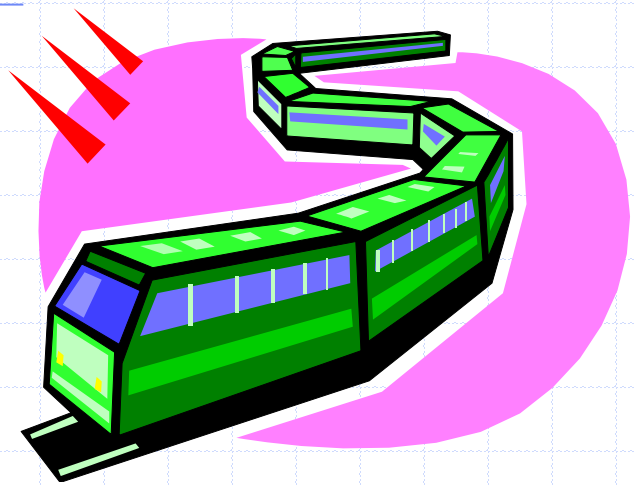
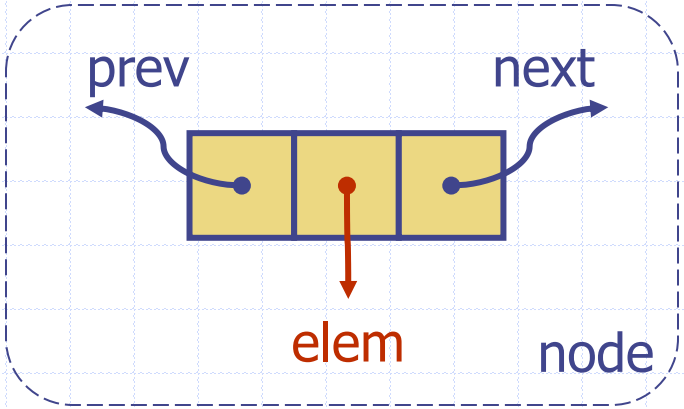
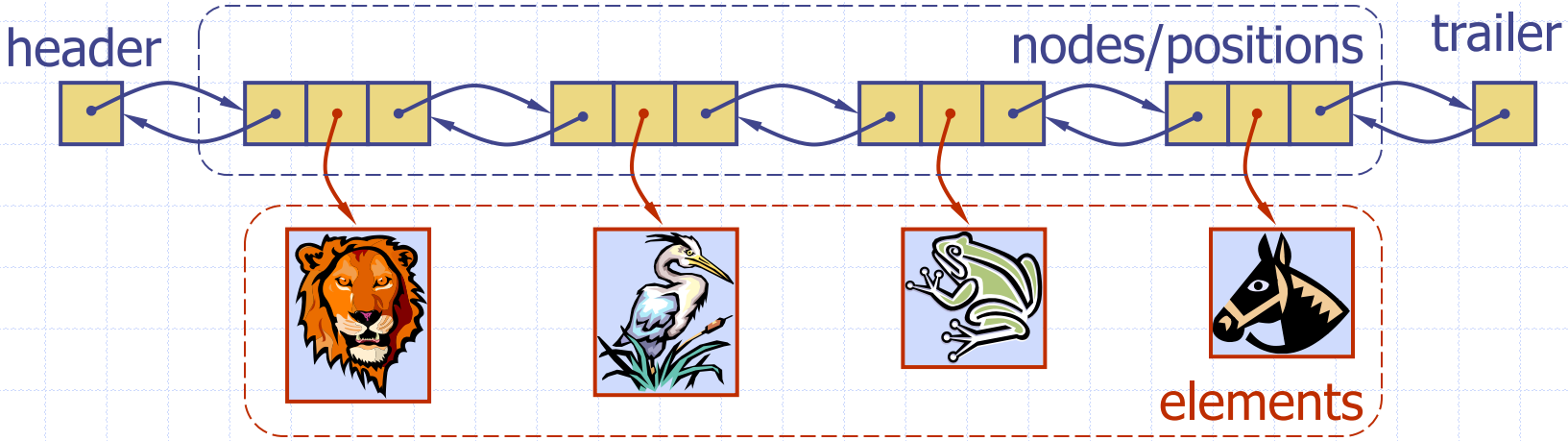
# Doubly-Linked Lists



