ME 57200 Aerodynamic Design

Lecture #17: Flow over Finite Wings

Dr. Yang Liu

Steinman 253

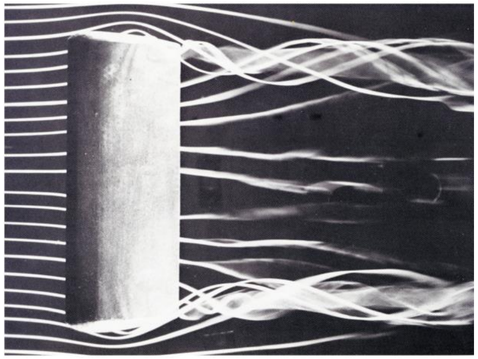
Tel: 212-650-7346

Email: yliu7@ccny.cuny.edu

- Project Assignment #1 is due on Friday, 4/12

Incompressible Flow over Finite Wings

3-D flow over a finite wing

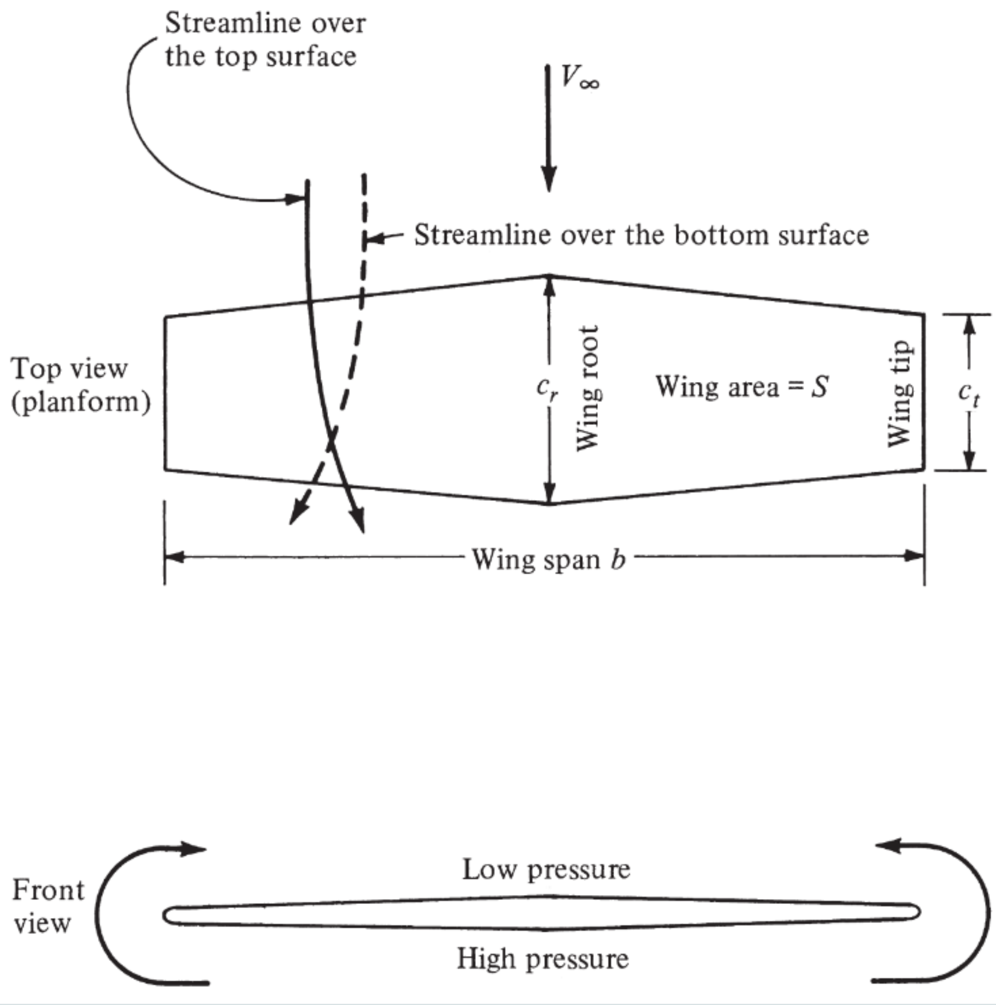


The net imbalance of the pressure distribution creates the lift

As a by-product of the pressure imbalance, the flow near the wing tips tends to curl around the tips, being forced from the high-pressure region underneath the tips to the low-pressure region on top.

- On the top surface of the wing, there is a spanwise component of flow from the tip toward the wing root, the streamlines bend toward the root.
- On the bottom surface of the wing, there is a spanwise component of flow from the root toward the wing tip, the streamlines bend toward the tip.

Incompressible Flow over Finite Wings



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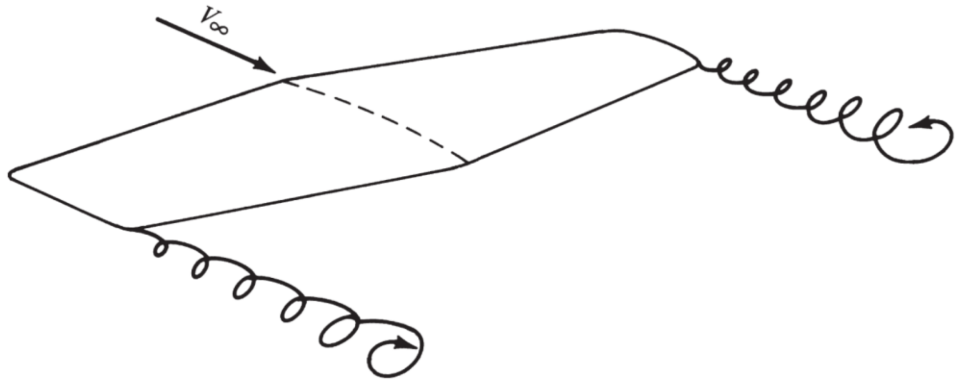
Wing-tip vortices

The flow "leak" around the wing tips leads to the formation of wing-tip vortices



Downwash: the velocity component in the downward direction at the wing due to the wing-tip vortices

