12.5 Planetary Flyby (Gravity-Assist) Trajectories

Useful in interplanetary missions to obtain a velocity change without expending propellant.

EXAMPLES

Pioneer 10 - launched in March 1972; flyby of Jupiter Dec. 1973; escaped solar system.

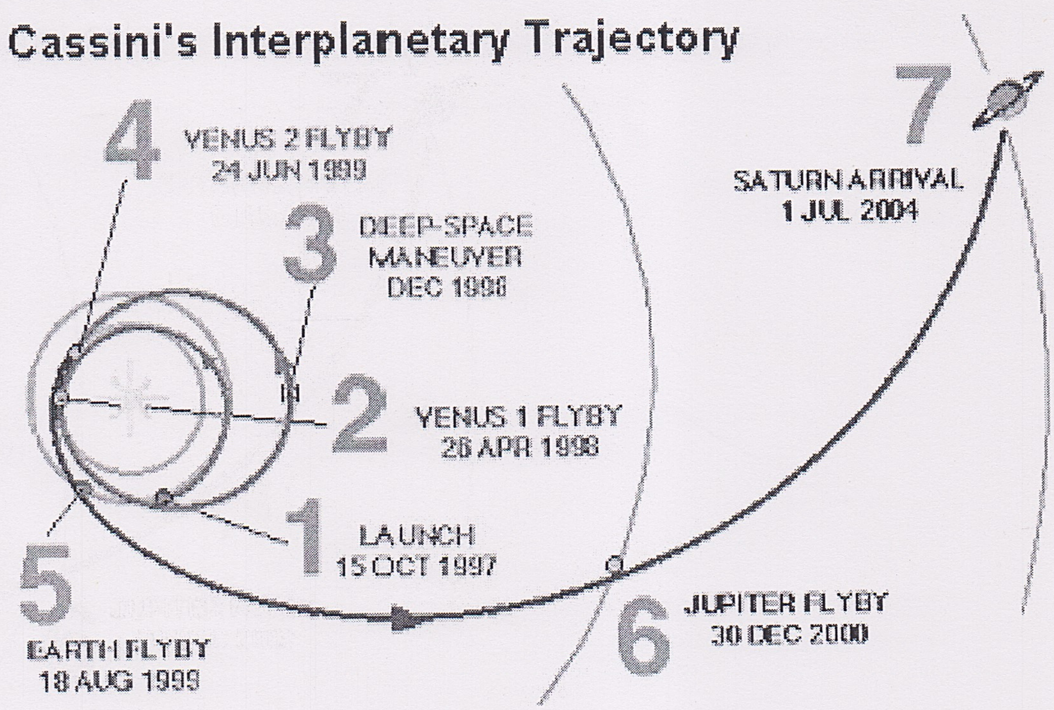
Pioneer 11 - launched in April 1973; used a flyby of Jupiter in Dec. 1974 to propel it to the first ever flyby of Saturn in Sept. 1979; escaped solar system.

Voyager 1 - launched in Sept. 1977; used a flyby of Jupiter in March 1979 to flyby Saturn in Nov. 1980 which in turn propelled it out of the ecliptic plane and out of the solar system.

Voyager 2 - launched in August 1977; flew by Jupiter (July 1979), Saturn (August 1981), Uranus (January 1986), Neptune (August 1989) after which it departed the solar system.

Galileo - launched in October 1989; used gravity assist flybys of Venus (Feb. 1990), earth (Dec 1990) and earth again (Dec 1992) before being placed in orbit around Jupiter (Dec. 1995).

