1 Model 1: Two-dimensional Cellular Automata Model

1.1 Introduction

In order to discover the influence of size, shape and merging pattern, we propose a two-dimensional Cellular Automata (CA) model. Our model is based on the one-dimensional *Nagel Schreckenberg* model, which was first presented in 1992 and successfully demonstrated many features of the traffic flow.

Compared with a one-dimensional CA model, a two-dimensional one is more complex but feasible enough to simulate the real traffic flow. Therefore, the results from a two dimensional CA model is relatively more accurate. Basically, the CA model can be regarded as an effective method to simulate features of traffic jams by showing how interactions between nearby vehicles cause the deceleration.

1.2 Assumptions

- We assume that all the drivers are selfish and short-sighted. To be specific, drivers always adopt measures to move to the Roads Leading to Exit (RLE), wherever they are.
- We assume that once a vehicle leaves the exit of the toll plaza, it will accelerate, namely no congestion outside the plaza.
- We assume that both the choices towards tollbooths and the coming time of vehicles satisfies random distribution.

1.3 Model Establishing

In our two-dimensional CA model, each cell is evenly distributed in the shape of square. So the toll plaza, or rather, the whole departure area consists of plenty of square cells. An independent cell or an adjacent cell cluster can denote both empty roads or vehicles according to their corresponding sizes, such as length and width. We can assign a certain speed to each cell-formed vehicle. But each speed value must be an integer, ranging from 0 to an identified v_{max}. In addition, time is also discretized. Each time step is defined as the time that a car takes to travel past the length of 10 cars at the speed of the restricted value.During a step interval, vehicles are set to perform the following actions sent by corresponding instructions in order. Another important rule is that vehicles always perform their updated actions at the same time.

Several actions are further explained as follows:

- **Acceleration:**

It reflects a characteristic that vehicles tends to travel as fast as possible. Here, this action obeys a rule as:

if $v_t < v_{max}$, then $v \rightarrow min(v_{t+1}, v_{max})$

- **Deceleration:**

  This action guarantee no collision with the vehicle ahead. It satisfies:

$$v_t \rightarrow min(v_t, dx)$$

- **Random decelerationïijŽ**

  It embodies behavioral discrepancy of drivers. The introduction of random deceleration basically reflects the overreaction in decelerative processes. In conclusion, it is a key factor that causes congestion. Likewise, it meets following principle:

  If $v_t > 0$, then $v_t \rightarrow v_{t-1}$ with probability p_v

- **Steering:**

  It describes the trends that divers are more willing to turn to the Roads Leading to Exit (RLE) if they are not on the RLE at this moment. The corresponding rule is:

  If (y>W_L), then $v_y = 1$ with probability p_y

- **Lane changing:** When a vehicle gets stuck, the driver is likely to convert his or her lane to a n earby one (only the one closer to the RLE) if that lane is empty. It is also set to satisty:

  If $dy > 1$ and $v = 0$, then $vy = 1$ with probability p_d

- **Horizontal velocity:**

$$v_x = \sqrt{v^2 - vy^2}$$

- **Motion:**

  It indicates that the position of a vehicle is shifted by its speed v_x and v_y, thus, $x \rightarrow x + dx, y \rightarrow y + dy$

- **Incoming vehicles:**

  Each vehicle will travel from the start of a certain tollbooth with the probability of p_{in}. This probability can denote traffic density in some ways.

  Where,

Table 1: Add caption

v_t	The speed of current moment
v_{max}	The maximum speed allowed
v_{t+1}	The speed of next moment
dx	The nearest distance between two nearby vehicles in their horizontal direction
dy	The nearest distance between two nearby vehicles in their vertical direction
p_v	The probability of randomlization
p_y	The probability of sheering
p_d	The probability of lane changing
p_{in}	The probability of incoming vehicles
v_x	The horizontal velocity
v_y	The vertical velocity
W_L	The exit width of the toll plazaÂă

1.4 Simulation and discussion

We convert our thoughts and design above into program instructions via Python, and simulate a toll plaze with 8 tollbooths and 3 lanes of travel.

Common vehicles are usually $4-4.5m$ long and $1.65-1.85m$ wide, so a vehicle will take up 2 cells. According to the *Green Book, 1994*, an appropriate design of a toll plaze would be a trapezoid with a 168-meter-long recovery zone and a 612-meter-long departure zone. The width of a tollbooth along with a toll island is usually $5.5m$, while that of each lane is $3.5-4m$. We set the length of each cell equal to $2m$: $l_{car} = 4m$. So the parameters are as follows:

$$
\begin{aligned}
&l_{veh} = 2m \\
&W_{veh} = 1m \\
&W_B = W_b B = 3 \times 8 = 24m \\
&W_L = W_l L = 2 \times 3 = 6m \\
&L_r = 84m \\
&L_d = 306m
\end{aligned}
$$

Figure 1 shows the relationship between current throughput and traffic flow density with different p_v.

The density in the correspondence with the maximal throughput q_{max} is defined as critical density d_c. We can distinguish that the curves in Figure 1 are divided into two sections by that critical density. If the flow density is lower than d_c, it will be a free flow (as Figure shows);otherwise, a crowded flow (as Figure shows).We can conclude from the simulations above that the RLE are the most crowded part once traffic density increases to cause congestion. In this case, drivers always consider turning to the RLE as soon as possible, which causes unevenly distribution of traffic density in the departure zone.

As mentioned above, p_v is the probability of random deceleration. It varies

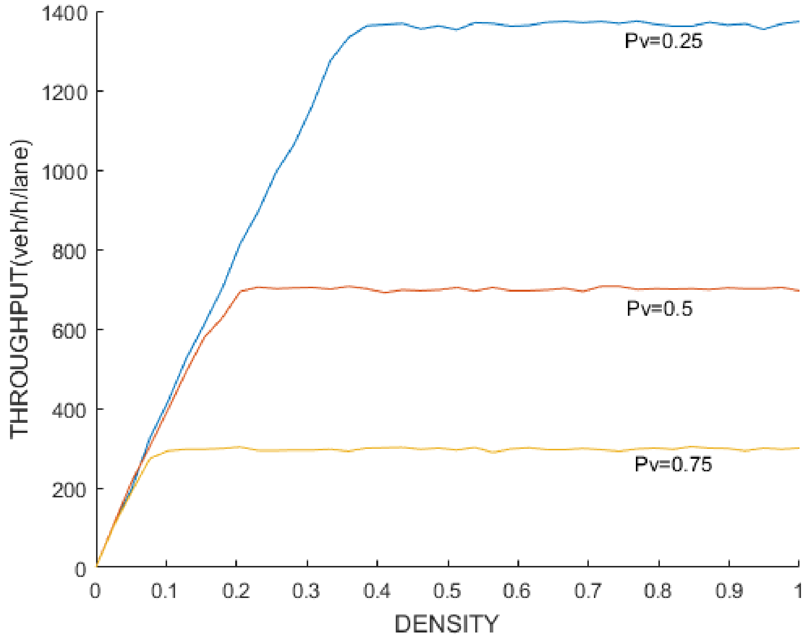

Figure 1: The relationship between throughput and density with different p_v

among different drivers. It is not difficult to conclude that the smaller the value is, the larger q_{max} is accessible. At the same time, d_c will increase correspondingly. In the following chapters, we choose to adopt $p_v = 0.5$ as our basis, because the q_{max} at this time is is consistent with the actual value, approximately 700 $veh/h/lane$. And more detailed simulation results will also be displayed.