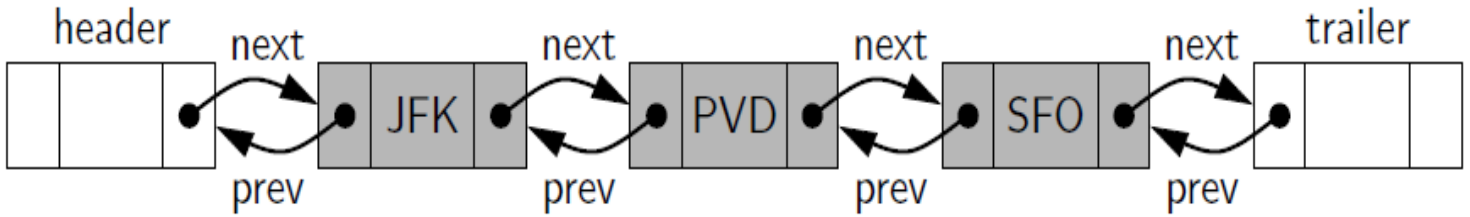
# Doubly Linked List

- A doubly linked list provides a natural implementation of the Node List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes

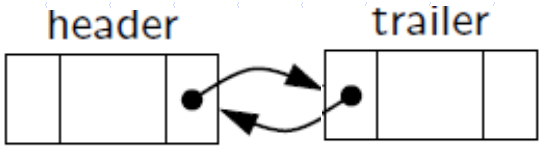


# Header and Trailer Sentinels




When Empty:



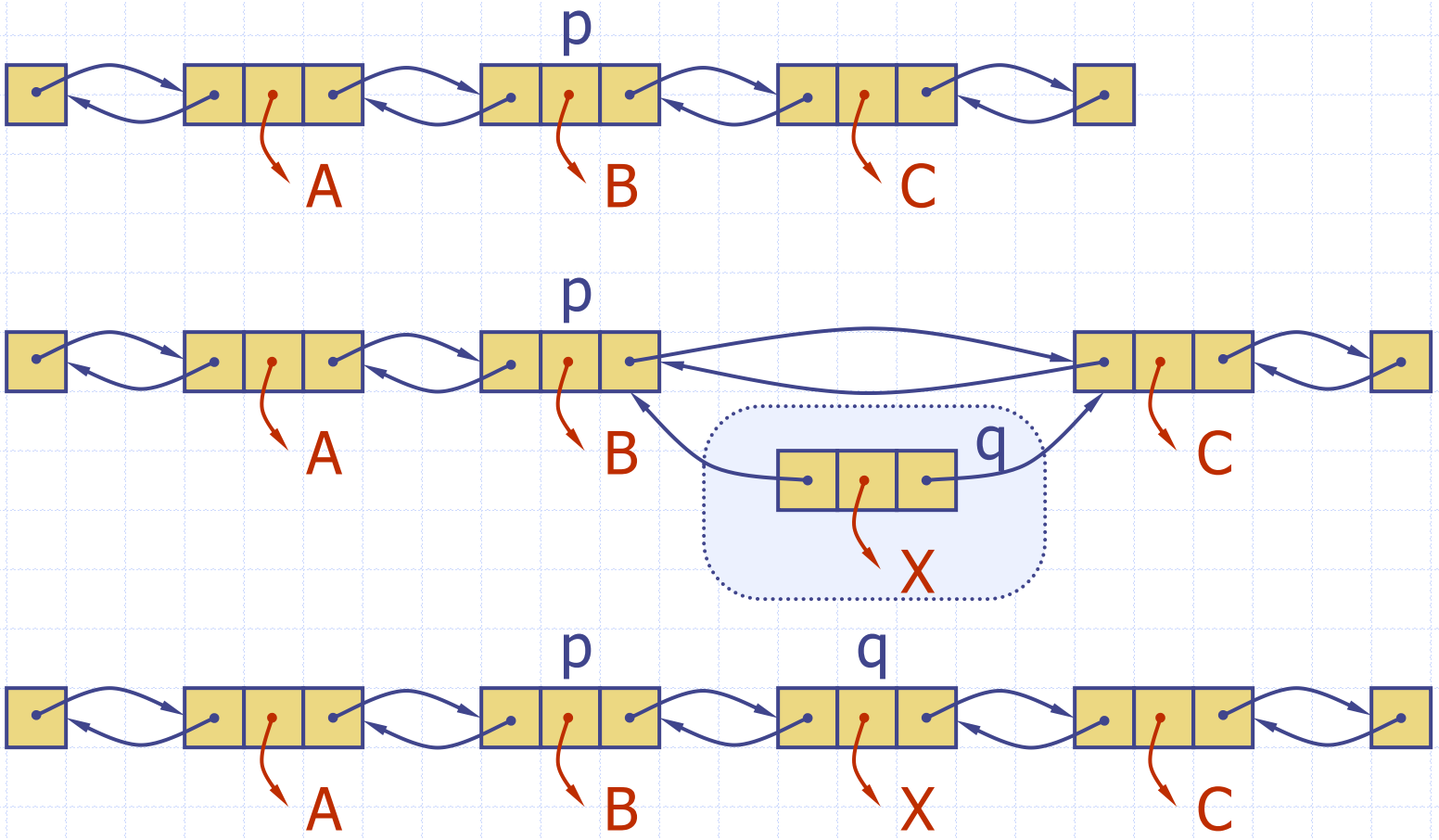
# Node Class for Double Link List

```
class _Node:
"""Lightweight, nonpublic class for storing a doubly linked node."""
    __slots__= _element , _prev , _next # streamline memory

    def __init__(self, element, prev, next): # initialize node's fields
        self._element = element # user's element
        self._prev = prev # previous node reference
        self._next = next # next node reference
```

