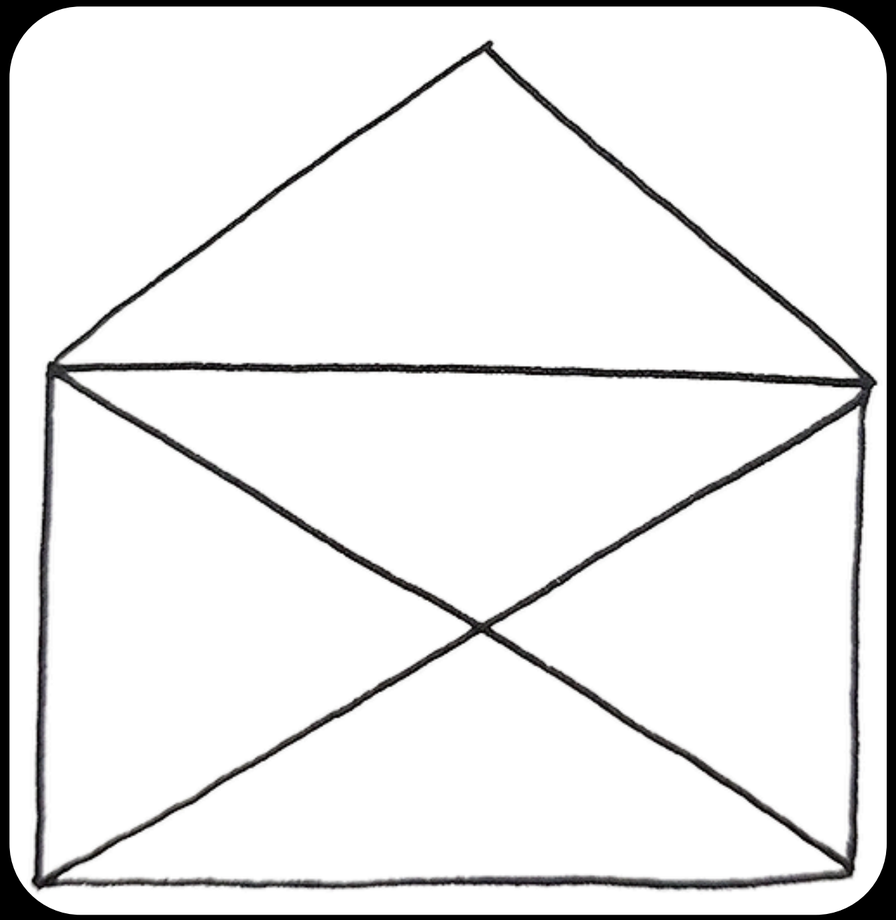
Una introducción
<Muy Algorítmica>
a la Teoría de
GRAFOS

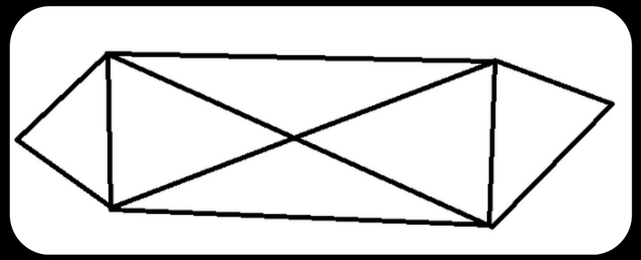
Presentado por:

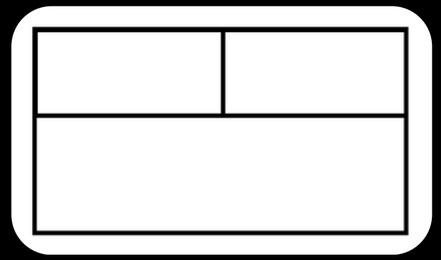
- Jerónimo Hoyos Botero.
- Daniel Fernando Verdecia Goyeneche

¿Puedes dibujar la casita de un solo trazo?



Otros ejemplos:





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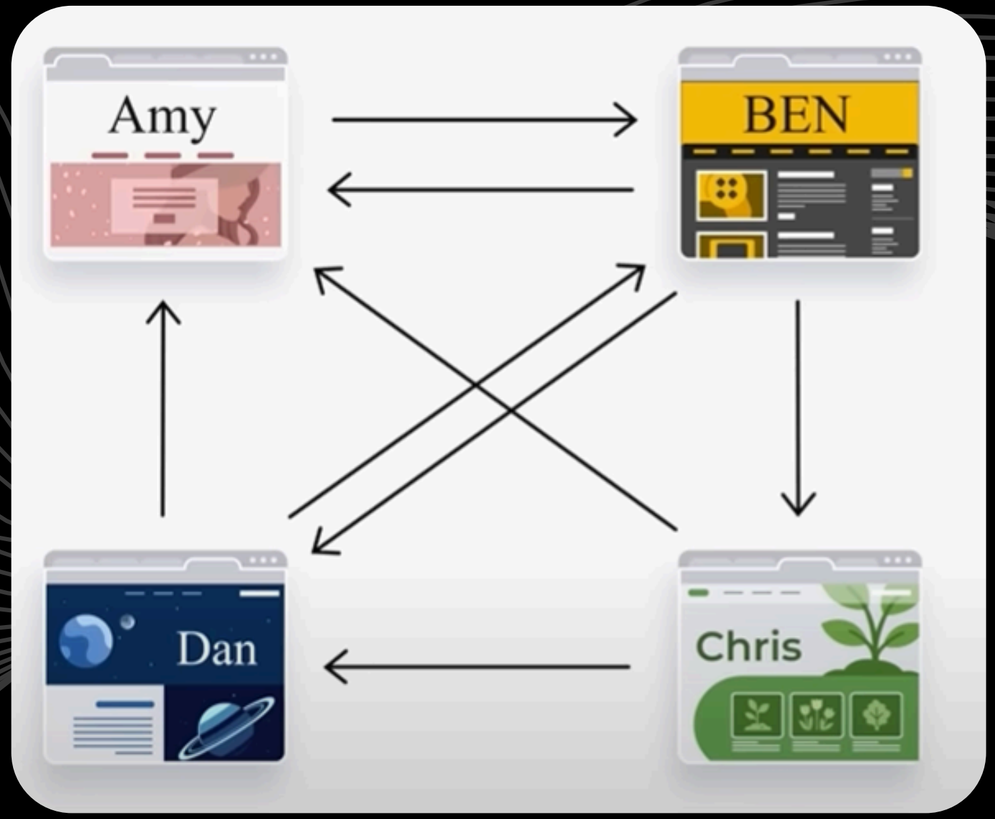
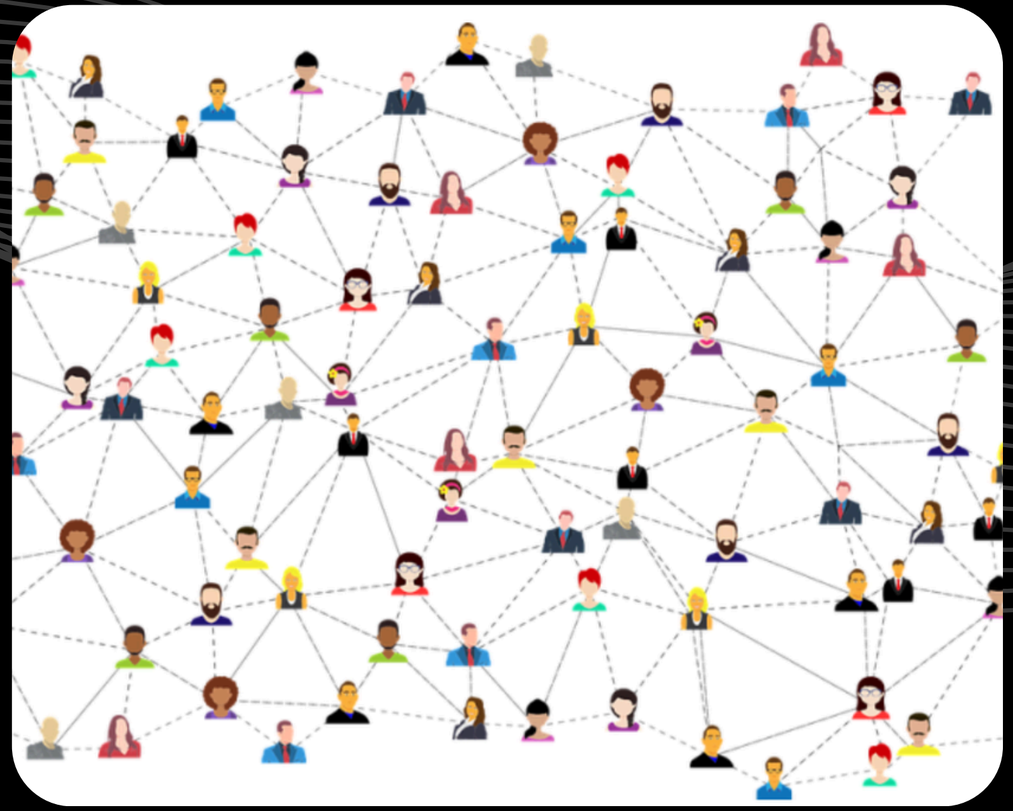
¡Está en todos lados!



Páginas WEB



Personas

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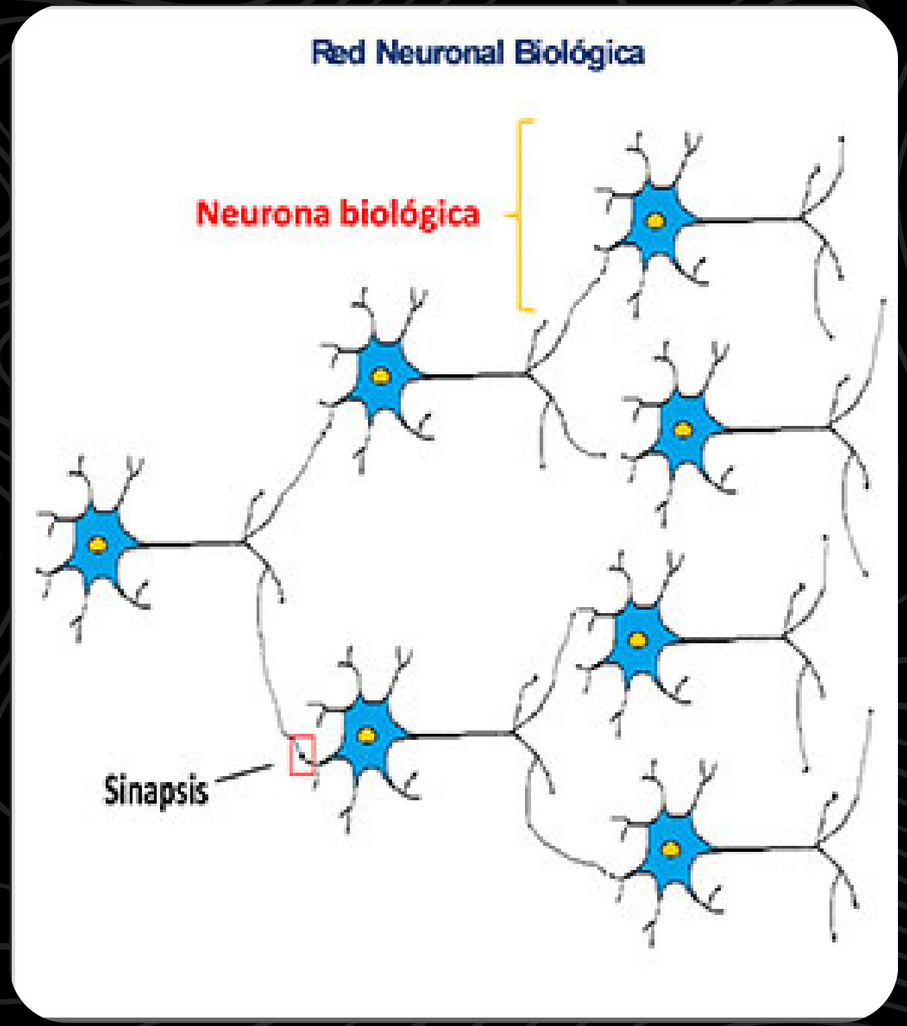
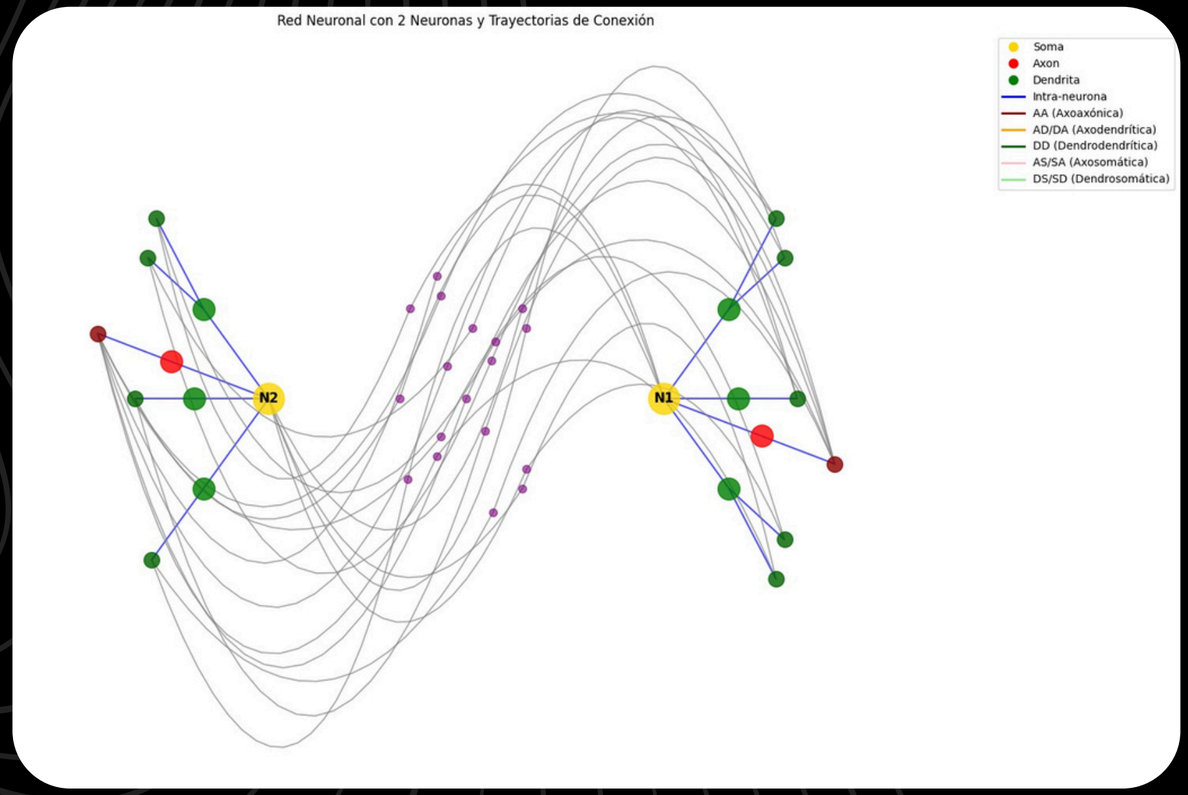
Analogía Fisiológica-Grafos





¡Investigación en proceso por semilleristas de neuroco!

