ME 57200 Aerodynamic Design

Lecture #14: Incompressible Flow over Airfoils

Dr. Yang Liu

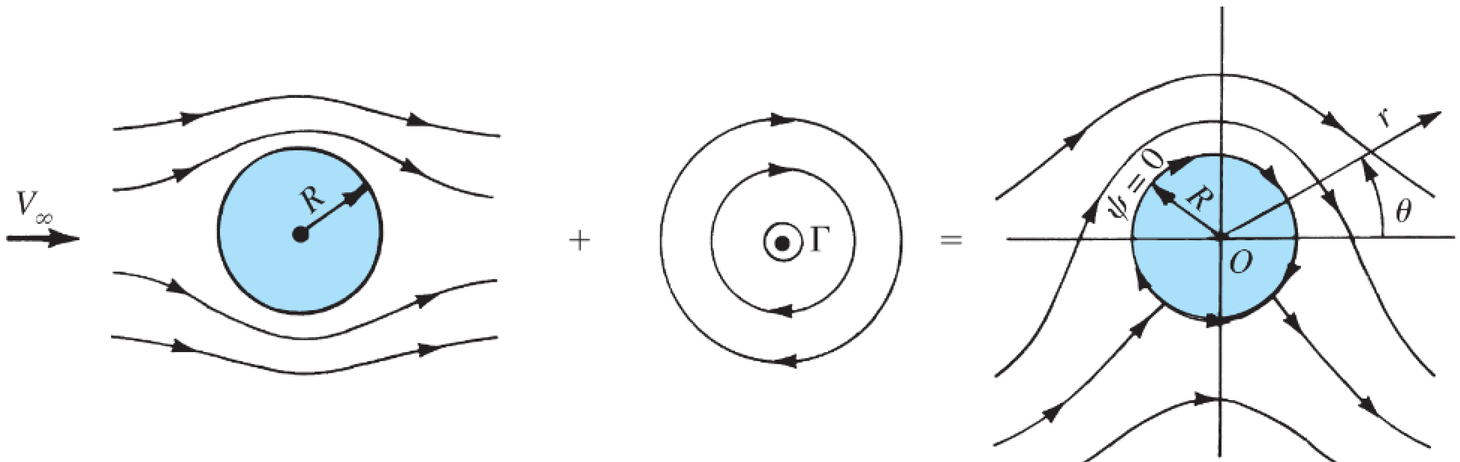
Steinman 253

Tel: 212-650-7346

Email: yliu7@ccny.cuny.edu

Lifting Flow over a Cylinder

How to synthesize a lifting flow over a cylinder with the elementary flows?



Nonlifting flow over a cylinder

Vortex of strength Γ

Lifting flow over a cylinder

What is the stream function of the synthesized flow?

$$\boxed{\psi = (V_\infty r \sin\theta)\left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi} \ln \frac{r}{R}}$$

Lifting Flow over a Cylinder

$$\boxed{\psi = (V_\infty r \sin\theta)\left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi}\ln\frac{r}{R}}$$

What are the velocities in the flow field?

$$V_r = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta$$

$$V_\theta = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta - \frac{\Gamma}{2\pi r}$$

How to locate the stagnation points in the flow?

$$V_r = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta = 0$$

$$V_\theta = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta - \frac{\Gamma}{2\pi r} = 0$$

