1) Consider the vertical ascent of a rocket having initial mass m_0 in a constant gravity field g, powered by an engine with specific impulse I_{sp} burning fuel at a constant rate $\dot{m}_e$. Obtain an expression for the altitude $h(t)$ during powered flight.

Using (9.64) in the notes

$$m\frac{d\bar{V}}{dt} = \bar{F}_{ext} - \dot{m}_e\bar{c}$$

with

$$\bar{V} = V\hat{j} \qquad \bar{F}_{ext} = -mg\hat{j} \qquad \bar{c} = -c\hat{j} = -I_{sp}\,g\,\hat{j}$$

$$m\frac{dV}{dt} = -mg + \dot{m}_e I_{sp}\,g$$

Since $\dot{m}_e = \text{const}$, during powered flight $m = m_0 - \dot{m}_e t$

$$\frac{dV}{dt} = \frac{\dot{m}_e I_{sp}\,g}{m} - g = \frac{\dot{m}_e I_{sp}\,g}{m_0 - \dot{m}_e t} - g$$

$$V(t) = \dot{m}_e I_{sp}\,g\int_0^t \frac{dt'}{m_0 - \dot{m}_e t'} - gt$$

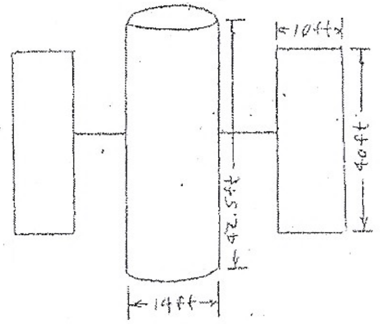
$$V(t) = -I_{sp}\,g\ln\left(\frac{m_0 - \dot{m}_e t}{m_0}\right) - gt$$

$$\frac{dh}{dt} = -I_{sp}\,g\ln\left(\frac{m_0 - \dot{m}_e t}{m_0}\right) - gt$$

$$h(t) = -I_{sp}\,g\int_0^t \ln\left(\frac{m_0 - \dot{m}_e t'}{m_0}\right)dt' - \frac{gt^2}{2}$$

$$\boxed{h(t) = \frac{I_{sp}\,g}{\dot{m}_e}\left[\dot{m}_e t - (m_0 - \dot{m}_e t)\ln\left(\frac{m_0}{m_0 - \dot{m}_e t}\right)\right] - \frac{gt^2}{2}}$$

2) The Hubble Space Telescope, launched on April 24, 1990, is currently in a 289 nautical mile high circular orbit. The telescope is a cylindrical shaped structure 14 ft diameter x 42.5 ft long. Two solar panels each measuring 10 ft x 40 ft are mounted radially on opposite sides of the cylindrical body. The total weight of the spacecraft is 27,000 lbs. Estimate the orbital lifetime of the satellite.



$$A_{surface} = \left[\pi(14)(42.5) + 2\cdot\frac{\pi(14)^2}{4} + 4(10)(40)\right] = 3777\ ft^2$$

$$A = \frac{1}{4}A_{surface} = 944\ ft^2$$

(Assuming all orientations equally probable).

$$\frac{W}{C_D A} = \frac{27{,}000}{(2)(944)} = 14.3\ \frac{lbf}{ft^2}$$

From Orbital Lifetime Chart, for $h = 289$ n-mi and $e = 0$

$$\frac{L_t}{W/C_D A} = 60\ \frac{days}{lbf/ft^2}$$

$$L_t = (60)(14.3) = \underline{\underline{860\ days}} = \underline{\underline{2.3\ years}}$$

Solar activity (sun spots) increases the density of the upper atmosphere which increases atmospheric drag. Solar activity is currently at a low level so that even without a reboost, the HST is predicted to remain in orbit until the end of the decade.

