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1. $\lim_{n\to\infty} \frac{(2n-3)^2(2n+3)}{(2-3n)} = \lim_{n\to\infty} \frac{(4n^2-12n+9)(2n+3)}{(2-3n)(4-12n+9n^2)} =$

$= \lim_{n\to\infty} \frac{8n^3+12n^2-24n^2-36n+18n+27}{8-24n+18n^2-12n+36n^2-27n^3} = \lim_{n\to\infty} \frac{8n^3-12n^2+18n+27}{8-36n+54n^2-27n^3} \Big/ n^3 = \lim_{n\to\infty} \frac{\frac{8n^3}{n^3}-\frac{12n^2}{n^3}+\frac{18n}{n^3}+\frac{27}{n^3}}{\frac{8}{n^3}-\frac{36n}{n^3}+\frac{54n^2}{n^3}-\frac{27n^3}{n^3}} =$

$= \lim_{n\to\infty} \frac{8-\frac{12}{n}+\frac{18}{n^2}+\frac{27}{n^3}}{\frac{8}{n^3}-\frac{36}{n^2}+\frac{54}{n}-27} = \frac{8-0-0+0}{0-0+0-27} = -\frac{8}{27}$ ($\frac{12}{n}, \frac{18}{n^2}, \frac{27}{n^3} \to 0$)

2. $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{\frac{n-3}{2}} = \lim_{n\to\infty} \sqrt{\left(1+\frac{1}{n}\right)^n} \cdot \sqrt{\left(1+\frac{1}{n}\right)^{-3}} = \sqrt{e} \cdot \sqrt{1^{-3}} = \sqrt{e}$ ($e$, $0$)

3. $a_n = \frac{2n-1}{n}$ a) $\lim_{n\to\infty} \frac{2n-1}{n} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\frac{n}{n}-0} = \frac{2-0}{1} = 2$

b) $\varepsilon = 0{,}25$ $a+\varepsilon = 2+0{,}25 = 2{,}25$

$a-\varepsilon = 2-0{,}25 = 1{,}75$ $O_\varepsilon(2) = (1{,}75;\ 2{,}25)$

$a_6 = \frac{2\cdot 6-1}{6} = \frac{11}{6} = 1{,}83$ Odg.: Šesti člen leži v tej $\varepsilon$-okolici.

4. $3+6x+12x^2+24x^3+\ldots \longrightarrow \frac{3}{1}-\frac{12}{5}+\frac{48}{25}-\frac{192}{125}+\ldots$

a) $x = -\frac{2}{5}$ $k = -\frac{4}{5}$

$a_1 = 3$ $S = \frac{a_1}{1-k} = \frac{3}{1+\frac{4}{5}} = \frac{3}{\frac{9}{5}} = \frac{3}{1}\cdot\frac{5}{9} = \frac{15}{9} = \frac{5}{3}$

b) $-5 = \frac{3}{1-k} \quad /\cdot(1-k)$

$-5+5k = 3$

$5k = 8 \quad /:5$

$k = \frac{8}{5}$ Odg.: Ne more biti, saj $-1 < k < 1$.

