Rigid Motions

A **rigid motion** is the action of taking an object and moving it to a different location without altering its shape or size.

Reflections, **rotations**, **translations**, and **glide reflections** are all examples of rigid motions.

In fact, every rigid motion is one of these four kinds.

Properties of Reflections

1. Every reflection is completely determined by its **axis of reflection.**
2. Every reflection is completely determined by a **single** point-image pair P and P' (provided that $P \neq P'$).
3. The **fixed points** of a reflection are exactly the points on its **axis of reflection.**
4. Reflections are **improper** rigid motions (they reverse clockwise and counterclockwise orientations).
5. Applying the same reflection twice gives the identity motion.

Properties of Rotations

1. Every rotation is completely determined by its **rotocenter** and the **angle of rotation**.
2. A 360° rotation is equivalent to the identity motion. (So are720°, 1040°, $\ldots$)
3. A rotation that is not the identity has **exactly one fixed point**: the **rotocenter**.
4. Rotations are **proper** rigid motions (they preserve clockwise/counterclockwise orientations).
5. A rotation is determined by any **two** point-image pairs P, P' and Q, Q'.

Translations

A **translation** consists of "dragging an object in a specified direction and by a specified amount" (Tannenbaum)

For example:

- "Move every point up three feet."
- "Move every point six miles north-northwest and two centimeters down."

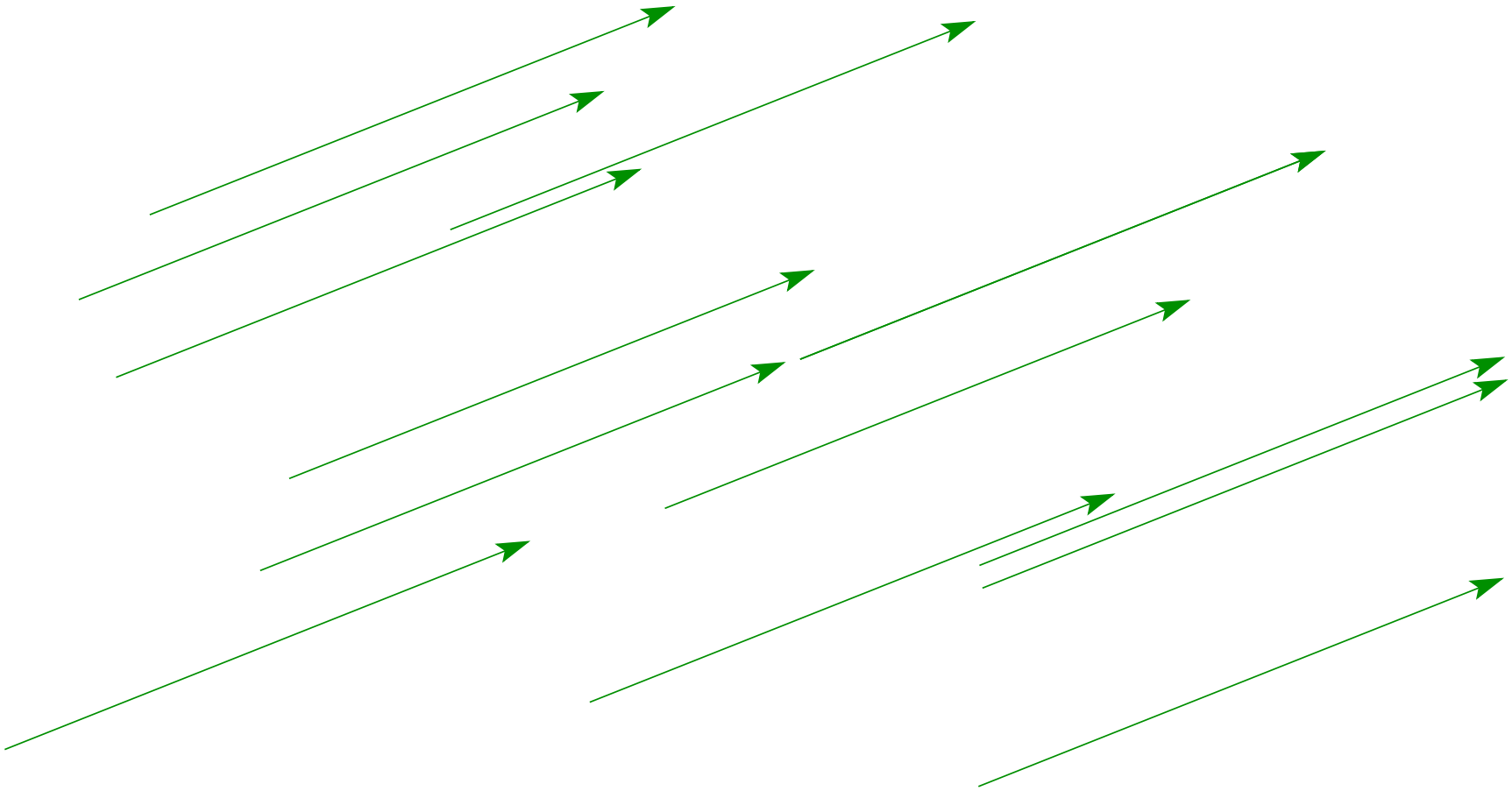
The direction and amount of dragging can be combined into a piece of information called the **vector of translation**.

What's a Vector?

