City College of New York

ENGR 55500/G5300 REACTOR THERMAL-HYDRAULICS

Assignment #1 Solutions Total = 46 marks

1. To build a containment wall for a nuclear reactor, concrete has been poured to form a 1.2m thick slab. The hydration of the concrete results in the equivalent of a constant heat source of $q_o''' = 100\ \text{W/m}^3$. If both surfaces of the concrete slab are kept at 16 °C, determine the maximum temperature, T_{max}, that would be reached, assuming a steady state condition. The thermal conductivity of the wet concrete may be taken as 0.84 W/mK.

Solution Total = 10 marks

$$k\frac{\partial^2 T}{\partial x^2} + \dot{q}_G = \rho c \frac{\partial T}{\partial t}$$

For steady state, $\frac{\partial T}{\partial t} = 0$ therefore:

$$k\frac{d^2 T}{dx^2} + \dot{q}_G = 0$$

This is subject to the following boundary conditions:

1. By symmetry, dT/dx = 0 at x = 0

2. $T = T_s$ at $x = L$

Also note that for this problem $\dot{q}_G$ is a constant.
Integrating the conduction equation:

$$\frac{dT}{dx} = -\frac{\dot{q}_G}{k}x + C_1$$

The constant C_1 can be evaluated using the first boundary condition:

$$0 = -\frac{\dot{q}_G}{k}(0) + C_1 \quad \Rightarrow \quad C_1 = 0$$

Integrating once again:

$$T = -\frac{\dot{q}_G}{2k}x^2 + C_2$$

The constant C_2 can be evaluated using the second boundary condition:

$$T_s = -\frac{\dot{q}_G}{2k}L^2 + C_2 \quad \Rightarrow \quad C_2 = T_s + \frac{\dot{q}_G}{2k}L^2$$

Therefore, the temperature distribution in the dam is:

$$T = T_s + \frac{\dot{q}_G}{2k}(L^2 - x^2)$$

8 marks

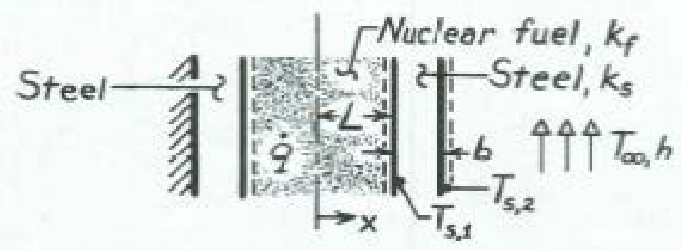
The maximum temperature occurs at x = 0:

$$T_{max} = T_s + \frac{\dot{q}_G}{2k}(L^2 - (0)^2) = 16°C + \frac{100\ \text{W/m}^3}{2(0.84\ \text{w/m K})}(0.6\ \text{m})^2 = 37°C$$

2 marks

2. A nuclear fuel element of thickness, 2L, is covered with a steel cladding of thickness b. Heat generated within the nuclear fuel at a rate q_o''' (W/m^3) is removed by a coolant at T_∞, which flows past the surface at x = L+b and is characterized by a heat transfer coefficient h. The other surface at x = -L-b is well insulated, and the fuel and steel have thermal conductivities of k_f and k_s, respectively.
   a) Obtain an expression for the temperature distribution T(x) in the nuclear fuel. Express your answer in terms of q_o''', k_f, L, b, k_s, h and T_∞.
   b) Sketch the temperature distribution T(x) for the entire system from x = -L-b to x = L+b. At what x does the maximum temperature occur?

Total = 10 marks



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k_f}=0 \qquad (-L \le x \le +L)$$

is $$T=-\frac{\dot{q}}{2k_f}x^2+C_1x+C_2.$$

The insulated wall at x = -(L+b) dictates that the heat flux at x = -L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in}=\dot{E}_{out}=0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L}=-\frac{\dot{q}}{k_f}(-L)+C_1=0 \qquad \text{or} \qquad C_1=-\frac{\dot{q}L}{k_f}$$

$$T=-\frac{\dot{q}}{2k_f}x^2-\frac{\dot{q}L}{k_f}x+C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g=q_{cond}=q_{conv}$, or on a unit area basis.

$$\dot{q}(2L)=\frac{k_s}{b}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right).$$

Hence,

$$T_{s,1}=\frac{\dot{q}(2Lb)}{k_s}+T_{s,2} \qquad \text{where} \qquad T_{s,2}=\frac{\dot{q}(2L)}{h}+T_\infty$$

$$T_{s,1}=\frac{\dot{q}(2Lb)}{k_s}+\frac{\dot{q}(2L)}{h}+T_\infty.$$

Hence from Eq. (1),

$$T(L)=T_{s,1}=\frac{\dot{q}(2Lb)}{k_s}+\frac{\dot{q}(2L)}{h}+T_\infty=-\frac{3}{2}\frac{\dot{q}\left(L^2\right)}{k_f}+C_2$$

which yields

$$C_2=T_\infty+\dot{q}L\left[\frac{2b}{k_s}+\frac{2}{h}+\frac{3}{2}\frac{L}{k_f}\right]$$

