

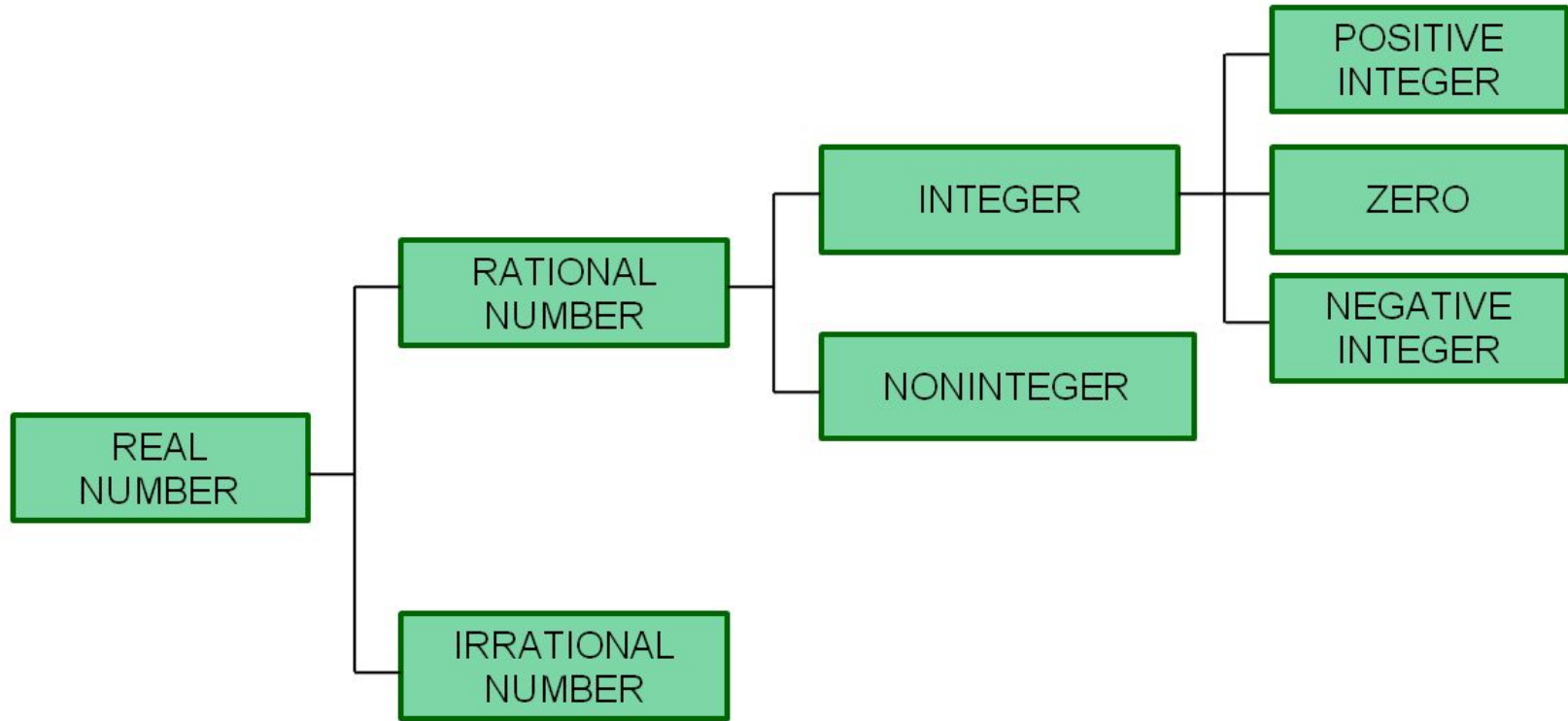
MODULE 5:

GENERAL MATHEMATICS

At the end of the module, you should be able to:

1. Classify numbers into different sets in the real number system and identify the property illustrated in a given statement;
2. Perform Operations on whole numbers and solve problems involving whole numbers;
3. Identify the greatest common factor (GCF) and the least common multiple (LCM) of two or more numbers;
4. Perform Operations on integers and solve problems involving integers;
5. Solve problems on fractions and percentages; and
6. Apply ratio and proportion concepts in word problems.

☐ REAL NUMBER SYSTEM



SECTION 5.1: REAL NUMBERS

RATIONAL NUMBER

→ number that can be expressed as ratio or quotient of two integers

→ either repeating or terminating decimal

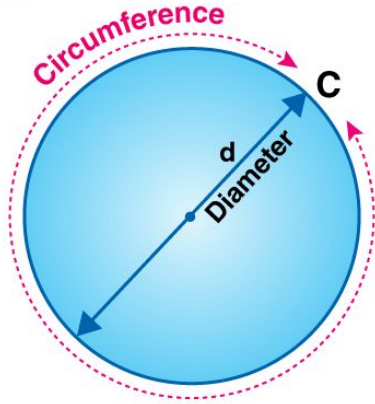
Examples: $\frac{1}{2}$, $\frac{3}{4}$, 6, etc.