12. $a_n = \frac{10 \cdot (-1)^n \cdot (n-1)}{n+1}$

$a_1 = \frac{10 \cdot (-1)^1 \cdot (1-1)}{1+1} = \frac{0}{2} = 0$

$a_2 = \frac{10 \cdot (-1)^2 \cdot (2-1)}{2+1} = \frac{10}{3}$

$a_3 = \frac{10 \cdot (-1)^3 \cdot (3-1)}{3+1} = \frac{-20}{4} = -5$

$a_4 = \frac{30}{5} = 6$

$a_5 = \frac{-40}{6} = \frac{-20}{3}$

$a_6 = \frac{50}{7}$

$a_7 = \frac{-60}{8} = -\frac{15}{2}$

$a_8 = \frac{70}{9}$

$a_9 = \frac{-80}{10} = -8$

$a_{10} = \frac{90}{11}$

$a_{11} = \frac{-100}{12} = \frac{-25}{3}$

$a_{12} = \frac{110}{13}$

$a_{13} = \frac{-120}{14} = \frac{-60}{7}$

a) $n = \binom{13}{2} = 78$

$m = \binom{4}{2} = 6$

$P(A) = \frac{6}{78} = 0,08$

b) $n = 78$

$m = \binom{4}{1}\binom{9}{1} + \binom{4}{2} = 4 \cdot 9 + 6 = 42$

$P(B) = \frac{42}{78} = 0,54$

c) $n = 78$

$m = \binom{12}{1} \cdot \binom{1}{1} = 12 \cdot 1 = 12$

$P(C) = \frac{12}{78} = 0,15$

d) $n = 78$

$m = \binom{6}{2} = 15$

$P(D) = \frac{15}{78} = 0,19$

8. $T(3,1)$
$A(2,-1)$

$y = a \cdot (x-r)^2 + q$
$-1 = a \cdot (2-3)^2 + 1$
$-1 = a + 1$
$-2 = a$

$y = -2(x-3)^2 + 1$
$y = -2(x^2 - 6x + 9) + 1$
$y = -2x^2 + 12x - 17$

$f(x) = -2x^2 + 12x - 17$

9. $y = x^2 - 6x + 5$
$2x - y - 7 = 0 \rightarrow y = 2x - 7$

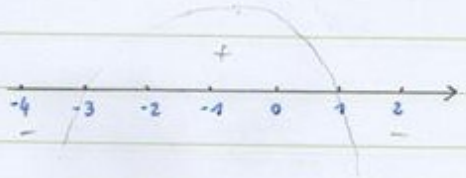
$2x - 7 = x^2 - 6x + 5$
$x^2 - 8x + 12 = 0$
$(x-6)(x-2) = 0$
$x_1 = 6 \quad x_2 = 2$

$A(6,5) \quad B(2,-3)$

$12 - 7 = y$
$y = 5$

$4 - 7 = y$
$y = -3$

10. $-2(x+1)^2 \leq -8$
$-2(x^2 + 2x + 1) + 8 \leq 0$
$-2x^2 - 4x + 6 \leq 0$
$-2(x^2 + 2x - 3) \leq 0$
$-2(x+3)(x-1) \leq 0$
$x_1 = -3 \quad x_2 = 1$




$x \in (-\infty, -3] \cup [1, \infty)$

Subject:

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**7.** $(2^{x-8})^{x-2} = \frac{1}{32}$

$2^{(x-8)\cdot(x-2)} = 2^{-5}$

$2^{x^2-10x+16} = 2^{-5}$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x_1 = 7 \quad x_2 = 3$

7, 3, ...

$a_1 = 7$

$d = -4$

$a_n = a_1 + (n-1)\cdot d$

$a_6 = 7 + (6-1)\cdot(-4)$

$a_6 = 7 - 20$

$a_6 = -13$

**8.** $\sqrt{2^{3x}}, 2^{2x+1}, 2\cdot 4^{\frac{3x}{2}} \longrightarrow 2^{\frac{3x}{2}}, 2^{2x+1}, 2^{3x+1}$

$a = \sqrt{a\cdot b}$

$2^{2x+1} = \sqrt{2^{\frac{3x}{2}}\cdot 2^{3x+1}}$

$2^{2x+1} = \sqrt{2^{\frac{9x}{2}+1}} \quad /^2$

$2^{4x+2} = 2^{\frac{9x}{2}+1}$

$4x + 2 = \frac{9x}{2} + 1$

$-\frac{1}{2}x = -1$

$x = 2$

**9.** 100, 97, 94, ...

$a_1 = 100$

$d = -3$

$a_n = a_1 + (n-1)\cdot d$

$a_n = 100 + (n-1)\cdot(-3)$

$a_n = 100 - 3n + 3$

$a_n = 103 - 3n$

$a_n < 31$

$103 - 3n < 31$

$-3n < -72 \quad /:(-3)$

$n > 24$

ODG: Od vključno 24 člena.

