

$$1. \quad a_n = 8 \cdot \left(\frac{1}{4}\right)^{\frac{n}{2}-2}$$

$$a_1 = 8 \cdot \left(\frac{1}{4}\right)^{\frac{1}{2}-2}$$

$$a_1 = 8 \cdot \left(\frac{1}{4}\right)^{-\frac{3}{2}}$$

$$a_1 = 8 \cdot \sqrt{4^3}$$

$$a_1 = 8 \cdot 8$$

$$\underline{a_1 = 64}$$

$$a_2 = 8 \cdot \left(\frac{1}{4}\right)^{1-2}$$

$$a_2 = 8 \cdot \left(\frac{1}{4}\right)^{-1}$$

$$a_2 = 8 \cdot 4$$

$$\underline{a_2 = 32}$$

$$a_4 = 8 \cdot \left(\frac{1}{4}\right)^{2-2}$$

$$a_4 = 8 \cdot \left(\frac{1}{4}\right)^0$$

$$a_4 = 8 \cdot 1$$

$$\underline{a_4 = 8}$$

$$a_{100} = 8 \cdot \left(\frac{1}{4}\right)^{50-2}$$

$$a_{100} = 8 \cdot \left(\frac{1}{4}\right)^{48}$$

$$a_{100} = 2^3 \cdot 2^{-2 \cdot 48}$$

$$\underline{a_{100} = 2^{-93}}$$

$$8 \cdot \left(\frac{1}{4}\right)^{\frac{n}{2}-2} = 2^{-27}$$

$$2^3 \cdot 2^{-2 \cdot (\frac{n}{2}-2)} = 2^{-27}$$

$$2^3 \cdot 2^{-n+4} = 2^{-27}$$

$$3-n+4 = -27$$

$$\underline{n = 34}$$

$$2. \quad a_1 = \frac{1}{2}$$

$$a_2 = \frac{1}{4}$$

$$\underline{a_{n+2} = 2a_n + 8a_{n+1}}$$

$$a_3 = 2a_1 + 8a_2 = 2 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4} = 1 + 2 = \underline{3}$$

$$a_4 = 2a_2 + 8a_3 = 2 \cdot \frac{1}{4} + 8 \cdot 3 = \frac{1}{2} + 24 = \underline{24\frac{1}{2}}$$

$$a_5 = 2a_3 + 8a_4 = 2 \cdot 3 + 8 \cdot 24\frac{1}{2} = 6 + 196 = \underline{202}$$

$$3. \quad a_1 + a_2 = 13$$

$$\underline{S_{20} = 670}$$

$$a_1 + a_2 = 13$$

$$a_1 + a_1 + d = 13$$

$$2a_1 + d = 13$$

$$2a_1 + d = 13 \quad / \cdot (-1)$$

$$\underline{2a_1 + 19d = 67}$$

$$-2a_1 - d = -13$$

$$\underline{2a_1 + 19d = 67}$$

$$18d = 54$$

$$\underline{d = 3}$$

$$2a_1 + 3 = 13$$

$$2a_1 = 10$$

$$\underline{a_1 = 5}$$

$$S_n = \frac{n}{2} \cdot (a_1 + a_n)$$

$$S_{20} = \frac{20}{2} \cdot (a_1 + a_{20})$$

$$670 = 10 \cdot (a_1 + a_{20})$$

$$670 = 20a_1 + 190d \quad / : 10$$

$$67 = 2a_1 + 19d$$

$$a_n = a_1 + (n-1) \cdot d$$

$$a_n = 5 + (n-1) \cdot 3$$

$$a_n = 5 + 3n - 3$$

$$\underline{a_n = 2 + 3n}$$

$$4. 1 \cdot 23 = 23$$

$$2 \cdot 23 = 46$$

$$3 \cdot 23 = 69$$

$$4 \cdot 23 = 92$$

$$5 \cdot 23 = 115$$

$$6 \cdot 23 = 138$$

⋮

$$43 \cdot 23 = 989$$

$$a_1 = 115$$

$$d = 23$$

$$a_n = 989$$

$$n = 39$$

$$43 - 4$$

$$\frac{n}{2} \cdot (a_1 + a_n) = \frac{39}{2} \cdot (115 + 989) =$$

$$= \underline{\underline{21528}}$$

$$5. a_1 = 15$$

$$d = -3$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 15 + (n-1) \cdot (-3)$$

$$a_n = 15 - 3n + 3$$

$$a_n = 18 - 3n$$

$$S_n < -2000$$

$$\frac{n}{2} \cdot (a_1 + a_n) < -2000$$

$$\frac{n}{2} \cdot (15 + 18 - 3n) < -2000 \quad | \cdot 2$$

$$n \cdot (33 - 3n) < -4000$$

$$33n - 3n^2 + 4000 < 0 \quad | \cdot (-1)$$

$$3n^2 - 33n - 4000 > 0$$

$$D = b^2 - 4ac$$

$$D = 1089 + 48000$$

$$D = 49089$$

$$\sqrt{D} = 221,56$$

$$n_1 = \frac{-b + \sqrt{D}}{2a} = \frac{33 + 221,56}{6} = 42,43 \checkmark$$

1. VEŠJE NI ŠTEVILO

$$n_2 = \frac{-b - \sqrt{D}}{2a} = \frac{33 - 221,56}{6} = -31,43 //$$

Odg.: Štetiti moramo najmanj 43 členov.