IRRATIONAL NUMBER

→ number that cannot be expressed as ratio or quotient of two integers

→ both nonrepeating and nonterminating decimal

Examples: π , e , $\sqrt{2}$, etc.



Circumference ≈ 745.30201
Diameter = 237.237

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159.....$$

SECTION 5.1: REAL NUMBERS

INTEGER \rightarrow nonfraction

Examples: $-4, 0, 5$, etc.

\Rightarrow POSITIVE INTEGER (also called natural number or counting number)

Example: $1, 2, 3, 4, \dots$

\Rightarrow ZERO

\Rightarrow NEGATIVE INTEGER

Examples: $-1, -3, -5, -7, \dots$

Note: The set of whole numbers is the set that results when zero is combined with positive integers.

SECTION 5.1: REAL NUMBERS

NON INTEGER → fractions in the lowest terms

Examples: $\frac{2}{3}$, $\frac{3}{5}$, etc.

OTHER SETS OF NUMBERS

NUMBER	Description	Example
Even Number	Positive integer that is a multiple of 2	4, 20, 82, etc.
Odd Number	Positive integer that is not a multiple of 2	3, 5, 27, etc.
Prime Number	Positive integer other than 1 that has 1 and the integer itself as the only two distinct factors	7, 11, 29, etc.
Composite Number	a counting number which can be expressed as the product of numbers other than 1 and itself.	4, 6, 20, etc.

PROPERTIES OF REAL NUMBERS

A. Closure Property

A.1) Addition: When two real numbers are added, the result is still a real number.

A.2) Multiplication: When two real numbers are multiplied, the result is still a real number.

B. Commutative Property

B.1) Addition: $a + b = b + a$ Example: $2+3 = 5 = 3+2$

B.2) Multiplication: $ab = ba$ Example: $(2)(3)=6=(3)(2)$

C. Associative Property

C.1) Addition: $(a + b) + c = a + (b + c)$ Example $(2+3)+4=9=2+(3+4)$

C.2) Multiplication: $(ab)c = a(bc)$ Example: $(2 \times 3)4=24=2(3 \times 4)$

D. Distributive Property: $a(b + c) = ab + ac$

E. Identity Property

E.1) Addition: $a + 0 = a$ Example: $7+0=7$

$0 + a = a$ Example: $0+7=7$

E.2) Multiplication: $a(1) = a$ Example: $12(1)=12$

$(1)a = a$ Example: $(1)12=12$

F. Inverse Property

F.1) Addition: $a + (-a) = 0$ Example: $4+(-4)=0$

$(-a) + a = 0$ Example: $-10+10=0$

F.2) Multiplication: $a(1/a) = 1$ Example: $5/4 (\frac{4}{5}) = 1$

$(1/a)a = 1$

G. Multiplication Property of Zero

→Any number multiplied by zero is zero.

SECTION 5.2: OPERATIONS ON WHOLE NUMBERS

FUNDAMENTAL OPERATIONS

A. ADDITION

Augend – the number that precedes the operation

Addend – the number that follows the operation

Sum – the result

Parts of Addition

The diagram shows the equation $2 + 3 = 5$ in large blue font. Below the number 2 is a red arrow pointing up to it, with the label "Addend" below the arrow. Below the number 3 is a red arrow pointing up to it, with the label "Addend" below the arrow. Below the number 5 is a red arrow pointing up to it, with the label "Sum or Total" below the arrow.

$$2 + 3 = 5$$

Addend Addend Sum or Total

SECTION 5.2: OPERATIONS ON WHOLE NUMBERS

B. SUBTRACTION

Minuend – the number from which another is to be subtracted

Subtrahend – the number to be subtracted

Difference – the result

$$6 - 1 = 5$$

The diagram illustrates the subtraction equation $6 - 1 = 5$. Below the equation, three blue arrows point upwards to the numbers 6, 1, and 5. Each arrow is positioned above a label in a rectangular box: 'Minuend' under the 6, 'Subtrahend' under the 1, and 'Difference' under the 5.

