Orbital Mechanics – Equations and Algorithms – By: Matt Rulli (Revision 1)

Kepler's Laws #1) Orbits are elliptical with the orbited body at one focus. #2) The radial vector sweeps out equal areas in equal time. #3) $T^2 \propto a^3$	$\mu_{earth} = 3.986 \times 10^5 \ \frac{km^3}{s^2}$ $radius_{earth} = 6.37812 \times 10^3 \ km$ $mass_{earth} = 5.974 \times 10^{24} \ kg$	$\mu_{sun} = 1.327 \times 10^{11} \ \frac{km^3}{s^2}$ $radius_{sun} = 6.9599 \times 10^5 \ km$ $mass_{sun} = 1.989 \times 10^{30} \ kg$	$AU = 1.495978 \times 10^8 \ km$ $G = 6.6743 \times 10^{-11} \ \frac{N \cdot m^2}{kg^2}$ $F_{12} = \frac{G m_1 m_2}{r}$	Where: $\vec{\boldsymbol{a}} = a_x \cdot \hat{\imath} + a_y \cdot \hat{\jmath} + a_z \cdot \hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = b_x \cdot \hat{\imath} + b_y \cdot \hat{\jmath} + b_z \cdot \hat{\boldsymbol{k}}$ Dot Product: $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} = (a_x b_x) + (a_y b_y) + (a_z b_z)$ Cross Product: $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = (a_y b_z - a_z b_y) \cdot \hat{\imath} + (a_z b_x - a_x b_z) \cdot \hat{\jmath} + (a_x b_y - a_y b_x) \cdot \hat{\boldsymbol{k}}$

