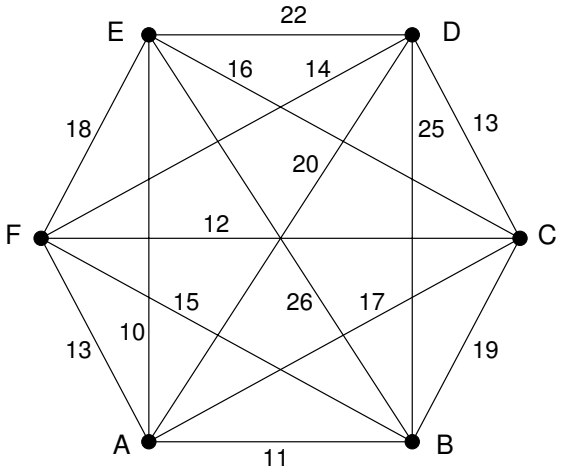
Exam Instructions

- **Make sure that you are in your assigned seat and that the color of your test corresponds to the color of your answer sheet.**

Test color	Answer sheet color
Blue	Blue
Yellow	Purple
Pink	Red
Green	Green

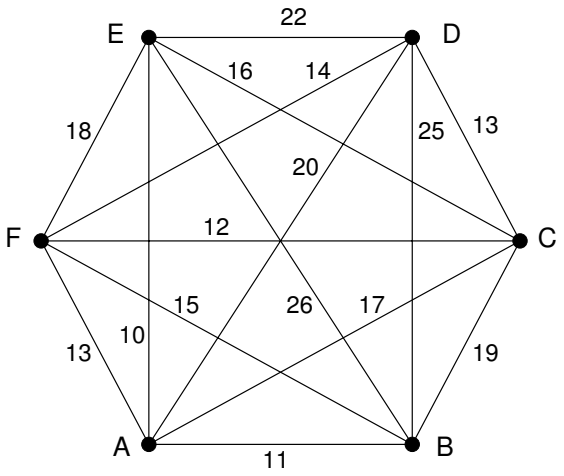
- **Enter your first and last name and your 7-digit KU ID number on the answer sheet. Write the letters at the top of the columns and fill in the bubbles below them.** *If you enter the wrong ID number, your grade may not be recorded!*
- **Use a #2 pencil.** Darken the oval for your answer completely. Do not mark any other ovals or make any stray marks on the answer sheet. If you change an answer, erase the old answer *completely*.
- **There are a total of 28 problems**, each worth 5 points, for a total of 140 points. You get ten points for free; however, *you will forfeit those ten points if there is any delay or extra work in recording your grade resulting from not following the instructions.*
- **Check that your test consists of three sheets of paper, and that there is text on every page.** If you need scratch paper, raise your hand and a TA will provide it.
- **No aids other than a calculator are permitted.** In particular, books and notes must be put away and cell phones must be turned off.
- **Have your KU ID card out on your desk for the TAs to check.**
- **No one may leave the room after 3:35.** All exams will be collected at 3:50.

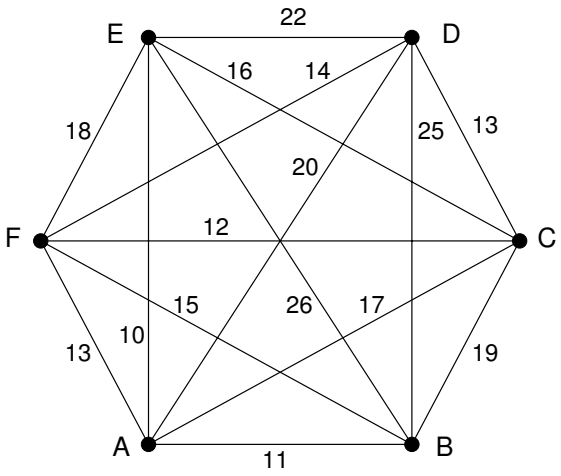
Problems 1–11 refer to the weighted K_6 shown below. Three copies of the graph are provided for your scratchwork. The graph is shown in two equivalent ways: as a figure and as a table.



Weights of edges

	A	B	C	D	E	F
A		11	17	20	10	13
B	11		19	25	26	15
C	17	19		13	16	12
D	20	25	13		22	14
E	10	26	16	22		18
F	13	15	12	14	18	





In order to answer Problems 1–11, first find:

- The nearest-neighbor tour using vertex F as the reference point
- The cheapest-link tour (keep track of the order you chose the edges!)
- The minimum spanning tree

Problems 1–4 refer to the **nearest-neighbor tour** of the network on the previous page. Write the tour with F as the reference point:

F — — — — — F

To answer these problems, fill in the oval on the answer sheet with the letter of your answer (A, B, C, D, or E.)

Problem #1 What is the second vertex in the tour?

Problem #2 What is the third vertex in the tour?

Problem #3 What is the fourth vertex in the tour?

Problem #4 What is the fifth vertex in the tour?

Problems 5–8 concern the **cheapest-link tour** of the network on the previous page.

Problem #5 What was the second edge chosen?

A. AB B. BC C. CD D. DE E. EF

Problem #6 What was the fourth edge chosen?

A. AB B. BC C. CD D. DE E. EF

Problem #7 What was the last edge chosen?

A. AB B. BC C. CD D. DE E. EF

Problem #8 What is the total weight of the cheapest-link tour?

A. 95 B. 72 C. 86 D. 78 E. 83

Problems 9–11 concern the **minimum spanning tree (MST)** of the network shown two pages ago.

Problem #9 What is the degree of vertex C in the MST?

A. 1 B. 2 C. 3 D. 4 E. 5

Problem #10 How many vertices are there of degree 1 in the MST?

