Digitalna vezja
UL, FRI

P3 – Minimizacija, NAND, NOR

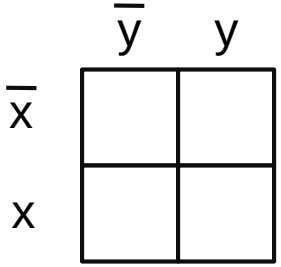
Vsebina

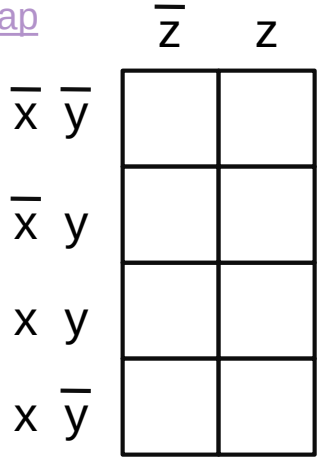
- Minimizacija
  - Minimalna disjunktivna normalna oblika (MDNO)
  - Minimalna konjunktivna normalna oblika (MKNO)
  - Minimalna normalna oblika (MNO)

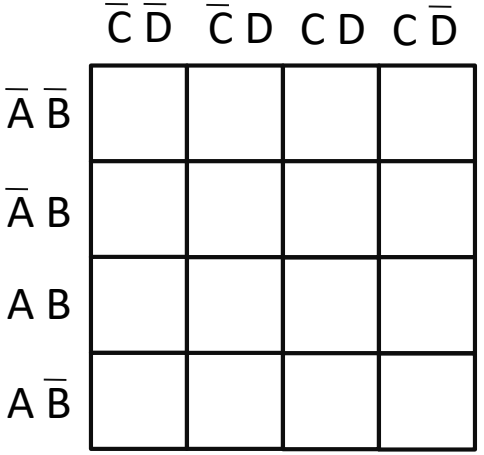
- Funkcijsko polni sistemi
  - Operatorji NAND
  - Operatorji NOR

Minimizacija logičnih funkcij

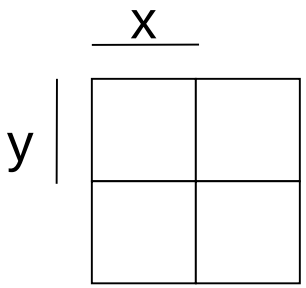
Karnaughjev diagram:
http://en.wikipedia.org/wiki/Karnaugh_map

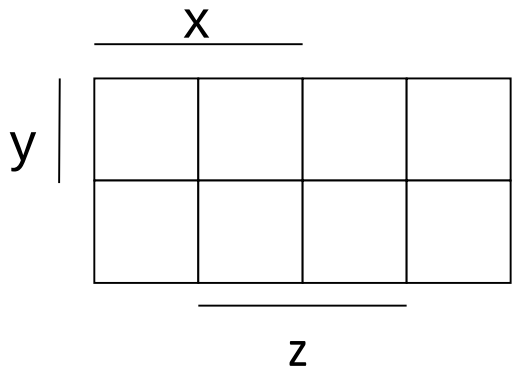


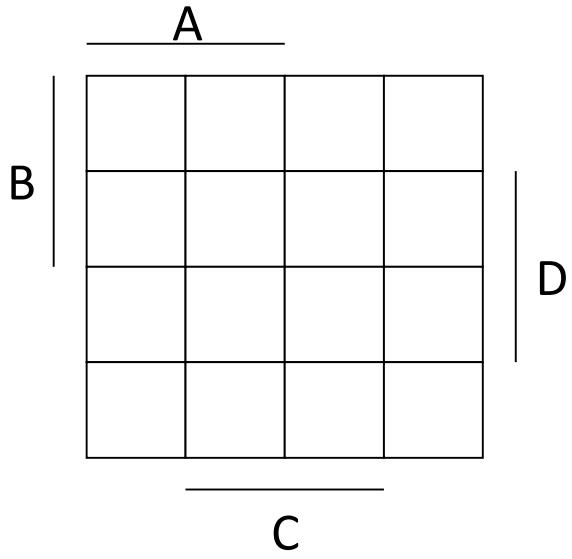




Veitchev diagram
http://de.wikipedia.org/wiki/Karnaugh-Veitch-Diagramm



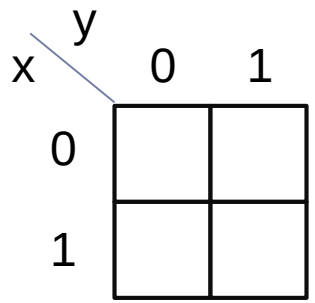




- Tabelarični in grafični zapis mintermov (n=2, n=3)

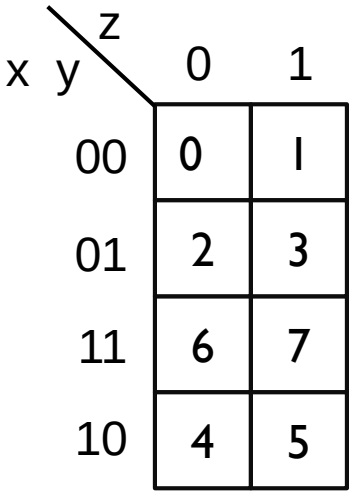
x	y	m_i	Mintermi
0	0	0	$\bar{x}.\bar{y}$
0	1	1	$\bar{x}.y$
1	0	2	$x.\bar{y}$
1	1	3	$x.y$

