ME 572 Aerodynamic Design
HW #1 (Due at 11:59 pm on Friday, Feb 02)

Problem 1 [10 pt]

Consider an airfoil at 12° angle of attack. The normal and axial force coefficients are 1.2 and 0.03, respectively. Calculate the lift and drag coefficients. Please schematically show the relations between the normal and axial forces and lift and drag forces. (Round answers to two decimal places.)

Solution:

$$c = c_n \cos\alpha - c_a \sin\alpha$$

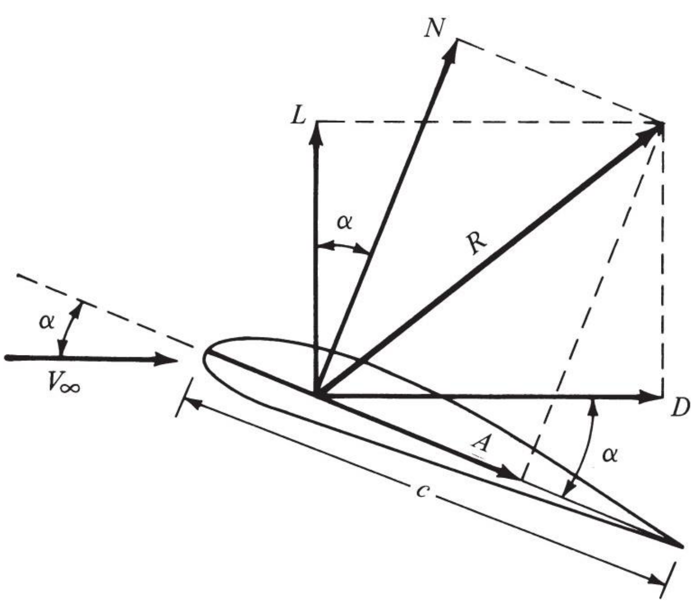
$$= (1.2)\cos 12^\circ - (0.3)\sin\alpha = \boxed{1.18}$$

4 Points

$$c_d = c_n \sin\alpha + c_a \cos\alpha$$

$$= (1.2)\sin 12^\circ + (0.3)\cos\alpha = \boxed{0.279}$$

4 Points



2 Points

Problem 2 [20 pt]

Consider an infinitely thin flat plate with a 1 m chord at an angle of attack of 10° in a supersonic flow. The pressure and shear stress distributions on the upper and lower surfaces are given by $p_u = 4 \times 10^4(x - 1)^2 + 5.4 \times 10^4$, $p_l = 2 \times 10^4(x - 1)^2 + 1.73 \times 10^5$, $\tau_u = 288x^{-0.2}$, and $\tau_l = 779x^{-0.2}$, respectively, where x is the distance from the leading edge in meters and p and τ are in newtons per square meter. Calculate the normal and axial forces, the lift and drag, moments about the leading edge. (Round answers to two decimal places.)

Solution:

For a flat plate, $\theta = 0$.

Normal force:

$$N' = \int_0^c (p_l - p_u)\,\mathrm{dx} = \int_0^1 \left(-2 \times 10^4(x - 1)^2 - 1.19 \times 10^5\right)\mathrm{dx}$$

4 Points

$$N' = -2 \times 10^4\left[\frac{x^3}{3} - x^2 + x\right]_0^1 + \left[1.19 \times 10^5 x\right]_0^1 = 1.12 \times 10^5\ \mathrm{N}$$

Axial force:

$$A' = \int_0^c (\tau_l - \tau_u)\,dx = \int_0^1 \left(779x^{-0.2} + 288\,x^{-0.2}\right)dx$$

$$A' = \left[1334x^{0.8}\right]_0^1 = 1334\ \mathrm{N}$$

4 Points

Lift:

$$L' = N'\cos\alpha - A'\sin\alpha$$

$$L' = (1.12 \times 10^5\ \mathrm{N})\cos 10^{\circ} - (1334\ \mathrm{N})\sin 10^{\circ}$$

4 Points

$$L' = 1.1 \times 10^5\ \mathrm{N}$$

Drag:

$$D' = N'\sin\alpha + A'\cos\alpha$$

$$D' = (1.12 \times 10^5\ \mathrm{N})\sin 10^{\circ} + (1334\ \mathrm{N})\cos 10^{\circ}$$

4 Points

$$D' = 2.08 \times 10^4\ \mathrm{N}$$

Moment about the leading edge:

$$M'_{\mathrm{LE}} = \int_0^c (p_u - p_l)\,x\,dx = \int_0^1 \left(2 \times 10^4(x - 1)^2 - 1.19 \times 10^5\right)x\,dx$$

4 Points

$$M'_{\mathrm{LE}} = 2 \times 10^4\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1 - \left[0.595 \times 10^5 x^2\right]_0^1 = -5.78 \times 10^4\ \mathrm{N{\cdot}m}$$