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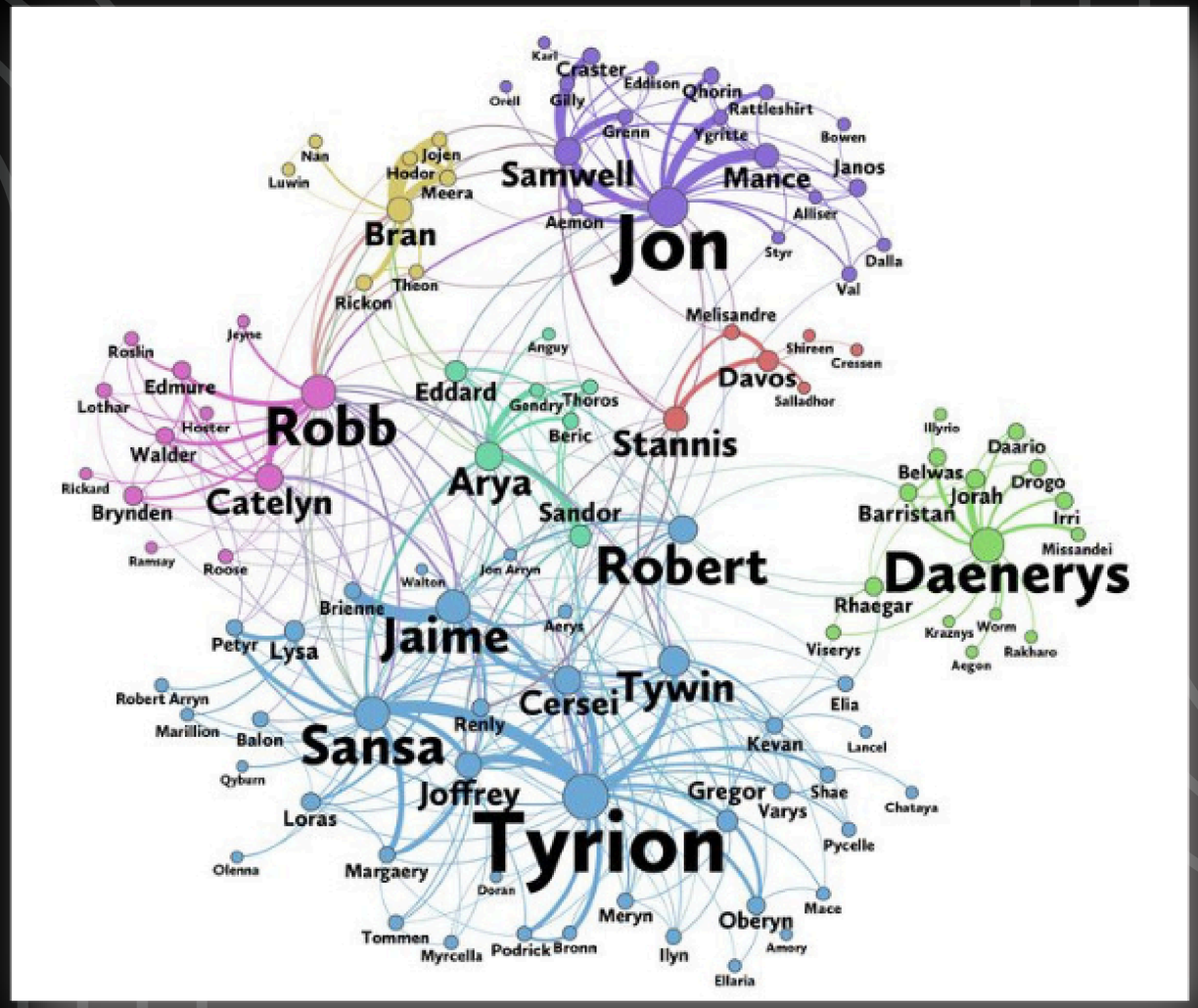
¿Quién es el protagonista de GOT?



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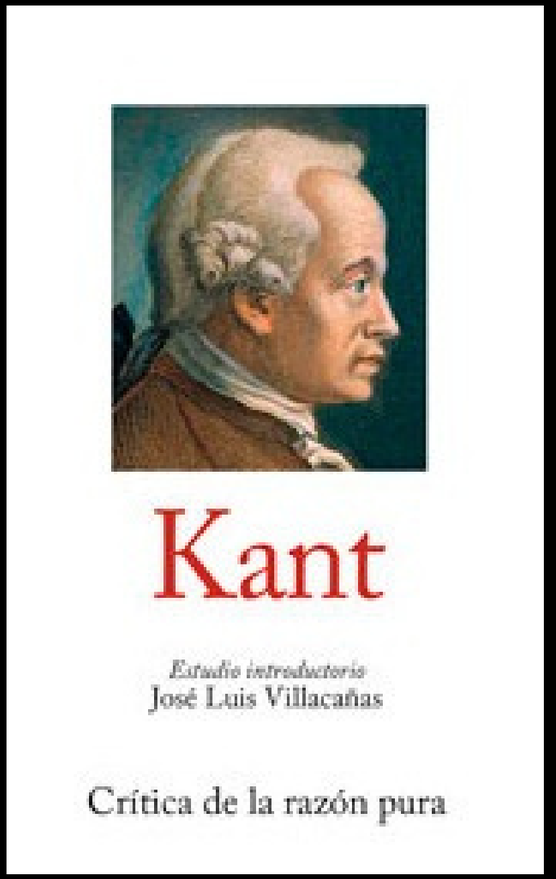
KÖNIGSBERG



Tierra de
Immanuel Kant

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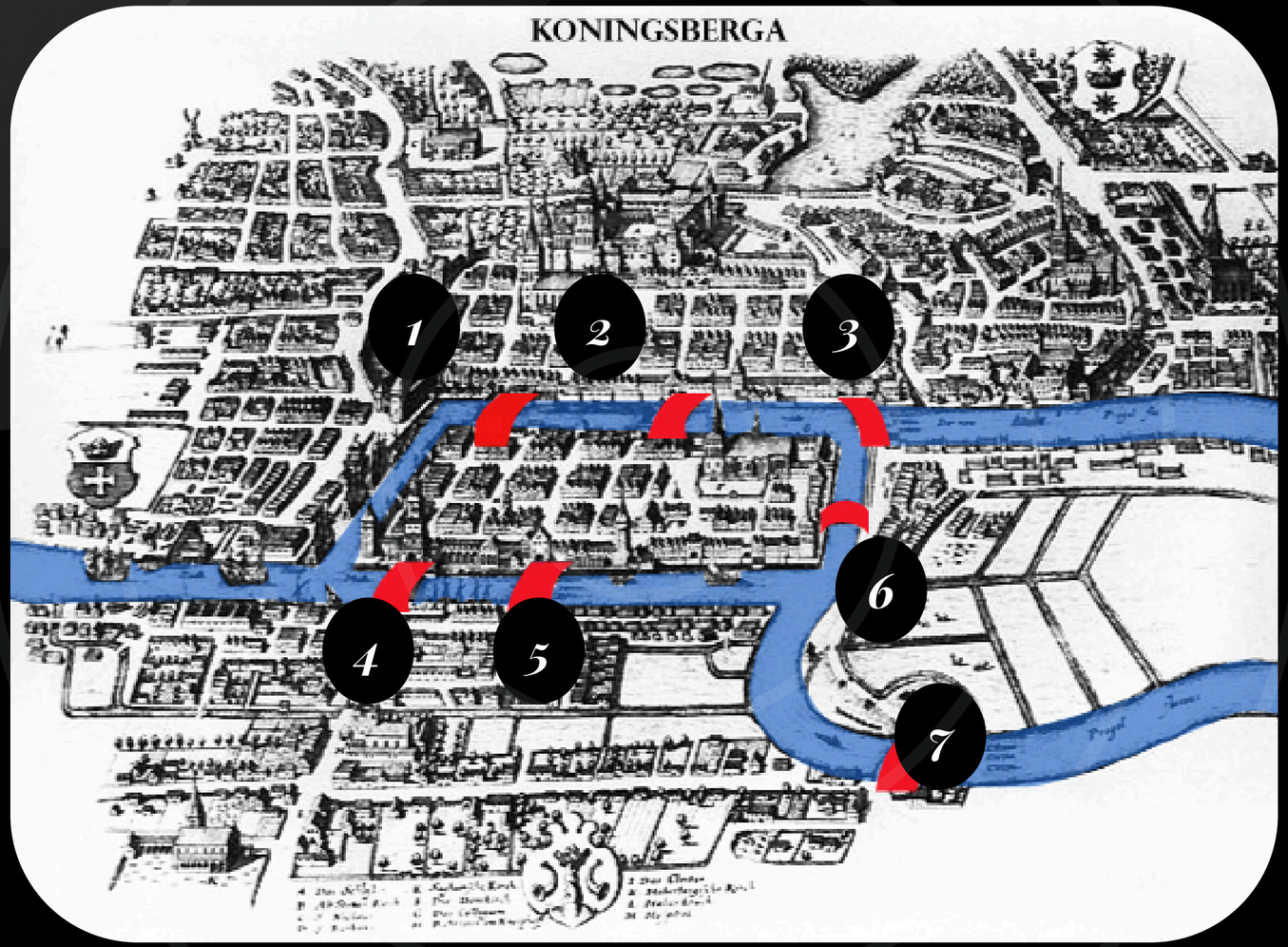
¿Se puede hacer un camino donde se pasen solo una vez cada puente?



Posibilidades

- [1,2,3,4,5,6,7]
- [6,7,2,1,5,4,3]
- [7,6,2,6,3,4,1]
- -
- -

1x2x3x4x5x6x7

7!=5040

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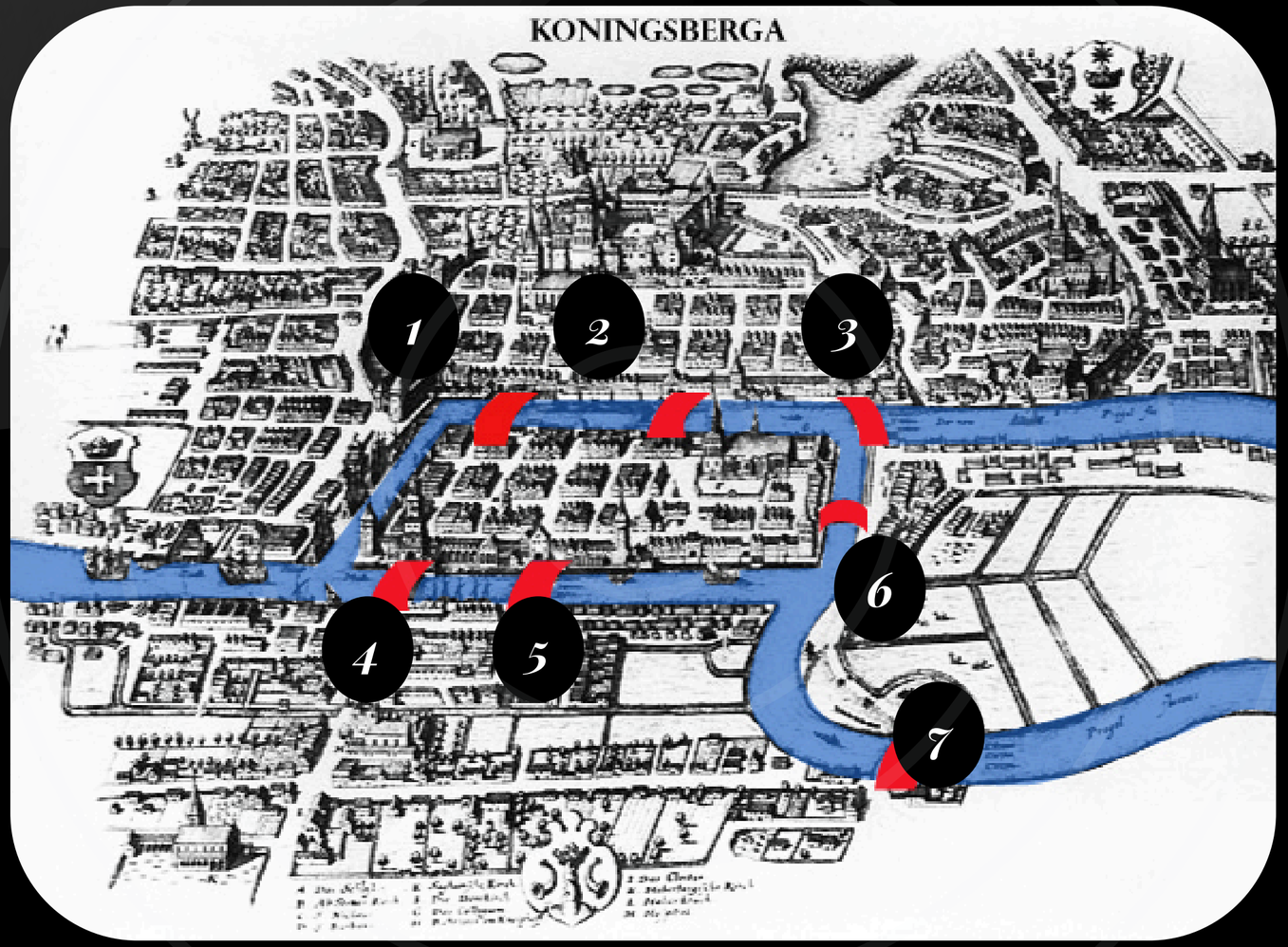
¿Se puede hacer un camino donde se pasen solo una vez cada puente?



Leonhard Euler

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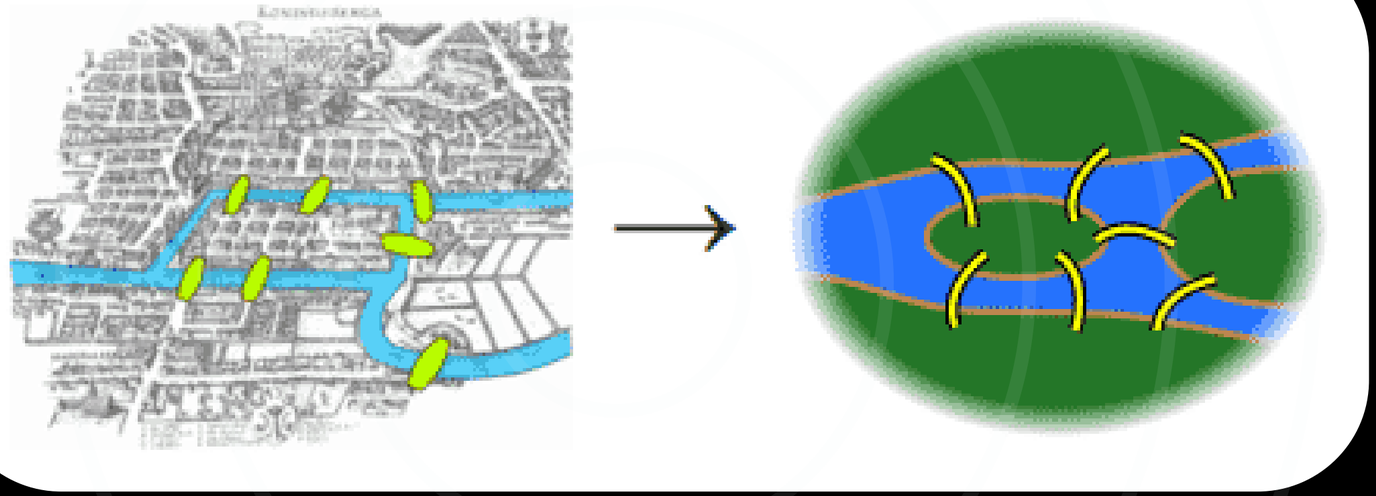
¿Se puede hacer un camino donde se pasen solo una vez cada puente?



Leonhard Euler

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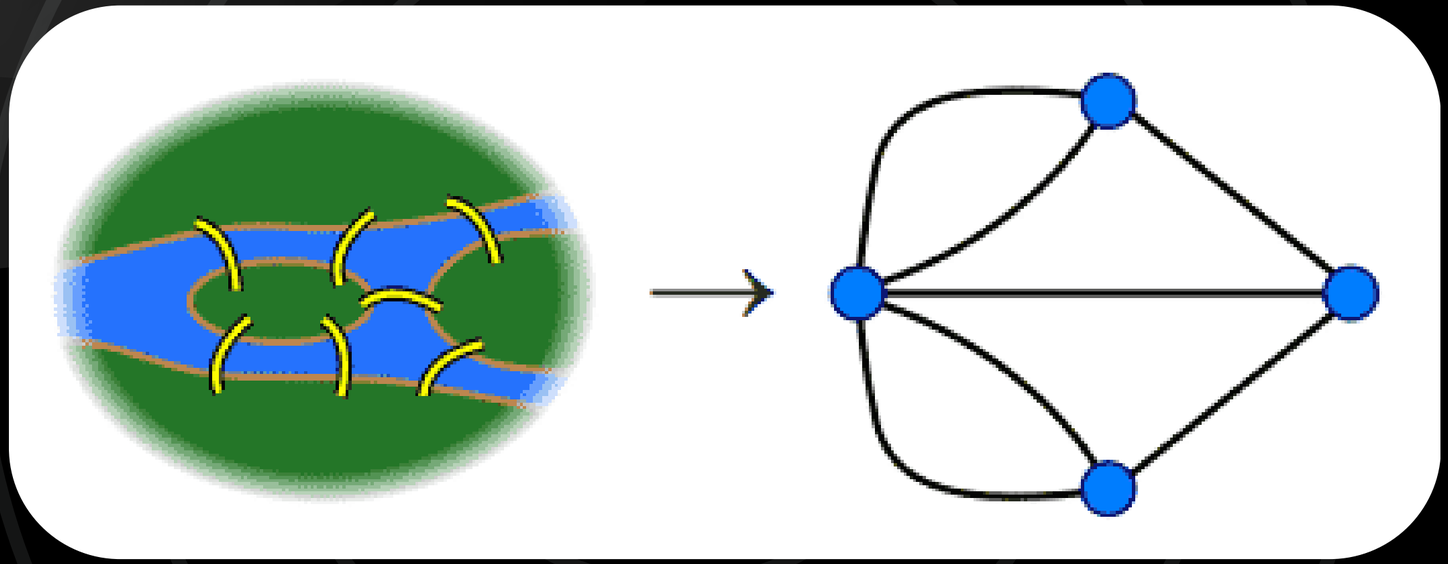
¿Se puede hacer un camino donde se pasen solo una vez cada puente?



