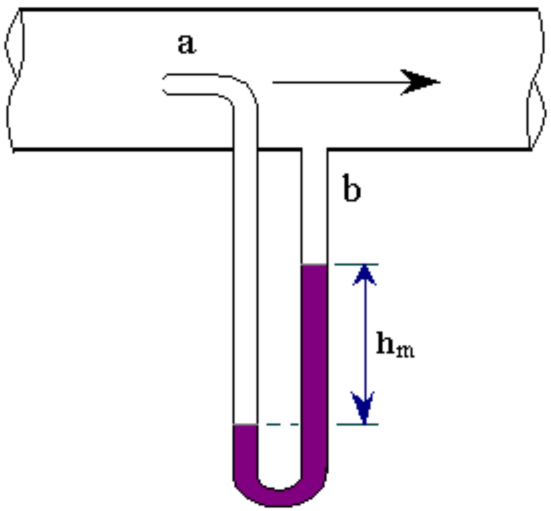
Pitot Tube



Static pressure (b):

$$P_b = P_{static} = P_a + gh(\rho_a - \rho_b)$$

Pitot Tube



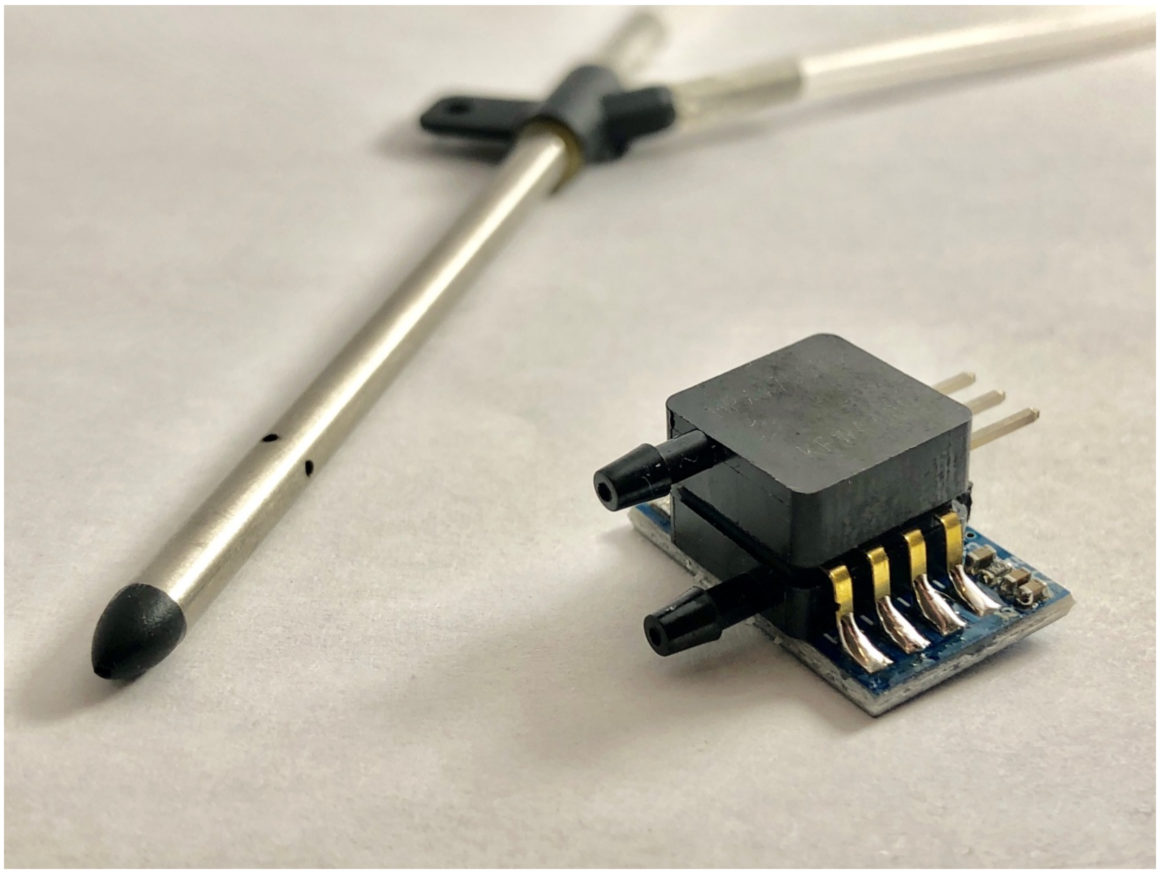
Velocity:

$$P_a = P_{static} + \frac{1}{2}\rho_a v_a^2$$

$$v_a = \sqrt{\frac{2\left(P_a - P_{static}\right)}{\rho_a}}$$

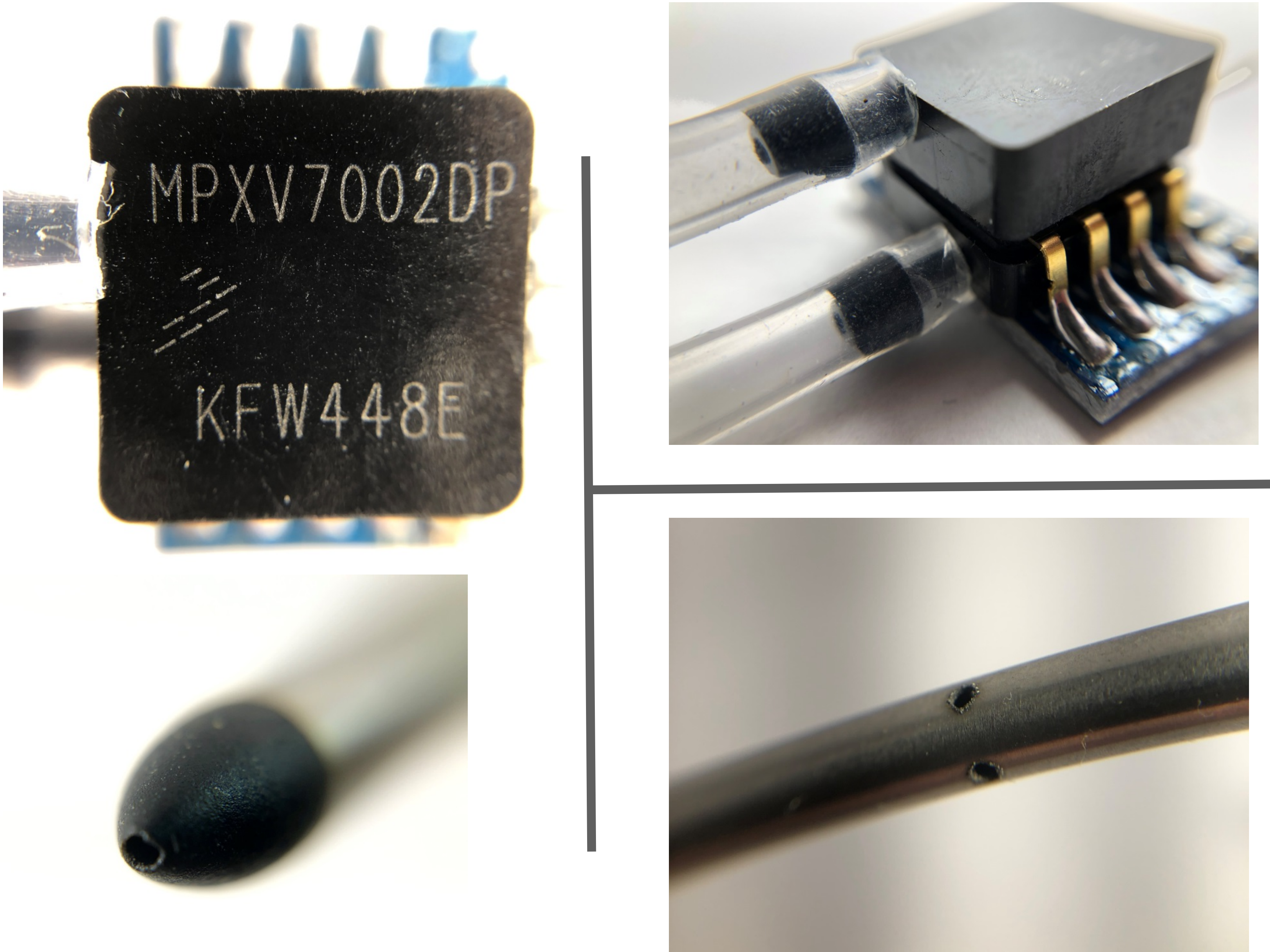
$$v_a = \sqrt{\frac{2gh(\rho_b - \rho_a)}{\rho_a}}$$

Pitot Tube Example



Velocity:

$$v_a = \sqrt{\frac{2\left(P_a - P_{static}\right)}{\rho_a}}$$


MPXV7002DP
KFW448E

Pitot Tube Example

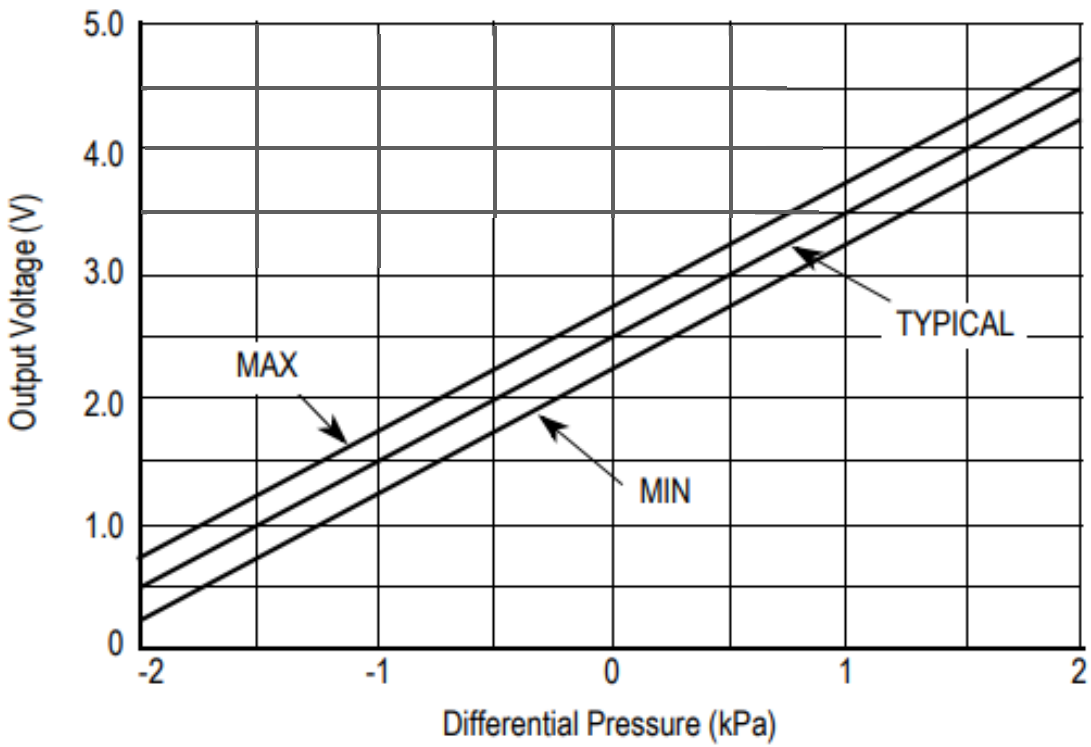


Figure 4. Output versus Pressure Differential

Velocity:

$$v_a = \sqrt{\frac{2\left(P_a - P_{static}\right)}{\rho_a}}$$

Create a relationship between differential pressure and voltage output

Pitot Tube Example

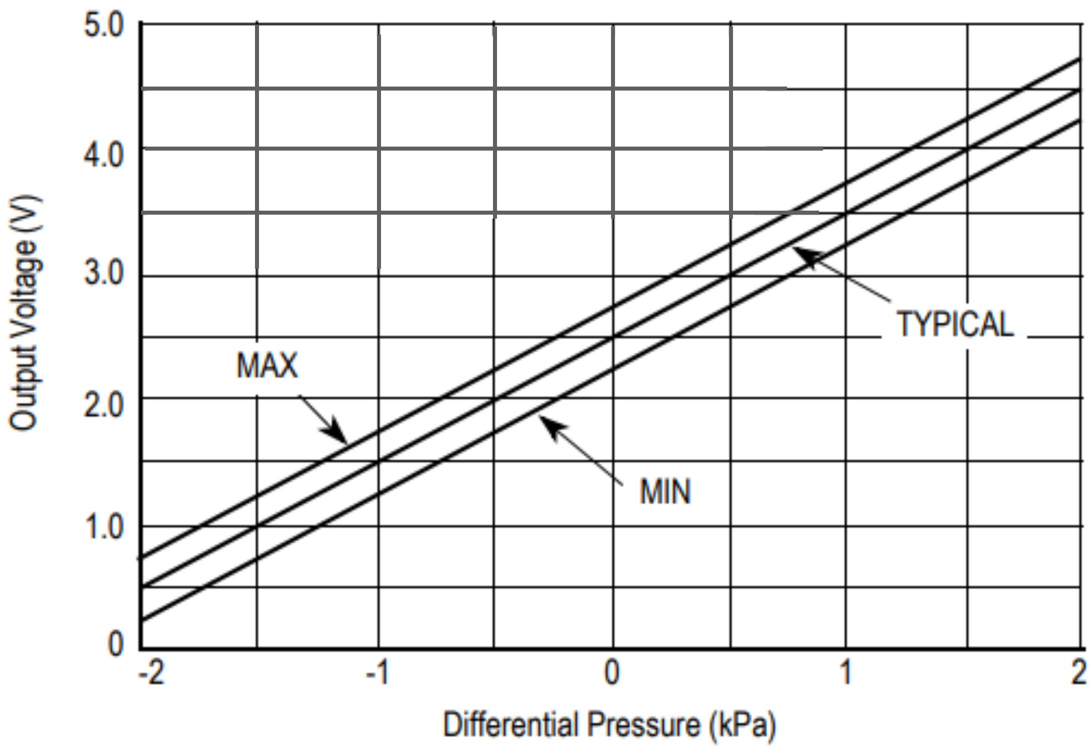


Figure 4. Output versus Pressure Differential

$$y = mx + b$$

$$y = 2.5,\ x = 0$$

$$y = 0.5,\ x = -2$$

Pitot Tube Example

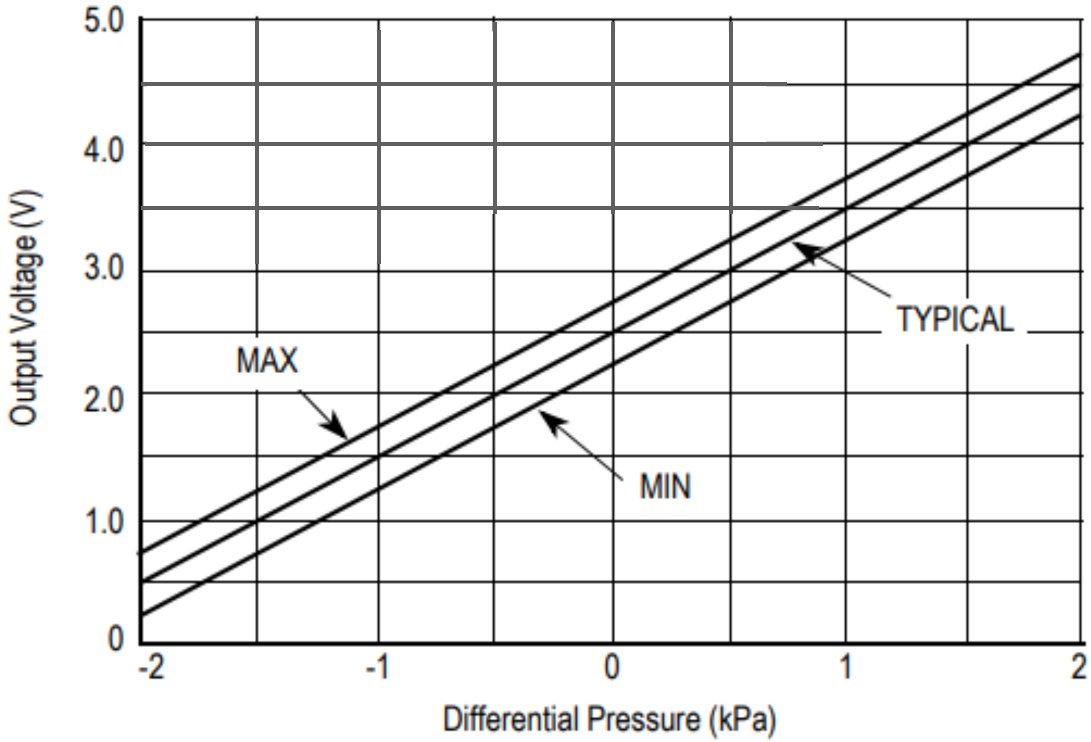


Figure 4. Output versus Pressure Differential

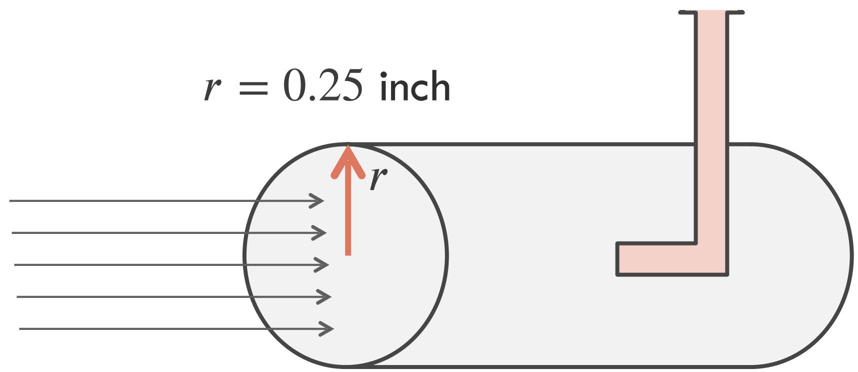
$$y = x + 2.5$$

$$V_{out} = \Delta P + 2.5$$

$$1000\Delta P = V_{out} - 2.5$$

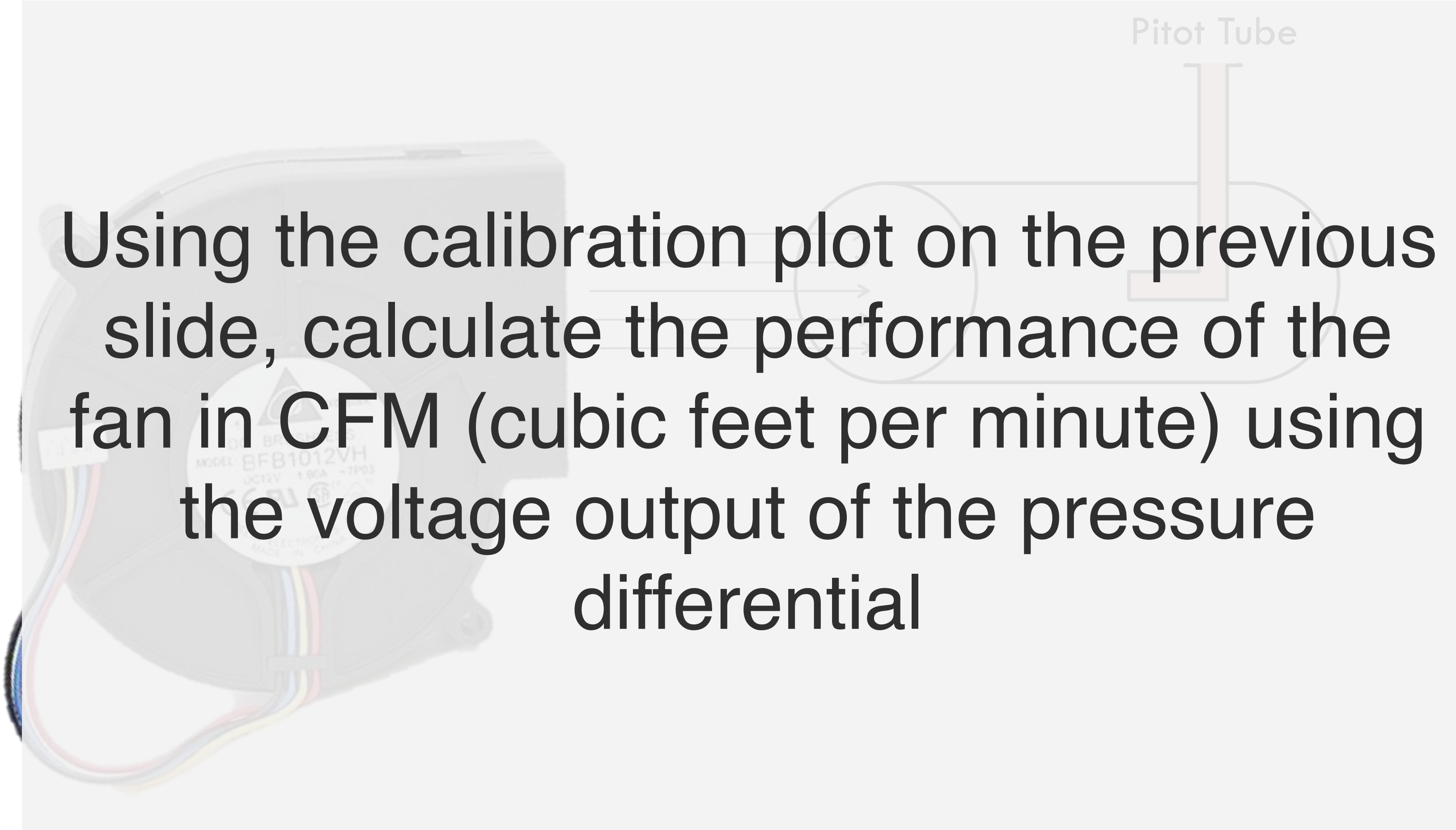
$$v_a = \sqrt{\frac{2000(V_{out} - 2.5)}{\rho_a}}$$

Pitot Tube Example



Pitot tube is measuring voltage difference between static port and velocity inlet port

Pitot Tube Example



Using the calibration plot on the previous slide, calculate the performance of the fan in CFM (cubic feet per minute) using the voltage output of the pressure differential

Arduino Bit Conversion

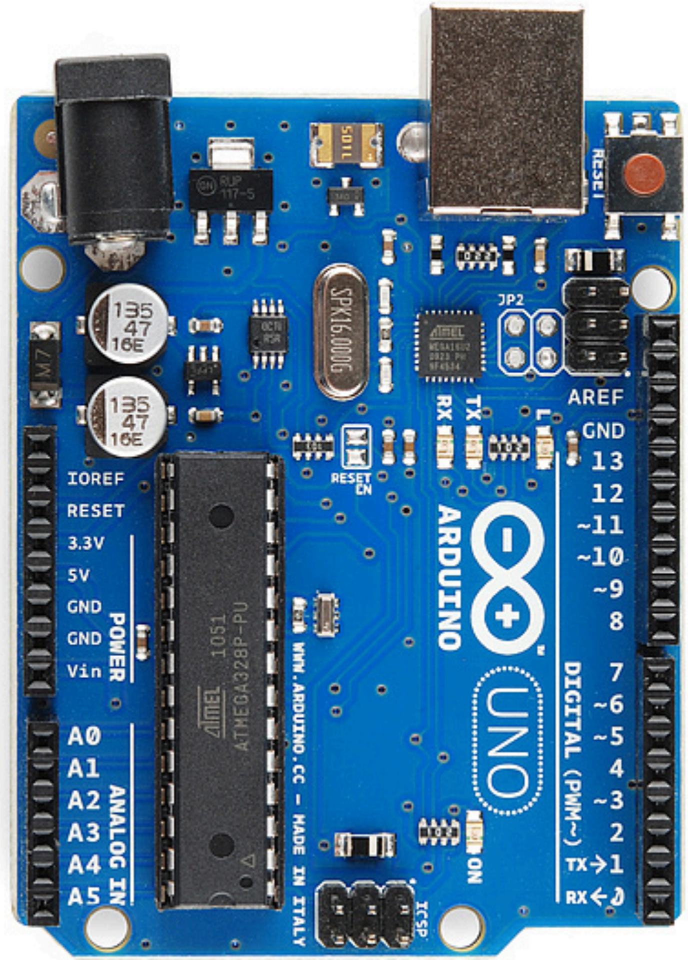
- Readings are 10-bits
- Voltage range of ADC is 0-5V

<u>Arduino Board</u>

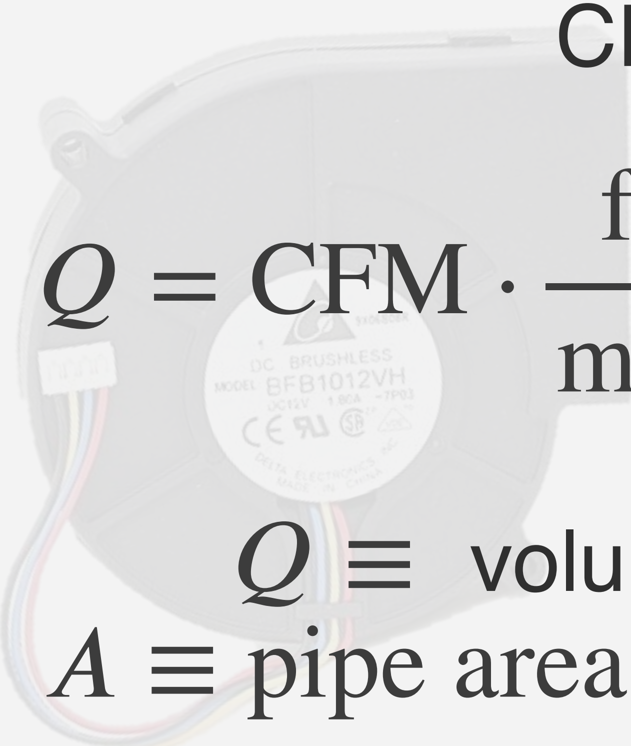
ADC Resolution = 10-bit

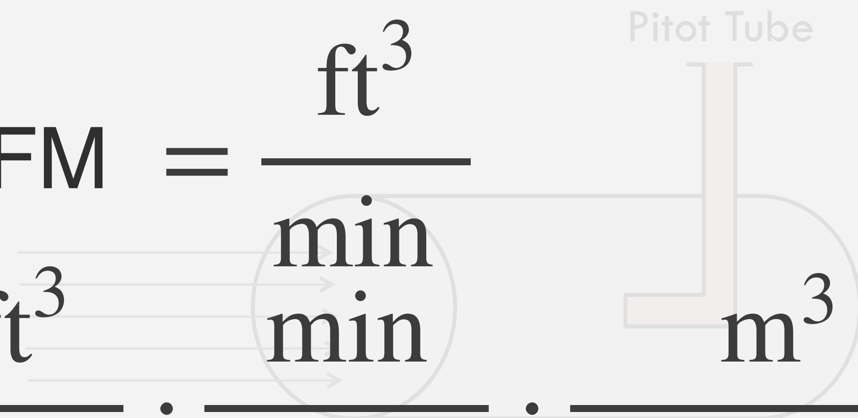
Measurement Range = 0V-5V

$$Q = \frac{5V}{2^{10}} = 4.9\,\mathrm{mV/bit}$$



Pitot Tube Example





$$\text{CFM} = \frac{\text{ft}^3}{\text{min}}$$

$$Q = \text{CFM} \cdot \frac{\text{ft}^3}{\text{min}} \cdot \frac{\text{min}}{60\text{sec}} \cdot \frac{\text{m}^3}{3.28^3 \cdot \text{ft}^3}$$

$$Q \equiv \text{ volumetric flow rate} = vA$$

$$A \equiv \text{pipe area} \quad v \equiv \text{mean flow velocity}$$

Pitot Tube Example

$$\text{CFM} = vA \cdot 60 \cdot 3.28^3 \text{CFM} = 2117.25\pi r^2 \cdot v r = 0.25 \text{ in} \,.\, 1 \text{ in} = 0.0254 \text{ m}$$

$$r = 0.00635 \text{ m}$$

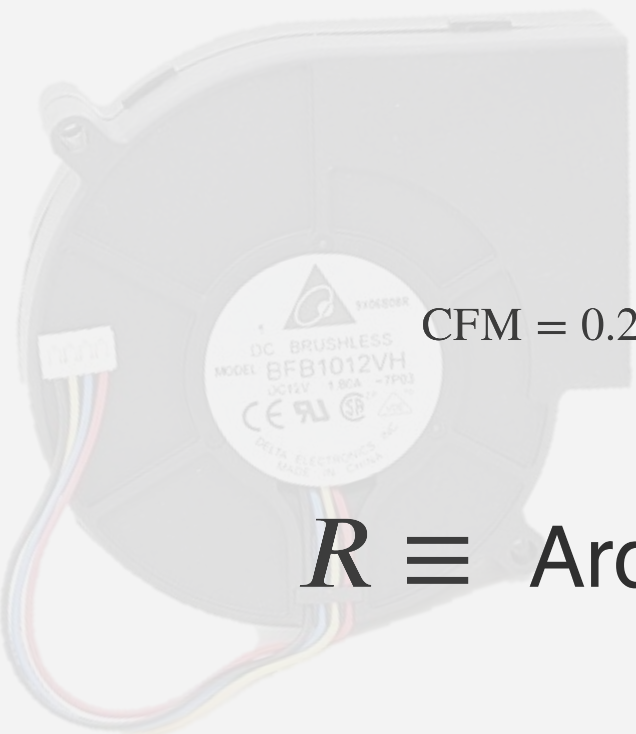
Pitot Tube Example

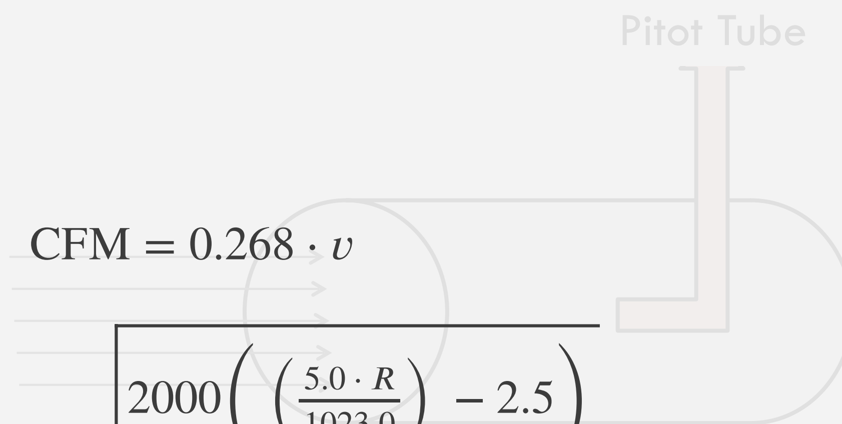
$$\mathrm{CFM} = vA \cdot 60 \cdot 0.3048^3 \mathrm{CFM} = 0.268 \cdot v$$

Pitot Tube Example

$$\text{CFM} = 0.268 \cdot v \text{CFM} = 0.268 \cdot \sqrt{\frac{2\Delta P}{\rho}}$$

Pitot Tube Example



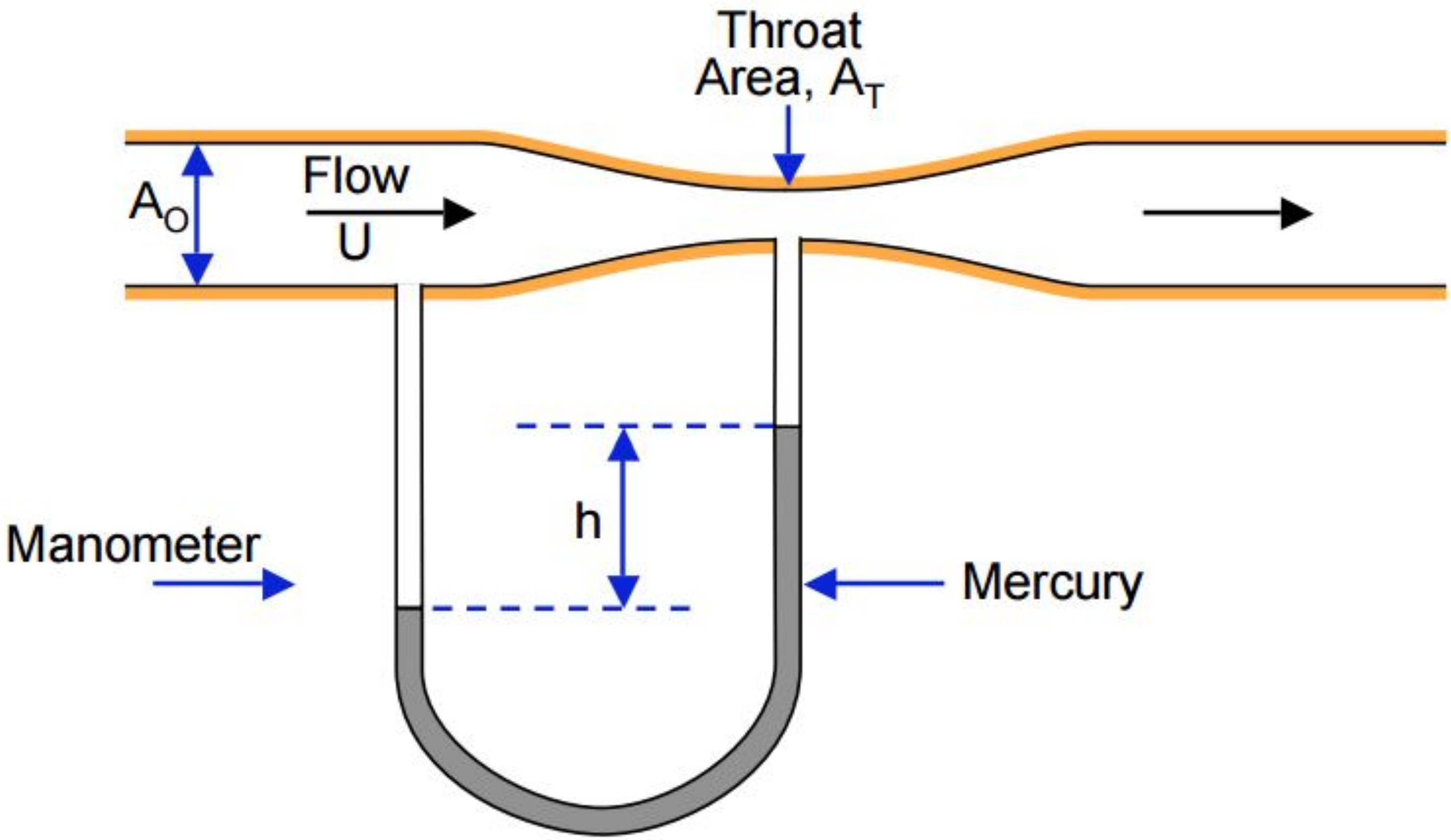


$$\text{CFM} = 0.268 \cdot v$$

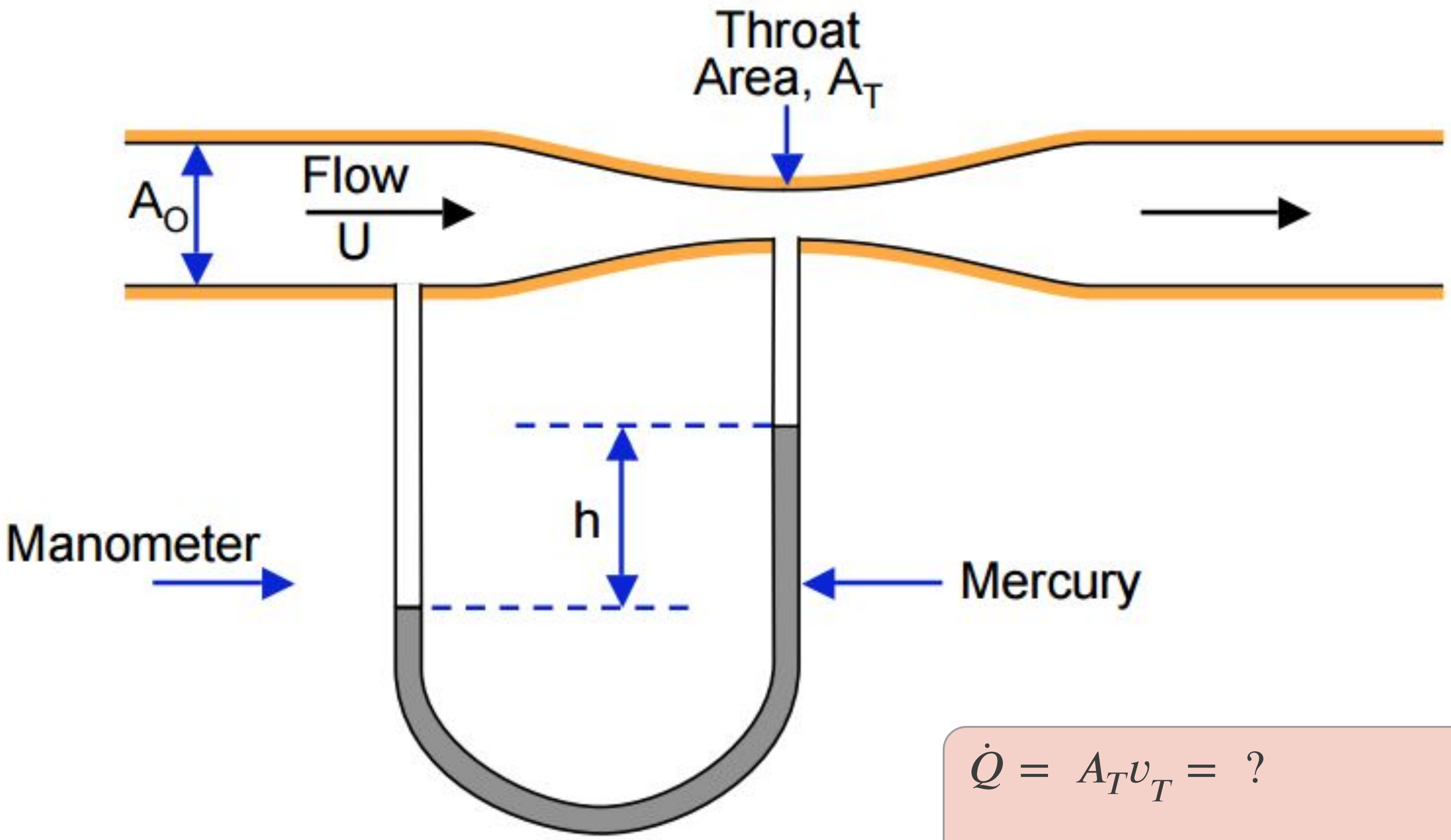
$$\text{CFM} = 0.268 \cdot \sqrt{\frac{2000\left(\left(\frac{5.0 \cdot R}{1023.0}\right) - 2.5\right)}{\rho_a}}$$