3) The International Space Station is in a nearly circular orbit of 420 km altitude and inclination of 51.6°. What is the spacing in kilometers between successive ground tracks at the equator, (a) assuming the earth is perfectly spherical, and (b) including the effect of the earth's oblateness. Check your results by performing a 10-hour simulation using GMAT with and without the J2 propagator. Print out the output of your simulations and comment on your results.

a) Spherical Earth

$h_c = 420$ km $\qquad i = 51.6°$

$a = r_c = h_c + r_e = 420 + 6368 = 6788$ km

The orbital period is

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(6788)^3}{398600}} = 5566 \text{ sec}$$

The earth's angular velocity is

$$\omega_e = \frac{2\pi \text{ rad}}{23 \text{ hr } 56 \text{ min } 4 \text{ sec}} = \frac{2\pi \text{ rad}}{86{,}164 \text{ sec}} = 7.29212 \times 10^{-5} \frac{\text{rad}}{\text{sec}}$$

In one orbital period, the earth rotates through angle α.

$$\alpha = \omega_e T = \left(7.29212 \times 10^{-5} \frac{\text{rad}}{\text{sec}}\right)(5566 \text{ sec}) = 0.4059 \text{ rad} = 23.26°$$

The spacing between successive ground tracks at the equator is:

$$\Delta S_{ze} = r_e \alpha = (6368)(0.4059) = \underline{2585 \text{ km}}$$

Comparison using GMAT

$$\Delta S_{ze} = (6368)(80.8925 - 57.5885)\left(\frac{\pi}{180}\right) = \underline{2585 \text{ km}}$$

b) Oblate Earth

$$\dot{\Omega}_{avg} = -\frac{\frac{3}{2}\sqrt{\mu}\, J_2 R^2}{(1-e^2)^2 a^{7/2}} \cos i$$

$$= -\frac{\frac{3}{2}\sqrt{398600}\,(0.0010826)(6378)^2}{(1-0^2)^2\,(6788)^{7/2}} \cos 51.6°$$

$$= -1.0053 \times 10^{-6} \frac{rad}{sec}$$

$$\dot{\Omega}_{avg} = 1.0053 \times 10^{-6} \frac{rad}{sec} \quad \text{(to the west)}$$

The spacing between successive ground tracks at the equator is:

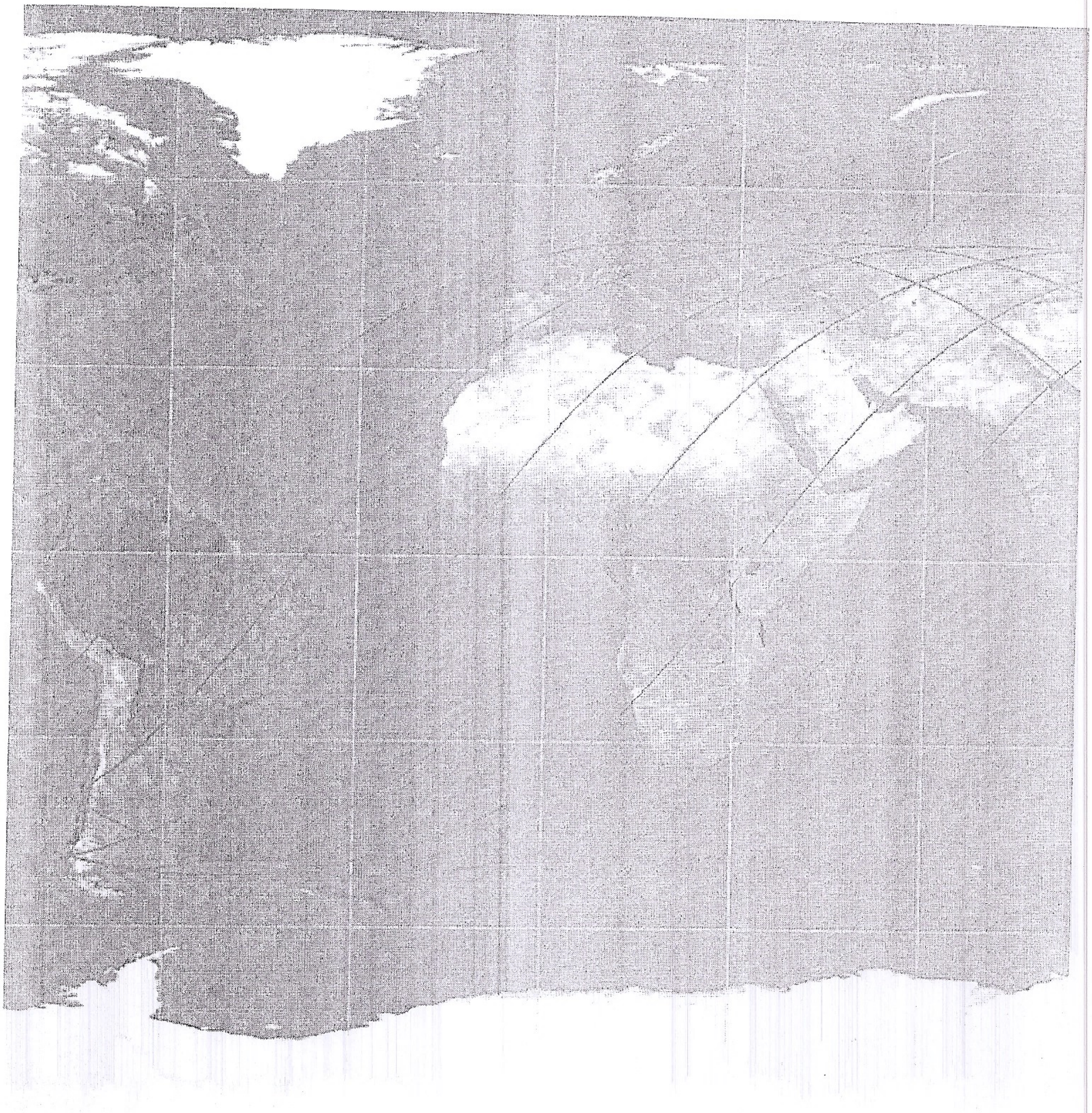
$$\Delta S_{J2} = r_e(\omega_e + \dot{\Omega}_{avg})\,T$$

