Large Scale Multi-vehicle Routing with Linear Programming

Zhemian Zhao

School of Management, Xiamen University

zhemian.zhao@gmail.com

ABSTRACT

Transportation activity is one of the main factors contributing to the overall logistics costs of a company. One way to minimize the cost of this activity is by optimizing the distribution routes. However, as the number of delivery points increases, the possible route combinations grow in a non-linear fashion, increasing the complexity of obtaining the optimal combination of distribution routes. In this paper, we have developed a linear programming model with the main purpose of verifying its feasibility at the level of a mathematical model, thus solving the problem presented in the case[1].

Keywords

- **Operations research**
- **Transportation**

1. INTRODUCTION

NHG is a chain store whose business scope covers 123 stores in six states in the northeast. Due to the increase in distribution demand, I need to re-estimate its internal transportation mileage and require the planned route to be the shortest. Among them, the information I have obtained is the matrix distance between 123 stores, the number of deliveries corresponding to the number of stores that need to be delivered each week, and the number of goods that need to be delivered every day. The case requires that all vehicles must depart from the distribution center and must return to the distribution center within two days at the latest, and store orders on different dates cannot be grouped on the same day's route. At the same time, the case gives us quite a lot of constraints. For example, the driver's duty time should be less than 14 hours or the driver's maximum driving time is 11 hours. The duty time includes the driver's driving time, waiting time, and unloading time; the vehicle load must be less than 3200 ft3; the unloading rate is 0.03 minutes/ft3, and the unloading time of fewer than 30 minutes is also counted as 30 minutes; the vehicle speed is set at 40 mph, and the service time is 8:00-18:00.

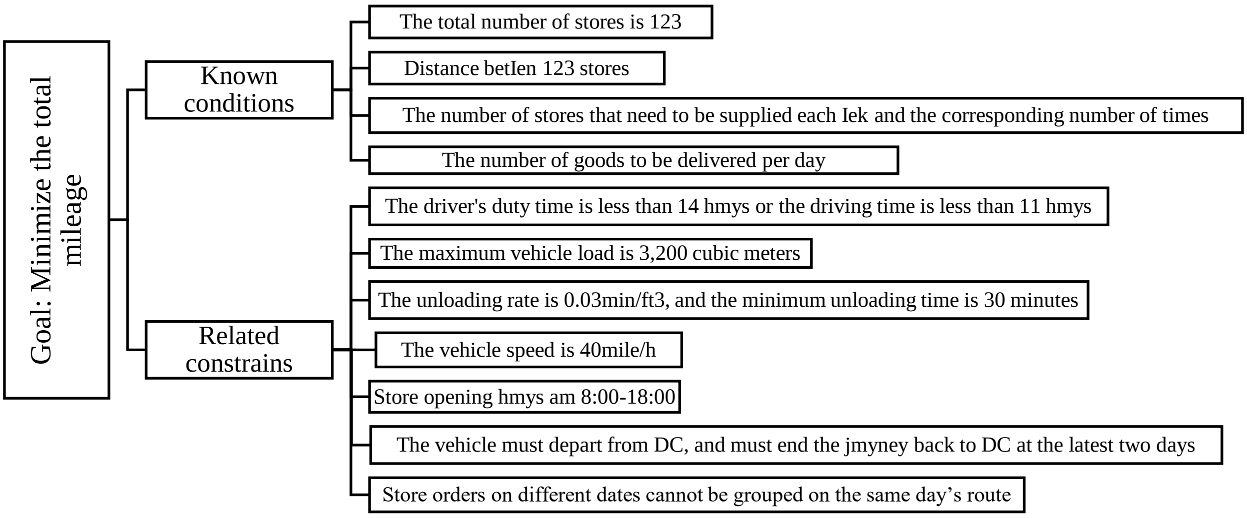


Figure 1. Problem Introduction

2. PROBLEM ANALYSIS

2.1 Problem Analysis

(1) How to determine variables and establish objective functions

From the description of this question, I get a lot of information to consider. First of all, when determining a variable, what needs to be satisfied is the route planning of the order of the distribution store. This condition can be expressed by defining the variable as to whether to pass through one store to another, so the variable should be a binary variable0. The second is the specific date of the week and the batches of different cars, so I need to consider adding other elements to distinguish the variables.