# Insertion

- Insert a new node, q, between p and its successor.



# More Insertations Examples

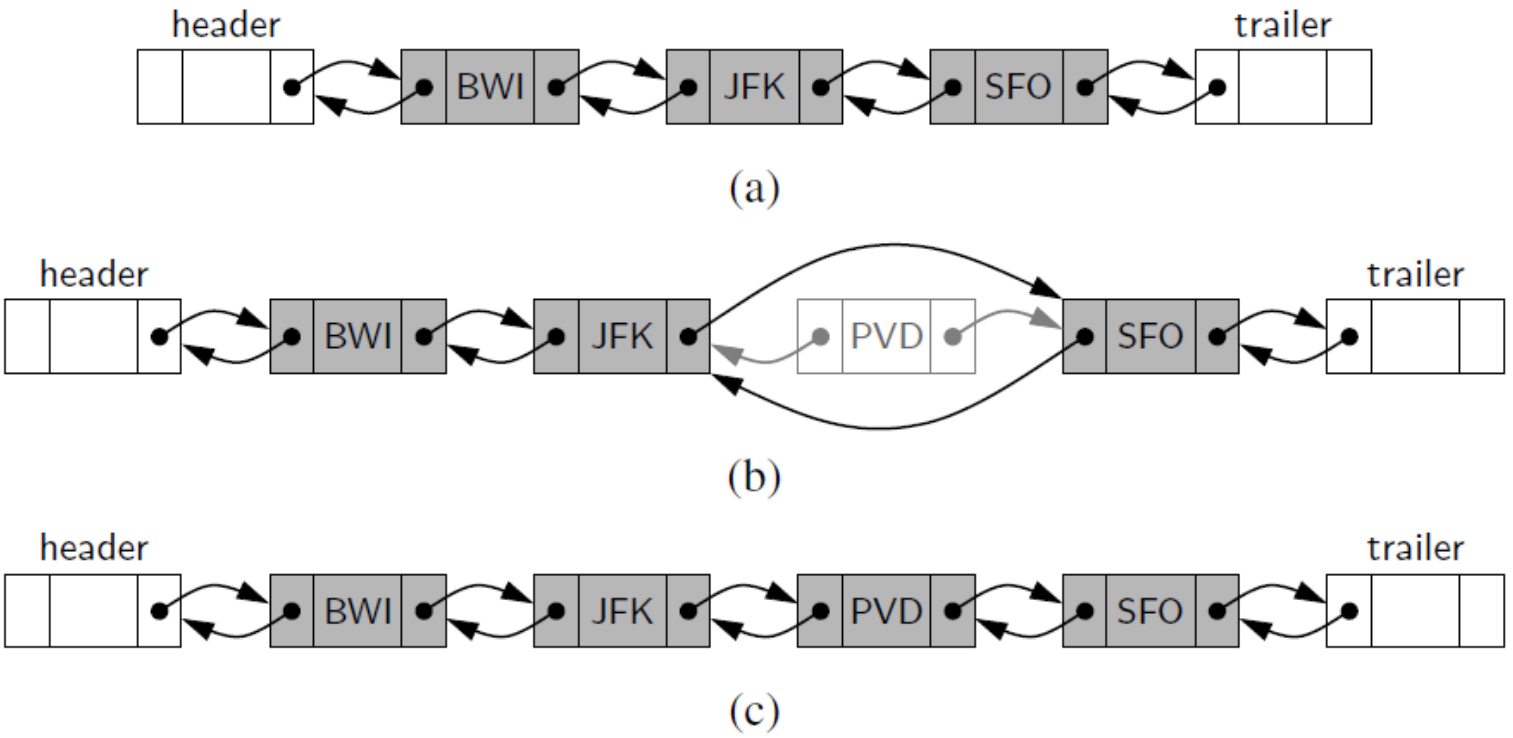



**Figure 7.11:** Adding an element to a doubly linked list with header and trailer sentinels: (a) before the operation; (b) after creating the new node; (c) after linking the neighbors to the new node.

