Lecture 2: Uninformed Search

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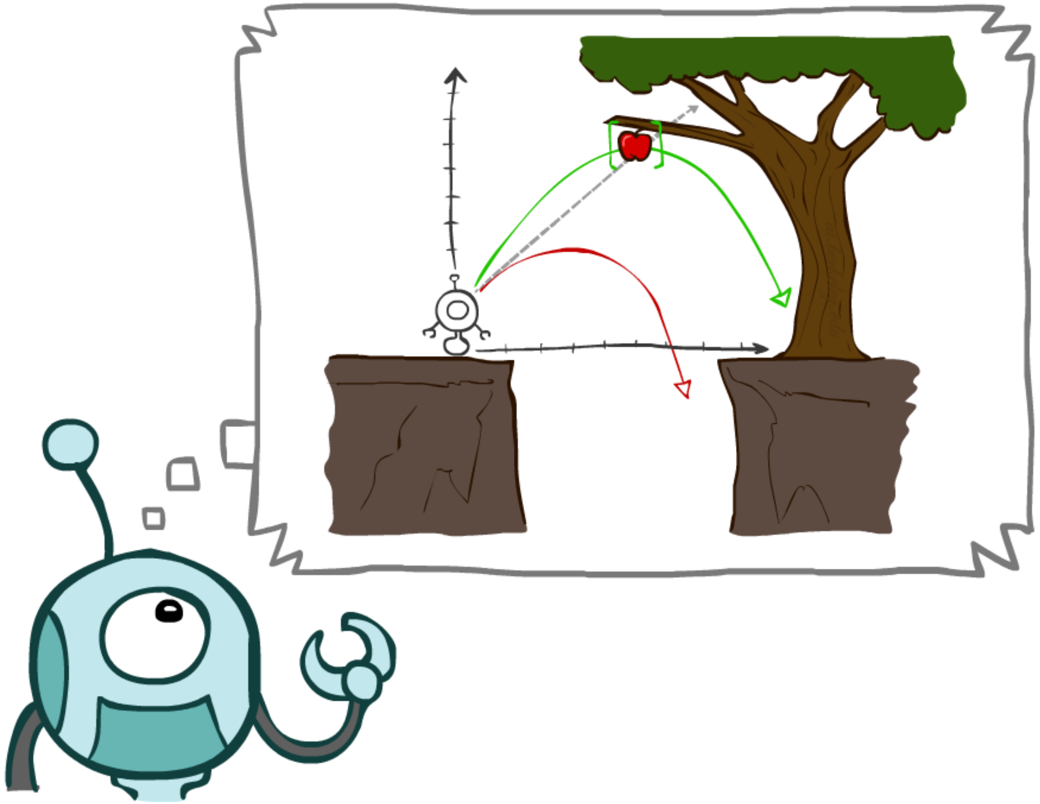
https://shuaili8.github.io/Teaching/CS410/index.html

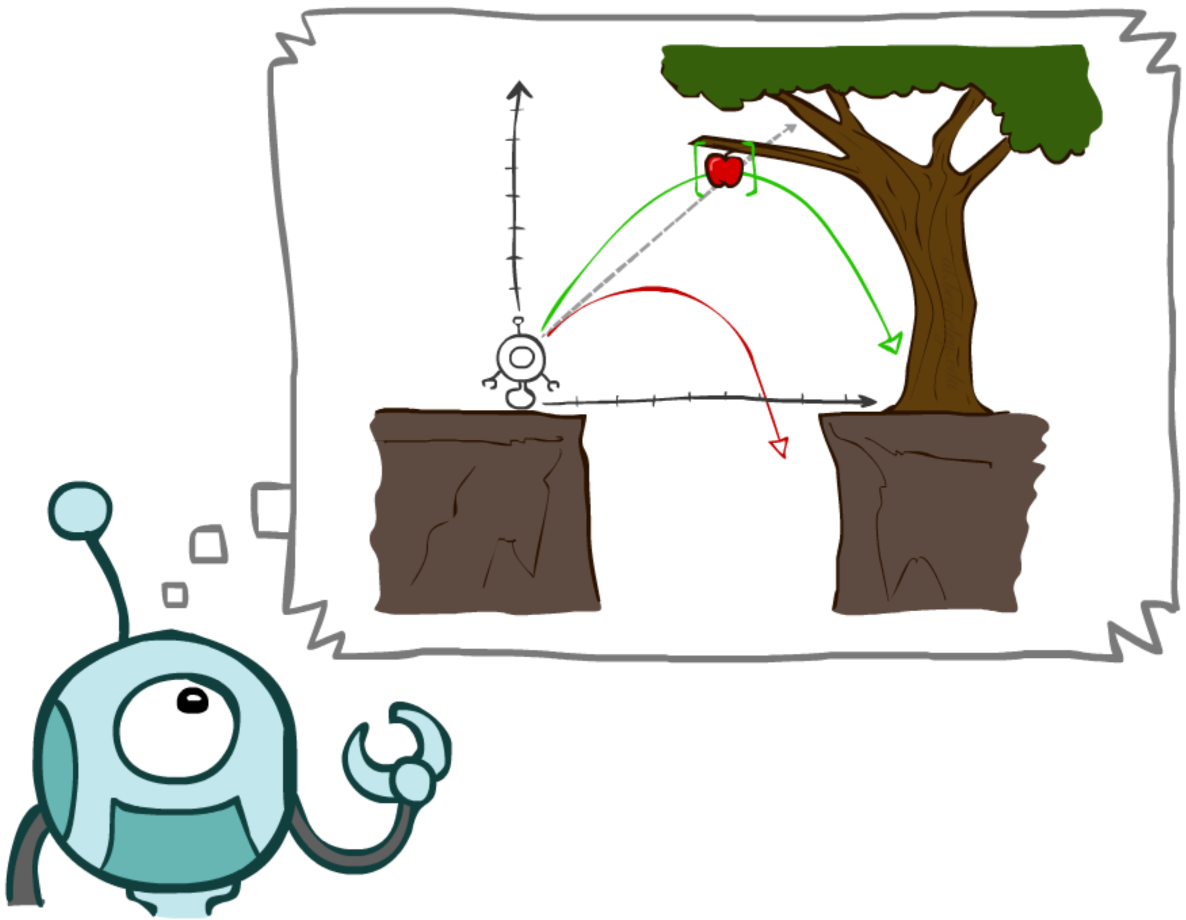
Part of slide credits: CMU AI & http://ai.berkeley.edu

Today

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
    - Depth-First Search
    - Breadth-First Search
    - Uniform-Cost Search

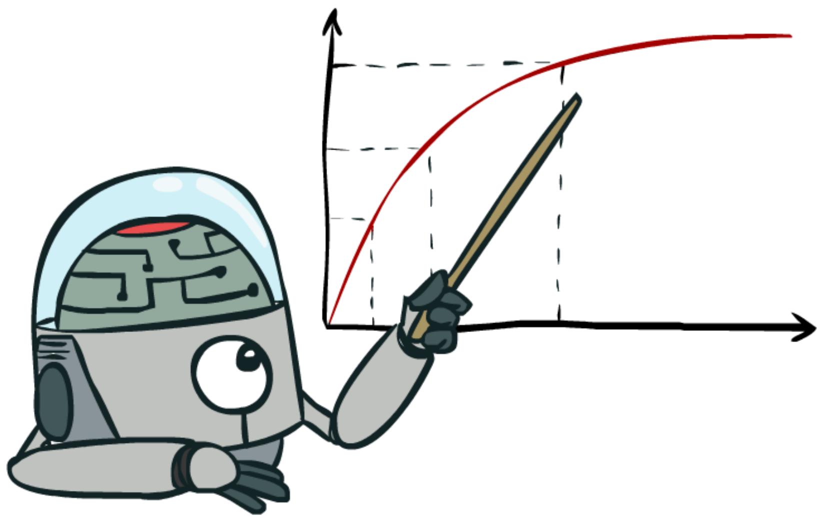


Agents that Plan



Rationality

- What is rational depends on:
  - Performance measure
  - Agent’s prior knowledge of environment
  - Actions available to agent
  - Percept/sensor sequence to date

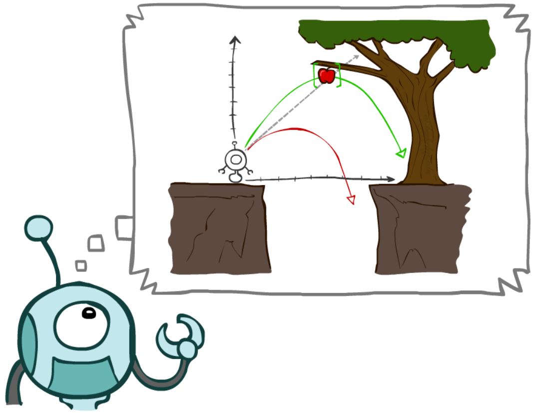


- Being rational means **maximizing your expected utility**

Rational Agents

- Are rational agents ***omniscient***? 无所不知的
  - No – they are limited by the available percepts
- Are rational agents ***clairvoyant***? 透视的
  - No – they may lack knowledge of the environment dynamics
- Do rational agents ***explore*** and ***learn***?
  - Yes – in unknown environments these are essential
- So rational agents are not necessarily successful, but they are ***autonomous*** (i.e., control their own behavior)

Planning Agents

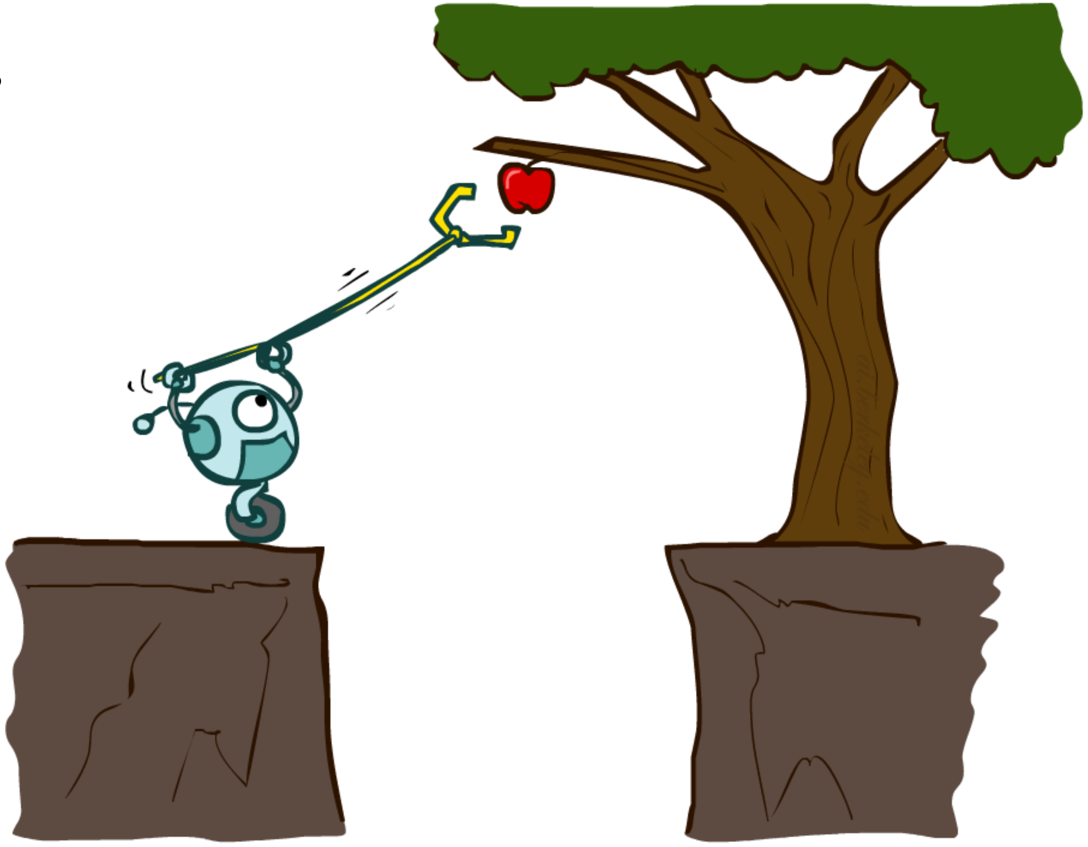


- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized or predicted) consequences of actions
  - Must have a transition model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world WOULD BE
- Spectrum of deliberativeness:
  - Generate complete, optimal plan offline, then execute
  - Generate a simple, greedy plan, start executing, replan when something goes wrong

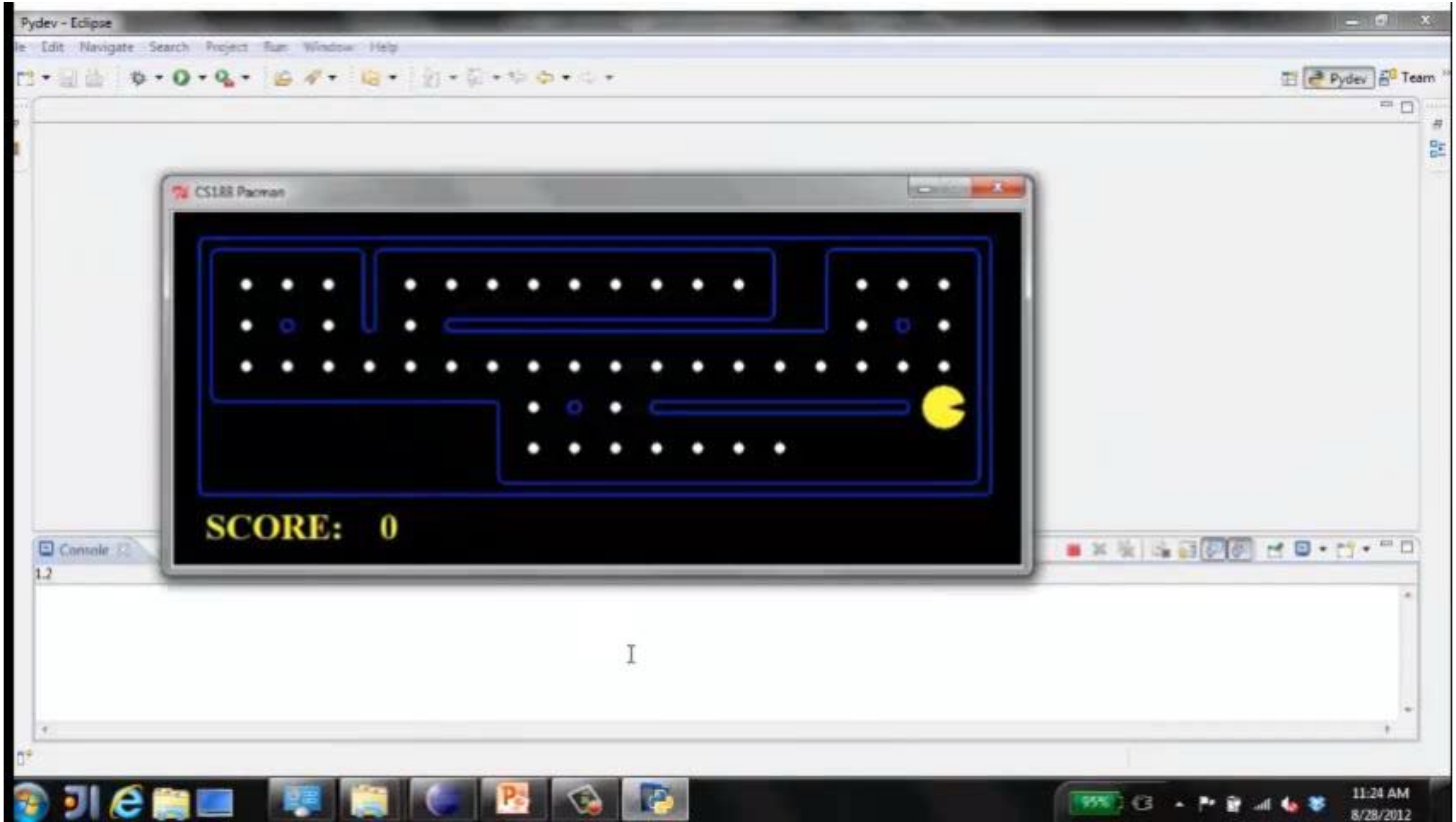
- Optimal vs. complete planning
- Planning vs. replanning