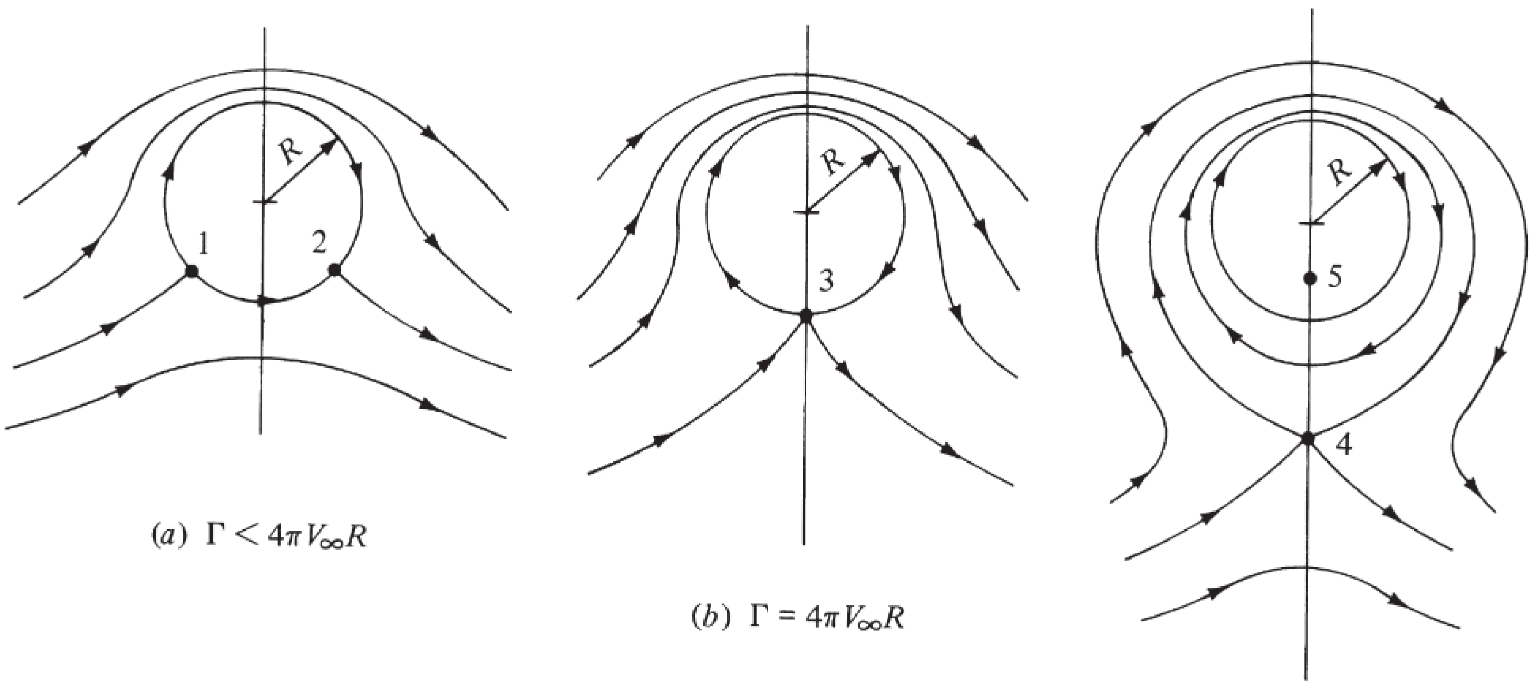
Lifting Flow over a Cylinder

$$\theta = \arcsin\left(-\frac{\Gamma}{4\pi V_\infty R}\right)$$

$$r = R.$$

$$\theta = \pi/2 \text{ or } -\pi/2.$$

$$r = \frac{\Gamma}{4\pi V_\infty} \pm \sqrt{\left(\frac{\Gamma}{4\pi V_\infty}\right)^2 - R^2}$$



(a) $\Gamma < 4\pi V_\infty R$

(b) $\Gamma = 4\pi V_\infty R$

(c) $\Gamma > 4\pi V_\infty R$

Lifting Flow over a Cylinder

What are the velocities at the cylinder surface?

$r = R$: $$V = V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}$$

What are the pressures at the cylinder surface?

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(-2\sin\theta - \frac{\Gamma}{2\pi R V_\infty}\right)^2$$

$$C_p = 1 - \left[4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right]$$

Lifting Flow over a Cylinder

How to obtain the aerodynamic drag coefficient?

$$c_d = c_a = \frac{1}{c}\int_{\mathrm{LE}}^{\mathrm{TE}} (C_{p,u} - C_{p,l})\, dy$$

$$c_d = -\frac{1}{2}\int_0^{\pi} C_p \cos\theta\, d\theta - \frac{1}{2}\int_{\pi}^{2\pi} C_p \cos\theta\, d\theta$$

$$c_d = -\frac{1}{2}\int_0^{2\pi} C_p \cos\theta\, d\theta$$

$$\boxed{c_d = 0}$$

Lifting Flow over a Cylinder

How to obtain the aerodynamic lift coefficient?

$$c_l = c_n = \frac{1}{c}\int_0^c C_{p,l}\,dx - \frac{1}{c}\int_0^c C_{p,u}\,dx$$

$$c_l = -\frac{1}{2}\int_\pi^{2\pi} C_{p,l}\sin\theta\,d\theta + \frac{1}{2}\int_\pi^0 C_{p,u}\sin\theta\,d\theta$$

$$c_l = -\frac{1}{2}\int_0^{2\pi} C_p\sin\theta\,d\theta$$

$$c_l = \frac{\Gamma}{RV_\infty} \qquad L' = q_\infty S c_l = \frac{1}{2}\rho_\infty V_\infty^2 S c_l$$

$$L' = \frac{1}{2}\rho_\infty V_\infty^2 2R \frac{\Gamma}{RV_\infty} \qquad \boxed{L' = \rho_\infty V_\infty \Gamma}$$

Lifting Flow over a Cylinder

Example: Consider the lifting flow over a circular cylinder with a diameter of 0.5 m. The freestream velocity is 25 m/s, and the maximum velocity on the surface of the cylinder is 75 m/s. The freestream conditions are those for a standard altitude of 3 km. Calculate the lift per unit span on the cylinder.

$\rho = 0.90926 \text{ kg/m}^3.$

The maximum velocity occurs at the top of the cylinder, where $\theta = 90°$

Lifting Flow over a Cylinder

Example: Consider the lifting flow over a circular cylinder with a diameter of 0.5 m. The freestream velocity is 25 m/s, and the maximum velocity on the surface of the cylinder is 75 m/s. The freestream conditions are those for a standard altitude of 3 km. Calculate the lift per unit span on the cylinder.

