CS47100 Homework 3

Due date: 5 am, US Eastern Time, April 10

This homework will involve conceptual exercises and programming exercises. Instructions at the end of the document present details on how to turn in the assignment on Gradescope and turnin.

Uncertainty (14 Pts)

1. (8 pts) Basic Probability

   (a) You are a witness of a night-time hit-and-run accident involving a bus in Toronto. All buses in Toronto are white or silver. You swear, under oath, that the bus was white. Extensive testing shows that under the dim lighting conditions, discrimination between a white and a silver bus is 80% reliable.

      i. Is it possible to calculate the most likely color of the bus involved in the accident? Show why or why not. (Hint: distinguish carefully between the proposition that the bus is white and the proposition that it appears white.)

      ii. Now consider that you are given an additional information stating that 7 out of 10 buses in Toronto are silver. Show whether or not it is now possible to to calculate the most likely color for the bus.

   (b) Sometimes we need to reason about a pair of variables X, Y conditioned on a set of evidence variables E. Using the definitions of conditional probabilities, prove the product rule and Bayes rule with additional conditions:

      i. $P(X, Y|E) = P(X|Y, E)P(Y|E)$

      ii. $P(Y|X, E) = P(X|Y, E)P(Y|E)/P(X|E)$

2. (6 pts) Probabilistic Inference and Independence

   (a) Let X_1, X_2 and Y be random variables. We want to calculate $P(Y|X_1, X_2)$ but we do not possess any independence/conditional independence information about these variables. For each of the following sets of distributions show how to calculate the desired query or explain why it is not possible:

      i. $P(X_1, X_2)$, $P(Y)$, $P(X_1|Y)$ and $P(X_2|X_1)$

      ii. $P(X_1, X_2)$, $P(Y)$ and $P(X_1, X_2|Y)$

      iii. $P(X_1|Y)$, $P(X_2|Y)$, $P(X_1, X_2)$ and $P(Y)$

      iv. $P(X_1)$, $P(X_2)$, $P(X_1, X_2)$ and $P(X_1, X_2|Y)$

      v. $P(X_1)$, $P(X_2)$, $P(X_1, X_2|Y)$ and $P(X_2|Y)$

   (b) Now suppose you know that $(X_1 \perp\!\!\!\perp X_2|Y)$. Now, which of the aforementioned sets are sufficient? Justify your answers.

Bayesian Networks (45 pts)

3. (15 pts) In your personal computer, the CPU fan will speed up when the CPU temperature sensor exceeds a given threshold. The CPU temperature sensor measures the temperature of the CPU. Consider Boolean variables A (true iff CPU fan speeds up), F_A (true iff CPU fan is faulty), F_G (true iff CPU temperature sensor is faulty) and multivalued discrete nodes G (CPU temperature

sensor reading) and T (actual CPU temperature). The sensor is more likely to fail (F_G is true) when the CPU temperature T increases. "iff" means if and only if.

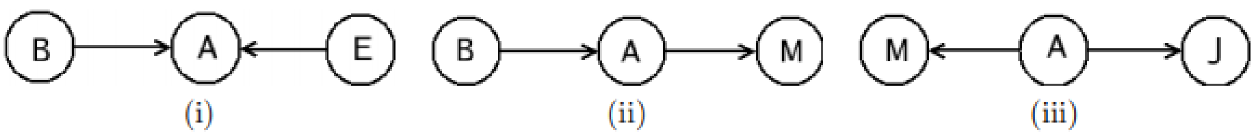
(a) Draw the graphical structure of the Bayesian network for this domain. The Bayesian network should involve A, F_A, F_G, G and T as nodes.

(b) Suppose the CPU temperature sensor reading G and the CPU temperature T have two possible values, normal and too-high. The probability that the CPU temperature sensor reading G matches the actual CPU temperature T is x when the sensor is not faulty, but is y when it's faulty. Write down the conditional probability table of G conditioned on its parents.

(c) Suppose the CPU fan will speed up if it's not faulty and the sensor senses too-high temperature of CPU. The fan won't speed up at all if it's faulty. Write down the conditional probability table of A conditioned on its parents.

4. (10 pts) What is the maximum number of edges in a Bayesian network (BN) with n nodes? Prove your answer by constructing a valid BN containing this number of edges. (Remember that the structure of a BN has to be a *Directed Acyclic Graph.*)

5. (10 pts) Let's look at the Bayesian networks below. We use the notation $X \perp Y$ to denote that the variable X is independent of Y, and $(X_1 \perp\!\!\!\perp X_2|Y)$ to denote X_1 is independent of X_2 conditioned on Y.



