Team 2 - Homework Two

Assignment 2: KJ 7.2; KJ 7.5 NAME DATE

Dependencies

```
# predictive modeling
libraries('mlbench', 'caret', 'AppliedPredictiveModeling')

# Formatting Libraries
libraries('default', 'knitr', 'kableExtra')

# Plotting Libraries
libraries('ggplot2', 'grid', 'ggfortify')
```

(1) Kuhn & Johnson 7.2

Friedman (1991) introduced several benchmark data sets create by simulation. One of these simulations used the following nonlinear equation to create data: $y = 10\sin(\pi x_1 x_2)20(x_3 - 0.5)^2 10x_4 5x_5 N(0, \sigma^2)$; where the x values are random variables uniformly distributed between [0, 1] (there are also 5 other non-informative variables also created in the simulation).

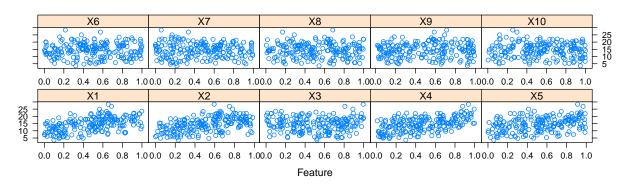
**The package mlbench contains a function called mlbench.friedman1 that simulates these data: **

```
set.seed(200)
trainingData <- mlbench.friedman1(200, sd = 1)

## We convert the 'x' data from a matrix to a data frame
## One reason is that this will give the columns names.

trainingData$x <- data.frame(trainingData$x)

## Look at the data using
featurePlot(trainingData$x, trainingData$y)</pre>
```



```
## or other methods.
## This creates a list with a vector 'y' and a matrix
```

```
## of predictors 'x'. Also simulate a large test set to
## estimate the true error rate with good precision:
testData <- mlbench.friedman1(5000, sd = 1)
testData$x <- data.frame(testData$x)</pre>
         (a) Tune several models on these data. For example:
set.seed(200)
knnModel <- train(x = trainingData$x,
                 y = trainingData$y,
                  method = "knn",
                  preProc = c("center", "scale"),
                  tuneLength = 10)
knnModel
k-Nearest Neighbors
200 samples
10 predictor
Pre-processing: centered (10), scaled (10)
Resampling: Bootstrapped (25 reps)
Summary of sample sizes: 200, 200, 200, 200, 200, 200, ...
Resampling results across tuning parameters:
     RMSE
               Rsquared
                          MAE
  5 3.554554 0.4895311 2.883670
  7 3.423303 0.5264402 2.767070
  9 3.361439 0.5525056 2.702852
  11 3.275234 0.5885952 2.632363
  13 3.245376 0.6099949 2.607849
  15 3.218637 0.6308597 2.576730
  17 3.229692 0.6380326 2.589076
  19 3.231915 0.6463092 2.595749
  21 3.228217 0.6591640 2.599618
  23 3.254794 0.6610119 2.628392
RMSE was used to select the optimal model using the smallest value.
The final value used for the model was k = 15.
knnPred <- predict(knnModel, newdata = testData$x)</pre>
## The function 'postResample' can be used to get the test set performance values
postResample(pred = knnPred, obs = testData$y)
     RMSE Rsquared
                          MAE
3.1750657 0.6785946 2.5443169
Model 1: KNN model with hyperparameter tuning
set.seed(100)
knn_model <- train(trainingData$x,
trainingData$y,
method = "knn",
 # Center and scaling will occur for new predictions too
```

```
preProc = c("center", "scale"),
tuneGrid = data.frame(.k = 1:50),
trControl = trainControl(method = "cv"))
knn_model
k-Nearest Neighbors
200 samples
10 predictor
Pre-processing: centered (10), scaled (10)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 180, 180, 180, 180, 180, 180, ...
Resampling results across tuning parameters:
 k
     RMSE
               Rsquared
                         MAE
  1 4.218081 0.4124441
                         3.500924
  2 3.546791 0.5197988 2.941632
  3 3.460792 0.5170308 2.841547
  4 3.304737 0.5584950 2.697459
  5 3.281638 0.5736341 2.695889
  6 3.201700 0.6056758 2.651120
  7 3.212544 0.6083203 2.677007
  8 3.175321 0.6293337 2.589611
  9 3.117499 0.6632514 2.555966
 10 3.118730 0.6619864 2.542520
 11 3.059105 0.6860779 2.486276
 12 3.110398 0.6794240 2.539603
 13 3.107626 0.6909955 2.504201
 14 3.098502 0.6981615 2.503473
 15 3.107763 0.7020139 2.517724
 16 3.092651 0.7141572 2.499280
 17 3.094870 0.7184612 2.502765
 18 3.106069 0.7205098 2.512843
 19 3.110156 0.7223952 2.522663
 20 3.129738 0.7248629 2.544792
 21 3.125325 0.7282228 2.555344
 22 3.152564 0.7265154 2.583124
 23 3.156426 0.7299184 2.590130
 24 3.156312 0.7315798 2.583509
 25 3.168639 0.7320840 2.600023
 26 3.192311 0.7301789 2.615933
 27 3.191953 0.7359772 2.626247
 28 3.236432 0.7229397 2.667947
 29 3.266908 0.7199837 2.696586
 30 3.264200 0.7279598 2.695916
 31 3.273003 0.7273885 2.702978
 32 3.282489 0.7276198 2.702930
 33 3.289932 0.7284479 2.707697
 34 3.303444 0.7281557 2.717839
 35 3.320845 0.7249744 2.727184
 36 3.331038 0.7257760 2.729478
```