Orbits — **Maneuvers and Transfers**

Elliptical $(0 < e < 1)\ (a^+)\ (\varepsilon^-)$	**Circular** $(e = 0)\ (a = r)\ (\varepsilon^-)$	**Energy and Velocity**	**Coplanar Hohmann Transfer** *Given r_i and r_f*	**Combined Hohmann Transfer (with plane change)**
$r = \frac{a(1-e^2)}{1 + e \cdot cos(f)} = \frac{p}{1 + e \cdot cos(f)}$	$v = \sqrt{\frac{\mu}{r}} \quad a = r_p = r_a$	$E = \frac{1}{2}\lVert\vec{v}\rVert^2 - \frac{\mu}{\lVert\vec{r}\rVert} = -\frac{\mu}{2a}$	$\Delta v_1 = v_{trans,p} - v_i = \sqrt{2\mu\left(\frac{r_f}{r_i(r_i + r_f)}\right)} - \sqrt{\frac{\mu}{r_i}}$	$\Delta v = \sqrt{v(t_k^-)^2 + v(t_k^+)^2 - 2v(t_k^-)v(t_k^+)\cos(\Delta\theta)}$
$r_p = a(1-e) = \frac{p}{1+e} = \frac{h^2}{\mu(1+e)}$	**Hyperbolic** $(e > 1)\ (a^-)\ (\varepsilon^+)$	$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \quad for \ v \leftrightarrow a$	$\Delta v_2 = v_f - v_{trans,a} = \sqrt{\frac{\mu}{r_f}} - \sqrt{2\mu\left(\frac{r_i}{r_f(r_i + r_f)}\right)}$	Plane Change on Initial Burn $v(t_k^-) = v_i = \sqrt{\frac{\mu}{r_i}} \quad v(t_k^+) = v_{trans,p} = \sqrt{2\mu\left(\frac{r_f}{r_i(r_i + r_f)}\right)}$
$r_a = a(1+e) = \frac{p}{1-e} = \frac{h^2}{\mu(1-e)}$	$\delta = \pi - 2\cos^{-1}\left(\frac{1}{e}\right)$	$v_p = \frac{(\mu p)^{\frac{1}{2}}}{r_p} = (\mu p)^{\frac{1}{2}}\left(\frac{1+e}{p}\right)$	For Transfer Orbit $a = \frac{r_i + r_f}{2} \quad e = 1 - \frac{r_p}{a} = \frac{r_f - r_i}{r_f + r_i}$	Plane Change on Final Burn $v(t_k^-) = v_{trans,a} = \sqrt{2\mu\left(\frac{r_f}{r_i(r_i + r_f)}\right)} \quad v(t_k^+) = v_f = \sqrt{\frac{\mu}{r_f}}$
$r_p = \frac{h}{v_p} = \frac{(\mu p)^{\frac{1}{2}}}{v_p}$	$\delta = 2\sin^{-1}\left(\frac{1}{e}\right)$	$v_{excess} = \lim_{r \to \infty} v(r) = \sqrt{\frac{-\mu}{a}}$	$\theta_{FPA} = \pi - \sin^{-1}\left(\frac{v_f \sin(\Delta\theta)}{\Delta v_{2_{Pln\ Chng}}}\right)$	**Combined Bielliptic Transfer (with plane change)**
$r_{parameter} = a(1-e^2) \quad \left[f = \frac{\pi}{2}\right]$	$\varepsilon = -\frac{\mu}{2a}$	$v_{escape} = \sqrt{\frac{2\mu}{r}}$	$\Delta t = \frac{T}{2} = \pi\sqrt{\frac{a^3}{\mu}} = \pi\sqrt{\frac{(r_i + r_f)^3}{8\mu}}$	$\Delta v = \sqrt{v(t_k^-)^2 + v(t_k^+)^2 - 2v(t_k^-)v(t_k^+)\cos(\Delta\theta)}$
$a = \frac{r_p + r_a}{2} = \frac{p}{1-e^2}$	$a = \frac{h^2}{\mu(1-e^2)}$	$v_{circular} = \sqrt{\frac{\mu}{r_c}}$	**Bielliptic Transfer Orbits**	Plane Change 1st Burn $v(t_k^-) = v_i = \sqrt{\frac{\mu}{r_i}} \quad v(t_k^+) = v_{t1,p} = \sqrt{2\mu\left(\frac{r_*}{r_i(r_i + r_*)}\right)}$
$b = a(1-e^2)^{\frac{1}{2}} = (r_p r_a)^{\frac{1}{2}}$	$r = \frac{h^2}{\mu(1 + e\cos f(t))} = \frac{a(e^2-1)}{1 + e\cos(f)}$	$\left.\begin{matrix} v_p = \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)} \\ v_a = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)} \end{matrix}\right\} v, a \to e$	$a_1 = \frac{r_i + r_*}{2} \quad e_1 = \frac{r_* - r_i}{r_* + r_i} \quad , \quad a_2 = \frac{r_f + r_*}{2} \quad e_2 = \frac{r_* - r_f}{r_* + r_f}$	Plane Change 2nd Burn $v(t_k^-) = v_{t1,a} = \sqrt{2\mu\left(\frac{r_*}{r_*(r_i + r_*)}\right)} \quad v(t_k^+) = v_{t2,a} = \sqrt{2\mu\left(\frac{r_f}{r_*(r_f + r_*)}\right)}$