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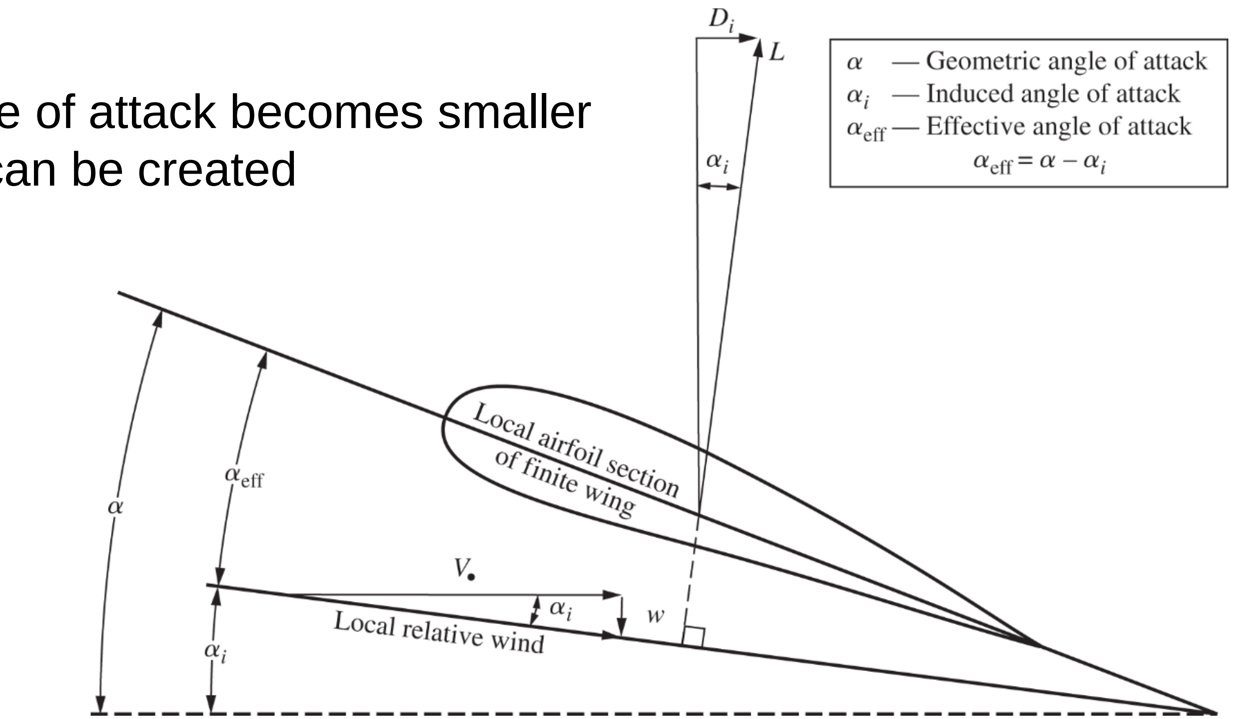
The presence of downwash, and its effect on inclining the local relative wind in the downward direction, has two import effects on the local airfoil section:

- The effective angle of attack becomes smaller
- An induced drag can be created



How is the lift distributed for a finite wing?
How to calculate the induced drag and the total lift?

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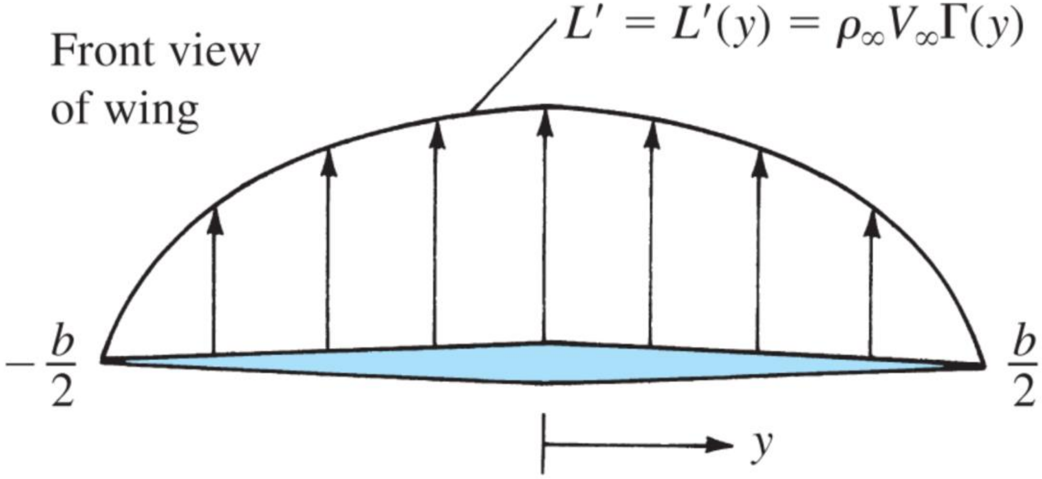
Lift distribution



- Most finite wings have a variable chord
- Many wings are geometrically twisted so that α is different at different spanwise locations
- Many wings have different airfoil sections along the span with different values of $\alpha_{L=0}$
- Zero lift at the tips?

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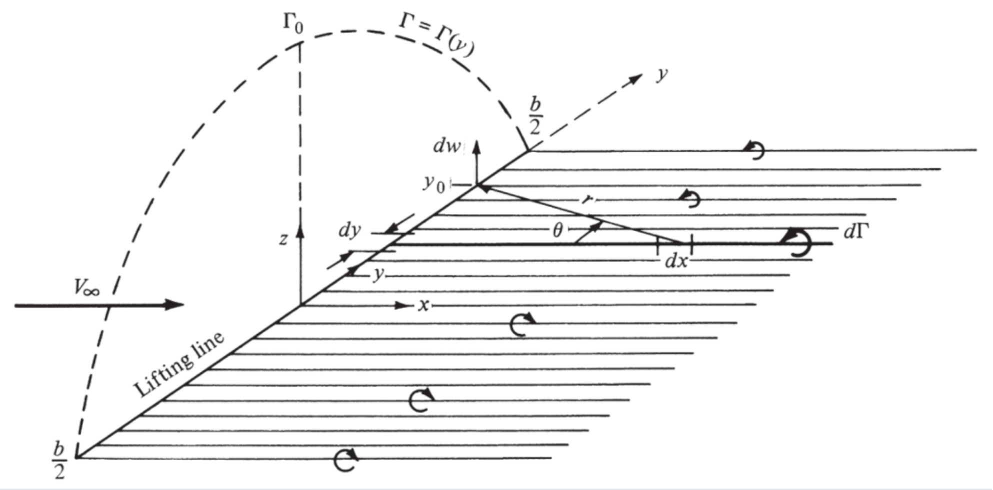
Prandtl's classical lifting-line theory



$$dw = -\frac{(d\Gamma/dy)\,dy}{4\pi(y_0 - y)}$$

$$\boxed{w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)\,dy}{y_0 - y}}$$

$$\boxed{\alpha_i(y_0) = \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)\,dy}{y_0 - y}}$$

