

# Lecture 7: Convolutional Neural Networks

Shuai Li

John Hopcroft Center, Shanghai Jiao Tong University

<https://shuaili8.github.io>

<https://shuaili8.github.io/Teaching/VE445/index.html>



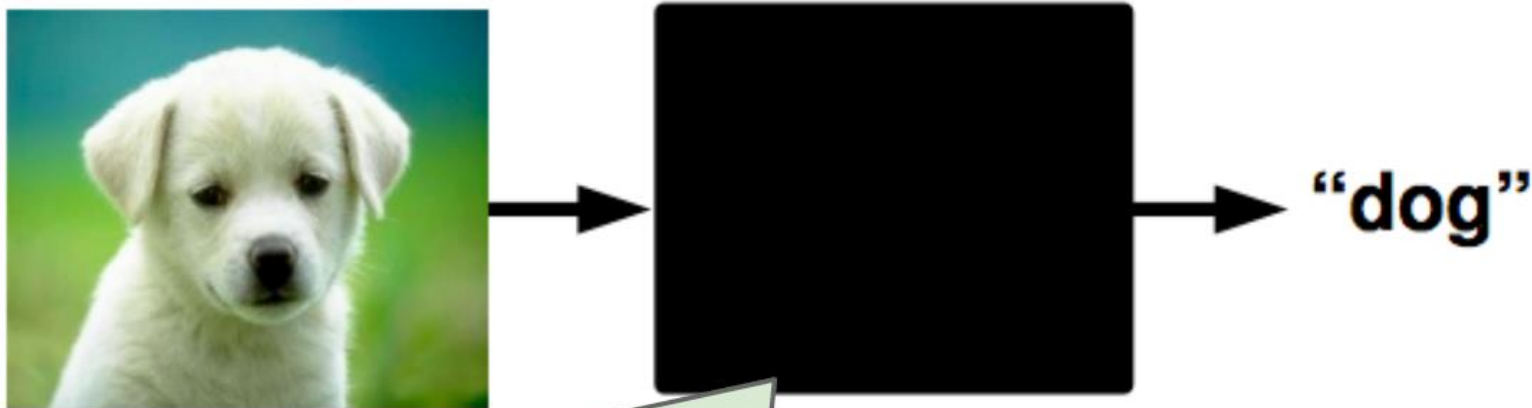
# Outline

- Motivation
- Convolution in math and NN
- Usage and parameters
- Training techniques
- Famous neural networks

# Motivation

# Previous pipeline of pattern recognition

- The black box in a traditional pattern recognition problem



Preprocessing

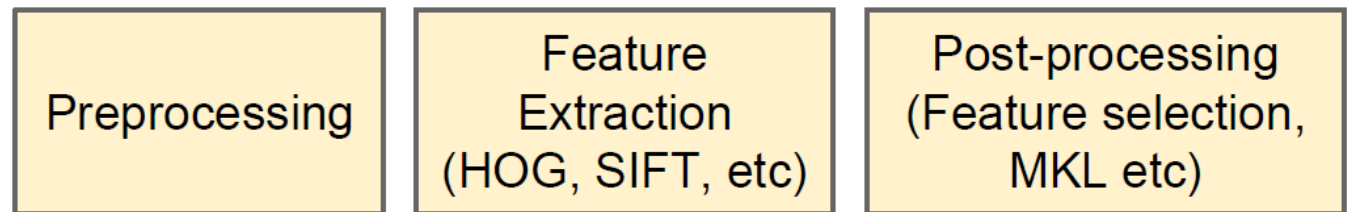
Feature  
Extraction  
(HOG, SIFT, etc)

Post-processing  
(Feature selection,  
MKL etc)

Classifier  
(SVM,  
boosting, etc)

# Hand engineered features

- Feature is of critical importance in machine learning, and there are many things to consider when design the features manually:
  - How to design a feature?
  - What is the best feature?
  - Time and money cost in feature engineering.
- Question: Can feature be learned automatically?

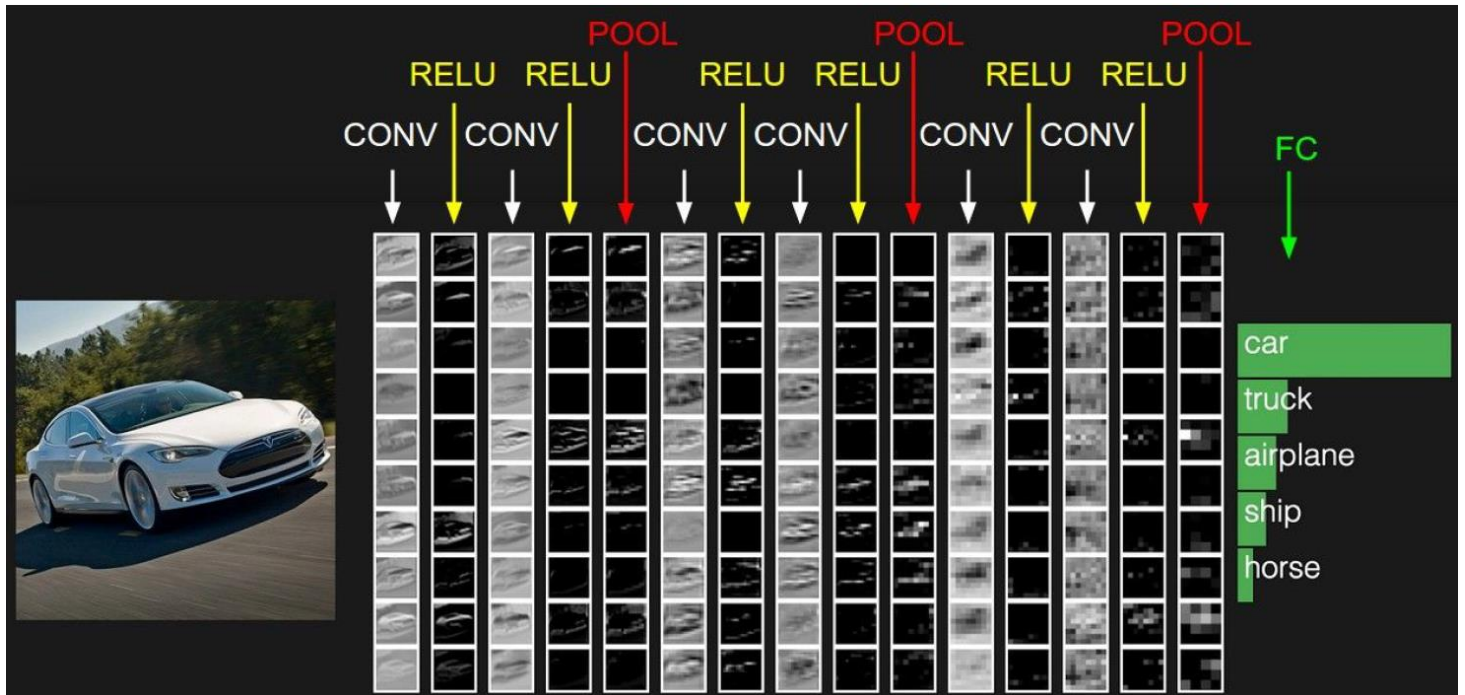


# Objective

- Learn features and classifier at the same time
- Learn an end-to-end recognition system
  - A non-linear map that takes raw pixels directly to labels

# Convolution neural networks

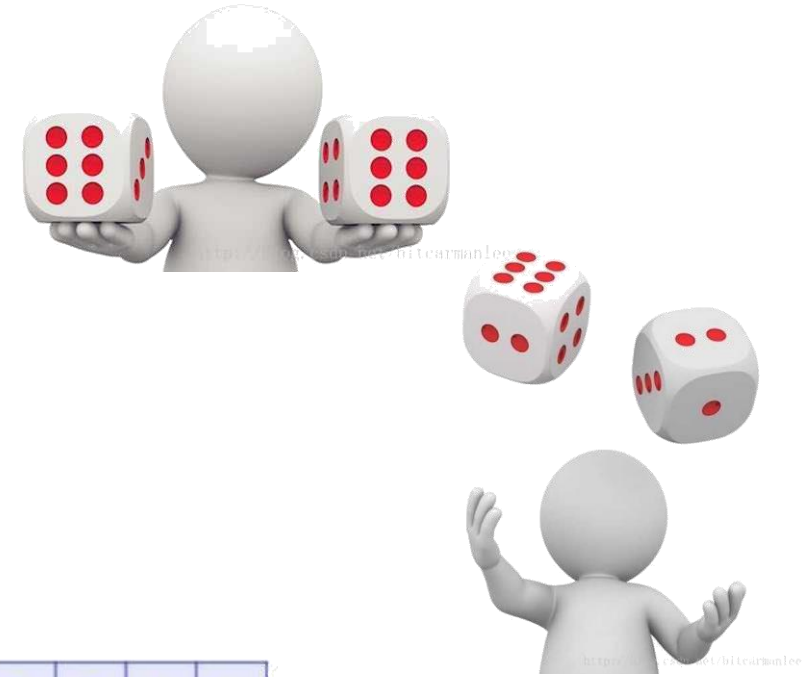
- Is an answer of an end-to-end recognition system
- Contains the following layers with flexible order and repetitions
  - Convolution layer
  - Activation layer (ReLU)
  - Pooling layer
- Example of CNN:



# Convolution in Math and NN

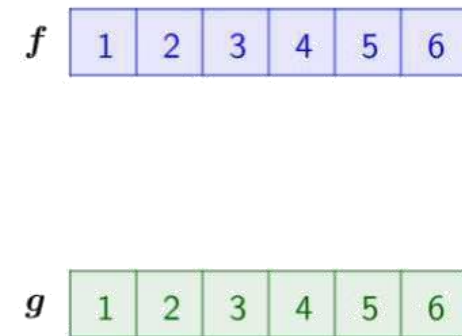
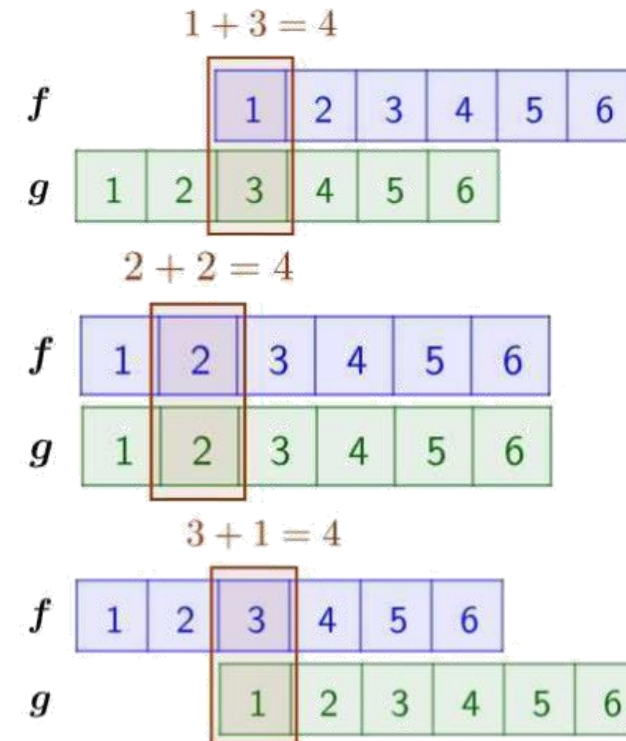


# Example of convolution in math

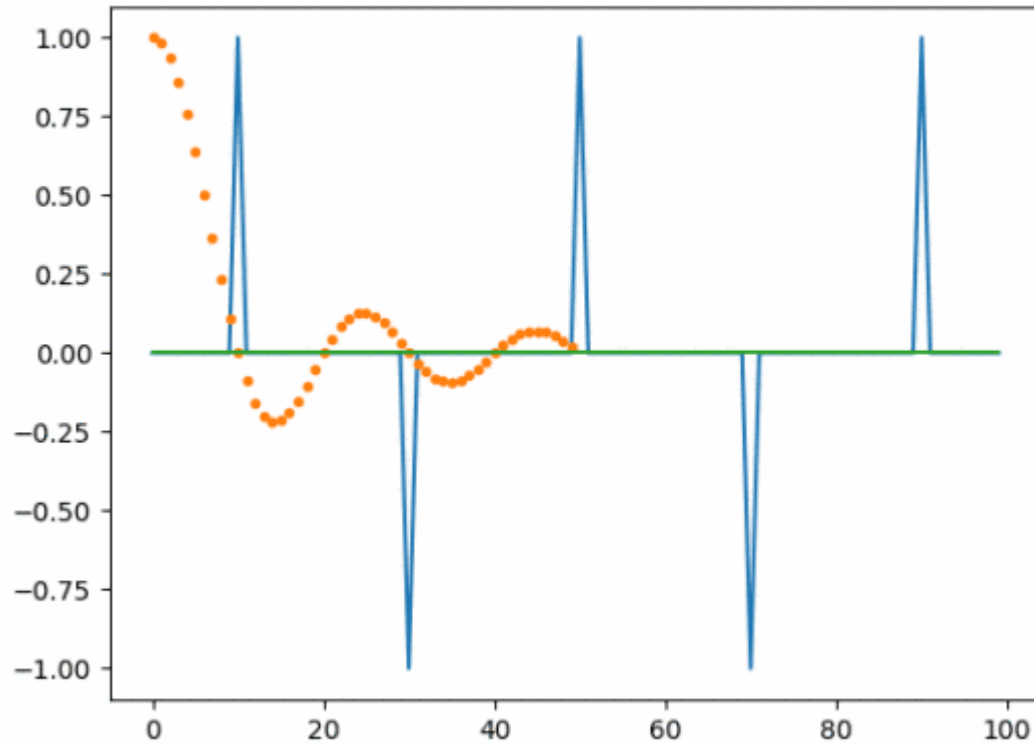


- Suppose we have two dice with probability over  $1, 2, \dots, 6$  are  $f$  and  $g$  respectively. We roll two dice. Then what is the probability that the sum of the two dice is 4?

