MODULE 2: MATHEMATICS AS LANGUAGE

LEARNING OUTCOMES

At the end of the module, you should be able to:

- Discuss the characteristics of mathematics as a language;
- Apply the basic concepts in math in differentiating expressions and sentences and in translating expressions using mathematical symbols;
- Evaluate statements in symbolic form and vice versa; and truth table for negation, conjunction, disjunction, and conditional.

SECTION 2.1: CHARACTERISTICS OF MATHEMATICS AS A LANGUAGE

- Math is not temporary.
- Math is devoid of emotional content.
- Math is precise.
- Math is concise.
- Math is powerful.

SECTION 2.2: MATHEMATICAL LANGUAGE: SYMBOLS AND CONCEPTS

As a language, math has its own sets of symbols; you have encountered most of these symbols way back in basic education. Earnhart & Adina (2018) made a summary of these types of symbols.

SYMBOLS IN MATHEMATICAL LANGUAGE

Shown in Table 1 are the different symbols, with some description and examples.

Table 1: Different Symbols in Mathematics

Symbols	Description	Examples
Numbers	symbols for quantity (equivalent to "nouns" in English	6, 23, 49, etc.
Operations	symbols applied to at least one number to obtain another number	+, -, ÷, x, √, !, etc.
Relations	symbols for comparing quantities; they functions as verb in math	<, >, ≤,≥,≠, =, etc.
Grouping symbols	symbols for associating groups of numbers and operators	( ), [ ], { }, | |,
Variables	symbols representing quantities; functions as pronouns	x, y, z, etc.
Special symbols	symbols that could stand for some conditions in math	∞, Ø, ∃, □, □, etc.

COMPARING ENGLISH AND MATHEMATICS

In mathematics, we usually distinguish between expression and sentence. An expression in mathematics is a number or a combination of a number, an operation, a grouping symbol, or variables that do not express a complete thought. A mathematical sentence on the other hand, expresses a complete thought (usually involving relation symbol) which can be regarded as true or false.

Burns(www.onemathematicalcat.org) made a distinction between English and Mathematics. Shown in the Table 2 on the next page is their distinction.

Table 2: Distinction between English and Mathematics

	English	Mathematics
Name for an object of interest	NOUN Examples: Andres (name), Manila (place), calculator (thing)	EXPRESSION Examples: 20, 6+11, ¾
A complete thought	SENTENCE Examples: The Philippines is in Asia. The Philippines is an archipelago.	SENTENCE Examples: 7+10 = 17 25 + 4 ≠ 30

TRANSLATING WORD PHRASES TO OPEN PHRASES

In translating word phrases (phrases expressed in ordinary language) to open phrases (expressions using mathematical symbols), you should be familiar with the words or phrases associated with the different operation. Listed in Table 3 are some of these words and/or phrases.

Table 3: Mathematical Operations and Words/Phrases Associated with these Operations

OPERATIONS	WORDS/PHRASES
Addition	added to, more than, larger than, plus, sum, increased by
Subtraction	decreased by, diminished by, less than, subtracted from, minus, etc.
Multiplication	times, twice, thrice, doubled, product, etc.
Division	divided by, etc.

Translate to open phrase:

six more than twice the number N.
Answer: 2N + 6

the sum when each of two numbers M and N is squared.

Answer: $M^2 + N^2$

Carlo's age five years ago if he is N years old at present.

Answer: N – 5

Translate to Math Phrase

1. The quotient of a number and 3
2. A number decreased by 4
3. The difference of 3 times a number and 5
4. The quotient of 3 and a number, increased by 2
5. The sum of a number and 5, multiplied by 3
6. The product of 3 and a number, increased by 5
7. A number increased by 12
8. Twelve less a number
9. Double the number, x
10. Half of the number x

SECTION 2.3: MATHEMATICAL LOGIC

PROPOSITION

A proposition, or a statement, is a declarative sentence which can be either true or false but not both.

Examples:

8 is a multiple of 4.

The sum of 8 and 12 is 20.

Note: For propositions, lowercase letters are used.

Example: *s*: 8 is an integer.

Nonexamples:

Enjoy the lovely weather! (This is not a proposition because this is not a declarative sentence.)

Divide 8 by 4. (This, too, is not a proposition because this is not a declarative sentence.)

TRUTH VALUES

This refers to the "truth" or "falsity" of a given proposition.

Note: T is for "Truth"; F for "Falsity".

If *s* is true, then the truth value of *s* is TRUE and is written T.

Examples:

The statement "2 is an even number" is TRUE.

The statement "1/2 < 1/3" is FALSE.

MATHEMATICAL LOGIC

In mathematical logic, there are basic propositions and the symbols used for each. Table 4 shows these propositions and symbols.

Table 4: Basic Propositional Logic and Their Symbolic Forms

Propositional Logic	Modifier/Connective	Symbol	Symbolic Form Involving Propositions	How It Is Read
Negation	Not	~	~s	It is not true that s. (s is false.)
Conjunction	And	∧	s ∧ t	s and t
Disjunction	Or	∨	s ∨ t	s or t
Conditional/ Implication	Implies(if…, then…)	→	s → t	If s, then t. (s implies t)

NEGATION

~s is the negation of s. It is the statement arrived at by the opposite statement of s. Table 5 shows the truth table for negation.

Example:

Let s: 3 divides 9.

Then ~s: It is not the case that 3 divides 9. This is also possible: ~s: 3 does not divide 9. Table 5: Negation(Truth Table)

s	~s
T	F
F	T

CONJUNCTION

The conjunction of the statements *s* and *t*, written *s* ^ *t*, is the statement that connects statements *s* and *t* by "and". Table 6 shows the truth table for conjunction. Table 6: Conjunction (Truth Table)

s	t	s ^ t
T	T	T
T	F	F
F	T	F
F	F	F

Let *s*: 2 is even and *t*: 2 is composite.

The truth value of *s* is T, while the truth value of *t* is F.

Then *s* ^ *t*: 2 is both even and composite. Its truth value is F.

DISJUNCTION

The disjunction of the statements *s* and *t*, written s v *t*, is the statement resulting from combining statements *s* and *t* using "or". Table 7 shows the truth table for disjunction.

Table 7: Disjunction (Truth Table)

s	t	s v t
T	T	T
T	F	T
F	T	T
F	F	F

CONDITIONAL/IMPLICATION

The statement "if *s*, then *t*" is called an implication, or a conditional, and is written $s \rightarrow t$. Table 8 shows the truth table for implication.

Note: In the statement $s \rightarrow t$, *s* is referred to as the hypothesis and *t* as the conclusion.

Table 8: Implication (Truth Table)

s	t	$s \rightarrow t$
T	T	T
T	F	F
F	T	T
F	F	T

Look at the table. When is the conditional FALSE?

Example:

Let s: A square is a polygon; and

t: A quadrilateral is a rectangle.

The truth value of s is T, while the truth value of t is F.

Then $s \rightarrow t$: If a square is a polygon, then a quadrilateral is a rectangle. Its truth value is F.

Exercises:

1) Which of the following are propositions?

A. *5 is an odd integer.*
B. *A number is more than 7*

2) Negate the statement "*7 is an even integer".*

3.) Determine the truth value of the statement:

9 > *11and 45 is a multiple of 5.*

4-5) Let, ***p: 2 is an even integer*** and ***q: 3 is a positive integer***.

4.) 2 is an even integer or 3 is a positive integer.

(a) write the symbolic form in terms of p, q
(b) find the truth values

5.) If 2 is an even integer, then 3 is a positive integer.

(a) write the symbolic form in terms of p, q
(b) find the truth values