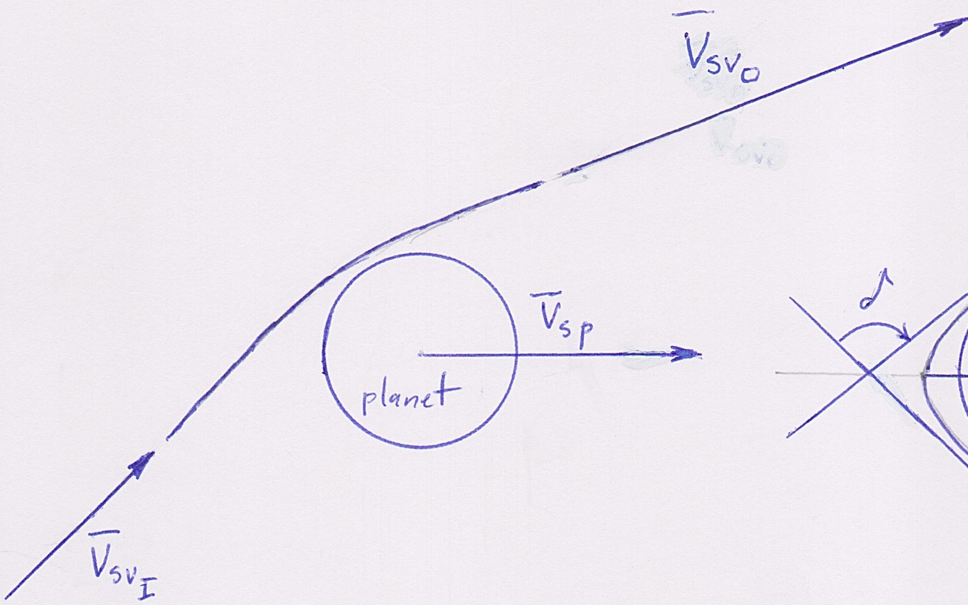
Cassini - launched in October 1997; used 4 flybys before being placed in orbit around Saturn 7 years later in July 2004.

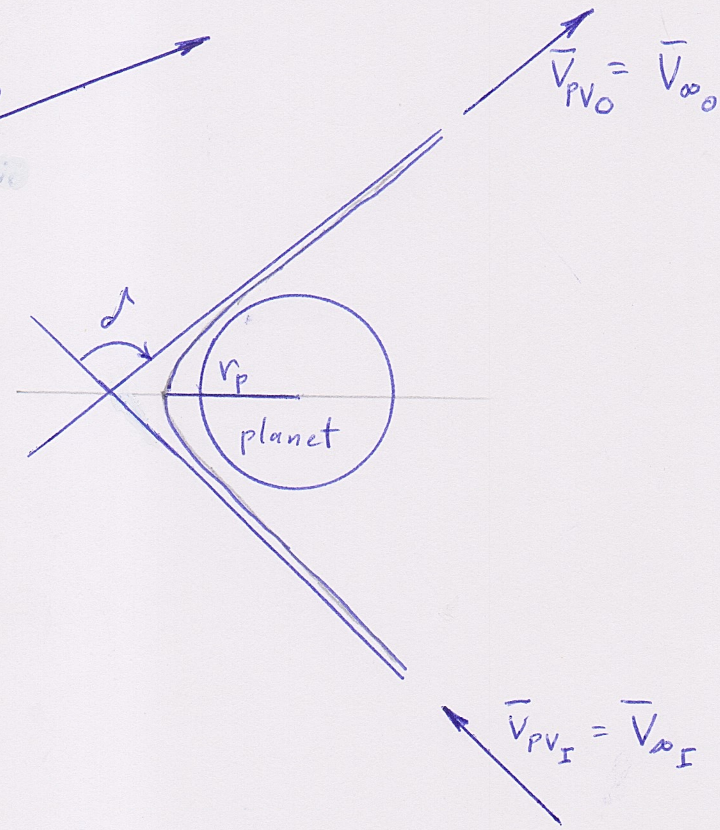


New Horizons - launched in January 2006; Jupiter flyby (Feb 2007); Pluto flyby (July 2015); Ultima Thule Flyby (Jan 2019) [asteroid 1 billion miles past Pluto]