[Demo: re-planning (L2D3)]

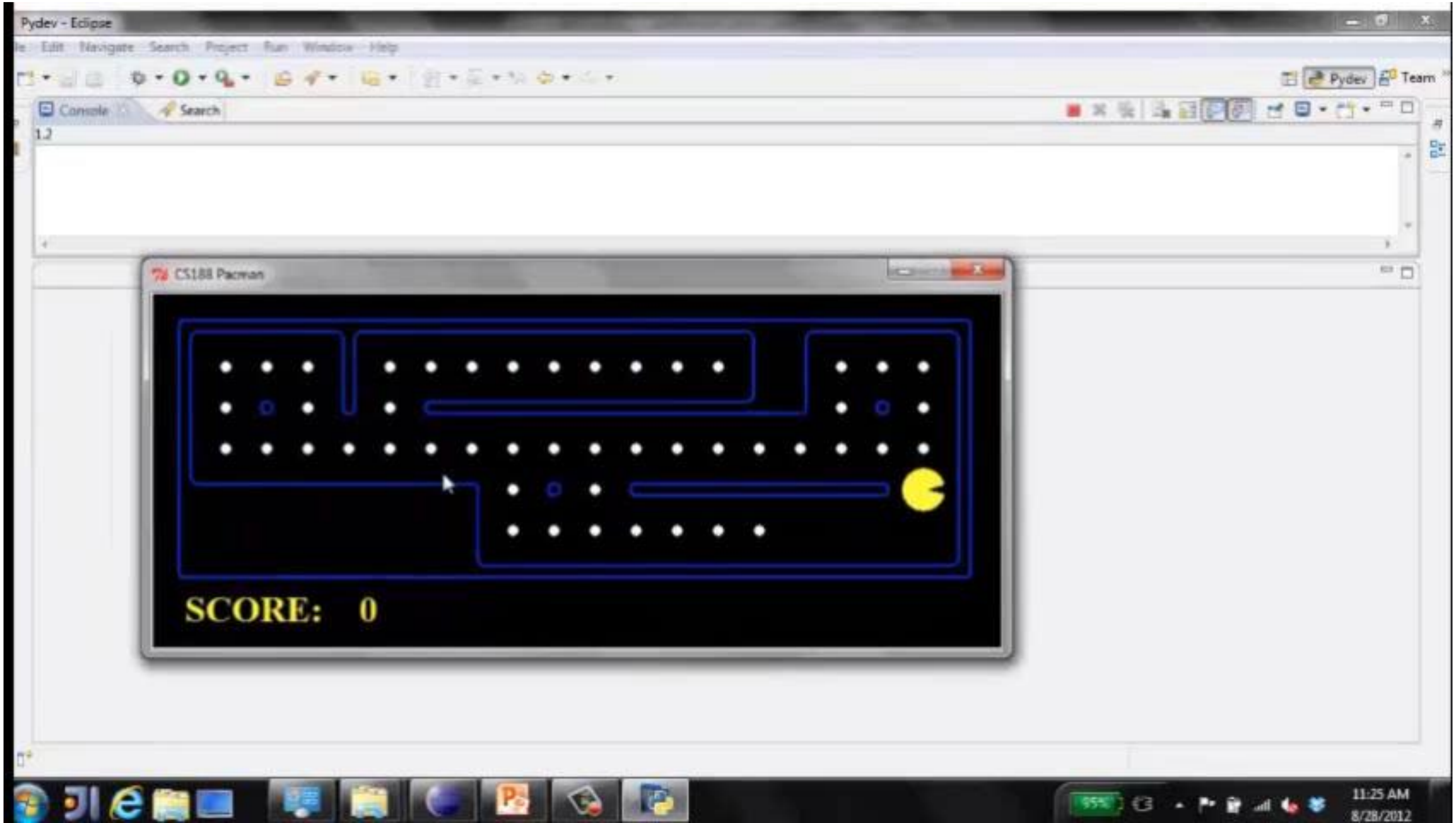
[Demo: mastermind (L2D4)]



Video of Demo Replanning



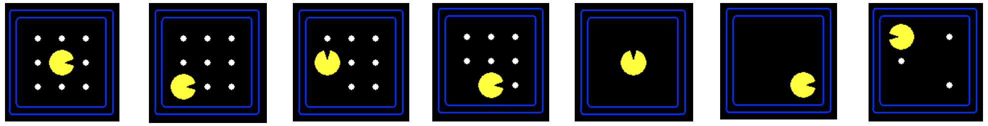
Video of Demo Mastermind



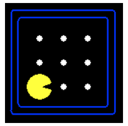
Search Problems

Search Problems

- A search problem consists of:
  - A state space

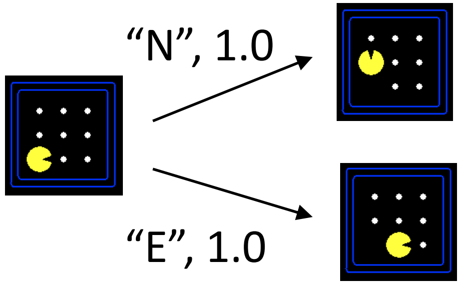


  - For each state, a set Actions(s) of successors/actions



{N, E}

  - A transition model T(s,a)

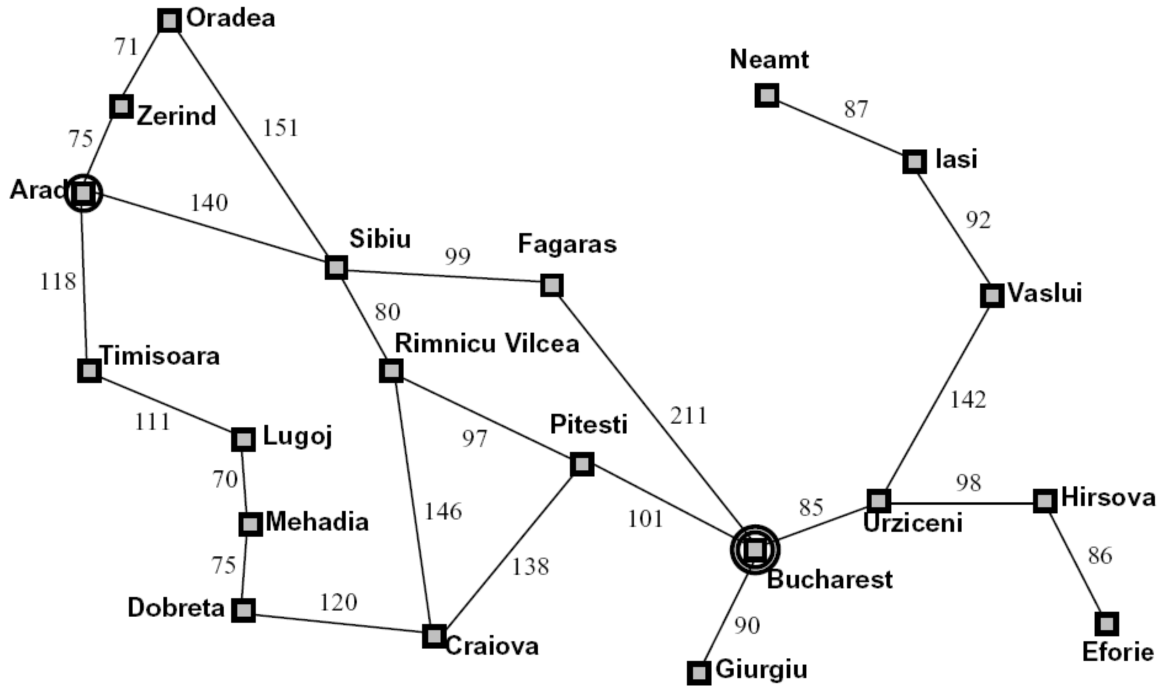


  - A step cost(reward) function c(s,a,s’)
  - A start state and a goal test

- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Search Problems Are Models

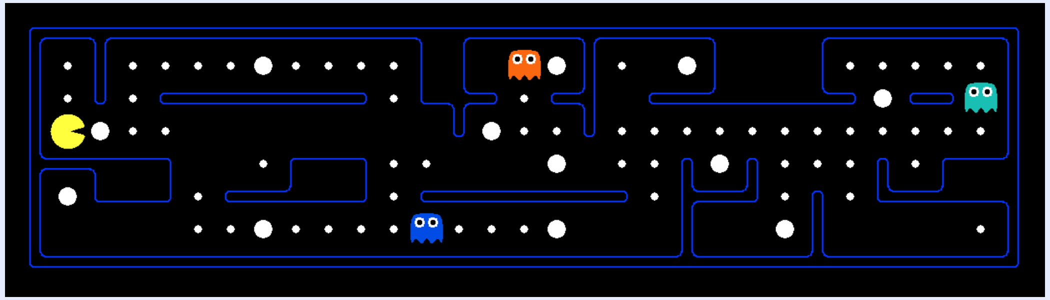
Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

What’s in a State Space?

The world state includes every last detail of the environment



A search state keeps only the details needed for planning (abstraction)

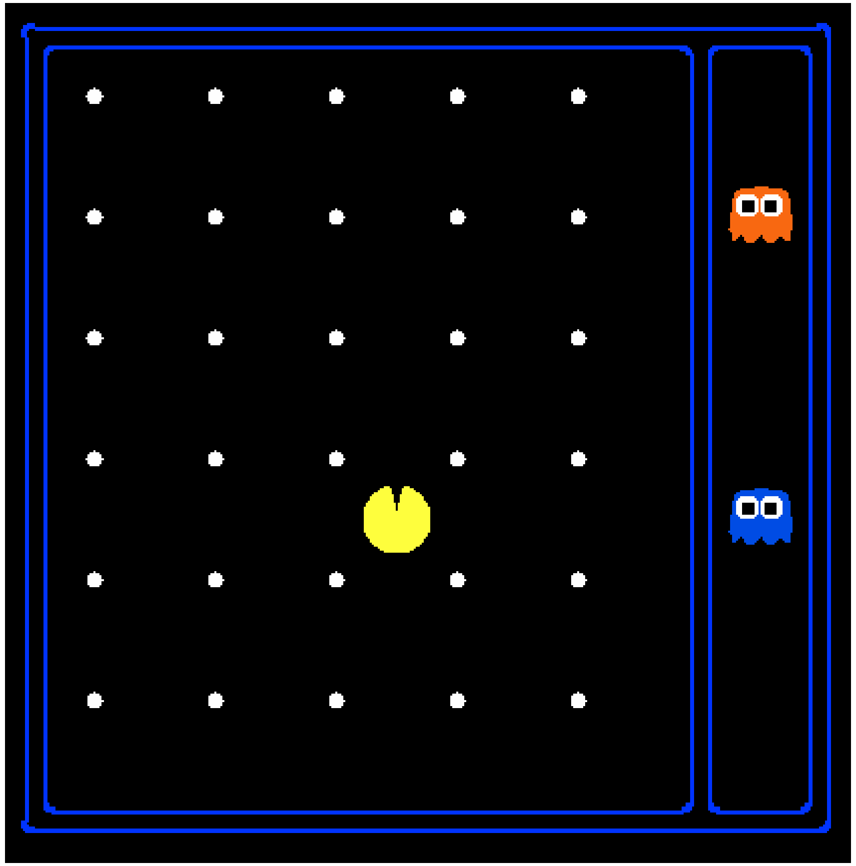
- Problem: Pathing
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END

- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

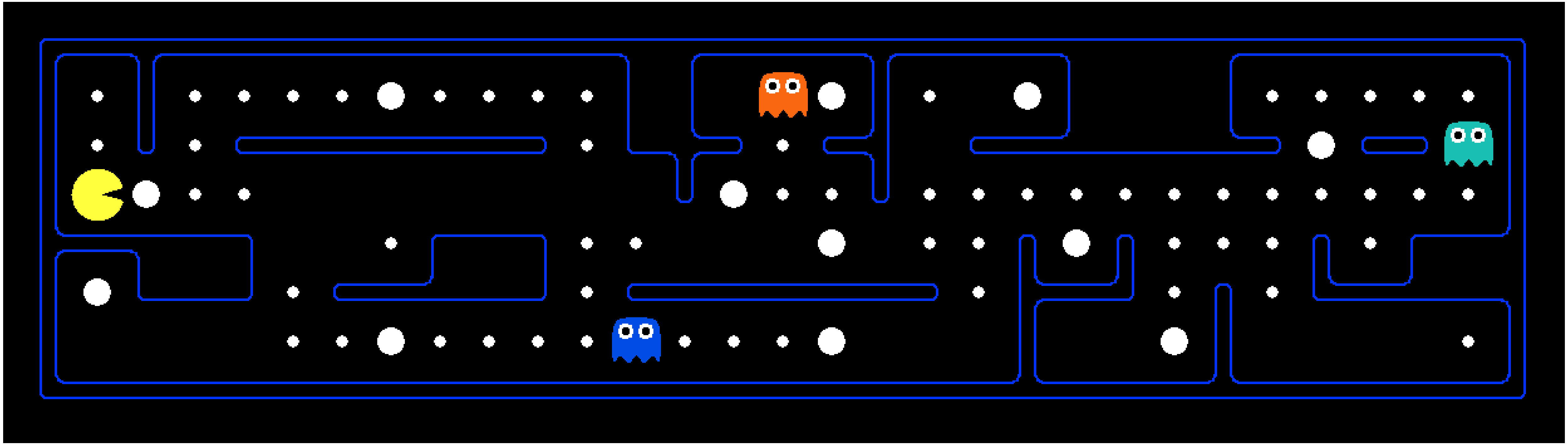
State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW

- How many
  - World states?
    $120 \times (2^{30}) \times (12^2) \times 4$
  - States for pathing?
    120
  - States for eat-all-dots?
    $120 \times (2^{30})$

