MODULE 1: MATHEMATICS AS A STUDY OF PATTERNS

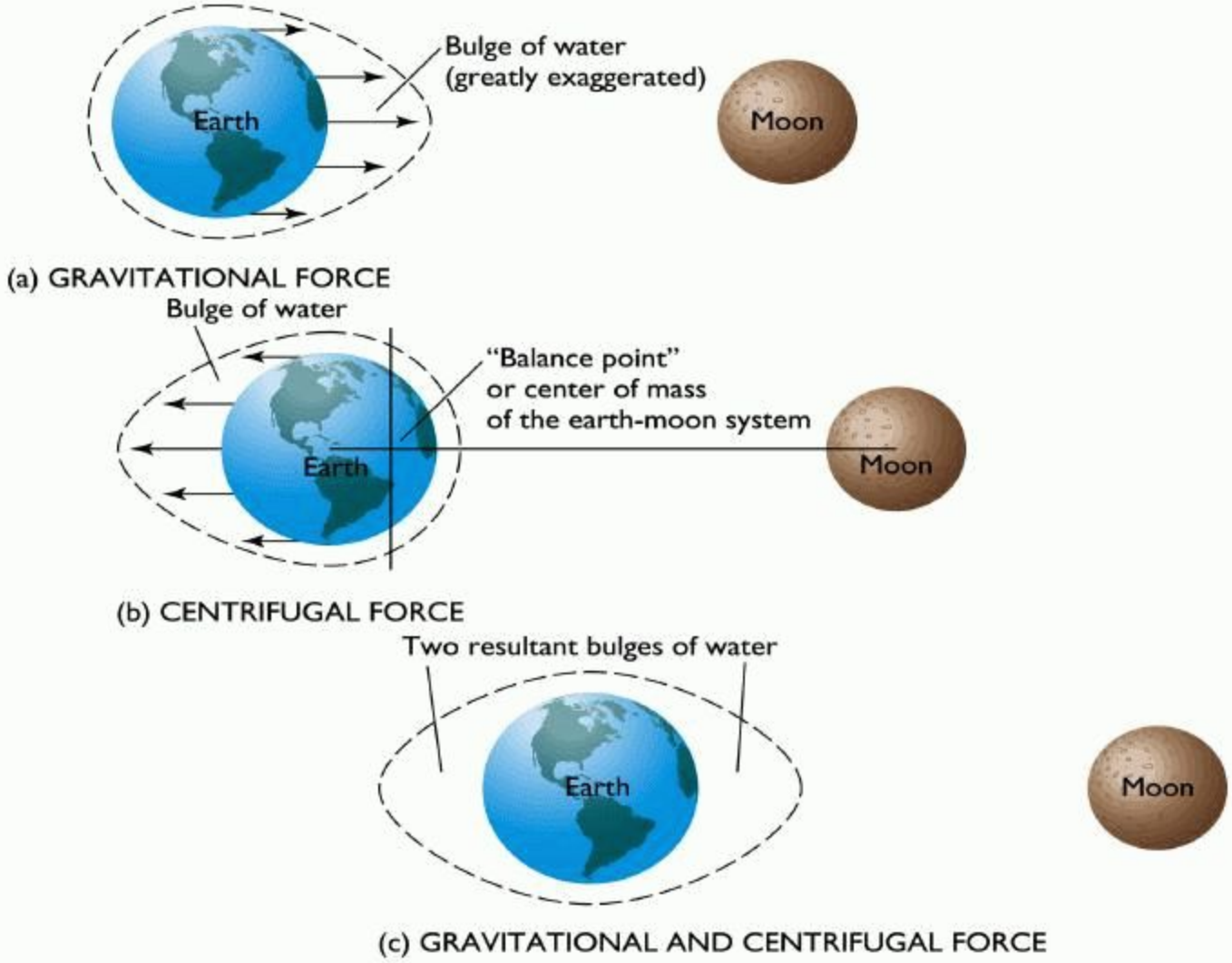
INTRODUCTION

We live in a world of patterns. The earth, with regularity, revolves around the sun in 365.25 days (approximately 365 days). We expect hot days during dry season, and rains and floods during wet season. Not only that, the earth, with regularity, too, rotates around its axis every 24 hours. We spend our daytime on worthwhile activities and our nighttime mostly on sleep.

