ME 57200 Aerodynamic Design

Lecture #18: Incompressible Flow over Delta Wing

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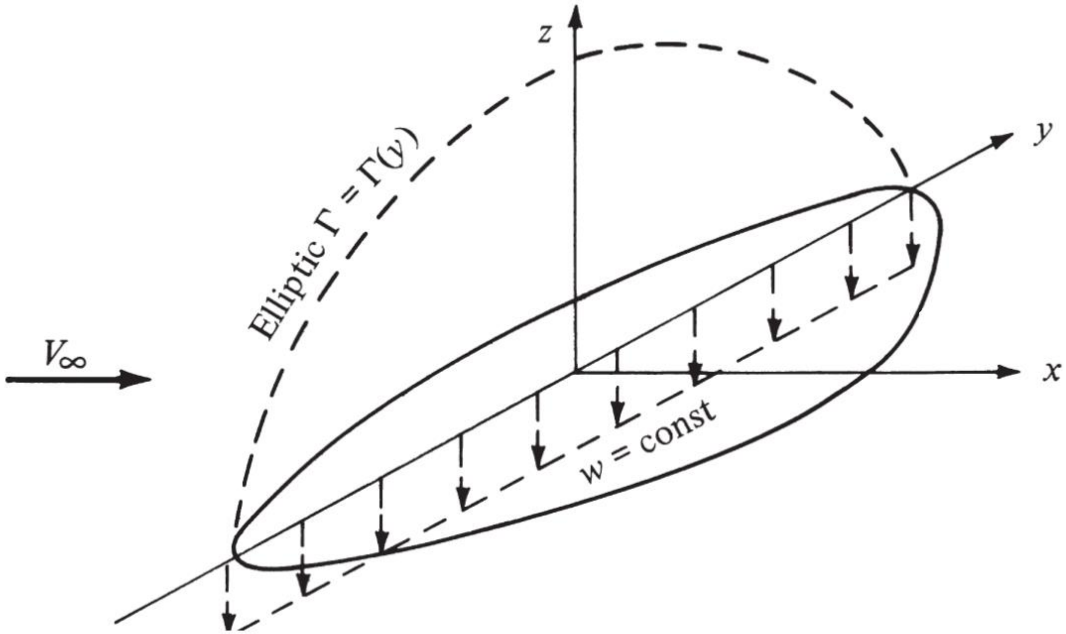
Elliptical Lift Distribution



How to design the wing to produce an elliptical lift distribution?

Incompressible Flow over Finite Wings

Elliptical Lift Distribution

Consider a wing with no geometric twist and no aerodynamic twist.

$$\alpha_i = -\frac{w}{V_\infty} = \frac{\Gamma_0}{2bV_\infty}$$

Induced angle of attack is also constant over the span for an elliptical lift distribution

Hence, $\alpha_{\text{eff}} = \alpha - \alpha_i$ is also constant along the span.

The local section lift coefficient c_l is given by

$$c_l = a_0\left(\alpha_{\text{eff}} - \alpha_{L=0}\right)$$

where $a_0 = 2\pi$ from the thin airfoil theory

c_l must be constant along the span

Incompressible Flow over Finite Wings

Elliptical Lift Distribution

The lift per unit span is $$L'(y) = q_\infty c c_l$$

The chord length can be solved: $$c(y) = \frac{L'(y)}{q_\infty c_l}$$

where q_∞ and c_l are constant along the span, while $L'(y)$ varies elliptically along the span

For an elliptic lift distribution, the chord must vary elliptically along the span; that is, the wing planform is elliptical

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General Lift Distribution

The circulation distribution along an elliptical finite wing is:

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2} \xrightarrow{y = -\frac{b}{2}\cos\theta} \Gamma(\theta) = \Gamma_0 \sin\theta$$

The circulation distribution along an arbitrary finite wing can be expressed by using a Fourier sine series:

$$\Gamma(\theta) = 2bV_\infty \sum_1^N A_n \sin n\theta$$

Geometric AoA

$$\boxed{\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_1^N A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_1^N nA_n \frac{\sin n\theta_o}{\sin\theta_0}}$$

At a given spanwise location, θ_0 is specified, b, $c(\theta_0)$, and $\alpha_{L=0}(\theta_0)$ are known quantities from the geometry and airfoil section of the finite wing.

