ME 51500/I5800 L
75 min.

Exam 2
Closed Book, Open Notes*

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*** Only original handwritten notes and homeworks are allowed. Photocopied notes and homework solution sheets are not permitted. Except for a hand calculator, no cell phone or electronic equipment of any kind is allowed.**

Show all work and give units in final answers.

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[50] 1. Consider the earth and Mars to be in coplanar circular orbits of radii 1 au and 1.524 au, respectively. For a transfer angle $\Delta\theta = 120°$

a) Calculate the semi-major axis, eccentricity, and transfer time of the minimum energy transfer ellipse.
b) Calculate the lead angle β_{12} of Mars at the time of launch with respect to the earth for interception with Mars to occur.
c) Draw an accurate labeled sketch which includes the earth and Mars orbits about the sun, the positions of earth and Mars at launch and at arrival, the transfer trajectory, and the angles $\Delta\theta$ and β_{12}

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[30] 2. Determine the minimum ΔV needed to change the inclination of a circular orbit about the earth having a 8000 km radius by

a) 25°
b) 50°

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[20] 3. Determine the required altitude for a 4" × 4" × 4" cubesat weighing 3 lb to remain in a circular orbit about the earth for 25 years.

Physical constants

The Earth

Mean Radius = 6368 km

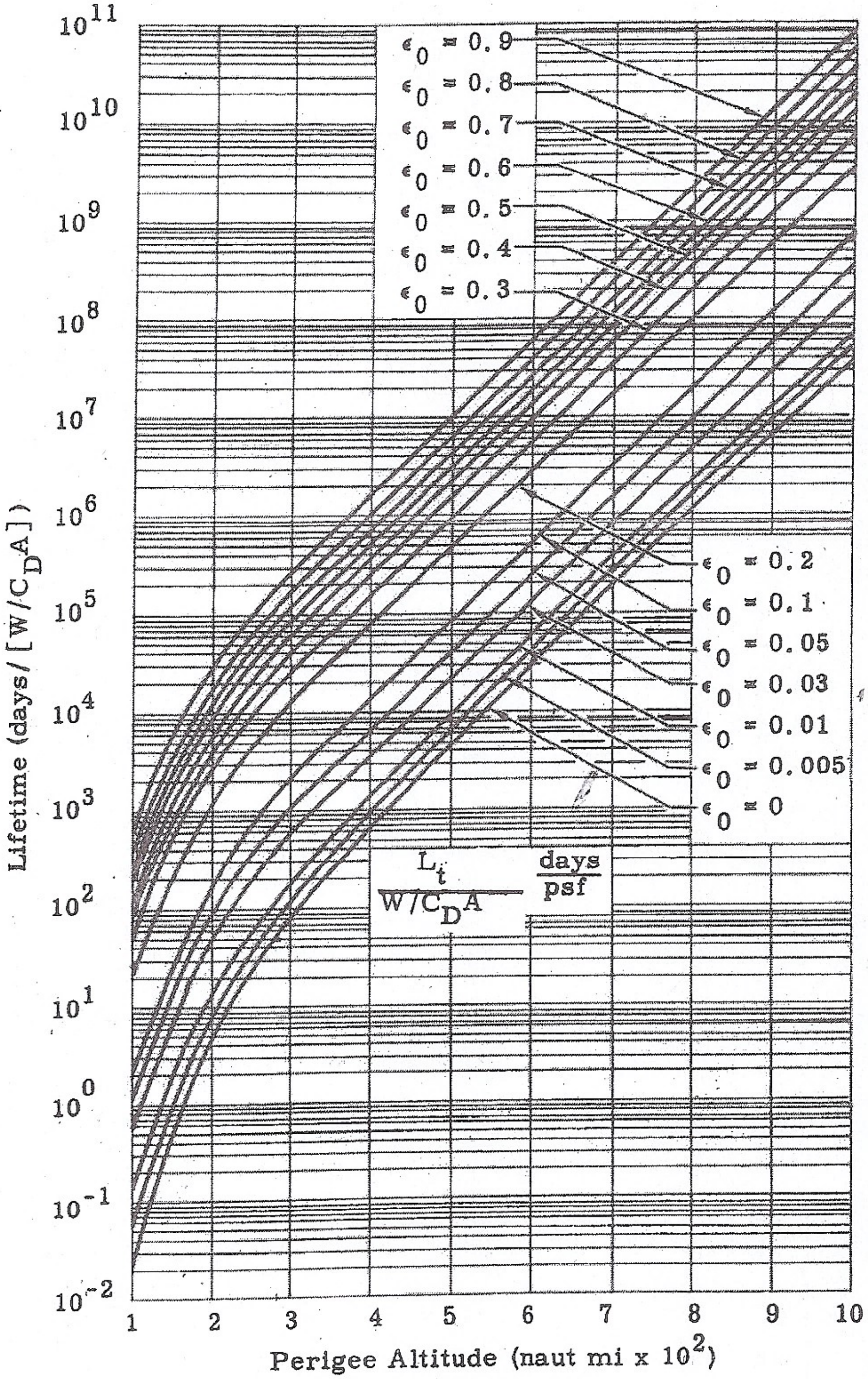
$\mu_{earth} = 3.986 \times 10^5$ km³/sec²

