Math 290 Test #3 **Name:** _____
November 9, 2006

Do Problems #1, #2, #4 and #5 on your own paper. Do Problem #3 on this sheet. Write your full name legibly on every sheet that you turn in. Staple all sheets together, with this sheet on top.

Problem #1 Let V be the set of all vectors $\mathbf{x} = (x_1, x_2, x_3, x_4)$ in R^4 such that $x_1 = x_4$ and $x_2 = -x_3$. (For example, $(3, 2, -2, 3)$ is in V, but $(3, 2, 2, 3)$ isn't.)

(#1a) [10 pts] What two properties do you need to check to prove that V is a vector space? Why don't you need to check the other eight properties in the definition of a vector space?

(#1b) [15 pts] Now prove that V is a vector space.

(#1c) [15 pts] Find a basis for V, and find its dimension.

Problem #2 Explain why the following sets are *not* vector spaces:

(#2a) [10 pts] Y, the set of all matrices (such as $\begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$).

(#2b) [10 pts] Z, the set of all vectors (x, y) in R^2 such that $x \geq y$ (such as $(1, -1)$ and (π, π)).

Problem #3 For each of the three given sets of vectors in R^3, determine whether it is (a) linearly independent; (b) a spanning set of R^3; (c) a basis of R^3. Indicate your answers by writing "yes" or "no" in the appropriate box of the grid. No explanation is necessary. [9x4 = 36 pts; no partial credit]

	Linearly independent?	Spanning set?	Basis?
$\{(-1,-1,2),\ (3,-1,3),\ (5,1,-1)\}$			
$\{(-1,-1,2),\ (3,-1,3),\ (5,1,0)\}$			
$\{(-1,-1,2),\ (3,-1,3)\}$			

Problem #4 [15 pts] Consider the set of vectors in R^3 given by

$$S = \{(q, 2, -2), \quad (5, q, -5), \quad (2, -2, q)\},$$

where q is some real number.

Determine all values of q for which S is *not* a basis of R^3. (Just giving the answer is not sufficient; you need to explain how you got it!)

Problem #5 Let A be the following matrix:

$$A = \begin{bmatrix} -1 & 3 & 7 \\ 3 & -3 & -3 \\ 1 & 2 & 8 \\ 4 & -3 & -1 \end{bmatrix}$$

(#5a) [10 pts] Find a basis for the row space of A.

(#5b) [10 pts] Find a basis for the column space of A.

(#5c) [10 pts] Find a basis for the nullspace of A.

(#5d) [9 pts] Find the rank and nullity of A.