	$\bar{y}$	y
$\bar{x}$	0	1
x	2	3



x	y	z	m_i	Mintermi
0	0	0	0	$\bar{x}.\bar{y}.\bar{z}$
0	0	1	1	$\bar{x}.\bar{y}.z$
0	1	0	2	$\bar{x}.y.\bar{z}$
0	1	1	3	$\bar{x}.y.z$
1	0	0	4	$x.\bar{y}.\bar{z}$
1	0	1	5	$x.\bar{y}.z$
1	1	0	6	$x.y.\bar{z}$
1	1	1	7	$x.y.z$

		$\bar{z}$	z
$\bar{x}$	$\bar{y}$	0	1
$\bar{x}$	y	2	3
x	y	6	7
x	$\bar{y}$	4	5



- Zapis logične funkcije (n=3) v Karnaughjevem diagramu

x y z	m_i	f(x.y.z)	$\overline{f(x.y.z)}$
0 0 0	0	1	0
0 0 1	1	0	1
0 1 0	2	0	1
0 1 1	3	1	0
1 0 0	4	0	1
1 0 1	5	0	1
1 1 0	6	0	1
1 1 1	7	1	0

$f(x,y,z) \rightarrow f_i=1$

vhodne kombinacije, kjer ima funkcija vrednost 1

	$\overline{z}$	z
$\overline{x}$ $\overline{y}$	1	
$\overline{x}$ y		1
x y		1
x $\overline{y}$		

$f(x,y,z) \rightarrow f_i= 0$

vhodne kombinacije, kjer ima funkcija vrednost 0

	$\overline{z}$	z
$\overline{x}$ $\overline{y}$		0
$\overline{x}$ y	0	
x y	0	
x $\overline{y}$	0	0

Negirana funkcija: $\overline{f(x,y,z)} \rightarrow f_i= 1$

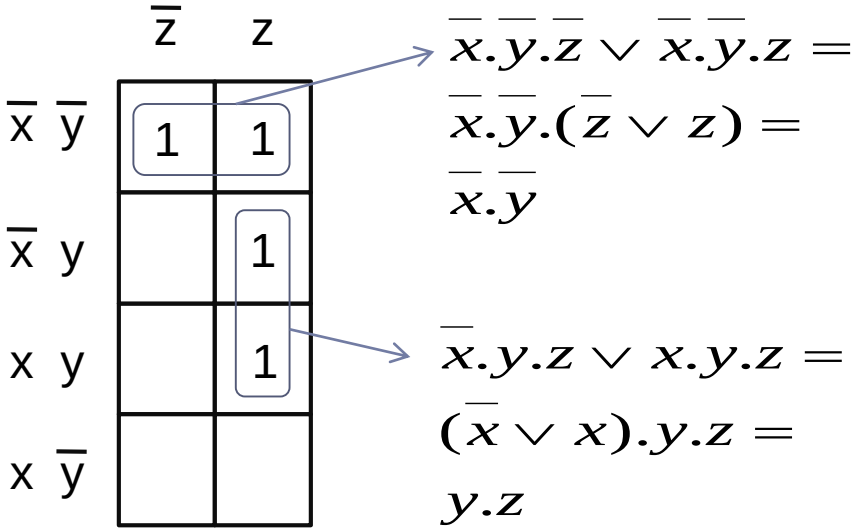
	$\overline{z}$	z
$\overline{x}$ $\overline{y}$		1
$\overline{x}$ y	1	
x y	1	
x $\overline{y}$	1	1

- **SOSEDNOST**

združevanje dveh konjunktivnih izrazov, ki se razlikujeta v eni spremenljivki za negacijo in nenegacijo (x in $\overline{x}$)

- Sosednost konjunkcij dolžine k=n, n-1,n-2,…

$$f(x,y,z) = \overline{x}.\overline{y}.\overline{z} \vee \overline{x}.\overline{y}.z \vee \overline{x}.y.z \vee x.y.z =$$
$$\overline{x}.\overline{y}.(\overline{z} \vee z) \vee y.z.(\overline{x} \vee x) =$$
$$\overline{x}.\overline{y} \vee y.z$$



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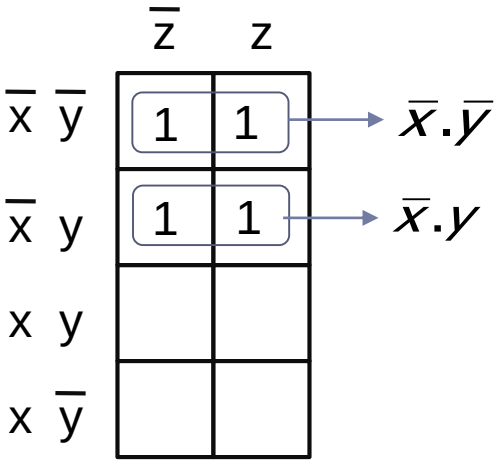
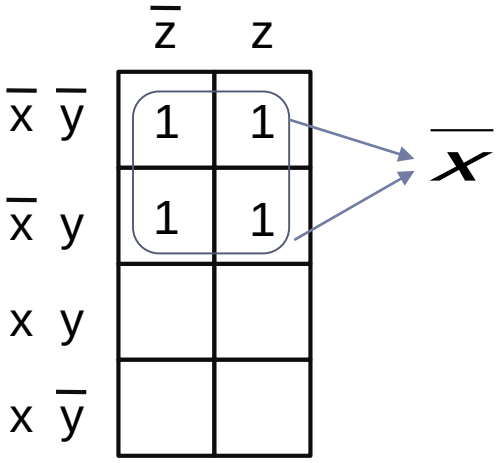
$$\overline{x}.\overline{y}.\overline{z} \vee \overline{x}.\overline{y}.z \vee \overline{x}.y.\overline{z} \vee \overline{x}.y.z =$$
$$\overline{x}.\overline{y}.(\overline{z} \vee z) \vee \overline{x}.y.(\overline{z} \vee z) =$$
$$\overline{x}.\overline{y} \vee \overline{x}.y = \overline{x}.(\overline{y} \vee y) = \overline{x}$$