Leonhard Euler

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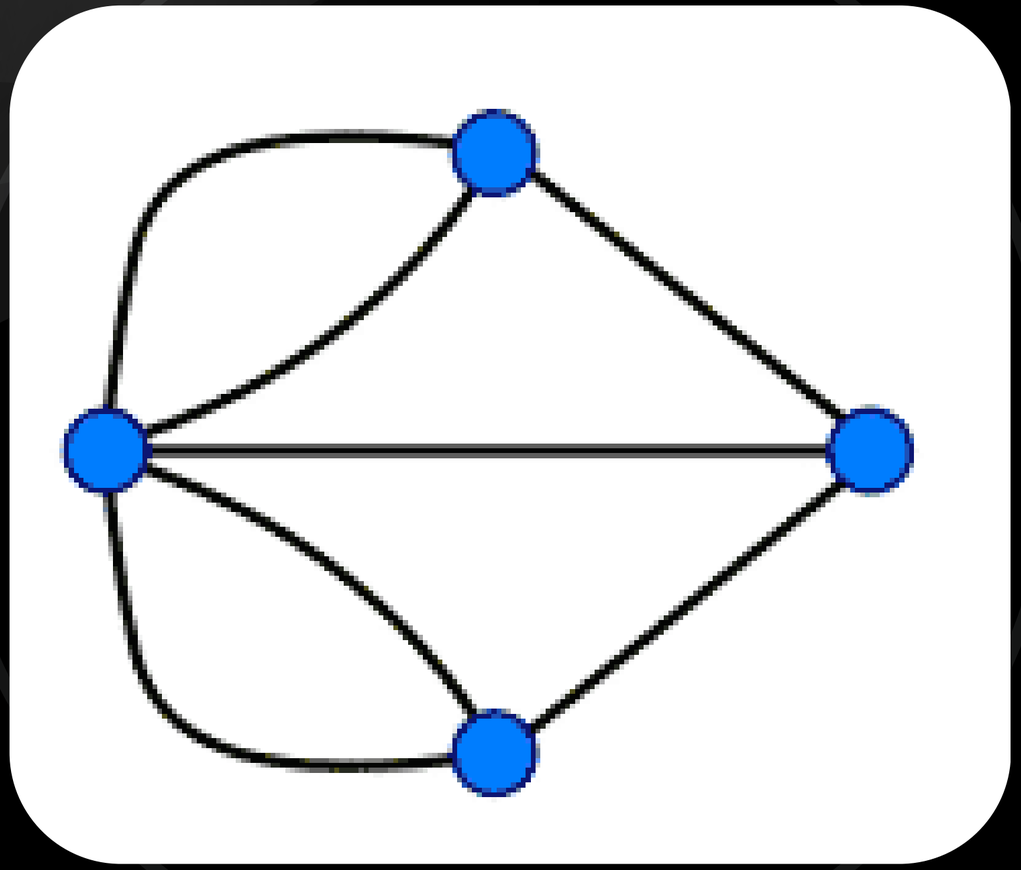
¿Se puede hacer un camino donde se pasen solo una vez cada puente?



Leonhard Euler

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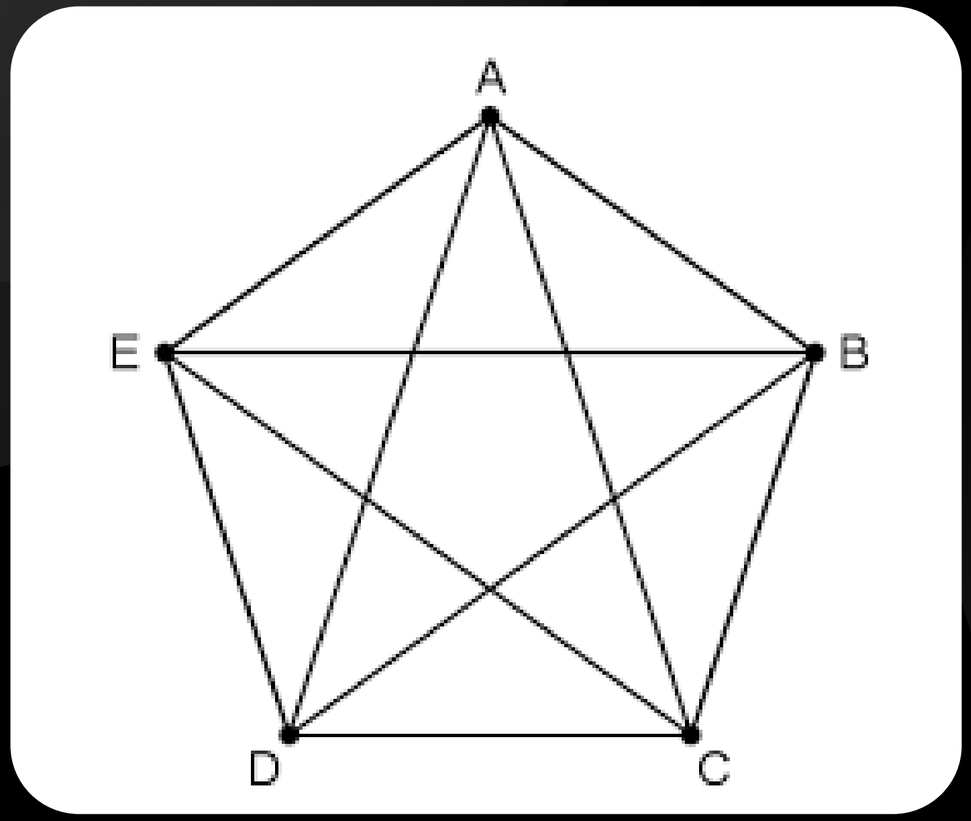
El Teorema de Euler



Es un Grafo Euleriano

Un grafo es Euleriano

Es **conexo** y todos sus **vértices** son **pares.**

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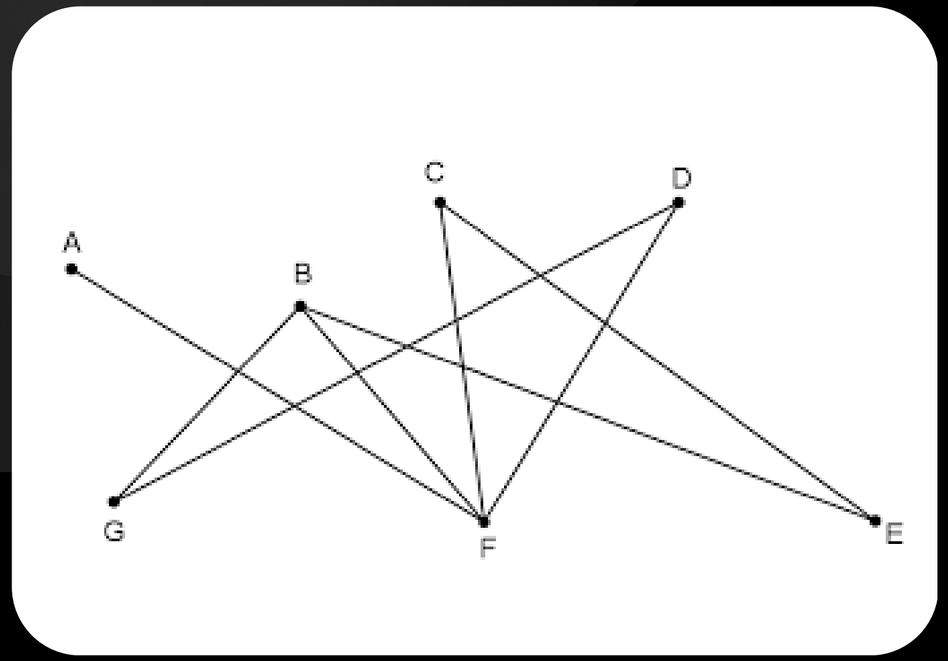
El Teorema de Euler

Circuito de Euler NO es lo mismo que Camino de Euler



Un grafo es Euleriano

Es **conexo** y todos sus **vértices** son **pares.**

No es un grafo de Euler

No tiene circuito Euleriano (no todos los vértices tienen grado par).

Sí tiene camino Euleriano.

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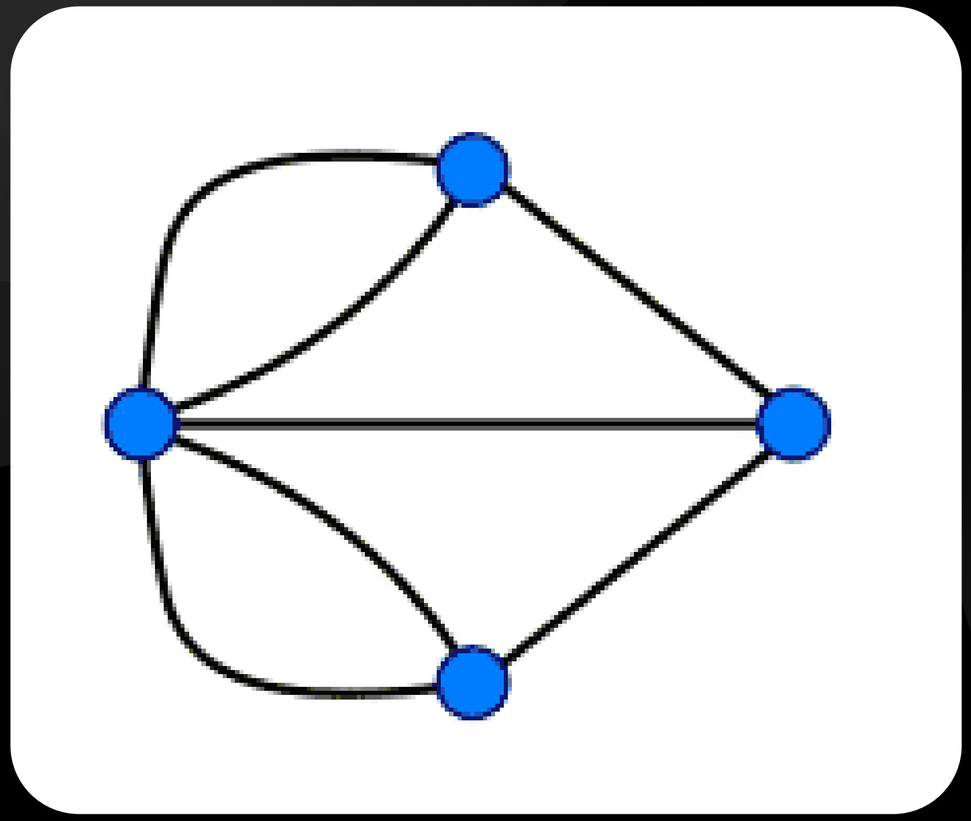
El Teorema de Euler



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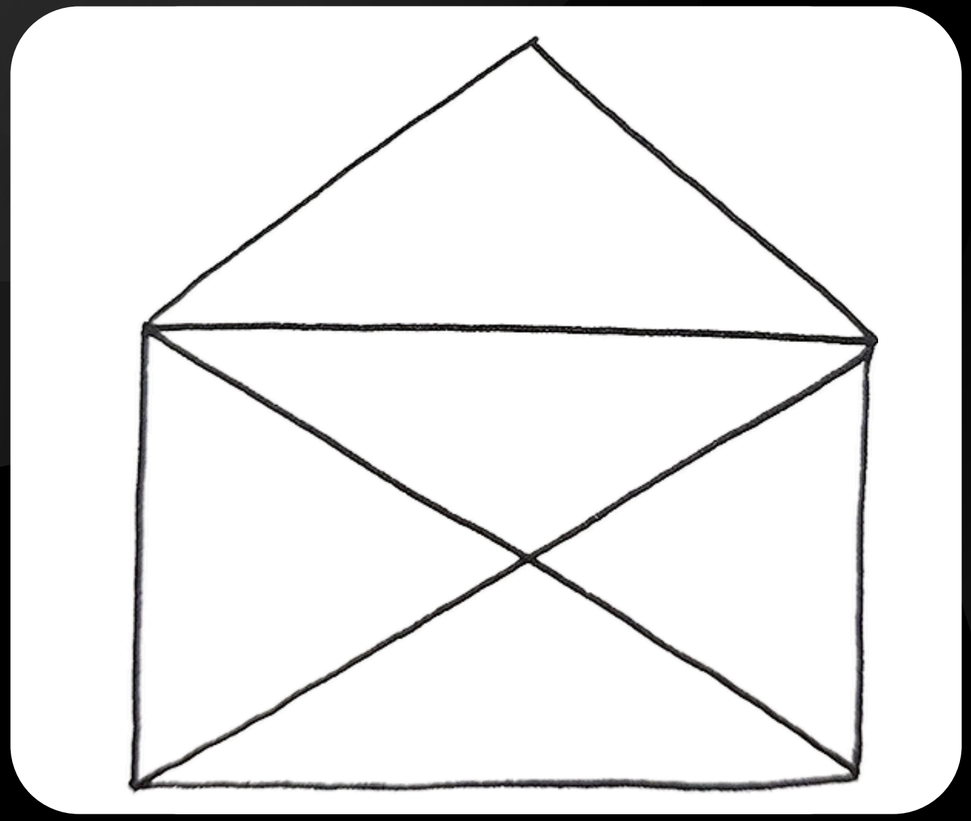
El Teorema de Euler



Es un Grafo Euleriano

Un grafo es Euleriano

Es **conexo** y todos sus **vértices** son **pares.**

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¿CÓMO LOS PUEDO USAR?

