12. Rocket Dynamics

Will consider the performance of a rocket in launching a payload or providing ΔV's needed for orbital maneuvers.

12.1. Rocket Payloads and Staging

The initial mass of a rocket m_0 can be decomposed as

$$m_0 = m_{PL} + m_p + m_s \qquad (12.1)$$

where

m_{PL} = mass of the payload

m_p = mass of propellant

m_s = structural mass (everything that is not payload or propellant mass)

Assuming all the propellant is consumed, the mass at engine burnout is

$$m_f = m_{PL} + m_s \qquad (12.2)$$

The <u>mass ratio</u> z is defined as

$$z = \frac{m_o}{m_s} = \frac{m_{PL} + m_p + m_s}{m_{PL} + m_s} \qquad (12.3)$$

The ΔV provided by the burn (also called the <u>characteristic velocity</u>) can be written from (9.66) as

$$\Delta V = c \ln z \qquad (12.4)$$

The <u>payload ratio</u> λ is defined as

$$\lambda = \frac{m_{PL}}{m_o - m_{PL}} = \frac{m_{PL}}{m_p + m_s} \qquad (12.5)$$

and the <u>structural coefficient</u> ε is defined as

$$\varepsilon = \frac{m_s}{m_p + m_s} \qquad (12.6)$$

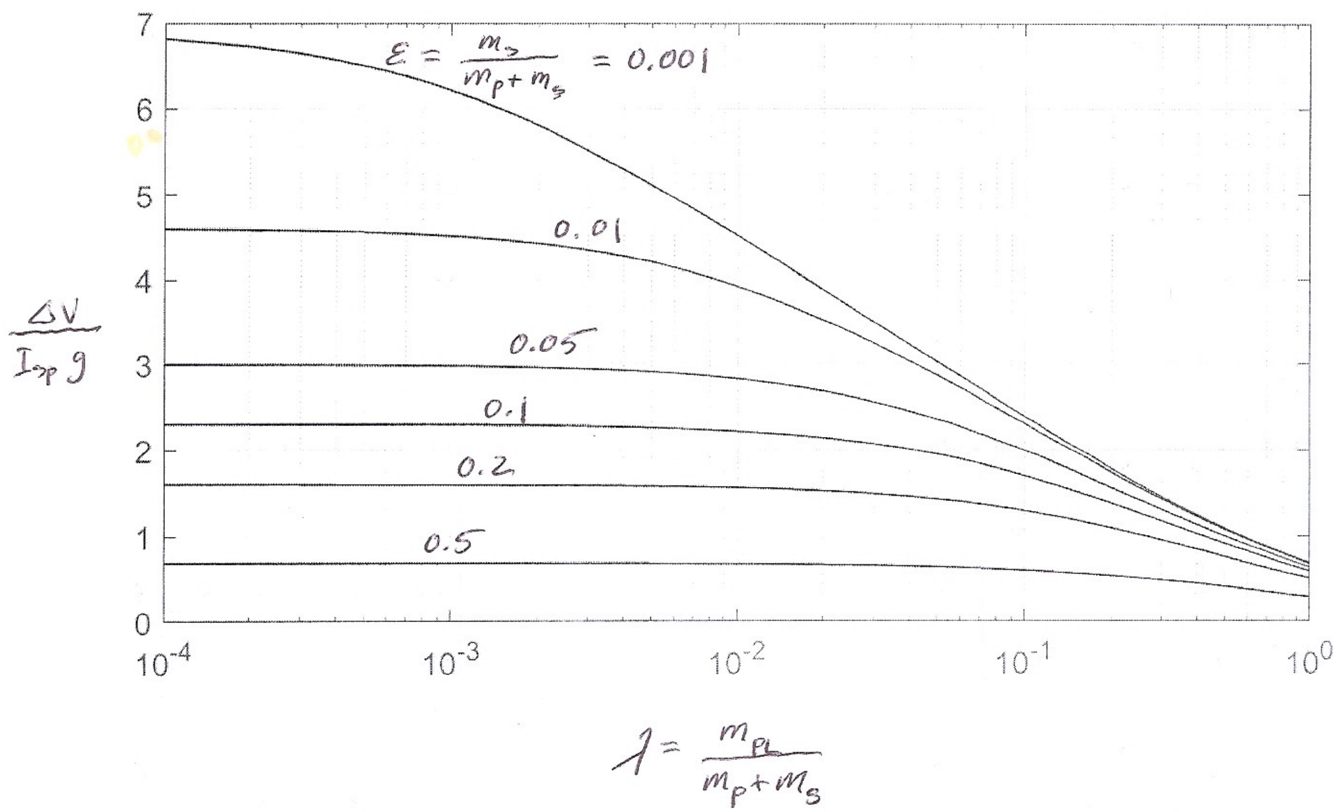
Using (12.5) & (12.6), eq. (12.3) can be written as

$$z = \frac{\lambda + 1}{\lambda + \varepsilon} \qquad (12.7)$$

Substituting (9.69) and (12.7) into (12.4) gives

$$\frac{\Delta V}{I_{sp}\, g} = \ln \frac{\lambda + 1}{\lambda + \varepsilon} \qquad (12.8)$$

A plot of (12.8) is shown below



- smallest payload weight (smallest λ) gives largest ΔV.
- want to maximize λ while keeping structure mass (value of ε) at a minimum.