$R \equiv$ Arduino 10-bit reading

Venturi Meter

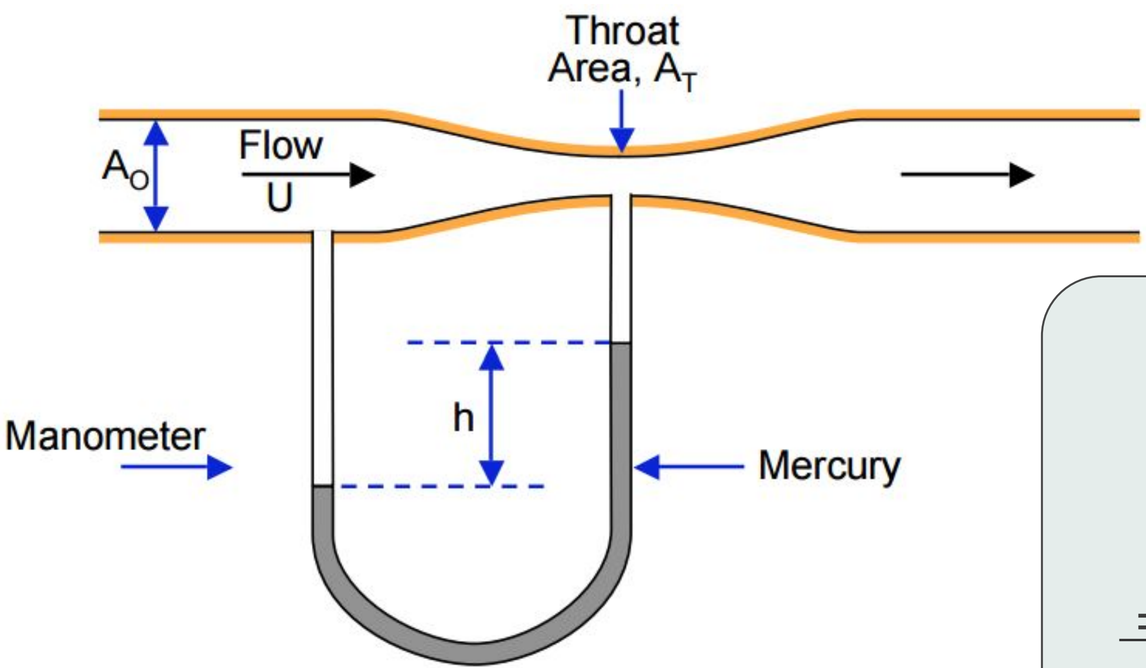


Venturi Meter



$$\dot{Q} = A_T v_T = ?$$

Venturi Meter

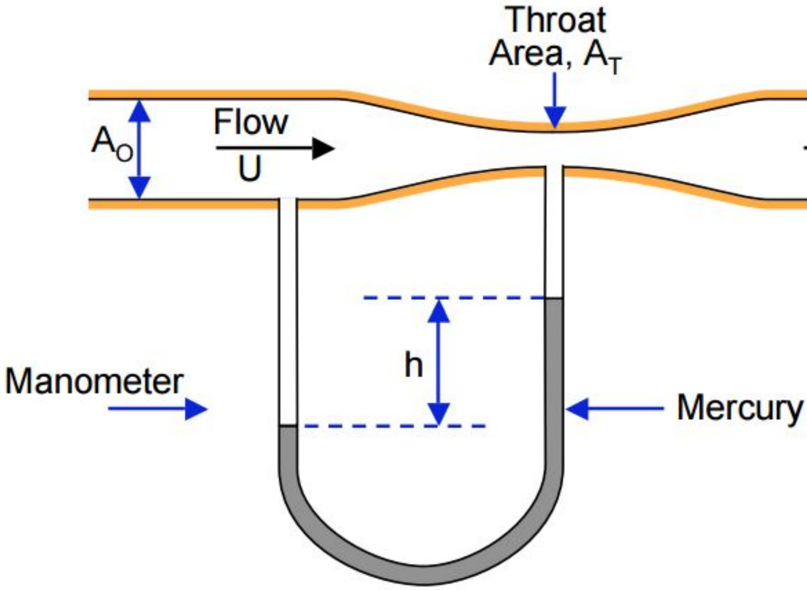


<u>Bernoulli's equation</u>

$$P_{0,tot} + \frac{1}{2}\rho_0 v_0^2 + \rho_0 g h_0$$
$$= P_{T,tot} + \frac{1}{2}\rho_T v_T^2 + \rho_T g h_T$$

Conservation equation:

$$A_0 v_0 = A_T v_T$$



<u>Bernoulli's equation</u>

$$P_{0,tot} + \frac{1}{2}\rho_0 v_0^2 + \rho_0 g h_0$$

$$= P_{T,tot} + \frac{1}{2}\rho_T v_T^2 + \rho_T g h_T$$

Conservation equation:

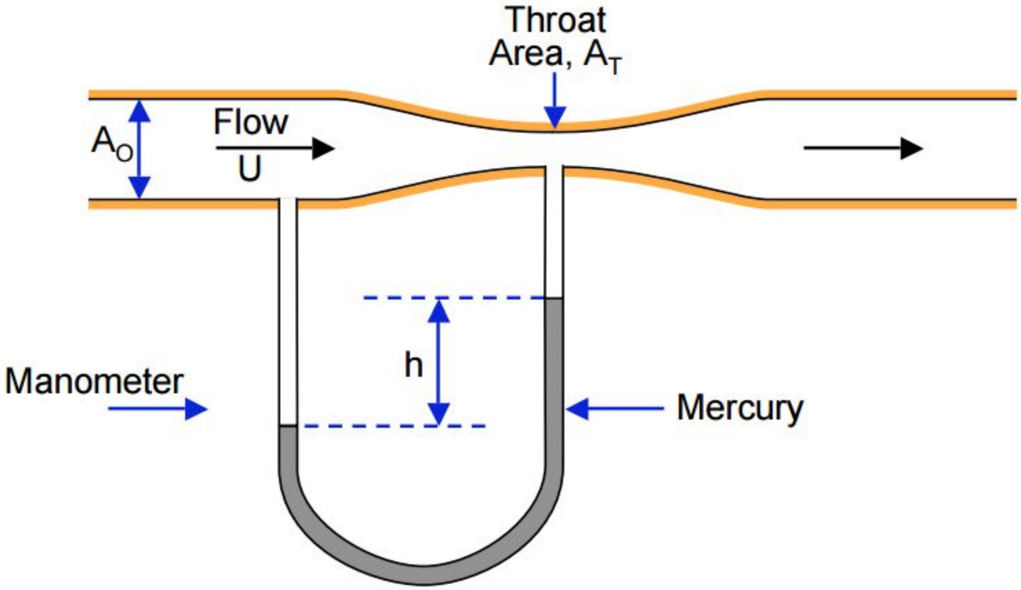
$$A_0 v_0 = A_T v_T$$

$$P_{0,tot} + \frac{1}{2}\rho_0 v_0^2 = P_{T,tot} + \frac{1}{2}\rho_0 v_T^2$$

$$P_0 - P_T = \frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0 v_0^2$$

$$P_0 - P_T = gh\left(\rho_m - \rho_0\right)$$

$$\frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0 v_0^2 = gh\left(\rho_m - \rho_0\right)$$



Bernoulli's equation

$$P_{0,tot} + \frac{1}{2}\rho_0 v_0^2 + \rho_0 g h_0 = P_{T,tot} + \frac{1}{2}\rho_T v_T^2 + \rho_T g h_T$$

Conservation equation:

$$A_0 v_0 = A_T v_T$$

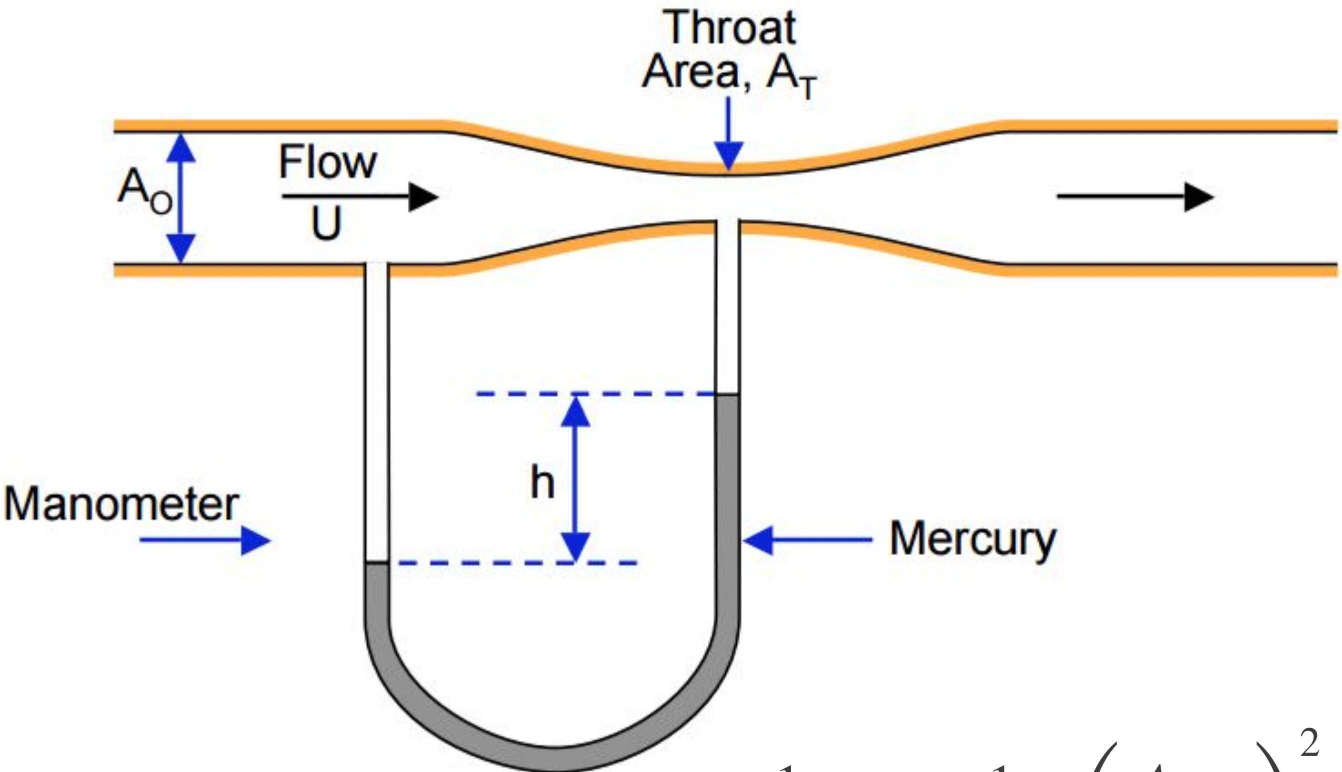
$$P_{0,tot} + \frac{1}{2}\rho_0 v_0^2 = P_{T,tot} + \frac{1}{2}\rho_0 v_T^2$$

$$P_0 - P_T = \frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0 v_0^2$$

$$P_0 - P_T = gh\left(\rho_m - \rho_0\right)$$

$$\frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0 v_0^2 = gh\left(\rho_m - \rho_0\right)$$

$$\frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0\left(\frac{A_T}{A_0}v_T\right)^2 = gh\left(\rho_m - \rho_0\right)$$



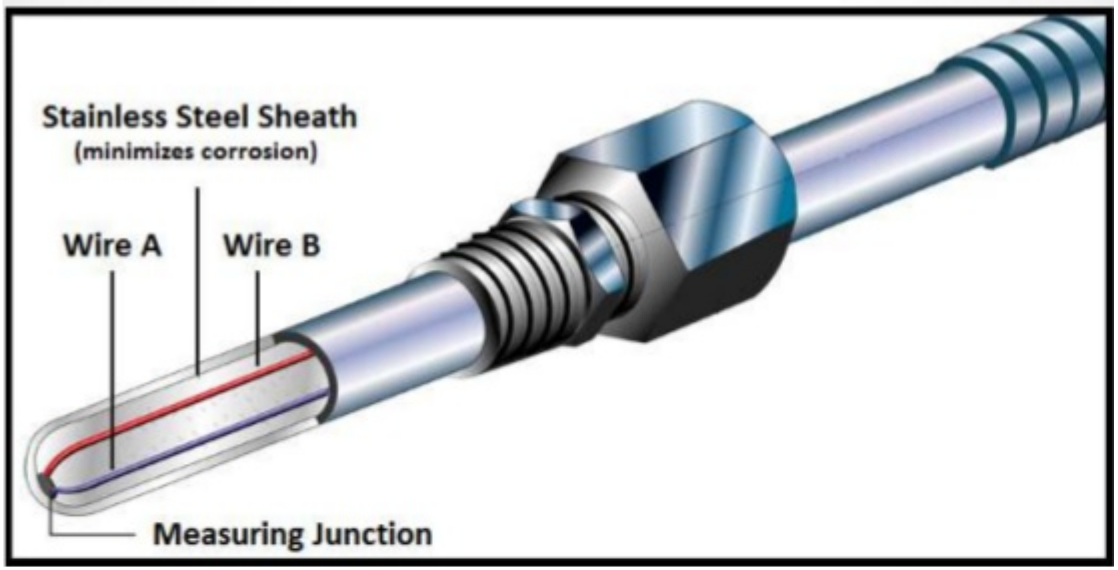
$$\frac{1}{2}\rho_0 v_T^2 - \frac{1}{2}\rho_0\left(\frac{A_T}{A_0}v_T\right)^2 = gh\left(\rho_m - \rho_0\right)$$

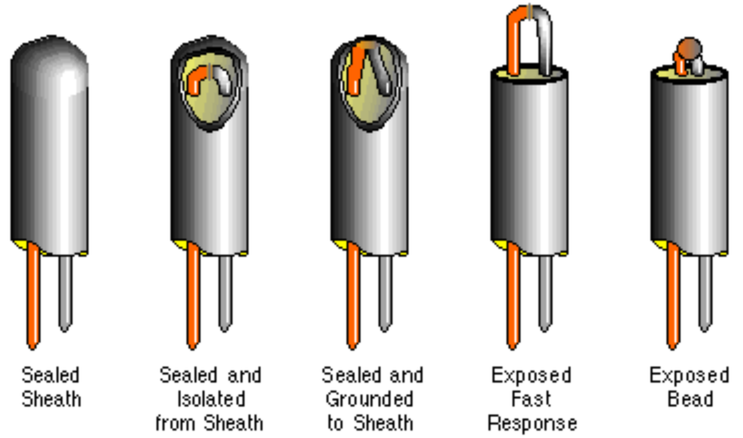
$$v_T^2\left(1 - \left(\frac{A_T}{A_0}\right)^2\right) = \frac{2gh\left(\rho_m - \rho_0\right)}{\rho_0}$$

$$\dot{Q} = A_T v_T = A_T\sqrt{\frac{2gh\left(\rho_m - \rho_0\right)}{\rho_0\left(1 - \left(\frac{A_T}{A_0}\right)^2\right)}}$$