SECTION 5.2: OPERATIONS ON WHOLE NUMBERS

C. MULTIPLICATION

Multiplicand – the number to be multiplied

Multiplier – the number of times another number is multiplied

Product – the result

$$\begin{array}{rcl} \text{Factors} & \left\{ \begin{array}{l} 14 \\ \times 2 \\ \hline \end{array} \right. & \begin{array}{l} \text{- multiplicand} \\ \text{- multiplier} \end{array} \\ \text{Product -} & 28 & \end{array}$$

SECTION 5.2: OPERATIONS ON WHOLE NUMBERS

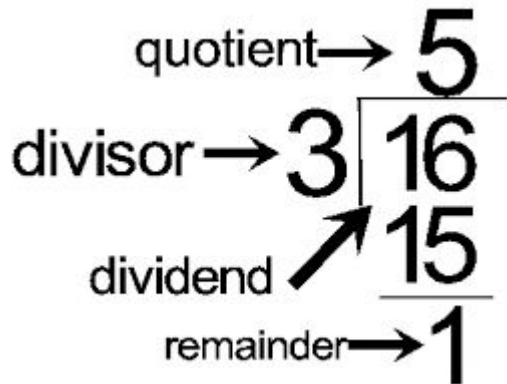
D. DIVISION

Dividend – the number to be divided

Divisor – the number that is used to divide another

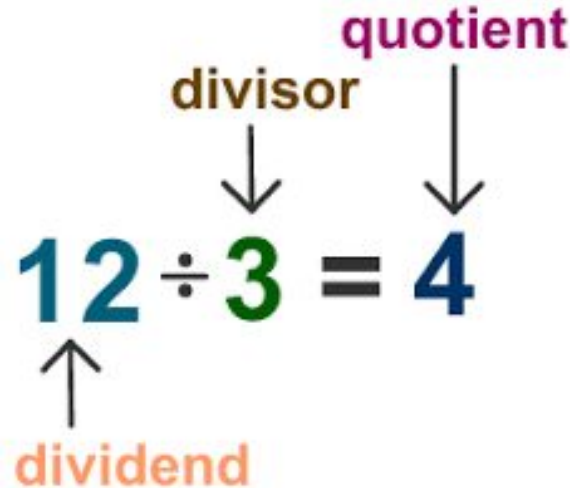
Quotient – the result

Remainder – the excess



A long division diagram showing 16 divided by 3. The divisor 3 is on the left, and the dividend 16 is on the right. A horizontal line is drawn above the 6. The quotient 5 is written above the line. A vertical line is drawn from the 5 down to the 6. The product 15 is written below the 16. An arrow points from the label 'dividend' to the 16. An arrow points from the label 'divisor' to the 3. An arrow points from the label 'quotient' to the 5. An arrow points from the label 'remainder' to the 1.

$$\begin{array}{r} \text{quotient} \rightarrow 5 \\ \text{divisor} \rightarrow 3 \overline{) 16} \\ \text{dividend} \nearrow 15 \\ \text{remainder} \rightarrow 1 \end{array}$$



A division equation diagram showing 12 divided by 3 equals 4. The dividend 12 is in blue, the divisor 3 is in green, and the quotient 4 is in blue. Arrows point from the labels 'dividend', 'divisor', and 'quotient' to their respective numbers in the equation.

$$\begin{array}{c} \text{dividend} \uparrow 12 \div 3 = 4 \downarrow \text{quotient} \\ \text{divisor} \downarrow \end{array}$$

MODULE 5: *Activity 1*

1. What is the most specific classification of real numbers that the following numbers belong to?

i. 5.67

VII. 4

ii. $-\sqrt{6}$

VIII. $\frac{6}{9}$

iii. $\frac{9}{5}$

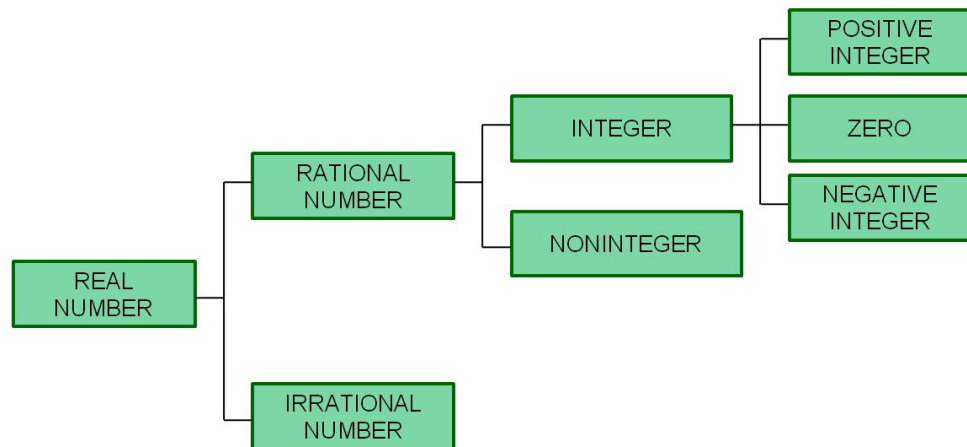
IX. π

iv. 0

v. -75

vi. $\sqrt{16}$

□ REAL NUMBER SYSTEM



2. Identify the property used.

I. $2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$

II. $(10 + 5) + 3 = 10 + (5 + 3)$

III. $10 + (4 + 11) = 10 + (11 + 4)$

IV. $(4 + 6)/2 \cdot 2/(4 + 6) = 1$

V. $12/13 + (-12/13) = 0$

3. Dianne has Php 24, 500. If she writes a check for Php 15, 355, how much does she have left in her account?
4. Wayne works as a tutor and is paid Php 245 per hour . If he tutors 18 hours per week, what is his weekly earnings?
5. How many 250-milliliter glasses can Sabrina fill from a 2-liter bottle of soda?
6. Erick bought 3 kilograms of mango sold for Php 60 per kilogram and 2 kilograms of banana sold for Php 50 per kilogram. How much was the change he got from the 500-peso bill he gave to the vendor?

