For office use only	Team Control Number	For office use only
T1 ________________	**0000**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Summary Sheet

The LaTeX Template for MCM Version v6.2.1

Summary

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Keywords: keyword1; keyword2

Contents

1 Introduction

1.1 Background

Lewis Mumford, a famous sociologist and literary critic, once said in a metaphorical manner, "Adding highway lanes to deal with traffic congestion is like loosening your belt to cure obesity." Fortunately, he did not experience the worse congestion around today's highway toll plaza.

Currently, with roaring number of vehicles, rising construction costs and constrained available areas, traffic jam becomes more and more serious but future toll-plaza construction opportunities are limited to improve this situation markedly. Figure 1 shows the congestion in the toll plaza near Tappan Zee Bridge.

Figure 1: Toll Plaza Congestion

Subject to the constraints referred above, neither increasing highway lanes nor building more tollbooths seems practical enough to relieve traffic jam around a toll plaza nowadays, particularly for some heavily-traveled roads such as the Garden State Parkway, New Jersey. Therefore, looking for some innovative design improvements on the geometric parameters of the extent toll plaza is an effective solution.

1.2 Restatement of the Problem

In this paper, we are required to explore if there is a better-than-ever toll plaza model with specific shape, size, and merging pattern. In this model, the prerequisite is that vehicles fan in from B tollbooth egress lanes down to L ($B > L$) lanes of traffic (i.e., the number of both tollbooths and the lanes after merging are fixed). We aim to construct a model that can optimize the arrangement according to the following conditions.

- Enhance the capability of the accident prevention(A).
- Maximize the throughput(T).
- Minimize the cost of the land and road construction(C).

Through our analysis, we determine if there are better solutions than any toll plaza in common use. Afterwards, the performance of our solution in light and heavy traffic and other various situations along with corresponding sensitivity analysis is discussed.

1.3 Our Work

2 Assumptions

3 Notations

4 Model

4.1 Time Cost and Construction Cost

4.1.1 Model establishing

The throughput and construct cost of a toll plaza are two contradictory indexes. Usually, the more throughput is wanted, the more construct cost is paid. In order to seek an optimal scheme to balance them both, we plan to establish an objective function in which the throughput is related to money consumption. So we introduce a variable called unit waiting time cost, $C_h(USD/h/veh)$. It signifies that if a vehicle queues for one hour at a toll plaza, it will cause C_h dollors loss. Briefly speaking, the overall cost of a toll plaza is defined as an aggregate of time cost and construct cost.

With regard to a toll plaza, suppose the average congestion time is h_g (h) and the average enter flow when congesting is $Q_g(veh/hs)$, then the total waiting time is,

$$h_d = \int_o^{h_g} (Q_g - Q_{max})(h_g - \tau)\, d\tau = 0.5{h_g}^2(Q_g - Q_{max})$$

Where, Q_{max} is maximal throughput of this toll plaza.

If the designed service time of the toll plaza is y years, then time cost during y years is,

$$C_t = 365 \times 24 h_d y C_h = 4380 h_g^2 C_h (Q_g - Q_c y$$

Because we only think about the design of the departure zone when calculating the construct cost, the cost of approach zone and tollbooths is not included. But such omission does not influence our conclusion. Construction can be calculated by a linear function concerning area, that is,

$$C_c = S_r \times C_r + S_l \times C_l$$

Where,

S_r	The area of the road
C_r	The cost of the road per unit area
S_l	The area of the occupied land
C_l	The cost of the occupied land per unit area

Our goal is to minimize the total cost C_s

$$C_s = C_t + C_c$$

This chapter mainly discusses the effects of throughput and construct cost on the toll plaza design, i.e., accident prevention is not included as a major research o bject. Instead, some basic security indicators are constraints when seeking the minimum C_s. In the next step, we are establishing more explicit relationships between size, shape as well as merging pattern, and cost along with throughput.

From our perspective, connecting all the considerations with cost directly or indirectly and make cost our major objective function is an explicit and effective plan. Thus, all our models are established out of this thought. In detail, we can determine an average waiting time by calculating the throughput of the toll plaza. In this way, we may then quantify the average waiting time as money consumption with an introduction of a uniform “waiting time cost“. Our goal is to look for the minimum cost (including time and construct cost) in the case of satisfying basic security conditions. In other words, the overall cost is our objective function and security factors are constraints towards the objective function. We can get better solutions by minimizing the overall cost.