A **vector** is an arrow that has

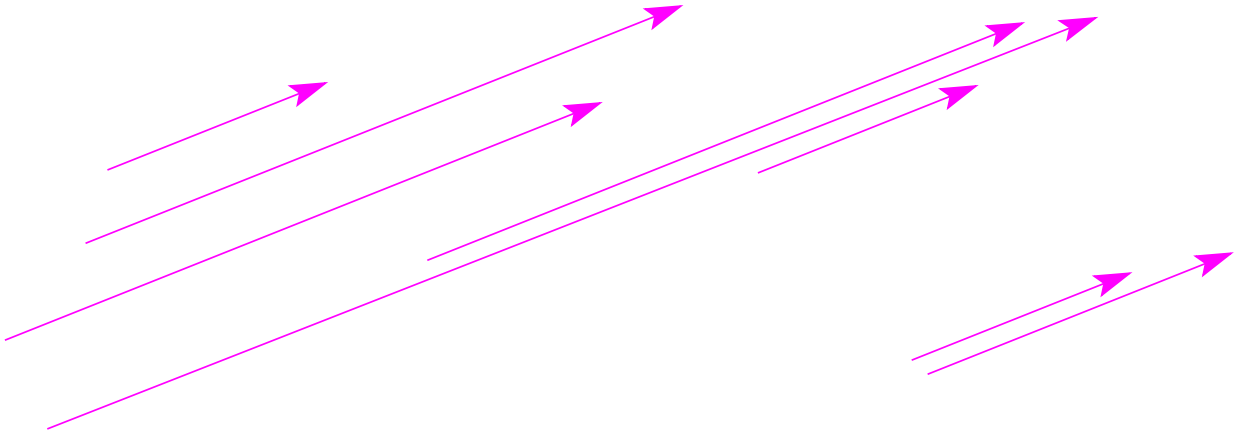
- a head and a tail,
- a fixed length,
- and a fixed direction,
- but not a fixed position.

Vectors

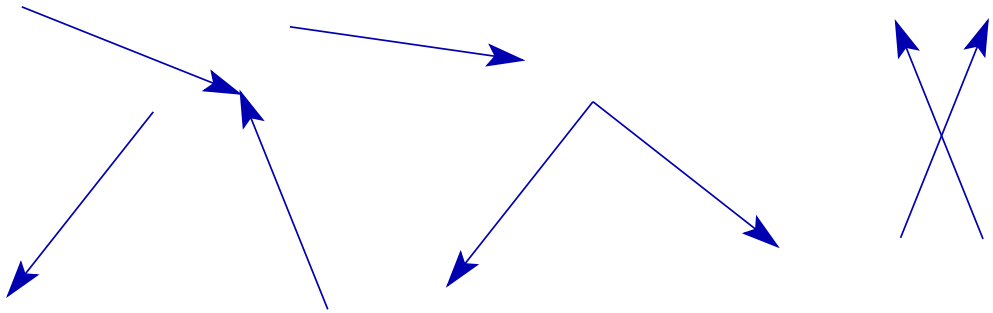


All these arrows represent the same vector!

Vectors



Not the same vector (different lengths)



Not the same vector (different lengths)

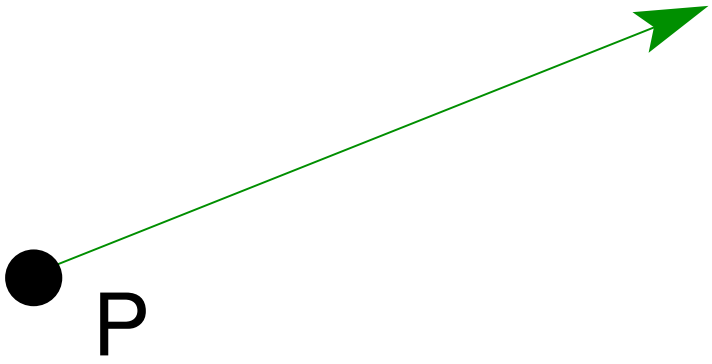
Translations

- Vectors are just what we need to describe translations, since we want to specify a direction and an amount of length to move every point.

- To find the image of a point P under a translation, place the translation vector with its tail at P and see where the head of the vector ends up.

Vectors

Vectors



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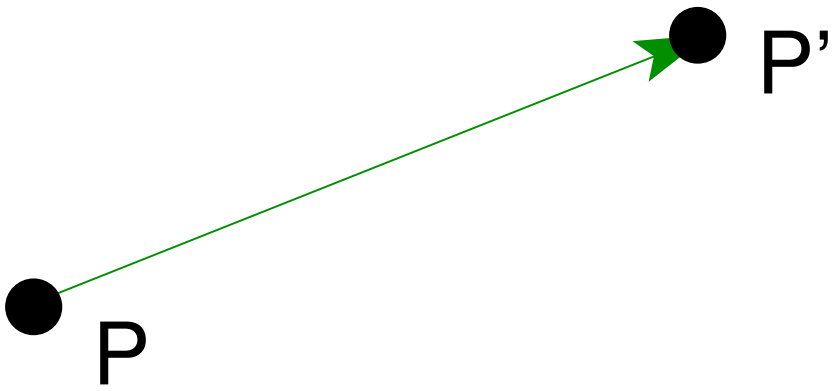