$$v_T = \sqrt{\frac{2gh\left(\rho_m - \rho_0\right)}{\ldots}}$$

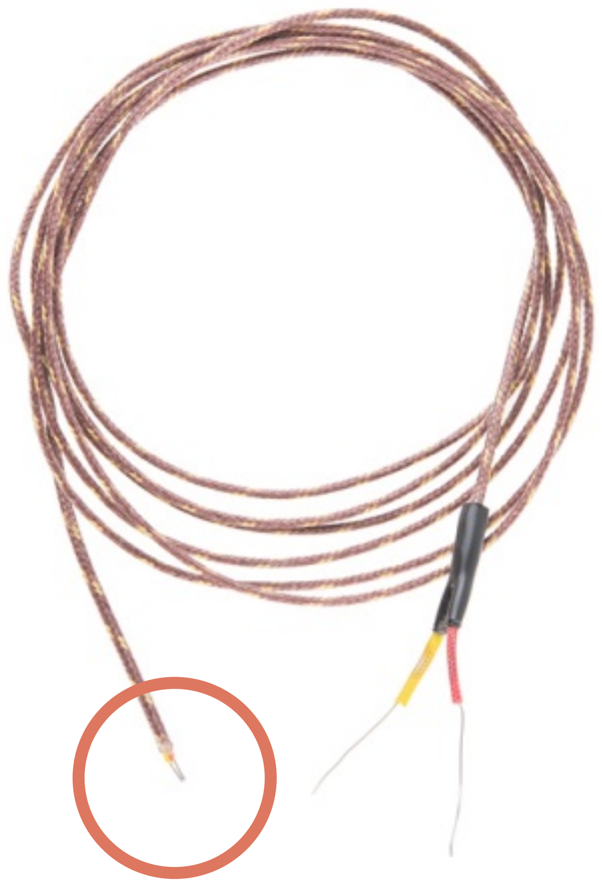
Thermocouples

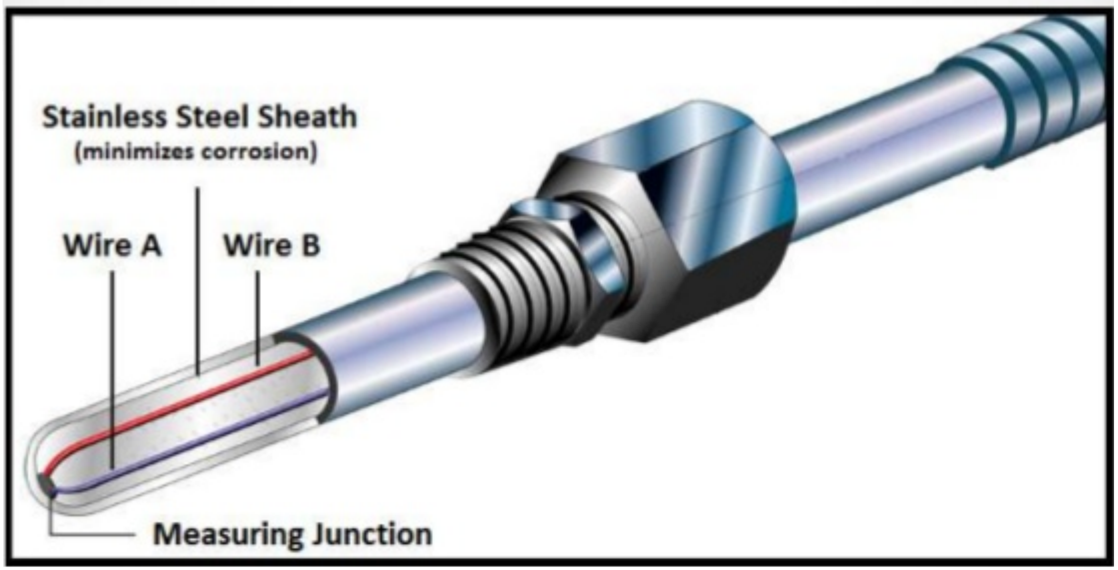


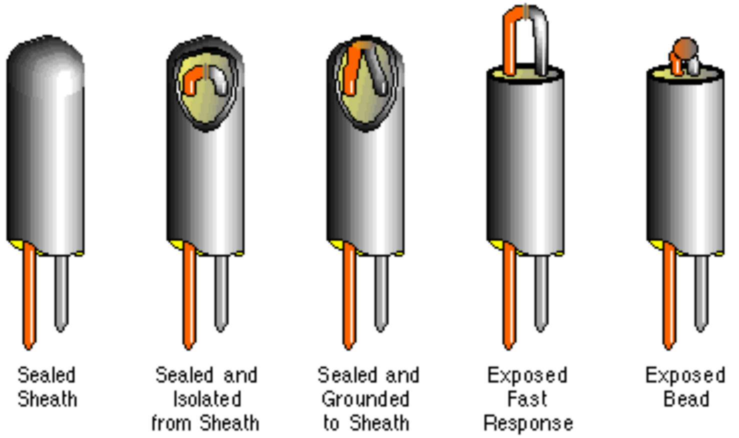


Thermocouple Sheath Options

Thermocouples

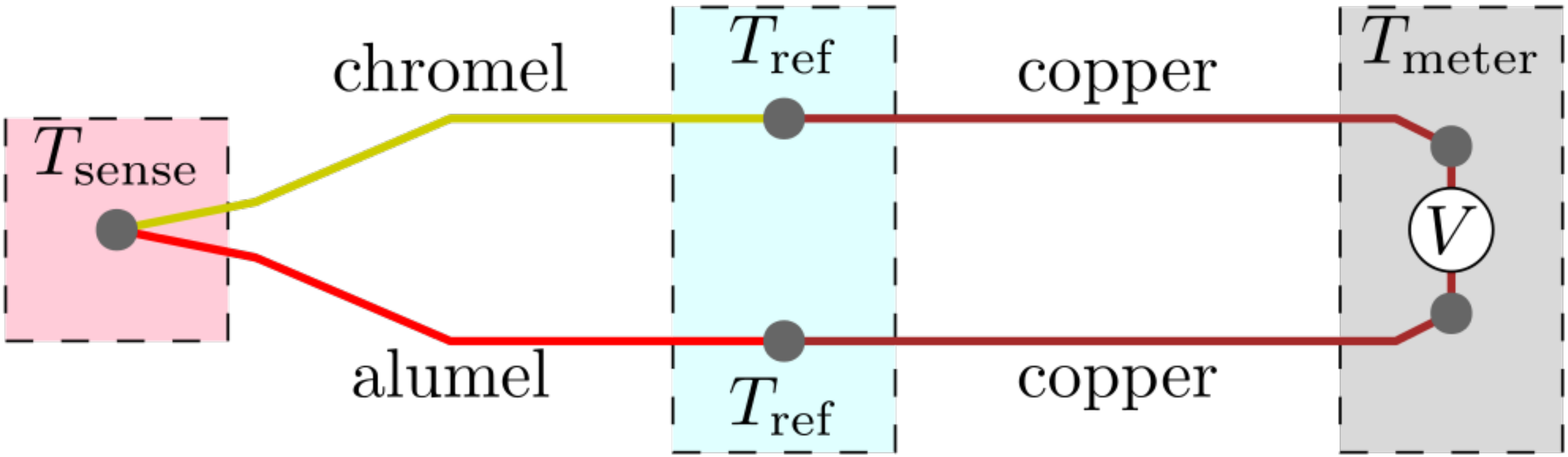






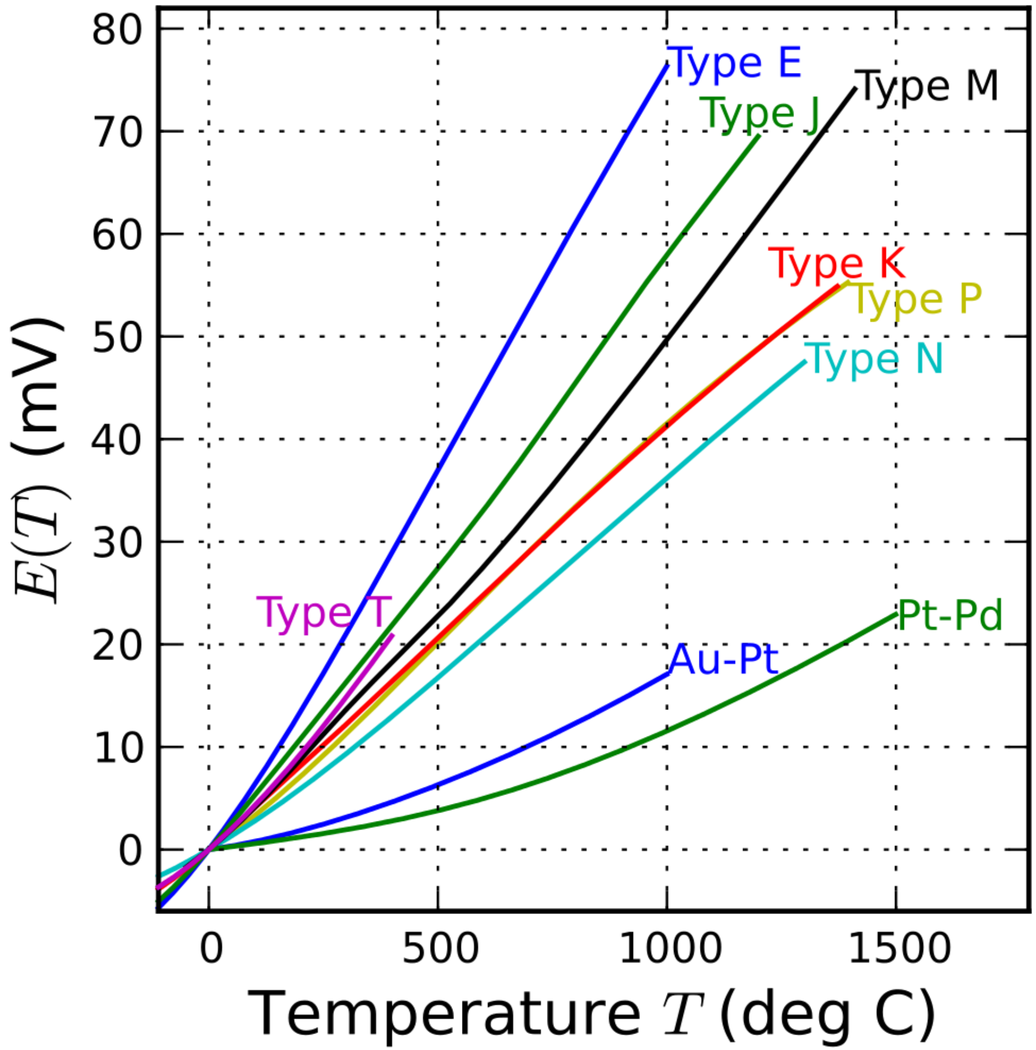
Thermocouple Sheath Options

Thermocouples



Temperature → Voltage

Thermocouples



$$V = E\left(T_{sense}\right) - E\left(T_{ref}\right)$$

$$E\left(T_{sense}\right) = V + E(T_{ref})$$

Measured

Reference junction sensor

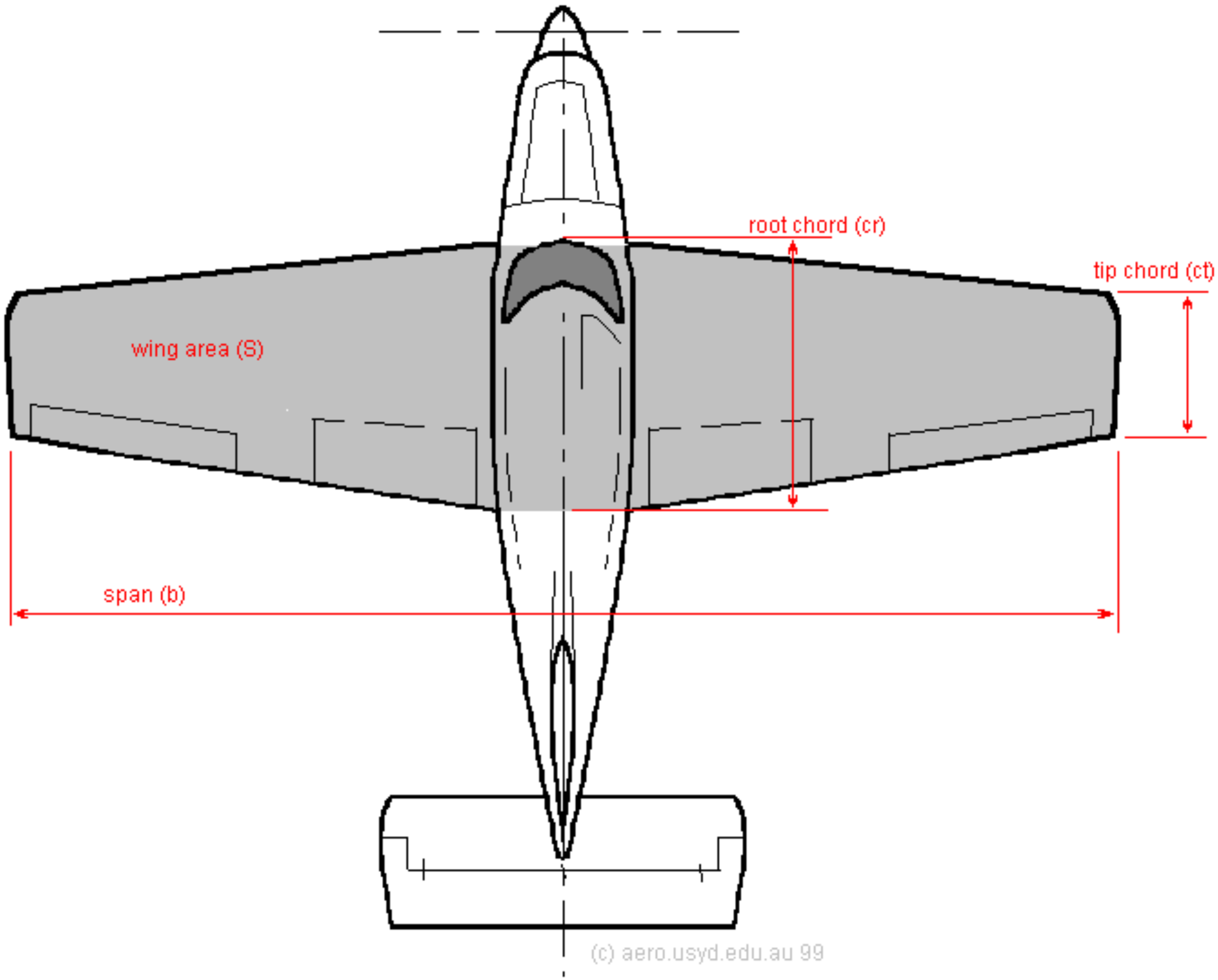
Engineering Refreshers

Engineering Refreshers

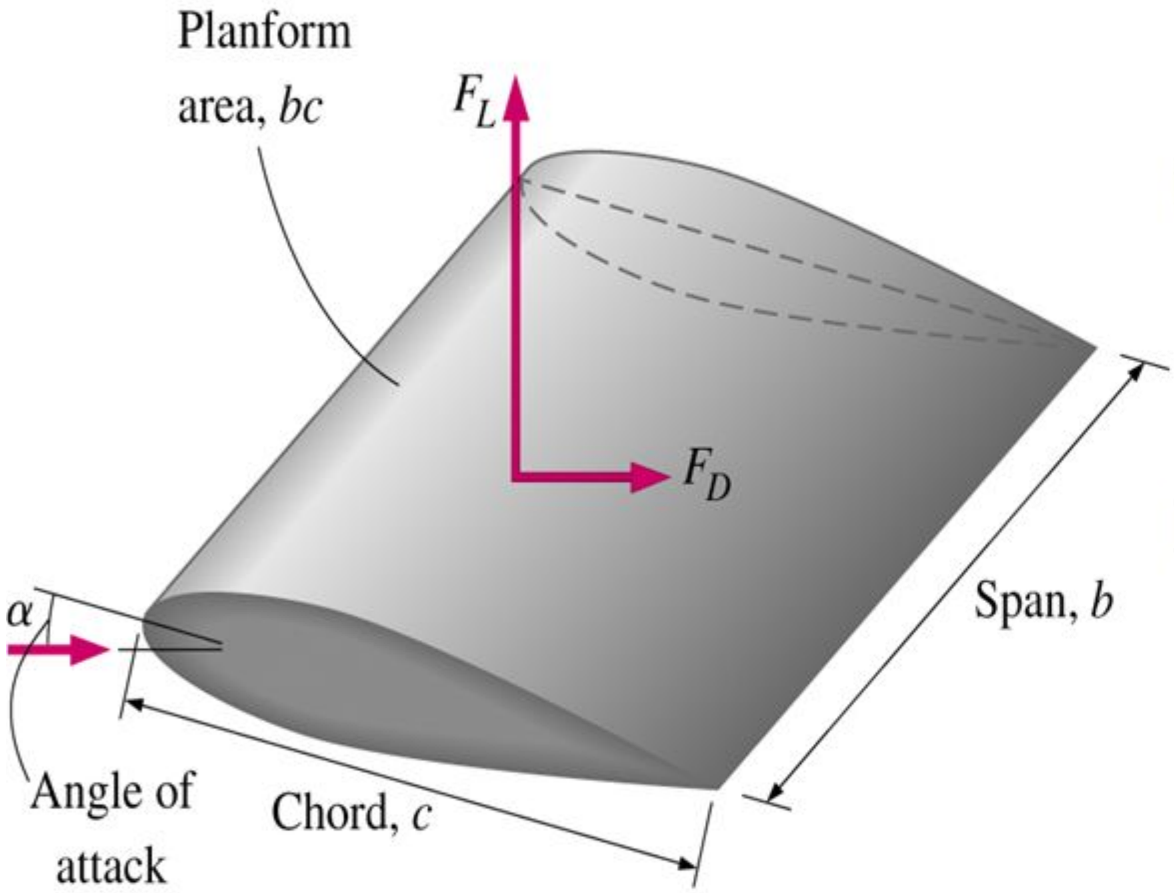
Laboratory Experiments

1. Wind Tunnel - aerodynamics
2. Viscous Pipe Flow – fluid dynamics
3. Heat Exchanger – heat transfer
4. Cross-Section Fin – heat transfer
5. Pelton Wheel – hydrodynamics, physics

