(a) $(\mathcal{R} \star \hat{X})(0) = \mathcal{R}_0 * \hat{X}_0 + \mathcal{R}_1 * \hat{X}_1 =$ 1*1.2+2*2.3 = 5.8

$(\mathcal{R} \star \hat{X})(1) = \mathcal{R}_0 * \hat{X}_1 + \mathcal{R}_1 * \hat{X}_2 =$ 1*2.3+2*0 = 2.3

$(\mathcal{R} \star \hat{X})(2) = \mathcal{R}_0 * \hat{X}_2 + \mathcal{R}_1 * \hat{X}_3 =$ 1*0+2*-1 = -2

(b) x = $\max(\mathcal{R} \star X) = 5.8$

ϕ(x, (0.5, 0.1)) = sigmoid (0.5 * 5.8 + 0.1) = sigmoid (3)

Sigmoid (3) $= \frac{1}{1+e^{-3}} \approx 0.953$

(c) H(y, p) = -y*log(p) – (1-y)*log(1-p), for binary cross-entropy

H(1, P) = -log(p)

λ(0.5,0.1) = H(1, Sigmoid(3)), sigmoid(3) from part (b) above

H(1, Sigmoid(3)) = -log(sigmoid(3)) ≈ -log (0.953) ≈ 0.0486

(d) use chain rule to compute partial derivative $\frac{\partial \lambda}{\partial m}$

Given $\lambda(m, b)$=H(1, Sigmoid($\hat{X}$, (m,b))), which expands to:

$\lambda(m, b) = -\log(\text{Sigmoid}(\hat{X}, (\text{m,b}))) = -\log(\frac{1}{1+e^{-(m*\max((\mathcal{R} \star \hat{X})+\text{b})}})$

Consider Sigmoid($\hat{X}$, (m,b)) as ϕ, and $(m * \max\left(\left(\mathcal{R} \star \hat{X}\right) + \text{b}\right)$ as z, then we have:

$\frac{\partial \lambda}{\partial m} = \frac{\partial \lambda}{\partial \phi} \times \frac{\partial \phi}{\partial z} \times \frac{\partial z}{\partial m}$

$\frac{\partial \lambda}{\partial \phi} = -\frac{1}{\phi}$; $\frac{\partial \phi}{\partial z} = \frac{e^{-z}}{(1+e^{-z})^2}$; $\frac{\partial z}{\partial m} = \max\left(\mathcal{R} \star \hat{X}\right)$

$\phi \approx 0.953$, as computed from part(b); z = 0.5 * 5.8 + 0.1 = 3; $\max\left(\mathcal{R} \star \hat{X}\right) = 5.8$

$-\frac{1}{\phi} \times \frac{e^{-z}}{(1+e^{-z})^2} \times \max\left(\mathcal{R} \star \hat{X}\right) \approx -\frac{1}{0.953} \times \frac{e^{-3}}{(1+e^{-3})^2} \times 5.8 \approx -0.275$

(e) use the same chain rule format to compute partial derivative $\frac{\partial \lambda}{\partial b}$, just replace the last term:

$\frac{\partial \lambda}{\partial m} = \frac{\partial \lambda}{\partial \phi} \times \frac{\partial \phi}{\partial z} \times \frac{\partial z}{\partial b}$

$\frac{\partial z}{\partial b} = 1$

$-\frac{1}{\phi} \times \frac{e^{-z}}{(1+e^{-z})^2} \times 1 \approx -\frac{1}{0.953} \times \frac{e^{-3}}{(1+e^{-3})^2} \approx -0.0474$