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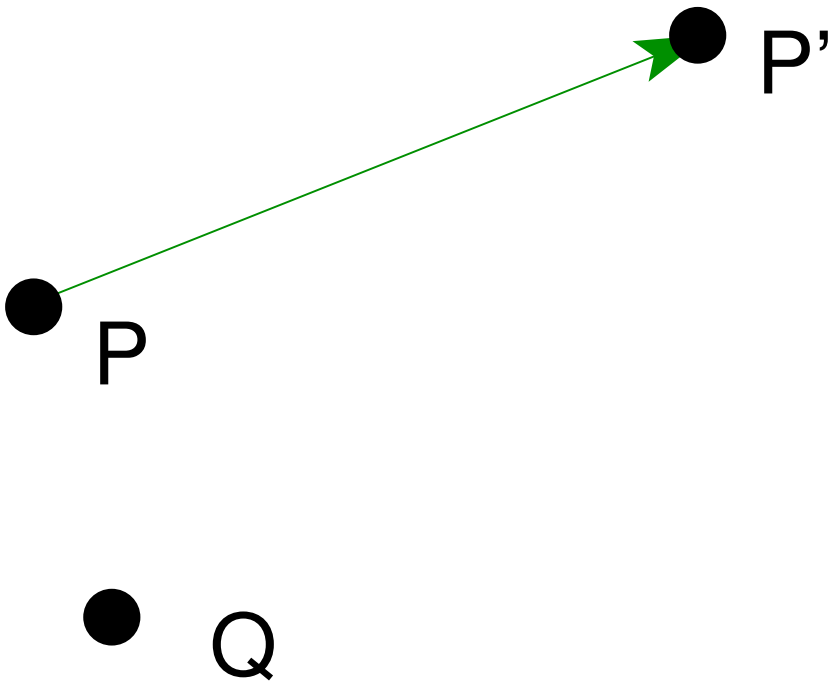

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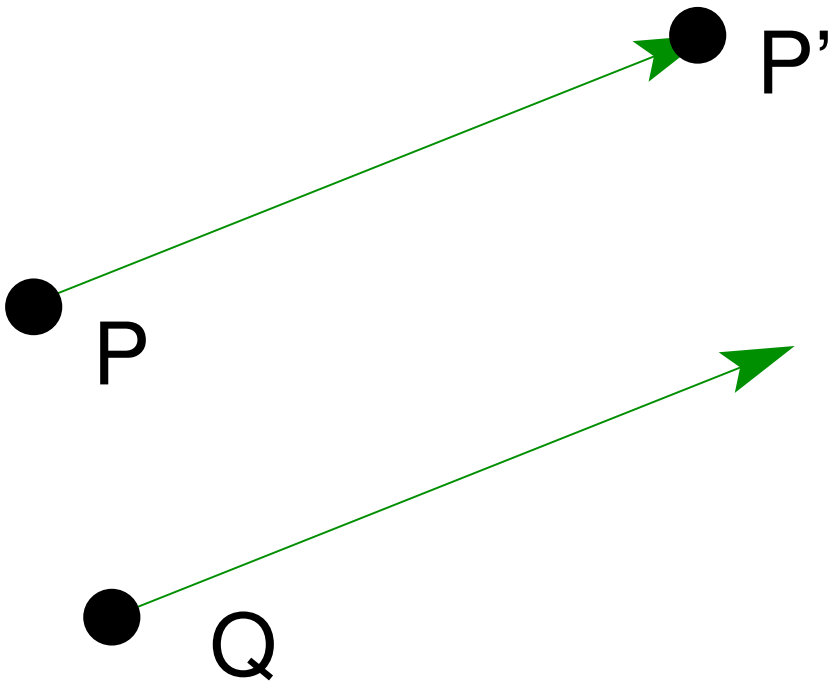

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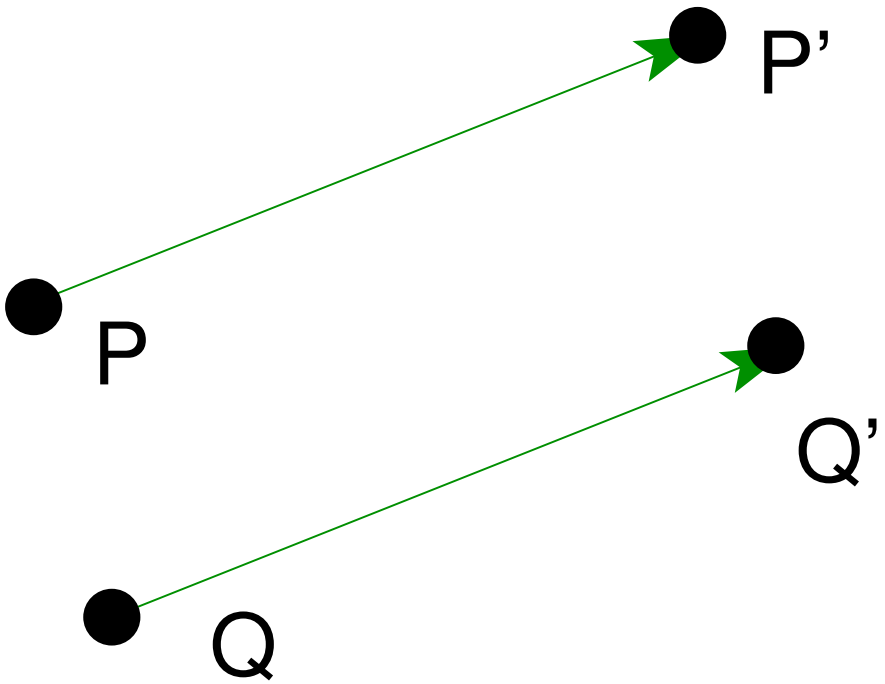