The moon also revolves around the earth, and goes into phases: new moon turns to full moon, and then the full moon wanes to new moon again. This occurrence occurs regularly every month (29.5 days to be exact).

With these phases comes the tide in our oceans and rivers that affect the lives of farmers and fishermen.

This module deals with the types and importance of patterns so common in mathematics, that have so many applications in our daily lives.


Bulge of water
(greatly exaggerated)
Earth
Moon
(a) GRAVITATIONAL FORCE
Bulge of water
"Balance point"
or center of mass
of the earth-moon system
Earth
Moon
(b) CENTRIFUGAL FORCE
Two resultant bulges of water
Earth
Moon
(c) GRAVITATIONAL AND CENTRIFUGAL FORCE

LEARNING OUTCOMES

At the end of the module, you should be able to:

1. Discuss the nature of mathematics as a study of pattern;
2. Differentiate patterns and predict a figure/number in a given sequence; and
3. Discuss the occurrence of mathematics in nature.

SECTION 1.1: MATHEMATICS AS PATTERNS

Stewart (1995), in his book Nature's Numbers, provides a rich discussion on mathematical understanding of nature. He begins with particular patterns and relates them to mathematics. Please read the following excerpt from his book.

https://docs.google.com/document/d/1_2tCpvUcLCj53h9x-O0hxQp8WKUtDInWzh9zpTMbEo0/edit?usp=sharing