SECTION 5.3: THE GREATEST COMMON FACTOR (GCF) AND THE LEAST COMMON MULTIPLE (LCM) OF TWO OR MORE NUMBERS

DIVISIBILITY, FACTORS, DIVISORS, AND MULTIPLES

These four terms used in mathematics can best be understood through an example. For the statement $6 \times 7 = 42$, the following are true:

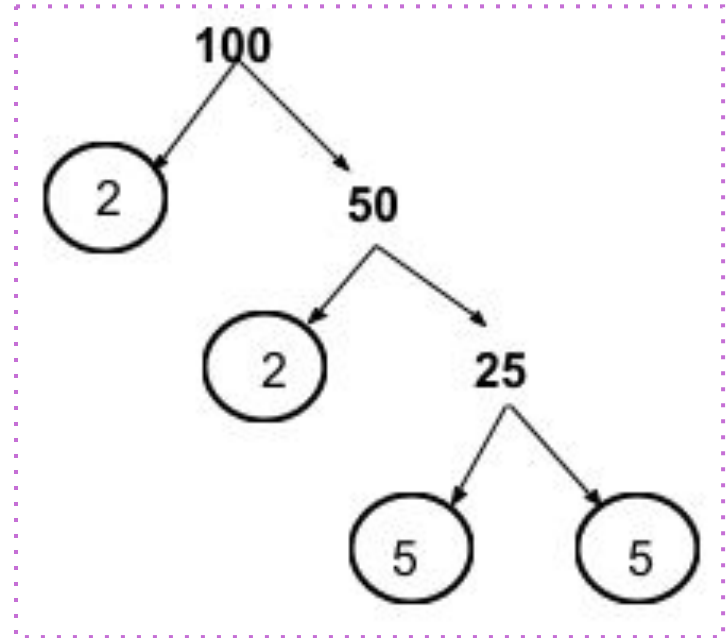
- 42 is divisible by 6 and 7. (The product is divisible by each of the factors.)
- 42 is a multiple by 6 and 7.
- 6 and 7 are factors of 42.
- 6 and 7 are divisors of 42.

PRIME FACTORIZATION OF A NUMBER

Every counting number, greater than 1, can be expressed as a product of prime factors in a unique way.

Example 1: Express 100 into its prime factors.

Solution: A tree diagram may be drawn to find the prime factors.



GREATEST COMMON FACTOR (GCF) OF TWO OR MORE NUMBERS

The greatest common factor (GCF) of two or more numbers is the largest number which is a factor of each of them. There are ways on how the GCF can be obtained.

Example 1: What is the GCF of 16 and 24?

Ways of Finding the GCF:

A. Factoring each number into primes, then multiplying the common factors:

$$\text{Factors of 16: } 16 = 2 \cdot 2 \cdot 2 \cdot 2$$

$$\text{Factors of 24: } 24 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$\text{GCF}(16, 24) = 2 \cdot 2 \cdot 2 = 8$$

B. Dividing each number by their common divisors, then multiplying these common divisors:

$$\begin{array}{r|l} 2 & 16 \quad 24 \end{array}$$

$$\begin{array}{r|l} 2 & 8 \quad 12 \end{array}$$

$$\begin{array}{r|l} 2 & 4 \quad 6 \end{array}$$

$$\begin{array}{l} 2 \quad 3 \end{array}$$

$$\text{GCF}(16, 24) = 2 \cdot 2 \cdot 2 = 8$$

Example 2: What is the GCF of 36, 54, and 72?

Example 2: What is the GCF of 36, 54, and 72?

Solution:

2		36	54	72
3		18	27	36
3		6	9	12
		2	3	4

Answer: The GCF is $2 \times 3 \times 3$ which is 18.

LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS

The least common multiple (LCM) of two or more numbers is the smallest counting number which is a multiple of each of them. Following are the ways on how to find the LCM of two numbers.

Example: Find the LCM of 16 and 24.