37 3.342847 0.7296750 2.740671

```
      38
      3.353494
      0.7305494
      2.745450

      39
      3.363513
      0.7335993
      2.748193

      40
      3.369217
      0.7349916
      2.752439

      41
      3.394484
      0.7291182
      2.779272

      42
      3.404124
      0.7292558
      2.783362

      43
      3.403433
      0.7342789
      2.780712

      44
      3.418392
      0.7341581
      2.794006

      45
      3.420892
      0.7410895
      2.790850

      46
      3.421315
      0.7485185
      2.801475

      47
      3.434838
      0.7498270
      2.816174

      48
      3.449996
      0.7487060
      2.831474

      49
      3.451778
      0.7525105
      2.833605

      50
      3.467105
      0.7471964
      2.844317
```

RMSE was used to select the optimal model using the smallest value. The final value used for the model was k = 11.

```
knn_Pred <- predict(knn_model, newdata = testData$x)

## The function 'postResample' can be used to get the test set performance values
knn_pv <- postResample(pred = knn_Pred, obs = testData$y)
knn_pv</pre>
```

```
RMSE Rsquared MAE 3.1222641 0.6690472 2.4963650
```

Unlike above approach where tuneLength = 10 to find 10 odd numbered Ks starting from 5, we will set tuneGrid running from k = 1 to 50 after CV process. RMSE on validation set was used to select the optimal model using the smallest value. The final value used for the model was k = 11 with RMSE on test set of 3.1222641.

Model 2: Neural Networks

```
# remove highly correlated predictors to ensure that the maximum absolute pariwise correlation between # we did not have any highly correlated predictors so let's keep the features as they are. findCorrelation(cor(trainingData$x), cutoff = .75)
```

integer(0)

```
# hyperparameter tuning for nnet
nnetGrid <- expand.grid(.size = c(1:10), .decay = c(0, 0.01, .1))

set.seed(100)
nnet_model <- train(trainingData$x, trainingData$y,
method = "nnet",
tuneGrid = nnetGrid,
trControl = trainControl(method="cv"),
## Automatically standardize data prior to modeling and prediction
preProc = c("center", "scale"),
linout = TRUE,
trace = FALSE,
MaxNWts = 10 * (ncol(trainingData$x) + 1) + 10 + 1,
maxit = 500)</pre>
```

