

Lecture 7: Convolutional Neural Networks

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<https://shuaili8.github.io>

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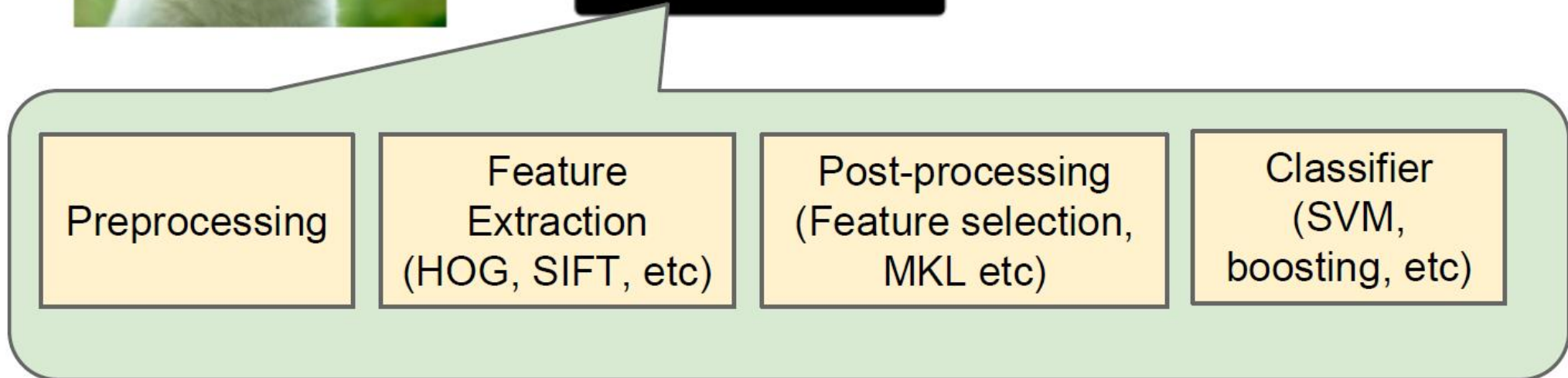
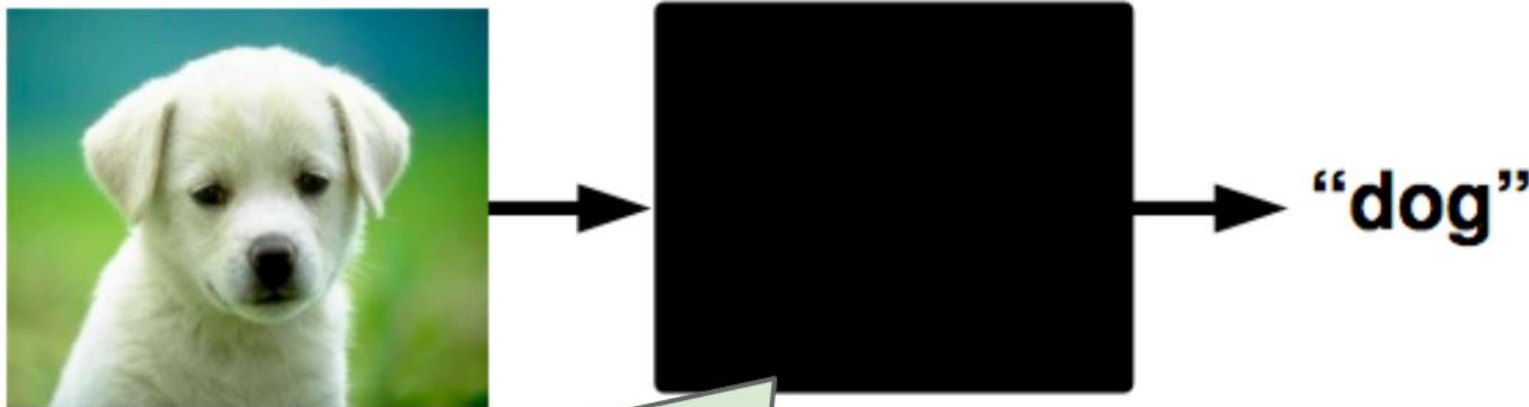
Outline

- Motivation
- Convolution in math and NN
- Usage and parameters
- Training techniques
- Famous neural networks

Motivation

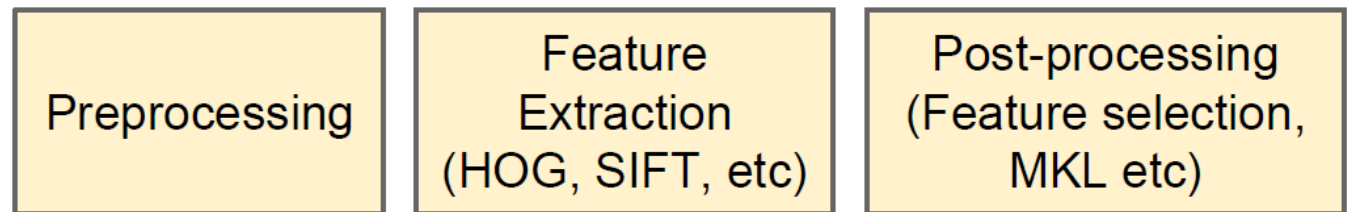
Previous pipeline of pattern recognition

- The black box in a traditional pattern recognition problem



Hand engineered features

- Feature is of critical importance in machine learning, and there are many things to consider when design the features manually:
 - How to design a feature?
 - What is the best feature?
 - Time and money cost in feature engineering.
- Question: Can feature be learned automatically?

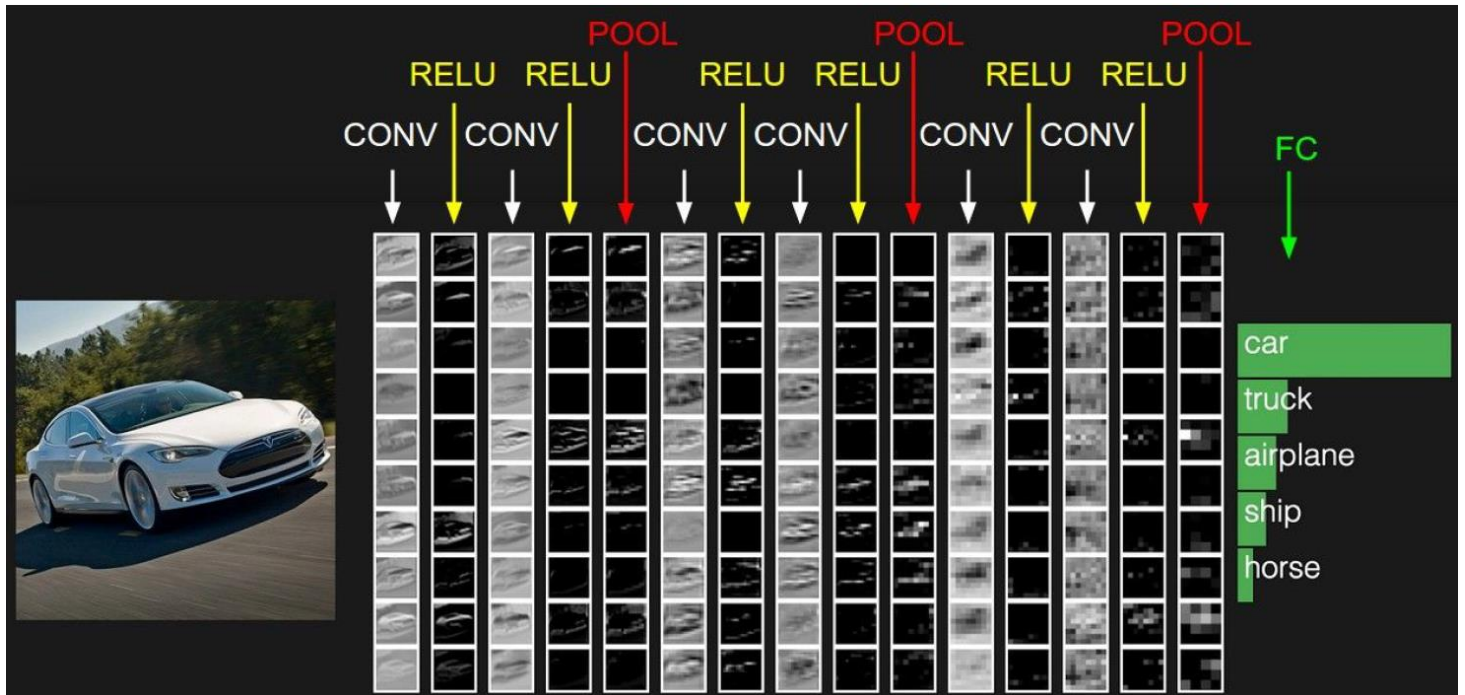


Objective

- Learn features and classifier at the same time
- Learn an end-to-end recognition system
 - A non-linear map that takes raw pixels directly to labels

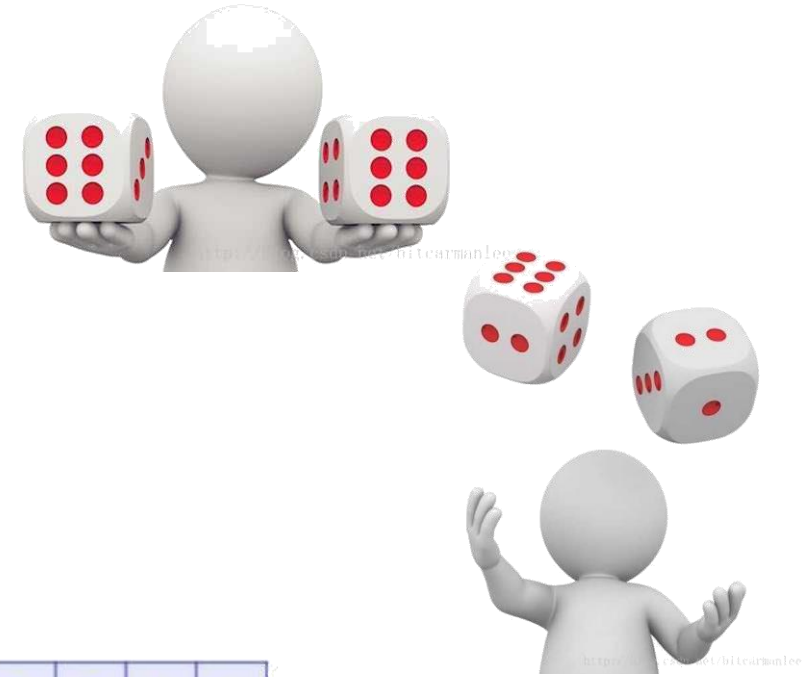
Convolution neural networks

- Is an answer of an end-to-end recognition system
- Contains the following layers with flexible order and repetitions
 - Convolution layer
 - Activation layer (ReLU)
 - Pooling layer
- Example of CNN:



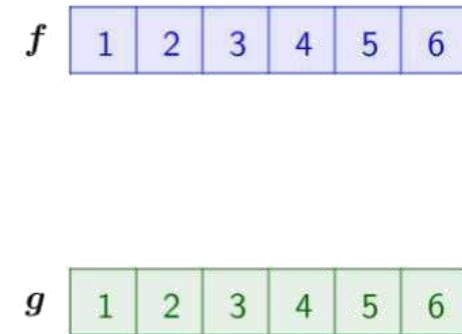
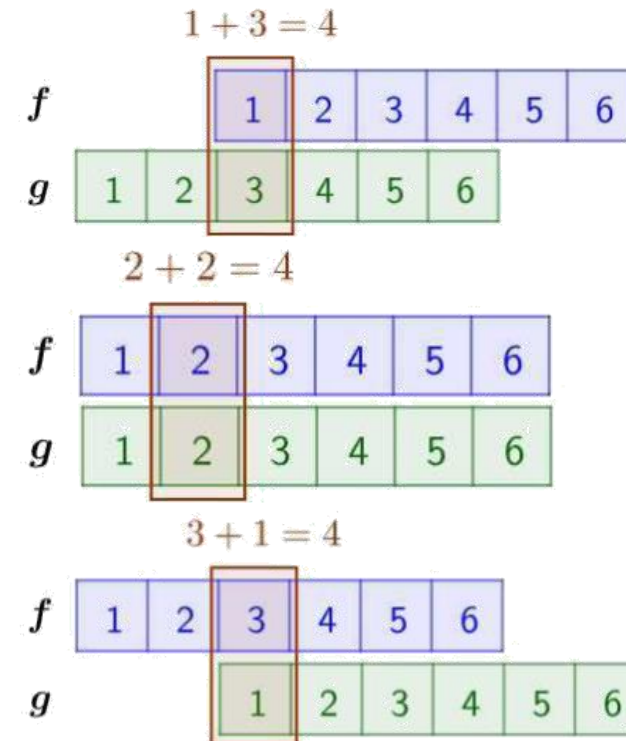
Convolution in Math and NN

Example of convolution in math

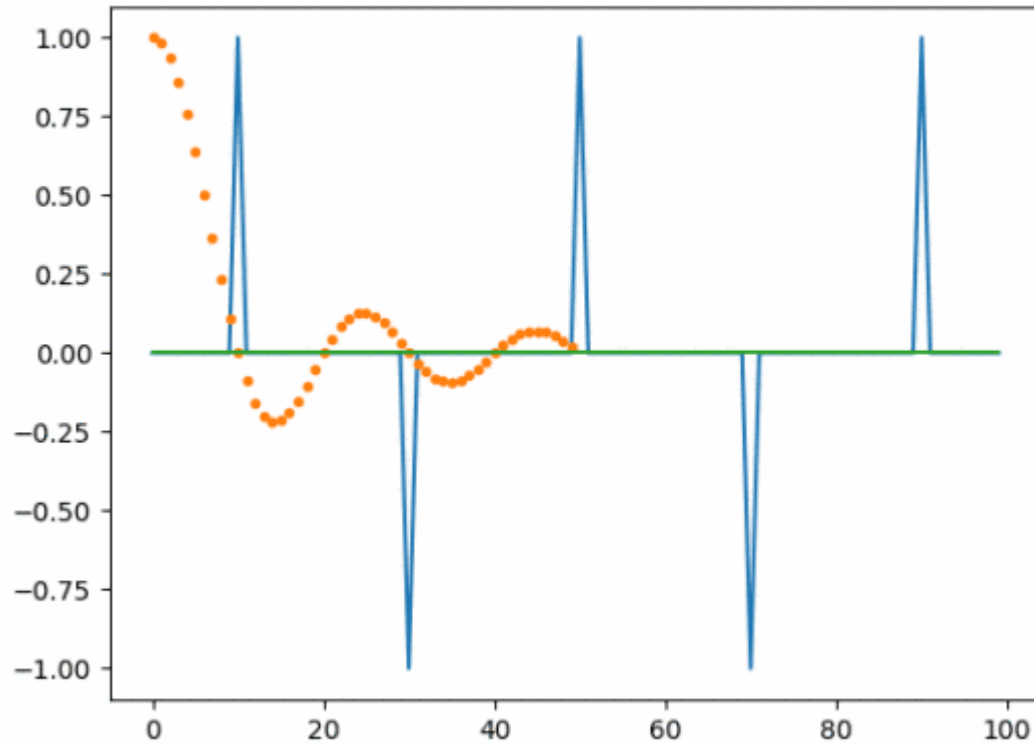


- Suppose we have two dice with probability over $1, 2, \dots, 6$ are f and g respectively. We roll two dice. Then what is the probability that the sum of the two dice is 4?

