Lecture Notes on Algebraic Combinatorics

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Foreword

The starting point for these lecture notes was my notes from Vic Reiner's Algebraic Combinatorics course at the University of Minnesota in Fall 2003. I currently use them for graduate courses at the University of Kansas. They will always be a work in progress. Please use them and share them freely for any research purpose. I have added and subtracted some material from Vic's course to suit my tastes, but any mistakes are my own; if you find one, please contact me at `jlmartin@ku.edu` so I can fix it. Thanks to those who have suggested additions and pointed out errors, including but not limited to: Kevin Adams, Nitin Aggarwal, Lucas Chaffee, Ken Duna, Josh Fenton, Darij Grinberg, Logan Godkin, Bennet Goeckner, Brent Holmes, Alex Lazar, Nick Packauskas, Billy Sanders, and Tony Se. Thanks to Marge Bayer for contributing the material on Ehrhart theory (§**??**).

1. Posets, Simplicial Complexes, and Polytopes

1.1. Posets.

Definition 1.1. A **partially ordered set** or **poset** is a set P equipped with a relation $\leq$ that is reflexive, antisymmetric, and transitive. That is, for all $x, y, z \in P$:

(1) $x \leq x$ (reflexivity).
(2) If $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetry).
(3) If $x \leq y$ and $y \leq z$, then $x \leq z$ (transitivity).

We say that x is **covered** by y, written $x \lessdot y$, if $x < y$ and there exists no z such that $x < z < y$. Two posets P, Q are **isomorphic** if there is a bijection $\phi : P \to Q$ that is order-preserving; that is, $x \leq y$ in P iff $\phi(x) \leq \phi(y)$ in Q.

We'll usually assume that P is finite.

Definition 1.2. A poset L is a **lattice** if every pair $x, y \in L$ has a unique **meet** $x \wedge y$ and **join** $x \vee y$. That is,

$$x \wedge y = \max\{z \in L \mid z \leq x, y\},$$
$$x \vee y = \min\{z \in L \mid z \geq x, y\}.$$

We'll have a lot more to say about lattices in Section **??**.