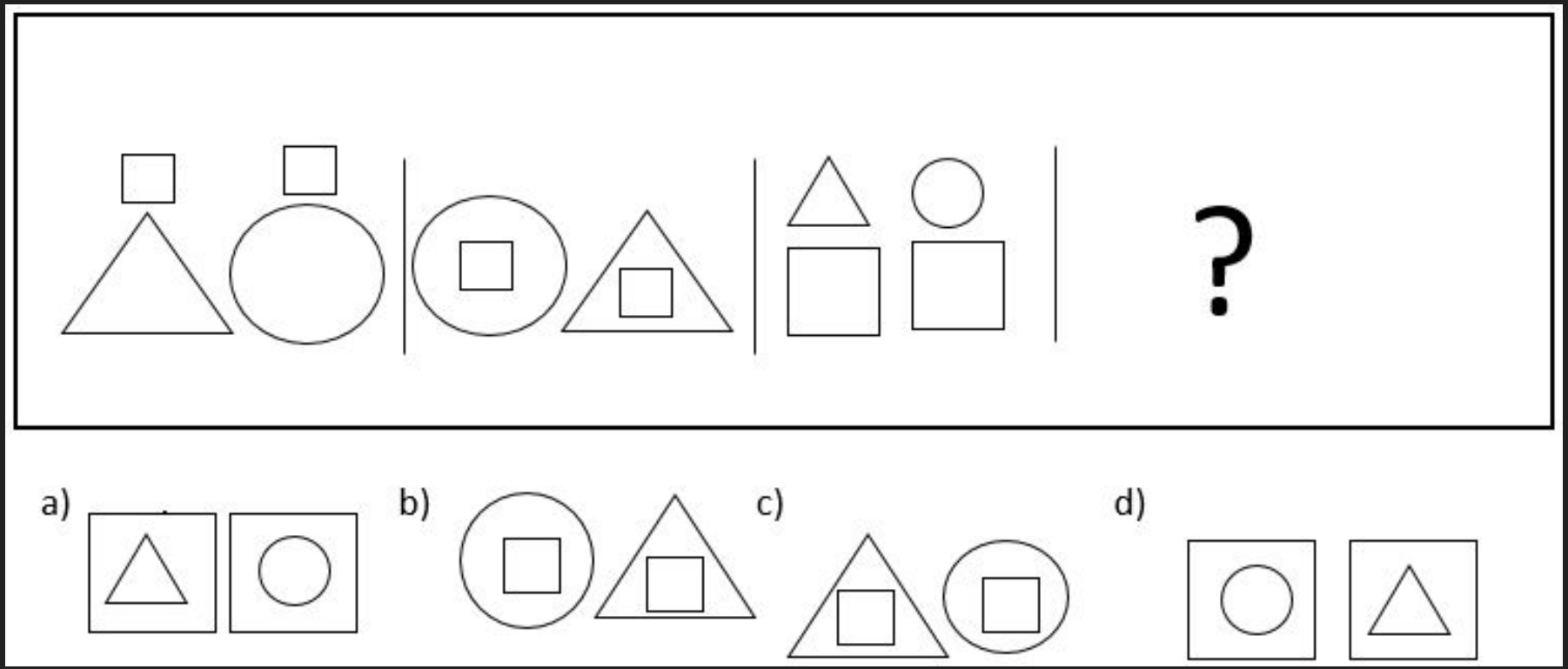
SECTION 1.2: TYPES OF PATTERNS

The British mathematician, G. H. Hardy, compared mathematicians to poets and painters, in their role as pattern-makers. Patterns made by mathematicians are made up of great and lasting ideas. Mathematics, then, may be considered as the study of patterns. Patterns are everywhere. And people are used to pattern-seeking behaviors throughout their life.

Nocon & Nocon (2018) mentions four types of patterns: logic patterns, number patterns, geometric patterns, and word patterns. Only two types will be discussed here.

Logical Patterns: Perhaps most of you have seen this when you took an aptitude test before. Logical patterns abound in logical reasoning questions in tests.

Example: What is the appropriate figure that fits the pattern?



Answering problems like this requires some degree of imagination on how the figures are positioned initially, and what logically follows from the given pattern. In the boxed figures, it can be noted that from the first pair, the figures on top are put inside the figure below but a switch in their position takes place. Following the same pattern, the right answer is (d).

Number Patterns. When given a sequence of numbers, your mind is wired to guess what the next number will be. Finding the next number may be done by comparing the differences or quotients between the previous two numbers. In more advanced mathematics, however, mathematicians resort to more complex analysis such as regression.

Example:

What is the next number in the sequence: 4, 7, 11, 16, 22, …?

Example:

What is the next number in the sequence: 4, 7, 11, 16, 22, …?

This is the pattern: 3 is added to 4 to get 7; 4 to 7 to get 11; 5 to 11 to get 16; and 6 to 16 to get 22. Hence, the next number increases by 1 from the previous numbers added. The next number therefore is 29.

Common Types of Patterns:

1. **Adding/subtracting the same number to the number that precedes it in a sequence**

Examples:

a) **12, 17, 22, 27, 32, …**

b) **98, 92, 86, 80, 74, …**

Common Types of Patterns:

2. Multiplying/dividing the previous number in a sequence by the same number to obtain the next one

Examples:

a) 4, 12, 36, 108, 324, …

b) 512, 256, 128, 64, 32, …

Common Types of Patterns:

3. A pattern also exists for the numbers added

Examples:

a) 5, 7, 11, 19, 35, …(Adding 2 to the first to get the next, then doubles of 2 are added to the succeeding numbers)

b) 20, 21, 24, 29, 36, …(Adding the odd numbers starting with 1 to the first to get the next and so on)