- $f * g(n) = \sum_i f(i)g(n - i)$



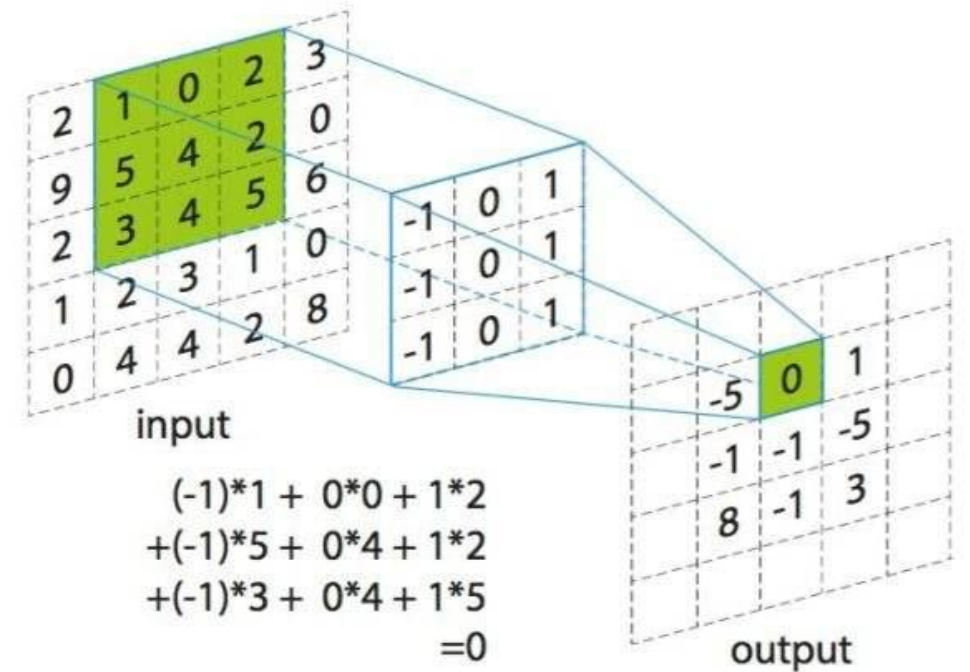
Convolution in math – continuous case



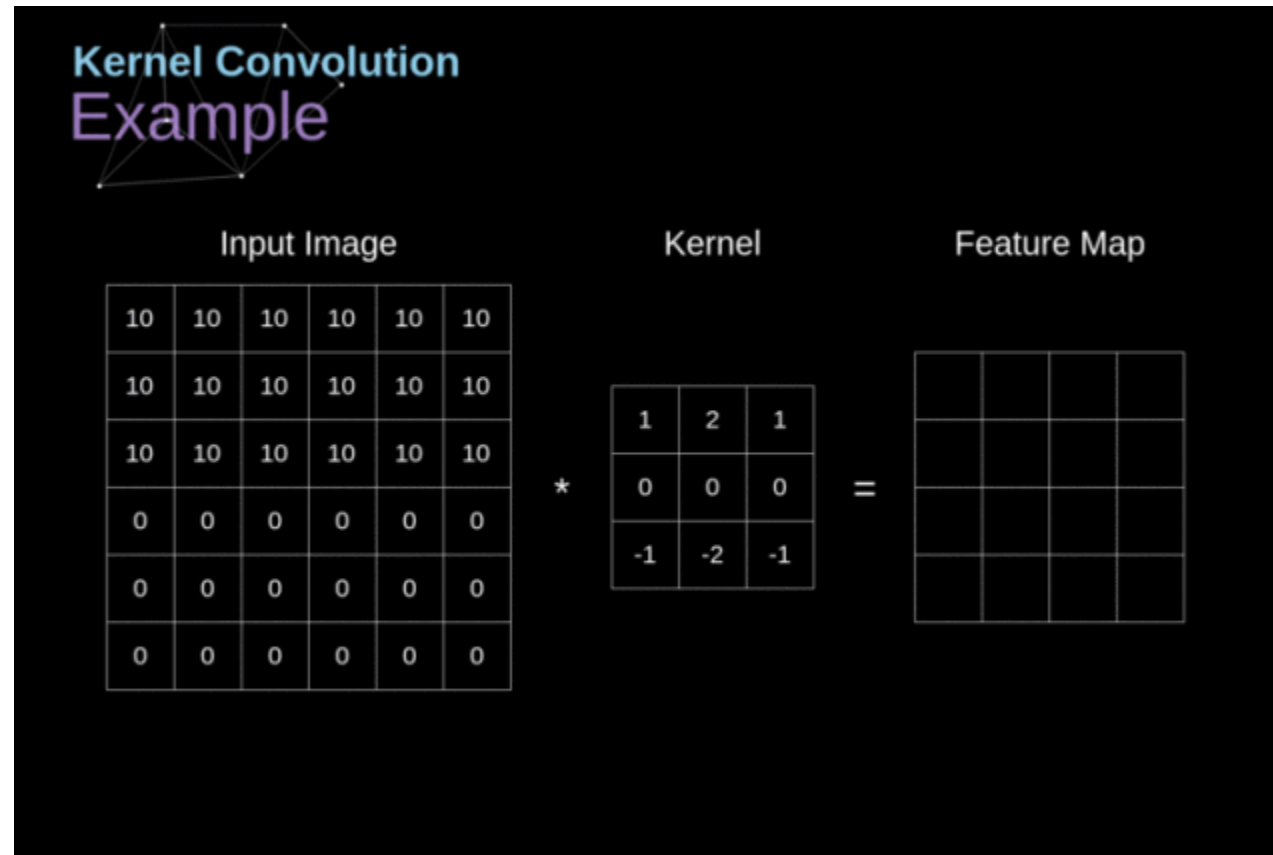
$$f * g(x) = \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau$$

Convolution in neural networks

- Given an input matrix (e.g. an image)
- Use a small matrix (called **filter** or **kernel**) to screening the input at every position of the input matrix
- Put the convolution results at corresponding positions

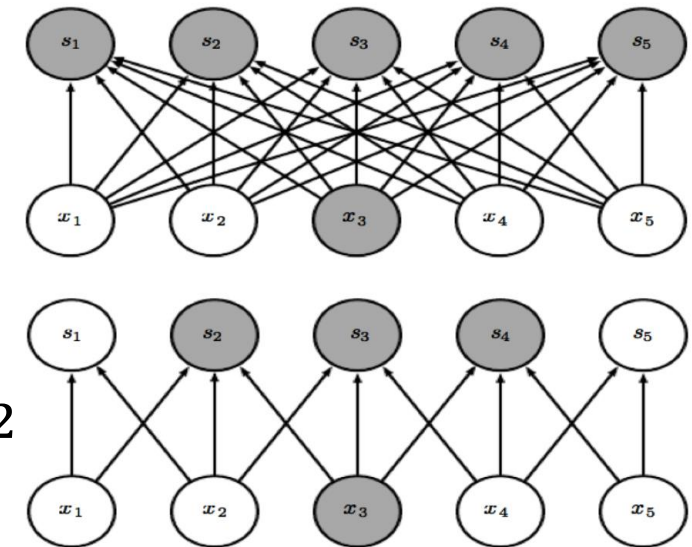


An animation example



Advantage – sparse connections

- Less computing burden
- In fully connected layer (top), every s is linked to every input x , so there are $5 \times 5 = 25$ connecting edges
- In the convolution layer (bottom) with filter width 3, e.g. s_2 is a weighted sum of x_1, x_2, x_3 , so there is no weight connecting s_2 and x_4, x_5 . In this example, there are 13 connecting edges



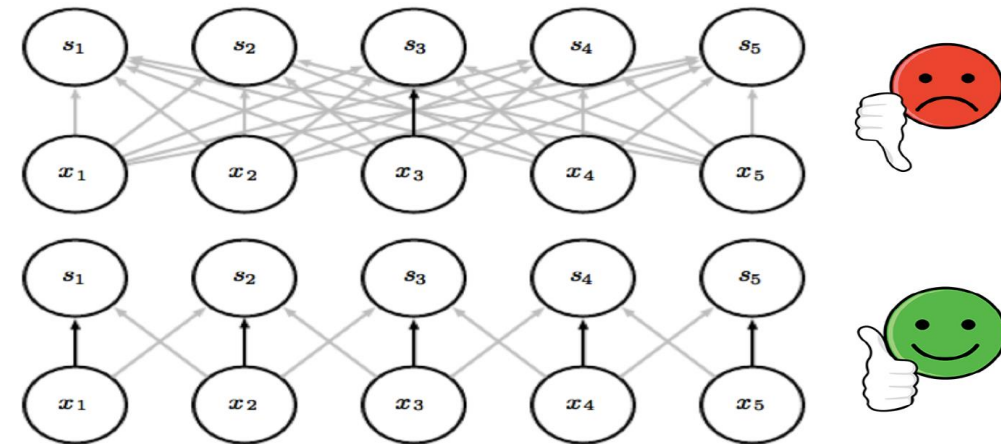
Advantage – weight sharing

- When moving the filter, we don't change the weights inside the filter and these weights are shared at different connecting edges
- In fully connected layer (top), there are **25** connecting edges. The weights on different edges are different parameters.
- In convolution layer (bottom), there are 13 connecting edges. But since

$$s_2 = w_1x_1 + w_2x_2 + w_3x_3$$

$$s_3 = w_1x_2 + w_2x_3 + w_3x_4$$

the number of different weights is even smaller. In this case, there are **3** different weights (**just the size of the filter!**). E.g. the weights on the black arrows are the same.

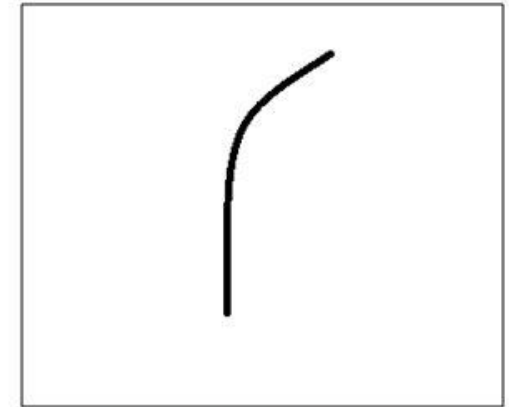


Interpretation of convolution

- Convolution can be used to find an area with **particular patterns!**
- Example:
 - The filter in the left represents the edge in the right, which is the back of a mouse

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

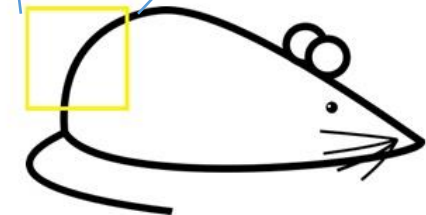
Pixel representation of filter



Visualization of a curve detector filter



Original image



Visualization of the filter on the image

Interpretation of convolution (cont.)

- When the filter moves to the back of the mouse, the convolution operation will generate a very large value



Visualization of the receptive field

0	0	0	0	0	0	30
0	0	0	0	50	50	50
0	0	0	20	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0

Pixel representation of the receptive field

*

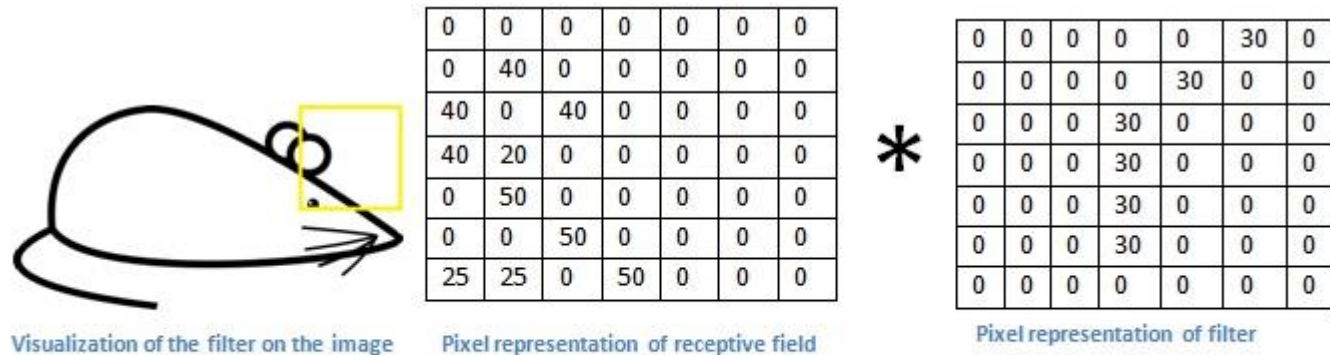
0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

Pixel representation of filter

Multiplication and Summation = $(50*30)+(50*30)+(50*30)+(20*30)+(50*30) = 6600$ (A large number!)

Interpretation of convolution (cont.)

- When the filter moves to other positions, it will generate small values



Multiplication and Summation = 0

Visualization

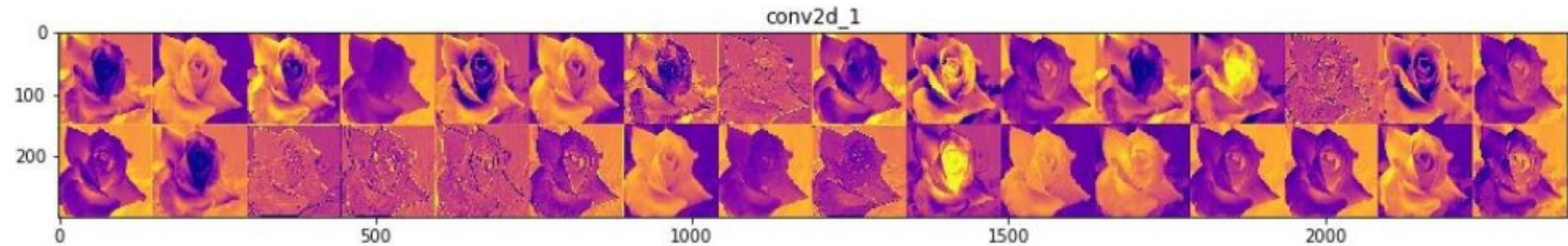
- Train the [InceptionV3](#) model (by Google) on the ImageNet dataset
- Then test it on a flower image from the test dataset
 - Example input image:



- Then let's look at the outputs of different convolutional layers

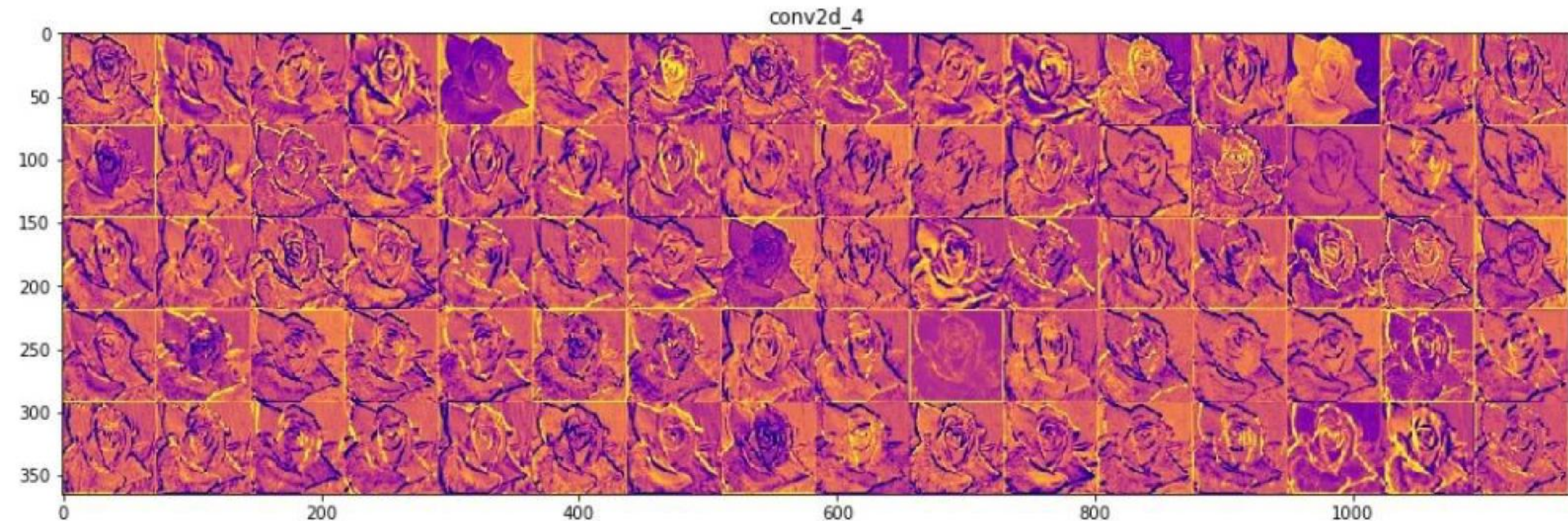
Visualization (cont.)

