Matching

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March 20, 2023

Many-to-One Matching (Agarwal, N., 2015)

Setup

- Residents match with programs positions
- Resident or program k's preference $\succsim_k$
- Number of residents in market t is N_t, $i \in \mathcal{N}_t$
- Number of programs in market t is J_t, $j \in \mathcal{J}_t$
- Number of positions offered by program j in each market t is c_{jt}
- Matching function $\mu_t : \mathcal{N}_t \to \mathcal{J}_t \cup \{0\}$

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Stable Match of Observation in t

- Individual Rationality
  - For residents match is better than unmatch

$$\mu_t(i) \succsim_i 0$$

  - For each program the number of matched residents should be smaller than positions and the matched residents are the best combination

$$|\mu_t^{-1}(j)| \leq c_{jt},\ \mu_t^{-1}(j) \succsim_j \mu_t^{-1}(j)\backslash\{i\},\ \forall i \in \mu_t^{-1}(j)$$

- No Blocking: if $j \succ_i \mu_t(i)$ then
  - Program j doesn't prefer to replace any of current residents with i

$$\forall i' \in \mu_t^{-1}(j),\ \mu_t^{-1}(j) \succsim_j (\mu_t^{-1}(j)\backslash\{i'\}) \cup \{i\}$$

  - If there still position in program j, program j simply prefer not match

$$|\mu_t^{-1}(j)| \leq c_{jt} \Rightarrow \mu_t^{-1}(j) \succsim_j \mu_t^{-1}(j) \cup \{i\}$$

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Resident preferences (Pure Characteristics Model)

$$u_{ijt} = z_{jt}\beta_i^z + w_{jt}\beta_i^w + \xi_{jt}$$

where z_{jt} is observed program traits, w_{jt} is salary offer, ξ_{jt} is unobserved program trait, and β_i is taste parameters.
The taste parameters can be flexible, in this paper

$$\beta_i^z = x_i\Pi + \eta_i,\ \beta_i^w = \beta^w$$

where x_i are observable characteristics of a resident and η_i is unobserved taste determinants.
There is no typical BLP set up $\varepsilon_{ijt} + \xi_{jt}$, because their distributions (variances) can't be separately identified since aggregate demand(market share) used to invert for product-level unobservables in consumer models is not observed in the dataset.

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Program preferences (Human Capital)
Programs can't observe the value of a team of residents, the paper models residency program preferences through a latent variable.

$$h_i = x_i\alpha + \varepsilon_i$$

where x_i is the characteristics of the resident, ε_i is normally distributed.

Normalization
Without generality the model can be written as

$$u_j = z_j\beta + \xi_j, \ h_i = x_i\alpha + \varepsilon_i$$

And assume $\mathbb{E}[u_j|z_j = 0] = 0$, $\mathbb{E}[h_i|x_i = 0] = 0$, ξ_j and ε_i are i.i.d. standard normal random variables.

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Endogeneity Issue

In previous setting

$$u_{ijt} = z_{jt}\beta_i^z + w_{jt}\beta_i^w + \xi_{jt}$$

but w_{jt} might correlated with program unobservables ξ_{jt}, which violates the previous normalization.

To solve this, the paper imposes a linear control function(IV)

$$w_{jt} = z_{jt}\gamma + r_{jt}\tau + \nu_{jt}$$

where r_{jt} is the instrument, further more, the paper model $\xi_{jt} = \kappa\nu_{jt} + \sigma\zeta_{jt}$

$$u_{ijt} = z_{jt}\beta_i^z + w_{jt}\beta_i^w + \kappa\hat{\nu}_{jt} + \sigma\zeta_{jt}$$

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