NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

2024 NGTS Next-Generation Transport Systems

Probability Distribution of Connected and Autonomous Vehicle Platoon Size in a Mixed Traffic Environment

Track: Connected and Automated Vehicles

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Connected and Autonomous Vehicles (CAVs)

Connectivity

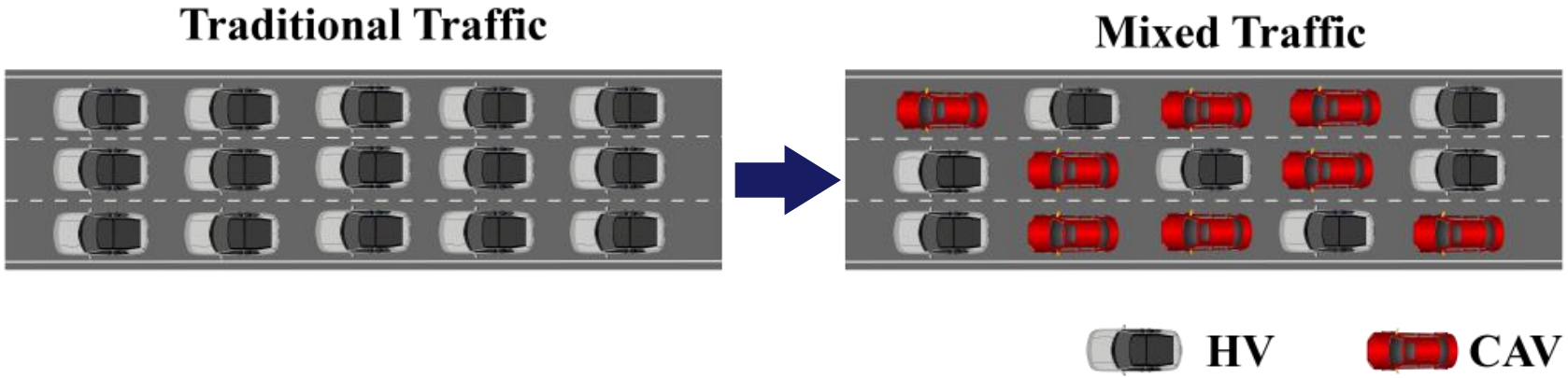
CAV

Autonomy

Platooning

Safety

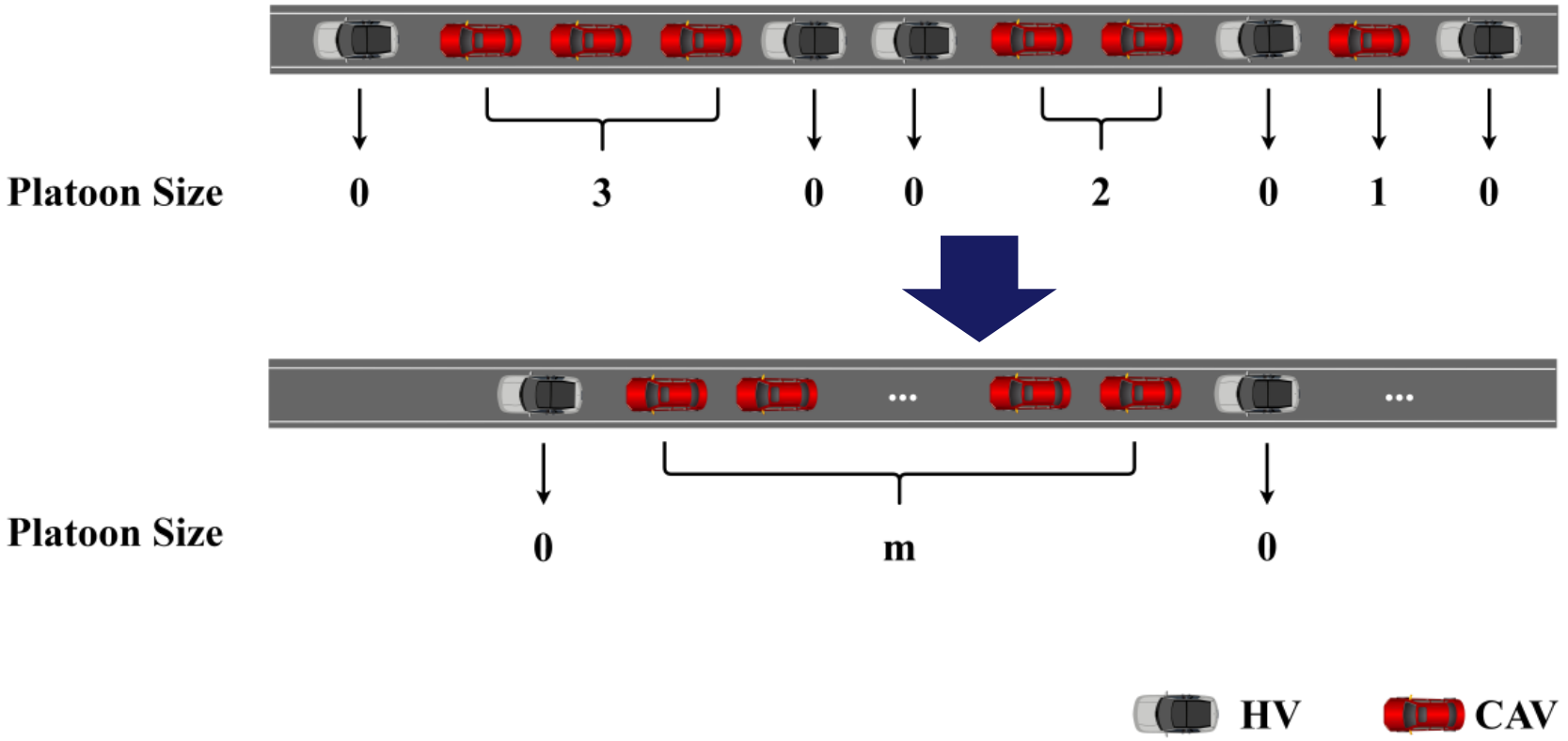
Mixed Traffic Environment



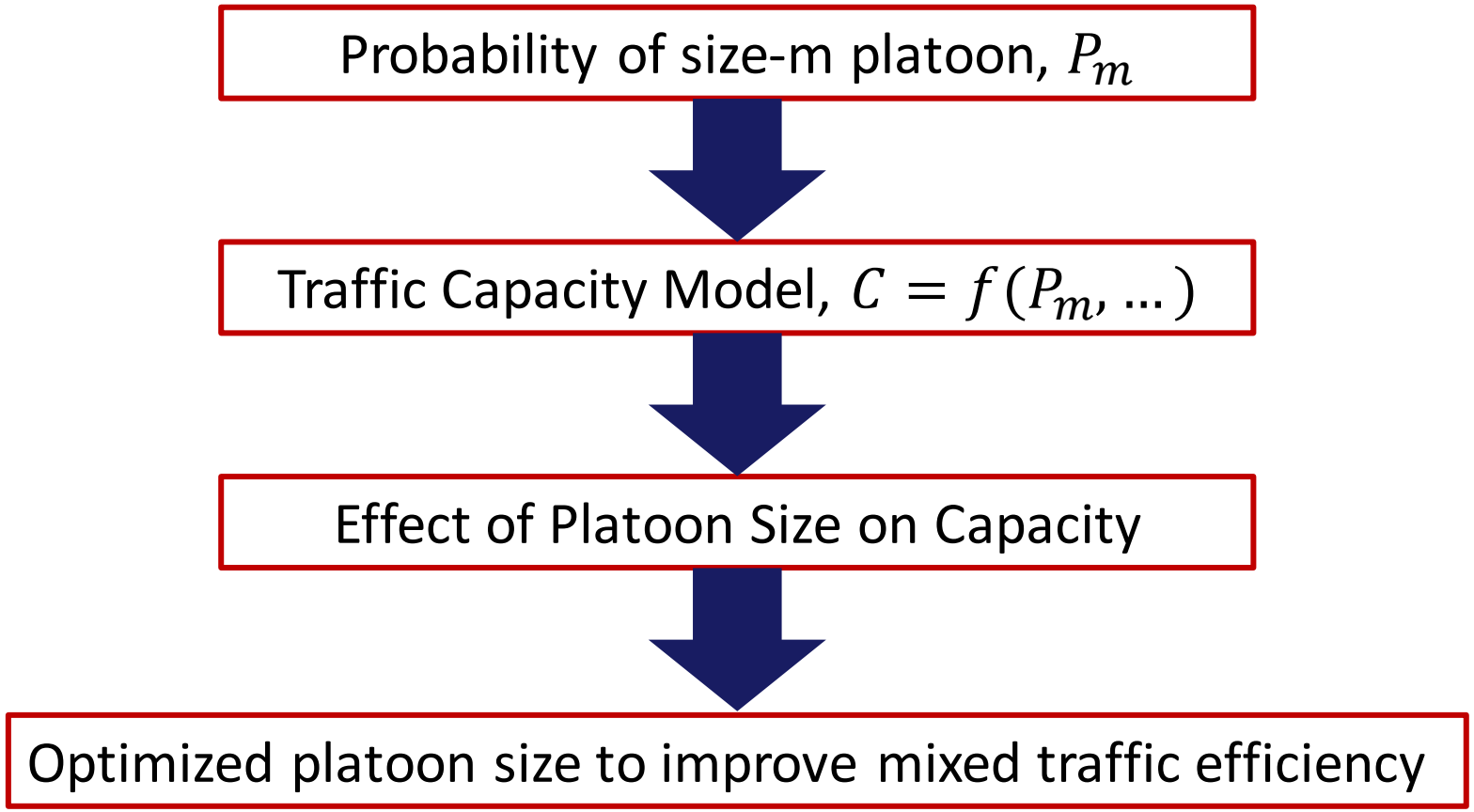
CAV features	Mixed Traffic
• Connectivity • Autonomy • Platooning • …	• Capacity • Safety • Fundamental Diagram • …