Geometric AoA

$$\boxed{\alpha(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0) + \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{(d\Gamma/dy)\,dy}{y_0 - y}}$$

effective angle induced angle

The solution $\Gamma = \Gamma(y_0)$ can be obtained!

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Prandtl's classical lifting-line theory

With the solution $\Gamma = \Gamma(y_0)$

- The lift distribution can be obtained from the Kutta-Joukowski theorem

$$L'(y_0) = \rho_\infty V_\infty \Gamma(y_0)$$

- The total lift can be obtained by integrating over the span

$$L = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y)\, dy$$

- The induced drag can be obtained

$$D_i' = L_i' \sin \alpha_i$$

$$D_i = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y)\, \alpha_i(y)\, dy$$

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Elliptical Lift Distribution

Consider a circulation distribution given by $\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$

$$\Gamma(b/2) = \Gamma(-b/2) = 0.$$

$$L'(y) = \rho_\infty V_\infty \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

$$w(y_0) = \frac{\Gamma_0}{\pi b^2} \int_{-b/2}^{b/2} \frac{y}{\left(1 - 4y^2/b^2\right)^{1/2} (y_0 - y)} dy$$

$$\boxed{w(\theta_0) = -\frac{\Gamma_0}{2b}}$$

Downwash is constant over the span for an elliptical lift distribution

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Elliptical Lift Distribution

$$\alpha_i = -\frac{w}{V_\infty} = \frac{\Gamma_0}{2bV_\infty}$$

Induced angle of attack is also constant over the span for an elliptical lift distribution

Aspect Ratio: $\mathrm{AR} \equiv \frac{b^2}{S}$

$$\boxed{\alpha_i = \frac{C_L}{\pi \mathrm{AR}}}$$

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}}$$

- The induced drag coefficient is directly proportional to the square of the lift coefficient
- The induced drag coefficient is inversely proportional to aspect ratio

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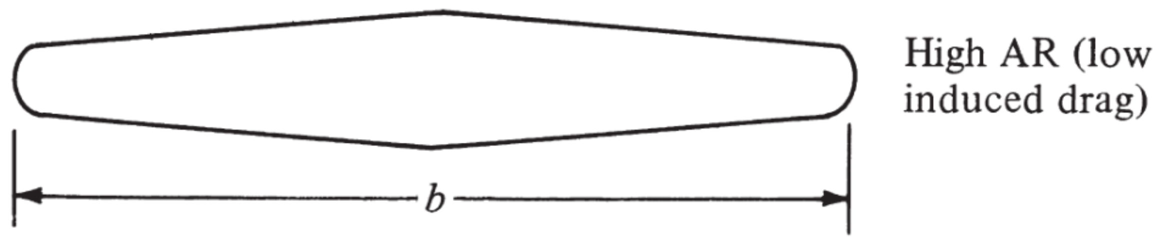
Elliptical Lift Distribution

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}}$$

$$\boxed{\mathrm{AR} = b^2/S}$$



- The AR of the 1903 Wright Flyer is 6
- The AR of conventional subsonic aircraft range typically from 6 to 8
- The AR of Lockheed U-2 high-altitude reconnaissance aircraft is 14.3

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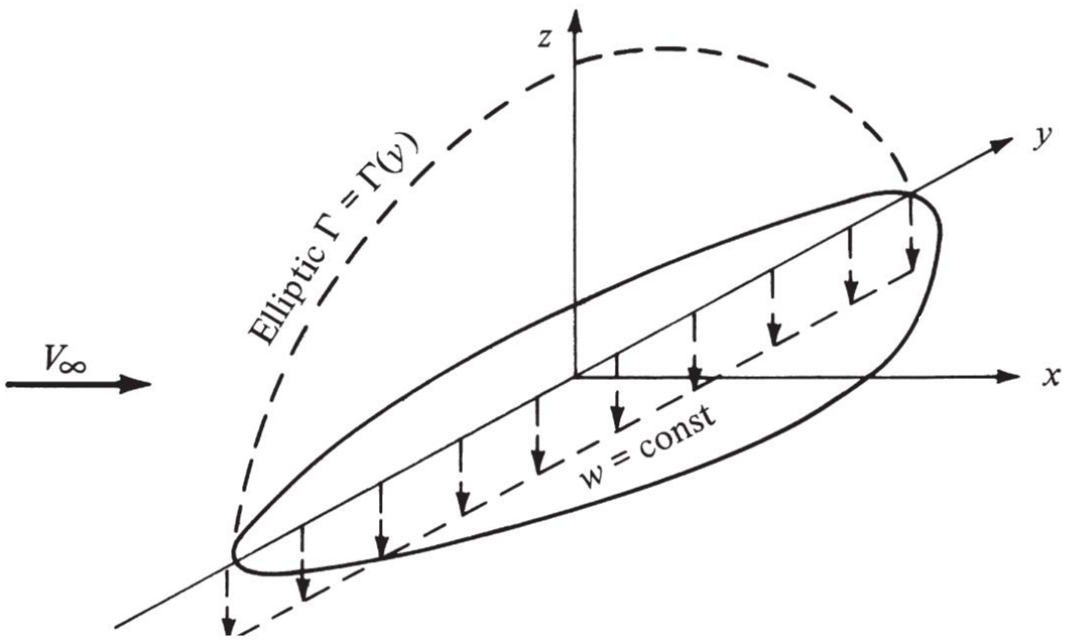
Elliptical Lift Distribution



How to design the wing to produce an elliptical lift distribution?