- $f * g(n) = \sum_i f(i)g(n - i)$



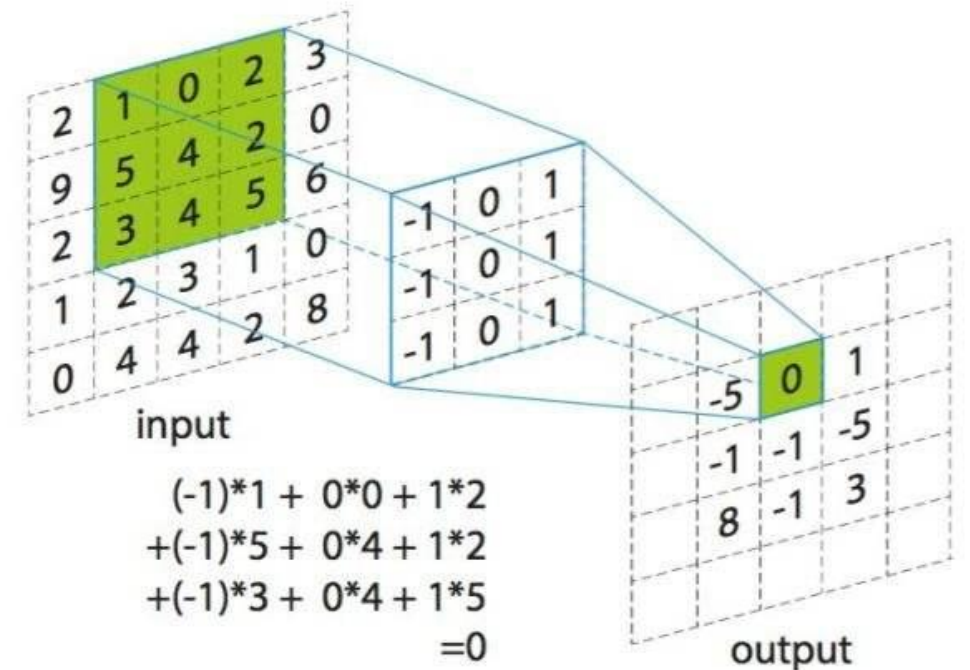
# Convolution in math – continuous case



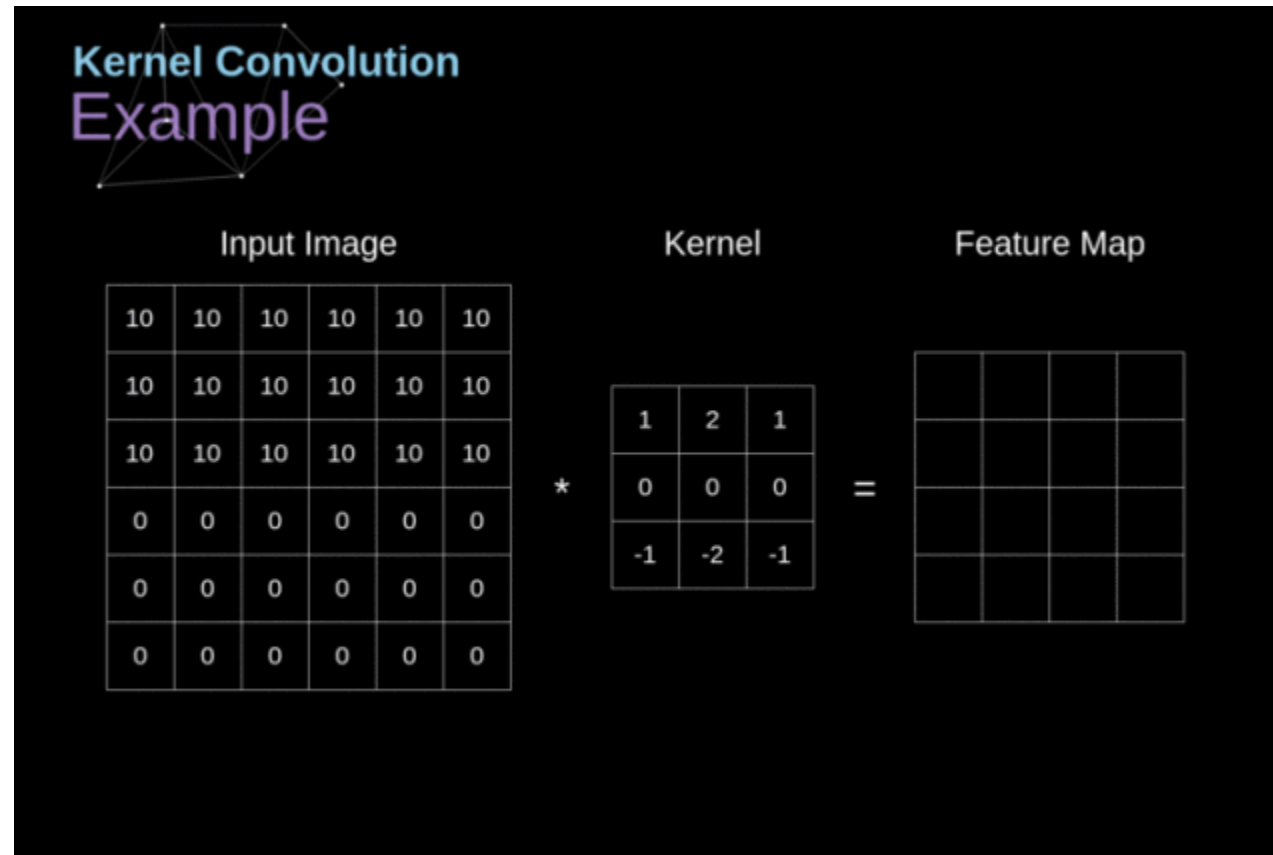
$$f * g(x) = \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau$$

# Convolution in neural networks

- Given an input matrix (e.g. an image)
- Use a small matrix (called **filter** or **kernel**) to screening the input at every position of the input matrix
- Put the convolution results at corresponding positions

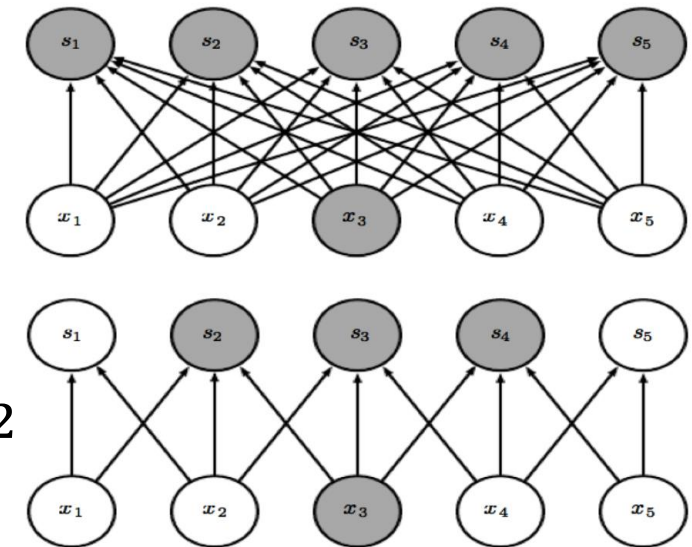


# An animation example



# Advantage – sparse connections

- Less computing burden
- In fully connected layer (top), every  $s$  is linked to every input  $x$ , so there are  $5 \times 5 = 25$  connecting edges
- In the convolution layer (bottom) with filter width 3, e.g.  $s_2$  is a weighted sum of  $x_1, x_2, x_3$ , so there is no weight connecting  $s_2$  and  $x_4, x_5$ . In this example, there are 13 connecting edges



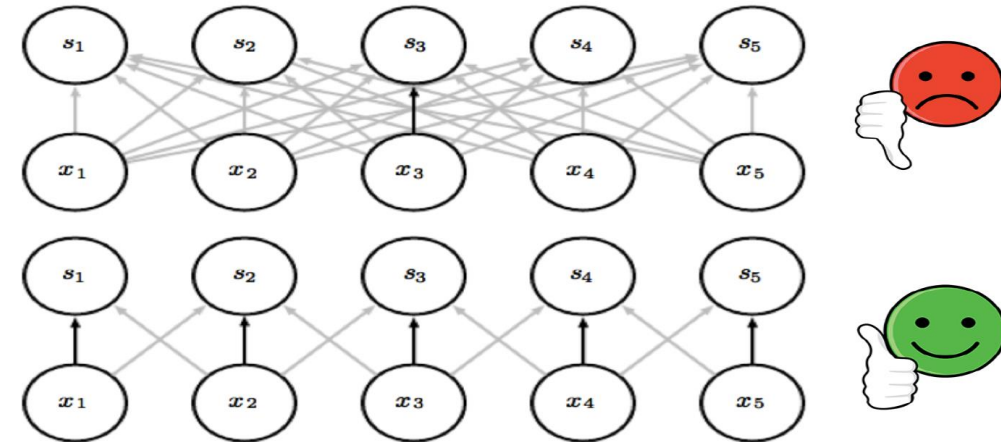
# Advantage – weight sharing

- When moving the filter, we don't change the weights inside the filter and these weights are shared at different connecting edges
- In fully connected layer (top), there are **25** connecting edges. The weights on different edges are different parameters.
- In convolution layer (bottom), there are 13 connecting edges. But since

$$s_2 = w_1x_1 + w_2x_2 + w_3x_3$$

$$s_3 = w_1x_2 + w_2x_3 + w_3x_4$$

the number of different weights is even smaller. In this case, there are **3** different weights (**just the size of the filter!**). E.g. the weights on the black arrows are the same.

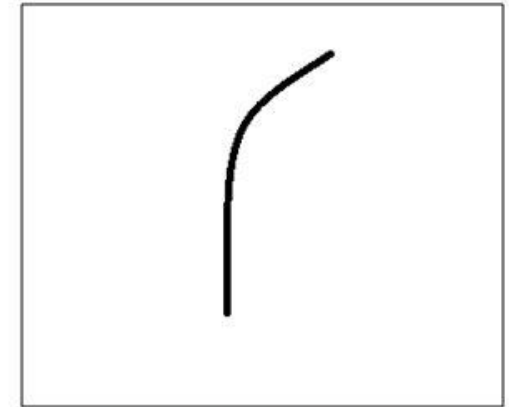


# Interpretation of convolution

- Convolution can be used to find an area with **particular patterns!**
- Example:
  - The filter in the left represents the edge in the right, which is the back of a mouse

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

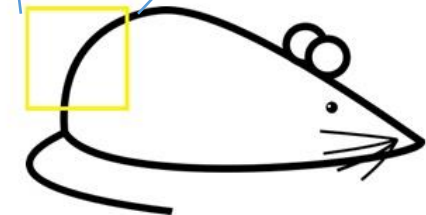
Pixel representation of filter



Visualization of a curve detector filter



Original image



Visualization of the filter on the image

# Interpretation of convolution (cont.)

- When the filter moves to the back of the mouse, the convolution operation will generate a very large value



Visualization of the receptive field

0	0	0	0	0	0	30
0	0	0	0	50	50	50
0	0	0	20	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0

Pixel representation of the receptive field

\*

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

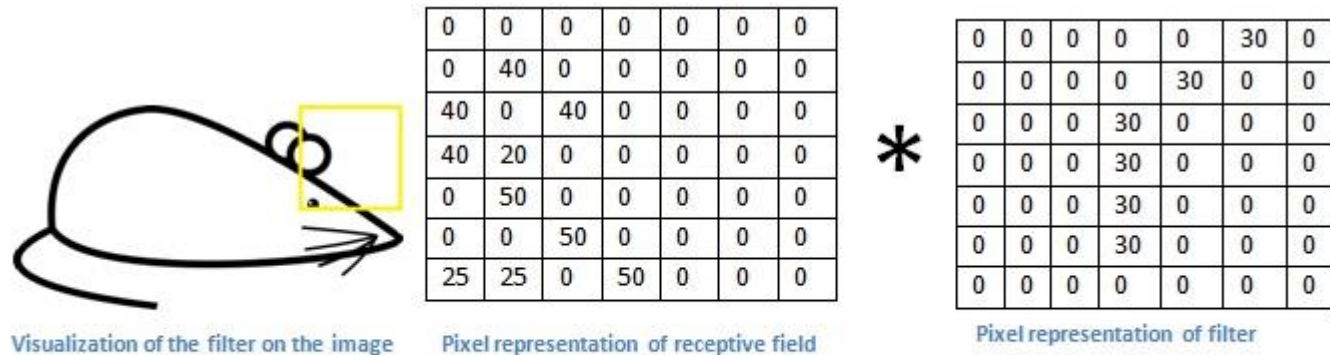
Pixel representation of filter

Multiplication and Summation =  $(50*30)+(50*30)+(50*30)+(20*30)+(50*30) = 6600$  (A large number!)



# Interpretation of convolution (cont.)

- When the filter moves to other positions, it will generate small values



Multiplication and Summation = 0

# Visualization

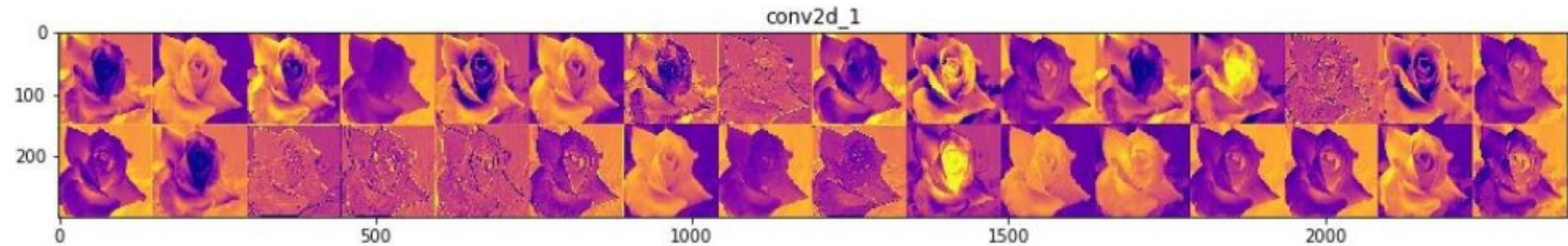
- Train the [InceptionV3](#) model (by Google) on the ImageNet dataset
- Then test it on a flower image from the test dataset
  - Example input image:



- Then let's look at the outputs of different convolutional layers

# Visualization (cont.)

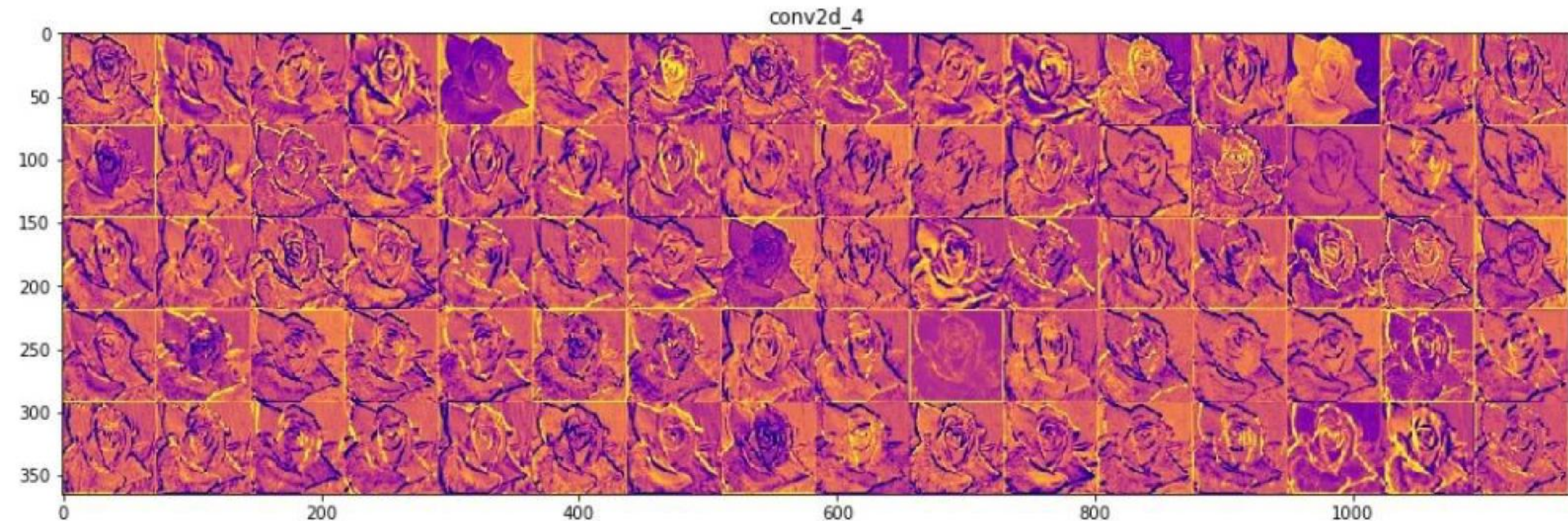
- The outputs of the filters in the first layer