CAV Platooning

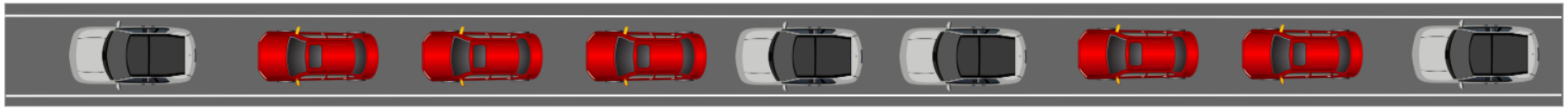
- Platoon: A group of CAVs driving together closely in a coordinated manner.
- Platoon size: number of CAVs in a CAV platoon.

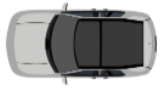


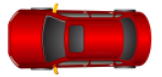
Probability Distribution of CAV Platoon Size



Model Assumption

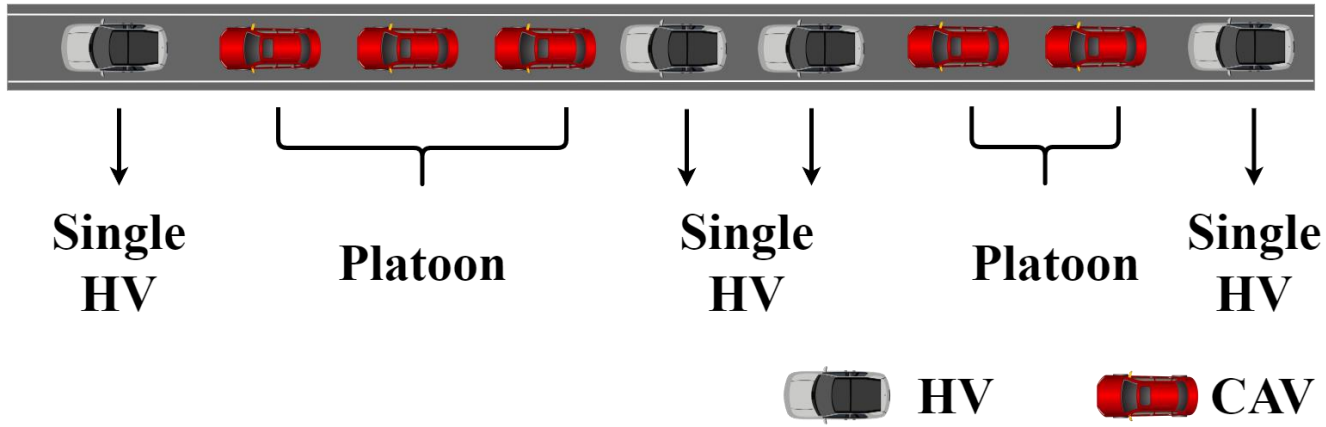



HV


CAV

1. The research focuses on the longitudinal behaviour of vehicles on the single-lane road. Therefore, the lateral vehicle actions, e.g., lane-changing and merging, are not considered.
2. The study concentrates on mixed traffic, which only includes CAVs and HVs. Pure CAVs or HVs on the road could be considered as special cases.

