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T1 ________________	**0000**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Summary Sheet

The LaTeX Template for MCM Version v6.2.1

Summary

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Keywords: keyword1; keyword2

Contents

1 Introduction

1.1 Background

Lewis Mumford, a famous sociologist and literary critic, once said in a metaphorical manner, "Adding highway lanes to deal with traffic congestion is like loosening your belt to cure obesity." Fortunately, he did not experience the worse congestion around today's highway toll plaza.

Currently, with roaring number of vehicles, rising construction costs and constrained available areas, traffic jam becomes more and more serious but future toll-plaza construction opportunities are limited to improve this situation markedly. Figure 1 shows the congestion in the toll plaza near Tappan Zee Bridge.

Figure 1: Toll Plaza Congestion

Subject to the constraints referred above, neither increasing highway lanes nor building more tollbooths seems practical enough to relieve traffic jam around a toll plaza nowadays, particularly for some heavily-traveled roads such as the Garden State Parkway, New Jersey. Therefore, looking for some innovative design improvements on the geometric parameters of the extent toll plaza is an effective solution.

1.2 Restatement of the Problem

In this paper, we are required to explore if there is a better-than-ever toll plaza model with specific shape, size, and merging pattern. In this model, the prerequisite is that vehicles fan in from B tollbooth egress lanes down to L ($B > L$) lanes of traffic (i.e., the number of both tollbooths and the lanes after merging are fixed). We aim to construct a model that can optimize the arrangement according to the following conditions.

- Enhance the capability of the accident prevention(A).
- Maximize the throughput(T).
- Minimize the cost of the land and road construction(C).

Through our analysis, we determine if there are better solutions than any toll plaza in common use. Afterwards, the performance of our solution in light and heavy traffic and other various situations along with corresponding sensitivity analysis is discussed.

1.3 Our Work

2 Assumptions

3 Notations

4 Model

4.1 Time Cost and Construction Cost

4.2 CA Model

5 Size

The size of the merge area can be determined by the following parameters:

- Total width of typical toll lanes (W_B).
- Length of the recovery zone (L_r).
- Length of total departure zone(L_d).
- Width of the exit(W_L).

Parameters hereinbefore are shown in Figure 2. For the number of travel lanes (L) is fixed, W_L is constant. Then we are considering the effect of the rest parameters separately. By simulating our model mentioned above via computer program, we can figure out how these parameters affect the maximal throughput of

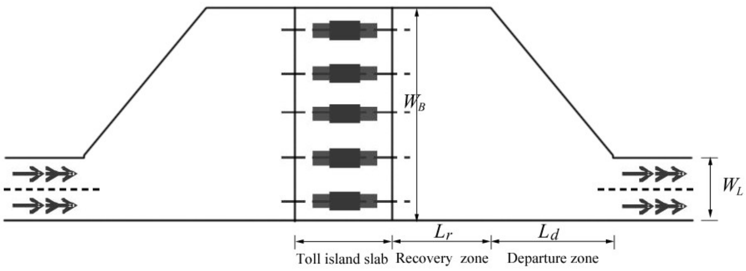

Figure 2: The Parameters

the merge area, that is, Q_{max}. Figure 3 shows the variation tendency of Q_{max} with the alteration of the width of each tollbooth W_b. Apparently $W_B = B \times W_b$. Figure 3 provides a result under the prerequisite that W_b ranges from 6 to 14 while other parameters are fixed. We utilize an appropriate Linear Fitting Function Model to

