Permutacije brez ponavljanja

$$P_n = n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$$

Permutacije s ponavljanjem

$$P_n^{k_1, k_2, \dots, k_r} = \frac{n!}{k_1! \cdot k_2! \cdot \dots \cdot k_r!}$$

Variacije brez ponavljanja

$$V_n^r = n \cdot (n-1) \cdot (n-2) \cdot ... \cdot (n-r+1) = \frac{n!}{(n-r)!}$$

Variacije s ponavljanjem

$$^{(p)}V_n^r = \underbrace{n \cdot n \cdot \dots \cdot n}_r = n^r$$

Kombinacije brez ponavljanja

$$C_n^r = \frac{V_n^r}{r!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

Lastnosti binomskih simbolov

$$\binom{n}{r} = \binom{n}{n-r} \qquad C_n^r = C_n^{n-r}$$

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1} \qquad C_n^r + C_n^{r+1} = C_{n+1}^{r+1}$$

$$\binom{n}{0} = 1 \qquad \binom{n}{n} = 1 \qquad \binom{n}{1} = n$$

Kombinacije s ponavljanjem

$$^{(p)}C_n^r = C_{n+r-1}^r = \binom{n+r-1}{r}$$

Vezane kombinacije

$$C_{n_1,n_2,\ldots,n_m}^{r_1,r_2,\ldots,r_m} = \binom{n_1}{r_1} \cdot \binom{n_2}{r_2} \cdot \ldots \cdot \binom{n_m}{r_m}$$

Binomski izrek

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

Kumulativna porazdelitev frekvenc

$$F_1 = 0$$
, $F_k = F_{k-1} + f_{k-1}$

Relativna frekvenca in kumulativna porazdelitev

$$f_k^o = \frac{f_k}{N}$$
, $F_1^o = 0$, $F_k^o = F_{k-1}^o + f_{k-1}^o$

Navadna in tehtana aritmetična sredina

$$\mu = \frac{1}{N} \sum_{i=1}^{N} y_i, \quad \mu = \frac{f_1 \cdot y_1 + \dots + f_r \cdot y_r}{f_1 + \dots + f_r} = \frac{1}{N} \sum_{i=1}^{r} f_i y_i$$

Geometrijska in harmonična sredina

$$G = \sqrt[N]{y_1 \cdot y_2 \cdot ... \cdot y_N}, \quad H = \left\{ \frac{1}{N} \sum_{i=1}^N \frac{1}{y_i} \right\}^{-1}$$

Varianca (negrupirani podatki)

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - \mu)^2 = \frac{1}{N} \sum_{i=1}^{N} y_i^2 - \mu^2$$

Varianca (grupirani podatki)

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^r f_i (y_i - \mu)^2 = \frac{1}{N} \sum_{i=1}^r f_i y_i^2 - \mu^2$$

Standardni odklon, variacijski razmik, koeficient variacije

$$\sigma = \sqrt{\sigma^2}$$
, $R_v = y_{max} - y_{min}$, $K_v = \frac{\sigma}{\mu}$

Bazni in verižni indeksi

$$I_{k/0} = 100 \cdot \frac{Y_k}{Y_0}, \quad J_k = 100 \cdot \frac{Y_k}{Y_{k-1}} \quad (k = 1, 2, ..., n)$$

Koeficient dinamike in povprečni koeficient dinamike

$$K_i = Y_i / Y_{i-1}$$
, $\overline{K} = \sqrt[N]{K_1 \cdot K_2 \cdot ... \cdot K_N} = \sqrt[N]{Y_n / Y_1}$

Kumulativna porazdelitev frekvenc

$$F_1 = 0$$
, $F_k = F_{k-1} + f_{k-1}$

Relativna frekvenca in kumulativna porazdelitev

$$f_k^o = \frac{f_k}{N}$$
, $F_1^o = 0$, $F_k^o = F_{k-1}^o + f_{k-1}^o$

Navadna in tehtana aritmetična sredina

$$\mu \; = \; \frac{1}{N} \sum_{i=1}^{N} \, y_i \, , \quad \mu \; = \; \frac{f_1 \cdot y_1 \, + \, \ldots \, + \, f_r \cdot y_r}{f_1 \, + \, \ldots \, + \, f_r} \; = \; \frac{1}{N} \sum_{i=1}^{r} \, f_i \, y_i$$

Geometrijska in harmonična sredina

$$G = \sqrt[N]{y_1 \cdot y_2 \cdot ... \cdot y_N}, \quad H = \{\frac{1}{N} \sum_{i=1}^N \frac{1}{y_i}\}^{-1}$$

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$$K_i = Y_i / Y_{i-1}$$
, $\overline{K} = \sqrt[N]{K_1 \cdot K_2 \cdot ... \cdot K_N} = \sqrt[N]{Y_n / Y_1}$

