ME 57200 Aerodynamic Design

Lecture #8: Basic Concepts in Aerodynamics

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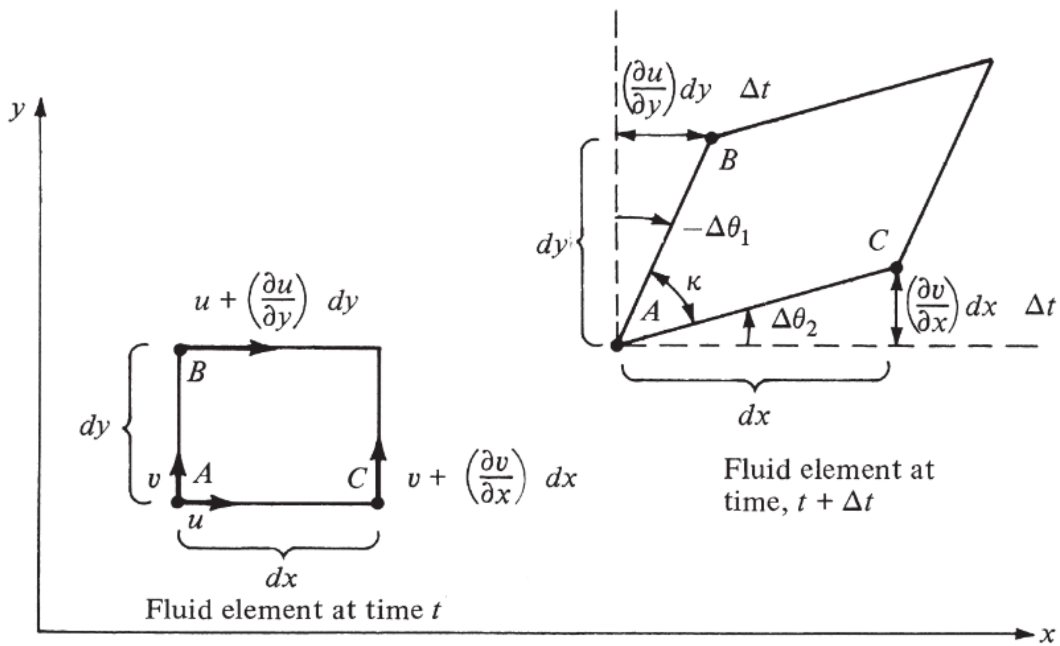
Midterm Exam

- Tuesday, 3/12, from 09:30 to 10:45 am at Shepard S-308.

- The exam is open-book and open-notes.

- 5 True/False Questions: 10 pt
- 4 Math-based Problems: 40 pt
- Total: 50 pt

Angular Velocity and Vorticity



$$\Delta\theta_1 = -\frac{\partial u}{\partial y}\Delta t$$

$$\Delta\theta_2 = \frac{\partial v}{\partial x}\Delta t$$

$$\frac{d\theta_1}{dt} = \lim_{\Delta t \to 0}\frac{\Delta\theta_1}{\Delta t} = -\frac{\partial u}{\partial y}$$

$$\frac{d\theta_2}{dt} = \lim_{\Delta t \to 0}\frac{\Delta\theta_2}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\omega_z = \frac{1}{2}\left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right) \Rightarrow \boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

Angular Velocity and Vorticity

$$\omega = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$$

$$\boxed{\omega = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}\right]}$$