SECTION 1.3: THE FIBONACCI SEQUENCE AND THE GOLDEN RATIO

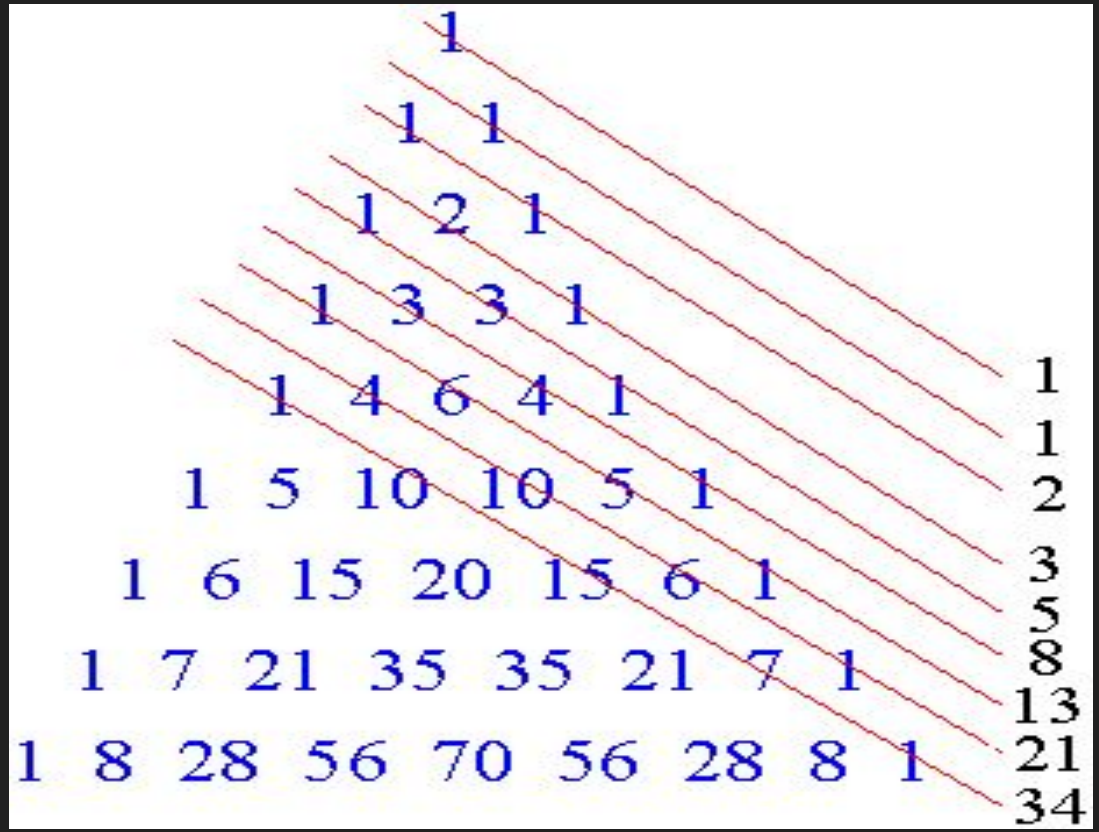
There seems to be a pattern in the numbers that occur in nature. The count of branches in a tree, spirals in a pineapple, and petals in a flower, among others, reveal numbers that belong to a sequence of numbers. This sequence of numbers was explored by Leonardo Pisano Bigollo, working under the name of Fibonacci. This sequence of numbers is very much related to the golden ratio, a number believed to be the standard of beauty in the arts.

FIBONACCI SEQUENCE

A sequence is a list of numbers that are arranged in a certain order. Each number in the list is called a term. If the first two numbers are both 1, and the next number is obtained by getting the sum of the previous two, it generates a sequence of numbers referred to as the Fibonacci sequence.

The following set, lists the first twelve elements in the sequence.
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …}. These numbers appear in nature.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ….