Mean distance from the sun = 1 au = 1.496×10^8 km

1 year = 365.24 days

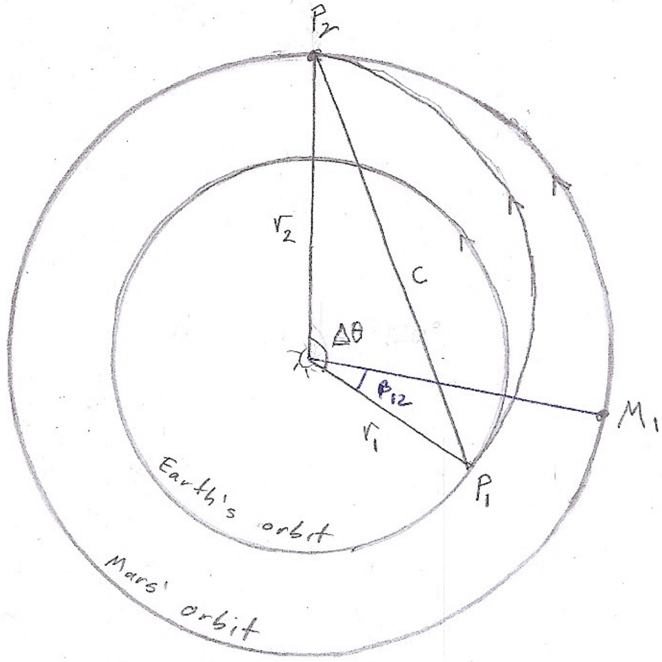
The Sun

$\mu_{sun} = 4\pi^2$ au³/yr² = 1.327×10^{11} km³/sec²



$\epsilon_0 = 0.9$
$\epsilon_0 = 0.8$
$\epsilon_0 = 0.7$
$\epsilon_0 = 0.6$
$\epsilon_0 = 0.5$
$\epsilon_0 = 0.4$
$\epsilon_0 = 0.3$
$\epsilon_0 = 0.2$
$\epsilon_0 = 0.1$
$\epsilon_0 = 0.05$
$\epsilon_0 = 0.03$
$\epsilon_0 = 0.01$
$\epsilon_0 = 0.005$
$\epsilon_0 = 0$
$\frac{L_t}{W/C_D A}$ $\frac{\text{days}}{\text{psf}}$
Lifetime (days/[$W/C_D A$])
10^{11} 10^{10} 10^{9} 10^{8} 10^{7} 10^{6} 10^{5} 10^{4} 10^{3} 10^{2} 10^{1} 10^{0} 10^{-1} 10^{-2}
1 2 3 4 5 6 7 8 9 10
Perigee Altitude (naut mi x 10^2)

1)



$r_1 = 1$ au

$r_2 = 1.524$ au

a) $c = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\Delta\theta}$

$= \sqrt{(1)^2 + (1.524)^2 - 2(1)(1.524)\cos 120°} = 2.20149$ au

$s = \dfrac{r_1 + r_2 + c}{2} = \dfrac{1 + 1.524 + 2.20149}{2} = 2.36275$ au

$a_m = \dfrac{s}{2} = \dfrac{2.36275}{2} = \underline{\underline{1.18137 \text{ au}}}$ [10]

From (8.24)

$\alpha_m = \pi$ radians

$\sin\left(\dfrac{\beta_m}{2}\right) = \left(\dfrac{s-c}{s}\right)^{1/2} = \left(\dfrac{2.36275 - 2.20149}{2.36275}\right)^{1/2} = 0.261249$

