Introduction to Survey Astrometry

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July 2020

(nearly) All Rubin Obervatory science in 1 table

Moment	Point sources	Resolved sources
0: Flux $f = \int dx\,dy\,I(x,y)$	Stellar evolution & populations, transients, distance scale, minor planet properties, Galactic structure	Galaxy evolution & populations Photometric redshifts
1: Centroid $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{1}{f} \int dx\,dy \begin{pmatrix} x \\ y \end{pmatrix} I(x,y)$	Galactic structure & dynamics Distance scale Minor planet orbits	*Nobody cares**
2: Size/Shape $\begin{pmatrix} M_r \\ M_+ \\ M_\times \end{pmatrix} = \int dx\,dy \begin{pmatrix} x^2+y^2 \\ x^2-y^2 \\ 2xy \end{pmatrix} I(x,y)$	*Nobody cares**	Weak gravitational lensing

**except as needed to facilitate galaxy shapes*

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**except as needed to facilitate galaxy shapes*

Getting a feel for angular scales…

For the Sun at a distance of:	100 pc	1 kpc	10 kpc
Parallax amplitude	10 mas	1 mas	100 uas
Proper motion, 20 km/s (disk star)	41 mas/yr	4 mas/yr	410 uas/yr
Proper motion, 200 km/s (halo star)	410 mas/yr	41 mas/yr	4 mas/yr
Angular diameter	100 uas	10 uas	1 uas
Reflex from a Jupiter in 5-year orbit	30 uas	3 uas	0.3 uas

- LSSTCam field of view: 12,600,000 mas
- LSSTCam PSF 1-sigma: ~300 mas
- LSSTCam pixel: 200 mas
- Single silicon atom: 11 uas

Astrometry overview

Measure (x,y) pixel center from image

(centroiding algorithms)

Map from pixel (x,y) to RA, Dec

(aka “world coordinate system”, WCS)

Relative astrometry (only measure relative stellar positions)

Absolute astrometry (need true RA,Dec)

The Gaia Data Release 2 (DR2) provides catalog of ~1B stellar positions, about 1 per sq arcmin, at <500 uas accuracy.
For wide-field imaging like LSST, this basically eliminates the distinction btwn absolute/relative.

Centroiding: simplest case

- Consider a 1-dimensional star and we record the arrival position x_i of each of N photons. The PSF is a distribution with width σ. We know from basic statistics that the uncertainty in the *mean* of the photon distribution is

$$\sigma_x = \frac{\sigma}{\sqrt{N}} = \frac{\sigma}{\nu}$$

- where ν is the "significance" or signal-to-noise ratio (S/N) of the total flux measurement of the star.

This is a good rule of thumb for any decent centroiding measurement, regardless of the shape of the PSF or source of noise!

Background-limited

- Now go to 2d case, …
- and pixelize our image in coordinates (x,y) so that the value $I(x,y) = I_{xy}$ is the total of counts in the area ΔA of the pixel, $\int dA\, I(x,y) \rightarrow \sum_{xy} I_{xy}$ - *e.g.* our basic centroid estimator is now $\bar{x} = \dfrac{\sum_{xy} xI_{xy}}{\sum_{xy} I_{xy}}$
- and add a fixed background noise variance *n* to each pixel, as would be left from the Poisson noise from subtracting a flat background contributing *n* counts per pixel. In this case the total standard deviation of the value of I_{xy} is $\sigma_{xy} = \sqrt{\hat{I}_{xy} + n}$, where $\hat{I}_{xy}$ is the ***true, noise-free*** value of the total flux in the pixel.
- Our previous example assumed *n=0*, i.e. "source dominated," as appropriate for bright stars or the brightest galaxies. Let's now go to the ***background limited*** case, $n \gg I_{xy}$. For Rubin *r* band on a dark night, the transition from source->bg limited occurs btwne 20<*r*<21 mag.

Centroids as root-finding

- The noise in our centroid measurement diverges when we integrate all photon counts! 🤮

X Centroid integral: $M_x = \int dxdy \cdot x \cdot I(x,y) \longrightarrow \sum_{x,y} x \cdot I_{xy}$

$\Rightarrow \mathrm{Var}(M_x) = \sum_{x,y} x^2 \cdot \mathrm{Var}(I_{xy}) = \sum_{x,y} x^2 \cdot n \longrightarrow \infty$ if summed over all pixels!

We need to restrict the area of our moment calculation!

Let's re-pose our centroid measurement first:

$$\bar{x} = \frac{\int dxdy \; x \cdot I(x,y)}{\int dxdy \cdot I(x,y)} = \frac{M_x}{M_f} \Rightarrow 0 = \int dxdy \; x \cdot I(x,y) - \bar{x} \int dxdy \cdot I(x,y)$$

$$\Rightarrow 0 = \int dx\,dy \,(x-\bar{x})\, I(x,y) \rightarrow \sum_{x,y} (x-\bar{x}) I_{xy}.$$

So we can define $\boxed{M_x(x_0) = \sum_{x,y} (x-x_0) \cdot I_{xy}}$

and define our centroid as the solution $M_x(\bar{x}) = 0$

similarly $M_y(\bar{y}) = 0$, or $\boxed{\begin{matrix} M_x(\bar{x},\bar{y}) = 0 \\ M_y(\bar{x},\bar{y}) = 0 \end{matrix}}$

Aperture centroids

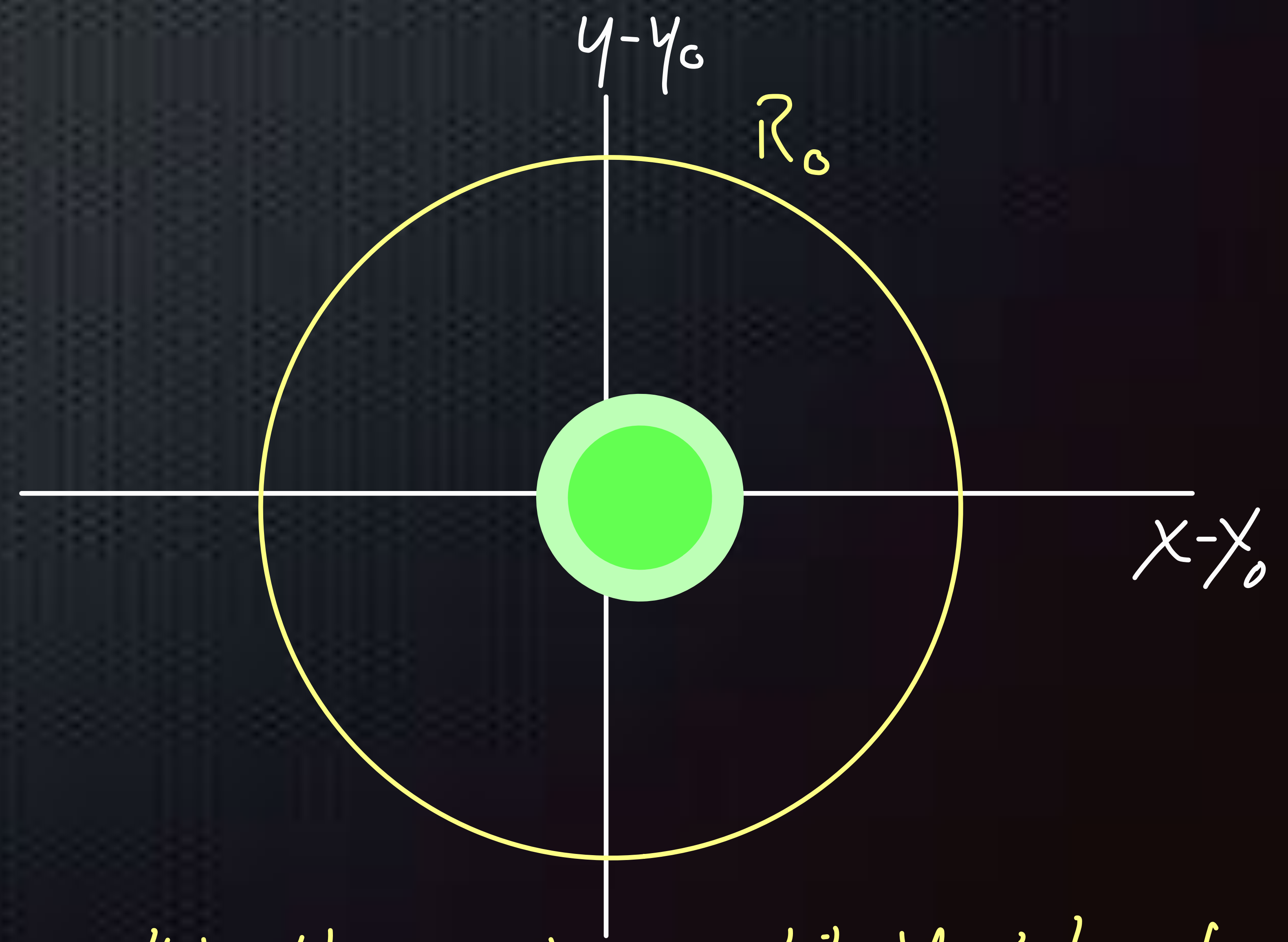
Let's fix our noise problem by introducing an aperture function $W(x,y)$

$$\begin{pmatrix} M_f \\ M_x \\ M_y \end{pmatrix} = \int dx\,dy \begin{pmatrix} 1 \\ x - x_0 \\ y - y_0 \end{pmatrix} \cdot I_{xy} \times W(x-x_0, y-y_0)$$

For example:

TOPHAT: $$W(x,y) = \begin{cases} 1 & r = \sqrt{x^2+y^2} \leq R \\ 0 & r > R \end{cases}$$

