Comparative analysis of Alamouti and Dominant Eigenmode Transmission schemes using OFDM in MIMO and MISO systems.

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***Abstract*—In this article, we present a comparison between an Alamouti scheme and a dominant eigenmode transmission scheme for Multiple-Input Multiple-Output (MIMO) and Multiple-Input Single-Output (MISO) systems. The Alamouti scheme is done using two Orthogonal Frequency Division Multiplexing (OFDM) methods: the space-frequency and space-time code division. The simulations are done using a scenario which is based on Ray tracing coming from the Sionna package from Python. Bit Error Rate (BER) - Signal-to-noise ratio (SNR) curves are showed to do clear comparisons between obtained results. This will reveal the strengths and weaknesses of each techniques.**

I. Introduction

In wireless communication, multi-antennas at the emission and reception (MIMO) and multiple input single output antennas (MISO) channels are often studied. Alamouti showed in 1998 [1] new kinds of scheme. The first one use 1 antennas at the receiver (RX) and 2 antennas at the transmission (TX) are used and the second one 2 at TX and 2 at RX. He did a comparison on the BER-SNR curves of the Maximal-Ratio Receive Combining (MRRC) scheme and the Alamouti scheme. In this article, we decided to do a comparison between the Alamouti scheme using Orthogonal Frequency-Division Multiplexing (OFDM, space-time and frequency code division) and the dominant eigenmode transmission scheme. This comparison is made for the MISO and MIMO systems. We will firstly investigate the Alamouti scheme (MIMO compared to the MISO and AWGN results), the analytical development and both OFDM implementations. Then, the dominant eigenmode transmission will be developed before showing the results and a discussion about them.

Those investigations were simulated inside a given scenario thanks to Ray Tracing. The parameters will be more detailed in the section IV.

II. OFDM - Alamouti

A. Basic Alamouti - Analytical development

1) MIMO Case:

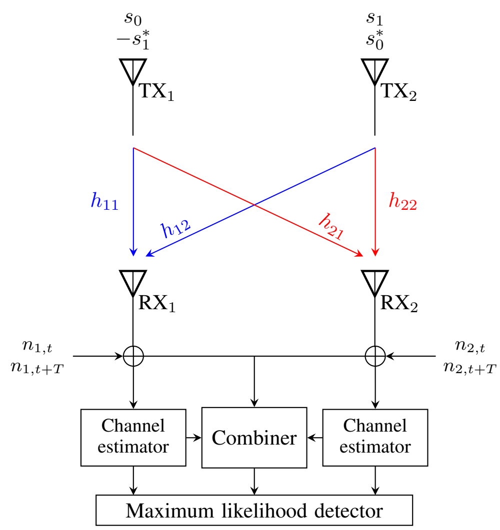

Fig. 1: 2×2 Alamouti scheme.

In this part, we will assume that the number of receiving antennas is the same as the one of the transmitting ones which is two. In the case of an Alamouti scheme, the channel knowledge is not required at the transmission. The channel shown in the Fig. 1 between each receiving and transmitting antennas can be expressed as

$$h_{nm} = \alpha_{nm}\, \mathrm{e}^{j\theta_{nm}}, \tag{1}$$

where θ_{nm} is the phase-shift between antennas RX n and TX m and α_{nm} is an amplitude coefficient. The channel is supposed to be constant between two consecutive symbols.

The received baseband signals in the case of the Alamouti 2×2 antennas are

$$\begin{cases} r_{1,t} = h_{11}s_0 + h_{12}s_1 + n_{1,t} \\ r_{1,t+T} = h_{11}(-s_1^*) + h_{12}s_0^* + n_{1,t+T} \\ r_{2,t} = h_{12}s_0 + h_{22}s_1 + n_{2,t} \\ r_{2,t+T} = h_{22}s_0^* + h_{21}(-s_1^*) + n_{2,t+T} \end{cases}, \tag{2}$$

where s_i are the sent messages and n_i are the noise. The n_i are assumed to be complex Gaussian variables.