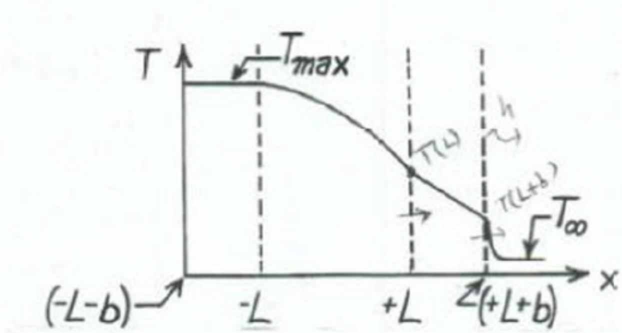
Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right] + T_\infty$$

a) 8 marks

(b) For the temperature distribution shown below,

$(-L-b) \le x \le -L$:	$dT/dx=0$, $T=T_{max}$
$-L \le x \le +L$:	$\lvert dT/dx \rvert \uparrow$ with $\uparrow x$
$+L \le x \le L+b$:	(dT/dx) is const.



b) 2 marks

3. Consider steady, one-dimensional heat conduction in a solid wall of thickness 15 cm without any internal heat generation. The thermal conductivity is not constant and varies with temperature as k = 2.0 +0.005T (W/mK), where T is in degrees Kelvin. If one surface of this wall is maintained at 150 °C and the other at 50 °C, determine the rate of heat conduction per square meter (W/m^2). Sketch the temperature distribution through the wall. Is it linear or non-linear?

No need to mark

$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$; $k = 2.0 + 0.005T$

Integrate to get $k\frac{dT}{dx}$ = constant

$= -q''$ from Fourier's Law

$-(2.0 + 0.005T)\frac{dT}{dx} = q''$

$-(2.0 + 0.005T)\,dT = q''dx$

Integrate again

$-2T - \frac{0.005}{2}T^2 = q''x + C$

Apply B.C.'s.

At $x=0$, $T = 150°C = 423K$

$-2(423) - 0.0025(423)^2 = C$

$\therefore C = -1293$

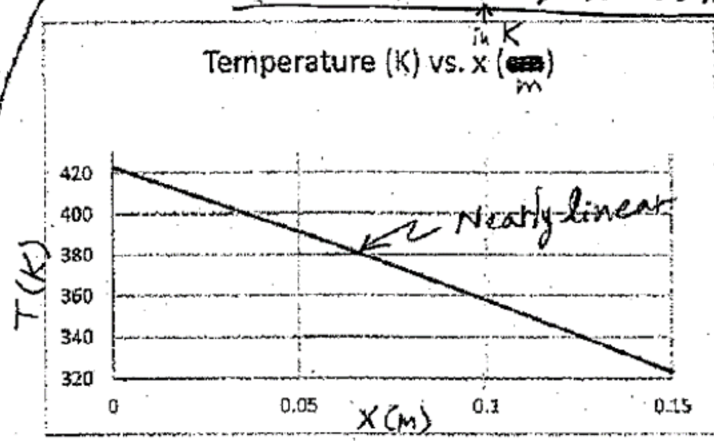
At $x = 0.15\,m$, $T = 50°C = 323K$

$-2\times 323 - 0.0025(323)^2 = q''\times 0.15 - 1293$

$q'' = 2575\ W/m^2$

$\therefore$ the temperature distribution is

$\underbrace{0.0025T^2 + 2.0T}_{\text{in } K} = \underbrace{1293 - 2575x}_{\text{in } m}$



4. Consider a shielding wall of thickness L for a nuclear reactor. The wall receives gamma-rays such that heat is generated within the wall according to the relation,

$$q''' = q_o''' e^{-\mu x}$$

where q_o''' is incident radiation flux (constant), μ is a gamma attenuation coefficient, and x is the distance from the inner surface. Using this relation, derive expressions for the temperature distribution in the wall,

(a) if both the inner and outer temperatures are maintained at T_1 at x = 0 and x = L,

(b) if the inner and outer temperatures are maintained at T_i at x = 0 and at T_o at x = L, respectively.

(c) For the temperature distribution obtained in (b), at what distance from the inner surface would the temperature be at a maximum?