$a_{50} = 103 - 3\cdot 50 = -47$

$S_{50} = \frac{50}{2}\cdot(a_1 + a_{50})$

$S_{50} = 25\cdot(100 - 47)$

$S_{50} = 25\cdot 53$

$S_{50} = 1325$

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1. $f(x) = ax^2 + bx - 4$

$x_1 = -2 \rightarrow B(-2,0)$

$A(4,-24)$

$0 = 4a - 2b - 4$

$-24 = 16a + 4b - 4$

$4a - 2b = 4 \quad /\cdot 2$

$16a + 4b = -20$

$8a - 4b = 8$

$16a + 4b = -20$

$24a = -12 \quad /:24$

$a = -\frac{1}{2}$

$-2 - 2b = 4$

$-2b = 6 \quad /:(-2)$

$b = -3$

2. $y = -0,5x^2 + x + 1,5$

$D = 1 + 3$

$D = 4$

$\sqrt{D} = 2$

$x_1 = \frac{-1+2}{-1} = -1$

$x_2 = \frac{-1-2}{-1} = 3$

NIČELNA: $y = a(x - x_1)(x - x_2)$

$y = -0,5(x+1)(x-3)$

$r = \frac{-1}{-1} = 1$

$q = \frac{-4}{-2} = 2$

TEMENSKA: $y = a(x-r)^2 + q$

$y = -0,5(x-1)^2 + 2$

3. $f(x) = -1(x+2)^2 + 1 \rightarrow r = -2 \quad q = 1 \quad T(-2,1)$

$f(x) = -1(x^2 + 4x + 4) + 1$

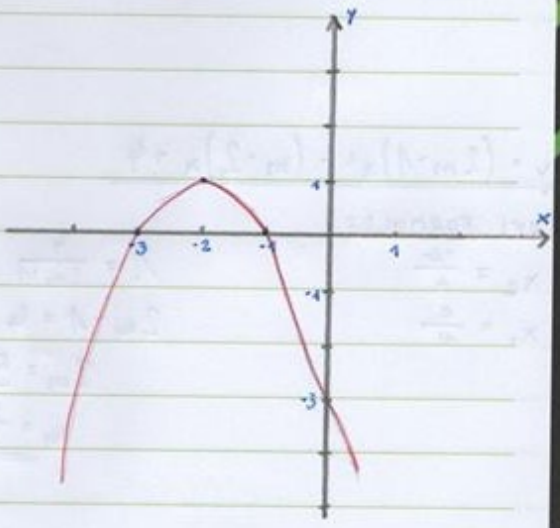
$f(x) = -x^2 - 4x - 3$

$f(x) = -1(x^2 + 4x + 3)$

$f(x) = -1(x+3)(x+1)$

$x_1 = -3 \quad x_2 = -1$

$N(0,-3)$



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10. $\frac{1}{2} + \frac{2}{x} + \frac{8}{x^2} + \dots$

$S = \frac{2}{3}$

$k = \frac{2}{x} : \frac{1}{2} = \frac{2}{x} \cdot \frac{2}{1} = \frac{4}{x}$

$S = \frac{a_1}{1-k}$

$\frac{2}{3} = \frac{\frac{1}{2}}{1-\frac{4}{x}}$

$2 - \frac{8}{x} = \frac{3}{2}$

$\frac{1}{2} = \frac{8}{x}$

$x = 16$

11. $\log_2(x+2),\ 3,\ \log_2(9x-2)$

$a = \frac{a+b}{2}$

$3 = \frac{\log_2(x+2) + \log_2(9x-2)}{2} \quad /\cdot 2$

$6 = \log_2(x+2)\cdot(9x-2)$

$\log_2 64 = \log_2(9x^2 - 2x + 18x - 4)$

$64 = 9x^2 + 16x - 4$

$9x^2 + 16x - 68 = 0$

$D = 256 + 2448$

$D = 2704$

$\sqrt{D} = 52$

$x_1 = \frac{-16+52}{18} = 2$ ✓

$x_2 = \frac{-16-52}{18} = \frac{-68}{18} = -\frac{34}{9}$ //

$PR_1$:

$\log_2(2+2) = \log_2 4 = 2$ ✓

$\log_2(18-2) = \log_2 16 = 4$ ✓

$PR_2$:

$\log_2\left(-\frac{34}{9}+2\right) = \log_2\left(-\frac{34}{9}+\frac{18}{9}\right) = \log_2 -\frac{16}{9}$ //

$\log_2(-34-2) = \log_2 -36$ //

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8. 0, 1, 2, 3, 5, 6, 7, 9 ← N=število manjših od 600

7 + 7 8 + 4 8 8 $N = 7 + 7 \cdot 8 + 4 \cdot 8 \cdot 8 = 319$

9. SLOVENIJA

$n = \binom{9}{3} = 84$

$m = 4$ $P(A) = \frac{m}{n} = \frac{4}{84} \doteq 0,048$

SAMO SOGLASNIKI SAMO SAMOGLASNIKI

$m_1 = \binom{5}{3} = 10$ $m_2 = \binom{4}{3} = 4$ $P(B) = 1 - \frac{10+4}{84} \doteq 0,83$

10. 5 knjig: 1 roman, 2 pesmi in 2 učbenika

a) $m = \underbrace{1}_{\text{ROMAN}} \cdot \underbrace{4!}_{\text{OSTALE}}$ $P(A) = \frac{1 \cdot 4!}{5!} = \frac{24}{120} = 0,2$