Ways of Finding the LCM of Two or More Numbers

- A. Factoring each number into primes, and then multiplying all the different prime factors

$$\text{Factors of 16: } 16 = 2 \cdot 2 \cdot 2 \cdot 2$$

$$\text{Factors of 24: } 24 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$\text{LCM}(16,24) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$$

- B. Dividing each number by their common divisor and multiplying these divisors including the quotients obtained

$$\begin{array}{r|rr} 2 & 16 & 24 \\ \hline 2 & 8 & 12 \\ \hline 2 & 4 & 6 \\ \hline & 2 & 3 \end{array}$$

$$\text{GCF}(16,24) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$$

Example 2: What is the LCM of 36, 54, and 72?

Example 2: What is the LCM of 36, 54, and 72?

To find the LCM, look first on all factors common to the three numbers, then the factors common to each pair of numbers. If a number cannot be divided by the factor common to the other two, just bring it down.

Solution:

Answer:

The LCM is $2 \bullet 3 \bullet 3 \bullet 2 \bullet 1 \bullet 3 \bullet 2$
which is 216.

2	36	54	72
3	18	27	36
3	6	9	12
2	2	3	4
	1	3	2

2	18	36	42
3	9	18	21
3	3	6	7
	1	2	7

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$$\begin{aligned}\text{LCM of } 18, 36 \text{ and } 42 &= 2 \times 3 \times 3 \times 2 \times 7 \\ &= 252\end{aligned}$$

1. What is the GCF and LCM of 9, 32, and 45?
2. Two buses leave the terminal at the same time. Bus A leaves every 15 minutes, and Bus B leaves every 20 minutes. How many minutes will it be before both buses leave the terminal again at the same time
3. Kiara baked 30 oatmeal cookies and 48 chocolate chip cookies to package in plastic containers for her friends at school. She wants to divide the cookies into identical containers so that each container has the same number of each kind of cookie. If she wants each container to have the greatest number of cookies possible, how many plastic containers does she need?

SECTION 5.4: OPERATIONS ON INTEGERS

ABSOLUTE VALUE OF A NUMBER

The absolute value of a number, denoted by the number between two bars, is defined as:

$$|N| = \begin{cases} N & \text{if } N > 0 \\ -N & \text{if } N < 0 \\ 0 & \text{if } N = 0 \end{cases}$$

Examples:

1. $|23| = 23$

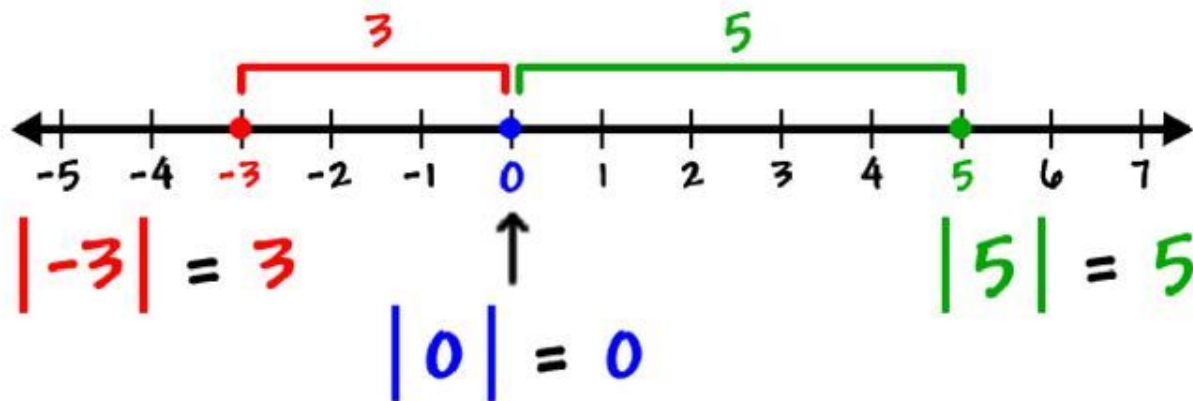
2. $|-18| = 18$

3. $|0| = 0$

Note: The absolute value of a number is always nonnegative. It represents the distance between two points in the number line whose coordinates are 0 and the number itself.

Absolute Value

Magnitude
& Distance



LAWS OF SIGNS

Addition

In adding two numbers with the same signs, add their absolute values and prefix the common sign.

Examples:

$$15 + 22 = 37$$
$$(-12) + (-43) = -55$$

In adding two numbers with different signs, subtract their absolute values and prefix the sign of the number with larger absolute value.

Examples:

$$(-25) + 10 = -15$$
$$49 + (-14) = 35$$

Multiplication

In multiplying two numbers with like signs, multiply their absolute values and the result must be positive.

Examples:

$$(11)(12) = 132$$

$$(-8)(-9) = 72$$

In multiplying two numbers with unlike signs, multiply their absolute values and the results must be negative.

Examples:

$$(15)(-4) = -60$$

$$(-8)(25) = -200$$

Division

In dividing two numbers with like signs, divide their absolute values and the result must be positive.

Examples:

$$54 \div 9 = 6$$

$$(-81) \div (-3) = 27$$

In dividing two numbers with unlike signs, divide their absolute values and the result must be negative.

