```python
# -*- coding: utf-8 -*-
"""
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"""
# See handwritten section for detailed derivation on partial derivatives and chain rules

"""
∂f/∂m1 at (10, 9, 8) = -23040*pi*exp(pi*cos(64*cos(36*cos(100))))*
                        sin(100)*sin(36*cos(100))*sin(64*cos(36*cos(100)))
                     = -45.41514711903361

∂f/∂m2 at (10, 9, 8) = 256*pi*exp(pi*cos(64*cos(36*cos(100))))*
                        sin(36*cos(100))*sin(64*cos(36*cos(100)))*cos(100)
                     = -0.8593337670347707

∂f/∂m3 at (10, 9, 8) = -8*pi*exp(pi*cos(64*cos(36*cos(100))))*
                        sin(64*cos(36*cos(100)))*cos(36*cos(100))
                     = -0.07971161119125678
"""
import math

n = [0,1,2,3] #the 0 is just for indexing purpose
m = [0,10,9,8] #the 0 is just for indexing purpose
x = 5

z1 = m[1]*x
v1 = math.cos(2**n[1]*z1)
z2 = m[2]*v1
v2 = math.cos(2**n[2]*z2)
z3 = m[3]*v2
v3 = math.cos(2**n[3]*z3)
f = math.exp(math.pi*v3)

dz1_dm1 = x
dv1_dz1 = -2**n[1]*math.sin(2**n[1]*z1)
dz2_dv1 = m[2]
dv2_dz2 = -2**n[2]*math.sin(2**n[2]*z2)
dz3_dv2 = m[3]
dv3_dz3 = -2**n[3]*math.sin(2**n[3]*z3)
df_dv3 = math.pi*math.exp(math.pi*v3)

df_dm1 = df_dv3*dv3_dz3*dz3_dv2*dv2_dz2*dz2_dv1*dv1_dz1*dz1_dm1
print("df/dm1: ", df_dm1)

dz2_dm2 = v1
df_dm2 = df_dv3*dv3_dz3*dz3_dv2*dv2_dz2*dz2_dm2
print("df/dm2: ", df_dm2)

dz3_dm3 = v2
df_dm3 = df_dv3*dv3_dz3*dz3_dm3
print("df/dm3: ", df_dm3)
```

$z_1 = m_1 \cdot x$

$v_1 = \cos(2^1 z_1) = \cos(2 m_1 x)$

$z_2 = m_2 \cdot v_1 = m_2 \cos(2 m_1 x)$

$v_2 = \cos(2^2 \cdot z_2) = \cos(4 m_2 \cdot \cos(2 m_1 x))$

$z_3 = m_3 \cdot v_2 = m_3 \cdot \cos(4 m_2 \cdot \cos(2 m_1 x))$

$v_3 = \cos(2^3 \cdot z_3) = \cos(8 m_3 \cos(4 m_2 \cos(2 m_1 x)))$

$f = \exp[\pi \cdot v_3]$

for df/dm_1: $\frac{df}{dv_3} \times \frac{dv_3}{dz_3} \times \frac{dz_3}{dv_2} \times \frac{dv_2}{dz_2} \times \frac{dz_2}{dv_1} \times \frac{dv_1}{dz_1} \times \frac{dz_1}{dm_1}$

$\frac{df}{dv_3} = \pi \cdot \exp[\pi \cdot v_3]$ $\frac{dv_3}{dz_3} = -2^3 \sin(2^3 \cdot z_3)$ $\frac{dz_3}{dv_2} = m_3$

$\frac{dv_2}{dz_2} = -2^2 \sin(2^2 \cdot z_2)$ $\frac{dz_2}{dv_1} = m_2$ $\frac{dv_1}{dz_1} = -2^1 \sin(2^1 \cdot z_1)$ $\frac{dz_1}{dm_1} = x$

multiply: $\pi \cdot \exp[\pi \cdot v_3] \cdot -8 \sin(8 z_3) \cdot m_3 \cdot -4 \sin(4 z_2) \cdot m_2 \cdot -2 \sin(2 z_1) \cdot x$

$= \pi \cdot \exp[\pi \cdot \cos(8 m_3 \cos(4 m_2 \cos(2 m_1 x)))] \cdot -8 \sin(8 m_3 \cos(4 m_2 \cos(2 m_1 x))) \cdot 8 \cdot -4 \sin(4 m_2 \cos(2 m_1 x)) \cdot 9 \cdot -2 \sin(2 m_1 x) \cdot 5$

$= -23040 \cdot \pi \cdot \exp[\pi \cdot \cos(64 \cos(36 \cos(100)))] \cdot \sin(64 \cos(36 \cos(100))) \cdot \sin(36 \cos(100)) \cdot \sin(100)$ //

for df/dm_2: $\frac{df}{dv_3} \times \frac{dv_3}{dz_3} \times \frac{dz_3}{dv_2} \times \frac{dv_2}{dz_2} \times \frac{dz_2}{dm_2}$, $\frac{dz_2}{dm_2} = v_1$

multiply: $\pi \cdot \exp[\pi \cdot v_3] \cdot -2^3 \sin(2^3 \cdot z_3) \times m_3 \times -2^2 \sin(2^2 \cdot z_2) \times v_1$

$= \pi \cdot \exp[\pi \cos(8 m_3 \cos(4 m_2 \cos(2 m_1 x)))] \cdot -8 \sin(8 m_3 \cos(4 m_2 \cos(2 m_1 x))) \cdot 8 \cdot -4 \sin(4 m_2 \cos(2 m_1 x)) \times \cos(2 m_1 x)$

$= 256 \pi \cdot \exp[\pi \cos(64 \cos(36 \cos(100)))] \cdot \sin(64 \cos(36 \cos(100))) \cdot \sin(36 \cos(100)) \cdot \cos(100)$ //

for df/dm_3: $\frac{df}{dv_3} \times \frac{dv_3}{dz_3} \times \frac{dz_3}{dm_3}$, $\frac{dz_3}{dm_3} = v_2$

multiply: $\pi \cdot \exp[\pi \cdot v_3] \cdot -2^3 \sin(2^3 \cdot z_3) \cdot v_2$

$= \pi \cdot \exp[\pi \cos(64 \cos(36 \cos(100)))] \cdot -8 \sin(64 \cos(36 \cos(100))) \cdot \cos(36 \cos(100))$

$= -8\pi \cdot \exp[\pi \cos(64 \cos(36 \cos(100)))] \cdot \sin(64 \cos(36 \cos(100))) \cdot \cos(36 \cos(100))$ //