ME 57200 Aerodynamic Design

Lecture #13: Elemental Flows

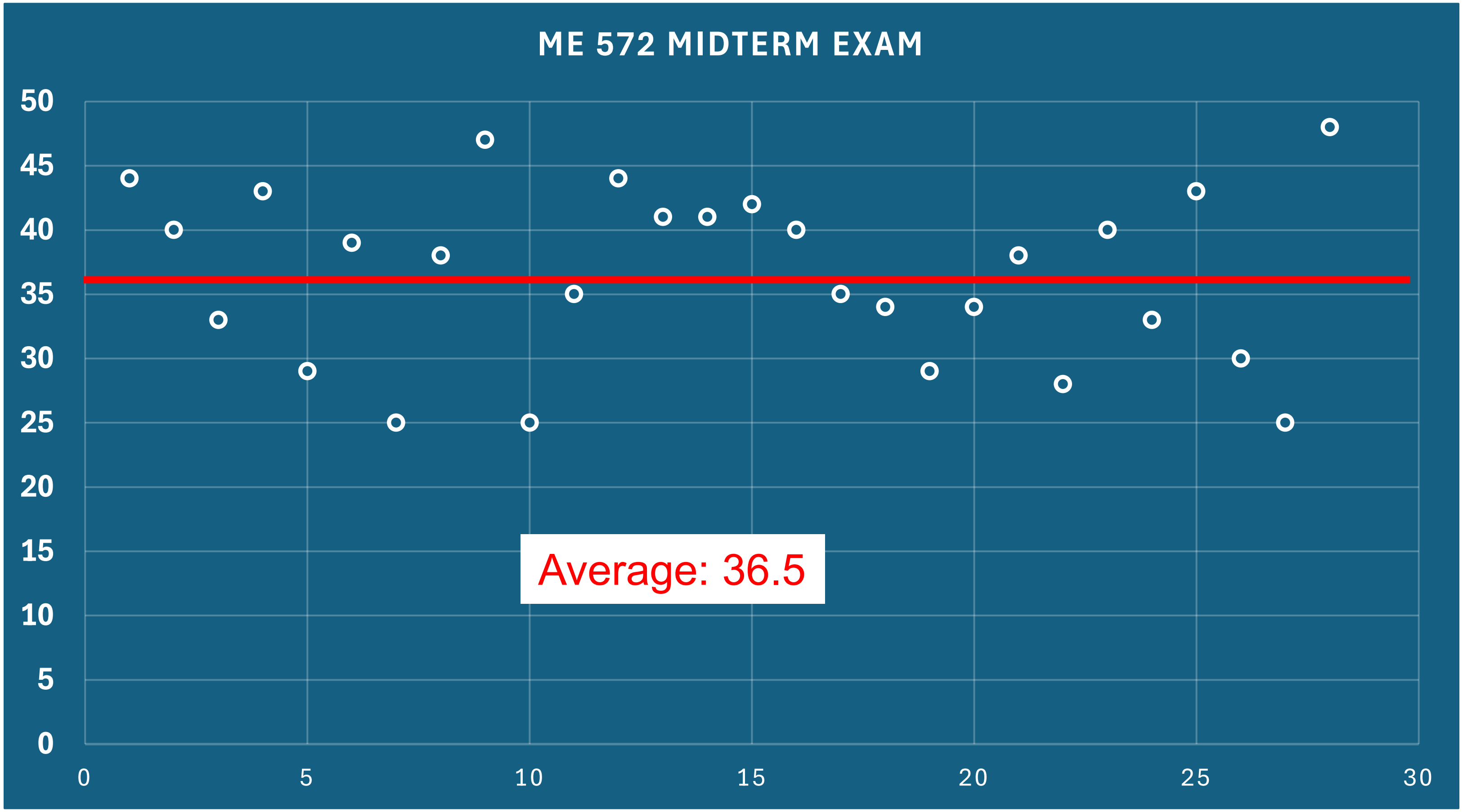
Dr. Yang Liu

Steinman 253

Tel: 212-650-7346

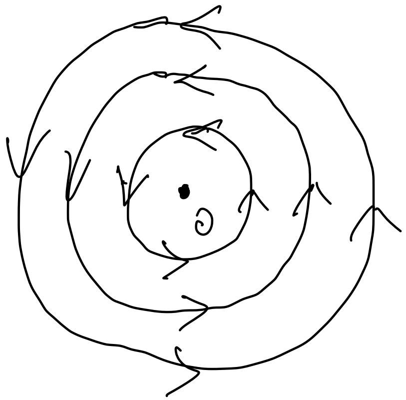
Email: yliu7@ccny.cuny.edu

Midterm Exam Score



Elementary Flows

Vortex Flow :



