$$\|u+v\|^2 = (u+v)^T(u+v)$$

$$= (u^T+v^T)(u+v)$$

$$= u^Tu + \boxed{u^Tv} + \boxed{v^Tu} + v^Tv$$

$$= \|u\|^2 - 2u^Tv + \|v\|^2$$

$$\|v\|^2 = \|-v\|^2$$

$$u^Tv = (u^Tv)^T = v^T u^{T^T} = v^Tu$$

$$\|u\|^2 = u^Tu$$

$$(AB)^T = B^TA^T$$

Want to prove

$$\boxed{|u^Tv| \le \|u\|\,\|v\|}$$

proof: Assume $u \neq 0,\ v \neq 0.$

$\alpha = \|u\|$, $\beta = \|v\|$

$$0 \le \|\beta u - \alpha v\|^2$$

$$= (\beta u - \alpha v)^T(\beta u - \alpha v)$$

$$= \|\beta u\|^2 - 2(\beta u)^T \alpha v + \|\alpha v\|^2$$

$$= \beta^2 \|u\|^2 - 2\beta\alpha u^T v + \alpha^2 \|v\|^2$$

$$= \|v\|^2 \|u\|^2 - 2\|v\|\,\|u\|\, u^T v + \|u\|^2 \|v\|^2$$

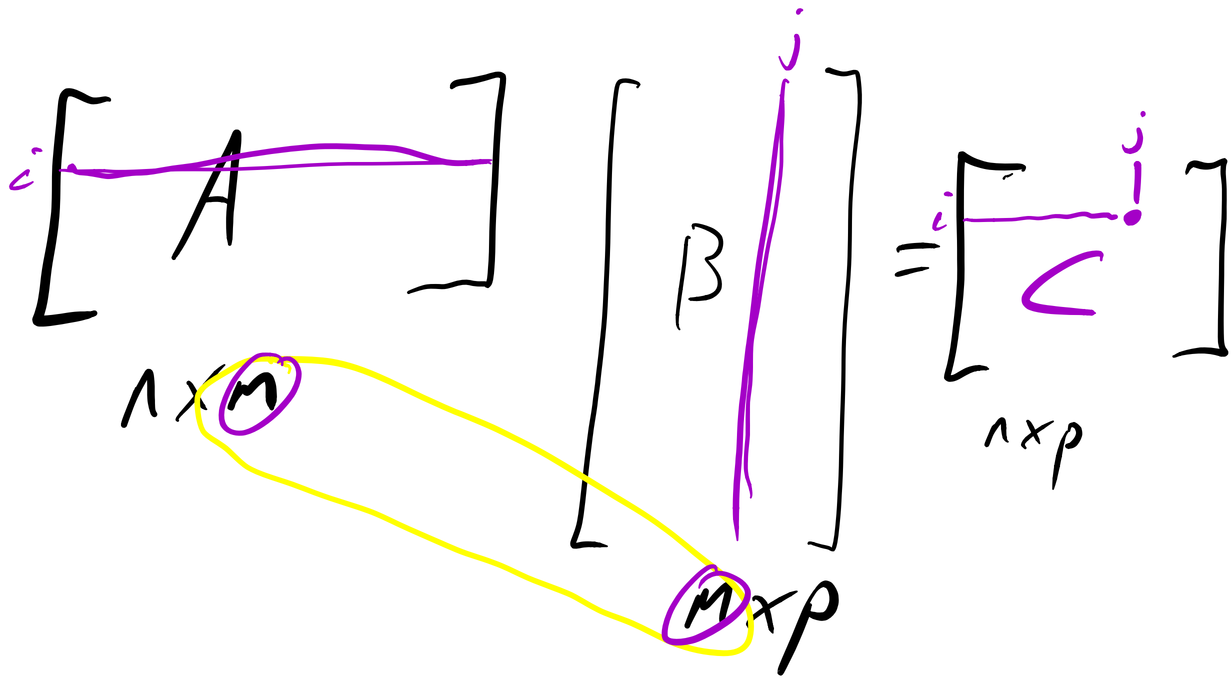
$$0 \le \|v\|\,\|u\| - 2u^T v + \|u\|\,\|v\|$$

$$u^T v \le \|u\|\,\|v\|$$

Repeat with $-u$, v

$$-\|u\|\,\|v\| \le u^T v$$

See [VMLS - pg 57]

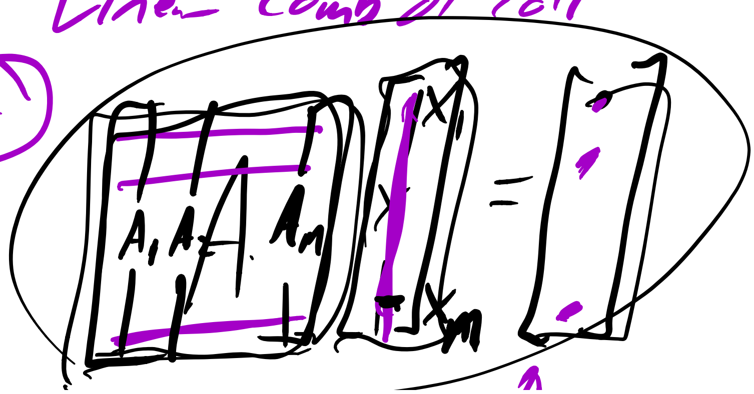


Way 1

$$C_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj}$$

Way 2

Linear comb of cols



Linear Comb' of cols'

$$= x_1 \begin{bmatrix} | \\ A_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ A_2 \\ | \end{bmatrix} + \cdots + x_n \begin{bmatrix} | \\ A_n \\ | \end{bmatrix}$$

$$\begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} | & | & | & & | \\ B_1 & B_2 & B_j & \cdots & B_p \\ | & | & | & & | \end{bmatrix} = \begin{array}{c} \\ i \end{array} \begin{array}{c} j \\ \begin{bmatrix} \quad & \; & \quad \end{bmatrix} \end{array}$$

Way 3

Dual of way 2

$$\begin{bmatrix} - X - \end{bmatrix} \begin{bmatrix} - B_1 - \\ - B_2 - \\ \vdots \\ - B_n - \end{bmatrix} = \begin{bmatrix} \text{Row}_{i,k} \end{bmatrix}$$

$$x_1 B_1 + x_2 B_2 + \cdots + x_n B_n$$

$$\begin{array}{c} \\ \\ i \\ \\ \\ \end{array}\begin{bmatrix} - A_1 - \\ - A_2 - \\ \vdots \\ \vdots \\ - A_n - \end{bmatrix}\begin{bmatrix} - B_1 - \\ - B_2 - \\ \vdots \\ - B_m - \end{bmatrix} = \begin{array}{c} \\ i \\ \\ \end{array}\begin{bmatrix} \\ \quad \\ \\ \end{bmatrix}$$

ith row is Lin comb of all B rows

Way 4

$\boxed{\text{Outer product}}$

$n=10$

inner product

Row

$$\underset{1\times 1}{\begin{bmatrix} 10 \\ \cancel{100} \\ 1000 \end{bmatrix}} \underset{1\times n}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} = \begin{bmatrix} 10 & 20 & 30 \\ 100 & 200 & 300 \\ 1000 & 2000 & 3000 \end{bmatrix}$$

Col

100 → 20

Rank I Matrix

$$\begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} C \end{bmatrix}$$

$$A_1 B_1 + A_2 B_2 + \cdots + \cancel{A_n B_n}$$

Low Rank Approx!...