The signals at the ouput of the combining block are

$$\tilde{s}_0 = h_{11}^* r_{1,t} + h_{12} r_{1,t+T}^* + h_{21}^* r_{2,t} + h_{22} r_{2,t+T}^*, \quad (3)$$
$$\tilde{s}_1 = h_{12}^* r_{1,t} - h_{11} r_{1,t+T}^* + h_{22}^* r_{2,t} - h_{21} r_{2,t+T}^*. \quad (4)$$

Using (2) in (3) and (4), the estimated signals can therefore be written as

$$\begin{aligned}\tilde{s}_0 &= (\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_{22}^2) s_0 + h_{11}^* n_{1,t} \\ &\quad + h_{12} n_{1,t+T}^* + h_{21}^* n_{2,t} + h_{22} n_{2,t+T}^*, \quad (5)\\ \tilde{s}_1 &= (\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_3^2) s_1 - h_{11} n_{1,t+T}^* \\ &\quad + h_{12}^* n_{1,t} - h_{21} n_{2,t+T}^* + h_{22}^* n_{2,t}. \quad (6)\end{aligned}$$

The maximum likelihood detector will then take a decision on these to get the estimated signal $\hat{s}_0$ and $\hat{s}_1$. In the case of a 2×2 Alamouti scheme, the provided diversity gain is $g_a = n_r = 4$ and there is no array gain.

2) MISO Case:

In the case of a 2×1 Alamouti scheme, which means that RX_2 is therefore not existing anymore, (2) becomes

$$\begin{cases} r_{1,t} = h_{11} s_0 + h_{12} s_1 + n_{1,t} \\ r_{1,t+T} = h_{11}(-s_1^*) + h_{12} s_0^* + n_{1,t+T} \end{cases}. \quad (7)$$

The signal at the output of the combining block are now

$$\tilde{s}_0 = h_{11}^* r_{1,t} + h_{12} r_{1,t+T}^*, \quad (8)$$
$$\tilde{s}_1 = h_{12}^* r_{1,t} - h_{11} r_{1,t+T}^*. \quad (9)$$

The estimated signals from (5) and (6) are now defined as

$$\tilde{s}_0 = (\alpha_{11}^2 + \alpha_{12}^2) s_0 + h_{11}^* n_{1,t} + h_{12} n_{1,t+T}^*, \quad (10)$$
$$\tilde{s}_1 = (\alpha_{11}^2 + \alpha_{12}^2) s_1 - h_{11} n_{1,t+T}^* + h_{12}^* n_{1,t}. \quad (11)$$

The provided received gain is g_a now equal to 1 and there is still no array gain.

B. OFDM - Space-frequency code division

In OFDM, symbols are transmitted using multiple subcarriers at a different modulated frequency. The duration of the OFDM symbol has a direct impact on the data rate.

To apply an Alamouti coding scheme to OFDM, two options are possible. The first one is presented in the Fig. 1, two different symbols s_0 and s_1 are sent through two different antennas TX_1 and TX_2. Those symbols are for the first tone. Once this is sent, a second tone $-s_1^*$ and s_0^* are sent respectively by TX_1 and TX_2. To not have interferences, a cyclic prefix needs to be used to ensure the orthogonality between the tones. We will assume here that the channel response between two tones is the identical, which means that the coherence bandwidth is bigger than the spacing between 2 OFDM frequencies. The other assumption is that the tones are independent. Furthermore, the number of sent symbols needs to be even for the first assumption to be sufficient. Indeed, if this amount was odd, the last 2 symbols would be divided between 2 different OFDM symbols. The additional assumption needed for the scheme to still perform would be that the coherence time should be then bigger than 2 OFDM symbols.

TABLE I: Ressource grid.

	TX_1	TX_2
f_1	s_0	s_1
f_2	$-s_1^*$	s_0^*
f_3	s_2	s_3
f_4	$-s_3^*$	s_2^*
$\vdots$	$\vdots$	$\vdots$
$f_{N_{SC}}$	$-s_{N_{SC}+1}^*$	$s_{N_{SC}}^*$

The Table I is showing the manner in which the OFDM symbols are going to be sent through both transmission antennas TX_1 and TX_2.

To decode the transmitted data, the same formulas from section II-A are reused. If the different assumptions are still valid, the bits should be recovered correctly. Only the receivers have to estimate the channel and to this purpose, pilots symbols could be added to the signal, either in frequency or time or a combination of both. In this paper, the pilot scheme is presented in figure 4.