In determining the objective function, the direction considered is to minimize the total path of all vehicles. Therefore, the variable itself and its corresponding path value should be processed and finally added to establish the objective function.

(2) How to use variables to describe the vehicle path

The most difficult and important problem in determining variables is how to use variables to describe the path of a vehicle. By combining the experience of solving problems in the past and the characteristics of this case, I add the starting point information and the arrival point information to the description of variables and combine the form of a matrix to carry out the practical operation.

(3) How to achieve cross-day delivery restrictions

Considering the assumptions in the case, The overnight route cannot span more than two days, I decided to introduce the idea of global optimization in two days as a unit of time. For example, vehicles on the first day can be delivered to stores with demand on the first day and stores with demand on the next day. In such a case, the first variable matrix I get is a 2*2 matrix. The next day's vehicles can be delivered to both the next day's shops and the third day's shops. Therefore, the first 2*2 variable matrix will include the matrix of the first day, the fourth day and the fifth day, and so on.

(4) How can delivery requirements be ensured for each store

According to the relevant description in the case, to meet the delivery demand of each store, the following constraints should be met in the final optimal distribution line: each store as the starting point can only have one destination. Each store as a destination can only have one starting point.

(5) How to ensure that there am as many cars as possible and how many am in use

In defining delivery vehicles, I first need to consider what the average daily demand for goods is for all stores and then get the average demand for delivery vehicles by dividing the demand by the maximum load of the vehicle. But I still need to consider the maximum demand quantity, so as to define the number of delivery vehicles.

(6) How to introduce time elements to describe travel/duty/business time-related restrictions

According to the case description, I need to consider three time limits: driving time, duty time, and business time. Since the assumption of cross-day delivery in the case must be satisfied, I assume that when the goods are being delivered for the last store on the first day, I take a direct rest in place and then continue to the first store on the next day. At the same time, the remaining driving time and duty time of the first day being given to the upper limit of driving time and duty time of the next day.

2.2 Solutions

(1) Data processing

First, I select the store that needs to be delivered every day and sort them according to the order time and zip serial number from small to large, then I establish the path matrix with store number as row and column. Taking the store corresponding to the column coordinate as the starting point and the store corresponding to the row coordinate as the destination, a series of (0,1) variables am set in it to indicate whether to pass through this path, and a table represents the driving path of a car.

Since each vehicle runs continuously for up to two days, the possible route of a vehicle is a matrix of 2×2(starting point number × destination number). Because the vehicle starting the next day can not be filled with the goods of the first day, there will be no variables in the cells of the third quadrant in the lower-left corner. Yellow ama for Monday departure, running on Monday and Tuesday vehicles. The first day is Monday and the next day is Tuesday. Green ama for Tuesday departure, running on Tuesday and Wednesday. The first day is Tuesday and the next day is Wednesday. Similar treatment on Thursday. The orange ama indicates vehicles leaving on Friday and running only on Friday. The division of the day of the week makes it easier to distinguish the order of which day a car travels in a certain path and makes it easier to limit the time of the two-day path.