4.2 CA Model

4.2.1 Introduction

In order to discover the influence of size, shape and merging pattern, we propose a two-dimensional Cellular Automata (CA) model. Our model is based on the one-dimensional *Nagel Schreckenberg* model, which was first presented in 1992 and successfully demonstrated many features of the traffic flow.

Compared with a one-dimensional CA model, a two-dimensional one is more complex but feasible enough to simulate the real traffic flow. Therefore, the results from a two dimensional CA model is relatively more accurate. Basically, the CA model can be regarded as an effective method to simulate features of traffic jams by showing how interactions between nearby vehicles cause the deceleration.

4.2.2 Assumptions

- We assume that all the drivers are selfish and short-sighted. To be specific, drivers always adopt measures to move to the Roads Leading to Exit (RLE), wherever they are.
- We assume that once a vehicle leaves the exit of the toll plaza, it will accelerate, namely no congestion outside the plaza.
- We assume that both the choices towards tollbooths and the coming time of vehicles satisfies random distribution.

4.2.3 Model Establishing

In our two-dimensional CA model, each cell is evenly distributed in the shape of square. So the toll plaza, or rather, the whole departure area consists of plenty of square cells. An independent cell or an adjacent cell cluster can denote both empty roads or vehicles according to their corresponding sizes, such as length and width. We can assign a certain speed to each cell-formed vehicle. But each speed value must be an integer, ranging from 0 to an identified v_{max}. In addition, time is also discretized. Each time step is defined as the time that a car takes to travel past the length of 10 cars at the speed of the restricted value.During a step interval, vehicles are set to perform the following actions sent by corresponding instructions in order. Another important rule is that vehicles always perform their updated actions at the same time.

Several actions are further explained as follows:

- **Acceleration:**

  It reflects a characteristic that vehicles tends to travel as fast as possible. Here, this action obeys a rule as:

  if $v_t < v_{max}$, then $v \to min(v_{t+1}, v_{max})$
- **Deceleration:**

  This action guarantee no collision with the vehicle ahead. It satisfies:

$$v_t \to min(v_t, dx)$$

- **Random decelerationïijŽ**

  It embodies behavioral discrepancy of drivers. The introduction of random deceleration basically reflects the overreaction in decelerative processes. In conclusion, it is a key factor that causes congestion. Likewise, it meets following principle:

  If $v_t > 0$, then $v_t \to v_{t-1}$ with probability p_v

- **Steering:**

  It describes the trends that divers are more willing to turn to the Roads Leading to Exit (RLE) if they are not on the RLE at this moment. The corresponding rule is:

  If (y>W_L), then $v_y = 1$ with probability p_y

- **Lane changing:** When a vehicle gets stuck, the driver is likely to convert his or her lane to a n earby one (only the one closer to the RLE) if that lane is empty. It is also set to satisty:

  If $dy > 1$ and $v = 0$, then $vy = 1$ with probability p_d

- **Horizontal velocity:**

$$v_x = \sqrt{v^2 - vy^2}$$

- **Motion:**

  It indicates that the position of a vehicle is shifted by its speed v_x and v_y, thus, $x \to x + dx, y \to y + dy$

- **Incoming vehicles:**

  Each vehicle will travel from the start of a certain tollbooth with the probability of p_{in}. This probability can denote traffic density in some ways.

  Where,

v_t	The speed of current moment
v_{max}	The maximum speed allowed
v_{t+1}	The speed of next moment
dx	The nearest distance between two nearby vehicles in their horizontal direction
dy	The nearest distance between two nearby vehicles in their vertical direction
p_v	The probability of randomlization
p_y	The probability of sheering
p_d	The probability of lane changing
p_{in}	The probability of incoming vehicles
v_x	The horizontal velocity
v_y	The vertical velocity
W_L	The exit width of the toll plaza

4.2.4 Simulation and discussion

We convert our thoughts and design above into program instructions via Python, and simulate a toll plaze with 8 tollbooths and 3 lanes of travel.