C. OFDM - Space-time code division

The second possibility for the Alamouti code in OFDM is to use space-time code (STC) division [2]. As observed in the Fig. 2, copies of the transmitted data are sent through multiple antennas (in the Fig. 2 TX_1 and TX_2) when no coding is applied. The data streams from both antennas are transmitted with a symbol delay between them. The Alamouti coding scheme is applied in the OFDM symbols instead of data symbols, as Fig. 3 depicts. The first assumption that we made in this part is that the number of OFDM symbols is even since Alamouti is working on two blocks at a time. The second assumption is that the coherence time needs to be higher than 2 OFDM symbols for the developments in section II-A to still be correct. If the pilot OFDM symbols (Fig. 4) are placed in an odd manner, the coherence time would need to be higher than 3 OFDM symbols, as the pilots symbols would divide the 2 Alamouti data symbols.

In the case of OFDM STC division, the code is applied in time and space domain to the data symbols after the OFDM modulation.

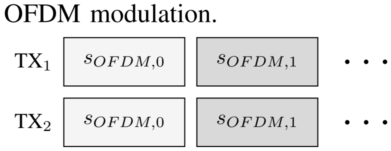

Fig. 2: Basic concept of the transmission of multiple symbols.

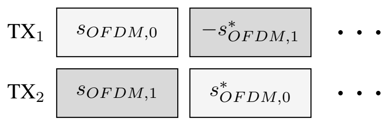

Fig. 3: OFDM STC division technique.

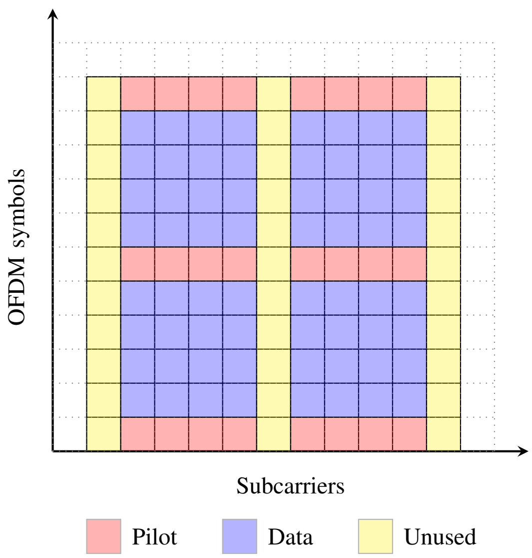

Fig. 4: Repartition of the subcarriers in an OFDM symbol.

The decoding of the scheme is applied at the receiver after having estimated the channel by using the pilot symbols. The different pilots could be sent using a scheme similar to Fig. 4.

In the simulation part of this article, perfect channel state information (CSI) was supposed and thus not pilots were sent.

III. Dominant eigenmode transmission

The second scheme that is studied is the dominant eigenmode transmission (DET). A main difference with the Alamouti scheme is that the channel is that we have a perfect transmit channel knowledge (CSIT). This knowledge at the transmission can come from channel estimation techniques or feedback mechanisms. This channel will be used to precode the information at the receiver and decode the received signal at the receiver. In the case of the dominant eigenmode transmission [3], the channel matrix can be expressed as

$$\mathbf{H} = \begin{bmatrix} e^{j\theta_{11}} & e^{j\theta_{12}} \\ e^{j\theta_{21}} & e^{j\theta_{22}} \end{bmatrix}, \tag{12}$$

where θ_{nm} are the phase-shifts.

After decomposing the channel matrix using the Singular Value Decomposition (SVD), the channel can be represented as

$$\mathbf{H} = \mathbf{U_H}\Sigma_\mathbf{H}\mathbf{V}_\mathbf{H}^H. \tag{13}$$

The right singular vector ($\mathbf{V}_\mathbf{H}^H$) will be used to precode the information bits. Indeed the sent signal is represented as

$$\mathbf{x} = \mathbf{v}_{max}s, \tag{14}$$

where $\mathbf{v}_{max}$ is the right singular vector associated to the maximum singular value.