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Elliptical Lift Distribution

Consider a wing with no geometric twist and no aerodynamic twist.

$$\alpha_i = -\frac{w}{V_\infty} = \frac{\Gamma_0}{2bV_\infty}$$

Induced angle of attack is also constant over the span for an elliptical lift distribution

Hence, $\alpha_{\text{eff}} = \alpha - \alpha_i$ is also constant along the span.

The local section lift coefficient c_l is given by

$$c_l = a_0 \left(\alpha_{\text{eff}} - \alpha_{L=0}\right)$$

where $a_0 = 2\pi$ from the thin airfoil theory

c_l must be constant along the span

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Elliptical Lift Distribution

The lift per unit span is $$L'(y) = q_\infty c c_l$$

The chord length can be solved: $$c(y) = \frac{L'(y)}{q_\infty c_l}$$

where q_∞ and c_l are constant along the span, while $L'(y)$ varies elliptically along the span

For an elliptic lift distribution, the chord must vary elliptically along the span; that is, the wing planform is elliptical

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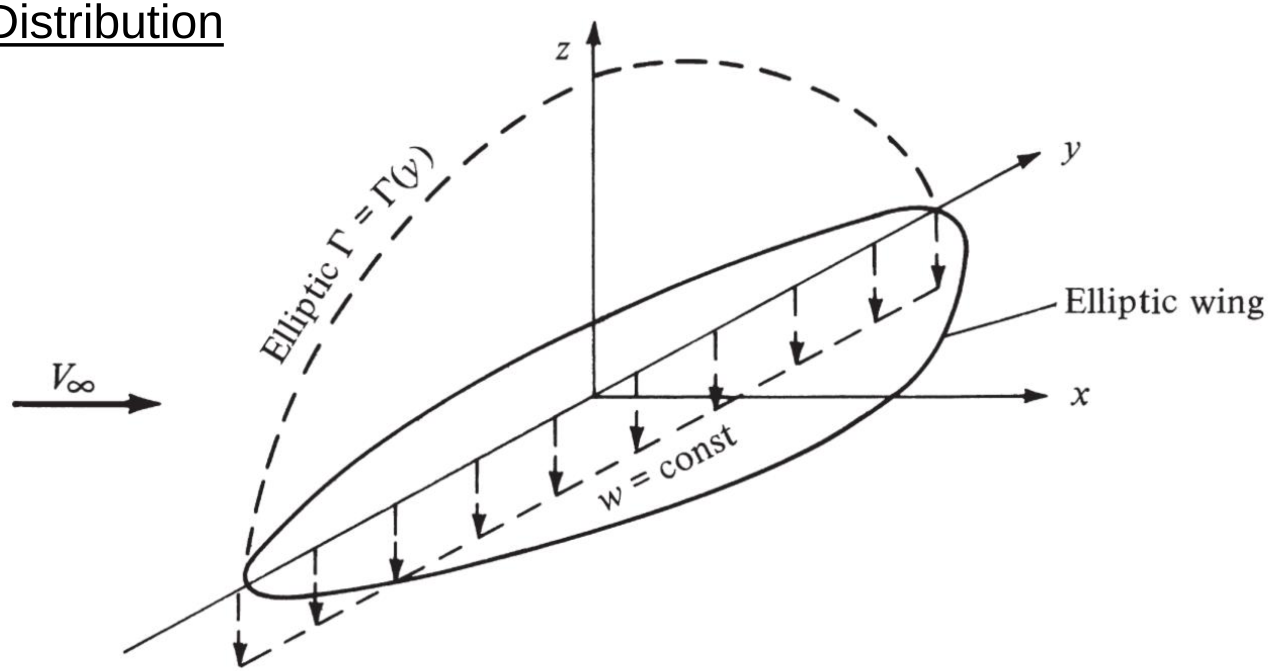
Elliptical Lift Distribution



- The elliptic lift distribution
- The elliptic planform
- Constant downwash

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General Lift Distribution

The circulation distribution along an elliptical finite wing is:

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2} \xrightarrow{y = -\frac{b}{2}\cos\theta} \Gamma(\theta) = \Gamma_0 \sin\theta$$

The circulation distribution along an arbitrary finite wing can be expressed by using a Fourier sine series:

$$\Gamma(\theta) = 2bV_\infty \sum_{1}^{N} A_n \sin n\theta$$

Geometric AoA

$$\boxed{\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_{1}^{N} A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_{1}^{N} nA_n \frac{\sin n\theta_o}{\sin\theta_0}}$$

At a given spanwise location, θ_0 is specified, b, $c(\theta_0)$, and $\alpha_{L=0}(\theta_0)$ are known quantities from the geometry and airfoil section of the finite wing.

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General Lift Distribution

$$\boxed{\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_1^N A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_1^N nA_n \frac{\sin n\theta_o}{\sin\theta_0}}$$

At a given spanwise location, it is algebraic equation with *N* unknowns, $A_1, A_2, \ldots, A_n$

We can choose *N* different spanwise stations to obtain a system of *N* independent algebraic equations with *N* unknowns.

The lift coefficient for the finite wing:

$$C_L = \frac{2}{V_\infty S}\int_{-b/2}^{b/2} \Gamma(y)\, dy = \frac{2b^2}{S}\sum_1^N A_n \int_0^\pi \sin n\theta \sin\theta d\theta$$

$$\int_0^\pi \sin n\theta \sin\theta \, d\theta = \begin{cases} \pi/2 & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases}$$

$$\boxed{C_L = A_1 \pi \frac{b^2}{S} = A_1 \pi \mathrm{AR}}$$

C_L depends only on the leading coefficient of the Fourier series expansion

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General Lift Distribution

The induced drag coefficient:

$$\begin{aligned} C_{D,i} &= \frac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y)\, \alpha_i(y)\; dy \\ &= \frac{2b^2}{S} \int_0^\pi \left(\sum_1^N A_n \sin n\theta \right) \alpha_i(\theta)\; \sin\theta\; d\theta \end{aligned}$$