$$\begin{cases} V_r = 0 \\ V_\theta = \dfrac{C}{r} \end{cases} \Rightarrow \begin{cases} \nabla \cdot \vec{V} = 0 & \text{"continuity"} \\ \nabla \times \vec{V} = 0 & \text{"Irrotational" except the origin} \end{cases}$$

Circulation around a given streamline in the vortex flow

$$\Gamma = -\oint_c \vec{V}\, d\vec{s} = -V_\theta (2\pi r)$$

$$\Rightarrow V_\theta = -\frac{\Gamma}{2\pi r}, \quad \left(V_\theta = \frac{C}{r}\right) \Rightarrow C = -\frac{\Gamma}{2\pi}$$

"Γ" is called vortex strength, is constant

Elementary Flows

Vortex flow is irrotational except at the origin

what happens at $r = 0$?

$$\Gamma = -\oint_C \vec{V} \, d\vec{s} = -\iint_S (\nabla \times \vec{V}) \, d\vec{s} = -2\pi C$$

As $r \to 0$ $\quad \iint_S (\nabla \times \vec{V}) \, d\vec{s} \longrightarrow |\nabla \times \vec{V}| \cdot ds = 2\pi C$

$$\Rightarrow |\nabla \times \vec{V}| = \frac{2\pi C}{ds}$$

$r \to 0$, $ds \to 0$ $\Rightarrow \boxed{|\nabla \times \vec{V}| \to \infty \text{ at } r = 0}$

Elementary Flows

Velocity Potential

$$\begin{cases} \dfrac{\partial \phi}{\partial r} = V_r = 0 \\ \dfrac{1}{r}\dfrac{\partial \phi}{\partial \theta} = V_\theta = -\dfrac{\Gamma}{2\pi r} \end{cases} \Rightarrow \boxed{\phi = -\frac{\Gamma}{2\pi}\theta} + \text{Const}$$

Stream Function

$$\boxed{\begin{cases} \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} = V_r = 0 \\ -\dfrac{\partial \psi}{\partial r} = V_\theta = -\dfrac{\Gamma}{2\pi r} \end{cases}} \Rightarrow \boxed{\psi = \frac{\Gamma}{2\pi}\ln r} + \text{Const} \quad (r \neq 0)$$

Elementary Flows

Example: You are standing at a location 20 feet from the center of a vortex with strength of $1.843\times10^4\,ft^2/s$. What is the flow speed you can feel?

$$V_\theta = -\frac{\Gamma}{2\pi r} = -\frac{1.843\times10^4\,ft^2/s}{2\pi\times 20ft} = 88\,ft/s = 60mph$$

Elementary Flows

Lifting flow over a cylinder

Add a vortex flow to the nonlifting flow

Elementary Flows

Stream Function

Nonlifting Flow : $\psi_1 = V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)$

Vortex Flow : $\psi_2 = \frac{\Gamma}{2\pi} \ln r + \text{Const}$

$$\text{Const} = -\frac{\Gamma}{2\pi} \ln R$$

$$\psi_2 = \frac{\Gamma}{2\pi} \ln \frac{r}{R}$$

$$\psi = \psi_1 + \psi_2 = \boxed{V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi} \ln \frac{r}{R}}$$

If $r = R$, $\psi = 0$ is a streamline of flow

Elementary Flows

$$\begin{cases} V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta \\ V_\theta = -\frac{\partial \psi}{\partial r} = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta - \frac{\Gamma}{2\pi r} \end{cases}$$

where are the stagnation points in the flow?