Common vehicles are usually $4-4.5m$ long and $1.65-1.85m$ wide, so a vehicle will take up 2 cells. According to the *Green Book, 1994*, an appropriate design of a toll plaze would be a trapezoid with a 168-meter-long recovery zone and a 612-meter-long departure zone. The width of a tollbooth along with a toll island is usually $5.5m$, while that of each lane is $3.5-4m$. We set the length of each cell equal to $2m$: $l_{car} = 4m$. So the parameters are as follows:

$$\begin{aligned} l_{veh} &= 2m \\ W_{veh} &= 1m \\ W_B &= W_b B = 3 \times 8 = 24m \\ W_L &= W_l L = 2 \times 3 = 6m \\ L_r &= 84m \\ L_d &= 306m \end{aligned}$$

Figure 2 shows the relationship between current throughput and traffic flow density with different p_v.

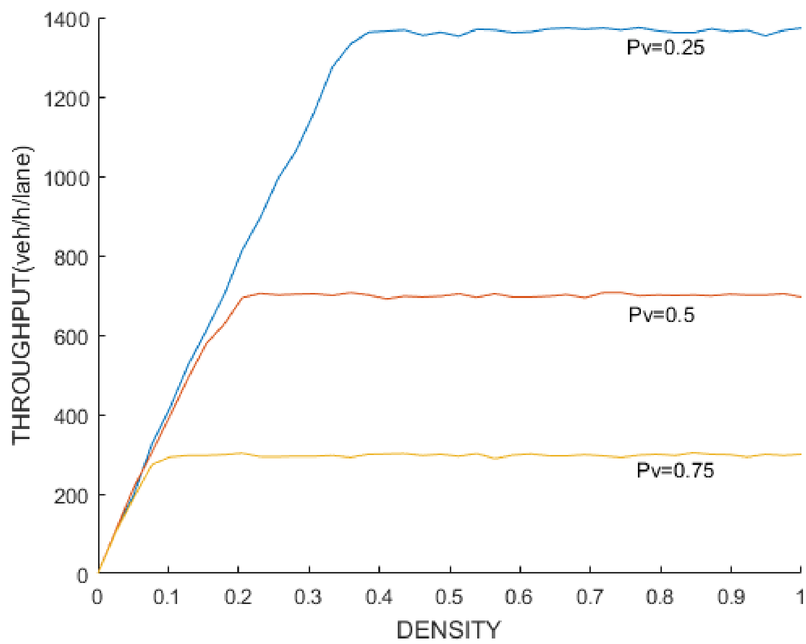

Figure 2: The relationship between throughput and density with different p_v

The density in the correspondence with the maximal throughput q_{max} is defined as critical density d_c. We can distinguish that the curves in Figure 2 are divided into two sections by that critical density. If the flow density is lower than d_c, it will be a free flow (as Figure shows);otherwise, a crowded flow (as Figure shows).We can conclude from the simulations above that the RLE are the most crowded part once traffic density increases to cause congestion. In this case, drivers always consider turning to the RLE as soon as possible, which causes unevenly distribution of traffic density in the departure zone.

As mentioned above, p_v is the probability of random deceleration. It varies among different drivers. It is not difficult to conclude that the smaller the value is, the larger q_{max} is accessible. At the same time, d_c will increase correspondingly. In the following chapters, we choose to adopt $p_v = 0.5$ as our basis, because the q_{max} at this time is is consistent with the actual value, approximately 700 $veh/h/lane$. And more detailed simulation results will also be displayed.

5 Size

The size of the merge area can be determined by the following parameters:

- Total width of typical toll lanes (W_B).
- Length of the recovery zone (L_r).
- Length of total departure zone(L_d).
- Width of the exit(W_L).