$p = a(1-e^2) = \frac{h^2}{\mu}$	$r_p = a(1-e) = \frac{p}{1+e} = \frac{h^2}{\mu(1+e)}$		$\Delta v_1 = v_{t1,p} - v_i = \sqrt{2\mu\left(\frac{r_*}{r_i(r_i + r_*)}\right)} - \sqrt{\frac{\mu}{r_i}}$	Plane Change 3rd Burn $v(t_k^-) = v_{t2,p} = \sqrt{2\mu\left(\frac{r_*}{r_f(r_f + r_*)}\right)} \quad v(t_k^+) = v_f = \sqrt{\frac{\mu}{r_f}}$
$M(t) = nt = \sqrt{\frac{\mu}{a^3}} \cdot t$	$M(t) = \sqrt{\frac{\mu}{-a^3}} \cdot t = e \cdot \sinh(H) - H$	**Angular Momentum**	$\Delta v_2 = v_{t2,a} - v_{t1,a} = \sqrt{2\mu\left(\frac{r_f}{r_*(r_f + r_*)}\right)} - \sqrt{2\mu\left(\frac{r_i}{r_*(r_i + r_*)}\right)}$	**Pure Inclination Change**
$\tan\left(\frac{E(t)}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{f(t)}{2}\right)$	$\tanh\left(\frac{H(t)}{2}\right) = \sqrt{\frac{e-1}{e+1}}\tan\left(\frac{f(t)}{2}\right)$	$\vec{h} = \vec{r} \times \vec{v}$	$\Delta v_3 = v_f - v_{t2,p} = \sqrt{\frac{\mu}{r_f}} - \sqrt{2\mu\left(\frac{r_*}{r_f(r_f + r_*)}\right)}$	$\Delta v = 2v\sin\left(\frac{\theta}{2}\right) \ \to \ For\ circular\ orbit$ Thrust direction $= \frac{\pi}{2} + \frac{\theta}{2} \ \to \ if\ \theta = \Delta i$
$\tan\left(\frac{f(t)}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E(t)}{2}\right)$	$\tan\left(\frac{f(t)}{2}\right) = \sqrt{\frac{e+1}{e-1}}\tanh\left(\frac{H(t)}{2}\right)$	$h = [\mu a(1-e^2)]^{\frac{1}{2}} \ (ellipse)$	$\Delta t = \pi\sqrt{\frac{a_1^3}{\mu}} + \pi\sqrt{\frac{a_2^3}{\mu}} \ \to \ Longest\ Time, Lowest\ Energy$	$\Delta v = \frac{2\sin\left(\frac{\Delta i}{2}\right)\sqrt{1-e^2}\cos(\omega + f)\,na}{1 + e\cos f} \ \to \ For\ generic\ inlination$
Parabolic $(e = 1)$	**Kepler's 3rd Law**	$h = r_p v_p = (\mu p)^{\frac{1}{2}}$	**Hohmann Rendezvous Maneuver** $\theta_0 = Relative\ Phase\ Angle\ between\ vehicles\ at\ \Delta v_1$	**Oberth Effect** Most efficient to burn fuel at periapsis – Does NOT hold for inclination change
$\varepsilon = -\frac{\mu}{2a} \ , \ v_{excess} = 0 \ , \ \delta = 180°$	$T = \frac{2\pi ab}{h} = 2\pi\sqrt{\left(\frac{a^3}{\mu}\right)}$	$h^2 = \mu(r + \vec{r} \cdot \vec{e})$	$\theta_0 = \pi - 2\pi\left(\frac{T_{trans}}{T_{tgt}}\right) = \pi - 2\pi\left(\frac{a_{trans}^3}{r_{tgt}^3}\right)^{\frac{1}{2}}$	$\Delta KE = \frac{1}{2}\Delta v^2 + v \cdot \Delta v \qquad KE_{propellant} = (v_{space\ craft} - \Delta v_{exhaust})^2$
$r_p = \frac{h^2}{2\mu} = a(1-e) = \frac{p}{2}$	$\frac{dA}{dt} = \frac{1}{2}r^2\frac{df}{dt} = \frac{h}{2} = constant$	$\vec{e} = \frac{1}{\mu}\left(\vec{v} \times \vec{h} - \mu\frac{\vec{r}}{\lVert\vec{r}\rVert}\right)$	$Transit\ time = \Delta t_{int} = \pi\sqrt{\frac{a_{trans}^3}{\mu}} = \pi\sqrt{\frac{(r_i + r_f)^3}{8\mu}}$	$PE_{propellant} = -\sqrt{\frac{\mu}{r}}$

