Part 2e) Prove that slope and intercept result from placing the derivative of the sum of square of residuals equal to zero.

Sum of square of residuals : $\sum_{i=1}^{n}(e_i)^2 = \sum_{i=1}^{n}(y_i-\hat{y}_i)^2 = \sum_{i=1}^{n}(y_i-(b_0+b_1x_i))^2$

Take the partial derivative with respect to b_0.

remember
derivative of sum = sum of derivatives

$$\frac{\partial}{\partial B_0}\left[\sum_{i=1}^{n}(y_i-(b_0+b_1x_i))^2\right]=0$$

$$\sum_{i=1}^{n}\left[\frac{\partial}{\partial B_0}(y_i-b_0-b_1x_i)^2\right]=0$$

$$\sum_{i=1}^{n}\left[2(y_i-b_0-b_1x_i)(-1)\right]=0$$

$$-2\sum_{i=1}^{n}\left[y_i-b_0-b_1x_i\right]=0$$

$$-2\left[\sum_{i=1}^{n}y_i-nb_0-b_1\sum_{i=1}^{n}x_i\right]=0$$

$$\Rightarrow nb_0=\sum_{i=1}^{n}y_i-b_1\sum_{i=1}^{n}x_i$$

$$b_0=\frac{\sum_{i=1}^{n}y_i}{n}-\frac{b_1\sum_{i=1}^{n}x_i}{n}$$

$$\Rightarrow \boxed{b_0=\bar{y}-b_1\bar{x}}$$

Take the partial derivative with respect to b_1.

$$\frac{\partial}{\partial B_1}\left[\sum_{i=1}^{n}(y_i-(b_0+b_1x_i))^2\right]=0$$

$$\sum_{i=1}^{n}\left[\frac{\partial}{\partial B_1}(y_i-b_0-b_1x_i)^2\right]=0$$

$$\sum_{i=1}^{n}\left[2(y_i-b_0-b_1x_i)(-x_i)\right]=0$$

$$-2\sum_{i=1}^{n}\left[x_i(y_i-b_0-b_1x_i)\right]=0$$

$$\sum_{i=1}^{n}\left[x_i(y_i-\bar{y}+b_1\bar{x}-b_1x_i)\right]=0$$

$$\sum_{i=1}^{n}\left[x_i(y_i-\bar{y})\right]-b_1\sum_{i=1}^{n}\left[x_i(x_i-\bar{x})\right]=0$$

$$b_1\sum_{i=1}^{n}x_i(x_i-\bar{x})=\sum_{i=1}^{n}x_i(y_i-\bar{y})$$

$$\Rightarrow b_1=\frac{\sum_{i=1}^{n}x_i(y_i-\bar{y})}{\sum_{i=1}^{n}x_i(x_i-\bar{x})}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\Rightarrow \boxed{b_1=\frac{S_{xy}}{S_x^2}}$$

Show:

num: $\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \sum_{i=1}^{n}x_i(y_i-\bar{y})$

$$=\sum_{i=1}^{n}x_i(y_i-\bar{y})-\sum_{i=1}^{n}\bar{x}(y_i-\bar{y})$$

$$=\quad\vdots\quad-\bar{x}\sum_{i=1}^{n}(y_i-\bar{y})$$

$$=\quad\vdots\quad-\bar{x}\left[\cancel{\sum_{i=1}^{n}y_i-n\bar{y}}\right]$$

0 since $\bar{y}=\frac{\Sigma y_i}{n}$

$$=\sum_{i=1}^{n}x_i(y_i-\bar{y})$$

den: $\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}x_i(x_i-\bar{x})$

$$=\sum_{i=1}^{n}(x_i-\bar{x})(x_i-\bar{x})$$

$$=\sum_{i=1}^{n}x_i(x_i-\bar{x})-\bar{x}\sum_{i=1}^{n}(x_i-\bar{x})$$

$$=\quad\vdots\quad-\bar{x}\left[\cancel{\sum_{i=1}^{n}x_i-n\bar{x}}\right]$$

0 since $\bar{x}=\frac{\Sigma x_i}{n}$

$$=\sum_{i=1}^{n}x_i(x_i-\bar{x})$$