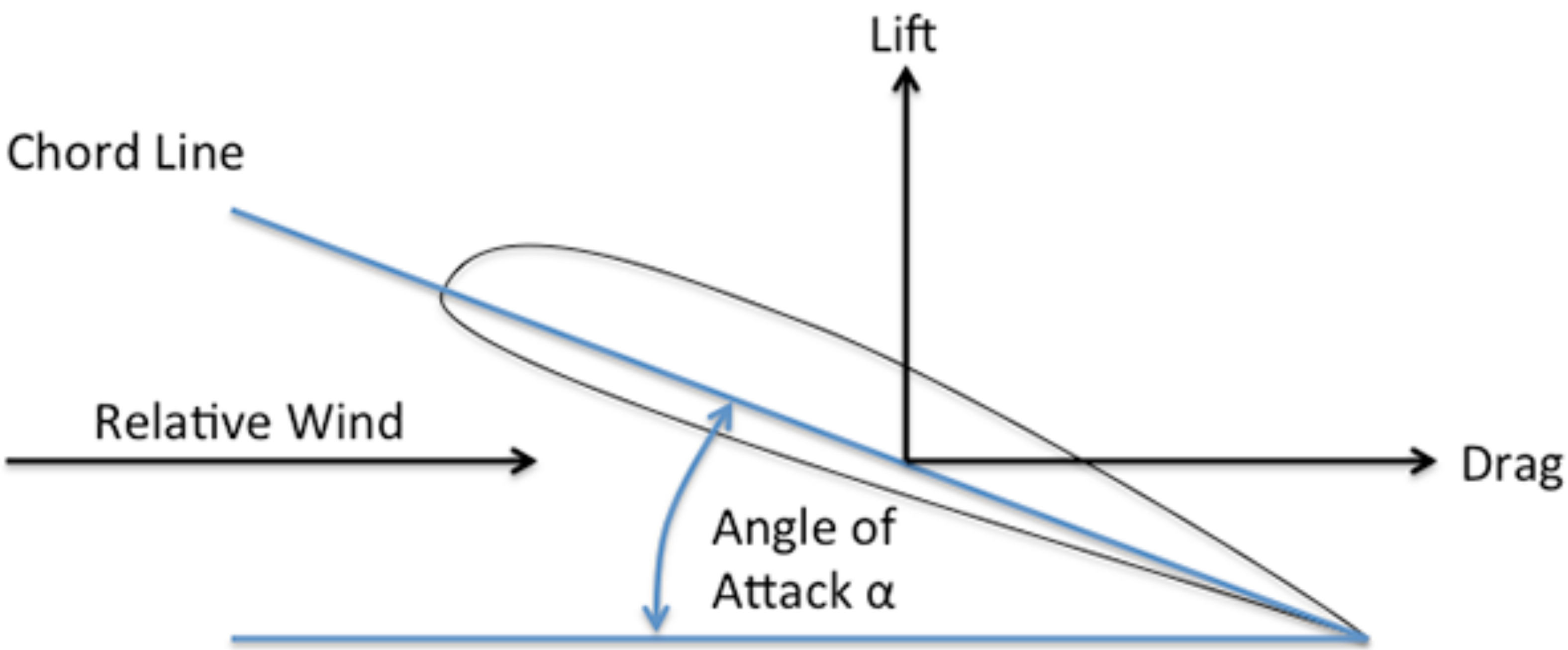
Aerodynamics



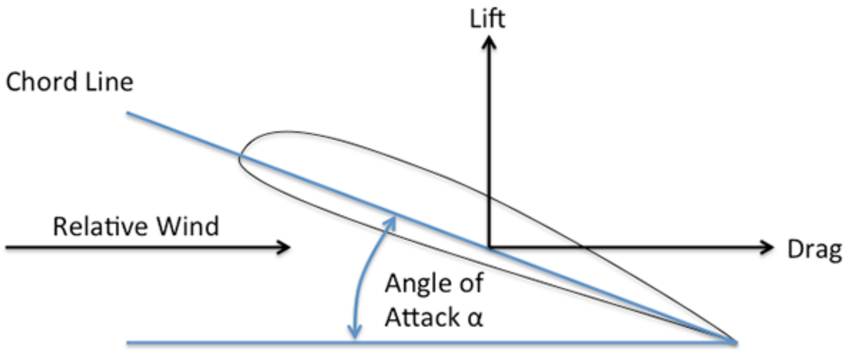
Aerodynamics

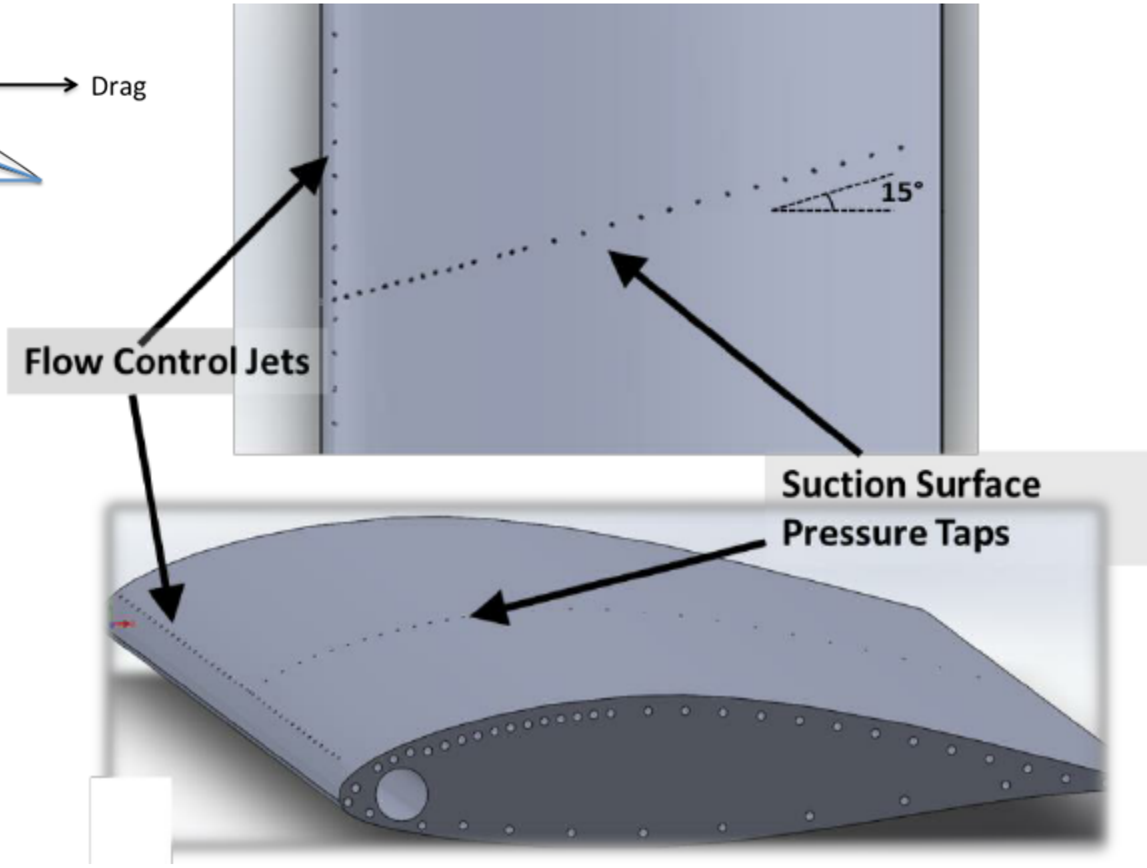


Aerodynamics



Aerodynamics





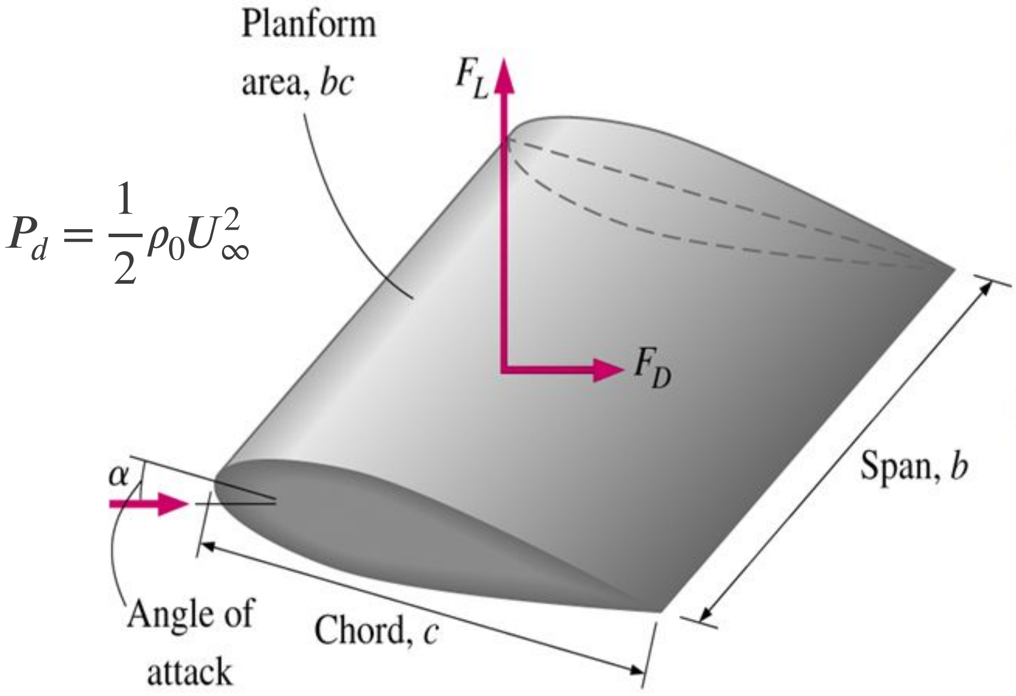
Aerodynamics

Performance of an Airfoil

$$C_L = \frac{L}{P_d S}$$

$$C_D = \frac{D}{P_d S}$$

$$C_P = \frac{P - P_\infty}{P_d S}$$



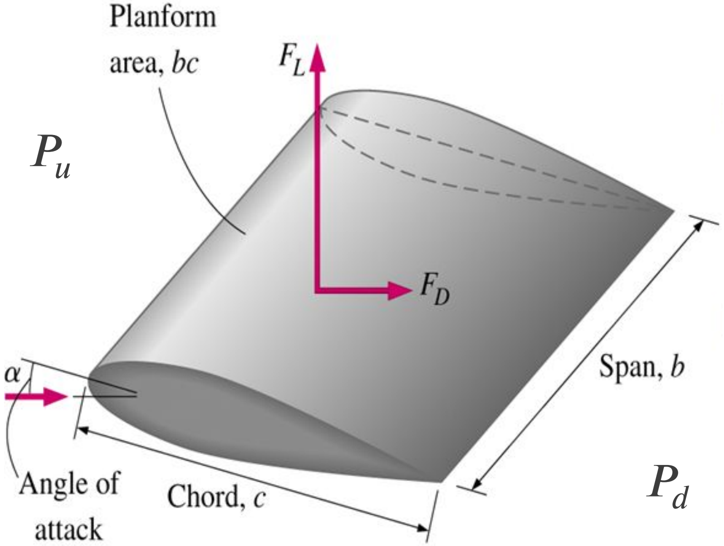
Aerodynamics

MATLAB's Trapz Function

Momentum deficit:

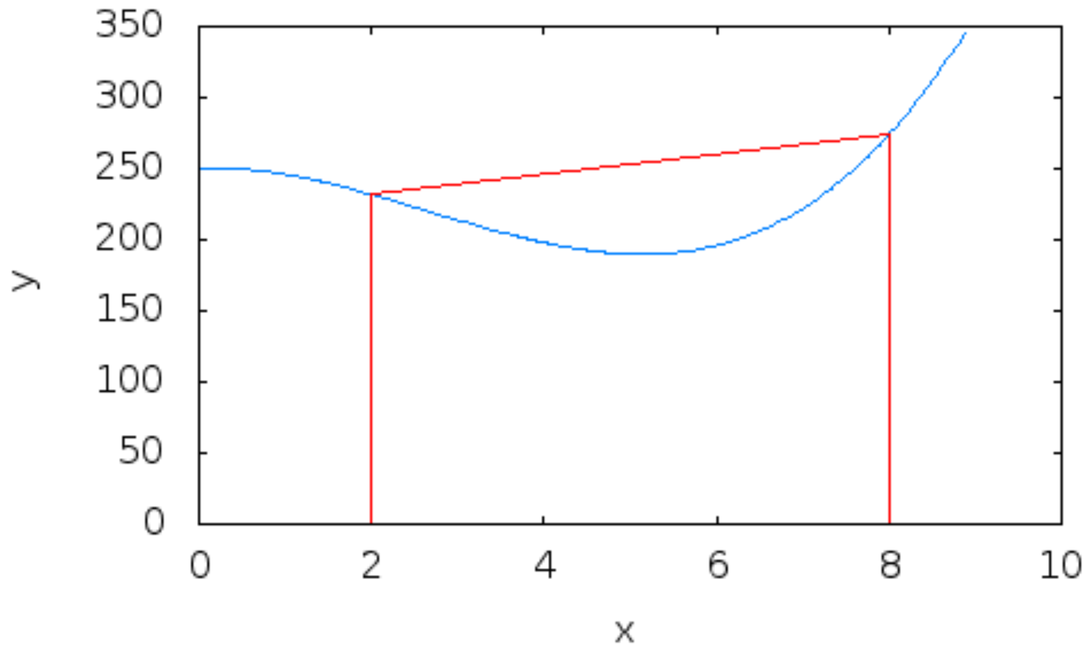
$$D' = \int_a^b (P_u + \rho u_0^2)dy - \int_c^d \left(P_d + \rho u^2\right)dy$$

$$\int_m^n f(x)\,dx = \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2}\Delta x_k$$

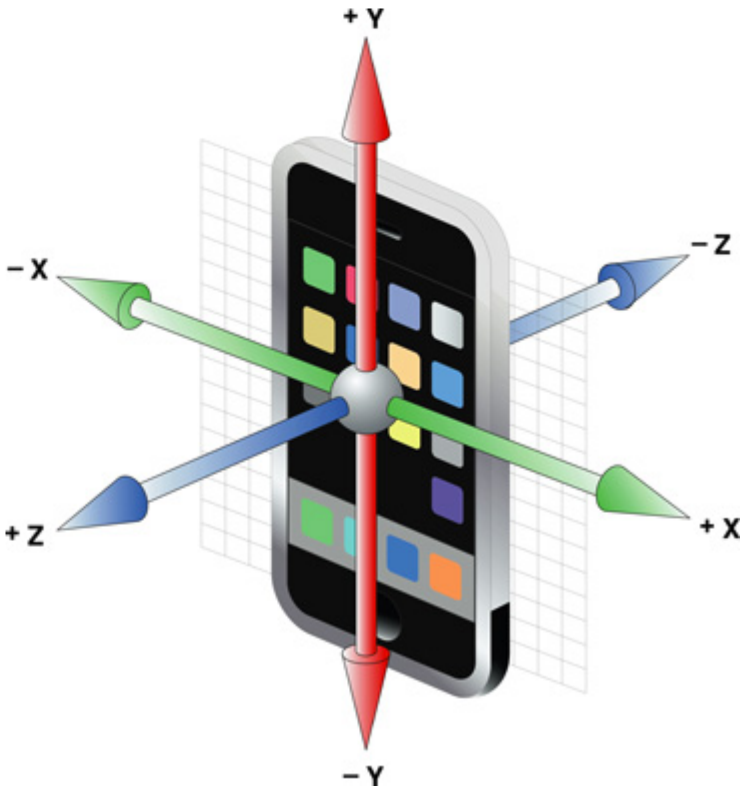


MATLAB Trapz

$$\int_{m}^{n} f(x)\,dx = \sum_{k=1}^{N} \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k$$



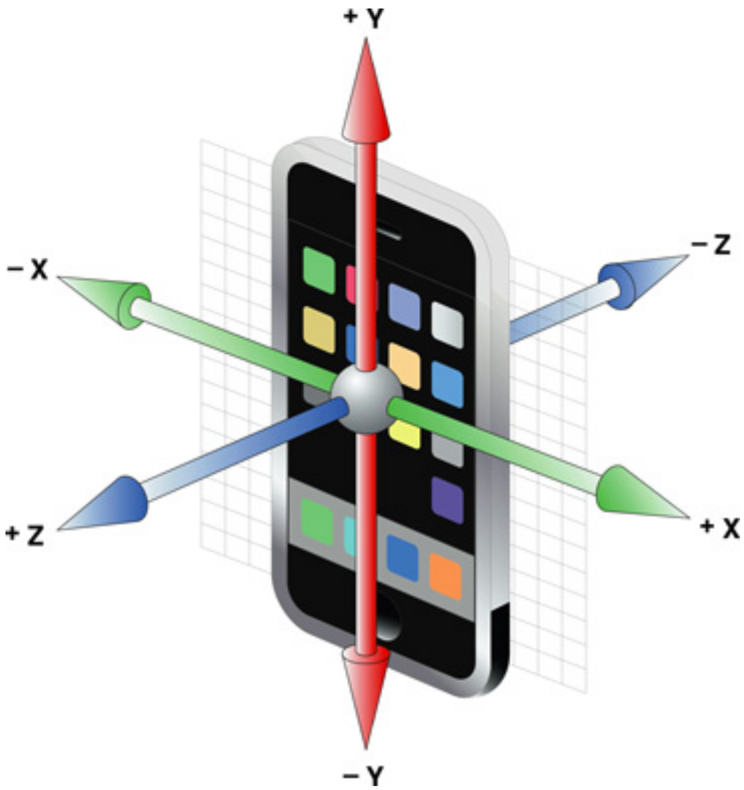
MATLAB Trapz - Example



$$\int_m^n f(x)\,dx = \sum_{k=1}^{N} \frac{f\left(x_{k-1}\right) + f\left(x_k\right)}{2} \Delta x_k$$

$$\int_m^n f(x)\,dx = \text{cumtrapz(data)}$$

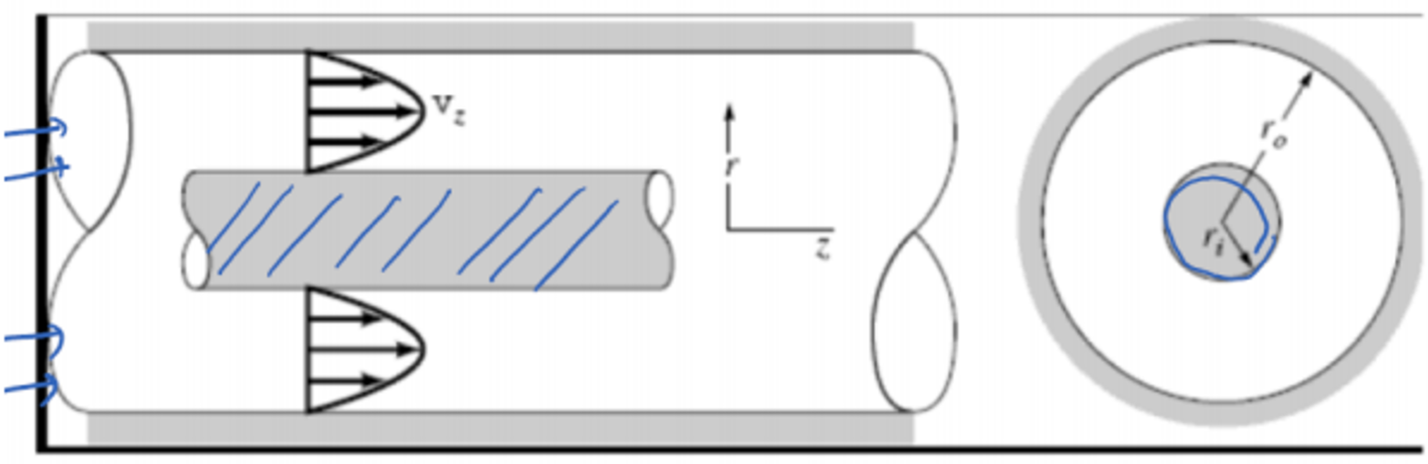
MATLAB Trapz - Example



$$a = \frac{dv}{dt} \quad \rightarrow \quad v = \frac{dx}{dt}$$

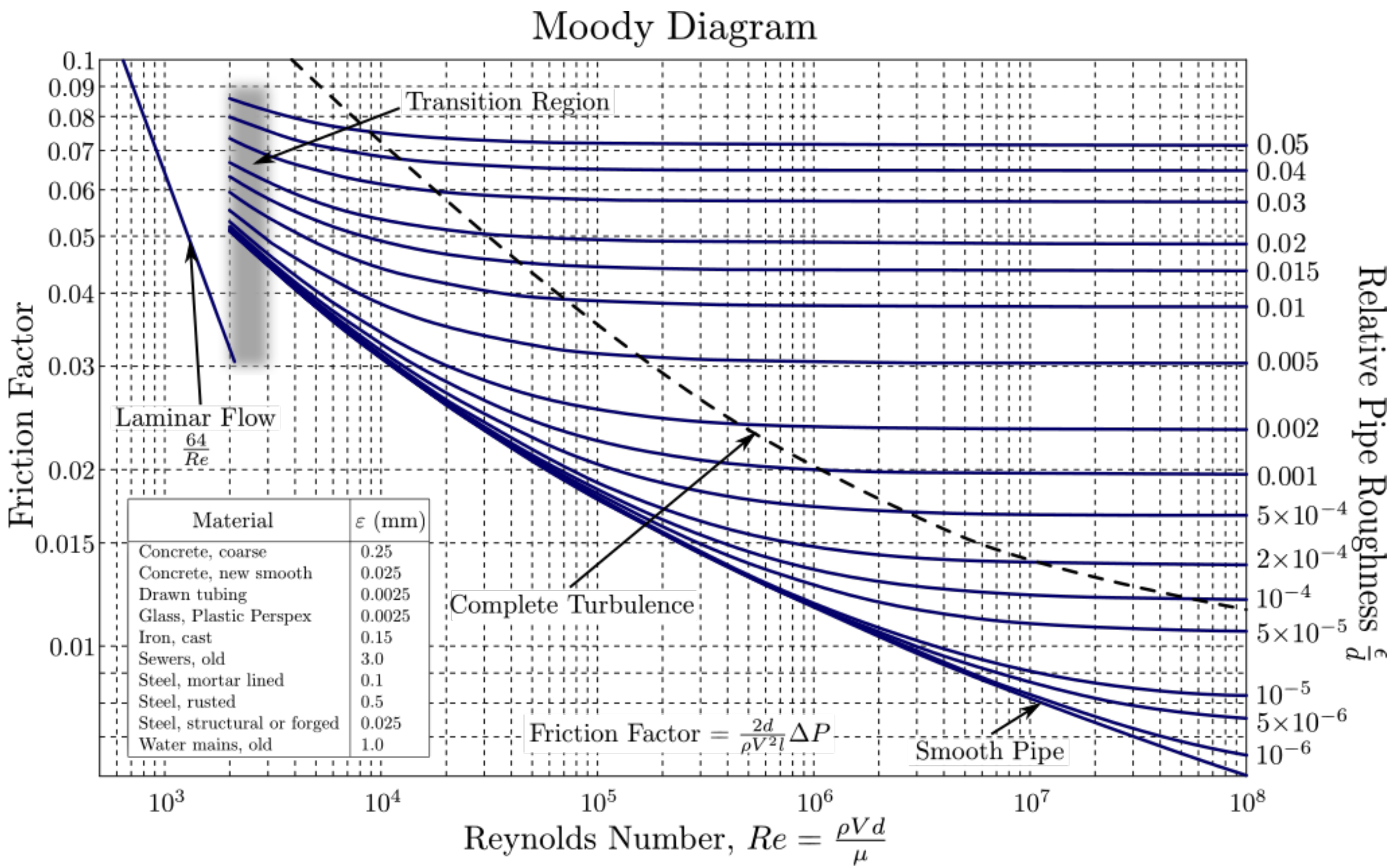
$$\int_{m}^{n} f(x)\, dx = \text{cumtrapz(data)}$$