A. 1 B. 2 C. 3 D. 4 E. 5

Problem #11 What is the total weight of the MST?

A. 58 B. 59 C. 60 D. 61 E. 62

Problem #12 How many spanning trees does K_N have?

A. $\frac{N(N-1)}{2}$ B. $\frac{N(N+1)}{2}$ C. $N-1$ D. $(N-1)!$ E. N^{N-2}

Problem #13 How many Hamilton circuits does K_N have?

A. $\frac{N(N-1)}{2}$ B. $\frac{N(N+1)}{2}$ C. $N-1$ D. $(N-1)!$ E. N^{N-2}

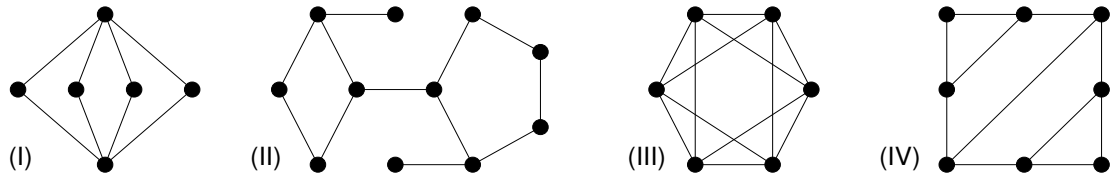
Problem #14 How many edges are there in each spanning tree of K_N?

A. $\frac{N(N-1)}{2}$ B. $\frac{N(N+1)}{2}$ C. $N-1$ D. $(N-1)!$ E. N^{N-2}

Problem #15 The brand-new Blazatron computer can generate all the Hamilton circuits of a 100-vertex complete graph in a mere 100 hours (a little over 4 days). How long will it take the Blazatron to generate all the Hamilton circuits of a 101-vertex complete graph?

A. 100 hours B. 101 hours C. 101 days D. Over a year E. Billions of years

Problems 16–18 are about the four networks shown below, labeled I, II, III and IV.



Problem #16 Which two of these four networks have Hamilton circuits?

A. I and III B. III and IV C. II and IV D. I and IV E. I and II

Problem #17 Which two of these four networks have Euler circuits?

A. I and III B. III and IV C. II and IV D. I and IV E. I and II

Problem #18 How many spanning trees does network II have?

A. 11 B. 9 C. 13 D. 12 E. 20

Problem #19 Which one of the following statements is true?

A. Every graph with a Hamilton circuit must have an Euler circuit, but not every graph with an Euler circuit must have a Hamilton circuit.

B. Every graph with a Hamilton circuit must have an Euler circuit, and every graph with an Euler circuit must have a Hamilton circuit.

C. Not every graph with a Hamilton circuit must have an Euler circuit, and not every graph with an Euler circuit must have a Hamilton circuit.

D. Not every graph with a Hamilton circuit must have an Euler circuit, but every graph with an Euler circuit must have a Hamilton circuit.

Problem #20 Fill in the blank to create a true statement: $\frac{2011!}{2010!} - \frac{2009!}{2008!} =$ _____

A. 4020 B. 2010 C. 2009 D. 2 E. 0

Problem #21 In the sequence of numbers $1, 1, 3, 16, 125, 1296, ...$, what comes next?

A. 12973 B. 16807 C. 13875 D. 10000 E. 22045

Problem #22 The Cheapest-Link Algorithm for finding a Hamilton circuit is...

A. optimal and efficient	B. not optimal, but efficient	C. optimal, but not efficient	D. neither optimal nor efficient

Problem #23 The Brute-Force Method for finding a Hamilton circuit is...

A. optimal and efficient	B. not optimal, but efficient	C. optimal, but not efficient	D. neither optimal nor efficient

Problem #24 Kruskal's Algorithm for finding a minimum spanning tree is...

A. optimal and efficient	B. not optimal, but efficient	C. optimal, but not efficient	D. neither optimal nor efficient

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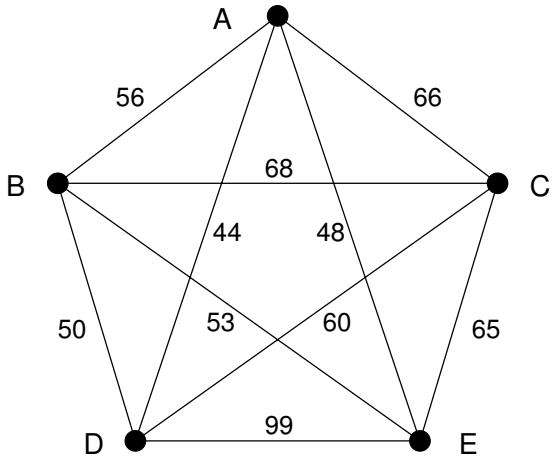
For Problems 25–28, use the Brute-Force Method to find the **optimal Hamilton circuit** in the weighted K_5 shown below. (Hint: With a little bit of thought, you can reduce the amount of work involved.) The weighted graph is shown both as a figure and as a table.



Weights of edges

	A	B	C	D	E
A		56	66	44	48
B	56		68	50	53
C	66	68		60	65
D	44	50	60		99
E	48	53	65	99	

Write the tour with A as the reference point: A __ __ __ __ A.
In order to answer Problems 25–28, fill in the oval on the answer sheet with the letter of your answer (A, B, C, D, or E). **These answers must all be correct in order to earn credit on any of them.**

Problem #25 What is the second vertex in the tour?

Problem #26 What is the third vertex in the tour?

Problem #27 What is the fourth vertex in the tour?

Problem #28 What is the fifth vertex in the tour?