Contents

1 Exercise 3.23

This exercise works on results from exercises 3.8 and 3.3. This is a summary of what is asked:

1.1 Part 1

Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

One thing I learned is that the sample space is easy to count for binary values as I can count them in binary as shown below:
$S = HHH, HHT, HTH, HTT, THH, THT, TTH, TTT$
$W = 3, 1, 1, -1, 1, -1, -1, -3$

1.2 Part 2

Find the probability distribution of the random variable W assuming that the coin is biased so that a head is twice as likely to occur as a tail.

At first, I thought I needed to find a function that represented the probability density function, here are my attempts in order:

Note: To make initial calculations easier, I let x = heads and tried to create the density function.

1. $(\frac{2}{3})^x * (\frac{1}{3})^{3-x}$ This is fine only when there is one outcome that lines up with the value of x that I have, eg. $x = 3$,$x = 0$.
2. $\binom{3}{x}(\frac{2}{3})^x * \binom{3}{3-x}(\frac{1}{3})^{3-x}$ This just didn't work
3. For the final (and correct) solution, I decided to think through my approach more. I know that the cases $P(0)$ and $P(3)$ are easier because

they are able to discard one heads/tails side of the formula, so to find a pattern, looking at $P(1)$ might be a good idea.

$P(1) = P(HTT) + P(THT) + P(TTH)$ $P(1) = 3(\frac{2}{3} * \frac{1}{3} * \frac{1}{3})$

From here, things became much clearer for me.

I want to multiply the probability of a certain outcome by the number of times that outcome occurs.

In other words, the probability of certain combinations by the combinations of those combinations (I'm sure this isn't academic language).

This led me to

$P(X = x) = \binom{3}{x}(\frac{2}{3})^x(\frac{1}{3})^{3-x}$

If I want to find $P(W = w)$ I can substitute $x = 3 + \frac{w-3}{2}$

Anyways, all this yields the following table (calculated in R):

```
library(MASS)

# x is number of heads in 3 tosses
prob_function <- function(x){
  return(choose(3, x)*(2/3)^x*(1/3)^(3-x))
}

w <- c(-3, -1, 1, 3)
x_inputs <- 3 + (w-3)/2

p_w_frac = fractions(sapply(x_inputs, prob_function))

print(w)
print(p_w_frac)
```

w	-3	-1	1	3
$P(W = w)$	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

1.3 Part 3

Find the cumulative distribution function of the random variable W. Using $F(w)$, find:

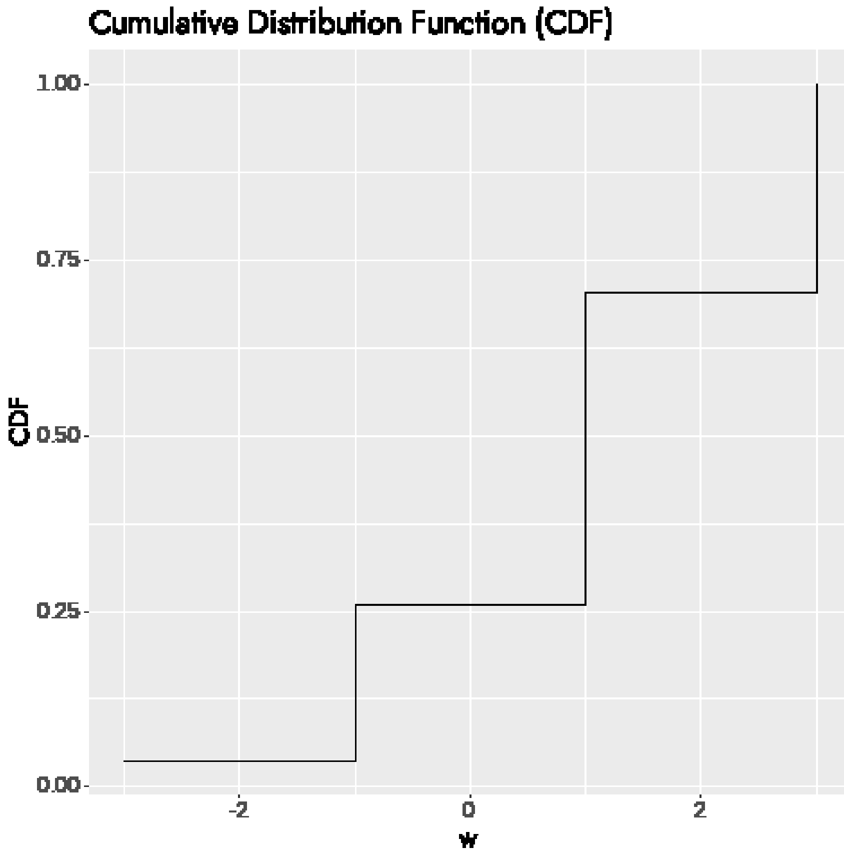
- $P(W > 0)$
- $P(-1 < 3)$

The cumulative distribution is fairly straightforward to compute:

$$f(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{27} & -3 \leq x < -1 \\ \frac{7}{27} & -1 \leq x < 1 \\ \frac{19}{27} & 1 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

1.3.1 Plotting cumulative distribution

```
library(ggplot2)
# Deals with font issues
library(showtext)
font_add_google("Jost", "jost")
showtext_auto()

p_w <- c(p_w_frac)
cdf <- cumsum(p_w)

df <- data.frame(w=w, cdf=cdf)

png("cdf.png")

df$highlight <- ifelse(df$w > 0, df$cdf, NA)

ggplot(df, aes(x = w, y = cdf)) +
  geom_step() +
  labs(x = "w", y = "CDF", title = "Cumulative Distribution Function (CDF)") +
  theme(axis.title = element_text(family="jost"), title = element_text(family="jost"))

dev.off()
```



1.3.2 Solve $P(w > 0)$

$1 - P(w \leq 0) = 1 - \frac{7}{27} = 2027$

1.3.3 Solve $P(-1 \leq w < 3)$

$P(w \leq 3) - P(w \leq 1) = \frac{2}{3}$