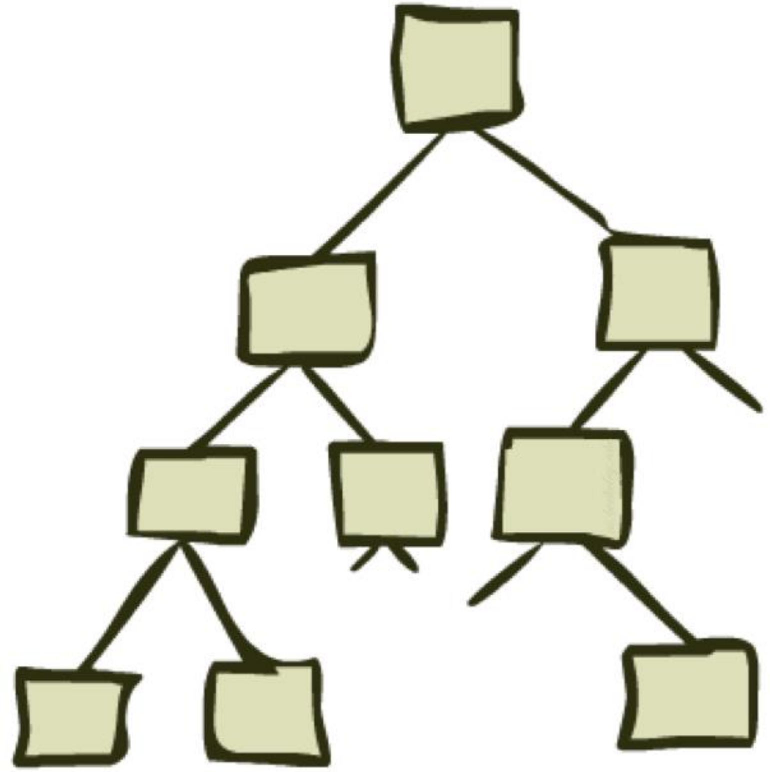


Safe Passage



- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

State Space Graphs and Search Trees

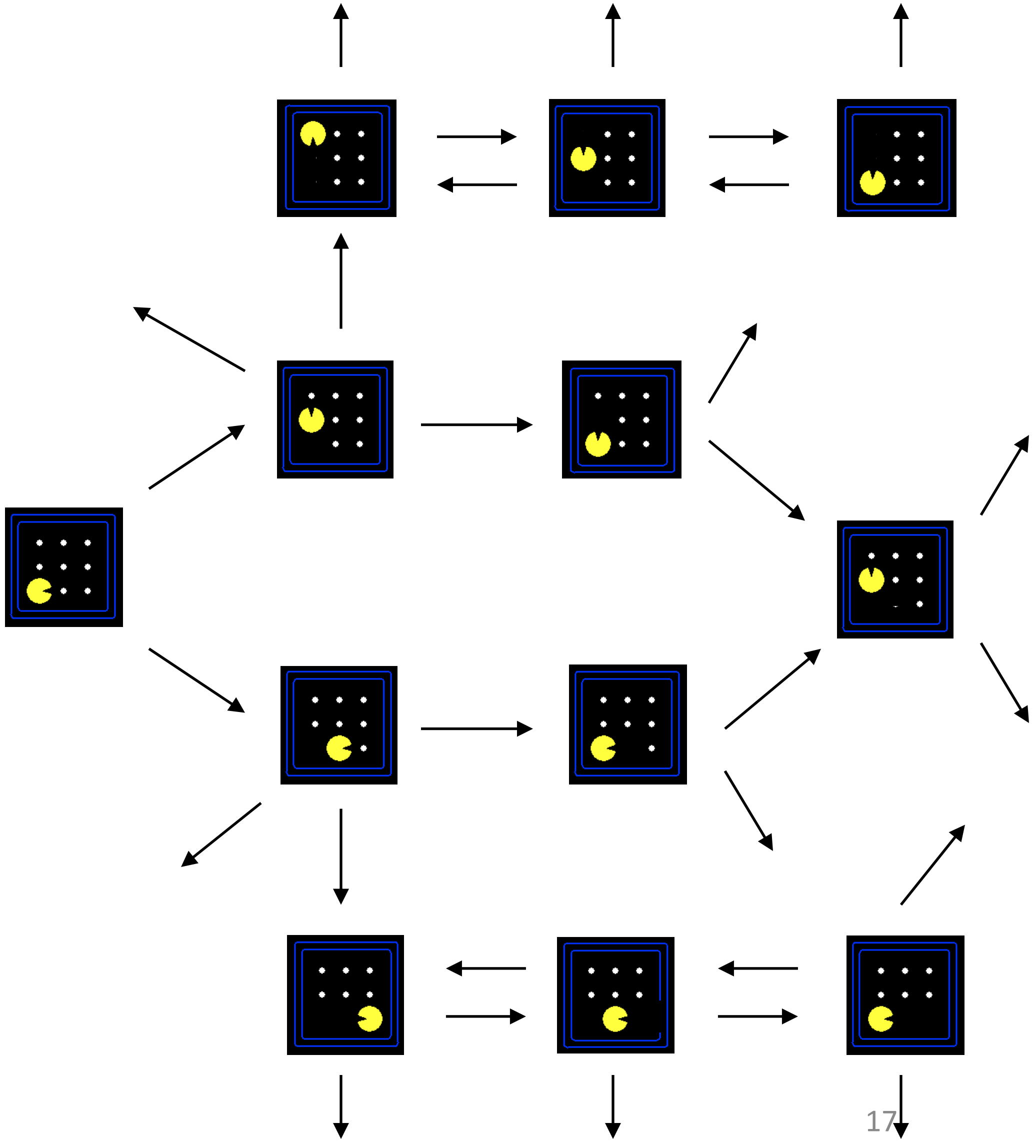


State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it’s too big), but it’s a useful idea



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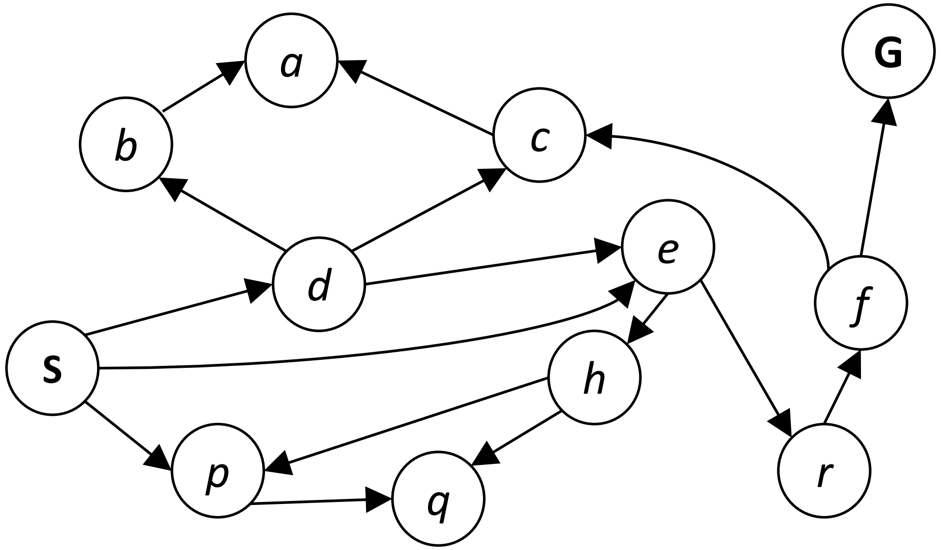
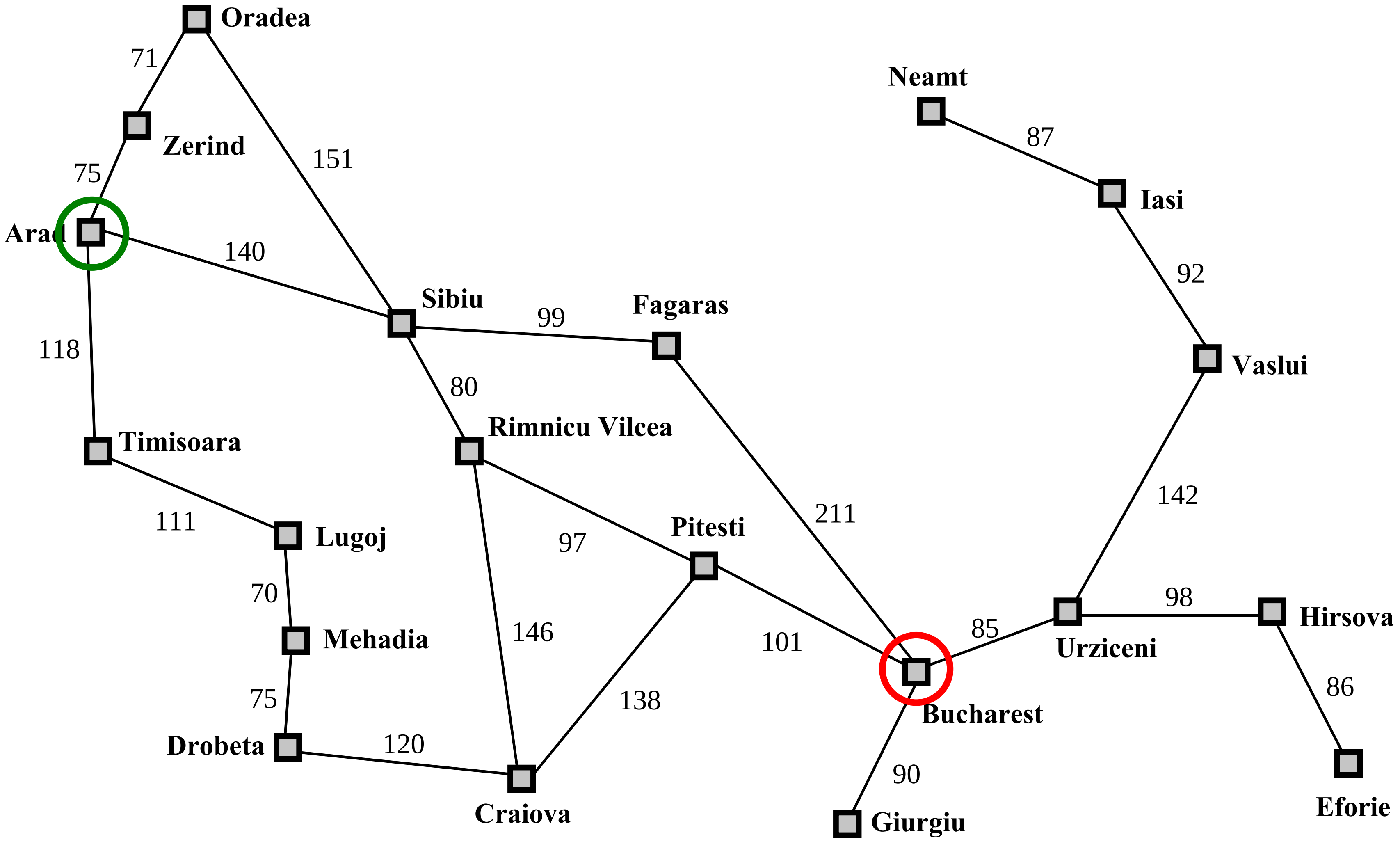
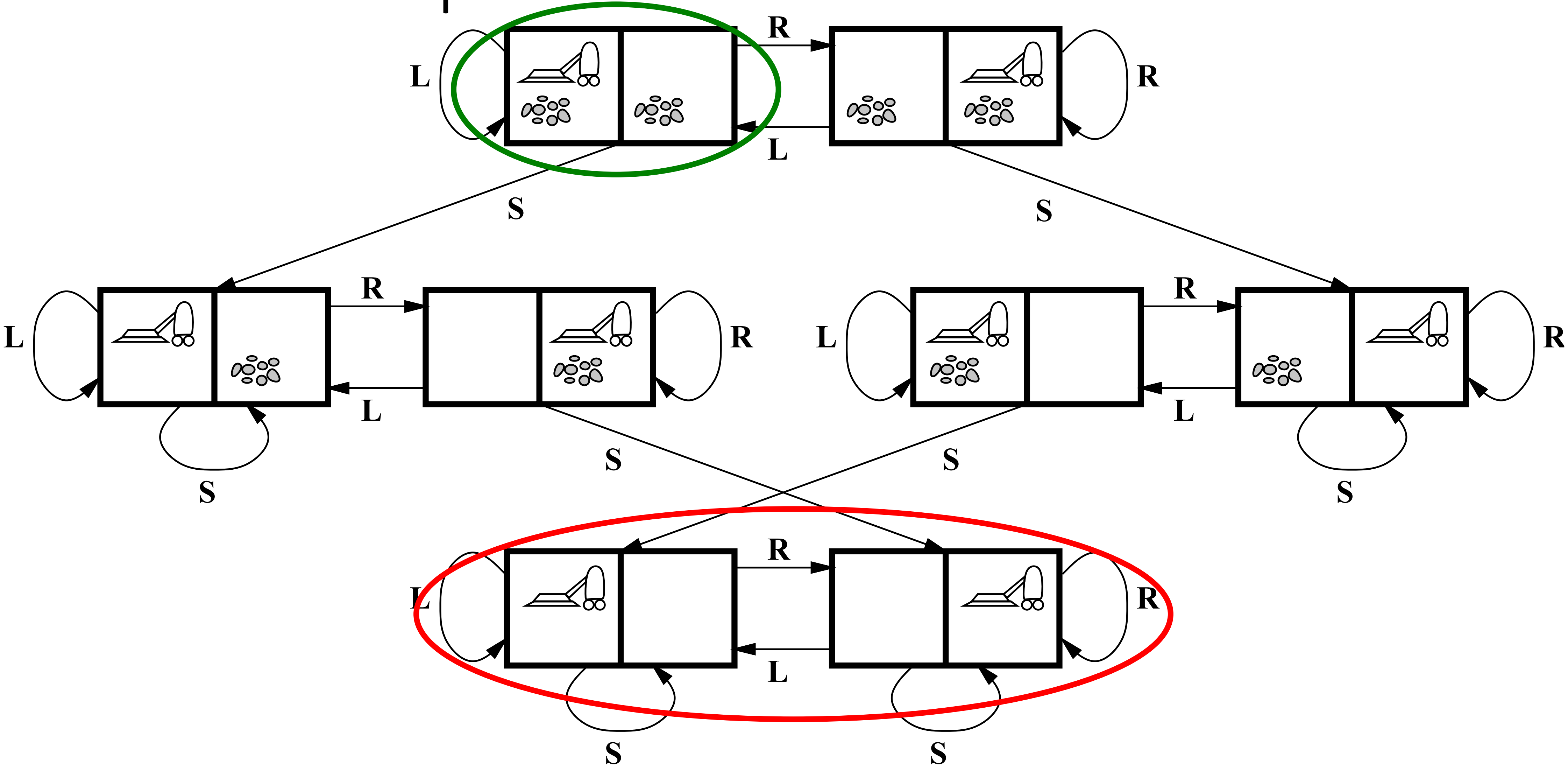


