厦門大學

Operation and research program case report

Activity Scheduling with Linear Programming

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Catalogue

1. Problem Description

1.1 Background description

In the face of today's fast-paced social life, time is one of the best chips. By using your time effectively, you can make your life more meaningful. Now Amy, a management science student at Xiamen University, needs to structure her weekly schedule for the second semester of her junior year. It is known that there are four compulsory courses, 7 optional courses, between the elective courses will have conflicts in class time. Her goal is to maximize the "total value" of her schedule.

1.2 Questions raised

Construct a reasonable schedule to maximize the total utility of student Amy's schedule. In addition to the known conditions, there are three additional constraints to be considered:

(1) The total credits of elective courses shall not be less than 8 points.

(2) Have at least one free time each day.

(3) Compulsory courses must be selected.

2. Question Analysis

2.1 How to select the model

According to the optimization problem of course scheduling and distribution described in the title, a simple linear optimization model can be developed and solved by Excel solver. The size and complexity of the problem can vary according to the course requirements and time constraints of students.

2.2 How to solve the model

(1) The decision variable that defines whether a course is selected or not is (0, 1). 0 means that the course is not selected, and 1 means that the course is selected.

(2) Define the utility scores of each course.

(3) The actual utility of the course to students can be obtained by multiplying the decision variable with the utility of the course.

(4) Add up the actual utility from Monday to Friday to get the total utility of each week. When the total utility reaches the maximum value, the schedule is the optimal schedule.

3. Basic Hypothesis And Explanation of The Model

3.1 Assumptions

(1) It is assumed that the effectiveness of each course can be quantified and evaluated subjectively.

(2) Assuming that the class time of each course is two hours, there is no overlap of class time of different courses

(3) Suppose that all courses are weekly, once a course is selected, the weekly schedule is consistent. For example, the situation of single and double weeks is not taken into consideration.

(4) It is assumed that there is no accident that interrupts the class schedule. For example, a

student is accidentally infected by COVID-19 and needs isolation.

3.2 Explanation of terms

(1) Utility

The state of being useful, profitable, or beneficial.

(2) Optimization model

An optimization model has three main components:

An objective function. This is the function that needs to be optimized.

A collection of decision variables. The solution to the optimization problem is the set of values of the decision.

A collection of constraints that restrict the values of the decision variables.

(3) A period of time

A class period is [n, N + 1] class, n ∈{1,3,5,7,9}.

(4) Compulsory course

Courses that must be selected.

(5) Elective course

Courses that can be selected according to your own needs.

4. Establishment and solution of the model

4.1 Description of symbols

Symbols	Description	Domain
C_i	Choose course i or not	0,1
Cr_i	Credits from course i	1,2,3
N_i	Number of times A course appeared in i week	0~10
U_{C_i}	The utility of course C_i	0~10
E_i	Choose to STUDY ENGLISH on the i day of the week	0,1
R_i	Whether to choose the i th SPORTS FITNESS activity within a week	0,1
FE_i	Whether to choose the i th FAMILY EDUCATION activity within a week	0,1
M	C_i is the set of i 's that belong to the compulsory course	/
O	C_i is the set of i 's that belong to the	/

	optional course	
P_j	A collection of courses in the j th period	/
D_j	A collection of courses on the j th day of the week	/

4.2 Establishment and solution of the model

4.2.1 Class schedule model : The basic idea

(1) Set the objective function

Before building the model, we have assumed that utilities can be measured and compared. Therefore, in a time frame of one week, we can take the sum of utilities as the objective function, among which the group of course plans with the largest sum of utilities is the optimal schedule.

$$Max \quad Z = \sum_{i=1}^{n} C_i N_i U_{C_i} \tag{1}$$

(2) Make a schedule of activities to be processed

Select all the activities under consideration and collect their schedule information. Use the letter C_i for the course name and represent the course arrangement in the form of a class schedule. Call it the "schedule of activities to be processed".

	Mon	*Tue*	*Wed*	*Thu*	*Fri*
1、2			C_1		
3、4	C_1	C_2	C_8	C_5+C_9	C_2
5、6	C_5+C_6		C_3	C_4	C_9+C_{11}
7、8				C_{10}	
9、10	C_7				

Among them， C_1 , C_2 , C_3 , C_4 are compulsory courses，C_5 , C_6 , C_7 , C_8 , C_9 , C_{10} , C_{11} are elective courses. Each course has corresponding credits.