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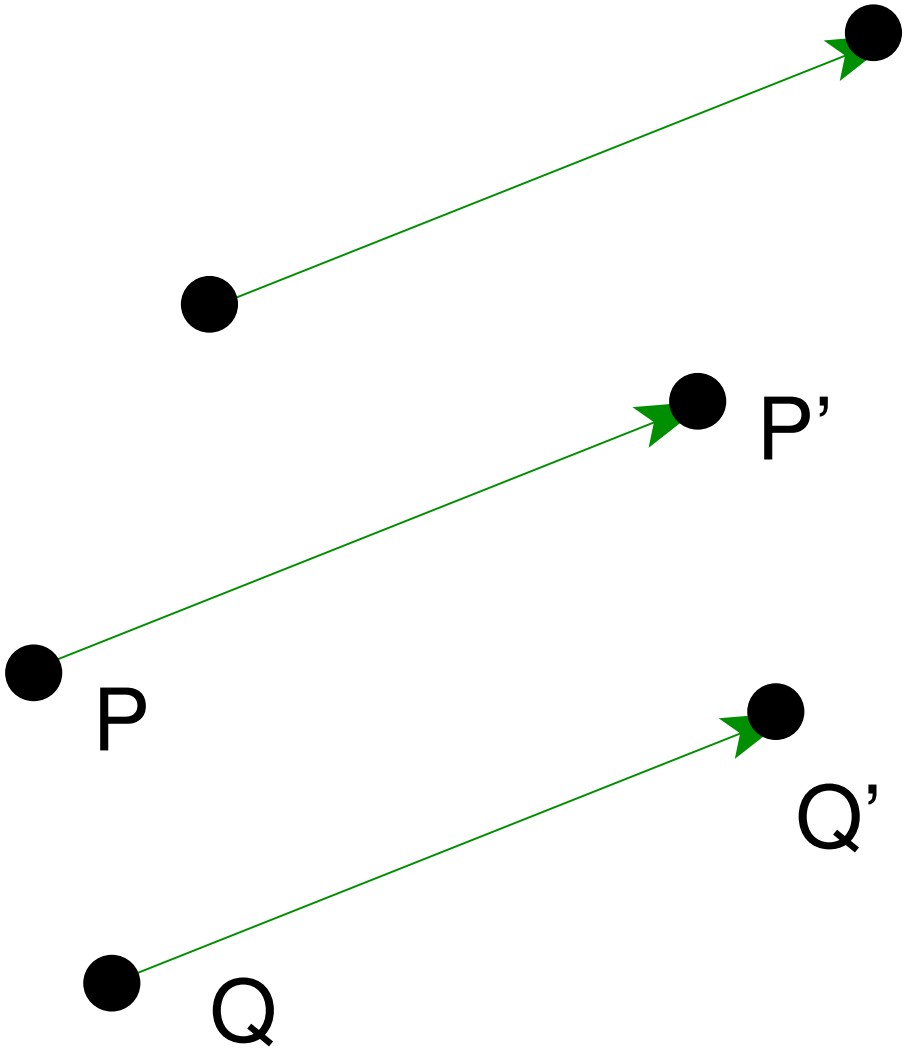