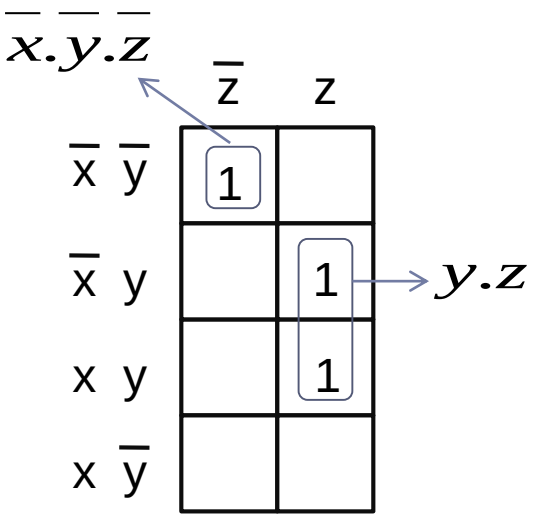


MDNO (n=3)

- Iskanje zapisa z najmanjšim številom logičnih vrat in povezav.

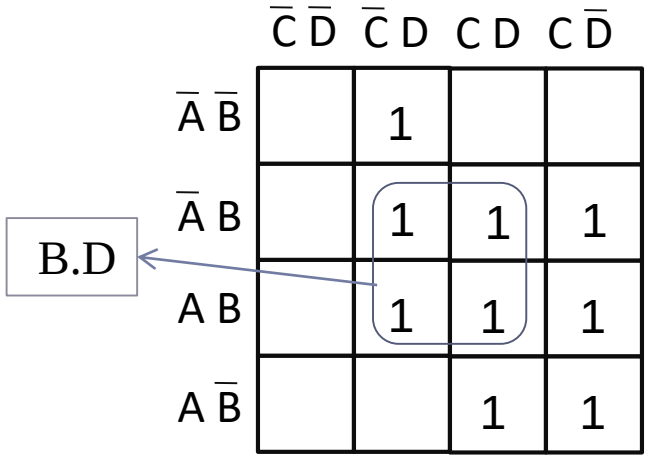
x	y	z	*f(x.y.z)*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

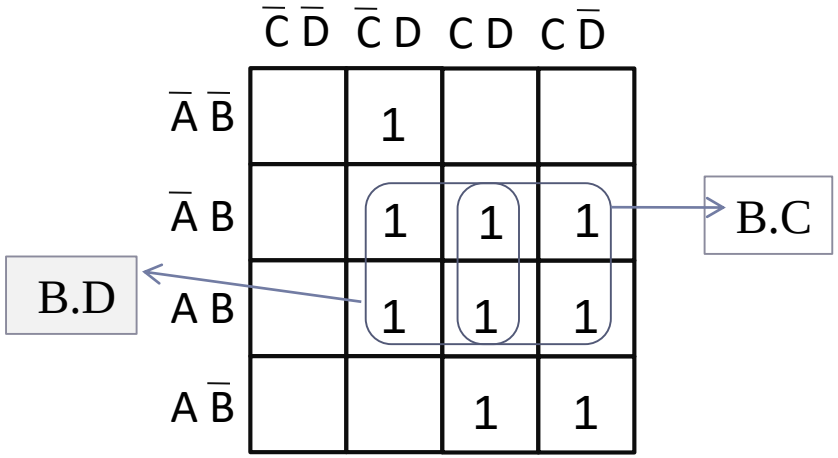
$$f(x,y,z) = \\ = \bar{x}.\bar{y}.\bar{z} \vee \bar{x}.y.z \vee x.y.z = \\ = \bar{x}.\bar{y}.\bar{z} \vee y.z(\bar{x} \vee x) = \\ = \bar{x}.\bar{y}.\bar{z} \vee y.z$$

