Exam Instructions

- **Make sure that you are in your assigned seat and that the color of your test corresponds to the color of your answer sheet.**

Test color	Answer sheet color
Blue	Blue
Yellow	Purple
Pink	Red
Green	Green

- **Enter your first and last name and your 7-digit KU ID number on the answer sheet.**

  If you enter the wrong ID number, your grade may not be recorded!

- **Use a #2 pencil.** Darken the oval for your answer completely. Do not mark any other ovals or make any stray marks on the answer sheet.

- **There are a total of 28 problems**, each worth 5 points, for a total of 140 points. You get ten points for free; however, *you will forfeit those ten points if there is any delay or extra work in recording your grade resulting from not following the instructions.*

- **Check that your test consists of four sheets of paper, and that there is text on every page.** If not, raise your hand and ask for a replacement test. You may use the rest of the paper (and the back of this sheet) for scratchwork. If you need additional scratch paper, raise your hand and a TA will provide it.

- **No aids other than a calculator are permitted.** In particular, books and notes must be put away and cell phones must be turned off.

- **Have your KU ID card out on your desk**. The TAs will be checking IDs during the test.

- **No one may leave the room after 3:35.** All exams will be collected at 3:50.

Four clowns (Nellie, Oddwaddle, Picciotto and Quackstein) are trying to divide a cream pie fairly using the Lone-Divider Method. The clowns' valuations of the four slices of pie are as follows. You may wish to work out the entire fair-division problem in order to answer the multiple-choice questions below.

	Slice #1	**Slice #2**	**Slice #3**	**Slice #4**
Nellie	\$3	\$2	\$6	\$9
Oddwaddle	\$6	\$6	\$6	\$6
Picciotto	\$5	\$9	\$6	\$8
Quackstein	\$1	\$9	\$3	\$3

Problem #1 Who was the divider?

A. Nellie B. Oddwaddle C. Picciotto D. Quackstein E. Can't tell

Problem #2 What is a fair share of the cream pie worth to Quackstein?

A. \$4 B. \$5 C. \$6 D. \$7 E. \$8

Problem #3 What is Picciotto's bid? (Assume that Picciotto bids honestly.)

A. $\{s_2\}$

B. $\{s_4\}$

C. $\{s_2,\ s_4\}$

D. $\{s_3,\ s_4\}$

E. $\{s_s,\ s_3,\ s_4\}$

Problem #4 Who gets Slice #4?

A. Nellie B. Oddwaddle C. Picciotto D. Quackstein E. Can't tell

The clowns' next task is to fairly divide a pile of booty consisting of a rainbow wig, a pair of pink-and-green striped socks, a pair of extremely large shoes, and a rubber chicken, using the Method of Sealed Bids. (The clowns have plenty of cash on hand.) As the circus manager, you will play the role of referee. The clowns' valuations of the items are below. You may wish to fill in the entire sealed-bids worksheet in order to answer the multiple-choice questions below.

	Nellie	**Oddwaddle**	**Picciotto**	**Quackstein**	
Wig	\$6	\$2	\$3	\$5	
Socks	\$2	\$2	\$6	\$3	
Shoes	\$22	\$18	\$14	\$24	
Chicken	\$6	\$2	\$5	\$8	
Total Valuation					
Fair Share					
Item Value					
First Settlement					Surplus:
Share of Surplus					
Final Settlement					

Problem #5 Which item(s) does Quackstein get?

A. Wig only B. Wig and chicken C. Socks only D. Shoes and chicken E. Nothing

Problem #6 What is the value of a fair share to Oddwaddle?

A. \$6 B. \$7 C. \$8 D. \$9 E. \$10

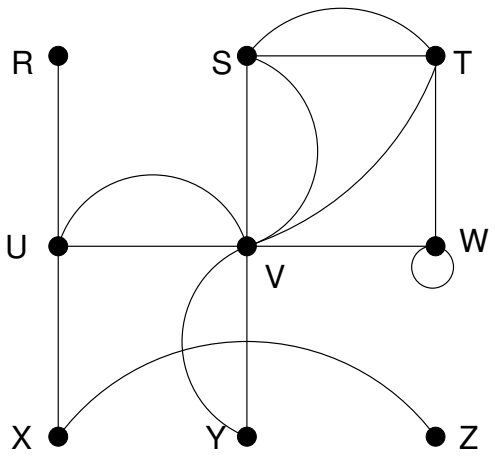
Problem #7 How much does Picciotto pay or receive in the first settlement?

A. Receives \$6 B. Receives \$3 C. Receives \$1 D. Pays \$22 E. Zero

Problem #8 How much does Nellie pay or receive in the final settlement?