Solution $q''' = q_o''' e^{-\mu x}$

Total = 10 marks

Heat conduction eq'n is $k \dfrac{d^2T}{dx^2} + q_o''' e^{-\mu x} = 0$

Integrate twice to get

$$T(x) = -\frac{q_o''' e^{-\mu x}}{k\mu^2} - C_1 x - C_2$$

a) If $T(x=0) = T(x=L) = T_1$,

$$T_1 = -\frac{q_o'''}{k\mu^2} - C_2$$

$$\therefore C_2 = -\left(T_1 + \frac{q_o'''}{k\mu^2}\right)$$

At $x = L$,

$$\cancel{T_1} = -\frac{q_o''' e^{-\mu L}}{k\mu^2} - C_1 L + \cancel{T_1} + \frac{q_o'''}{k\mu^2}$$

$$C_1 = \frac{q_o'''}{k\mu^2 L}\left(1 - e^{-\mu L}\right)$$

$$T(x) = T_1 + \frac{q_o'''}{k\mu^2}\left[\left(1 - e^{-\mu x}\right) - \frac{x}{L}\left(1 - e^{-\mu L}\right)\right]$$

a) 2 marks

b) Apply B.C.'s. $T(x=0) = T_i$ and $T(x=L) = T_o$

$$\Rightarrow T(x) = T_i + (T_o - T_i)\frac{x}{L} + \frac{q_o'''}{\mu^2 k}\left[\frac{x}{L}\left(e^{-\mu L} - 1\right) - \left(e^{-\mu x} - 1\right)\right]$$

b) 6 marks

c) Maximum temp is found at $\left.\dfrac{dT}{dx}\right|_{x = x_{max}} = 0$

$$x_{max} = -\frac{1}{\mu} \ln\left[\frac{\mu k}{q_o''' L}(T_i - T_o) + \frac{1}{\mu L}\left(1 - e^{-\mu L}\right)\right]$$

c) 2 marks

5. Consider a nuclear fuel shaped as a long slab of thickness, 7.5 cm, and with k = 12 W/m°C. The fuel generates heat internally at a rate of 10^5 W/m^3. One side of the wall at x = 0 is insulated and the other side at x = 7.5 cm is exposed to a convection environment with a heat transfer coefficient of h = 500 W/m^2°C and a fluid temperature of 90 °C. Determine the temperature profile in the slab and calculate the maximum temperature in °C assuming steady one-dimensional heat conduction.

Total = 10 marks

Heat conduction eq'n is

$$k\frac{d^2T}{dx^2} + g''' = 0 \qquad (1)$$

B.C.'s are : (i) $\frac{dT}{dx} = 0$ at $x = 0$ insulated surface

(ii) $-k\frac{dT}{dx} = h(T - T_f)$ at $x = L$ Convection

Integrate eq'n (1) once

$$\frac{dT}{dx} = -\frac{g'''}{k}x + C_1$$

Apply B.C. (i) $\quad 0 = 0 + C_1 \Rightarrow C_1 = 0$

Integrate again to get $T(x) = -\frac{g'''}{2k}x^2 + C_2$

Apply B.C. (ii) $\quad -k\left(-\frac{g'''}{k}L\right) = h\left(-\frac{g'''}{2k}L^2 + C_2 - T_f\right)$

$$\Rightarrow C_2 = \frac{g'''L}{h} + \frac{g'''}{2k}L^2 + T_f$$

$$\therefore T(x) = -\frac{g'''}{2k}x^2 + g'''L\left(\frac{1}{h} + \frac{L}{2k}\right) + T_f$$

$$= T_f + \frac{g'''}{2k}\left(L^2 + \frac{2kL}{h} - x^2\right)$$

7 marks

The max. temperature occurs at $x = 0$

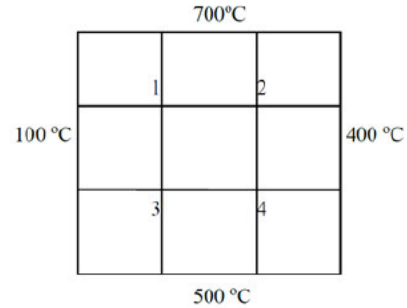
$$\therefore T_{max} = T_f + \frac{g'''}{2k}\left(L^2 + \frac{2kL}{h}\right)$$

$$= 90 + \frac{10^5}{2\times 12}\left((0.075)^2 + \frac{2\times 12\times 0.075}{500}\right)$$

$$= 128\,°C$$

3 marks

6. For the square solid without any heat generation shown on the right, numerically solve the steady state 2-D heat conduction equation for the temperatures $T_1 - T_4$. Thermal conductivity is k = 1.5 W/mK and each square mesh is 2 cm wide. The four surfaces are kept at constant temperatures as shown.



Total = 6 marks

Solution:

Since $T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0$

2 marks for this equation

For node 1, 700+100+T2+T3 – 4T1 = 0
For node 2, 700+T1+T4+400 – 4T2 = 0
For node 3, T1+100+500+T4 – 4T3 = 0
For node 4, T2+T3+500+400 – 4T4 = 0

$T_1 = 412.5$ °C
$T_2 = 487.5$ °C
$T_3 = 362.5$ °C
$T_4 = 437.5$ °C

4 marks (1 mark for each correct answer)

[redacted]