Minimum Δv to Change Inclination

Given: a, e, Ω, ω, i *Find r_p and r_a:* $r_a = a(1+e) \qquad r_p = a(1-e)$ *Find node furthest from perigee:* *Node A:* $f_A(t) = 2\pi - \omega$ *Node B:* $f_B(t) = \pi - \omega$ *Find h:* $\sqrt{2\mu}\sqrt{\frac{r_a r_p}{(r_a + r_p)}}$	*Compute r_A and r_B:* $r_A = \frac{h^2}{\mu}\frac{1}{1 + e\cos(f_A)}$ $r_B = \frac{h^2}{\mu}\frac{1}{1 + e\cos(f_B)}$ *Compute perpendicular velocities:* $v_{\perp_A} = \frac{h}{r_A} \qquad v_{\perp_B} = \frac{h}{r_B}$	*Compute minimum Δv:* *If $v_{\perp_B} < v_{\perp_A}$:* $\Delta v_{min} = 2v_{\perp_B}\sin\left(\frac{\Delta i}{2}\right)$ *If $v_{\perp_A} < v_{\perp_B}$:* $\Delta v_{min} = 2v_{\perp_A}\sin\left(\frac{\Delta i}{2}\right)$	$\theta_{FPA} = \pi - \sin^{-1}\left(\frac{v(t_k^+)\sin(\Delta\theta)}{\Delta v_{plane\ change}}\right)$	