Parameters hereinbefore are shown in Figure 3. For the number of travel lanes

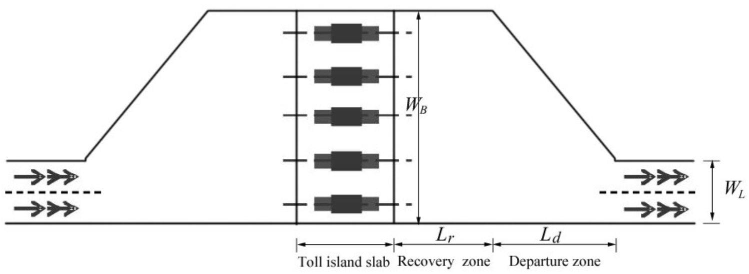

Figure 3: The Parameters

(L) is fixed, W_L is constant. Then we are considering the effect of the rest parameters separately. By simulating our model mentioned above via computer program, we can figure out how these parameters affect the maximal throughput of the merge area, that is, Q_{max}. Figure 4 shows the variation tendency of Q_{max} with the alteration of the width of each tollbooth W_b. Apparently $W_B = B \times W_b$. Figure 4 provides a result under the prerequisite that W_b ranges from 6 to 14 while other parameters are fixed. We utilize an appropriate Linear Fitting Function Model to

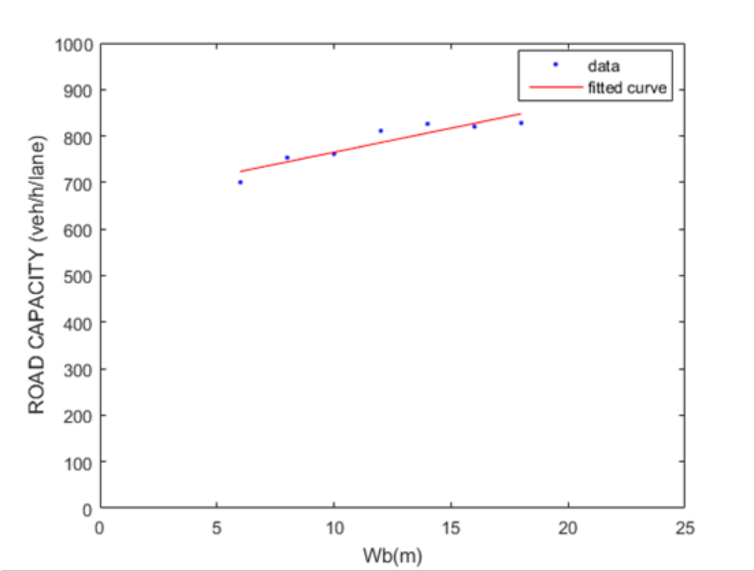

Figure 4: The Linear Fitting Image of Q_{max} and W_b

address the data, and then get the fitting function of Q_{max} and W_b:

$$Q_{max} = p_1 \times W_b + p_2$$

Where,

$$p_1 = 10.35, p_2 = 661.7$$

The simulation result indicates that Q_{max} would only be affected by the total width of typical toll lanes (W_B) in a small degree. However, increasing W_b will markedly result in a rise in construction costs. For L_r, the linear fitting image is showed in Figure 5 and the variance of Q_{max} is 36.7188. We can see that L_r causes almost no effect on the merge area capacity. In the Linear Fitting Function, the coefficient $p_3 = 0$.

Both linear fitting image, and function of Q_{max} and L_d are shown below. There is a negative correlation between Q_{max} and L_d. Nevertheless, the relationship is so faint that enlarging Q_{max} by changing L_d is not functional.

$$Q_{3max} = p_5 \times L_d + p_6$$

Where,

$$p_5 = -0.1248, p_6 = 801.1.$$

From discussion above, the size does cause impact on Q_{max}, while the impact is not that obvious. In addition, L_d and W_B should never be constructed too small because it may cause potential safety problems and result in higher accident rate. For ensuring safety, the departure taper rates T_r must be limited into $[T_{rmin}, T_{rmax}]$. In summary, in order to determine the optimal size of a toll plaza, the problem can be transformed into a linear minimization problem with the form:

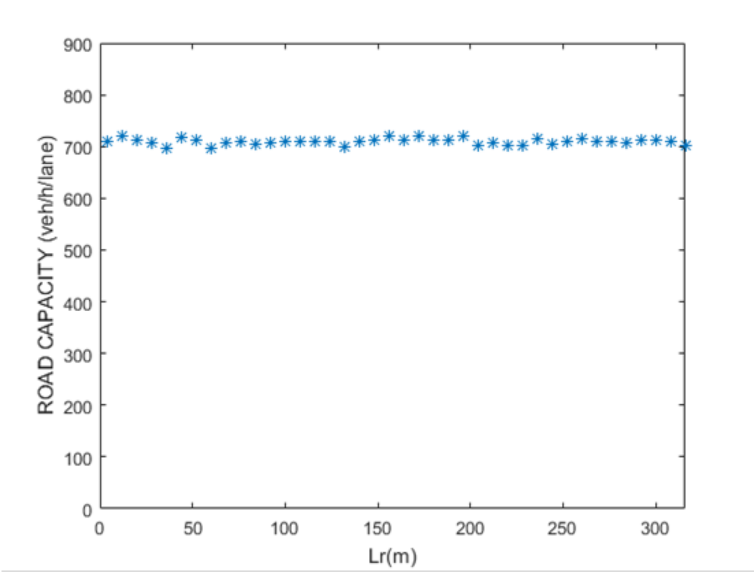

Figure 5: The Linear Fitting Image of Q_{max} and L_r

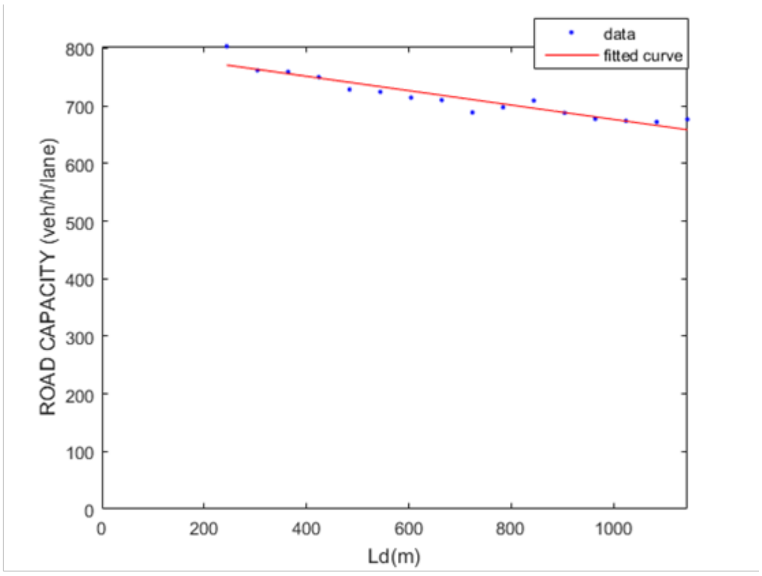

Figure 6: The Linear Fitting Image of Q_{max} and L_d

$$
\begin{aligned}
\textbf{minimize} \quad & C = S(C_{road} + C_{construct}) + 4380h_g^2\mathrm{C}_h(Q_g - Q_{\max}) \\
\textbf{s.t} \quad & Q_{max} \,|\, \mathrm{W}_b = 3m, L_d = 612m, L_r = 168m = 709XL \\
& \frac{dQ_{max}}{dW_b} = p_1 = 10.35 \\
& \frac{dQ_{max}}{dL_r} = p_3 = 0 \\
& \frac{dQ_{max}}{dL_d} = p_5 = -0.1248 \\
& T_{rmin} < T_r < T_{rmax} \\
& W_b > W_{bmin}
\end{aligned}
$$

Here W_{bmin} signifies the minimal width of the toothbooths, and the area of toll plaza

$$S = 13W_b(L_r + 0.5L_d) + 0.5LW_LL_d$$

The departure taper rate

$$T_r = \frac{L_d}{13W_b - LW_L}$$

For example, there is a toll plaza with three lanes and eight tollbooths. To solve the problem, we can make assumptions as following:

- The limited speed is 30 km/h.
- The lifespan planned reaches to 10 years.
- The average daily congestion time $h_g = 1h$.
- The average congestion flow $Q_q = 2300veh/h$.
- The land price locally $C_{land} = 85USD/m^2$
- The cost of highway construction $C_{road} = 357USD/m_2$

According to *1994 Green Book* taper rate for lane addition in a 3-lane section ,T_r should arrange from 8 to 15. Commonly, it takes 1 USD as the cost for each per son to wait one hour.

On the basis of these conditions, the optimal solution of linear programming is

$$W_b = 5 \tag{1}$$
$$L_r = 47m \tag{2}$$
$$L_d = 265.5m \tag{3}$$

the total cost

$$C = 8,167,645USD$$

6 Shape

We propose two types of the plaza shape: series type and parallel type.

