Cryptocurrency Price Bubble Detection Using Log-Periodic Power Law Model and Wavelet Analysis

Junhuan Zhang, Haodong Wang, Jing Chen, and Anqi Liu

***Abstract*—In this article, we establish a method to detect and formulate price bubbles in the cryptocurrency markets. This method identifies abnormal crashes through violations of the exponential decaying property. Confirmations of bubble bursts within these anomalies are obtained through wavelet analysis. By decomposing the cryptocurrency price into the high-frequency and low-frequency factors, we distinguish the price regimes versus the periods with bubbles and crashes in both time and frequency domains. In addition, we apply the log-periodic power law model to fit the bubble formation. In the analysis of eight cryptocurrencies—Bitcoin, Ethereum, Litecoin, Antshares, Ethereum Classic, Dash, Monero, and OmiseGO—from 15 May 2018 to 28 November 2022, we identify 24 bubbles. Some of them exhibit a significant and strong exponential growth pattern.**

***Index Terms*—Cryptocurrency, financial crises, log-periodic power law (LPPL), price bubbles, wavelet analysis.**

I. Introduction

In 2008, Nakamoto published "Bitcoin: A Peer-to-Peer Electronic Cash System," proposing a new, decentralized paradigm for money that provides an alternative to the current centralized banking system. As a result, Bitcoin (BTC) was launched and started to operate as "a currency" in the following year. A few years later, multiple cryptocurrency exchanges launched and BTC, together with a variety of new cryptocurrencies swiftly gain strong investment attention.[1] BTC has remained the most dominant cryptocurrency for trading and investment and has been associated with many episodes of extreme price movements that triggered turbulence in both cryptocurrency and general financial market. For instance, after the BTC price first climbed above \$20 000 on 30 November 2020, it soon reached a historical high of \$68 521 on 5 November 2021. However, this climax only lasted for a few days before a sharp stumble to around \$42 000. Since, the BTC prices are widely witnessed with a general descending trend accompanied with frequent and strong short-term fluctuation until it fell below the \$20 000 mark, to \$19 420 on June 18 2022, representing more than \$200 billion market capitalization vaporization[2]—this had sparked Ethereum (ETH) price to slide sharply and caused concerns of wider cryptocurrency market crash. Renowned scholars such as the Nobel Prize winner for Economics, Prof. R. J. Shiller, stated that BTC portrays the best example of speculative bubble and irrational exuberance.[3] Many professionals echo Shiller's view: some suggest a cryptocurrency bubble could eventually result in hyperinflation[4] and some even call the BTC investment an epidemic that would expose the originally healthy and functional monetary system and financial market to a range of threats and instability.[5]

While cryptocurrencies differ from fiat currencies, crypto exchange prices show some similar statistical mechanisms to the traditional financial assets. For example, using a network approach, Liang et al. [1] concluded that the crypto market is close to the stock market from a systemic perspective, but it is more fragile. This similarity can be explained by investors' unchanged trading habits in this new market. Liu et al. [2] observed groups of traders in the BTC market act alike to market makers, high-frequency traders, fundamental investors, etc. Ha and Lee [3] replicated properties of BTC exchange microstructure in an agent-based model that includes strategic traders and their social interactions. Nevertheless, the majority of participants in the cryptocurrency market are nonprofessional investors who act irrationally, emotionally, and impulsively, thus leading to more sentiment-induced or speculative bubbles and crashes [2], [4], [5], [6]. Understanding the life cycle of such a bubble traces back to the fundamental drivers of crypto prices. In an empirical study [7], it has been verified that, in the long-run, the BTC capitalization follows the Metcalfe's law—price is proportional

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[1]Bitstamp, founded in 2011, is usually considered as the first cryptocurrency exchange.

[2]https://www.forbes.com/sites/siladityaray/2022/06/18/price-of-bitcoin-drops-below-20000-as-crypto-crash-continues/

[3]https://qz.com/1067557/robert-shiller-wrote-the-book-on-bubbles-he-says-the-best-example-right-now-is-bitcoin/

[4]https://techtelegraph.co.uk/how-a-bubble-in-bitcoin-could-lead-to-hyperinflation/

[5]https://www.bloomberg.com/opinion/articles/2021-06-09/don-t-call-bitcoin-a-bubble-it-s-an-epidemic

to the square of the number of active users. This finding aligns with the economic model established by Ciaian et al. [8], which implies that the BTC price decreases with the velocity and the stock of coins and increases with the size of BTC economy and its purchasing power. Ciaian et al. [8] also pointed out that many investors, albeit are not active users in the crypto economy, involve in the market based on their credibility in the security of the blockchain system and exchanges. They tend to demand and hold cryptocurrencies due to their belief that the "coins" will be much more valuable and can always be accepted by exchanges safely, leading to herding behaviors. This explains why events such as the futures exchange (FTX) hack, commodity futures trading commission (CFTC) lawsuit against Binance, liquidation of the stablecoin TerraUSD, and bans on crypto derivatives trading, among others, often precede crypto market crashes (see examples in [9]).

As the crypto economy expands, exchanges such as Binance, Coinbase, and Bybit feature well-established market structures, high trading volumes and reliable liquidity providers, facilitating smooth trading in the crypto market. Considering cryptocurrencies as a medium of exchanges among fiat currencies in the financial market, the traditional stochastic pricing processes such as stochastic volatility models and jump diffusion models should apply in the absence of server sentiment trading activities [10], [11]. Some empirical studies shed light on the impact of social media and market sentiment on crypto bubbles; see, for example, [12], [13], and [14]. The crypto market is also known for its unregulated speculative environment, foresting sentiment-driven trading that spreads such as a pandemic and eventually creates bubbles [15]. Hence, regarding technical interpretation of crypto bubbles, the authors in [16] and [17] introduced a stochastic attention factor into the stochastic volatility model and demonstrated the theoretical contribution of sentiment-based trading to bubble behaviors. In a laboratory environment, another study [18] verified the presence of super-exponential bubbles are caused by excess speculation in markets where asset prices lack fundamental values, such as the crypto market. Considering the faster-than-exponentially growing bubbles and volatile swings in the crypto market, several studies extended the standard augmented Dickey–Fuller (ADF) unit root test to approaches such as supremum-, generalized supremum-, and backward supremum-ADF to detect crypto bubbles [19], [20]. The authors in [21], [22], and [23] found that various factors, ranging from fundamental sources to speculative and technical ones, influence crypto prices in different time–frequency domains; and social attention has a long-term impact. It is worth noting that the idea of using wavelet decomposition in this article aligns with the findings on trading behaviors and sentiment impact under different frequencies mentioned in the aforementioned literature.

Our study is motivated by the theoretical background of cryptocurrencies' price and bubble formation mentioned previously, as well as the benefits of crypto bubble analysis offers in helping investors avoid the illusion of wealth and irrational exuberance. We construct a drawdown-wavelet-LPPL process to detect and comprehend the full life cycles of crypto bubbles. To timestamp the peak or bursting of bubbles, we use the bubble detection technique relates to finding outliers in the ε-drawdown [24], [25]. The outliers can be defined according to individual requirements, for example, Johansen and Sornette [25] took the top 2% samples, and Johansen [26] defined a threshold based on the historical volatility. But the common approach involves assessing how significantly the "drawdown" diverges from the theoretical tail model—a stretched exponential distribution or an exponential law [24], [25], [27]. The effectiveness of ε-drawdown bubble detection technique in the crypto market is well substantiated by the presence of superexponential bubbles and theoretical stochastic prices, while only one study [28] has applied this method in the existing literature. Given the volatile nature of the crypto market, we think the method has not been widely adopted due to its potential tendency to overcount normal consecutive decreases, such as those seen during market downturns, as bubble crashes. We involve a multilevel wavelet decomposition to address this issue.

The rationale of applying wavelet analysis lies on the observation that various types of traders, strategies, and response times inherently generate time-localized oscillations in price. Several studies adopted the wavelet coherence measure to examine comovements among different cryptocurrency prices [5], [29], [30], [31]. Multiresolution analysis of memory, fractal dynamic, and other spectral properties of the crypto market were also facilitated by wavelet transform analysis [32]. We find that Daubechies 4 (db4) is commonly used in decomposing financial time-series data; see, for example, [30], [31], and [33]. Rao et al. [34] confirmed that, when extracting prediction signals from financial data, db4 performs the best among members of the Daubechies family dbN, where $N = 1, 2, \ldots, 6$. According to these studies and the cryptocurrency price trends, we think that the support of db4 is small enough to detect closely spaced features. By removing low-frequency trends and high-frequency noise, wavelet analysis becomes a useful tool for exposing bubble formations among normal market fluctuations. Results from the db4 wavelet analysis and the ε-drawdown can jointly confirm full bubble cycles. Moreover, finding the bubble formation regime is a key step that allows us to go beyond identifying bubbles to further explore the modeling of the entire life cycle of bubbles.

The authors in [35] and [36] suggested that the financial bubble pattern is a power law decorated with log-periodic oscillations, leading to the construction of the log-periodic power law (LPPL) model. Geraskin and Fantazzini [37] comprehensively illustrated details of the LPPL model from its theoretical model construction to empirical calibrations (also see [38]), resulting in a series of applications of diagnosing bubbles in financial markets (for example, [39], [40], [41], [42], and [43]). Several variants of the LPPL model have also been developed to accommodate markets experiencing higher volatility and sharper fluctuations within shorter time frames [38], [44]. In addition, Papastamatiou and Karakasidis [45] developed the DS-LPPLS (a.k.a., Didier Sornette log-periodic power law singularity) model that detects both positive and negative bubbles. The applications of LPPL models in the cryptocurrency market is limited. Only a few have studied BTC market bubbles through incorporating with methods such as ε-drawdown, extended ADF-based, the generalized Metcalfe's Law, and the price-electricity cost ratio [7], [28], [46], [47]. To model the superexponential bubble formation and rapid price surge explosion, we employ LPPL models with an exponential kernel. These models feature shape and decay parameters that describe the formation and development of bubbles.

To summarize, we propose a step-wise approach to fill the prominent gap of crypto bubble analysis. Our analytical procedure provides accurate times and scales of crypto bubbles using the ε-drawdown and the db4 wavelet analysis, then formulates the growth of bubbles through the LPPL model. By investigating a variety of highly traded cryptocurrencies in the market, including BTC, ETH, Litecoin (LTC), Antshares (NEO), Ethereum Classic (ETC), Dash (DASH), Monero (XMR), and OmiseGo, we not only demonstrate the effectiveness of our methodology but also contribute to providing a comprehensive bubble profile of the cryptocurrency market and comparing it to traditional markets. To capture activities of different types of investors in the time–frequency domain, we use hourly data for the empirical analysis. Such high-frequency data ensures that we do not miss short-lived bubbles with small vibrations. We identify 24 bubbles in the eight different cryptocurrencies. According to the LPPL model fitting results, we conclude that crypto bubbles tend to have weaker periodicity a much faster pace of expansion than stock bubbles.