- The outputs of the filters in the first layer



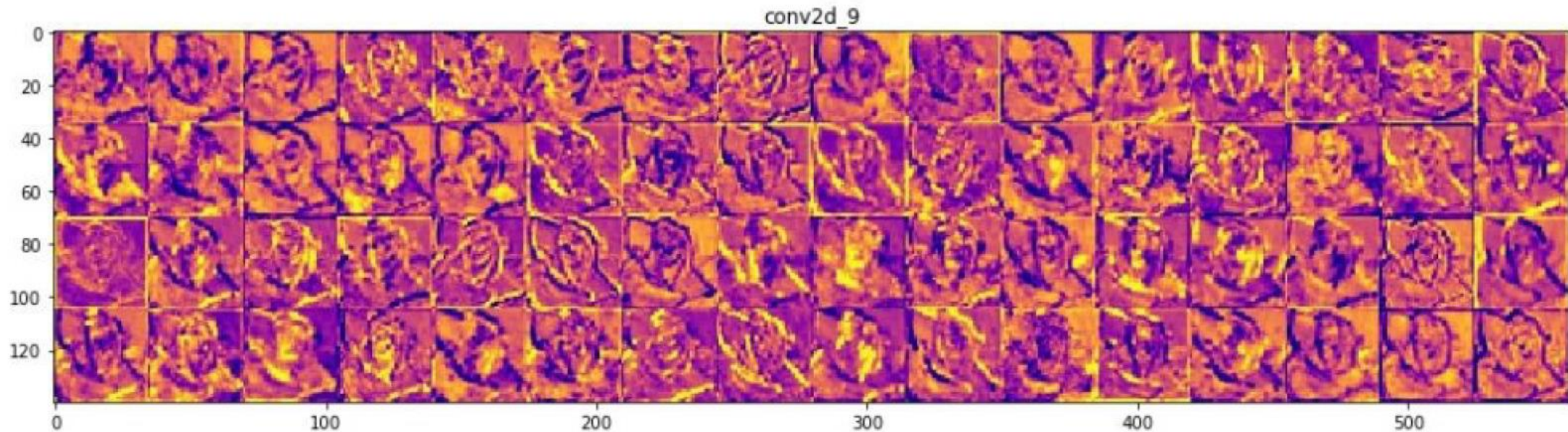
Visualization (cont.)

- The outputs of the filters in the fourth layer



Visualization (cont.)

- The outputs of the filters in the ninth layer



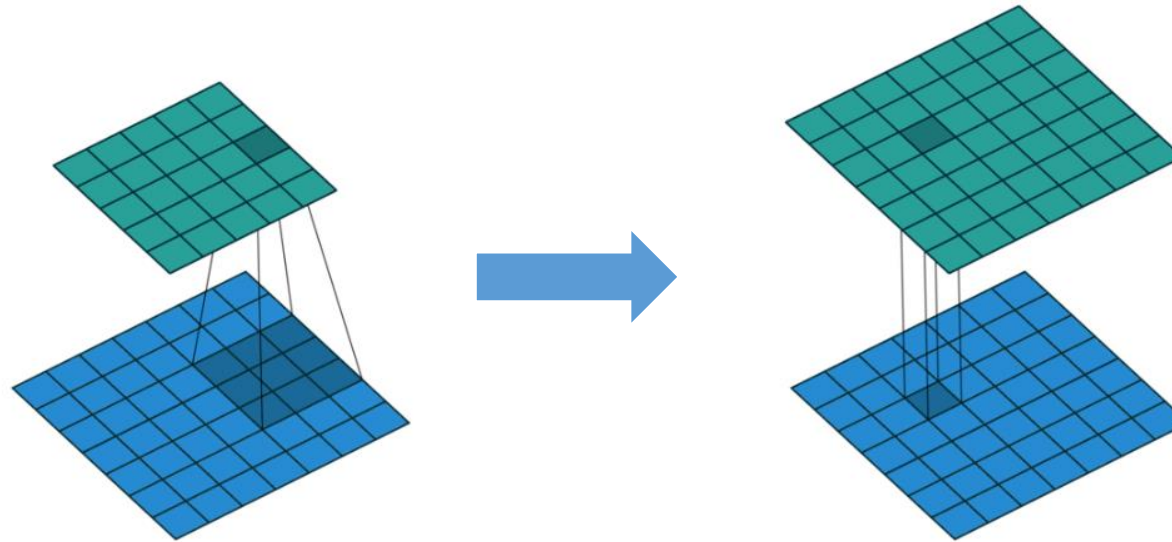
Visualization (cont.)

- Summary
 - The filters in the deeper layers characterize more abstract patterns
 - Layers that are deeper in the network visualize more training data specific features
 - While the earlier layers tend to visualize general patterns like edges, texture, background

Usage and parameters

1×1 convolution

- Here we introduce a special filter, which as a size of 1×1



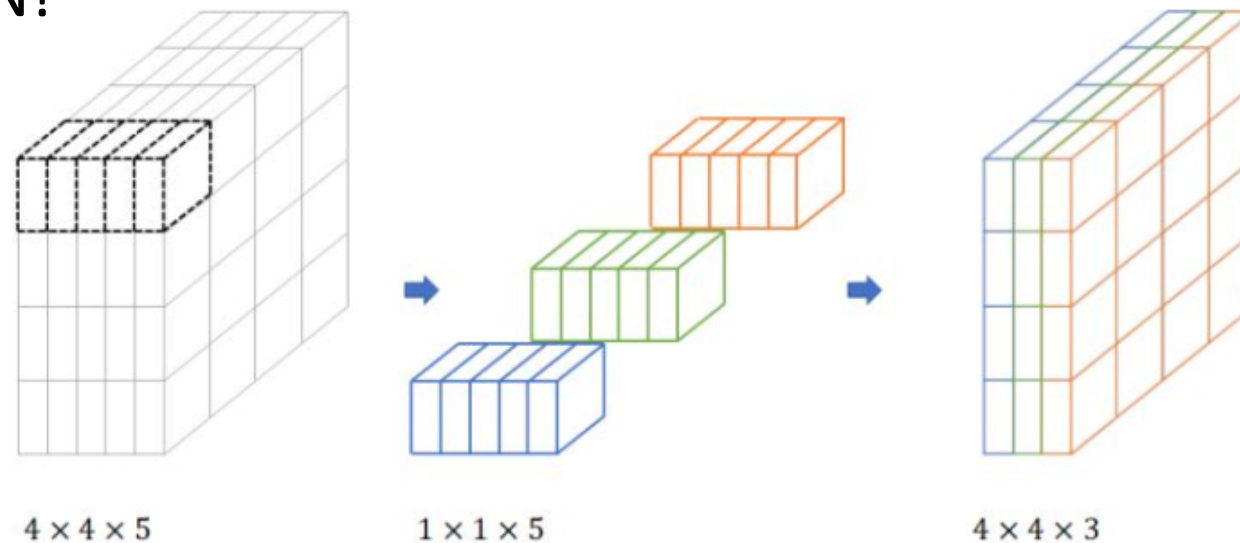
Convolution with large kernel

Convolution with 1×1 kernel

- The 1×1 convolution cannot detect edges with any shape, so is it really useful? Or is it redundant?

1×1 convolution (cont.)

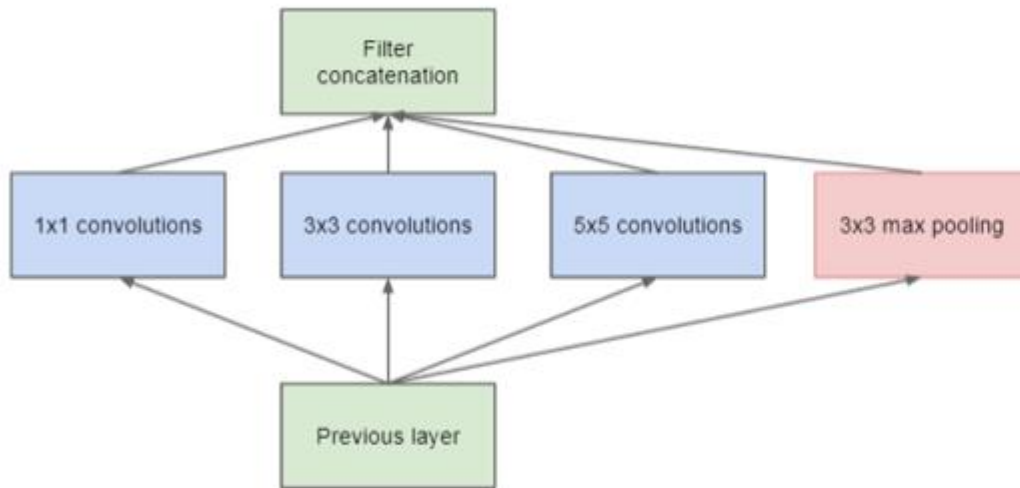
- The 1×1 convolution is very useful as it **can reduce the computation complexity** in CNN!



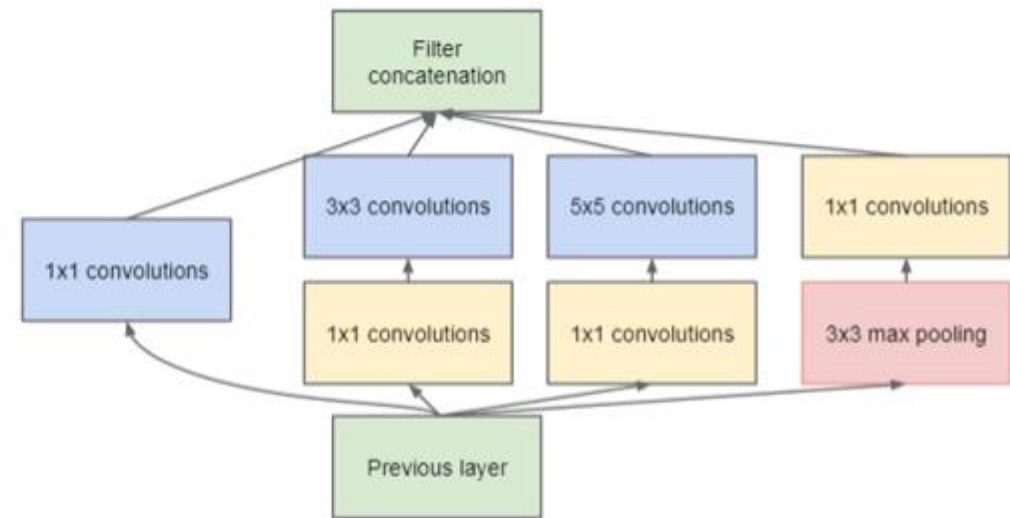
- Help reduce the number of channels
- In the example above, the size of the input data is reduced from $4 \times 4 \times 5$ to $4 \times 4 \times 3$
- Usually we assume the depth of the filter is the same with depth of data

1×1 convolution (cont.)

- 1×1 convolution filter is usually followed by 3×3 or other bigger filters. In this way, the computational complexity is greatly reduced
- This architecture is used in Google's inception model

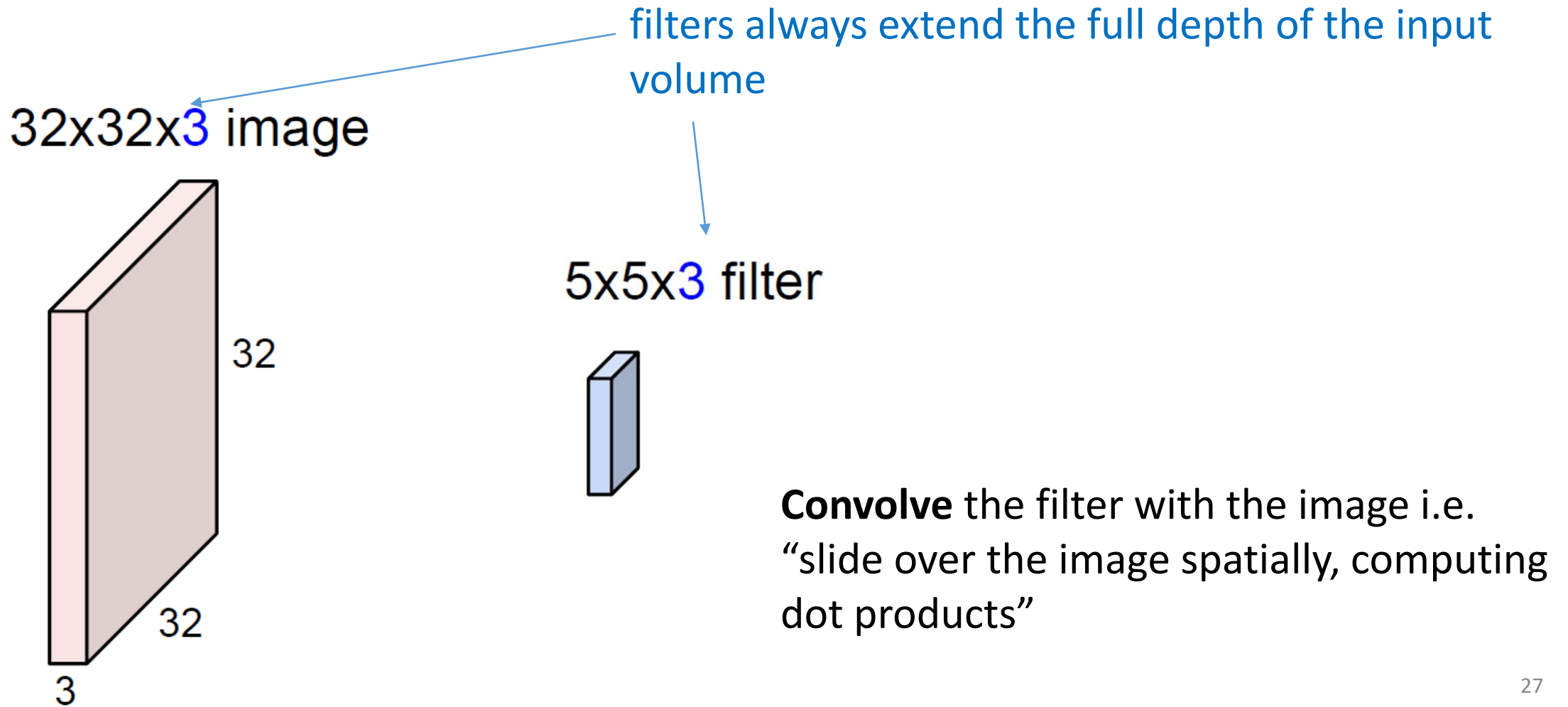


(a) Inception module, naïve version



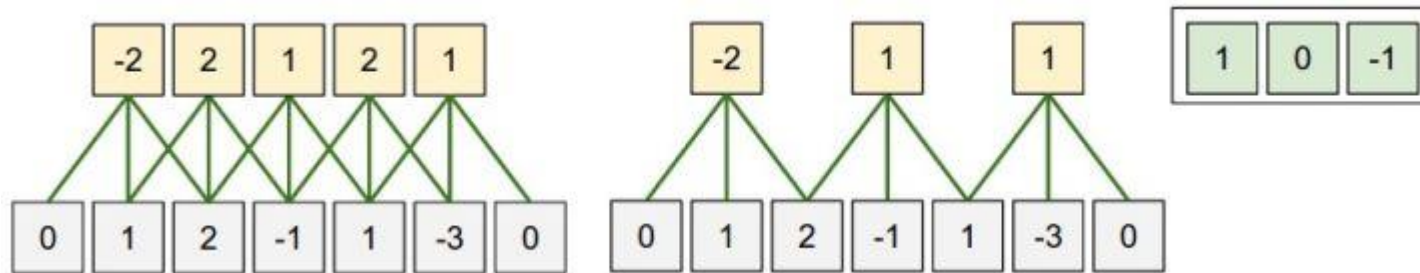
(b) Inception module with dimension reductions

