Exercise Set 6

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Note: This exercise sheet does not require or benefit from the use of Tamarin.

Preliminary Definitions

In the following exercises we make use of the equational theories defined below. These theories model pairs of terms, hashes, symmetric encryption and asymmetric encryption. Recall from the lectures that that an equational theory is compromised of a function signature and a set of equations. A function signature is a set of function symbols and their arities. The equations are (implicitly) universally quantified.

$$\left.\begin{array}{l}\mathsf{fst}/1,\ \mathsf{snd}/1,\ \mathsf{pair}/2\\ \\ \mathsf{fst}(\mathsf{pair}(x,y)) = x\\ \mathsf{snd}(\mathsf{pair}(x,y)) = y\end{array}\right\} := E_p$$

$$\left.\begin{array}{l}\mathsf{h}/1\\ \\ \emptyset\end{array}\right\} := E_h$$

$$\left.\begin{array}{l}\mathsf{senc}/2,\ \mathsf{sdec}/2\\ \\ \mathsf{sdec}(\mathsf{senc}(m,k),k) = m\end{array}\right\} := E_{se}$$

$$\left.\begin{array}{l}\mathsf{aenc}/2,\ \mathsf{adec}/2,\ \mathsf{pk}/1\\ \\ \mathsf{adec}(\mathsf{aenc}(m,\mathsf{pk}(sk)),sk) = m\end{array}\right\} := E_{ae}$$

Where convenient, we make use of common shorthand for these functions. We write $\langle x, y, z\rangle$ for $\mathsf{pair}(x, \mathsf{pair}(y, z))$ and so on. We write $\{m\}_k$ and $\{m\}_{\mathsf{pk}(a)}$ for $\mathsf{senc}(m, k)$ and $\mathsf{aenc}(m, \mathsf{pk}(a))$ respectively. We also overload notation and write $E_x \cup E_y$ for the combination of two equational theories (with disjoint signatures) formed by the union of their respective signatures and sets of equations.

Part 1: Term Rewriting Systems

1.1 Equalities

Under the equational theory $E = E_p \cup E_h \cup E_{se} \cup E_{ae}$, determine which of the following equations hold. The symbols $x, y, z, a, b, c, d, k, m$ are constants.

$$\begin{aligned}
\mathsf{pair}(\mathsf{fst}(\mathsf{pair}(x,y)), \mathsf{snd}(\mathsf{pair}(x,y))) &=_? \mathsf{pair}(x,y) \\
\mathsf{pair}(\mathsf{fst}(\mathsf{pair}(x,y)), z) &=_? \mathsf{pair}(x,z) \\
\mathsf{sdec}(\mathsf{senc}(\mathsf{pair}(a,b), \mathsf{pair}(c,d)), \mathsf{pair}(c,d)) &=_? \mathsf{pair}(a,b) \\
\mathsf{pair}(\mathsf{senc}(k,m), \mathsf{aenc}(k, \mathsf{pk}(a))) &=_? \mathsf{pair}(\mathsf{aenc}(k, \mathsf{pk}(a)), \mathsf{senc}(k,m))) \\
\mathsf{sdec}(\mathsf{aenc}(m, \mathsf{pk}(a)), \mathsf{pk}(a))) &=_? m
\end{aligned}$$

1.2 Deductions

Using E as defined in the previous problem and letting a, b, i, N_z, K_z be constants. Consider an adversary who has learned the following set of terms I through observing network communication or compromising agents in a system:

$$I := \begin{cases} \mathsf{pk}(a), \quad \mathsf{pk}(b), \quad \mathsf{pk}(i), \quad \mathsf{sk}(i), \quad \mathsf{h}(K_2), \quad K_1, \quad \{N_3\}_{\mathsf{pk}(i)} \\ \{\langle \mathsf{h}(N_1), N_3 \rangle\}_{\mathsf{pk}(b)}, \quad \{\langle N_1, N_2 \rangle\}_{\mathsf{h}(K_1)}, \quad \{N_4\}_{\mathsf{h}(\langle N_1, K_2 \rangle)}, \end{cases}$$

For each term t in the set T below, show how the adversary could deduce t by applying function symbols from E to the elements in I or explain why t cannot be deduced by the adversary.

$$\mathsf{h}(\langle \mathsf{h}(N_1), N_3 \rangle) \qquad N_4 \qquad \{\langle N_2, \mathsf{h}(N_3) \rangle\}_{\mathsf{pk}(a)}$$

1.3 Diffie-Hellman Groups

We now consider E_{dh}, the equational theory for Diffie-Hellman groups. It defines the function symbols $inv/1$, $1/0$, $exp/2$ and $*/2$. We use $exp(g, a)$, written g^a to denote exponentiation in the group, $*$ to denote multiplication in the exponent, 1 to denote the identity element and inv to represent a multiplicative inverse. These symbols are governed by the following equations:

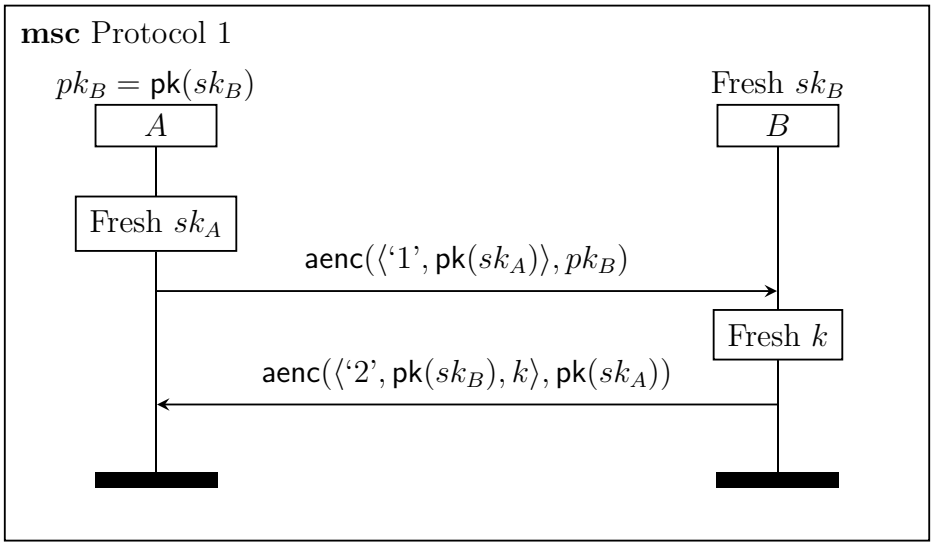
$$\begin{aligned}
(x^y)^z &= x^{(y*z)} \\
x^1 &= x \\
x * y &= y * x \\
(x * y) * z &= x * (y * z) \\
x * 1 &= x \\
x * inv(x) &= 1
\end{aligned}$$

Explain how the resulting equational theory encodes the Discrete Logarithm hardness assumption. How could this theory be extended to model an attacker who could break the discrete logarithm?

Part 2: Transition Systems and Traces

2.1 Encoding a Protocol as Multiset Rewriting Rules

Protocol 1 is described in the MSC diagram below. Translate Protocol 1 into multiset rewriting rules. Your translation should require **at most** five rewrite rules.



2.2 Extracting a Protocol from an LTS

The following multiset rewriting rules describes a two party key agreement protocol between an Initiator and a Responder. Each rule is written in the form:

$$[Premise]\mapsto$$
$$[Conclusion]$$

Recall that persistent facts, prefixed with !, are **not** removed after being consumed in a rule. Fresh facts $\mathsf{Fr}(x)$ are automatically instantiated with a unique constant. Variables that are preceded by a \$ sign are publicly known (e.g. agent names: $\$A$, $\$B$, etc.).
By inspecting the multiset rewriting rules, draw the corresponding MSC diagram.

$$[\mathsf{Fr}(sk_x)]\mapsto$$
$$[\mathsf{Agent}(\$X, sk_x), !\mathsf{PublicKey}(\$X, \mathsf{pk}(sk_x))]$$

$$[\mathsf{Agent}(\$A, sk_A), !\mathsf{PublicKey}(\$B, pk_B)]\mapsto$$
$$[\mathsf{Out}(\langle \mathsf{pk}(sk_A), pk_B, `Params' \rangle), \mathsf{Init1}(\$A, sk_A, pk_B)]$$

$$[\mathsf{Agent}(\$B, sk_B), \mathsf{Fr}(k_1), \mathsf{In}(\langle pk_A, \mathsf{pk}(sk_B), `Params' \rangle)]\mapsto$$
$$[\mathsf{Out}(\mathsf{aenc}(\langle `1', k_1, \mathsf{pk}(sk_B)\rangle, pk_A)), \mathsf{Resp1}(\$B, sk_B, pk_A, k_1)]$$

$$[\mathsf{Init1}(\$A, sk_A, pk_B), \mathsf{In}(\mathsf{aenc}(\langle `1', k_r, pk_B\rangle, \mathsf{pk}(sk_A))), \mathsf{Fr}(k_2)]\mapsto$$
$$[\mathsf{Out}(\mathsf{aenc}(\langle `2', \mathsf{h}(k_r), k_2, \mathsf{pk}(sk_A)\rangle, pk_B)), \mathsf{Init2}(\$A, sk_A, pk_B, \langle \mathsf{h}(k_r, k_2), \mathsf{pk}(sk_A), pk_B\rangle)]$$

$$[\mathsf{Resp1}(\$B, sk_B, pk_A, k_1), \mathsf{In}(\mathsf{aenc}(\langle `2', \mathsf{h}(k_1), k_i, pk_A\rangle, \mathsf{pk}(sk_B)))]\mapsto$$
$$[\mathsf{Out}(\mathsf{senc}(`ACK', \langle \mathsf{h}(\langle k_1, k_i\rangle), pk_A, \mathsf{pk}(sk_B)\rangle), \mathsf{Resp2}(\$B, sk_B, pk_A, \mathsf{h}(\langle k_1, k_i\rangle)))]$$

$$[\mathsf{Init2}(\$A, sk_A, pk_B, k_f), \mathsf{In}(\mathsf{senc}(`ACK', k_f))]\mapsto$$
$$[\mathsf{Init3}(\$A, pk_B, k_f)]$$