Week 1

Scope of Machine Learning

- Database mining
- Applications that can't be implemented
- Self-customizing programs
- Understand human learning

***Definition*: Machine Learning**

A computer program is said to *learn* from *Experience* E with some *Task* T and some performance measure P if its performance on T, measured by P, improves with *Experience* E

Types of Machines Learning

1. Supervised Learning

2. Unsupervised Learning
3. Reinforced Learning
4. Recommender systems

Supervised Learning

In supervised learning, we are given a data set and already know what our correct output should look like, having the idea that there is a relationship between the input and the output.

Supervised learning problems are categorized into "regression" and "classification" problems. In a regression problem, we are trying to predict results within a continuous output, meaning that we are trying to map input variables to some continuous function. In a classification problem, we are instead trying to predict results in a discrete output. In other words, we are trying to map input variables into discrete categories. Here is a description on Math is Fun on Continuous and Discrete Data.

Definition: **Regression Problem**

Predict results within a continuous output

Definition: **Classification Problem**

Predict results in a discrete output

Question: Infinite Number of Features?

How do we make computers handle this? -> Support Vector Machine (dealt with later)

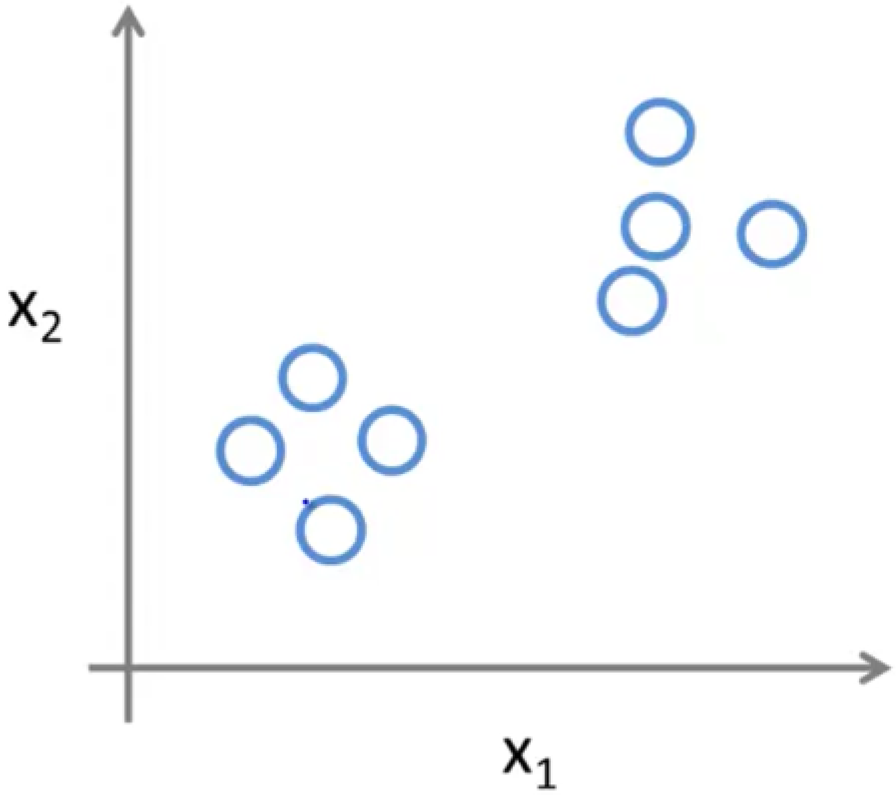
Unsupervised Learning

Unsupervised learning allows us to approach problems with little or no idea what our results should look like. We can derive structure from data where we don't necessarily know the effect of the variables.

We can derive this structure by *clustering the data based* on relationships among the variables in the data.

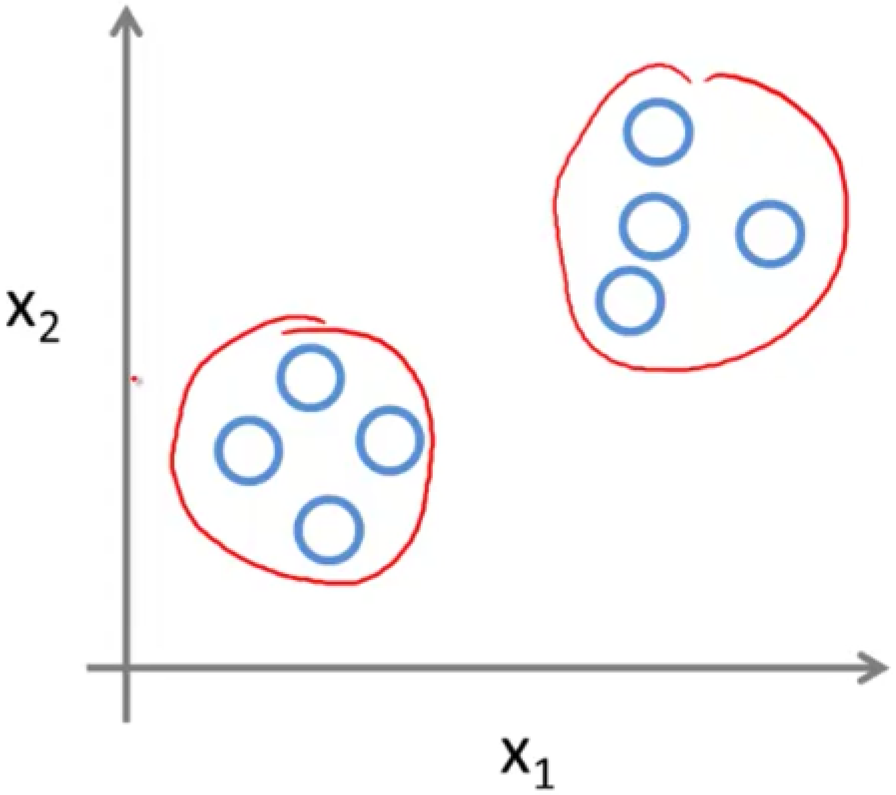
With unsupervised learning there is no feedback based on the prediction results.