$\rho = 0.90926 \text{ kg/m}^3.$

The maximum velocity occurs at the top of the cylinder, where $\theta = 90°$

$$V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R} \qquad V_\theta = -2V_\infty - \frac{\Gamma}{2\pi R} \qquad \Gamma = -2\pi R(V_\theta + 2V_\infty)$$

Γ is positive in the clockwise direction, and V_θ is negative in the clockwise direction

$$V_\theta = -75\text{m/s}$$

$$\Gamma = -2\pi R(V_\theta + 2V_\infty) = -2\pi(0.25)[-75 + 2(25)]$$

$$\Gamma = -2\pi(0.25)(-25) = 39.27 \text{ m}^2/\text{s}$$

$$L' = \rho_\infty V_\infty \Gamma$$

$$L' = (0.90926)(25)(39.27) = \boxed{892.7 \text{ N/m}}$$

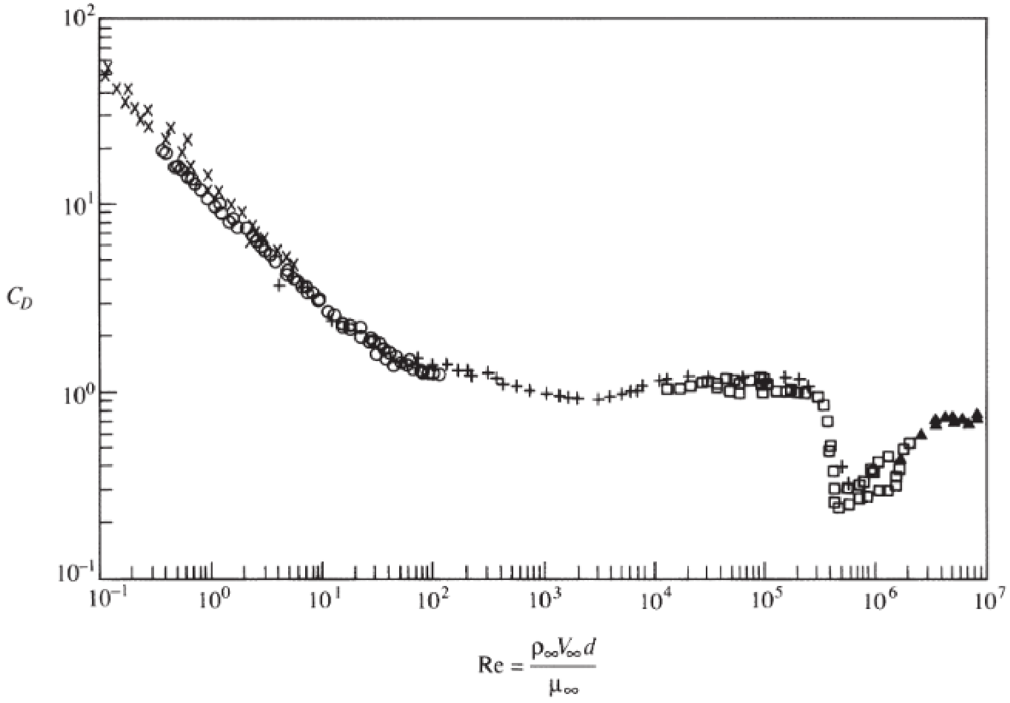
Flow over a Cylinder – the Real Case

Consider the real flow over a circular cylinder.

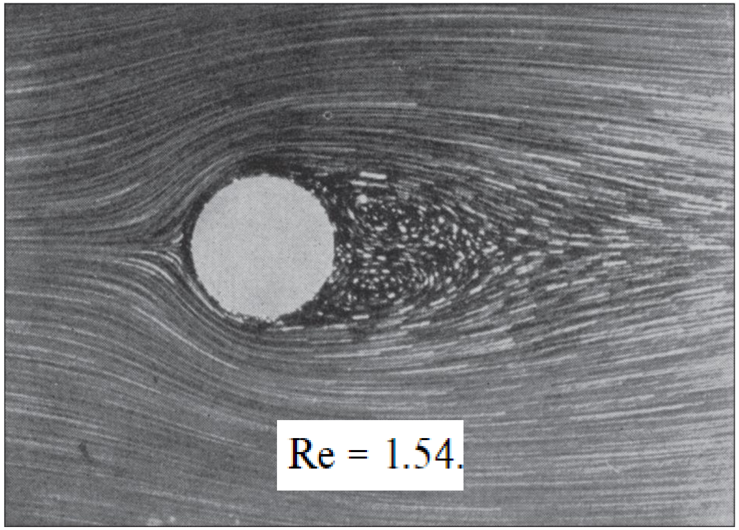
Would the flow be different?

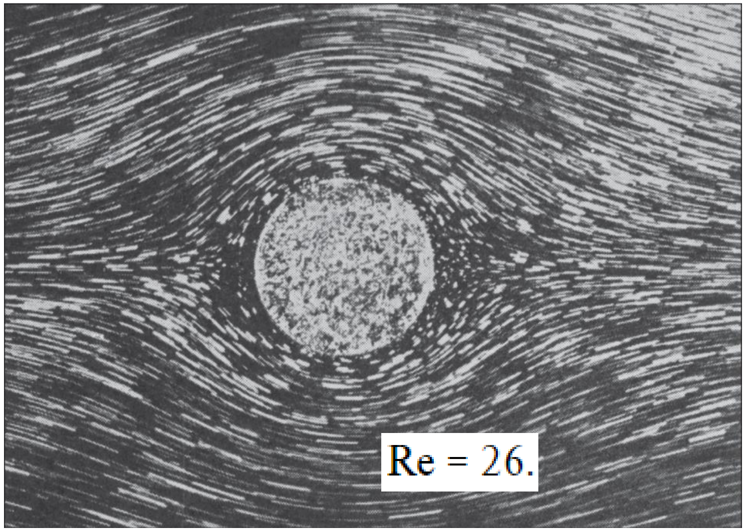
Where are the differences from?