6.1 Series Type

Literally, this type is to connect two or more merge areas in series. Here, we only consider connecting two merge areas. Furthermore, we might as well suppose $B = 8$ and $L = 3$. Specially, vehicles fan in from eight tollbooth egress lanes

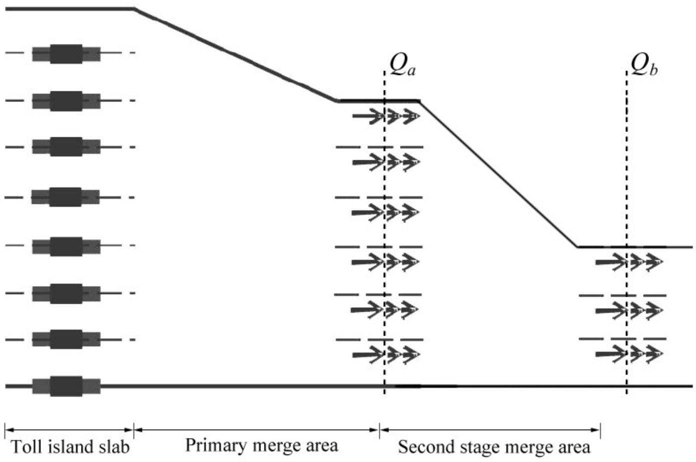

Figure 7: Series Type

down to six lanes of traffic, then fan in from six lanes of traffic to three, as Figure 7 shows.

According to the Buckets Effect (BE),

$$Q_{smax} = min\{Q_{amax}, Q_{bmax}\}$$

Q_{amax}, Q_{bmax} and Q_{smax} respectively signify the maximal throughput of the primary merge area, second stage merge area and the whole series-type toll plaza. We can get Table 1 from simulation results, which indicates the value of the maximal throughput for each traffic line(Q_{emax}) with different B and L ($B > L$).

Table 1: Q_{max} with B and L

Q_{max}	B=1	B=2	B=3	B=4	B=5	B=6	B=7	B=8	B=9	B=10
L=1		882	845	832	796	771	772	736	689	640
L=2			815	789	773	755	720	718	686	659
L=3				755	758	734	724	709	684	671
L=4					724	700	715	695	694	673
L=5						716	695	690	688	673
L=6							695	688	682	667
L=7								682	676	670
L=8									676	660
L=9										651

As for the example shown in Figure 7,

$$Q_{smax} = min\{688 \times 6, 734 \times 3\}$$

For a simple toll plaza with the same number of B and L,

$$Q_{max} = 709 \times 3 = 2127$$

Therefore

$$Q_{smax} > Q_{max}$$

Moreover, since Q_{emax} is becoming large as B or L d ecrease, we can prove that the merge area in series type would have a larger capacity for any B and L (BL). Thus, connecting two or more merge area in series is a practical and optimized scheme.

6.2 Parallel Type

That is, divide the merge area transversely and put them together in parallel. Similarly, if we suppose $B = 8$ and $L = 3$ again, the toll plaza can be divided into two portions as Figure 8 shows. Since the two areas are juxtaposed,

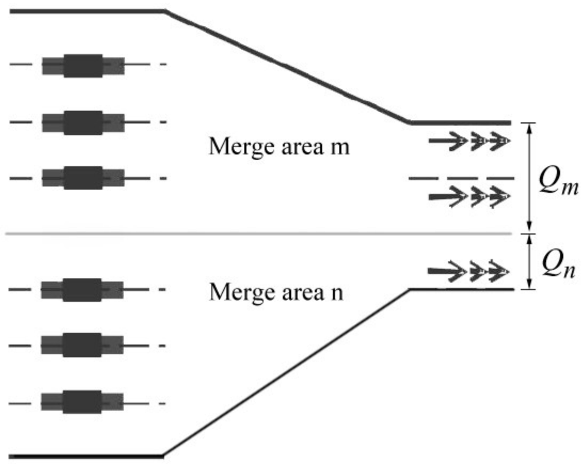

Figure 8: Parallel Type

$$Q_{pmax} = Q_{mmax} + Q_{nmax}$$

Q_{mmax}, Q_{nmax} and Q_{pmax} respectively signify the maximal throughput of the merge area m, merge area n and the whole parallel-type toll plaza. Similar to the analysis of the series type, Q_{emax} is becoming large with the increasing of B or L. Thus, this solution could enlarge the maximal throughput efficaciously.