Pipe Flow



$$\frac{u(r)}{u_{max}} = \left(1 - \frac{r^2}{R^2}\right)$$

Pipe Flow - Losses



Material	ε (mm)
Concrete, coarse	0.25
Concrete, new smooth	0.025
Drawn tubing	0.0025
Glass, Plastic Perspex	0.0025
Iron, cast	0.15
Sewers, old	3.0
Steel, mortar lined	0.1
Steel, rusted	0.5
Steel, structural or forged	0.025
Water mains, old	1.0

Pipe Flow Experiment

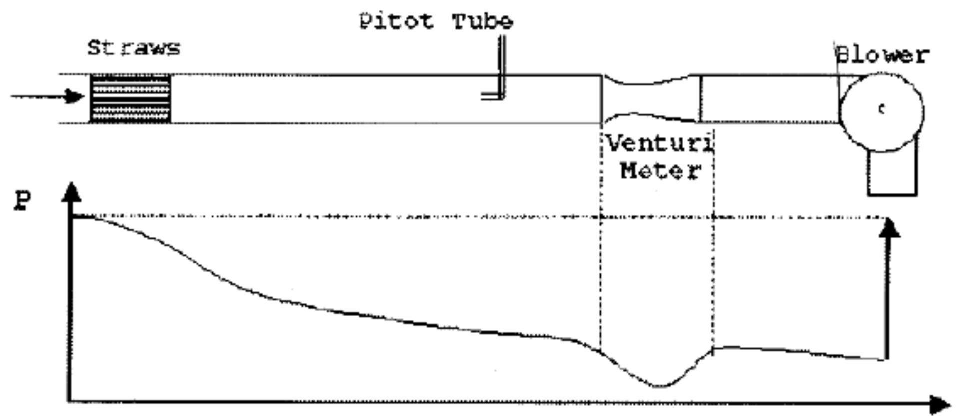
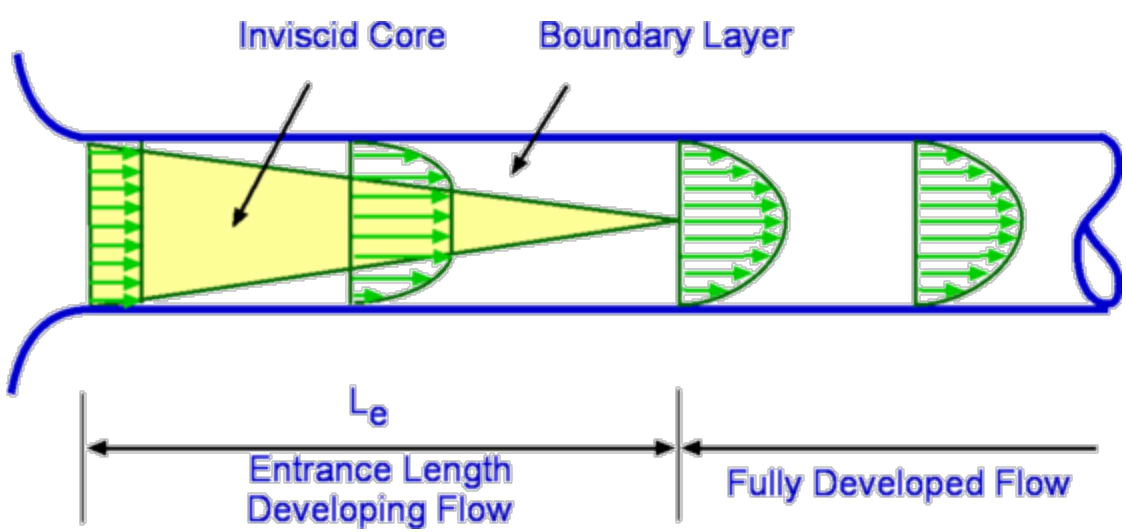


Figure 6.4. *Experimental apparatus and pressure diagram*



Pipe Flow Experiment

Re will be the most important information here, along with location of local velocity measurement.

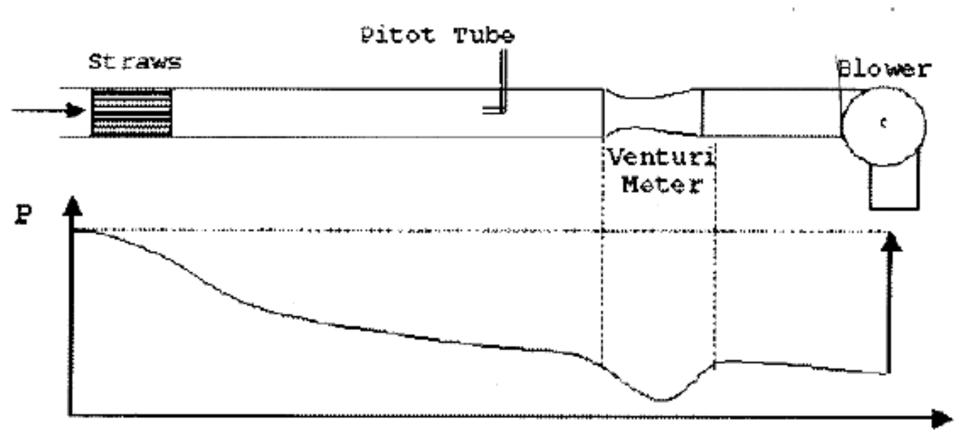
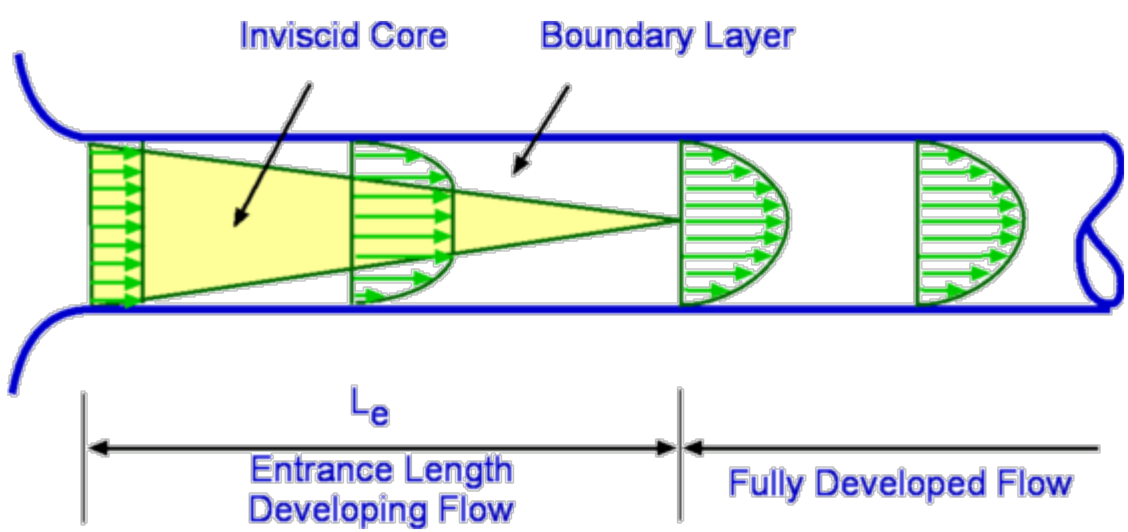


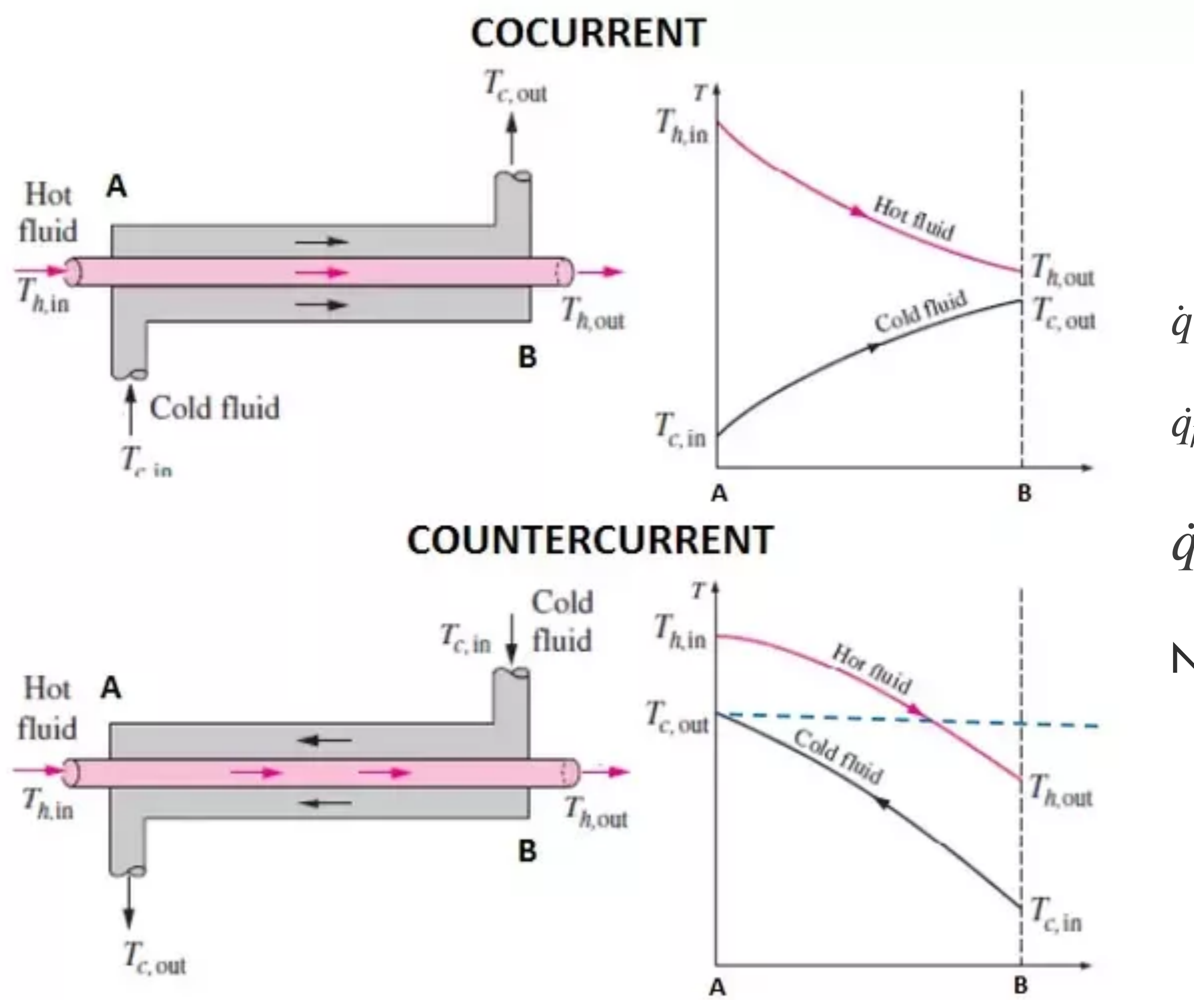
Figure 6.4. *Experimental apparatus and pressure diagram*

Friction factor (laminar):

$$f = \frac{64}{Re}$$



Heat Transfer – Heat Exchangers



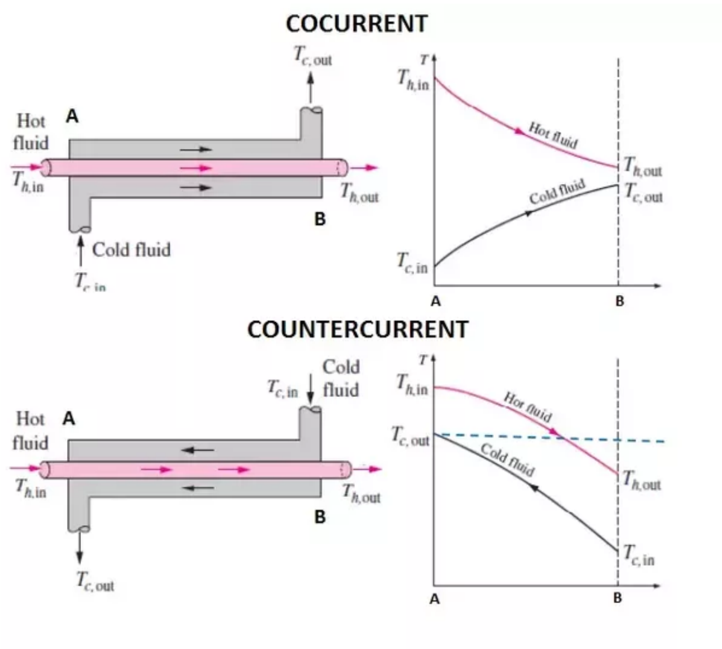
$$\dot{q} = hA(\Delta T)$$

$$\dot{q}_h = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o})$$

$$\dot{q}_c = \dot{m}_c c_{p,c}(T_{c,i} - T_{c,o})$$

No losses $\rightarrow$ $\dot{q} = \dot{q}_h = \dot{q}_c$

Heat Transfer – Heat Exchangers



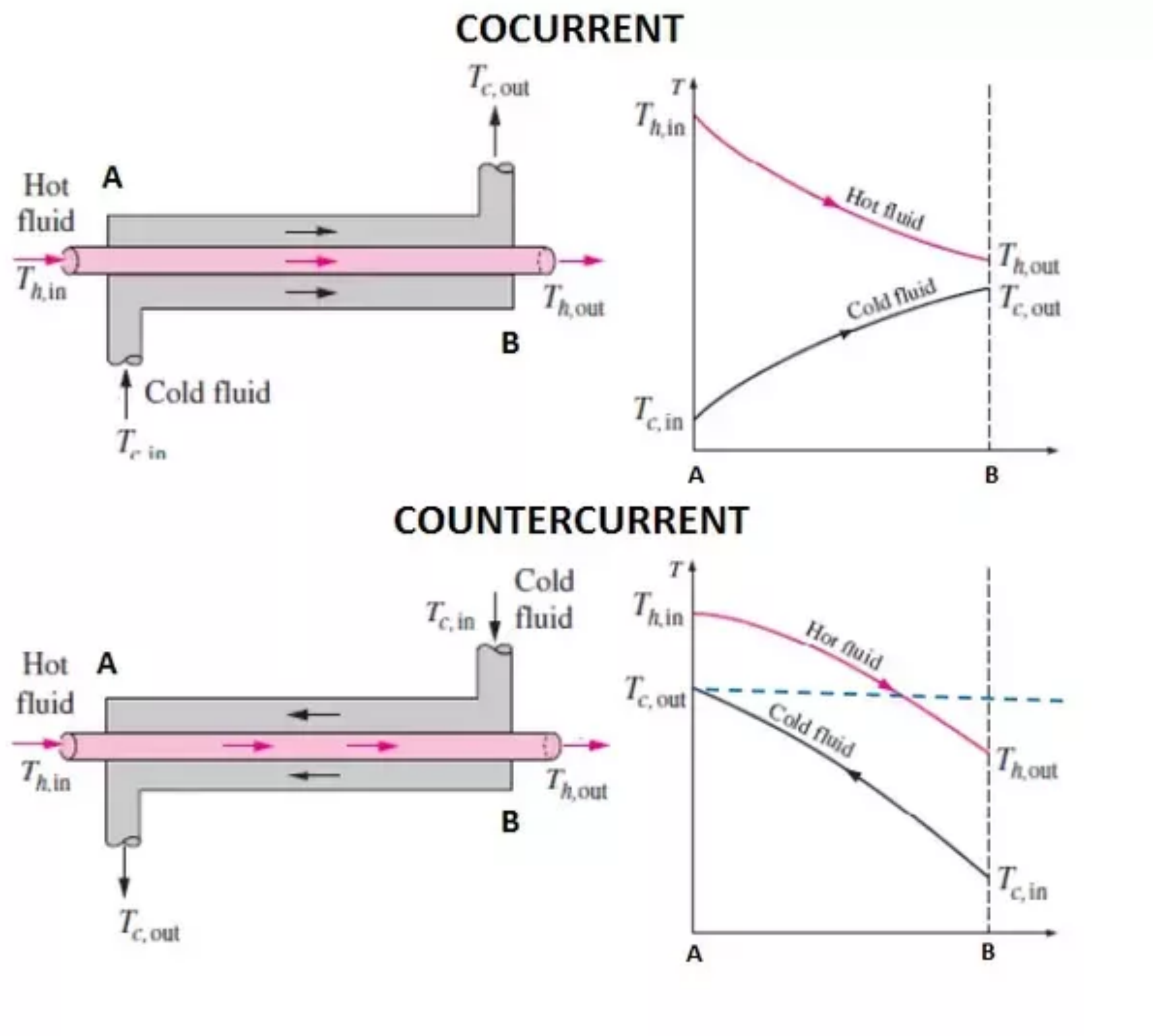
<u>Methods for Calculating $\dot{q}$</u>

$$\underline{\dot{q} = hA\left(\Delta T_{lm}\right)}$$

$\Delta T_{lm} \equiv$ logarithmic mean temperature difference (LMTD)

$$\Delta T_{lm} = LMTD = \frac{\Delta T_A - \Delta T_B}{\ln \frac{\Delta T_A}{\Delta T_B}}$$

Heat Transfer – Heat Exchangers

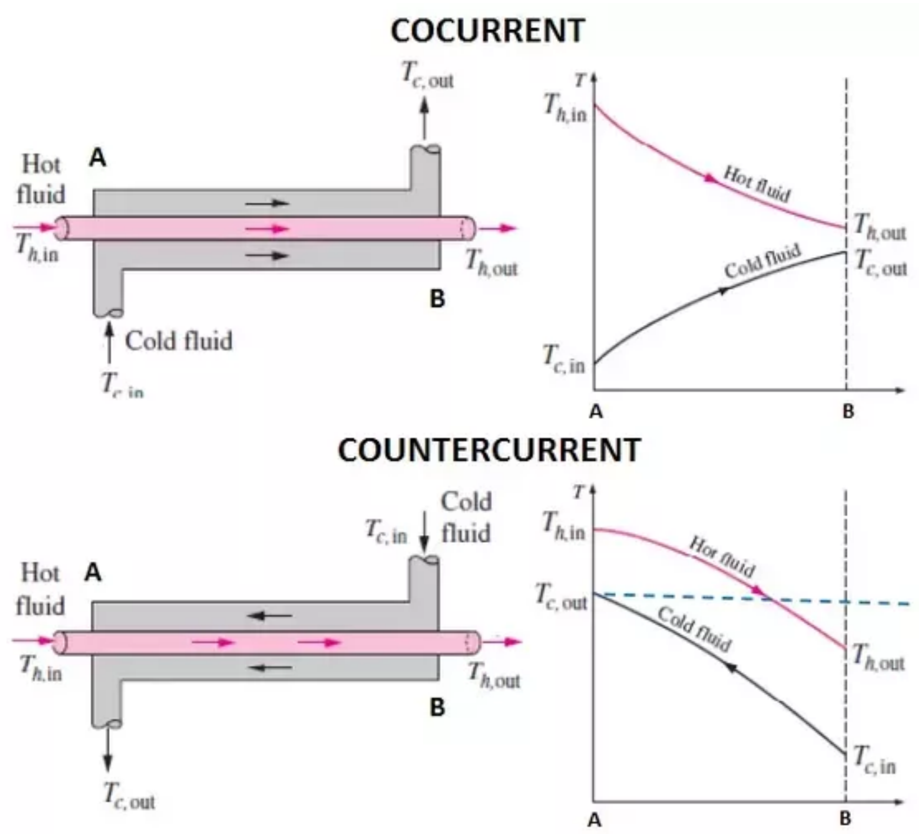


$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{\ln\frac{\Delta T_A}{\Delta T_B}}$$

What is ΔT_{lm} for parallel?

