Problem: Given an image $(u, v) \in [0, W) \times [0, H)$ sampled from 360 view by $\lambda(u) = 2\pi \frac{u}{W} - \pi$ $\phi(v) = \frac{\pi}{2} - \pi \frac{v}{H}$

1 Part 1

General Strategy outline:

- First, we want to transform the image into pixel on the viewing sphere in 3D space obtaining (λ_w, ϕ_w).
- Then, rotate the sphere to position relative to the camera, obtaining (λ_c, ϕ_c).
- Sample (u', v') within latitude range and longtitude range $(\pi/4, -\pi/4)$ from (λ_c, ϕ_c).

1.1 Transforming image into 3D Dome

The affine transformation $\tau : (u, v) \to (\lambda_w, \phi_w)$ can be seen as linear transformation in homogenous P^2 space, denoted by matrix multiplication:

$$\begin{bmatrix} \frac{2\pi}{W} & 0 & -\pi \\ 0 & \frac{-\pi}{H} & \frac{\pi}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = S \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_w \\ \phi_w \\ 1 \end{bmatrix}$$

Computing the inverse allows us to sample points on the 3D sphere using the information in the image by

$$\begin{bmatrix} u(\lambda_w) \\ v(\phi_w) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{W}{2\pi} & 0 & \frac{W}{2} \\ 0 & \frac{-H}{\pi} & \frac{H}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_w \\ \phi_w \\ 1 \end{bmatrix} = S^{-1} \begin{bmatrix} \lambda_w \\ \phi_w \\ 1 \end{bmatrix}$$

1.2 Rotating sphere to camera perspective

Applying rotation matrices on the world coordinates to obtain the camera view, we have:

$$\begin{aligned} \begin{bmatrix} \lambda_c \\ \phi_c \\ 1 \end{bmatrix} &= P^{-1} R_{yaw}(30°) R_{pitch}(10°) R_{roll}(25°) P \begin{bmatrix} \lambda_w \\ \phi_w \\ 1 \end{bmatrix} \\ &= R \begin{bmatrix} \lambda_w \\ \phi_w \\ 1 \end{bmatrix} \end{aligned}$$

Where R is the camera rotation matrix and P is the matrix transforming polar coordinates to cartesian coordinates. We let x be the horizonal aspect of the view, y be the vertical, and z be the depth. According to the rotations conventions, we have

$$R_{yaw}(30°) = \begin{bmatrix} \cos 30° & 0 & -\sin 30° \\ 0 & 1 & 0 \\ \sin 30° & 0 & cos 30° \end{bmatrix} \qquad \text{y axis}$$

$$R_{pitch}(10°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 10° & -\sin 10° \\ 0 & \sin 10° & \cos 10° \end{bmatrix} \qquad \text{x axis}$$

$$R_{roll}(25°) = \begin{bmatrix} \cos 25° & -\sin 25° & 0 \\ \sin 25° & \cos 25° & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{z axis}$$

1.3 Sampling from Camera coordinates

We sample $(u_c, v_c) \in [0, W) \times [0, H)$ with $\lambda_c(u_c) = \frac{\pi}{2W} u_c - \frac{\pi}{4}$, $\phi_c(v_c) = \frac{\pi}{2H} v_c - \frac{\pi}{4}$. This way, the range of sampled angles for $\lambda_c, \phi_c \in (-\frac{\pi}{4}, \frac{\pi}{4})$, which corresponds to a 90° viewing angle.

1.4 Putting it together

. Therefore, we can construct the perspective view image (new) from the original image (old) by composing transformations of each step. Giving us

$$New(u_c, v_c) = Old(S^{-1}R^{-1}\begin{bmatrix} \frac{\pi}{2W}u_c - \frac{\pi}{4} \\ \frac{\pi}{2H}v_c - \frac{\pi}{4} \\ 1 \end{bmatrix})$$

2 Part 2

Starting from any orientation, you can yaw 90,−90, and 180 to obtain 3 faces. And pitch up 90 and down 90 to obtain the other 2. Therefore, let R be as defined in part 1. Then the question is to find r, p, y such that $R(r, p, y) = R_\Delta R(25, 10, 30)$ where $R_\Delta \in \{R_{yaw}(90), R_{yaw}(-90), R_{yaw}(180), R_{pitch}(90), R_{pitch}(-90)\}$

3 Part 3: Sampling Jacobian and solid-angle element

Solid-angle element on the unit sphere. The catesian position of unit sphere points are parametrized by

$$\boldsymbol{p}(\lambda, \phi) = \begin{bmatrix} \cos\phi\cos\lambda \\ \sin\phi \\ \cos\phi\sin\lambda \end{bmatrix}.$$

longitude $\lambda \in [-\pi, \pi)$ and latitude $\phi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ via Then

$$\partial_\lambda \boldsymbol{p} = \begin{bmatrix} -\cos\phi\sin\lambda \\ 0 \\ \cos\phi\cos\lambda \end{bmatrix}, \qquad \partial_\phi \boldsymbol{p} = \begin{bmatrix} -\sin\phi\cos\lambda \\ \cos\phi \\ -\sin\phi\sin\lambda \end{bmatrix}.$$

The differential area (and hence solid angle on the unit sphere) is

$$d\Omega \;=\; \|\partial_\lambda \boldsymbol{p} \times \partial_\phi \boldsymbol{p}\|\, d\lambda\, d\phi \;=\; \cos\phi\; d\lambda\, d\phi.$$

Thus the solid-angle element maps from ERM angles as

$$\boxed{d\Omega = \cos\phi\; d\lambda\, d\phi}.$$

From image pixels (u, v) to solid angle. With the equirectangular mapping

$$\lambda(u) = 2\pi\frac{u}{W} - \pi, \qquad \phi(v) = \frac{\pi}{2} - \pi\frac{v}{H},$$

the Jacobian from (u, v) to (λ, ϕ) is constant:

$$\frac{\partial(\lambda, \phi)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial\lambda}{\partial u} & \frac{\partial\lambda}{\partial v} \\ \frac{\partial\phi}{\partial u} & \frac{\partial\phi}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{2\pi}{W} & 0 \\ 0 & -\frac{\pi}{H} \end{vmatrix} = -\frac{2\pi^2}{WH}.$$

Taking the absolute value for area,

$$d\lambda\, d\phi = \frac{2\pi^2}{WH}\, du\, dv,$$

so a pixel area $du\, dv$ subtends solid angle

$$\boxed{d\Omega \;=\; \cos\phi(v)\; \frac{2\pi^2}{WH}\; du\, dv}.$$

Equivalently, the per-pixel solid angle is

$$\boxed{\Delta\Omega(u,v) \;\approx\; \cos\bigl(\phi(v)\bigr)\,\frac{2\pi^2}{WH}}.$$

Hence each ERM texel covers *larger* solid angle near the equator ($\phi \approx 0$) and *shrinks* to zero near the poles ($|\phi| \to \frac{\pi}{2}$). At the poles, each pixel covers a tiny solid angle, thus prone to amplify the effects of flickers. Therefore, smoothing out signals or antialiasing near the poles is very important.