Math 290 Test #2
October 10, 2006

Do all problems on your own paper. Write your full name legibly on every sheet that you turn in. You may keep this sheet.

Problem #1 Let A be the following matrix:

$$\begin{bmatrix} 8 & -10 & 7 \\ 1 & -1 & 1 \\ 3 & -4 & 3 \end{bmatrix}$$

#1a [20 pts] Find A^{-1} by adjoining an identity matrix to A and performing row-reduction. Clearly indicate every step of your calculation. Giving the answer alone without any calculation will earn no credit.

#1b [20 pts] Use your answer to #1a to solve the system of equations

$$\begin{cases} 8x - 10y + 7z = 4 \\ x - y + z = -1 \\ 3x - 4y + 3z = 2. \end{cases}$$

Problem #2 [20 pts] Suppose that A and B are invertible $n \times n$ matrices such that $AB = BA$. Is it always true that $A^{-1}B^{-1} = B^{-1}A^{-1}$? Either explain why the equation is always true, or give an example of matrices A, B that illustrate why it is not always true.

Problem #3 [20 pts] Evaluate the following determinant by hand. You may use expansion by cofactors, row and column operations, or a combination. Indicate clearly all the steps in your calculation. Giving the answer alone without any calculation will earn no credit.

$$\begin{vmatrix} 3 & 4 & -2 & 0 \\ 1 & -2 & 8 & 3 \\ 0 & 0 & -2 & 5 \\ 0 & 0 & 3 & 0 \end{vmatrix}$$

Problem #4 I'm having a lot of problems with the wireless card in my computer. If it doesn't work one day, there is only a 30% chance that it will work the next day. On the other hand, if it works one day then the probability that it stops working on the following day is only 10%.

#4a [10 pts] If the computer is working today, what is the probability that it is still working two days from now?

#4b [10 pts] If the computer is working today, what is the probability that it was working yesterday?

In this problem (and this problem only!), you don't need to show the details of your matrix calculations such as products and inverses.

Problem #5 [15 pts] Let M be an invertible $n \times n$ matrix. Recall that

$$M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M),$$

where $\operatorname{adj}(M)$ is the adjoint matrix of M.

Use this fact to calculate $\det(\operatorname{adj}(M))$ in terms of $\det(M)$ and n.

Problem #6 [15 pts] Explain why there does *not* exist any square matrix A such that $\det(A^2) = -4$.

Problem #7 [20 pts] Find the volume of the tetrahedron with vertices at $(0, 0, 0)$, $(0, 0, -1)$, $(0, 4, 2)$, and $(2, 3, -1)$.

Bonus Problem Let S be an $n \times n$ matrix that is symmetric and invertible. Prove that S^{-1} is also symmetric.