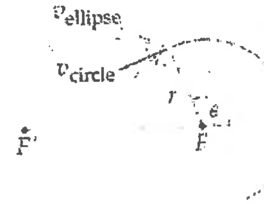
1) Curtis

2.18 Determine the true anomaly θ of the point(s) on an elliptical orbit at which the speed equals the speed of a circular orbit with the same radius (i.e., $v_{ellipse} = v_{circle}$). See the figure

{Ans.: $\theta = \cos^{-1}(-e)$, where e is the eccentricity of the ellipse}



Using the result of HW 2, prob. 1, for the elliptic orbit

$$V_{ellipse} = \frac{\mu}{h}\sqrt{1 + 2e\cos\theta + e^2} \qquad (1)$$

For the circular orbit, at the point of intersection

$$V_c = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{\frac{h^2/\mu}{1+e\cos\theta}}} = \frac{\mu}{h}\sqrt{1+e\cos\theta} \qquad (2)$$

Equate (1) to (2)

$$\cos\theta = -e \qquad (3)$$

or

$$\boxed{\theta = \cos^{-1}(-e)}$$

2) Curtis

2.19 Calculate the flight path angle at the locations found in Problem 2.18.

{Ans.: $\gamma = \tan^{-1}\left(e/\sqrt{1-e^2}\right)$}

Using eq (6.26) in the class notes

$$\tan\gamma = \frac{e\sin\theta}{1+e\cos\theta} \qquad (4)$$

Using eq. (3) from prob. (1)

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-e^2} \qquad (5)$$

Sub. (3) & (5) into (4)

$$\tan\gamma = \frac{e\sqrt{1-e^2}}{1+e(-e)}$$

$$\tan\gamma = \frac{e}{\sqrt{1-e^2}}$$

or

$$\boxed{\gamma = \tan^{-1}\left(\frac{e}{\sqrt{1-e^2}}\right)}$$

3) Curtis

2.20 An unmanned satellite orbits the earth with a perigee radius of 10,000 km and an apogee radius of 100,000 km. Calculate:

(a) the eccentricity of the orbit;
(b) the semimajor axis of the orbit (km);
(c) the period of the orbit (h);
(d) the specific energy of the orbit (km^2/s^2);
(e) the true anomaly (degrees) at which the altitude is 10,000 km;
(f) v_r and $v_\perp$ (km/s) at the points found in part (e);
(g) the speed at perigee and apogee (km/s).

{Partial Ans.: (c) 35.66 h; (e) 82.26°; (g) 8.513 km/s, 0.8513 km/s}

a) $$e = \frac{r_A - r_P}{r_A + r_P} = \frac{100,000 - 10,000}{100,000 + 10,000} = \underline{\underline{0.8182}}$$

b) $$a = \frac{r_A + r_P}{2} = \frac{100,000 + 10,000}{2} = \underline{\underline{55,000 \text{ km}}}$$

c) $$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(55,000)^3}{3.986\times10^5}} = 128,368 \text{ sec} = \underline{\underline{35.66 \text{ hr}}}$$

d) $$\varepsilon = -\frac{\mu}{2a} = -\frac{3.986\times10^5}{2(55,000)} = \underline{\underline{-3.624 \frac{km^2}{sec^2}}}$$

e) $r = 10000 + 6378 = 16{,}378 \text{ km}$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \Rightarrow \cos\theta = \frac{\frac{a}{r}(1-e^2) - 1}{e}$$

$$\cos\theta = \frac{\frac{55{,}000}{16{,}378}\left(1-(0.8182)^2\right) - 1}{0.8182} = 0.1345$$

$$\boxed{\theta = 1.436 \text{ rad} = 82.27^\circ}$$

f) $h = \sqrt{\mu p} = \sqrt{\mu a(1-e^2)} = \sqrt{(3.986\times10^5)(55{,}000)(1-(0.8182)^2)}$

$= 85{,}127 \ \frac{\text{km}^2}{\text{sec}}$

$$V_r = \frac{he\sin\theta}{a(1-e^2)} = \frac{(85{,}127)(0.8182)\sin 82.27^\circ}{55{,}000\left(1-(0.8182)^2\right)} = 3.796 \ \frac{\text{km}}{\text{sec}}$$

$$V_\theta = \frac{h}{r} = \frac{85{,}127}{16{,}378} = 5.198 \ \frac{\text{km}}{\text{sec}}$$

