You've now used Geometer's Sketchpad to construct a square. There are many ways to do this. Let's look at a way I might do this by the seat of my pants.

I start by drawing two points P, Q and connecting them with a line L. I now draw two circles C_1, C_2, each centered at one of the points and containing the other. Let's give the names R and S to the other points where L meets the two circles.
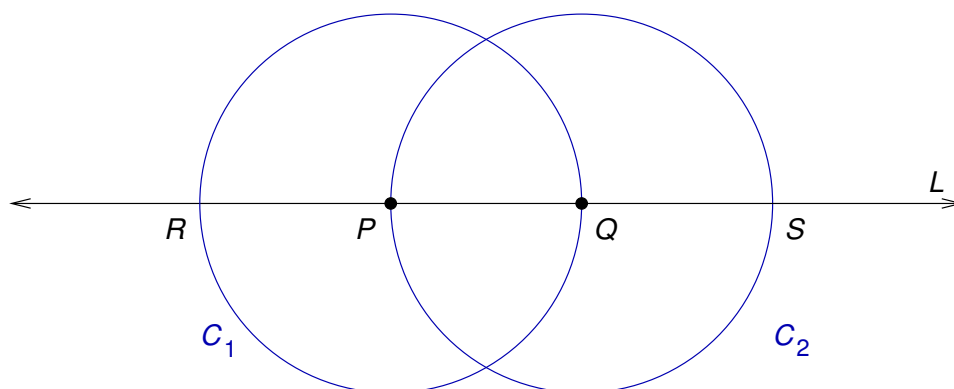

