3.1) $\phi(x) = x + \sin(x)$

$$\phi(\alpha) = \alpha$$
$$\alpha + \sin(\alpha) = \alpha$$
$$\sin(\alpha) = 0$$

fixed points $\Rightarrow \alpha = n\pi \quad n \in \mathbb{Z}$

3.2) show $\exists L < 1 : |\phi(x_1) - \phi(x_2)| \leq L|x_1 - x_2| \quad \forall x_1, x_2 \in [\frac{\pi}{2}, \frac{3\pi}{2}]$

$$\frac{|\phi(x_1) - \phi(x_2)|}{|x_1 - x_2|} \leq L$$

$\phi(x) = x + \sin(x)$

$\phi'(x) = 1 + \cos(x)$

$$\phi'(x) \leq L$$
$$\phi'(x) < 1$$
$$1 + \cos(x) < 1$$
$$\cos(x) < 0 \Rightarrow x \in]\pi/2, 3\pi/2[$$

since $\phi'(x) < 1 \quad \forall x \in]\pi/2, 3\pi/2[$

$\exists$ a globally attracting unique fixed point in $]\pi/2, 3\pi/2[$

we found that fixed point to be π in number 1

so $\phi(x)$ converges to π for any $x_0 \in]\pi/2, 3\pi/2[$

3.3 $\phi(x) = x + \sin(x)$

$\phi'(x) = 1 + \cos(x) \Rightarrow 1 + \cos(\pi) = 0$

$\phi''(x) = -\sin(x) \Rightarrow . \sin(\pi) = 0$

$\phi'''(x) = -\cos(x) \Rightarrow -\cos(\pi) = 1$

ϕ has a convergence order: $p+1 = 3$

$\frac{d^k \phi(\alpha)}{d(\alpha)^k}$ for $k = 1, 2$

$p = 2$