

Question 1 – Prove Kepler’s second law (numerically)

a) The object chosen for this analysis is a comet orbiting the sun called Swift-Tuttle (109P). JPL Horizons [1] was utilized to obtain vector and orbital element data for a time range from June 15th, 1737, to August 23rd, 1862 – about 125 years. According to Nasa’s small-body lookup database [2], the eccentricity of Swift-Tuttle’s orbit is around 0.96, which is highly eccentric. Some of the Swift-Tuttle’s orbital elements are shown in **Table 1**.

Swift-Tuttle Orbital Elements	
Eccentricity	0.96
Inclination (w.r.t. the ecliptic plane) [deg]	113.45
Perihelion [AU]	0.96
Aphelion [AU]	51.22

Table 1: Orbital Elements of Swift-Tuttle's orbit.

With the range and true anomalies of the Swift-Tuttle at specific time times obtained from JPL Horizons, we can pick a smaller time range to prove Kepler’s 2nd Law, which is that “A line connecting a planet to the Sun sweeps out equal areas in equal times”. In this case, instead of a planet we will be using a comet. A date range of 240 months (20 years) was chosen. The equation to numerically estimate the swept area of an ellipse is given by **Equation 1**.

$$\Delta A = \frac{1}{2} \mathbf{r}' * (\mathbf{r} \sin(\Delta\theta)) \quad (1)$$

where \mathbf{r}' is the final range (distance from comet to sun), \mathbf{r} is the initial range, and $\Delta\theta$ is the difference in the true anomalies from the initial and final time, which can be defined as:

$$\Delta\theta = \theta' - \theta \quad (2)$$

The first 20-year range was from 1739 to 1759 (starting and ending on February 2nd), and the swept area was approximated to **4931.61 AU²**. The second 20-year range was from 1829 to 1849 (starting and ending on February 2nd), and the swept area was approximated to **4967.84 AU²**. These values are, although varying by 31 AU² in calculated area, are close enough to prove Kepler’s 2nd Law. The percent error from the second and first area is a mere **0.73%**. Reasons for this error is due to data uncertainties and the nature of numerical methods being only approximations of a true value.

b) **Figure 1** shows four plots. The first plot (top left) shows Swift-Tuttle's orbit around the sun with earth's orbit as reference. The second plot (top right) is a zoomed in version of the first plot to better see earth's orbit compared to Swift-Tuttle's. We can see that the perihelion of the comet is in fact a little smaller than the radius of earth's orbit, which is 1 AU. The third plot (bottom left) represents the calculated swept area for the first-time range, and the fourth plot (bottom right) represents the calculated swept area for the second-time range. The reason for plotting the orbit in 3D is because the inclination of Swift-Tuttle's orbit is off of the Sun-Earth ecliptic plane. The full orbit of the comet cannot be seen in just two dimensions.