The induced angle of attack

$$\begin{aligned} \alpha_i(y_0) &= \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)\, dy}{y_0 - y} \\ &= \frac{1}{\pi} \sum_1^N n A_n \int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_0} d\theta \end{aligned} \Longrightarrow \alpha_i(\theta) = \sum_1^N n A_n \frac{\sin n\theta}{\sin\theta}$$

$$\Longrightarrow \quad C_{D,i} = \frac{2b^2}{S} \int_0^\pi \left(\sum_1^N A_n \sin n\theta \right) \left(\sum_1^N n A_n \sin n\theta \right) d\theta$$

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General Lift Distribution

The induced drag coefficient:

$$C_{D,i} = \frac{2b^2}{S} \int_0^{\pi} \left(\sum_1^N A_n \sin n\theta \right) \left(\sum_1^N nA_n \sin n\theta \right) d\theta$$

$$\int_0^{\pi} \sin m\theta \ \sin k\theta = \begin{cases} 0 & \text{for } m \neq k \\ \pi/2 & \text{for } m = k \end{cases} \longrightarrow$$

$$\begin{aligned} C_{D,i} &= \frac{2b^2}{S} \left(\sum_1^N nA_n^2 \right) \frac{\pi}{2} = \pi \text{AR} \sum_1^N nA_n^2 \\ &= \pi \text{AR} \left(A_1^2 + \sum_2^N nA_n^2 \right) \\ &= \pi \text{AR} A_1^2 \left[1 + \sum_2^N n \left(\frac{A_n}{A_1} \right)^2 \right] \end{aligned}$$

$$\delta = \sum\nolimits_2^N n(A_n/A_1)^2 \geq 0 \longrightarrow$$

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi \text{AR}}(1 + \delta)}$$

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General Lift Distribution

Define a span efficiency factor, *e*

$$e = (1 + \delta)^{-1} \leq 1.$$

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1 + \delta)} \quad \Longrightarrow \quad \boxed{C_{D,i} = \frac{C_L^2}{\pi e \mathrm{AR}}}$$

- When $\delta = 0$ and $e = 1$, $\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}}$ Elliptical lift distribution
- The lift distribution which yields minimum induced drag is the ***elliptical lift distribution***

Incompressible Flow over Finite Wings

Supremarine Spitfire, a famous British World War II fighter

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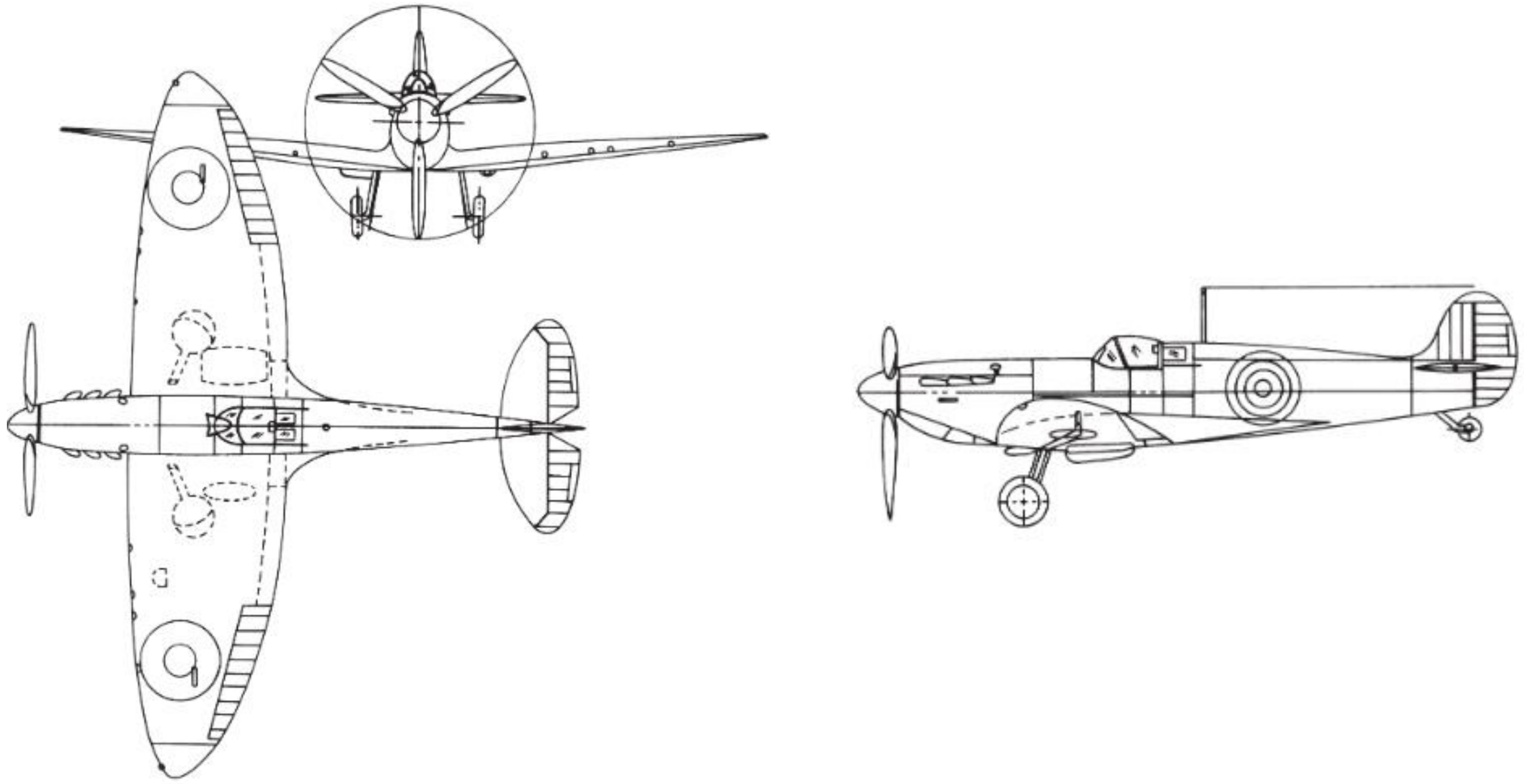