Notice how the diagram doesn't have defined labels.

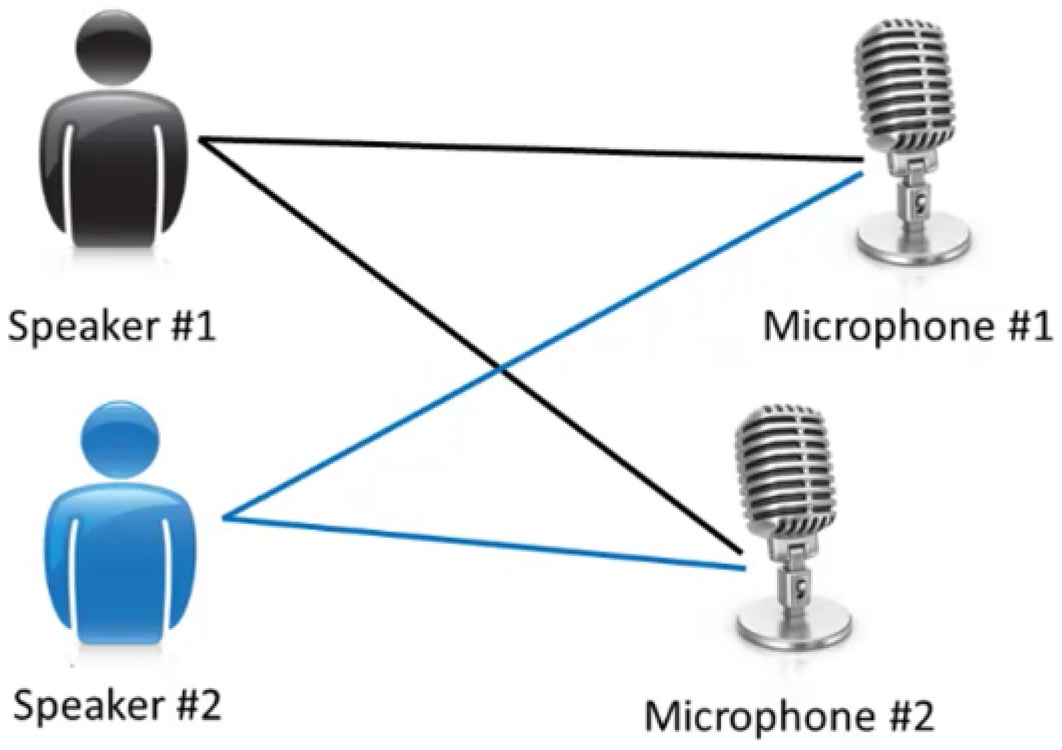


Algorithm Example: Clustering

Clusters data according to similar properties it defines by it self



Application Example: **Cocktail Problem**



By clustering identity by itself the 2 separate frequencies -> we can isolate the individual speakers

This can be done with the following code:

```
[W,s,v] = svd((repmat(sum(x.*x,1),size(x,1),1).*x)*x);
## Where svd -> single value devision
```

Model and Cost Function

Linear Regression with one variable - Univariate Linear Regression

- $h_\theta = \theta_0 + \theta_1 x$
- $h : X \rightarrow Y$

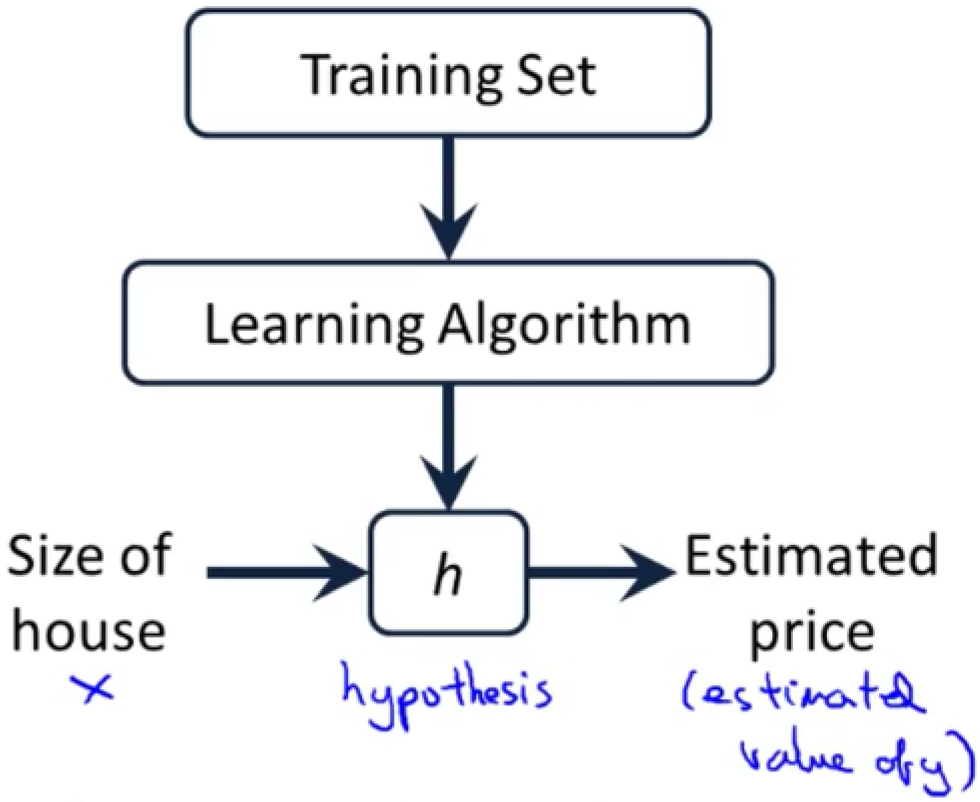
Properties:

- Supervised Learning
- Regression Problem

Notation:

$$
\begin{gathered}
m = \text{training examples} \\
x = \text{input var / feature} \\
y = \text{output var / target} \\
(x, y) - \text{one training example} \\
(x^{(i)}, y^{(i)}) - \text{ith training example} \\
i - \text{training set} \\
X - \text{space of input values} \\
Y - \text{space of output values}
\end{gathered}
$$

Structure:



Cost Function

***Definition*: Parameters**

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Formalized Cost function for Linear Regression

How do we determine the parameters?

Choose θ_0, θ_1 so that h is close to y for our training example (x, y)

Squared Error Function / Mean squared error:

This is the most common function used for regression problems but there are others.

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta x^{(i)} - y^{(i)} \right)^2$$

Vectorised **- Cost Function**

$$J(\theta) = \frac{1}{2m} (X\theta - \vec{y})^T (X\theta - \vec{y})$$

Minimize the Square Error Function

$$min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$$

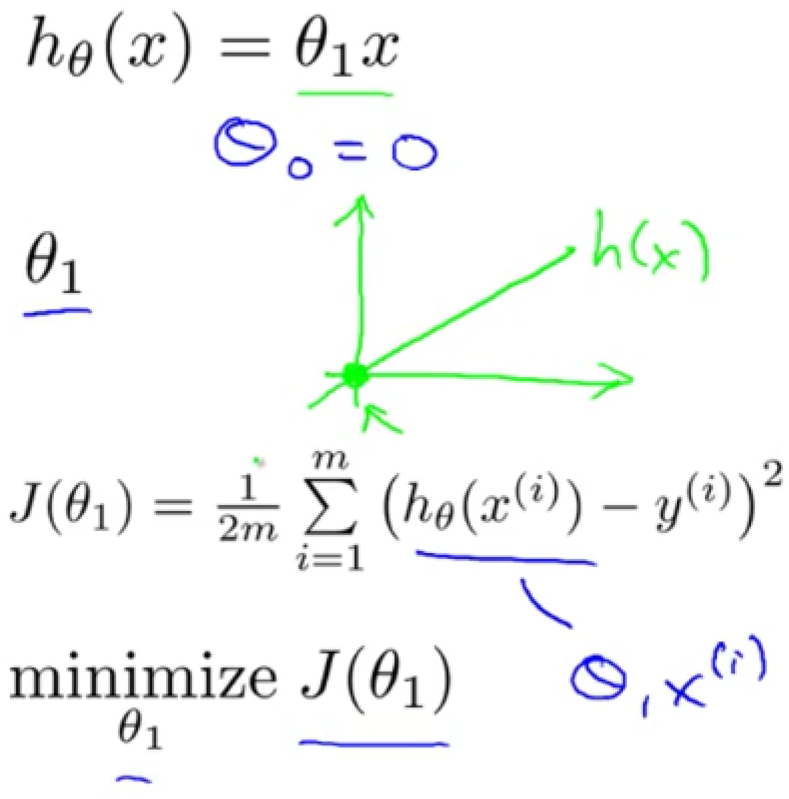
Where J is our Cost Function

NOTE:

- $\frac{1}{2m}$ - Average term - the 2 is there to simplify the derivative
- $\sum_{i=1}^{m} \left(h_\theta x^{(i)} - y^{(i)} \right)^2$ - Squared error difference term

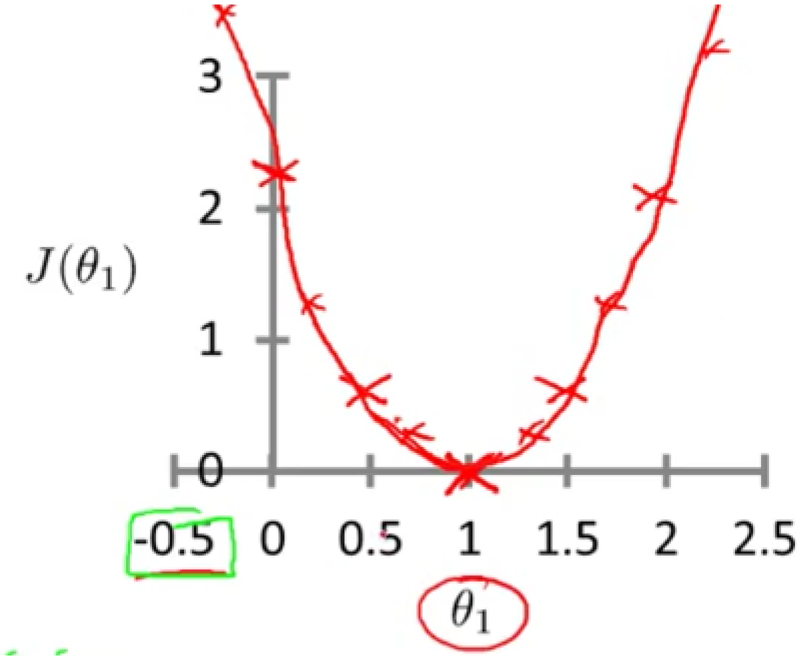
Intuition 1

Simplified problem for example:



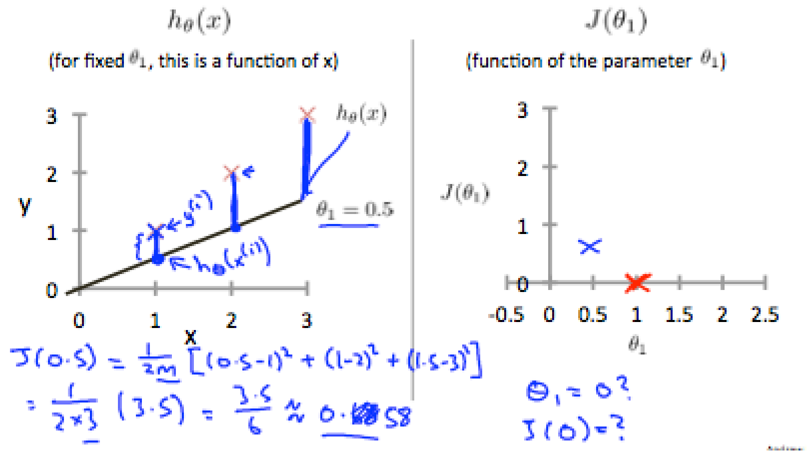
Determine the cost for θ_1

We can see that the function minimizes at $\theta_1 = 1$

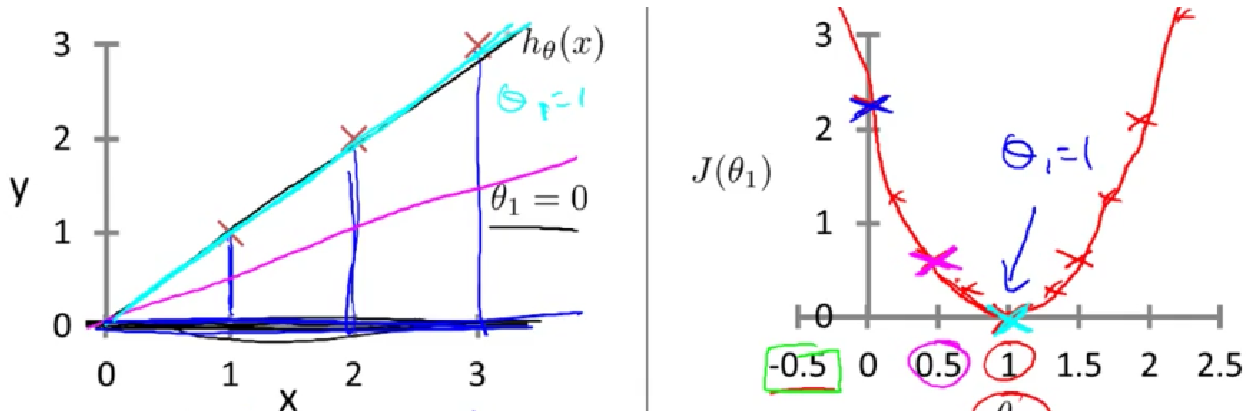


Messy comparison for intuition

Here a single iteration was done, the blue line shows the "error distance" the hypothesis function was from the actual plots (red x's)



The graphs bellow: Look at the cost function J value (colour- RHS) corresponding to the same colour plot of the hypothesis function h_θ on the LHS



Intuition 2

Example is done for the following

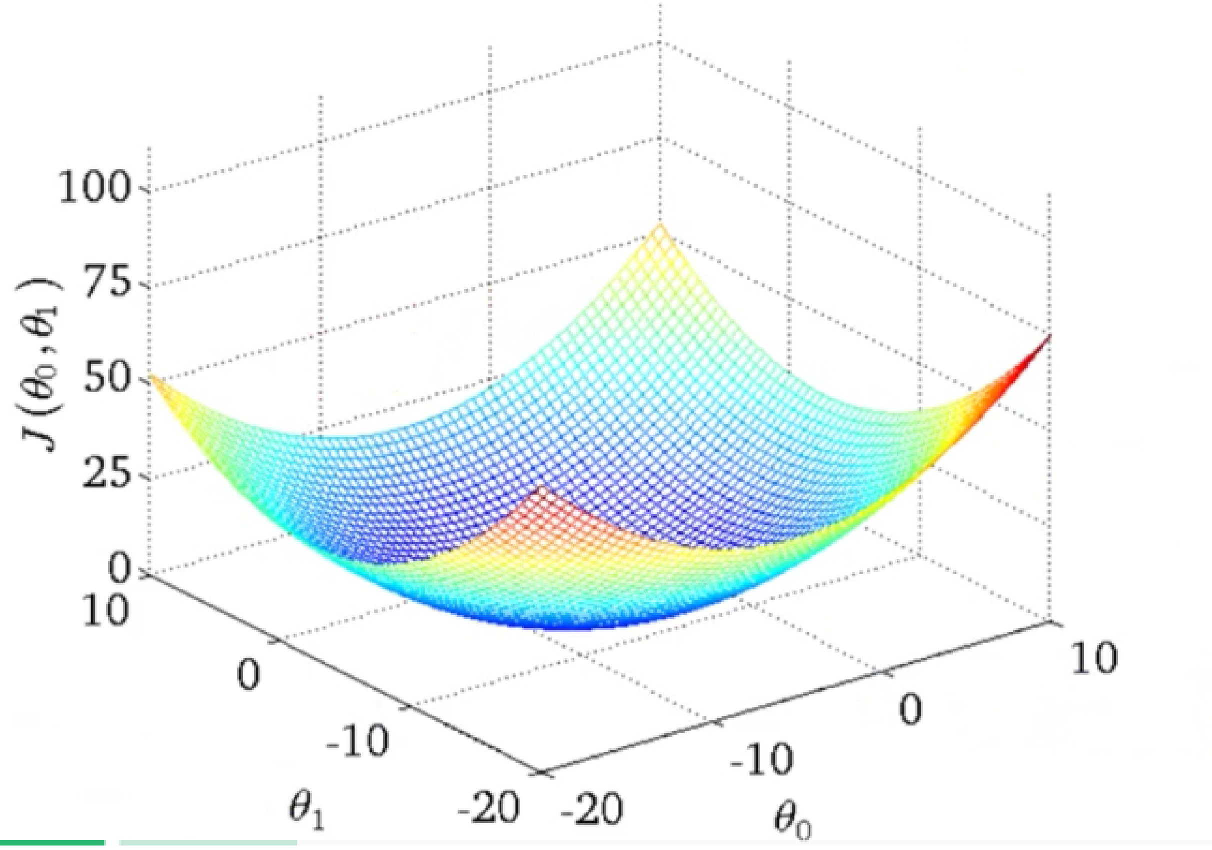
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

