# Matching

Junrui Lin

NYU MSQE

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## Risk Preference Estimation (Cohen and Einav, 2007)

**Setup**

- $x_i$: vector of characteristics individual $i$ reports to the insurance company
- $p_{it} = f_t(x_i)$: regular premium function that unknown for us, stable over time
- $d_{it} = \min\{\frac{1}{2}p_{it},\ cap_t\}$: regular deductible level
- $w_i$: wealth of individual $i$
- $(p_i^h, d_i^h)$: insurance contract, high premium, high deductible
- $(p_i^l, d_i^l)$: insurance contract, low premium, low deductible
- $t_i$: duration of the policy
- $u_i(w)$: individual's vNM utility function
- $\lambda_i$: claim rate, known to individual (assume no moral harzard)

# Risk Preference Estimation (Cohen and Einav, 2007)

**Model**

The expected utility from a contract $(p, d)$ is

$$v(p,d) := (1-\lambda t)u(w-pt) + (\lambda t)u(w-pt-d)$$

when individual is indifference between $(p^h, d^h)$ and $(p^l, d^l)$

$$\lambda = \lim_{t \to 0} \frac{\frac{1}{t}\left(u(w-p^h t) - u(w-p^l t))\right)}{\left(u(w-p^h t) - u(w-p^h t-d^h) - u(w-p^l t) + u(w-p^l t-d^l)\right)}$$
$$= \frac{(p^l-p^h)u'(w)}{u(w-d^l)-u(w-d^h)}$$

Using Taylor expansion

$$\frac{p^l-p^h}{\lambda}u'(w) \approx (d^h-d^l)u'(w) - \frac{1}{2}(d^h-d^l)(d^h+d^l)u''(w)$$

# Risk Preference Estimation (Cohen and Einav, 2007)

**Model** (cont.)
Let $\Delta d = d^h - d^l$, $\Delta p = p^l - p^h$, and $\bar{d} = \frac{1}{2}(d^h + d^l)$, then previous Taylor expansion will be

$$\frac{\Delta p}{\lambda} u'(w) = \Delta d u'(w) - \bar{d} \Delta d u''(w) \Rightarrow r := \frac{-u''(w)}{u'(w)} = \frac{\frac{\Delta p}{\lambda \Delta d} - 1}{\bar{d}}$$

where $r$ is coefficient of absolute risk aversion at wealth level $w$.

**Note**: the indifferent set $(r^*(\lambda), \lambda)$ and $(\lambda^*(r), r)$ are interchangeable and have closed form.

# Risk Preference Estimation (Cohen and Einav, 2007)

**Model Estimation**

We want to estimate the joint distribution of $(\lambda_i, r_i)$, conditional on observables $x_i$. Assuming $(\lambda_i, r_i)$ is bivariate lognormal

$$\ln \lambda_i = x_i'\beta + \varepsilon_i,\ \ln r_i = x_i'\gamma + \nu_i$$

$$\begin{bmatrix} \varepsilon_i \\ \nu_i \end{bmatrix} \overset{\text{i.i.d.}}{\sim} N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\lambda^2 & \rho\sigma_\lambda\sigma_r \\ \rho\sigma_\lambda\sigma_r & \sigma_r^2 \end{bmatrix} \right)$$

$\lambda$ and $r$ are not directly observed, we only observe claims and deductible choice.

Assume claims follows Poisson process

$$claims_i \sim Poisson(\lambda_i t_i)$$

When choosing deductible plans, they will choose low deductible iff $r_i \geq r_i^*(\lambda_i)$

$$Pr(\text{low deductible}) = Pr\left( \exp(x_i'\gamma + \nu_i) > \frac{\frac{\Delta p_i}{\exp(x_i'\beta + \varepsilon_i)\Delta d_i} - 1}{\bar{d}_i} \right)$$

Then it's fairly easy to write out the likelihood or use simulation method

# Asymmetric Information (Einav et al. 2010)

**Setup**

# Asymmetric Information (Einav et al. 2010)