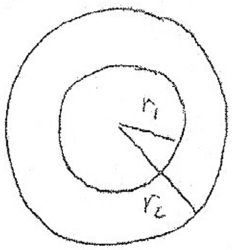
1) Two satellites are in coplanar circular orbits around the earth. Their orbit radii are r_1 and r_2. How long is it before they are separated by 90° if their radius vectors are initially coincident?



$$V_{c1} = \sqrt{\frac{\mu}{r_1}} \qquad V_{c2} = \sqrt{\frac{\mu}{r_2}}$$

$$n_1 = \frac{V_{c1}}{r_1} = \sqrt{\frac{\mu}{r_1^3}} \qquad n_2 = \frac{V_{c2}}{r_2} = \sqrt{\frac{\mu}{r_2^3}}$$

$$(n_1 - n_2)\,t = \frac{\pi}{2}$$

$$t = \frac{\pi/2}{n_1 - n_2} = \frac{\pi/2}{\sqrt{\frac{\mu}{r_1^3}} - \sqrt{\frac{\mu}{r_2^3}}}$$

$$\boxed{t = \frac{\pi}{2\sqrt{\mu}}\,\frac{(r_1 r_2)^{3/2}}{r_2^{3/2} - r_1^{3/2}}}$$

Note: The period required for the satellites to return to the separation angle they had orignally is given by

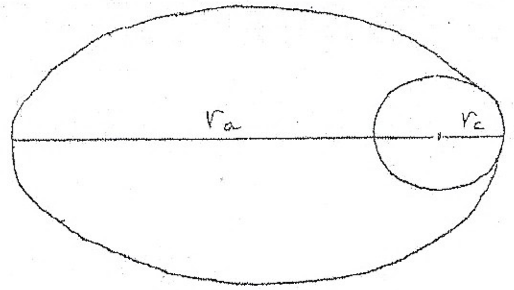
$$t = \frac{2\pi}{n_1 - n_2} = \frac{2\pi}{\sqrt{\mu}}\,\frac{(r_1 r_2)^{3/2}}{r_2^{3/2} - r_1^{3/2}}$$ and is called the <u>synodic period.</u>

2) A communications satellite launched from Cape Canaveral is placed into a 160 nautical mile high circular parking orbit inclined at 28.5° to the equator.

   a) As the vehicle crosses the equator, the upper stage is reignited to place the satellite into a highly elliptic 19,432 x 160 nautical mile transfer orbit. Determine the velocity increment needed for this maneuver.

   b) After a final checkout of the satellite, the apogee kick motor is fired on the third apogee to place the satellite into geosynchronous orbit. Determine the velocity increment needed for this maneuver.

a)



$$r_p = r_c = 160 \text{ n-mi} \left(\frac{6{,}076 \text{ ft}}{1 \text{ n-mi}}\right)\left(\frac{1 \text{ km}}{3{,}281 \text{ ft}}\right) + 6368 \text{ km} = 6664 \text{ km}$$

$r_a = 42{,}164$ km (geo-synchronous radius)

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{42{,}164 - 6664}{42{,}164 + 6664} = 0.7270$$

$$\Delta V_p = \sqrt{\frac{\mu}{r_c}}\left(\sqrt{1+e} - 1\right) = \sqrt{\frac{3.986 \times 10^5}{6664}}\left(\sqrt{1+0.7270} - 1\right)$$

$$\boxed{\Delta V_p = 2.430 \ \frac{\text{km}}{\text{sec}} = 7{,}970 \ \frac{\text{ft}}{\text{sec}}}$$

b) At apogee of transfer ellipse, orbit must be circularized and plane must be changed by $\Delta\alpha = 28.5°$

$$V_{ca} = \sqrt{\frac{\mu}{r_a}} = \sqrt{\frac{3.986 \times 10^5}{42{,}164}} = 3.0747 \ \frac{\text{km}}{\text{sec}}$$

$$V_a = V_{ca}\sqrt{1-e} = (3.0747)\sqrt{1-0.7270} = 1.6065 \text{ km/sec}$$

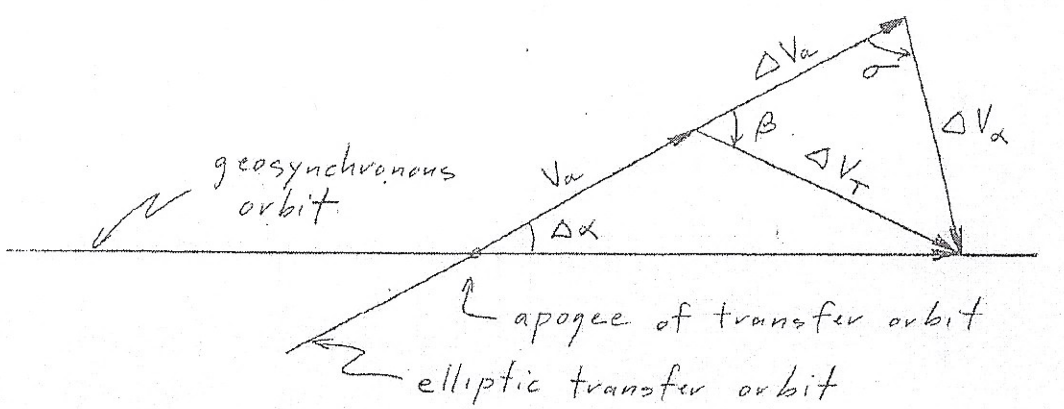
To circularize orbit:

$$\Delta V_a = V_{ca} - V_a = 3.0747 - 1.6065 = 1.468 \text{ km/sec}$$

To change plane:

