ME 55600/I0200
Homework #4: The Marangoni Effect

Consider a liquid layer of thickness h on a solid plate. The liquid is exposed to ambient air, but the surface tension varies longitudinally along the free surface in the x direction, namely $\gamma = \gamma(x)$. The surface tension gradient is affected by a corresponding temperature gradient resulting in fluid motion along the interface.

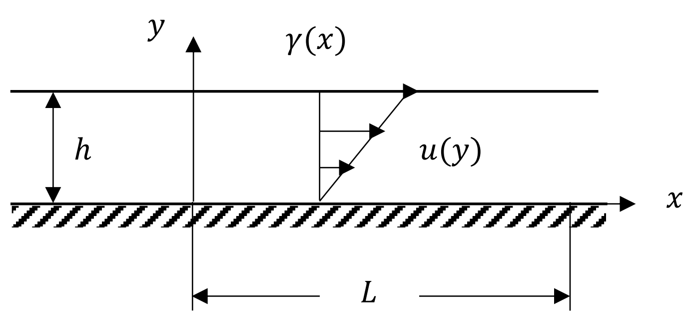
(a) Write the governing equation and boundary conditions for the liquid layer velocity $u(y)$ and determine the velocity profile.

(b) The dependence of the surface tension on the temperature T is given by the relations:

$$\gamma = \gamma_0 - kT(x)$$

where $T(x)$ is a linear temperature distribution, γ_o is the surface tension at temperature T_o and k is a positive constant.

Determine the induced velocity profile in terms of the temperature gradient for a layer of length L with $T(L) = T_L$, and the volumetric flow rate.



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Governing Equations (including gravity, but it can be neglected):

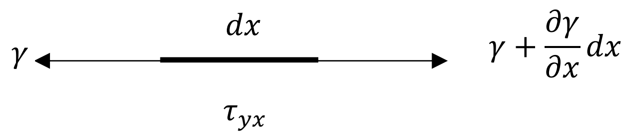
$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{\partial p}{\partial y} = -\rho g \quad (2)$$

Boundary Condition

(i) at $y = 0, u = 0.$

(ii) at $y = h$:



Using force balance on element dx, neglecting the stress on the ambient side gives at $y = h$

$$\tau_{yx} = \frac{\partial \gamma}{\partial x}$$

or

$$\mu \left. \frac{\partial u}{\partial y} \right|_{y=h} = \frac{\partial \gamma}{\partial x}$$

and for the pressure

$$p(x, h) = P_{atm}$$

Solution of (2): $$p = -\rho g y + f(x)$$

From B.C. on Pressure: $f(x) = p_{atm} + \rho g h,$ then $p = p_{atm} + \rho g(h - y),$

hence $\frac{\partial p}{\partial x} = 0.$

Solving (1): $$u = c_1 y + c_2$$

From B.C. (i): $c_2 = 0,$ from (ii): $\mu c_1 = \frac{\partial \gamma}{\partial x},$ so $c_1 = \frac{1}{\mu}\frac{\partial \gamma}{\partial x},$ and the solution for the velocity profile is:

$$u = \frac{1}{\mu}\frac{\partial \gamma}{\partial x}$$

Since $\gamma = \gamma_0 - kT$ and $T = T_0 + (T_L - T_0)\frac{x}{L}$ then, $\frac{\partial \gamma}{\partial x} = \frac{\partial \gamma}{\partial T}\frac{\partial T}{\partial x} = -\gamma \frac{T_L - T_0}{L}$

Therefore,

$$u = \frac{\gamma(T_0 - T_L)}{\mu L} y$$

and the flow rate is:

$$Q = \int_0^h u dy = \frac{\gamma h^2}{2\mu L}(T_0 - T_L)$$