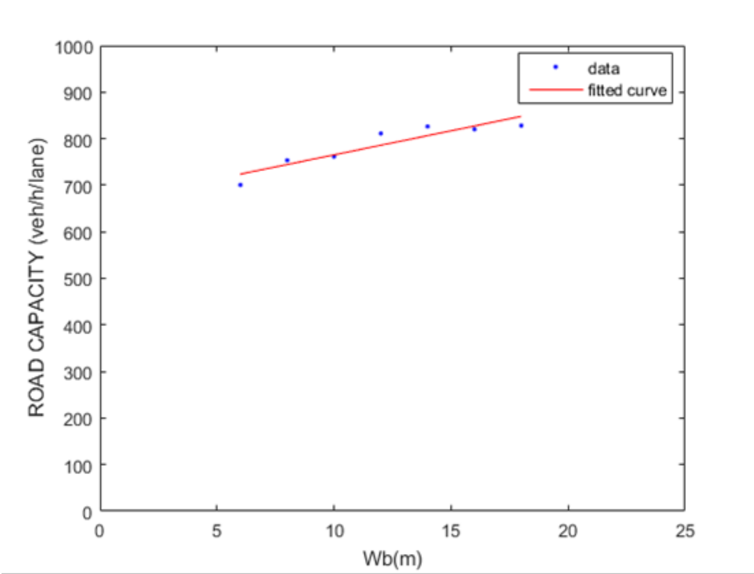

Figure 3: The Linear Fitting Image of Q_{max} and W_b

address the data, and then get the fitting function of Q_{max} and W_b:

$$Q_{max} = p_1 \times W_b + p_2$$

Where,

$$p_1 = 10.35, p_2 = 661.7$$

The simulation result indicates that Q_{max} would only be affected by the total width of typical toll lanes (W_B) in a small degree. However, increasing W_b will markedly result in a rise in construction costs. For L_r, the linear fitting image is showed in Figure 4 and the variance of Q_{max} is 36.7188. We can see that L_r causes almost no effect on the merge area capacity. In the Linear Fitting Function, the coefficient $p_3 = 0$.

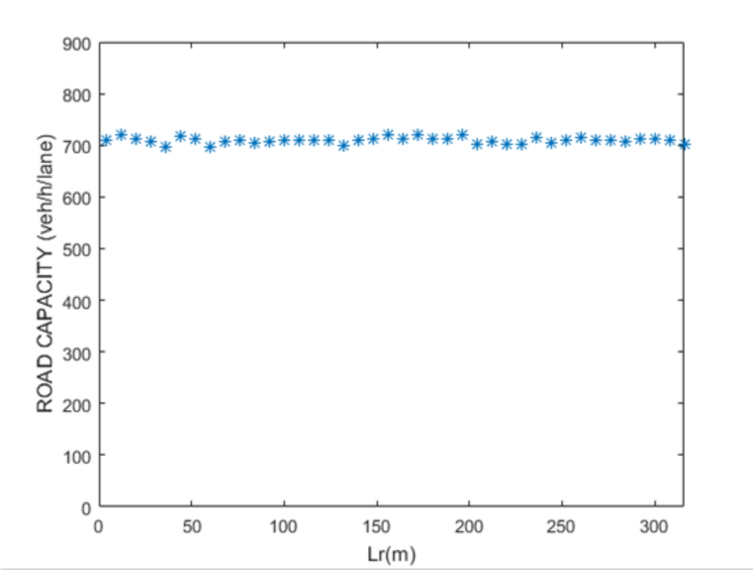

Figure 4: The Linear Fitting Image of Q_{max} and L_r

Both linear fitting image, and function of Q_{max} and L_d are shown below. There is a negative correlation between Q_{max} and L_d. Nevertheless, the relationship is so faint that enlarging Q_{max} by changing L_d is not functional.