The rest of this article is organized as follows. Section II sets up the bubble crash identification method using the ε-drawdown-wavelet-LPPL model; Section III explains cryptocurrency data used in this study; Section IV reports results including bubble regimes, bubble formation patterns, and timestamps of bubble bursts; Section V discusses periodicity and growth parameters in the LPPL model for crypto bubbles and the differences from the stock market; and finally, Section VI concludes this article.

II. Methodologies

To detect a bubble and examine its lifecycle, we apply three modeling techniques sequentially, namely ε-drawdown outlier detection, Daubechies wavelet analysis, and the LPPL model. In the drawdown-wavelet-LPPL process, the ε-drawdown method is used to find all possible bubble bursting times; the wavelet analysis is performed to identify the intervals where the actual bubbles exist; and the LPPL model helps pin down the exact time of the bubble burst and other lifecycle features including growth and growth rate of the bubble. To ensure accurate results from this analysis, the timestamps of bubble bursts are cross-checked with the results from both ε-drawdown and LPPL model fitting.

A. Timestamping Bubble Bursts: The ε-Drawdown Method

A "drawdown," in the context of financial markets, generally means the degree of price drops from a local maximum to the next local minimum. The authors in [24] and [25] investigated statistical properties of drawdowns and proposed a cumulative stretched distribution $N(x) = A \cdot \exp(-(\frac{|x|}{\chi})^z)$, such that outliers emerge in the presence of market shocks. A special case $z = 1$ leads to a pure exponential distribution, aligning with the assumption of normal-like and uncorrelated price variations. In this article, we find the outliers in the drawdowns that deviate the exponential law, [as described in (2)], which could potentially indicate the bubble crashes.

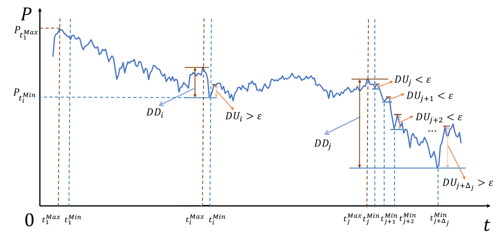


Fig. 1. ε-drawdown search illustration.

Due to the consistent vibrations of the market, calibrating drawdowns are challenging. We use the "loc max vs loc min" approach applied in [48], which measures the loss from the last local maximum to the next local minimum. ε represents the drawdown threshold. According to Johansen and Sornette [24], outliers in price drawdown, fitting to the exponential distribution, indicate the timing of a bubble burst. We apply this method to different cryptocurrency price series that are collected at hourly frequency. To find the sequential downward trends, we ignore the small price increases that occur after the local maximum but within the scale of ε. We follow [49] to decide on the drawdown threshold ε by examining the volatility of a cryptocurrency's log-returns.

Define the price series $\mathbb{P} = \{P_t : t = 0, 1, 2, \ldots, T\}$. We start from the first local maximum $P_{t_1^{\text{Max}}}$ and search for the first minimum price before an increase exceeding ε is observed. This local minimum price is denoted by $P_{t_1^{\text{Min}}}$. Then, we find the next maximum price and repeat this procedure to locate a series of local maximums and minimums $\{(P_{t_i^{\text{Max}}}, P_{t_i^{\text{Min}}}) : i = 1, 2, \ldots, m\}$ where t_i^{Max} and t_i^{Min} are the times of the ith local maximum and local minimum, respectively; and m is the number of drawdowns counted based on each set of local maximum and local minimum.

This drawdown searching process is illustrated in Fig. 1. From the first local maximum $P_{t_1^{\text{Max}}}$ on the left, we move the $P_{t_1^{\text{Min}}}$ forward till the point that a drawup (DU), defined as the degree of price increase from the local minimum to the next local maximum, exceeds the threshold ε. For example, the $\text{DU}_i > \varepsilon$ in the middle part of Fig. 1 indicates the local minimum $P_{t_i^{\text{Min}}}$. In the process, there could be local "bouncing backs" (e.g., the $\text{DU}_{j+1}, \text{DU}_{j+2}, \ldots$ on the right of Fig. 1). When the local DU_{j+k} is below the threshold ε, it suggests that the price will continue to decline and the search for the local minimum needs to carry on (e.g., from $\text{DU}_j, \text{DU}_{j+1}, \text{DU}_{j+2}, \ldots$ until $\text{DU}_{j+\Delta j} > \varepsilon$). With $\{(P_{t_i^{\text{Max}}}, P_{t_i^{\text{Min}}}) : i = 1, 2, \ldots, N_0\}$ identified, the drawdown is calculated as follows:

$$\text{DD}_i = -\ln\left(\frac{P_i^{\text{Max}}}{P_i^{\text{Min}}}\right), i = 0, 1, 2, \ldots, m. \tag{1}$$

According to the research of [25] and [49], $\{\text{DD}_i : i = 1, 2, \ldots, N_0\}$ should follow the exponential law as follows:

$$N(x) = N_0 \cdot \mathrm{e}^{-|x|/\text{DD}_c} \tag{2}$$

where $N(x)$ is the number of drawdowns with the value x (in this article, x is always negative), N_0 is the total number of drawdowns, DD_c is a scale parameter. Obviously, it represents the cumulative distribution function of the exponential distribution. Hence, DD_c denotes the standard deviation of drawdowns.

Outliers mainly come from market crashes and will not follow exponential law. We take the largest δ-quantile as outliers. To avoid their influence on the evaluation of (2), find the DD_c using the remaining normal drawdown samples. The logarithmic version [see (3)] of the exponential law is used to confirm the goodness-of-fit. We present the choice of δ and detailed results in Section IV-A.

$$\ln(N(x)) = \ln(N_0) - \frac{1}{\mathrm{DD}_c} \cdot |x|. \tag{3}$$

It must be noted that the outliers are not necessarily linked to bubble bursts—some of them may be shocks in price downturns (or negative bubbles). But we can say that these timestamps potentially indicate crashes of bubbles after abnormal price drops. We further distinguish the outliers for bubble bursts through cross-checking with the LPPL model in Section II-C.

B. Extracting Bubble Regimes: The db4 Wavelet Analysis

We use wavelet analysis to transform the price series at different frequencies, which further helps us find the bubble regimes. Wavelets are defined by the wavelet function $\psi(t)$, acting as a high-pass filter, and the scaling function $\phi(t)$, acting as a low-pass filter, in the time domain. The discrete wavelet transform (DWT) technique decomposes a signal $\boldsymbol{s}$ (or $\boldsymbol{a_0}$) into a low-frequency "approximation coefficients" $\boldsymbol{a_1}$ and a high-frequency "detail coefficients" $\boldsymbol{d_1}$, then $\boldsymbol{a_1}$ is further decomposed to $\boldsymbol{a_2}$ and $\boldsymbol{d_2}$, and so on. Given the number of coefficients (also called support width) m, this procedure is written as follows:

$$\begin{aligned} a_{k+1,t} &= \sum_m \phi(m-2t+2)a_{k,t} \\ d_{k+1,t} &= \sum_m \psi(m-2t+2)a_{k,t}. \end{aligned} \tag{4}$$

We use the db4 wavelet for three reasons. First, it is relatively smooth in time domain, which is an important feature when detecting abrupt changes in the data. Second, we need the number of vanishing moments to exceed the degree of the polynomial of the signal. By eyeballing the price trends of the cryptocurrencies, we believe that a vanishing moment of 4 is sufficient. Third, existing literature also confirmed that db4 is a suitable wavelet for financial price decomposition [30], [31], [33], [34]. Fig. 2(a)–(d) shows the scaling function, the wavelet function, the low-pass filter, and the high-pass filter of the db4 wavelet, respectively.

Remark II.1: If the original signal is fast mean-reverting, then $\boldsymbol{d_n}$ converges to 0 as $n \to \infty$. If the original signal is slowly mean-reverting or monotonically increasing (or decreasing), then $\boldsymbol{d_n}$ explodes as $n \to \infty$.

Now, we contextualize the application of DWT under the context of our research, wherein the logarithmic price of a cryptocurrency serves as the original signal. We know that an orderly market under good economic conditions should show a long-term trend and short-term reversal price pattern, which is a combination of the two cases in Remark II.1. Hence, for a healthy price movement, with an appropriate selection of the decomposition level n, we should observe decomposed properties as follows.

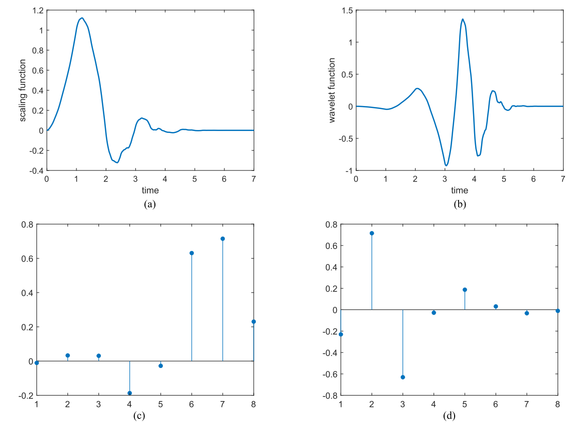


Fig. 2. Daubechies 4 (db4) wavelet. (a) Scaling function. (b) Wavelet function. (c) Low-pass filter. (d) High-pass filter.

1) $\boldsymbol{a_n}$ shows the trend.
2) The first few levels of high-frequency components, e.g., $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$, are noises with cyclic patterns. This is because trend and reversal produce opposite noises.
3) $\boldsymbol{d_n}$ is a white noise near zero. This is because in relatively lower frequency scale, trend and reversal noises cancel each other out.

However, when a bubble is formed, the market tends to have an extremely long-term, sharp price increase. Using Remark II.1, $\boldsymbol{d_n}$ of this trend stands out from the normal near-zero coefficients. The authors in [50] and [51] also find that the maxima of the continuous wavelet transform can reveal the structure of the time series. Hence, we detect peaks of $\boldsymbol{d_n}$ to approximate times of price trend changes, which is likely to be bubble crashes. In other words, the maxima of $\boldsymbol{d_n}$ splits the price series into several regimes. From one peak to the following one, we say it is one price regime. According to the results of LPPL model fitting (see Sections II-C and IV-B), we can confirm whether it is a bubble regime or a stable price regime.