Supremarine Spitfire, a famous British World War II fighter

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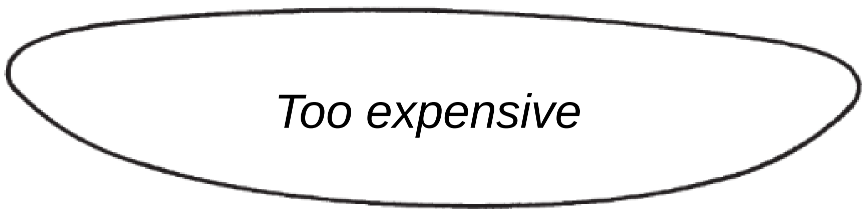
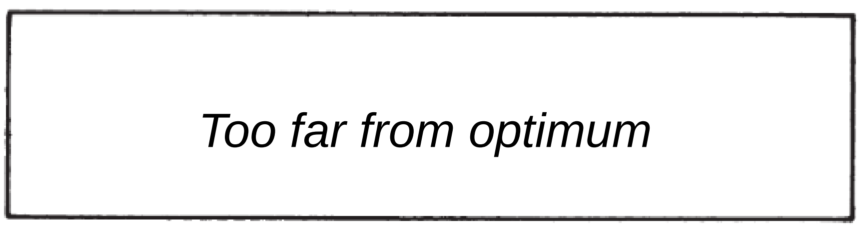
However,…

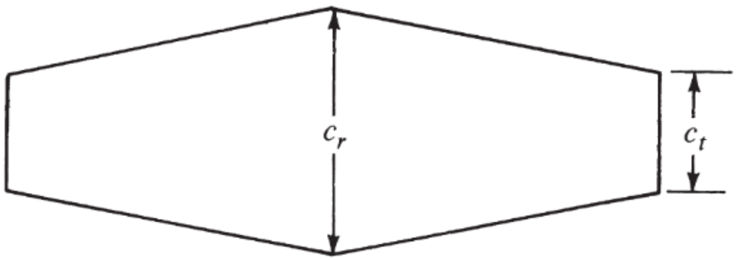
Elliptic planforms are more expensive to manufacture.



Elliptic wing



Rectangular wing



Tapered wing

Taper Ratio: c_t / c_r

- The lift distribution closely approximates the elliptic case
- Most conventional aircraft employ tapered rather than elliptical wing planforms

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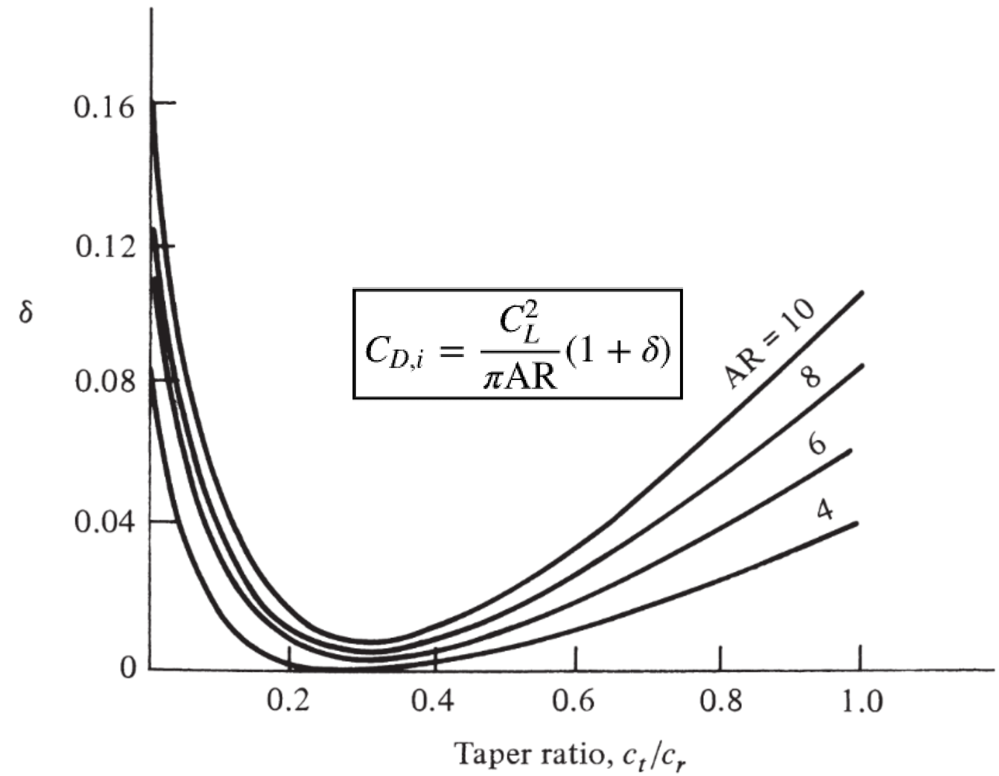
Variation of Induced drag factor as a function of taper ratio