THE GOLDEN RATIO

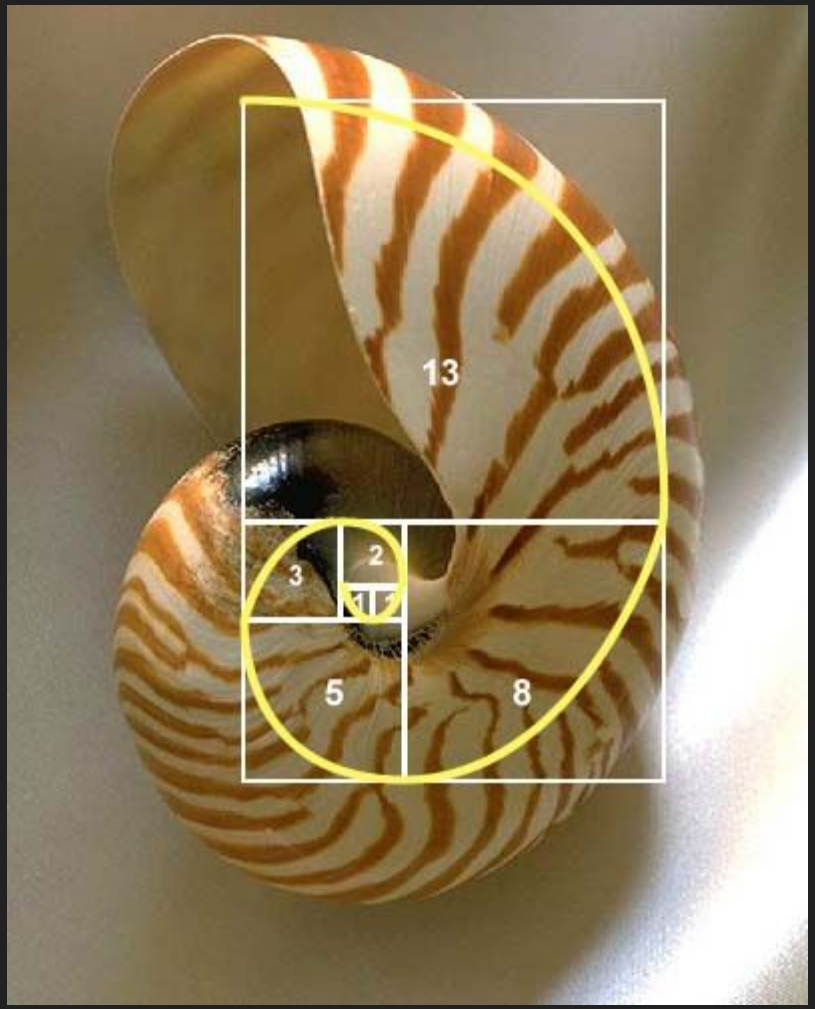
When two consecutive numbers in this sequence are divided (larger by the smaller, except for the first two), the ratio approaches a certain value called the golden ratio.

$$\frac{1}{1} = 1.000,\ \frac{2}{1} = 2.000,\ \frac{3}{2} = 1.500,\ \frac{5}{3} = 1.667,\ \frac{8}{5} = 1.600,\ \frac{13}{8} = 1.625,\ \frac{21}{13} = 1.615,$$

$$\frac{34}{21} = 1.619,\ \frac{55}{34} = 1.618,\ \frac{89}{55} = 1.618,\ \frac{144}{89} = 1.618,\ \frac{233}{144} = 1.618,$$

THE GOLDEN RATIO

Based on the values, the ratio approaches 1.618 which is an approximate value of the phi-function. The relation between the Fibonacci sequence and the golden ratio is explored in the required video in the next activity.


13
3
2
1
1
5
8

Fibonacci sequence in our hand allows for it to form a perfect curl when we clench our fist.

SUMMARY

The module discussed the nature of math as a study of patterns. These patterns occur in nature. In fact, most numbers that occur in nature belong to a certain set of numbers called the Fibonacci sequence. This sequence in turn is very much related to the Golden ratio.