Math 290 Test #1 Solution Set
September 7, 2006

(#1) Write down the augmented matrix corresponding to the given system of equations:

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 3 & 1 & 1 & 10 \\ 4 & 1 & -1 & 13 \\ 0 & 1 & 7 & 1 \end{bmatrix}$$

Now apply Gaussian elimination to transform the matrix into row-echelon form. Start by subtracting 4 $\times$ row #2 from row #3 to get

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 3 & 1 & 1 & 10 \\ 0 & 1 & 7 & 1 \\ 0 & 1 & 7 & 1 \end{bmatrix}$$

Subtract row #3 from row #4:

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 3 & 1 & 1 & 10 \\ 0 & 1 & 7 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Subtract 3 $\times$ row #1 from row #3:

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 7 & 1 \\ 0 & 1 & 7 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Subtract row #2 from row #3:

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 7 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix is now in row-echelon form (in fact, reduced row-echelon form). It represents the equations

$$x - 2z = 3 \quad \text{and} \quad y + 7z = 1.$$

The pivot variables are x and y, and z is a free variable. So we can express the solution set parametrically:

$$x = 3 + 2t, \quad y = 1 - 7t, \quad z = t, \qquad \text{for any real number } t.$$

(#2a)

$$3\begin{bmatrix}2 & 1\\ 2 & 4\end{bmatrix} - 2\begin{bmatrix}1 & 6\\ 0 & 7\end{bmatrix} = \begin{bmatrix}6 & 3\\ 6 & 12\end{bmatrix} - \begin{bmatrix}2 & 12\\ 0 & 14\end{bmatrix}$$
$$= \begin{bmatrix}4 & -9\\ 6 & -2\end{bmatrix}.$$

(#2b)

$$\begin{bmatrix}7 & -2 & 3\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 3\\ 4 & 0\end{bmatrix} = \begin{bmatrix}(7)(1)+(-2)(0)+(3)(4) & (7)(-1)+(-2)(3)+(3)(0)\end{bmatrix} = \begin{bmatrix}19 & -13\end{bmatrix}.$$

(#2c)

$$\begin{bmatrix}1 & -1\\ 3 & 2\\ 5 & 6\end{bmatrix}^T + I_2\begin{bmatrix}-1 & 3 & 4\\ 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 3 & 5\\ -1 & 2 & 6\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 3 & 4\\ 0 & 0 & 0\end{bmatrix}$$
$$= \begin{bmatrix}1 & 3 & 5\\ -1 & 2 & 6\end{bmatrix} + \begin{bmatrix}-1 & 3 & 4\\ 0 & 0 & 0\end{bmatrix}$$
$$= \begin{bmatrix}0 & 6 & 9\\ -1 & 2 & 6\end{bmatrix}.$$

(#2d) $I_2 I_3$ is undefined because the number of columns of I_2 (namely 2) does not equal the number of rows of I_3 (namely 3), so the matrices cannot be multiplied.

(#2e) $O_{58}O_{85} = O_{55}$ (the 5×5 zero matrix).

(#3) The equation of a parabola is $y = ax^2 + bx + c$, where a, b, c are real numbers. In this case the conditions that the points $(1, -2)$, $(4, 1)$, and $(-2, 4)$ lie on the parabola mean respectively that