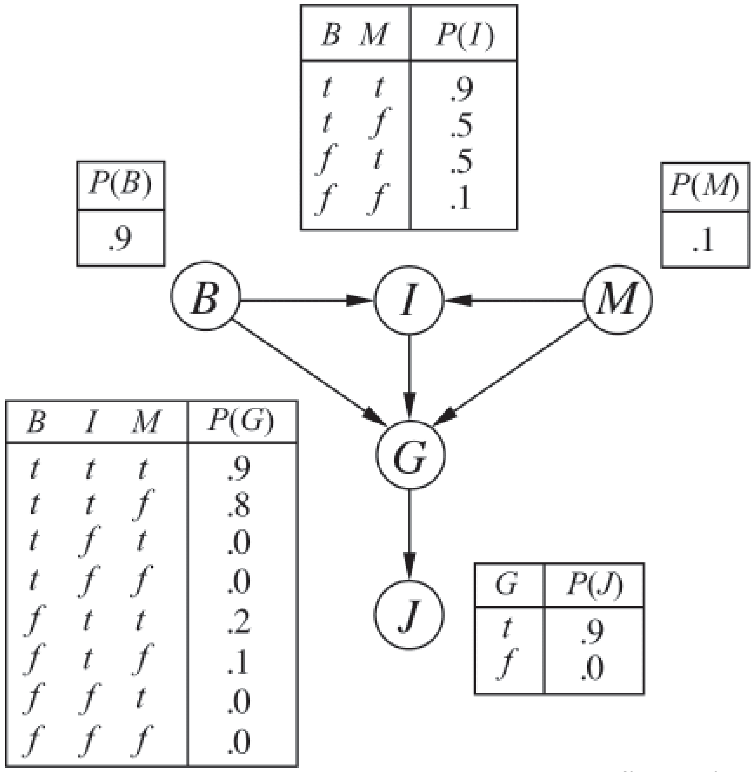
(a) For each of these three network, write the factored joint distribution. For example, for a BN with structure $X \to Y$, the factored joint distribution is $P(X, Y) = P(X)P(Y|X)$.

(b) Using the joint distribution you wrote down for Fig(i), write down a formula for $P(B, E)$.

(c) Now prove that $B \perp E$.

(d) Similarly, prove that $B \perp\!\!\!\perp M|A$ in the Bayesian network of Fig(ii), and $M \perp\!\!\!\perp J|A$ in the Bayesian network of Fig(iii).

6. (10 pts) Consider the following Bayesian network



$P(B)$
.9

B	M	$P(I)$
t	t	.9
t	f	.5
f	t	.5
f	f	.1

$P(M)$
.1

B	I	M	$P(G)$
t	t	t	.9
t	t	f	.8
t	f	t	.0
t	f	f	.0
f	t	t	.2
f	t	f	.1
f	f	t	.0
f	f	f	.0

G	$P(J)$
t	.9
f	.0

The conditional probability tables in this question are slightly different from those used in class. For example, the first line of the uppermost table means that $P(I = t|B = t, M = t) = 0.9$, which implicitly implies that $P(I = f|B = t, M = t) = 0.1$. Similarly, the upper-left table $P(B) = 0.9$ means that $P(B = t) = 0.9$ and $P(B = f) = 0.1$.

(a) Calculate the conditional probability table of $P(J|B, I, M)$ (i.e., calculating the conditional probabilities for all value assignments of J, B, I, M).

(b) Calculate the conditional probability table of $P(B|J)$.

(c) Suppose I observe $J = f$, which value B is more likely to take?

Probability (18 pts)

7. (6 pts) One lottery ticket costs \$2. There are two possible prizes: a \$20 payoff with probability 1/50 and a $\$2,000,000$ payoff with probability 1/2,000,000.

   (a) (2 pts) What is the expected payoff of buying one ticket?

   (b) (3 pts) Suppose Tom used \$100 to buy 50 lottery tickets. Let random variable X be the payoff that Tom gets from buying these 50 tickets. What is the expectation and variance of X? Notice that Tom is not very smart. He may buy the same lottery number multiple times. In other words, you can treat the outcomes of the 50 tickets independent with each other.

   (c) (1 pt) Sociological studies show that people with low income buy a disproportionate number of lottery tickets. Do you think this is because they are worse decision makers or because they have a different utility function? Consider that many people like the feeling of winning. For example, people imagine themselves becoming action heros while watching an adventure movie, which, in itself, is enjoyable.

8. (6 pts) A used-car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the tests, decide if he will buy the used car. We will assume that the buyer is deciding whether to buy car c_1. There is time to carry out at most one test. Suppose t_1 is the test and it costs \$50. A car can be in good shape (quality q^+) or bad shape (quality q^-), and the test might indicate what shape the car is in. Car c_1 costs \$1500 and its market value is \$2,000 if it is in good shape; if not, \$700 in repairs will be needed to make it in good shape. The buyer's prior estimate is that c_1 has a 70% chance of being in good shape.