$$= (6378)(7.29212 \times 10^{-5} + 1.0053 \times 10^{-6})(5566)$$

$$\Delta S_{J2} = \underline{\underline{2624 \text{ km}}}$$

Comparison using GMAT

$$\Delta S_{J2} = (6378)(80.5689 - 57.0408)\left(\frac{\pi}{180}\right) = \underline{\underline{2619 \text{ km}}}$$



Spherical Earth

MySat.Earth.Latitude	MySat.Earth.Longitude
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21.24535487989204	120.1572326016647
25.61628059458683	124.1313672420766
29.84545734191407	128.427592333925
33.89230672330352	133.1244477430315
37.70755994474154	138.3076968439324
41.2310968095631	144.0657081580405
44.39060293227668	150.4794335521566
47.1020475896153	157.6047193081618
49.27309510904653	165.4451571799303
50.81195663087577	173.922305366788
51.64126341005633	-177.1440147690336
51.71428478200976	-168.024983330442
51.02672284514714	-159.0328851377867
49.61787362431712	-150.4516814926808
47.56048088976998	-142.4808680361098
44.94479199596881	-135.2174334497718
41.86387252652919	-128.6711706191702
38.40378170194572	-122.7939595270861
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30.63228936721782	-112.7250837253989
26.43455094558229	-108.3588529057256
22.08781652405462	-104.3289584868101
17.62689834658091	-100.5634537650992
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-14.70278375955725	-76.81816490530227
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-23.64554877365685	-69.33704583811709
-27.94369474793075	-65.19644285461249
-32.07889481079266	-60.69193239874554
-36.00616554681979	-55.74063793952089
-39.67044279142223	-50.25465557001635
-43.00512545683814	-44.14784542668289
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-48.35954779459058	-29.83063943161406
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-51.35625781807411	-12.87173981915597
-51.77620446055174	-3.799173785754105
-51.43189680671183	5.289222286993977
-50.34347072066498	14.08781371199835
-48.57024421704548	22.35423823441807

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21.64880321939496	97.25400570913953
26.0083076774146	101.2544768801172
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34.25061024237616	110.321436328167
38.04196508454282	115.5534902097227
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-28.32803400731881	-88.05619634150267
-32.44630387436381	-83.51447278335074
-36.35203449489283	-78.51813314748959
-39.98923319017693	-72.97897507571869

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-46.17470945294176	-59.9486730473046
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-51.77652367327237	-26.22026886881224
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22.05148298386292	74.35306530291349
26.39927987473463	78.38047797938171
30.59842958313829	82.74363461138554
34.60700159974101	87.52262007269017
38.3738994102837	92.80415839659503
41.83678459409395	98.67612375220426

Oblate Earth

MySat.Earth.Latitude	MySat.Earth.Longitude
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3.186929189585646	106.3478947048537
7.617173057450067	109.5165063217221
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