Model Assumption

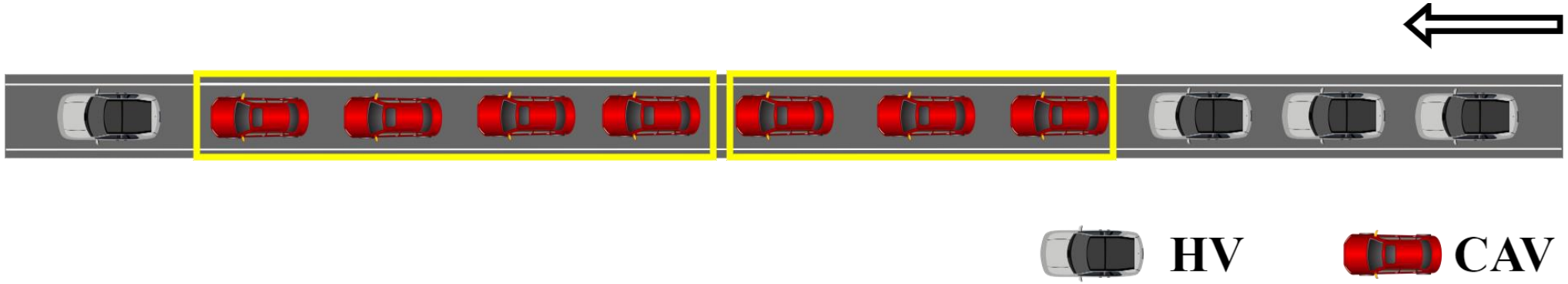


3. With the help of Cooperative Adaptive Cruise Control (CACC) and Vehicle-to-Vehicle (V2V) communication, CAVs are assumed to be able to form the platoon automatically. Correspondingly, HVs are assumed to be unable to form the platoon.
4. Single CAV is considered as size-1 CAV platoon. Single HV is considered as size-0 CAV platoon because no CAV inside the platoon.

Model Assumption

5. CAV platoon size cannot be infinite because of the limitation of technologies, e.g., communication failures and message delivery delay. When a platoon exceeds the maximum platoon size (MPS), the next CAV in line will serve as the leader of a new platoon [2].

- E.g., MPS = 4,



[2] W. Yu, X. Hua, D. Ngoduy, W. Wang, "On the assessment of the dynamic platoon and information flow topology on mixed traffic flow under connected environment," Transportation Research Part C: Emerging Technologies, vol. 154, p. 104265, 2023.

Probability Distribution of CAV Platoon Size

Limited Assumption[1]	Refutation
$P_0 = 1 - P_{CAV}$ P_0: Probability of size-0 platoon P_{CAV}: Penetration rate of CAV • Commonly interpreted as the proportion of CAVs on the road $1 - P_{CAV}$: Penetration rate of HV	Suppose that there are N vehicles forming M platoons, then, $MP_0 = N(1 - P_{CAV})$ As adjacent CAVs may form 1 platoon, N is larger than M. Consequently, P_0 is larger than $(1 - P_{CAV})$.

[1] J. Zhou, F. Zhu, "Analytical analysis of the effect of maximum platoon size of connected and automated vehicles," Transportation Research Part C: Emerging Technologies, vol. 122, p. 102882, 2021.

Probability Distribution Method

Given a specific number of vehicles, N, and CAV penetration rate, P_{CAV}, all possible sequences are enumerated.