Tiny search graph for a tiny search problem

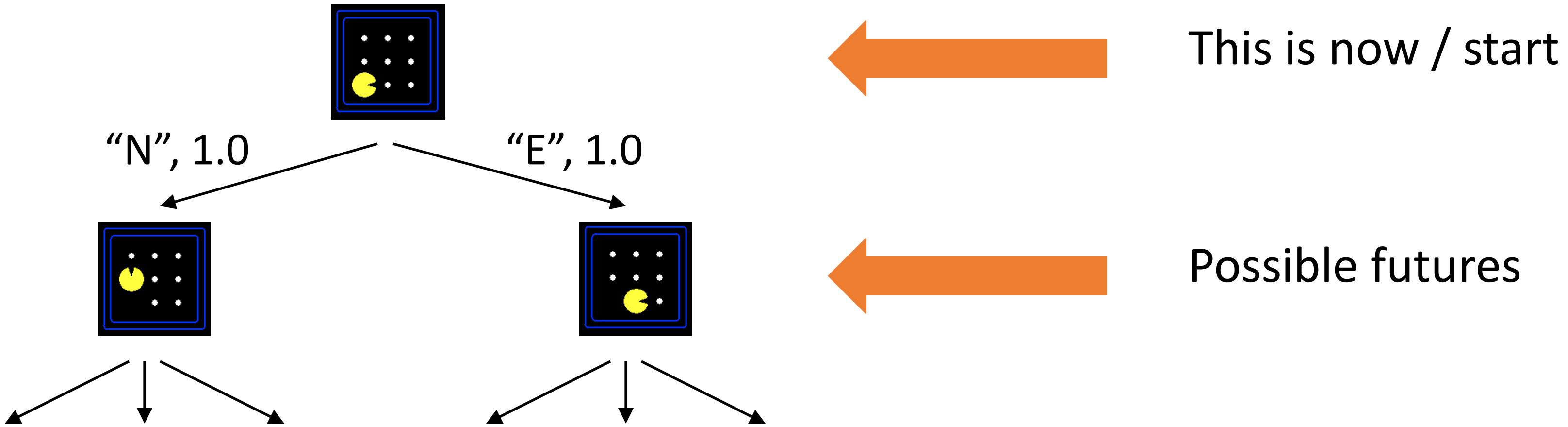
More Examples



More Examples



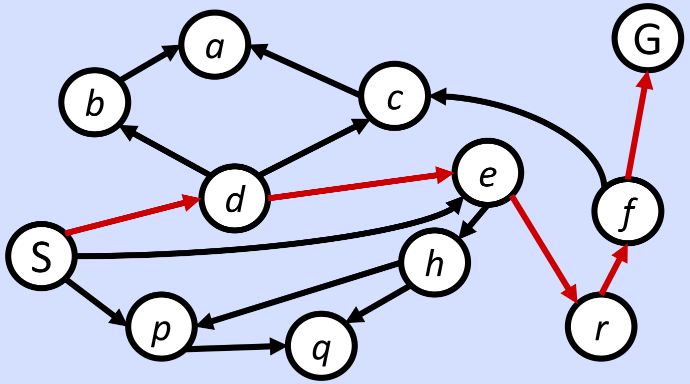
Search Trees



- A search tree:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees

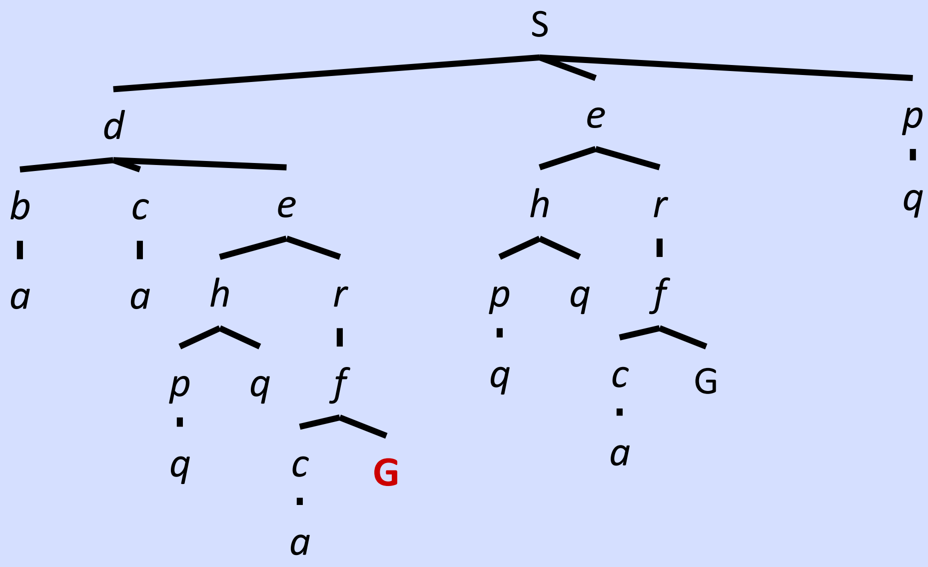
State Space Graph



Each NODE in in the search tree is an entire PATH in the state space graph.

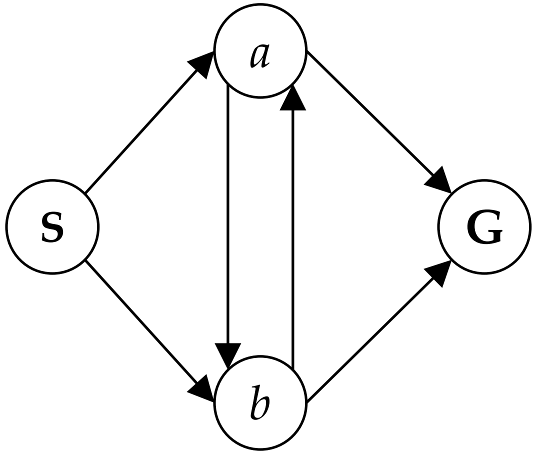
We construct both on demand – and we construct as little as possible.

Search Tree

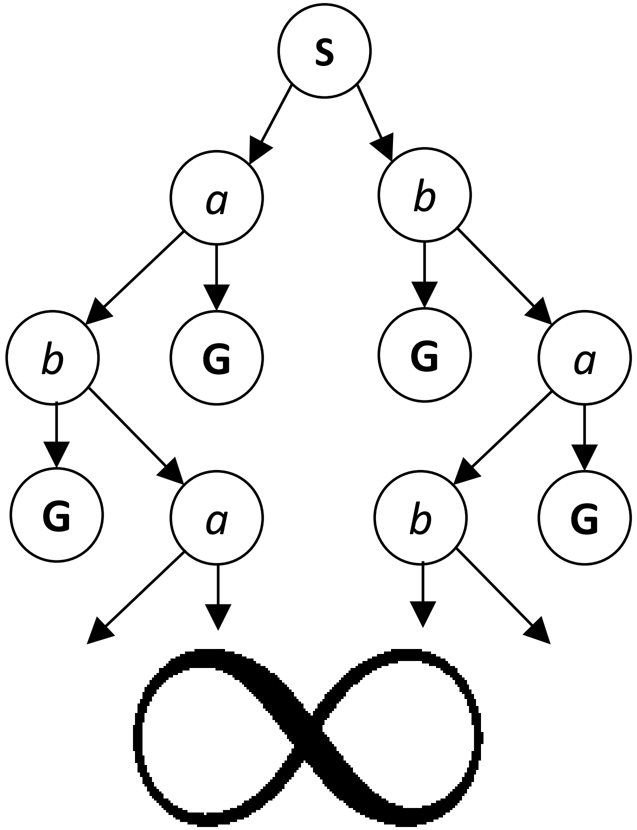


State Space Graphs vs. Search Trees

Consider this 4-state graph:

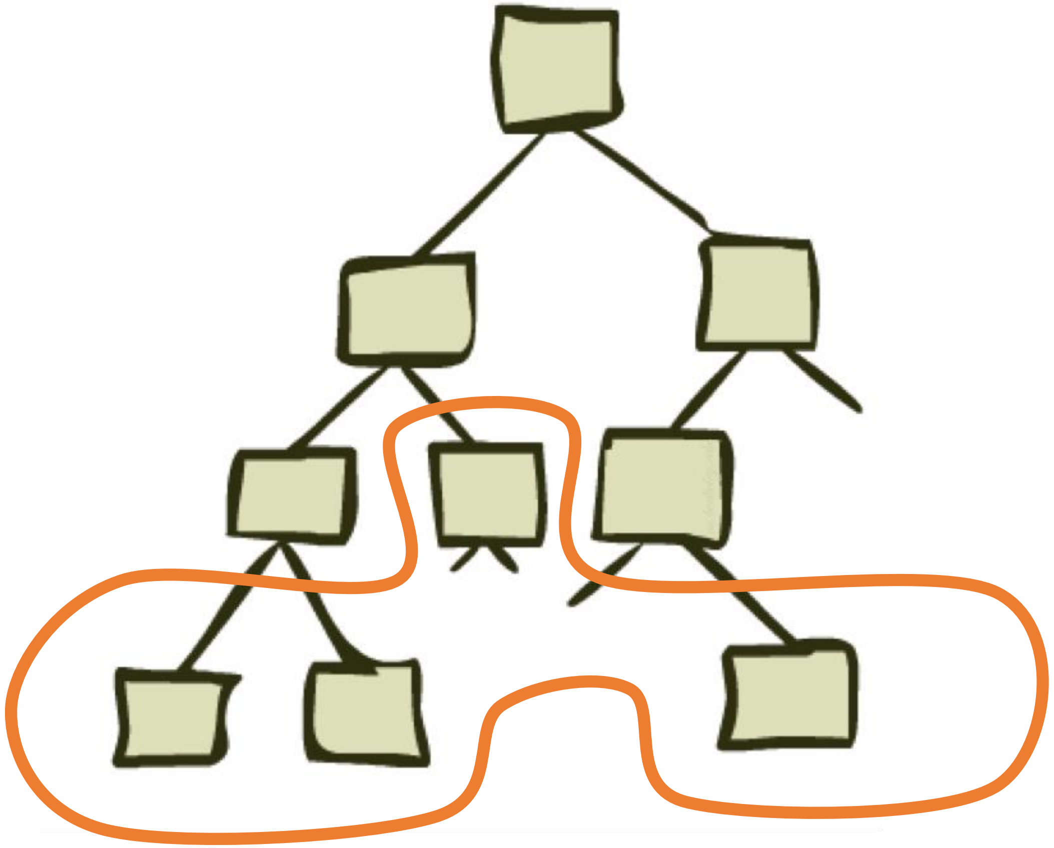


How big is its search tree (from S)?

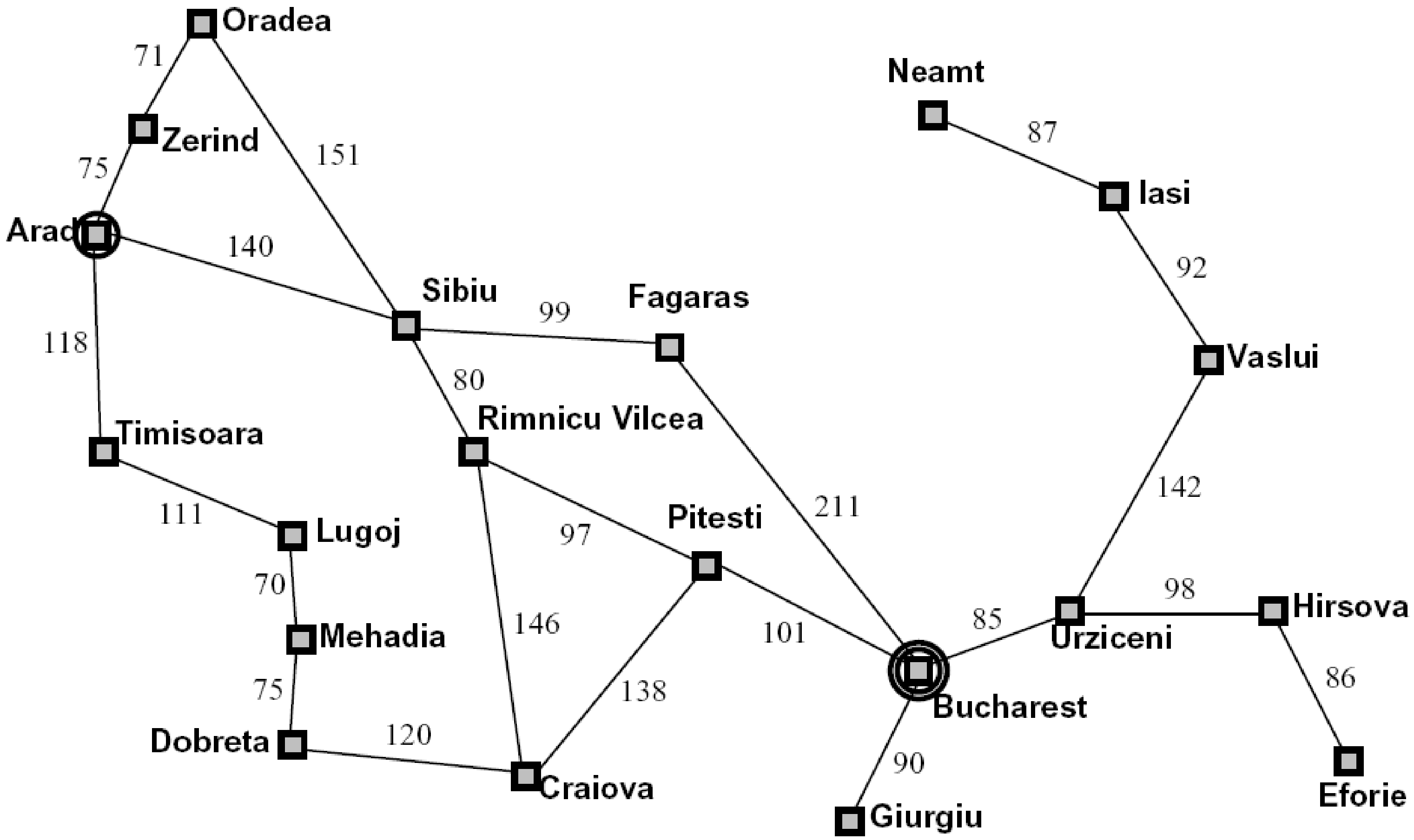


Important: Lots of repeated structure in the search tree!

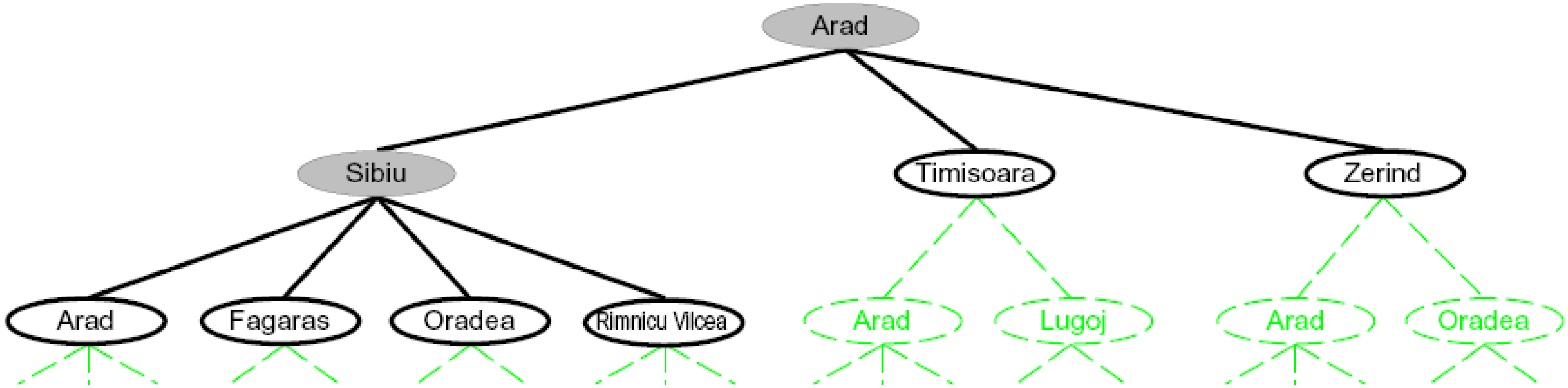
Tree Search



Search Example: Romania



Searching with a Search Tree



- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
    - Fringe
    - Expansion
    - Exploration strategy

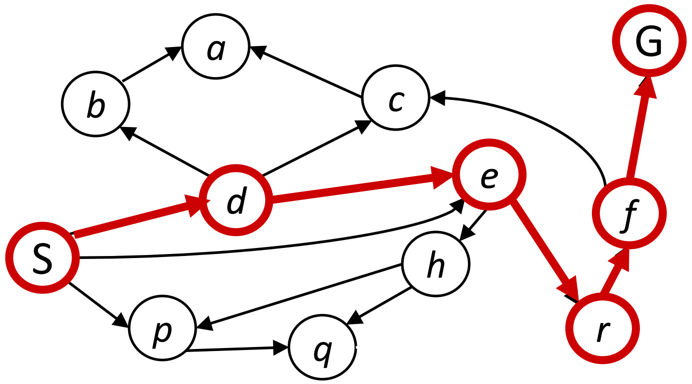
- Main question: which fringe nodes to explore?