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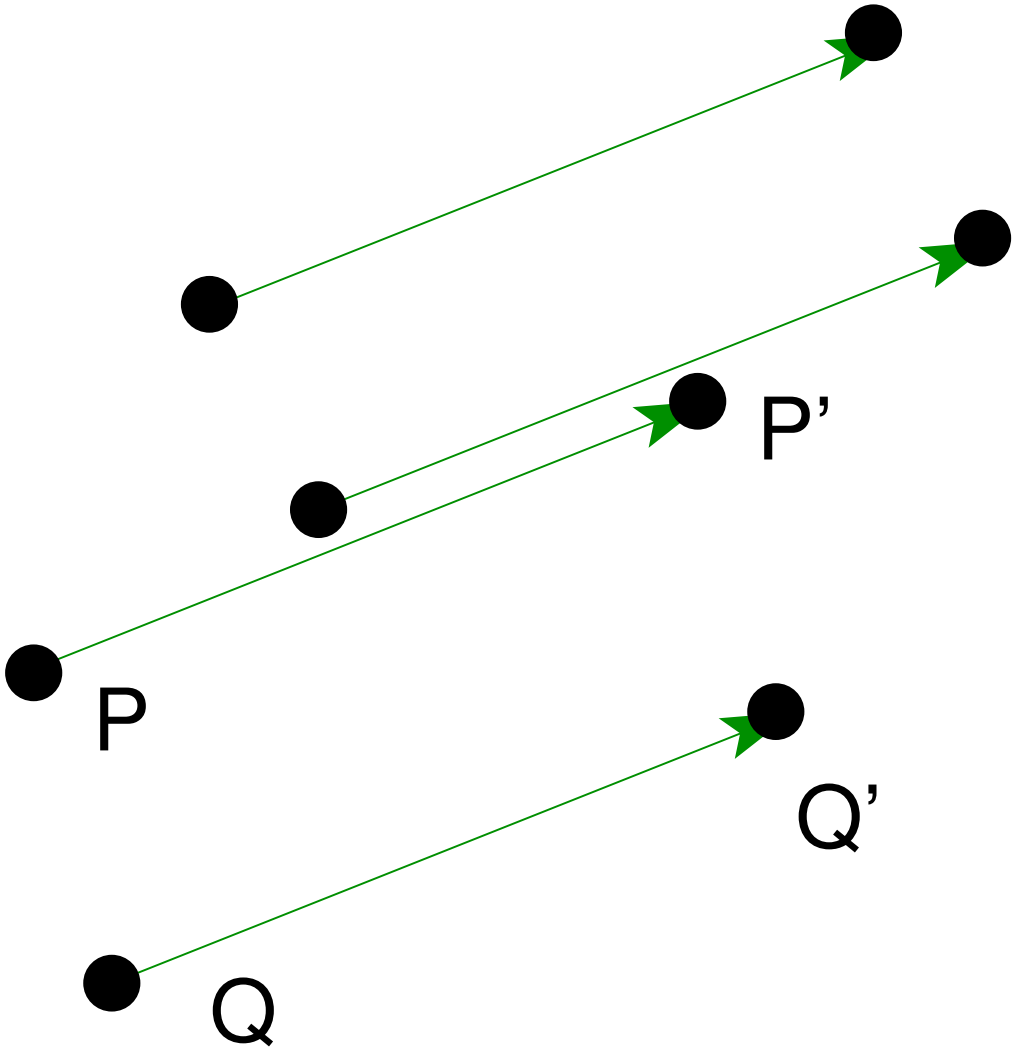
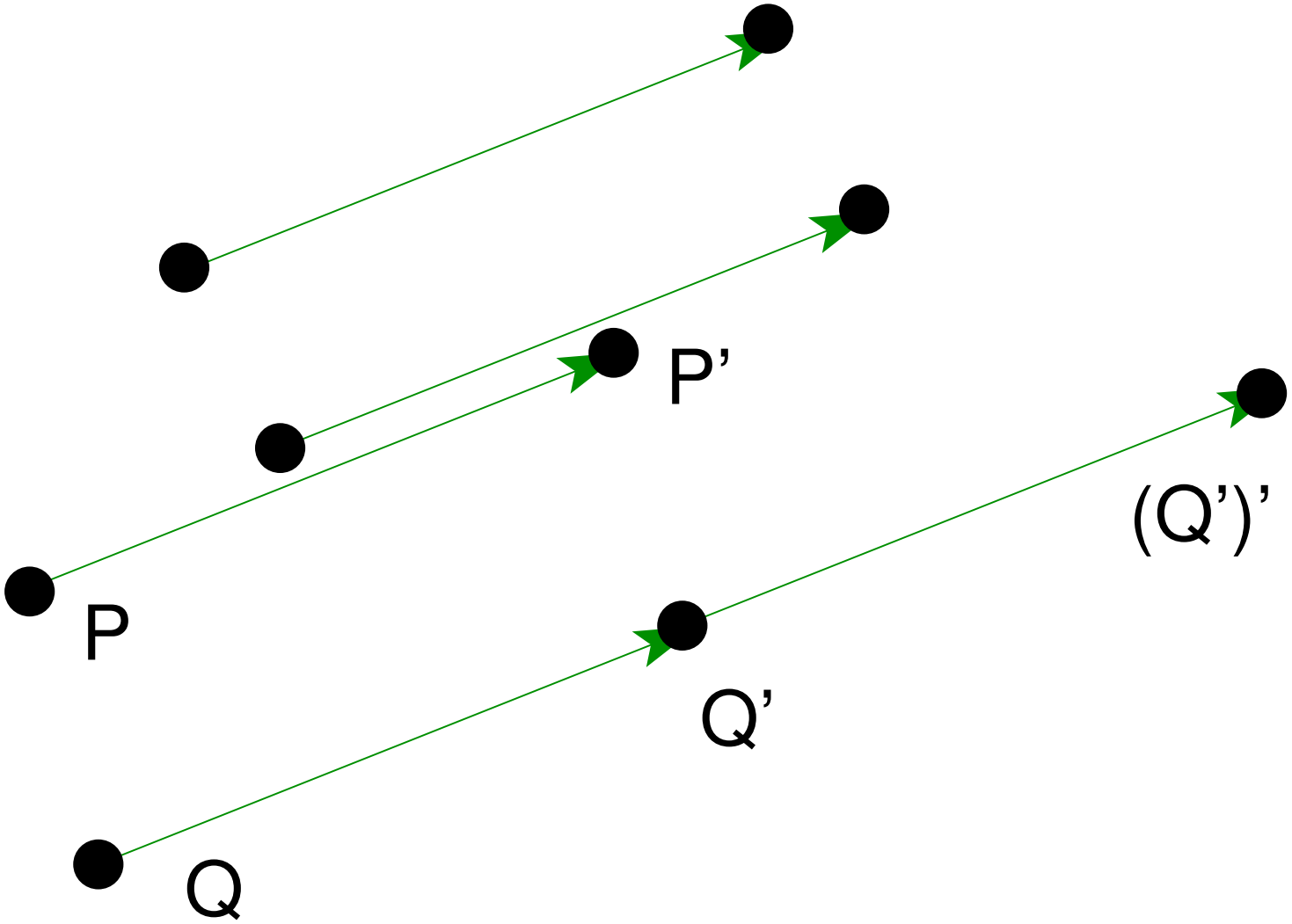