C. Fitting the Bubble Formation: The LPPL Model

The LPPL model is a widely adopted modeling technique for financial bubbles. It involves parameters of bubble growth rate, oscillation frequency, and length of time it takes to form a bubble [see (5)]. The study in [52] serve as a crucial reference that explains the construction and calibration intricacies of the LPPL model. We recap mathematical rationale for the model in Appendix A and start with the LPPL equation (5). The LPPL model is given as

$$\ln(P_t) \approx A + B\,(t_c - t)^{\beta} + C\,(t_c - t)^{\beta} \cos\left(\omega \ln\left(t_c - t\right) - \phi\right) \tag{5}$$

where a, b, β, and ϕ are parameters, and t_c is the time when the bubble burst. Besides, we have $B = -\kappa a/\beta$ and $C = -\kappa ab/\sqrt{\beta^2 + \omega^2}$. To interpret this model from the perspective of the bubble regime, $\ln(P_{t_c}) = A$ given by $t = t_c$, hence, A is the logarithm of the price when the bubble bursts. B shows the price trend in the bubble formation. $B < 0$ indicates $\kappa > 0$ so that represents a positive bubble, and vice versa. Moreover, β represents the speed of the price increases, ω reflects the periodicity of price changes, and C is the strength of periodic

TABLE I
BASIC DATA INFORMATION

Cryptocurrency	Symbol	Start Date/Time	End Date/Time	Num. Obs.
Bitcoin	BTC	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Ethereum	ETH	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Litecoin	LTC	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Antshares	NEO	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Ethereum Classic	ETC	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Dash	DASH	2018-05-15 06-AM	2022-11-28 01-AM	39 779
Monero	XMR	2018-05-15 06-AM	2022-11-28 01-AM	39 779
OmiseGO	OMG	2018-05-15 06-AM	2022-11-28 01-AM	39 779

fluctuation. Solving the large parameter set of the LPPL model is computationally complex and at risk of overfitting. Also, nonlinear optimization tends to fall into local optimization with the increasing number of parameters to be estimated. Filimonov and Sornette [52] addressed this issue by rearranging (5) to as follows, which reduced the number of nonlinear parameters and increase the number of linear parameters:

$$\ln(P_t) \approx A + B\left(t_c - t\right)^{\beta} + C_1\left(t_c - t\right)^{\beta}\cos\left(\omega \ln\left(t_c - t\right)\right) + C_2\left(t_c - t\right)^{\beta}\sin\left(\omega \ln\left(t_c - t\right)\right) \tag{6}$$

where $C_1 = C\cos\phi$ and $C_2 = C\sin\phi$.

To summarize, the parameter set is $\{\omega, \beta, t_c, A, B, C_1, C_2\}$. Recall that we extract the bubble regimes using wavelet analysis (in Section II-B). For each bubble regime, we fit the LPPL model to present different bubble formation processes under different market conditions. Regarding parameter constraints, Sornette and Johansen [53] noted that a positive value of β can guarantee that A is a finite value, whereas $\beta > 1$ may result in an unrealistic prediction of the asset prices. The common range of β for the stock market is 0.33 ± 0.18 [36]. A few studies gave the range of ω for the stock market, e.g., 6.36 ± 1.56 in [36] and $6 \leq \omega \leq 13$ in [52]. Xiong et al. [47] mentioned that the constraint for the BTC market in their experiment was $2 < \omega < 20$. In our model calibrations, we take these restrictions into account but do not fully "copy" the setting of previous studies. It is worth noting that the fitted LPPL model may just describe fast-changing trends and reversal price patterns under volatile market conditions rather than a real bubble. This is because we face difficulty in distinguishing between these two scenarios when using wavelet analysis to extract bubble regimes. Hence, we identify valid LPPL models for bubbles through the timestamps found by ε-drawdown (in Section II-A). If the predicted time t_c and the ε-drawdown timestamp are within one day, we believe that the bubble regime and the LPPL model are good fits for a real bubble formation.

III. DATA

Our crypto price data are from the Bitfinex—one of the exchanges that contributes the largest cryptocurrency trading volume. We select the top-eight cryptocurrencies paired with USD by trading volumes: BTC, ETH, LTC, NEO, ETC, DASH, XMR, and OmiseGO (OMG) (see Table I). In order to perform robust bubble identification and wavelet analysis, as well as to show some microscopic properties of crypto trading, we use hourly data to ensure sufficient observations that cover intraday and high-frequency trading activities of these typical cryptocurrency markets. The data are from 15 May 2018 6:00:00 to 28 November 2022 1:00:00.[6]

We present basic statistics of cryptocurrencies' daily log-returns and compare those with major Chinese equity indexes,[7] including the Hang Seng index (HSI), Shanghai stock exchange composite index (SSEC), and Shenzhen component index (SZI), in Table II. The same time horizon are used for these indexes.It is evident that the kurtosis of cryptocurrencies are substantially higher than stock indexes. All crypto assets exhibit fat-tail, or more accurately heavy-tail features, which suggests the cryptocurrency markets are more likely to experience extreme events. The much greater maximum and lower minimum returns in cryptocurrency markets, compared to those in stock markets, further support that the cryptocurrency market is much more volatile. Most markets have negative skewness, except DASH and HSI. Most cryptocurrencies are much more negatively skewed than the stock indexes, indicating the opportunities to grab quick returns if investing in them. The BTC and ETH are more aligned with the stock indexes in the skewness, which shows that their risk and return profiles are becoming similar to traditional assets. While most skewness are small, we can still say that the returns of these assets are nearly symmetric. The two exceptions are LTC and NEO, which show extremely strong negative skewness, -2.2268 and -2.937, respectively. Due to the less than 5 years of data coverage and the common market downturns of cryptocurrencies in those years, we cannot conclude that returns of LTC and NEO consistently skew to the left. All cryptocurrencies and stock indexes have comparable levels of volatility. In the cryptocurrency market, the volatility of BTC is the smallest, followed by ETH. Connecting to the discussion of kurtosis, what we find interesting is that although the risk levels of these markets are similar, the chances of

[6]The DASH price data are only available from 15 May 15 2018 to 15 November 2020. In order to ensure consistency of the time span, we obtain the final price series by concatenating the DASH/USDT price from 15 May 2020 to 28 November 2022 on the Binance with the former prices from the Bitfinex. We are aware of the FTX liquidity crisis, within which Binance played an explicit role. However, this event has not affected us to obtain data. In fact, our results (e.g., Fig. 7) may have provided insights into the bubble burst linked to this liquidity event.

[7]Other indices can surely be selected. We choose these Chinese equity indexes for pure experimental purposes, especially since it is known that the cryptocurrency trading is heavily prevalent in China, onshore before some major regulatory intervention and offshore afterward.

TABLE II
STATISTICS OF LOG RETURNS

	Kurtosis	Skewness	Variance	Mean	Maximum	Minimum
BTC	45.464	−0.404	0.000062	0.000016	0.181	−0.190
ETH	28.241	−0.602	0.000103	0.000012	0.168	−0.231
LTC	96.590	−2.268	0.000133	−0.000017	0.221	−0.452
NEO	138.120	−2.937	0.000168	−0.000059	0.199	−0.588
ETC	44.979	−0.685	0.000168	0.000000	0.265	−0.327
DASH	40.779	0.234	0.000146	−0.000060	0.335	−0.289
XMR	71.342	−1.683	0.000123	−0.000012	0.246	−0.379
OMG	73.222	−0.914	0.000222	−0.000066	0.399	−0.495
HSI	6.529	0.236	0.000211	−0.000457	0.087	−0.066
SSEC	7.672	−0.585	0.000136	−0.000033	0.056	−0.080
SZI	5.610	−0.488	0.000223	0.000030	0.054	−0.088

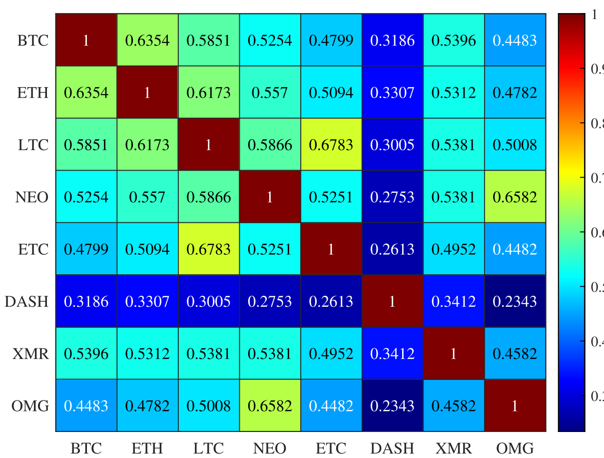


Fig. 3. Correlation coefficients of the cryptocurrencies.

extreme events in the cryptocurrency market are prominently higher than those in the stock market.

Fig. 3 shows the correlation coefficient matrix of the cryptocurrencies. It suggests that most of these cryptocurrencies, except DASH and OMG, have strong positive correlations. This explains the contagion effects among the cryptocurrency system, which raise concerns about systemic crashes. For example, when BTC prices crashed in 2021, ETH was also greatly affected and the impact spread over the wider cryptocurrency markets.

IV. Results

We present the results of bubble detection and the LPPL models in this section.

A. Outliers of ε-Drawdown

We identify outliers of drawdown given $\delta = 2\%$, which has been verified as a suitable choice for financial markets by Johansen and Sornette [24],[25]. The parameters of the exponential law for each cryptocurrency are provided in Table III. To recap, N_0 is the number of drawdowns, and DD_c is the standard deviation of the drawdowns. We exclude drawdowns smaller than 1% as they fall within one standard deviation of returns and should therefore be considered normal price movements. Fig. 4 verifies the fitting to the exponential law and shows outliers of ε-drawdown of each cryptocurrency.