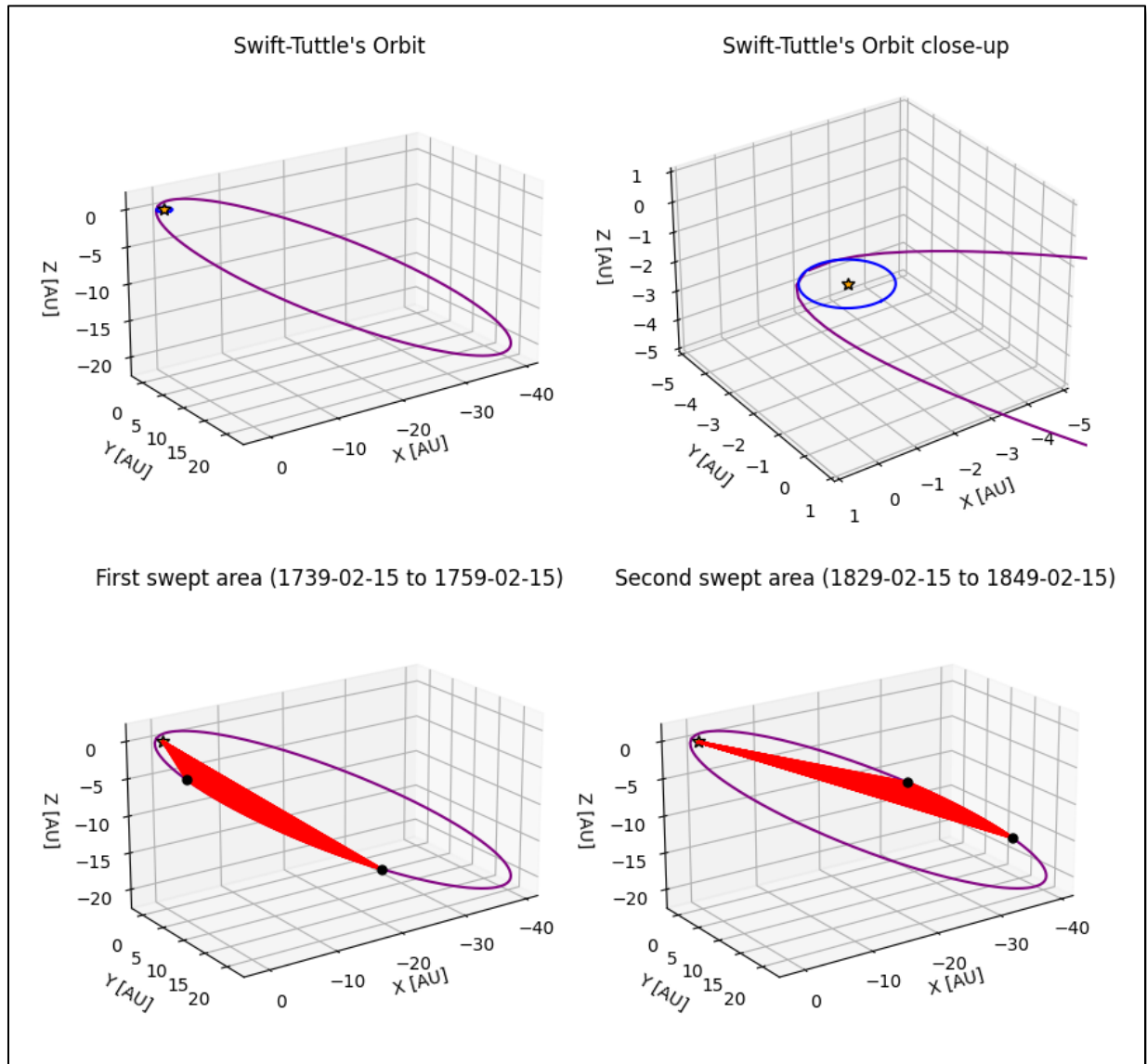


Figure 1: Plots of Swift-Tuttle's orbit. The view angle is defined as 20 deg elevation, 55 deg azimuth, and 0 deg roll.

Question 2 – (Not) to Scale

The image shown in **Figure 2** shows the sun and the planets of the solar system (image was pulled from Google). Although the order of the planets are correct, the sizes and the distances are highly inaccurate and not scaled properly.