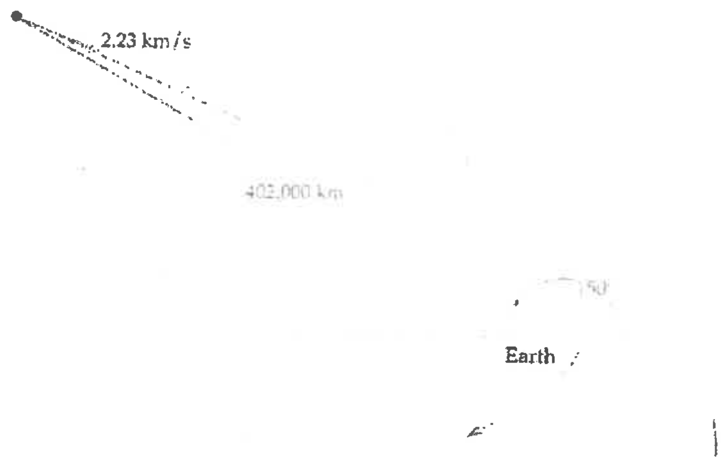
g) $V_p = \frac{h}{r_p} = \frac{85{,}127}{10{,}000} = 8.513 \ \frac{\text{km}}{\text{sec}}$

$$V_A = \frac{h}{r_A} = \frac{85{,}127}{100{,}000} = 0.8513 \ \frac{\text{km}}{\text{sec}}$$

4) Curtis

2.37 A meteoroid is first observed approaching the earth when it is 402,000 km from the center of the earth with a true anomaly of 150°, as shown in the figure below. If the speed of the meteoroid at that time is 2.23 km/s, calculate:
(a) the eccentricity of the trajectory;
(b) the altitude at closest approach;
(c) the speed at the closest approach.

{Ans.: (a) 1.086; (b) 5088 km; (c) 8.516 km/s}



a) $$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{(2.23)^2}{2} - \frac{3.986\times10^5}{402{,}000} = 1.4949 \ \frac{km^2}{sec^2}$$

Since $\mathcal{E} > 0$, trajectory is a hyperbola

$$\mathcal{E} = \frac{\mu}{2a} \Rightarrow a = \frac{\mu}{2\mathcal{E}} = \frac{3.986\times10^5}{2(1.4949)} = 133{,}320 \text{ km}$$

$$r = \frac{a(e^2-1)}{1+e\cos\theta}$$

$$r + re\cos\theta = ae^2 - a$$

$$ae^2 - (r\cos\theta)e - (r+a) = 0$$

$$e = \frac{r\cos\theta \pm \sqrt{r^2\cos^2\theta + 4a(r+a)}}{2a}$$

$$= \frac{(402,000)\cos 150° \pm \sqrt{(402,000)^2\cos^2 150° + 4(133,320)(402,000+133,320)}}{2(133,320)}$$

$$= \frac{-348,142 \pm 637,713}{266,640} = \begin{cases} 1.086 \\ -3.697 \end{cases}$$

since e must be > 1 for a hyperbola

$$\boxed{e = 1.086}$$

b) $r = \dfrac{a(e^2-1)}{1+e\cos\theta}$

$r = r_p$ at $\theta = 0°$

$$r_p = \frac{a(e^2-1)}{1+e\cos 0°} = \frac{a(e^2-1)}{1+e} = a(e-1)$$

$$= 133,320(1.086-1) = 11,466 \text{ km}$$

Altitude at perigee $= 11,466 - 6378 = \underline{\underline{5088 \text{ km}}}$

c) $h = \sqrt{\mu p} = \sqrt{\mu a(e^2-1)} = \sqrt{(3.986\times10^5)(133,320)((1.086)^2-1)}$

$h = 97,639 \ \dfrac{km^2}{sec}$

$$V_p = \frac{h}{r_p} = \frac{97,639}{11,466} = \underline{\underline{8.516 \ \frac{km}{sec}}}$$