Neural Network

200 samples 10 predictor

Pre-processing: centered (10), scaled (10) Resampling: Cross-Validated (10 fold)

```
Resampling results across tuning parameters:
 size
       decay
              RMSE
                        Rsquared
                                   MAE
  1
       0.00
              2.803056
                        0.6666327
                                   2.258691
  1
       0.01
              2.427596
                        0.7621743
                                   1.887808
  1
       0.10
              2.435471 0.7608686
                                   1.890066
  2
       0.00
              2.489139 0.7430098 1.959395
  2
       0.01
              2.580965 0.7303017
                                   2.026386
   2
       0.10
              2.697160
                        0.7081857
                                   2.145994
  3
       0.00
              2.175481 0.8114073
                                   1.736758
   3
       0.01
              2.247174 0.7923587
                                   1.847530
              2.574029 0.7395256
  3
       0.10
                                   2.056691
  4
       0.00
              2.338196 0.7863493
                                   1.857565
  4
       0.01
              2.382258 0.7789730
                                   1.875869
   4
       0.10
              2.441937 0.7640421 1.906016
  5
       0.00
                                   2.815675
              4.095639 0.6461716
   5
       0.01
              2.611351 0.7275315
                                   2.058338
   5
       0.10
              2.527299 0.7479751
                                   2.043740
  6
       0.00
              4.360771 0.5655777
                                   2.776134
  6
       0.01
              2.740151 0.7334422
                                   2.120769
  6
       0.10
              2.723467 0.7037555
                                   2.191856
  7
       0.00
              8.107456 0.6013541
                                   3.614263
  7
       0.01
              2.634878 0.7306846
                                   2.082135
  7
       0.10
              2.634554
                        0.7272090
                                   2.034430
  8
       0.00
              9.294033 0.4764955
                                   3.995120
  8
       0.01
              3.239373 0.6734412
                                   2.543492
              2.857949
  8
       0.10
                        0.7043441
                                   2.285976
  9
       0.00
              7.318398
                        0.5467571
                                   3.670600
  9
       0.01
              3.541729 0.5379448
                                   2.671080
  9
       0.10
              3.187206 0.6050659
                                   2.544700
 10
       0.00
              4.083062 0.5181907
                                   2.838082
 10
       0.01
              3.525950
                        0.5879990
                                   2.836265
 10
       0.10
              3.010387 0.6719106 2.387353
```

Summary of sample sizes: 180, 180, 180, 180, 180, 180, ...

RMSE was used to select the optimal model using the smallest value. The final values used for the model were size = 3 and decay = 0.

```
nnet_Pred <- predict(nnet_model, newdata = testData$x)

## The function 'postResample' can be used to get the test set performance values
nnet_pv <- postResample(pred = nnet_Pred, obs = testData$y)
nnet_pv</pre>
```

RMSE Rsquared MAE 2.3950120 0.7740742 1.7970761

We found nnet with $\mathtt{size} = 3$ (number of units in the hidden layer) and $\mathtt{decay} = 0$ (parameter for weight decay) is the optimal model based on RMSE on validating set. RMSE on test set was 2.395012.

```
Model 3: Neural Networks Using Model Averaging
# hyperparameter tuning for aunnet
nnetGrid2 \leftarrow expand.grid(.size = c(1:10), .decay = c(0, 0.01, .1), .bag = FALSE)
set.seed(100)
avnnet_model <- train(trainingData$x, trainingData$y,</pre>
method = "avNNet",
tuneGrid = nnetGrid2,
trControl = trainControl(method="cv"),
## Automatically standardize data prior to modeling and prediction
preProc = c("center", "scale"),
linout = TRUE,
trace = FALSE,
MaxNWts = 10 * (ncol(trainingData\$x) + 1) + 10 + 1,
maxit = 500)
avnnet_model
Model Averaged Neural Network
200 samples
10 predictor
Pre-processing: centered (10), scaled (10)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 180, 180, 180, 180, 180, 180, ...
Resampling results across tuning parameters:
```

```
size decay RMSE
                     Rsquared
                               MAF.
1
     0.00
           2.451722 0.7625351
                               1.896021
1
     0.01
           2.427658 0.7621763
                               1.887707
1
     0.10 2.435387 0.7608838
                               1.890037
2
     0.00
           2.458303 0.7556215 1.947825
2
     0.01
           2.447743 0.7533484
                               1.884509
2
     0.10
           2.483261 0.7506467 1.924237
 3
     0.00
           2.086103 0.8297667 1.625836
3
     0.01
           2.155718 0.8096710 1.676070
3
           2.227768 0.8003473 1.734663
     0.10
4
     0.00
           2.036944 0.8294847 1.636994
4
     0.01 2.102926 0.8232439 1.623997
4
           2.032960 0.8301759 1.604834
     0.10
5
     0.00
          2.314361 0.7871277 1.717315
5
     0.01 2.131569 0.8131597 1.670786
5
     0.10
           2.192707 0.8054839 1.778156
6
     0.00
           2.593804 0.7448382 1.966474
6
     0.01
           2.167612 0.8108275 1.674682
6
     0.10 2.042238 0.8227937 1.645410
7
     0.00 4.989121 0.5232150 3.110796
7
     0.01
           2.238776 0.7948069
                               1.754542
7
     0.10
           2.202599 0.7926956 1.751645
8
     0.00
          5.839958 0.5788055 3.266803
8
     0.01
           2.392621 0.7727856 1.893142
8
     0.10
           2.271702 0.7860966 1.790783
     0.00
           5.138515 0.4191673 3.248740
```

```
9 0.01 2.463813 0.7534985 1.927194
9 0.10 2.250755 0.7930757 1.820346
10 0.00 3.400511 0.6426513 2.428742
10 0.01 2.456849 0.7404324 1.988814
10 0.10 2.437919 0.7608963 1.971687
```