Examples:

$$120 \div (-10) = -12$$

$$(-144) \div 18 = -8$$

SERIES OF OPERATIONS ON INTEGERS

Example 1: $4(5 - 8) \div 3(8 - 10)$
 $= 4(-3) \div 3(-2)$
 $= 8$

Example 2: $16 \div (5 - 7)3 - 3(1 - 4)$
 $= 16 \div (-2)3 - 3(-3)$
 $= 16 \div (-8) - 3(-3)$
 $= -2 + 9$
 $= 7$

$$9 - 6 \div 2 \times 3 + 1 = ?$$

()

\sqrt{x} x^2

\times

OR

\div

$+$

OR

$-$

Parentheses

Exponents

Multiply

Divide

Add

Subtract

P

E

M

D

A

S

APPLICATIONS

Example 1: A vendor profited Php 500 on the first day, had a loss of Php 60 on the next day, gained Php 150 on the third; and had a loss of Php 200 on the fourth. Did he gain or lose? By how much?

APPLICATIONS

Example 1: A vendor profited Php 500 on the first day, had a loss of Php 60 on the next day, gained Php 150 on the third; and had a loss of Php 200 on the fourth. Did he gain or lose? By how much?

Solution: You may solve this by assigning a plus sign to gains and negative sign to losses.

Hence, $500 - 60 + 150 - 200 = \text{Php } 390$.

Answer: The vendor profited Php 390 for his four days of business.

APPLICATIONS

Example 2: On a particular day in Japan, the temperature reached 25°C in daytime and dropped to -5°C on nighttime. Determine the difference between the high and the low temperatures on that day?

APPLICATIONS

Example 2: On a particular day in Japan, the temperature reached 25°C in daytime and dropped to -5°C on nighttime. Determine the difference between the high and the low temperatures on that day?

Solution: Subtract the low temperature from high temperature. So, the difference is $25^{\circ}\text{C} - (-5^{\circ}\text{C})$ which is equal to 30°C .

Answer: On that day, a difference of 30°C between high and low temperatures has been registered.

SECTION 5.5: FRACTION, DECIMAL AND PERCENT

TERM	DESCRIPTION/DEFINITION	EXAMPLE
Proper fraction	the numerator is less than the denominator	$\frac{3}{5}$
Improper fraction	the numerator is greater than or equal to the denominator	$\frac{7}{4}$
Unit fraction	the numerator is 1	$\frac{1}{9}$
Complex fraction	a fraction that contains another fraction in the numerator, in the denominator, or in both	$\frac{1 + \frac{1}{3}}{2 - \frac{2}{3}}$

SECTION 5.5: FRACTION, DECIMAL AND PERCENT

Equivalent fraction	fractions with the same value	$\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$
Fraction in the lowest terms	fraction whose numerator and denominator have no common factor except 1	$\frac{5}{6}$
Similar fractions	fractions with the same denominator	$\frac{5}{8}$ and $\frac{7}{8}$
Dissimilar fractions	fractions with different denominators	$\frac{4}{5}$ and $\frac{6}{7}$

EXPRESSING MIXED NUMBER TO IMPROPER FRACTION

RULE: Multiply the whole number part of the mixed number by the denominator of its fractional part, and then the resulting product must be added to the numerator of its fractional part. The resulting number is written over a denominator which is the same as denominator of the fractional part to form the improper fraction.

Example: Express $6\frac{5}{8}$ as improper fraction.

Solution:

$$6\frac{5}{8} = \frac{(6 \times 8) + 5}{8} = \frac{48 + 5}{8} = \frac{53}{8}$$

REDUCING FRACTION TO THE LOWEST TERMS

To reduce a fraction to its lowest terms, remove factors common to both the numerator and the denominator.

Example: Reduce $\frac{45}{60}$ to the lowest terms.

Solution: The GCF of both terms (numerator and denominator) is 15. So, $\frac{45}{60} = \frac{15 \times 3}{15 \times 4} = \frac{3}{4}$

EXPRESSING IMPROPER FRACTION TO MIXED NUMBER

To express an improper fraction to a mixed number, divide the numerator of the improper fraction by its denominator. The resulting integral quotient is the whole number part of the mixed number; the remainder is the numerator of its fractional part; and the divisor is the denominator of its fractional part.

Example: Express $\frac{119}{11}$ as a mixed number.

Solution: Divide 119 by 11. The resulting quotient is 10, the remainder is 9, and the divisor is 11.

$$\text{Hence, } \frac{119}{11} = 10\frac{9}{11}$$

PERFORMING DIFFERENT OPERATIONS ON FRACTIONS

ADDITION AND SUBTRACTION OF FRACTIONS

Similar Fractions

Add or subtract the numerators, and then write the result over the common denominator.