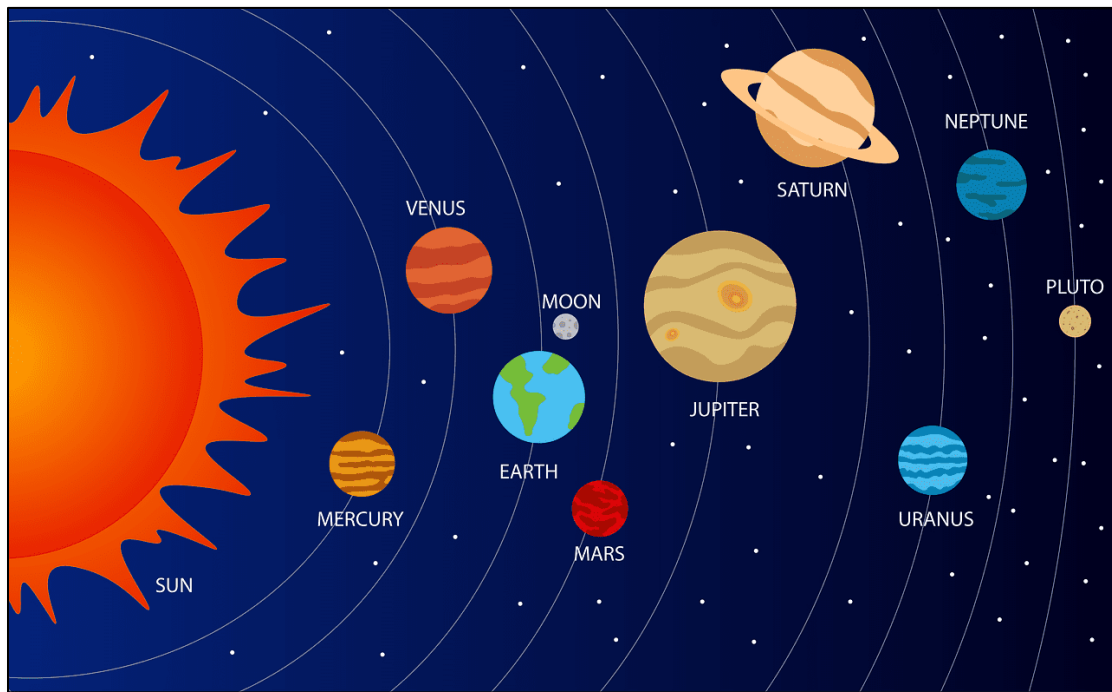


Figure 2: Model of Solar System planets and Sun

The sizes of the planets are obviously not scaled (Earth should not be more than half the size of Jupiter). To put it into perspective, the distance between the Earth and Sun is 1 AU. From NASA's JPL spec sheet [3], the distances of the planets and the sun are shown in **Table 2**.

Distances of planets to the Sun [AU]	
Mercury	0.4
Venus	0.7
Earth	1.0
Mars	1.5
Jupiter	5.2
Saturn	9.6
Uranus	19.2
Neptune	30.0
Pluto	39.5 (average)

Table 2: Distances of solar system planets to the sun, in astronomical units. Pluto is included as well.

We can compare the distance from the sun of Jupiter compared to that of Mars – almost 5 times in magnitude, yet in **Figure 2** their orbits are relatively close. The jump from Uranus and Pluto is large as well, yet in the figure the distances between their orbits, and thus their distances to the sun are almost equivalent when comparing the planets' distances to their neighbors' distances to the sun.

Question 3 – IM1 Mission

a) The IM1 mission launched on February 15th, 2024, at approximately 06:06:47 TDB. The IM1 vector data was obtained from JPL Horizons [1], with data from launch until February 22nd, 2024, and approximately 22:50:09 TDB. The trajectory of IM1 and the Moon's orbit, both with respect to the Earth-Moon barycenter, is shown in **Figure 3**.