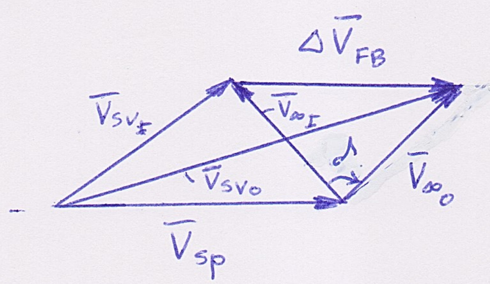
How it Works

Velocities relative to Sun | **Velocities relative to Planet**





$$|\bar{V}_{\infty_O}| = |\bar{V}_{\infty_I}|$$



From the velocity diagram

$$\Delta\bar{V}_{FB} = \bar{V}_{SV_O} - \bar{V}_{SV_I} = \bar{V}_{\infty_O} - \bar{V}_{\infty_I} \qquad (12.21)$$

$$\Delta V_{FB} = 2 V_{\infty} \sin\frac{\delta}{2} \qquad (12.22)$$

Using the result of HW 4, prob. 2 $\left[\sin\frac{\delta}{2} = \frac{1}{1+\psi}\right]$

$$\Delta V_{FB} = \frac{2V_\infty}{1+\psi}$$

or

$$\frac{\Delta V_{FB}}{V_s} = \frac{2\,V_\infty/V_s}{1+\left(\frac{V_\infty}{V_s}\right)\left(\frac{r_p}{r_s}\right)} \qquad (12.23)$$

where r_s is the planet radius and V_s is the the speed required for a circular orbit on the planet's surface.

A flyby behind the planet (in its motion about the sun) results in an increase in the heliocentric speed of the vehicle ($V_{sv_o} > V_{sv_I}$).

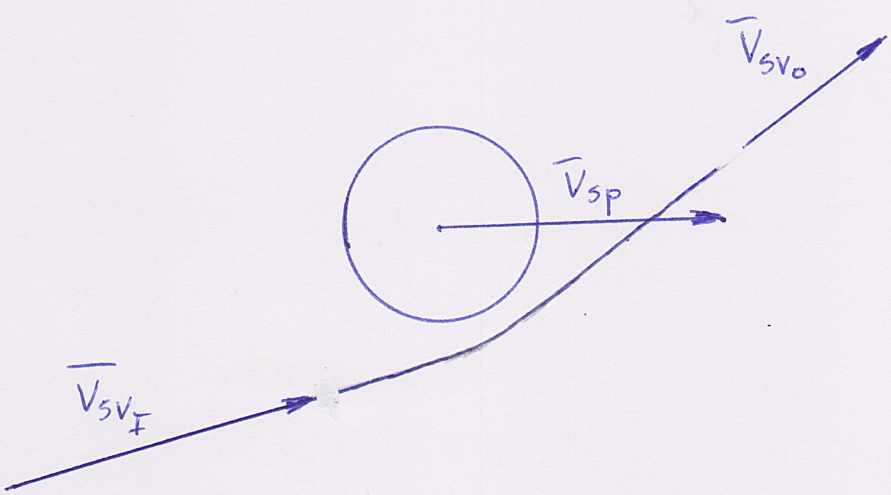
Heliocentric KE of the vehicle increases.

Where does the energy come from?

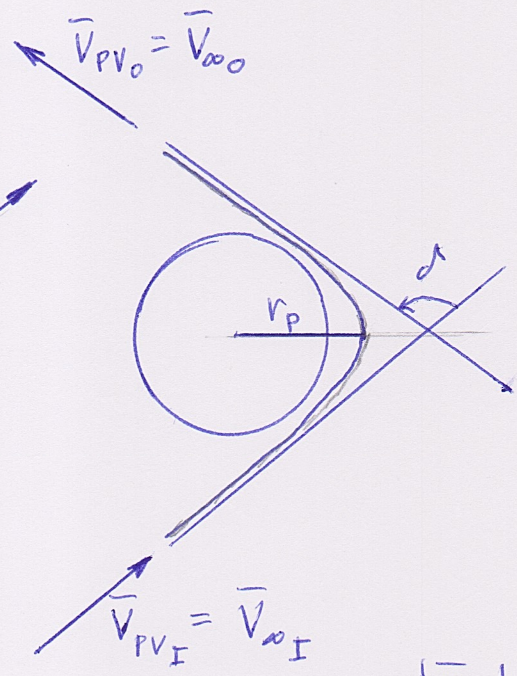
<u>Ans</u>: Decrease in KE of planet.