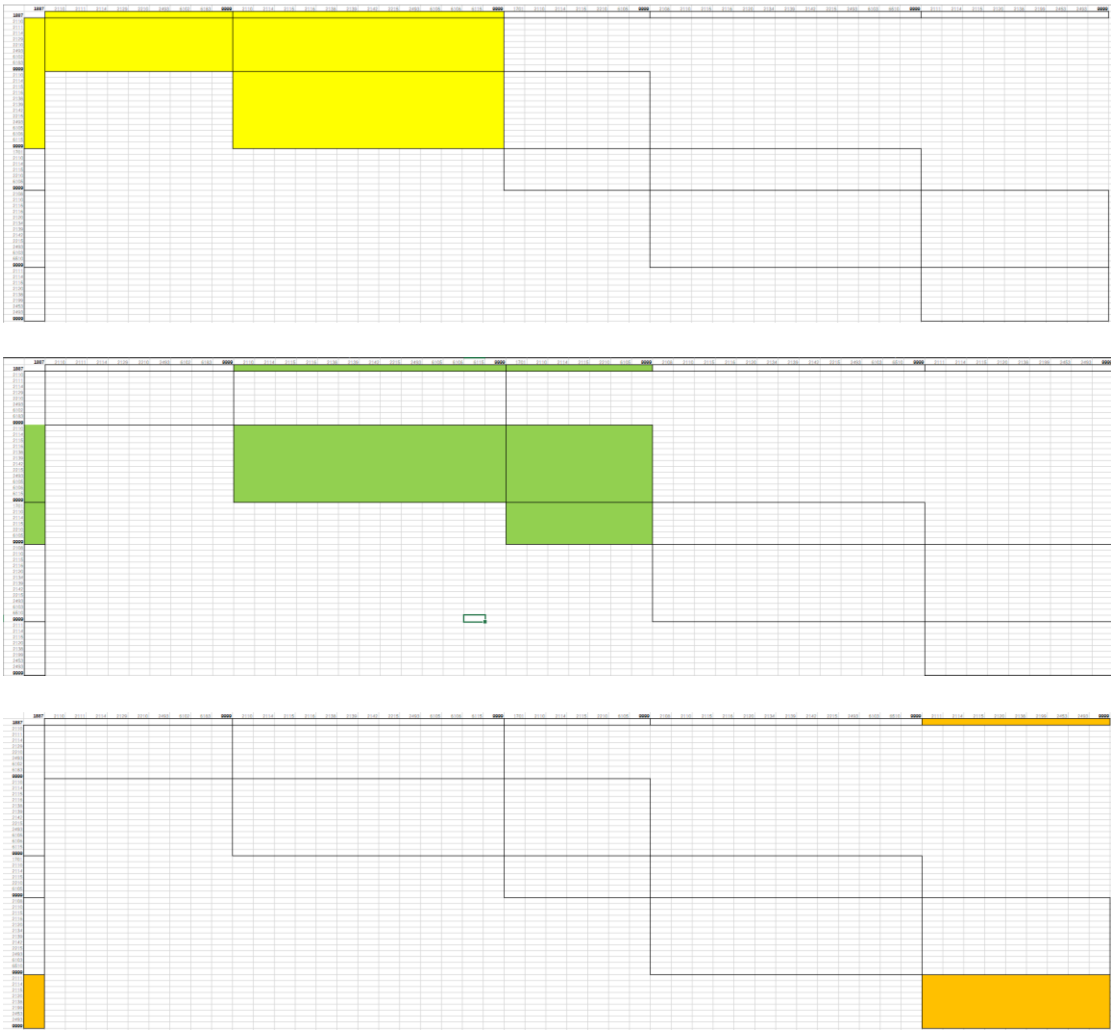


Figure 2. Data Processing in Excel

(2) Model implementation

The final driving distance of all vehicles is obtained by multiplying and adding all the vehicle path matrices and distance matrix by the SUMPRODUCT() function in the Excel software, which is the objective function. The driving paths of all vehicles being separated according to the number of days, and then they are summarized into a matrix of total vehicle paths, and each shop with demand on the day is required to be passed so that the delivery demand can be met.

Set up enough path matrix to indicate that there am enough cars. If you give priority to driving used vehicles, the vehicles that have not been used for five days am redundant, and the rest am necessary vehicles.