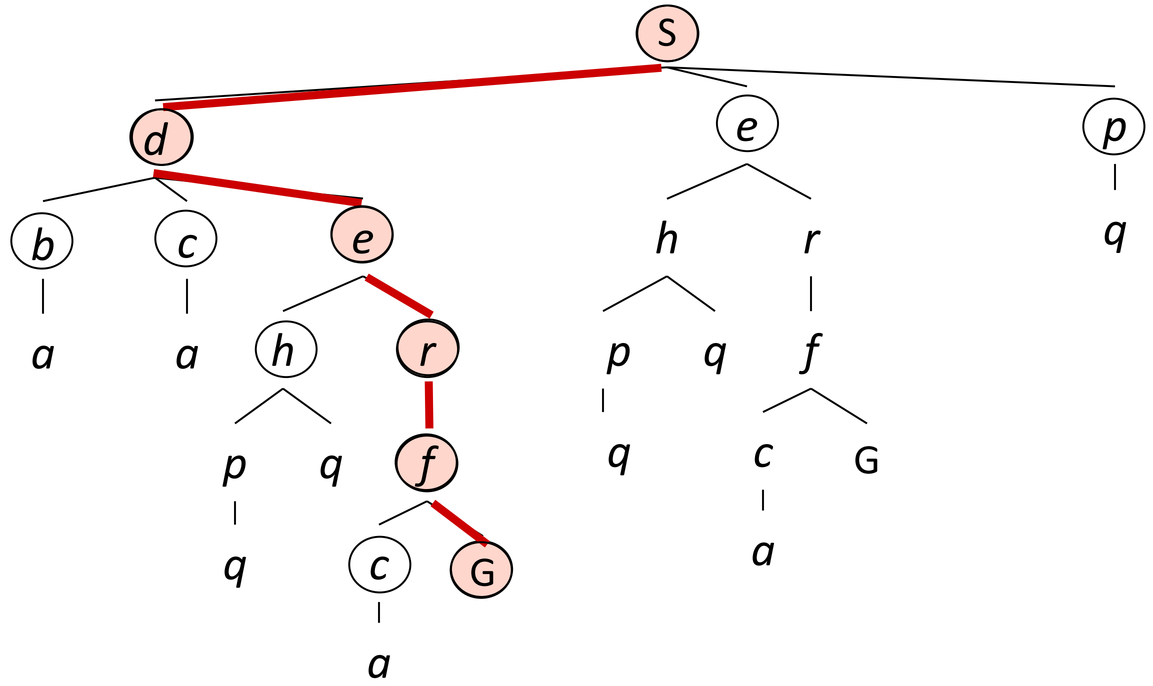
General Tree Search 2

function TREE_SEARCH(problem) returns a solution, or failure

```
    initialize the frontier as a specific work list (stack, queue, priority queue)
    add initial state of problem to frontier
    loop do
        if the frontier is empty then
            return failure
        choose a node and remove it from the frontier
        if the node contains a goal state then
            return the corresponding solution
        for each resulting child from node
            add child to the frontier
```

Example: Tree Search





~~s~~

~~s → d~~

s → e

s → p

s → d → b

s → d → c

~~s → d → e~~

s → d → e → h

~~s → d → e → r~~

~~s → d → e → r → f~~

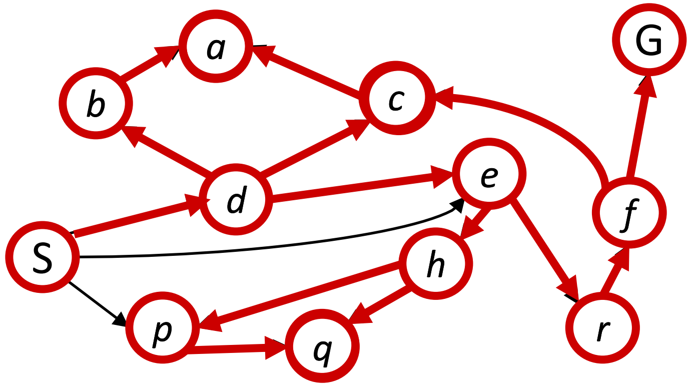
s → d → e → r → f → c

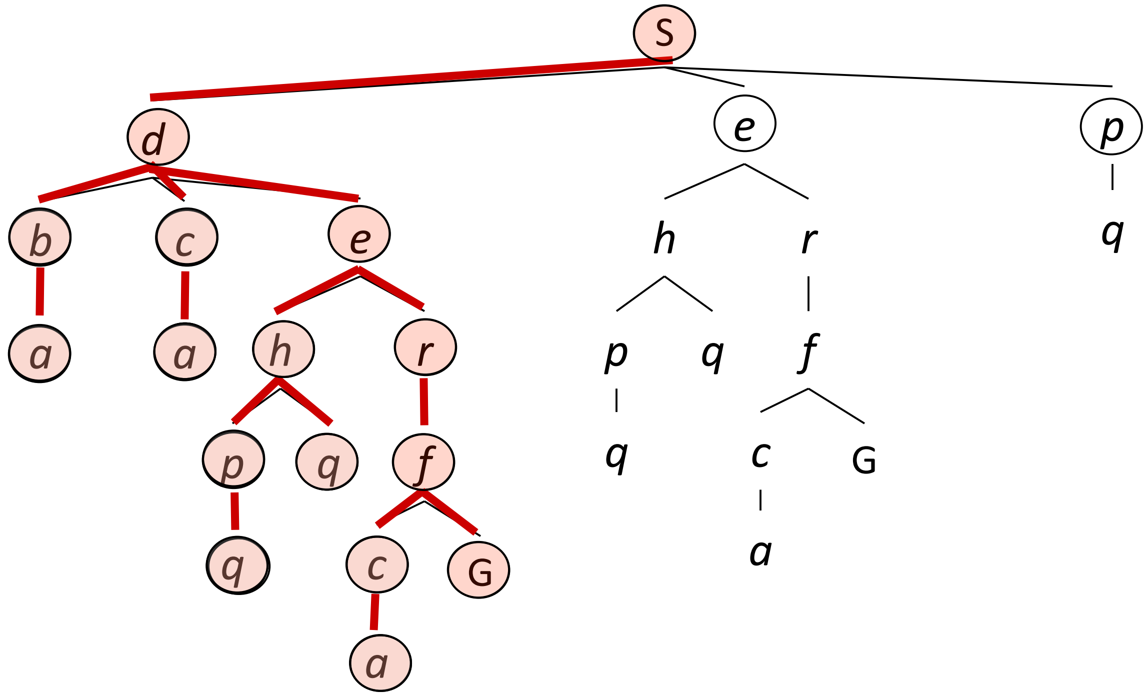
~~s → d → e → r → f → G~~

Depth-First (Tree) Search

Strategy: expand a deepest node first

Implementation: Fringe is a LIFO stack

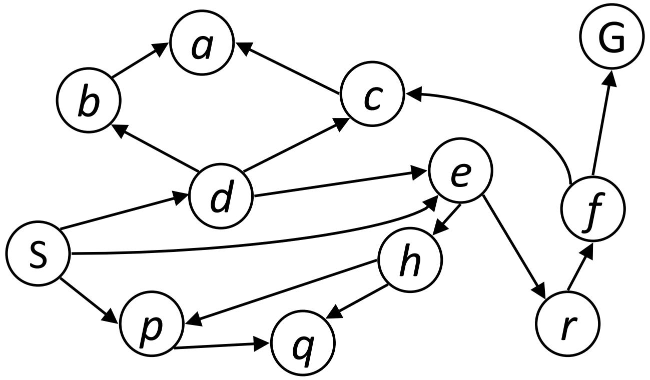


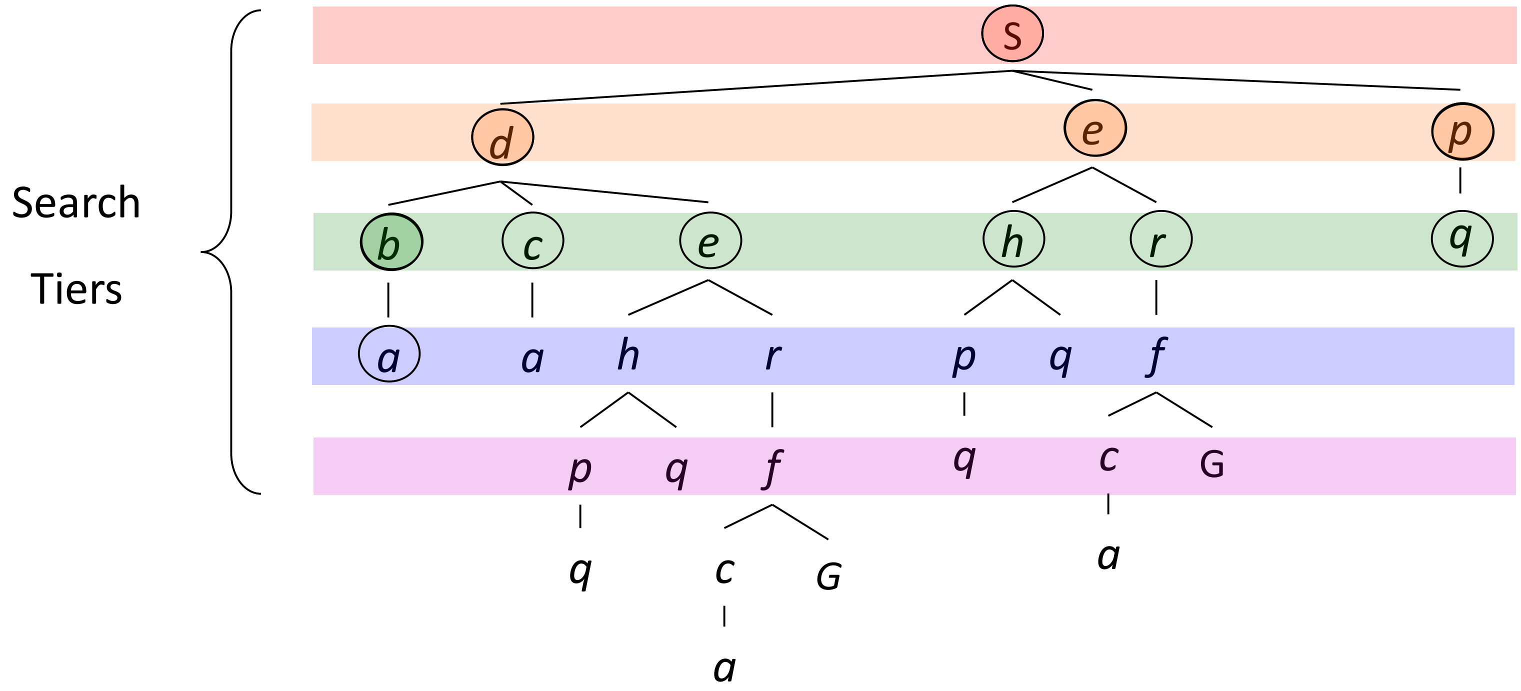


Breadth-First (Tree) Search

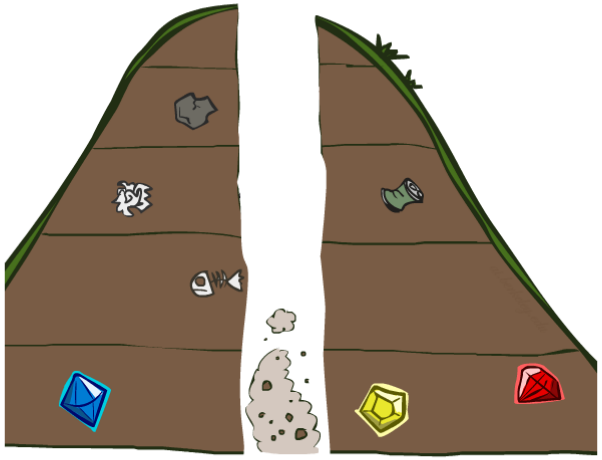
Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue



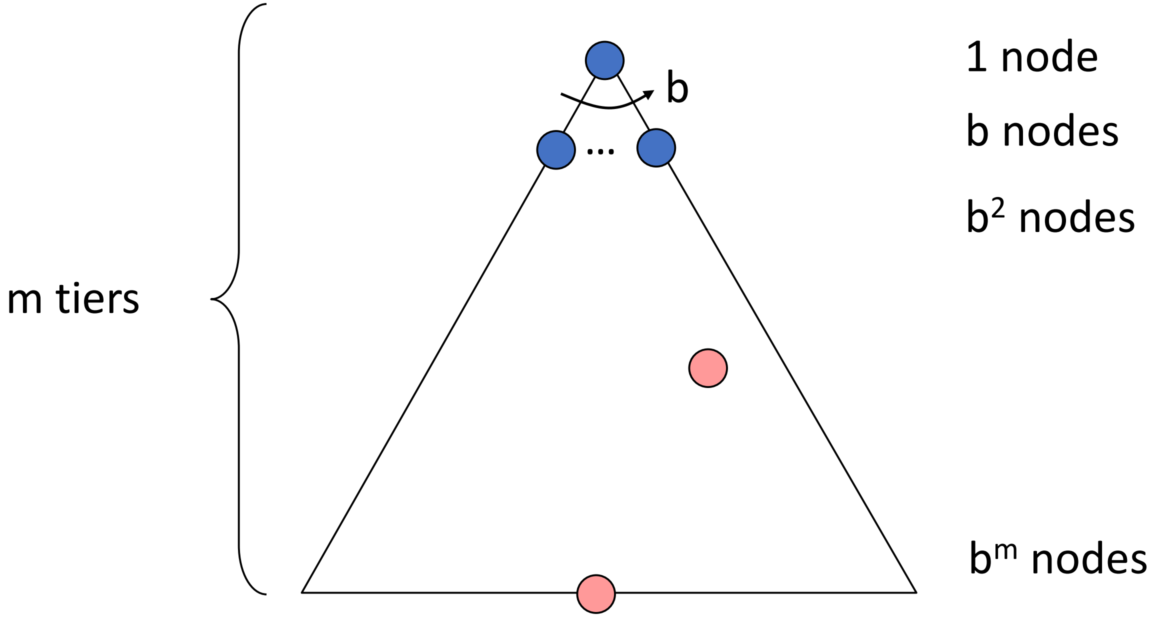


Search Algorithm Properties



- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

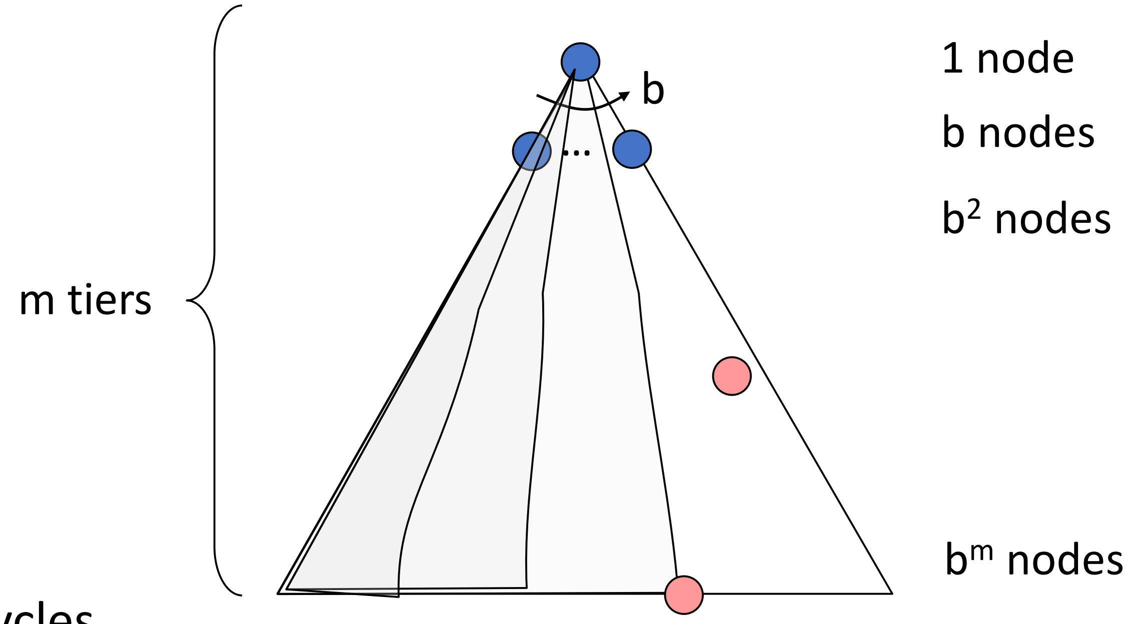
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths



- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots + b^m = O(b^m)$

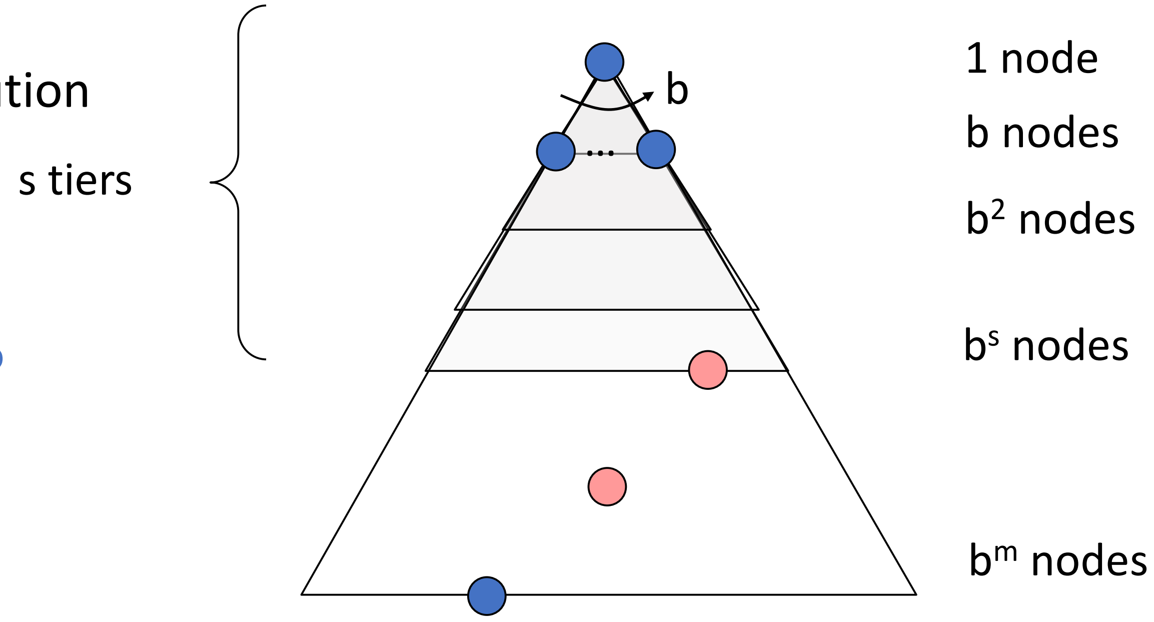
Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so $O(bm)$
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
  - No, it finds the “leftmost” solution, regardless of depth or cost

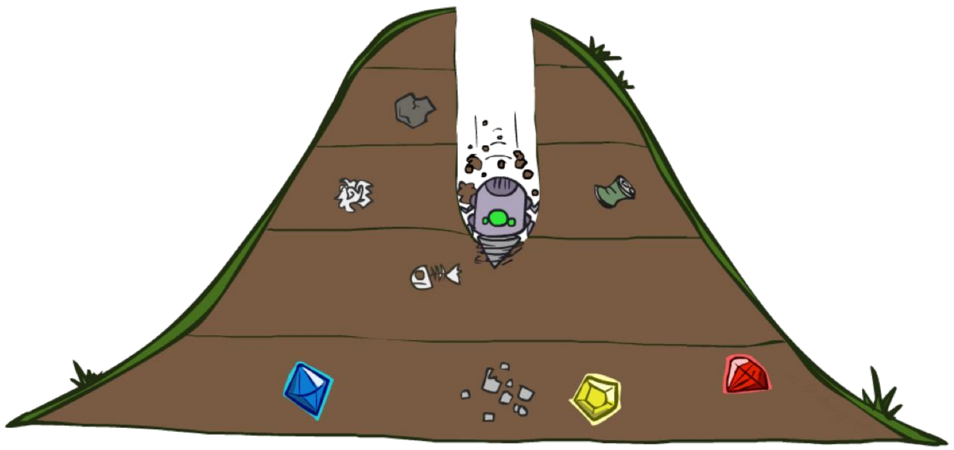


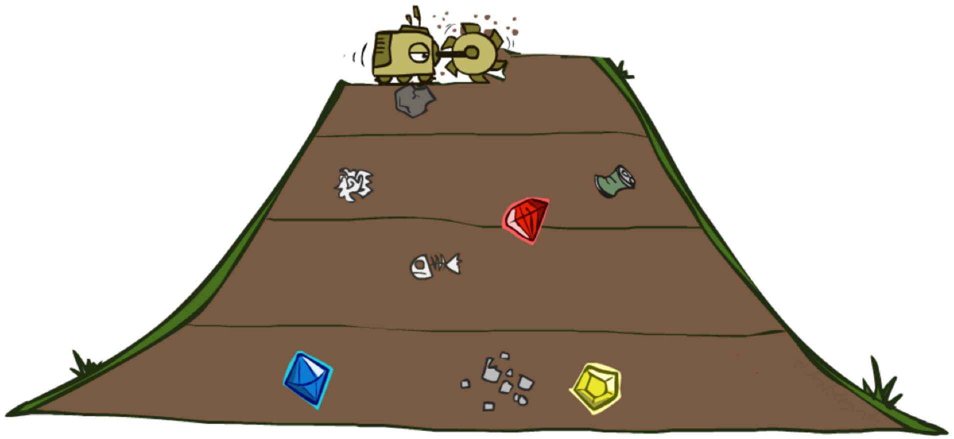
Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^s)$
- Is it complete?
  - s must be finite if a solution exists
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



DFS vs BFS





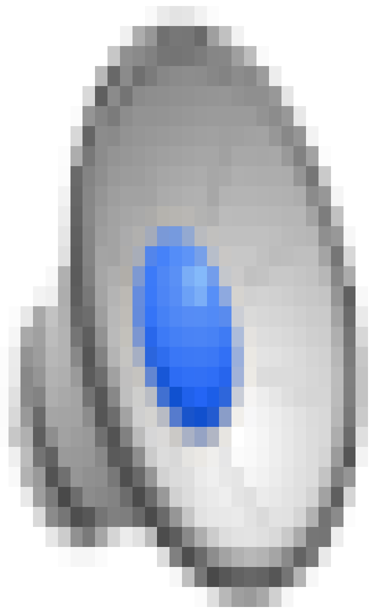
DFS vs BFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?

[Demo: dfs/bfs maze water (L2D6)]

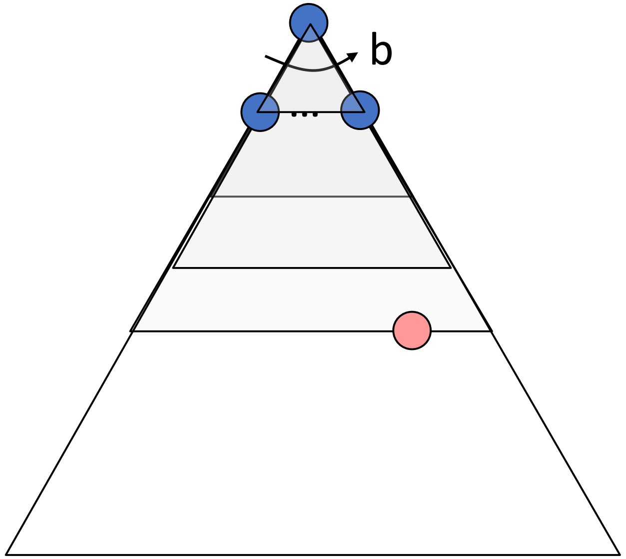
Video of Demo Maze Water DFS/BFS (part 1)

Video of Demo Maze Water DFS/BFS (part 2)



Iterative Deepening

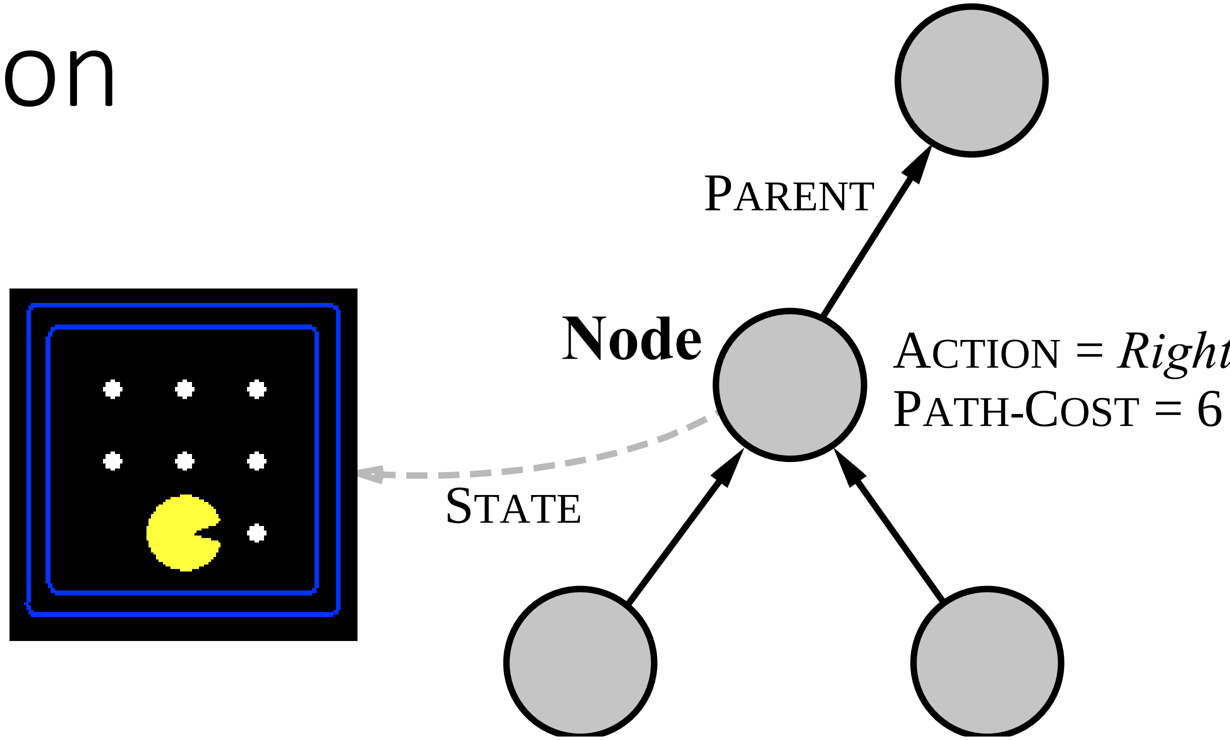
- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..



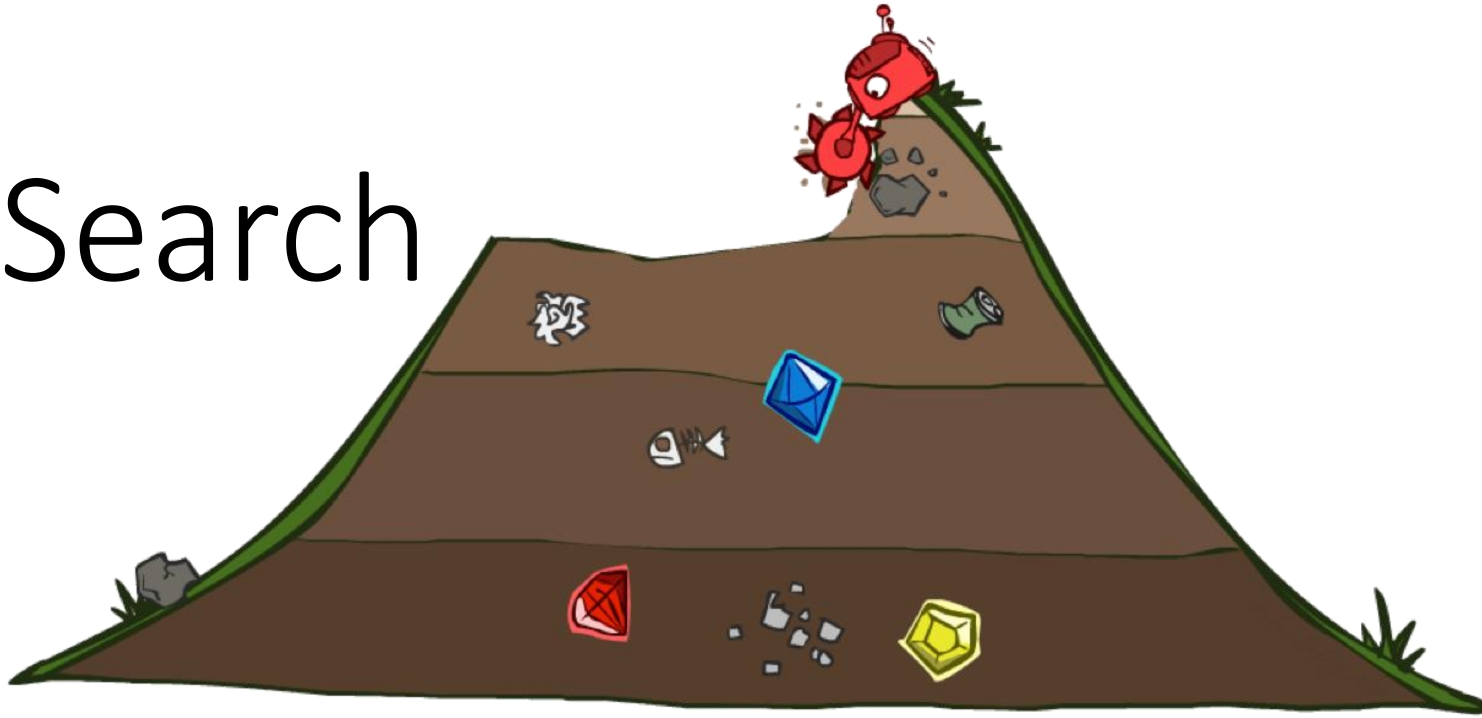
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!