$$\begin{cases} V_r = 0 \\ V_\theta = 0 \end{cases} \Rightarrow \begin{cases} r = R \\ \theta = \arcsin\left(-\frac{\Gamma}{4\pi V_\infty R}\right) \end{cases}$$

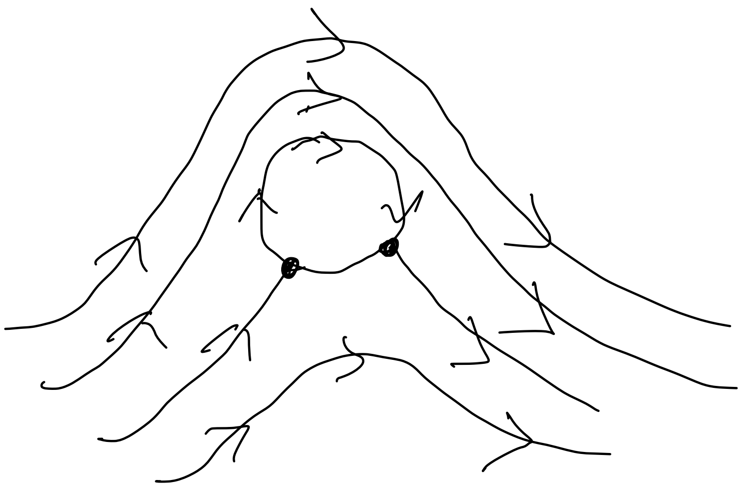
$$\sin\theta = -\frac{\Gamma}{4\pi V_\infty R}$$

Elementary Flows

For Γ. is positive.

1. $\left|\frac{\Gamma}{4\pi V_\infty R}\right| < 1$

in the bottom half of the cylinder



2. $\left|\frac{\Gamma}{4\pi V_\infty R}\right| = 1$

only one stagnation point $(R, -\frac{\pi}{2})$

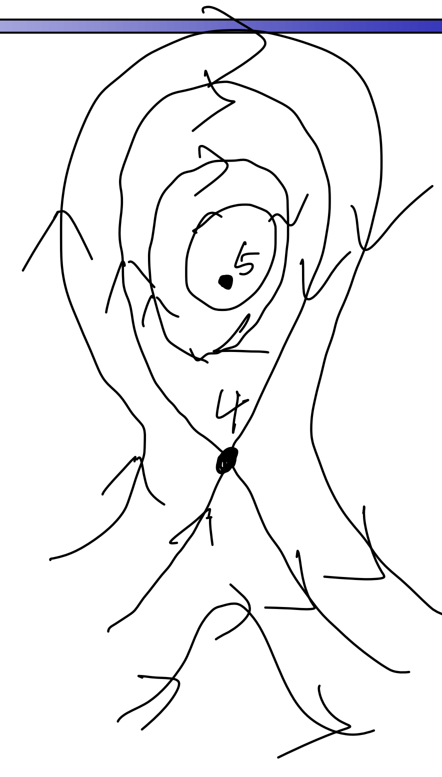
Elementary Flows

3- $\boxed{\left|\frac{\Gamma}{4\pi V_\infty R}\right| > 1}$

Not at $r=R$ any more.

satisfied only when we have $\boxed{\theta = \frac{\pi}{2} \text{ or } -\frac{\pi}{2}}$

and $\boxed{r = \frac{\Gamma}{4\pi V_\infty} \pm \sqrt{\left(\frac{\Gamma}{4\pi V_\infty}\right)^2 - R^2}}$



Elementary Flows

The velocity on the surface of the cylinder $(r = R)$

$$\Rightarrow \quad \underbrace{V = V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}}$$

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(-2\sin\theta - \frac{\Gamma}{2\pi R V_\infty}\right)^2$$

$\underline{C_d \text{ - } C_a}$ $\qquad\qquad C_d = 0$

Elementary Flows

Lift coefficient

$$C_l = \frac{\Gamma}{R V_\infty}$$

$$\Rightarrow L' = C_l \cdot \frac{1}{2} \rho_\infty V_\infty^2 \cdot S \qquad S = 2R(1)$$

$$\Rightarrow \boxed{L' = \rho_\infty V_\infty \Gamma}$$

In-Class Quiz