<u>Vorticity</u>: twice of the <u>angular velocity</u> $\xi \equiv 2\,\omega$

$$\boxed{\xi = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}}$$

$$\boxed{\xi = \nabla \times \mathbf{V}}$$

Angular Velocity and Vorticity

$$\boxed{\xi = \nabla \times \mathbf{V}}$$

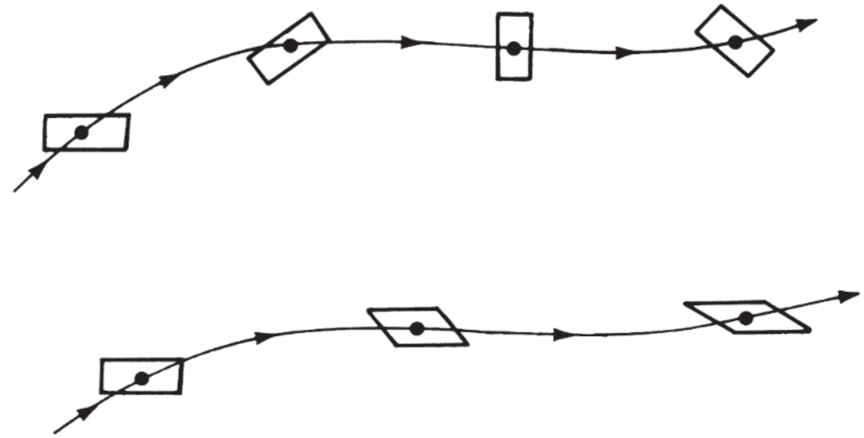
In a velocity field, the curl of the velocity is equal to the vorticity.

1. If $\nabla \times \boldsymbol{V} \neq 0$ at every point in a flow, the flow is called *rotational*. This implies that the fluid elements have a finite angular velocity.

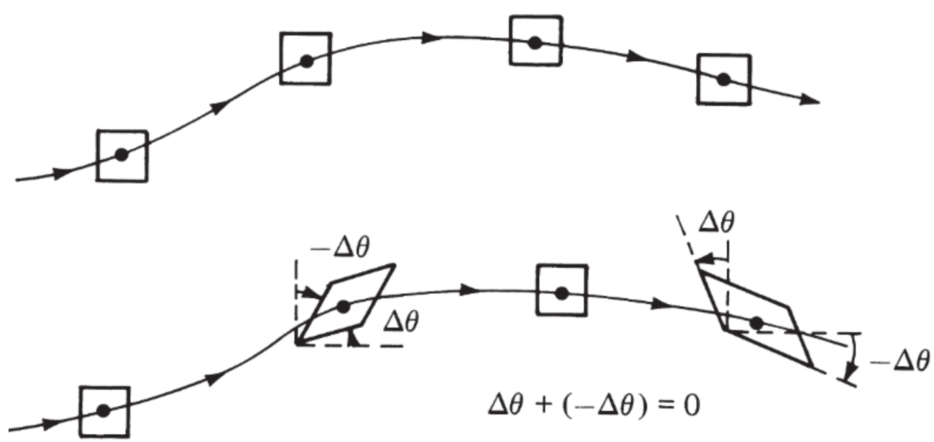
2. If $\nabla \times \boldsymbol{V} = 0$ at every point in a flow, the flow is called *irrotational*. This implies that the fluid elements have no angular velocity; rather, their motion through space is a pure translation.

Angular Velocity and Vorticity

Rotational Flow



Irrotational Flow



Angular Velocity and Vorticity

For 2-D flow

$$\xi = \xi_z \mathbf{k} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}$$

If the flow is irrotational, $$\boxed{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0}$$

The *condition of irrotationality for two-dimensional flow*

In-Class Example

Consider the velocity field given by $u = y/(x^2 + y^2)$ and $v = -x/(x^2 + y^2)$.

calculate the vorticity.

Solution:

$$\vec{\xi} = \nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{y}{x^2+y^2} & \frac{-x}{x^2+y^2} & 0 \end{vmatrix}$$

$$= \vec{i}\,[0-0] - \vec{j}\,(0-0) + \vec{k}\left[\frac{(x^2+y^2)(-1)+x(2x)}{(x^2+y^2)^2} - \frac{(x^2+y^2)-y(2y)}{(x^2+y^2)^2}\right]$$

$$= 0\vec{i} + 0\vec{j} + 0\vec{k} = 0$$

Irrotational except at the origin (0, 0)

Circulation

Circulation: a fundamental tool to calculate aerodynamic lift

Consider a closed curve "C" in the flow field, let $\vec{V}$ and $\vec{ds}$ be the velocity and directed line segment.