What is ΔT_{lm} for counter?

Heat Transfer – Heat Exchangers

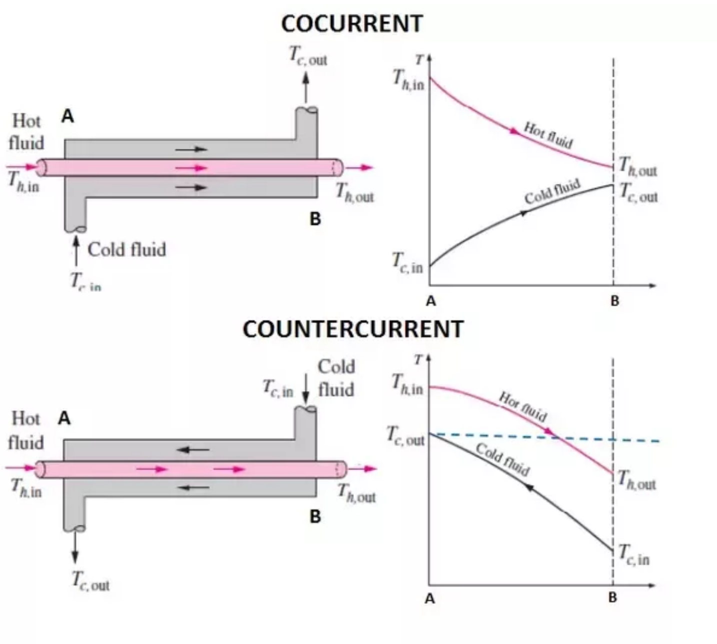


$$\Delta T_{lm} = \frac{\Delta T_A - \Delta T_B}{\ln \frac{\Delta T_A}{\Delta T_B}}$$

$$\Delta T_{lm} = \frac{\left(T_{h,i} - T_{c,i}\right) - \left(T_{h,o} - T_{c,o}\right)}{\ln\left(T_{h,i} - T_{c,i}\right) - \ln(T_{h,o} - T_{c,o})}$$

$$\Delta T_{lm} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left(T_{h,i} - T_{c,o}\right) - \ln(T_{h,o} - T_{c,i})}$$

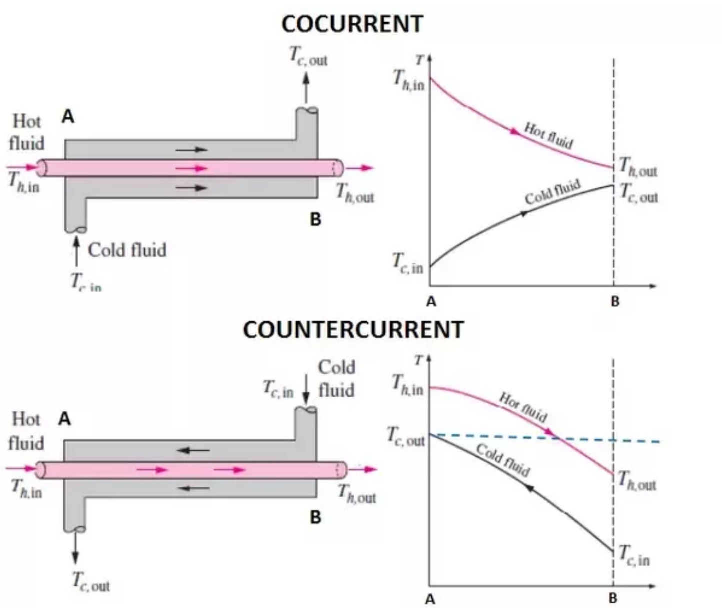
Heat Transfer – Heat Exchangers



<u>Method for Calculating $\dot{q}$</u>

$$\dot{q} = hA \cdot LMTD = hA\frac{\Delta T_A - \Delta T_B}{\ln\frac{\Delta T_A}{\Delta T_B}}$$

Heat Transfer – Heat Exchangers



<u>Efficiency is also important:</u>

$$\underline{\epsilon = \frac{\dot{q}}{\dot{q}_{max}}}$$

$$\underline{\dot{q}_{max} = C_{min}(T_{h,i} - T_{c,i})}$$

where C_{min} is the minimum between:

$m_c c_{p,c}$ or $m_h c_{p,h}$

Heat Transfer – Heat Exchangers

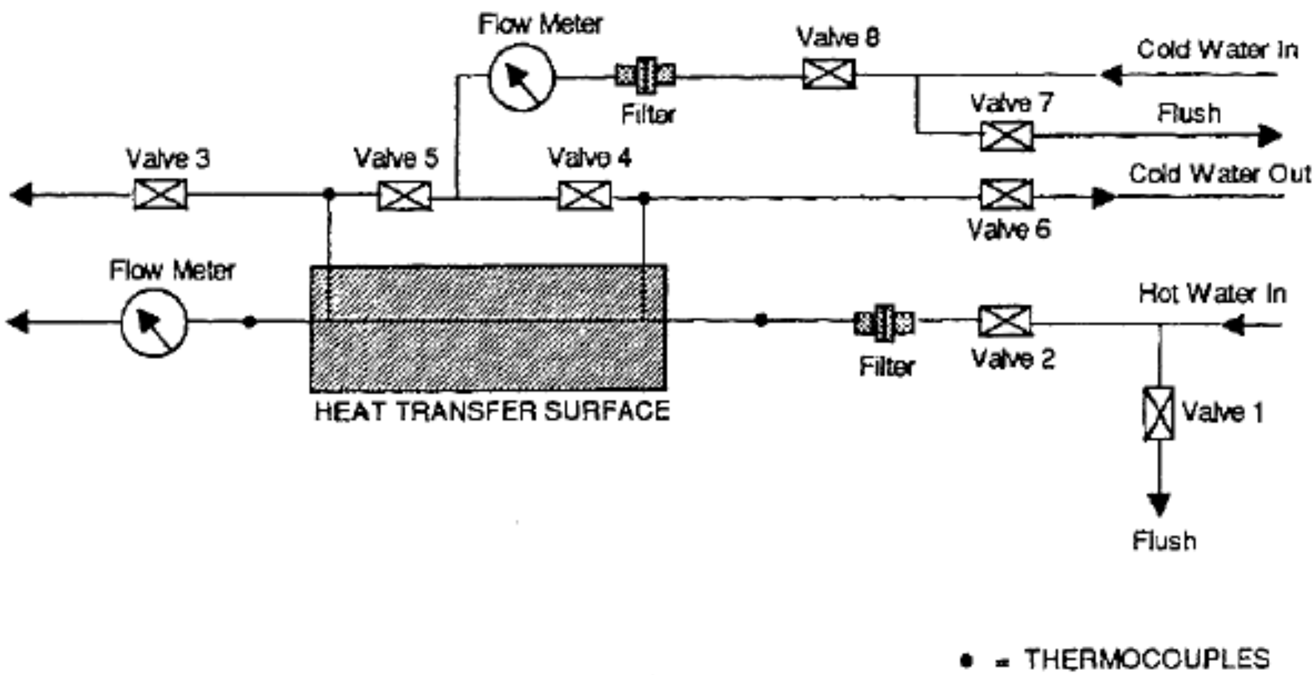


Figure 7.2. *Heat exchanger experimental setup*.

Heat Transfer – Heat Exchangers

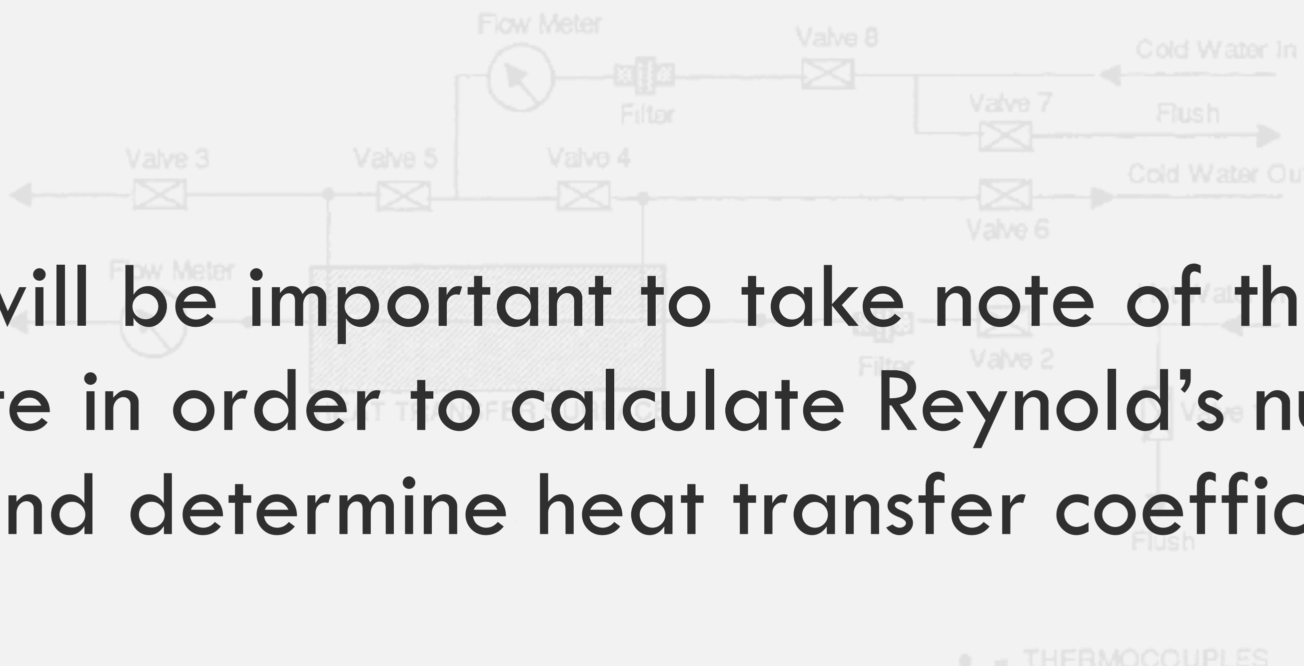
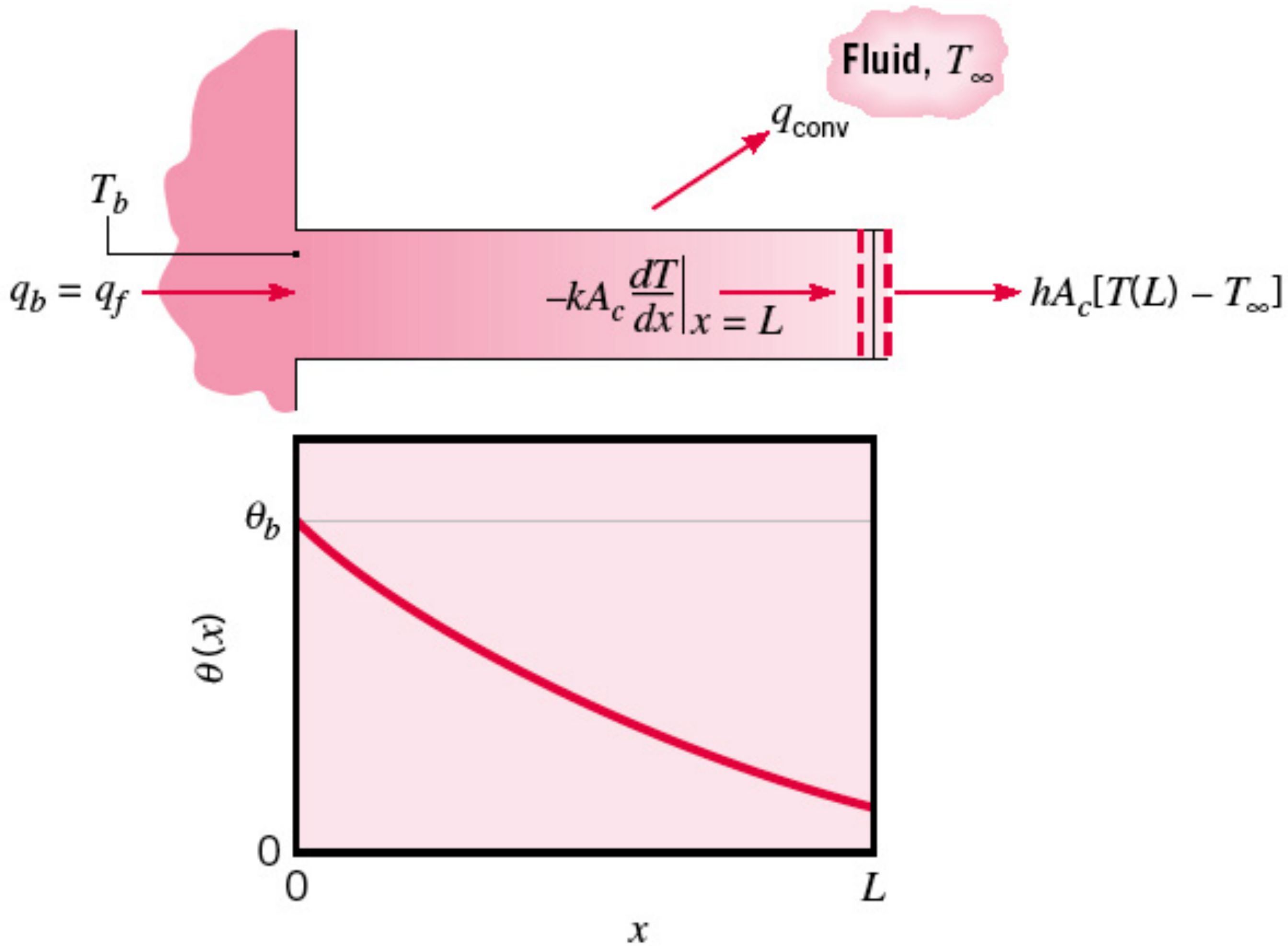


Figure 7.2. *Heat exchanger experimental setup.*

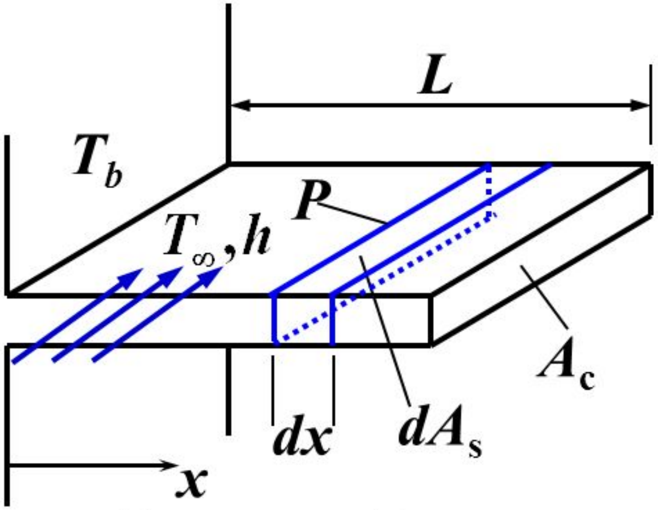
It will be important to take note of the flow rate in order to calculate Reynold’s number and determine heat transfer coefficients

Heat Transfer - Fins



Heat Transfer - Fins

Fins of Uniform Cross-Sectional Area



P: fin perimeter

$A_c(x)$ = constant,
and $dA_s = Pdx$

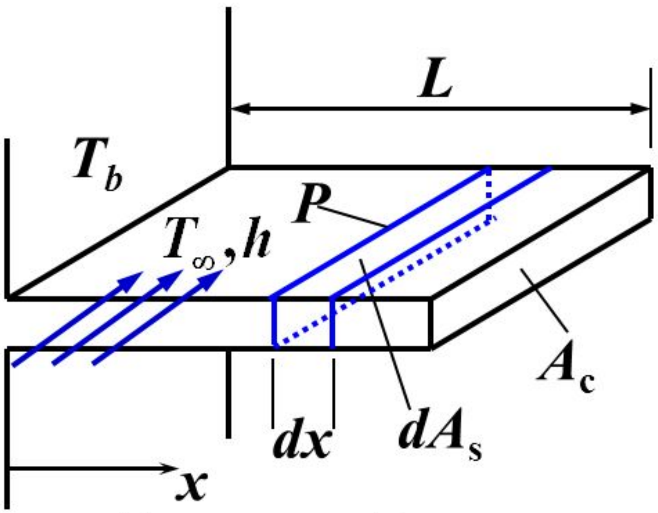
$$\frac{d}{dx}\left[A_c(x)\frac{dT}{dx}\right]-\frac{h}{k}\frac{dA_s}{dx}\left[T(x)-T_\infty\right]=0$$

$$\frac{d^2T}{dx^2}-\frac{hP}{kA_c}\left[T(x)-T_\infty\right]=0$$

Steady state

Heat Transfer - Fins

Fins of Uniform Cross-Sectional Area



P: fin perimeter

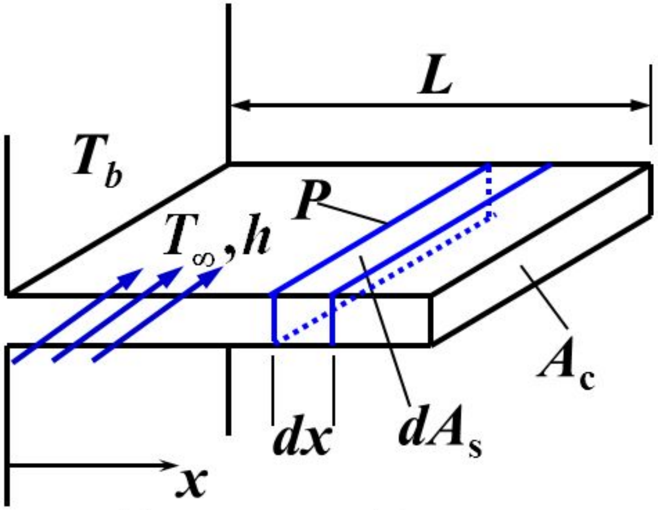
$A_c(x)$ = constant,
and $dA_s = Pdx$

$$\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{hC}{kA}\left[T(x,t) - T_\infty\right] = \frac{\frac{1}{\alpha}\partial T(x,t)}{\partial t}$$