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IMPLEMENTACIÓN

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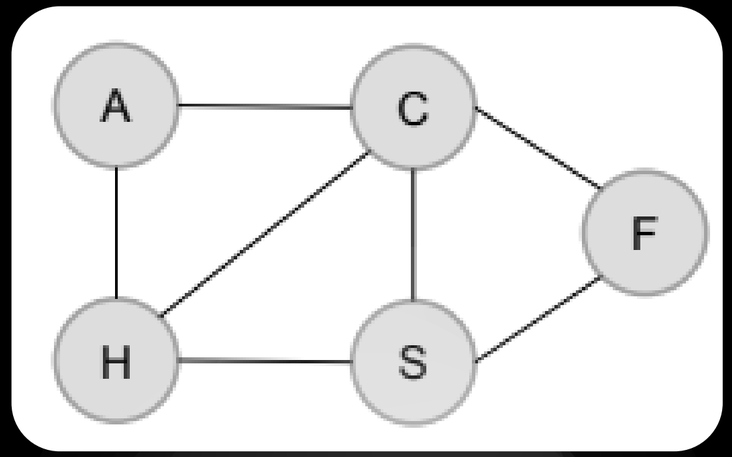
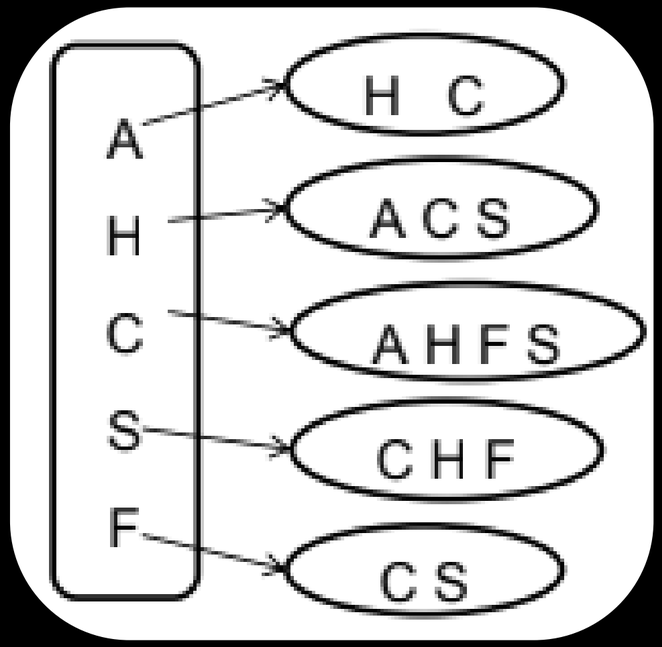


Diccionarios



Matriz Adyacencia

	A	C	F	S	H
A	-	1	0	0	1
C	1	-	1	1	1
F	0	1	-	1	0
S	0	1	1	-	1
H	1	1	0	1	-

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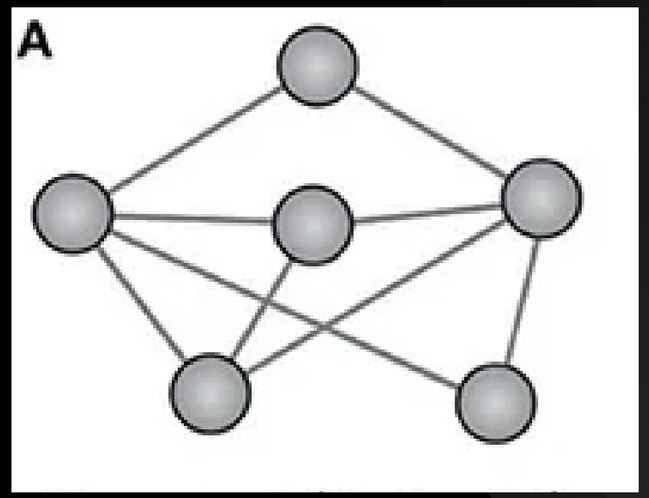
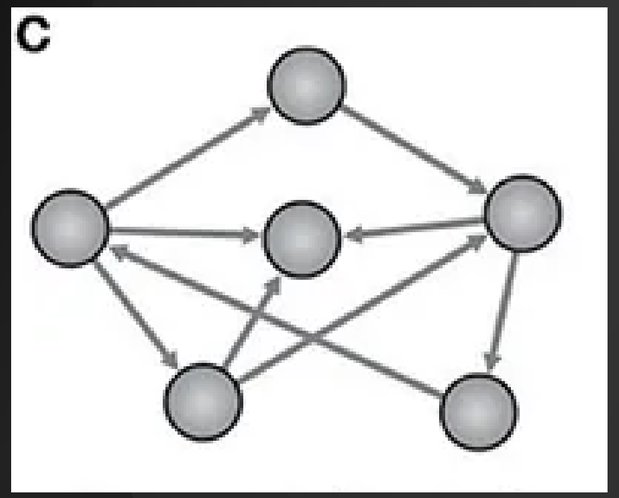
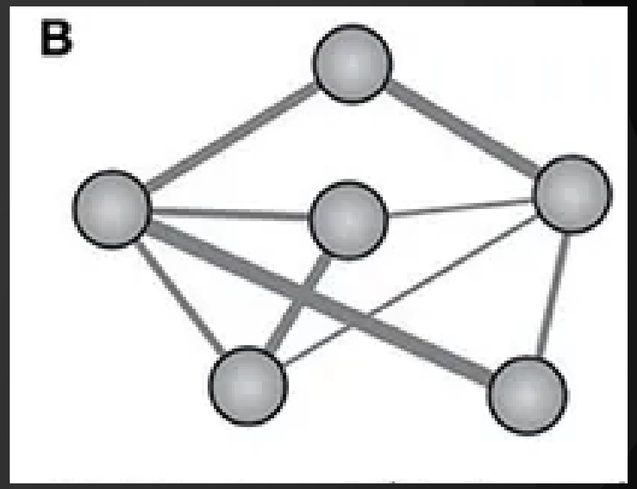
Algunos tipos de Grafos:



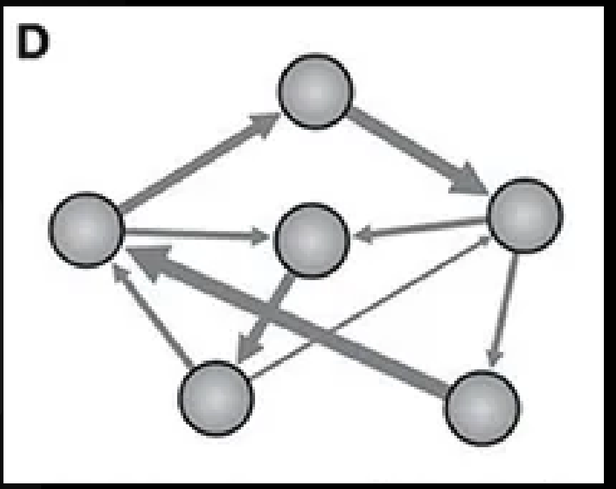
Grafo binario



Grafo dirigido



Grafo con pesos



Grafo con pesos y dirigido

MÉTRICAS

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Métricas Globales

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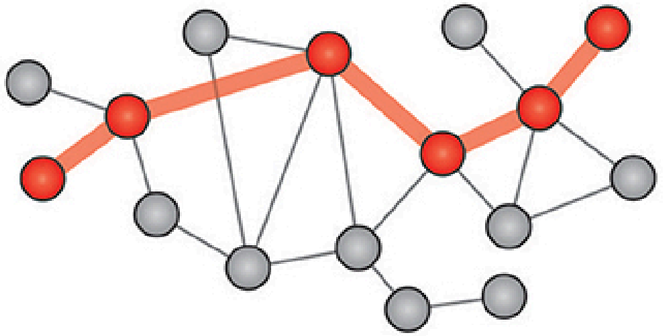
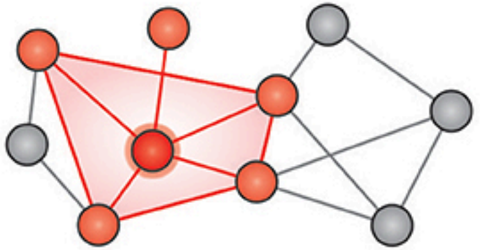
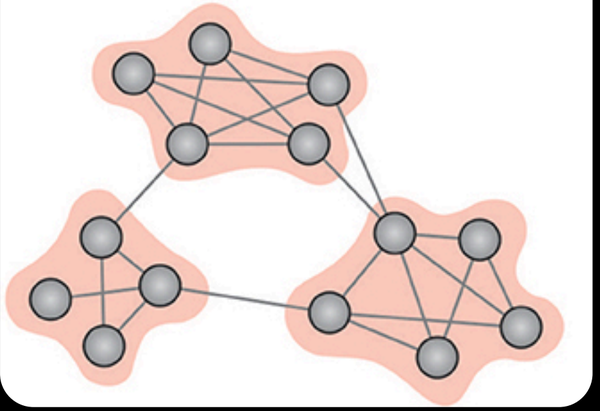
Shortest path



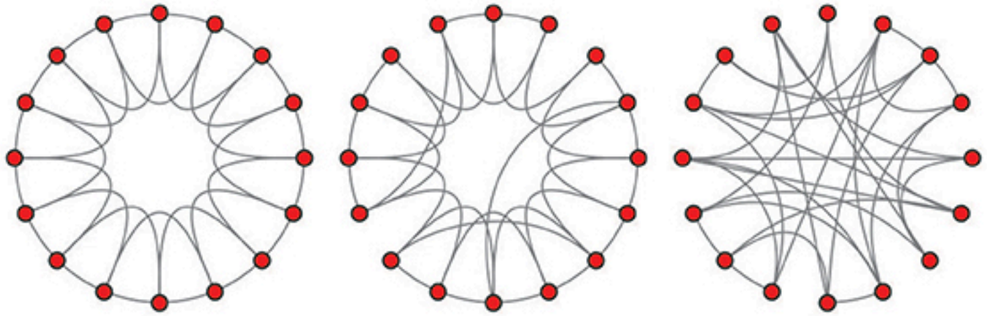
Clustering coefficient



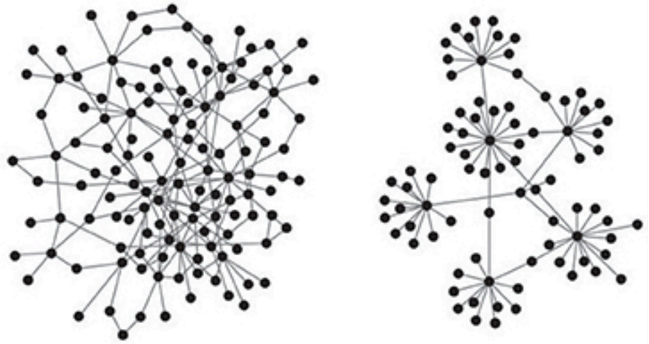
Modularity



Regular Small-World Random



Assortative Disassortative



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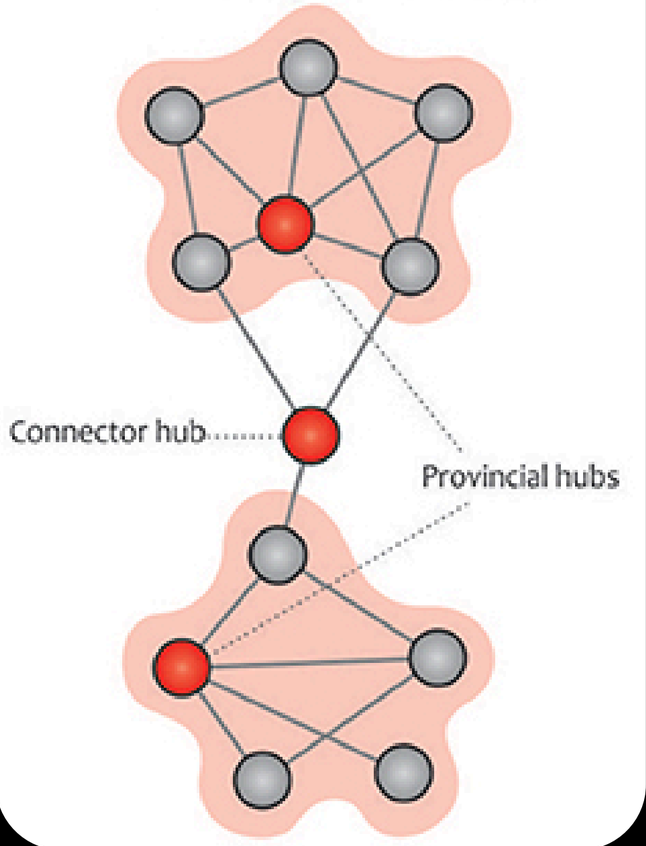
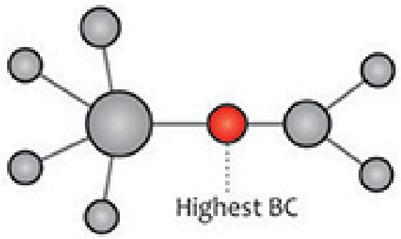
Métricas Locales

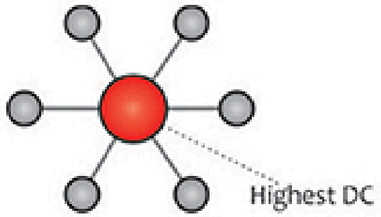
Centrality and hubs



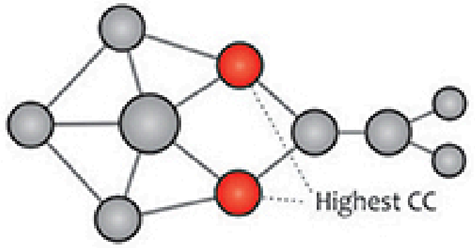
Betweenness centrality



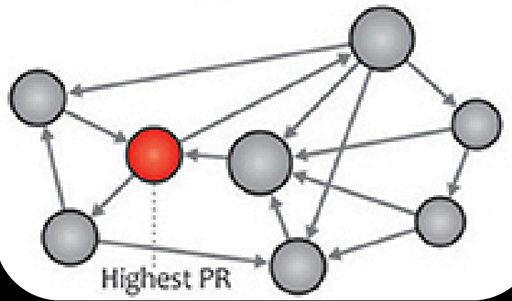
Degree centrality



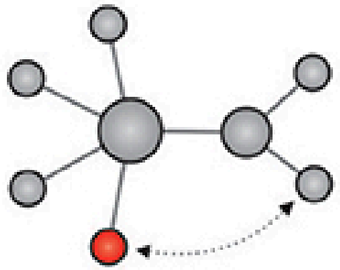
Closeness centrality



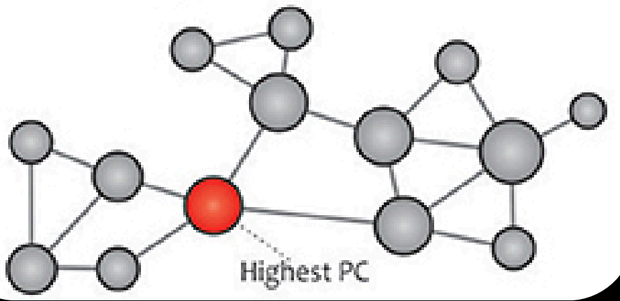
PageRank



Eigenvector centrality



Participation coefficient



<ALGORITMOS>

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Recorridos de Grafos

El objetivo es, dado un nodo de partida:

1) explorar tantos nodos restantes como sea posible pero

2) no explorar ningún nodo más de una vez.

BFS (Breadth-First Search) o búsqueda en amplitud

Explora los nodos por “capas”

Puede implementarse con una cola

DFS (Depth-First Search) o búsqueda en profundidad

Explora los nodos “agresivamente”

Puede implementarse con una pila

Recorridos de Grafos

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<BFS>

```
BFS(grafo G, nodo a):
   for each nodo u:
      u.visited = false
   a.visited = true
   q = new queue
   q.push(a)
   while (q is not empty):
      u = q.pop()
      for each arista (u, v):
         if v.visited = false:
            v.visited = true
            q.push(v)
```

<DFS>

```
DFS(grafo G, nodo a):
   for each nodo u:
      u.visited = false
   s = new stack
   s.push(a)
   while (s is not empty):
      u = s.pop()
      if u.visited = false:
         u.visited = true
         for each arista (u, v):
            if v.visited = false:
               s.push(v)
```

Cantidad Mínima de saltos

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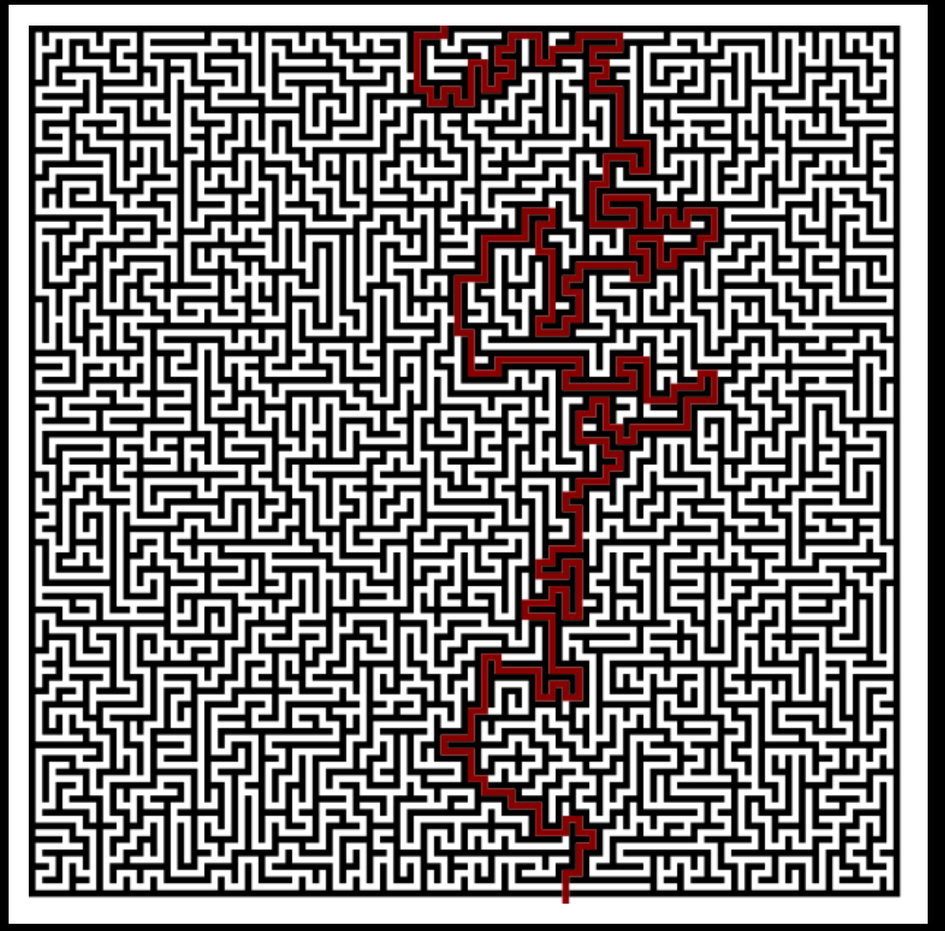
El objetivo es, dado un nodo de partida, determinar cuál es la mínima cantidad de “saltos” o aristas por las que se debe pasar para llegar al resto de nodos

```
countHops(grafo G, nodo a):
    for each nodo u:
        u.visited = false
    a.visited = true
    a.hops = 0
    q = new queue
    q.push(a)
    while (q is not empty):
        u = q.pop()
        for each arista (u, v):
            if v.visited = false:
                v.visited = true
                v.hops = u.hops + 1
                q.push(v)
```

