A) Explicit Solution Method

Case 1

$\Delta X = \frac{1}{10}$

$\Delta t = \frac{1}{1000}$

$r = 0.1$

		i = 0 x = 0	i = 1 0·1	i = 2 0·2	i = 3 0·3	i = 4 0·4	i = 5 0·5	i = 6 0·6
(j = 0)	t = 0·000	0	0·2000	0·4000	0·6000	0·8000	1·0000	0·8000
(j = 1)	0·001	0	0·2000	0·4000	0·6000	0·8000	0·9600	0·8000
(j = 2)	0·002	0	0·2000	0·4000	0·6000	0·7960	0·9280	0·7960
(j = 3)	0·003	0	0·2000	0·4000	0·5996	0·7896	0·9016	0·7896
(j = 4)	0·004	0	0·2000	0·4000	0·5986	0·7818	0·8792	0·7818
(j = 5)	0·005	0	0·2000	0·3999	0·5971	0·7732	0·8597	0·7732
⋮								
(j = 10)	0·01	0	0·1996	0·3968	0·5822	0·7281	0·7867	0·7281
⋮								
(j = 20)	0·02	0	0·1938	0·3781	0·5373	0·6486	0·6891	0·6486

A

j		Finite-difference solution (x = 0·3)	Analytical solution (x = 0·3)	Difference	Percentage error
5	t = 0·005	0·5971	0·5966	0·0005	0·08
10	t = 0·01	0·5822	0·5799	0·0023	0·4
20	t = 0·02	0·5373	0·5334	0·0039	0.7
100	t = 0·10	0·2472	0·2444	0·0028	1·1

B

j		Finite-difference solution (x = 0·5)	Analytical solution (x = 0·5)	Difference	Percentage error
5	t = 0·005	0·8597	0·8404	0·0193	2·3
10	t = 0·01	0·7867	0·7743	0·0124	1·6
20	t = 0·02	0·6891	0·6809	0·0082	1·2
100	t = 0·10	0·3056	0·3021	0·0035	1·2

C

Case 2

$\Delta X = \frac{1}{10}$

$\Delta t = \frac{5}{1000}$

$r = 0.5$

	i = 0 x = 0	1 0·1	2 0·2	3 0·3	4 0·4	5 0·5	6 0·6
T = 0·000	0	0·2000	0·4000	0·6000	0·8000	1·0000	0·8000
0·005	0	0·2000	0·4000	0·6000	0·8000	0·8000	0·8000
0·010	0	0·2000	0·4000	0·6000	0·7000	0·8000	0·7000
0·015	0	0·2000	0·4000	0·5500	0·7000	0·7000	0·7000
0·020	0	0·2000	0·3750	0·5500	0·6250	0·7000	0·6250
⋮							
0·100	0	0·0949	0·1717	0·2484	0·2778	0·3071	0·2778

D

	Finite-difference solution (x = 0·3)	Analytical solution (x = 0·3)	Difference	Percentage error
t = 0·005	0·6000	0·5966	0·0034	0·57
t = 0·01	0·6000	0·5799	0·0201	3·5
t = 0·02	0·5500	0·5334	0·0166	3·1
t = 0·1	0·2484	0·2444	0·0040	1·6

r = 0.5 still gives stable and convergent solution

E

Case 3

$\Delta X = \frac{1}{10}$

$\Delta t = \frac{1}{100}$

$r = 1$

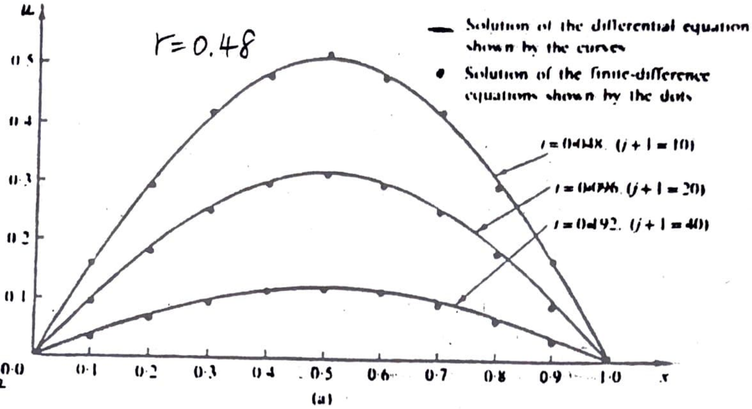
	i = 0 x = 0	1 0·1	2 0·2	3 0·3	4 0·4	5 0·5	6 0·6
t = 0·00	0	0·2	0·4	0·6	0·8	1·0	0·8
0·01	0	0·2	0·4	0·6	0·8	0·6	0·8
0·02	0	0·2	0·4	0·6	0·4	1·0	0·4
0·03	0	0·2	0·4	0·2	1·2	−0·2	1·2
0·04	0	0·2	0·0	1·4	−1·2	2·6	−1·2

unrealistic

r = 1.0 gives unstable and divergent solution

F

case 4



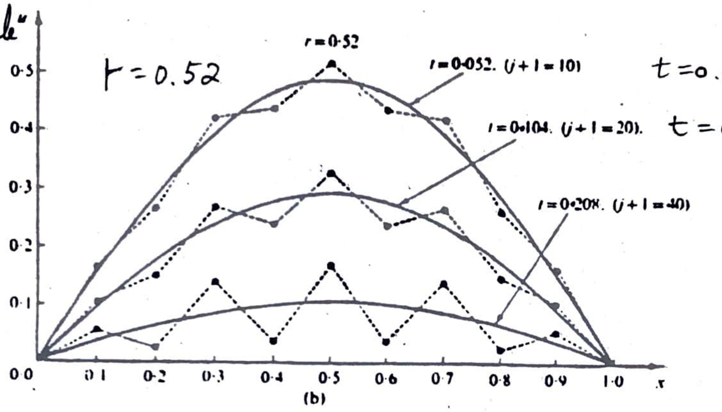
t = 0.048 j = 9
t = 0.096 j = 19
t = 0.192 j = 39

F(1)

if $r \le \frac{1}{2}$, stable

if $r > \frac{1}{2}$, unstable

Case 5



t = 0.052, j = 9
t = 0.104, j = 19
t = 0.192, j = 39

F(2)

CASE B) Crank-Nicolson (Implicit) Scheme

1) Gauss Method

$\Delta x = \frac{1}{10}$

$\Delta t = \frac{1}{100}$

$r = 1$

	x = 0	0·1	0·2	0·3	0·4	0·5
t = 0·00	0	0·2	0·4	0·6	0·8	1·0
j = 1 – t = 0·01	0	0·1989	0·3956	0·5834	0·7381	0·7691
t = 0·02	0	0·1936	0·3789	0·5400	0·6461	0·6921
·	0					
·	·					
·	·					
t = 0·10	0	0·0948	0·1803	0·2482	0·2918	0·3069
Analytical solution t = 0·10	0	0·0934	0·1776	0·2444	0·2873	0·3021

G

	Finite-difference solution (x = 0·5)	*Analytical solution (x = 0·5)*	*Difference*	*Percentage error*
t = 0·01	0·7691	0·7743	−0·0052	−0·7
t = 0·02	0·6921	0·6809	+0·0112	+1·6
t = 0·10	0·3069	0·3021	0·0048	+1·6

H