$$6. 4x-1, 2x-4, x+3$$

$$a) a = \frac{a+b}{2}$$

$$2x-4 = \frac{4x-1+x+3}{2} \quad | \cdot 2$$

$$4x-8 = 4x-1+x+3$$

$$\underline{\underline{x = -10}}$$

$$b) a = \sqrt{a \cdot b}$$

$$(2x-4) = \sqrt{(4x-1) \cdot (x+3)} \quad |^2$$

$$4x^2 - 16x + 16 = 4x^2 + 12x - x - 3$$

$$-27x = -19$$

$$\underline{\underline{x = \frac{19}{27}}}$$

$$2 = \sqrt{4 \cdot k}$$

$$5^{-2x+1} = \sqrt{5^{-2x+2} \cdot 5^{3x-2}} \quad /^2$$

$$5^{-4x+2} = 5^{2x+2} \cdot 5^{3x-2}$$

$$-4x+2 = 2x+2+3x-2$$

$$-9x = -2$$

$$\underline{\underline{x = \frac{2}{9}}}$$

8. $a_n = \frac{2n-1}{n}$

$$a_1 = \frac{2 \cdot 1 - 1}{1} = \frac{2-1}{1} = 1$$

$$a_2 = \frac{2 \cdot 2 - 1}{2} = \frac{4-1}{2} = \frac{3}{2}$$

$$a_{n+1} - a_n > 0$$

$$\frac{2n+1}{n+1} - \frac{2n-1}{n} > 0$$

$$\frac{2n^2+n}{n(n+1)} - \frac{2n^2+n-1}{n(n+1)} > 0$$

$$\underline{\underline{\frac{1}{n(n+1)} > 0}}$$

9. $a_1 = 4$

$$a_5 = 324$$

$$a_5 = a_1 \cdot k^4$$

$$324 = 4 \cdot k^4$$

$$81 = k^4 \quad / \sqrt[4]{}$$

$$k_1 = 3 \quad \longrightarrow \quad 4, 12, 36, 108, 324$$

$$k_2 = -3 \quad \longrightarrow \quad \underline{\underline{4, -12, 36, -108, 324}}$$

10. $f(x) = 16x^3 - 42x^2 + 21x - 2$

$$\frac{c}{d} = \pm 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, 2$$

16	-42	21	-2
2	32	-20	2
16	-10	1	0

$$x_1 = 2$$

$$a_1 = 2$$

$$a_2 = \frac{1}{2}$$

$$a_3 = \frac{1}{8}$$

$$a_2 = a_1 \cdot k$$

$$\frac{1}{2} = 2 \cdot k \quad / :2$$

$$k = \frac{1}{4}$$

$$16x^2 - 10x + 1 = 0$$

$$D = 100 - 64$$

$$D = 36$$

$$\sqrt{D} = 6$$

$$a_n = a_1 \cdot k^{n-1}$$

$$\underline{\underline{a_n = a_1 \cdot \left(\frac{1}{4}\right)^{n-1}}}$$

$$x_1 = \frac{10+6}{32} = \frac{1}{2}$$

$$x_2 = \frac{10-6}{32} = \frac{1}{8}$$

11. $a_1 = 3$

$a_n = 41$

$S_n = 440$

$S_n = \frac{n}{2} \cdot (a_1 + a_n)$

$440 = \frac{n}{2} \cdot (3 + 41)$

$440 = \frac{n}{2} \cdot 44 \quad / : 44$

$10 = \frac{n}{2} \quad / \cdot 2$

$n = 20$

$a_n = a_1 + (n-1) \cdot d$

$41 = 3 + (20-1) \cdot d$

$41 = 3 + 19d$

$38 = 19d$

$d = 2$

3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41

12.

13. $n=4$

$u_1+1, u_2, u_3+2 \rightarrow \text{GEO. ZAP.}$

$$\begin{array}{ccc} \downarrow & \downarrow & \\ u_1+4 & u_1+2 \cdot 4+2 & \\ & \parallel & \\ & u_1+10 & \end{array}$$

$$(u_1+4)^2 = \sqrt{(u_1+1) \cdot (u_1+10)} / ^2$$

$$u_1^2 + 8u_1 + 16 = u_1^2 + 11u_1 + 10$$

$$3u_1 = 6$$

$$u_1 = 2$$

$u_1, u_2, u_3 \rightarrow \text{ARI. ZAP.}$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 & 2+4 & 2+2 \cdot 4 \\ & \parallel & \parallel \\ & 6 & 10 \end{array}$$

2, 6, 10