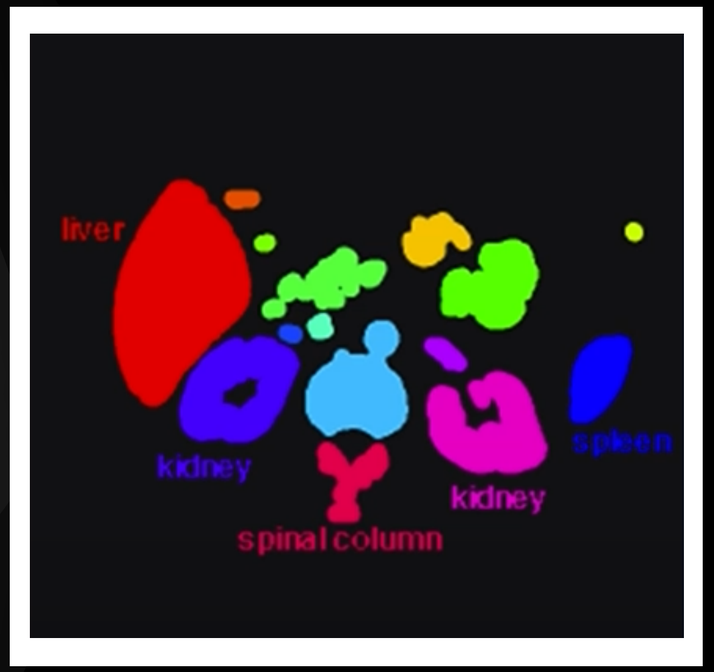


Resolver un laberinto

Cantidad Mínima de saltos

El objetivo es, dado un grafo, ¿cuántos subgrafos conectados hay?

```
connectedComponents(grafo G):
   for each nodo u:
      u.visited = false
   c = 0
   for each nodo a:
      if a.visited = false:
         c += 1
         s = new stack
         s.push(a)
         while (s is not empty):
            u = s.pop()
            if u.visited = false:
               u.visited = true
               for each arista (u, v):
                  if v.visited = false:
                     s.push(v)
```



Este algoritmo aporta la segmentación de imagenes médicas

EL CAMINO MÁS CORTO

Input: Un grafo simple conexo y con pesos en el que todos los pesos son positivos.
Output: $L(z)$, la longitud del camino más corto de a a z

```
1:  function DIJKSTRA(w, a, z, L)
2:      for all x ∈ V do
3:          L(x) = ∞
4:      L(a) = 0
5:      T = conjunto de todos los vértices
6:      while z ∈ T do
7:          Tomar v ∈ T con mínimo L(v).
8:          T = T − {v}
9:          for x ∈ T adyacente con v do
10:             L(x) = mín{L(x), L(v) + w(v, x)}
11:     return L(z)
```

ALGORITMO DE DIJKSTRA

El algoritmo de Dijkstra es **la base de todos los algoritmos** de caminos más cortos. Representa una introducción fundamental para entender **cómo funcionan** los algoritmos en grafos, por eso lo estudiaremos hoy.

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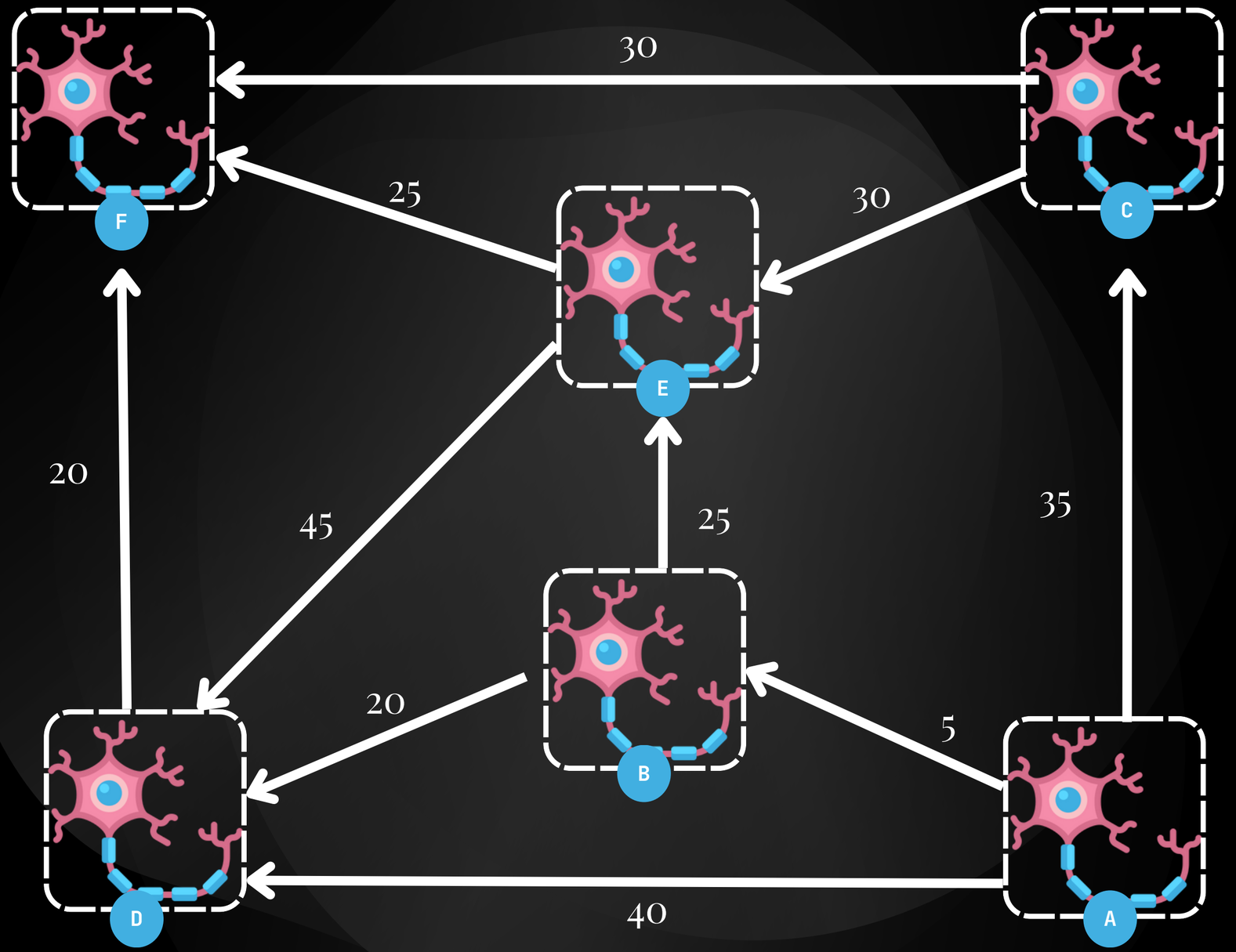
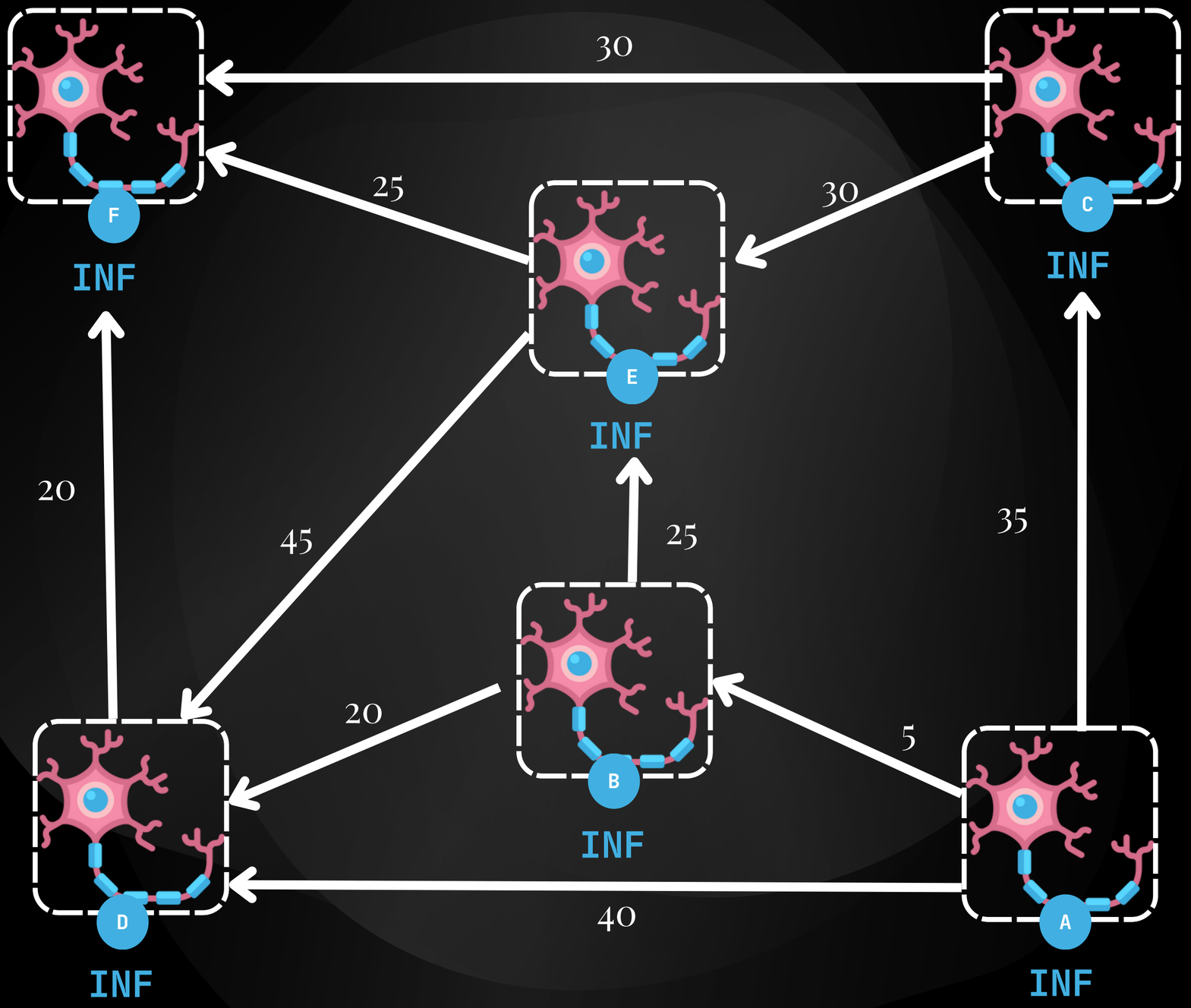
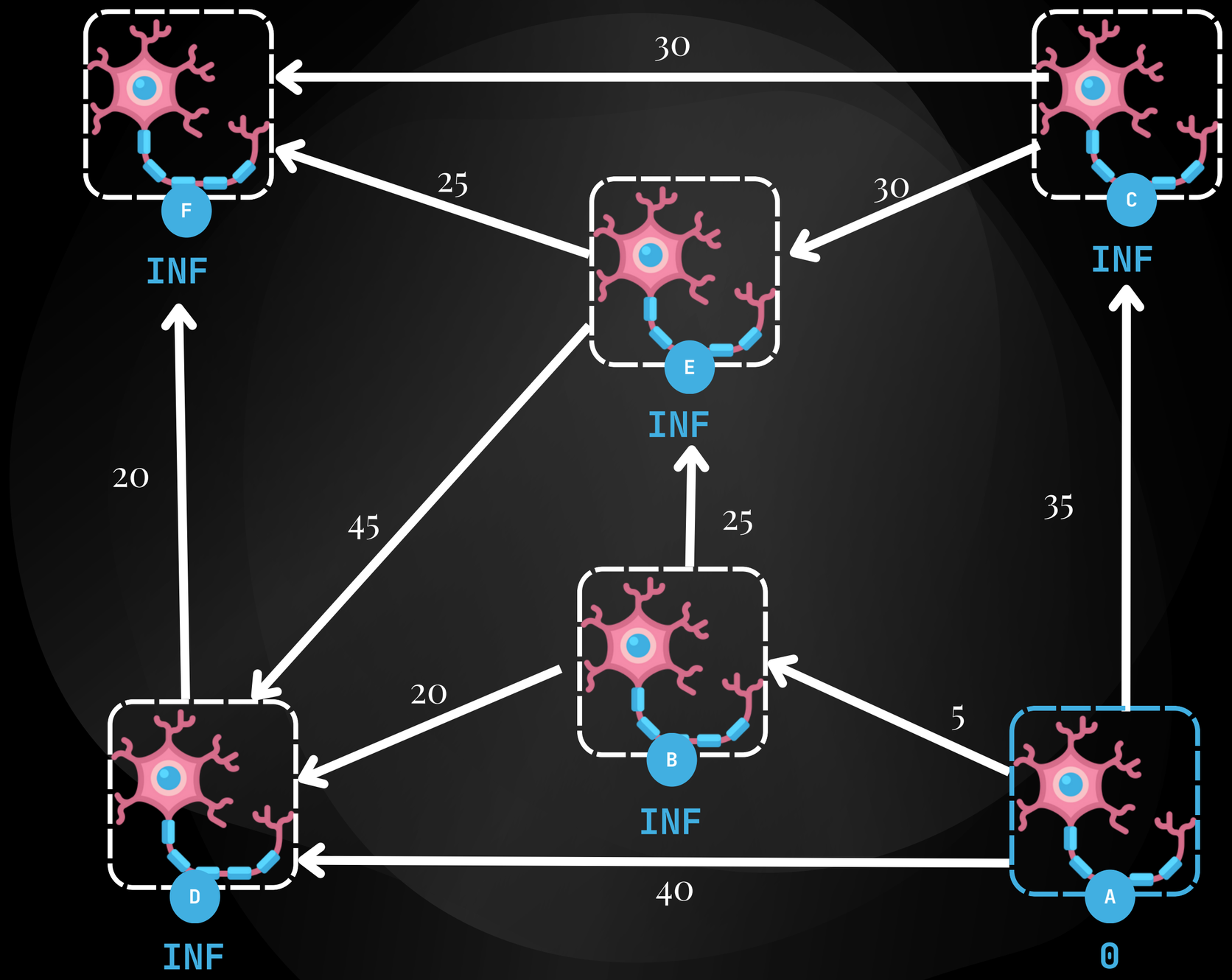
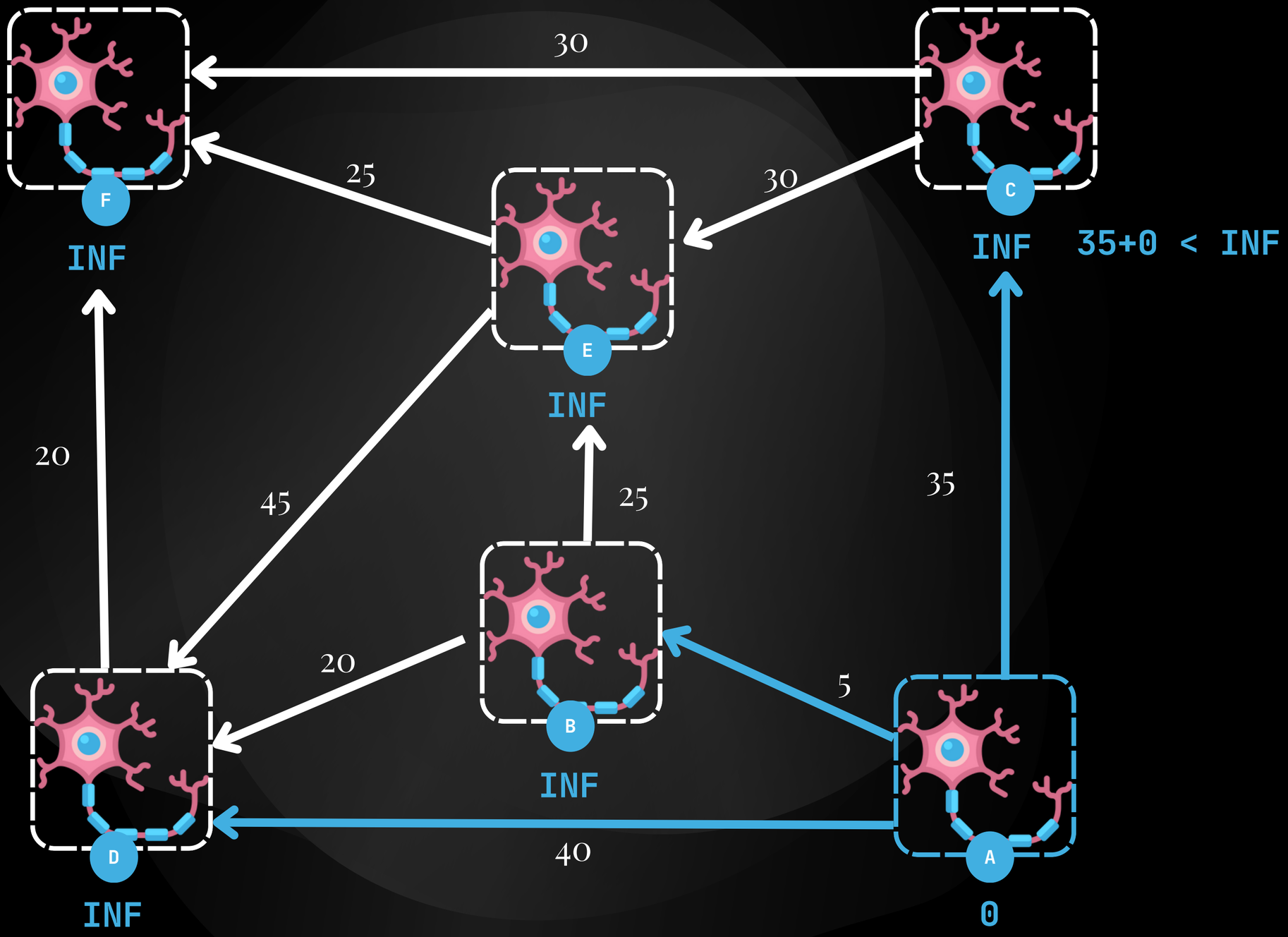
¿Cuál es el camino más corto entre A y F?