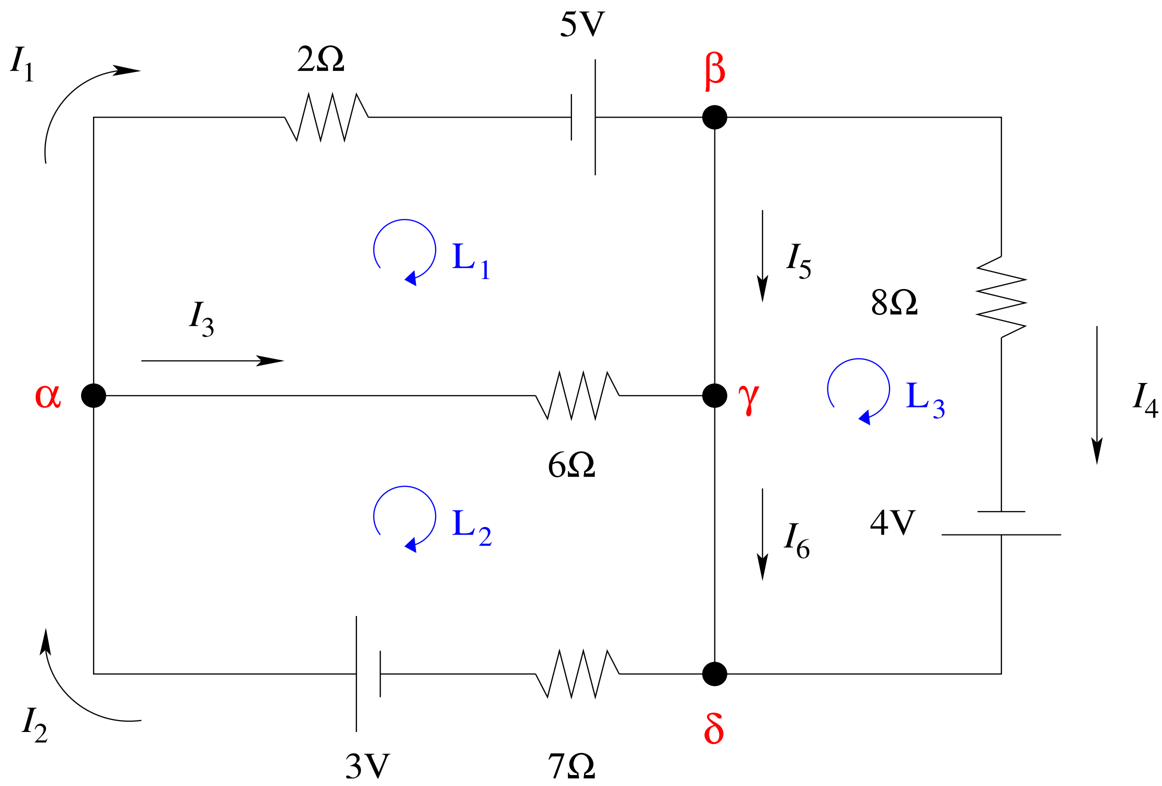
$$\begin{aligned} a + b + c &= -2,\\ 16a + 4b + c &= 1,\\ 4a - 2b + c &= 4. \end{aligned}$$

Solving that system of equations will give the coefficients a, b, c in the equation of the parabola.

(#4) See next page.

(#5) The matrix product is of the form AA^T (that is, the second matrix being multiplied is the transpose of the first). That means that it has to be symmetric (because $(AA^T)^T = (A^T)^T A^T = AA^T$). But the matrix on the right-hand side is not symmetric (for example, the entry in row 4 and column 3 does not equal the entry in row 3 and column 4), so the equation must be wrong.

(#4) First, here is the electrical circuit with the currents I_5 and I_6 indicated. I've also indicated all the junction points by red Greek letters $\alpha, \beta\gamma\delta$, and the three loops by blue L_1, L_2, L_3.



The equations governing current flow are as follows. First, for every junction point, the total current flowing in must equal the total current flowing out. This gives the following four linear equations:

$$\begin{aligned} I_2 &= I_1 + I_3, & (\alpha) \\ I_1 &= I_4 + I_5, & (\beta) \\ I_3 + I_5 &= I_6, & (\gamma) \\ I_4 + I_6 &= I_2. & (\delta) \end{aligned}$$

Second, the net change in voltage around every loop in the circuit must equal zero. (Remember that voltage changes only when there is a battery or a resistor!) So we have the following additional equations (reading around every loop in the indicated direction, starting at junction point γ):

$$\begin{aligned} -6I_3 + 2I_1 + 5 &= 0, & (L_1) \\ 7I_2 + 3 + +6I_3 &= 0, & (L_2) \\ 8I_4 + 4 &= 0. & (L_3) \end{aligned}$$