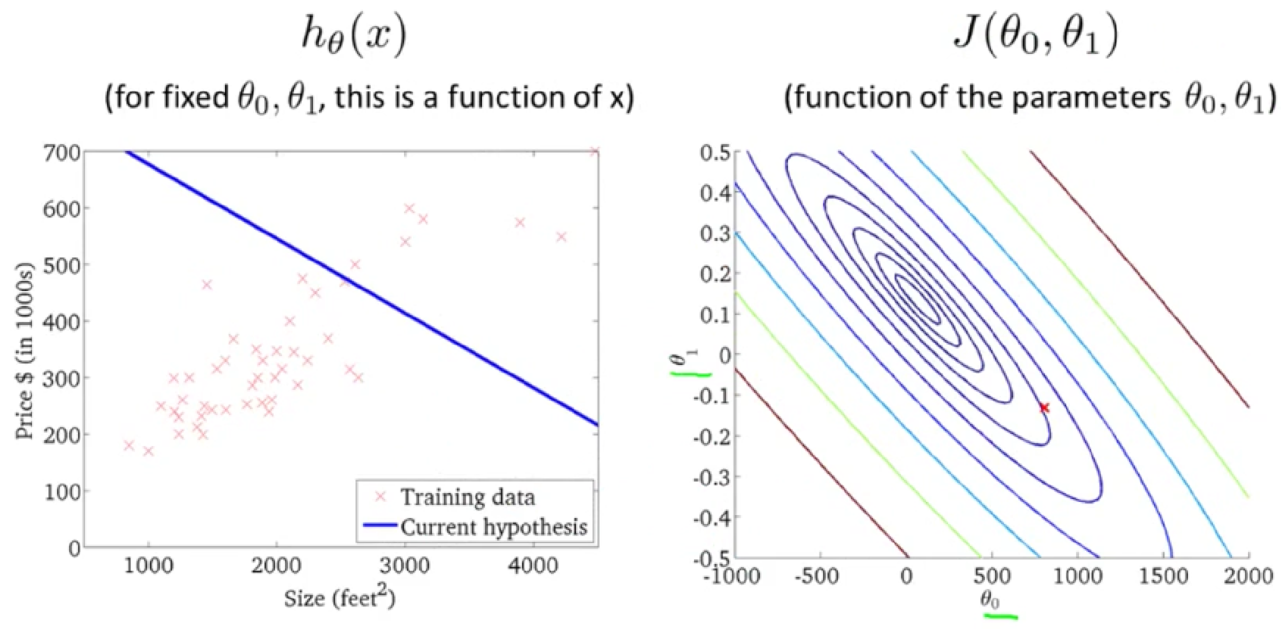
3d surface plot of $J(\theta_1, \theta_2)$ according to the different parameter values