Tuning parameter 'bag' was held constant at a value of FALSE RMSE was used to select the optimal model using the smallest value. The final values used for the model were size = 4, decay = 0.1 and bag = FALSE.

```
avnnet_Pred <- predict(avnnet_model, newdata = testData$x)

## The function 'postResample' can be used to get the test set performance values
avnnet_pv <- postResample(pred = avnnet_Pred, obs = testData$y)
avnnet_pv</pre>
```

```
RMSE Rsquared MAE 2.1306481 0.8202697 1.5982639
```

We found nnet with size = 4 (number of units in the hidden layer) and decay = 0.1 (parameter for weight decay) is the optimal model based on RMSE on validating set. RMSE on test set was 2.1306481.

Model 4: Multivariate Adaptive Regression Splines

```
# hyperparameter tuning for MARS
marsGrid <- expand.grid(.degree = 1:3, .nprune = 2:38)

set.seed(100)
mars_model <- train(trainingData$x, trainingData$y,
  method = "earth",
  tuneGrid = marsGrid,
  trControl = trainControl(method="cv"))

#mars_model
summary(mars_model)</pre>
```

```
coefficients
(Intercept)
                                    20.378441
h(0.621722-X1)
                                   -15.512132
h(X1-0.621722)
                                     9.177132
h(0.601063-X2)
                                   -17.940676
h(X2-0.601063)
                                    10.064356
h(X3-0.281766)
                                    11.590022
h(0.447442-X3)
                                   14.641640
h(X3-0.447442)
                                   -12.924806
h(X3-0.606015)
                                    13.416764
h(0.734892-X4)
                                   -10.074386
h(X4-0.734892)
                                     9.687149
h(0.850094-X5)
                                    -5.385762
h(0.218266-X1) * h(X2-0.601063)
                                   -55.372637
h(X1-0.218266) * h(X2-0.601063)
                                   -27.542831
h(X1-0.621722) * h(X2-0.295997)
                                   -26.527403
```

```
h(0.649253-X1) * h(0.601063-X2)
                                   26.129827
Selected 16 of 18 terms, and 5 of 10 predictors
Termination condition: Reached nk 21
Importance: X1, X4, X2, X5, X3, X6-unused, X7-unused, X8-unused, ...
Number of terms at each degree of interaction: 1 11 4
GCV 1.61518
               RSS 210.6377
                               GRSa 0.934423
                                                RSq 0.9568093
mars Pred <- predict(mars model, newdata = testData$x)</pre>
## The function 'postResample' can be used to get the test set performance values
mars_pv <- postResample(pred = mars_Pred, obs = testData$y)</pre>
mars_pv
     RMSE Rsquared
                          MAE
1.1492504 0.9471145 0.9158382
We found MARS with degree = 2 (Maximum degree of interaction (Friedman's mi)) and nprune = 17
(aximum number of terms (including intercept) in the pruned model) is the optimal model based on RMSE
on validating set. RMSE on test set was 1.1492504.
Model 5: Support Vector regression
set.seed(100)
svm_model <- train(trainingData$x, trainingData$y,</pre>
method = "svmRadial",
preProc = c("center", "scale"),
tuneLength = 14,
trControl = trainControl(method="cv"))
#svm model
svm_model
Support Vector Machines with Radial Basis Function Kernel
200 samples
10 predictor
Pre-processing: centered (10), scaled (10)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 180, 180, 180, 180, 180, 180, ...
Resampling results across tuning parameters:
           RMSE
                     Rsquared
                                MAE
     0.25 2.534788 0.7882081 2.034824
     0.50 2.292127 0.8029516 1.819981
     1.00 2.091598 0.8284381 1.657402
     2.00 1.967193 0.8457471 1.546737
     4.00 1.883133 0.8561761 1.482054
    8.00 1.863807 0.8588797 1.468328
    16.00 1.834215 0.8633819 1.456738
   32.00 1.836471 0.8632508 1.459909
   64.00 1.836471 0.8632508 1.459909
   128.00 1.836471 0.8632508 1.459909
   256.00 1.836471 0.8632508 1.459909
```