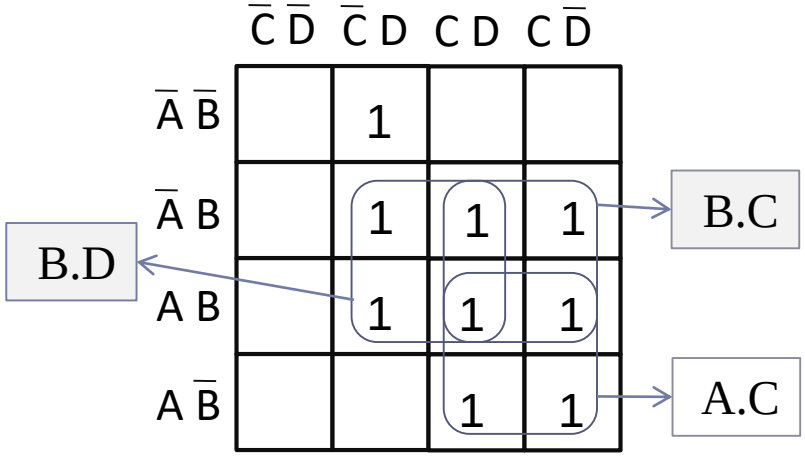


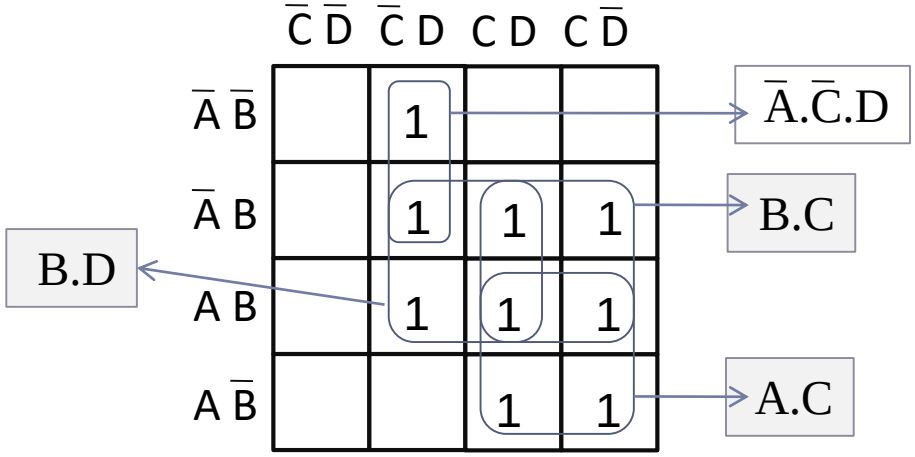
$$f(x,y,z) = \bar{x}.\bar{y}.\bar{z} \vee y.z$$

MDNO (n=4)









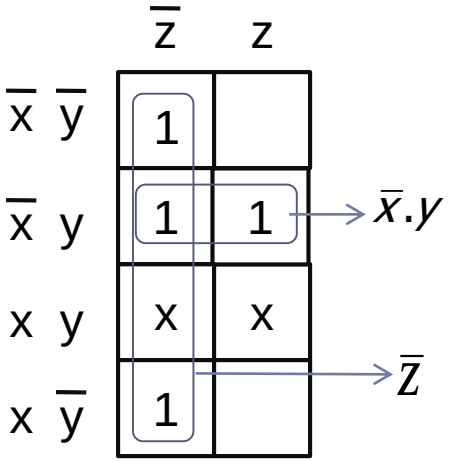
Minimalna disjunktivna normalna oblika: $f(A,B,C,D) = B.D \vee A.C \vee B.C \vee \bar{A}.\bar{C}.D$

MDNO – nepopolne log. funkcije

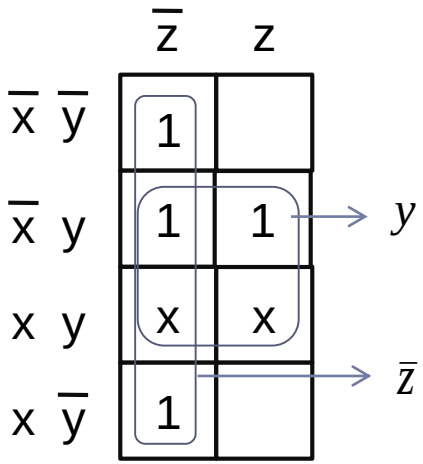
- Logična funkcija je podana z nedoločeno vrednostjo izhodov (x) pri vhodni kombinaciji x=y=1.
- Imenujemo jo nepopolna logična funkcija ali funkcija z redundancami.
- Iskanje zapisa z najmanjšim številom logičnih vrat in povezav poteka tako, da minterme, ki imajo nedoločeno vrednost upoštevamo pri združevanju na osnovi sosednosti, če omogočajo enostavnejšo obliko logične funkcije.

f(x,y,z): f_6=f_7= x -> vrednost ima lahko 0 ali 1)

x	y	z	f(x,y,z)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	X (0,1)
1	1	1	X (0,1)



$f(x,y,z) = \bar{x}.y \vee \bar{z}$



$f(x,y,z) = y \vee \bar{z}$

MKNO (n=4)

Minimalna konjunktivna normalna oblika (MKNO):

1. Minimiziramo negirano funkcijo ($\overline{f}$) : $f_i = 0$
2. Zapišemo MDNO negirane funkcije
3. Levo in desno stran enačbe negiramo
4. Po DeMorganovem izreku pretvorimo desno stran. Dobimo MKNO

Funkcija f(A,B,C,D)