Algorithms

Given	r_p and T		Orbital Elements *Given $\vec r_1$ and $\vec v_1$:*	Propagate Orbit in Time *Given $\vec r_1, \vec v_1$, t, and elements:*		
a	$T = 2\pi\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}} \rightarrow a = \left[\mu\left(\frac{T}{2\pi}\right)^2\right]^{\frac{1}{3}}$		$r_1 = \sqrt{\vec r_{1x}^{\,2} + \vec r_{1y}^{\,2} + \vec r_{1z}^{\,2}}$ or $r_1 = \sqrt{\vec r_1 \cdot \vec r_1}$ $v_1 = \sqrt{\vec v_{1x}^{\,2} + \vec v_{1y}^{\,2} + \vec v_{1z}^{\,2}}$ or $v_1 = \sqrt{\vec v_1 \cdot \vec v_1}$	[ELLIPTICAL] $E(t) = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{f}{2}\right)\right)$	[HYPERBOLIC] $H(t) = 2\tanh^{-1}\left(\sqrt{\frac{e-1}{e+1}}\tan\left(\frac{f(t)}{2}\right)\right)$	
r_a	$a = \frac{r_p + r_a}{2} \rightarrow r_a = 2a - r_p$		$v_r = \frac{\vec v_1 \cdot \vec r_1}{r}$ $v_r > 0$ *away from peri* $v_r < 0$ *towards peri*	$M(t) = E(t) - e\sin(E(t))$	$M(t) = e\sinh(H(t)) - H(t)$	
	r_p and v_p		$\vec h = \vec r_1 \times \vec v_1$ $h = \sqrt{\vec h_x^2 + \vec h_y^2 + \vec h_z^2}$ or $h = \sqrt{\vec h \cdot \vec h}$	$t_1 = \frac{E(t) - e\sin(E(t))}{\frac{2\pi}{T}}$	$t_1 = \frac{e\sinh(H(t)) - H(t)}{\sqrt{\frac{\mu}{-a^3}}}$	
h	$r_p = \frac{h}{v_p} \rightarrow h = r_p v_p$		$i = \cos^{-1}\left(\frac{\vec h_z}{h}\right) \rightarrow$ *if* $i > \frac{\pi}{2}$ *Orbit retrograde*	$\Delta t = t_1 + t$ *where* t = *coast time* $n_{rotation} = \frac{\Delta t}{T}$ $t_2 = T \cdot (n_{rotation} - round(n_{rotation}))$	$t_2 = t_1 + t$ *where* t = *coast time*	**Earth-Centered Inertial**
p	$p = \frac{h^2}{\mu}$		$\vec N = [0\ 0\ 1] \times \vec h$ $N = \sqrt{\vec N_x^2 + \vec N_y^2 + \vec N_z^2}$ or $N = \sqrt{\vec N \cdot \vec N}$	$M(t_2) = \sqrt{\frac{\mu}{a^3}} \cdot t_2$ [*MUST be between* 0 *and* 2π]	$M(t_2) = \sqrt{\frac{\mu}{-a^3}} \cdot t_2$	$\hat x \rightarrow \hat I =$ ***FPOA*** $\hat y \rightarrow \hat J =$ ***RHR*** $\hat z \rightarrow \hat K =$ ***N.Pole***
e	$r_p = \frac{p}{1+e} \rightarrow e = \frac{p}{r_p} - 1$		$\Omega = \cos^{-1}\left(\frac{\vec N_x}{N}\right) \rightarrow$ *if* $\vec N_y < 0$, $\Omega = 2\pi - \Omega$	$E_1 = M$ $E_{k+1} = E_k - \left(\frac{M - E_k + e\sin(E_k)}{e\cos(E_k) - 1}\right)$	$H_1 = M$ $H_{k+1} = H_k + \frac{(M - e \cdot \sinh(H_k) + H_k)}{e \cdot \cosh(K_k) - 1}$	
a	$p = a(1-e^2) \rightarrow a = \frac{p}{(1-e^2)}$		$\vec e = \frac{1}{\mu}\left(\left(v^2 - \frac{\mu}{r}\right) \cdot \vec r_1 - r \cdot v_r \cdot \vec v_1\right)$ $e = \sqrt{\vec e_x^2 + \vec e_y^2 + \vec e_z^2}$ or $e = \sqrt{\vec e \cdot \vec e}$	*Repeat while:* $\lvert M - E_k + e\sin(E_k)\rvert >$ *desired error*	*Repeat while:* $\left\lvert\frac{M - e \cdot \sinh(H_k) + H_k}{e\cosh(H_k) - 1}\right\rvert >$ *desired error*	**Perifocal Coordinates**
	Elliptical (0 < e < 1)	**Hyperbolic (e > 1)**	$\omega = \cos^{-1}\left(\frac{\vec N \cdot \vec e}{N \cdot e}\right) \rightarrow$ *if* $\vec e_z < 0$, $\omega = 2\pi - \omega$	$f(t_2) = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E_k}{2}\right)\right)$	$f(t_2) = 2\tan^{-1}\left(\sqrt{\frac{e+1}{e-1}}\tanh\left(\frac{H_k}{2}\right)\right)$	$\hat x \rightarrow \hat P = \frac{\vec e}{e}$ $\hat y \rightarrow \hat Q =$ ***RHR*** $\hat z \rightarrow \widehat W = \frac{\vec h}{h}$
$f(t)$	$f(t) = \cos^{-1}\left(\left(\frac{1}{e}\right)\left[\frac{p}{r} - 1\right]\right)$	$f(t) = \cos^{-1}\left(\left(\frac{1}{e}\right)\left[\frac{p}{r} - 1\right]\right)$	$f = \cos^{-1}\left(\frac{\vec e \cdot \vec r_1}{e \cdot r}\right) \rightarrow$ *if* $v_r < 0$, $f = 2\pi - f$	$r_2 = \frac{a(1-e^2)}{1 + e\cos(f(t_2))}$	$r_2 = \frac{a(e^2-1)}{1 + e\cos(f)}$	
$E(t)$	$E(t) = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{f}{2}\right)\right)$	$H(t) = 2\tanh^{-1}\left(\sqrt{\frac{e-1}{e+1}}\tan\left(\frac{f}{2}\right)\right)$	$r_p = \frac{h^2}{\mu}\left(\frac{1}{1+e}\right)$ *and* $r_a = \frac{h^2}{\mu}\left(\frac{1}{1-e}\right)$	$v_2 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$	$v_2 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$	
$M(t)$	$M(t) = E(t) - e \cdot \sin(E(t))$	$M(t) = e\sinh(H(t)) - H(t)$	$a = \frac{r_p + r_a}{2} = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1}$	$\vec r_{2_{ECI}} = \begin{bmatrix} r_2(\cos(\Omega)\cos(\omega + f(t_2)) - \sin(\Omega)\sin(\omega + f(t_2))\cos(i)) \\ r_2(\sin(\Omega)\cos(\omega + f(t_2)) + \cos(\Omega)\sin(\omega + f(t_2))\cos(i)) \\ r_2(\sin(\omega + f(t_2))\sin(i)) \end{bmatrix}$		$\vec r_{2_{PQW}} = \begin{bmatrix} r_2\cos(f(t_2)) \\ r_2\sin(f(t_2)) \\ 0 \end{bmatrix}$
t	$M(t) = \sqrt{\frac{\mu}{a^3}} \cdot t \rightarrow t = \frac{M(t)}{\sqrt{\frac{\mu}{a^3}}}$	$M(t) = \sqrt{\frac{\mu}{-a^3}}\, t \rightarrow t = \frac{M(t)}{\sqrt{\frac{\mu}{-a^3}}}$	$T = \left(\frac{2\pi}{\sqrt{\mu}}\right)a^{\frac{3}{2}}$	$\vec v_{2_{ECI}} = \begin{bmatrix} \left(-\frac{\mu}{h}\right)(\cos(\Omega)(\sin(\omega + f(t_2)) + e\sin(\omega)) + \sin(\Omega)(\cos(\omega + f(t_2)) + e\cos(\omega))\cos(i)) \\ \left(-\frac{\mu}{h}\right)(\sin(\Omega)(\sin(\omega + f(t_2)) + e\sin(\omega)) - \cos(\Omega)(\cos(\omega + f(t_2)) + e\cos(\omega))\cos(i)) \\ \left(\frac{\mu}{h}\right)(\cos(\omega + f(t_2)) + e\cos(\omega))\sin(i) \end{bmatrix}$		$\vec v_{2_{PQW}} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\sin(f(t_2)) \\ \sqrt{\frac{\mu}{p}}(e + \cos(f(t_2))) \\ 0 \end{bmatrix}$