Circulation is defined as

$$\Gamma = -\oint \vec{V} \cdot \vec{ds}$$

The negative line integral of the velocity around a closed curve

Circulation is dependent only on
- the velocity field
- the choice of curve "C"

Circulation

Stokes' theorem: $-\oint_C \vec{V}\,d\vec{s} = -\iint_S (\nabla \times \vec{V})\,d\vec{s}$

$$\Rightarrow \quad \underset{\sim}{\Gamma} = -\iint_S (\nabla \times \vec{V})\,d\vec{s} = -\iint_S \vec{\xi}\,d\vec{s}$$

$\cancel{*}$: The circulation about a curve "c" is equal to the vorticity integrated over the open surface bounded by "c"

For irrotational flow $\Rightarrow \Gamma = 0$

Circulation

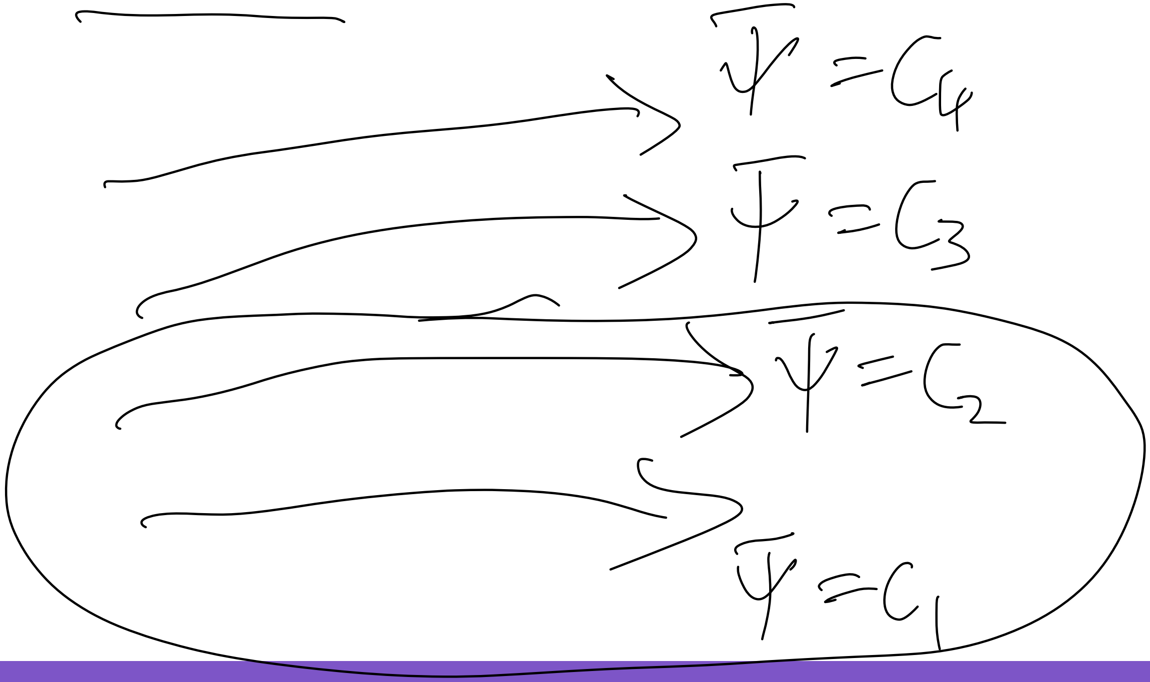
Circulation

Stream Function

Differential equation for a streamline : $\frac{dy}{dx} = \frac{v}{u}$

$u = u(x, y)$, $v = v(x, y)$

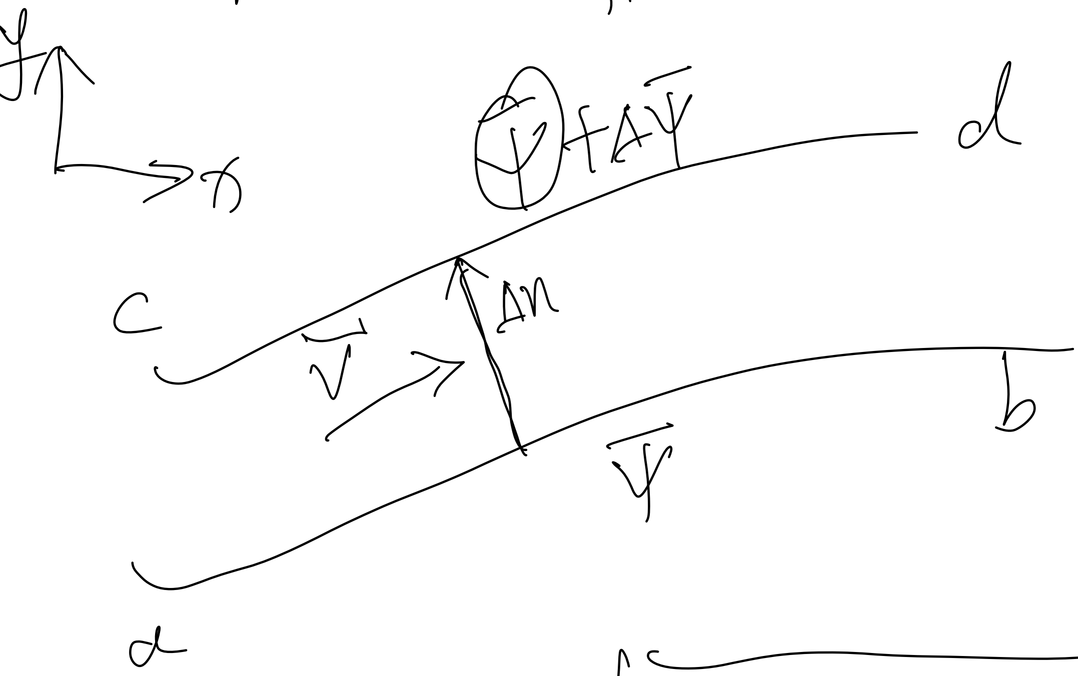
$\Psi(x, y) = C$ (C is arbitrary constant)



$\Delta\Psi = C_2 - C_1$

Stream Function

Consider the mass flow rate inside a stream-tube



$$\Delta\Psi = \rho V \Delta n$$

$$\Rightarrow \frac{\Delta\Psi}{\Delta n} = \rho \cdot V$$

$$\Rightarrow \boxed{\frac{\partial\Psi}{\partial n} = \rho V}$$

$$\Delta\Psi = \rho u \Delta y + \rho v(-\Delta x)$$