6.3 An example

For the convenience of discussion, we still take a toll plaza with three lanes and eight tollbooths as an example. If we adopt the method of 8-6-3 series type, from what has been discussed above, Q_{max} increases from 2127 to 2212 veh/h with increasing rate of 3.9%. Therefore, the cost time decreases by 1.4%.

However, at the same time, the area S_r and S_d increases. The total area will increase by

$$\Delta S = 4W_bL_d + W_L(6L_r + 3L_d)$$

We can solve out that the total area will increase by 73.4%.Thus, the construction cost will also increase by 73.4%. Under normal conditions , construction cost and time cost are of the same order of magnitude, so the series cannot solve the problem practically.

On the other hand, if we adopt the method of parallel type ,we can find that Qmax increase by 13.3%, which is fully significant. And we adopt the structure shown in figure 9 simultaneously. Such shape would cause the total area to increase by the following formula:

$$\Delta S = 6W_L(L_D + L_r + L_A)$$

Where L_A is the length of approving zone of the toll plaza, which increases by

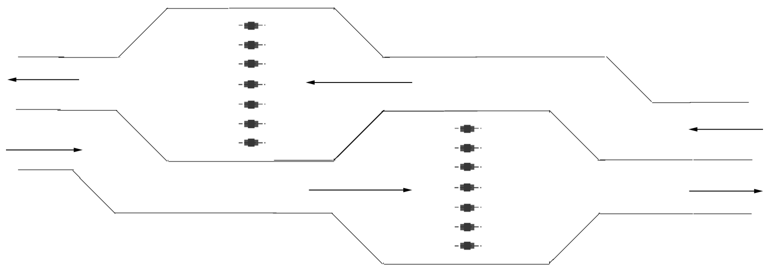

Figure 9: Parallel Type

21.3% approximately. Hence it is necessary to construct toll plaza in this way when the traffic congestion is serious and the cost time is extremely large.

7 Merging Pattern

Here, we devise a real-time merging control system for toll plaza based on the previous work by M. Papageorgiou et al. Through our improvement, it can be specially used for the toll plaza we are discussing. In addition, this system can effectively maximize the throughput by maintaining the occupancy of departure area close to a critical value. Figure 10 illustrates the framework of this system.

Merge area

As a matter of fact, the merge area is equal to the departure zone as referred to above. Typically, it is an approximately trapezoidal area where the vehicles leave from the booths on a total of B lanes and finally fit into L lanes of the exit. Here, we focus on the flow-density variation with the occupancy increasing in

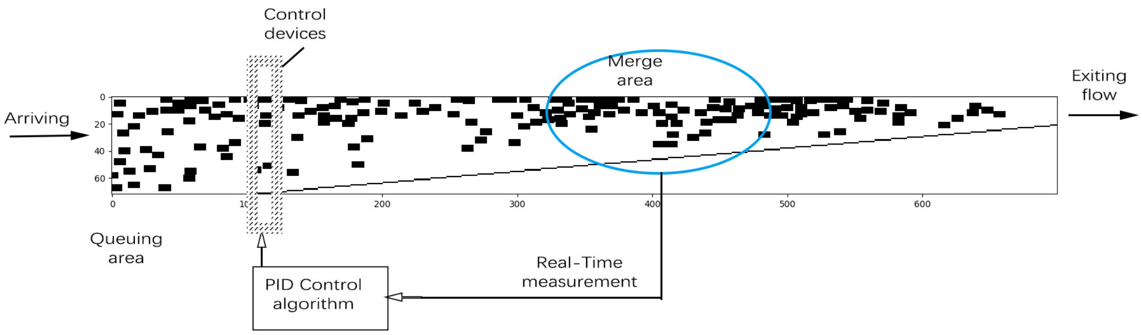

Figure 10: The Framework of This System

the merge area. Eventually,according to our CA model's simulation, we obtain a diagram to describe this functionary relationship, which is shown in Figure 11.