A flyby in front of the planet would cause a decrease in the vehicle's heliocentric speed.

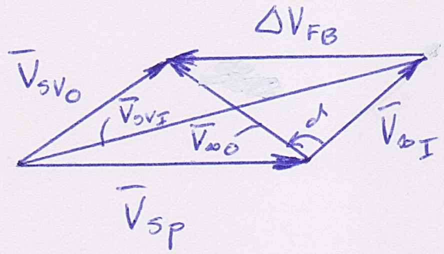
Velocities relative to Sun



Velocities relative to Planet



$$|\bar{V}_{\infty O}| = |\bar{V}_{\infty I}|$$



EXAMPLE (prob. 9.4)

For a hyperbolic flyby of a planet,

a) Determine the values of periapse radius r_p and hyperbolic excess speed V_∞ that will yield the maximum possible magnitude of ΔV_{FB}. Express your answer for r_p in terms of the planet radius r_s and include the constraint $r_p \geq r_s$.

b) Determine this maximum value in terms of V_s and determine numerical values for the corresponding turn angle and eccentricity.

a) Examining (12.23)

$$\frac{\Delta V_{FB}}{V_s} = \frac{2\left(\frac{V_\infty}{V_s}\right)}{1+\left(\frac{V_\infty}{V_s}\right)^2\left(\frac{r_p}{r_s}\right)}$$

Decreasing r_p increases ΔV_{FB}

smallest possible $\boxed{r_p = r_s}$

To determine V_∞ for maximum ΔV_{FB}, set $\frac{d(\Delta V_{FB})}{dV_\infty} = 0$

With $r_p = V_s$

$$\Delta V_{FB} = \frac{2V_\infty}{1 + \left(\frac{V_\infty}{V_s}\right)^2}$$

$$\frac{d(\Delta V_{FB})}{dV_\infty} = \frac{\left[1 + \left(\frac{V_\infty}{V_s}\right)^2\right] 2 - 2V_\infty \, 2\left(\frac{V_\infty}{V_s}\right)\frac{1}{V_s}}{\left[1 + \left(\frac{V_\infty}{V_s}\right)\right]^2} = 0$$

$$1 + \left(\frac{V_\infty}{V_s}\right)^2 - 2\left(\frac{V_\infty}{V_s}\right)^2 = 0$$

$$1 - \left(\frac{V_\infty}{V_s}\right)^2 = 0$$

$$\left(\frac{V_\infty}{V_s}\right)^2 = 1$$

$$\boxed{V_\infty = V_s}$$

b) $$\frac{\Delta V_{FB}}{V_s} = \frac{2\left(\frac{V_\infty}{V_s}\right)}{1 + \left(\frac{V_\infty}{V_s}\right)^2\left(\frac{r_p}{V_s}\right)}$$

$$\frac{\Delta V_{FB\,max}}{V_s} = \frac{2(1)}{1+(1)(1)} = 1$$

$$\boxed{\Delta V_{FB_{max}} = V_s}$$

Turn angle for $\Delta V_{FB\,max}$

$$\sin\frac{\delta}{2} = \frac{1}{1+\psi} \qquad \psi = \left(\frac{V_\infty}{V_s}\right)^2\left(\frac{V_p}{V_s}\right) = (1)^2(1) = 1$$

$$\sin\frac{\delta}{2} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{\delta}{2} = 30^\circ$$