512.00 1.836471 0.8632508 1.459909

```
1024.00 1.836471 0.8632508 1.459909
2048.00 1.836471 0.8632508 1.459909
```

Tuning parameter 'sigma' was held constant at a value of 0.0552698 RMSE was used to select the optimal model using the smallest value. The final values used for the model were sigma = 0.0552698 and C = 16.

```
svm_Pred <- predict(svm_model, newdata = testData$x)

## The function 'postResample' can be used to get the test set performance values
svm_pv <- postResample(pred = svm_Pred, obs = testData$y)
svm_pv</pre>
```

```
RMSE Rsquared MAE 2.0490047 0.8297577 1.5586106
```

Since the nature of the equation of the data is non-linear, we will use symRadial as kernal function for regression. The final values used for the model were sigma = 0.0552698 and C = 16 with RMSE on test set of 2.0490047.

(b) Which models appear to give the best performance? Does MARS select the informative predictors (those named X1-X5)?

MARS appears to give the best performance based on RMSE, R squared and MAE on test set. The summary out put of mars_model gives us that Importance: X1, X4, X2, X5, X3, X6-unused, X7-unused, X8-unused, X9-unused, MARS does select the informative predictors X1-X5 only.

```
# summary mars
summary(mars_model)
```

| | coefficients |
|----------------|--------------|
| (Intercept) | 20.378441 |
| h(0.621722-X1) | -15.512132 |
| h(X1-0.621722) | 9.177132 |
| h(0.601063-X2) | -17.940676 |
| h(X2-0.601063) | 10.064356 |
| h(X3-0.281766) | 11.590022 |
| h(0.447442-X3) | 14.641640 |
| h(X3-0.447442) | -12.924806 |
| | |

```
h(X3-0.606015)
                                    13.416764
h(0.734892-X4)
                                   -10.074386
h(X4-0.734892)
                                     9.687149
h(0.850094-X5)
                                    -5.385762
h(0.218266-X1) * h(X2-0.601063)
                                   -55.372637
h(X1-0.218266) * h(X2-0.601063)
                                   -27.542831
h(X1-0.621722) * h(X2-0.295997)
                                   -26.527403
h(0.649253-X1) * h(0.601063-X2)
                                    26.129827
Selected 16 of 18 terms, and 5 of 10 predictors
Termination condition: Reached nk 21
Importance: X1, X4, X2, X5, X3, X6-unused, X7-unused, X8-unused, ...
Number of terms at each degree of interaction: 1 11 4
GCV 1.61518
               RSS 210.6377
                               GRSq 0.934423
                                                 RSq 0.9568093
```

(2) Kuhn & Johnson 7.5

Exercise 6.3 describes data for a chemical manufacturing process. Use the same data imputation, data splitting, and pre-processing steps as before and train several nonlinear regression models.

(a) Which nonlinear regression model gives the optimal resampling and test set performance?