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General Lift Distribution

$$\boxed{\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_{1}^{N} A_n \sin n\theta_0 + \alpha_{L=0}(\theta_0) + \sum_{1}^{N} nA_n \frac{\sin n\theta_o}{\sin \theta_0}}$$

At a given spanwise location, it is algebraic equation with N unknowns, $A_1, A_2, \ldots, A_n$

We can choose N different spanwise stations to obtain a system of N independent algebraic equations with N unknowns.

The lift coefficient for the finite wing:

$$C_L = \frac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y)\, dy = \frac{2b^2}{S} \sum_{1}^{N} A_n \int_0^\pi \sin n\theta \sin\theta d\theta$$

$$\int_0^\pi \sin n\theta \sin\theta \, d\theta = \begin{cases} \pi/2 & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases} \longrightarrow \boxed{C_L = A_1 \pi \frac{b^2}{S} = A_1 \pi \mathrm{AR}}$$

C_L depends only on the leading coefficient of the Fourier series expansion

Incompressible Flow over Finite Wings

General Lift Distribution

The induced drag coefficient:

$$C_{D,i} = \frac{2b^2}{S}\int_0^{\pi}\left(\sum_1^N A_n \sin n\theta\right)\left(\sum_1^N nA_n \sin n\theta\right)d\theta$$

$$\int_0^{\pi} \sin m\theta \ \sin k\theta = \begin{cases} 0 & \text{for } m \neq k \\ \pi/2 & \text{for } m = k \end{cases} \longrightarrow$$

$$\begin{aligned} C_{D,i} &= \frac{2b^2}{S}\left(\sum_1^N nA_n^2\right)\frac{\pi}{2} = \pi \mathrm{AR}\sum_1^N nA_n^2 \\ &= \pi \mathrm{AR}\left(A_1^2 + \sum_2^N nA_n^2\right) \\ &= \pi \mathrm{AR} A_1^2\left[1 + \sum_2^N n\left(\frac{A_n}{A_1}\right)^2\right] \end{aligned}$$

$$\delta = \sum_2^N n(A_n/A_1)^2 \geq 0 \longrightarrow \boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1+\delta)}$$

Incompressible Flow over Finite Wings

General Lift Distribution

Define a span efficiency factor, *e*

$$e = (1 + \delta)^{-1} \leq 1.$$

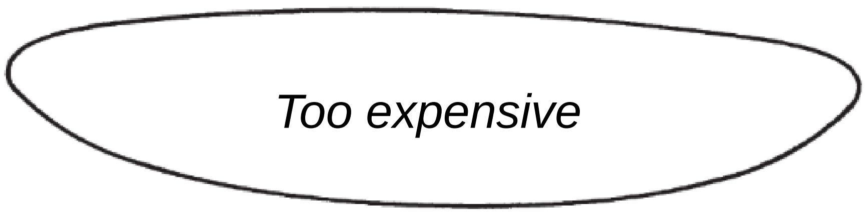
$$\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1 + \delta)} \quad \Longrightarrow \quad \boxed{C_{D,i} = \frac{C_L^2}{\pi e \mathrm{AR}}}$$

- When $\delta = 0$ and $e = 1$, $\boxed{C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}}$ Elliptical lift distribution
- The lift distribution which yields minimum induced drag is the ***elliptical lift distribution***