# More Insertations

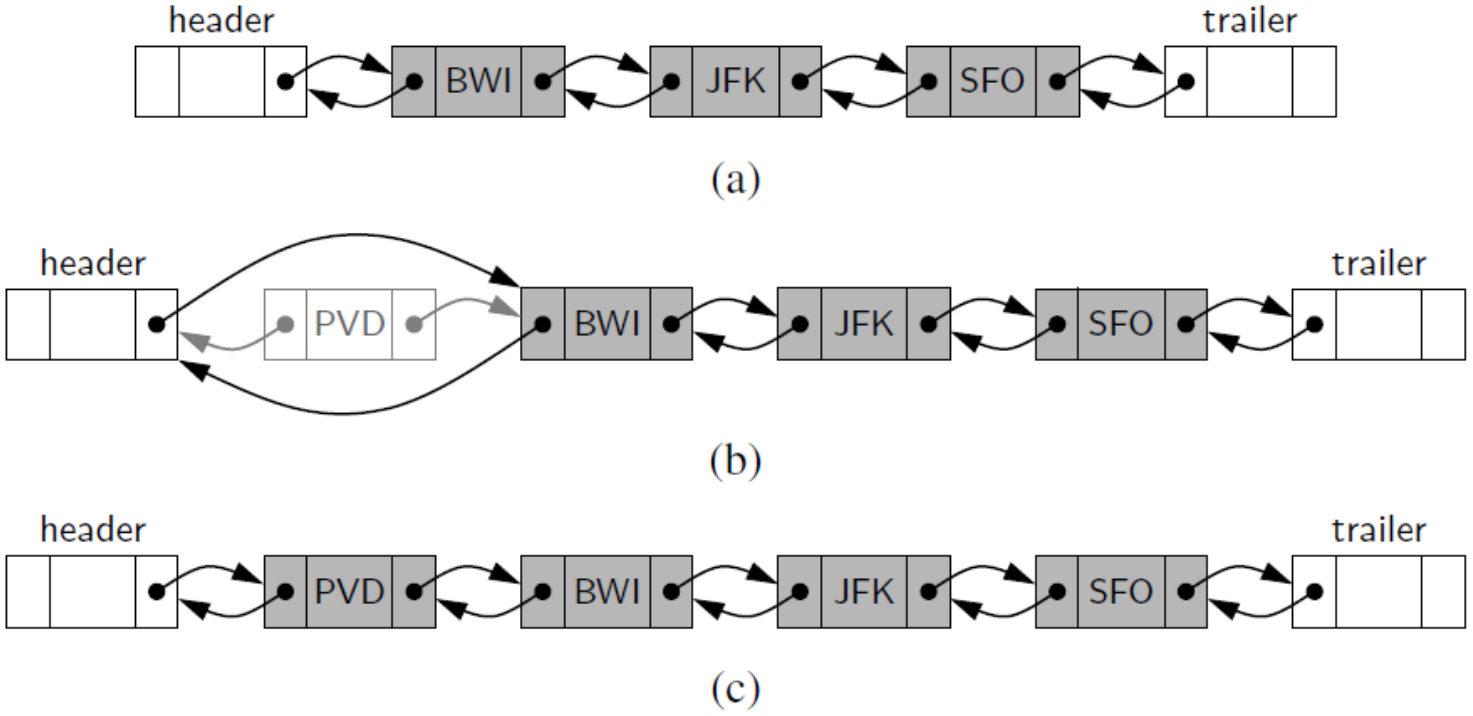
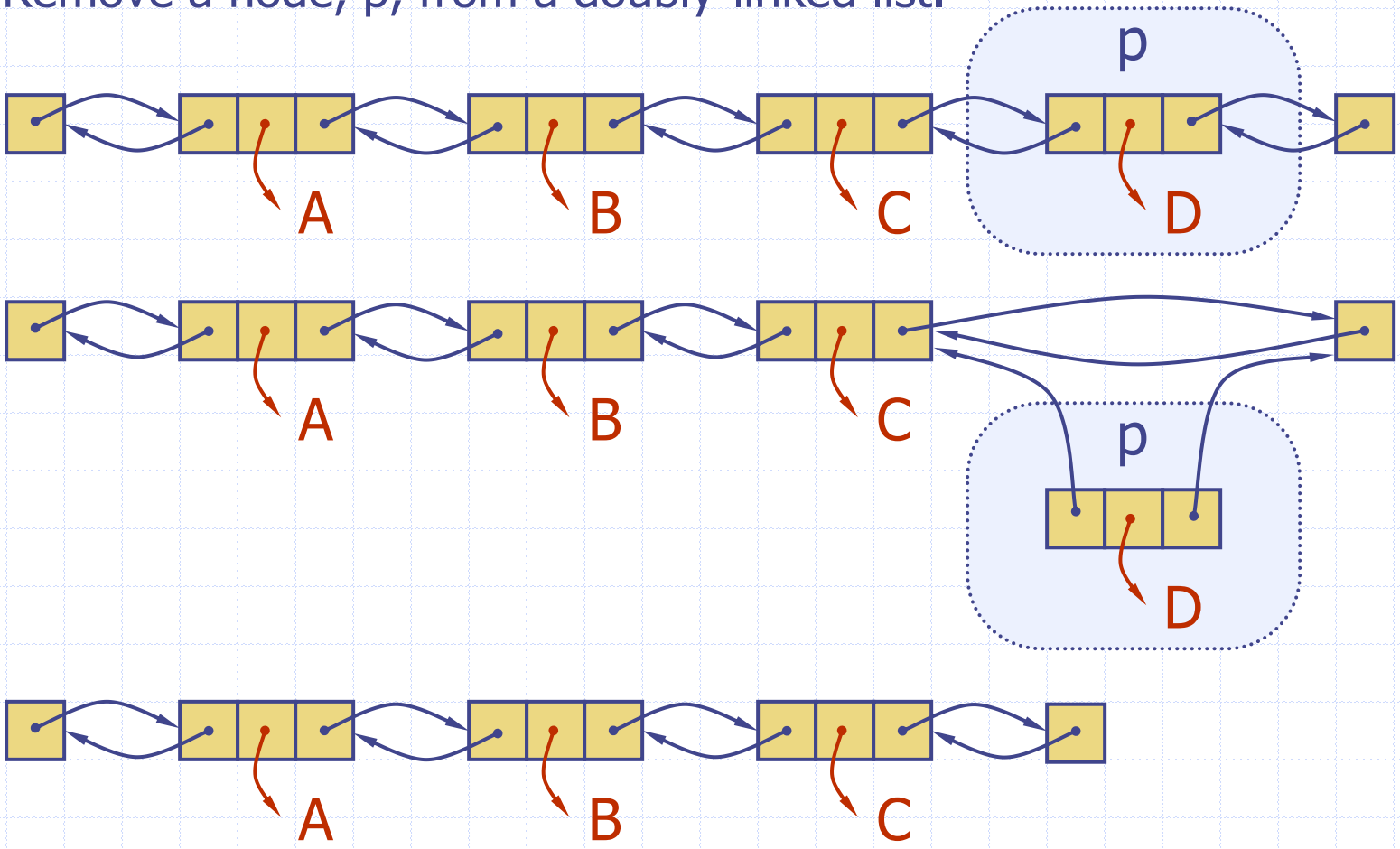



**Figure 7.12:** Adding an element to the front of a sequence represented by a doubly linked list with header and trailer sentinels: (a) before the operation; (b) after creating the new node; (c) after linking the neighbors to the new node.