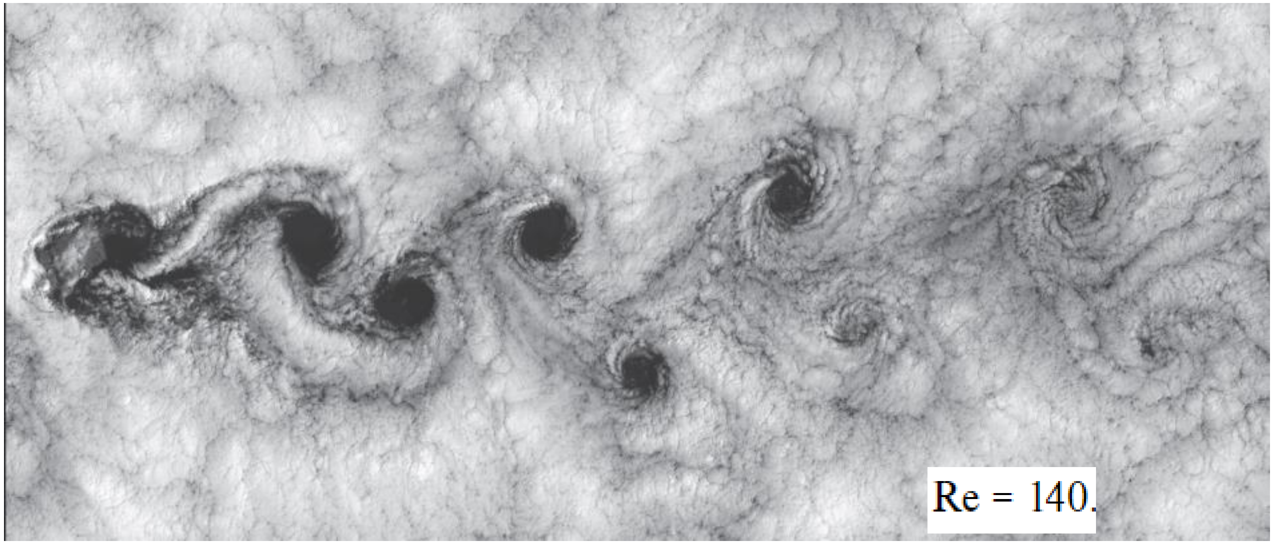
Friction



Flow over a Cylinder – the Real Case

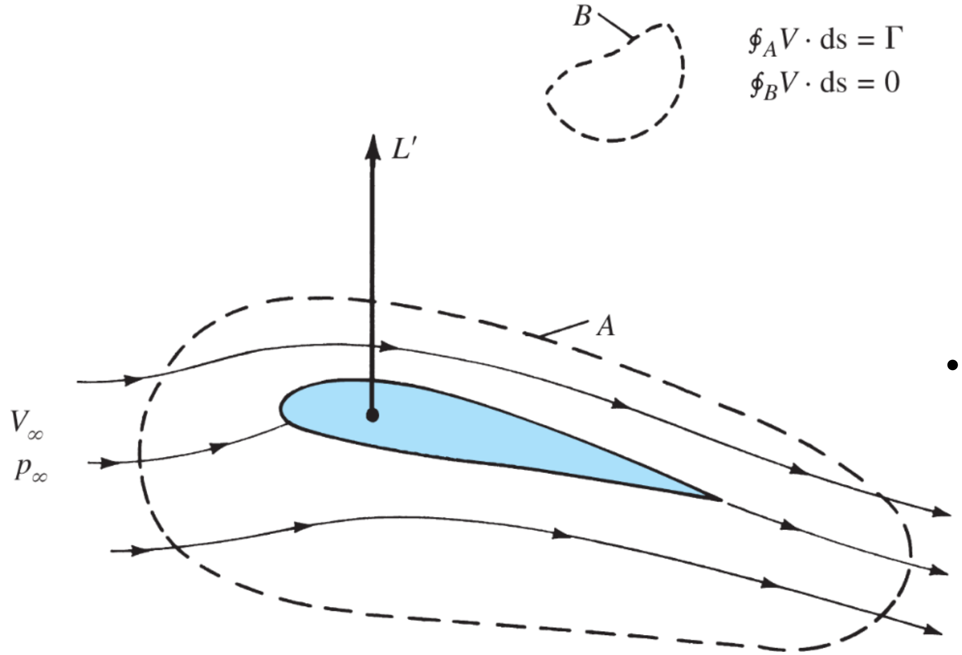






The Kutta-Joukowski Theorem

Consider the incompressible flow over an airfoil section. Let curve A be a curve in the flow enclosing the airfoil.