Lambert's Problem: Interception

Given $r_i(t_1)$, $v_i(t_1)$ and $r_t(t_1)$, $v_t(t_1)$ and TOF: Determine **Orbital Elements** of interceptor and target orbits. (see procedure above)	**Propagate** target orbit to find location of interception point by iterating Newton's method. (see procedure above)	**Lambert's Problem:** Parameters $c = \lvert\vec r_{tgt}(t_2) - \vec r_{int}(t_1)\rvert$ $s = \frac{c + r_{int}(t_1) + r_{tgt}(t_2)}{2}$ **Check minimum TOF!** $TOF \geq \Delta t_p = \frac{\sqrt{2}}{3}\sqrt{\frac{s^3}{\mu}}\left(1 - \left(\frac{s-c}{s}\right)^{\frac{3}{2}}\right)$	Set initial value for a: $a_{min} = \frac{s}{2}$ $a_{max} = ks \rightarrow$ (*where* $k \geq 2+$) $a = \frac{a_{max} + a_{min}}{2}$	Iterate g(a): $g(a) = \sqrt{\frac{a^3}{\mu}}(\alpha - \beta - (\sin(\alpha) - \sin(\beta)))$ $\alpha = 2\sin^{-1}\left(\sqrt{\frac{s}{2a}}\right)$, $\beta = 2\sin^{-1}\left(\sqrt{\frac{s-c}{2a}}\right)$	Check: *If* $g(a) > TOF \rightarrow a_{min} = a$ *If* $g(a) < TOF \rightarrow a_{max} = a$ Recalculate "a" with new min/max value and repeat previous step until desired tolerance is met.	Velocity vectors: $A = \sqrt{\frac{\mu}{4a}}\cot\left(\frac{\alpha}{2}\right)$, $B = \sqrt{\frac{\mu}{4a}}\cot\left(\frac{\beta}{2}\right)$ $\hat u_i = \frac{\hat r_i(t_1)}{r_i(t_1)}$, $\hat u_t = \frac{\hat r_t(t_2)}{r_t(t_2)}$, $\hat u_c = \frac{\hat r_t(t_2) - \hat r_i(t_1)}{c}$ $\vec v_{trans}(t_1) = (B + A)\,\hat u_c + (B - A)\,\hat u_i$ $\vec v_{trans}(t_2) = (B + A)\,\hat u_c - (B - A)\,\hat u_t$ $\Delta v_{int} = \vec v_{trans}(t_1) - \vec v_i(t_1)$

Lambert's Problem: Targeting

Given: r_1 , r_2 , *angle, and TOF* $\rightarrow c = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\Delta f)}$ Solve for semi-perimeter (s), and then find (a) using bisection method above.	Given: $\vec r_1$, $\vec r_2$, *and* v (*magnitude*) *at one point,* Find position magnitude at that point and solve for (a) using vis-viva: $a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1} \rightarrow \alpha, \beta \rightarrow g(a) = \Delta t$			