EXAMPLE

Consider a one-stage chemical rocket with

$$m_{PL} = 20,000 \text{ kg}$$

$$m_s = 40,000 \text{ kg}$$

$$\underline{m_p = 240,000 \text{ kg}}$$

$$m_o = 300,000 \text{ kg}$$

and $I_{sp} = 300$ sec

For this vehicle

$$Z = \frac{m_o}{m_{PL} + m_s} = \frac{300,000}{20,000 + 40,000} = 5$$

$$\varepsilon = \frac{m_s}{m_p + m_s} = \frac{40,000}{240,000 + 40,000} = \frac{1}{7} = 0.143$$

$$\lambda = \frac{m_{PL}}{m_p + m_s} = \frac{20,000}{240,000 + 40,000} = \frac{1}{14} = 0.0714$$

$$\Delta V = I_{sp} g \ln Z = (300)(0.00981) \ln(5) = \underline{\underline{4.73 \ \frac{\text{km}}{\text{sec}}}}$$

To place a satellite in a circular orbit at altitude of 160 km requires a burnout speed of

$$V_{bo} = \sqrt{\frac{\mu}{r_c}} = \sqrt{\frac{3.986 \times 10^5}{6368 + 160}} = 7.81 \frac{km}{sec}$$

Even neglecting gravitational and drag forces, this ΔV is not sufficient to orbit the satellite.

Now consider a two-stage chemical rocket with equal values of z, ε, λ and I_{sp} in both stages. These are called <u>similar stages</u>

Assume further that $m_{PL} = 20{,}000$ kg and $m_o = 300{,}000$ kg (as before) where m_o is the sum of the individual masses from both stages

$$m_o = m_{S1} + m_{P1} + m_{S2} + m_{P2} + m_{PL}$$

If the empty first stage is jettisoned, the initial mass of the second stage is the payload mass of the first stage

$$m_{o2} = m_{S2} + m_{P2} + m_{PL}$$

Equal payload ratios requires

$$\lambda = \lambda_1 = \lambda_2 = \frac{m_{o2}}{m_o - m_{o2}} = \frac{m_{PL}}{m_{o2} - m_{PL}}$$

or

$$m_{02} = \sqrt{m_0 \, m_{PL}}$$

Equal structural coefficients for the 2 stages requires

$$\varepsilon = \varepsilon_1 = \varepsilon_2 = \frac{m_{s1}}{m_0 - m_{02}} = \frac{m_{s2}}{m_{02} - m_{PL}}$$

with $\varepsilon = \frac{1}{7}$ (as before), this gives

$$m_{02} = 77,460 \text{ kg}$$

$$\left.\begin{array}{l} m_{s1} = 31,791 \text{ kg} \\ m_{s2} = 8209 \text{ kg} \end{array}\right\} 40,000 \text{ kg (as before)}$$

$$\left.\begin{array}{l} m_{p1} = 190,749 \text{ kg} \\ m_{p2} = 49,251 \text{ kg} \end{array}\right\} 240,000 \text{ kg (as before)}$$

$$z_1 = \frac{m_0}{(m_{s1} + m_{s2}) + m_{p2} + m_{PL}} = \frac{300,000}{40,000 + 190,749 + 20,000} = 2.75$$

$$z_2 = \frac{m_{02}}{m_{s2} + m_{PL}} = \frac{77,460}{8209 + 20,000} = 2.75 \checkmark$$

$$\lambda_1 = \frac{m_{02}}{m_0 - m_{02}} = \frac{77,460}{300,000 - 77,460} = 0.348$$

$$\lambda_2 = \frac{m_{PL}}{m_{02} - m_{PL}} = \frac{20,000}{77,460 - 20,000} = 0.348 \checkmark$$

Note that the mass ratio of each individual stage is less than that of the single-stage rocket but

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = 2\, I_{sp}\, g \ln z$$

$$= 2(300)(0.00981) \ln 2.75$$

$$= \underline{\underline{5.95 \ \frac{km}{sec}}}$$

is significantly larger than the single-stage value but still not enough to achieve orbital velocity.

Question: What makes the two-stage ΔV 26% larger when exactly the same amount of propellant, structure and payload are included?

A 3-stage rocket with the same conditions would produce

$$\Delta V = \underline{\underline{6.29 \text{ km/sec}}}$$

The limiting value of ΔV using an infinite number of stages is given by

$$\lim_{n \to \infty} \Delta V = I_{sp}\, g\, (1-\varepsilon) \ln \left[\frac{m_0}{m_{PL}} \right]$$

$$= (300)(0.00981)\left(1-\frac{1}{7}\right) \ln\left[\frac{300{,}000}{20{,}000}\right]$$

$$= 6.83 \ \frac{km}{sec}$$ (still shy of the required orbital speed)

Options to improve performance

- Take advantage of earth's rotation speed.
  At the equator

$$V_0 = R\omega_{earth} = (6378)\left(\frac{2\pi}{86{,}164}\right) = 0.465 \ \frac{km}{sec}$$

For other launch sites, multipy by $\cos\Lambda$ (Λ = latitude)

- Decrease value of structural coefficient. In this example $\varepsilon = \frac{1}{7} = 0.142$. Using lighter composite materials have achieved $\varepsilon = 0.07$.

- Increase I_{sp}. Using liquid hydrogen as fuel, can get $I_{sp} = 450$ sec. (e.g. SLS).

- Decrease value of payload mass. In this example, 6.67% of the total mass was payload.