A. Receives \$9 B. Receives \$6 C. Receives \$4 D. Pays \$19 E. Zero

Problems 9–16 are about the graph represented by the figure below.



Problem #9 How many vertices of degree 2 does this graph have?

A. Zero B. One C. Two D. Three E. Four

Problem #10 How many odd vertices does this graph have?

A. Zero B. One C. Two D. Three E. Four

Problem #11 How many bridges does this graph have?

A. Zero B. One C. Two D. Three E. Four

Problem #12 How many loops does this graph have?

A. Zero B. One C. Two D. Three E. Four

Problem #13 Prof. Nitram is trying to calculate an Euler circuit of the graph on the previous page, using Fleury's algorithm. He starts walking as follows:

$$R,\ U,\ V,\ W,\ W,\ T,\ S,\ V,\ U,\ X,\ Z.$$

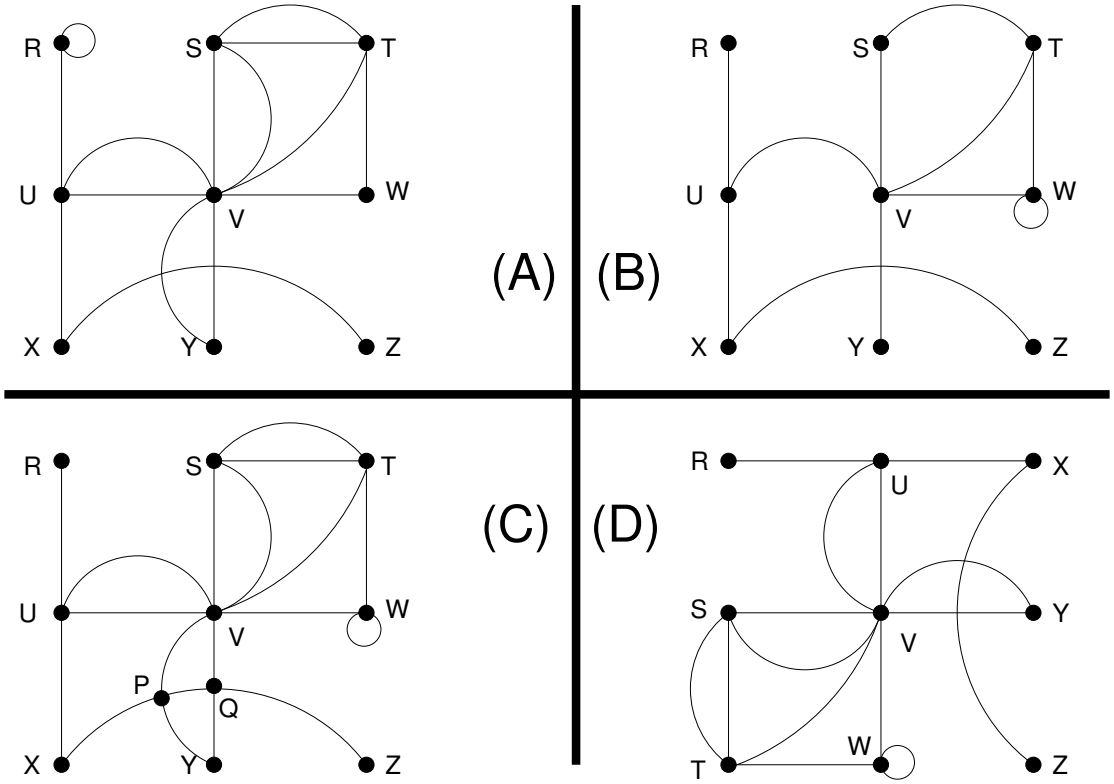
At this point, he realizes he has made a mistake. Which step was the mistake?

A. Starting at vertex R B. Edge RU C. Edge VU D. Edge WW E. Edge XZ

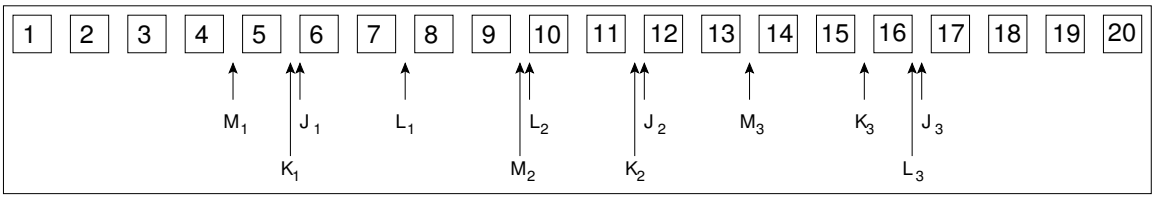
Problem #14 If Prof. Nitram fixes his mistake and successfully completes the Euler path, at which vertex will he finish?

