STRATHMORE UNIVERSITY
STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
STA 8405- PROBABILITY AND STOCHASTIC PROCESSES.
Take-away Assignment. July-Sept Semester
Date: Deadline 13th August, 2022. Prepared by Dr. Collins Odhiambo

Instruction: Answer all Questions

Question 1

a) Clearly distinguish the following terms
   i. Stochastic Process and Deterministic Data
   ii. Discrete and Continuous stochastic process.
   iii. σ-algebra and Filtration
   iv. Random Walk and Gambler's Ruin Problem
   v. Counting Process and Branching Process
   vi. Birth and Death Process
   vii. Static Simulation Model and Dynamic Simulation Model
   viii. Mathematical Model and Simulation Model
   ix. Monte Carlo Simulation and Queuing System
   x. Markov and Poisson processes
   xi. Queuing and Renewal processes
   xii. Martingale and Brownian motion
   xiii. Hidden Markov model and semi-Markov process

(13 marks)

b) Collins bought a share of stock for $12, and it is believed that the stock price moves (day by day) as a simple random walk with p = 0.58. What is the probability that Collins' stock reaches the high value of $35 before the low value of $8?

(3 marks)

c) Explain clearly the difference between the following terms as used in Markov Chains
   i. Communicating class and absorption state
   ii. Recurrence and nonrecurrence state
   iii. Periodicity and aperiodic
   iv. Ergodic chain and transient state
   v. Reducible and irreducible

(10 marks)

d) Consider an M/M/1 model at steady state, with μ as the service mechanism rate and λ as the arrival rate. Let $P_n(t)$ = P [n customers in the system at time t*] (Probability that there are n customers at time t).* Derive Lq, Ls, Wq and Ws.

(20 marks)

e) Clearly specify five components of a Hidden Markov Model

(5 marks)

f) Use Chapman Kolmogorov postulates to derive the Poisson Process. Also derive the mean and variance of the Poisson process.

(20 marks)

g) A certain stock price has been observed to follow a pattern. If the stock price goes up one day, there's a 25% chance of it rising tomorrow, a 35% chance of it falling, and a

40% chance of it remaining the same. If the stock price falls one day, there's a 25% chance of it rising tomorrow, a 50% chance of it falling, and a 25% chance of it remaining the same. Finally, if the price is stable on one day, then it has a 50-50 change of rising or falling the next day.

i. Generate the transition matrix
ii. Draw the Markov chain using R
iii. Determine if the chain is Ergodic
iv. Find the limiting distribution of the transition matrix

(8 marks)

h) A telephone attendant receives 110 calls during the busy hour. Each call takes, on average, 2.1 minutes to process.

a) What percentage of the attendant's time is devoted to answering calls?
b) How long must people wait, on average, before their call is processed?

(7 marks)

i) Jobs arrive to a computer system (consisting of a CPU and an I/O device) according to a Poisson process with rate 8 jobs per minute. Once in the system, a job requires on average 30 seconds of CPU time and 9 minutes of I/O time, in which the CPU and I/O time required by the jobs are exponentially distributed.

i. What is the probability that a job will have to wait before being processed by the devices? (Hint: replace the CPU and I/O subsystem as equivalent to single server)
ii. What proportion of time is the system busy?
iii. On average, how many jobs are waiting in line to be processed?
iv. On average, how long will a job spend in the system?
v. What is the probability that exactly 10 jobs arrive to the system in one minute?

(10 marks)

j) Consider a Markov chain with two possible states, S = {0, 1}. In particular, suppose that the transition matrix is given by

$$p = \begin{bmatrix} 1-\propto & \propto \\ \beta & 1-\beta \end{bmatrix}$$

Show that

$$P^n = \frac{1}{\propto +\beta}\begin{bmatrix} \beta & \propto \\ \beta & \propto \end{bmatrix} + \frac{(1-\propto -\beta)^n}{\propto +\beta}\begin{bmatrix} \propto & -\propto \\ -\beta & \beta \end{bmatrix}$$

(4 marks)