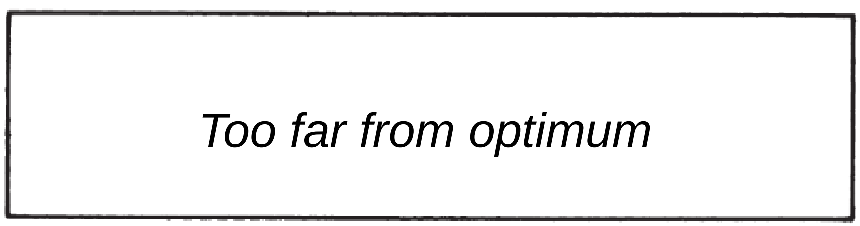
Incompressible Flow over Finite Wings

However,…

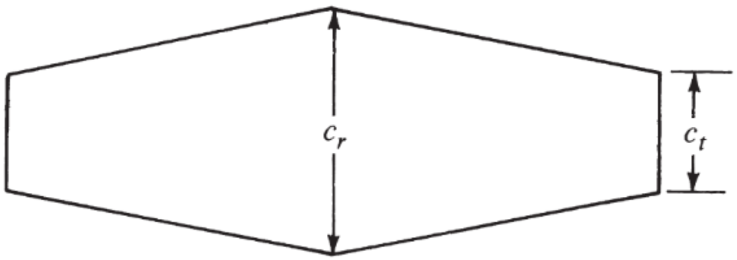
Elliptic planforms are more expensive to manufacture.



Elliptic wing



Rectangular wing



Tapered wing

Taper Ratio: c_t / c_r

- The lift distribution closely approximates the elliptic case
- Most conventional aircraft employ tapered rather than elliptical wing planforms

Incompressible Flow over Finite Wings

Differences between airfoil and finite-wing properties

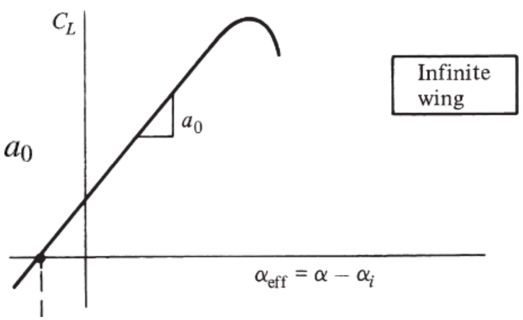
- A finite wing generates induced drag due to downwash effects

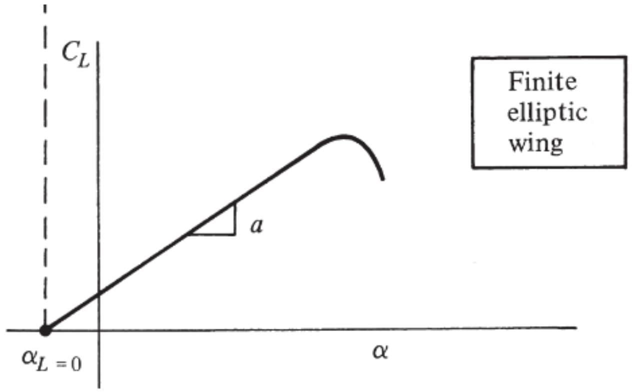
In practice, we always observe the geometric angle of attack.

Since $\alpha > \alpha_{\text{eff}}$, the observed lift curve is less inclined.

- A finite wing has a reduced lift slope

$$\frac{dC_L}{d(\alpha - \alpha_i)} = a_0$$





Incompressible Flow over Finite Wings

Differences between airfoil and finite-wing properties

$$\frac{dC_L}{d(\alpha - \alpha_i)} = a_0 \qquad C_L = a_0(\alpha - \alpha_i) + \text{const}$$

For an elliptic wing $$C_L = a_0\left(\alpha - \frac{C_L}{\pi \text{AR}}\right) + \text{const}$$

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + a_0/\pi \text{AR}}$$

For a finite wing of general planform

$$a = \frac{a_0}{1 + (a_0/\pi \text{AR})(1+\tau)}$$

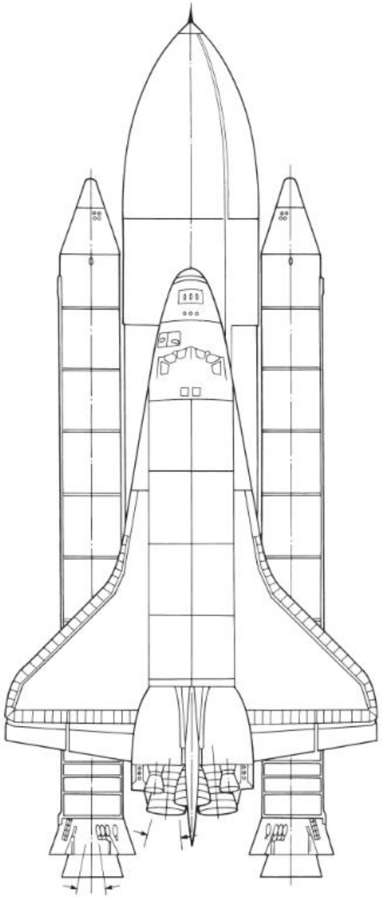
where τ is a function of the Fourier coefficients, A_n, ranges between 0.05 and 0.25