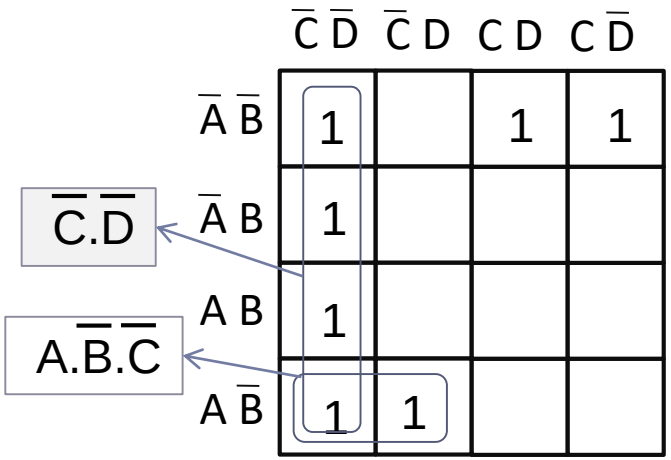
	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$		1		
$\overline{A}\,B$		1	1	1
$A\,B$		1	1	1
$A\,\overline{B}$			1	1

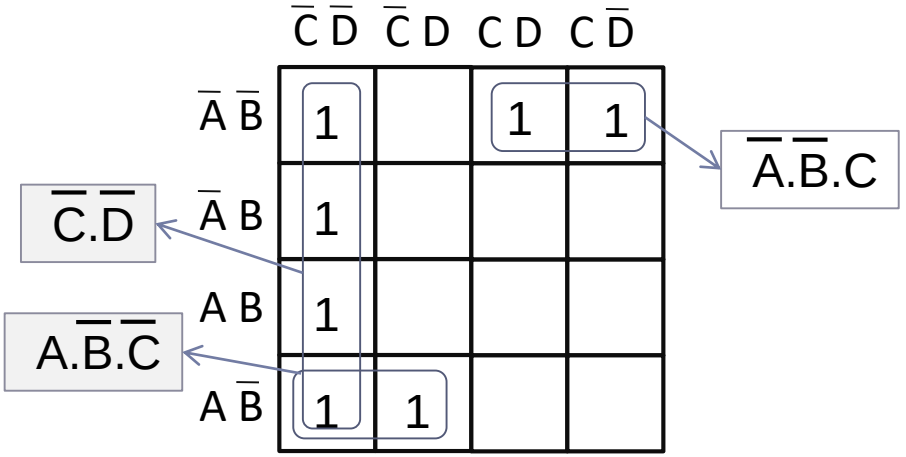
$\overline{f\ (A,B,C,D)}$

1.

$\overline{C}.\overline{D}$

	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$	1		1	1
$\overline{A}\,B$	1			
$A\,B$	1			
$A\,\overline{B}$	1	1		





2. $\bar{f}(A,B,C,D) = \overline{C}.\overline{D} \vee A.\overline{B}.\overline{C} \vee \overline{A}.\overline{B}.C$

3. $\bar{\bar{f}}(A,B,C,D) = \overline{\overline{C}.\overline{D} \vee A.\overline{B}.\overline{C} \vee \overline{A}.\overline{B}.C}$

4. $f(A,B,C,D) = (C \vee D).(\overline{A} \vee B \vee C).(A \vee B \vee \overline{C})$

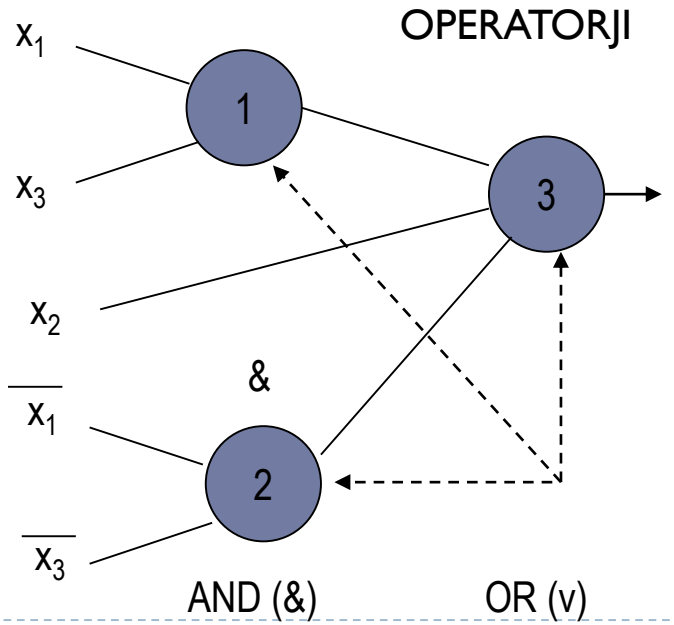
MKNO

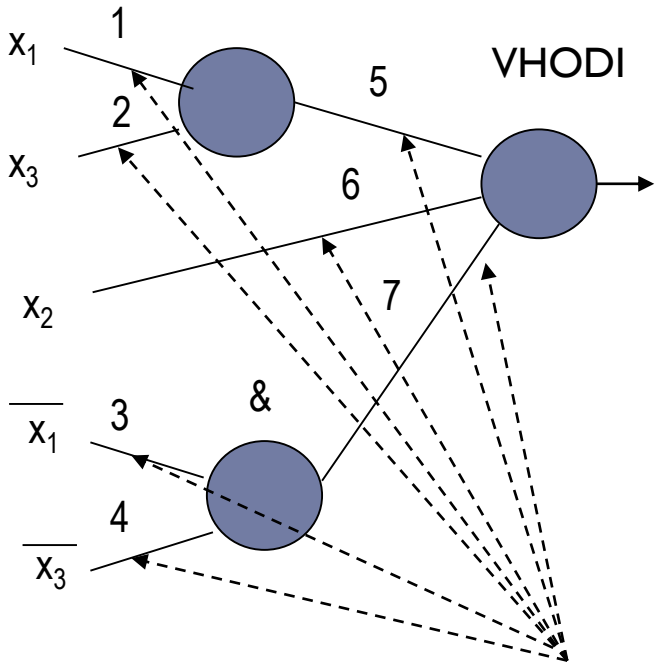
MNO

1. Poiščemo MDNO in MKNO
2. Določimo: [Število operatorjev / število vhodov]

MNO dobimo tako, da izberemo enostavnejšo obliko preklopne funkcije glede na operatorje in nato glede na vhode, če je število enih ali drugih različno.