   (a) Draw the decision network that represents this problem.

   (b) Calculate the expected net gain from buying c_1, given no test.

   (c) We know that there is a probability that a car pass (or fail) a test given the fact that it's in good (or bad) shape. We have the following information:
   $P(pass(c_1, t_1)|q^+(c_1)) = 0.8$ which is the probability that a car pass the test given it's in good shape.
   $P(pass(c_1, t_1)|q^-(c_1)) = 0.35$ which is the probability that a car pass the test given it's in bad shape.
   Use Bayes' theorem to calculate the probability that the car is in good (or bad) shape given the fact that it passed (or failed) its test. In other words, we ask you to compute $P(q^+(c_1)|pass(c_1, t_1))$, $P(q^-(c_1)|pass(c_1, t_1))$, $P(q^+(c_1)|\neg pass(c_1, t_1))$ and $P(q^-(c_1)|\neg pass(c_1, t_1))$.

   (d) Calculate the value of information of the test, and derive an optimal conditional plan for the buyer.

9. (6 pts) Consider the definition of *value of information* from Chapter 16 in Russel and Norvig, prove that the value of information is nonnegative.

Programming Assignment (40 pts)

Note: For the programming assignments we will use the Pacman project designed for the course CS188 at UC Berkeley.
`http://ai.berkeley.edu/tracking.html`

Please remember that solutions to any assignment should be your own. Using other people solutions, within or outside Purdue goes against the course academic honesty policy. The TAs will be using code similarity measures to detect plagiarism cases when grading the assignment.

In this assignment, we will use the Pacman projects 4. In project 4, you will design inference methods for Pacman agents to use to locate and eat invisible ghosts.

Please work on the provided code. Do not download code from the Berkeley site.

As in HW1 and HW2, the assignment includes an autograder for you to grade your answers on your machine.

For Project 4,

- **Files you will need to edit**: You will fill in portions of `bustersAgents.py` and `inference.py` during the assignment. You should submit these two files with your code and commemts. Please do not change the other files in this distribution.
- **Files you might want to look at**:
  - `busters.py`: The main entry to Ghostbusters (replacing Pacman.py)
  - `bustersGhostAgents.py`: New ghost agents for Ghostbusters
  - `distanceCalculator.py`: Computes maze distances
  - `game.py`: Inner workings and helper classes for Pacman
  - `ghostAgents.py`: Agents to control ghosts
  - `graphicsDisplay.py`: Graphics for Pacman
  - `graphicsUtils.py`: Support for Pacman graphics
  - `keyboardAgents.py`: Keyboard interfaces to control Pacman.
  - `layout.py`: Code for reading layout files and storing their contents
  - `util.py`: Utility functions

More information about how to start projects 4 is provided on the Pacman website. Please look at `http://ai.berkeley.edu/tracking.html`

TODO:

- Complete Project 4, Questions 1-3 described on the Berkeley site. Submit your modified versions of `inference.py` and `bustersAgents.py` for grading.
- **Bonus Points** Complete Project 4, Questions 4-5 for bonus points.

Points Distribution: Questions 1-3 counts for 40 pts. Questions 4-5 counts for 28 pts (bonus).

Submission Instructions:

Upload your answers to the **conceptual** questions as a **typed pdf** in gradescope: `https://www.gradescope.com/courses/81160`

- For your pdf file, use the naming convention `username_hw#.pdf`. For example, your TA with username *simac* would name his pdf file for HW3 as `simac_hw3.pdf`.
- To make grading easier, please start a new page in your pdf file for each subquestion. Hint: use a `\newpage` command in LaTeX after every question ends. For example, for HW3, use a `\newpage` command after each of part (a)-(d) of Question 5.
- After uploading to gradescope, mark each page to identify which question is answered on the page. (Gradescope will facilitate this.)
- Follow the above convention and instruction for future homeworks as well.

After logging into data.cs.purdue.edu, please follow these steps to submit your **coding** assignment:

1. Make a directory named `username` and copy all of your files there.
2. While in the upper level directory (if the files are in `/homes/yexiang/hw3/yexiang`, go to `/homes/yexiang/hw3`), execute the following command:

   ```
   turnin -c cs471 -p HW3 username
   ```

   For example, your professor would use: `turnin -c cs471 -p HW3 yexiang` to submit his work. Keep in mind that old submissions are overwritten with new ones whenever you execute this command.

   You can verify the contents of your submission by executing the following command:

   ```
   turnin -v -c cs471 -p HW3
   ```

   Do not forget the -v flag here, as otherwise your submission would be replaced with an empty one.

Your submission should include the following files:

1. Your modified python file `bustersAgents.py` and `inference.py`.