- If the airfoil is producing lift, the velocity field around the airfoil will be such that the line integral of velocity around A will be finite, that is, the circulation is finite

$$\Gamma \equiv \oint_A \mathbf{V} \cdot \mathbf{ds}$$

- The lift per unit span on the airfoil will be give by *Kutta-Joukowski theorem*

$$\boxed{L' = \rho_\infty V_\infty \Gamma}$$

The Kutta-Joukowski Theorem

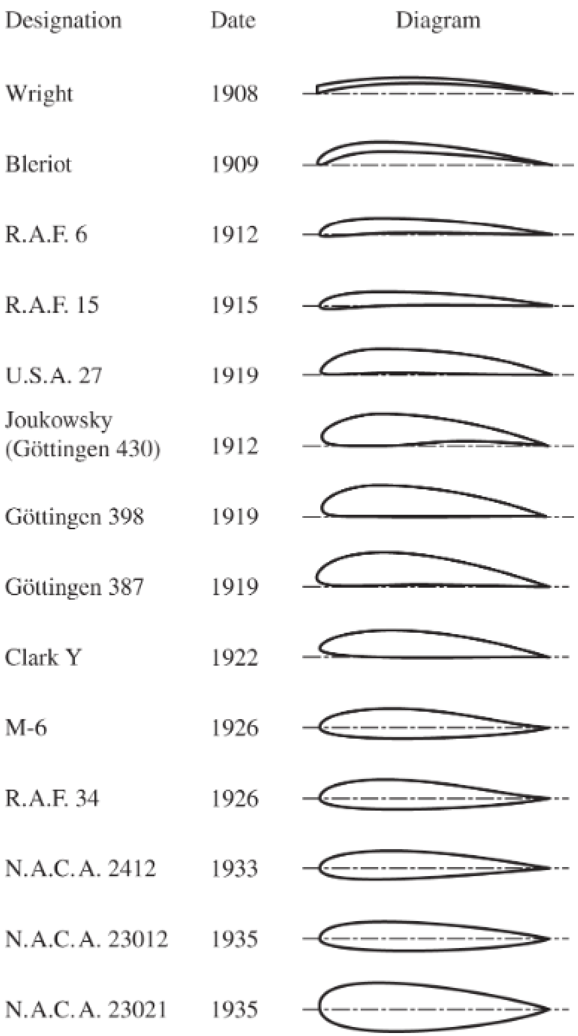
Kutta-Joukowski theorem: Lift per unit span on a two-dimensional body is directly proportional to the circulation around the body.

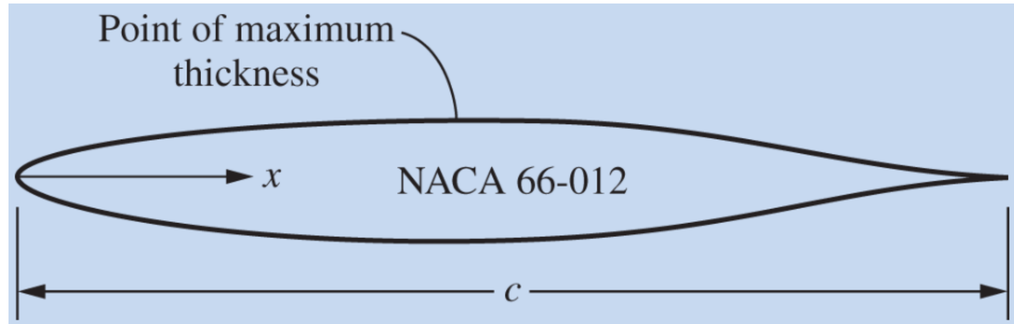
$$\boxed{L' = \rho_\infty V_\infty \Gamma}$$

- The value of Γ must be evaluated around a closed curve that encloses the body.
- The curve can be arbitrary, but it must have the body inside it.

How can we calculate the circulation for a given body (airfoil) in a given incompressible, inviscid flow?