TABLE III
PARAMETERS OF THE EXPONENTIAL LAW

	$\ln(N_0)$	$1/\mathrm{DD}_c$	N_0	DD_c	RMSE
BTC	7.40792	43.03410	1649	0.02324	0.39752
ETH	7.46965	31.69424	1754	0.03155	0.30969
LTC	7.50329	31.65235	1814	0.03159	0.33576
NEO	7.55538	27.50106	1911	0.03636	0.26752
ETC	7.53636	26.94411	1875	0.03711	0.28160
DASH	7.54908	28.39999	1899	0.03521	0.31638
XMR	7.49665	31.95773	1802	0.03129	0.30379
OMG	7.58579	24.74334	1970	0.04041	0.36706

The largest drawdown of BTC is close to 36% and it is around 42% for ETH. Compared with other cryptocurrencies, these are the smallest ones. The consecutive price decreases of most of the other cryptocurrencies have reached over 60%. In particular, other cryptocurrencies, such as XMR and OMG, have experienced several severe market fluctuations. This finding indicates that the BTC price is relatively stable among the entire cryptocurrency market.

B. Bubble Regimes and Bubble Formations

Recall that we use the detailed coefficient $\boldsymbol{d_n}$ from the n-level DWT filter bank to determine the bubble regimes. We face trade-offs when selecting the level of the filter bank. A higher level (i.e., larger n) is helpful for smoothing out noises and provides a more robust result, however, it is weak in the time domain and may miss some short-lived bubbles. The largest n we can apply in this study is 15 according to the number of observations. After examining the results of selections of n, we find the ten-level filter bank gives the most reliable bubble regime detection.

Fig. 5 presents decomposition of the BTC price by "db4" wavelet, in which "s" is the log-price, "d1," "d2,"..., "d10" are the ten-level high-frequency factors of the log-price, and "ca10" is the smoothed trend. We observe that the high-frequency factors from the first few levels' decomposition give some insights on the periods of stable market and abnormal upturns or downturns. However, these components are still too volatile to extract bubble regimes. The splits of regimes become clearer in $\boldsymbol{d_9}$ and $\boldsymbol{d_{10}}$. In this study, we choose the local maximum from the component $\boldsymbol{d_{10}}$. To ignore local maximum from small vibrations during the period of stable market, we set a restriction on the bubble regimes—a minimum of 1500 h (approximately 2 months). The bubble regimes for BTC are shown in Fig. 6(a) and (b). Furthermore, the price decomposition of

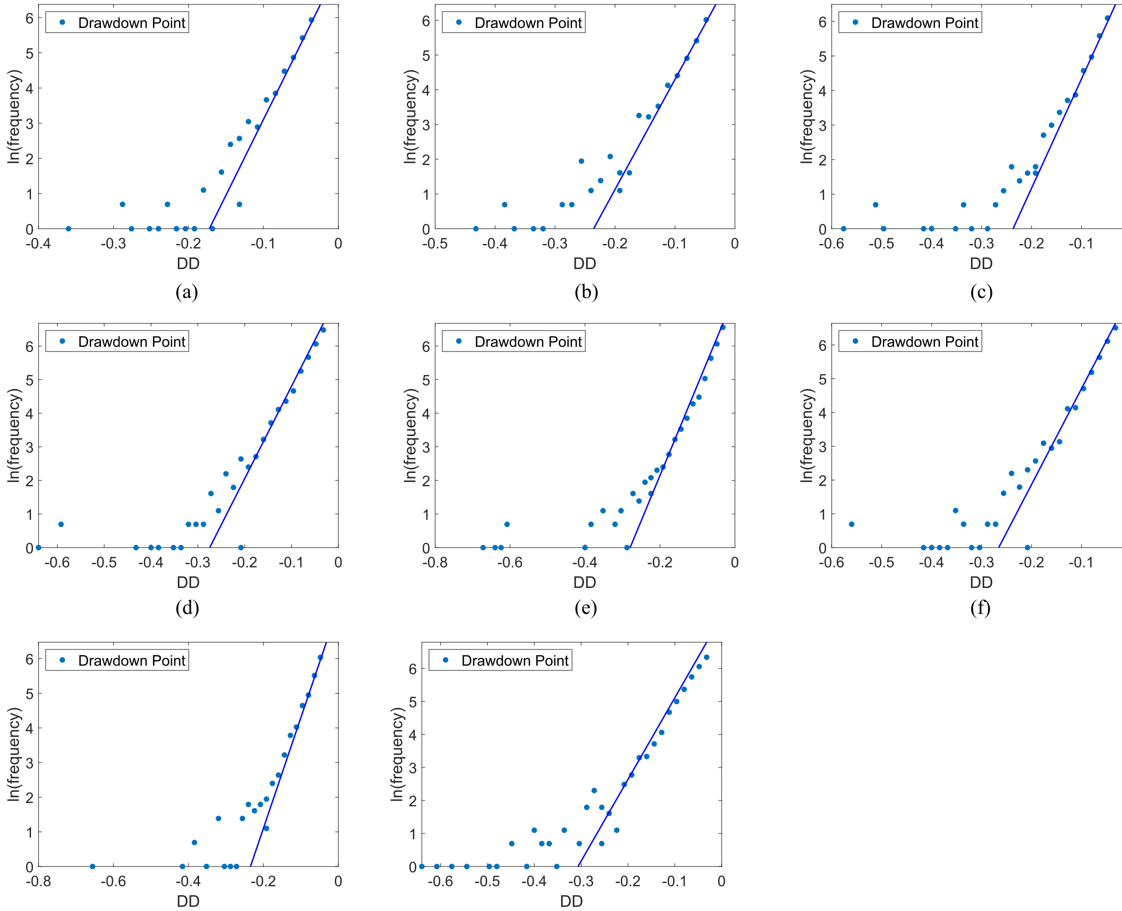


Fig. 4. Outliers of ε-drawdowns. (a) Outliers of BTC. (b) Outliers of ETH. (c) Outliers of LTC. (d) Outliers of NEO. (e) Outliers of ETC. (f) Outliers of DASH. (g) Outliers of XMR. (h) Outliers of OMG.

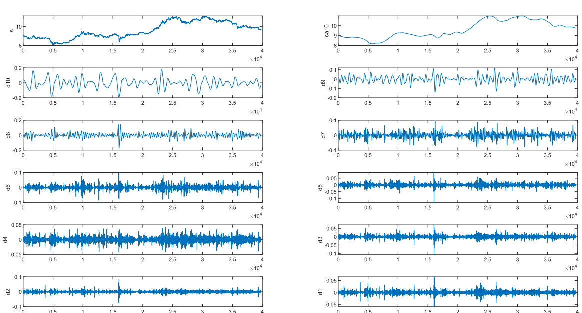


Fig. 5. Wavelet analysis of BTC.

the other seven cryptocurrencies except for BTC is shown in Appendix B (see Fig. 18), and the bubble regimes are shown in Appendixes C and D (see Figs. 19 and 20, respectively). We use t_i to denote the starting time of the ith bubble regime. Let $\mathbb{P}_i$ denote the price series from the beginning of the ith bubble regime to the beginning of the $(i+1)$th bubble regime, i.e., $\mathbb{P}_i = \{P_{t_i}, P_{t_i+1}, \ldots, P_{t_i+k}, \ldots, P_{t_{i+1}}\}$. Hence, each interval ends with the initial price of the next interval. The timestamps of some large crashes in Fig. 6(b) are obvious, while we still need to utilize the LPPL model to obtain comprehensive descriptions of the bubble growth. Most of the bubbles start to form at the beginning of each regime. We search for the timing of bubble burst by fitting the LPPL model to each subset of the price series $\mathbb{P}_i$. The calibration method is in [54]. To enhance the efficiency of model fitting and find realistic growth patterns, we constrain $0.01 \leq \beta \leq 0.99$ (also see [41]). Note that after LPPL model fitting, we confirm that some bubbles are shorter lived than the bubble regimes we find by wavelet analysis.

We use t_{DD} to denote the timestamp of the crash found by the ε-drawdown. Here, we set a constraint on t_c in the LPPL model: $|t_c - t_{\mathrm{DD}}| < t_{\mathrm{acc}}$. When taking $t_{\mathrm{acc}} = 24$, we ensure that the bubble burst time "errors" between the LPPL model and the ε-drawdown method are within 24 h, or a day. Given this restriction, the LPPL models of BTC bubbles are in Table IV. Corresponding model fitting results on BTC price movements in each bubble regime are shown in Fig. 7 and the specific bubble burst time given by t_c is presented as the title of each subfigure. To better observe the price changes of cryptocurrencies following the burst of bubbles, we extend the prices for 168 periods (equivalent to a week) from the moment of the bubble burst and present them together in the figures.

C. Robustness Checks

We analyze the impact of different choices of δ, Daubechies wavelet types, and t_{acc} on the results.

Firstly, we expand the choice of δ to 1%, 1.5%, 2.5%, and 3% and check the goodness-of-fit for the exponential law. By

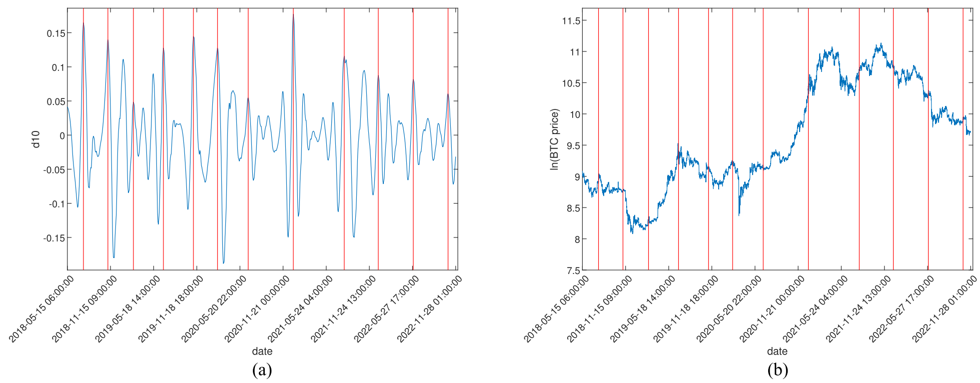


Fig. 6. Bubble regimes of BTC. (a) $\boldsymbol{d_{10}}$ component. (b) Logarithmic price.

