Mathematics 141 Extra Problems (11/22/05)

Doing some or all of these problems can earn you up to 25 honors points. Since you have some time to complete this work, I'll consider the clarity of your solutions as well as their substance. In particular, I will award little or no credit for illegible, unjustified or disorganized work.

Submit your answers on a separate sheet (or sheets) of paper. The problems are due Monday, November 28 in class.

Problem #1.

(a) Find $\int \tan^2 x \, dx$. (Hint: Use a trigonometric identity.)

(b) Find $\int \tan^4 x \, dx$. (Hint: Your answer from (a) will be helpful.)

(c) Find $\int \tan^6 x \, dx$.

(d) Based on your answers to (a)...(c), conjecture (that is, make an intelligent guess at) a formula for

$$\int \tan^{2n} x \, dx$$

for all positive integers n. (Hint: Once you have done this, calculate $\int \tan^8 x \, dx$ by hand and see if it matches your pattern.)

Problem #2.   Show that the area of an ellipse with horizontal radius r and vertical radius s is πrs. (Hint: If the ellipse is centered at the origin, its equation is $x^2/r^2 + y^2/s^2 = 1$.)

Problem #3.   Let r be a real number. Evaluate the indefinite integral

$$\int e^{rx} \cos x \, dx.$$

Your answer should involve both x and r. (Hint: It may help to look at Bonus Problem (a) from Quiz #11. Also, remember that you can check your answer by differentiation.)