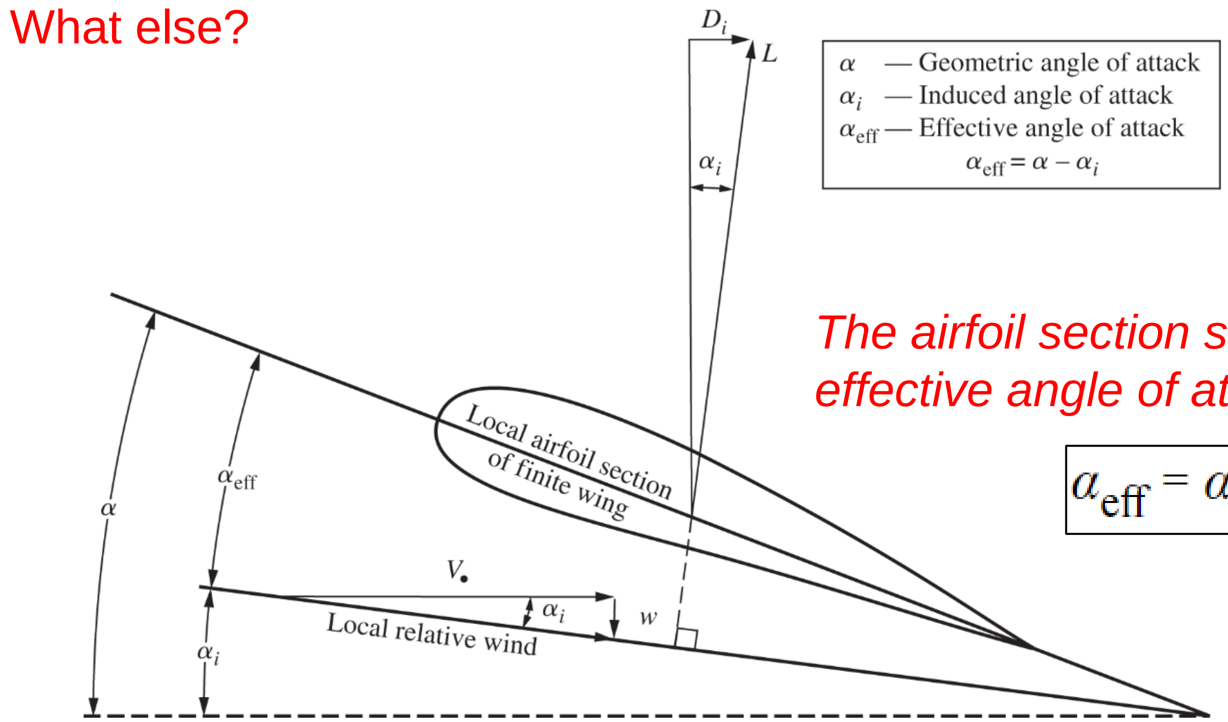


- A tapered wing can be designed with an induced drag coefficient reasonably close to the minimum value.
- AR has a much stronger effect on $C_{D,I}$ than the value of δ

Incompressible Flow over Finite Wings

Differences between airfoil and finite-wing properties

- A finite wing generates induced drag due to downwash effects
- What else?



The airfoil section senses a smaller effective angle of attack

$$\boxed{\alpha_{\text{eff}} = \alpha - \alpha_i.}$$

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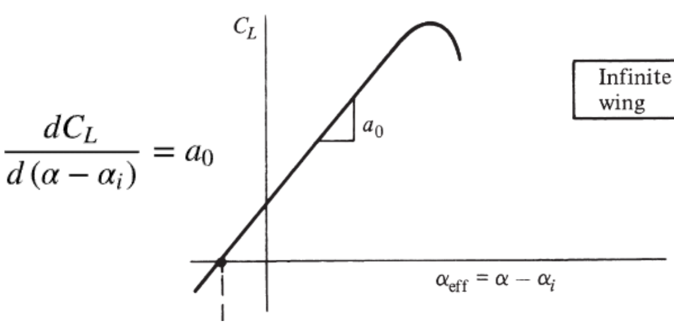
Differences between airfoil and finite-wing properties

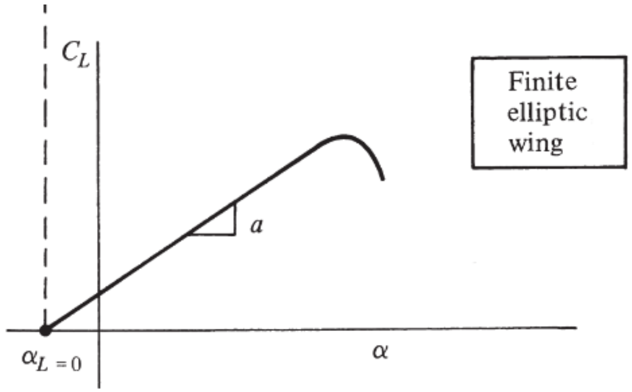
- A finite wing generates induced drag due to downwash effects
- What else?

In practice, we always observe the geometric angle of attack.

Since $\alpha > \alpha_{\text{eff}}$, the observed lift curve is less inclined.

- A finite wing has a reduced lift slope





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Differences between airfoil and finite-wing properties

$$\frac{dC_L}{d(\alpha - \alpha_i)} = a_0 \qquad\qquad C_L = a_0(\alpha - \alpha_i) + \text{const}$$

For an elliptic wing $$C_L = a_0\left(\alpha - \frac{C_L}{\pi \mathrm{AR}}\right) + \text{const}$$

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + a_0/\pi \mathrm{AR}}$$

For a finite wing of general planform

$$a = \frac{a_0}{1 + (a_0/\pi \mathrm{AR})(1+\tau)}$$

where τ is a function of the Fourier coefficients, A_n, ranges between 0.05 and 0.25