$\Rightarrow \dfrac{\beta_m}{2} = 0.264136 \Rightarrow \beta_m = 0.528632$ radians

From (8.19)

$$P = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha+\beta}{2}\right) = \frac{4(1.18137)(2.36275-1)(2.36275-1.524)}{(2.20149)^2} \cdot \sin^2\left(\frac{\pi + 0.528632}{2}\right)$$

$= 1.03839$ au

$$p = a(1-e^2) \Rightarrow e = \left(1 - \frac{p}{a}\right)^{1/2}$$

$$e = \left(1 - \frac{1.03839}{1.18137}\right)^{1/2} = \underline{\underline{0.347892}} \quad [10]$$

$$t_m = \sqrt{\frac{s^3}{8\mu}}\left(\pi - \beta_m + \sin\beta_m\right)$$

$$= \sqrt{\frac{(2.36275)^3}{8(4\pi^2)}}\left(\pi - 0.528632 + \sin(0.528632)\right)$$

$$t_m = \underline{\underline{0.637063 \text{ year}}} \quad [10]$$

b) The time it takes Mars to travel from M_1 to P_2 is

$$\tau = \frac{\Delta\theta - \beta_{12}}{n_2} = \frac{\Delta\theta - \beta_{12}}{\sqrt{\frac{\mu}{r_2^3}}}$$

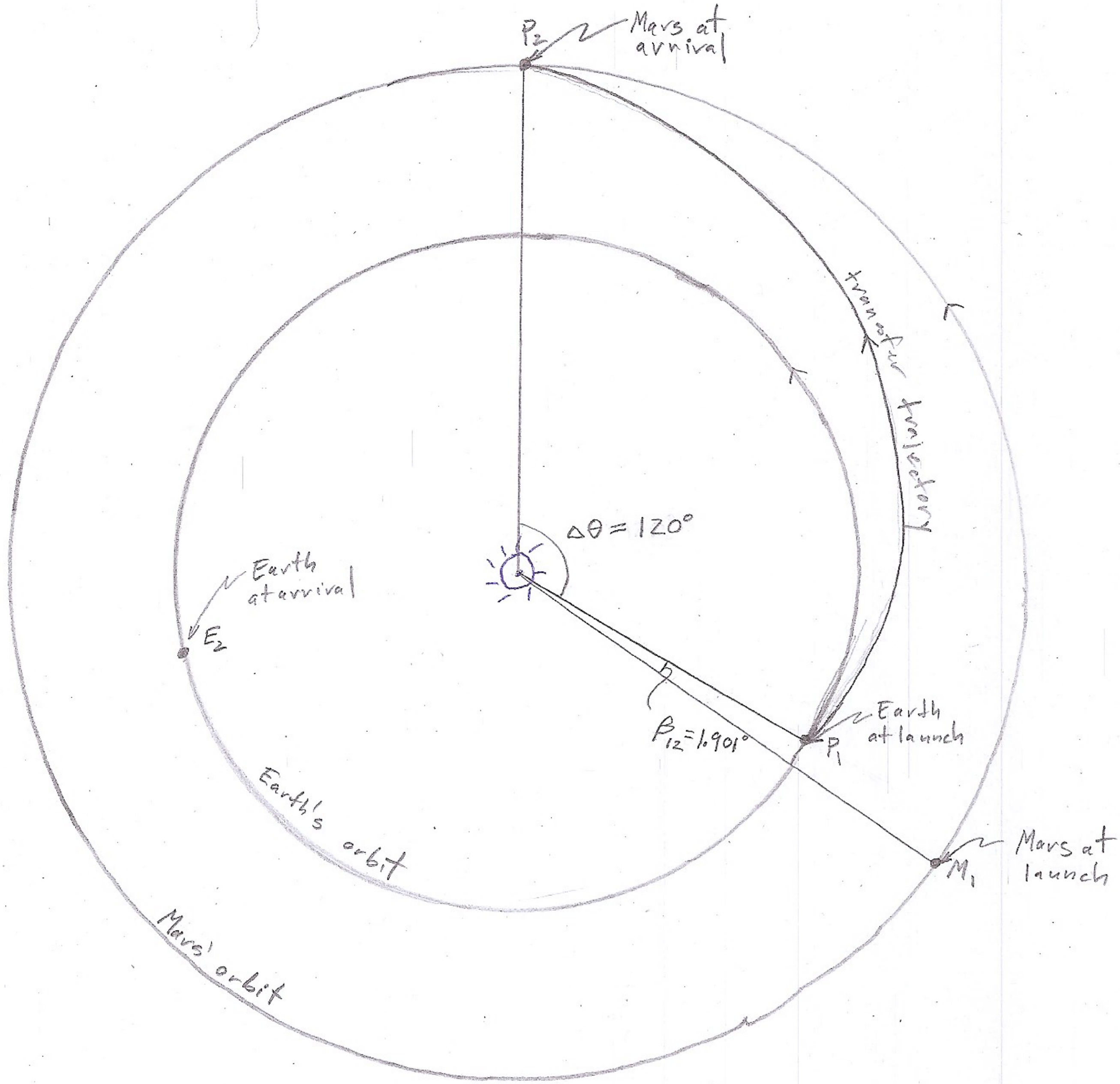
For interception with Mars, require $\tau = t_m$. Solve for β_{12}.

