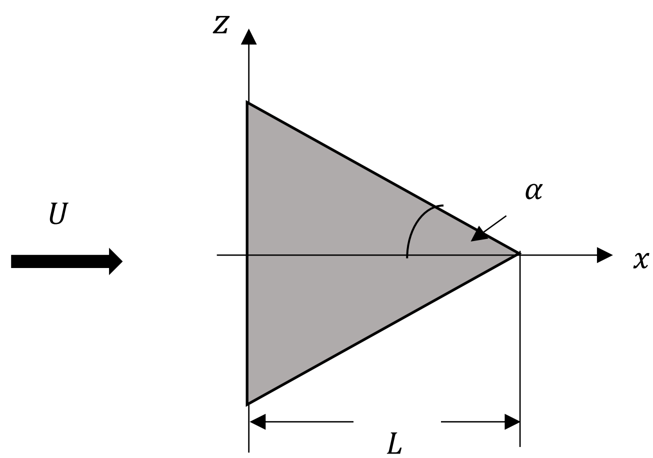
ME 55600/I0200
HW #6 Boundary Layer Theory

1. Ignoring edge effects determine the drag force over a flat isosceles triangle shown in the figure below where the equation of the upper side is given by

$$\frac{Z}{L} = \left(1 - \frac{x}{L}\right)\tan\alpha$$



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$$\tau_w = 0.332\mu U\sqrt{\frac{U}{\nu x}}$$

$$F_D = \int \tau_w dA = \int_0^L dx\left[2\int_0^{L\left(1-\frac{x}{L}\right)\tan\alpha} \tau_w dz\right]$$

$$= \int_0^L dx\left[2\int_0^{L\left(1-\frac{x}{L}\right)\tan\alpha} 0.332\mu U\sqrt{\frac{U}{\nu x}}dz\right]$$

$$= 0.664\mu U\sqrt{\frac{U}{\nu}}L\tan\alpha\int_0^L\left(1-\frac{x}{L}\right)\frac{1}{\sqrt{x}}dx$$

$$F_D = 0.886\mu UL\tan\alpha\sqrt{Re_L}$$

2. Use a parabolic velocity profile to determine the boundary layer thickness in a uniform flow of air over a flat plate.

(a) Find the boundary layer thickness, δ, at $x = 10\ cm$ for a fee stream velocity of $10\ m/s$.

(b) Determine the drag force on the plate and compare the result with Blasius Solution.

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(a) $\frac{u}{U} = a + b\eta + c\eta^2 \qquad \eta = \frac{y}{\delta} \qquad U = const$

Use Boundary Conditions to solve for the constant coefficients.

$$\begin{array}{lll} u(0) = 0 & a = 0 & \\ u(\delta) = U & b + c = 1 & c = -1 \\ \frac{\partial u}{\partial y}(\delta) = 0 & b + 2c = 0 & b = 2 \end{array}$$

The velocity profile is:

$$\frac{u}{U} = \eta(2-\eta)$$

Evaluate the momentum thickness the wall stress:

$$\frac{\Theta}{\delta} = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta = \int_0^1 \eta(2-\eta)[1-\eta(2-\eta)]d\eta = \frac{2}{15}$$

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \mu\frac{1}{\delta}\frac{\partial u}{\partial \eta}\bigg|_{\eta=0} = \frac{2\mu U}{\delta}$$

Substitute into Von Karman integral Equation:

$$\frac{2\mu U}{\delta} = U^2 \frac{2}{15}\frac{d\delta}{dx} \quad \text{or} \quad \delta d\delta = \frac{15\nu}{U} dx$$

Integrate:

$$\frac{\delta^2}{2} = \frac{15\nu x}{U} \qquad \delta = 5.48\sqrt{\frac{\nu x}{U}}$$

$$\delta = 5.48\sqrt{\frac{1.5 \times 10^{-5} 0.1}{10}} = 0.002\, m$$

(b) Integrate the shear stress at the wall with the result for δ:

$$F_D = \int_0^1 \tau_w dx = 0.729 \mu U \sqrt{Re_L}$$

Blasius coefficient is 0.664

By the way, the same approach with a third order polynomial generates a coefficient of 0.802. Higher polynomial is not necessarily more accurate.

3. The free stream velocity is given by

$$U(x) = \frac{4}{11}\sqrt{x}$$

Assume a linear velocity profile inside the boundary layer and determine the drag force acting on the plate of length L.

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The free stream velocity is given by

$$U(x) = \frac{4}{11}\sqrt{x}$$

Assuming a linear profile

$$\frac{u}{U} = a + b\eta$$

With the Boundary Conditions

$u(0) = 0 \quad a = 0$
$u(\delta) = U \quad b = 1$

The velocity profile becomes:

$$\frac{u}{U} = \eta$$

Evaluate the momentum thickness, the displacement thickness, and the shear stress at the wall:

$$\frac{\Theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta = \frac{1}{6}$$

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\eta = \frac{1}{2}$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu \frac{1}{\delta}\frac{\partial u}{\partial \eta}\bigg|_{\eta=0} = \frac{\mu U}{\delta}$$

Substitute into Von-Karman integral equation

$$\frac{\mu U}{\rho \delta} = U^2 \frac{1}{6}\frac{d\delta}{dx} + \left(\frac{\delta}{3} + \frac{\delta}{2}\right) U \frac{dU}{dx}$$

Substitute $U(x)$ and rearrange:

$$\frac{d\delta^2}{dx} + \frac{5}{x}\delta^2 = \frac{33\nu}{\sqrt{x}}$$

Using an Integrating Factor solution:

$$\frac{d}{dx}(\delta^2 x^5) = 33\nu x^{9/2}$$

Then $\qquad \delta^2 x^5 = 6\nu x^{11/2} \quad$ or $\quad \delta = \sqrt{6\nu}x^{1/4}$

Using the solution for δ in the shear stress, we get:

$$\tau_w = \frac{\mu U}{\delta} = \mu \frac{\frac{4}{11}\sqrt{x}}{\sqrt{6\nu}x^{\frac{1}{4}}} = \frac{4}{11}\sqrt{\frac{\mu\rho}{6}}x^{\frac{1}{4}}$$

Integrate to determine the Drag Force:

$$F_D = \int_0^L \tau_w dx = 0.12\sqrt{\mu\rho}L^{5/4}$$

<u>Integrating Factor solution</u>

$$\frac{dy}{dx} + a(x)y = h(x)$$

$$\frac{d}{dx}(py) = ph$$

where

$$p = e^{\int a dx}$$

Then

$$y = \frac{1}{p}\int phdx + \frac{Const}{p}$$