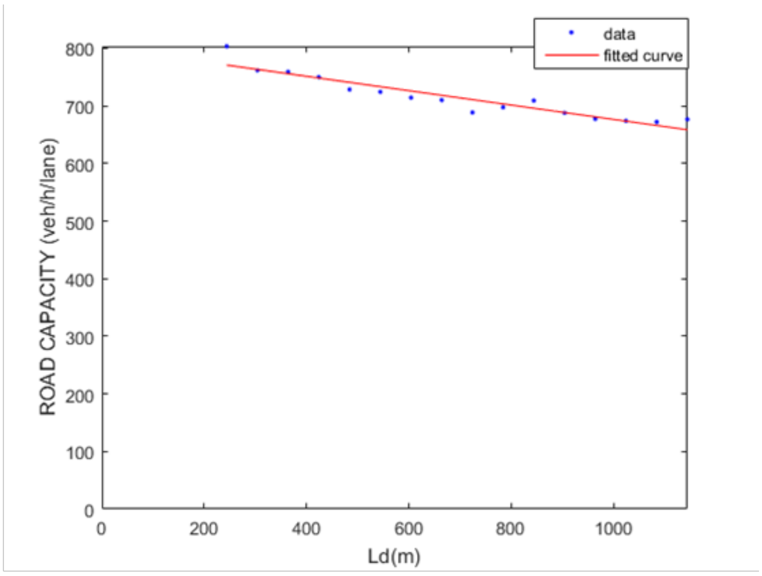

Figure 5: The Linear Fitting Image of Q_{max} and L_d

$$Q_{3max} = p_5 \times L_d + p_6$$

Where,

$$p_5 = -0.1248, p_6 = 801.1.$$

From discussion above, the size does cause impact on Q_{max}, while the impact is not that obvious. In addition, L_d and W_B should never be constructed too small because it may cause potential safety problems and result in higher accident rate. For ensuring safety, the departure taper rates T_r must be limited into $[T_{rmin}, T_{rmax}]$. In summary, in order to determine the optimal size of a toll plaza, the problem can be transformed into a linear minimization problem with the form:

$$
\begin{aligned}
\textbf{minimize} \quad & C = S(C_{road} + C_{construct}) + 4380 h_g^2 \mathrm{C}_h (Q_g - Q_{\max}) \\
\textbf{s.t} \quad & Q_{max} \mid \mathrm{W}_b = 3m, L_d = 612m, L_r = 168m = 709XL \\
& \frac{dQ_{max}}{dW_b} = p_1 = 10.35 \\
& \frac{dQ_{max}}{dL_r} = p_3 = 0 \\
& \frac{dQ_{max}}{dL_d} = p_5 = -0.1248 \\
& T_{rmin} < T_r < T_{rmax} \\
& W_b > W_{bmin}
\end{aligned}
$$

Here W_{bmin} signifies the minimal width of the toothbooths, and the area of toll plaza

$$S = 13W_b(L_r + 0.5L_d) + 0.5LW_L L_d$$

The departure taper rate

$$T_r = \frac{L_d}{13W_b - LW_L}$$

For example, there is a toll plaza with three lanes and eight tollbooths. To solve the problem, we can make assumptions as following:

- The limited speed is 30 km/h.
- The lifespan planned reaches to 10 years.
- The average daily congestion time $h_g = 1h$.
- The average congestion flow $Q_q = 2300veh/h$.
- The land price locally $c_{land} = 85USD/m^2$
- The cost of highway construction $c_{road} = 357USD/m_2$

According to *1994 Green Book* taper rate for lane addition in a 3-lane section ,T_r should arrange from 8 to 15. Commonly, it takes 1 USD as the cost for each per son to wait one hour.

On the basis of these conditions, the optimal solution of linear programming is

$$W_b = 5 \quad (1)$$
$$L_r = 47m \quad (2)$$
$$L_d = 265.5m \quad (3)$$

the total cost

$$C = 8,167,645USD$$

6 Shape

7 Merging Pattern

Here, we devise a real-time merging control system for toll plaza based on the precious work by M. Papageorgiou et al. Through our improvement, it can be specially used for the toll plaza we are discussing. In addition, this system can effectively maximize the throughput by maintaining the occupancy of departure area close to a critical value. Figure 6 illustrates the framework of this system.

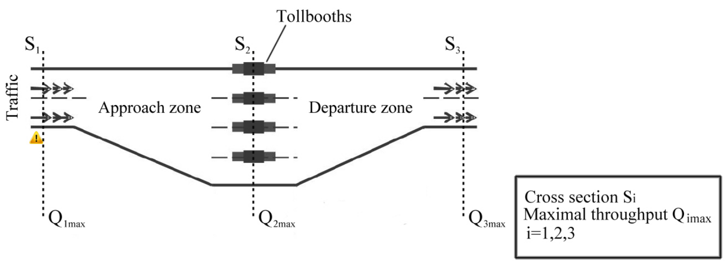

Figure 6: The Framework of This System

Elements

Merge area

As a matter of fact, the merge area is equal to the departure zone as referred to above. Typically, it is an approximately trapezoidal area where the vehicles leave from the booths on a total of B lanes and finally fit into L lanes of the exit. Here, we focus on the flow-density variation with the occupancy increasing in the merge area. Eventually, we obtain a diagram to describe this functionary relationship, which is shown in Figure 7.