# Visualization (cont.)

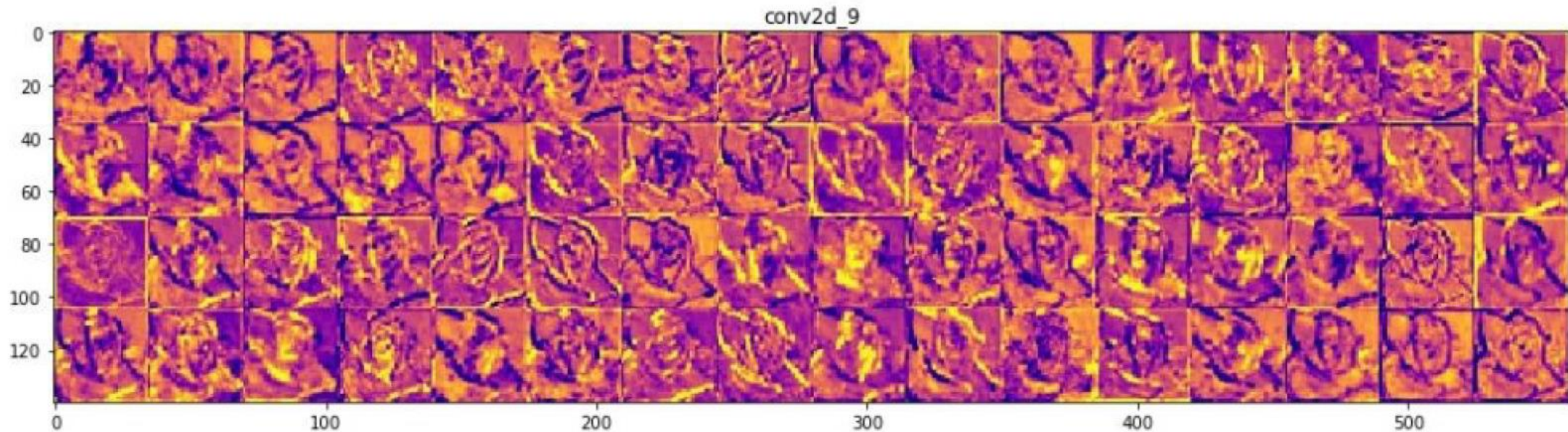
- The outputs of the filters in the fourth layer





# Visualization (cont.)

- The outputs of the filters in the ninth layer



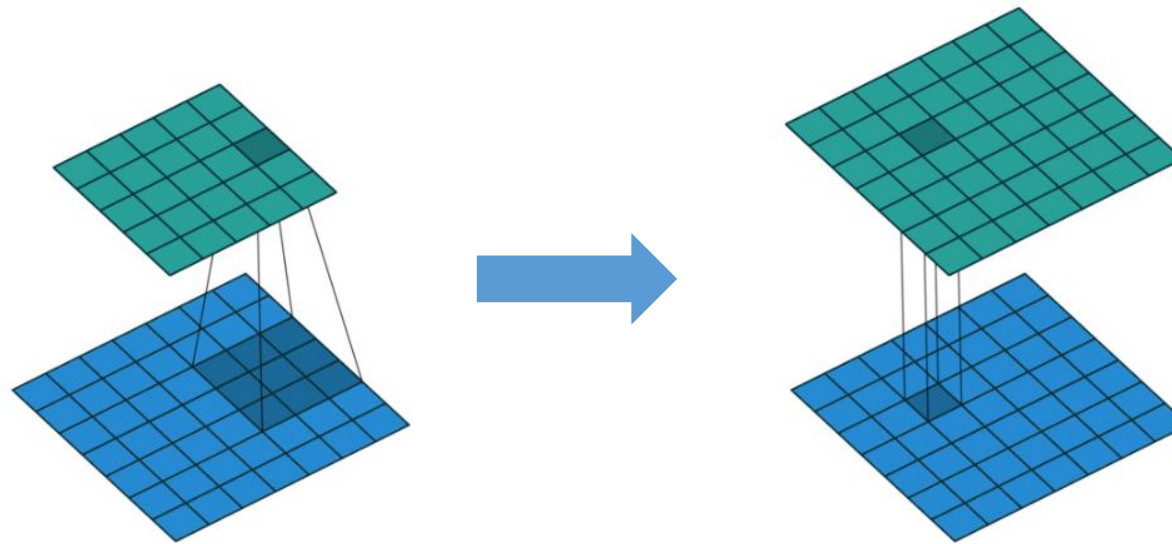
# Visualization (cont.)

- Summary
  - The filters in the deeper layers characterize more abstract patterns
  - Layers that are deeper in the network visualize more training data specific features
  - While the earlier layers tend to visualize general patterns like edges, texture, background

# Usage and parameters

# $1 \times 1$ convolution

- Here we introduce a special filter, which as a size of  $1 \times 1$



Convolution with large kernel

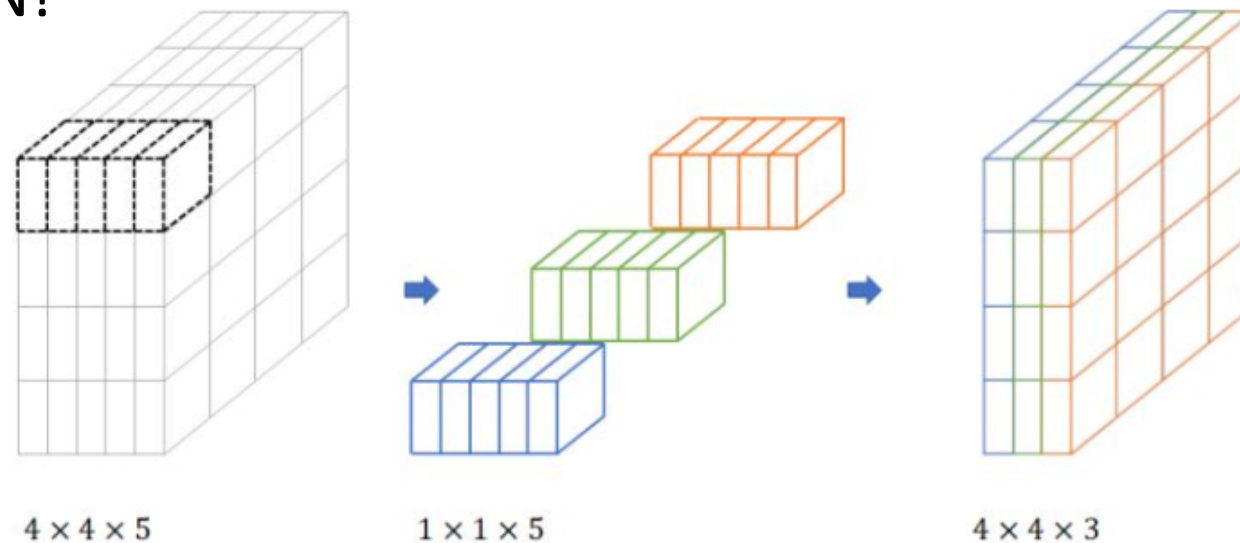
Convolution with  $1 \times 1$  kernel

- The  $1 \times 1$  convolution cannot detect edges with any shape, so is it really useful? Or is it redundant?



# $1 \times 1$ convolution (cont.)

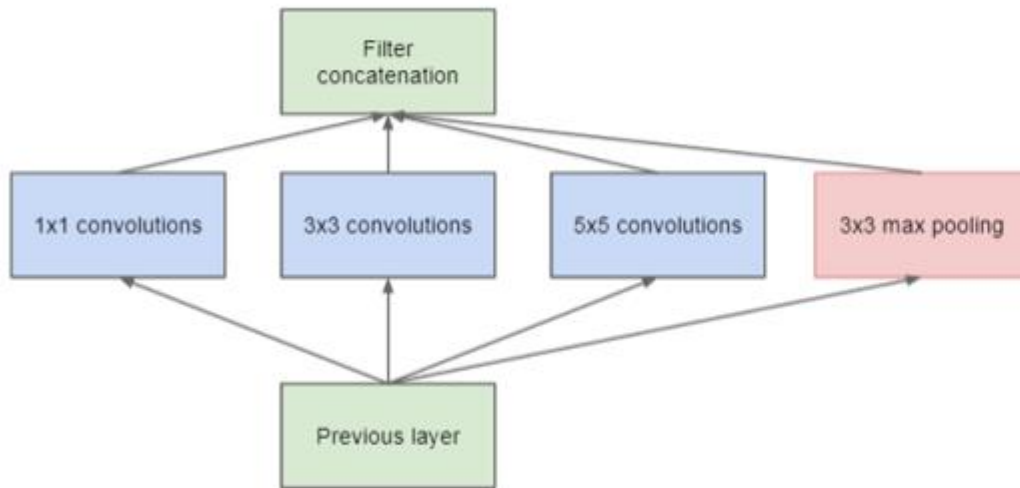
- The  $1 \times 1$  convolution is very useful as it **can reduce the computation complexity** in CNN!



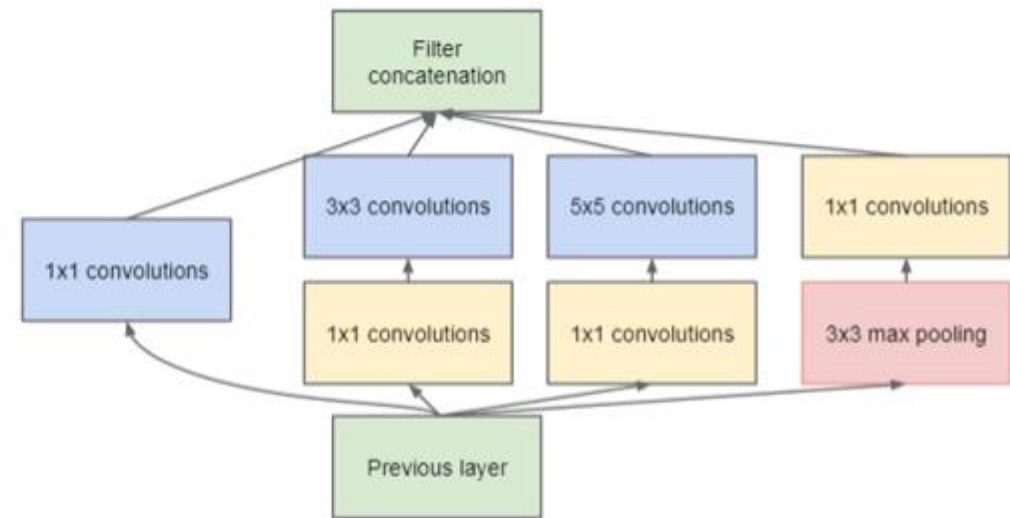
- Help reduce the number of channels
- In the example above, the size of the input data is reduced from  $4 \times 4 \times 5$  to  $4 \times 4 \times 3$
- Usually we assume the depth of the filter is the same with depth of data

# $1 \times 1$ convolution (cont.)

- $1 \times 1$  convolution filter is usually followed by  $3 \times 3$  or other bigger filters. In this way, the computational complexity is greatly reduced
- This architecture is used in Google's inception model

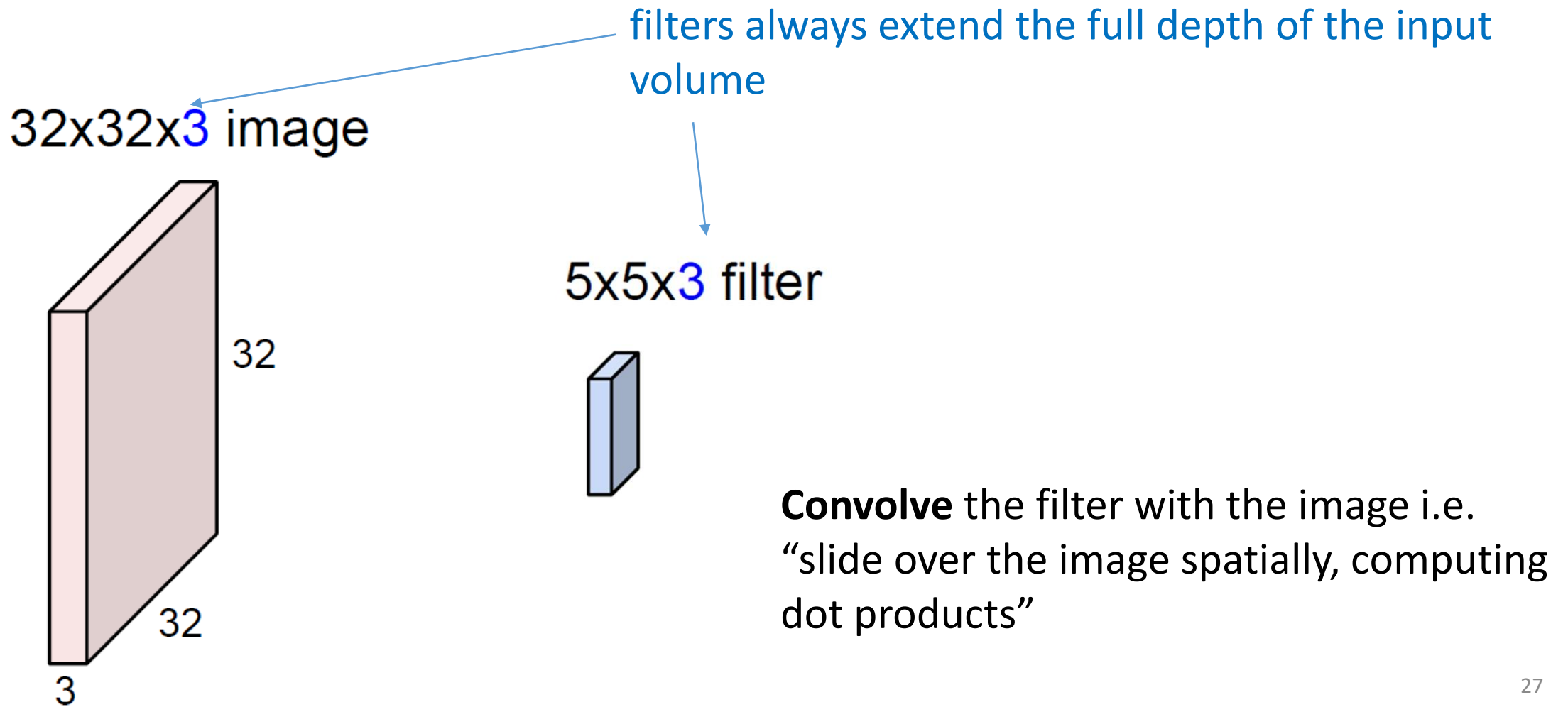


(a) Inception module, naïve version



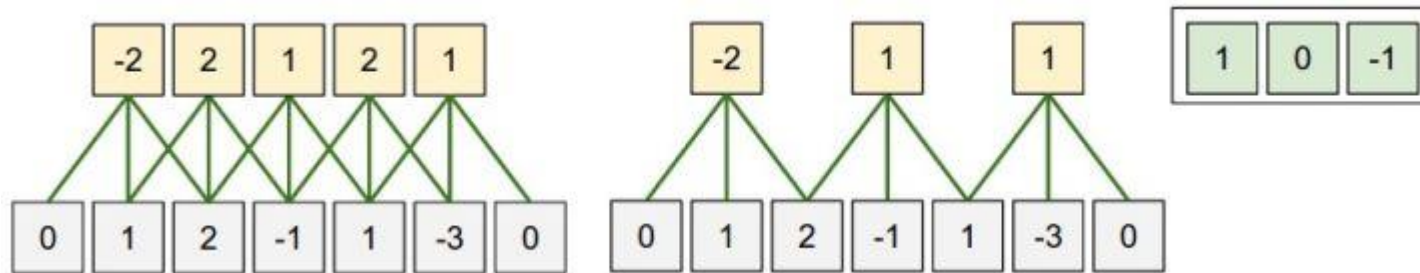
(b) Inception module with dimension reductions