Filter depth



Stride

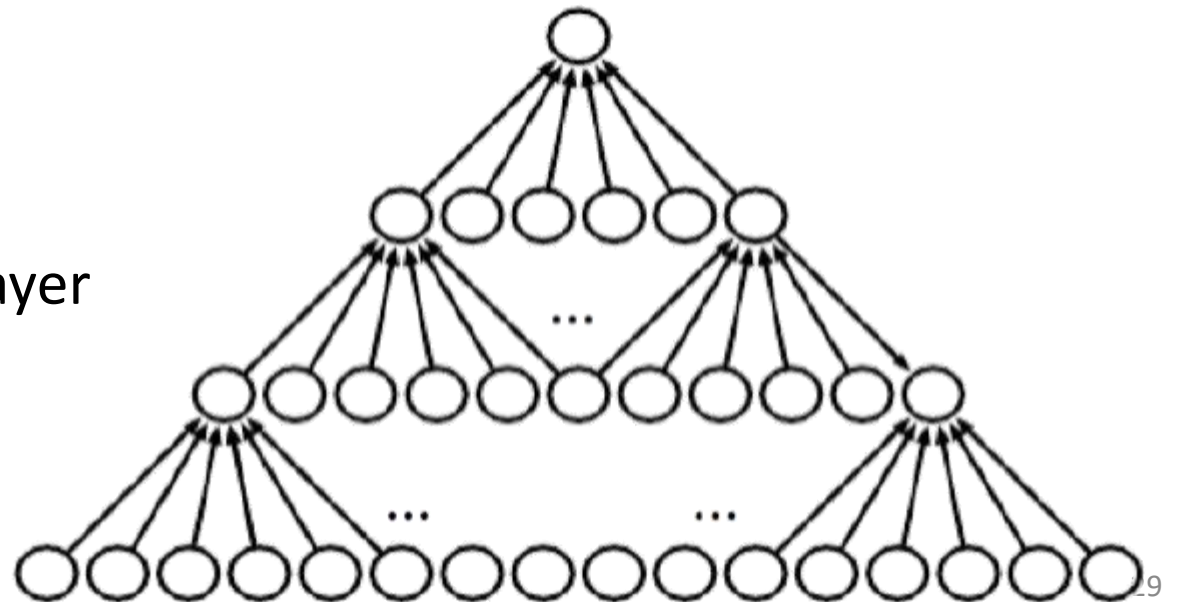
- The distance that the filter is moved in each step
- Examples of stride=1 and stride=2



Padding

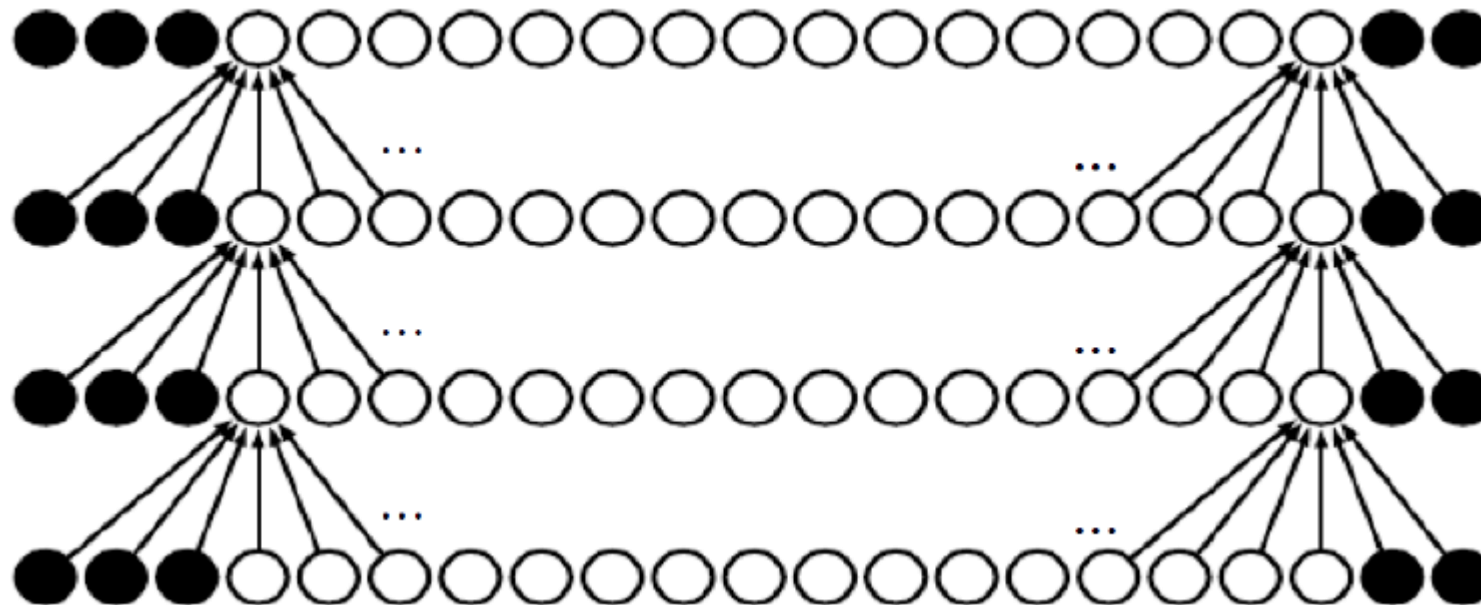
- A solution to the problem of data shrinking
- Data shrinking: as convolution can only happen within the border of the input data, the size of the data will become smaller as the network becomes deeper

- 1D Example:
 - suppose the filter width is 6,
the data will shrink 5 pixel each layer



Padding (cont.)

- Add numbers (usually zero, called **zero padding**) around the input data to make sure that the size of the output data is the same as that of the input data
- Left padding = 3, right padding = 2
- Usually $\text{left padding} + \text{right padding} = \text{filter width} - 1$



Example

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?

Example (cont.)

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?
- **7x7 output!**

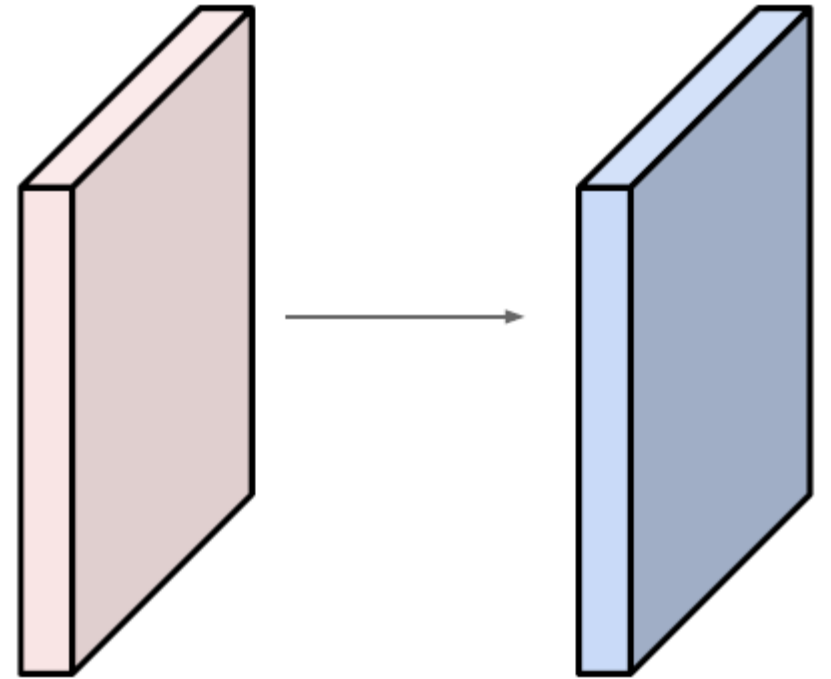
Example (cont.)

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?
- **7x7 output!**
- In general, convolution layers with stride 1, filters of size $F \times F$, and zero-padding with $(F-1)/2$. (will **preserve** size spatially)
- e.g. $F = 3 \Rightarrow$ zero pad with 1
 $F = 5 \Rightarrow$ zero pad with 2
 $F = 7 \Rightarrow$ zero pad with 3

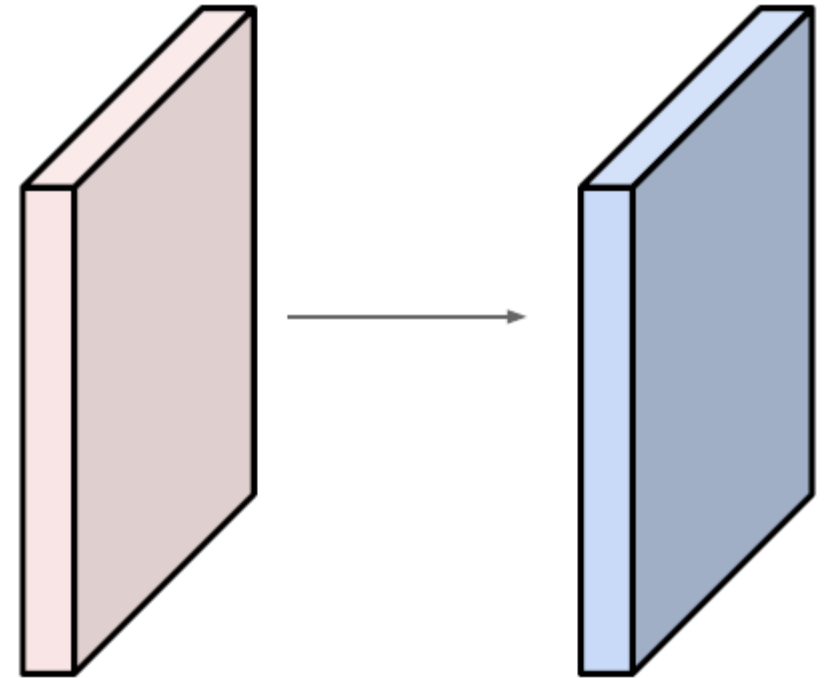
Example 2

- Input volume: $32 \times 32 \times 3$
- 10 5×5 filters with stride 1, pad 2
- Output volume size: ?



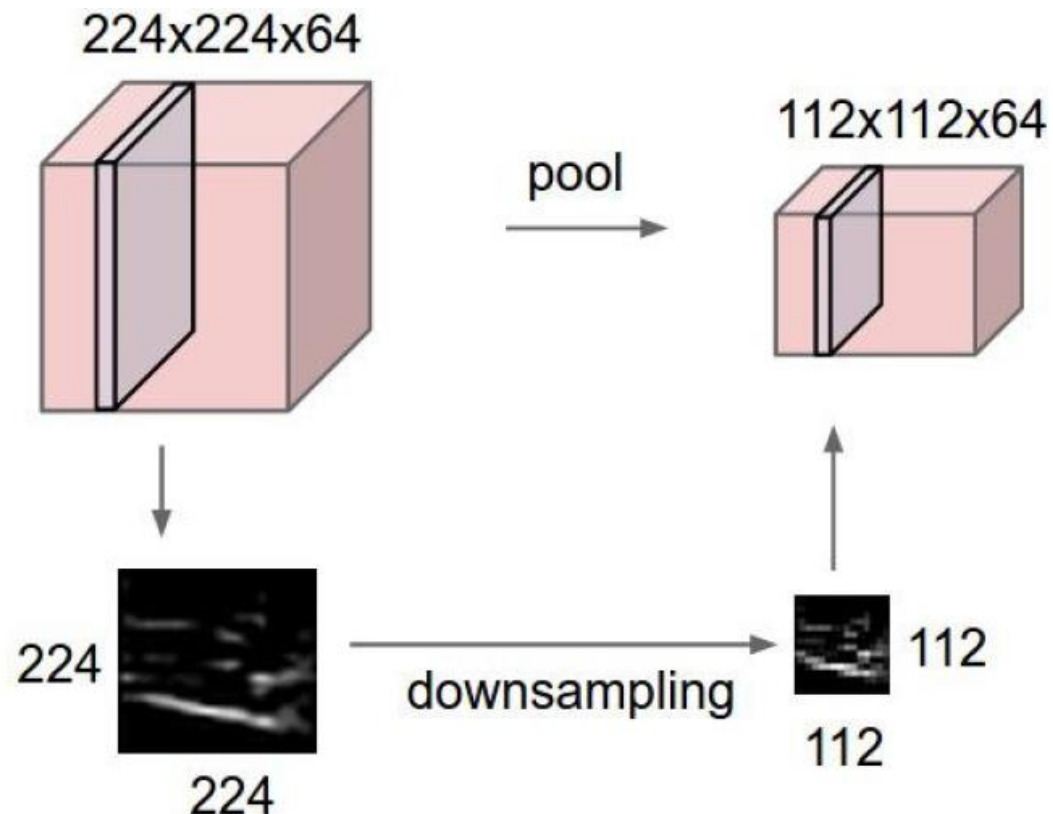
Example 2 (cont.)