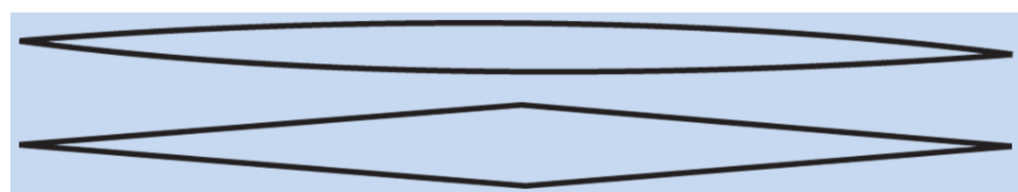
Incompressible Flow over Airfoils





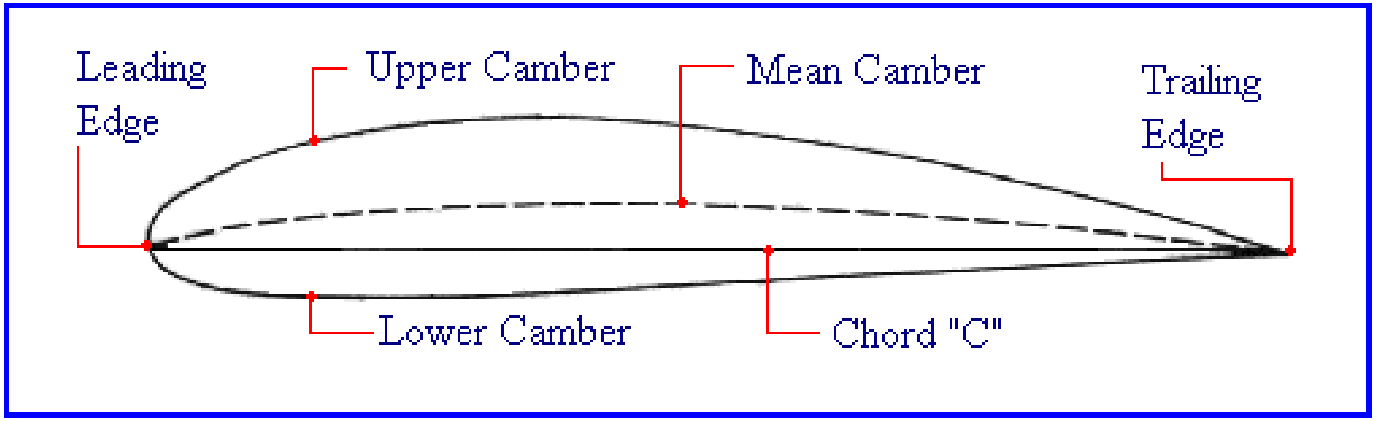
Laminar-flow airfoil (1938): to encourage laminar flow in the boundary layer over the airfoil to reduce skin friction drag

Supercritical airfoil (1965): designed for efficient flight near Mach one



Supersonic airfoil: designed for supersonic flow

Airfoil Characteristics

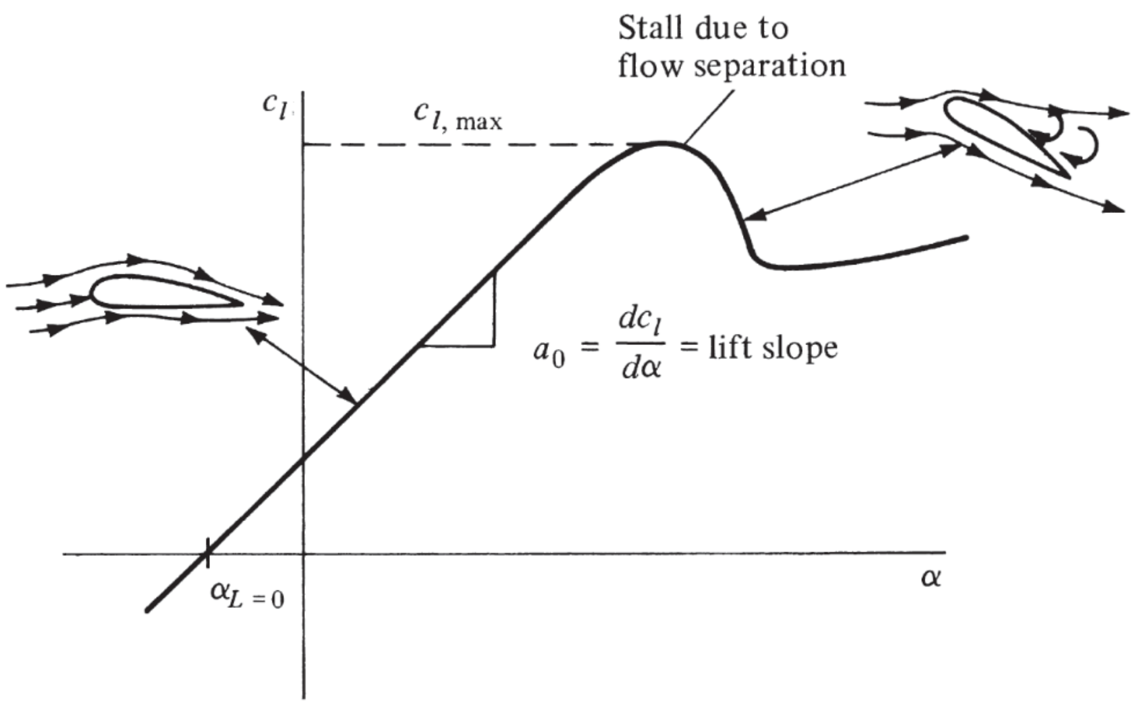


How to obtain the lift or lift coefficient of an airfoil?

- <u>Experimental measurements</u>
- <u>Calculation: Circulation theory of lift</u>
- Numerical simulations