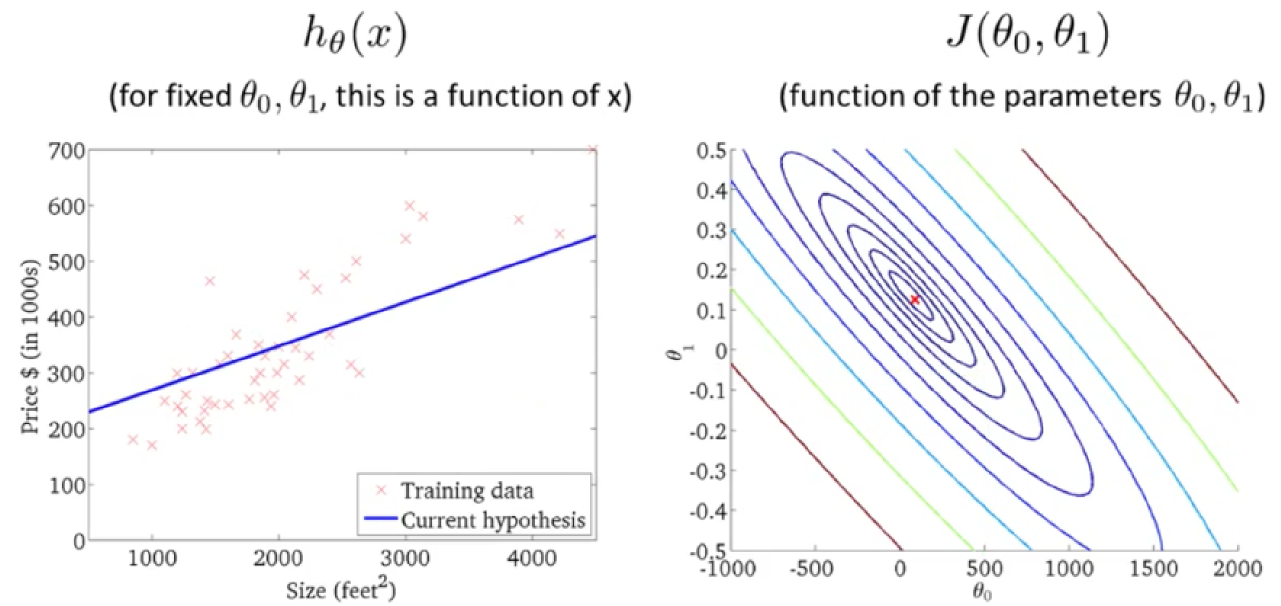


Contour plots

- Think of it as cross sections of the 3d plot.
- How close the lines in the contour plots are to one another illustrates the gradient of the function. Color illustrates the value of J , Red->Blue, darker Blue-> closer to 0
- Taking any color and going along the 'circle', one would expect to get the same value of the cost function.

Now look at x on the RHS and the corresponding plot on the LHS





Parameter Learning

Algorithm Gradient Descent

General algorithm to optimize functions.

$$min_{\theta_0 \ldots \theta_n} J(\theta_0 \ldots \theta_n)$$

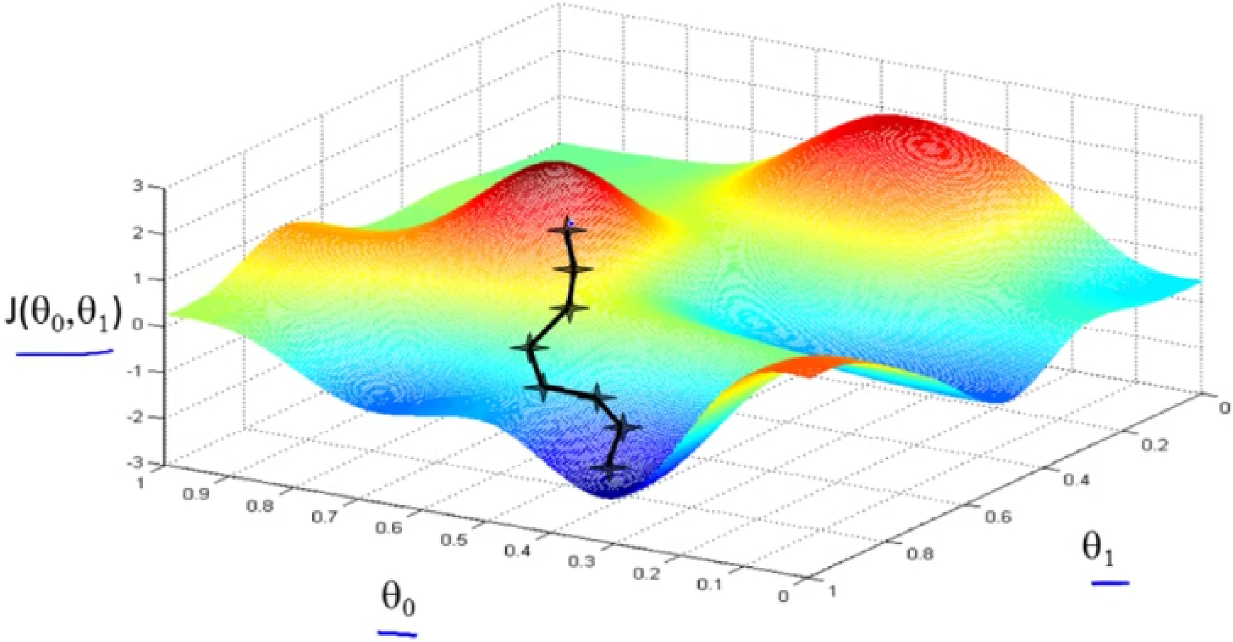
Outline

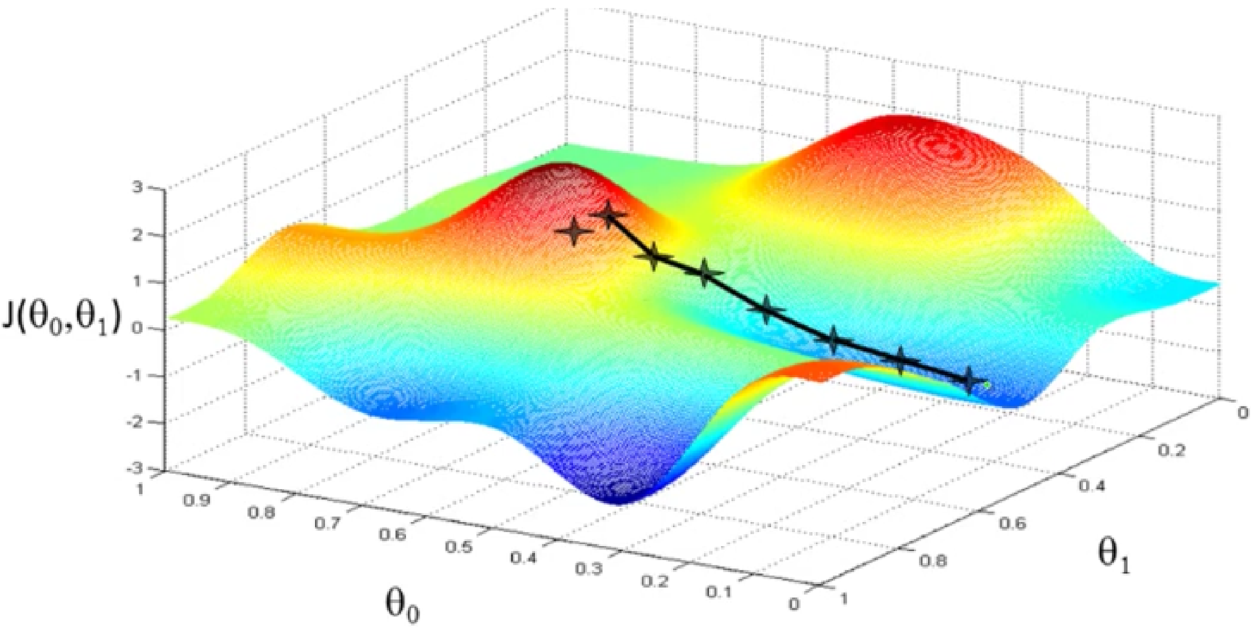
- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we find the minimum