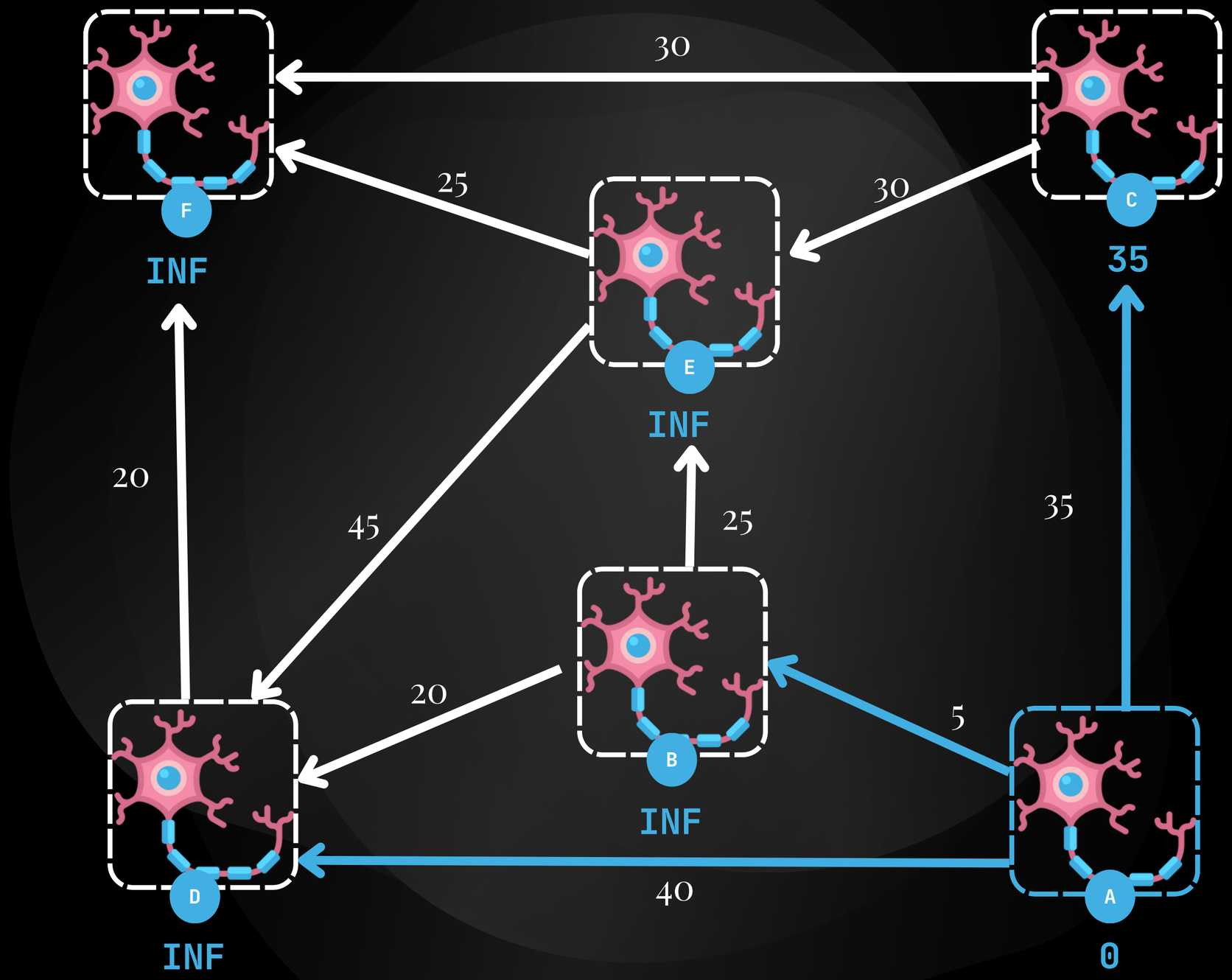
Edsger Dijkstra




30
25
30
20
45
25
35
20
5
40
F
INF
C
INF
E
INF
B
INF
D
INF
A
INF


30
25
30
20
45
25
35
20
5
40
F
INF
C
INF
E
INF
B
INF
D
INF
A
0


30
25
30
F
C
INF
INF
35+0 < INF
E
INF
20
45
25
35
20
5
B
INF
D
A
40
INF
0

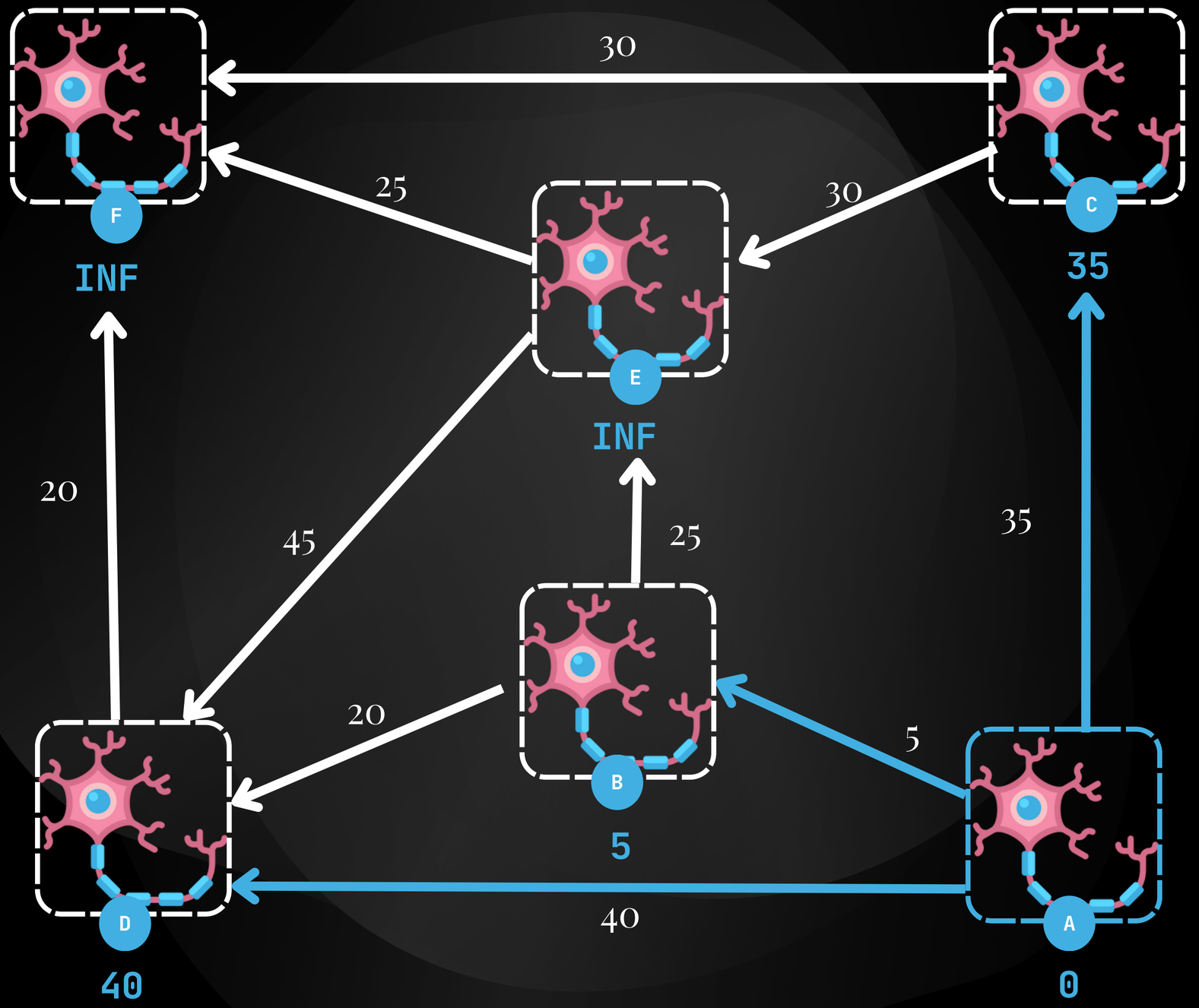

30
25
30
F
INF
C
35
E
INF
20
45
25
35
20
5
B
INF
D
INF
40
A
0

Predecesores

A → B

A → C

A → D



Fila prioridad

B = 5

C = 35

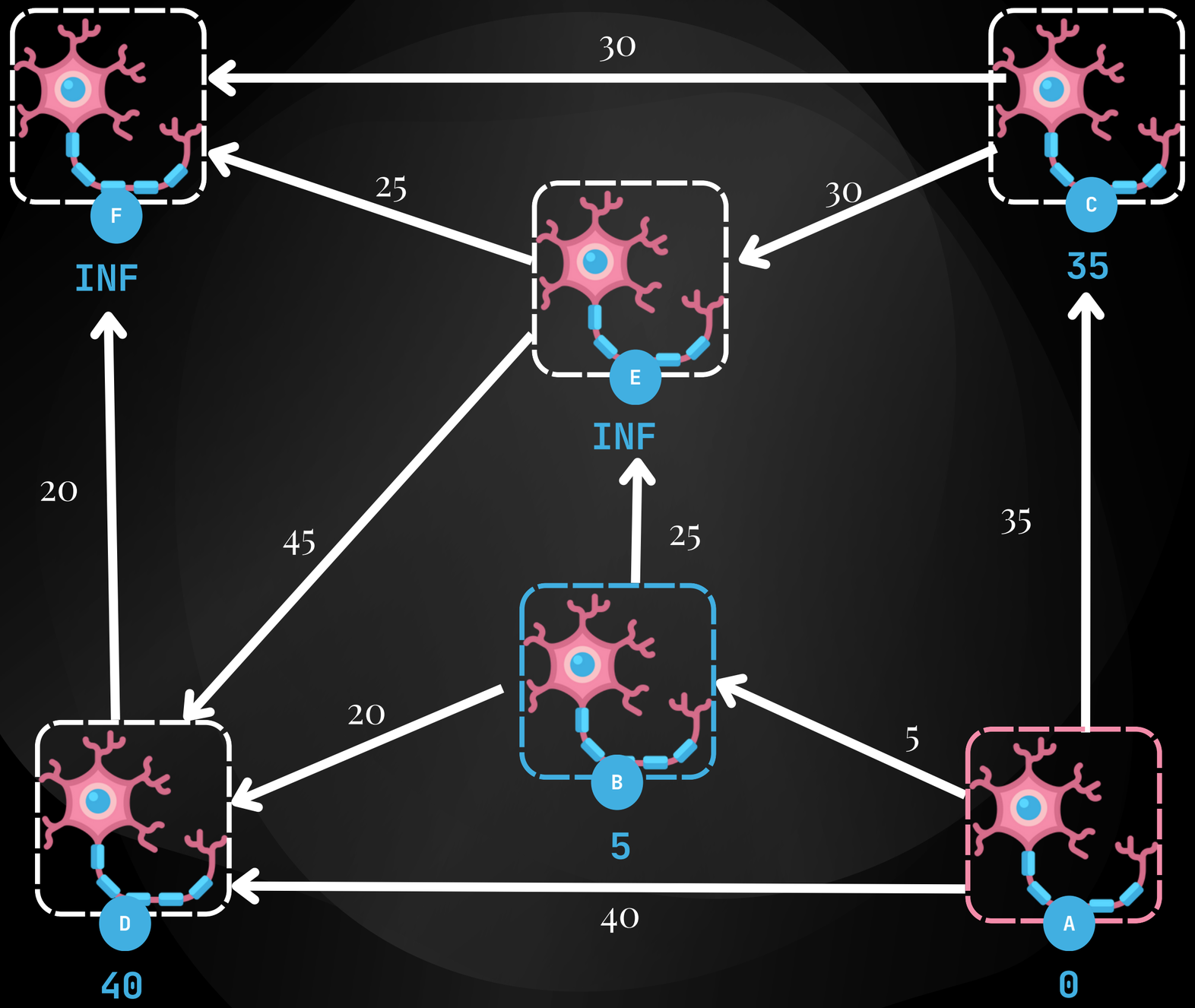
D = 40

Predecesores

A → B

A → C

A → D



Fila prioridad

B = 5

C = 35

D = 40