k-Nearest Neighbors

```
144 samples
56 predictor
```

Pre-processing: centered (56), scaled (56)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 129, 131, 128, 129, 129, 130, ...
Resampling results across tuning parameters:

```
k
   RMSE
             Rsquared
                        MAE
1
   1.562947
             0.4104086
                        1.211757
2
  1.394114
             0.4526620
                        1.103294
3
   1.285798 0.5202328
                        1.022250
4
  1.261951 0.5511694
                       1.005031
  1.290154 0.5346045
                        1.016046
6
  1.285469 0.5467205
                        1.015962
7
   1.277158
             0.5691222
                        1.014170
8
   1.280950
            0.5672804
                        1.022692
  1.260191
             0.5951063
                        1.008777
9
10 1.268148
             0.5927028
                        1.022267
11 1.274834 0.5762849
                        1.040308
12 1.283879 0.5691015
                        1.046630
13 1.300146
            0.5565749
                        1.063440
14 1.294960
             0.5643696
                        1.054302
15
   1.315419
             0.5533099
                        1.071270
16 1.322033 0.5432361
                        1.078412
17
  1.333203
            0.5374531
                        1.078099
18 1.345231
             0.5267062
                        1.089399
19 1.354941 0.5240723
                        1.095199
20 1.361053 0.5260280
                        1.100479
21 1.357930 0.5291765 1.095387
```

```
22 1.360462 0.5363972 1.100225
23 1.369994 0.5307997 1.105951
24 1.381541 0.5262263 1.116182
25 1.390061 0.5225105 1.124752
26 1.390820 0.5240722 1.125271
27 1.401032 0.5200972 1.133713
28 1.406645 0.5247034 1.137767
29 1.417484 0.5124727 1.145324
30 1.422851 0.5160566 1.149150
31 1.424039 0.5158770 1.155165
32 1.431312 0.5132871 1.164555
33 1.441439 0.5029494 1.170119
34 1.445577 0.5033464 1.173391
35 1.446068 0.4997379 1.173717
36 1.451050 0.4987082 1.175739
37 1.450722 0.5073898 1.176310
38 1.452006 0.5093112 1.180577
39 1.458957 0.5080230 1.185948
40 1.462371 0.5046533 1.190404
41 1.470614 0.5009788 1.197493
42 1.476481 0.4945194 1.201055
43 1.478755 0.4983923 1.199669
44 1.484498 0.4951622 1.203632
45 1.490334 0.4982518 1.208023
46 1.491714 0.5057906 1.210398
47 1.496813 0.5022176 1.215699
48 1.500448 0.5047809 1.219058
49 1.506809 0.5048347 1.225709
50 1.509769 0.5034198 1.228911
```

RMSE was used to select the optimal model using the smallest value. The final value used for the model was k=9.

Neural Network

144 samples
35 predictor

Pre-processing: centered (35), scaled (35)

Resampling: Cross-Validated (10 fold)

Summary of sample sizes: 129, 131, 128, 129, 129, 130, ...

Resampling results across tuning parameters:

| size | decay | RMSE | Rsquared | MAE |
|------|-------|----------|-----------|----------|
| 1 | 0.00 | 1.755205 | 0.2066616 | 1.455577 |
| 1 | 0.01 | 1.531002 | 0.4448691 | 1.192716 |
| 1 | 0.10 | 1.373454 | 0.5175944 | 1.109371 |
| 2 | 0.00 | 2.212948 | 0.1773784 | 1.727733 |
| 2 | 0.01 | 1.864348 | 0.3910941 | 1.542705 |
| 2 | 0.10 | 1.715287 | 0.4696443 | 1.333888 |
| 3 | 0.00 | 4.213947 | 0.2706156 | 3.025339 |
| 3 | 0.01 | 3.402447 | 0.1312987 | 2.650608 |
| 3 | 0.10 | 2.274638 | 0.3535430 | 1.809244 |
| 4 | 0.00 | 3.753927 | 0.1694120 | 2.927575 |

```
4
     0.01
            3.032265 0.2730534
                                2.293653
4
     0.10
            2.662301 0.2596654 2.073559
     0.00
5
            3.193235 0.2142524 2.523091
5
     0.01
            2.847820 0.2693243
                                2.167825
5
     0.10
            2.843172 0.2010863
                                2.136950
6
     0.00
            3.141860 0.3508280 2.459695
6
     0.01
            2.535074 0.1739787
                                2.089114
6
     0.10
            2.009758 0.3223722 1.549843
                                3.377113
7
     0.00
            5.113950 0.1935983
7
     0.01
            2.849091 0.2545805
                                2.129850
7
     0.10
            2.114057 0.2762636
                                1.594852
8
     0.00
            4.294256 0.1767868
                                3.299536
8
     0.01
            2.795623 0.1388297
                                2.169489
8
     0.10
            2.275122 0.2983466 1.713263
9
     0.00
            9.006573 0.2327137 5.799823
9
     0.01
            2.730389
                     0.2517318
                                2.214361
9
     0.10
            2.237043 0.3159019
                                1.714727
10
     0.00
            8.689726 0.3538469
                                5.208147
10
     0.01
            4.329962 0.1650993
                                3.119468
10
     0.10
            2.410800
                     0.2083370
                                1.808763
```

RMSE was used to select the optimal model using the smallest value. The final values used for the model were size = 1 and decay = 0.1.