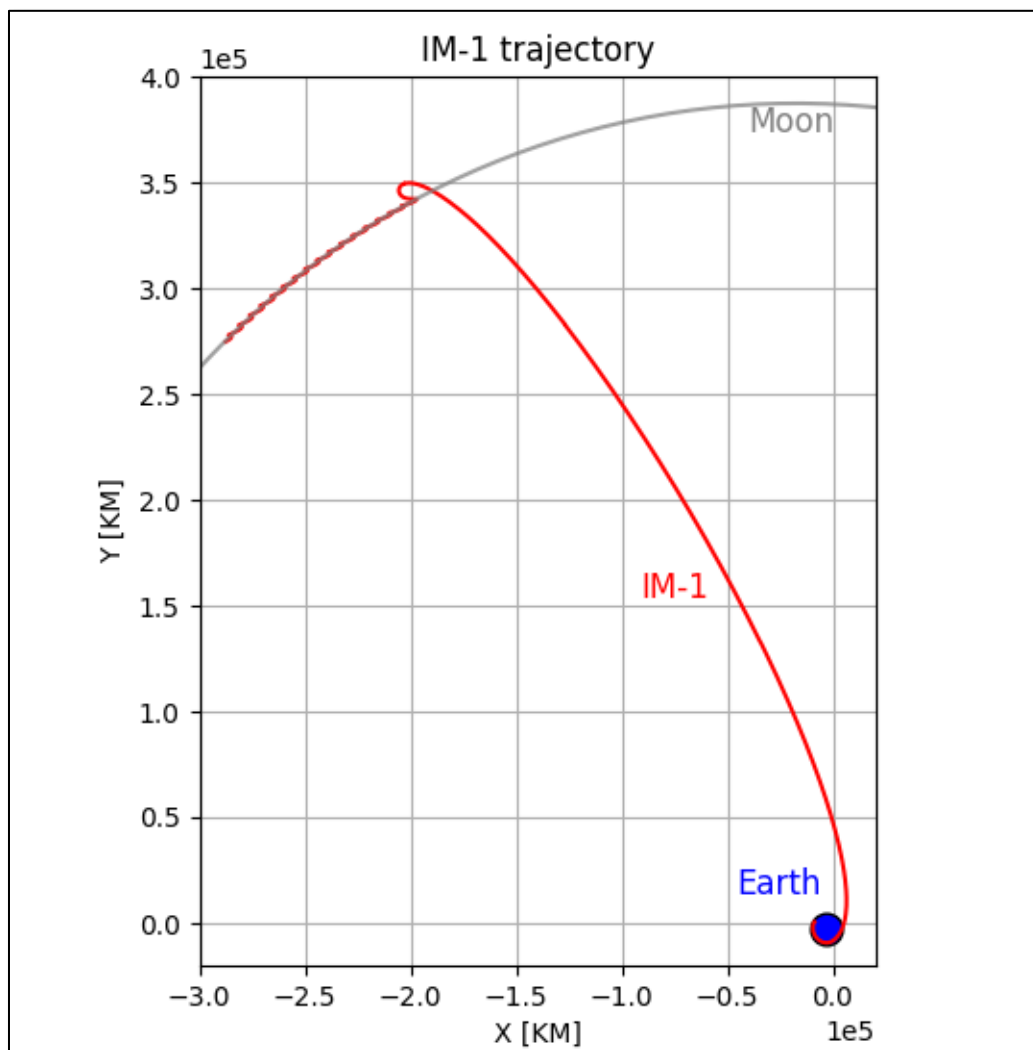


Figure 3: Trajectory of IM1 mission from the Earth to the Moon.

b) **Figure 4** shows the same IM1 mission, but from a 3D point of view. It can be seen that the trajectory of IM1 is not entirely coplanar to the ecliptic plane of the Earth and Moon.

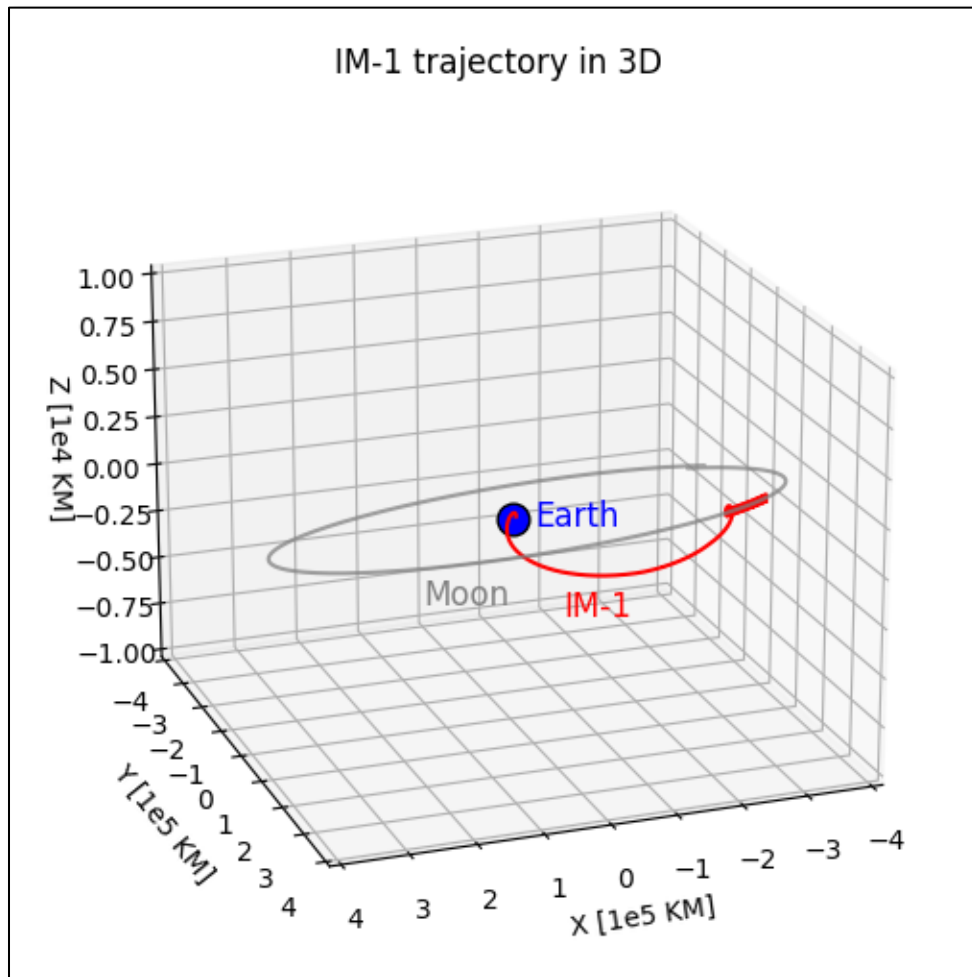


Figure 4: Trajectory of IM1 mission from the Earth to the Moon in 3D. The view angle is defined as 20 deg elevation, 70 deg azimuth, and 0 deg roll.