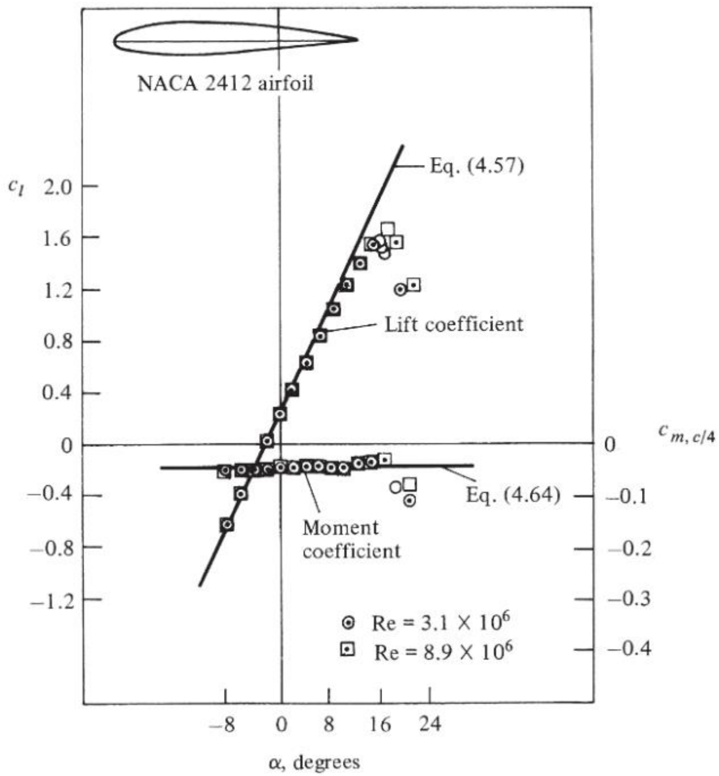
Airfoil Characteristics

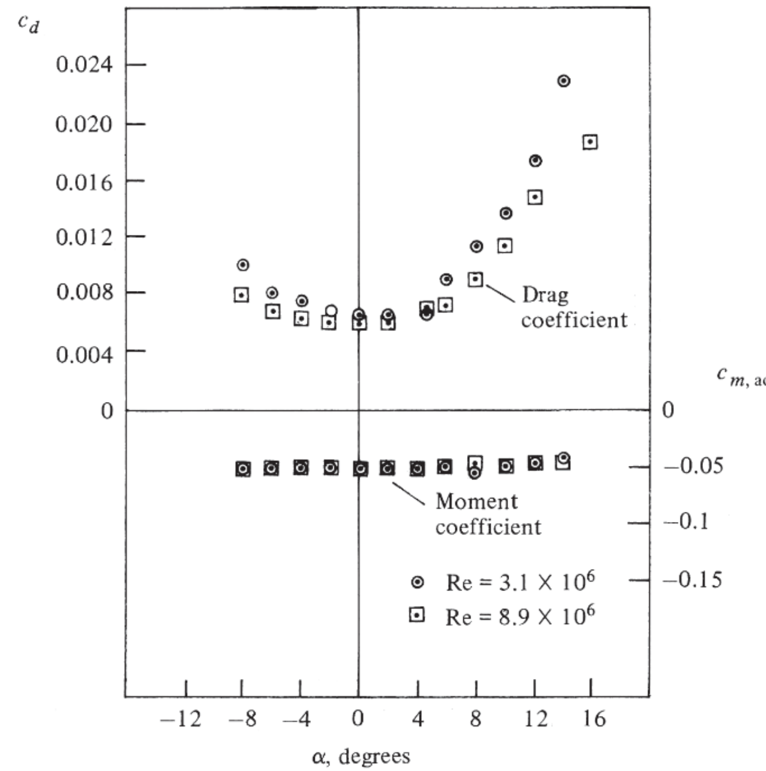
- Experimental measurements: Some typical results through wind tunnel measurements.



Airfoil Characteristics

- <u>Experimental measurements</u>: Some typical results through wind tunnel measurements.





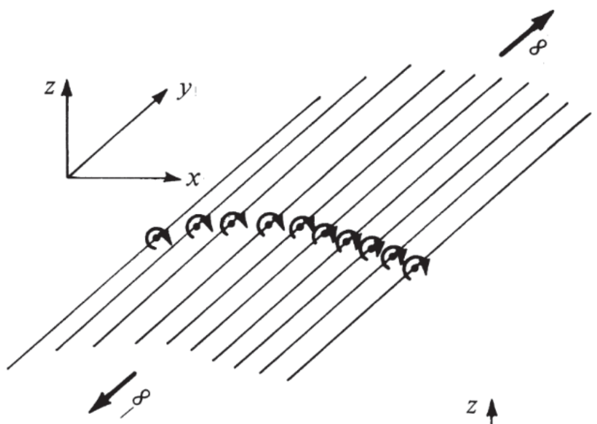
Can we calculate the $c_{l,max}$ with the inviscid flow airfoil theory?

Theoretical Solutions for Flow over Airfoils

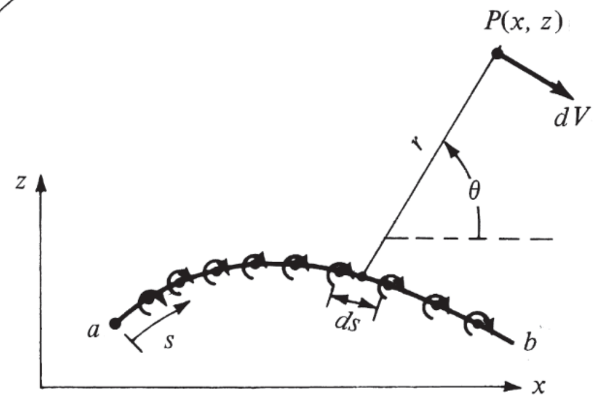
- <u>Calculation: Circulation theory of lift</u>

Consider an infinite number of straight vortex filaments side by side, where the strength of each filament is infinitesimally small.

- Let s be the distance measured along the vortex sheet.
- Define $\gamma = \gamma(s)$ as the strength of the vortex sheet, per unit length along s.