- Input volume: $32 \times 32 \times 3$
- 10 5×5 filters with stride 1, pad 2
- Output volume size: ?
- Filter size 5, pad = $(F - 1)/2$,
so keep the size 32×32 spatially
- 10 5×5 filters means 10 $5 \times 5 \times 3$ filters
- So $32 \times 32 \times 10$



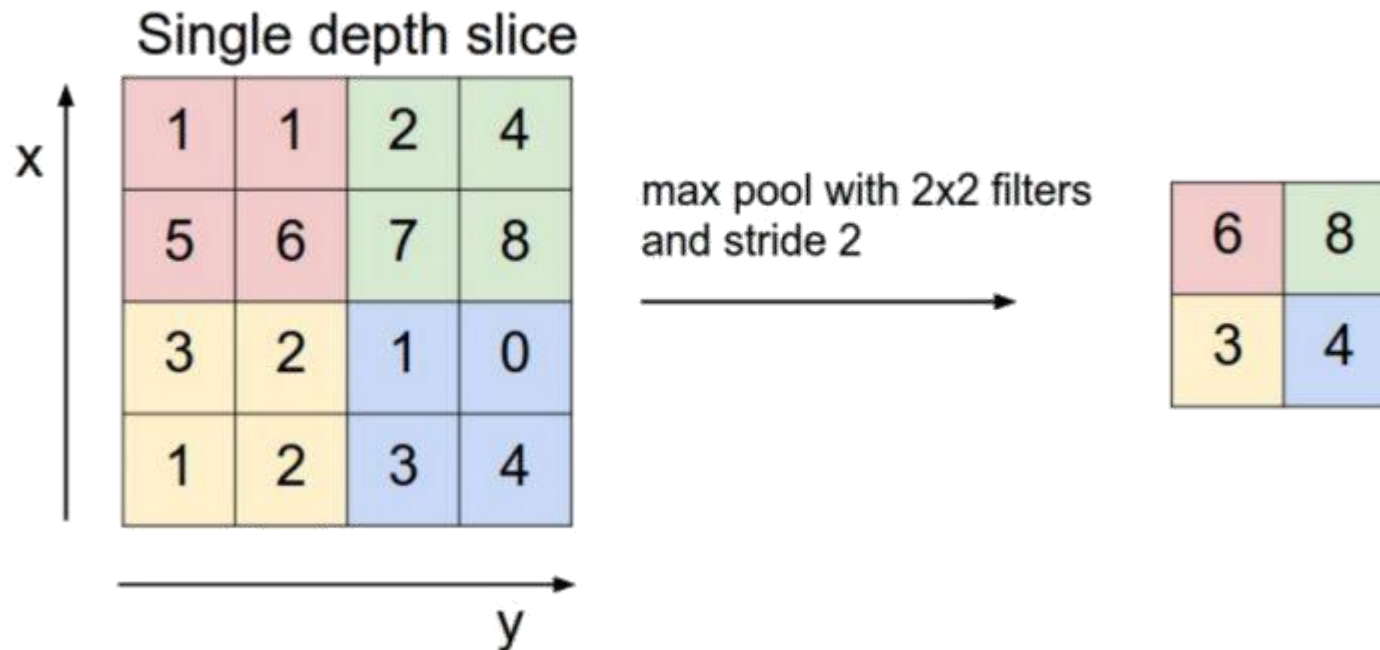
Pooling layer

- Make the representations denser and more manageable
- Operate over each activation map independently:



Example of pooling layer

- Pooling of size 2×2 with stride 2



- Most common pooling operations are **max** and **average**

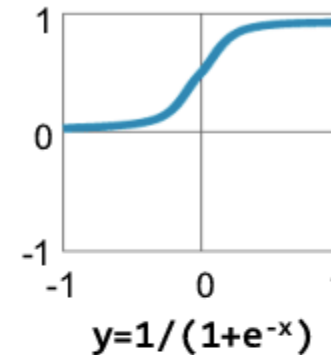
Activation functions (Review)

- Sigmoid: $\sigma(z) = \frac{1}{1+e^{-z}}$
- Tanh: $\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$
- ReLU (Rectified Linear Unit):
 $\text{ReLU}(z) = \max(0, z)$

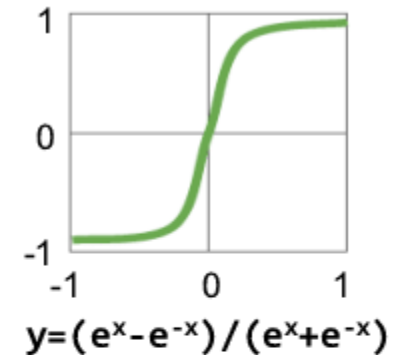
**Traditional
Non-Linear
Activation
Functions**

Most popular in fully
connected neural network

Sigmoid

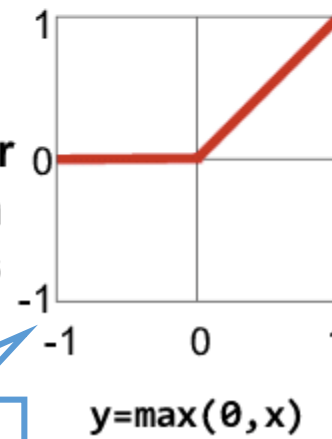


Hyperbolic Tangent

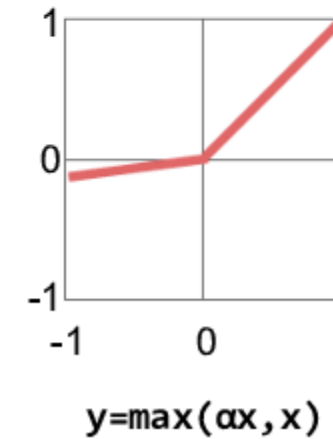


**Modern
Non-Linear
Activation
Functions**

**Rectified Linear Unit
(ReLU)**

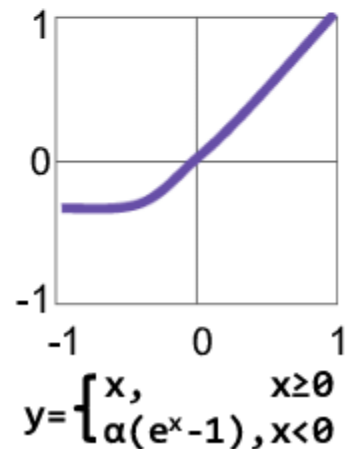


Leaky ReLU



$\alpha = \text{small const. (e.g. 0.1)}$

Exponential LU

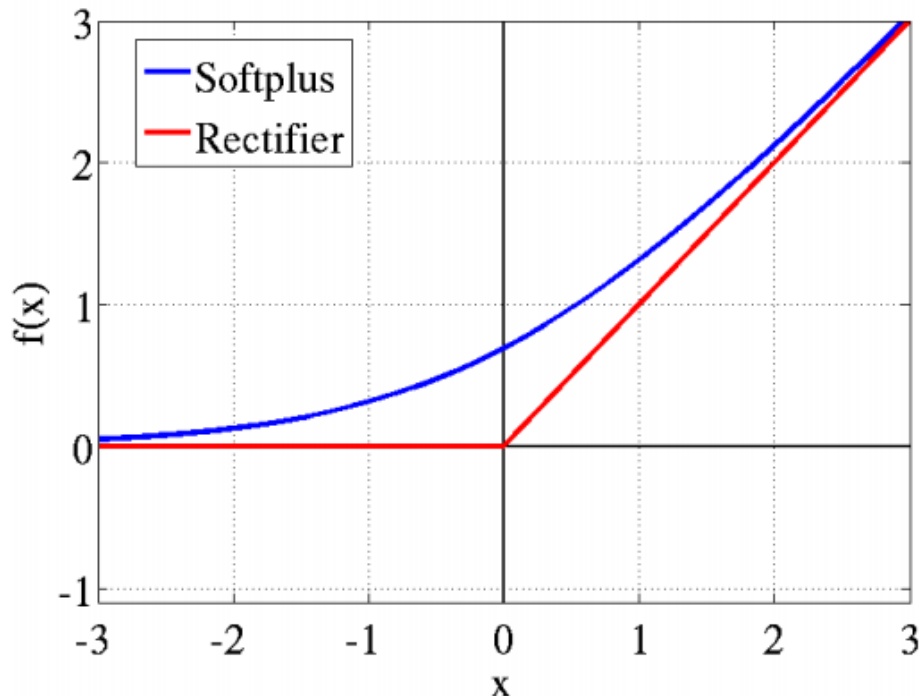


Most popular in
deep learning

ReLU activation function

- ReLU (Rectified linear unit) function

$$\text{ReLU}(z) = \max(0, z)$$



- Its derivative

$$\text{ReLU}'(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$$

- ReLU can be approximated by softplus function

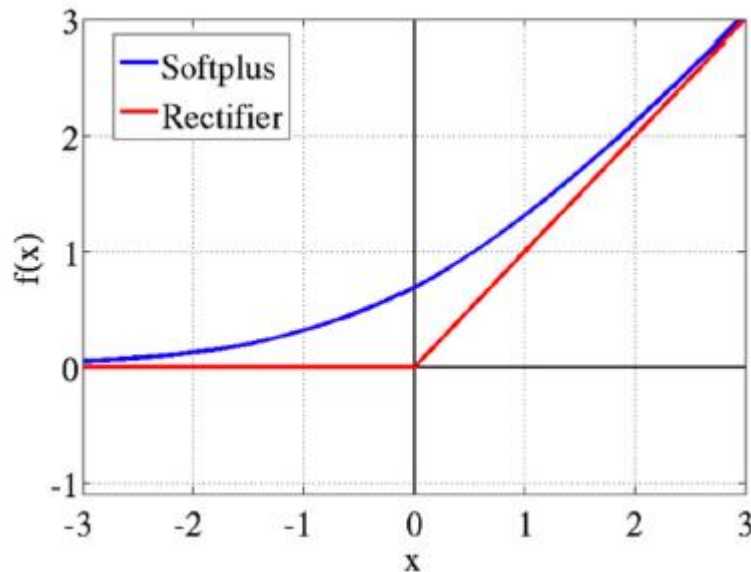
$$f_{\text{Softplus}}(x) = \log(1 + e^x)$$

- ReLU's gradient doesn't vanish as x increases
- Speed up training of neural networks
 - Since the gradient computation is very simple
 - The computational step is simple, no exponentials, no multiplication or division operations (compared to others)
- The gradient on positive portion is larger than sigmoid or tanh functions
 - Update more rapidly
 - The left "dead neuron" part can be ameliorated by Leaky ReLU

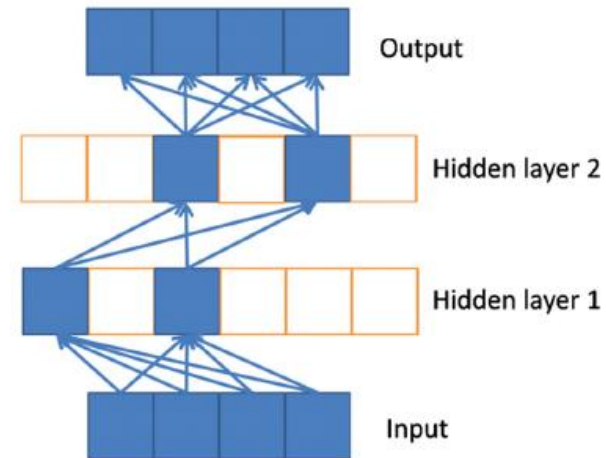
ReLU activation function (cont.)

- ReLU function

$$\text{ReLU}(z) = \max(0, z)$$



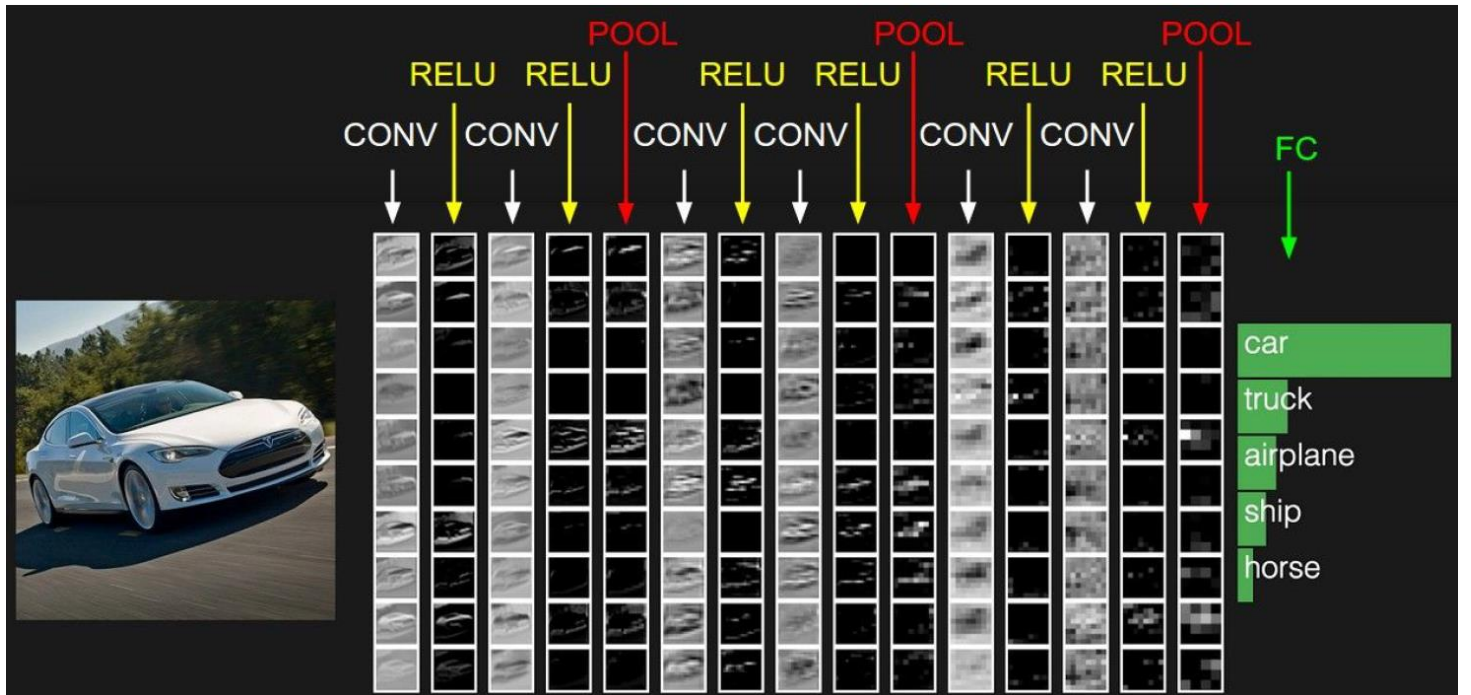
- The only non-linearity comes from the path selection with individual neurons being active or not
- It allows sparse representations:
 - for a given input only a subset of neurons are active



Sparse propagation of activations and gradients

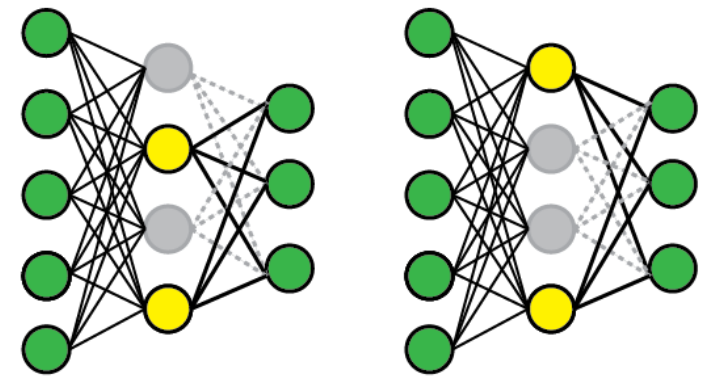
A typical CNN structure

- Convolution/Activation (ReLU)/Pooling layers appear in flexible order and flexible repetitions