Model Averaged Neural Network

144 samples
35 predictor

Pre-processing: centered (35), scaled (35)

Resampling: Cross-Validated (10 fold)

Summary of sample sizes: 129, 131, 128, 129, 129, 130, ...

Resampling results across tuning parameters:

| size | decay | RMSE | Rsquared | MAE |
|------|-------|----------|-----------|----------|
| 1 | 0.00 | 1.589451 | 0.3851410 | 1.271954 |
| 1 | 0.01 | 1.376260 | 0.5016400 | 1.103760 |
| 1 | 0.10 | 1.298216 | 0.5536547 | 1.053173 |
| 2 | 0.00 | 1.614710 | 0.4230801 | 1.274696 |
| 2 | 0.01 | 1.435679 | 0.5191818 | 1.116245 |
| 2 | 0.10 | 1.486560 | 0.4888622 | 1.185787 |
| 3 | 0.00 | 1.893573 | 0.3390867 | 1.482814 |
| 3 | 0.01 | 1.947427 | 0.2967394 | 1.543828 |
| 3 | 0.10 | 1.702341 | 0.4794429 | 1.298251 |
| 4 | 0.00 | 2.183278 | 0.2198895 | 1.802826 |
| 4 | 0.01 | 1.837276 | 0.3676411 | 1.438189 |
| 4 | 0.10 | 1.841018 | 0.3934255 | 1.451878 |
| 5 | 0.00 | 1.788868 | 0.3797231 | 1.487272 |
| 5 | 0.01 | 1.677605 | 0.4073755 | 1.350633 |
| 5 | 0.10 | 1.790401 | 0.4160895 | 1.340008 |
| 6 | 0.00 | 2.183750 | 0.3741136 | 1.680205 |
| 6 | 0.01 | 1.474744 | 0.5088465 | 1.228068 |
| 6 | 0.10 | 1.842838 | 0.4122807 | 1.340819 |
| 7 | 0.00 | 2.535701 | 0.3775290 | 2.006750 |

```
7
     0.01
           1.678518 0.4521095 1.327766
7
     0.10
           1.808841 0.3698767 1.370640
     0.00
8
          3.327318 0.2290038 2.309492
8
     0.01 1.619979 0.4401995 1.268287
8
     0.10
           1.789302 0.4025716 1.399899
9
     0.00 4.247876 0.2434059 2.861125
9
     0.01 1.699004 0.4937072 1.303050
9
     0.10 1.967303 0.2727353 1.492640
10
     0.00 7.011010 0.2117344 4.342906
10
     0.01
           2.038958 0.3644072 1.468109
10
     0.10
          1.748484 0.3668079 1.353886
```

Tuning parameter 'bag' was held constant at a value of FALSE RMSE was used to select the optimal model using the smallest value. The final values used for the model were size = 1, decay = 0.1 and bag = FALSE.

| | coefficients |
|--|--------------|
| (Intercept) | 40.118477 |
| h(ManufacturingProcess09-46.65) | -27.147162 |
| h(33.1-ManufacturingProcess13) | 2.520578 |
| h(ManufacturingProcess13-33.1) | -0.686221 |
| h(ManufacturingProcess32-154) | 0.301393 |
| h(ManufacturingProcess33-61) | -0.209534 |
| BiologicalMaterial12 * h(ManufacturingProcess09-46.65) | 1.325649 |

Selected 7 of 51 terms, and 5 of 56 predictors
Termination condition: RSq changed by less than 0.001 at 51 terms
Importance: ManufacturingProcess32, ManufacturingProcess13, ...
Number of terms at each degree of interaction: 1 5 1
GCV 1.26095 RSS 143.4681 GRSq 0.6380526 RSq 0.7100031

Support Vector Machines with Radial Basis Function Kernel

144 samples
56 predictor

Pre-processing: centered (56), scaled (56) Resampling: Cross-Validated (10 fold)

Summary of sample sizes: 129, 131, 128, 129, 129, 130, ...