Vectors

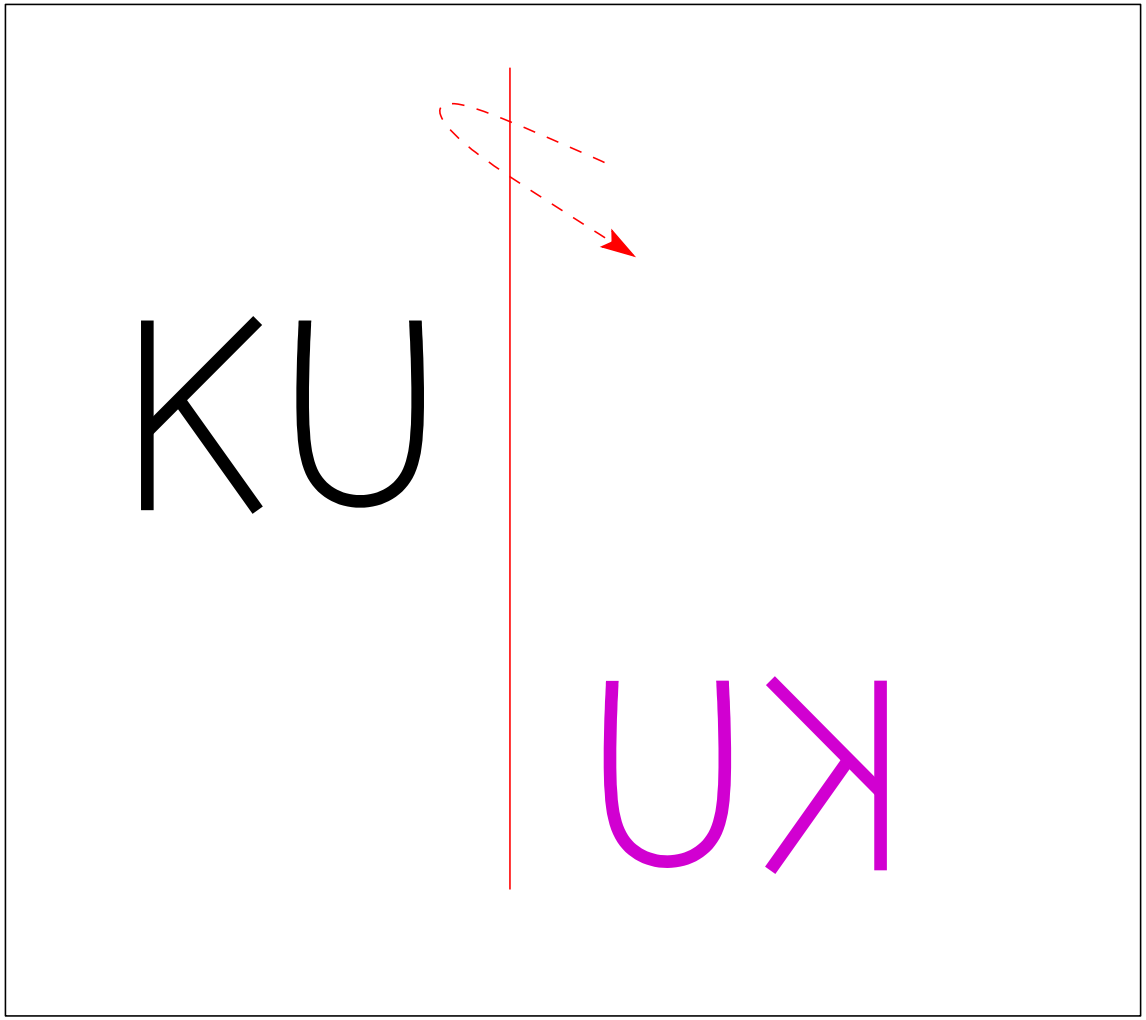


Properties of Translations

1. Every translation is completely determined by a **vector of translation, v.**
2. In a translation other than the identity, every point gets moved, so there are **no fixed points**.
3. Translations are **proper** rigid motions (they preserve clockwise/counterclockwise orientations).
4. A translation is determined by any **one** point-image pair P, P'.
5. The effect of a translation can be reversed by a translation in the **opposite direction** (with vector $-v$).