Relativna frekvenca dogodka
$$f^{o}(A) = \frac{f(A)}{n}$$

Klasična definicija verjetnosti
$$P(A) = \frac{m}{n}$$

Lastnosti verjetnosti - aksiomi Kolmogorova

NENEGATIVNOST
$$(\forall A)(A \in PG \implies P(A) \ge 0)$$

NORMIRANOST
$$P(G) = 1$$

ADITIVNOST
$$A \cap B = N \Longrightarrow P(A \cup B) = P(A) + P(B)$$

Posledice

$$A \subset B \Longrightarrow P(A) \leq P(B)$$

 $P(A) + P(A') = 1$, $P(A) = 1 - P(A')$
 $P(A) \leq 1$, $P(N) = 0$
 $P(A \cup B) = P(A) + P(B) - P(AB)$

Pogojna verjetnost, verjetnost produkta, neodvisnost

$$P(A/B) = \frac{P(AB)}{P(B)}, \quad P(AB) = P(B)P(A/B) = P(A)P(B/A)$$

$$A$$
 in B neodvisna $\iff P(A) = P(A/B)$, $P(B) = P(B/A)$
 A in B neodvisna $\iff P(AB) = P(A) P(B)$

$$P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1) P(A_2/A_1)...P(A_n/(A_1 \cap ... \cap A_{n-1}))$$

V celoti neodvisni :
$$P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1) P(A_2)...P(A_n)$$

Obrazec za popolno verjetnost

$$P(A) = P(H_1) P(A/H_1) + P(H_2) P(A/H_2) + ... + P(H_n) P(A/H_n)$$

Bayesov obrazec

$$P(H_i/A) = \frac{P(H_i) P(A/H_i)}{P(A)}$$
 $(i = 1, 2, ..., n)$

Bernoullijev obrazec

$$P(n; p; k) = \binom{n}{k} p^k (1 - p)^{n - k} \qquad (k = 0, 1, 2, ..., n)$$

Najverjetnejša frekvenca v Bernoullijevem zaporedju

$$np-q$$
 ni celo število $\implies k_o = [np-q]+1$ $np-q$ je celo število $\implies k_o = np-q$ in $k_o = np-q+1$

Pascalov obrazec

$$P(\text{frekvenca } k \vee n - \text{tem poskusu}) = \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

Posplošeni Bernoullijev obrazec

$$P(n; p_1, p_2, ..., p_m; k_1, k_2, ..., k_m) = \frac{n!}{k_1! \cdot k_2! \cdot ... \cdot k_m!} p_1^{k_1} \cdot p_2^{k_2} \cdot ... \cdot p_m^{k_m}$$

Matematično upanje (končna zaloga vrednosti)

$$E(X) = \sum_{k=1}^{n} p_k x_k$$

Disperzija in standardni odklon

$$D(X) = E\{[X - E(X)]^2\}$$

$$D(X) = \sum_{i=1}^{n} p_i [x_i - E(X)]^2 = \{\sum_{i=1}^{n} p_i x_i^2\} - [E(X)]^2$$

$$\sigma = \sqrt{D(X)}$$

Binomska porazdelitev

$$X \sim b(n,p) \Longrightarrow E(X) = n p, \quad \sigma(X) = \sqrt{n p q}$$

Neenačba Čebiševa

$$P(|X - E(X)| < t) \ge 1 - \frac{D(X)}{t^2}$$

 $P(|X - E(X)| \ge t) \le \frac{D(X)}{t^2}$

(B)

$$B/A$$
)

$$A_{n-1}))$$

$$P(A_n)$$

$$(A/H_n)$$

Normalna porazdelitev

$$y = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-a)^2}{2\sigma^2}}$$

Standardizacija

$$X \sim N(a, \sigma) \implies Z = \frac{X - a}{\sigma} \sim N(0, 1)$$

Standardizirana normalna porazdelitev

$$\begin{split} y &= \frac{1}{\sqrt{2\pi}} \, e^{-\frac{x^2}{2}} \, = \, \varphi(x) \\ \Phi(z) &= \text{ploščina pod } \varphi(x) \, \text{ od } x = 0 \, \text{do } x = z \, , \, \, \Phi(-z) \, = \, -\Phi(z) \\ X \sim N(0,1) \implies P(\alpha \leq Z \leq \beta) \, = \, \Phi(\beta) \, - \, \Phi(\alpha) \end{split}$$

Poljubna normalna porazdelitev

$$X \sim N(a, \sigma) \implies P(\alpha \le X \le \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right)$$

Verjetnosti za značilne intervale

$$X \sim N(a, \sigma) \implies P(a - \sigma \le X \le a + \sigma) = 0.6826$$

 $P(a - 2\sigma \le X \le a + 2\sigma) = 0.9544$
 $P(a - 3\sigma \le X \le a + 3\sigma) = 0.9974$

Bernoullijev zakon velikih števil

$$P\left(\left|\frac{X}{n} - p\right| < t\right) \ge 1 - \frac{p(1-p)}{nt^2}$$

X: frekvenca dogodka, p: njegova verjetnost, n: število poskusov