<u>Transient</u>

Heat Transfer - Fins

Fins of Uniform Cross-Sectional Area



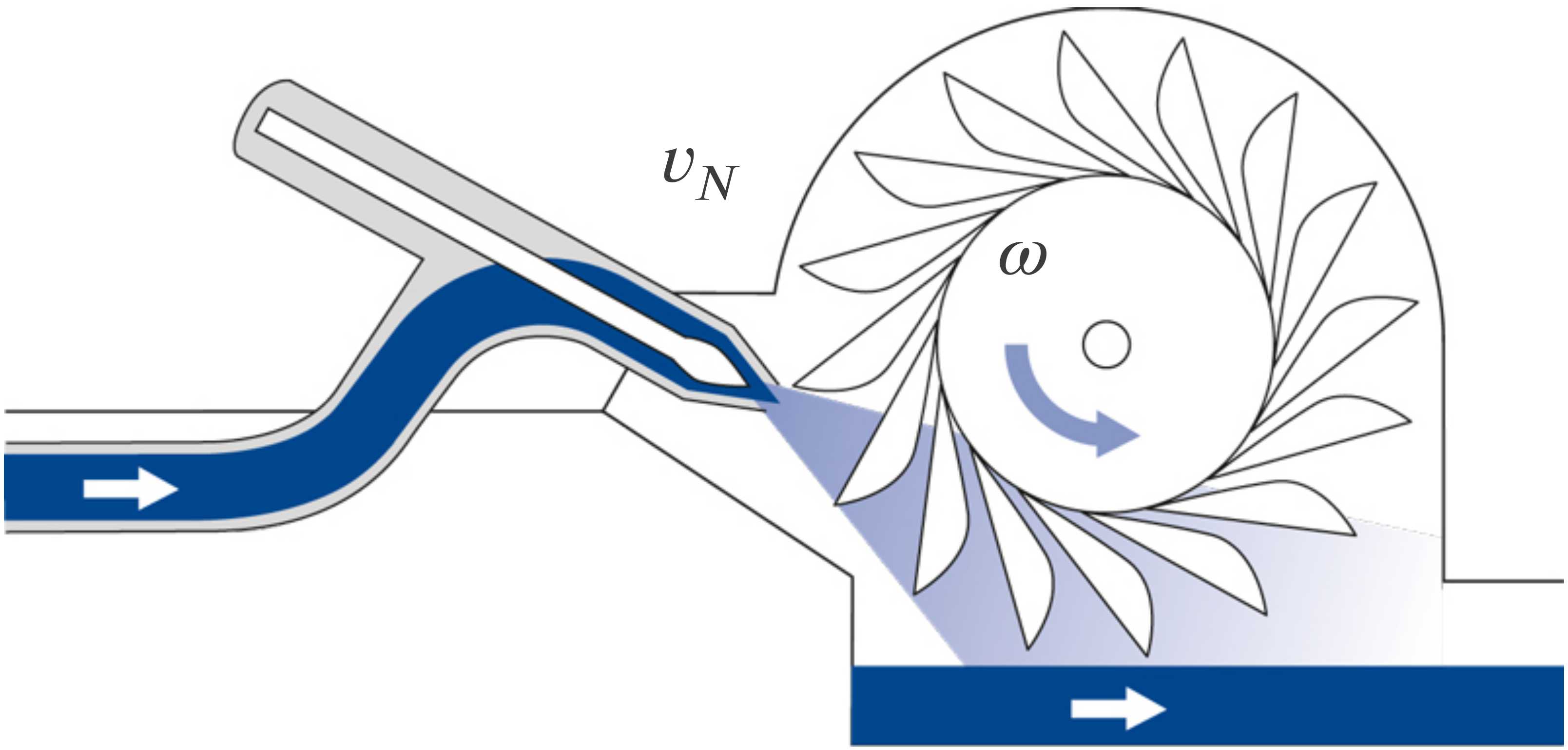
P: fin perimeter

$A_c(x)$ = constant,

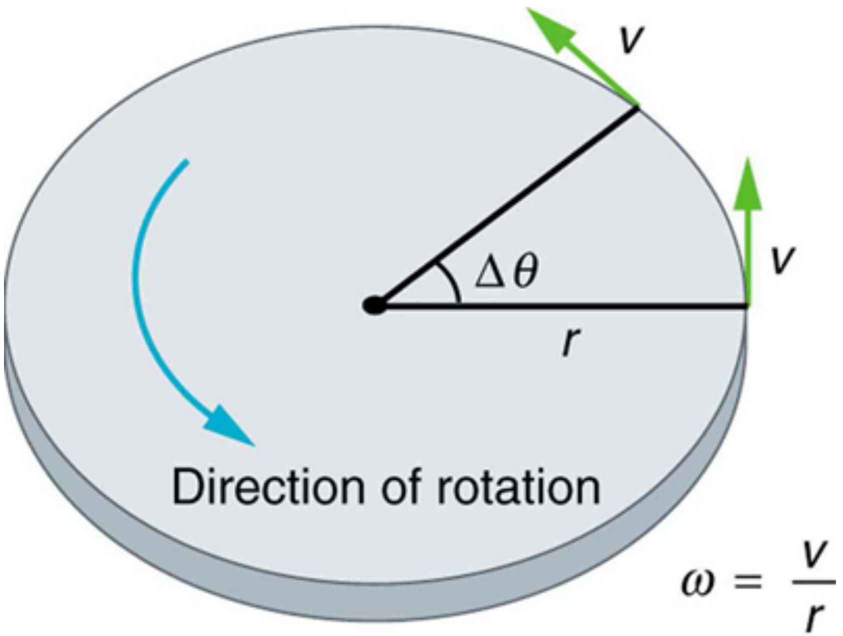
and $dA_s = Pdx$

$$T(x,t) = T_{amb} + \frac{q''_o}{k}\left\{\left[\frac{e^{m(x-2L)} + e^{-mx}}{m(1-e^{-2mL})}\right] - 2e^{-m^2\alpha t}\left[\frac{1}{2m^2L} + L\sum_{n=1}^{\infty}\frac{\cos\left(\frac{n\pi}{L}x\right)e^{-\frac{n^2\pi^2}{L^2}\alpha t}}{m^2L^2+n^2\pi^2}\right]\right\}$$

Hydrodynamics



Hydrodynamics



Velocity relative to moving buckets

$$V_r = v_N - \omega r$$

Friction factor k_1:

$$V_{r,losses} = k_1(v_N - \omega r)$$

Angular dependence:

$$\Delta V_r = V_r + V_{r,losses}\cos\theta = (v_N - \omega r) + k_1(v_N - \omega r)\cos\theta$$

$$\Delta V_r = (v_N - \omega r)(1 + k_1\cos\theta)$$

Hydrodynamics

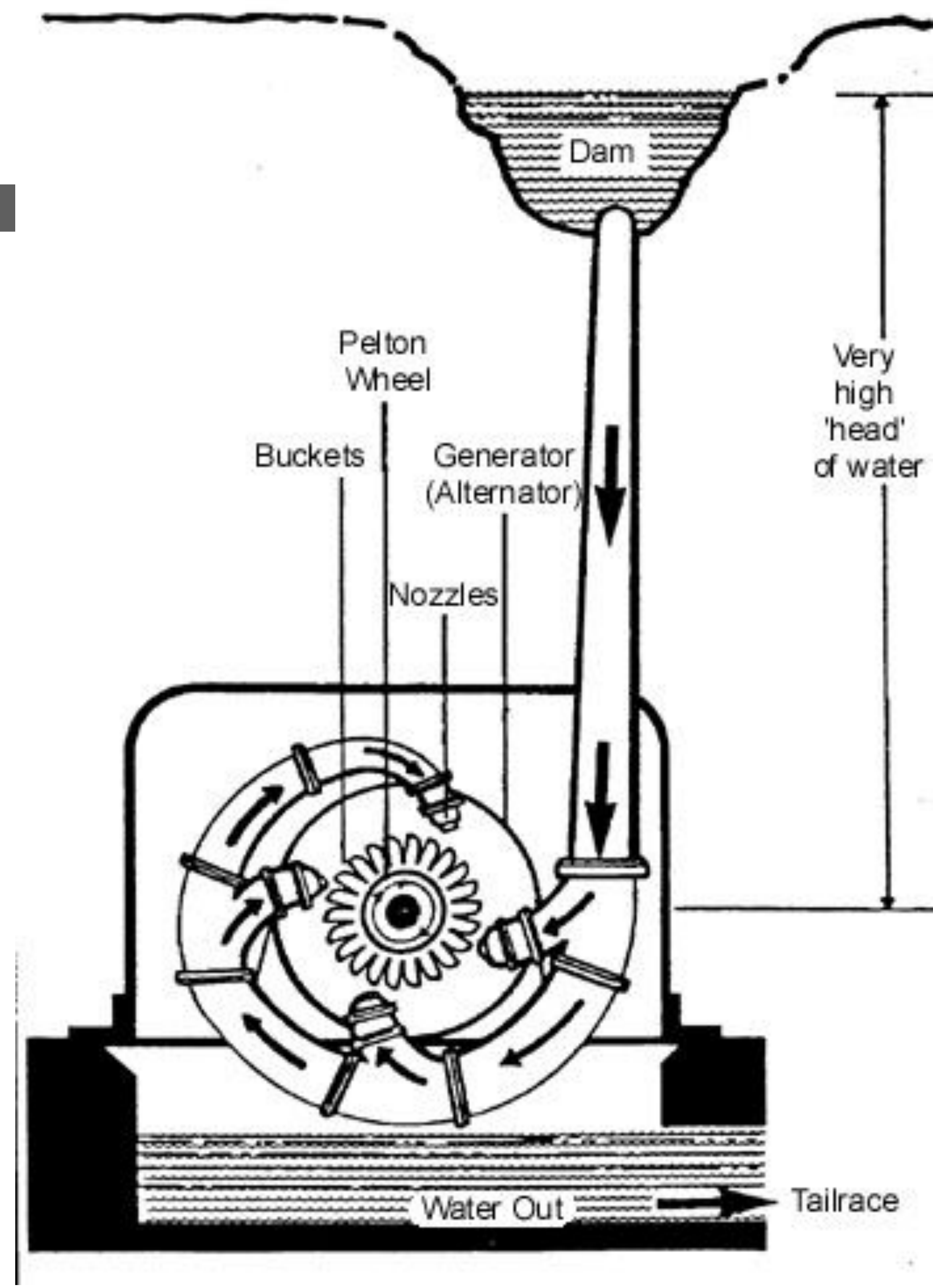
Nozzle Flow

$$\frac{1}{2}\rho v_N^2 + \rho g h = 0$$

$$v_N = \sqrt{2gh}$$

Assuming loss:

$$v_N = C_v\sqrt{2gh}$$



Hydrodynamics

Power Output

$$P = \frac{\text{work}}{\text{time}} = \frac{dW}{dt} = \frac{d}{dt}(\boldsymbol{F} \cdot \boldsymbol{r})$$

$$P = \boldsymbol{F} \cdot \boldsymbol{v}$$

$$\boldsymbol{F} = ?$$

Hydrodynamics

Power Output

$$P = \frac{\text{work}}{\text{time}} = \frac{dW}{dt} = \frac{d}{dt}(\boldsymbol{F} \cdot \boldsymbol{r})$$

$$P = \boldsymbol{F} \cdot \boldsymbol{v}$$

$$\boldsymbol{F} = m\boldsymbol{a} = m\frac{dv}{dt} = \frac{m\Delta \mathrm{V}}{\Delta t}$$

$$m = \dot{m}\Delta t = v_N A\rho\Delta t$$

$$\boldsymbol{F} = \rho A v_N \Delta V$$

$$P = \rho A v_N \Delta V \omega r$$

Hydrodynamics

Power Output

$$P = \rho A v_N \Delta V \omega r$$

$$P = \ \omega r \rho A v_N (v_N - \omega r)(1 + k_1 \cos\theta)$$

$$P = \ \omega r \rho A C_v \sqrt{2gh}(C_v \sqrt{2gh} - \omega r)(1 + k_1 \cos\theta)$$

What is P_{max} and when is it reached? Try:

$$P = v_r \rho A v_N (v_N - v_r)(1 + c)$$

Hydrodynamics

What is P_{max} and when is it reached? Try:

$$P = v_r \rho A v_N (v_N - v_r)(1 + c)$$

$$\frac{\partial P}{\partial t} = 0$$

$$P = \rho A(1 + c)\left(v_r v_N^2 - v_r^2 v_N\right)$$

$$\frac{\partial P}{\partial v_r} = \frac{\partial\left(\rho A(1 + c)\left(v_r v_N^2 - v_r^2 v_N\right)\right)}{\partial v_r} = 0$$

Hydrodynamics

$$\frac{\partial P}{\partial v_r} = \frac{\partial\left(\rho A(1+c)\left(v_r v_N^2 - v_r^2 v_N\right)\right)}{\partial v_r} = 0$$

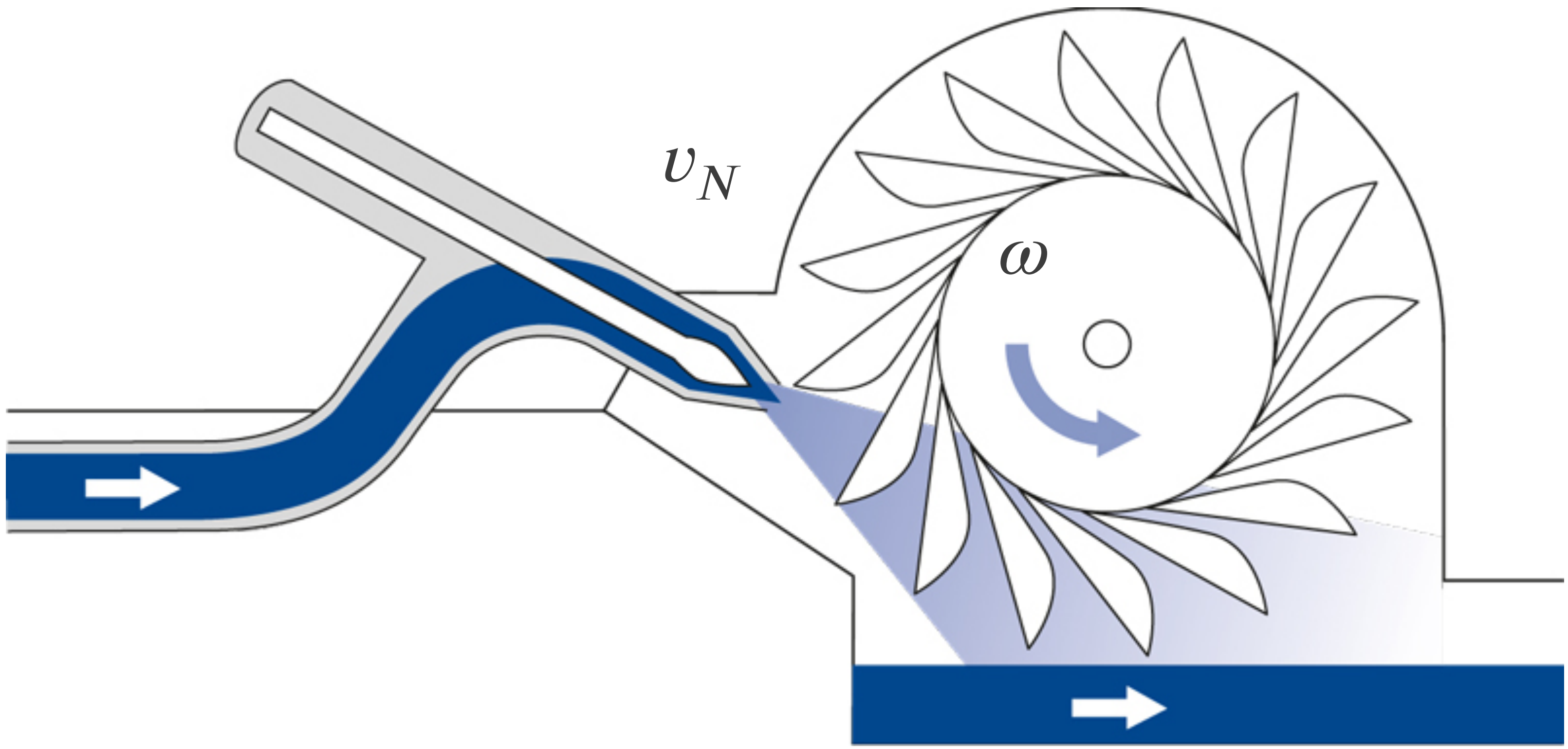
$$v_N^2 - 2v_r v_N = 0$$

$$v_r = \frac{v_N}{2}$$

$$P_{max} = \rho A(1+c)\left(\frac{v_N}{2}v_N^2 - \left(\frac{v_N}{2}\right)^2 v_N\right)$$

$$P_{max} = \frac{\rho A(1+c)v_N^3}{4} = \frac{\rho A}{2}v_N^3\Big|$$

Hydrodynamics



Next Week Labs Start

Prepare for lab by reviewing respective material

Group Laboratory Assignments

	Lab for the Week				
Lab Group	Week 1	Week 2	Week 3	Week 4	Week 5
1	Wind Tunnel	Pipe	Heat X	X-Fin	Pelton
2	Pelton	Wind Tunnel	Pipe	Heat X	X-Fin
3	X-Fin	Pelton	Wind Tunnel	Pipe	Heat X
4	Heat X	X-Fin	Pelton	Wind Tunnel	Pipe
5	Pipe	Heat X	X-Fin	Pelton	Wind Tunnel

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