After going through the channel, the received signals are

$$\mathbf{r} = \mathbf{Hx} = \mathbf{U_H}\Sigma_\mathbf{H}\mathbf{V}_\mathbf{H}^H\mathbf{v}_{max}s. \tag{15}$$

The signal after the combining block can be written as

$$z = \mathbf{u}_{max}^H\mathbf{r} = \sigma_{max}s, \tag{16}$$

where σ_{max} is the maximum singular value of $\mathbf{H}$. The diversity gain in this case is $g_d = n_r n_t = 4$ and the array gain, since $n_t = n_r = n$ is $g_a = 4n = 8$ for MIMO.

TABLE II: Parameters used for Simulation.

General parameters	Values
Constellation	QPSK
num. of streams per TX	1
num. of TX	[1,2]
num. of streams per RX	1
num. of RX	[1,2]
num. of OFDM symbols	2
OFDM pilots[1]	-
Size of the FFT	76
Spacing between subcarriers	15 kHz

IV. Simulation

To achieve the simulation part, the Sionna package [4] from python is used. The different parameters for the scenario will be described below. Then the results will be shown and discussed.

A. Parameters

The general values used for the simulations are resumed in the Table II below.

For the scenario, the used computation for the path is a *fibonacci* method instead of an *exhaustive* method. The *exhaustive* will try all possibilities whereas the *fibonacci* method uses a shoot-and-bounce approach. The reflection, diffraction and scattering are neglected in this scenario. The maximum depth which is the number of bounces is here set to 5. See [4] for more explanations about the path computations.

[1]We have for hypothesis that the channel knowledge is perfect so no pilots are sent. In practice they would be needed.

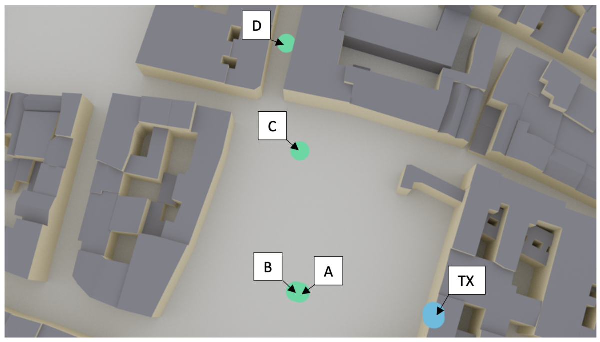

Fig. 5: Scenario - Frauenkirche in Munich.

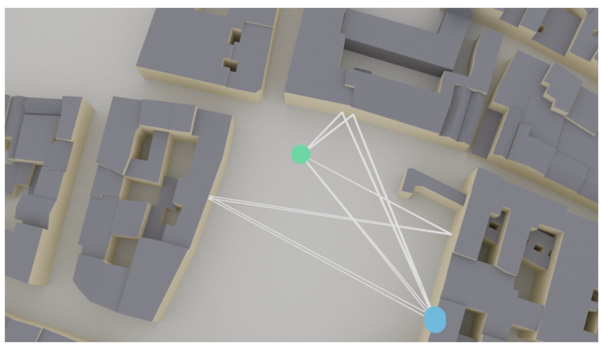

Fig. 6: Scenario - Ray Tracing at C.

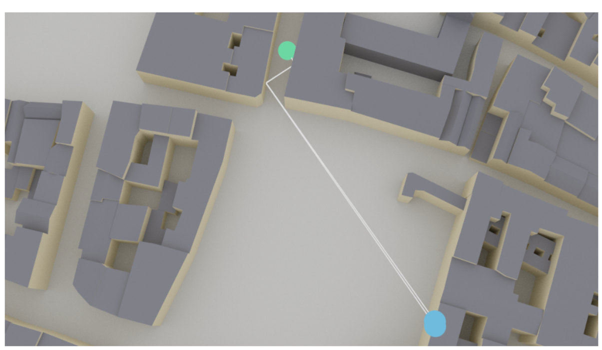

Fig. 7: Scenario - Ray Tracing at D.

B. Scenario

The subcarriers spacing (Δ_f) is equal to $15\,\mathrm{kHz}$. This gives an OFDM symbol duration (T_{OFDM}) of $6.667\,\mu\mathrm{s}$.

Figure 5 presents the studied scenario: the area around the Frauenkirche in Munich. The green points going from A to D represents the positions of the receivers, while the blue points called TX are the emitted. The number of emitter and receiver will vary with the studied case (MIMO or MISO). The transmitters are positioned on the top of a building, while each receiver has a fixed height of $1.5\,\mathrm{m}$.