Primer: $f_{MDNO}(x_1, x_2, x_3) = x_1x_3 \vee x_2 \vee \overline{x}_1\overline{x}_3$ [3,7]





Naloga 1: PDNO, Booleova algebra, MDNO

Zapis vsote produktov (PDNO)

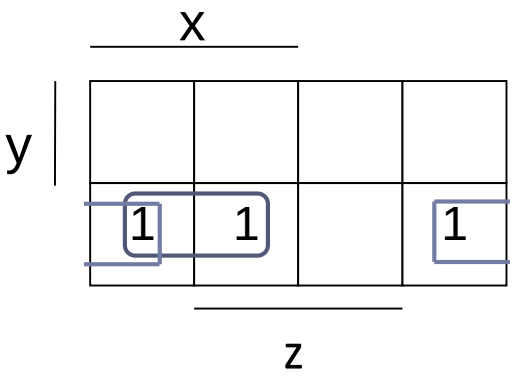
Poenostavitev logične funkcije

- Booleova algebra
- MDNO - Veitchev diagram

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$f = \bar{x}.\bar{y}.\bar{z} \vee x.\bar{y}.\bar{z} \vee x.\bar{y}.z$$

$$f = \bar{x}.\bar{y}.\bar{z} \vee x.\bar{y}.\bar{z} \vee x.\bar{y}.z =$$
$$= \bar{y}.\bar{z}(x \vee \bar{x}) \vee x.\bar{y}.z =$$
$$= \bar{y}.\bar{z} \vee x.\bar{y}.z =$$
$$= \bar{y}.(\bar{z} \vee x.z) =$$
$$= \bar{y}.(\bar{z} \vee x)(\bar{z} \vee z) =$$
$$= \bar{y}.\bar{z} \vee \bar{y}.x =$$
$$= \bar{y}.\bar{z} \vee x.\bar{y}$$



$$f = \bar{y}.\bar{z} \vee x.\bar{y}$$

Naloga 2: PKNO, MKNO

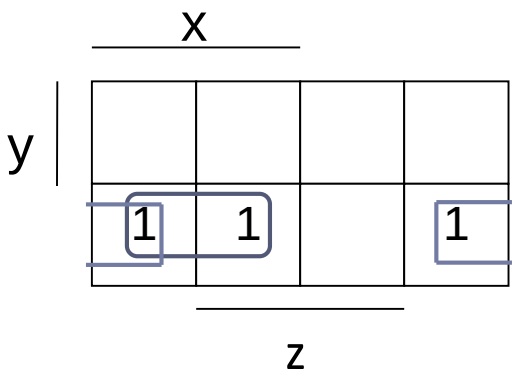
Zapis produkta vsot (PKNO)

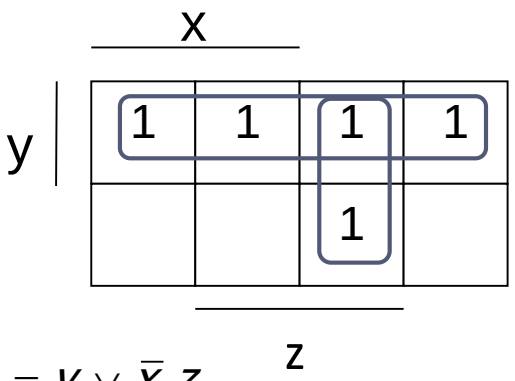
Poenostavitev logične funkcije - Veitchev diagram

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$$f == (x \vee y \vee \bar{z}).(x \vee \bar{y} \vee z).$$
$$(x \vee \bar{y} \vee \bar{z}).(\bar{x} \vee \bar{y} \vee z).(\bar{x} \vee \bar{y} \vee \bar{z})$$

$f(x,y,z)$ $\qquad$ $\overline{f(x,y,z)}$



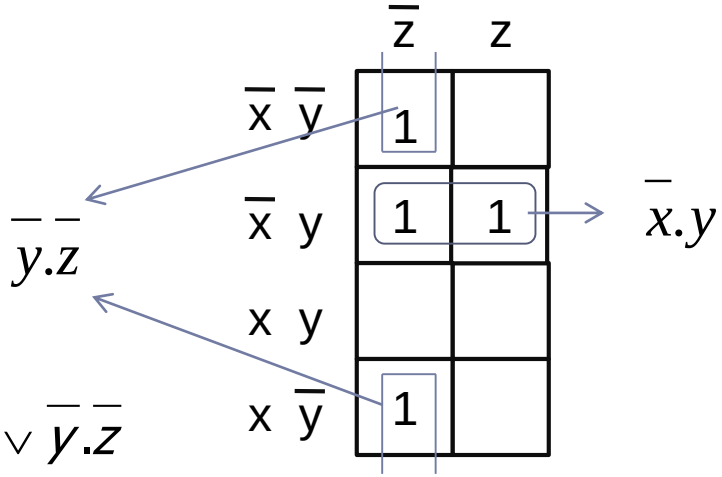


$$\bar{f} = y \vee \bar{x}.z$$
$$\bar{\bar{f}} = \overline{y \vee \bar{x}.z}$$
$$f = \bar{y}.(x \vee \bar{z})$$

