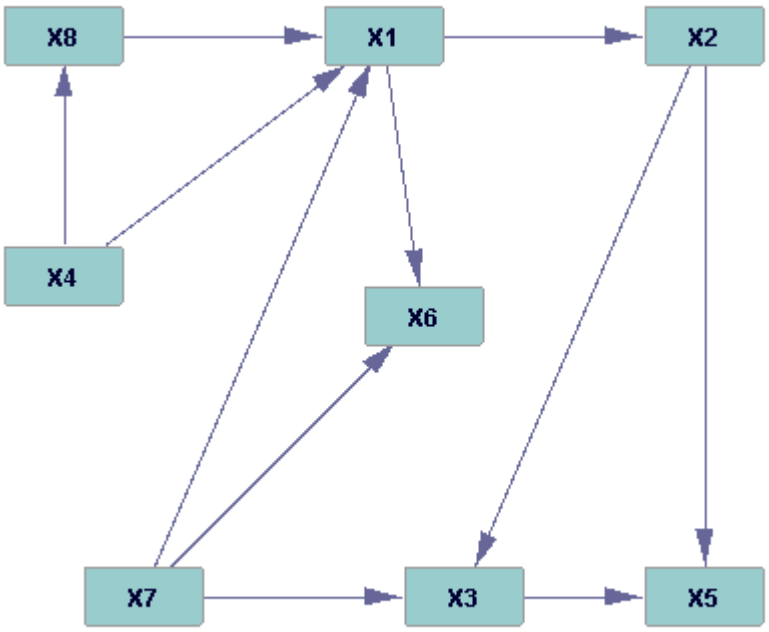
Problem Set 2

Markov Decomposition,
d-separation under Observation and Intervention



Easy Problems

1) List all active paths between X4 and X2.
2) Are X4 and X2 d-separated?
3) Write down the factorized joint distribution of the graph.
4) Are X8 and X6 d-separated conditional on X1?
5) Are there any two variables that are unconditionally d-separated?
6) How many actives paths are there between X4 and X2 if we condition on X6 and X3? List them.
7) How many actives paths are there between X4 and X2 if we condition on X6 and X5? List them.
8) How many actives paths are there between X4 and X2 if we condition on X6, X3 and X5? List them.
9) How many active paths are there between X4 and X8 if we condition on X5?
10) How many active paths are there between X4 and X8 if we condition on X7?
11) Which variables do we have to condition on to d-separate X7 and X2?
12) Which (non-trivial) set of variables do we have to condition on to d-separate the sets {X4,X7} and {X2, X5}?

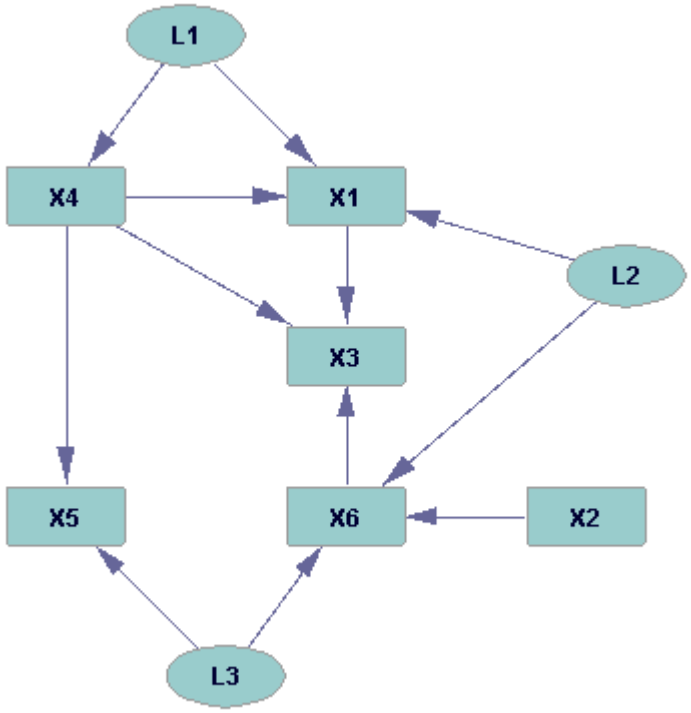
Medium Problems

13) Suppose all variables are binary and suppose we did not know the graphical structure. How many values would we need to specify for the joint distribution over the variables?
14) Suppose the variables are binary and we know the graphical structure. How many values do we need to specify for the joint distribution?
15) Is there any edge in the graph whose direction you could flip without changing the implied independence relations?
16) Draw the post-manipulation graph if you intervene on X4 and X7.
17) Draw the post-manipulation graph if you intervene on X1.
18) If you intervene on X1, are X7 and X8 d-connected conditional on X5? Would they be if there were no intervention on X1?
19) Given an intervention on X1, are X1 and X7 d-connected given X6?

Harder Problems

20) Suppose there were a latent common cause of X2 and X5. Would we be able to discover this latent variable just by looking at the conditional independence relations? That is, would we be able to distinguish the structure with that latent common cause from the structure without the latent common cause?
21) Are X7 and X2 d-separated conditional on X1 and X6? If so, can you d-connect them by *increasing* the size of the conditioning set by one variable? Can you d-connect them by increasing the conditioning set by an additional 3 variables? By 4?
22) You are not allowed to intervene on X7 and X2. What is the smallest set of variables you have to intervene on so that X7 and X2 are unconditionally d-separated in the post manipulation graph?
23) We call an intervention *fat-hand* if it intervenes on two variables at once in a synchronous fashion. A fat hand intervention induces a (spurious) correlation between the two variables it intervenes on. You can think of a fat hand intervention as an unmeasured common cause that breaks all other arrows into the variables it intervenes on. If there were a fat-hand intervention on X7, X2 and X3, would X5 and X1 be d-separated?

Consider the following graph. Circular variables are unmeasured latent variables, you cannot condition on them.



Easy Problems

1) Are X1 and X6 d-separated?
2) Are X1 and X5 d-separated conditional on X6?
3) What do we have to condition on to d-separate X5 and X6?

Medium Problems

4) Draw the post manipulation graph for an intervention on X6 and specify whether or not X6 is d-separated from each other observed variable.

Harder Problems

5) Why are we most worried about latent common causes? What about latent common effects and latent intermediary variables?
6) Conditional on X6, are X2 and X5 d-separated? Would they be d-separated if L3 did not exist?