A Note on Implementation

- Nodes have
  - state, parent, action, path-cost
- A child of node by action a has
  - state = Transition(node.state, a)
  - parent = node
  - action = a
  - path-cost = node.path_cost + step_cost(node.state, a, self.state)
- Extract solution by tracing back parent pointers, collecting actions

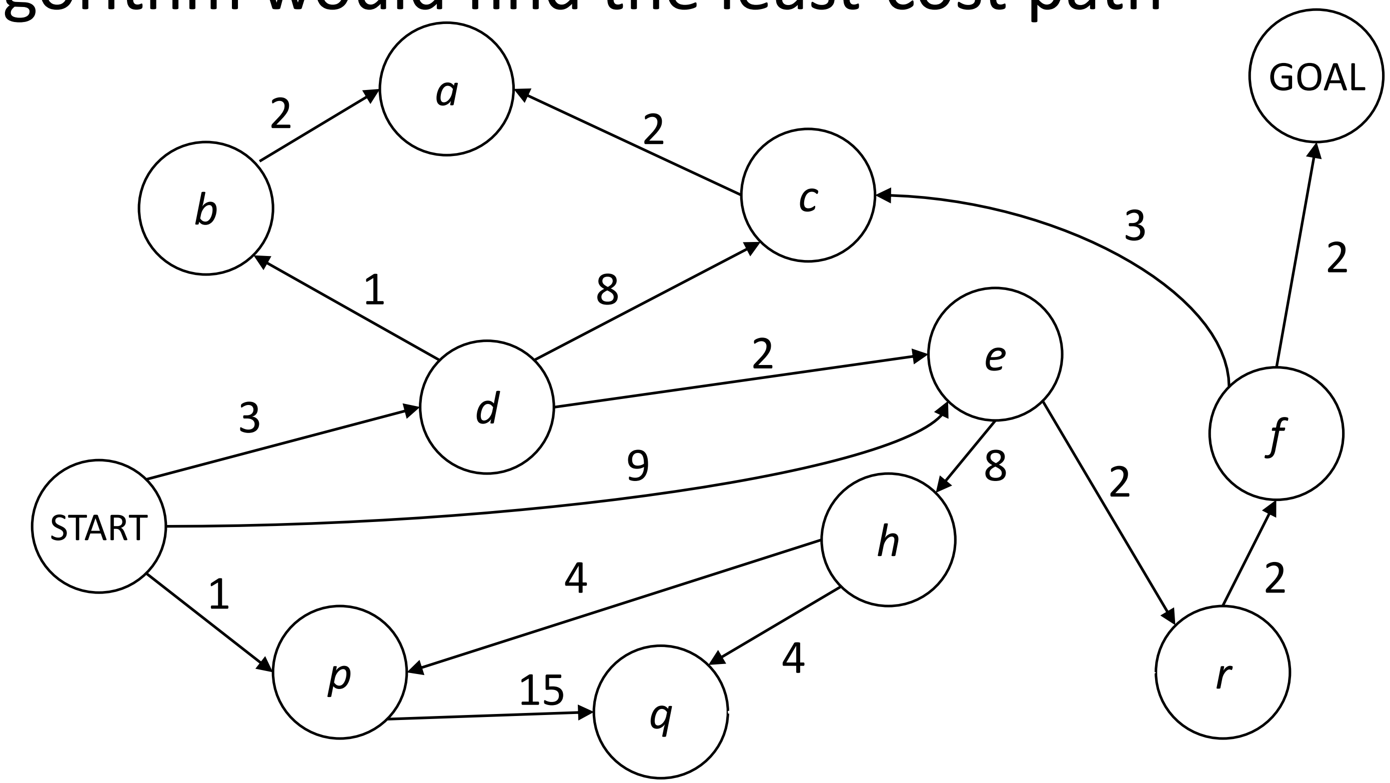


Uniform Cost Search



Finding a Least-Cost Path

- BFS finds the shortest path in terms of number of actions, but not the least-cost path
- A similar algorithm would find the least-cost path

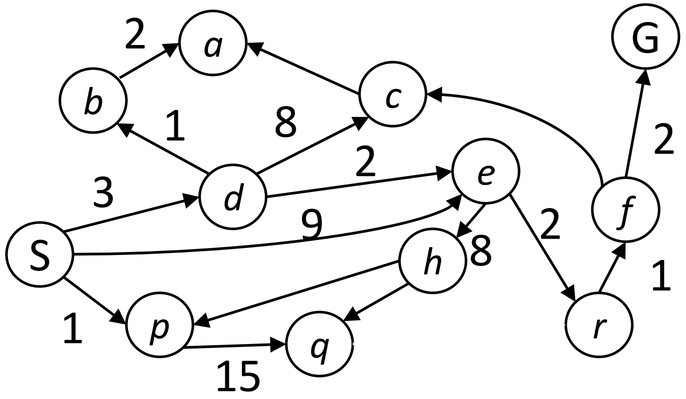


How?

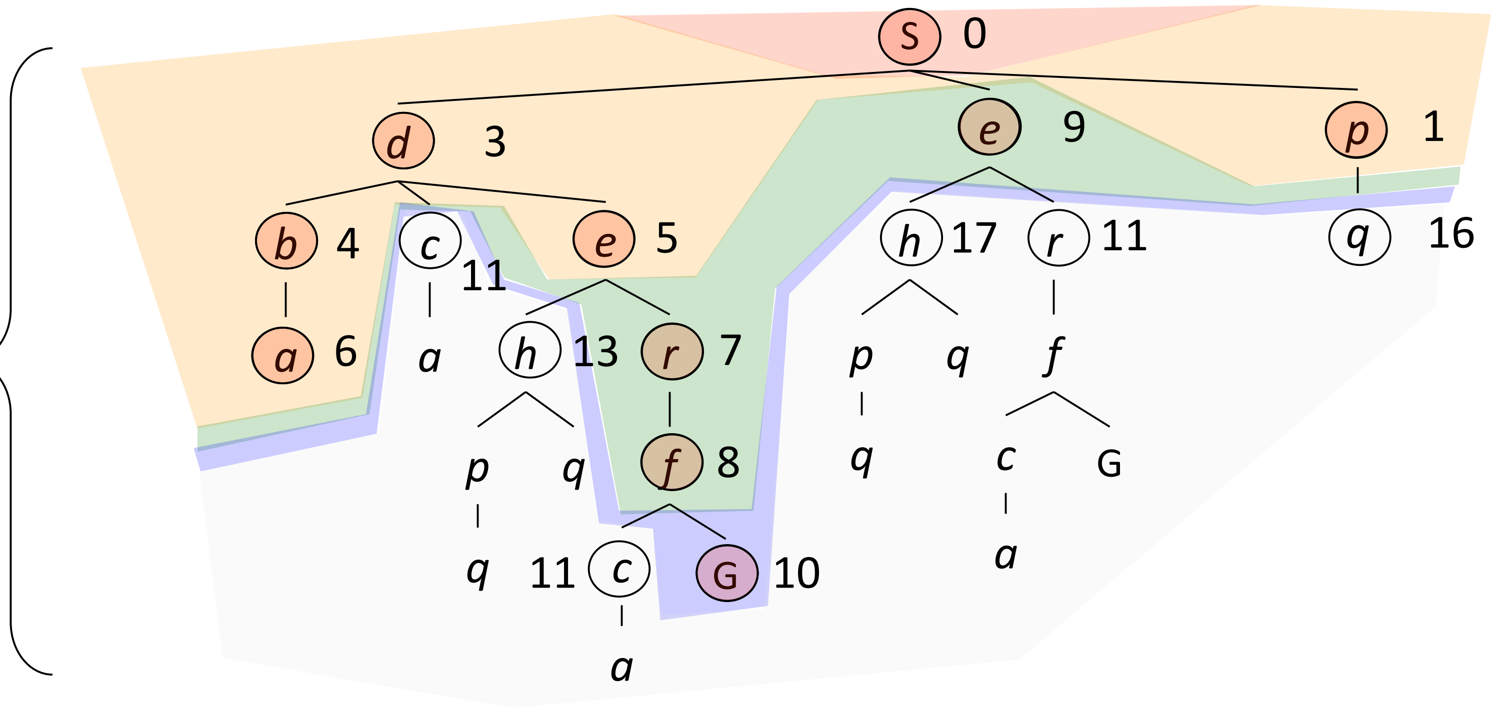
Uniform Cost Search

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)



Cost contours

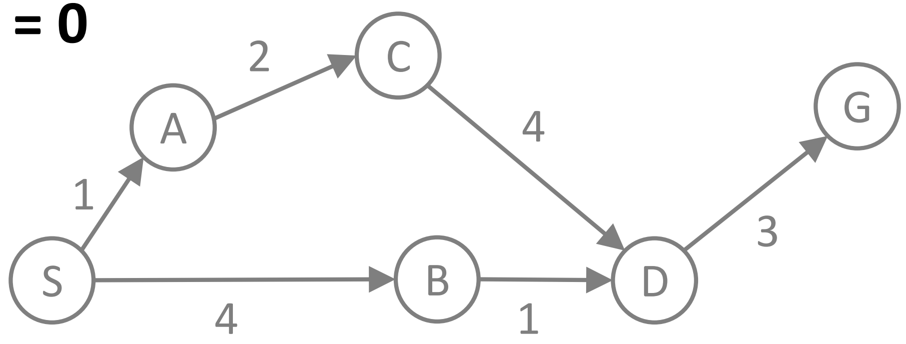


Uniform Cost Search 2

function UNIFORM-COST-SEARCH(problem) returns a solution, or failure

initialize the frontier as a **priority queue using node's path_cost as the priority**

add initial state of problem to frontier **with path_cost = 0**



loop do

if the frontier is empty then

return failure

choose a node (with minimal path_cost) and remove it from the frontier

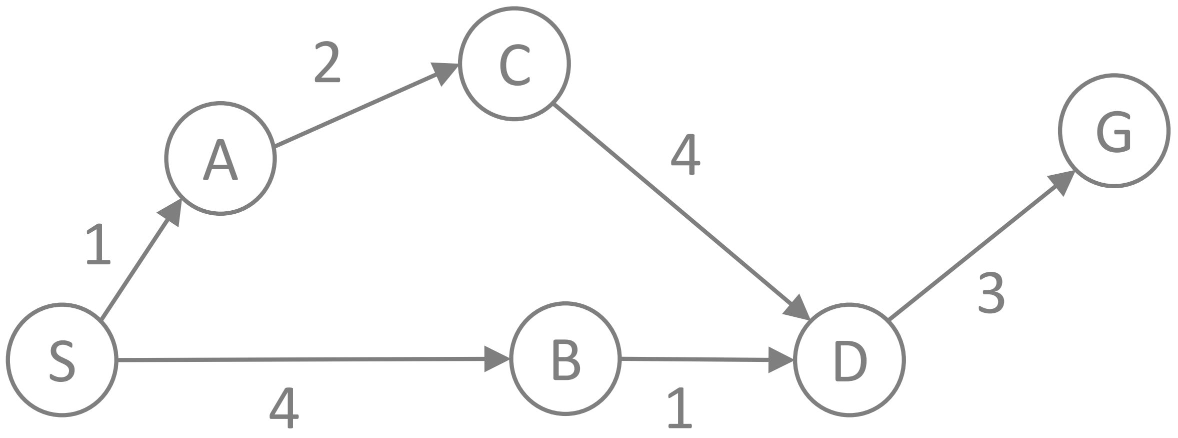
if the node contains a goal state then

return the corresponding solution

for each resulting child from node

add child to the frontier with **path_cost** = **path_cost**(node) + cost(node, child)

Walk-through UCS



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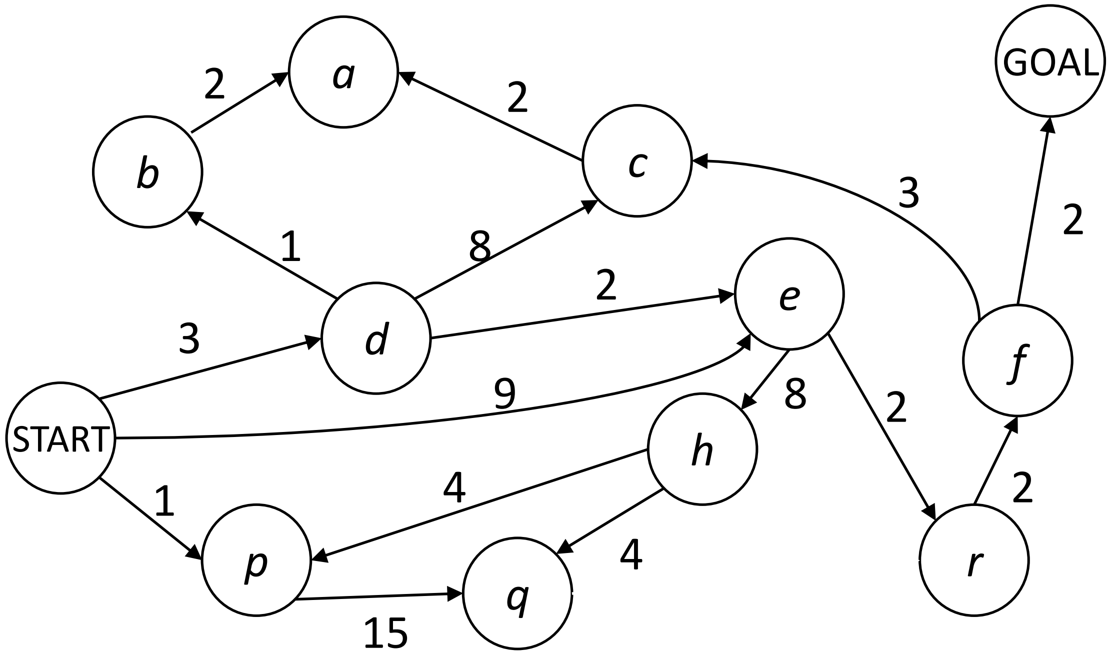
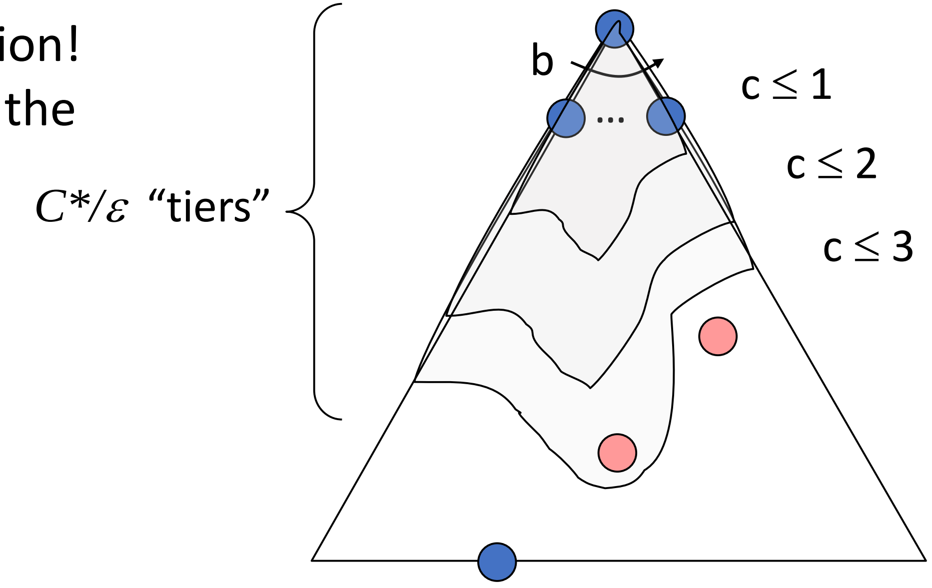


Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε, then the "effective depth" is roughly C^*/ε
  - Takes time O($b^{C^*/\varepsilon}$) (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so O($b^{C^*/\varepsilon}$)
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next via A*)



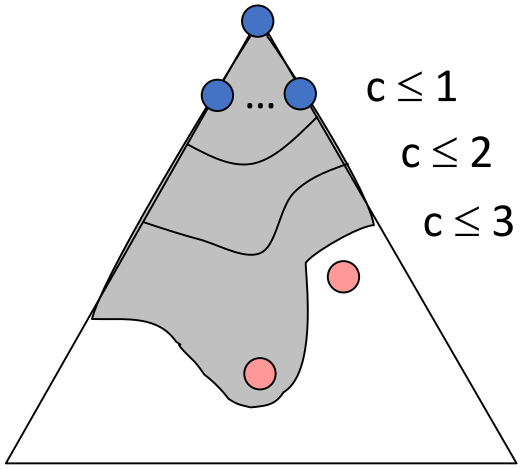
Uniform Cost Issues

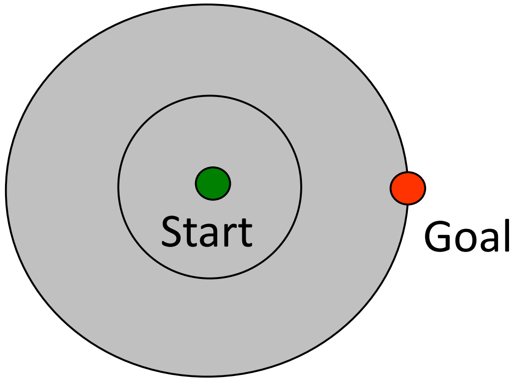
- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

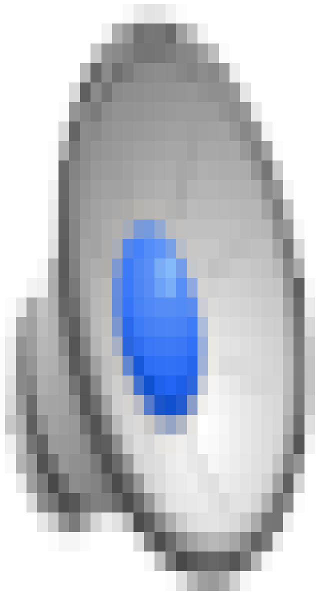
- We’ll fix that soon!



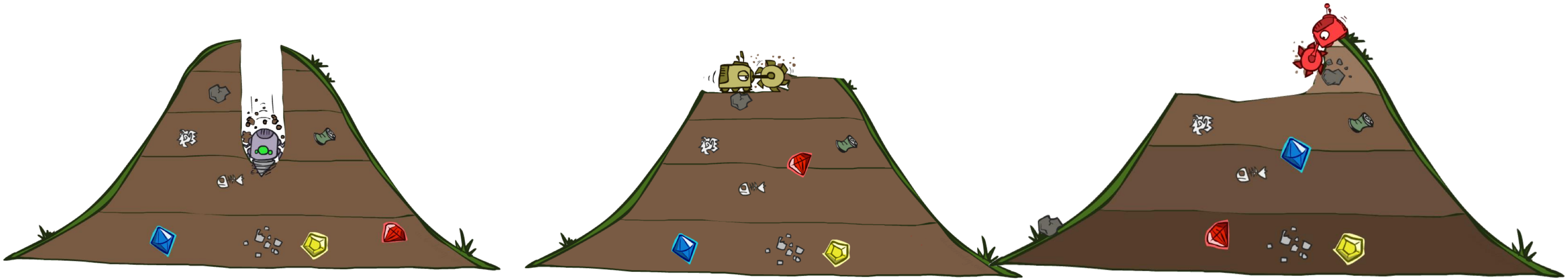


[Demo: empty grid UCS (L2D5)]
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

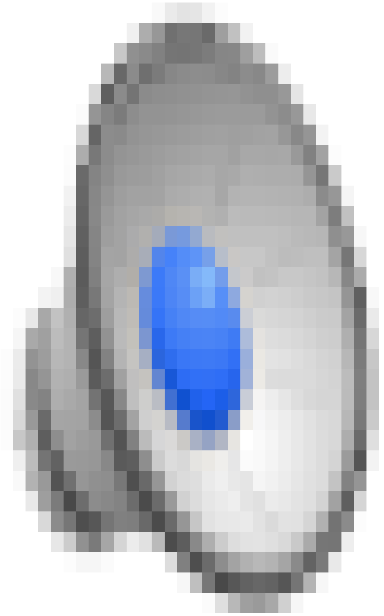
Video of Demo Empty UCS (same cost)



DFS, BFS, or UCS?



Video of Demo Maze with Deep/Shallow Water (part 1)

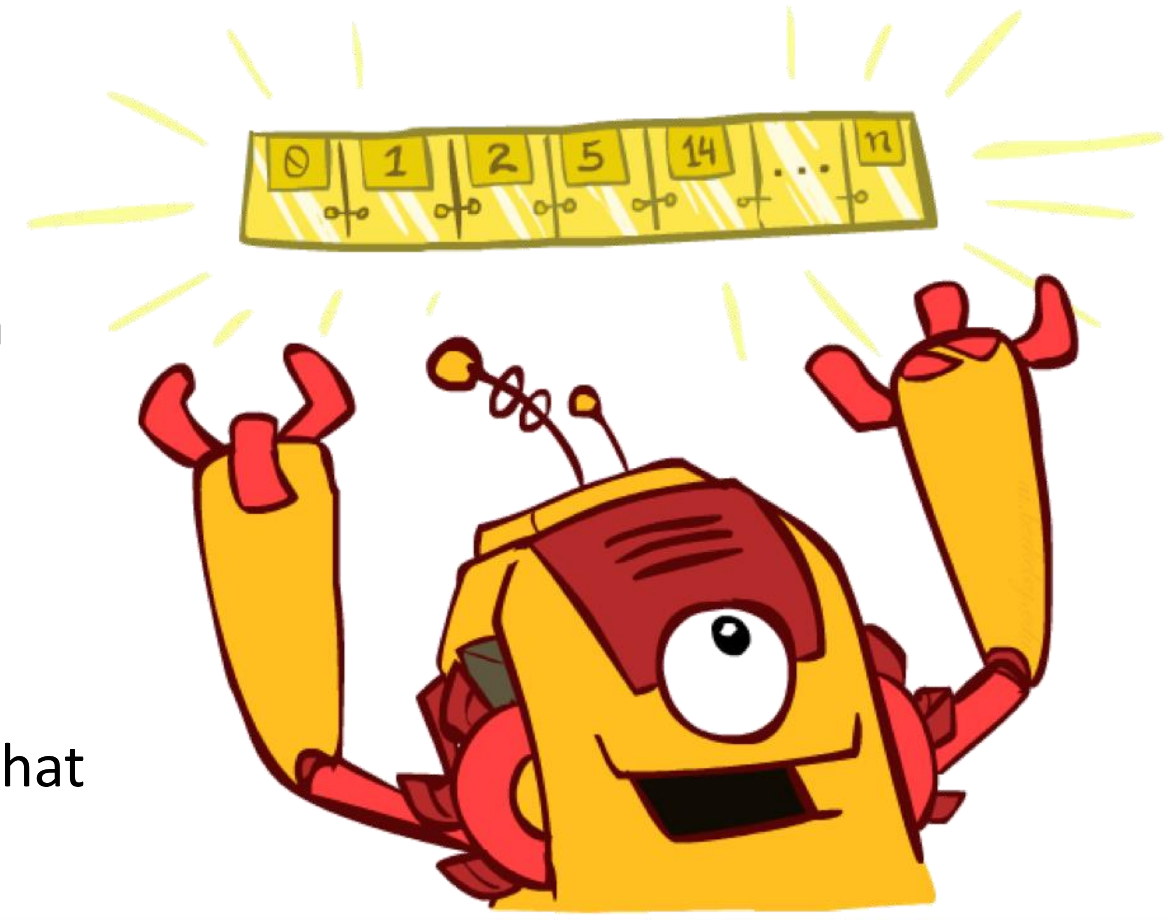


Video of Demo Maze with Deep/Shallow Water (part 2)

Video of Demo Maze with Deep/Shallow Water (part 3)

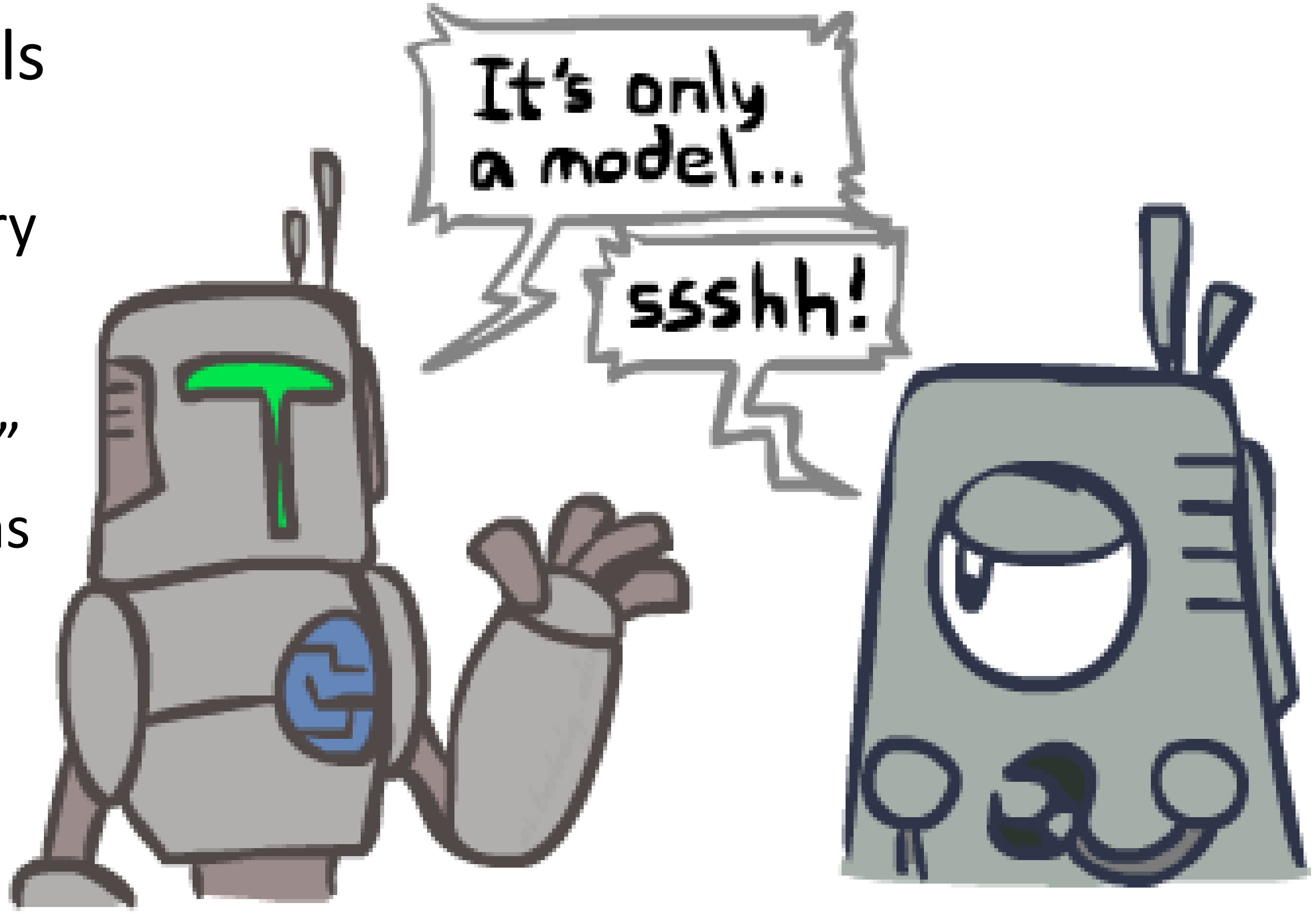
The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object

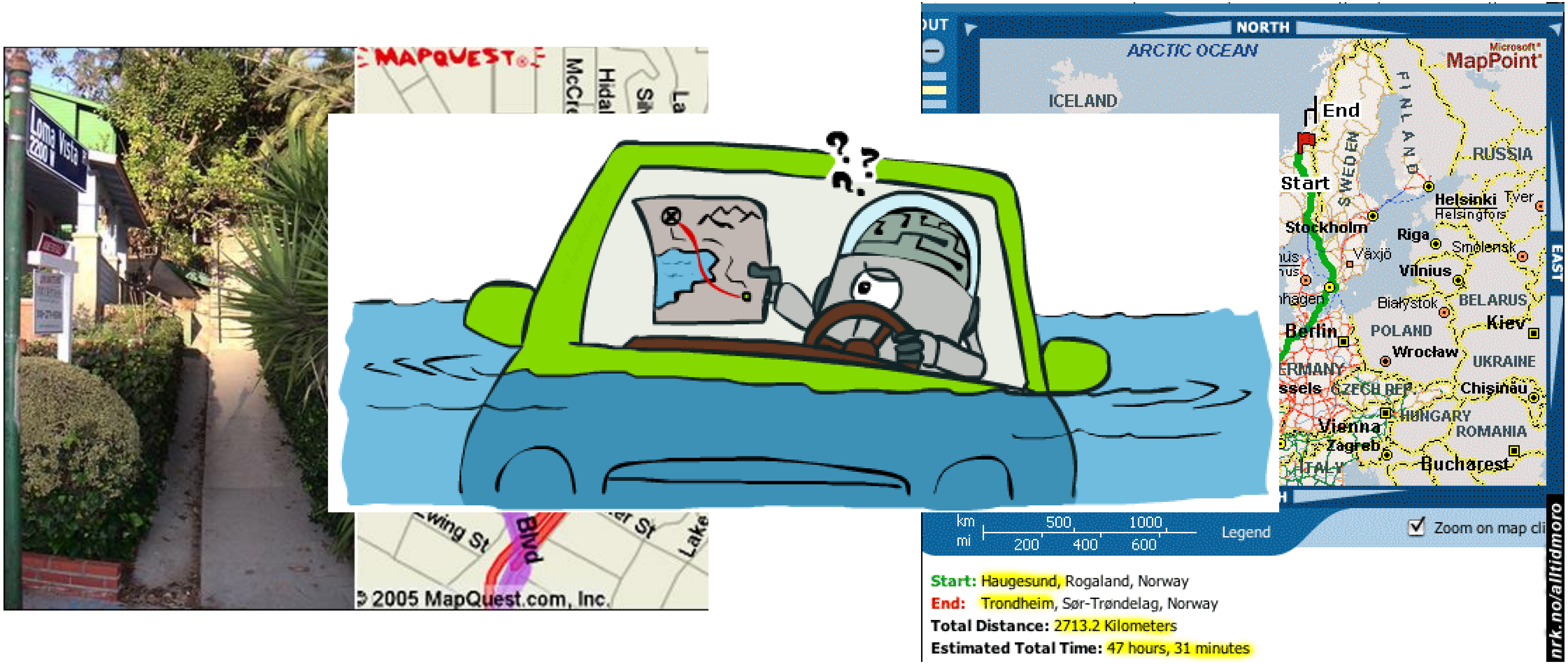


Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Search Gone Wrong?



Summary

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- Rational agents
- Search problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

Questions?