$$\boxed{\delta = 60^\circ}$$

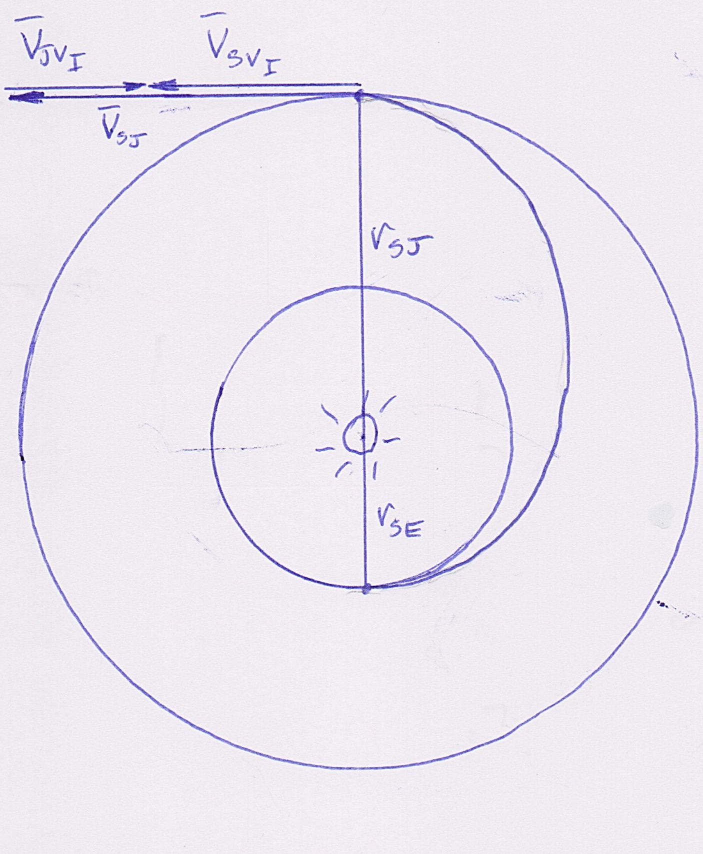
Eccentricity for $\Delta V_{FB\,max}$

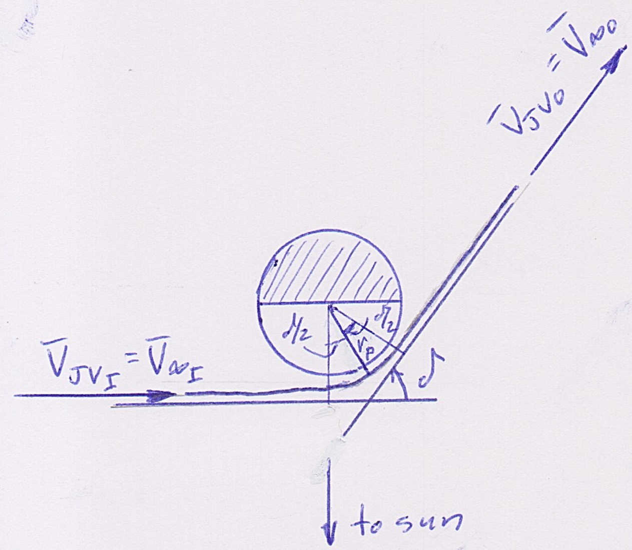
$$e = 1+\psi = 1+1$$

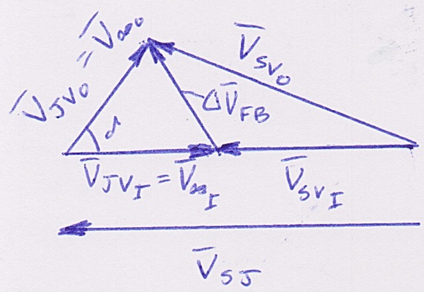
$$\boxed{e = 2}$$

EXAMPLE

Suppose a spacecraft approaches Jupiter on a Hohmann transfer ellipse from earth. If the spacecraft flies by Jupiter at an altitude of 200,000 km on the sunlit side of the planet, determine the orbital elements of the post-flyby trajectory and the delta-V imparted to the spacecraft by Jupiter's gravity. Assume that all orbits lie in the same (ecliptic) plane.