# Filter depth



# Stride

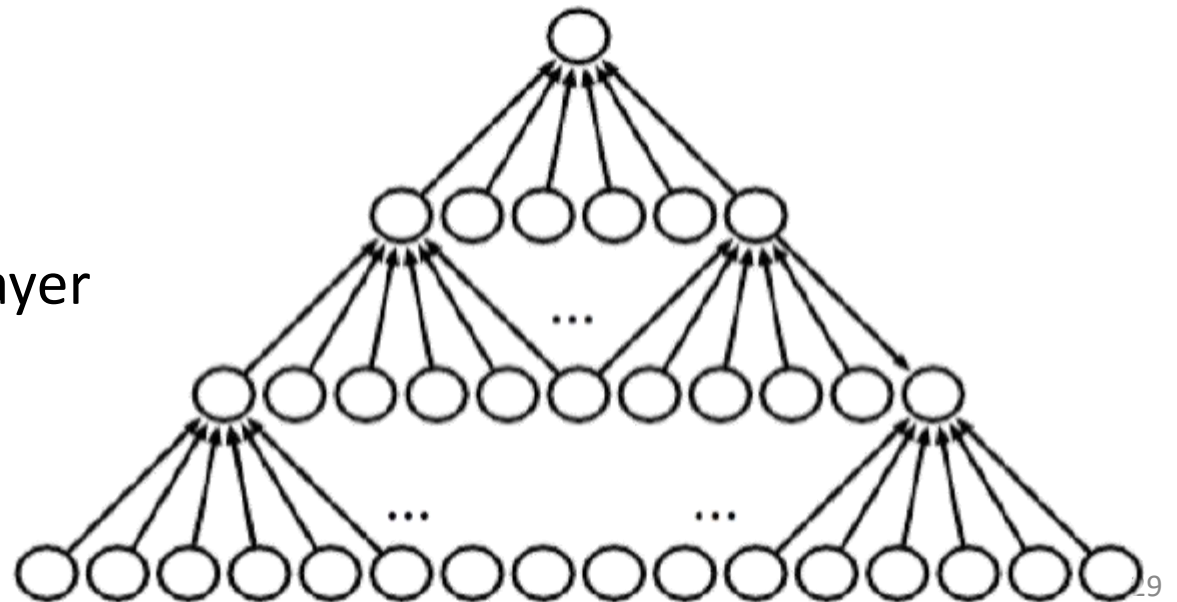
- The distance that the filter is moved in each step
- Examples of stride=1 and stride=2



# Padding

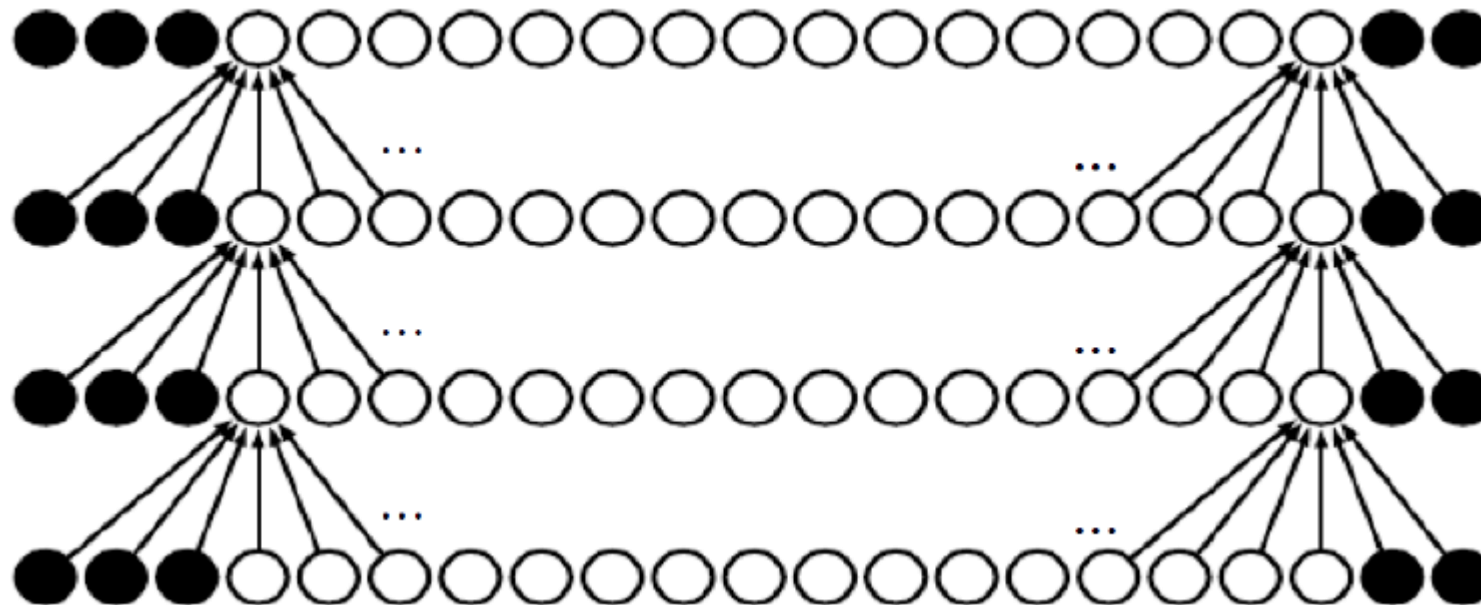
- A solution to the problem of data shrinking
- Data shrinking: as convolution can only happen within the border of the input data, the size of the data will become smaller as the network becomes deeper

- 1D Example:
  - suppose the filter width is 6,  
the data will shrink 5 pixel each layer



# Padding (cont.)

- Add numbers (usually zero, called **zero padding**) around the input data to make sure that the size of the output data is the same as that of the input data
- Left padding = 3, right padding = 2
- Usually  $\text{left padding} + \text{right padding} = \text{filter width} - 1$



# Example

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?

## Example (cont.)

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?
- **7x7 output!**



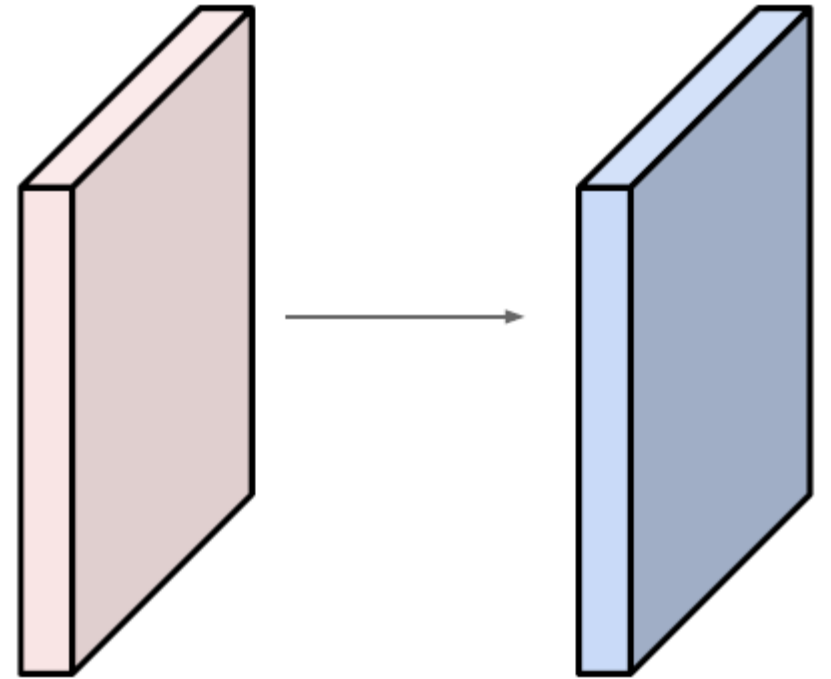
# Example (cont.)

0	0	0	0	0	0			
0								
0								
0								
0								

- Input 7x7 (white area)
- **3x3** filter, applied with **stride 1**
- **pad with 1 pixel** border (dark area)
- => what is the size of the output?
- **7x7 output!**
- In general, convolution layers with stride 1, filters of size  $F \times F$ , and zero-padding with  $(F-1)/2$ . (will **preserve** size spatially)
- e.g.  $F = 3 \Rightarrow$  zero pad with 1  
 $F = 5 \Rightarrow$  zero pad with 2  
 $F = 7 \Rightarrow$  zero pad with 3

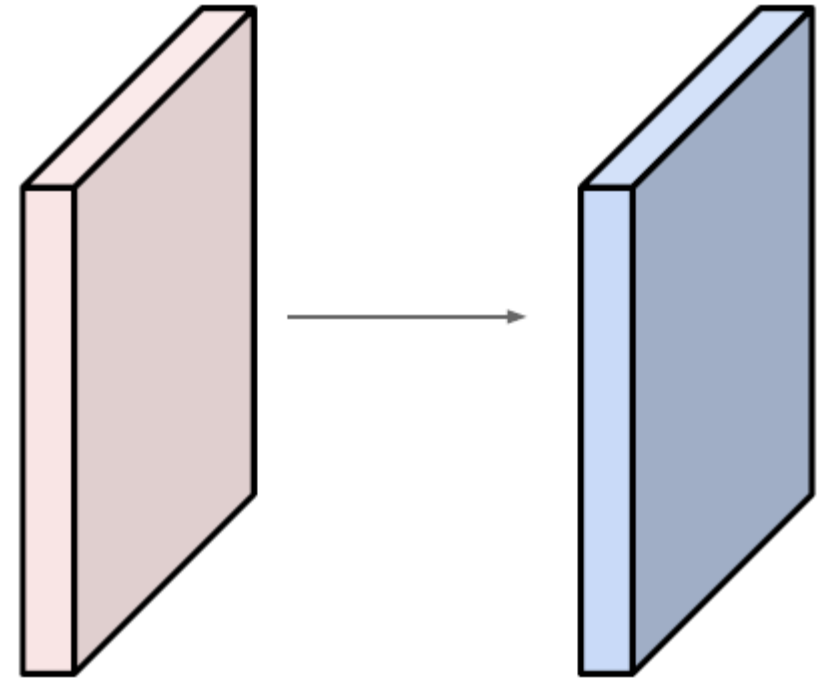
## Example 2

- Input volume:  $32 \times 32 \times 3$
- 10  $5 \times 5$  filters with stride 1, pad 2
- Output volume size: ?



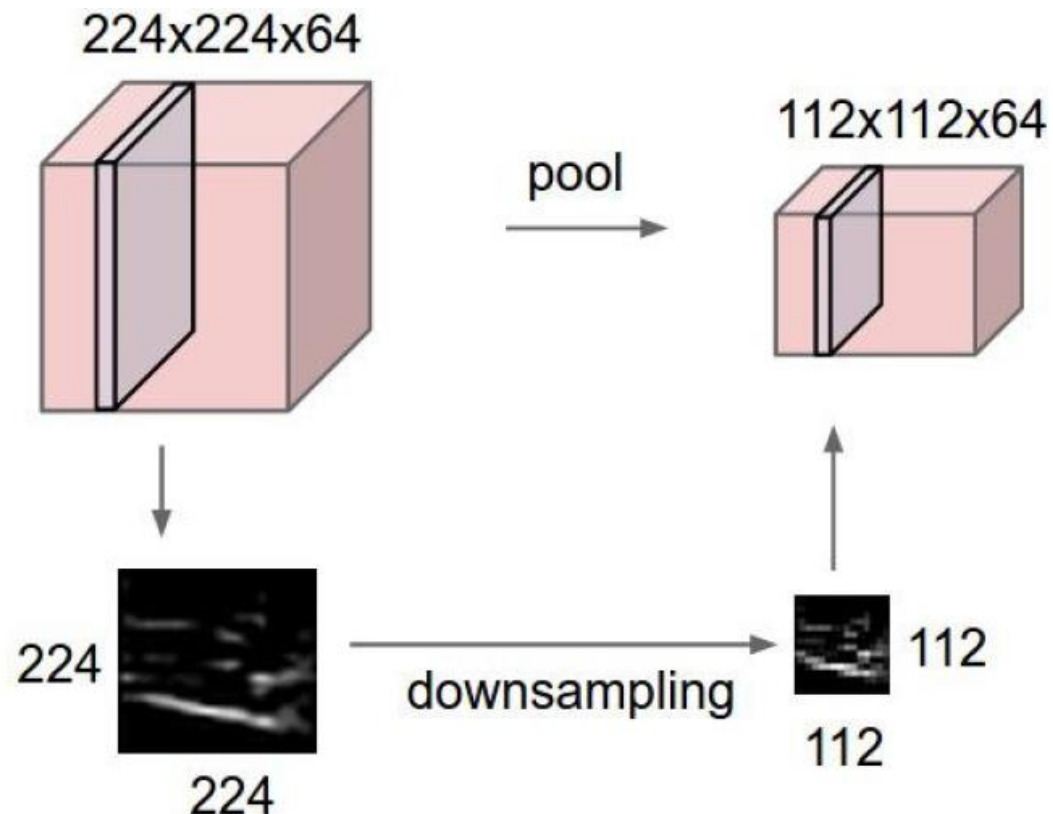
## Example 2 (cont.)