Examples:

$$\text{a) } \frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\text{b) } \frac{9}{10} - \frac{3}{10} = \frac{6}{10} = \frac{3}{5}$$

ADDITION AND SUBTRACTION OF FRACTIONS

Dissimilar Fractions

The LCD is the LCM of the denominators.

Change each fraction to an equivalent fraction using LCD.

Add or subtract the numerators and write the result over the LCD.

Reduce the answer to the lowest terms.

Examples:

$$\begin{aligned} \text{a) } \frac{5}{9} + \frac{5}{12} &= \frac{20}{36} + \frac{15}{36} = \frac{35}{36} \\ \text{b) } \frac{8}{15} - \frac{3}{10} &= \frac{16}{30} - \frac{9}{30} = \frac{7}{30} \end{aligned}$$

ADDITION AND SUBTRACTION OF FRACTIONS

Mixed Numbers

Add (or subtract) the fractional parts and add (or subtract) the whole number parts. In the case of mixed numbers, if the fractional part of the minuend is lower than that of the subtrahend, the procedure involves borrowing 1 from the whole number part of the minuend.

Examples:

$$\begin{aligned} \text{a) } 8\frac{3}{8} + 2\frac{1}{6} &= 8\frac{9}{24} + 2\frac{4}{24} = 10\frac{13}{24} \\ \text{b) } 7\frac{1}{2} - 2\frac{3}{4} &= 7\frac{2}{4} - 2\frac{3}{4} = 6\frac{6}{4} - 2\frac{3}{4} = 4\frac{3}{4} \end{aligned}$$

$$\text{a) } 8\frac{3}{8} + 2\frac{1}{6} =$$

$$\text{b) } 7\frac{1}{2} - 2\frac{3}{4} =$$

MULTIPLICATION OF FRACTIONS

Multiply the numerator and multiply the denominator, and then write the product of the numerator over the product of the denominator. Reduce to the lowest terms.

Note: If one of the factors is a mixed number, express it first as an improper fraction before performing multiplication.

Examples:

$$\text{a) } \frac{5}{12} \times \frac{3}{10} = \frac{15}{120} = \frac{1}{8}$$

$$\text{b) } 3\frac{1}{2} \times 1\frac{1}{7} = \frac{7}{2} \times \frac{8}{7} = \frac{56}{14} = 4$$

DIVISION OF FRACTIONS

Dividing two fractions is the same as multiplying the first fraction by the reciprocal of the second fraction. The first step to dividing fractions is to find the reciprocal (reverse the numerator and denominator) of the second fraction. Next, multiply the two numerators. Then, multiply the two denominators.

$$\frac{4}{11} \div \frac{5}{9} = \frac{4}{11} \times \frac{9}{5} = \frac{36}{55}$$

1. $\frac{7}{9} + \frac{3}{27} =$

2. $\frac{9}{25} - \frac{1}{5} =$

3. $5\frac{4}{5} + 6\frac{2}{3} =$

4. $\frac{9}{25} \div \frac{1}{5} =$

5. $1\frac{4}{5} \div 2\frac{2}{3} =$

FRACTION, DECIMAL, AND PERCENT

Fractions, decimals, and percent are three related concepts in mathematics. In fact, you can even convert a number in fractional form to the other two forms, and vice versa. A fraction is an indicated division. A decimal is an implied fraction with a denominator of 10, 100, 1000, etc. A percent is just a fraction with an implied denominator of 100.

Example: The following values are equivalent.

- a) fraction: $\frac{1}{2}$
- b) decimal: 0.5
- c) percent: 50%

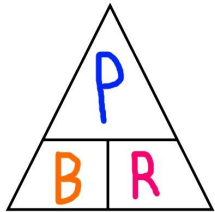
$$\begin{array}{r} 75 \\ 4 \overline{) 3.00} \\ \underline{-28} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$



PERCENT to
DECIMAL to
FRACTION

PERCENTAGE, BASE, AND RATE

Related to the concept of comparing the parts to the whole are the concepts of percentage, base, and rate. Simply put, the base is the basis of comparison. The rate is the percent indicating the number or quantity for every 100. The percentage is the number in the desired situation or condition. The formulas are given below:



FORMULA

Computing the percentage: $Percentage = Base \times Rate$

Computing the base: $Base = \frac{Percentage}{Rate}$

Computing the rate: $Rate = \frac{Percentage}{Base}$

Example 1: What is 10% of 60?

Given: $b = 60$
 $r = 10\% = 0.1$

Find P: $P = 60 \times 0.1 = 6$

Example 2: 20 is 5% of what number?

Given: $P = 20$
 $r = 5\% = 0.05$

Find b: $b = \frac{P}{r} = \frac{20}{0.05} = \frac{2000}{5} = 400$

Example 3: 10 is what percent of 40?