GAUSSIAN: $$W(x,y) = e^{-R^2/2\sigma_w^2}$$

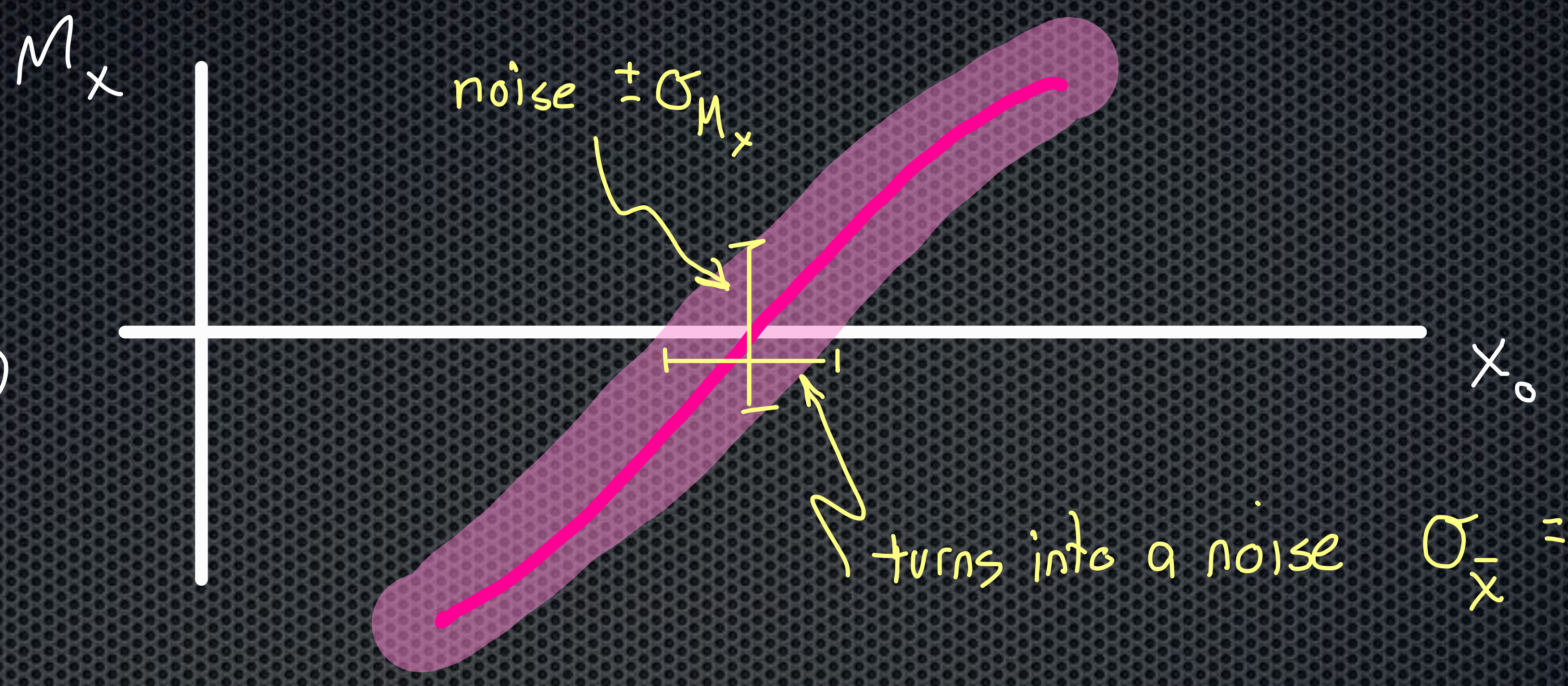


slide the aperture until the integrals for M_x and M_y are zero!

Uncertainties in aperture centroids

Aperture method turns into an iterative solution for the root (in 2d) of

$$M_x(x_0, y_0) = M_y = 0$$



$$\sigma_{\bar{x}} = \frac{\sigma_{M_x}}{|dM_x/dx_0|}$$

$$\sigma_{M_x}^2 = \sum_{x,y} (x-x_0)^2 W^2(x-x_0, y-y_0) \cdot \mathrm{Var}(I_{xy}) = n \cdot \sum_{x,y} x^2 W^2(x,y)$$

FOR TOPHAT: $\equiv n \cdot 2\pi \cdot \int_0^R r\,dr \cdot \frac{r^2}{2} = n \cdot \frac{\pi R^4}{4}$

What about $\frac{dM_x}{dx_0} = \frac{d}{dx_0} \int_{r<R_0} dx\,dy\; I(x,y)\, W(x-x_0, y-y_0) \cdot (x-x_0)$?

For tophat with $R \gg \mathrm{PSF}$, $\left|\frac{dM_x}{dx_0}\right| = f$, $\sigma_x = \frac{R^2 \cdot \sqrt{\pi n}}{2f}$

we want smaller aperture!

But when $R \ll \mathrm{PSF}$, $\frac{dM_x}{dx_0} \to 0$ and σ_x blows up!

Just like aperture photometry, our best aperture choice for precision is $R \approx R_{PSF}$,

$$\Rightarrow \sigma_x \approx R_{PSF} \cdot \frac{\sqrt{\pi R_{PSF}^2 \cdot n}}{f}$$ ← this is the noise on the aperture flux, σ_f

(Homework to try a more exact version)!

$$= R_{PSF} \cdot \frac{\sigma_f}{f} = \frac{R_{PSF}}{\nu_{aper}}, \quad \nu_{aper} = \frac{S}{N} = \frac{f}{\sigma_f}.$$ (our rule of thumb!)

What's the best possible $W(x,y)$ to minimize σ_x??

Not the tophat! A Gaussian with $\sigma_W = \sigma_{PSF}$ is better, significantly.

XWIN_IMAGE, YWIN_IMAGE in SExtractor use this. * Recommended! *

If the PSF is Gaussian with σ_{PSF}, then the optimal photometry aperture is also this Gaussian ("matched aperture") and we can show the following:

$$M_f = \int dx\,dy \cdot f \cdot PSF \times W = \frac{1}{2} f \; ; \quad \frac{\partial M_x}{\partial x} = f/4$$

$$\sigma_{M_f}^2 = n \int dx \cdot dy \cdot W^2 = \pi \sigma^2 n, \quad \sigma_{M_x}^2 = n \cdot \int dx\,dy \cdot W^2 X^2 = \frac{\pi \sigma^4 n}{2}$$

$$\nu_{opt}^2 = \frac{M_f^2}{\sigma_{M_f}^2} = \frac{4\pi\sigma^2 n}{f^2}$$

$$\boxed{\sigma_x = \sigma_{PSF} \cdot \sqrt{\frac{8\pi\sigma_{PSF}^2 \cdot n}{f^2}} = \sqrt{2} \cdot \frac{\sigma_{PSF}}{\nu_{opt}}}$$

Gaussian PSF Formulae

Cramer-Rao and the Fisher matrix

But what's the best possible? Always good to know this for a measurement!

Let's learn an important theorem: Cramer-Rao inequality.

- If you are fitting data D (= our image!) to a model M that has parameters Θ_i, then the uncertainty σ_i on Θ_i for any unbiased estimator satisfies
  $\Theta_i = (\bar{x}, \bar{y})$

$$\sigma_i^2 \geq (F_{ii})^{-1}$$

- where F is the Fisher information matrix defined by

$$F_{ij} = \left\langle - \frac{\partial^2 \log P(D|\Theta)}{\partial\Theta_i \, \partial\Theta_j} \right\rangle$$

- More generally, the best possible covariance matrix for multiple Θ's is $C_{opt} = F^{-1}$

Cramer-Rao bounds for centroids

What's the Cramer-Rao bound on σ_x?

Data: $D = \{I_{xy}\}$, pixel values

model: $\hat{I}_{xy} = f \cdot PSF(x-x_0, y-y_0) \Rightarrow$ parameters $\Theta = \{f, x_0, y_0\}$

Probability: (approximate Poisson as Gaussian) $p(D|\Theta) \propto \prod_{x,y} \exp\left[-\frac{1}{2}\frac{(I_{xy} - f\,PSF(x-x_0,y-y_0))^2}{\sigma_{xy}^2}\right]$

$\sigma_{xy}^2 = f \cdot PSF + n$

$$F_{xx} = \left\langle -\frac{\partial^2 \log P}{\partial x_0^2}\right\rangle = \left\langle \frac{1}{2}\sum_{x,y}\frac{\partial^2}{\partial x_0^2}\left(I_{xy} - f\cdot PSF(x-x_0,y-y_0)\right)^2 / \sigma_{xy}^2\right\rangle$$

$$\boxed{= \sum_{x,y}\frac{f^2}{\sigma_{xy}^2}\cdot\left[\frac{\partial}{\partial x}PSF(x-x_0,y-y_0)\right]^2}$$

(+ another term that vanishes)

Source limited: : $\sigma_{xy}^2 = f\cdot PSF \Rightarrow F_{xx} = f\sum_{xy}\left(\frac{\partial}{\partial x}PSF\right)^2/PSF \rightarrow \boxed{f\cdot\int dx\,dy\,\left(\frac{d}{dx}PSF\right)^2/PSF}$

Background limited: $\sigma_{xy}^2 = n \Rightarrow F_{xx} = f^2/n \cdot \sum_{xy}\left[\frac{\partial}{\partial x}PSF(x,y)\right]^2 \longrightarrow \boxed{\frac{f^2}{n}\int dx\,dy\left[\frac{d}{dx}PSF(x,y)\right]^2}$

Fisher info on centroid for generic PSF

Model-fitting centroids

For PSF-fitting photometry, we maximized $p(D|\Theta)$ or minimized

$$\chi^2 = \sum_{x,y} \left[I_{xy} - f\,\mathrm{PSF}(x-x_0, y-y_0) \right]^2 / \sigma_{xy}^2 .$$

← let's assume background limit here

To minimize χ^2 w.r.t. x_0, we require

$$0 = \frac{\partial \chi^2}{\partial x_0} = \frac{2}{n} \cdot \sum_{xy} \left[I_{xy} - f \cdot \mathrm{PSF}(x-x_0, y-y_0) \right] \cdot \frac{\partial}{\partial x} \mathrm{PSF}$$

$$\Rightarrow \quad \sum_{x,y} I_{xy} \cdot \frac{\partial}{\partial x} \mathrm{PSF}(x-x_0, y-y_0) = f \sum_{xy} \mathrm{PSF} \cdot \frac{\partial}{\partial x} \mathrm{PSF} = 0$$

↑ integrate by parts to show this.

PSF-fitting estimate of $\bar{x}$ is exactly the same as aperture astrometry

with $\quad W(x,y) = \frac{\partial}{\partial x} \mathrm{PSF}(x,y) \times \frac{1}{x}$

← for a Gaussian, $\frac{1}{x} \cdot \frac{\partial}{\partial x} \mathrm{PSF}$

$\frac{1}{x} \frac{\partial}{\partial x} \mathrm{PSF} \propto \mathrm{PSF}$!

solve $\quad \sum_{x,y} I_{xy} \cdot (x-x_0)\, W(x-x_0, y-y_0) = 0$

Optimal results for Gaussian PSF's

For Gaussian PSF: $\frac{d}{dx}(\mathrm{PSF}) = \frac{-x}{\sigma^2_{\mathrm{PSF}}} \times \frac{1}{2\pi\sigma^2_{\mathrm{PSF}}} e^{-r^2/2\sigma^2_{\mathrm{PSF}}}$

Source Limited	Background Limited
$F_{xx} = \frac{f}{\sigma^2_{\mathrm{PSF}}}$	$F_{xx} = \frac{f^2}{n} \cdot \frac{1}{8\pi\sigma^4_{\mathrm{PSF}}}$
$\Rightarrow \sigma_x = \frac{\sigma_{\mathrm{PSF}}}{\sqrt{f}}$	$\Rightarrow \sigma_x = \sigma_{\mathrm{PSF}} \cdot \sqrt{\frac{8\pi\sigma^2_{\mathrm{PSF}} n}{f^2}}$
Our first guess is the best possible!	Gaussian aperture photometry is = best possible!

PSF-fitting attains the Cramer-Rao bound!

For PSF-fitting, σ_x is the shift that will increase χ^2 by 1 -

$$\sigma_x^{-2} = \left.\frac{\partial^2 \chi^2}{\partial x_0^2}\right|_{\bar{x}} = \sum_{xy} \frac{f^2}{\sigma_{xy}^2} \times \left[\frac{\partial PSF}{\partial x_0}\right]^2 = F_{xx}$$

* This is the same as Cramer-Rao bound - so PSF fitting is the best possible centroid algorithm!