Heliocentric orbital elements (before flyby)

$$a_I = \frac{r_{SJ} + r_{SE}}{2} = \frac{5.2028 + 1}{2} = 3.1014\text{ au}\left(\frac{1.495978 \times 10^8\text{ km}}{1\text{ au}}\right)$$

$$= 4.63963 \times 10^8\text{ km}$$

$$e_I = \frac{r_{SJ} - r_{SE}}{r_{SJ} + r_{SE}} = \frac{5.2028 - 1}{5.2028 + 1} = 0.677565$$

$$V_{SJ} = \sqrt{\frac{\mu_{sun}}{r_{SJ}}} = \sqrt{\frac{1.327 \times 10^{11}}{(5.2028)(1.495978 \times 10^8)}} = 13.0573\ \frac{\text{km}}{\text{sec}}$$

$$V_{SV_I} = V_{SJ}\sqrt{1 - e_I} = 13.0573\sqrt{1 - 0.677565} = 7.41437\ \frac{\text{km}}{\text{sec}}$$

$$V_{JV_I} = V_{SJ} - V_{SV_I} = 13.0573 - 7.41437$$

$$= 5.64293\ \frac{\text{km}}{\text{sec}} = V_\infty$$

$$r_p = r_J + 200{,}000\text{ km} = 71{,}446 + 200{,}000 = 271{,}446\text{ km}$$

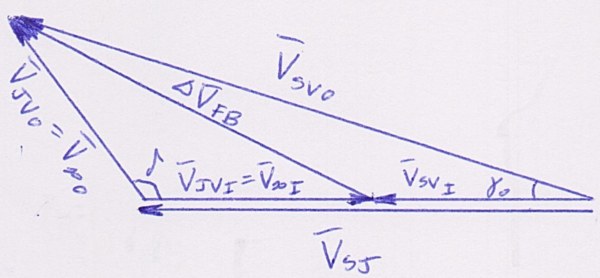
$$V_s = \sqrt{\frac{\mu_J}{r_J}} = \sqrt{\frac{(317.938)(3.986 \times 10^5)}{(11.209)(6378)}} = 42.1031\ \frac{\text{km}}{\text{sec}}$$

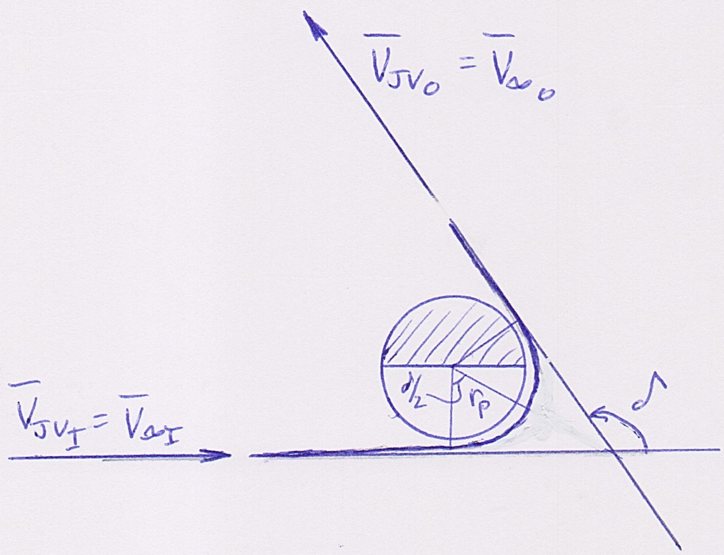
$$\psi = \left(\frac{V_\infty}{V_s}\right)^2 \left(\frac{r_{p_I}}{r_s}\right) = \left(\frac{5.64293}{42.1031}\right)^2 \left(\frac{271,446}{71,446}\right) = 0.068247$$

$$\sin\frac{\delta}{2} = \frac{1}{1+\psi} = \frac{1}{1+0.068247} = 0.936144$$

$$\frac{\delta}{2} = 68.6604^\circ \qquad \delta = 137.321^\circ$$

Correction of velocity diagram





$$\Delta V_{FB} = \left[V_{\infty_I}^2 + V_{\infty_O}^2 - 2V_{\infty_I}V_{\infty_O}\cos\delta\right]^{1/2}$$