$$\boxed{\text{as AR} \to \infty,\ a \to a_0}$$

Incompressible Flow over Delta Wings

The Delta Wing

The Convair F-102A



The Space Shuttle

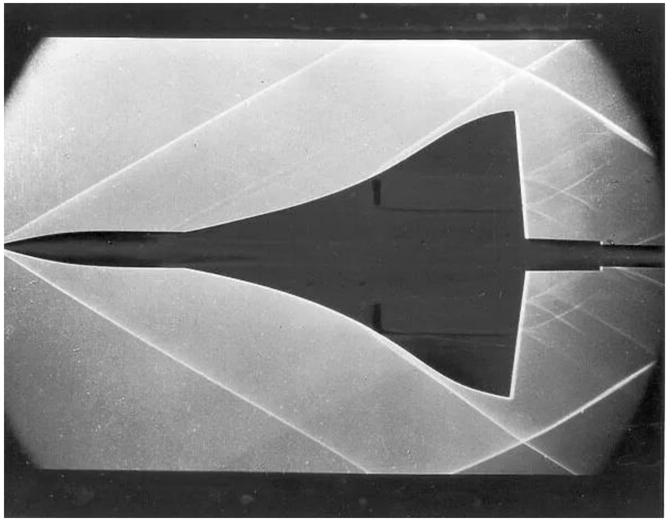
Incompressible Flow over Delta Wings

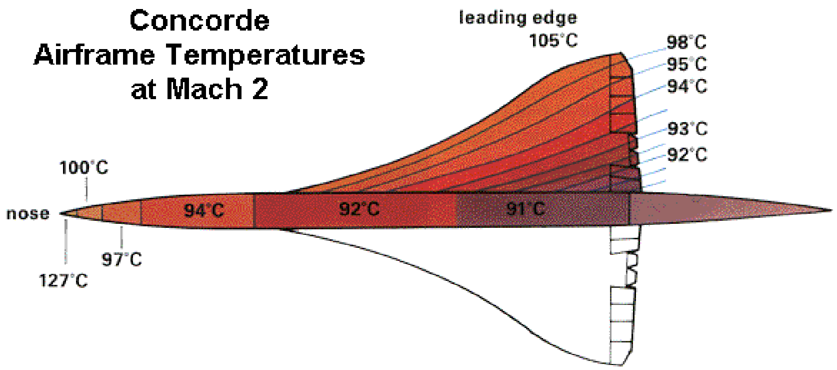
<u>The Delta Wing – Designed for Supersonic Flows</u>

- Delta wings produce significantly less wave drag due to their low wing thickness, which means that it produces less relative drag than a normal wing while supersonic.
- The large root chord gives the delta wing a high internal fuel volume even though the wings relative thickness is low.
- A delta wing is naturally stable in pitch, therefore it does not require a separate tail surface.
- Deltas can operate at extremely high angles of attack, which give it unparalleled amounts of maneuverability at high speeds.

Flow over Delta Wings

High-speed flow





- Energy transformations and temperature changes are important considerations
  - Science of Thermodynamics

Incompressible Flow over Delta Wings

The Delta Wing

- Designed specifically for high-speed airplanes
- Why are we discussing it in this topic, which deals with low-speed incompressible flow over finite wings?