Summary of centroid measurement:

- The Cramer-Rao theorem or Fisher matrix tells us that the best possible centroid accuracy is attained by PSF-fitting, with:

$$\sigma_x^{-2} = \sum_{x,y} \frac{f^2}{\sigma_{xy}^2} \left(\frac{\partial}{\partial x} \mathrm{PSF} \right)^2$$

- In the background-limited case, PSF-fitting is equivalent to doing aperture centroiding with a weight function $W \propto x \frac{\partial PSF}{\partial x}$
- For a Gaussian PSF, this optimal weight is the exact same Gaussian as the PSF itself.
- For most ground-based PSF's, the precision of a Gaussian-aperture PSF is quite close to optimal, and you can do this without the trouble of knowing the PSF. SExtractor offers this.
- For most PSFs and decent centroiding algorithm, the centroid accuracy is within a modest factor of

(RMS error in centroid) = (RMS width of PSF) / (S/N ratio of flux measurement)

Scaling of astrometric accuracy

- The number of photons received from a star with flux $f_\star$ using a telescope with collecting area $A_{\rm tel}$, a filter with bandwidth $\Delta\nu/\nu = \Delta\lambda/\lambda$, detector with quantum efficiency ϵ, and integration time t is $f = N_\gamma \propto \epsilon f_\star A_{\rm tel}\, t_{\rm exp}\,(\Delta\nu/\nu)\,.$
- The background noise from sky of brightness b is $n \propto \epsilon\, b\, A_{\rm tel}\, t_{\rm exp}\,(\Delta\nu/\nu)\,.$
- So in the ***source-dominated*** limit, the astrometric error per coordinate scales as
- $$\sigma_x \propto \frac{\sigma_{\rm PSF}}{\sqrt{\epsilon f_\star A_{\rm tel} t_{\rm exp}(\Delta\nu/\nu)}} \propto 10^{0.2 m_\star}$$
- In the background-limited regime, the astrometric error per coordinate scales as
- $$\sigma_x \propto \frac{\sigma^2_{\rm PSF}}{f_\star}\sqrt{\frac{b}{\epsilon A_{\rm tel} t_{\rm exp}(\Delta\nu/\nu)}} \propto 10^{0.4 m_\star}$$
- In the bg limited case, one exposure with 0.5” seeing is as valuable as 2 exposures with 0.6” seeing or 4 exposures at 0.71” seeing! Also the accuracy degrades more rapidly with flux once we get into the background-limited regime.

Back to Rubin Observatory:

- From https://smtn-002.lsst.io/:
  - Gaussian-equivalent FWHM at *r* band is 0.83”, so $\sigma_{\mathrm{PSF}} = 830/2.36 = 350$ mas
  - “Effective area” of Gaussian PSF is 1.54 sq arcsec
  - Zenith dark sky in *r*: 21.2 mag per sq arcsec, so brightness of sky in the effective area is $21.2 - 2.5\log_{10} 1.54 = 20.7$ (divides source-limited and bg-limited domains)
  - Magnitude zeropoint in *r* is 28.13 (1 e/s) so # photons at this mag is $30 \times 10^{0.4*(28.13-20.7)} = 28{,}000$, and the S/N is about sqrt of this, 167.
  - The nominal shot-noise error at mag 20.7 is thus about $\sqrt{2}\sigma_{\mathrm{PSF}}/\nu_{\mathrm{opt}} \approx 3$ mas
- Gaia expected performance on mean position expected to be between 17 uas (G<15) - 380 uas (G=20). G and *r* are similar for most stars. So at G=20 limit of Gaia, a single LSST visit has about 2 mas RMS error, roughly 5x as larger as Gaia end-of-mission. Since LSST has >>25 visits over its lifetime, LSST precision can exceed Gaia at its faint end - and of course go >4 mag deeper!

Part 2: World Coordinate Systems

Our job is to produce a map from (x,y) pixel coordinates to (α, δ) sky coordinates in a known system (typically the International Celestial Reference System, ICRS). We have to "undo" the journey of the photon from its origin outside the solar system to a pile of electrons on the CCD. That journey encounters these deflections/distortions:

1. Gravitational deflection by solar system members
2. Stellar aberration due to Earth's motion
3. Refraction toward the vertical by $n>1$ in Earth's atmosphere
    a. - wavelength-dependent = "Differential chromatic refraction, DCR)
4. Further ***stochastic*** deflection by turbulence in the atmosphere
5. Projection of spherical sky onto flat focal plane about axis of telescope
6. Radial ("pincushion," "barrel") or other distortions by telecope optics
    a. - wavelength-dependent = "lateral color" if there are refracting elements
7. Locations/orientations of the CCDs in the focal plane
8. Deflection of photo-electrons (actually holes) by lateral electric fields in CCDs
9. Departures of the CCD pixels from a perfect square grid

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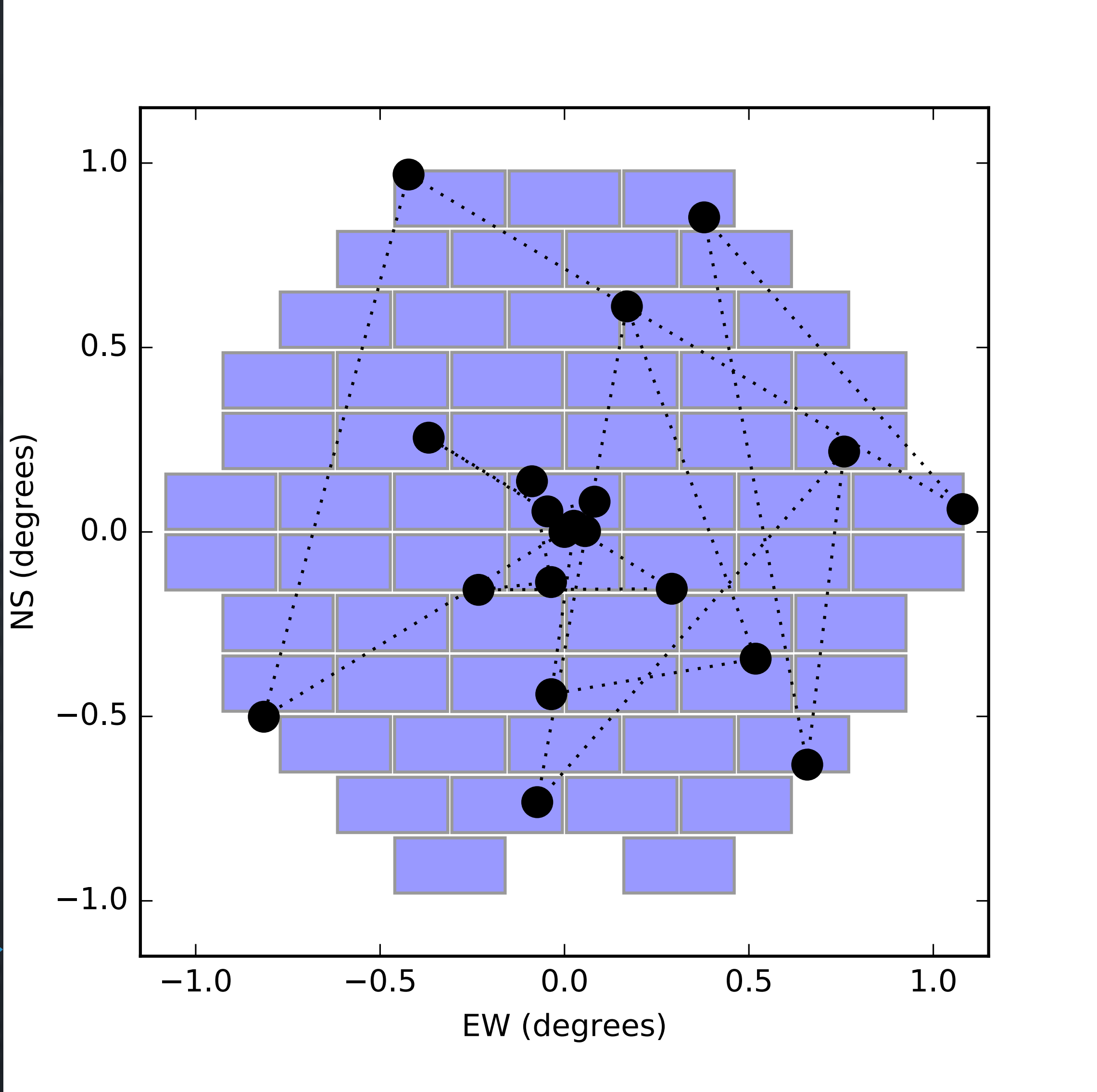
OMG, how will we figure all that out?

- We have a **LOT** of information to use to solve for the astrometric solution (WCS) of each exposure:
  - External consistency: There are O(10,000) Gaia stars on every LSSTCam exposure!
  - Internal consistency: A star should have the same RA,Dec when measured on different parts of the focal plane in different exposures!
- With these constraints, we can quickly constrain any model for WCS that is “smooth” on scales of several arcminutes, *i.e.* any degree of freedom would affect 10’s or more of the Gaia or internal-consistency star.



For the astrometric calibration of DECam, we take a “star flat” sequence of exposures with this dither pattern that moves a given star all around the focal plane.

“Standard” WCS solutions

Most of our sources of astrometric distortion are smooth functions which can be accuractely modeled by low-order polynomials (typically 3rd or 4th order across each CCD of DECam or LSSTCam). These polynomials are readily constrained using straightforward least-squares fit to reference stars and internal agreement. This will take care of most of our distortions:

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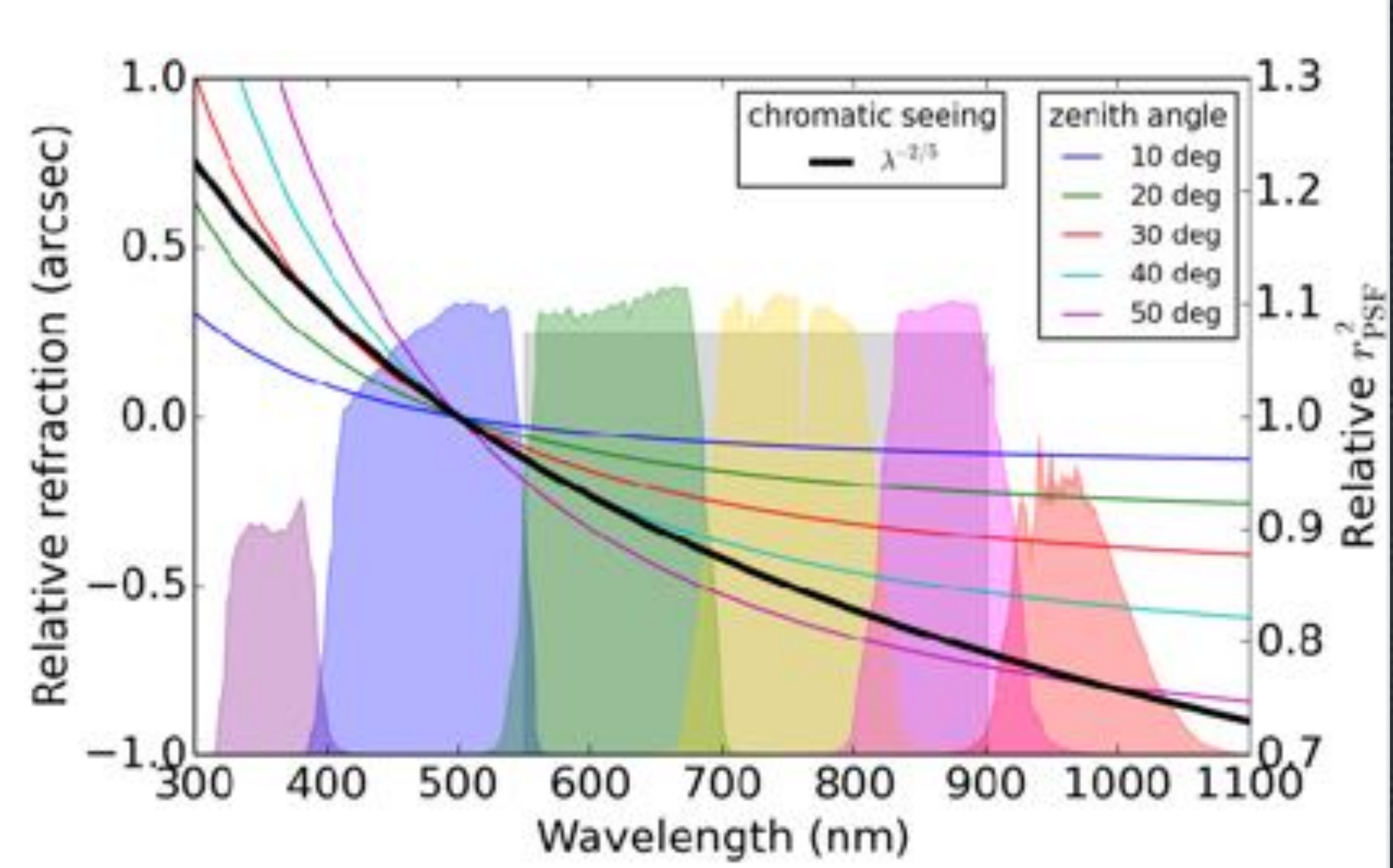
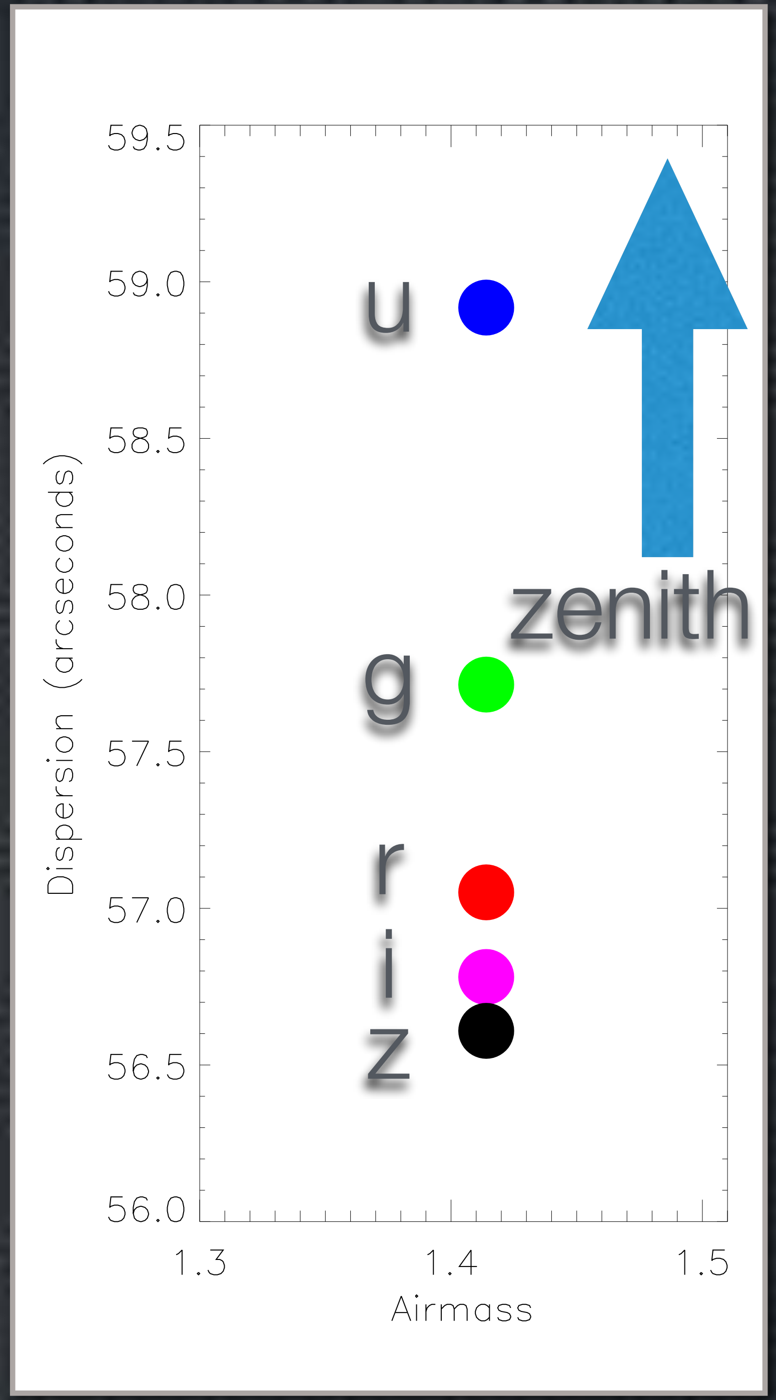
FITS WCS standards

- Most images that you get back from an observatory are in FITS format.
- There is a FITS standard for encoding WCS equations into keyword/value pairs in the FITS image header. This standard is built to allow polynomial distortion functions.
- DES images (and most others) are distributed with WCS solutions in their headers for you to use.
- DS9 knows how to read and use them so you can see RA,Dec as you move around the pixels of an image
- AstroPy includes software to read and use these WCS's. We'll practice with it in the notebooks!

Differential chromatic refraction



Plot from Josh Meyer



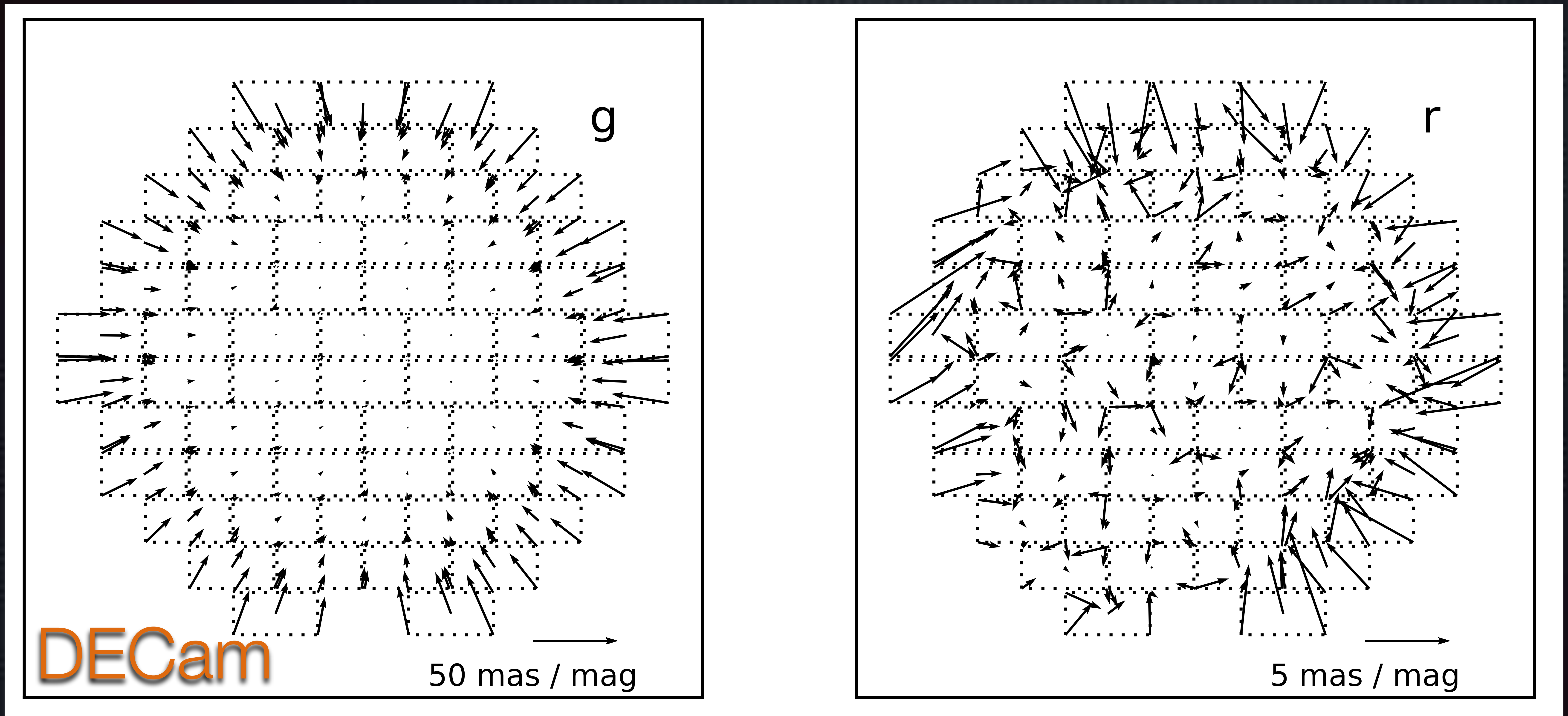
Plot from Kaczmarczik, Richards, & Mehta (2009)

In DECam *g* band, 45 deg from zenith: 45 mas per mag of *g-i*

- Apparent position will depend on the slope of each star's spectrum across the filter band (i.e. color)
- DCR is a *large* effect compared to our ~mas statistical errors for bright stars
- It grows rapidly into the blue - *g* and *u* are strong affected
- It is ***not*** in the typical WCS information. But it's a huge issue for ground-based astrometry.
- Always points toward or away from zenith.

Also: lateral color from glass in the optics

- Similar characteristics to DCR (e.g. worse in blue), but points to center of focal plane.



- Chromatic effects are smooth and can be accurately fit vs stellar color using Gaia/internal constraints
- But if you are using the data, you must be aware of this and *very* careful about the SED's of your sources!