- Input volume:  $32 \times 32 \times 3$
- 10  $5 \times 5$  filters with stride 1, pad 2
- Output volume size: ?
- Filter size 5, pad =  $(F - 1)/2$ ,  
so keep the size  $32 \times 32$  spatially
- 10  $5 \times 5$  filters means 10  $5 \times 5 \times 3$  filters
- So  $32 \times 32 \times 10$



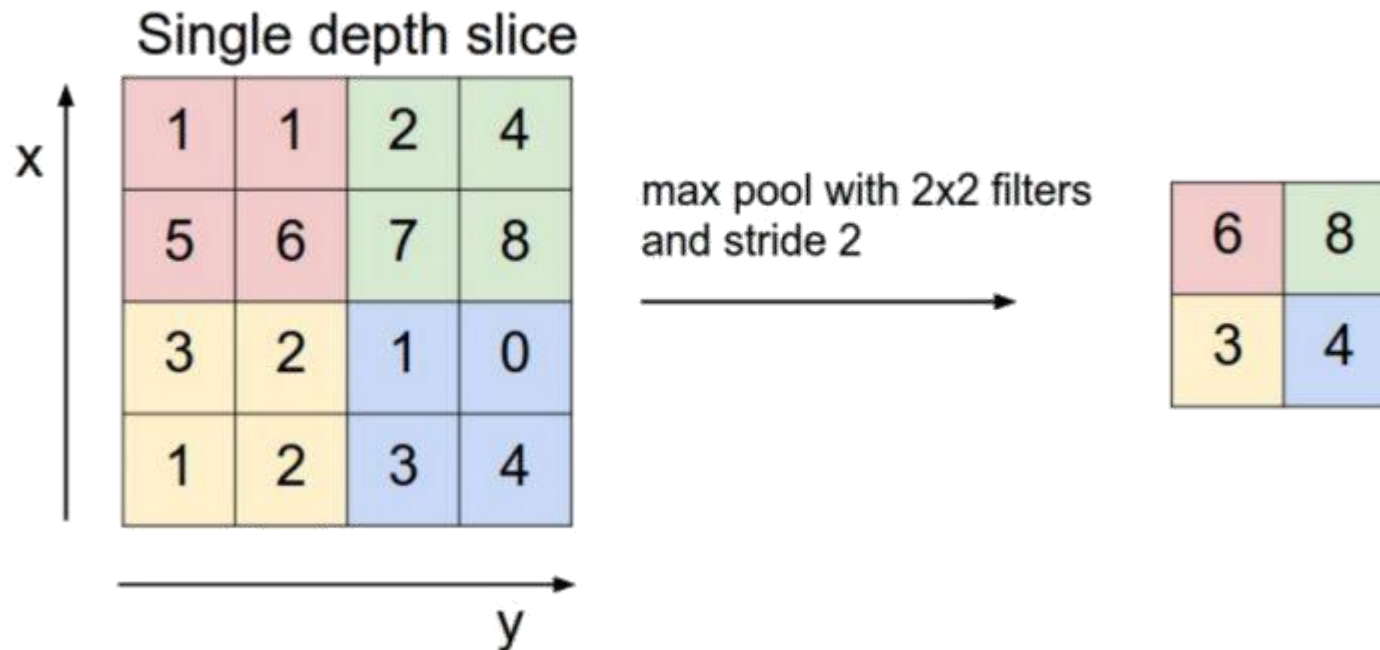
# Pooling layer

- Make the representations denser and more manageable
- Operate over each activation map independently:



# Example of pooling layer

- Pooling of size  $2 \times 2$  with stride 2



- Most common pooling operations are **max** and **average**

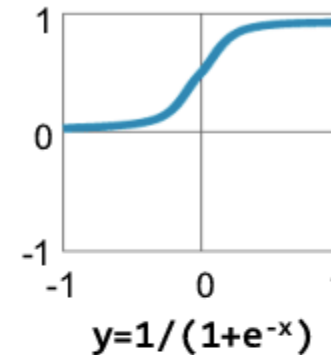
# Activation functions (Review)

- Sigmoid:  $\sigma(z) = \frac{1}{1+e^{-z}}$
- Tanh:  $\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$
- ReLU (Rectified Linear Unit):  
 $\text{ReLU}(z) = \max(0, z)$

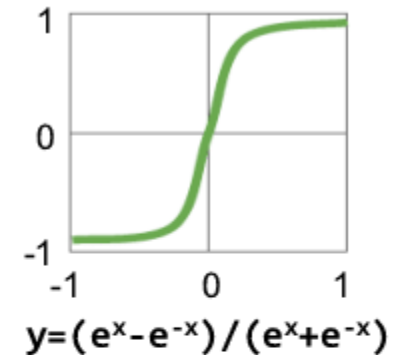
**Traditional  
Non-Linear  
Activation  
Functions**

Most popular in fully  
connected neural network

**Sigmoid**

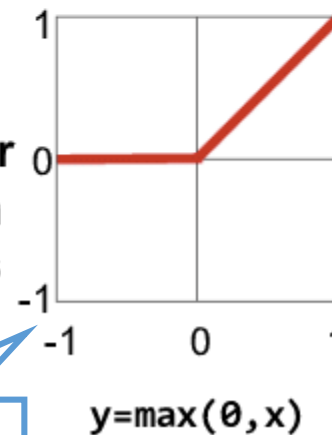


**Hyperbolic Tangent**

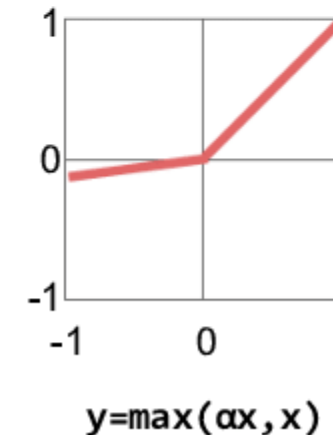


**Modern  
Non-Linear  
Activation  
Functions**

**Rectified Linear Unit  
(ReLU)**

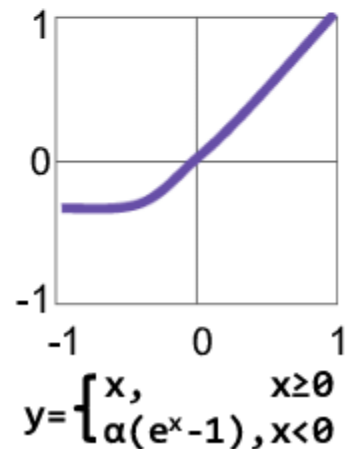


**Leaky ReLU**



$\alpha = \text{small const. (e.g. 0.1)}$

**Exponential LU**

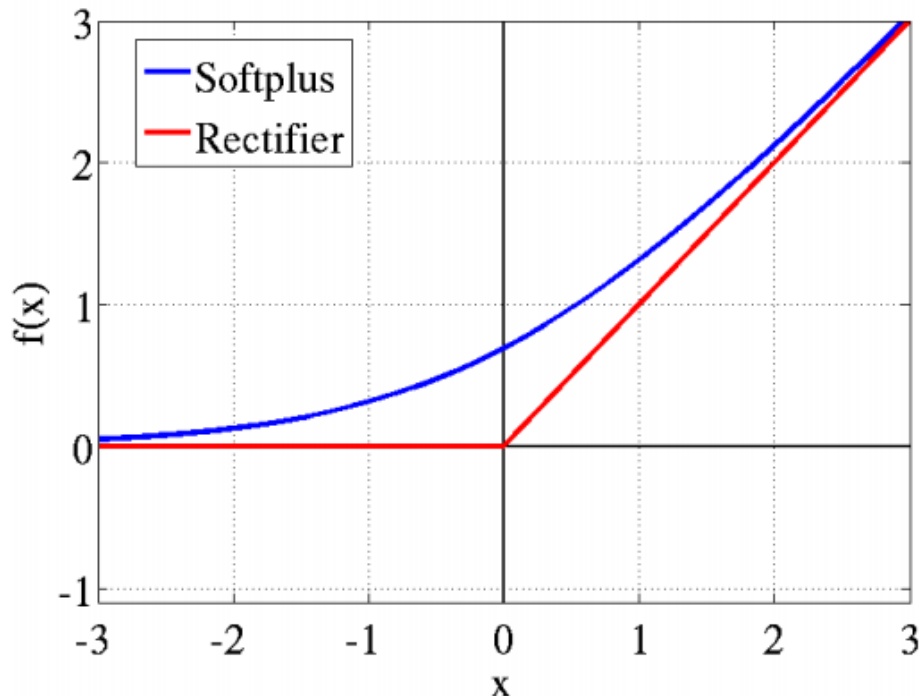


Most popular in  
deep learning

# ReLU activation function

- ReLU (Rectified linear unit) function

$$\text{ReLU}(z) = \max(0, z)$$



- Its derivative

$$\text{ReLU}'(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$$

- ReLU can be approximated by softplus function

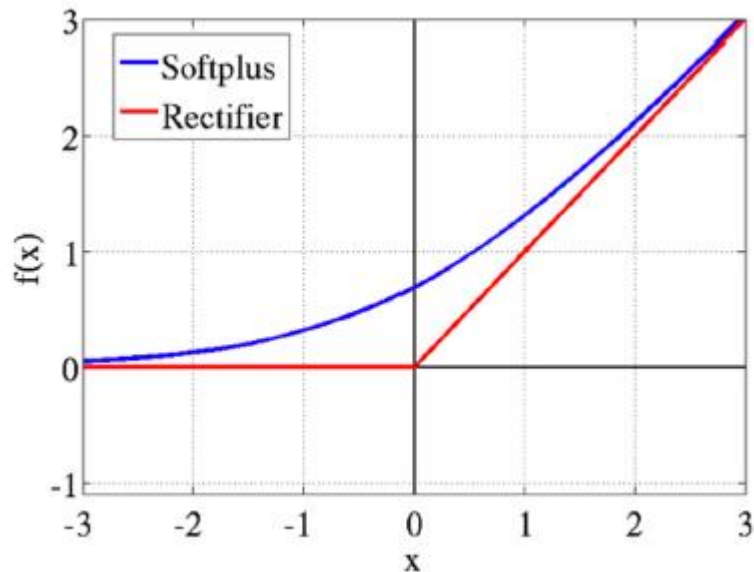
$$f_{\text{Softplus}}(x) = \log(1 + e^x)$$

- ReLU's gradient doesn't vanish as x increases
- Speed up training of neural networks
  - Since the gradient computation is very simple
  - The computational step is simple, no exponentials, no multiplication or division operations (compared to others)
- The gradient on positive portion is larger than sigmoid or tanh functions
  - Update more rapidly
  - The left "dead neuron" part can be ameliorated by Leaky ReLU

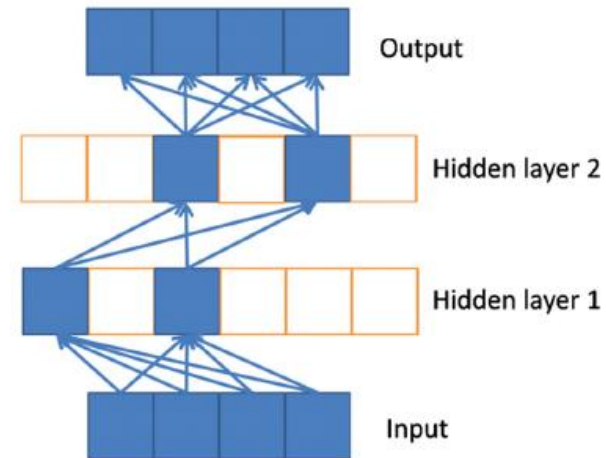
# ReLU activation function (cont.)

- ReLU function

$$\text{ReLU}(z) = \max(0, z)$$



- The only non-linearity comes from the path selection with individual neurons being active or not
- It allows sparse representations:
  - for a given input only a subset of neurons are active

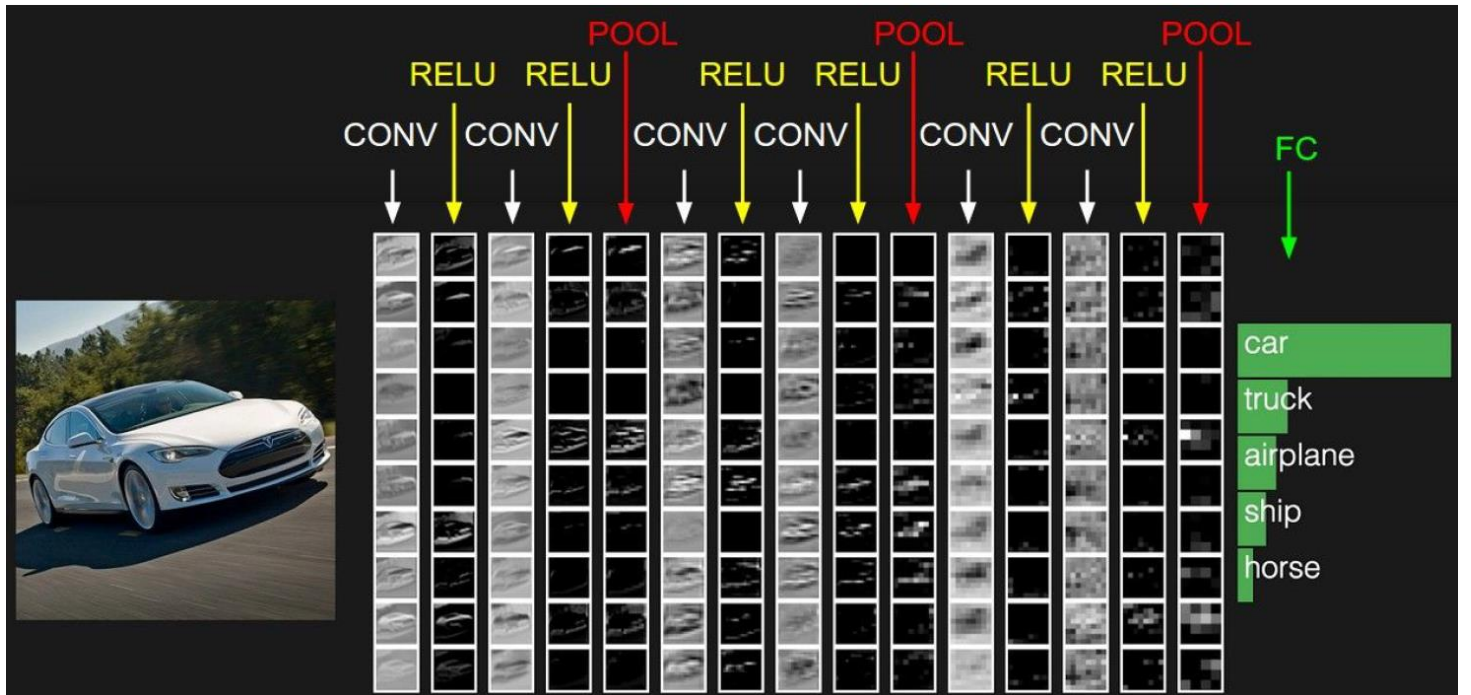


Sparse propagation of activations and gradients



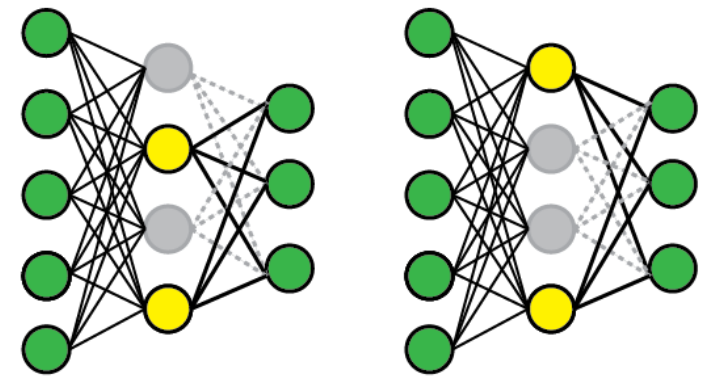
# A typical CNN structure

- Convolution/Activation (ReLU)/Pooling layers appear in flexible order and flexible repetitions