Here are basic limits:

Ⅰ. Course variable C_i is equal to 0 or 1

$$(C_i - 0)(C_i - 1) = 0,\ \forall i = 1,2...11 \tag{2}$$

Ⅱ.There can be at most one class in a period of time

$$\sum C_i \le 1,\ \forall i \in P_j, j = 1,2...25 \tag{3}$$

(3) Constructing utility functions

Firstly, R_i^1 is obtained based on personal subjective views of the course, R_i^2 is obtained based on other people's objective evaluation of the course, and then weight α is set. Finally, the following formula is used to calculate the real utility score of each course. The scores of the above three variables were distributed between 0 and 10.

$$U_{c_i} = \alpha R_i^1 + (1-\alpha) R_i^2 \tag{4}$$

(4) Set limits for the schedule, taking into account your specific needs and preferences

Ⅰ. Compulsory courses must be chosen

$$C_i = 1,\ \forall i \in M \tag{5}$$

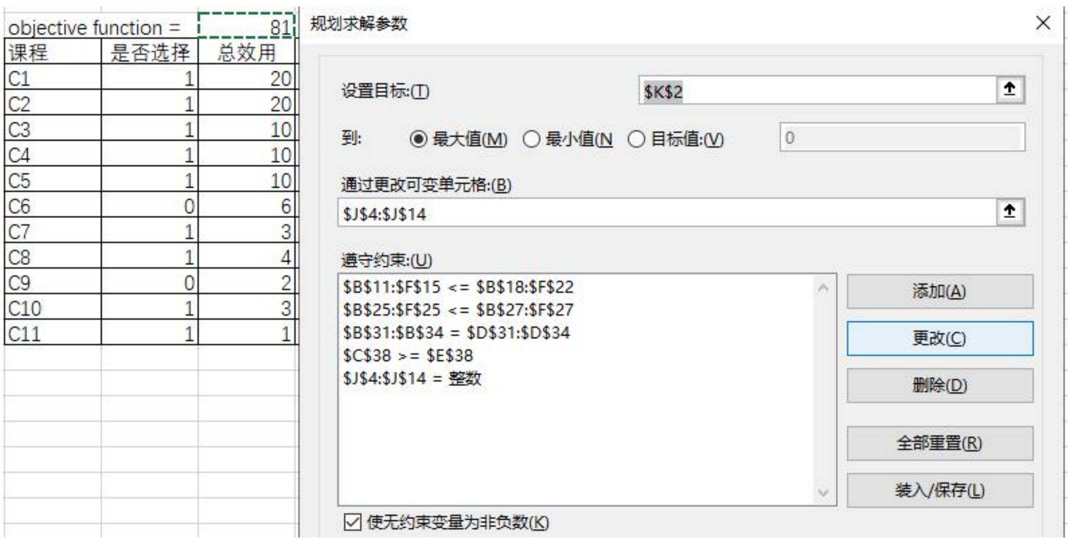
II. Have at least one free time each day

$$\sum C_i \le 4,\ \forall i \in D_j, j = 1,2...5 \tag{6}$$

III. The total credits of elective courses should not be less than 8 points

$$\sum C_i Cre_i \ge 8,\ \forall i \in O \tag{7}$$

(4) Using the programming function attached to Excel to solve the problem



Please refer to the document: Activity schedule. XLSX /sheet 'Class schedule' for details.

4.2.2 Model upgrade and expansion--From Class schedule model to Activity schedule model

(1) Enter the newly added activity

In fact, given the existing assumptions, any activity A can be regarded as A course C, and we can still subjectively assign utility values to each activity; In the process of making the optimal activity schedule, the basic principle is still to maximize the total utility:

$$Max \quad Z = \sum_{i=1}^{n} A_i N_i U_{A_i} \tag{8}$$

(2) Special treatment for activities with the same name but inconsistent content

However, different from the above curriculum arrangement, the extra attention should be paid to the fact that there are some special situations: within a week, there can be multiple activities with the same name but without mutual influence. For example, students who chose courses that appear more than once a week need to attend each class without any absence, but the students who made the daily plan to do morning reading English, can be absent occasionally.

Therefore, in order to distinguish the two activities, we do this by giving them different names. For example, we name Monday morning English reading as A_j and Tuesday morning English reading as A_{j+1}, and their values (0 or 1) will not affect each other.

(3) Example: model construction and solution

In order to clearly distinguish the different types of activities, we use E_i for English morning reading, R_i for running, and FE_i for tutoring activities. The activity schedule after joining the new activity is as follows:

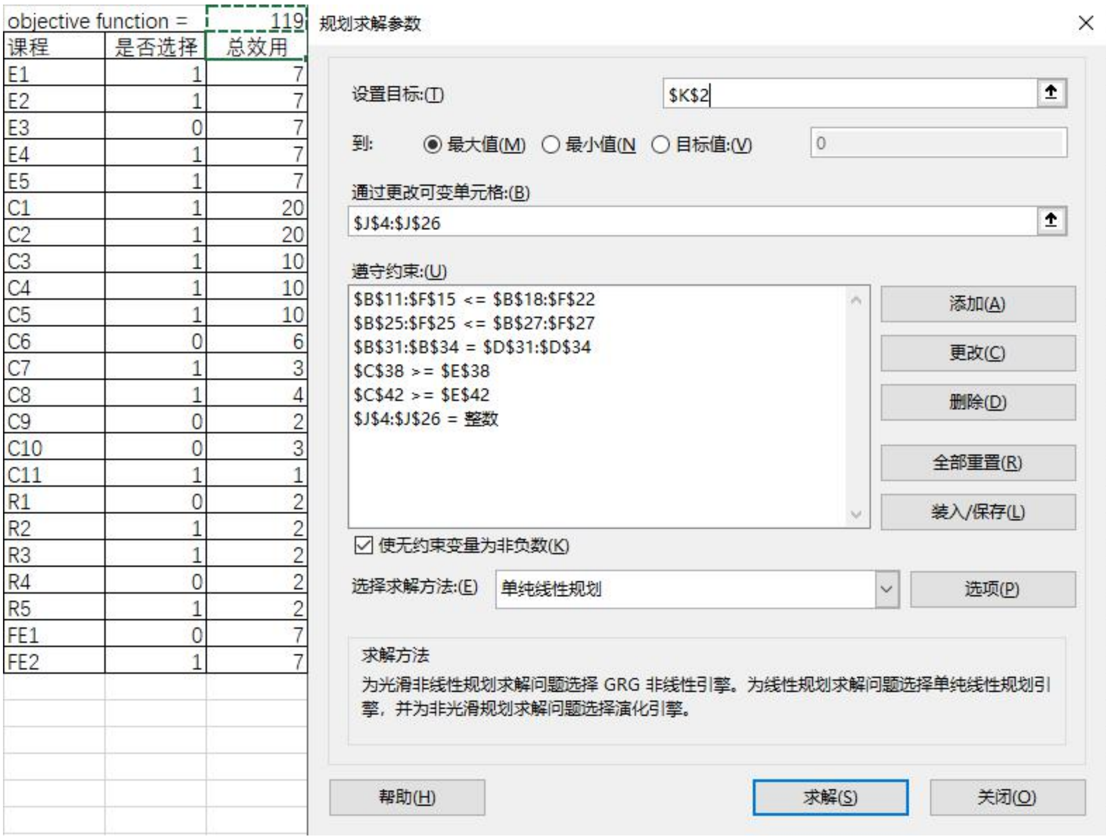
	Mon	*Tue*	*Wed*	*Thu*	*Fri*
1、2	E_1	E_2	C_1+E_i	E_4	E_5
3、4	C_1	C_2	C_8	C_5+C_9	C_2
5、6	C_5+C_6		C_3	C_4	C_9+C_{11}
7、8				C_{10}	
9、10	C_7+R_1	R_2+FE_1	R_3	R_4+FE_2	R_5

New restrictions：

Ⅰ. Run at least 3 times a week

$$\sum_{i=1}^{5} R_1 \geq 3 \tag{9}$$

Planning to solve:



objective function =		119
课程	是否选择	总效用
E1	1	7
E2	1	7
E3	0	7
E4	1	7
E5	1	7
C1	1	20
C2	1	20
C3	1	10
C4	1	10
C5	1	10
C6	0	6
C7	1	3
C8	1	4
C9	0	2
C10	0	3
C11	1	1
R1	0	2
R2	1	2
R3	1	2
R4	0	2
R5	1	2
FE1	0	7
FE2	1	7

Please refer to the document: Activity schedule.xlsx/sheet 'Activity schedule' for details.

5. Model optimization

5.1 Improve the accuracy of the time scale

Increase the time precision to a more detailed precision – minutes for more detailed activity planning

1、2	English reading
3、4	Course A
5、6	Course B
7、8	Running
9、10	Private tutor

→

7:00-7:30	Washing up
7:30-8:00	breakfast
8:00-9:00	English reading
9:00-9:40	Previewing course
9:40-10:10	Taking a break
10:10-11:50	Course A

5.2 Consider the case where the time of the event is partially overlapped

Tuesday

<table>
<tr><td>….</td><td>….</td><td>….</td></tr>
<tr><td>16:40-17:30</td><td>Running</td><td rowspan="3">Course B</td></tr>
<tr><td>17:30-18:00</td><td>Taking a shower</td></tr>
<tr><td>18:00-18:20</td><td rowspan="2">Paper reading</td></tr>
<tr><td>18:20-7:00</td><td>Dinner</td></tr>
</table>

To make such a change, we need to add a new constraint to the base constraint: the selected schedule cannot have any overlap of time.

5.3 Consider the extension of the time span (More than a week)

If we extend the time span, the only thing changing is the objective function, but in fact we can take into account a lot of situations by doing that, such as considering single-week or bi-week courses, considering events that happen once a month, once a month, etc.

Mon	…	Fri
…	…	…
…	…	…
…	…	…
…	…	…

1st week	…	nth week
…	…	…
…	…	…
…	…	…
…	…	…
…	…	…

5.4 Consider a temporary revision of the schedule of activities

In the actual operation, it often happens that the plan cannot catch up with the changes. Therefore, we need to design the midway revision of the schedule, so as to deal with some unexpected events that could not be predicted in advance:

In the face of a general emergency, modify/add/delete the information in the "schedule of activities to be processed" and modify/add/delete the corresponding restrictions. It is worth noting that the activities that have occurred cannot be modified at all, and some restrictions need to be added so that their variables are equal to 1.