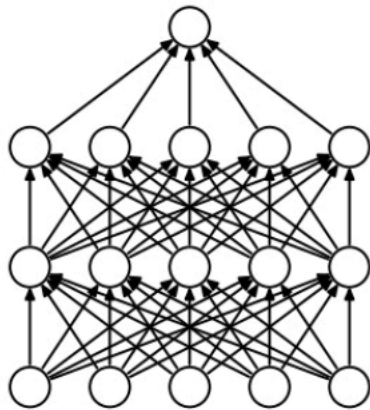


Training Techniques

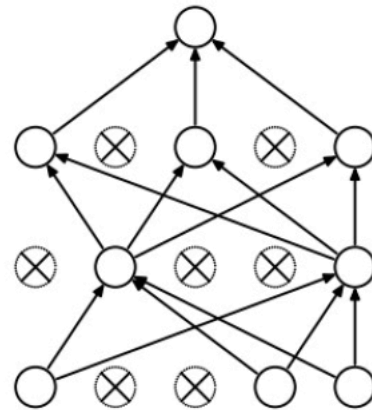
Dropout



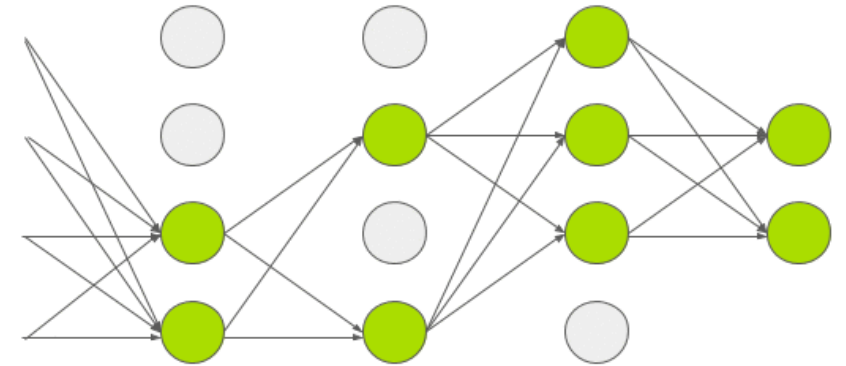
- Dropout randomly ‘drops’ units from a layer on each **training** step, creating ‘sub-architectures’ within the model
- It can be viewed as a type of sampling a small network within a large network
- Prevent neural networks from overfitting



(a) Standard Neural Net

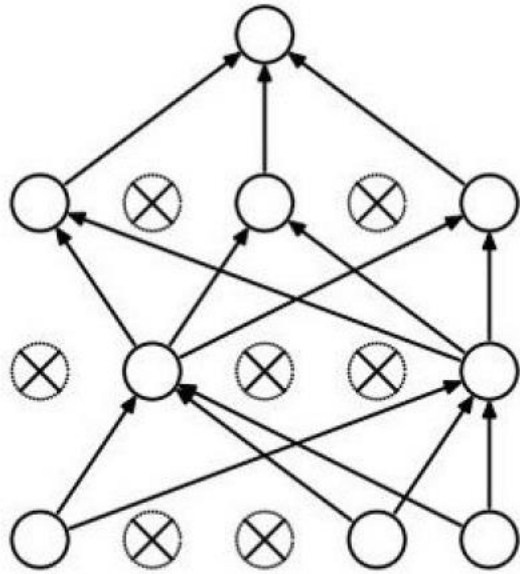


(b) After applying dropout.



Dropout (cont.)

- Forces the network to have a redundant representation
- Increase robustness in prediction power



Weights initialization

- If the weights in a network start too small,
 - then the signal shrinks as it passes through each layer until it's too tiny to be useful
- If the weights in a network start too large,
 - then the signal grows as it passes through each layer until it's too massive to be useful

Weights initialization (cont.)

- All zero initialization
- Small random numbers
- Draw weights from a Gaussian distribution
 - with the standard deviation of $\sqrt{\frac{2}{n}}$
 - n is the number of inputs to the ending neuron



Batch normalization

- Batch training:
 - Given a set of data, each time a small portion of data are put into the model for training
- Extreme example
 - Suppose we are going to learn some pattern of people, and the input data are people's weights and heights
 - Unluckily, women and men are divided into two batches when we randomly split the data
 - As the weights and heights of women are very different from these of men, the neural network have to make **huge changes** to the weight when we switch the batch during training, which will cause slow convergence or even divergence

Batch normalization (cont.)

- The problem in the example is called *Internal Covariate Shift*
- The solution to Internal Covariate Shift is batch normalization
- Suppose $Z_j^{(i)}$ is the i^{th} input for the j^{th} neuron in the input layer

$$\mu_j = \frac{1}{m} \sum_{i=1}^m Z_j^{(i)}$$

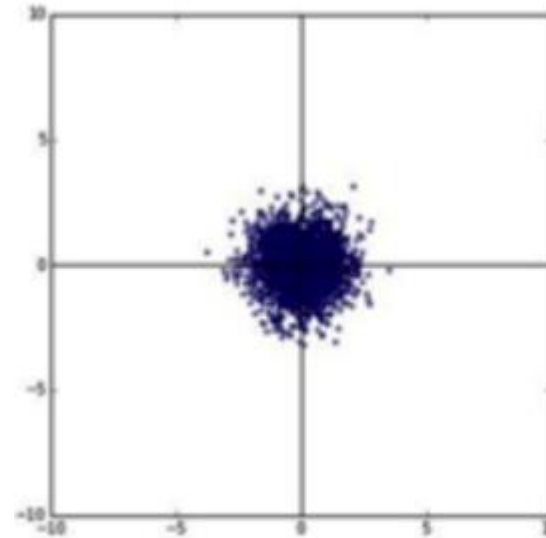
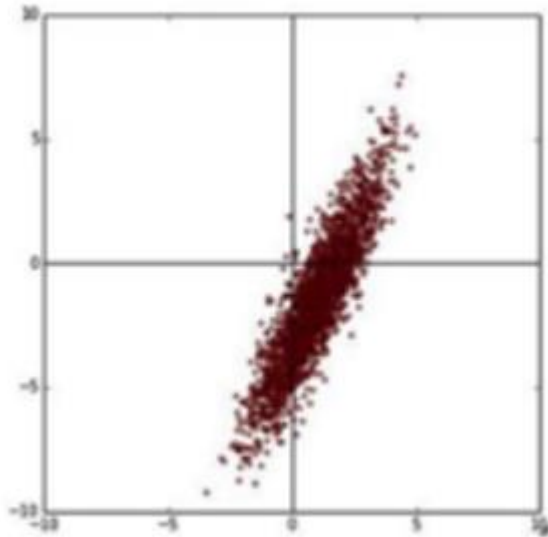
$$\sigma_j^2 = \frac{1}{m} \sum_{i=1}^m (Z_j^{(i)} - \mu_j)^2$$

$$\hat{Z}_j = \frac{Z_j - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

- Or normalize the whole input layer together

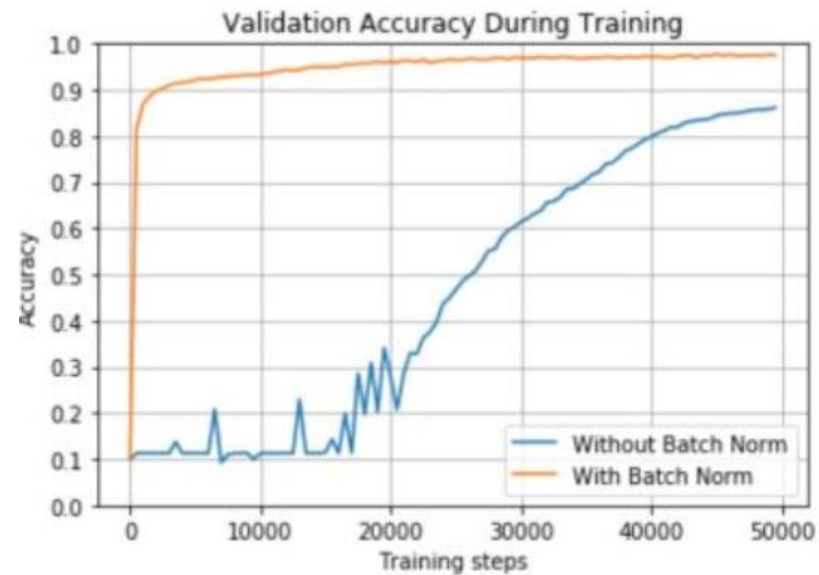
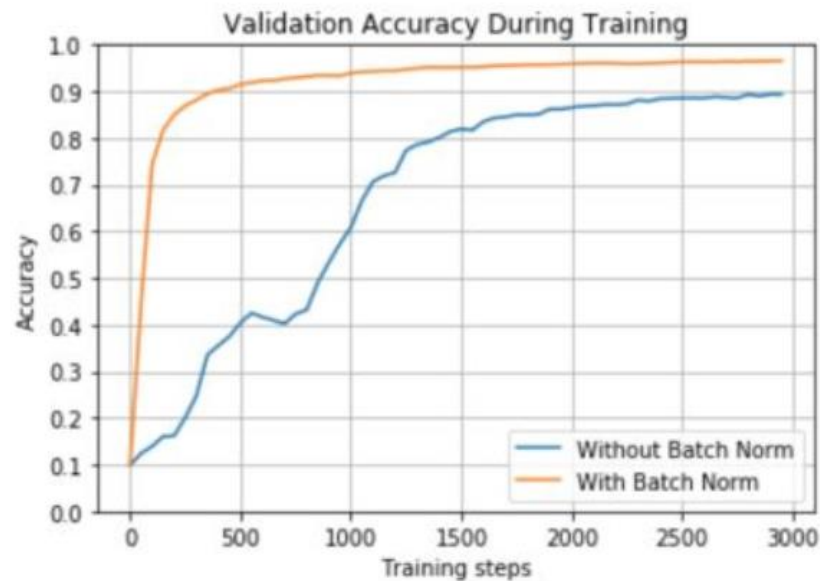
Batch normalization (cont.)

- 2D example



Example of batch normalization

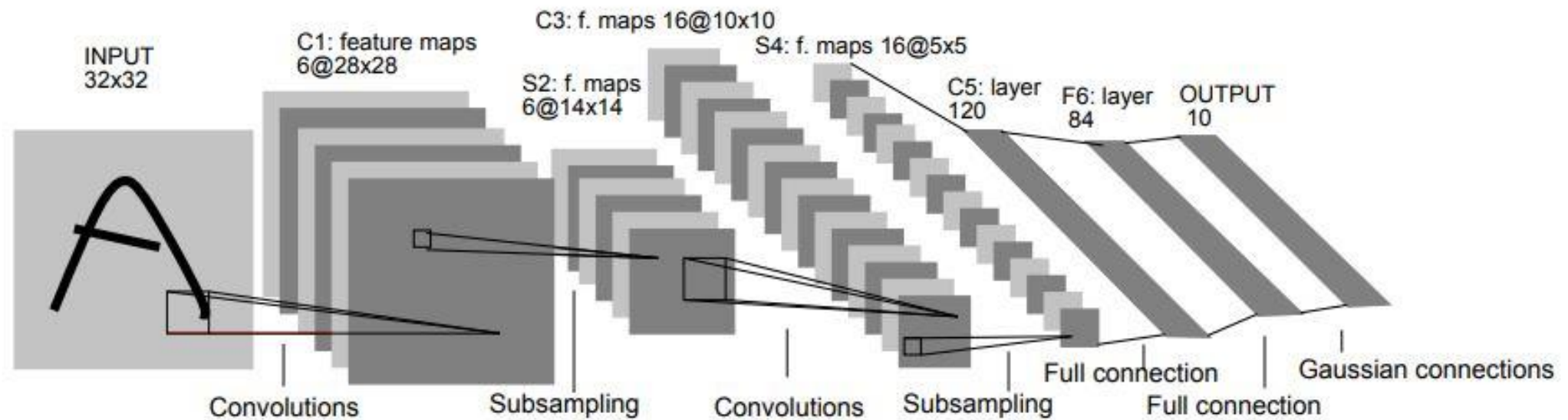
- Without batch normalization
 - Slow convergence and fluctuation



Famous Neural Networks

LeNet

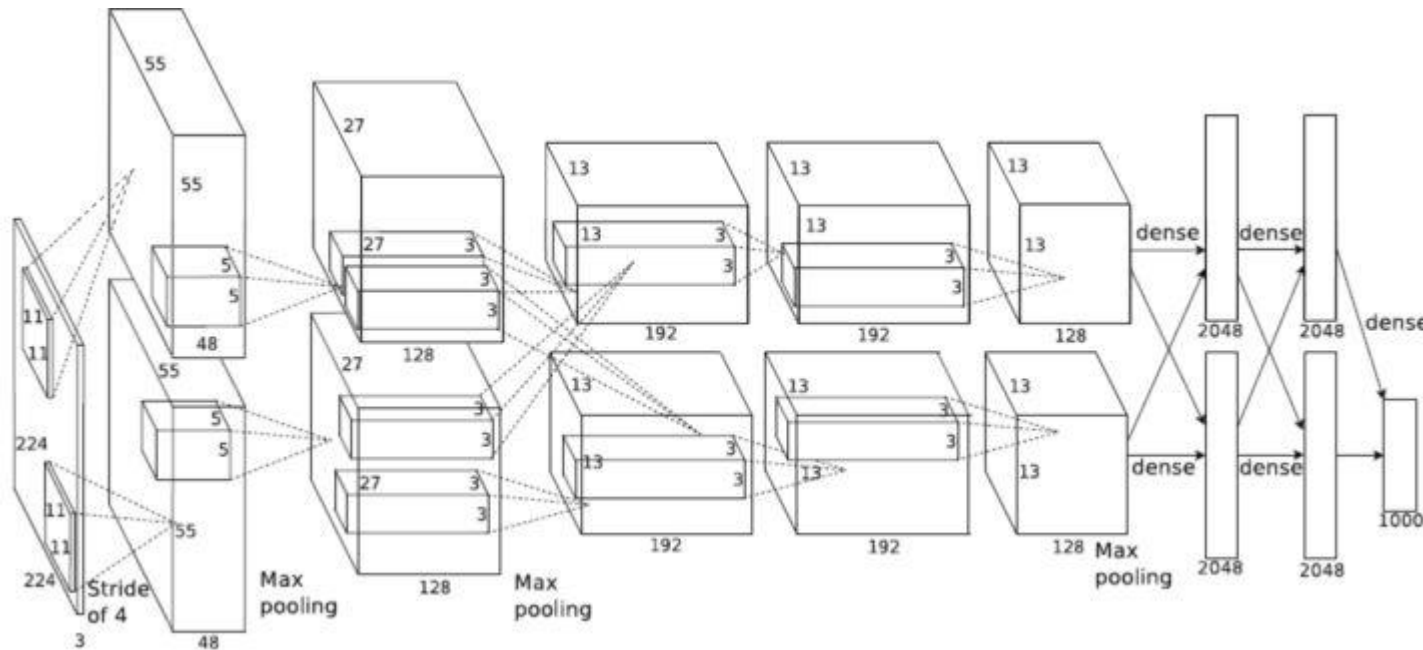
- LeNet [LeCun et al., 1998]