Vortex sheet in perspective



Edge view of sheet

<u>The strength of an infinitesimal portion ds of the sheet is γds</u>

<u>The induced velocity at point P</u>

$$dV = -\frac{\gamma\, ds}{2\pi r}$$

Theoretical Solutions for Flow over Airfoils

- Calculation: Circulation theory of lift

The strength of an infinitesimal portion ds of the sheet is γds

The induced velocity at point P $$dV = -\frac{\gamma\, ds}{2\pi r}$$

The increment in velocity potential dφ $$d\phi = -\frac{\gamma\, ds}{2\pi}\theta$$

The velocity potential at P due to the entire vortex sheet from a to b

$$\boxed{\phi(x, z) = -\frac{1}{2\pi}\int_a^b \theta\gamma\, ds}$$

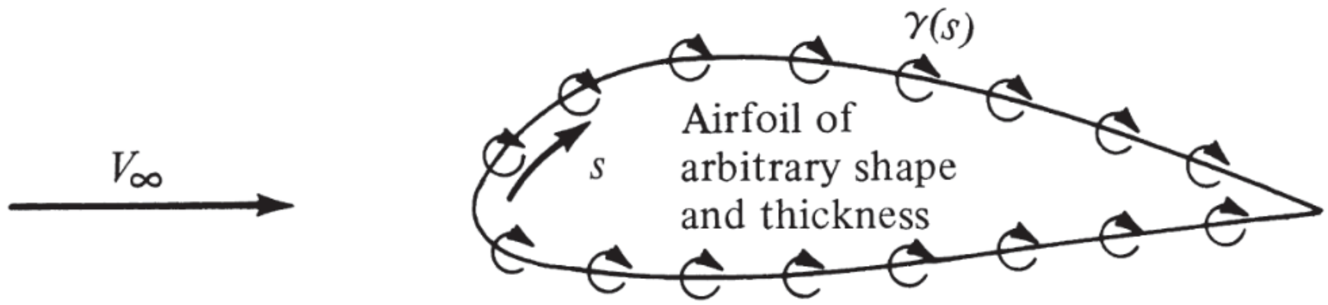
The circulation around the vortex sheet is the sum of the strengths of the elemental vortices from a to b

$$\boxed{\Gamma = \int_a^b \gamma\, ds}$$

Theoretical Solutions for Flow over Airfoils

- <u>Calculation: Circulation theory of lift</u>

Consider an airfoil of arbitrary shape and thickness in a freestream with velocity V_∞. Replace the airfoil surface with a vortex sheet of variable strength $\gamma(s)$



- The circulation around the airfoil will be given by

$$\Gamma = \int \gamma \, ds$$

- The resulting lift is given by the Kutta-Joukowski theorem:

$$L' = \rho_\infty V_\infty \Gamma$$

Theoretical Solutions for Flow over Airfoils

- Calculation: Circulation theory of lift

Consider an airfoil of arbitrary shape and thickness in a freestream with velocity V_∞. Replace the airfoil surface with a vortex sheet of variable strength $\gamma(s)$

The need for analytical solutions for $\gamma = \gamma(s)$

No general analytical solution exists…

- The circulation around the airfoil will be given by

$$\Gamma = \int \gamma \, ds$$

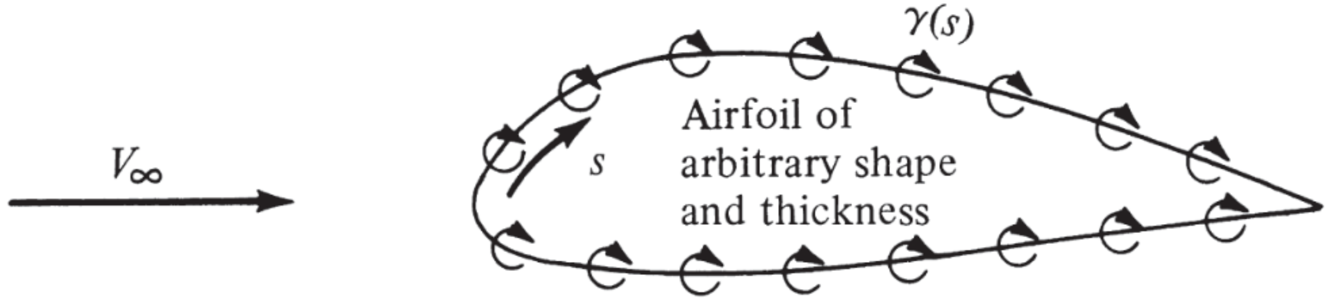
- The resulting lift is given by the Kutta-Joukowski theorem:

$$L' = \rho_\infty V_\infty \Gamma$$

Theoretical Solutions for Flow over Airfoils

- <u>Calculation: Circulation theory of lift</u>

Consider an airfoil of arbitrary shape and thickness in a freestream with velocity V_∞.
Replace the airfoil surface with a vortex sheet of variable strength $\gamma(s)$



- The concept of replacing the airfoil surface with a vortex sheet is more than just a mathematical device.
- In real life, there is a thin boundary layer on the surface, due to the action of friction between the surface and the airflow, in which the large velocity gradients produce substantial vorticity.
- There is a distribution of vorticity along the airfoil surface due to viscous effects.