TABLE IV
LPPL MODELS FOR BTC BUBBLES ($t_{\text{ACC}} = 24$)

t_{DD}	t_c	ω	β	A	B	C1	C2
2708	2726.54	6.974	0.923	8.915	−0.0006	0.000031	−0.000077
9781	9768.41	2.759	0.445	9.454	−0.0299	0.004644	−0.003286
17411	17398.00	3.690	0.617	9.234	−0.0077	0.001374	−0.000187
24352	24349.20	4.885	0.857	10.986	−0.0023	−0.000108	0.000302

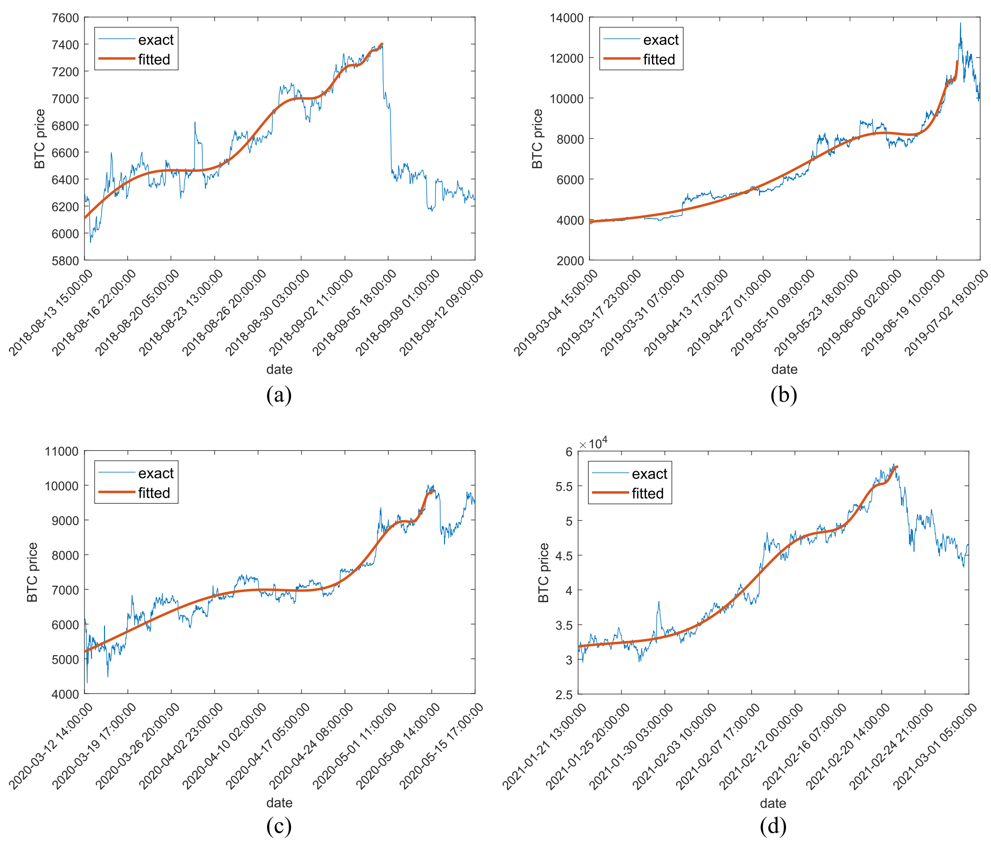
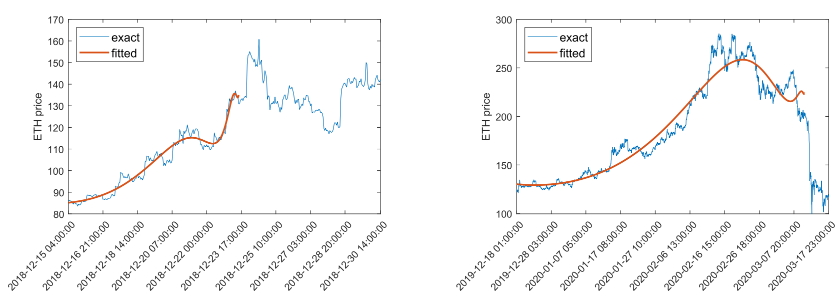


Fig. 7. LPPL models fitting to BTC bubble regimes. (a) 5 September 2018 9:00. (b) 26 June 2019 19:00. (c) 8 May 2020 17:00. (d) 22 February 2021 5:00.

examining the values of root mean square error (RMSE) in Tables III and VI, we find that these choices do not generate better fitting than $\delta = 2\%$. We observe that as δ increases (i.e., δ equals 2.5% and 3%), the number of samples used for parameter fitting decreases, resulting in an increase in RMSE values. In such cases, a larger δ value could also lead to normal price fluctuations being falsely categorized as outliers. Conversely, the smaller δ (i.e., 1% and 1.5%) results in a bit volatile RMSE. Overall, $\delta = 2\%$ manages to maintain all RMSE below 0.4, which is the best consistent result. The other four δ values (i.e., $1\%, 1.5\%, 2.5\%$, and 3%) all result in some RMSE values greater than 0.4, which are highlighted in bold in Table III. Moreover,

TABLE V
LPPL Model Parameters for Crypto Bubbles

	t_{DD}	t_c	ω	β	A	B	C1	C2
BTC	2708	2726.54	6.974	0.923	8.915	–0.0006	0.000031	–0.000077
BTC	9781	9768.41	2.759	0.445	9.454	–0.0299	0.004644	–0.003286
BTC	17411	17398.00	3.690	0.617	9.234	–0.0077	0.001374	–0.000187
BTC	24352	24349.20	4.885	0.857	10.986	–0.0023	–0.000108	0.000302
ETH	5361	5342.08	3.307	0.262	5.164	–0.1592	0.000036	0.020058
ETH	16001	15978.00	2.138	0.914	5.410	0.0000	0.000607	0.000010
LTC	15917	15894.00	8.738	0.658	4.158	–0.0027	–0.000533	0.000740
LTC	22206	22214.00	4.496	0.310	4.520	–0.0614	–0.016009	–0.007373
LTC	30694	30681.50	5.137	0.682	5.445	–0.0023	0.000684	0.000791
LTC	34507	34490.00	3.884	0.800	4.691	0.0005	0.000287	–0.000585
NEO	25647	25624.00	4.295	0.626	4.382	–0.0111	–0.000609	–0.001580
NEO	31139	31134.50	2.329	0.318	3.520	0.0324	–0.002364	0.010395
ETC	11927	11906.00	2.822	0.551	1.826	0.0008	–0.000685	–0.001851
ETC	15881	15866.70	4.368	0.798	2.109	–0.0005	0.001277	–0.000667
ETC	24322	24319.60	2.967	0.660	2.934	–0.0132	–0.000950	0.003365
DASH	5361	5338.04	2.210	0.729	4.548	–0.0060	–0.001757	0.002789
DASH	24131	24108.00	3.418	0.616	4.882	–0.0035	–0.000099	–0.001080
DASH	39304	39312.40	4.540	0.063	4.978	–0.9020	0.006514	0.022440
XMR	23562	23541.60	4.093	0.441	5.123	–0.0083	0.000798	0.005234
XMR	27159	27136.00	3.841	0.782	5.551	0.0008	0.000089	0.001288
OMG	5351	5332.14	2.741	0.495	0.562	–0.0230	–0.000262	–0.010207
OMG	16001	15978.00	2.740	0.742	–0.128	0.0007	–0.000373	–0.002036
OMG	26118	26095.40	3.453	0.386	2.228	–0.0198	–0.020069	0.004624
OMG	29703	29680.40	2.441	0.265	3.050	–0.1984	–0.022961	0.019910



(a) (b)

Fig. 8. LPPL models fitting to ETH bubble regimes. (a) 23 December 2018 14:00. (b) 10 March 2020 23:00.

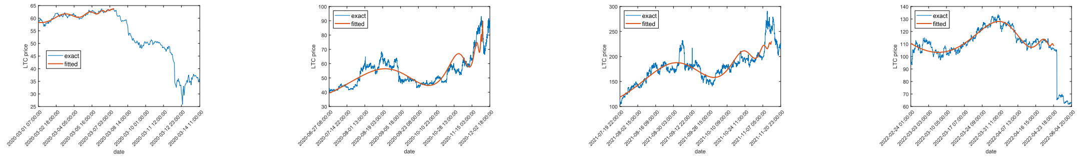
the additional outliers given by these choices do not indicate new bubble regimes from results of the wavelet analysis. So following the study in [25], we set the value of δ to 2% in this study.

We then investigate the use of different types of Daubechies wavelets similar to the test conducted by Rao et al. [34]. We find that db5 and db6 wavelets have too large vanishing moments to construct price trends, which means they are unable to produce reliable price regime split results. While the db3 wavelets are also effective in finding the price regimes, there are no additional bubble regimes after matching to the ε-drawdown outlier timestamps.

Finally, we increase the t_{acc} to 36,48 (hours) to test the robustness of the life cycles of bubbles and LPPL model fitting. When $t_{\text{acc}} > 24$, loosening the constraint on t_c does not lead to the discovery of more valid bubbles. In the identification of bubbles for the other seven cryptocurrencies, increasing the value of t_{acc} also does not result in the detection of additional cryptocurrency bubbles. From Table IV and Fig. 7, we confirm that, overall, the LPPL model fits well to cyclical and exponential bubble growth patterns and can capture the time when bubbles burst and the peak prices. We also observe that some bubbles have obvious periodicity [e.g., Fig. 7(a)], while others are nearly monotonic [e.g., Fig. 7(b)]. For those that have stronger periodicity, we should get larger values of ω.

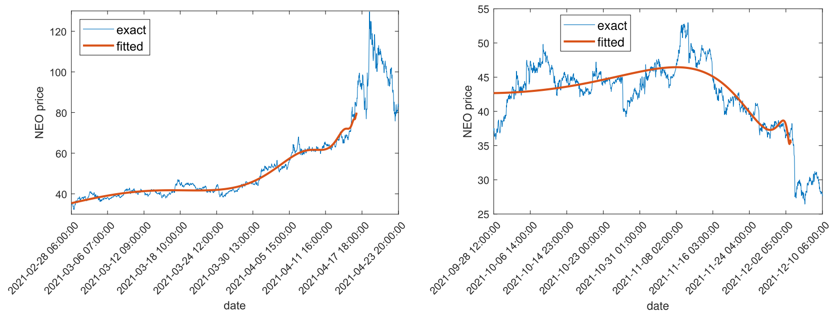
Overall, we apply the same method to all eight cryptocurrencies. In total, we find 24 bubbles (see Table V). Figs. 8–14 illustrate the detected bubbles for ETH, LTC, NEO, ETC, DASH, XMR, and OMG, respectively. Generally speaking, the growth of some bubbles has a specific cyclical nature and multiple peaks, troughs, and eventually collapses. The other part of the bubble generation process has no apparent periodicity. The bubble growth process only reflects exponential growth, which is somewhat different from the situation in the stock market.

The bubble burst times t_{DD} in Table V indicate three sequential bubble crashes. The first event starts with the OMG price bubble burst at $t_{\text{DD}} = 5351$ (i.e., 23 December 2018 4:00), closed followed by the bursts of ETH and DASH within a day.