Naloga 3: MNO

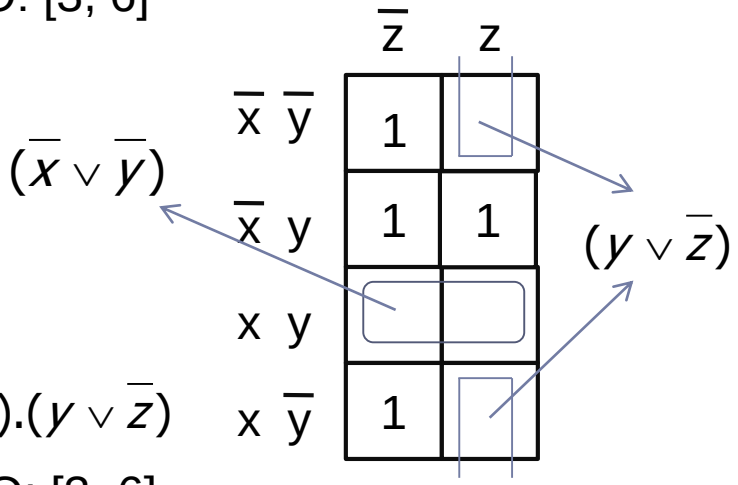
- Zapišite: MDNO, MKNO, MNO

x	y	z	f(x.y.z)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0



$$f(x,y,z) = \bar{x}.y \vee \bar{y}.\bar{z}$$

MDNO: [3, 6]



PDNO:

$$f(x,y,z) = \bar{x}.\bar{y}.\bar{z} \vee \bar{x}.y\bar{z} \vee \bar{x}.y.z \vee x.\bar{y}.\bar{z}$$

MNO=MDNO=MKNO

$$f(x,y,z) = (\bar{x} \vee \bar{y}).(y \vee \bar{z})$$

MKNO: [3, 6]

Funkcijsko polni sistemi (NAND, NOR)

- Operatorji: NOT, AND, OR
  - Vsaka logična funkcija z n spremenljivkami je določena z njimi.

- Nabor operatorjev je funkcijsko poln, če je z njim mogoče zapisati vse osnovne logične funkcije v Booleovi algebri (NOT, AND, OR)
  - (NAND) $x \uparrow y = \overline{x.y} = \bar{x} \vee \bar{y}$

$$\bar{x} = \overline{x.x} = x \uparrow x$$
$$x.y = \overline{\overline{x.y}} = \overline{x \uparrow y} = \overline{(x \uparrow y).(x \uparrow y)} = (x \uparrow y) \uparrow (x \uparrow y)$$
$$x \vee y = \overline{\overline{x \vee y}} = \overline{\bar{x}.\bar{y}} = \bar{x} \uparrow \bar{y} = (x \uparrow x) \uparrow (y \uparrow y)$$

  - (NOR) $x \downarrow y = \overline{x \vee y} = \bar{x}.\bar{y}$

$$\bar{x} = \overline{x \vee x} = x \downarrow x$$
$$x.y = \overline{\overline{x.y}} = \overline{\bar{x} \vee \bar{y}} = \bar{x} \downarrow \bar{y} = (x \downarrow x) \downarrow (y \downarrow y)$$
$$x \vee y = \overline{\overline{x \vee y}} = \overline{x \downarrow y} = \overline{(x \downarrow y) \vee (x \downarrow y)} = (x \downarrow y) \downarrow (x \downarrow y)$$

Dvonivojske funkcije: PSNO, PKNO

- NAND: $x \uparrow y = \overline{x.y} = \bar{x} \vee \bar{y}$
- NOR: $x \downarrow y = \overline{x \vee y} = \bar{x}.\bar{y}$
- PDNO $\rightarrow$ PSNO (Popolna Shefferjeva normalna oblika)
- PKNO $\rightarrow$ PPNO (Popolna Pierceova normalna oblika)

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$f = \bar{x}.y.z \vee x.\bar{y}.\bar{z} \vee x.y.\bar{z}$$

$$f = \overline{\overline{\bar{x}.y.z \vee x.\bar{y}.\bar{z} \vee x.y.\bar{z}}} =$$

$$f = \overline{\overline{(\bar{x}.y.z)}.\overline{(x.\bar{y}.\bar{z})}.\overline{(x.y.\bar{z})}}$$

$$f = (\bar{x} \uparrow y \uparrow z) \uparrow (x \uparrow \bar{y} \uparrow \bar{z}) \uparrow (x \uparrow y \uparrow \bar{z})$$

$$f = (x \vee y \vee z).(x \vee y \vee \bar{z}).(x \vee \bar{y} \vee z).(\bar{x} \vee y \vee \bar{z}).(\bar{x} \vee \bar{y} \vee \bar{z})$$

$$f = \overline{\overline{(x \vee y \vee z).(x \vee y \vee \bar{z}).(x \vee \bar{y} \vee z).(\bar{x} \vee y \vee \bar{z}).(\bar{x} \vee \bar{y} \vee \bar{z})}}$$