A. Vertex R B. Vertex U C. Vertex W D. Vertex Y E. Vertex Z

Problem #15 Which of these figures represents the same graph as the one on the previous page?



Four small children (Jamie, Kim, Leslie, and Mason) are trying to split a bag of candy using the Method of Markers. There are a total of twenty pieces of candy. They begin by arranging the pieces of candy in a line and numbering them 1 through 20, then place their markers as shown below. (The arrow lengths have no special significance other than to make the figure easier to read.)



Problem #16 Who gets piece #1?

A. Jamie B. Kim C. Leslie D. Mason E. It is left over

Problem #17 Who gets piece #6?

A. Jamie B. Kim C. Leslie D. Mason E. It is left over

Problem #18 Who gets piece #14?

A. Jamie B. Kim C. Leslie D. Mason E. It is left over

Problem #19 Who gets piece #20?

A. Jamie B. Kim C. Leslie D. Mason E. It is left over

Problem #20 Which one of the following statements is true about every fair-division method we have studied?

A. If you bid honestly, you might not receive a fair share if other players bid insincerely.

B. If you bid honestly, you are guaranteed to receive at least a fair share even if other players bid insincerely.

C. If you bid insincerely, you will definitely not receive an exact fair share.

D. If you bid insincerely, you will always increase your own share.

The NCAA Division I Women's Basketball season can be represented as a graph with a vertex for each team. Each edge represents a game played between the two corresponding teams; if two teams play more than once, then there are multiple edges. Problems 22–24 are about this graph.

Problem #21 The degree of a vertex represents...

A. the number of games the team plays.

B. the number of different opponents that team faces.

C. the number of teams in the same conference.

D. the number of games the team wins.

Problem #22 An isolated vertex would represent...

A. a team that plays no games at all

B. a team in a remote location such as Alaska or Hawaii

C. a team that wins all its games

D. a team that plays all its games against the same opponent

E. a team that only plays games within its own conference

Problem #23 The teams in the Medium 13 Conference play only against each other; no team from the conference plays any team outside the conference. This is an example of...

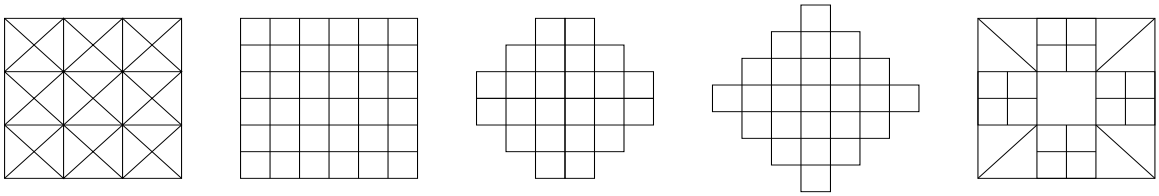
A. an Euler circuit

B. isolated vertices

C. an even vertex

D. a connected component

Problem #24 Which of these figures has a unicursal tracing?



(A) (B) (C) (D) (E)

Problem #25 Which one of these statements is *definitely false*?

A. There are an *odd* number of Facebook users who each have an *odd* number of friends.

B. There are an *odd* number of Facebook users who each have an *even* number of friends.

C. There are an *even* number of Facebook users who each have an *even* number of friends.

D. There are an *even* number of Facebook users who each have an *odd* number of friends.

Problem #26 A graph is connected and has eight vertices, of degrees

$$7,\ 6,\ 6,\ 6,\ 5,\ 4,\ 3,\ 2.$$

Which of these statements is true?

A. The graph has an Euler circuit but not an Euler path.

B. The graph has an Euler path but not an Euler circuit.

C. The graph has both an Euler circuit and an Euler path.

D. The graph has neither an Euler circuit nor an Euler path.

E. No such graph can possibly exist.

Problem #27 A graph is connected and has eight vertices, of degrees

$$7,\ 6,\ 6,\ 5,\ 5,\ 4,\ 3,\ 2.$$

Which of these statements is true?

A. The graph has an Euler circuit but not an Euler path.

B. The graph has an Euler path but not an Euler circuit.

C. The graph has both an Euler circuit and an Euler path.

D. The graph has neither an Euler circuit nor an Euler path.

E. No such graph can possibly exist.

Problem #28 A graph has a total of 22 vertices, of which ten have degree 8 and twelve have degree 5. How many edges does the graph have?

A. 35

B. 70

C. 140

D. 280

E. No such graph can possibly exist.