Probability and Stochastic Processes Assignment 2

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A

On mixing the 2 samples the resulting mean and variance will be a function of the weighted means and variances of the samples respectively.

$$Var(x) = E(X^2) - [E(X)]^2$$

$$E(X^2) = var(X) + [E(X)]^2$$

$$=> [\delta_1\sigma_1^2 + \delta_2\sigma_2^2] + [\delta_1\mu_1^2 + \delta_2\mu_2^2]$$

Thus:

$$=> E(X^2) = \delta_1(\mu_1^2 + \sigma_1^2) + \delta_2(\mu_2^2 + \sigma_1^2)$$

$$E(X) = \mu_1\delta_1 + \mu_2\delta_2$$

$$=> E(X^2) = (\mu_1\delta_1 + \mu_2\delta_2)^2$$

$$=> E(X^2) = \mu_1^2\delta_1^2 + \mu_2^2\delta_2^2 + 2\delta_1\delta_2\mu_1\mu_2$$

$$Var(X) = [\delta_1(\mu_1^2 + \sigma_1^2) + \delta_2(\mu_2^2 + \sigma_2^2)] - [\mu_1^2\delta_1^2 + \mu_2^2\delta_2^2 + 2\delta_1\delta_2\mu_1\mu_2]$$

$$=> \delta_1\mu_1^2 + \delta_1\sigma_1^2 + \delta_2\mu_2^2 + \delta_2\sigma_2^2 - \mu_1^2\delta_1^2 - \mu_2^2\delta_2^2 - 2\delta_1\delta_2\mu_1\mu_2$$

$$(\mu_1 - \mu_2)^2 = \mu_1^2 - 2\mu_1\mu_2 + \mu^2$$

Therefore:

$$=> \delta_1\sigma_1^2 + \delta_2\sigma_2^2 + \delta_1\delta_2(\mu_1 - \mu_2)^2$$

B

Recall that:

$$Var(X) = \delta_1\sigma_1^2 + \delta_2\sigma_2^2 + \delta_1\delta_2(\mu_1 - \mu_2)^2$$

Given that they are poisson processes $var(X) = E(X) = \lambda_1$ and λ_2:

$$\delta_1\lambda_1 + \delta_2\lambda_2 + \delta_1\delta_2(\lambda_1 - \lambda_2)^2$$

And:

$$\text{E(mix poisson)} = \delta_1\lambda_1 + \delta_2\lambda_2$$

Thus:

$$=> E(X) + \delta_1\delta_2(\lambda_1 - \lambda_2)^2$$

which prooves that:

$$var(X) > E(X)$$