The small (scale) stuff:

1. ~~Gravitational deflection by solar system members~~
2. ~~Stellar aberration due to Earth's motion~~
3. ~~Refraction toward the vertical by *n*>1 in Earth's atmosphere~~
   a. ~~—wavelength-dependent = "Differential chromatic refraction, DCR)~~
4. Further ***stochastic*** deflection by turbulence in the atmosphere
5. ~~Projection of spherical sky onto flat focal plane about axis of telescope~~
6. ~~Radial ("pincushion," "barrel") or other distortions by telecope optics~~
   a. ~~—wavelength-dependent = "lateral color" if there are refracting elements~~
7. ~~Locations/orientations of the CCDs in the focal plane~~
8. Deflection of photo-electrons (actually holes) by lateral electric fields in CCDs
9. Departures of the CCD pixels from a perfect square grid

Varies rapidly over space **and** time

Vary rapidly over space

…so we can map/constrain models of the latter by combining information from many exposures, millions of stars' residuals

Stray electric fields in CCDs

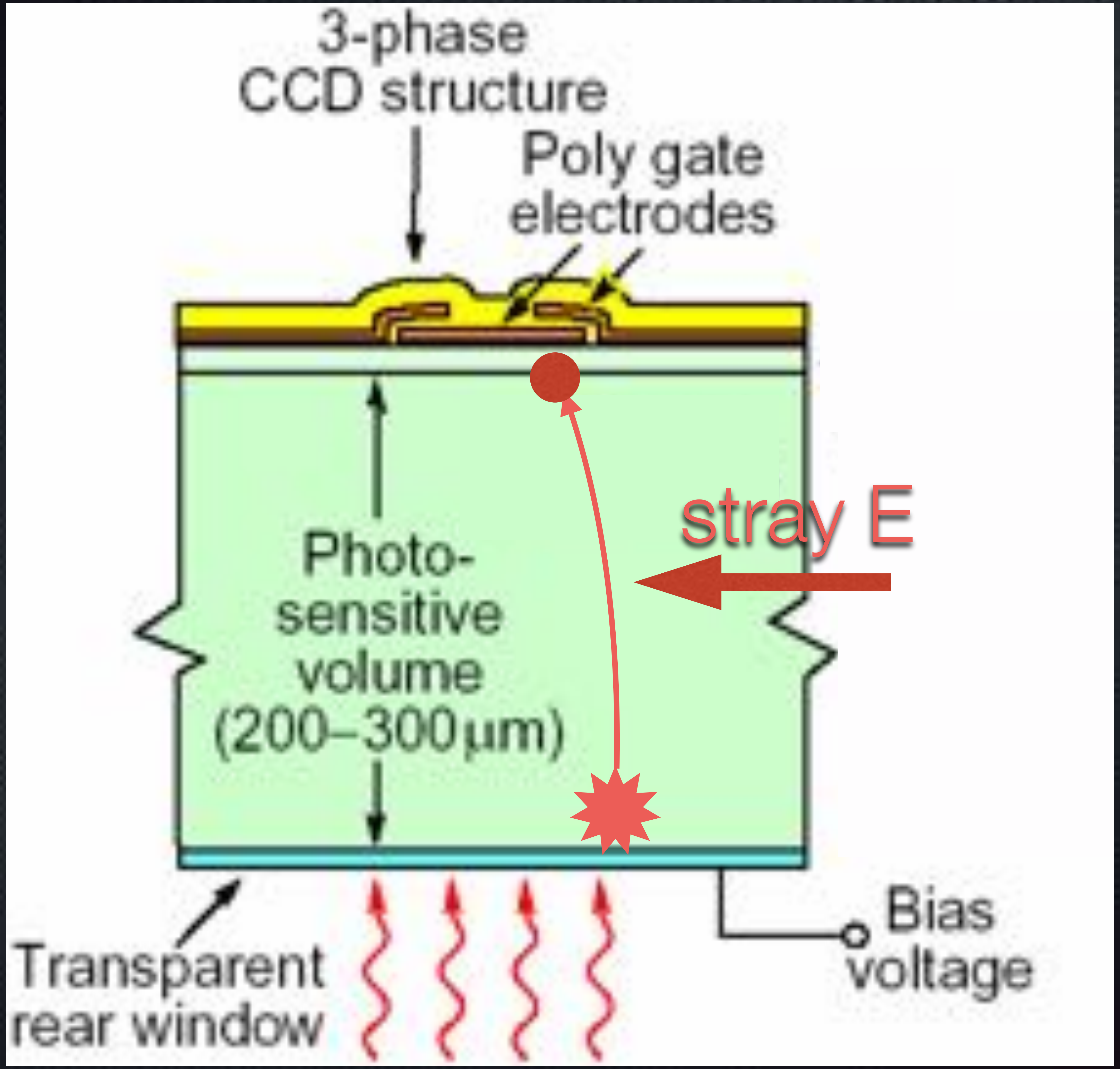


Diagram from Holland et al. (2004)

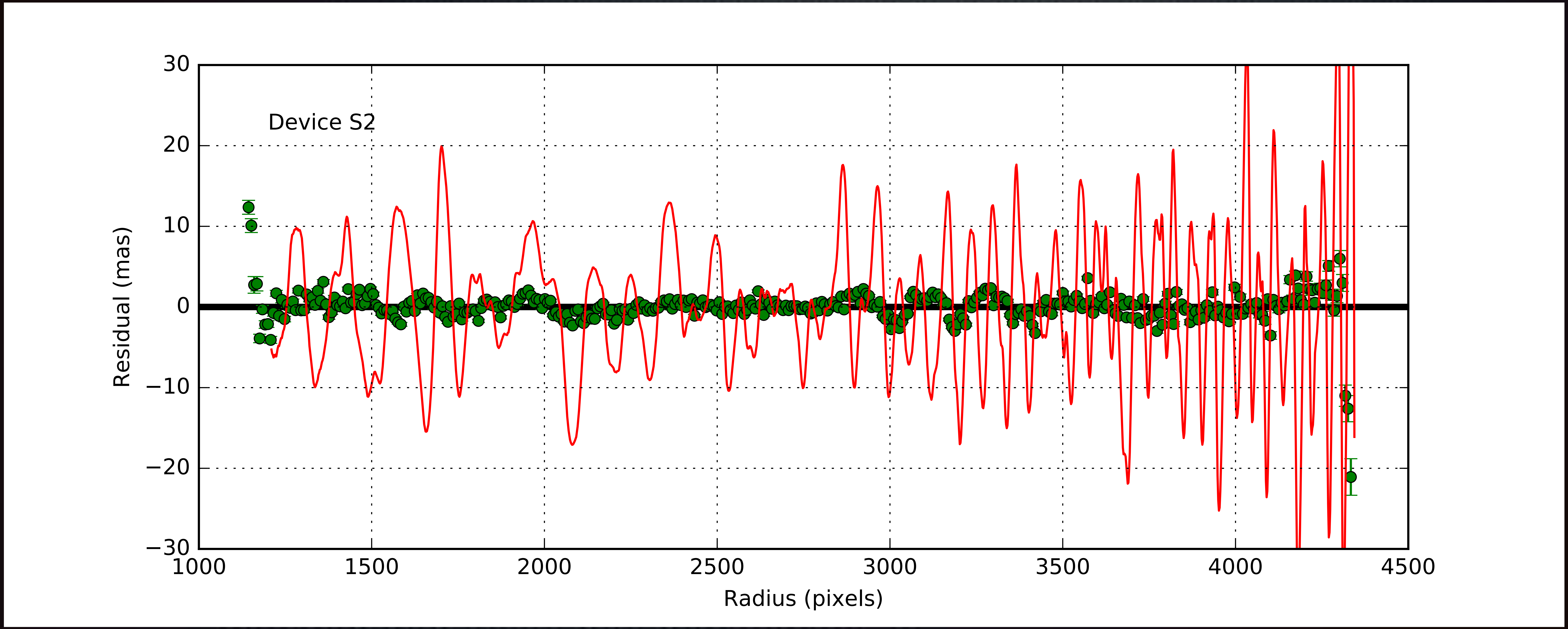
E.g. “tree rings”

From Plazas, Bernstein, & Sheldon (2014)

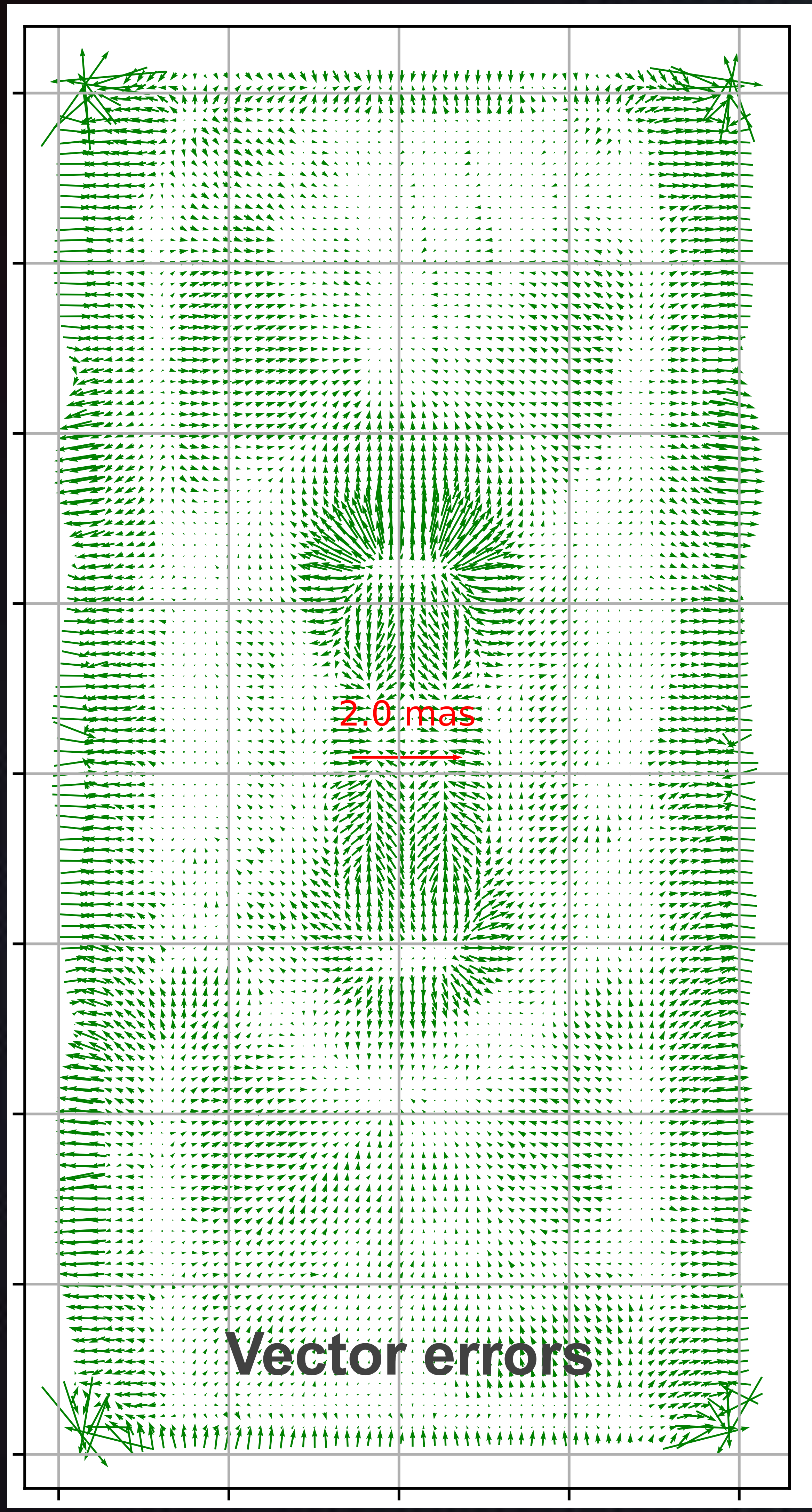
E.g. “tree rings”

From Plazas, Bernstein, & Sheldon (2014)

