AY640 Computational Experiment 2:
Thermal Radiation

Discussion: Friday March 4, 2022

In this experiment, you will use SiRTIPy to numerically integrate the radiative transfer equation over a range of frequencies through a uniform thermal medium.

1. Download `experiment2.py` from the Blackboard class website. Read through it so you understand what each line does.

2. Part A: Pure Thermal Emission

   In this section, there is no incident radiation, just the glow of a 100K medium of length 3×10^{11} cm. We will assume for simplicity that the absorption coefficient $\alpha_\nu = 1.5 \times 10^{-10}$ cm^{-1} is a constant, independent of frequency, but make sure that the emission coefficient j_ν has the correct form so that the source function S_ν is correct for thermal emission (i.e. $S_\nu = B_\nu$).

   Look for places marked FIXME in Part A of the code. You will need to:

   (a) Write (or copy from Experiment 1) your `alpha_constant()` function, which defines an absorption coefficient α_ν that is a constant.

   (b) Finish the definition of the `j_thermal()` function to make sure that the source function is the blackbody spectrum (RL equation 1.37). Note that within `j_thermal()`, you can find out the value of the absorption coefficient by calling `alphafunc()` and the value of the Planck spectrum by calling `sirtipy.blackbody_intensity()` — see the code comments for details.

   Then run the code ("`python experiment2.py`"). One output file shows the spectrum at various points throughout the medium, and the other shows the intensity I_ν versus optical depth τ along the ray at various frequencies. Look at them! Compare them to the analytic solution for the constant source function (RL equation 1.30). Do they look like you would expect? Can you follow how the two plots are showing the same story? At what optical depth does the spectrum effectively reach its final value? Does it depend on frequency?

3. Part B: Background Radiation Passing Through A Thermal Medium

   In this part, instead of just a glowing 100K medium, we have a 100K medium that is in front of a 10,000K object. Therefore, the incident radiation is a 10,000K blackbody spectrum.

   Look at Part B of the code, which will tell you to copy the Part A code and then make the appropriate changes to define the correct input spectrum. Make sure you also change the plot titles and file names to be consistent!

   You will get new plots showing the spectrum at various points through the medium, and the intensity curves. Look at them! How are they different from Part A? How are they similar? Ask yourself the same questions as in Part A. Think about the meaning of "optically thick" in the context of what you see at different frequencies. Flip back and forth between the Part A and Part B plots, noticing both the shape and normalization when they are at their most optically thick — what do you notice? If you observed the $\tau = 30$ spectrum using a UV ($\log \nu \sim$ 15–16.5) and/or optical ($\log \nu \sim$ 14.5–15) telescope, what would you infer about the object you were observing? If you observed it using a mid- or far-IR telescope ($\log \nu \sim$ 12–13.5), what would you infer?

4. Part C: Measurements of Spectral Temperature

   We discussed several different characteristic temperatures of the Planck spectrum. Here we will measure them for a number of the spectra that came out in Parts A and B to see how they can differ from each other in cases when the spectrum isn't a pure Planck function:

(a) **Brightness Temperature**: RL equation (1.60). To measure this, you will need to decide on a frequency that is clearly on the Rayleigh-Jeans tail of all of the spectra, find out which index it is on the frequency axis, and then find the intensity for that frequency at the desired location.

(b) **Color Temperature**: This is less well-defined. We will try two different definitions:

i. *Wien Displacement Law*: Find the frequency $\nu_{\rm max}$ where the intensity is at its maximum, and use RL equation (1.56).

ii. *Mean Emission Frequency*: Find the mean emission frequency by numerically evaluating

$$\langle \nu \rangle = \frac{\int_0^\infty \nu I_\nu \, d\nu}{\int_0^\infty I_\nu \, d\nu}$$

and noting that for a Planck spectrum,

$$\langle \nu \rangle = \frac{\int_0^\infty \nu B_\nu(T) \, d\nu}{\int_0^\infty B_\nu(T) \, d\nu} \approx 3.83 \, \frac{kT}{h}.$$

You can use the `trapz` function to do the numerical integral, as explained in the code comments.

(c) **Effective Temperature**: RL equation (1.63). You will need to numerically integrate the spectrum to find the total flux (again using `trapz`). You can assume that the light is all going outward through a surface, i.e. that $I_\nu = 0$ if $\theta < 0$, and use the result we derived in class for the relationship between I and F in this case.

Uncomment the PART C section of the code. Follow the FIXMEs in the code comments to do the calculations listed above. Plot how each temperature evolves with optical depth, both for the Part A and Part B experiments. Analyze this plot in the context of the spectra you observed in Parts A and B why do the different characteristic temperatures evolve the way they do? Some of these measurements are a bit crude; can you estimate the uncertainty based on the plots?

5. Part D: Play.

Try doing something else of your choice! For example, you could see how Parts B and C are affected if you change the temperature of the background radiation and/or the thermal medium, or you could see what happens if the absorption coefficient is a function of ν (note that if you do this, you will need to be careful to use the correct element of `tau` if you need it — this is not necessary for constant α_ν, so I've been slightly sloppy about it), or you could see what happens if the incident radiation spectrum looks very different from a blackbody, or you could see what happens if the medium does not have a constant temperature (for example, a gradient).

Everyone must bring all of their plots, not just the discussion leaders. We will discuss everyone's part D plot.