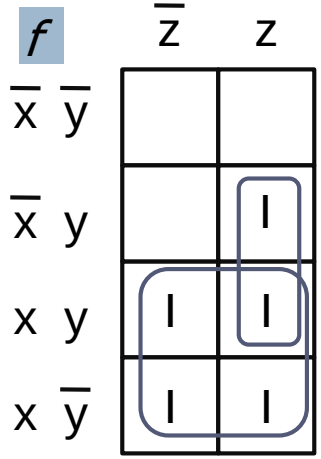
$$f = \overline{\overline{(x \vee y \vee z)} \vee \overline{(x \vee y \vee \bar{z})} \vee \overline{(x \vee \bar{y} \vee z)} \vee \overline{(\bar{x} \vee y \vee \bar{z})} \vee \overline{(\bar{x} \vee \bar{y} \vee \bar{z})}}$$

$$f = (x \downarrow y \downarrow z) \downarrow (x \downarrow y \downarrow \bar{z}) \downarrow (x \downarrow \bar{y} \downarrow z) \downarrow (\bar{x} \downarrow y \downarrow \bar{z}) \downarrow (\bar{x} \downarrow \bar{y} \downarrow \bar{z})$$

MDNO → NAND, NOR

- Zapis funkcije f(x,y,z) v MDNO
- Zapis MDNO z operatorji NAND (MSNO) in NOR

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



MDNO :

$f = x \vee yz$

NAND

$$\overline{\overline{f}} = \overline{\overline{x \vee y.z}} = \overline{\bar{x}.(\overline{y.z})} = \bar{x} \uparrow (\overline{y.z}) = \bar{x} \uparrow (y \uparrow z)$$

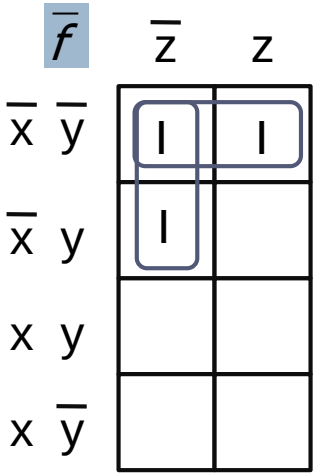
NOR

$$\overline{\overline{f}} = \overline{\overline{x \vee yz}} = \overline{x \downarrow (\overline{x.y})} = \overline{x \downarrow (\bar{y} \vee \bar{z})} = \overline{x \downarrow (\bar{y} \downarrow \bar{z})}$$

MKNO → NOR, NAND

- Zapis funkcije f(x,y,z) v MKNO
- Zapis MKNO z operatorji NOR (MPNO) in NAND

x	y	z	f	$\overline{f}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0



MKNO

$$\overline{f} = \overline{x}.\overline{z} \vee \overline{x}.\overline{y}$$

$$\overline{\overline{f}} = \overline{\overline{x}.\overline{z} \vee \overline{x}.\overline{y}}$$

$$f = (\overline{\overline{x}.\overline{z}}).(\overline{\overline{x}.\overline{y}}) = (x \vee z)(x \vee y)$$

NOR

$$\overline{\overline{f}} = \overline{\overline{(x \vee z)(x \vee y)}} = \overline{(\overline{x \vee z}) \vee (\overline{x \vee y})} = (x \downarrow z) \downarrow (x \downarrow y)$$

NAND

$$\overline{\overline{f}} = \overline{\overline{(x \vee z)(x \vee y)}} = \overline{(x \vee z) \uparrow (x \vee y)} = \overline{(\overline{x \vee z}) \uparrow (\overline{x \vee y})} =$$

$$\overline{(\overline{x}.\overline{z}) \uparrow (\overline{x}.\overline{y})} = \overline{(\overline{x} \uparrow \overline{z}) \uparrow (\overline{x} \uparrow \overline{y})}$$

Naloga 1: Nepopolne logične funkcije – NAND, NOR

- Zapišite: PDNO za redundance x=0
- MDNO, MKNO, MNO

x	y	z	f(x.y.z)
0	0	0	0
0	0	1	x
0	1	0	x
0	1	1	1
1	0	0	x
1	0	1	1
1	1	0	1
1	1	1	0

$$f(x,y,z) =$$
$$\bar{x}.y \vee x.\bar{y} \vee x.\bar{z} =$$
$$= (\bar{x} \uparrow y) \uparrow (x \uparrow \bar{y}) \uparrow (x \uparrow \bar{z})$$

f	$\bar{z}$	z
$\bar{x}$ $\bar{y}$		x
$\bar{x}$ y	x	1
x y	I	
x $\bar{y}$	x	I

$$\bar{f} = \bar{x}.\bar{y} \vee x.y.z$$
$$\bar{\bar{f}} = \overline{\bar{x}.\bar{y} \vee x.y.z}$$

$$f(x,y,z)$$
$$= (x \vee y).(\bar{x} \vee \bar{y} \vee \bar{z})$$
$$= (x \downarrow y) \downarrow (\bar{x} \downarrow \bar{y} \downarrow \bar{z})$$

$\bar{f}$	$\bar{z}$	z
$\bar{x}$ $\bar{y}$	1	x
$\bar{x}$ y	x	
x y		1
x $\bar{y}$	x	