Fig. 6 shows the results of the Ray Tracing computation with 1 receiver and 2 emitters. The depth[2] is fixed to 5 and a non-exhaustive[3] method of calculation is used. When the user (UE) is located at position D, the channel in the Fig. 7 will change compare to the previous one.

For each test, 2 OFDM symbols will be sent, one will be affected by the first channel (i.e. at position A) and the second one by the next second channel (i.e. at position B). This will simulate the move of a UE from the first point to the second. The UE will thus observe of a variation of channel due to this displacement. The speed needed to achieve such displacements are far than being reachable, but this allows to find difficult scenarios for the system and find limits of it.

Three different scenarios are presented. The first one is going from A to B. A small variation of the channel between the points is considered and the performances should be high with each coding. The second one is a displacement from A to C. In this case, the channel variation is important but as the place is not composed of many elements, the line-of-sight (LoS) should still be the highest path and contribute the most to the channel. Finally, the third displacement is from C to D. In this last case, the LoS disappear for position D. This should lead to a decrease in the performances for the space-time code division as the hypothesis of a constant channel is not verified any more.

C. Results

1) Small channel variation (A-B): With the small variation of the channel present in this test, the expected results are obtained. Indeed, no difference between the three MIMO[4] cases is observed in the Fig. 8.

For the MISO[5], the performance between the dominant eigenmode transmission and the first Alamouti implementation is also similar. Only the first implementation was done because the results should follow the same trend as the MIMO case.

The diversity gain can be observed in the figures 8-9-10 by the slope of the curves. In all the presented coding schemes, this gain is hard to approximated at low BER. The array gain on the other hand is represented by the shift of the curves.

In this situation, the distance between positions A and B are shown in the Table III while the duration of the one OFDM symbol is only $6.667\,\mu\mathrm{s}$. The speed of the user is then also computed in the Table III if there is no null packet (NP) time. A more realistic situation would

[2]The depths is the maximal number of bounce allowed to reach the receiver from the transmitter. For an easier observation of the paths in the figure, a depth of 2 was used

[3]This method is used to reduce the number of paths needed to be computed and is done because of limited computation power

[4]The MIMO case will refer to the 2TX-2RX situation

[5]The MISO case will refer to the 2TX (transmitters)-1RX (receiver) situation

TABLE III: Distances and speeds computed.

Situation	Distance [m]	Speed [m/s]
$A \to B$	2.06	15 461
$A \to C$	70.00	525 000
$C \to D$	57.42	430 711

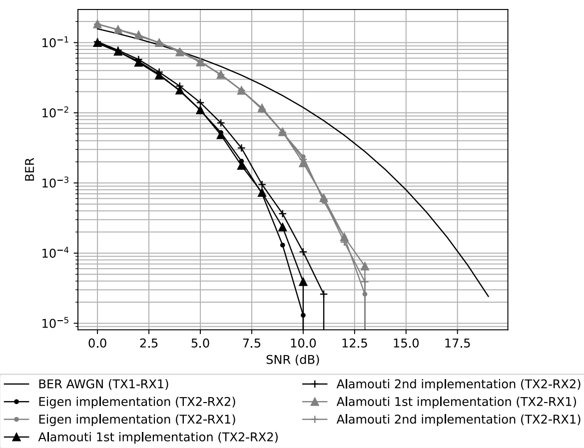

Fig. 8: BER - Scenario $A \to B$.

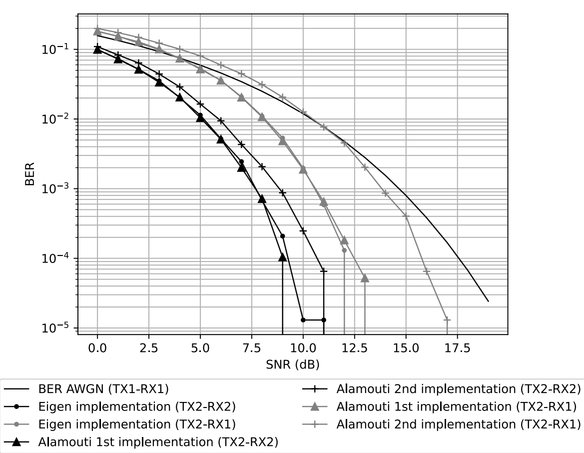

Fig. 9: BER - Scenario $A \to C$.

be one where the emitter doesn't send any information for a certain time.

