PROBABILITY AND STOCHASTIC PROCESSES CAT 1

Jeremy Gachanja

7/13/2022

1. Peter is in jail and has 20 dollars; he can get out on bail if he has 64 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollar, he wins A dollar with probability 0.55 and loses A dollar with probability 0.45. Find the probability that he wins 8 dollars before losing all of his money if

a. he bets 5 dollars each time (timid strategy).

```
i_1 = 5
n = 8
p=0.55
q=0.45
p_winning = (1-(q/p)^i_1)/(1-(q/p)^n)
print(p_winning)
```

```
## [1] 0.7924987
```

From the above via the timid approach he has a 0.79 chance of wining his freedom

b. he bets, each time, as much as possible but not more than necessary to

bring his fortune up to 64 dollars (bold strategy).

solution

$$P_{20} = 0.55P_{40} + 0.45P_0$$

$$P_{40} = 0.55P_{64} + 0.45P_{16}$$

$$P_{16} = 0.55P_{32} + 0.45P_0$$

$$P_{32} = 0.55P_{64} + 0.45P_0$$

The only known probabilities are P_{64}, P_0 which are 1 and 0 respectively. We can therefore estimate P_{40}, P_{16}, P_{32} recursively from the equation (i) to (iv)

```
p_32 = 0.55*(1)+0.45*(0)
p_16 = 0.55*(p_32)+0.45*(0)
p_40 = 0.55*(1)+0.45*(p_16)
```

```
p_20 = 0.55*(p_40)+0.45*(0)

print(p_20)
```

```
## [1] 0.3773688
```

Therefore the probability of wining using the bold approach would be 0.3773

c. Which strategy gives Smith the better chance of getting out of jail?

solution

The timid appraoch gives him higher chances of wining his freedom with the probability of 0.79

2. Derive the formulas for the Gambler's Ruin Problem for situation when p=q and not p=q . Also describe how to becoming infinitely rich or getting ruined in the same Gambler's Ruin setting [Hint: Check several literatures on google and other text books]

Deriving the formula for $p \neq q$

We start from the foundational equation that $q_z = pq_{z+1} = qq_{z-1}$

Here we can generalise that the solution to this equation can be found such that where A, B are real numbers then we can r

Recall that the above equation can be generalised to $pP_i + qP_i = pP_{i+1} + qP_{i-1}$

Picking the like terms together we get $pP_{i+1} - pP_i = qP_i - qP_{i-1}$

$$p(P_{i+1} - P_i) = q(P_i - P_{i-1})$$

$$P_{i+1} - P_i = \frac{q}{p}(P_i - P_{i-1})$$

In this case lets substitute i with numbers, such that i = 1. This would lead to:

$$P_2 - P_1 = \frac{q}{p}(P_1 - P_0)$$

Recall that $P_0 = 0$ As such the formula reduces to:

$$P_2 - P_1 = \frac{q}{p}(P_1)$$

Consequently we can expand this to i = 2, such that:

$$P_3 - P_2 = \frac{q}{p}(P_2 - P_1) \text{ Recall that } P_2 - P_1 = \frac{q}{p}(P_1)$$

This would thus lead to $P_3 - P_2 = \frac{q}{p}(\frac{q}{p}(P_1)) \rightarrow P_3 - P_2 = (\frac{q}{p})^2 P_1$

$$\text{This can be generalised to} P_{i+1} - P_i = (\frac{q}{p})^i (P_{i+1} - P_{i-1})$$

Given that $q_0 = 1$ and $q_a = 0$ then 1 will be come for i = 0

$$A + B = 1$$

for i = a, 1 will become

$$B(\frac{q}{p})^a = q_a$$

3. Tom starts with 5, and p = 0.63: What is the probability that:

a.Tom obtains a fortune of N = 12 without going broke?

solution

```
n = 12
p=0.63
q=1-p
i = 5
p_win = (1-(q/p)^i)/(1-(q/p)^n)

print(p_win)
```

```
## [1] 0.9316965
```

b. What is the probability that Tom will become infinitely rich?

solution

```
p_infinite_w = 1-(q/p)^5
print(p_infinite_w)
```

```
## [1] 0.9301276
```

From the above output, the chances of becoming infinitely rich are 0.93

c.If Tom instead started with i = 2, what is the probability that he would go broke?

```
n = 12
p=0.63
q=1-p
i = 2
p_win = (1-(q/p)^i)/(1-(q/p)^n)

print(1-p_win)
```

```
## [1] 0.3438182
```

From the above output, the chances of being ruined are 0.34

4. Collins bought a share of stock for 10, and it is believed that the stock price moves (day by day) as a simple random walk with p = 0.6. What is the probability that Collins' stock reaches the high value of $25 before the low value of 4?

```
p = 0.6
q=1-p
i=10
n=25
a=n-i
b=4
p_stock_movement = (1-(q/p)^b)/(1-(q/p)^(a+b))
print(p_stock_movement)
```

```
## [1] 0.8028313
```

From the output above, the chances of the stock going up to 25 before going down to 4 is 0.802