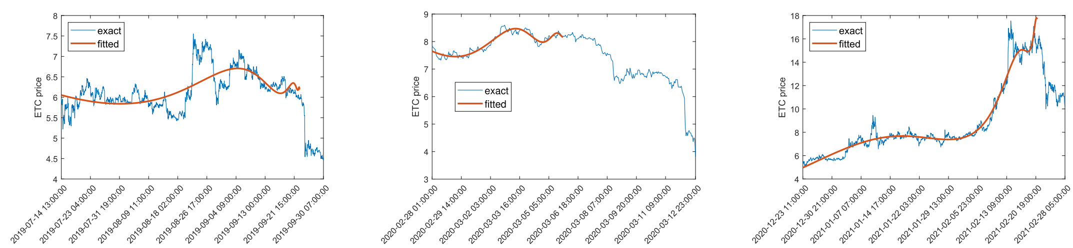
(a) (b) (c) (d)

Fig. 9. LPPL models fitting to LTC bubble regimes. (a) 7 March 2020 11:00. (b) 25 November 2020 18:00. (c) 13 November 2021 23:00. (d) 28 May 2022 20:00.



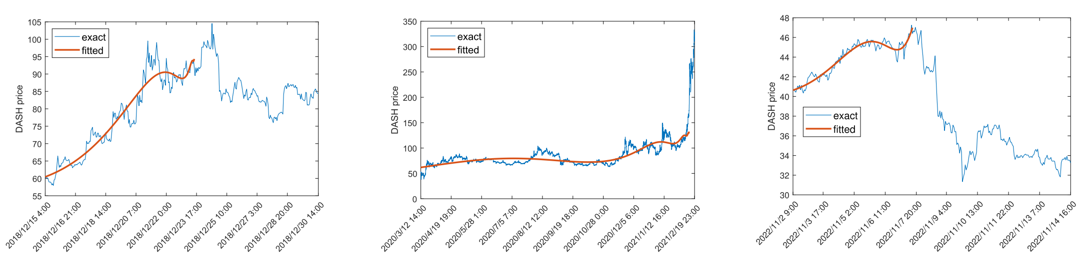
(a) (b)

Fig. 10. LPPL models fitting to NEO bubble regimes. (a) 16 April 2021 20:00. (b) 3 December 2021 6:00.



(a) (b) (c)

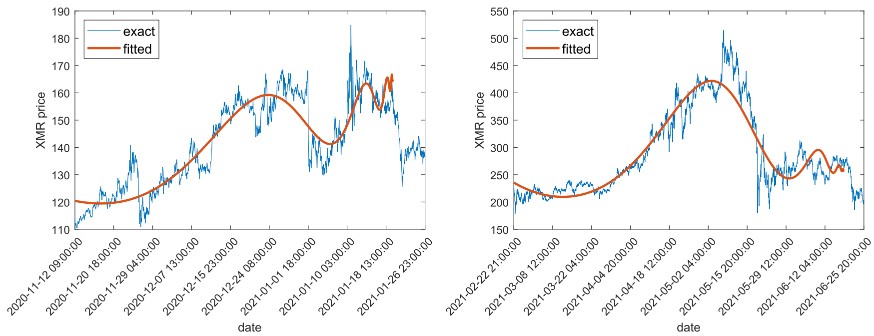
Fig. 11. LPPL models fitting to ETC bubble regimes. (a) 23 September 2019 7:00. (b) 5 March 2020 23:00. (c) 21 February 2021 5:00.



(a) (b) (c)

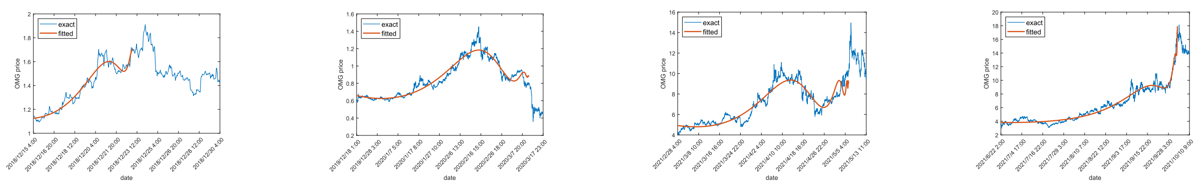
Fig. 12. LPPL models fitting to DASH bubble regimes. (a) 23 December 2018 14:00. (b) 12 February 2021 23:00. (c) 7 November 2022 16:00.

The second event starts with the ETC price bubble burst at $t_{\mathrm{DD}} = 15881$ (i.e., 5 March 2020 23:00), LTC, ETH, and OMG also experienced bubble crashes within five days. Interestingly, a BTC bubble starts to form right after this, and finally, crashed two months later at $t_{\mathrm{DD}} = 17411$ (i.e., 8 May 2020 17:00). But NEO, DASH, and XMR do not exhibit any bubbles in the near future. The third event is "initiated" by a DASH bubble burst, at $t_{\mathrm{DD}} = 24131$ (i.e., 12 February 2021 23:00), closely followed by ETC and BTC. Similar to the second event, the NEO market starts to build a bubble that lasts for 2 months. It is worth emphasizing that not all bubbles have leaders or laggers. Most bubbles are individual market events, such as BTC at $t_{\mathrm{DD}} = 9781$, which does not have contagion effect or spillover to the broader cryptoeconomy. As further analysis of the mechanism of sequential bubble formations and bursts or lead–lag effects is beyond the scope of this study, we will not discuss them in this section. We further compare the entire process from bubble formation to the rupture of all identified bubbles, as shown in Fig. 15. We mark the moments of bubble bursts, corroborating the occurrence of sequential bubble crashes. We specifically investigate the two key parameters ω and β in the LPPL model that control the shape of bubble growth.



(a) (b)

Fig. 13. LPPL models fitting to XMR bubble regimes. (a) 19 January 2021 23:00. (b) 18 June 2021 20:00.



(a) (b) (c) (d)

Fig. 14. LPPL models fitting to OMG bubble regimes. (a) 23 December 2018 4:00. (b) 10 March 2020 23:00. (c) 6 May 2021 11:00. (d) 3 October 2021 9:00.

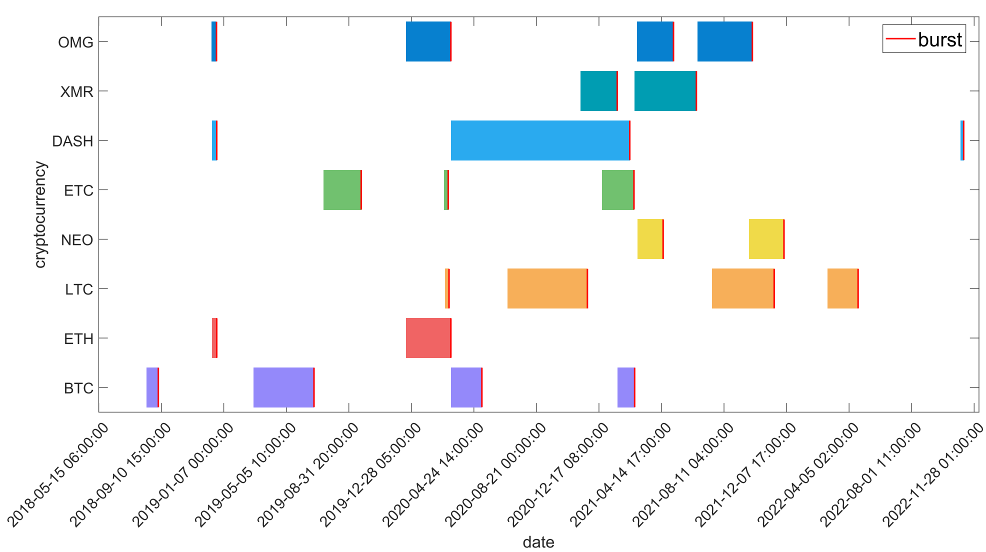


Fig. 15. Periods of cryptocurrency bubble formation and bursts.

Histograms of these two parameters are shown in Figs. 16 and 17. Among all the cryptocurrency bubbles, 21 out of 24 have ω falling in $(2, 5)$, indicating weaker periodicity compared with the stock bubbles. Meanwhile, more than 60% of the cryptocurrency bubbles have $\beta > 0.5$, which reveals a faster pace of bubble expansion than in the stock market. Regarding specific parameter ranges for the stock market bubbles, see [36] and [52]. Although the investigation of differences between the cryptocurrency market and traditional financial markets is beyond the scope of this study, we further discuss potential explanations for these findings in Section V. In summary, the range of the parameters ω and β for cryptocurrency bubbles are $2 < \omega < 9$ and $0 < \beta < 1$.

V. Discussion

Considering the underlying trading mechanism and the market microstructure of cryptocurrencies, we speculate about various factors that may contribute to the weak periodicity and strong growth of crypto bubbles.

First of all, the cryptocurrency market lacks an efficient price correction mechanism. This market opens 24/7. In the absence of after-hours trading, there may be no mechanism to halt or temper the accumulation of buying power following a rapid price surge. In contrast, transactions in the stock market can only be executed during the market's open hours, which, to some extent, prevents the herding effect or positive feedback from holding

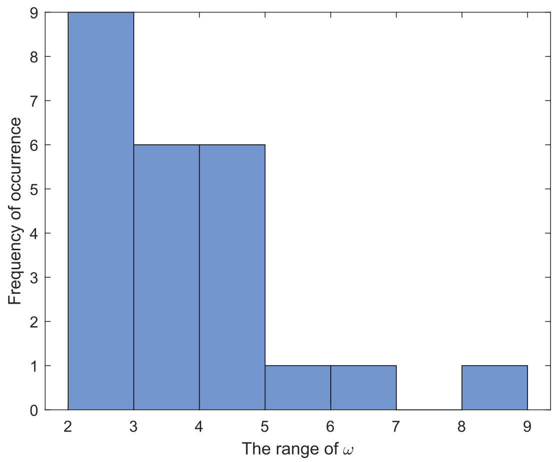


Fig. 16. Histogram of Parameter ω.