Property:

- Only find the *local optimal solution*

- The bellow graphs show this





Algorithm

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \text{ for } j = 0, 1$$

Idea: Simultaneously update θ_0, θ_1

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Terms

- α - learning rate
  - control how big a step we take for each iteration
- $\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$ - gradient term
- $\alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$ - gradient descent term

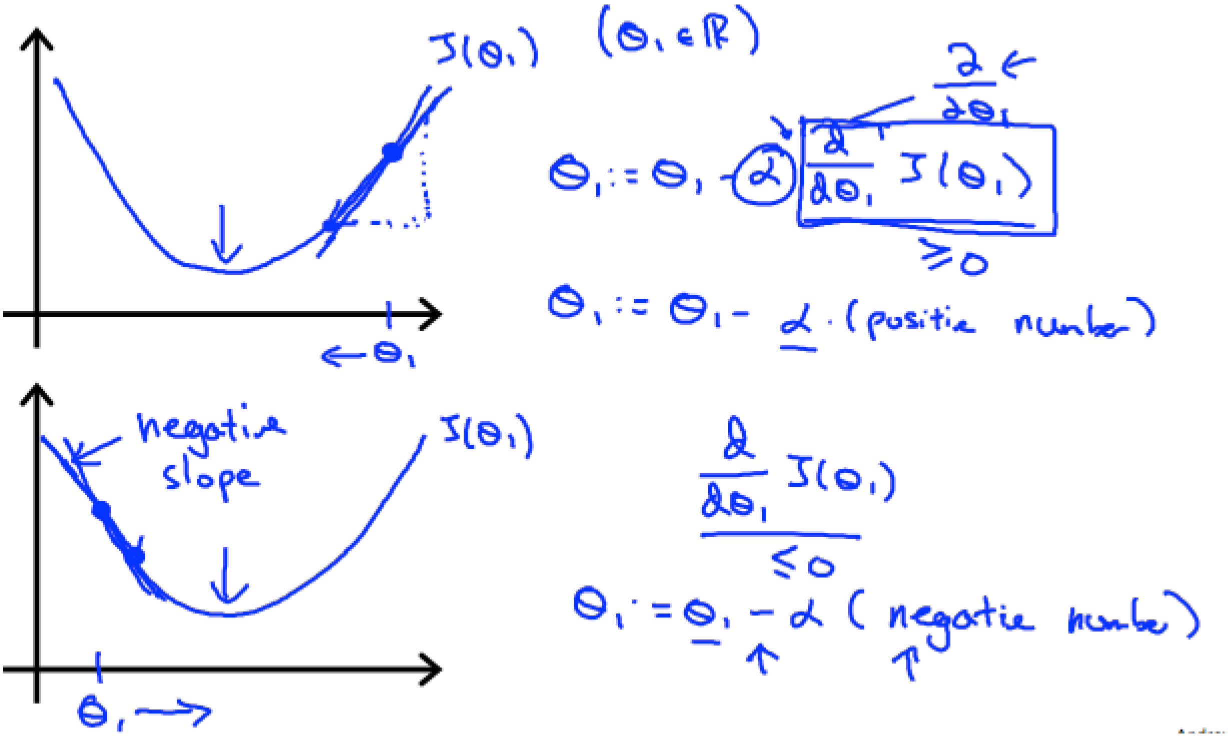
Notation:

- $a := b$ "Assignment"
- $a = b$ "Truth assertion"