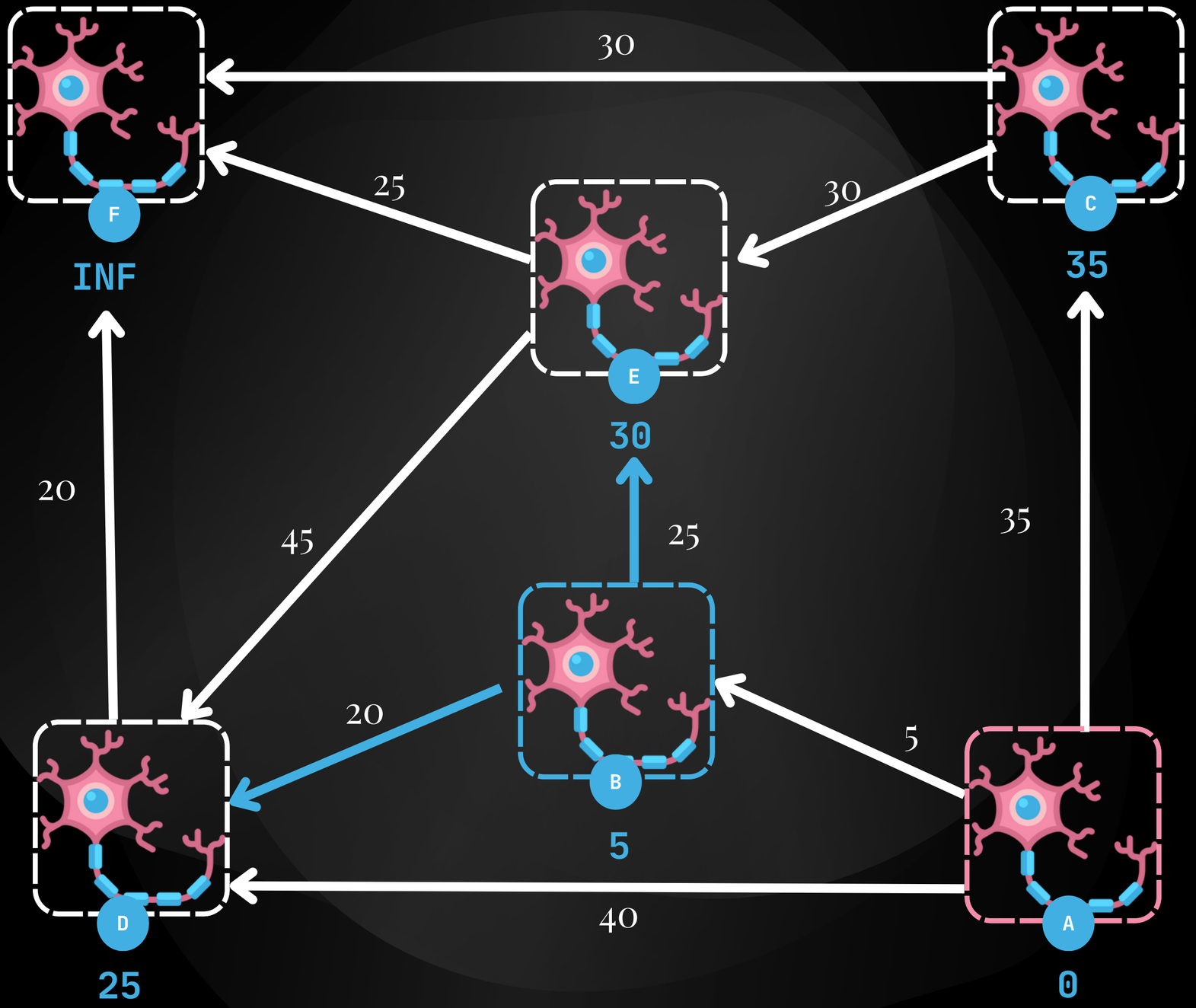
Predecesores

A → B

A → C

B → D

B → E



Fila prioridad

D = 25

E = 30

C = 35

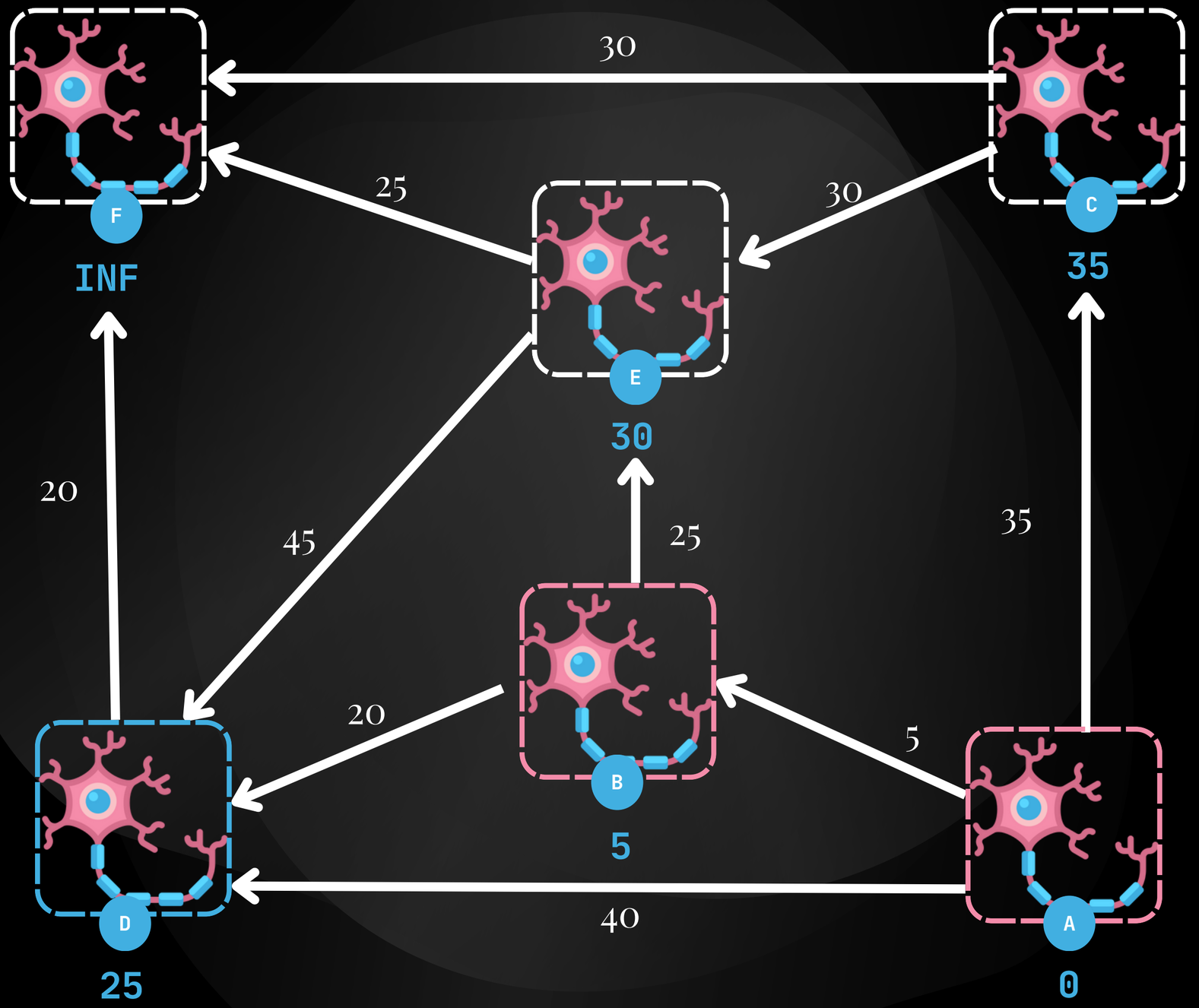
Predecesores

A → B

A → C

B → D

B → E



Fila prioridad

D = 25

E = 30

C = 35

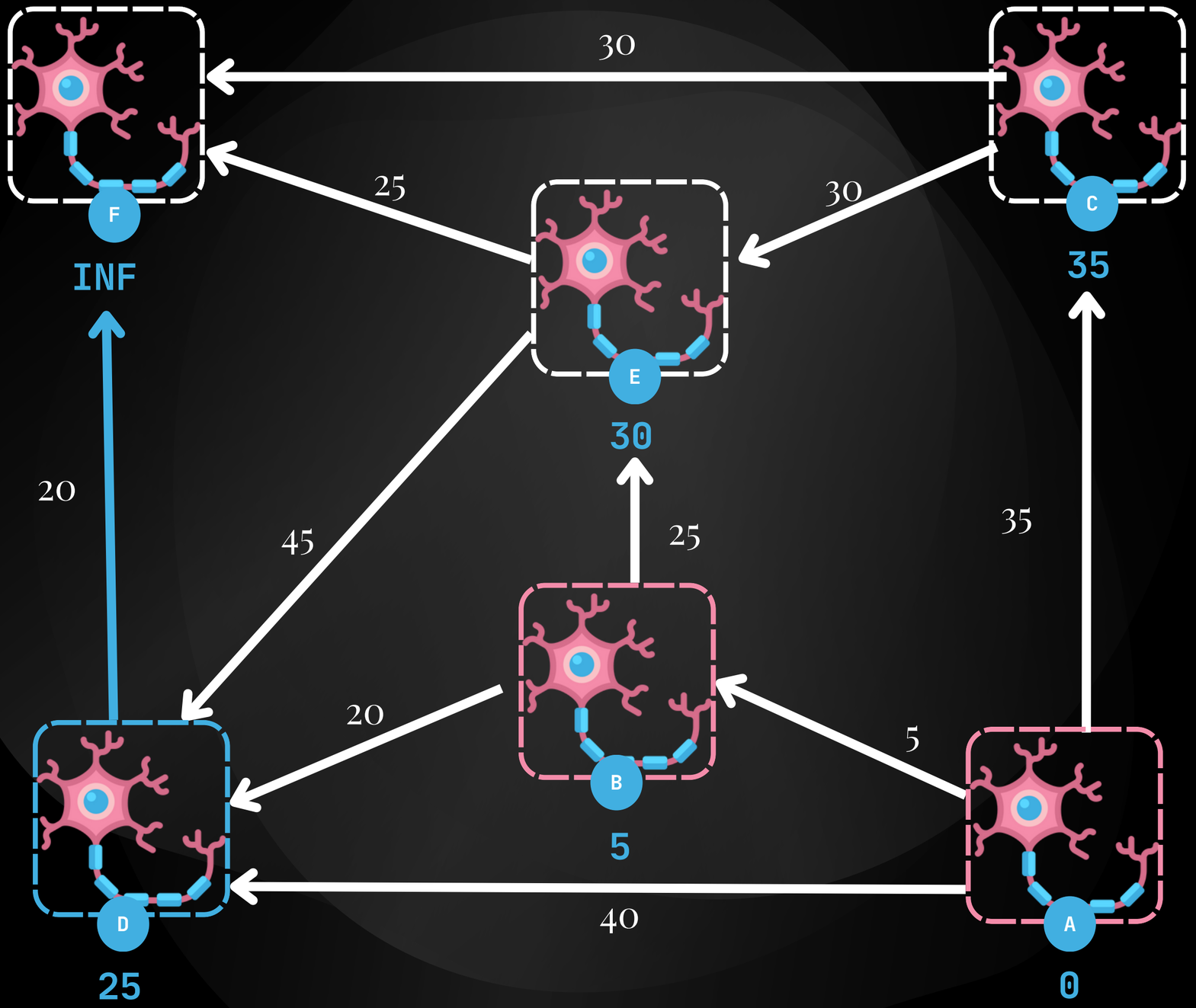
Predecesores

A → B

A → C

B → D

B → E



Fila prioridad

D = 25

E = 30

C = 35

Predecesores

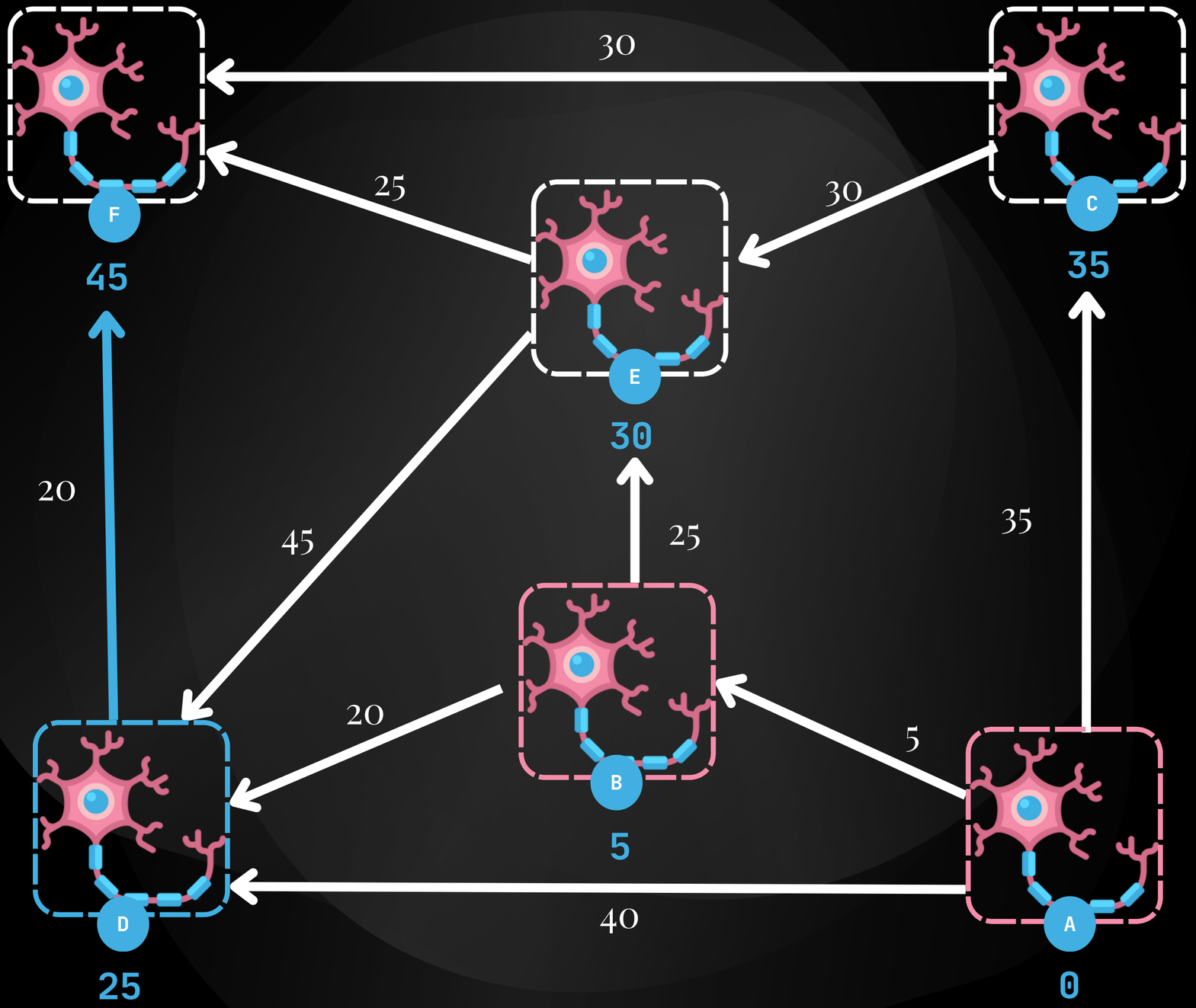
A → B

A → C

B → D

B → E

D → F



Fila prioridad

E = 30

C = 35

F = 45

Predecesores

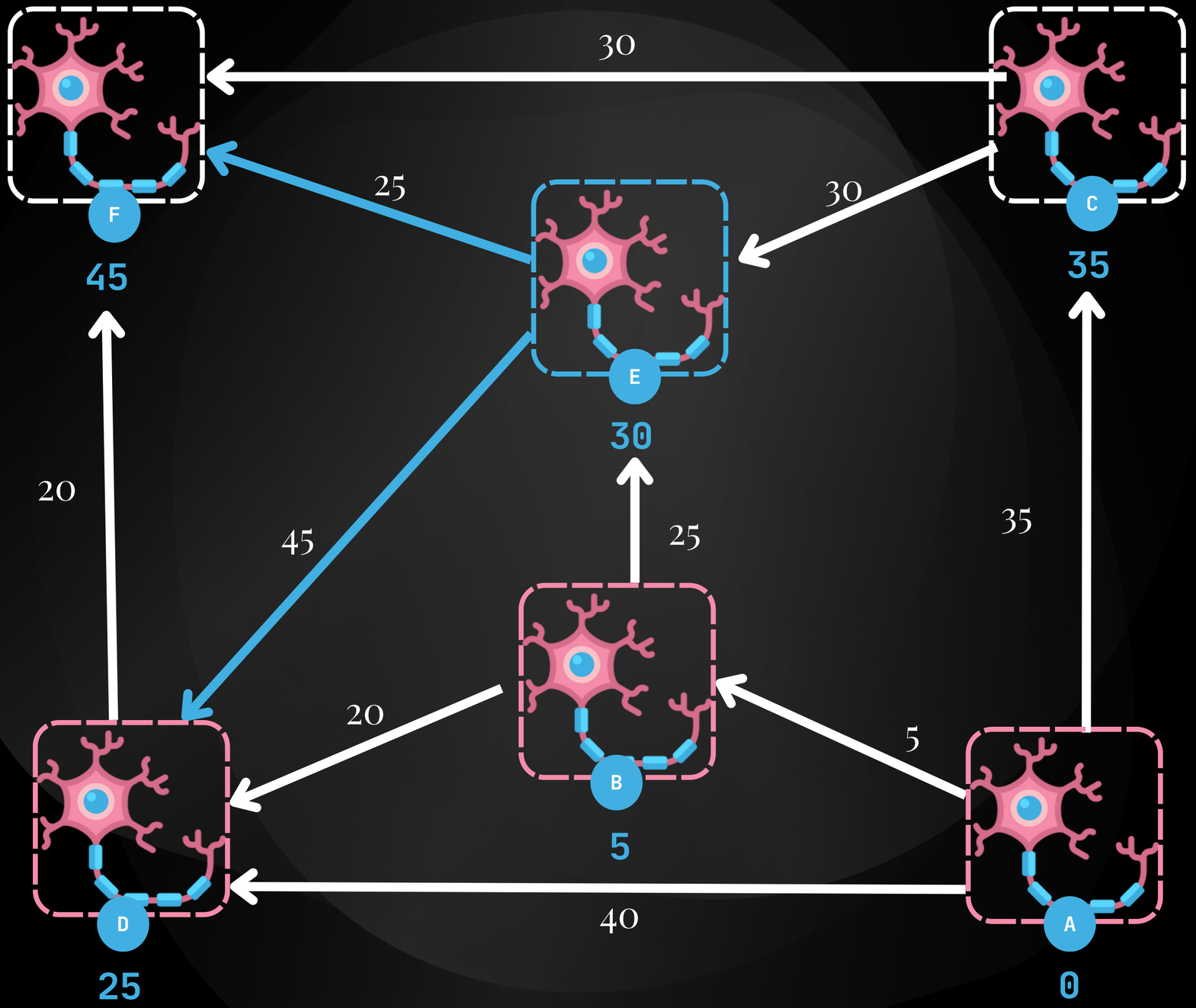
A → B

A → C

B → D

B → E

D → F