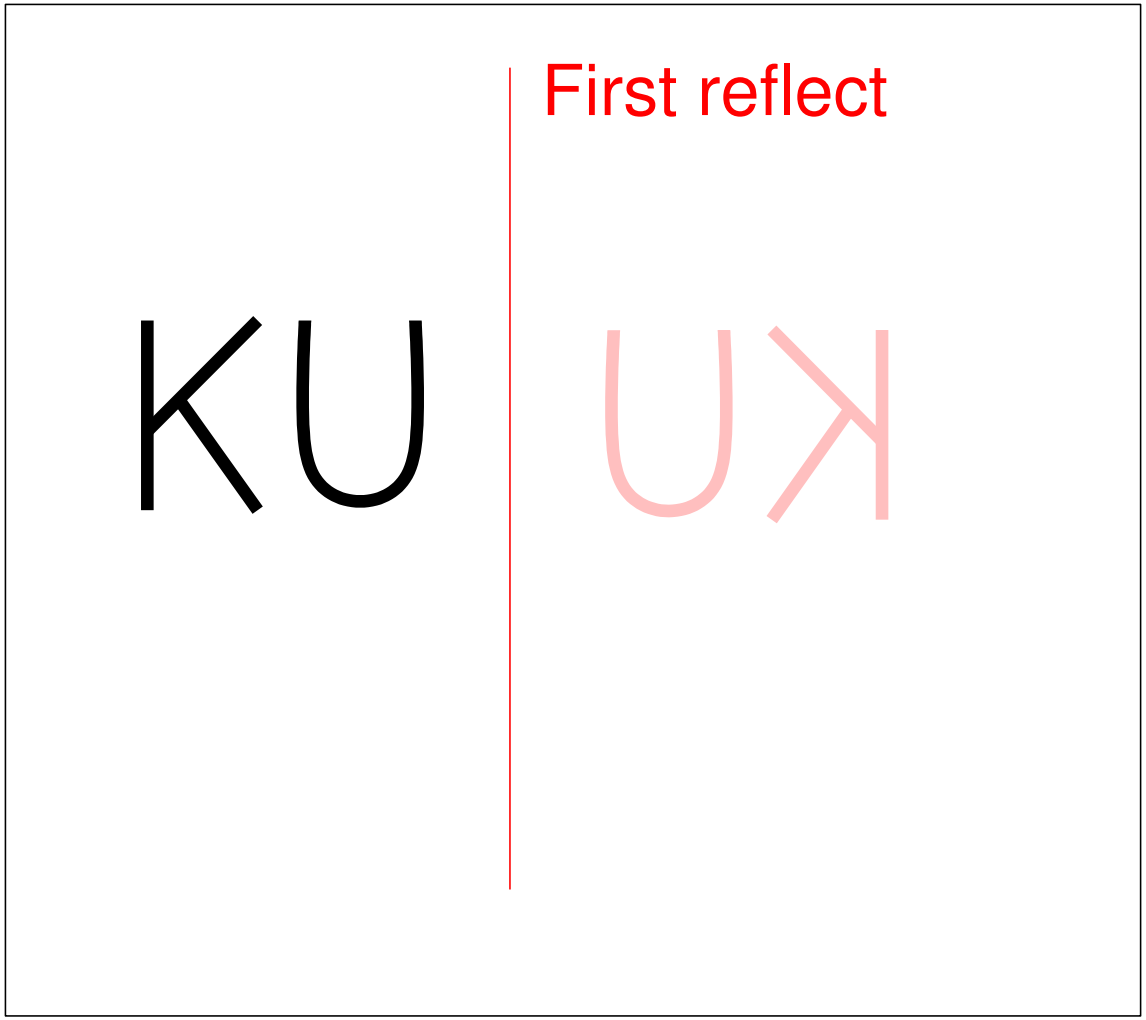
Glide Reflections

Glide Reflections

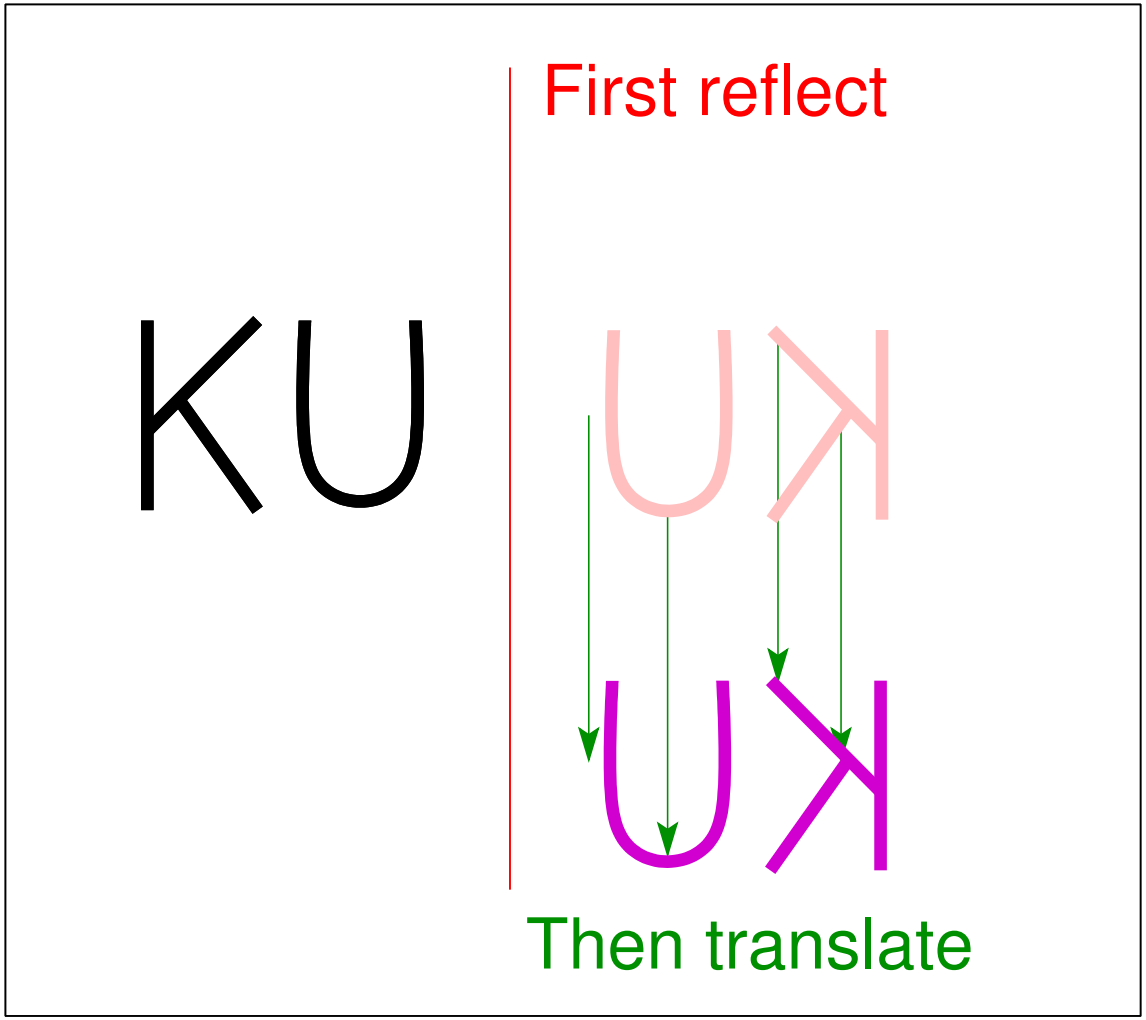


Glide reflection

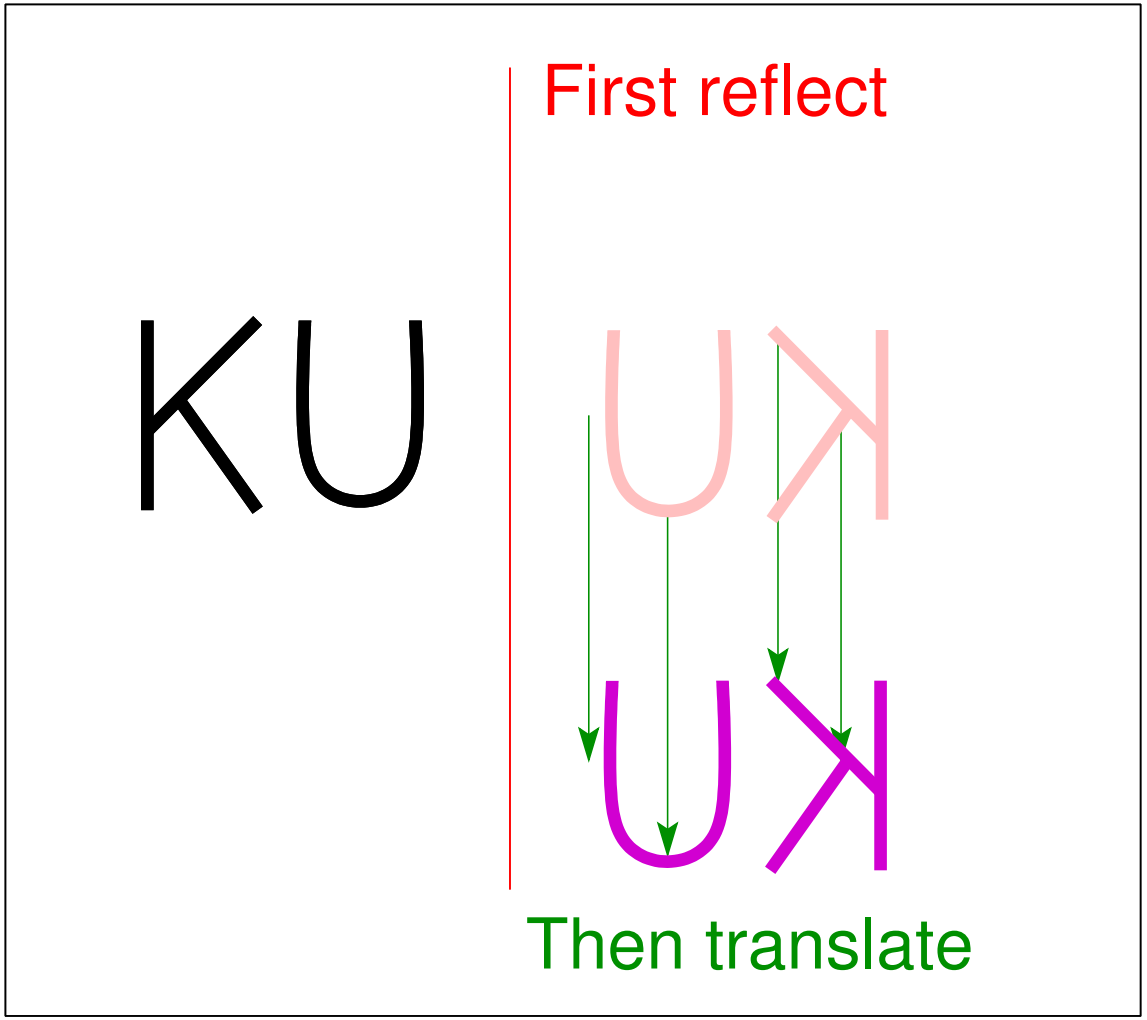
Glide Reflections



Glide Reflections



Glide Reflections



Note: vector of translation is **parallel** to axis of reflection

Properties of Glide Reflections

1. Every glide reflection is completely determined by a **line of reflection L** *and* a **vector of translation v**, which must be parallel to L.

2. In a glide reflection, every point gets moved, so there are **no fixed points**.

3. Glide reflections are **improper** rigid motions (they reverse clockwise/counterclockwise orientations).

4. A glide reflection is determined by any **two** point-image pair P, P' and Q, Q'.

Summary: Comparison of Rigid Motions

