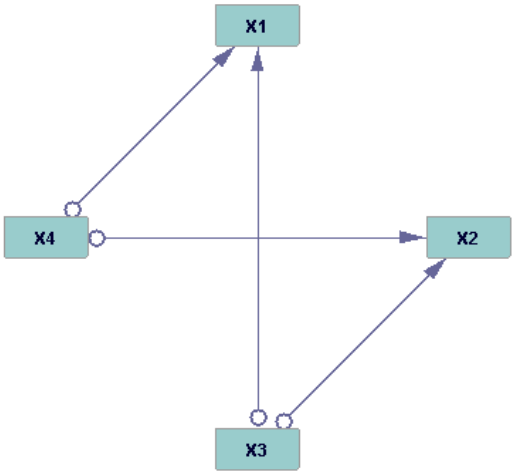
Problem Set 3

Search

Easy Problems

1) How many observational Markov equivalence classes (OMEs) are there of two variables? How many graphs are in each OME?
2) Suppose you have 3 variables (no latents) and there are no (conditional) independence constraints. What size is the observational Markov equivalence class?
3) Suppose you have 3 variables (no latents) and there are no independence constraints. You decide to perform an intervention on X1. What is the worst case size of the post manipulation Markov equivalence class?
4) Suppose you have 3 variables and there are no independence constraints. However, there may be latent variables. What size is the observational Markov equivalence class now?
5) What is the practical advantage of searching by considering conditional independencies in the order of the size of the conditioning set (i.e. smallest conditioning set first)?

Consider the below graph that was generated using a search algorithm (FCI) that allows for the presence of latent variables.



6) Briefly describe what the search output represents.
7) What is the size of the OME for this search output?
8) Suppose we now found out that there are no latent variables, how would the search output have to be represented? What is the size of the OME then?

Medium Problems

9) Given your answer to question 2, how would we have to perform interventions in order to recover the true graph uniquely?
10) Suppose you have 3 variables, no latent variables. You know that only the following independence constraint holds:
    a. X1 _||_ X3

    Can you recover the causal graph uniquely?
11) Suppose you have 3 variables, no latent variables. You know that only the following independencies hold:
    a. X1 _||_ X3
    b. X1 _||_ X2
    c. X1 _||_ X3 | X2
    d. X1 _||_ X2 | X3

    Can you recover the causal graph uniquely?
12) In the previous question, do we need to know the independence constraints c and d? Why or why not?
13) Suppose you have 4 variables, no latent variables. You know that only the following independencies hold:
    a. X1 _||_ X2
    b. X2 _||_ X4 | {X3, X1}

    Can you recover the causal graph uniquely?
14) Suppose you have 4 variables, no latent variables. You know that only the following independencies hold:
    a. X1 _||_ X4 | {X2, X3}
    b. X2 _||_ X3 | {X1, X4}

    Draw all the causal graphs that satisfy ALL these and ONLY these independence constraints. What conclusions do you draw from the result?
15) Suppose you have 3 variables, no latent variables. You know that no *unconditional* independencies hold. Would knowledge of a conditional independence constraint help you reduce the equivalence class of graphs satisfying this (lack of) independence constraints? If so, which constraint would you test? If not, why not?

Harder Questions

16) Suppose you have 5 variables, no latent variables. You know that the following independencies hold:
    a. X1 _||_ X2
    b. {X1,X2} _||_ {X4, X5} | X3

    Draw all the causal graphs that satisfy these independence constraints. What size is the Markov equivalence class? Would knowledge of additional independencies help in reducing the size of the equivalence class?
17) Suppose you have 4 variables, no latent variables. You find that only the following independence relations hold:
    a. X1 _||_ X4
    b. X2 _||_ X4
    c. X1 _||_ {X3, X4} | X2

    Describe the observational Markov equivalence class of graphs.
18) Suppose you have 4 variables. You know that ONLY the following independencies hold:
    a. X1 _||_ {X3, X4}
    b. X2 _||_ X4

    Draw all the causal graphs that satisfy these and only these independence constraints. What is going on?
19) Given your answer to question 9, what is the worst case number of interventions you have to perform in order to uniquely recover the true causal graph among 3 variables if there are no independence constraints? What about the best case (again you know there are no independence constraints)?
20) Suppose you have 3 variables, no latent variables. You intervene on X1 and find the following independence relations:
    a. X1 _||_ X2
    b. X1 _||_ X3
    c. X3 _||_ X2
    d. X1 _||_ X2 | X3
    e. X1 _||_ X3 | X2
    f. X3 _||_ X2 | X1

    What is your post manipulation Markov equivalence class of graphs?
21) Suppose we have 5 variables. How many possible independence tests are there? Can you generalize your result to N variables?
22) Give an example of a causal structure with 5 variables that contains at least one edge where we can stop early and do not have to consider all possible independence tests.
23) Give an example of an observational Markov equivalence class over 5 variables that contains at least 6 graphs where we can stop early and do not have to consider all possible independence tests.