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Informatics II
Exercise 3

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Algorithmic Complexity and Correctness

Task 1. Algorithm `whatDoesItDo(A,k)` gets an array `A[1...n]` of n integers as an input.

```
Algo: WHATDOESITDO(A,k)
sum = 0;                              why k+1 last check once i=k
for $i = 1$ to $k$ do
    mini = i;
    for $j = i + 1$ to $length(A)$ do
        if $A[j] < A[mini]$ then
            mini = j;
    sum = sum + A[mini];
    swp = A[i];
    A[i] = A[mini];
    A[mini] = swp;
return sum
```

# times exec.	Cost
1	c 1
k + 1	c 2
k	c 3
n - (k+1) + 1	c 4
	c 5
	c 6
k	c 7
k	c 8
k	c 9
k	c 10
1	c 11

a) Implement the algorithm as a C program that reads the elements of A and prints the result.

b) Describe what the algorithm does.

c) Do an exact analysis of the running time of the algorithm.

d) Determine the best and the worst case of the algorithm. What is the running time and asymptotic complexity in each case?

e) What influence has the parameter k in the asymptotic complexity?

Asymptotic Complexity

Task 2. Calculate the asymptotic tight bound for the following functions and rank them by their order of growth (lowest first). Clearly work out the calculation steps in your solution.

$$f_1(n) = n^n + 2^{2n} + \overset{\text{constant}}{\cancel{13^{124}}} \qquad n^n + 4^n \quad \in \Theta(n^n)$$

$$f_2(n) = \log\left(14(n-1)n^{3n+2}\right) \qquad \log(14) + \log(n-1) + (3n+2)\log(n) \in \quad \Theta(n \log(n))$$

$$f_3(n) = 4^{log_2 n} \qquad (2^{\log_2 n})^2 \Rightarrow n^2$$

$$f_4(n) = 12\sqrt{n} + 10^{223} + \log 5^n \qquad 12\, n^{0.5} + 10^{233} + n \log 5 \quad \in \Theta(n)$$

$$f_5(n) = n^2 \log(n+1) + n \log n^2 + 0.5n \qquad n^2 \log(n+1) + 2n\log n + 0.5n$$

$$f_6(n) = 7n^4 + 100n \log n + \sqrt{32} + n \qquad \in O(n^4)$$

$$f_7(n) = \log\left(\min\left(n, \sqrt{n}\right)\right) \qquad \log(n^{0.5}) \in \Theta\, 0.5 \log(n)$$

$$f_8(n) = \log^2(n) + 50\sqrt{n} + \log(n) \qquad \in \Theta(n^{0.5})$$

$$f_9(n) = (n+3)! \qquad \in \Theta(n+3)!$$

$$f_{10}(n) = 2\log(6^{\log n^2}) + \log(\pi n^2) + n^3$$

$$2 \log n^2 \cdot 2\log(6)$$

$$4\log(n) \cdot \log(6) + \log(\pi) + 2\log n + n^3 \qquad \in n^3$$

Special Case Analysis

$$4\log(n)$$

Task 3. Given two strings A and B, develop an algorithm that checks if B is a substring of A and, if so, returns the number of occurrences of B in A.

a) Specify all the special cases that need to be considered and provide examples of the input data for each of them.

b) Write a C program implementing your algorithm and make sure it runs for all the special cases you provided. Include a function `int substrings(char A[], char B[])` which returns the number of occurrences of B in A. Your program should print the number of occurrences along with the starting and ending index of each occurrence. Here is an example output corresponding to the call `substrings([nitrite], [it])`:

```
A = [nitrite]
B = [it]
Indices = (2,3) (5,6)
Repetitions = 2
```

Attention! You are not allowed to use string-functions and/or `string.h`.