Input: 28*28*1 image, Conv filters of size 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

AlexNet [Krizhevsky et al. 2012]

- AlexNet:



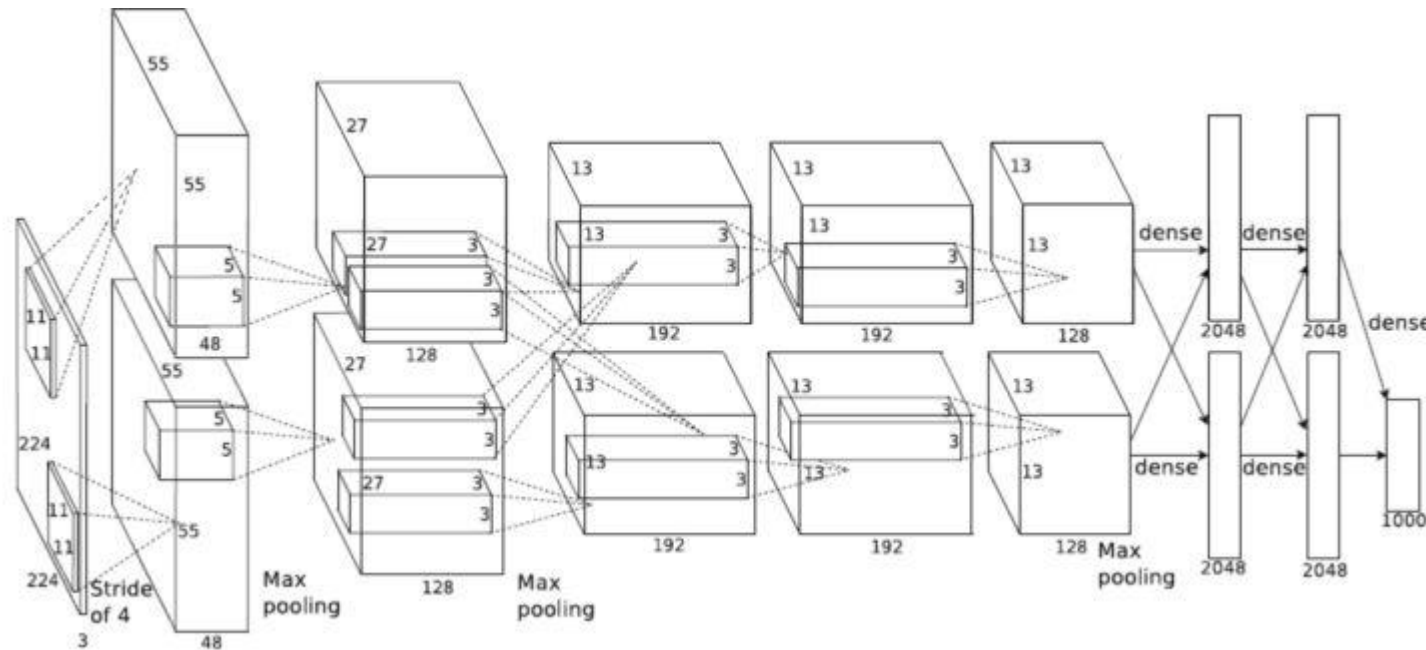
Input: 227x227x3 images

First layer (CONV1): 96 11x11 kernels applied at stride 4

Q: what is the output volume size?

AlexNet (cont.)

- AlexNet:



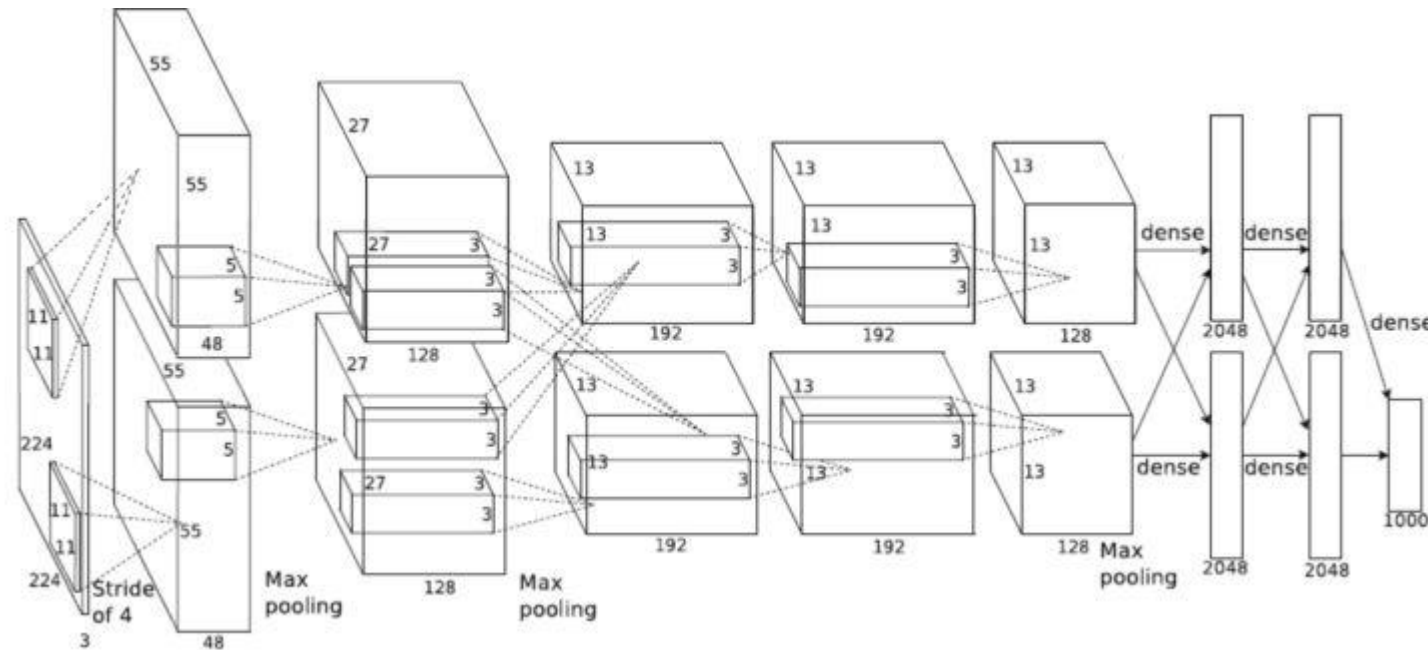
Input: 227x227x3 images

First layer (CONV1): 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint: $(227-11)/4+1 = 55$ [55x55x96]

AlexNet (cont.)

- AlexNet:



Input: 227x227x3 images

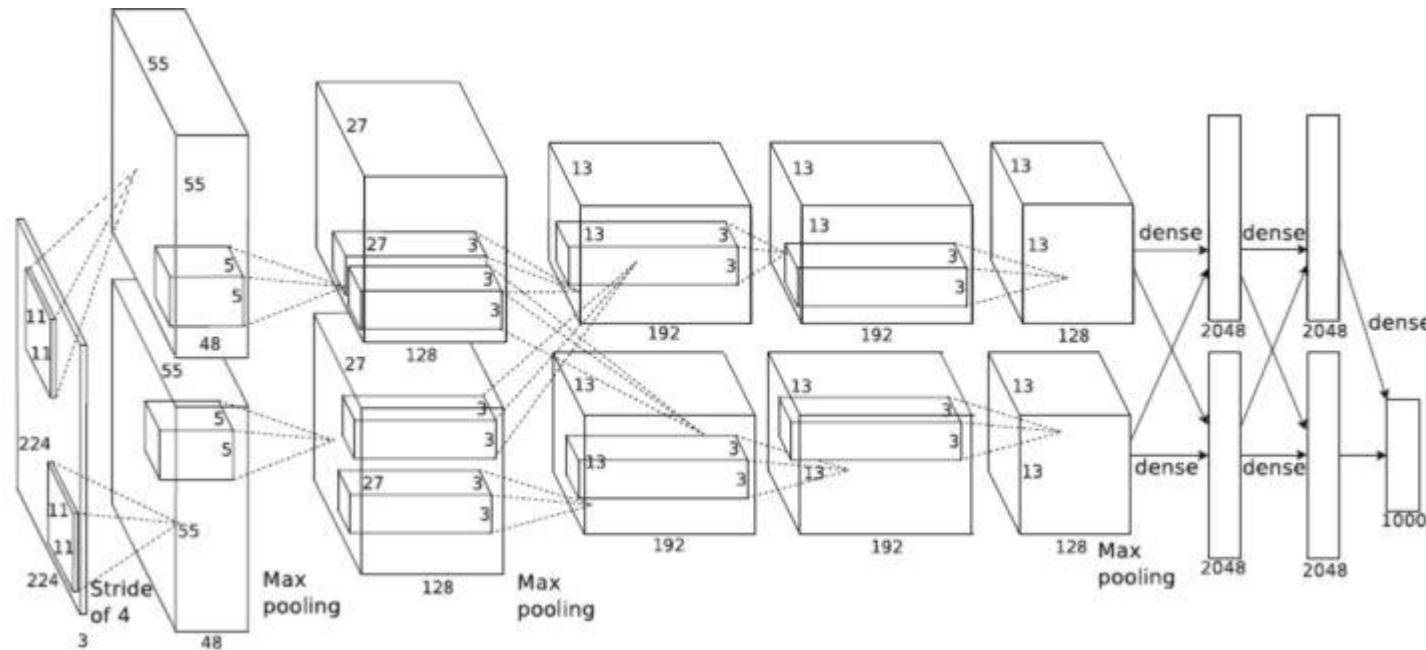
First layer (CONV1): 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint: $(227-11)/4+1 = 55$ [55x55x96]

Q: What is the total number of parameters in this layer?

AlexNet (cont.)

- AlexNet:



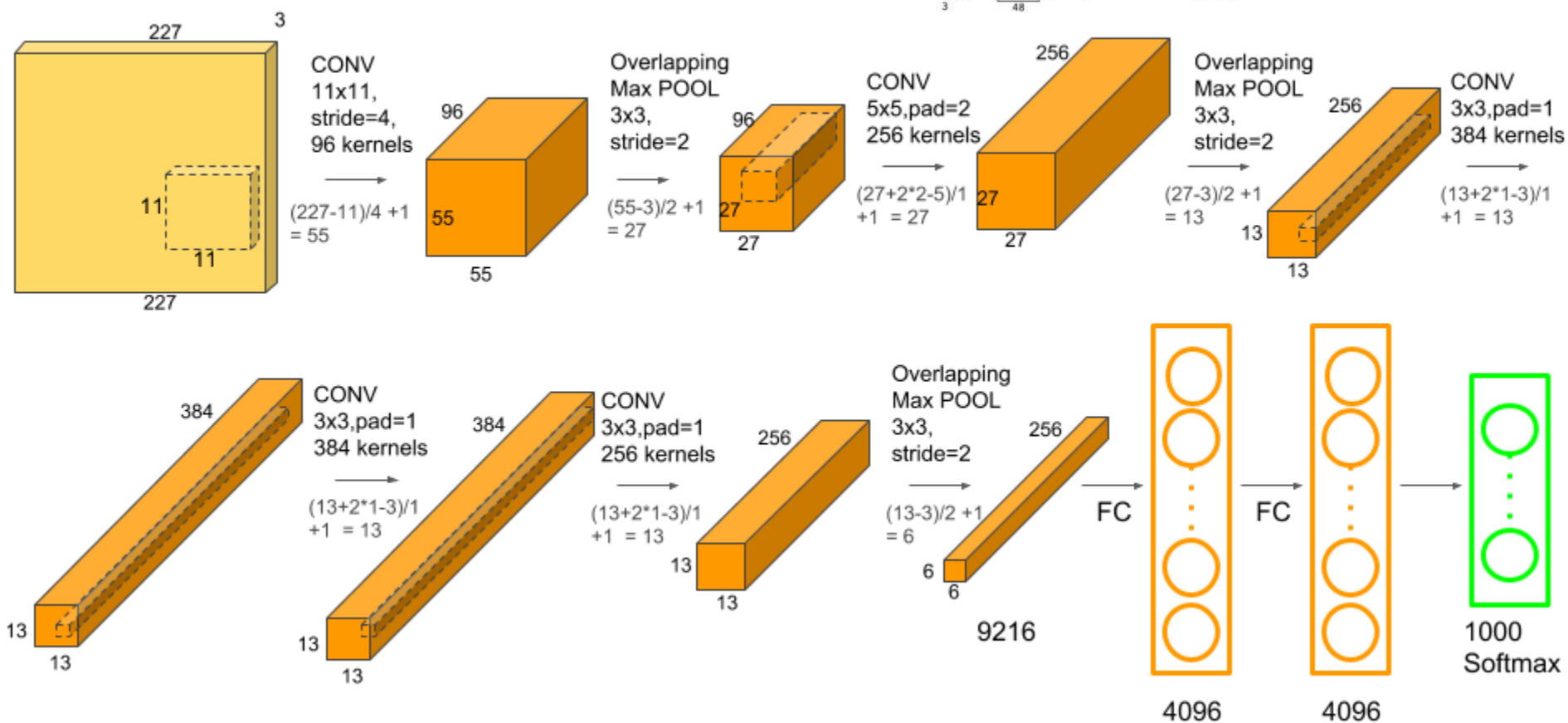
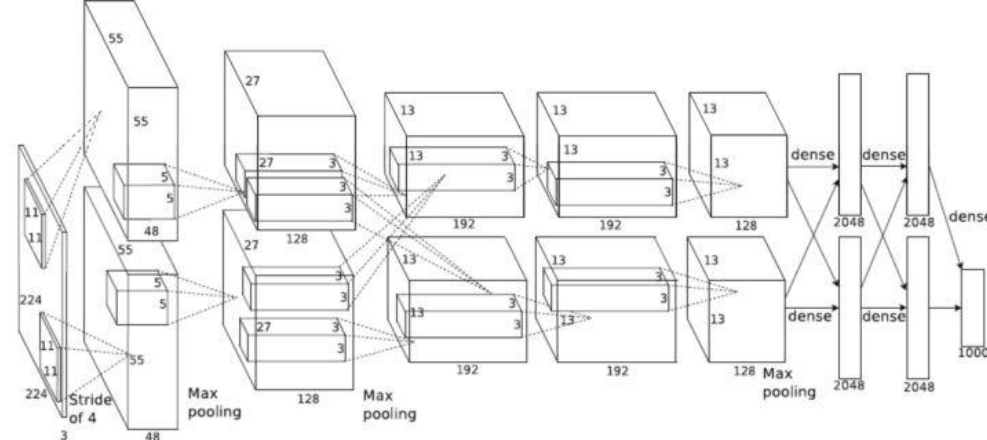
Input: 227x227x3 images

First layer (CONV1): 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint: $(227-11)/4+1 = 55$ [55x55x96]

Q: What is the total number of parameters in this layer? $(11*11*3)*96 = 35K$

AlexNet (cont.)