	Rotation	**Reflection**	**Translation**	**Glide reflection**
Defined by	Rotocenter, angle	Axis of reflection	Vector of translation	Axis, vector
Fixed points	Rotocenter only	All points on axis	None	None
Proper?	Proper	Improper	Proper	Improper
#PIP	2	1	2	1

"#PIP" = # of point-image pairs needed to determine

Rigid Motions and Symmetries

Every rigid motion is either a reflection, rotation, translation, or a glide reflection.

Meanwhile, a **symmetry** of an object is just a **rigid motion that moves it onto itself.**

Mathematicians can classify geometric figures by how many, and what kinds, of symmetries they have.

Rigid Motions and Symmetries

Rigid Motions and Symmetries

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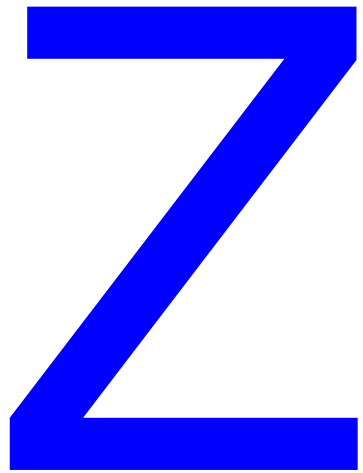
Rigid Motions and Symmetries



- These two figures have the same **symmetry type**.