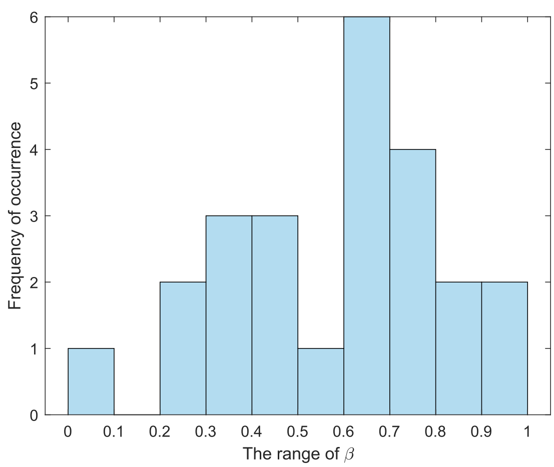


Fig. 17. Histogram of Parameter β.

overnight. On the other hand, unlike foreign exchange rates, which are also traded 24/7, cryptocurrencies are not backed by any government or central bank. Essentially, when a crypto price rockets, it develops a monotonic rapid growth pattern rather than a cyclic, constrained growth.

The second reason lies in the capability to model short-lived bubbles and their microscopic behaviors using hourly data frequency. The successive price increase for several hundreds of hours (i.e., a few days) may be considered reasonable. However, if the data frequency is daily, we should not accept a model showing a successive price increase for hundreds of days. Therefore, if daily data are used for this experiment, we may obtain weaker growth and stronger cyclic parameters.

Finally, the cryptocurrency market is immature, and there are no universally adopted methods to value cryptocurrencies. Investors are primarily speculative, hoping to gain profits from the rapid price increase and the perpetual market fluctuations and targeting short-term holdings. Due to these irrational trading activities, a large amount of hot money flows into the cryptocurrency market rapidly when prices rise, which promotes sustained and rapid price growth. Such a scenario is uncommon in a fair and orderly market.

VI. Conclusion

The rapid fluctuations in cryptocurrency prices suggested a volatile market with immature patterns of growth and decline, potentially indicating the presence of financial bubbles. Extensive literature on cryptocurrency bubbles asserted the existence of bubbles and highlighted that the market is not ordered. While some behavioral aspects of cryptocurrency trading suggested that traditional modeling techniques were applicable, the statistical characteristics of the cryptocurrency market differ significantly from those of traditional stock markets. This study contributed to enhancements in bubble studies to better align with the unique dynamics of the cryptocurrency market. Using the intraday hourly data allowed us to adapt to the unique microstructure and nuances of the cryptocurrency market.

In this article, we established a bubble modeling technique through combining the wavelet analysis and the LPPL model. By applying the drawdown-wavelet-LPPL bubble analysis process to eight cryptocurrencies, we not only verified that this is a robust methodology for the cryptocurrency market, but also explored the properties of crypto bubble formation. We found 24 bubble regimes of eight cryptocurrencies and fit the corresponding LPPL growth models. We found that, comparing with stock market bubbles, crypto bubbles exhibited weaker periodicity and faster expansion. The typical LPPL parameter ranges were $\omega \in (2, 5)$ and $\beta \in (0.5, 1)$. We also observed three sequential bubble crash events starting with the OMG market, the ETC market, and the DASH market, respectively. Another interesting finding was that the DASH market sometimes isolated from others, as revealed not only by smaller correlations but also by bubbles occurring while others remain in stable price regimes. To conclude, our comprehensive bubble analysis using the drawdown-wavelet-LPPL process presented several typical bubbles in the cryptocurrency market and contributed to enriching both the technical modeling and empirical understanding of cryptocurrency markets.

Appendix A
Mathematical Details of the LPPL Model

The Johansen–Ledoit–Sornette model in the following equation includes the bubble crash term $\mathrm{d}j$: $j = 0$ before the crash and $j = 1$ after the crash:

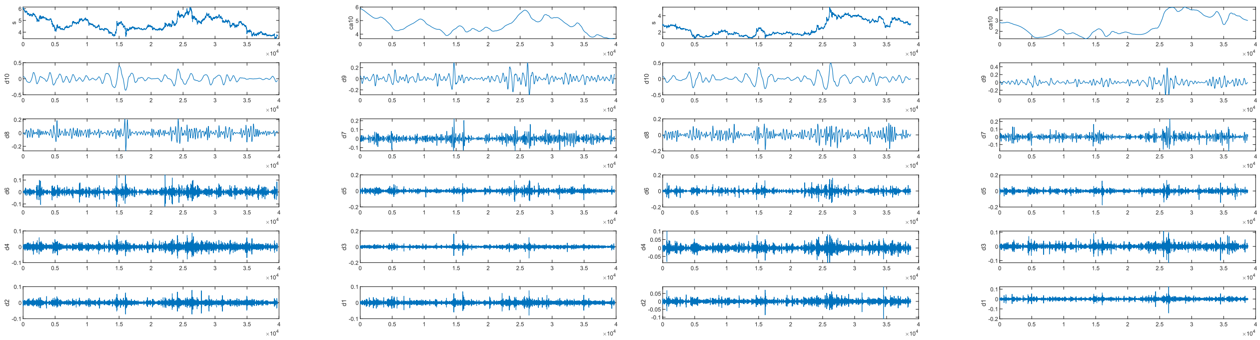
$$\mathrm{d}\ln(P_t) = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t - \kappa \mathrm{d}j \tag{7}$$

where μ_t is the drift term, σ_t is the volatility, and W_t is the standard Brownian motion. The expectation of $\mathrm{d}j$ is $\mathbb{E}[\mathrm{d}j] = h_t \mathrm{d}t$, where h_t is the crash hazard rate modeled as follows:

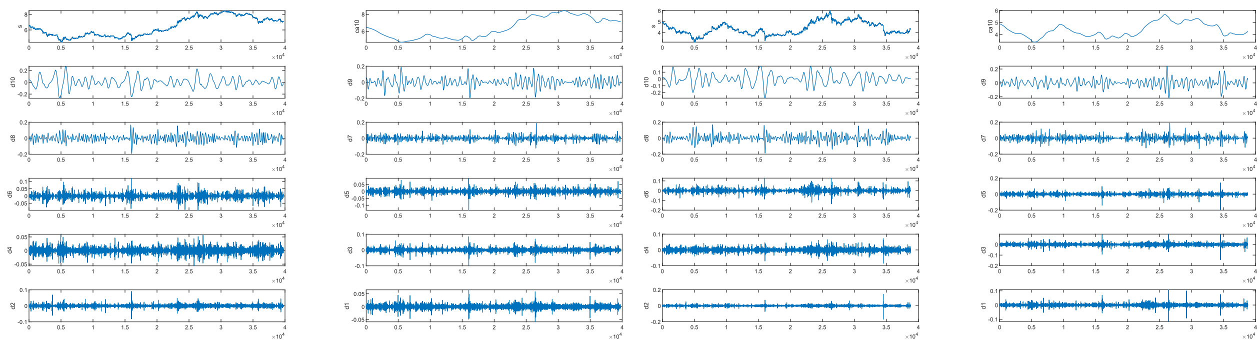
$$h_t = a\,(t_c - t)^{\beta - 1}\,(1 + b\cos(\omega \ln(t_c - t) - \phi)) \tag{8}$$

where a, b, β, and ϕ are parameters, and t_c is the time when the bubble burst.

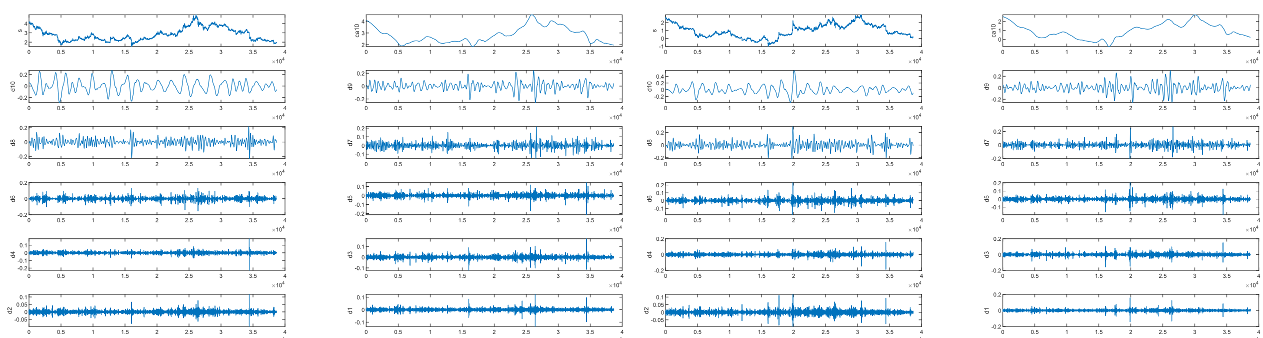
APPENDIX B
RESULT OF WAVELET ANALYSIS AND HIGH-FREQUENCY FACTOR EXTRACTION



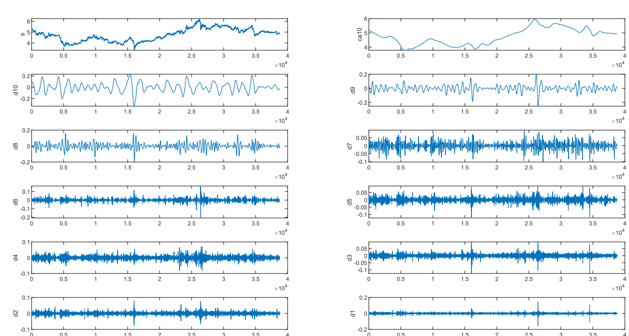
(a) (b)



(c) (d)



(e) (f)



(g)

Fig. 18. (a) Wavelet analysis and high-frequency factor extraction of DASH. (b) Wavelet analysis and high-frequency factor extraction of ETC. (c) Wavelet analysis and high-frequency factor extraction of ETH. (d) Wavelet analysis and high-frequency factor extraction of LTC. (e) Wavelet analysis and high-frequency factor extraction of NEO. (f) Wavelet analysis and high-frequency factor extraction of OMG. (g) Wavelet analysis and high-frequency factor extraction of XMR.