3. FORMULATION OF VRP MODEL

3.1 Denotation

Table 1. Denotation

	Description	Domain
v_i	The i-th vertex, where v_0 is the starting point, and is sorted from small to large according to the order time and zip sequence number. Stores that need orders for many days can appear repeatedly	$i \in \{0,1,...,N_5\}$
d_{ij}	The distance between v_i and v_j $d_{ii} = Big\ M = 10000$, $d_{ij} = d_{ji}$	$i, j \in \{0,1,...,N_5\}$
q_{ij}	The amount of goods required by the store at the destination v_j for the day	$i, j \in \{0,1,...,N_5\}$
K	Maximum number of distribution vehicles available daily (adequate vehicle supply)	
N_m	The total number of stores that need to be delivered in the first m days within a week is $N_m - 1$ (the stores that require multiple orders am considered multiple times), and the N_m -th point is the virtual address inserted manually, $N_0 = 0$	$m \in \{1,...,5\}$
x_{ijk}^m	Whether the k -th vehicle that departs on the m -th day from Monday to Friday passes through arc $< v_i, v_j >$ after departure	$i, j \in \{0,1,...,N_5\}$ $k \in \{1,...,K\}$
I^m	The collection of coordinate i of the store to be delivered on the m -th day from Monday to Friday E.g, $I^2 = \{N_1+1, N_1+2,...,N_2\}$	$m \in \{1,...,5\}$

3.2 Formulation of Vehicle Routing Problem Model

(1) How to establish objective function

Objective function: total path minimization for all vehicles

$$\min F(x) = \sum_{k=1}^{K}\sum_{m=1}^{5}\sum_{i=0}^{N_m}\sum_{j=0}^{N_m} d_{ij} x_{ijk}^{m} \tag{1}$$

(2) How to ensure continuous vehicle travel

① Vehicle exit and return restrictions: each vehicle needs to start from the starting point once and return to the starting point once

$$\begin{aligned} &\sum_{i=N_{m-1}+1}^{N_m} x_{iok}^{m} = 1, \forall k \in \{1,2,...,K\} \\ &\sum_{j=N_{m-1}+1}^{N_{m+1}} x_{ojk}^{m} = 1, \forall k \in \{1,2,...,K\} \end{aligned} \tag{2}$$

② Continuous vehicle routing restriction: the number of times any store serves as a destination equals the number of times it serves as a starting point

$$\sum_{j=N_{m-1}+1}^{N_m} x_{ejk}^{m} = \sum_{i=N_{m-1}+1}^{N_m} x_{iek}^{m}, \forall e \in I^m, k \in \{1,2,...,K\} \tag{3}$$

③ About the itinerary plan supplement: set up the virtual shop, as the place where the surplus vehicles go

Since I don't know the number of vehicles required by the optimal solution, I can only set K to the largest value as possible in the solution process, but I need to know which cars need to be used and which cars am redundant. Therefore, I have added a virtual store to the list of stores I^m that need to deliver goods every day.

$$\begin{aligned} &d_{oN_m} = d_{N_m o} = 0 \\ &d_{N_m j} = Big\ M = 10000 \\ &d_{iN_m} = Big\ M = 10000 \\ &\forall i \in \{1,2,...,N_5\},\ \ j \in \{1,2,...,N_5\} \end{aligned} \tag{4}$$

The distance between the virtual store and the original place is set to zero, and the distance between the virtual store and other stores is set to big M. these two restrictions together make the redundant vehicles must go to the virtual store. In other words, when there is at least one redundant vehicle departure (to the virtual store), the remaining vehicles belong to the vehicles necessary to achieve the optimal solution.