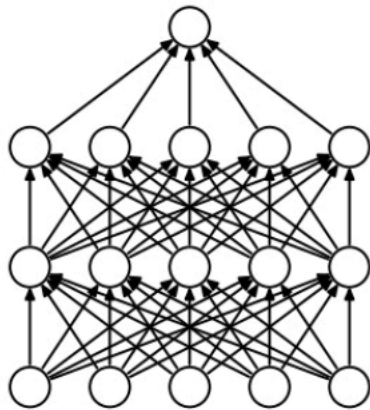


# Training Techniques

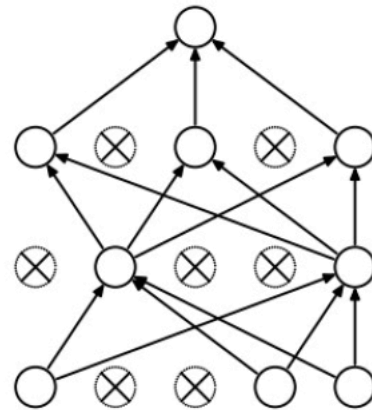
# Dropout



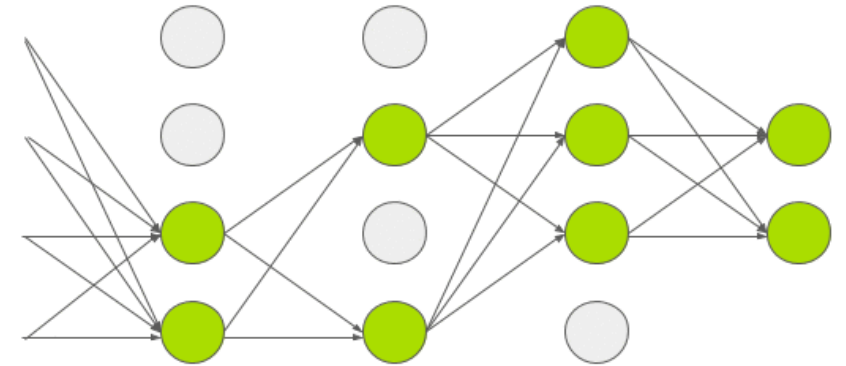
- Dropout randomly ‘drops’ units from a layer on each **training** step, creating ‘sub-architectures’ within the model
- It can be viewed as a type of sampling a small network within a large network
- Prevent neural networks from overfitting



(a) Standard Neural Net

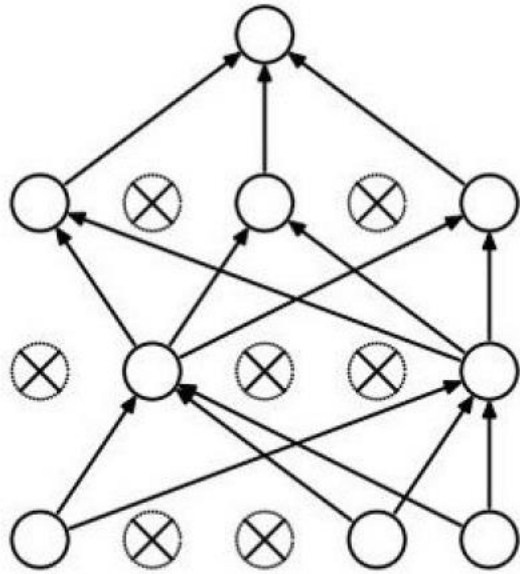


(b) After applying dropout.



# Dropout (cont.)

- Forces the network to have a redundant representation
- Increase robustness in prediction power



# Weights initialization

- If the weights in a network start too small,
  - then the signal shrinks as it passes through each layer until it's too tiny to be useful
- If the weights in a network start too large,
  - then the signal grows as it passes through each layer until it's too massive to be useful

# Weights initialization (cont.)

- All zero initialization
- Small random numbers
- Draw weights from a Gaussian distribution
  - with the standard deviation of  $\sqrt{\frac{2}{n}}$ 
    - $n$  is the number of inputs to the ending neuron



# Batch normalization

- Batch training:
  - Given a set of data, each time a small portion of data are put into the model for training
- Extreme example
  - Suppose we are going to learn some pattern of people, and the input data are people's weights and heights
  - Unluckily, women and men are divided into two batches when we randomly split the data
  - As the weights and heights of women are very different from these of men, the neural network have to make **huge changes** to the weight when we switch the batch during training, which will cause slow convergence or even divergence

# Batch normalization (cont.)

- The problem in the example is called *Internal Covariate Shift*
- The solution to Internal Covariate Shift is batch normalization
- Suppose  $Z_j^{(i)}$  is the  $i^{\text{th}}$  input for the  $j^{\text{th}}$  neuron in the input layer

$$\mu_j = \frac{1}{m} \sum_{i=1}^m Z_j^{(i)}$$

$$\sigma_j^2 = \frac{1}{m} \sum_{i=1}^m (Z_j^{(i)} - \mu_j)^2$$

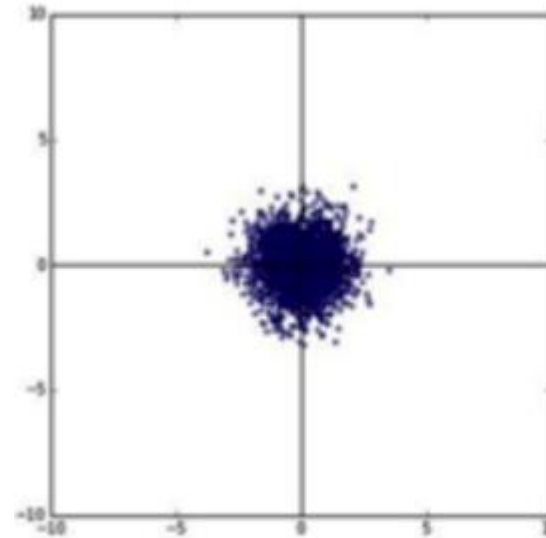
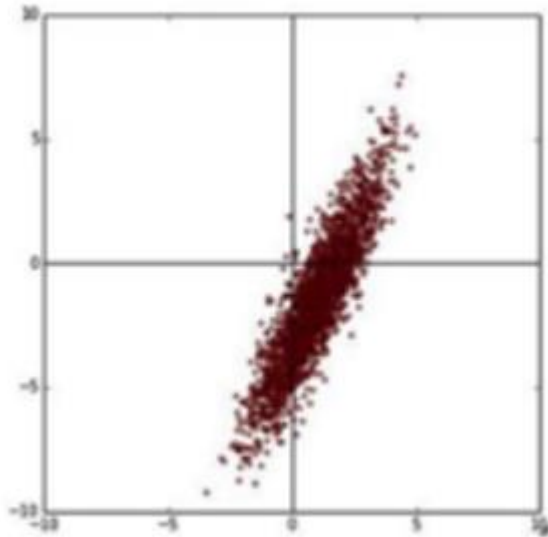
$$\hat{Z}_j = \frac{Z_j - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

- Or normalize the whole input layer together



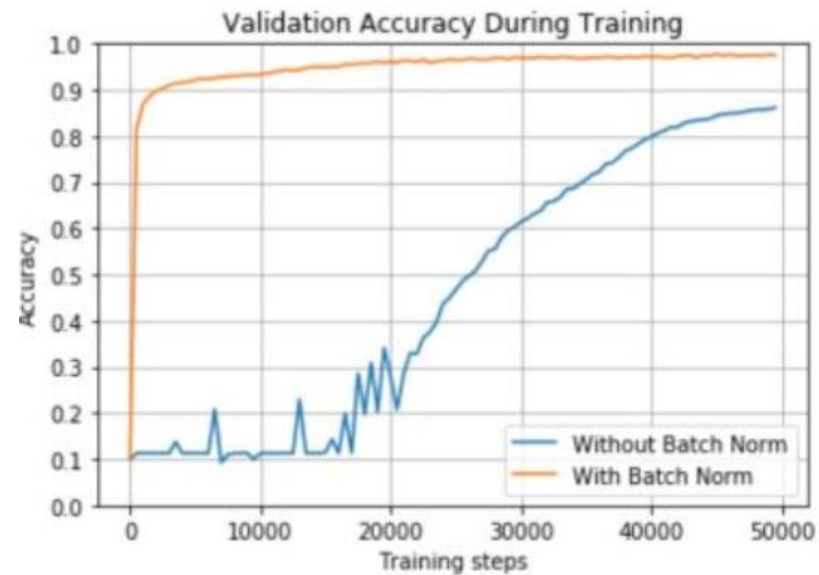
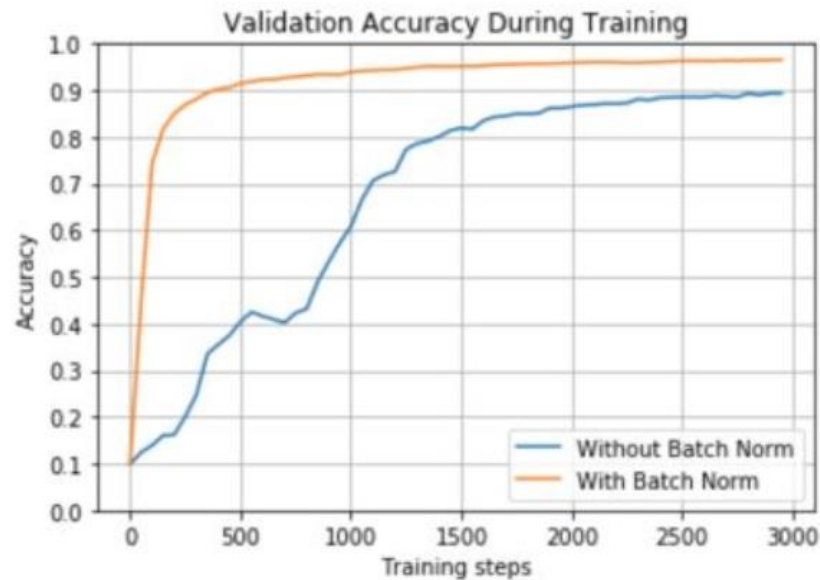
# Batch normalization (cont.)

- 2D example



# Example of batch normalization

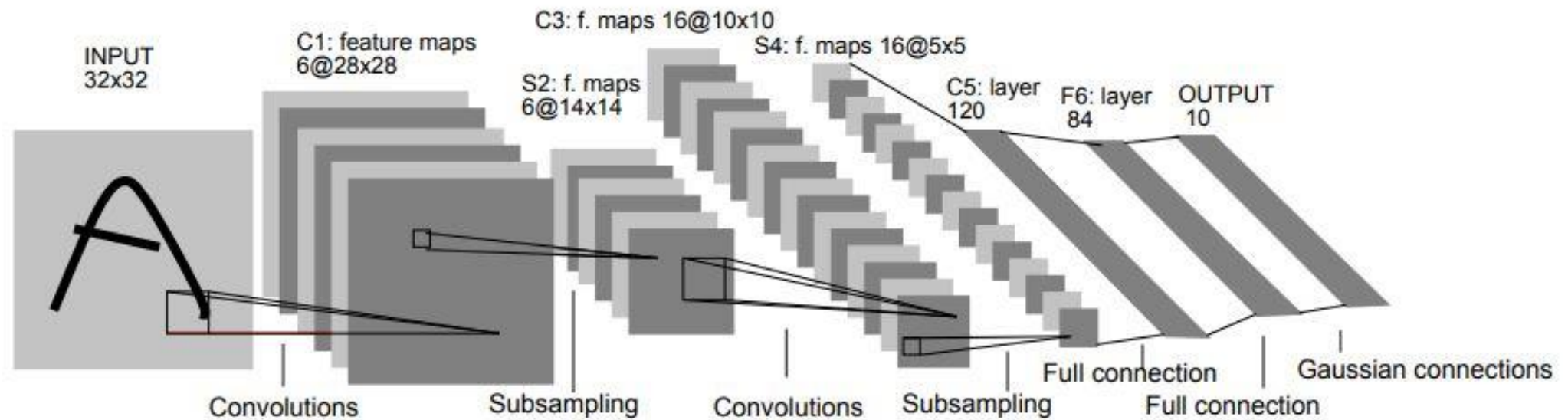
- Without batch normalization
  - Slow convergence and fluctuation



# Famous Neural Networks

# LeNet

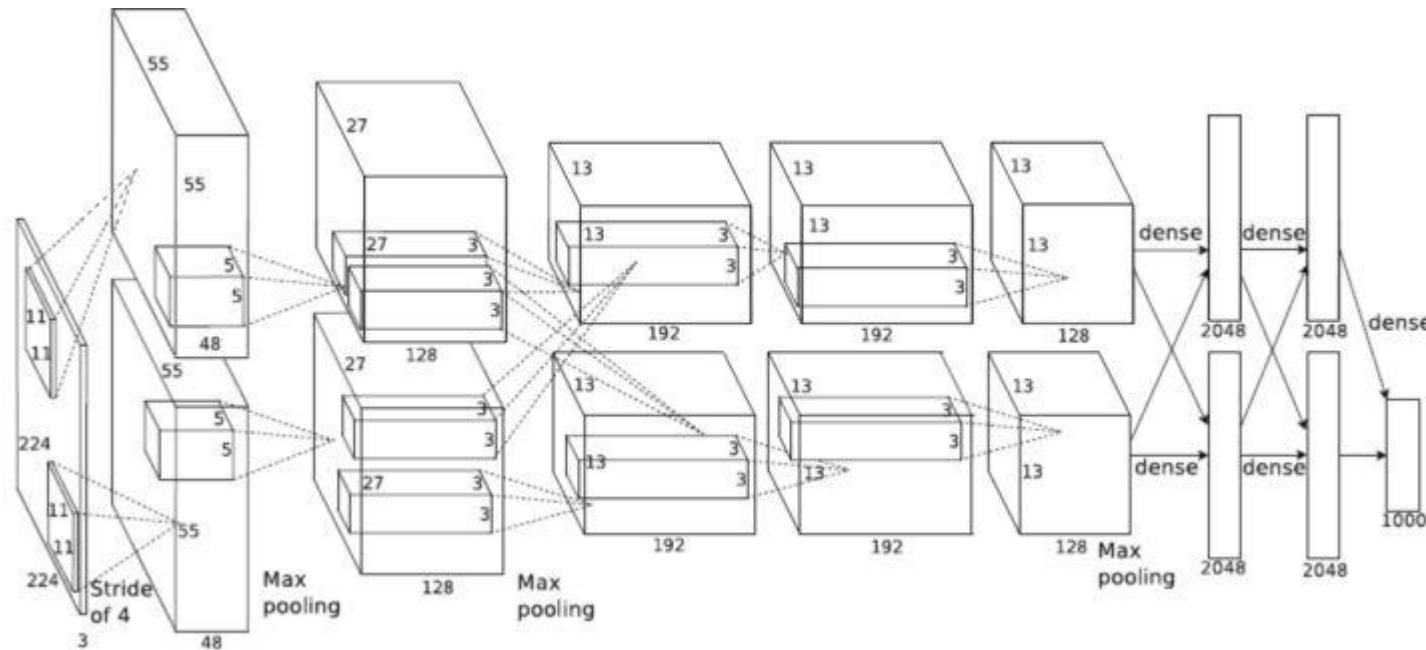
- LeNet [LeCun et al., 1998]



Input: 28\*28\*1 image, Conv filters of size 5x5, applied at stride 1  
Subsampling (Pooling) layers were 2x2 applied at stride 2  
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

# AlexNet [Krizhevsky et al. 2012]

- AlexNet:



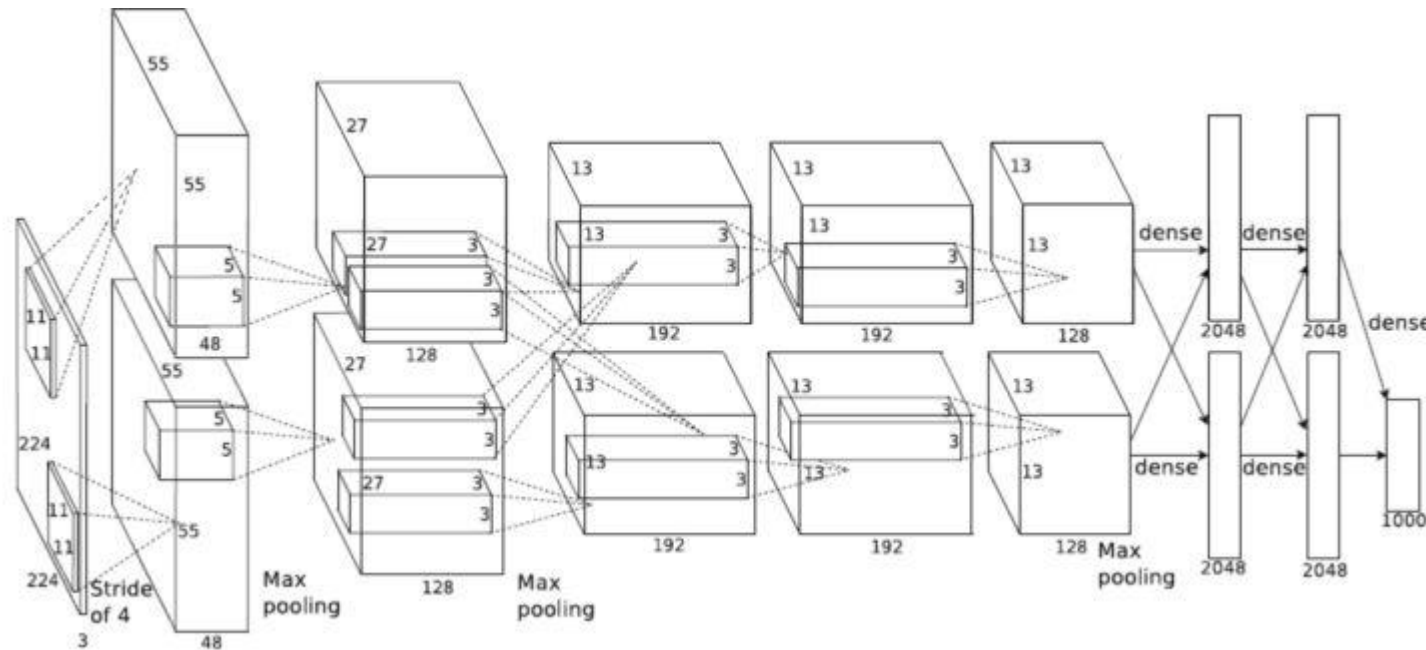
Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 kernels applied at stride 4

Q: what is the output volume size?

# AlexNet (cont.)

- AlexNet:



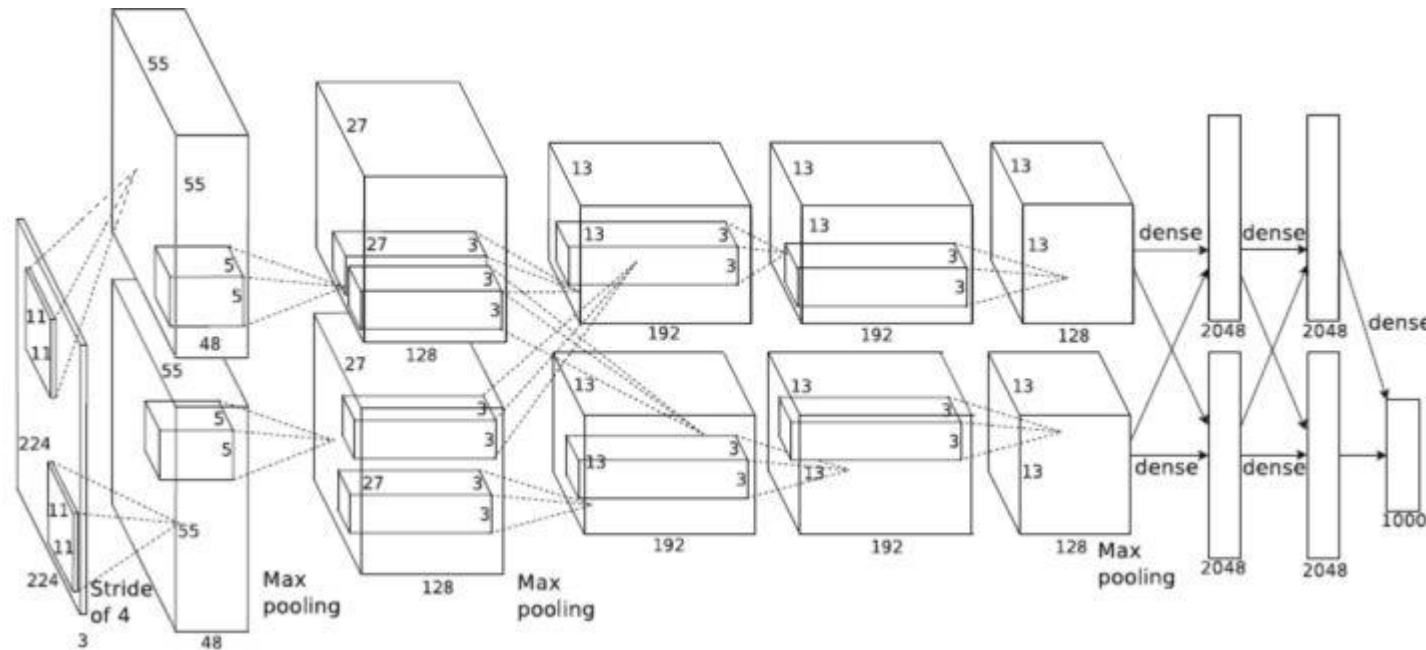
Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint:  $(227-11)/4+1 = 55$  [55x55x96]

# AlexNet (cont.)

- AlexNet:



Input: 227x227x3 images

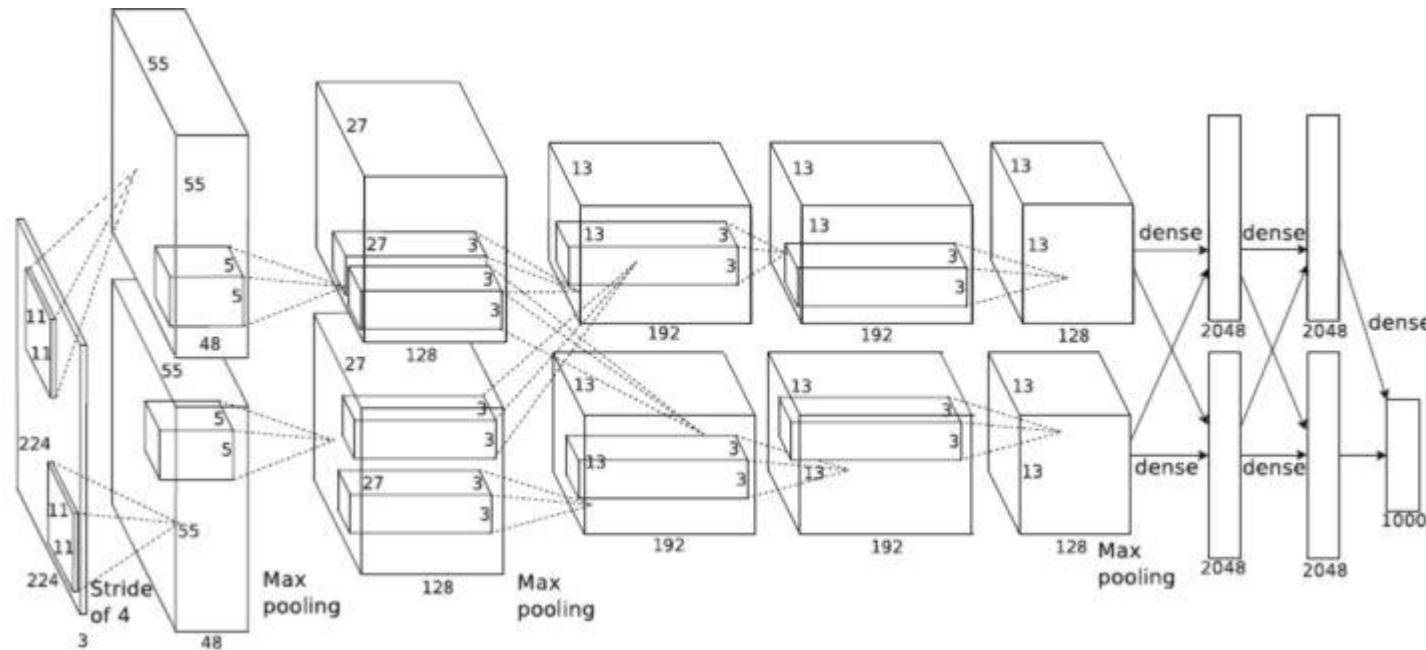
**First layer (CONV1):** 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint:  $(227-11)/4+1 = 55$  [55x55x96]

Q: What is the total number of parameters in this layer?

# AlexNet (cont.)

- AlexNet:



Input: 227x227x3 images

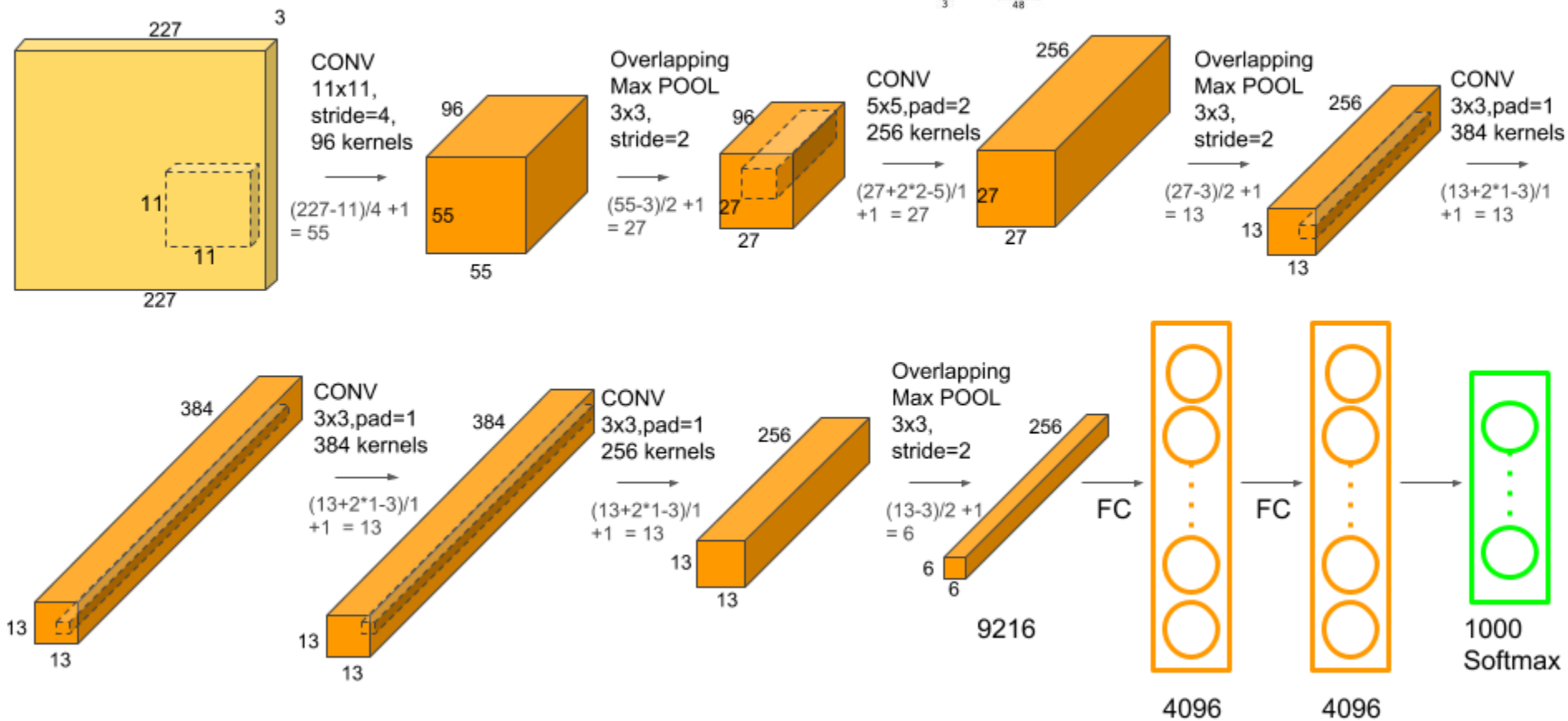
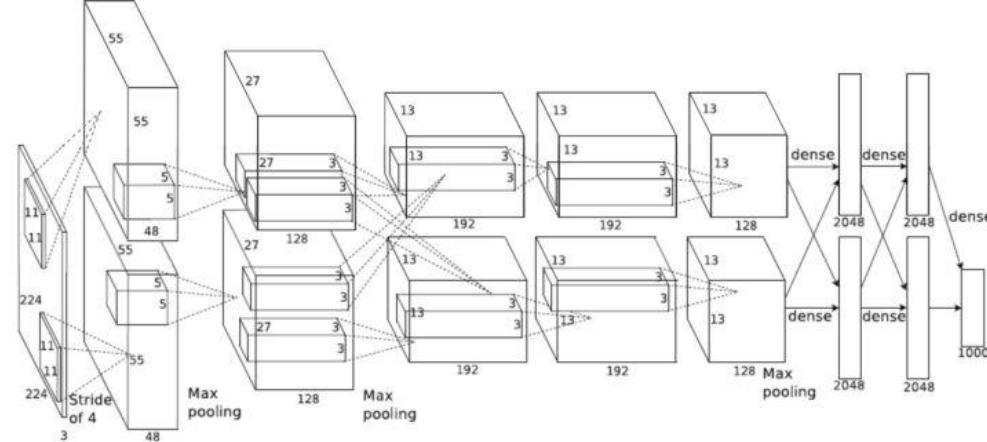
**First layer (CONV1):** 96 11x11 kernels applied at stride 4

Q: what is the output volume size? Hint:  $(227-11)/4+1 = 55$  [55x55x96]