Incompressible Flow over Delta Wings

The Delta Wing

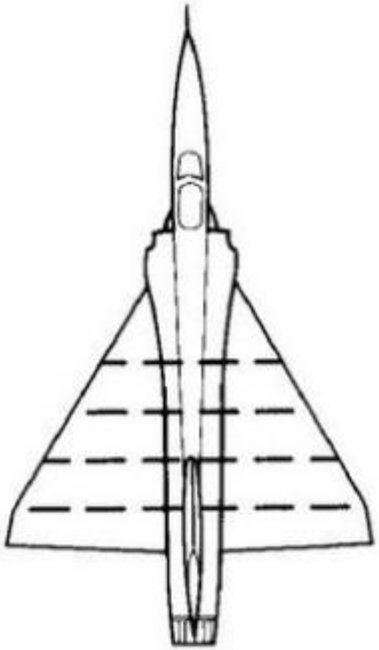
- Takeoff and landing at low speeds.
- In most cases, these aircraft spend the most of their flight time at subsonic speeds.
- Early work by Alexander Lippisch in Germany during the 1930s

Incompressible Flow over Delta Wings

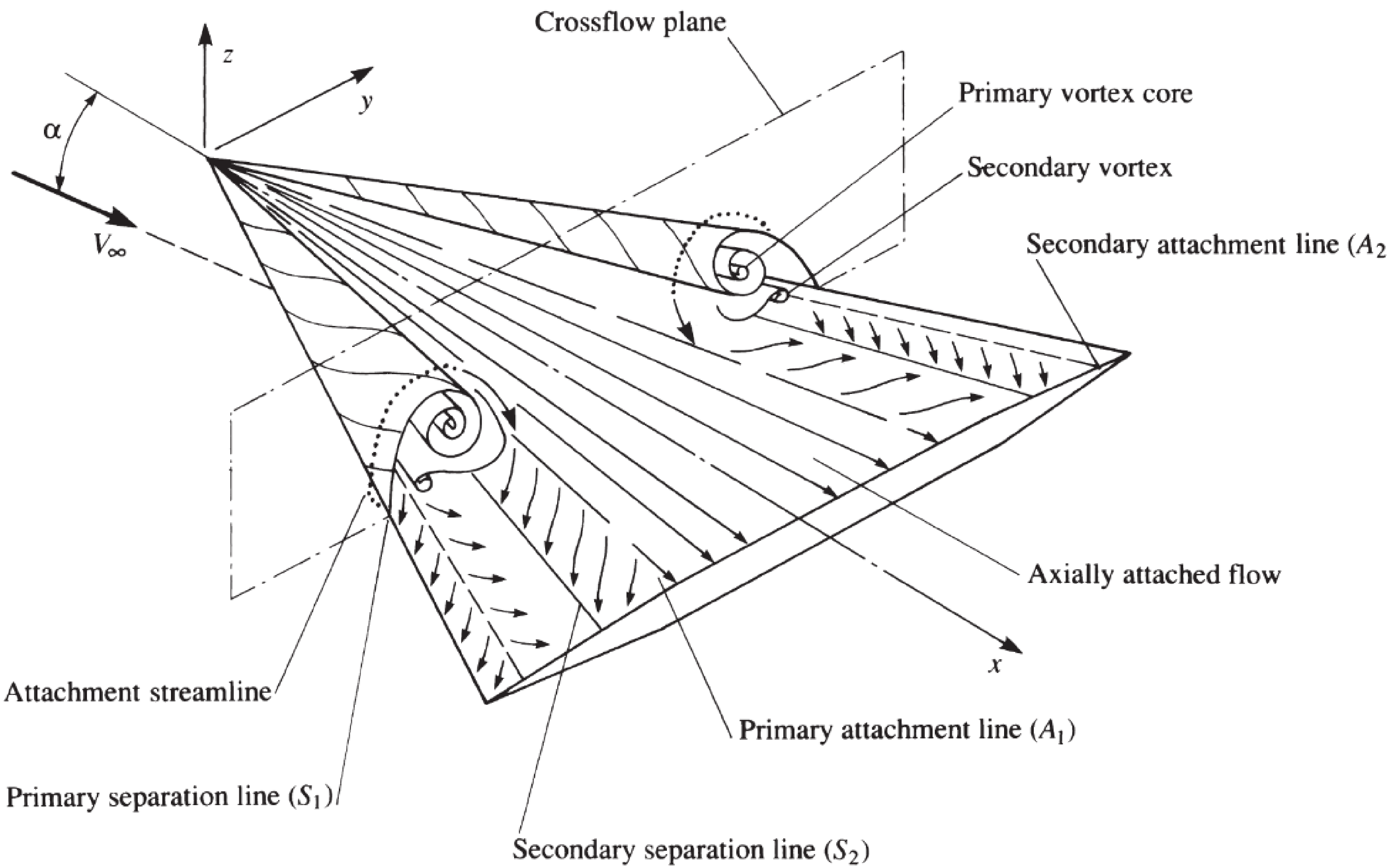
Subsonic Flow Pattern over a Delta Wing

Discussion:

- Flow patterns over a delta wing?