I might as well use the segment $\overline{PQ}$ as one of the sides of the square. Where are the other sides? It would certainly help to be able to construct a line through P that is perpendicular to L. Aha! If I bisect the segment $\overline{QR}$, I should obtain the line I'm looking for. To perform that bisection, I make my compass a bit bigger and draw two arcs centered at Q and the other point of the diameter (let's call it R). Then I draw the line M through the two points where those arcs intersect.
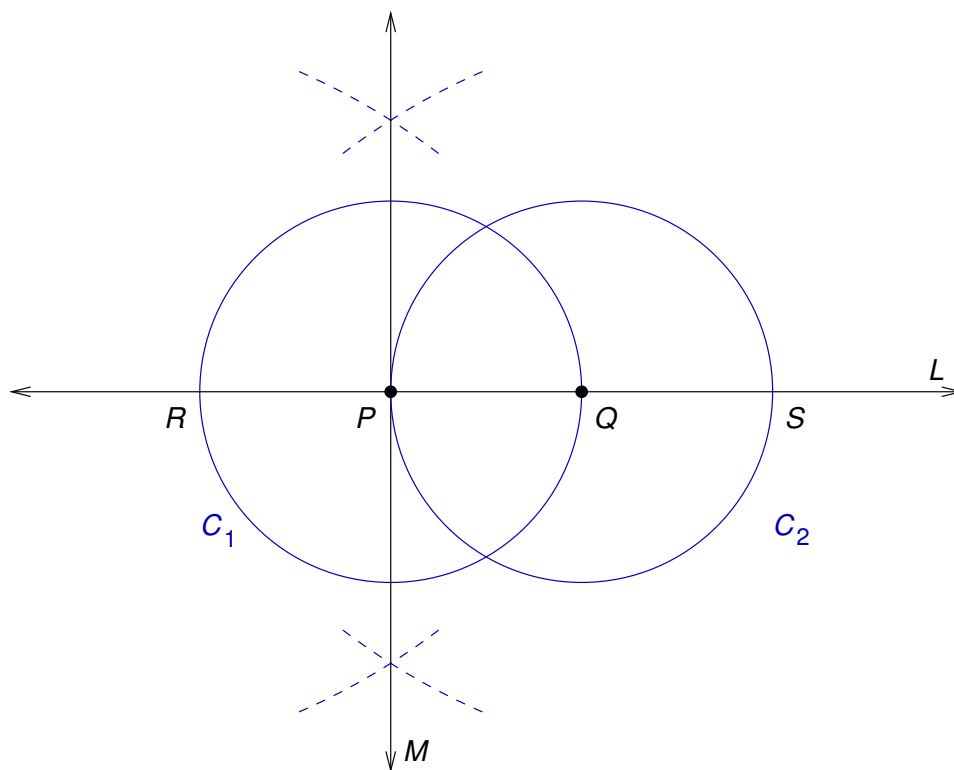
As I had hoped for, M passes through P and is perpendicular to L. I mark the point T where M meets C_1; this point T ought to be a third corner of the square.
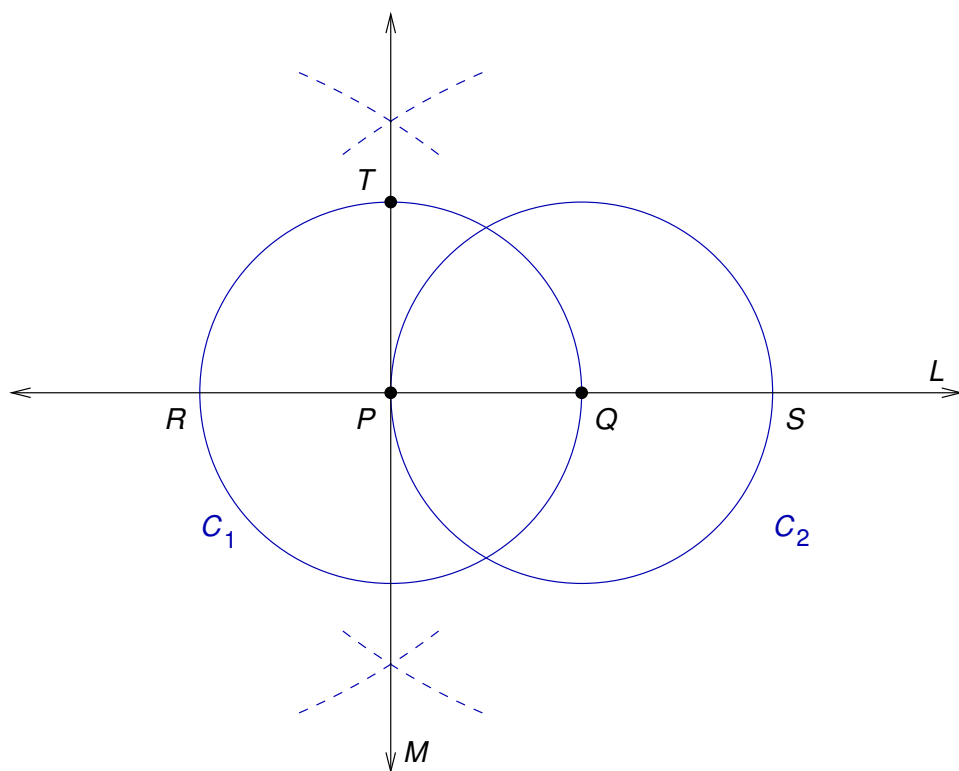
I could find the fourth point of the square by bisecting $\overline{PS}$, but there's actually an easier way. That point ought to lie on C_2 and be equidistant from Q and T. So I put the center of my compass on T, put the other half of the compass on P, and draw a circle C_3. The other point — let's call it U — where this circle meets C_2 is the fourth and final point of my square $PQUT$.
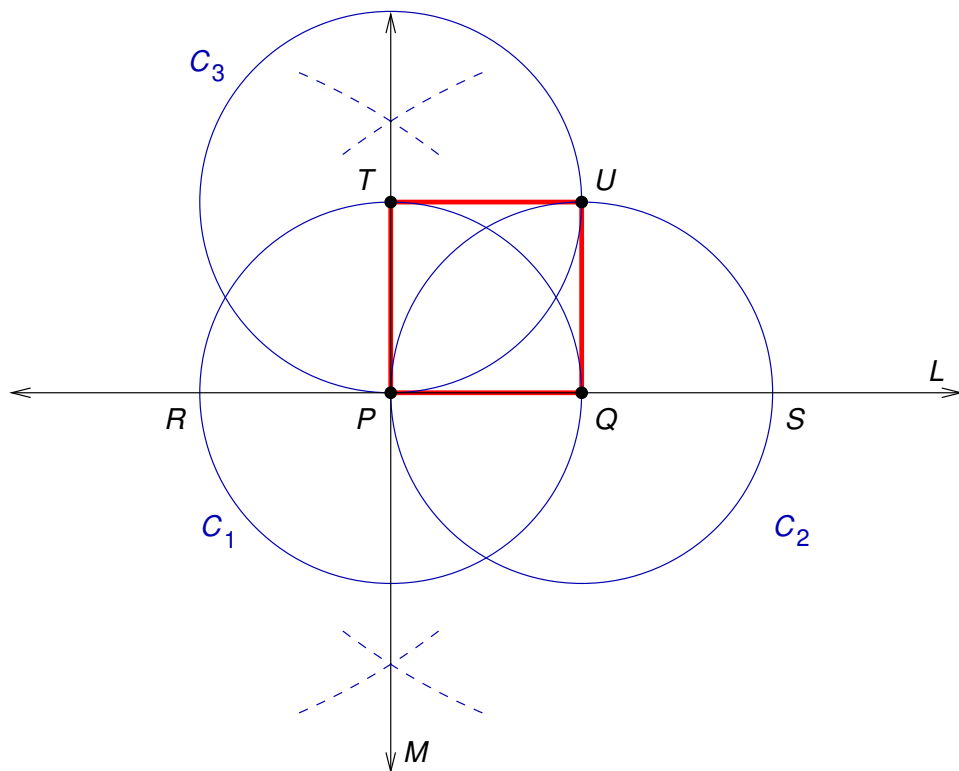
This is not too complicated a construction, as these things go. On the other hand, it still has several ingredients, and it is no trivial task to explain why they all fit together. For example:

Why is $\angle QPT$ a right angle? Why does M pass through P? Why is M perpendicular to L? Why are PQ and QU equal? Why are PQ and TU equal? Why is $\angle TUQ$ a right angle? Why is $PQUT$ a square?

We can't just say that $\angle QPT$ is a right angle because it looks like one. What if it's a little bit off, just not enough for the naked eye to detect? How do we know that our construction definitely produces an honest-to-goodness right angle?

It's tempting, but wrong, to give a reason like, "Well, $PQ = QU$ because the quadrilateral $PQUT$ is a square." That explanation leads to circularity: "But why is it a square?" "Because all four sides are equal and all the angles are right." "Why is that the case?" "Because it is a square." And so on.