ME 572 Aerodynamic Design
HW #3 (Due at 11:59 pm on Friday, Feb 23)

Problem 1 [10 pt]
Consider a velocity field where the x and y components of velocity are given by $u = cx/(x^2 + y^2)$ and $v = cy/(x^2 + y^2)$, respectively, where c is a constant. Find the equations of the streamlines and describe the streamlines pattern.

Solution:

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{v}{u} = \frac{cy/\left(x^2+y^2\right)}{cx/\left(x^2+y^2\right)} = \frac{y}{x}$$

2 Points

$$\frac{dy}{y} = \frac{dx}{x}$$

2 Points

$$\ln y = \ln x + c_1$$

2 Points

$$\ln y = \ln c_2 x$$

2 Points

$$\boxed{y = c_2 x}$$

The streamlines are straight lines emanating from the origin.

2 Points

Problem 2 [10 pt]

Consider a velocity field where the x and y components of velocity are given by $u = cy/(x^2 + y^2)$ and $v = -cx/(x^2 + y^2)$, respectively, where c is a constant. Find the equations of the streamlines and describe the streamlines pattern.

Solution:

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{x}{y}$$

2 Points

$$y\,dy = -x\,dx$$

2 Points

$$y^2 = -x^2 + \text{constant}$$

2 Points

$$\boxed{y^2 + x^2 = \text{constant}}$$

2 Points

The streamlines are concentric with their centers at the origin.

2 Points

Problem 3 [10 pt]
Consider a velocity field where the radial and tangential components of velocity are V_r = 0 and V_θ = cr, respectively, where c is a non-zero constant. Please mathematically prove if this flow field is irrotational or rotational?

Solution:

Given: $V_r = 0$ and $V_\theta = cr$
From the equation

$$\nabla \times \vec{V} = \frac{1}{r}\begin{vmatrix} e_r & re_\theta & e_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix}$$

2 Points

$$\nabla \times \vec{V} = \vec{e_z}\left[\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right]$$

2 Points

$$\nabla \times \vec{V} = \vec{e_z}\left[\frac{\partial(\frac{c}{r})}{\partial r} + \frac{cr}{r} - \frac{1}{r}\frac{\partial(0)}{\partial \theta}\right]$$

2 Points

$$\nabla \times \vec{V} = \vec{e_z}\,(c + c - 0)$$

$$\nabla \times \vec{V} = 2c\,\vec{e_z}$$

2 Points

Since $\nabla \times \vec{V} \neq 0$ at every point in a flow field, the flow is rotational. **2 Points**