Universal Statistical Simulator: Quantum Galton Board Summary

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1 Introduction

Imagine simulating millions of particle trajectories in seconds—quantum computing makes this possible with the Quantum Galton Board (QGB). The "Universal Statistical Simulator" paper by Carney and Varcoe [1] presents a quantum circuit that simulates a Galton Board, a device modeling random walks yielding a binomial distribution, approximating a Gaussian for large layers. Prepared for the Womanium & WISER Quantum Program 2025, this summary explores the QGB's circuit design, exponential speed-up, and applications to partial differential equations (PDEs) and sustainability. By leveraging quantum algorithms, the QGB addresses complex computational challenges, aligning with the hackathon's theme and UN SDG 7 for affordable and clean energy [2].

2 Quantum Galton Board Circuit Design

The Quantum Galton Board (QGB) simulates a classical Galton Board's random walk, where a ball cascades through pegs to produce a binomial distribution. It uses three quantum gates: Hadamard (H), controlled-SWAP (CSWAP), and controlled-NOT (CNOT). The circuit's core unit, a "quantum peg," comprises three working qubits and one control qubit.

Initialized to $|0\rangle$, a designated "ball" qubit is set to $|1\rangle$ via an X gate, marking the starting position. A Hadamard gate transforms the control qubit to $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, creating a superposition. The CSWAP gate, controlled by this qubit, swaps two working qubits' states, simulating 50% left-right branching. A CNOT gate, with the ball qubit as control, adjusts probabilities and resets the control qubit. Entanglement across qubits ensures all possible paths are explored simultaneously, producing a superposition state like $\frac{1}{\sqrt{2^n}} \sum_{\text{paths}} |\text{path}\rangle$.

For n layers, the circuit uses $2n$ qubits and $O(n^2)$ gates, generating all 2^n trajectories. Measurement on position qubits yields $n+1$ outcomes, matching the classical binomial distribution, approximating a Gaussian for large n.

3 Exponential Speed-Up

The Quantum Galton Board (QGB) achieves an exponential speed-up by computing all 2^n trajectories for an n-layer Galton Board in one quantum computation [1]. Using $2n$ qubits and $O(n^2)$ gates ($2n^2 + 5n + 2$, including n Hadamard, n reset, one X, and up to $4 \cdot \frac{n(n+1)}{2}$ peg gates), it leverages superposition and entanglement to explore all paths simultaneously, forming states like $\frac{1}{\sqrt{2^n}} \sum_{\text{paths}} |\text{path}\rangle$. Classical Monte Carlo methods require $O(Nn)$ operations for N particles, with

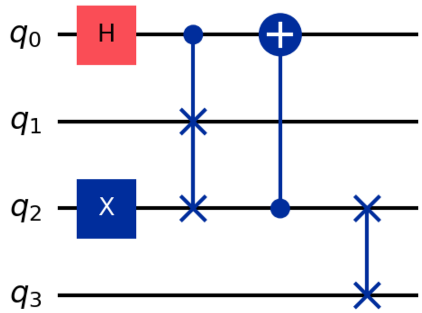

Figure 1: Quantum Galton Board circuit for one peg, showing Hadamard, CSWAP, and CNOT gates.

large $N \approx 2^n$ for accuracy, leading to exponential scaling. The QGB's low circuit depth ($< \frac{1}{2}$ of prior methods) enhances its suitability for NISQ devices, making it a powerful tool for complex simulations.

4 Relevance to PDEs and Sustainability

The Quantum Galton Board (QGB) connects to partial differential equations (PDEs) through its simulation of random walks, which underpin stochastic processes like diffusion. The heat equation, $\frac{\partial u}{\partial t} = \alpha \nabla^2 u$, yields Gaussian solutions that the QGB's binomial distribution approximates, enabling efficient sampling for quantum PDE solvers in fluid dynamics or material science [1]. For sustainability, the QGB optimizes simulations of particle transport in solar cells or fluid dynamics in wind turbines, reducing computational energy costs. These advancements support UN SDG 7 by promoting affordable and clean energy through efficient quantum algorithms [2].

5 Conclusion

The Quantum Galton Board (QGB), with its elegant circuit of Hadamard, CSWAP, and CNOT gates, delivers an exponential speed-up, computing 2^n trajectories with $O(n^2)$ resources. Its applications to PDEs like the heat equation and sustainable energy systems, such as solar cell optimization, align with UN SDG 7 [2]. This work positions our team to pioneer scalable quantum algorithms, advancing the hackathon's mission and Jerison's vision for a quantum hardware company.

References

[1] Mark Carney and Ben Varcoe. Universal statistical simulator. *arXiv preprint arXiv:2202.01735*, 2022.

[2] United Nations. Sustainable development goal 7: Affordable and clean energy, 2023.