VGGNet

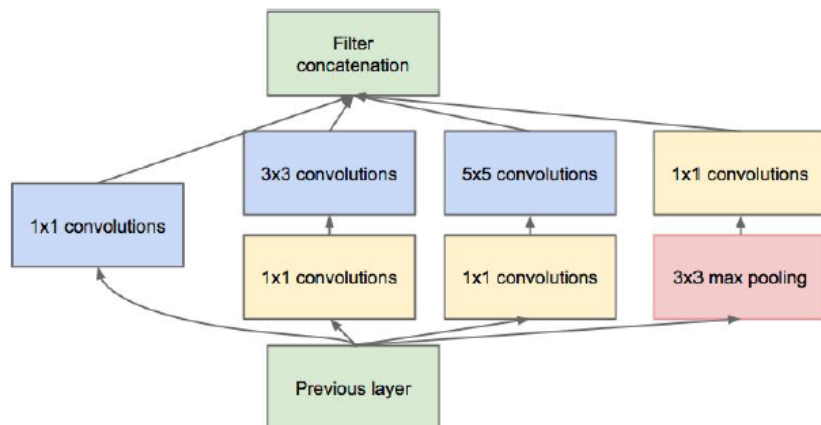
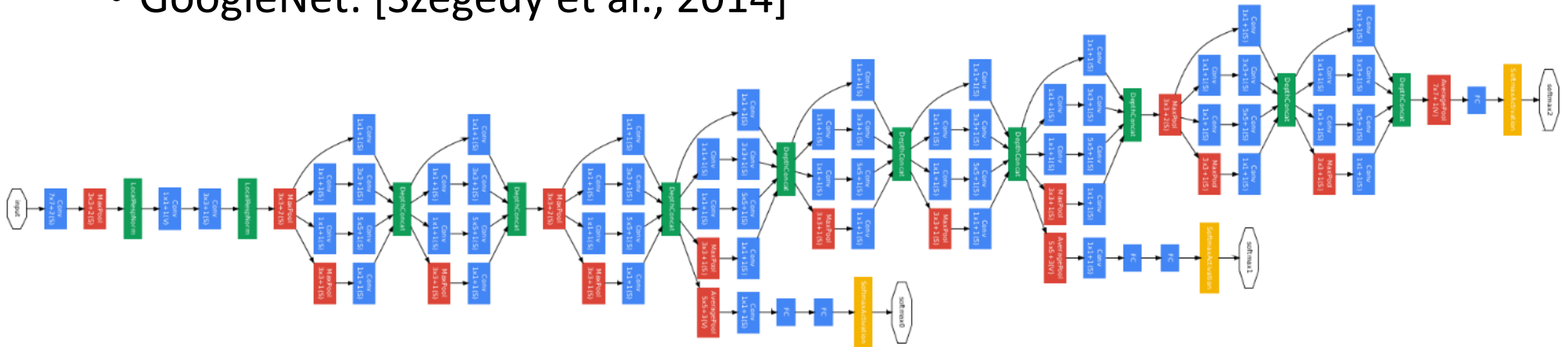
- VGGNet: [Simonyan and Zisserman, 2014]
- Only 3x3 CONV stride 1, pad 1 and 2x2 MAX POOL stride 2
- 11.2% top 5 error in ILSVRC 2013
 - ImageNet Large Scale Visual Recognition Challenge 2013
- 7.3% top 5 error

Best model

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 LRN	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 conv1-256	conv3-256 conv3-256 conv3-256	conv3-256 conv3-256 conv3-256 conv3-256
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512 conv3-512
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512 conv3-512
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

GoogleNet

- GoogleNet: [Szegedy et al., 2014]



ILSVRC 2014 winner (6.7% top 5 error)

GoogleNet (cont.)

- GoogleNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Fun features:

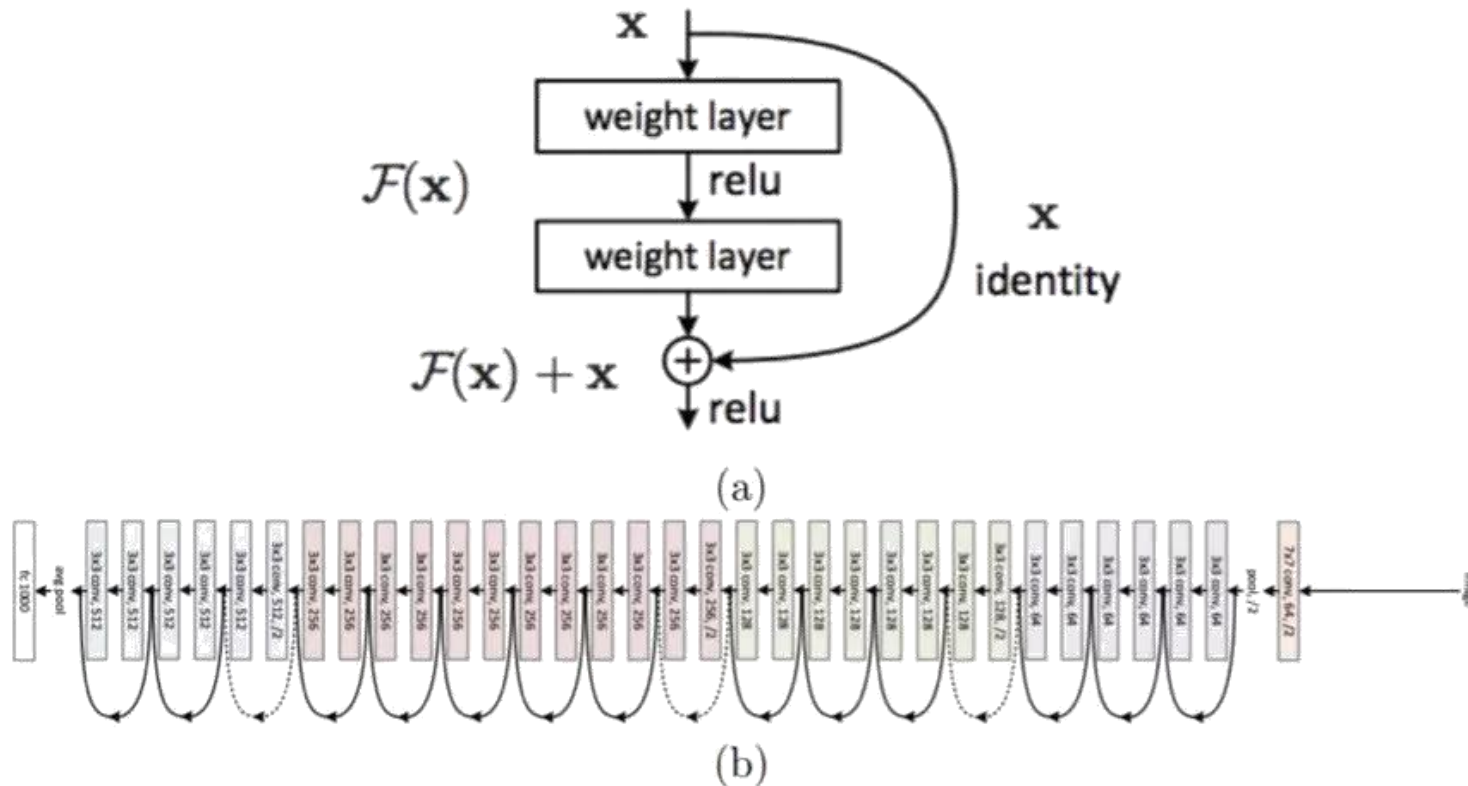
- Only 5 million parameters! (Removes FC layers completely)

Compared to AlexNet:

- 12X less parameters
- 2x running speed
- Error rate 6.67% (vs. 16.4%)

ResNet

- ResNet [He et al., 2015]
 - Residual networks
 - Solves the problem of drifting by adding the original input to later layers



ResNet (cont.)

- ResNet: ILSVRC 2015 winner (3.6% top 5 error)

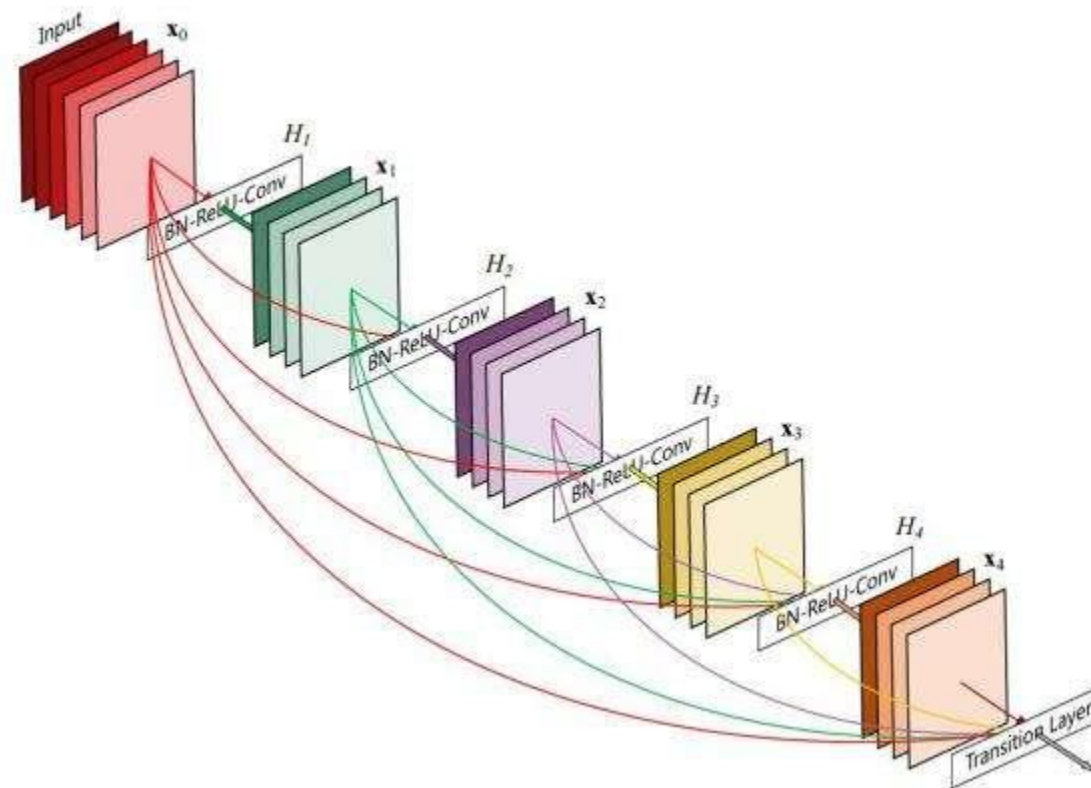
MSRA @ ILSVRC & COCO 2015 Competitions

- **1st places in all five main tracks**

- ImageNet Classification: “*Ultra-deep*” (quote Yann) **152-layer** nets
- ImageNet Detection: **16%** better than 2nd
- ImageNet Localization: **27%** better than 2nd
- COCO Detection: **11%** better than 2nd
- COCO Segmentation: **12%** better than 2nd

DenseNet

- DenseNet

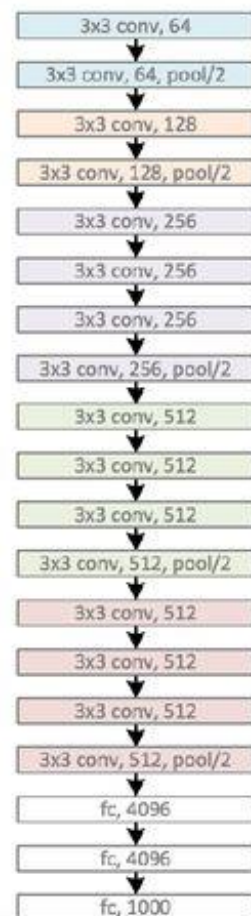


Revolution of Depth

AlexNet, 8 layers
(ILSVRC 2012)



VGG, 19 layers
(ILSVRC 2014)

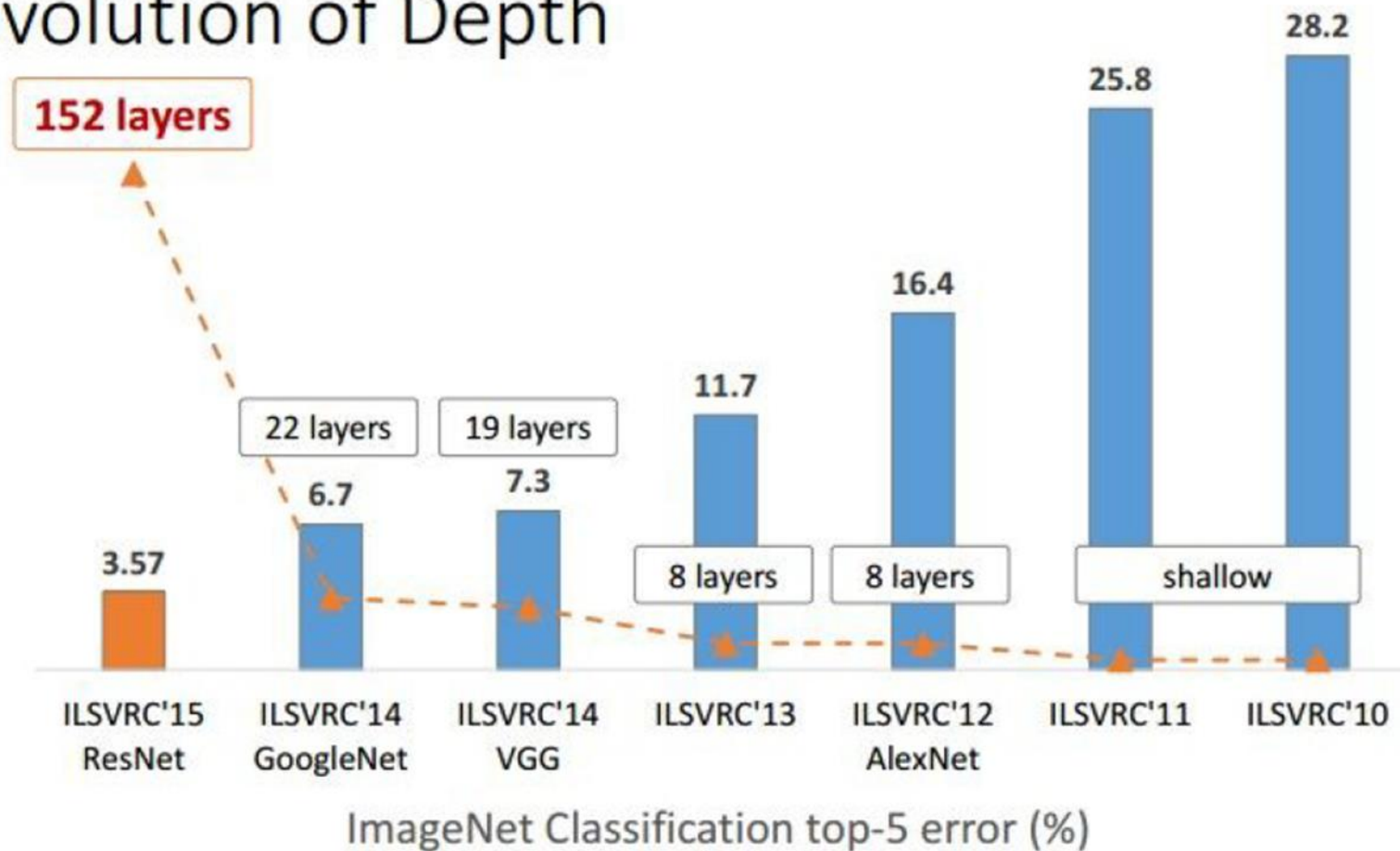


GoogleNet, 22 layers
(ILSVRC 2014)



Revolution of depth

Revolution of Depth



Architecture comparison