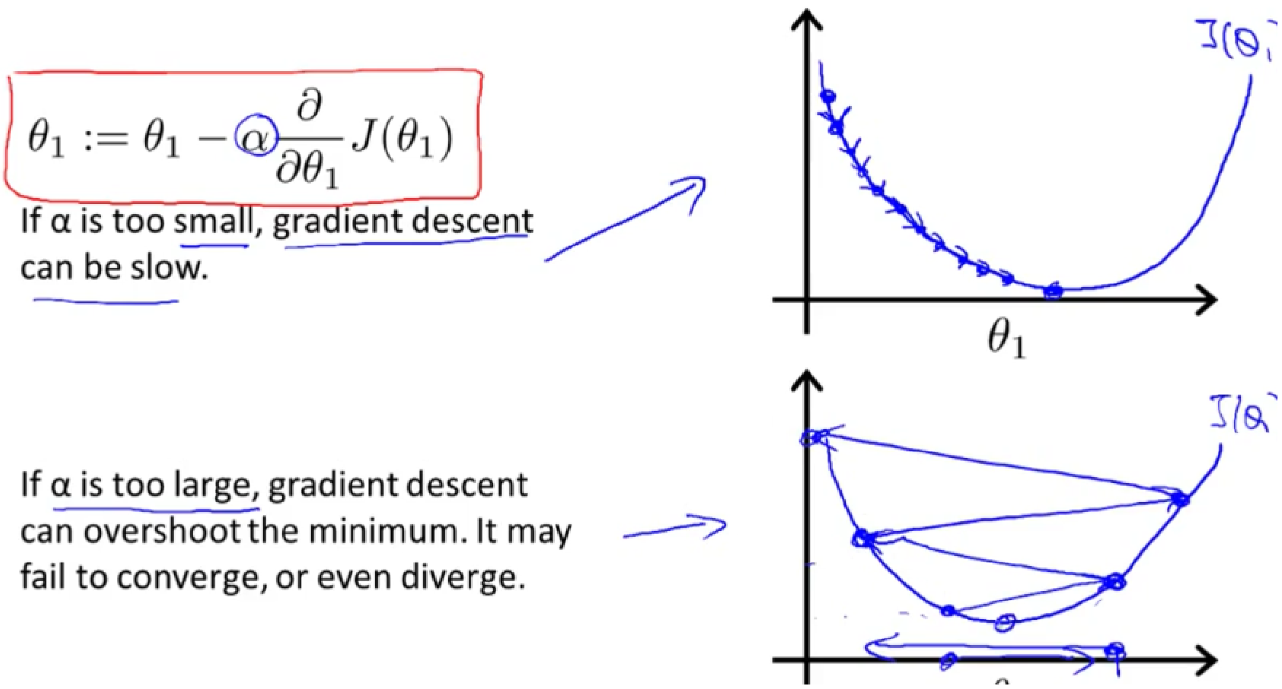
Intuition

Gradient Term

Regardless of the slope's sign for $\frac{d}{d\theta_1} J(\theta_1)$, θ_1 eventually converges to its minimum value. The following graph shows that when the slope is negative, the value of θ_1 increases and when it is positive, the value of θ_1 decreases.



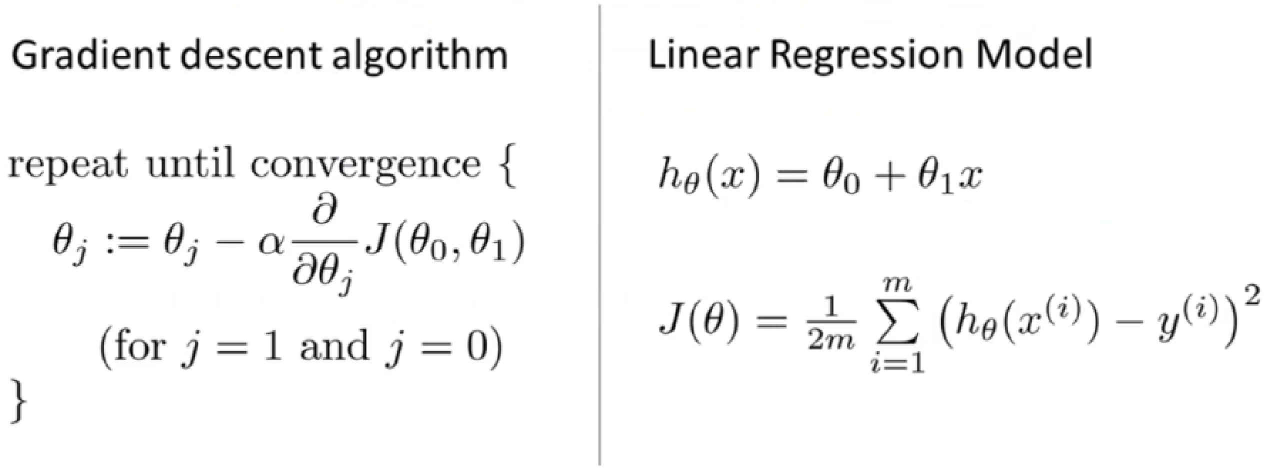
We should adjust our parameter α to ensure that the gradient descent algorithm converges in a reasonable time



Convergence

As the gradient term gets closer to the local optimal point, its value get closer to 0, then the gradient descent term as a whole gets smaller for every iteration.

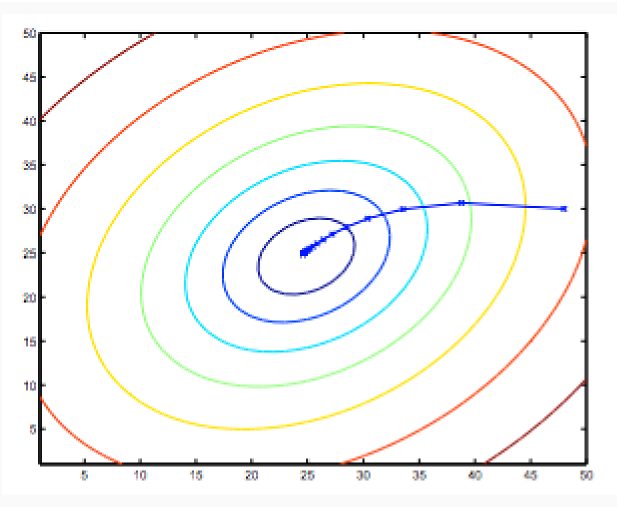
Gradient Descent for Linear Regression



Finding the partial derivative terms:

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \\
&\quad \theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)} \\
&\}
\end{aligned}
$$

NOTE: Linear regression problems are *convex function*s i.e. it has one minimum -> we don't have to worry about local / global min.



Definition: **Batch**

Each step of gradient descent uses all the training examples i.e. the above algorithm is also know as *batch gradient descent*

NOTE

- Normal Equations method: method to numerically find the minimum solution
- Gradient Descent Method scales better for larger data sets