

- (a) Recursively compute H_t for $t=2$, then use $H_{t=2}$ to compute $Z_{t=2}$
See below recursion algorithm, which computes $H_{t=2} = 0.125$

```
In [1]: wh = 0.5
        t = 2
        w0 = 0.6

        def compute_H(wh, t):
            if t == -1:
                Ht = 1
            else:
                Ht = wh*compute_H(wh, t-1)
            return Ht

        compute_H(wh,t)
```

Out[1]: 0.125

```
In [2]: def compute_Z(w0,wh,t):
        Z = w0*compute_H(wh,t)
        return Z

        compute_Z(w0,wh,t)
```

Out[2]: 0.075

$Z_{t=2} = 0.075$

- (b) From the equation we know that $Z_2 = w_0 \times H_2$, then we have $\frac{\partial Z_2}{\partial w_0} = H_2$

From part (a) I have already computed $H_2 = 0.125$, so:

$$\frac{\partial Z_2}{\partial w_0} = \mathbf{0.125}$$

- (c) Expand the equation to express Z_2 as function of w_h

$$Z_2 = w_0 \times H_2 = w_0 \times w_h \times H_1$$

$$\text{Then we have } \frac{\partial Z_2}{\partial w_h} = w_0 \times H_1$$

Compute $H_{t=1}$ using the same recursion algorithm (change t to 1) then multiply by constant w_0 :

```
In [3]: wh = 0.5
        t = 1
        w0 = 0.6
        compute_H(wh,t)
```

Out[3]: 0.25

```
In [4]: compute_H(wh,t)*w0
```

Out[4]: 0.15

$$\frac{\partial Z_2}{\partial w_h} = \mathbf{0.15}$$