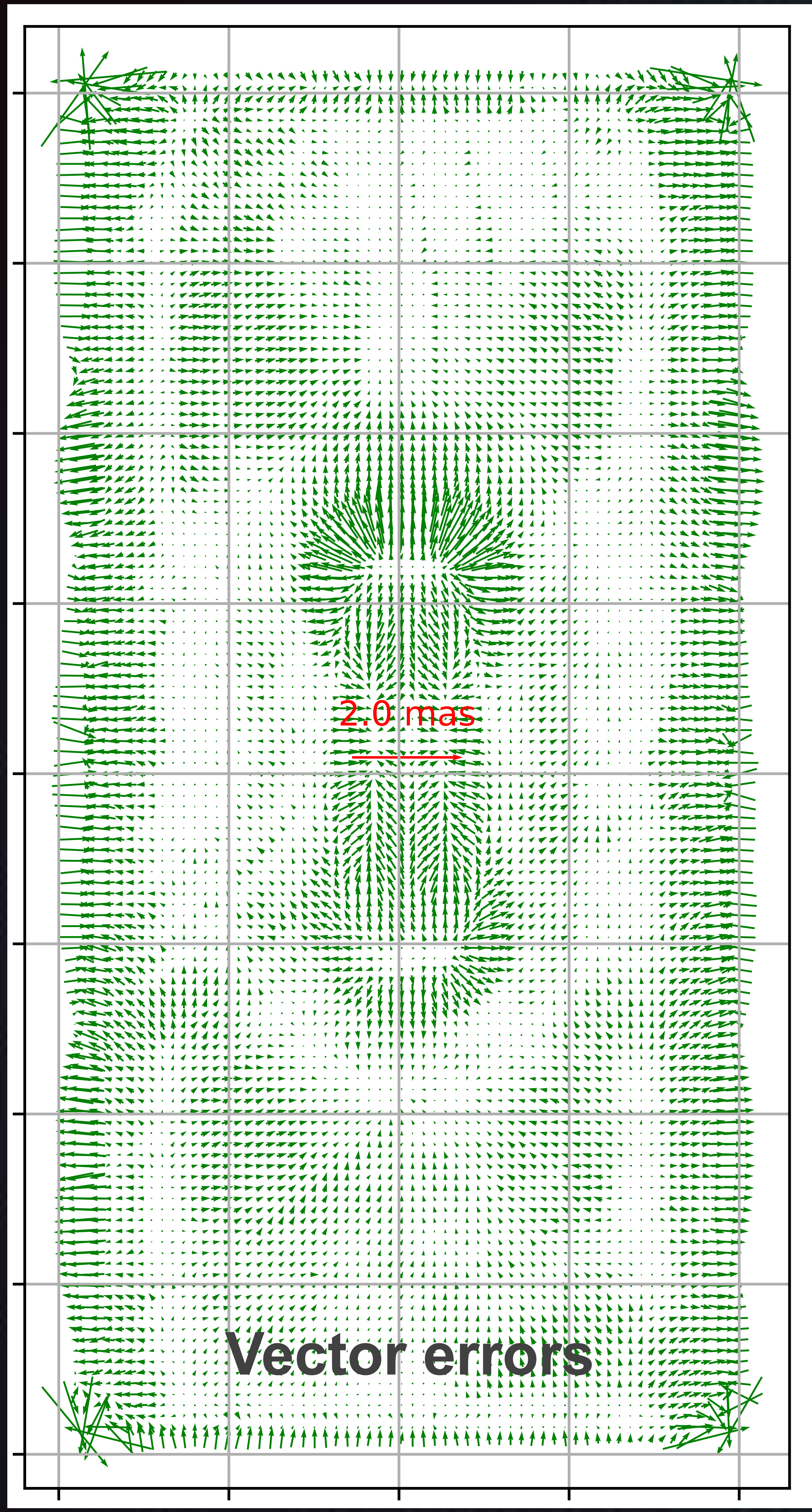
For DECam, we think we have modelled all static WCS features to ~1 mas RMS

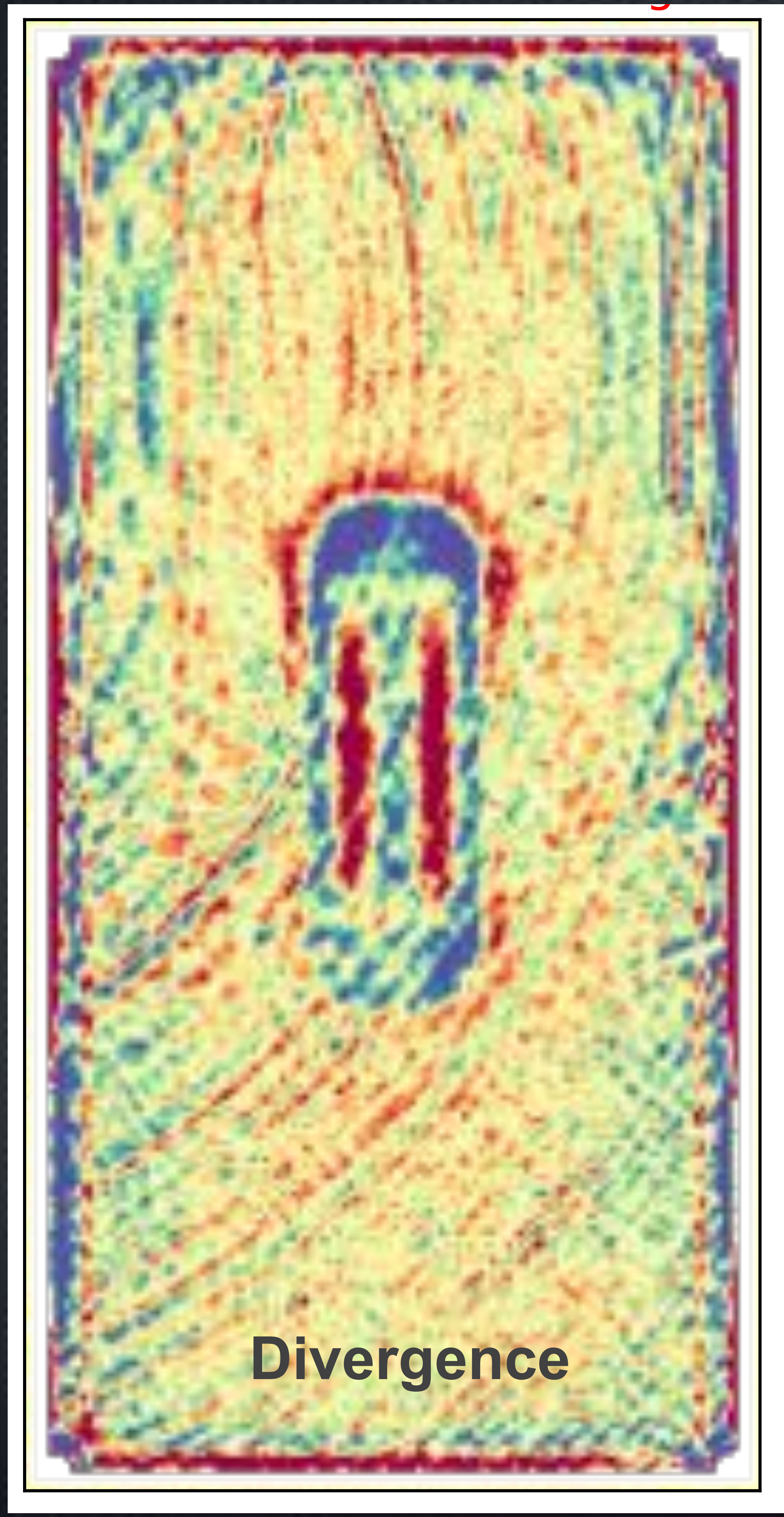


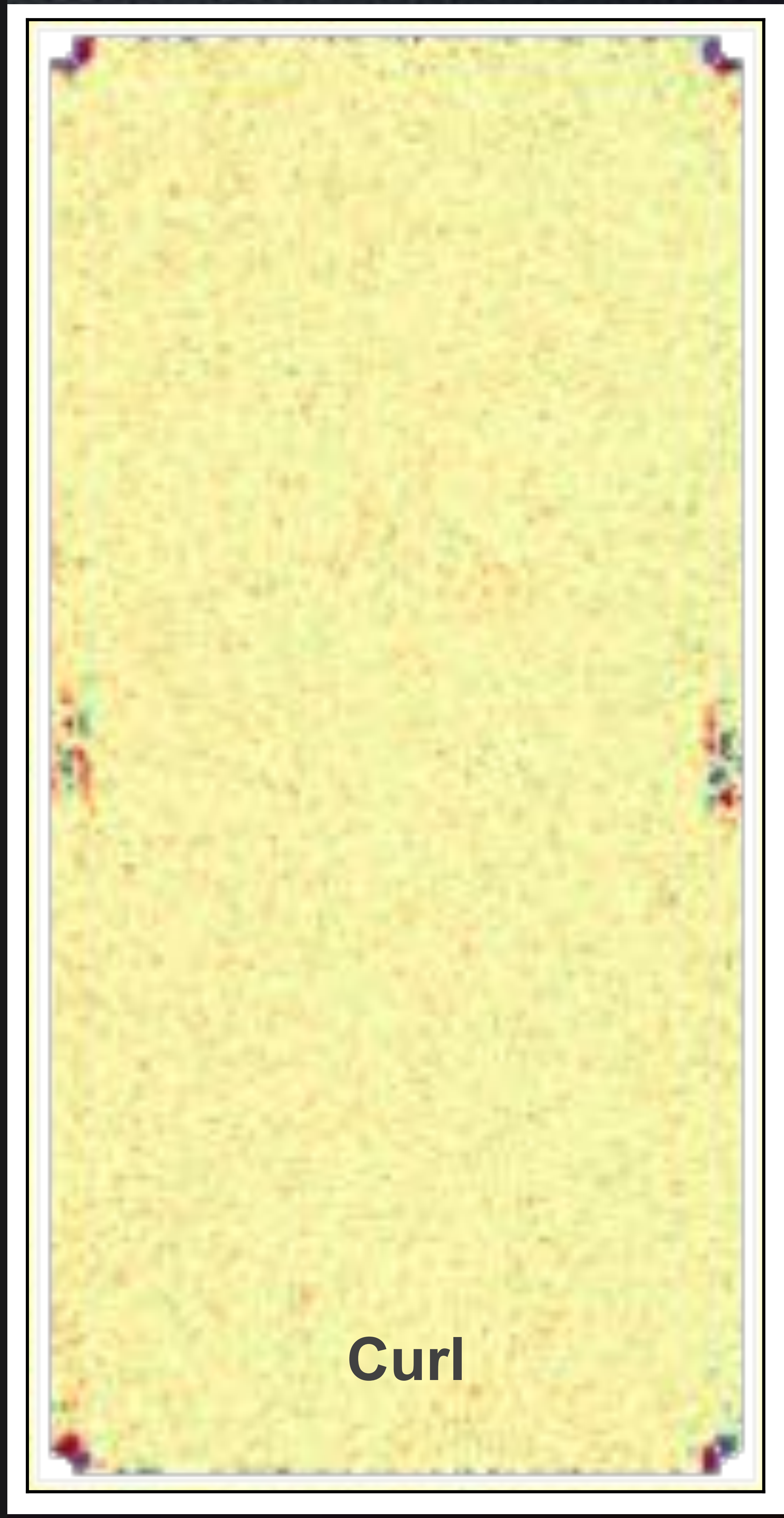
Mean astrometric residuals for all DES CCDs:



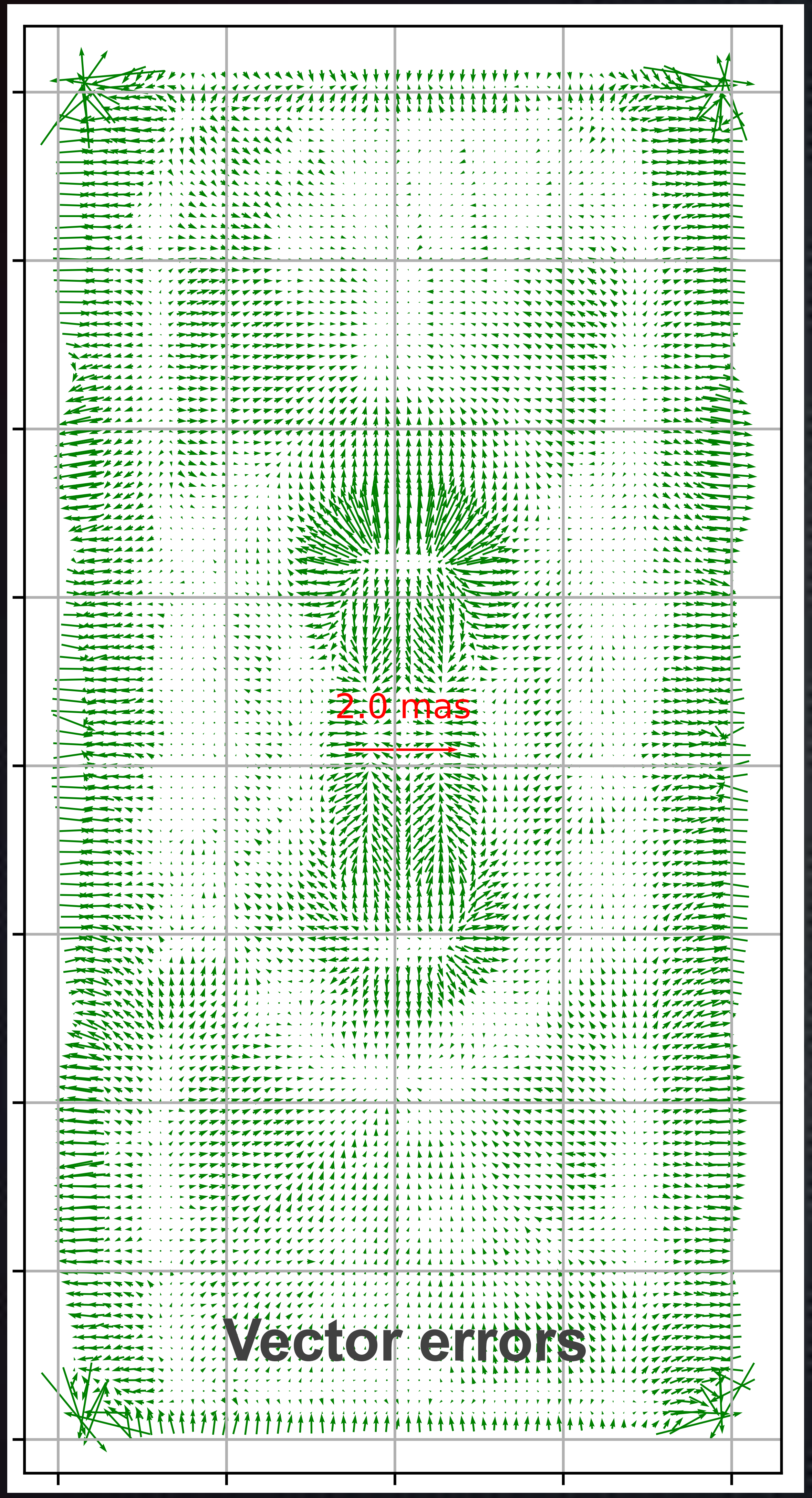
Mean astrometric residuals for all DES CCDs:

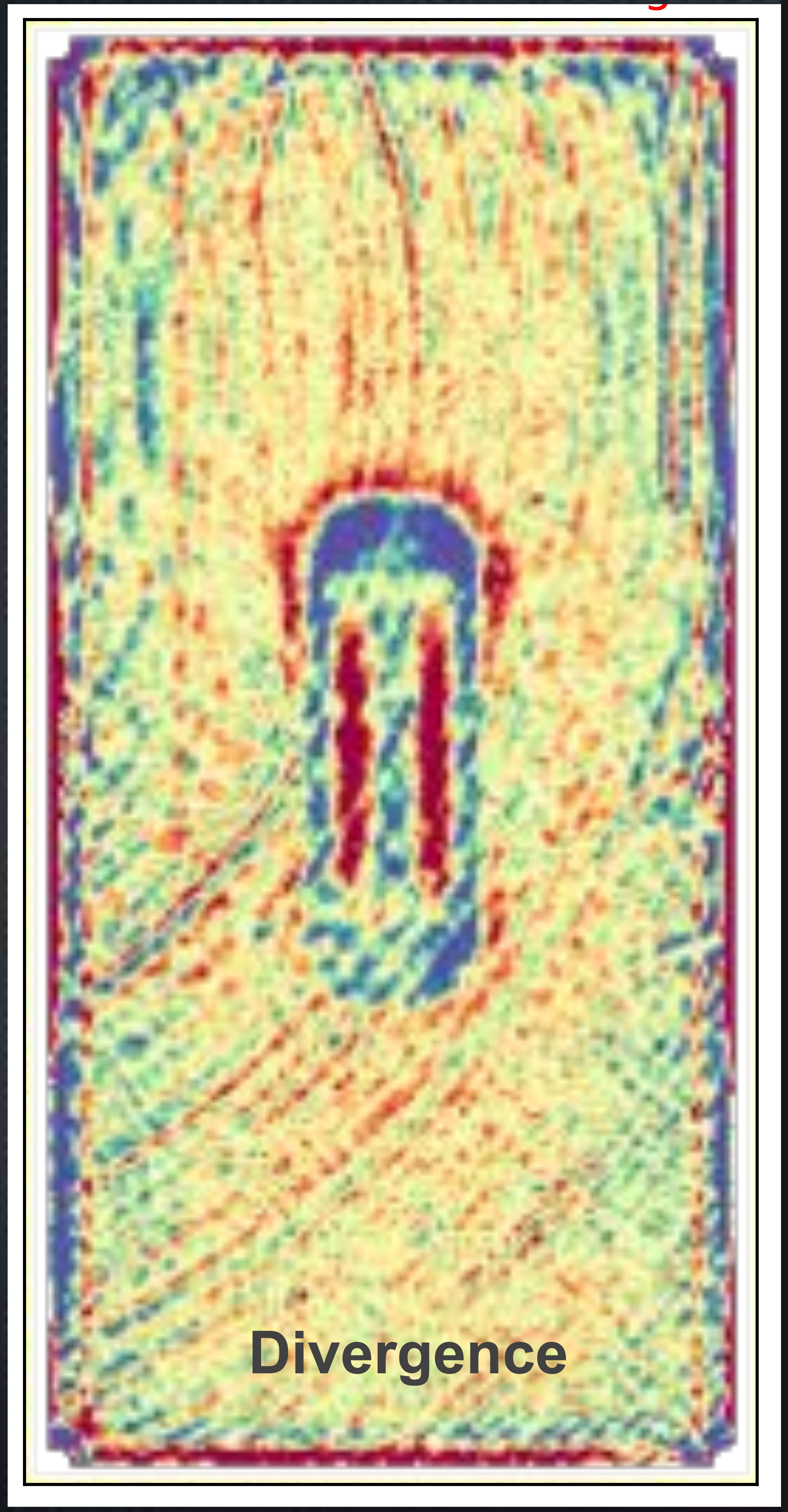






Mean astrometric residuals for all DES CCDs:



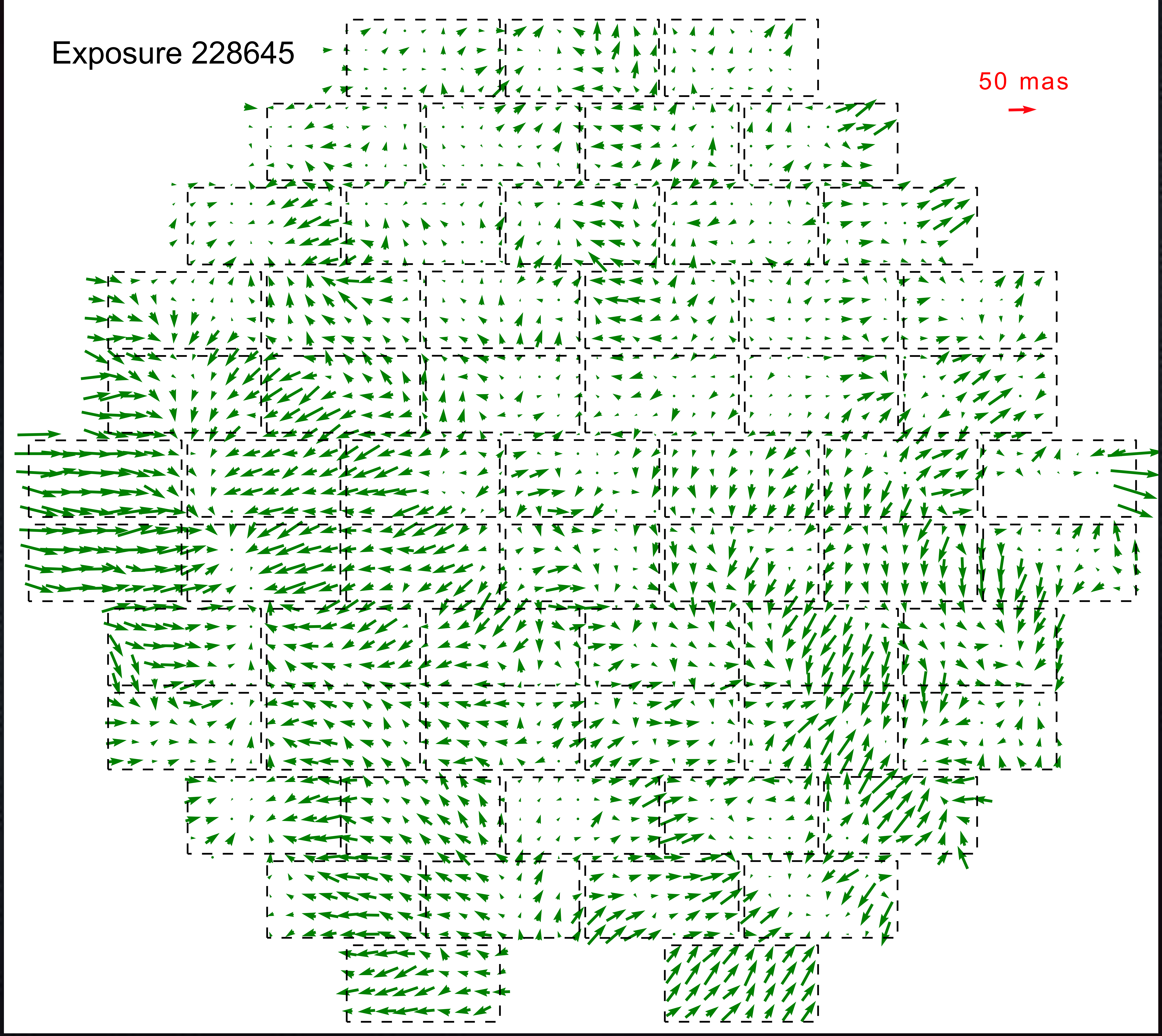


What's left?