$$\boxed{\text{as AR} \to \infty,\ a \to a_0}$$

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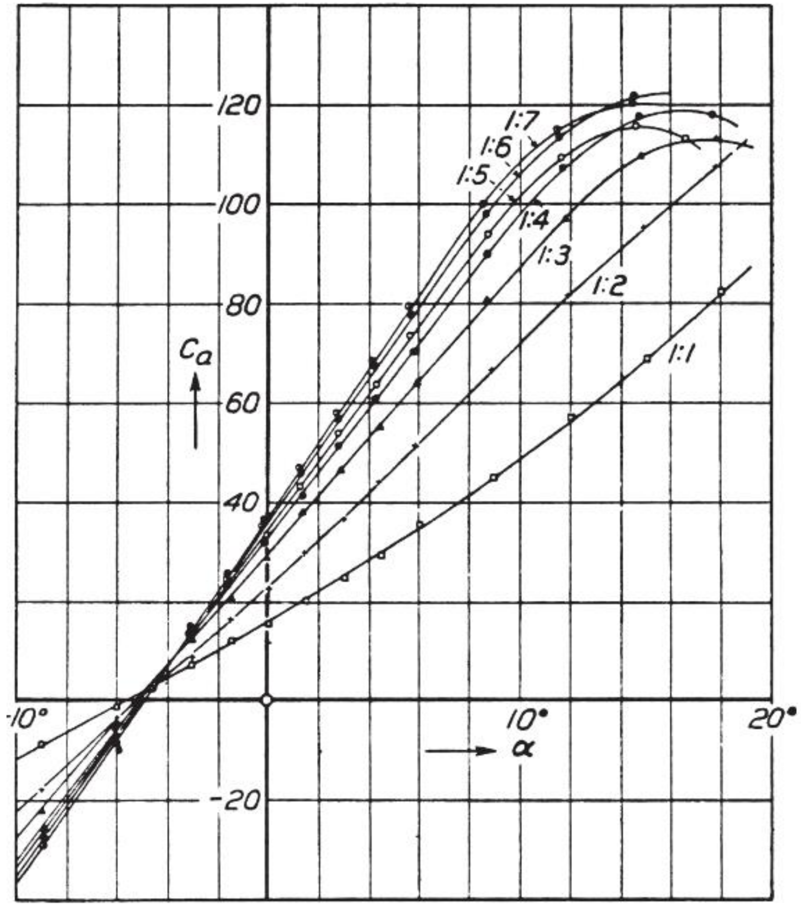
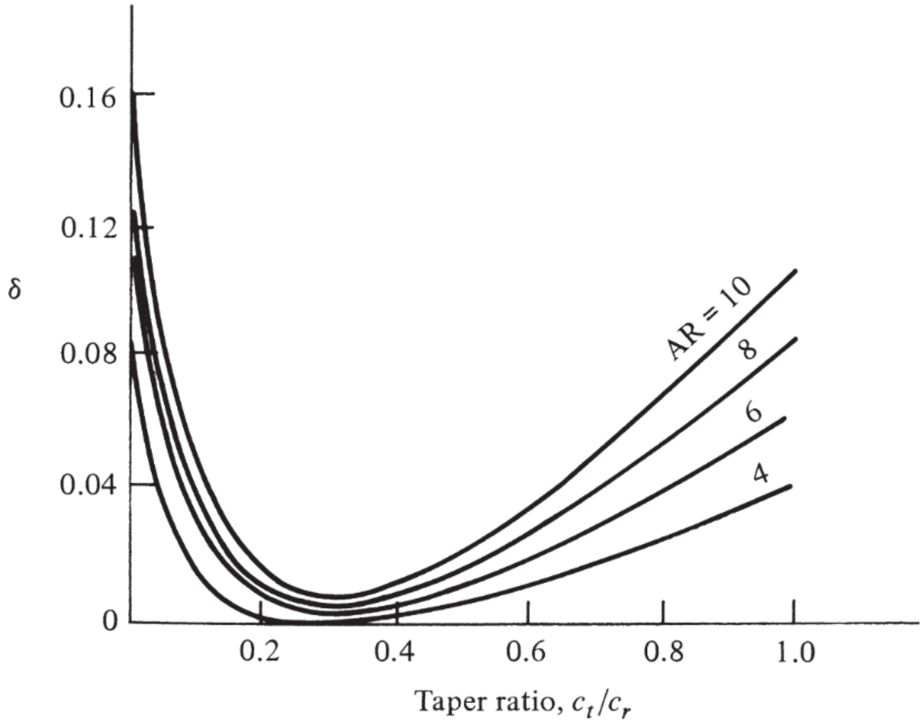
Variation of lift coefficient with angle of attack for a rectangular wing at different aspect ratios



Incompressible Flow over Finite Wings

<u>Example Practice</u>

Consider a finite wing with an aspect ratio of 8 and a taper ratio of 0.8. The airfoil section is thin and symmetric. Calculate the lift and induced drag coefficients for the wing when it is at an angle of attack of 5°. Assume that $\delta = \tau$



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Example Practice

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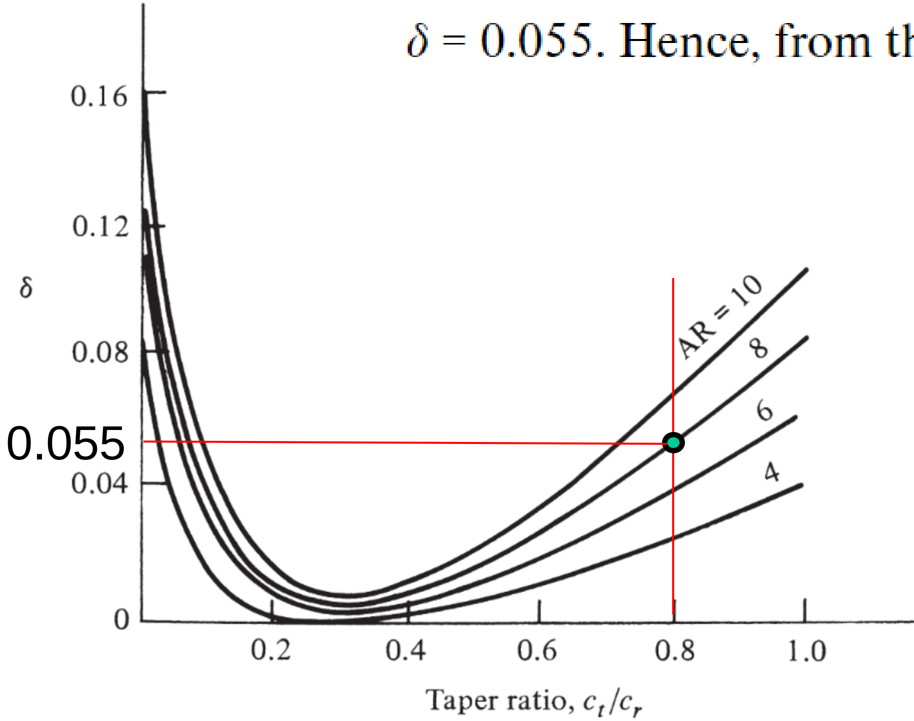

$\delta = 0.055$. Hence, from the stated assumption, τ also equals 0.055.

$$a = \frac{a_0}{1 + a_0/\pi \mathrm{AR}(1+\tau)} == 4.97\ \mathrm{rad}^{-1}$$

$$= 0.0867\ \mathrm{degree}^{-1}$$

Since the airfoil is symmetric, $\alpha_{L=0} = 0°$. Thus,

$$C_L = a\alpha = \left(0.0867\ \mathrm{degree}^{-1}\right)\left(5°\right) = \boxed{0.4335}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1+\delta) = \frac{(0.4335)^2\,(1+0.055)}{8\pi} = \boxed{0.00789}$$