$n = 5!$

b) $m = \underbrace{2!}_{\text{PESMI}} \cdot \underbrace{3!}_{\text{OSTALE}}$ $P(B) = \frac{2! \cdot 3!}{5!} = \frac{12}{120} = 0,1$

c) $m =$ 1 1 2

11. 20 vrtk - 3 prazne

a) VSE DOBITNE 2 DOBITNI
$n = \binom{17}{3} + \binom{17}{2} \cdot \binom{3}{1} = 1088$ $P(A) = \frac{1088}{1140} = 0,95$
$m = \binom{20}{3} = 1140$ PRAZNA

b) $\binom{3}{3}$ 3 IZBEREJO 3 PRAZNE $= 1$ $P(B) = \frac{1}{1140} \doteq 0,0009$
$\binom{20}{3} = 1140$

5. 11 mest : 4 Renaulti, 2 Opla in 5 Toyot

a) $N = 11! = 39\,916\,800$

b) $\frac{2!}{OP}\ \frac{4!}{RE}\ \frac{5!}{TO} + \frac{2!}{OP}\ \frac{5!}{TO}\ \frac{4!}{RE} + \frac{5!}{TO}\ \frac{4!}{RE}\ \frac{2!}{OP} +$

$+ \frac{5!}{TO}\ \frac{2!}{OP}\ \frac{4!}{RE} + \frac{4!}{RE}\ \frac{2!}{OP}\ \frac{5!}{TO} + \frac{4!}{RE}\ \frac{5!}{TO}\ \frac{2!}{OP}$

$N = (4! \cdot 5! \cdot 2!) \cdot 3! = 34560$

c) $\frac{5!}{TOYOTE}\ \frac{6!}{OSTALI}$ $\quad N = 5! \cdot 6! = 86400$

d) $\_\_\_\ \frac{2!}{OPLA}\ \_\_\_$ $\quad N = 2! \cdot 9! = 725\,760$

OSTALI $9!$

6. 5 Gor., 4 Prim. in 6 Dol. → 6 članov

a) $N = \binom{15}{6} = 5005$ $\quad$ b) $N = \binom{5}{2} \cdot \binom{4}{2} \cdot \binom{6}{2} = 10 \cdot 6 \cdot 15 = 900$

c) $N_1 = \binom{6}{3} \cdot \binom{10}{3} = 10 \cdot 120 = 1200$
$N_2 = \binom{5}{4} \cdot \binom{10}{2} = 5 \cdot 45 = 225$ $\quad N = 1200 + 225 + 10 = 1435$
$N_3 = \binom{5}{5} \cdot \binom{10}{1} = 1 \cdot 10 = 10$

d) $N = \binom{11}{6} = 462$

7. FUNKCIJA

a) $8\ \ 7\ \ 6\ \ 5\ \ 4$ $\quad N = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$

b) $8\ \ 8\ \ 8\ \ 8\ \ 8$ $\quad N = 8^5 = 32768$

c) $\binom{5}{1}\ \ 7\ \ 6\ \ 5\ \ 4$ $\quad N = 5 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 4200$

4. $-x^2+bx-81=0$

$D=0$

$b^2-4\cdot(-1)\cdot(-81)=0$

$b^2-324=0$

$b^2=324$

$b_1=18$

$b_2=-18$

5. $\sqrt{x+7}+3=3x$

$\sqrt{x+7}=3x-3\ /^2$

$x+7=9x^2-18x+9$

$9x^2-19x+2=0$

$D=361-72$

$D=289$

$\sqrt{D}=17$

$x_1=\frac{19+17}{18}=2$ ✓

$x_2=\frac{19-17}{18}=\frac{1}{9}$ //

PR. 1: $\sqrt{9}+3=6$

$\sqrt{9}=3$ ✓

PR. 2: $\sqrt{\frac{64}{9}}=-\frac{26}{9}$ //

6. $a:b=5:12 \rightarrow a=5x=15\,cm$

$b=12x=36\,cm$

$hip=39\,cm$

$39^2=(5x)^2+(12x)^2$

$1521=25x^2+144x^2$

$1521=169x^2\ /:169$

$9=x^2$

$x=3$

7. $y=(2m-1)x^2-(m-2)x+4$

VIETOVI FORMULI:

$x_1+x_2=\frac{-b}{a}$

$x_1\cdot x_2=\frac{c}{a}$

$1=\frac{4}{2m-1}$

$2m-1=4$

$2m=5$

$m=\frac{5}{2}$