# Deletion

- Remove a node, p, from a doubly-linked list.



# More Deletions

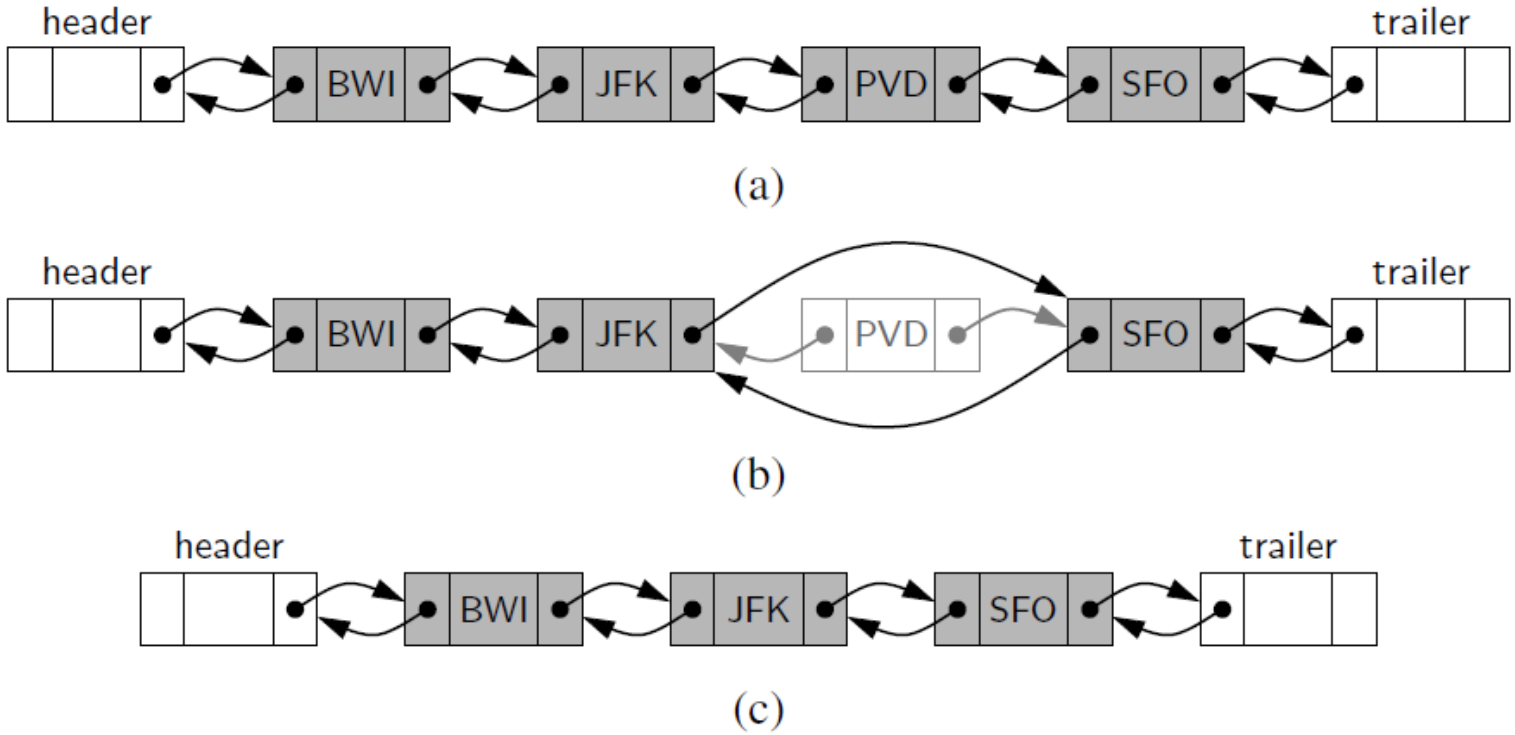



**Figure 7.13:** Removing the element PVD from a doubly linked list: (a) before the removal; (b) after linking out the old node; (c) after the removal (and garbage collection).