Incompressible Flow over Delta Wings

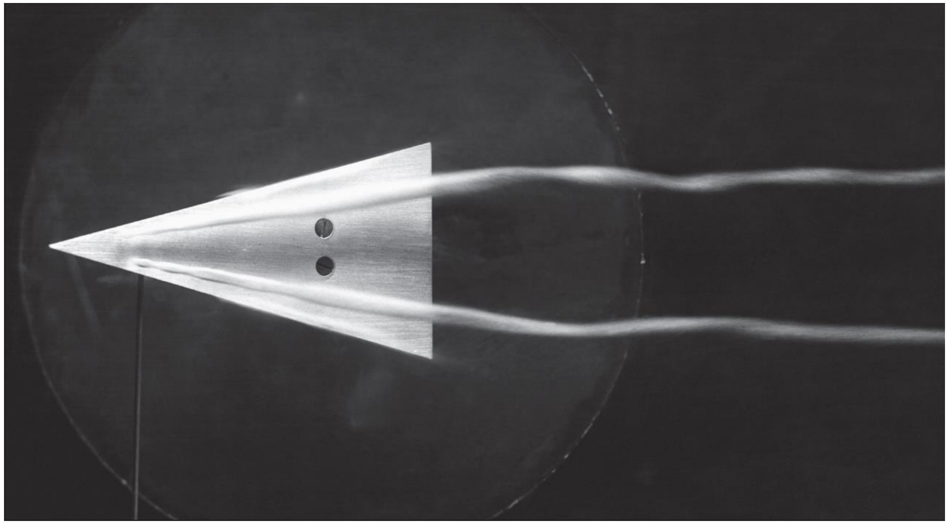


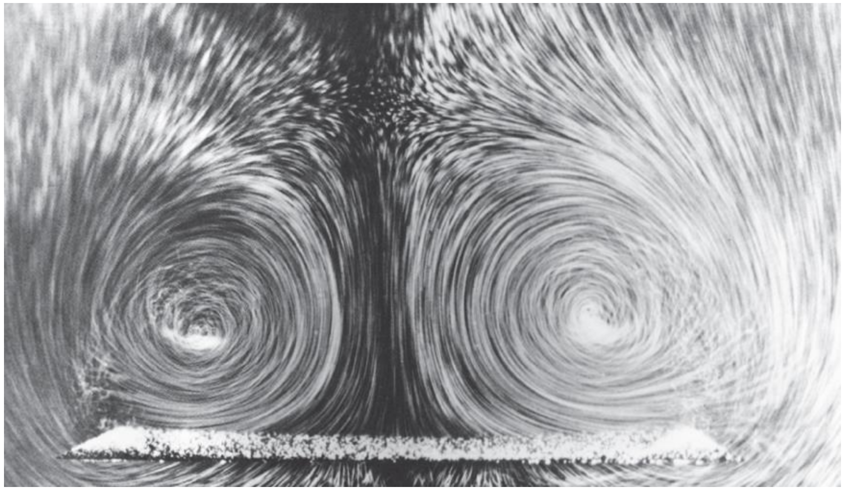
Incompressible Flow over Delta Wings

Two vortex patterns at the highly swept leading edges.

- The pressure on the bottom surface is higher than that on the top surface
- The flow curl from the bottom to the top at the leading edges
- Since the leading edge is sharp, the flow will separate along the entire length.
- The separated flow curls into a primary vortex above the wing just inboard of each leading edge.
- A secondary vortex is formed underneath the primary vortex.
- Inboard of the leading-edge vortices, the surface streamlines are attached.