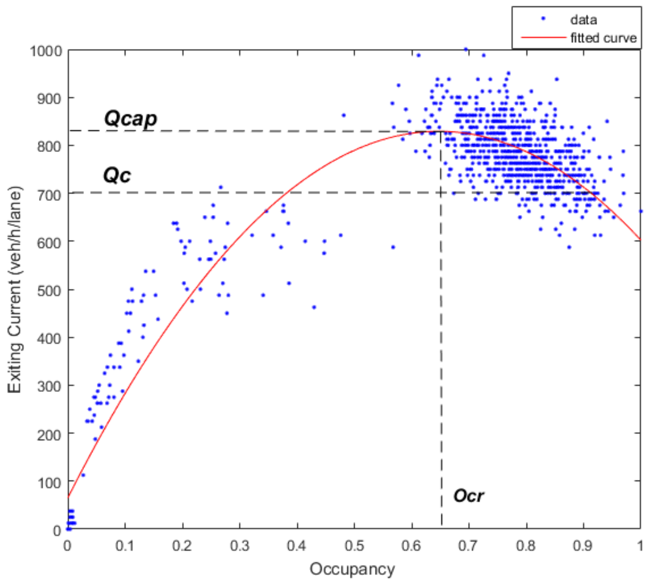

Figure 11: The Functionary Relationship

After noticing that X-axis is occupancy o (%), while Y-axis represents the exit flow q_{out}, we can tell from the diagram:

- When o is small, merging conflicts are scarce. the exit flow is correspondingly low and it increases linearly with density increasing
- As o increases, merging conflicts may increase, but q_{out} also increases as well until, for a specific value o_{cr}, the exit flow reaches the capacity q_{cap}.
- If o increases beyond o_{cr}, merging conflicts become more frequent, leading to a serious congestion. Consequently, a capacity drop happens.

Therefore, we can conclude that the occupancy of the merge area can directly influence the exit flow, or rather, the throughput. And we can regulate the occupancy under the goal to maintain $o \approx o_{cr}$ by controlling the merging pattern with the assistant of a control algorithm and feedback. From a macroscopic perspective, the maximum throughput can be achieved by a certain merging pattern design. As a result, our goal is to model this design.

Feedback control based on PID controller

We are inspired by the commonly used PID control algorithm in industrial control systems and decide to deploy traffic lights to individual lanes as control devices.

However, the most crucial task is to determine the form of feedback control.

Our goal is to ensure that occupancy is maintained at around o_{cr} regardless of how serious the traffic congestion is. What's different from common PID control system is that the traffic system is not a Continuous system,and The control loops are very long compared to other systems. We suppose that the feedback control is activated at each discrete time interval witch usually set as 1 2 min. After activation, it will collect latest measurements of occupancy o, and send data-converted instructions to control devices under the purpose of maintaining $o \approx o_{cr}$.

The PID system can be expressed as:

$$q\left(n\right) = K_p e\left(n-1\right) + K_i \sum_{j=1}^{n-1} e\left(n-1\right) + \frac{d}{dx}\left[e\left(n-1\right) - e\left(n-2\right)\right]$$

$$e\left(n\right) = \hat{o}\left(n\right) - o\left(n\right)$$

Where,

n	The discrete time index
$q(n)$	The controlled entering flow (veh/h) to be implemented in a new time step n
$o(n)$	The measured occupancy of merge area in this time step
$\hat{o}$	The desired value of occupancy (can be set as o_{cr})
K_p, K_i, K_d	coefficients for the proportional, integral, and derivative terms, always positive

In addition, the occupancy measurement should best be placed at or just upstream of the location where serious vehicle decelerations (congestion) appear first.

8 Conclusion

9 Sensitivity Analysis

9.1 The Performance of Our Solution in Light and Heavy Traffic

9.2 Autonomous Vehicles

9.3 The Proportions of Different Tollbooths

10 Strengths and Weaknesses

10.1 Strengths

10.2 Weaknesses

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[2] Lamport, Leslie, LATEX: " A Document Preparation System ", Addison-Wesley Publishing Company, 1986.

Appendices

Appendix A First appendix

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
       aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
```

```
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===========================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===========================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```