# Doubly-Linked List in Python

```
1  class _DoublyLinkedBase:
2    """A base class providing a doubly linked list representation."""
3
4    class _Node:
5      """Lightweight, nonpublic class for storing a doubly linked node."""
6      (omitted here; see previous code fragment)
7
8    def __init__(self):
9      """Create an empty list."""
10     self._header = self._Node(None, None, None)
11     self._trailer = self._Node(None, None, None)
12     self._header._next = self._trailer                # trailer is after header
13     self._trailer._prev = self._header                # header is before trailer
14     self._size = 0                                    # number of elements
15
16   def __len__(self):
17     """Return the number of elements in the list."""
18     return self._size
19
20   def is_empty(self):
21     """Return True if list is empty."""
22     return self._size == 0
23
```

```
24   def _insert_between(self, e, predecessor, successor):
25     """Add element e between two existing nodes and return new node."""
26     newest = self._Node(e, predecessor, successor)   # linked to neighbors
27     predecessor._next = newest
28     successor._prev = newest
29     self._size += 1
30     return newest
31
32   def _delete_node(self, node):
33     """Delete nonsentinel node from the list and return its element."""
34     predecessor = node._prev
35     successor = node._next
36     predecessor._next = successor
37     successor._prev = predecessor
38     self._size -= 1
39     element = node._element                          # record deleted element
40     node._prev = node._next = node._element = None   # deprecate node
41     return element                                   # return deleted element
```

# Performance

- In a doubly linked list
  - The space used by a list with $n$ elements is $O(n)$
  - The space used by each position of the list is $O(1)$
  - All the standard operations of a list run in $O(1)$ time

# Positional List

- To provide for a general abstraction of a sequence of elements with the ability to identify the location of an element, we define a **positional list** ADT.
- A position acts as a marker or token within the broader positional list.
- A position p is unaffected by changes elsewhere in a list; the only way in which a position becomes invalid is if an explicit command is issued to delete it.
- A position instance is a simple object, supporting only the following method:
  - p.element( ): Return the element stored at position p.

# Positional Accessor Operations

| | |
|---|---|
| L.first( ): | Return the position of the first element of L, or None if L is empty. |
| L.last( ): | Return the position of the last element of L, or None if L is empty. |
| L.before(p): | Return the position of L immediately before position p, or None if p is the first position. |
| L.after(p): | Return the position of L immediately after position p, or None if p is the last position. |
| L.is_empty( ): | Return True if list L does not contain any elements. |
| len(L): | Return the number of elements in the list. |
| iter(L): | Return a forward iterator for the *elements* of the list. See Section 1.8 for discussion of iterators in Python. |

# Positional Update Operations

| | |
|---|---|
| L.add_first(e): | Insert a new element e at the front of L, returning the position of the new element. |
| L.add_last(e): | Insert a new element e at the back of L, returning the position of the new element. |
| L.add_before(p, e): | Insert a new element e just before position p in L, returning the position of the new element. |
| L.add_after(p, e): | Insert a new element e just after position p in L, returning the position of the new element. |
| L.replace(p, e): | Replace the element at position p with element e, returning the element formerly at position p. |
| L.delete(p): | Remove and return the element at position p in L, invalidating the position. |

# Usage of Positional Lists

**Example 7.1:** *The following table shows a series of operations on an initially empty positional list L. To identify position instances, we use variables such as p and q. For ease of exposition, when displaying the list contents, we use subscript notation to denote its positions.*

| Operation | Return Value | L |
|---|---|---|
| L.add_last(8) | p | $8_p$ |
| L.first( ) | p | $8_p$ |
| L.add_after(p, 5) | q | $8_p, 5_q$ |
| L.before(q) | p | $8_p, 5_q$ |
| L.add_before(q, 3) | r | $8_p, 3_r, 5_q$ |
| r.element( ) | 3 | $8_p, 3_r, 5_q$ |
| L.after(p) | r | $8_p, 3_r, 5_q$ |
| L.before(p) | None | $8_p, 3_r, 5_q$ |
| L.add_first(9) | s | $9_s, 8_p, 3_r, 5_q$ |
| L.delete(L.last( )) | 5 | $9_s, 8_p, 3_r$ |
| L.replace(p, 7) | 8 | $9_s, 7_p, 3_r$ |