APPENDIX C
RESULT OF HIGH-FREQUENCY FACTOR EXTRACTION "D10"

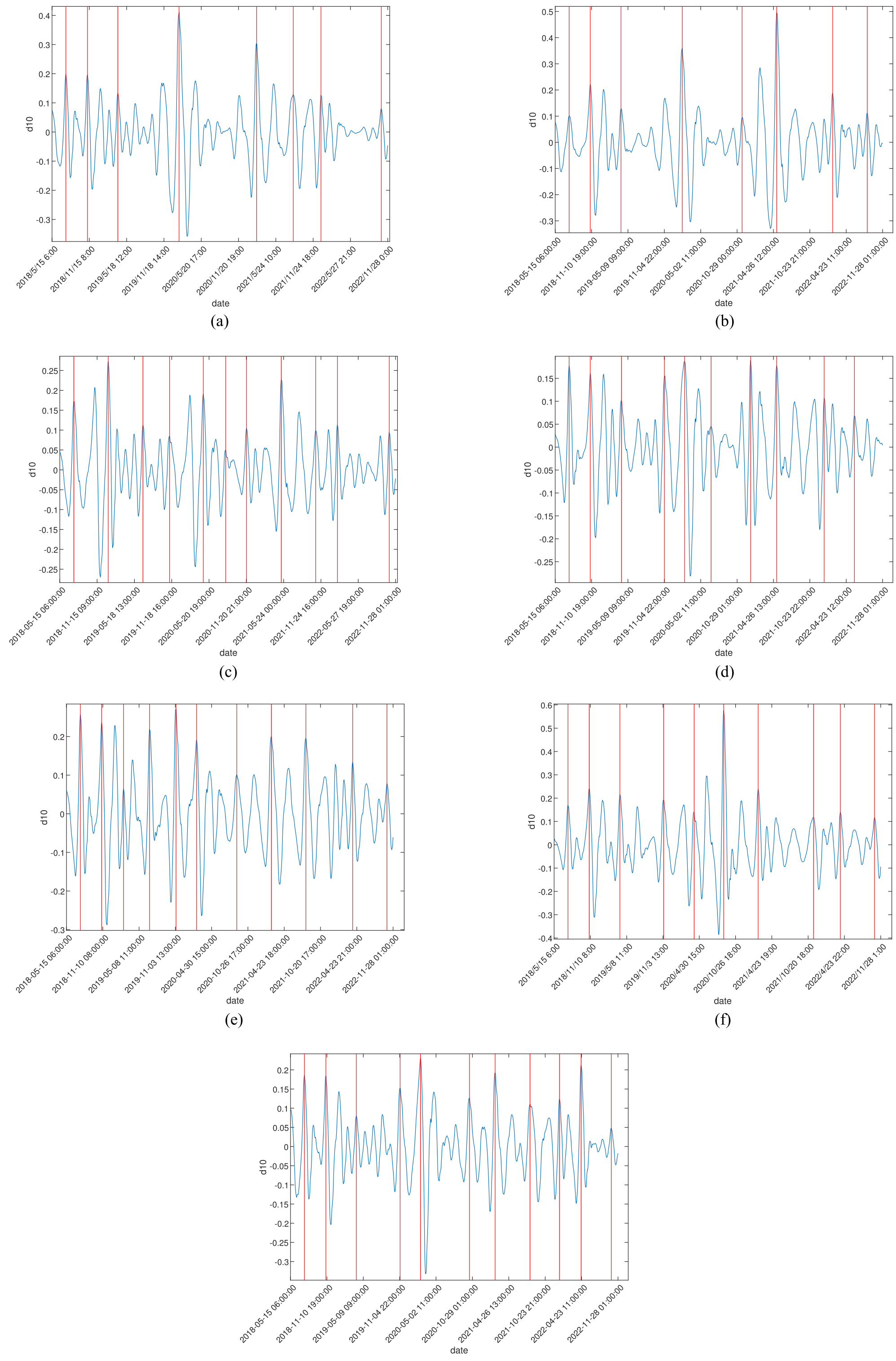


Fig. 19. (a) High-frequency factor extraction "d10" of DASH. (b) High-frequency factor extraction "d10" of ETC. (c) High-frequency factor extraction "d10" of ETH. (d) High-frequency factor extraction "d10" of LTC. (e) High-frequency factor extraction "d10" of NEO. (f) High-frequency factor extraction "d10" of OMG. (g) High-frequency factor extraction "d10" of XMR.

APPENDIX D
RESULT OF PRICE REGIMES

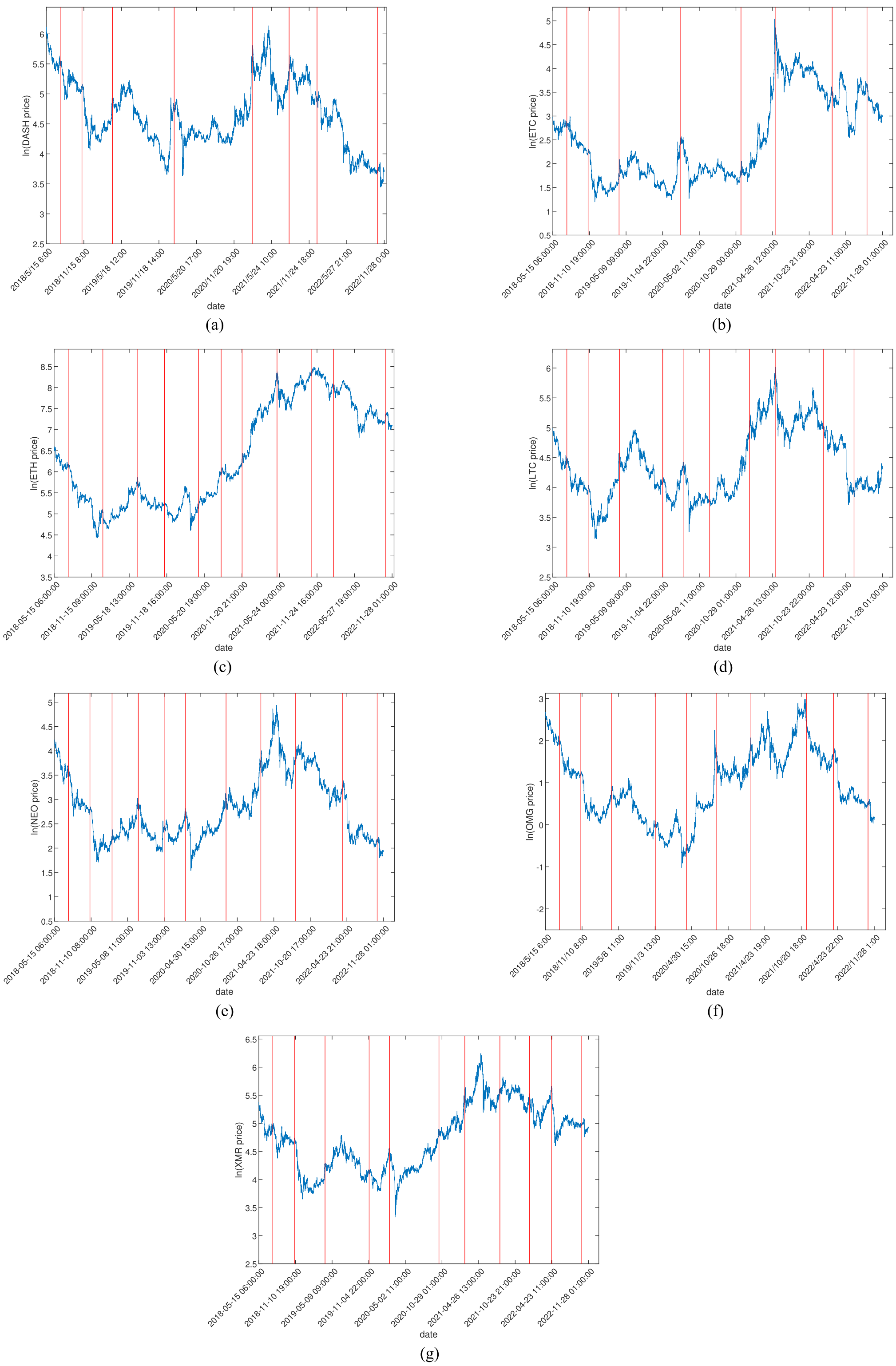


Fig. 20. (a) Price regime of DASH. (b) Price regime of ETC. (c) Price regime of ETH. (d) Price regime of LTC. (e) Price regime of NEO. (f) Price regime of OMG. (g) Price regime of XMR.

APPENDIX E
PARAMETERS OF THE EXPONENTIAL LAW

TABLE VI
PARAMETERS OF THE EXPONENTIAL LAW

	$\ln(N_0)$	$1/DD_c$	N_0	DD_c	RMSE	δ
BTC	7.41818	39.33515	1666	0.02542	0.28776	1%
ETH	7.47986	28.32095	1772	0.03531	0.36164	1%
LTC	7.51371	28.37219	1833	0.03525	0.27396	1%
NEO	7.56528	24.94305	1930	0.04009	0.36112	1%
ETC	7.54645	24.26581	1894	0.04121	**0.43996**	1%
DASH	7.55904	25.56332	1918	0.03912	0.38777	1%
XMR	7.50659	28.56308	1820	0.03501	0.29958	1%
OMG	7.59589	21.99107	1990	0.04547	**0.65043**	1%
BTC	7.41276	41.37778	1657	0.02417	0.31949	1.5%
ETH	7.47477	30.12593	1763	0.03319	0.32586	1.5%
LTC	7.50879	30.06532	1824	0.03326	0.27640	1.5%
NEO	7.56008	26.35956	1920	0.03794	0.29011	1.5%
ETC	7.54168	25.58877	1885	0.03908	0.37689	1.5%
DASH	7.55381	27.10193	1908	0.03690	0.30037	1.5%
XMR	7.50163	30.42591	1811	0.03287	0.27834	1.5%
OMG	7.59085	23.36207	1980	0.04280	**0.53491**	1.5%
BTC	7.40245	44.80789	1640	0.02232	**0.41536**	2.5%
ETH	7.46451	33.06034	1745	0.03025	**0.66743**	2.5%
LTC	7.49832	32.96246	1805	0.03034	**0.42567**	2.5%
NEO	7.55014	28.64994	1901	0.03490	0.19602	2.5%
ETC	7.53155	28.15216	1866	0.03552	0.23352	2.5%
DASH	7.54380	29.69925	1889	0.03367	0.34554	2.5%
XMR	7.49165	33.37846	1793	0.02996	0.38386	2.5%
OMG	7.58070	26.09039	1960	0.03833	**0.44172**	2.5%
BTC	7.39756	46.40594	1632	0.02155	**0.50759**	3%
ETH	7.45934	34.20314	1736	0.02924	**0.46431**	3%
LTC	7.49332	34.21873	1796	0.02922	**0.54331**	3%
NEO	7.54486	29.78598	1891	0.03357	0.27047	3%
ETC	7.52618	29.49500	1856	0.03390	0.25178	3%
DASH	7.53849	30.94190	1879	0.03232	0.39431	3%
XMR	7.48605	34.84109	1783	0.02870	**0.49903**	3%
OMG	7.57558	27.12916	1950	0.03686	0.23376	3%

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