2) Medium channel variation (A-C): In this scenario, the LoS component is present for both channels (A and C) but the channel will still change between the two positions.

The performances of the systems are degraded with the increase of the distance between the 2 positions for the second Alamouti as Fig. 9 shows. This is expected as the channel varies more and the hypothesis of a constant channel over multiple OFDM symbols is not respected anymore. This phenomenon is observed in the MIMO and MISO situation. For the other coding implementations, the performances stay the same and present the same results as Section IV-C1.

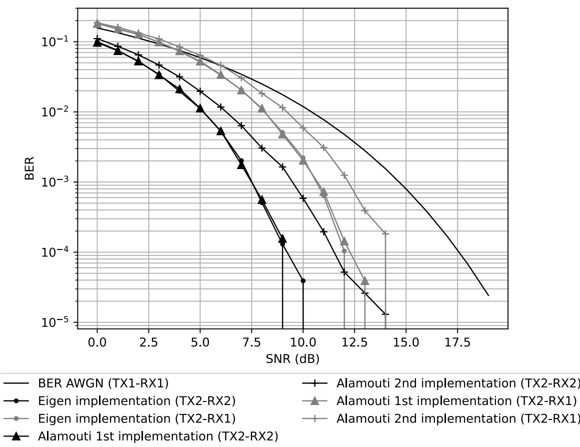

Fig. 10: BER - Scenario $C \to D$.

3) High channel variation (C-D): When the UE moves from C (Fig. 6) to D (Fig. 7), the LoS component is not present anymore and the channel applied to the first and second OFDM symbols is not similar. The Fig. 10 shows that the performances of the second Alamouti implementation are also greatly decreased for both MIMO and MISO. The others coding schemes are more resilient with this strong variation of channel. In a practical case, the channel would be estimated and in such situation, the estimation would perform badly, and thus the performances would be reduced.

An interesting aspect seems to be that the second MISO Alamouti coding scheme seems to be less resilient than the MIMO one, as the performance in Section IV-C2 are worse than in the results in this scenario.

In this scenario, as Table III shows, the speed is infeasible. Such a big channel variation could be present when the user enters a building or even when the user takes a street and no line of sight is present any more.

V. Conclusion

In summary, three main coding schemes were studied. The first one is the Alamouti with space-frequency code division. The hypothesis for using this scheme is to have a constant channel over two subcarriers and an even number of those allocated to the data transmission. Thus the channel's coherence bandwidth should be bigger than two subcarriers ($30\,\mathrm{kHz}$).This type of coding has been implemented both in MIMO (TX2-RX2) and MISO (TX2-RX1). The second one is the Alamouti with space-time code division. For this coding to work properly, the channel needs to be as constant as possible over two successive OFDM data symbols. This fix the coherence time to be over two OFDM symbols

(13.32 µs). Both of those codings require a channel estimation at the receiver. This has been discussed by the addition of pilots over some OFDM symbols, but a perfect CSI knowledge has been assumed. The first coding scheme is more efficient in most of the cases as the OFDM turns a frequency selective channel into multiple frequency flat ones and two successive subcarriers should be similar. On the other hand, the second one has the same performances as the first one when the channel stays quite constant. Nonetheless, in the case of a high variation of the channel (i.e. when entering a building, changing street,...), the second coding's performance decreases greatly. The third considered scheme is the dominant eigenmode transmission. This type of coding requires a channel knowledge both at the receiver and emitter. As we considered a perfect CSI, knowing it is not a drawback but if such a coding was implemented in reality, a CSI feedback from the receiver to the emitter should be considered. This would induce a channel which does not vary much over time in order for the fedback CSI to still be accurate. This coding has been implemented both for the MIMO and MISO system. For the second one, a decrease of the performance is observed as the channel variation increases.

In practice it may be more straightforward to directly implement and decode the Alamouti code on the OFDM symbols and not on the subcarriers. This could be an advantage to use the second Alamouti coding, even though it is not the best one when there is fast variation of the channel. Moreover, the different scenarios were quite extreme and in a more favourable one, the results could approach the one of the first scenario.

References

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