# Positional List in Python

```
1  class PositionalList(_DoublyLinkedBase):
2    """A sequential container of elements allowing positional access."""
3
4    #------------------------- nested Position class -------------------------
5    class Position:
6      """An abstraction representing the location of a single element."""
7
8      def __init__(self, container, node):
9        """Constructor should not be invoked by user."""
10       self._container = container
11       self._node = node
12
13     def element(self):
14       """Return the element stored at this Position."""
15       return self._node._element
16
17     def __eq__(self, other):
18       """Return True if other is a Position representing the same location."""
19       return type(other) is type(self) and other._node is self._node
20
21     def __ne__(self, other):
22       """Return True if other does not represent the same location."""
23       return not (self == other)           # opposite of __eq__
24
25   #------------------------------ utility method ------------------------------
26   def _validate(self, p):
27     """Return position's node, or raise appropriate error if invalid."""
28     if not isinstance(p, self.Position):
29       raise TypeError('p must be proper Position type')
30     if p._container is not self:
31       raise ValueError('p does not belong to this container')
32     if p._node._next is None:               # convention for deprecated nodes
33       raise ValueError('p is no longer valid')
34     return p._node
```

# Positional List in Python, Part 2

```
35    #------------------------------ utility method ------------------------------
36    def _make_position(self, node):
37      """Return Position instance for given node (or None if sentinel)."""
38      if node is self._header or node is self._trailer:
39        return None                                  # boundary violation
40      else:
41        return self.Position(self, node)             # legitimate position
42
43    #------------------------------ accessors ------------------------------
44    def first(self):
45      """Return the first Position in the list (or None if list is empty)."""
46      return self._make_position(self._header._next)
47
48    def last(self):
49      """Return the last Position in the list (or None if list is empty)."""
50      return self._make_position(self._trailer._prev)
51
52    def before(self, p):
53      """Return the Position just before Position p (or None if p is first)."""
54      node = self._validate(p)
55      return self._make_position(node._prev)
56
57    def after(self, p):
58      """Return the Position just after Position p (or None if p is last)."""
59      node = self._validate(p)
60      return self._make_position(node._next)
61
62    def __iter__(self):
63      """Generate a forward iteration of the elements of the list."""
64      cursor = self.first()
65      while cursor is not None:
66        yield cursor.element()
67        cursor = self.after(cursor)
```

# Positional List in Python, Part 3

```
68    #------------------------------ mutators ------------------------------
69    # override inherited version to return Position, rather than Node
70    def _insert_between(self, e, predecessor, successor):
71      """Add element between existing nodes and return new Position."""
72      node = super()._insert_between(e, predecessor, successor)
73      return self._make_position(node)
74
75    def add_first(self, e):
76      """Insert element e at the front of the list and return new Position."""
77      return self._insert_between(e, self._header, self._header._next)
78
79    def add_last(self, e):
80      """Insert element e at the back of the list and return new Position."""
81      return self._insert_between(e, self._trailer._prev, self._trailer)
82
83    def add_before(self, p, e):
84      """Insert element e into list before Position p and return new Position."""
85      original = self._validate(p)
86      return self._insert_between(e, original._prev, original)
87
88    def add_after(self, p, e):
89      """Insert element e into list after Position p and return new Position."""
90      original = self._validate(p)
91      return self._insert_between(e, original, original._next)
92
93    def delete(self, p):
94      """Remove and return the element at Position p."""
95      original = self._validate(p)
96      return self._delete_node(original)      # inherited method returns element
97
98    def replace(self, p, e):
99      """Replace the element at Position p with e.
100
101     Return the element formerly at Position p.
102     """
103     original = self._validate(p)
104     old_value = original._element           # temporarily store old element
105     original._element = e                   # replace with new element
106     return old_value                        # return the old element value
```