$$\Delta V_\alpha = 2V_{ca}\sin\tfrac{1}{2}\Delta\alpha = 2(3.0747)\sin\tfrac{1}{2}(28.5°) = 1.514 \tfrac{\text{km}}{\text{sec}}$$

$$\sigma = 90° - \frac{\Delta\alpha}{2} = 90° - \tfrac{1}{2}(28.5°) = 75.75°$$



Total velocity increment ΔV_T required at apogee of transfer orbit for plane change and orbit circularization is

$$\Delta V_T = \left[(\Delta V_a)^2 + (\Delta V_\alpha)^2 - 2(\Delta V_a)(\Delta V_\alpha)\cos\sigma\right]^{1/2}$$
$$= \left[(1.468)^2 + (1.514)^2 - 2(1.468)(1.514)\cos 75.75°\right]^{1/2}$$

$$\boxed{\Delta V_T = 1.831 \text{ km/sec} = 6{,}010 \text{ ft/sec}}$$

Direction of applied thrust β relative to direction of flight at apogee of transfer orbit is found from

$$\frac{\Delta V_\alpha}{\sin\beta} = \frac{\Delta V_T}{\sin\sigma}$$

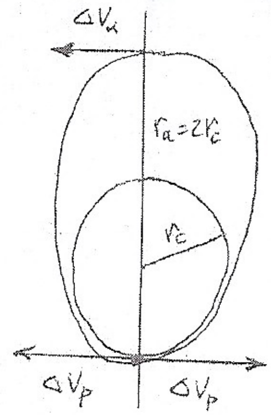
$$\sin\beta = \frac{\Delta V_\alpha}{\Delta V_T}\sin\sigma = \frac{1.514}{1.831}\sin 75.75^\circ = 0.8014$$

$$\boxed{\beta = 53.3^\circ}$$

3) Prussing and Conway, 7.11

a) Determine the cost ($\Delta V / V_0$) of a bi-elliptic transfer between circular orbits of equal radius in which the final orbit is inclined by 50° to the initial orbit and in which the radius at which the intermediate impulse is performed is twice that of the initial circular orbit.
b) Determine the flight time in units of the period of the initial circular orbit.
c) Determine the cost ($\Delta V / V_0$) of the same orbit plane change accomplished by a single impulsive maneuver.

a)



$$\Delta\alpha = 50^\circ$$

$$e = \frac{r_a - r_c}{r_a + r_c} = \frac{2r_c - r_c}{2r_c + r_c} = \frac{1}{3}$$

$$\Delta V_p = \sqrt{\frac{\mu}{r_c}}\left[\sqrt{1+e} - 1\right] = \sqrt{\frac{\mu}{r_c}}\left[\sqrt{1+\tfrac{1}{3}} - 1\right] = 0.154701\sqrt{\frac{\mu}{r_c}}$$

$$V_a = \sqrt{\frac{2\mu r_c}{r_a(r_c + r_a)}} = \sqrt{\frac{2\mu r_c}{2r_c(r_c + 2r_c)}} = \sqrt{\frac{\mu}{3r_c}}$$

$$\Delta V_\alpha = 2V_a \sin\tfrac{1}{2}\Delta\alpha = 2\sqrt{\frac{\mu}{3r_c}}\sin\tfrac{1}{2}(50^\circ) = 0.487998\sqrt{\frac{\mu}{r_c}}$$

$$\frac{\Delta V_{tot}}{V_c} = \frac{2\Delta V_p + \Delta V_\alpha}{V_c} = \frac{2(0.154701)\sqrt{\frac{\mu}{r_c}} + 0.487998\sqrt{\frac{\mu}{r_c}}}{\sqrt{\frac{\mu}{r_c}}}$$

$$\boxed{\frac{\Delta V_{tot}}{V_c} = 0.7974}$$

b) $$a = \frac{r_c + r_a}{2} = \frac{r_c + 2r_c}{2} = \frac{3}{2} r_c$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{\left(\frac{3}{2}r_c\right)^3}{\mu}} = 2\pi\left(\frac{3}{2}\right)^{3/2}\sqrt{\frac{r_c^3}{\mu}}$$

$$T_c = 2\pi\sqrt{\frac{r_c^3}{\mu}}$$

$$\frac{T}{T_c} = \frac{2\pi\left(\frac{3}{2}\right)^{3/2}\sqrt{\frac{r_c^3}{\mu}}}{2\pi\sqrt{\frac{r_c^3}{\mu}}} = \left(\frac{3}{2}\right)^{3/2} = \underline{\underline{1.83712}}$$

c) $$V_c = \sqrt{\frac{\mu}{r_c}}$$

$$\Delta V_\alpha = 2V_c \sin\tfrac{1}{2}\Delta\alpha$$

$$\frac{\Delta V_\alpha}{V_c} = 2\sin\tfrac{1}{2}\Delta\alpha = 2\sin\tfrac{1}{2}(50^\circ) = \underline{\underline{0.8452}}$$

4) The General Mission Analysis Tool (GMAT) is a powerful software developed by NASA and private industry for simulating spacecraft orbits and trajectories. A free version of the software is available which you can download from https://sourceforge.net/projects/gmat/ and install on your home computer. The software is also available on all computers in the ST-213 and ST-226 Computer Labs.

Start the software and on the Welcome page, view the tutorials and videos which are available to help you familiarize yourself with the software. As a start, view the video tutorial Part 3 and the written tutorial Simulating an Orbit (p. 18) which can be found at https://gmat.sourceforge.net/doc/R2012a/help-letter.pdf

Use GMAT to determine the ground track of a satellite in a geosynchronous orbit inclined at 30° with respect to the equator. Run the animation for a 24 hour period and observe the ground track. Print a full view copy of the 2-D window and submit it with this assignment.