Resampling results across tuning parameters:

| С | | RMSE | Rsquared | MAE |
|---|-------|----------|-----------|-----------|
| | 0.25 | 1.372028 | 0.5607359 | 1.1092751 |
| | 0.50 | 1.237001 | 0.6179908 | 1.0047103 |
| | 1.00 | 1.126298 | 0.6748623 | 0.9158092 |
| | 2.00 | 1.081110 | 0.6935348 | 0.8707261 |
| | 4.00 | 1.081172 | 0.6897600 | 0.8820878 |
| | 8.00 | 1.085143 | 0.6878698 | 0.8848315 |
| | 16.00 | 1.085143 | 0.6878698 | 0.8848315 |
| | 32.00 | 1.085143 | 0.6878698 | 0.8848315 |
| | 64.00 | 1.085143 | 0.6878698 | 0.8848315 |

```
128.00
        1.085143 0.6878698
                             0.8848315
 256.00
        1.085143 0.6878698 0.8848315
                  0.6878698
512.00
        1.085143
                             0.8848315
1024.00
        1.085143
                  0.6878698
                             0.8848315
2048.00
        1.085143
                  0.6878698
                             0.8848315
```

Tuning parameter 'sigma' was held constant at a value of 0.01632049 RMSE was used to select the optimal model using the smallest value. The final values used for the model were sigma = 0.01632049 and C = 2.

```
knn_pv2 nnet_pv2 avnnet_pv2 mars_pv2 svm_pv2

RMSE 1.4986649 1.6189836 1.6190778 1.4131453 1.2756876

Rsquared 0.3302198 0.3159727 0.3159386 0.3911259 0.5011139

MAE 1.2035764 1.3311308 1.3312383 1.0797807 1.0200509
```

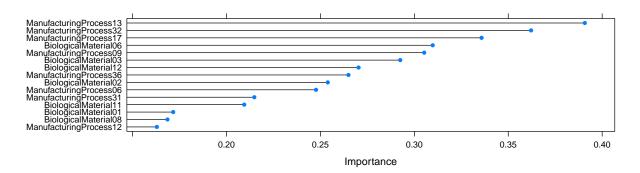
SVM regression gives the optimal performance based on RMSE, Rsquared and MAE on test set.

(b) Which predictors are most important in the optimal nonlinear regression model? Do either the biological or process variables dominate the list? How do the top ten important predictors compare to the top ten predictors from the optimal linear model?

In linear model, ManufacturingProcess32 was the most important predictor but in non-linear model, it is 2nd most important predictor - the most important predictor is ManufacturingProcess13.

In linear model, only 2 of top 10 were Biological where as in non-linear, 4 of them were.

```
# code
varimp <- varImp(svm_model2,scale=F,useModel = T)
plot(varimp, top=15, scales = list(y = list(cex = 0.8)))</pre>
```

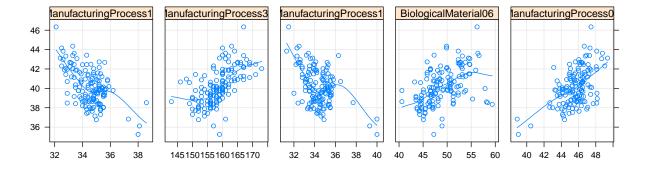


(c) Explore the relationships between the top predictors and the response for the predictors that are unique to the optimal nonlinear regression model. Do these plots reveal intuition about the biological or process predictors and their relationship

with yield?

From Bivariate plot and correlation matrix, we know that ManufacturingProcess32 has fairly positive relationship with Yield where as other 2 variables have fairly negative relationship. Among biological predictors, we know BiologicalMaterial06 is the most important with fairly strong positive relationship with Yield.

This information can help researchers to focus more on ManufacturingProcess32 and BiologicalMaterial06 if their goal is to increase Yield.



```
# corr_matrix
corr_top5 <- cor(df_train[, topVIP], df_train$Yield, method = 'pearson', use = 'pairwise.complete.obs')
corr_top5</pre>
```

[,1]
ManufacturingProcess13 -0.5645290
ManufacturingProcess32 0.6017957
ManufacturingProcess17 -0.4666781
BiologicalMaterial06 0.4673292
ManufacturingProcess09 0.5423679