Incompressible Flow over Delta Wings



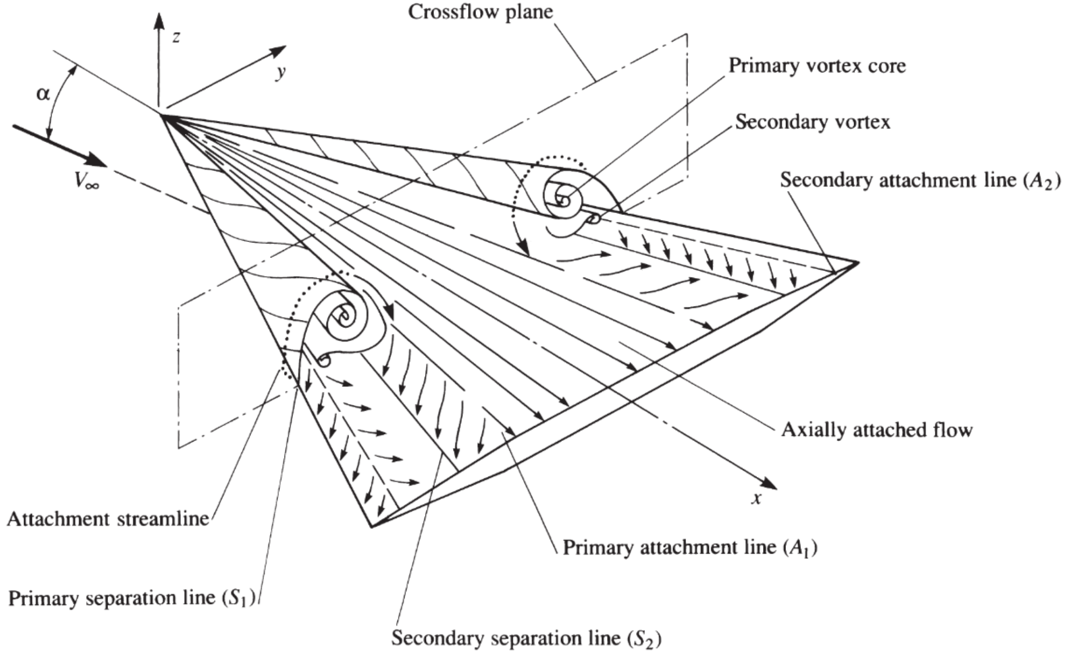


Incompressible Flow over Delta Wings

Subsonic Flow Pattern over a Delta Wing

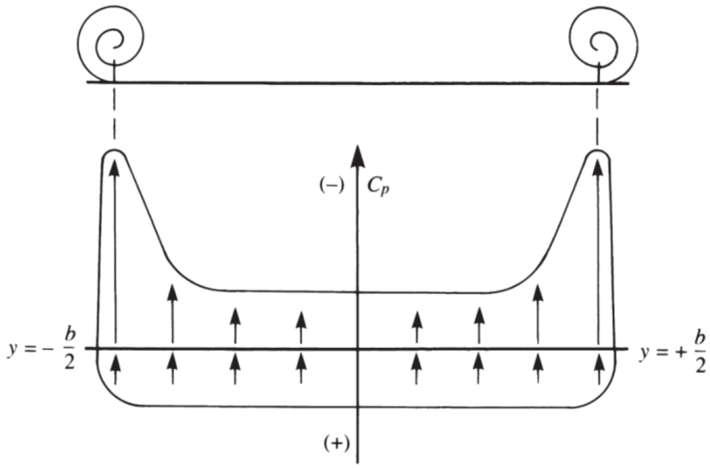
Discussion:

- Benefits or Disadvantages of Delta wing in subsonic flows?



Incompressible Flow over Delta Wings

Subsonic Flow Pattern over a Delta Wing



- Near the leading edges, the static pressure drops considerably, creating a strong "suction" on the top surface, which enhances the lift
- The large wing area causes more viscous drag for the same amount of lift compared to a high aspect ratio wing.
- Vortex lift at high angle of attack produces considerably more drag than a comparatively sized conventional wing.
- Highly unstable due to unsymmetrical vortices at high angle of attack.


DELTA
WING
FAST

Incompressible Flow over Finite Wings

The Delta Wing – HOMEWORK #7, due on Friday, 4/19

- Write a 2-page essay about the aerodynamics of the delta wing
    - The history of the delta wing
    - its design principles
    - its performance characteristics
        - High-speed vs. low-speed
    - …