Fila prioridad

E = 30

C = 35

F = 45

Predecesores

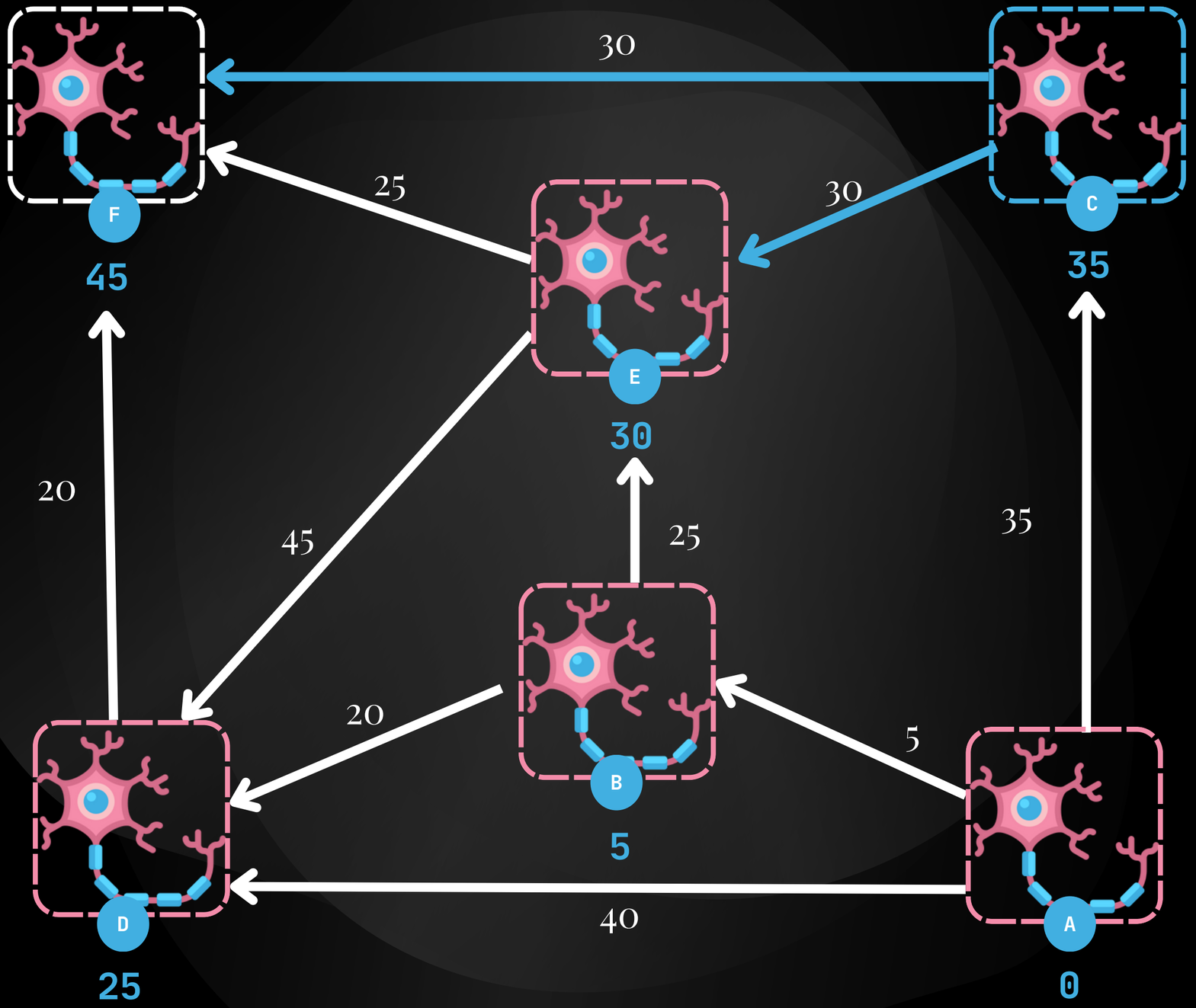
A → B

A → C

B → D

B → E

D → F



Fila prioridad

C = 35

F = 45

Predecesores

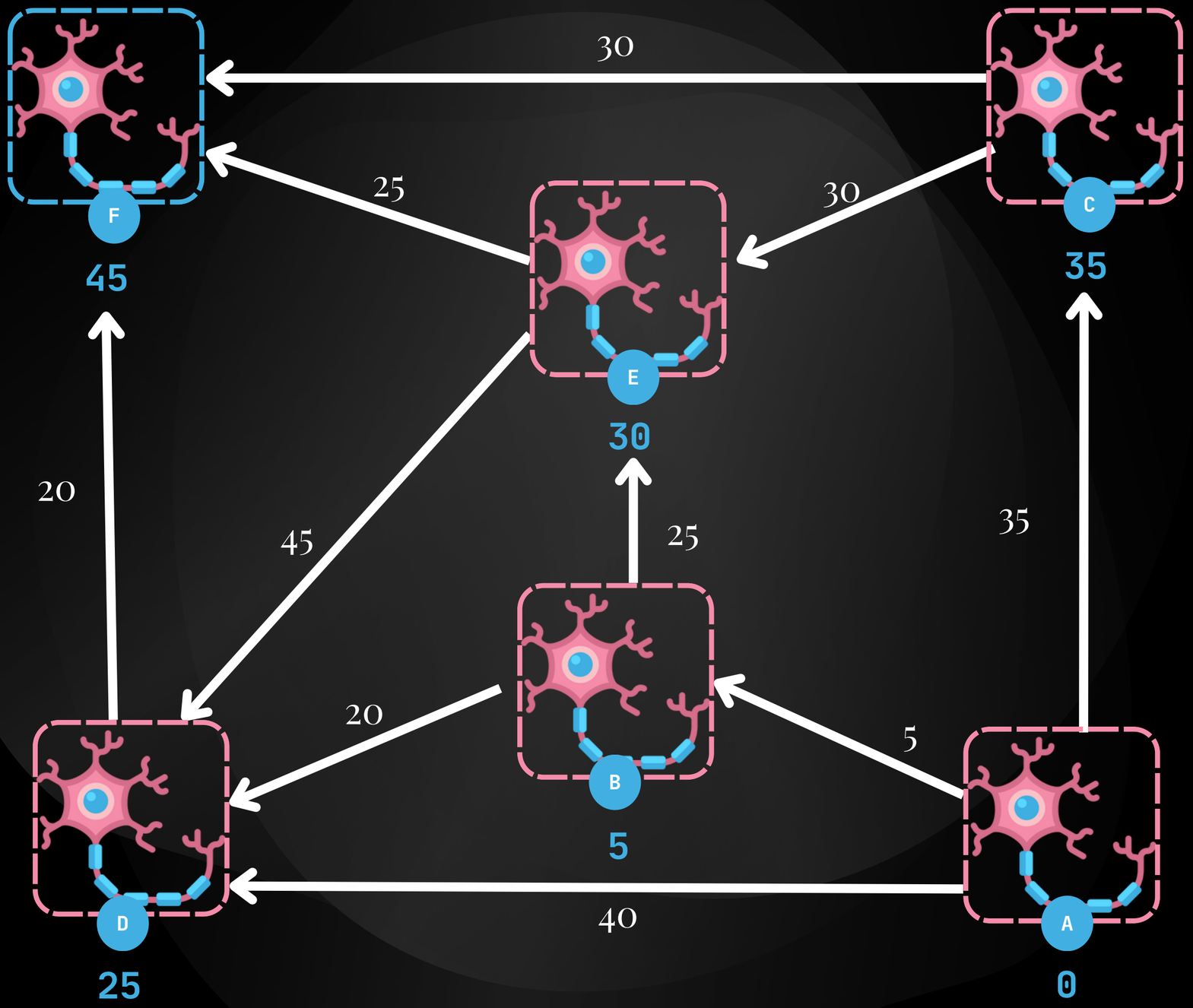
A → B

A → C

B → D

B → E

D → F



Fila prioridad

F = 45

Predecesores

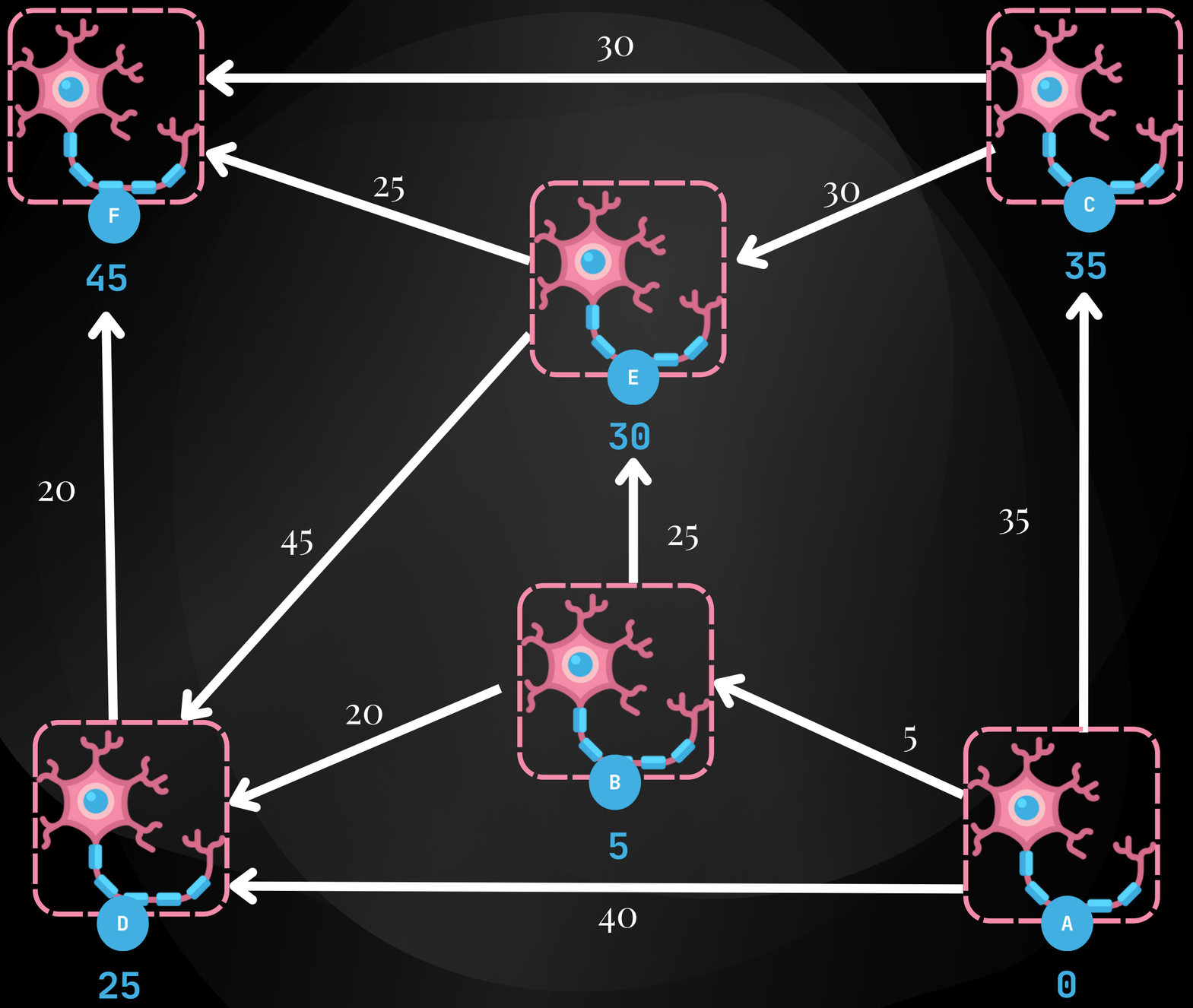
A → B

A → C

B → D

B → E

D → F



Fila prioridad

Hallamos el camino más corto entre A y F

Predecesores

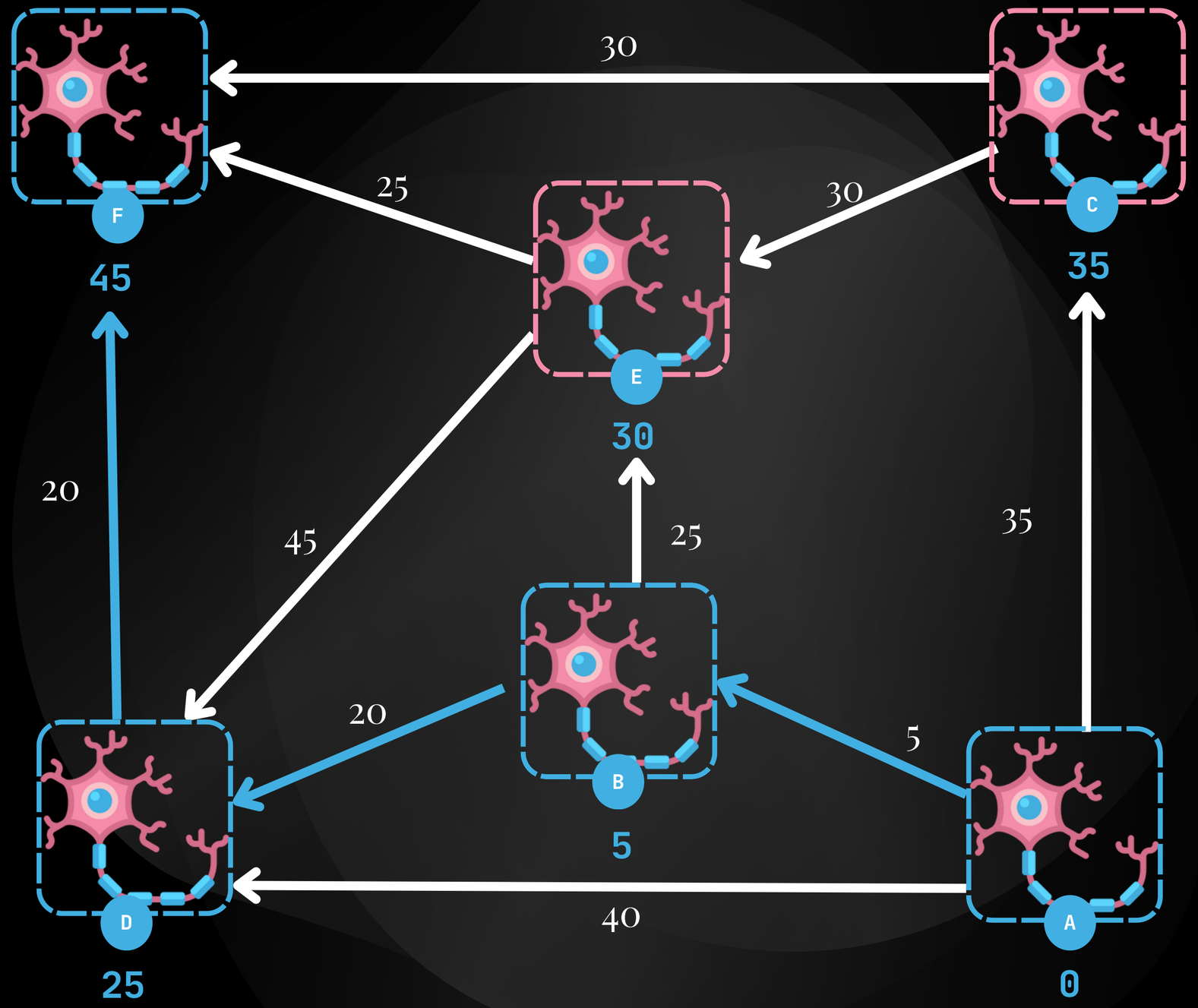
A → B

A → C

B → D

B → E

D → F



¡MUCHAS Gracias!

Reto

Presentación

UNIVERSIDAD NACIONAL DE COLOMBIA