$$= V_\infty\left[2(1-\cos\delta)\right]^{1/2}$$

$$= (5.64293)\left[2(1-\cos 137.321^\circ)\right]^{1/2}$$

$$\boxed{\Delta V_{FB} = 10.5117 \ \frac{km}{sec}}$$

$$V_{SV_I} = V_{SJ} - V_{JV_I} = 13.0573 - 5.64293 = 7.41437 \ \frac{km}{sec}$$

$$V_{SVo} = \left[V_{JVo}^2 + V_{SJ}^2 - 2 V_{JVo} V_{SJ} \cos \delta \right]^{1/2}$$

$$= \left[(5.64293)^2 + (13.0573)^2 - 2(5.64293)(13.0573) \cos 137.321° \right]^{1/2}$$

$$= 17.6259 \ \frac{km}{sec}$$

Compare with heleocentric escape speed at 5.2 au

$$V_{escape} = \sqrt{2}\, V_{SJ} = \sqrt{2}\,(13.0573) = 18.4658 \ \frac{km}{sec}$$

($V_{SVo} < V_{escape} \Rightarrow$ vehicle does not escape solar system)

Now find the heliocentric orbital elements after flyby

Flight path angle

$$\frac{V_{JVo}}{\sin \gamma_o} = \frac{V_{SVo}}{\sin \delta}$$

$$\sin \gamma_o = \frac{V_{JVo}}{V_{SVo}} \sin \delta = \frac{5.64293}{17.6259} \sin 137.321° = 0.217026$$

$$\gamma_o = 12.5344°$$

$$h_o = r_{SJ} \underbrace{V_{SV_o} \cos \gamma_o}_{V_{\theta o}}$$

$$= (5.2028 \cdot 1.495978 \times 10^8)(17.6259) \cos 12.5344°$$

$$= 1.33917 \times 10^{10} \ \frac{km^2}{sec}$$

$$V_{r_o} = V_{SV_o} \sin \gamma_o = (17.6259) \sin 12.5344°$$

$$= 3.82528 \ \frac{km}{sec}$$

Using

$$r = \frac{h^2/\mu}{1 + e\cos\theta} \Rightarrow e_o \cos\theta_o = \frac{h_o^2}{\mu_{sun} r_{SJ}} - 1$$

$$= \frac{(1.33917 \times 10^{10})^2}{(1.327 \times 10^{11})(5.2028 \cdot 1.495978 \times 10^8)} - 1$$

$$= +0.736354$$

$$V_r = \frac{\mu e}{h} \sin\theta \Rightarrow e_o \sin\theta_o = \frac{V_{ro} h_o}{\mu_{sun}}$$

$$= \frac{(3.82528)(1.33917 \times 10^{10})}{1.327 \times 10^{11}}$$

$$= +0.386036$$

$$\tan\theta_o = \frac{e_o \sin\theta_o}{e_o \cos\theta_o} = \frac{0.386036}{0.736354} = 0.524253$$

$$\theta_o = 27.6659^\circ \text{ or } 207.6659^\circ$$

θ_o must lie in the first quadrant since sine and cosine are both positive

The true anomaly in the new orbit about the sun after the flyby is

$$\theta_o = 27.6659^\circ$$

$$e_o \sin\theta_o = 0.386036$$

$$e_o = \frac{0.386036}{\sin\theta_o} = \frac{0.386036}{\sin 27.6659^\circ}$$

$$\boxed{e_o = 0.83141}$$

$$\frac{h_o^2}{\mu_{sun}} = a_o(1-e_o^2) \Rightarrow a_o = \frac{h_o^2}{\mu_{sun}(1-e_o^2)} = \frac{(1.33917\times10^{10})^2}{(1.327\times10^{11})(1-0.83141^2)}$$

$$\boxed{a_o = 4.37707\times10^9 \text{ km}}$$