Q: What is the total number of parameters in this layer?  $(11*11*3)*96 = 35K$



# AlexNet (cont.)



# VGGNet

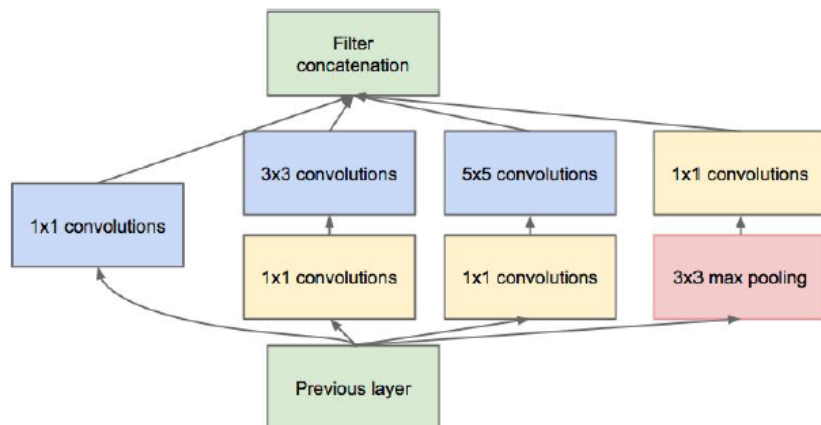
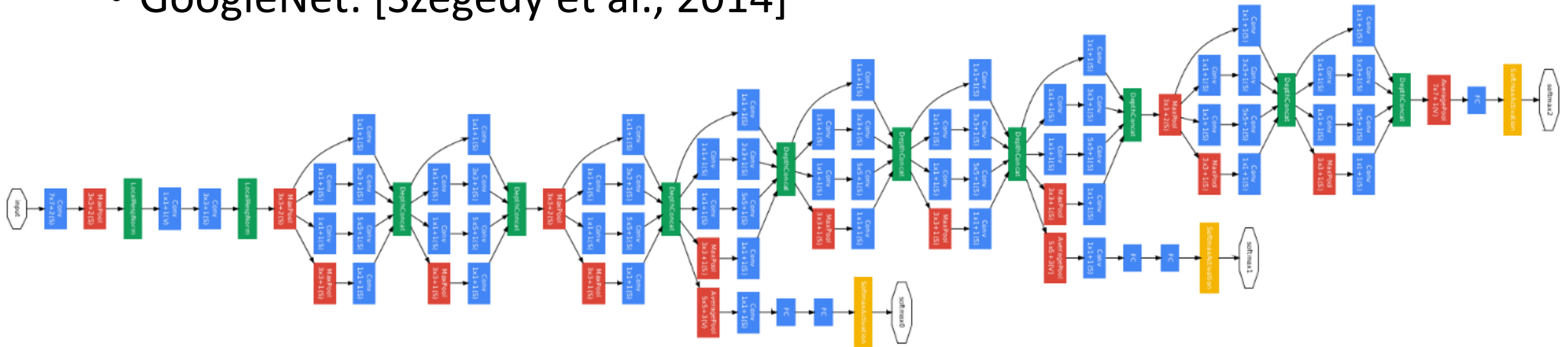
- VGGNet: [Simonyan and Zisserman, 2014]
- Only 3x3 CONV stride 1, pad 1 and 2x2 MAX POOL stride 2
- 11.2% top 5 error in ILSVRC 2013
  - ImageNet Large Scale Visual Recognition Challenge 2013
- 7.3% top 5 error

Best model

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 <b>LRN</b>	conv3-64 <b>conv3-64</b>	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 <b>conv3-128</b>	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 <b>conv1-256</b>	conv3-256 conv3-256 <b>conv3-256</b>	conv3-256 conv3-256 conv3-256 <b>conv3-256</b>
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 <b>conv1-512</b>	conv3-512 conv3-512 <b>conv3-512</b>	conv3-512 conv3-512 conv3-512 <b>conv3-512</b>
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 <b>conv1-512</b>	conv3-512 conv3-512 <b>conv3-512</b>	conv3-512 conv3-512 conv3-512 <b>conv3-512</b>
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

# GoogleNet

- GoogleNet: [Szegedy et al., 2014]



ILSVRC 2014 winner (6.7% top 5 error)

# GoogleNet (cont.)

- GoogleNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Fun features:

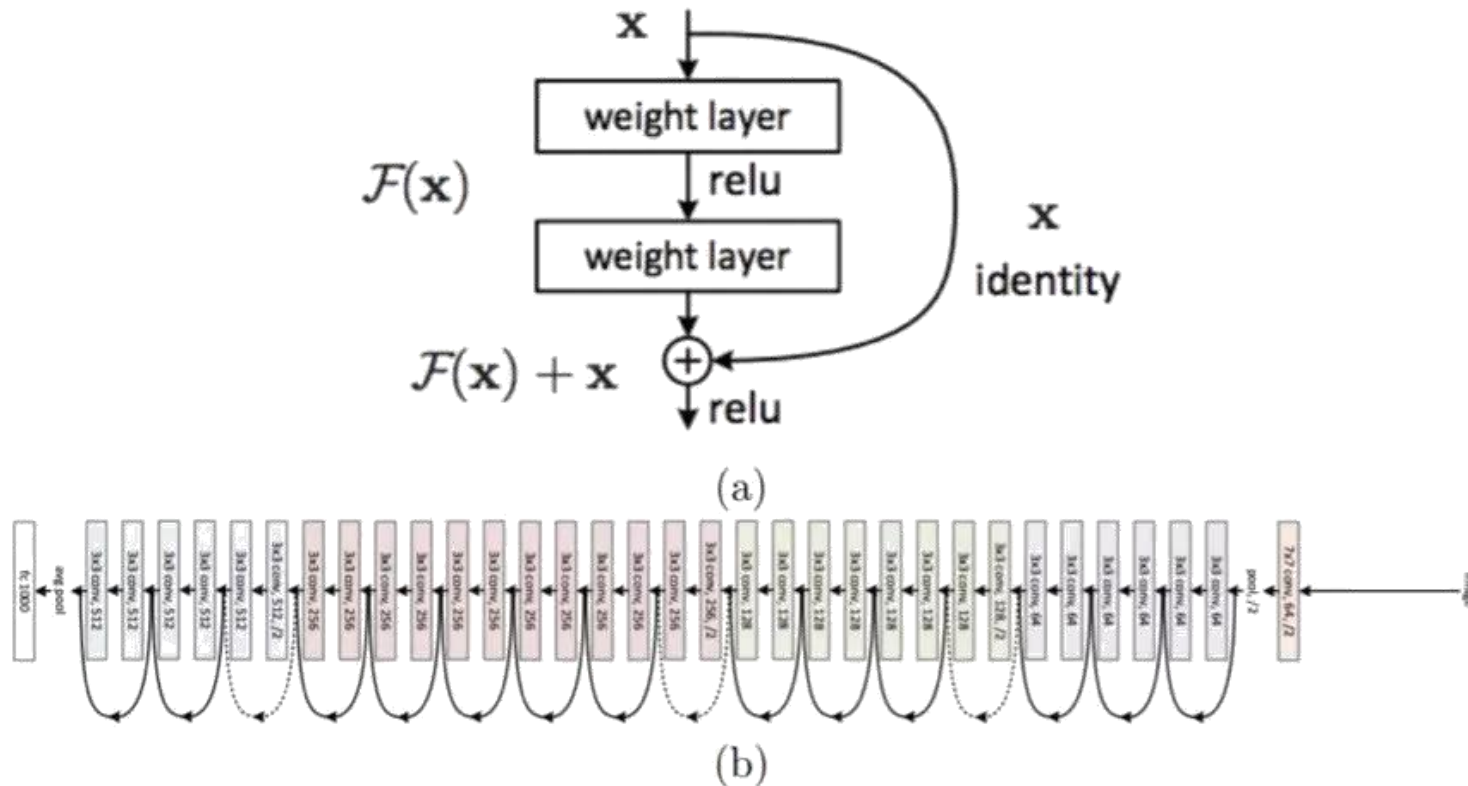
- Only 5 million parameters! (Removes FC layers completely)

Compared to AlexNet:

- 12X less parameters
- 2x running speed
- Error rate 6.67% (vs. 16.4%)

# ResNet

- ResNet [He et al., 2015]
  - Residual networks
  - Solves the problem of drifting by adding the original input to later layers





# ResNet (cont.)

- ResNet: ILSVRC 2015 winner (3.6% top 5 error)

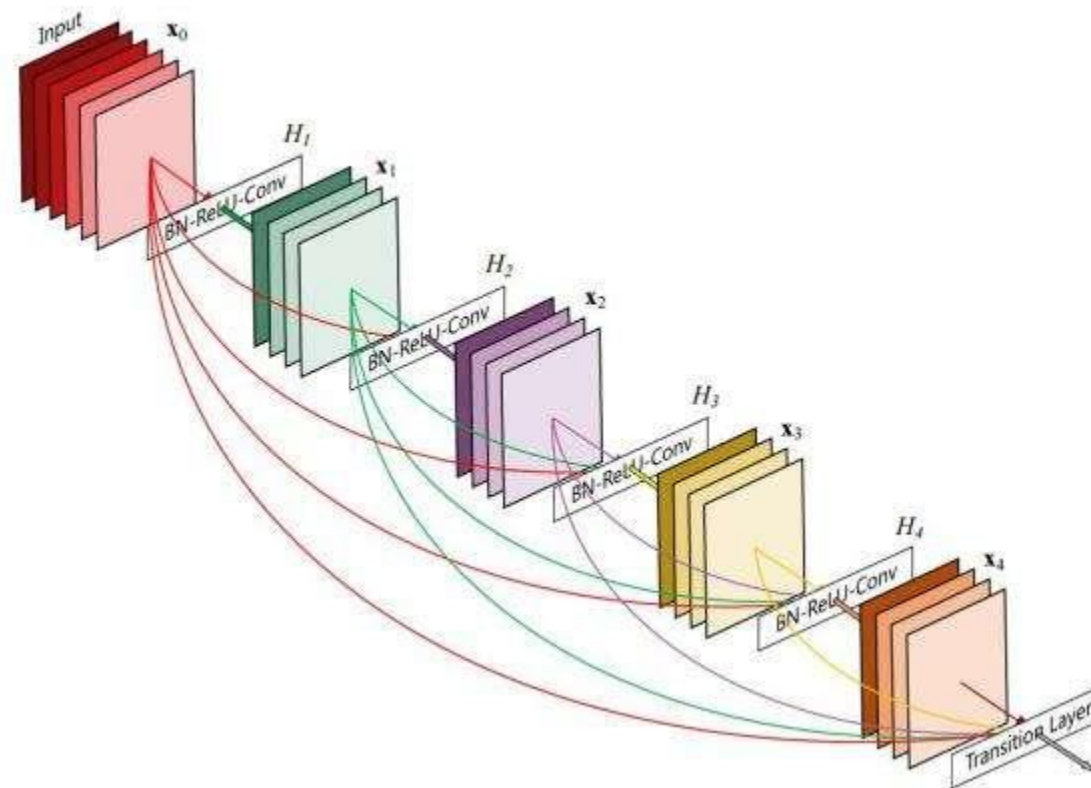
## MSRA @ ILSVRC & COCO 2015 Competitions

- **1st places in all five main tracks**

- ImageNet Classification: “*Ultra-deep*” (quote Yann) **152-layer** nets
- ImageNet Detection: **16%** better than 2nd
- ImageNet Localization: **27%** better than 2nd
- COCO Detection: **11%** better than 2nd
- COCO Segmentation: **12%** better than 2nd

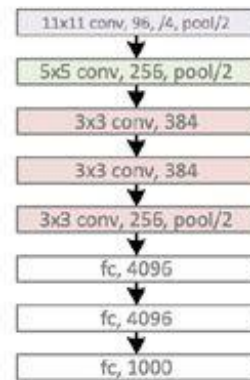
# DenseNet

- DenseNet

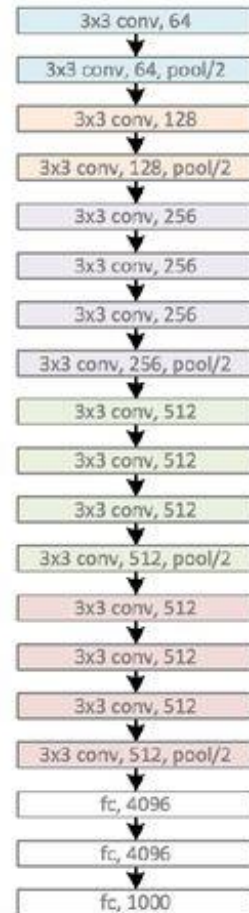


# Revolution of Depth

AlexNet, 8 layers  
(ILSVRC 2012)



VGG, 19 layers  
(ILSVRC 2014)



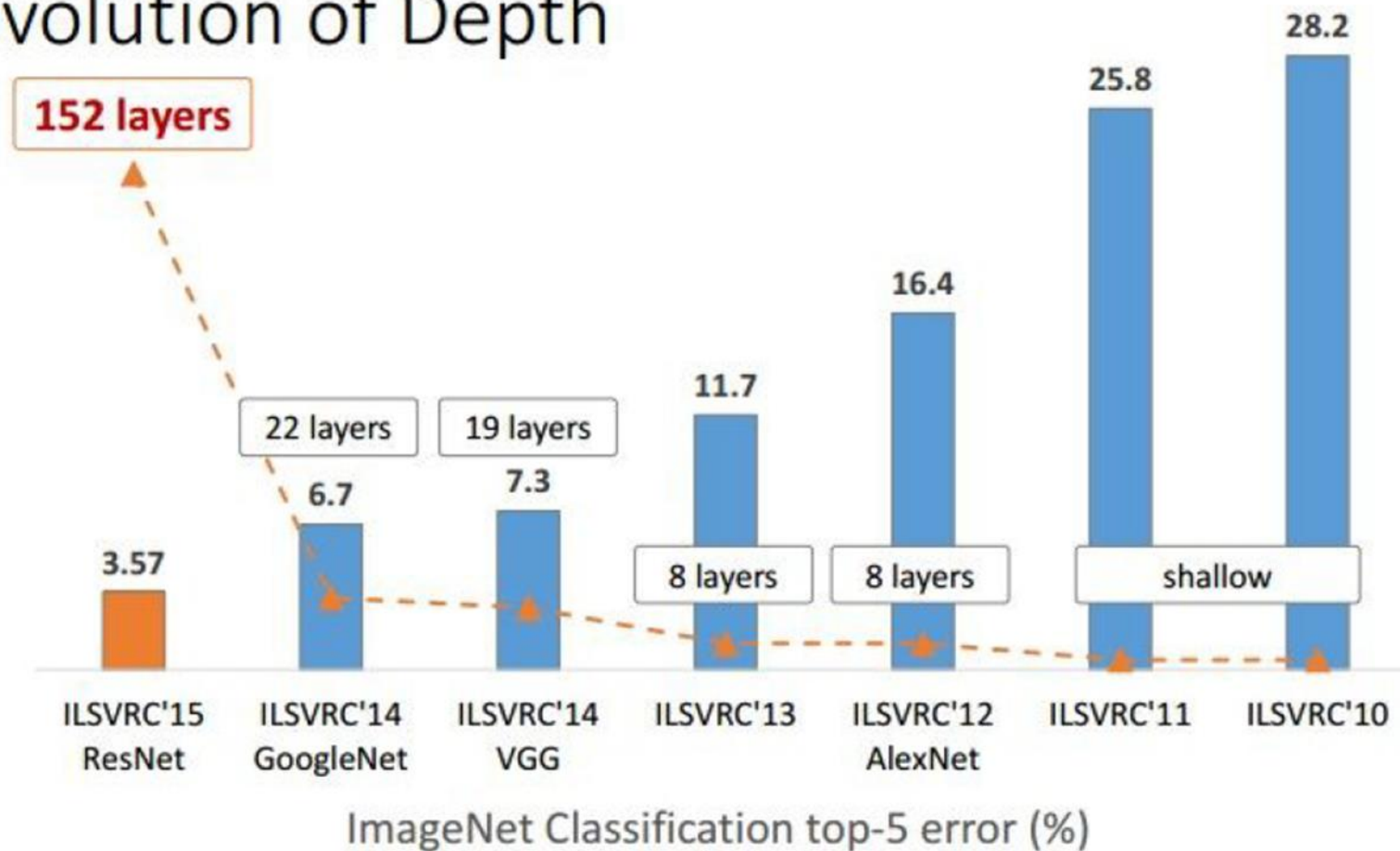
GoogleNet, 22 layers  
(ILSVRC 2014)





# Revolution of depth

## Revolution of Depth



# Architecture comparison