$$\Psi = \Psi(x, y),\quad \boxed{d\Psi = \frac{\partial\Psi}{\partial x}dx + \frac{\partial\Psi}{\partial y}dy}$$

$$\boxed{d\Psi = \rho u\, dy - \rho v\, dx}$$

Stream Function

$$\rho u\,dy - \rho v\,dx = \frac{\partial \Psi}{\partial y}dy + \frac{\partial \Psi}{\partial x}dx$$

$$\Rightarrow \begin{cases} \rho u = \dfrac{\partial \Psi}{\partial y} \\ \rho v = -\dfrac{\partial \Psi}{\partial x} \end{cases} \xrightarrow{\rho \text{ is constant}} \boxed{\begin{cases} u = \dfrac{\partial \psi}{\partial y} \\ v = -\dfrac{\partial \psi}{\partial x} \end{cases}}$$

$$\psi = \Psi/\rho$$

Stream Function

Velocity Potential

Irrotational flow : $\nabla \times \vec{V} = 0$

$\vec{V} = \nabla \phi$ "ϕ" : velocity potential

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} = \frac{\partial \phi}{\partial x}\vec{i} + \frac{\partial \phi}{\partial y}\vec{j} + \frac{\partial \phi}{\partial z}\vec{k}$

$$\Rightarrow \begin{cases} u = \frac{\partial \phi}{\partial x} \\ v = \frac{\partial \phi}{\partial y} \\ w = \frac{\partial \phi}{\partial z} \end{cases}$$

"For irrotational flow only"

Velocity Potential

Velocity Potential

Velocity Potential

In-Class Quiz