Given: $P = 10$
 $b = 40$

Find r: $r = \frac{P}{b} = \frac{10}{40} = 0.25 = 25\%$

APPLICATIONS

Example 1: Arthur ate half of a whole pizza on the table. Ben ate half of remaining pizza. What part of the pizza was left after the boys ate parts of it?

APPLICATIONS

Example 1: Arthur ate half of a whole pizza on the table. Ben ate half of remaining pizza. What part of the pizza was left after the boys ate parts of it?

Solution: The whole pizza may be represented by 1. So, the part eaten by the boys must be subtracted. If Arthur ate half of the pizza, this can be computed as $1 - \frac{1}{2} = \frac{1}{2}$. So, $\frac{1}{2}$ of the pizza was left. Ben ate half of the remaining half, so $\frac{1}{2} (\frac{1}{2}) = \frac{1}{4}$. $\frac{1}{4}$ of the entire pizza was eaten by Ben. Then subtracting the parts eaten by both Arthur and Ben: $1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. Hence, $\frac{1}{4}$ of the pizza was left.

Answer: $\frac{1}{4}$ of the pizza was left.

Example 3: In a class of 50, 80% of the students are right-handed.
How many are left-handed?

Example 3: In a class of 50, 80% of the students are right-handed. How many are left-handed?

Solution: This is a percentage problem. If 80% of the class are right-handed, then only 20% are left-handed. In the given problem, the base is 50, the rate is 20% or 0.2, and the percentage is missing.

To get the percentage, just multiply the base and the rate. It follows that $50(0.2) = 10$.

Answer: Ten students of the class are left-handed.

SECTION 5.6: RATIO, RATE, AND PROPORTION

RATIO vs. RATE

A ratio compares two quantities having the same unit by dividing one by the other. For example, if there are three girls for every two boys, in a group, then the ratio of girls to boys is 3:2. A rate compares two quantities that have different units by dividing one by the other. For example, if a man can walk 35 kilometers in 10 hours, then his rate is 35 km/10 hr or 3.5 km/hr.

RATIO vs. RATE

Example 1:

Express in simplest form: 28:42

Solution: $28:42 = \frac{28}{42} = \frac{2}{3} = 2:3$

Example 2:

Express in simplest form: $3\frac{1}{3} : 2\frac{1}{2}$

Solution: $3\frac{1}{3} : 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2} = \frac{10}{3} \times \frac{2}{5} = \frac{4}{3} = 4:3$

Example 3:

Express in simplest form: 20 minutes: 2 hours

Solution: $20 \text{ minutes} : 2 \text{ hours} = \frac{20 \text{ minutes}}{2 \text{ hours}} = \frac{20 \text{ minutes}}{120 \text{ minute}} = \frac{1}{6} = 1:6$

PROPORTION

When two ratios or two rates are equated, the resulting equation is a proportion. The concept being referred to here is direct proportion.

DIRECT PROPORTION

In this type, the ratios of two quantities that are being compared is a constant. As the first quantity increases, the second one also increases.

$$\blacktriangleright y = kx \text{ or } k = \frac{y}{x}$$

Example 1: Find N in this proportion: 2:3 = N:15

Solution:

$$\begin{aligned}\frac{2}{3} &= \frac{N}{15} \\ 3N &= 30 \\ \frac{3N}{3} &= \frac{30}{3} \\ N &= 10\end{aligned}$$

Example 2: Find N in this proportion: 2:(N + 2) = 6:15

Solution:

$$\begin{aligned}\frac{2}{N+2} &= \frac{6}{15} \\ 6(N+2) &= 30 \\ 6N + 12 &= 30 \\ 6N &= 30 - 12 \\ 6N &= 18 \\ \frac{6N}{6} &= \frac{18}{6} \\ N &= 3\end{aligned}$$

Example 3: If two pens cost Php 11.00, how much do 9 pens cost?

Solution: Let N = cost of 9 pens

Rate: $\frac{\text{Number of Pens}}{\text{Cost}}$

Equation:

$$\begin{aligned}\frac{2}{11} &= \frac{9}{N} \\ 2N &= 99 \\ N &= \text{Php } 49.50\end{aligned}$$

Answer: 9 pens cost Php 49.50.

INDIRECT PROPORTION

There is another type of proportion which is called indirect proportion. In this type, the product of two quantities that are being compared is a constant. As the first quantity increases, the second one decreases.

$$\blacktriangleright y = \frac{k}{x} \text{ or } k = xy$$

Example: To build a house for 60 days, it takes 20 men. How many will be needed to build one in only 15 days?

Let N = the number of men needed to build a house in 15 days

Solution:

$$M_1D_1 = M_2D_2$$

$$20(60) = \underline{N}(15)$$

$$1200 = 15N$$

$$\frac{1200}{15} = \frac{15N}{15}$$

$$80 = N$$

Answer: It will take 80 men to build the house in 15 days.

Grade 8

Maths



Direct and Inverse Proportions