After noticing that X-axis is occupancy o (%), while Y-axis represents the exit flow q_{out}, we can tell from the diagram:

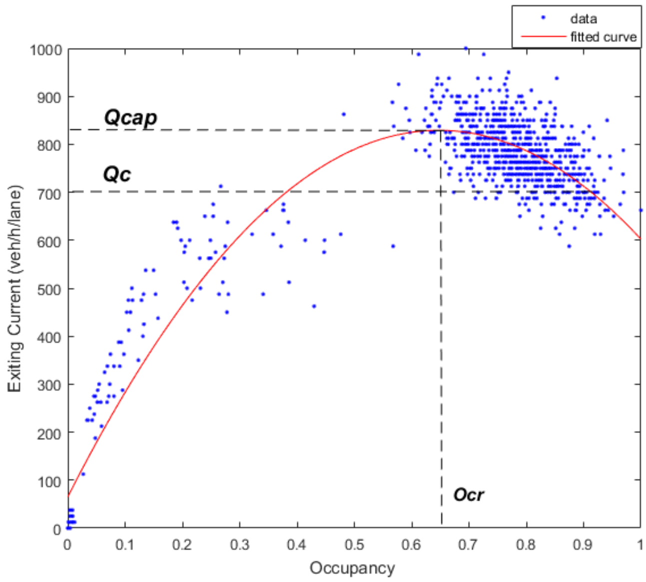

Figure 7: The Functionary Relationship

- When o is small, merging conflicts are scarce, and the exit flow is correspondingly low.
- As o increases, merging conflicts may increase, but q_{out} also increases as well until, for a specific value o_{cr}, the exit flow reaches the capacity q_{cap}.
- If o increases beyond o_{cr}, merging conflicts become more frequent, leading to a serious congestion. Consequently, a capacity drop happens.

Therefore, we can conclude that the occupancy of the merge area can directly influence the exit flow, or rather, the throughput. And we can regulate the occupancy under the goal to maintain $o \approx o_{cr}$ by controlling the merging pattern with the assistant of a control algorithm and feedback. From a macroscopic perspective, the maximum throughput can be achieved by a certain merging pattern design. As a result, our goal is to model this design.

Feedback control based on ALINEA

We are inspired by a scheme from a previous article (*Real-time merging traffic control with applications to toll plaza and work zone management*,2008), and decide to deploy traffic lights to individual lanes as control devices.

However, the most crucial task is to determine the form of feedback control.

We suppose that the feedback control is activated at each discrete time interval. After activation, it will collect latest measurements of occupancy o, and send data-converted instructions to control devices under the purpose of maintaining $o \approx o_{cr}$. Thus, we choose to apply ALINEA as our control algorithm.

ALINEA can be expressed as:

$$q\left(n\right) = q\left(n-1\right) + K_R\left[\hat{o} - o(n-1)\right]$$

Where,

n	The discrete time index
$q(n)$	The controlled entering flow (veh/h) to be implemented in a new time step n
$q(n-1)$	The existed entering flow (veh/h) in last time step
$o(n-1)$	The measured occupancy of merge area in last time step
$\hat{o}$	The desired value of occupancy (can be set as o_{cr})
K_R	A regulator parameter, always positive

In addition, the occupancy measurement should best be placed at or just upstream of the location where serious vehicle decelerations (congestion) appear first.

8 Conclusion

9 Sensitivity Analysis

9.1 The Performance of Our Solution in Light and Heavy Traffic

9.2 Autonomous Vehicles

9.3 The Proportions of Different Tollbooths

10 Strengths and Weaknesses

10.1 Strengths

10.2 Weaknesses

References

[1] D. E. KNUTH The TEXbook the American Mathematical Society and Addison-Wesley Publishing Company , 1984-1986.

[2] Lamport, Leslie, LATEX: " A Document Preparation System ", Addison-Wesley Publishing Company, 1986.

Appendices

Appendix A First appendix

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
       aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===========================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===========================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
```

```
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```