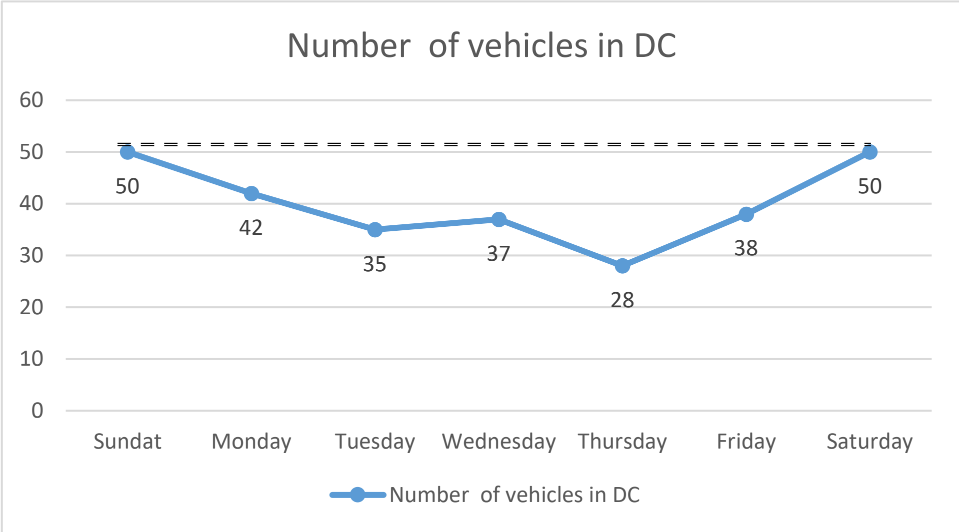


Figure 3. Number of Vehicles in DC

The line in the graph represents the maximum change in the number of vehicles in a week, which is also the number of vehicles required for the optimal solution.

(3) How to meet the delivery needs of all stores

All the vehicle schedule matrices am summed up to draw a vehicle weekly total travel matrix, which needs to meet the following requirements: each store has been used as the starting point once, and each store has been used as the target point once

$$\sum_{k=1}^{K}\sum_{i=0}^{N_5}\sum_{m=1}^{5} x_{ijk}^{m} = 1, \ j \in \{1, 2, \ldots, N_5\}$$
$$\sum_{k=1}^{K}\sum_{j=0}^{N_5}\sum_{m=1}^{5} x_{ijk}^{m} = 1, i \in \{1, 2, \ldots, N_5\} \quad (5)$$

(4) How to describe the capacity limits of freight vehicles

The total amount of goods required by all stores passing by each vehicle is less than the cargo capacity

$$\sum_{i=0}^{N_5}\sum_{j=0}^{N_5} q_{ij} x_{ijk}^{m} \leq 3200, \forall m \in \{1,2,3,4,5\}, k \in \{1,2\ldots, K\} \quad (6)$$

(5) How to add time element to the restriction

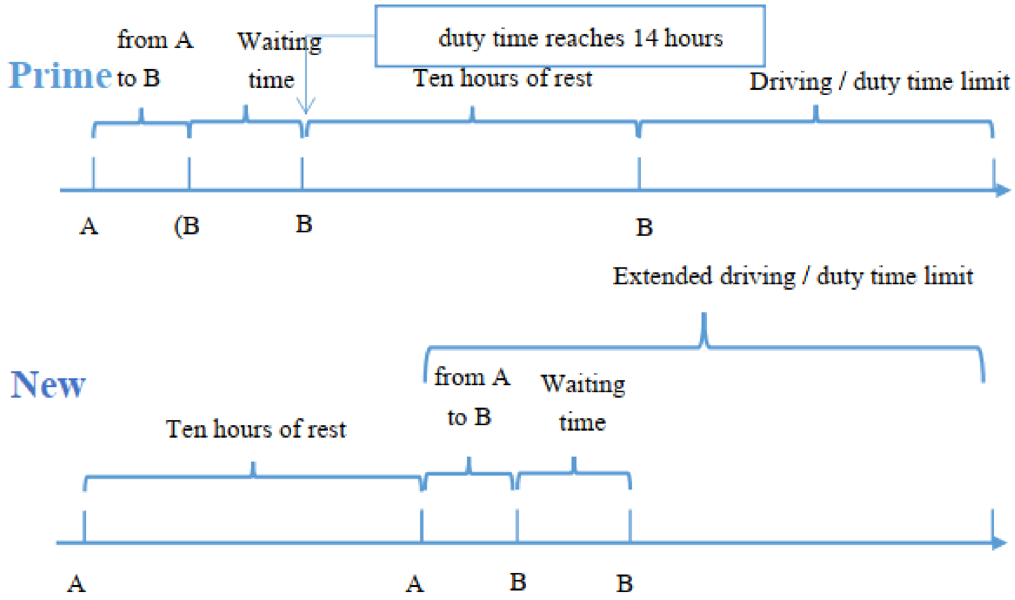
Any vehicle needs to meet three time constraints:

② The single driving time is less than 11 hours

② The single duty time is less than 14 hours

③ Unloading time belongs to store business hours

I separate the first day and the second day after departure (except for vehicles leaving on Friday); for vehicles that need to deliver goods for two days, assume that the first day is the last store (set as point a), when the goods am delivered, they am directly in place for a 10 hour rest, and then continue to drive to the first store on the second day (set as point B), at the same time, the remaining driving time and duty time of the first day am assigned to the upper limit of the corresponding time of the second day.



The size of the added value of the upper limit depends on the remaining driving time, the remaining duty time and the distance from the store A to the store B, which I will discuss in detail in the later part. Next I describe the time limit for the first day:

① First day driving time limit: driving time from DC to a < = 11h

$$drt_1 = \sum_{i=0}^{N_m} \sum_{j=0}^{N_m} d_{ij} x_{ijk}^m / 40 \le 11$$

$$\forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4, 5\} \qquad (7)$$

② Duty time limit of the first day: duty time = from DC to a (travel time + unloading time) < = 14h

Unloading time less than half an hour shall be calculated as half an hour

$$dut_1 = drt_1 + \sum_{i=0}^{N_m} \sum_{j=0}^{N_m} x_{ijk}^m Max\{0.0005q_{ij}, 0.5\}) \le 14$$

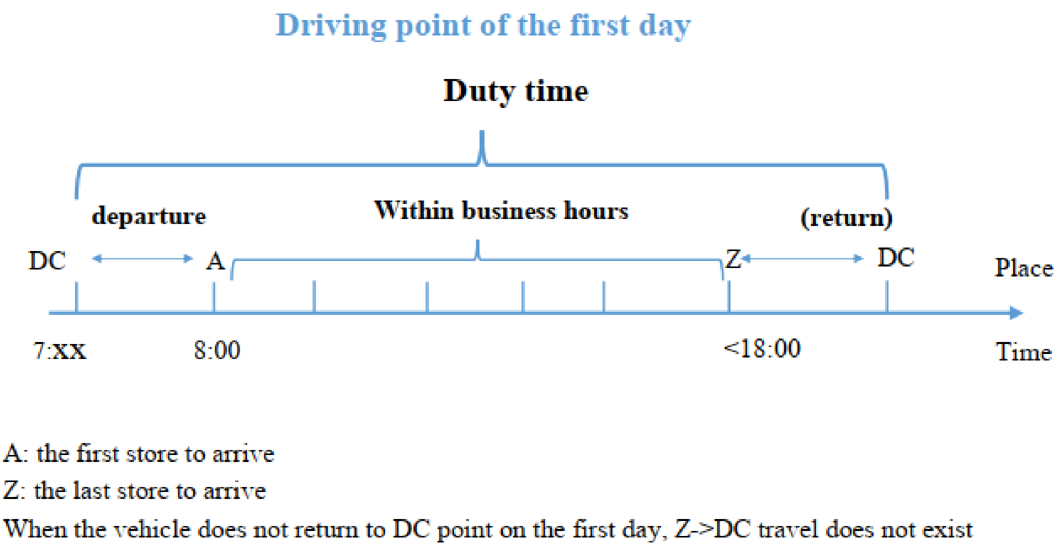
$$\forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4, 5\} \qquad (8)$$

③ The first day shop business hours limit: duty time - journey time from origin - journey time to return to origin < = 10

Unloading starts at 8 a.m. for the first store, and delivery to the last store must be completed by 6 ：00p.m.

$$dut_1 - \sum_{j=0}^{N_m} d_{oj} x_{ojk}^m / 40 - \sum_{i=0}^{N_m} d_{io} x_{iok}^m / 40 \le 18:00 - 8:00 = 10$$

$$\forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4, 5\} \qquad (9)$$



For the next day's time limit:

For convenience, use the following letter to indicate the relevant time

$$S_{drt} = surplus\ driving\ time\ on\ the\ 1th\ day = 11 - drt_1$$

$$S_{dut} = surplus\ duty\ time\ on\ the\ 1th\ day = 14 - dut_1 \qquad (10)$$

$$S_{dist} = Time\ from\ a\ to\ B = \sum_{i=N_m+1}^{N_{m+1}} \sum_{j=N_{m-1}+1}^{N_m} d_{ij} x_{ijk}^m / 40$$

$$\forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4\}$$

① Driving time limit for the following day:

$$drt_2 = drt_{1\text{-}2} - drt_1 = \le 11 + M, \forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4\} \qquad (11)$$

$$drt_{1\text{-}2} = \sum_{i=0}^{N_{m+1}} \sum_{j=0}^{N_{m+1}} d_{ij} x_{ijk}^m / 40, \quad \forall k \in \{1, 2, ..., K\}, m \in \{1, 2, 3, 4\}$$

$$M = \begin{cases} S_{dist} & S_{dist} \le S_{drt} \le S_{dut} or S_{dist} \le S_{dut} \le S_{drt} \\ S_{dut} & S_{dut} \le S_{drt} \le S_{dist} or S_{dut} \le S_{dist} \le S_{drt} \\ S_{drt} & S_{drt} \le S_{dut} \le S_{dist} or S_{drt} \le S_{dist} \le S_{dut} \end{cases}$$

Case 1: after arriving at the store B, start waiting until the duty time reaches the upper limit

Case 2: before arriving at the store B, the duty time is the first to reach the upper limit

Case 3: Before reaching the store B, the driving time is the first to reach the upper limit

② Duty time limit for the next day:

$$dut_2 = dut_{1\text{-}2} - dut_1 \le 14 + N, \forall k \in \{1,2,...,K\}, m \in \{1,2,3,4\} \tag{12}$$

$$dut_{1\text{-}2} = drt_{1\text{-}2} + \sum_{i=0}^{N_{m+1}} \sum_{j=0}^{N_{m+1}} x_{ijk}^{m} Max\{0.0005 q_{ij}, 0.5\},$$
$$\forall k \in \{1,2,...,K\}, m \in \{1,2,3,4\}$$

$$N = \begin{cases} S_{dut} & S_{dist} \le S_{drt} \le S_{dut} or S_{dist} \le S_{dut} \le S_{drt} \\ S_{dut} & S_{dut} \le S_{drt} \le S_{dist} or S_{dut} \le S_{dist} \le S_{drt} \\ S_{drt} & S_{drt} \le S_{dut} \le S_{dist} or S_{drt} \le S_{dist} \le S_{dut} \end{cases}$$

Case 1: after arriving at the store B, start waiting until the duty time reaches the upper limit

Case 2: before arriving at the store B, the duty time is the first to reach the upper limit

Case 3: Before reaching the store B, the driving time is the first to reach the upper limit

③ Restriction of shop opening on the second day

In the same way, the vehicle will begin unloading the first store at 8 a.m., using the formula of the same meaning as (9)

Among them, the next day on duty = two days on duty-the first day on duty

$$dut_2 - \sum_{i=N_{m-1}}^{N_m} \sum_{j=N_m+1}^{N_{m+1}} d_{oj} x_{ojk}^{m} / 40 - \sum_{i=N_m+1}^{N_{m+1}} d_{io} x_{iok}^{m} / 40 \le 10$$
$$\forall k \in \{1,2,...,K\}, m \in \{1,2,3,4\} \tag{13}$$

5. CONCLUSION

In this paper, based on the division of the distribution categories of NHG demand points, the large-scale multi-vehicle path problem is studied in detail. First of all, I deal with the data, and then according to the actual situation, I make reasonable assumptions on the problem and establish the optimization objectives of the problem. Finally, I abstract the relevant constraints of the problem, so as to establish a large-scale multi-vehicle path planning model, and introduce the linear programming of large-scale data to solve the model..

6. REFERENCES

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