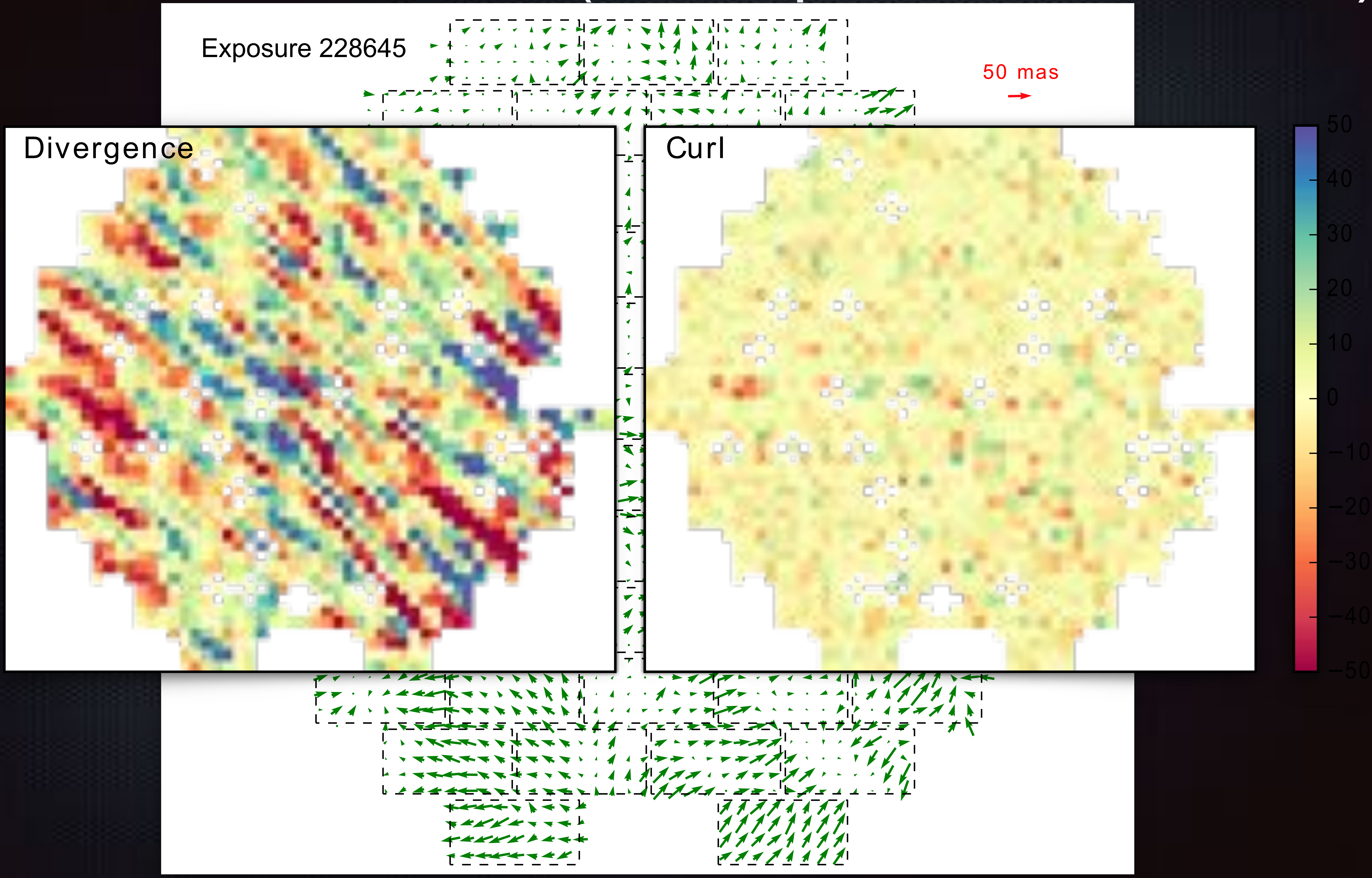
1. ~~Gravitational deflection by solar system members~~
2. ~~Stellar aberration due to Earth's motion~~
3. ~~Refraction toward the vertical by *n*>1 in Earth's atmosphere~~
    a. ~~—wavelength-dependent = "Differential chromatic refraction, DCR)~~
4. Further ***stochastic*** deflection by turbulence in the atmosphere
5. ~~Projection of spherical sky onto flat focal plane about axis of telescope~~
6. ~~Radial ("pincushion," "barrel") or other distortions by telecope optics~~
    a. ~~—wavelength-dependent = "lateral color" if there are refracting elements~~
7. ~~Locations/orientations of the CCDs in the focal plane~~
8. ~~Deflection of photo-electrons (actually holes) by lateral electric fields in CCD~~s
9. ~~Departures of the CCD pixels from a perfect square grid~~

Varies rapidly over space **and** time

Stochastic residuals (atmospheric turbulence)



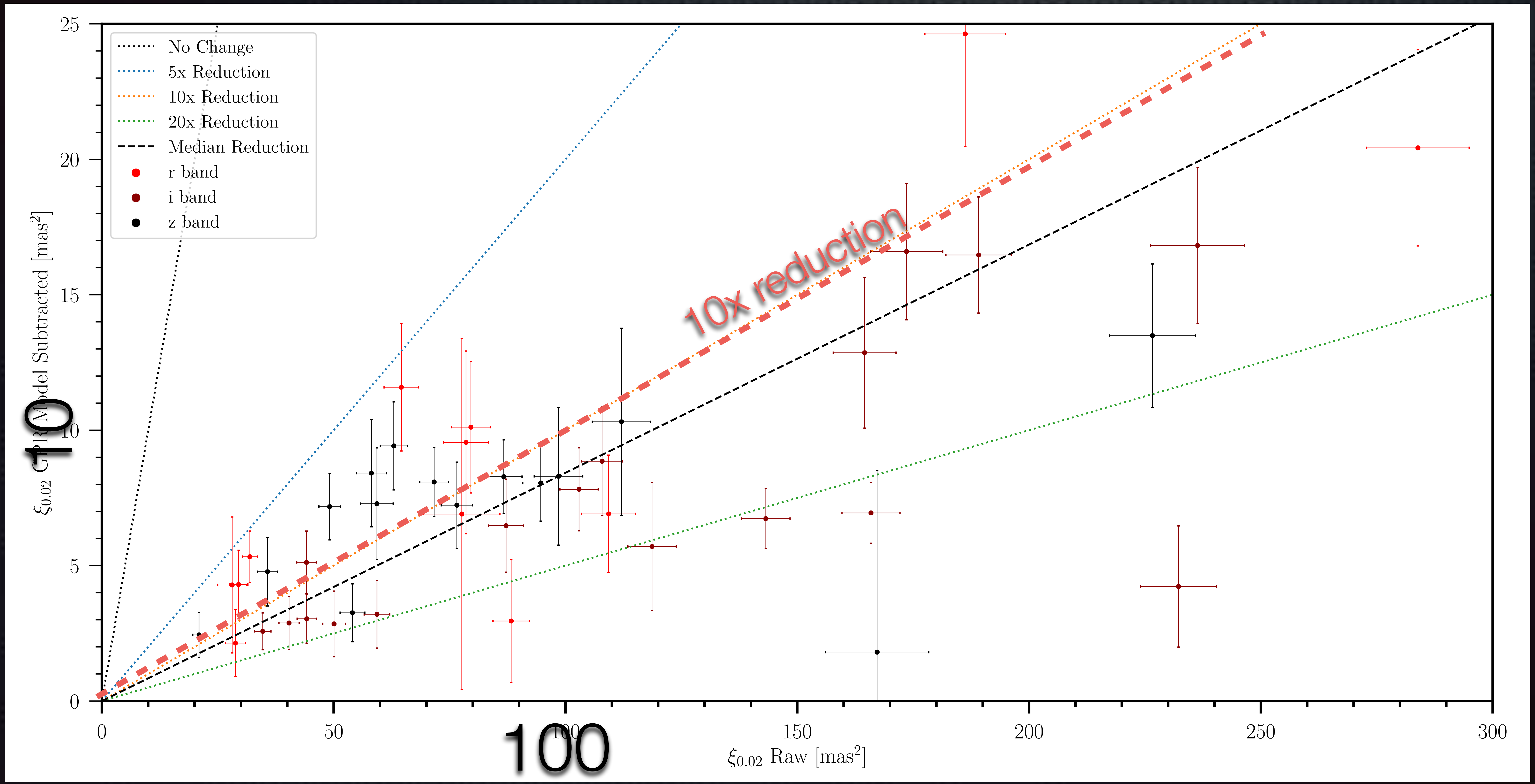
Stochastic residuals (atmospheric turbulence)



Atmospheric turbulence

- For DES exposures (90s, 4m telescope), typical star has ~10 mas RMS position change due to atmospheric turbulence.
- This will dominate the shot-noise errors in LSST exposures for stars up to 1-2 mags fainter than Gaia limit!!
- The turbulence pattern has a coherence length of about 10 arcminutes - maybe we can *measure* the turbulence using Gaia stars and use it to correct the rest of the image?
- Yes! Work by UPenn undergrad Willow Fortino (->U. Delaware) shows that Gaussian Process interpolation can reduce the turbulence RMS errors by about $\sqrt{10}\,\times$.

GP interpolation of atmospheric turbulence (Fortino sr thesis) (early results!)



RMS variance **after** GP (mas²)

RMS variance **before** GP (mas²)

Conclusions: Astrometry with Rubin

- We hope to produce Rubin astrometric positions limited by 2 unavoidable forms of noise:

Shot noise

- We have algorithms to reduce this to theoretical minimum level.
- Depends on source brightness
- ~2 mas per visit at r=20
- Depends on seeing, sky brightness

Atmospheric turbulence

- Working on best possible removal algorithm
- Independent of source brightness, dominant for bright stars,
- perhaps 2-3 mas per visit?
- Depends on atmospheric conditions

- *With its ~1000 measurements of each source, LSST will become the best source of dynamical information for all but the brightest stars, and could revolutionize orbital measurements in the Solar System!*
- From a user's perspective, a WCS function is provided (or images are remapped to a distortion-free grid).
- But chromatic corrections are essential to have accuracy close to expected precision! These may be the limiting factor in the full-survey accuracy, especially in g band.
-

Useful references

- Gaia expected performance and conversions from G to other bands
- Gaia DR2 primer
- Calculating LSST limiting magnitudes and SNR
- "Advanced Exposure Time Calculations" paper by GMB, 2002
- Dark Energy Survey astrometric methods (GMB et al, 2017)