The next plot was prepared with different vector data. Instead of using the Earth-Moon barycenter as the reference center, the Moon was used. This changes the relative vector positions of the Earth and IM1's trajectory. **Figure 5** shows the trajectory of both the Earth and IM1 with respect to the Moon center. It can be seen that at launch, the trajectory is more curved and eventually spirals toward the Moon, which is static in this coordinate reference system. Earth continues to move with respect to the moon.

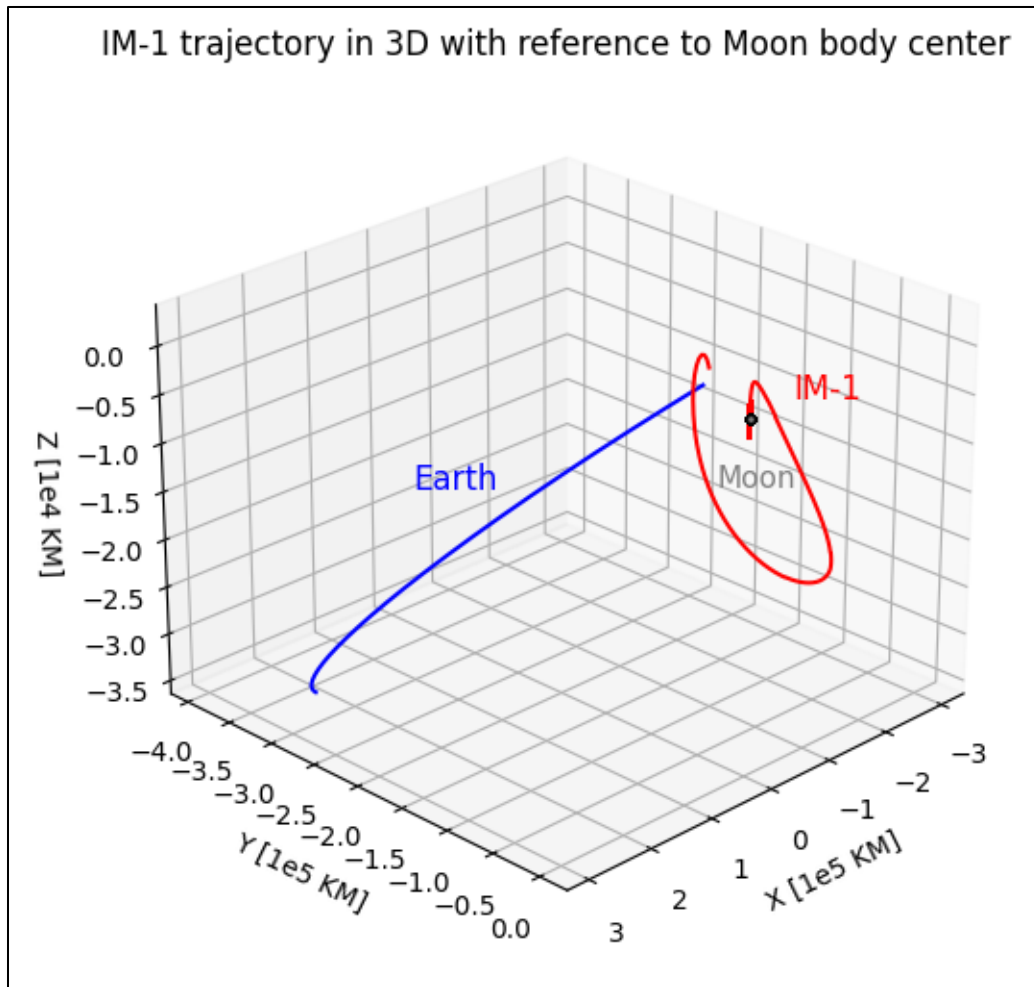


Figure 5: Trajectory of IM1 mission from the Earth to the Moon with respect to the Moon center. The view angle is defined as 25 deg elevation, 45 deg azimuth, and 0 deg roll.

References

- [1] [Horizons System \(nasa.gov\)](https://nasa.gov)
- [2] [Small-Body Database Lookup \(nasa.gov\)](https://nasa.gov)
- [3] [ssbeads_answerkey.pdf \(nasa.gov\)](#)
- [4] Source Code + Vector and Orbital Elements data for Q1 -
https://colab.research.google.com/drive/173MCQpEXn7Sr_fr0bualLHwxBQiqv8Yb?usp=sharing
https://drive.google.com/file/d/1B5F3zdSHRj5YaGfiCpyiAMysBt6CvqH_/view?usp=drive_link
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