$$\beta_{12} = \Delta\theta - n_2 t_m = \Delta\theta - \sqrt{\frac{\mu}{r_2^3}}\, t_m$$

$$= 120^\circ - \sqrt{\frac{4\pi^2}{(1.524)^3}}(0.637063)\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 120^\circ - 121.901^\circ$$

$$\boxed{\beta_{12} = -1.901^\circ}$$ (actually a lag angle)

c)



P_2
Mars at arrival
transfer trajectory
$\Delta\theta = 120°$
Earth at arrival
E_2
$\beta_{12} = 1.901°$
Earth at launch
P_1
Earth's orbit
Mars at launch
M_1
Mars' orbit

[10]

2) a) $0° < \Delta\alpha = 25° < 38.94°$

$$V_c = \sqrt{\frac{\mu}{r_c}} = \sqrt{\frac{3.986\times10^5}{8000}} = 7.05868 \text{ km/sec}$$ [5]

$$\Delta V_\alpha = 2V_c \sin\tfrac{1}{2}\Delta\alpha = 2(7.05868)\sin(\tfrac{1}{2}25°)$$

$$\boxed{\Delta V_\alpha = 3.05556 \text{ km/sec}}$$ [5]

b) $38.94° < \Delta\alpha = 50° < 60°$

$$r_a = r_c \frac{\sin\frac{1}{2}\Delta\alpha}{1-2\sin\frac{1}{2}\Delta\alpha} = 8000\frac{\sin(\frac{1}{2}50°)}{1-2\sin(\frac{1}{2}50°)} = 21{,}845.9 \text{ km}$$ [5]

$$e = \frac{r_a - r_c}{r_a + r_c} = \frac{21{,}845.9-8000}{21{,}845.9+8000} = 0.463913$$ [3]

$$\Delta V_p = V_c\left[\sqrt{1+e}-1\right] = 7.05868\left[\sqrt{1+0.463913}-1\right] = 1.48178 \tfrac{\text{km}}{\text{sec}}$$ [3]

$$V_a = \sqrt{\frac{2\mu r_c}{r_a(r_c+r_a)}} = \sqrt{\frac{2(3.986\times10^5)(8000)}{(21{,}845.9)(8000+21{,}845.9)}} = 3.12753 \tfrac{\text{km}}{\text{sec}}$$ [3]

$$\Delta V_\alpha = 2V_a\sin\tfrac{1}{2}\Delta\alpha = 2(3.12753)\sin(\tfrac{1}{2}50°) = 2.6435 \tfrac{\text{km}}{\text{sec}}$$ [3]

$$\Delta V_{total} = 2\Delta V_p + \Delta V_\alpha = 2(1.48178) + 2.6435$$

$$\boxed{\Delta V_{total} = 5.60706 \tfrac{\text{km}}{\text{sec}}}$$ [3]

3) $W = 3\,lb$ $\qquad L_t = 25$ years

$$A_s = 6\left(\frac{4}{12}\right)\left(\frac{4}{12}\right) = \frac{2}{3}\ ft^2$$

$$A = \frac{A_s}{4} = \frac{1}{6}\ ft^2$$ (Assuming all orientations are equally probable)

$$\frac{W}{C_D A} = \frac{3}{(2)\left(\frac{1}{6}\right)} = 9\ psf$$

$$\frac{L_t}{\frac{W}{C_D A}} = \frac{25\,(365.24)}{9} = 1015\ \frac{days}{psf}$$

$$\boxed{h = 420\ n\text{-}mi}$$