↓

Manipulate each vehicle sequence into platoon size sequence

↓

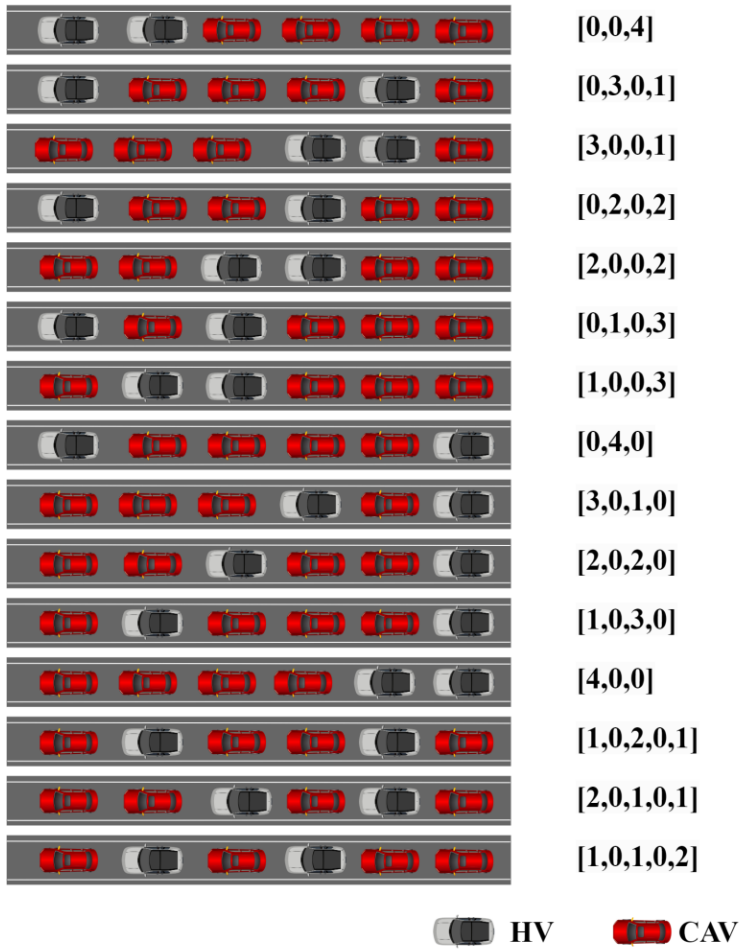
Formulate the frequency of each platoon size

↓

Calculate the relative frequency of each platoon size

↓

Derive the probability of each platoon size when the number of vehicle is sufficiently large



Frequency of CAV Platoon Size

$$N_m = \begin{cases} \binom{N}{NP_{CAV}} N(1-P_{CAV}), & m = 0; \\ (N - NP_{CAV} + 1)\binom{N-m-1}{NP_{CAV}-m}, & \text{otherwise.} \end{cases}$$

$$f_m = \begin{cases} \dfrac{N(1-P_{CAV})}{N(1-P_{CAV}^2) + P_{CAV}}, & m = 0; \\ \dfrac{(N - NP_{CAV} + 1)(1-P_{CAV})}{N(1-P_{CAV}^2) + P_{CAV}} \displaystyle\prod_{i=1}^{m} \frac{NP_{CAV} - i + 1}{N - i}, & \text{otherwise.} \end{cases}$$

$$N'_{m'} = \begin{cases} \binom{N}{NP_{CAV}} N(1-P_{CAV}), & m' = 0; \\ \displaystyle\sum_{i=0}^{C} N_{m'+iL}, \quad C = \lfloor \frac{NP_{CAV} - m'}{L} \rfloor, & m' \in \{1, 2, \ldots, L-1\}; \\ \displaystyle\sum_{j=1}^{D-1} \sum_{i=0}^{L-1} j N_{jL+i} + \sum_{i=0}^{NP_{CAV}-DL} D N_{DL+i}, \quad D = \lfloor \frac{NP_{CAV}}{L} \rfloor, & m' = L. \end{cases}$$

- N_m: frequency of size-m CAV platoon without MPS.
- f_m: relative frequency of size-m CAV platoon without MPS.
- N'_m: frequency of size-m CAV platoon when MPS is L.

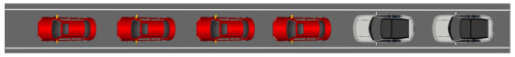
Probability of CAV Platoon Size

$$P_m = \lim_{N\to\infty} f_m = \begin{cases} \dfrac{1}{1+P_{CAV}}, & m = 0; \\ \dfrac{P_{CAV}^m(1-P_{CAV})}{1+P_{CAV}}, & \text{otherwise.} \end{cases}$$

$$P'_{m'} = \begin{cases} \dfrac{1-P_{CAV}^L}{1-P_{CAV}^L+P_{CAV}}, & m' = 0; \\ \dfrac{(1-P_{CAV})P_{CAV}^{m'}}{1-P_{CAV}^L+P_{CAV}}, & m' \in \{1, 2, \dots L-1\}; \\ \dfrac{P_{CAV}^L}{1-P_{CAV}^L+P_{CAV}}, & m' = L. \end{cases}$$

- P_m: Probability of size-m CAV platoon without MPS.
- P'_m: Probability of size-m CAV platoon when MPS is L.

Verification for Platoon Size Frequency

Combination Type	Platoon Size
	[0,0,4]
	[0,3,0,1]
	[3,0,0,1]
	[0,2,0,2]
	[2,0,0,2]
	[0,1,0,3]
	[1,0,0,3]
	[0,4,0]
	[3,0,1,0]
	[2,0,2,0]
	[1,0,3,0]
	[4,0,0]
	[1,0,2,0,1]
	[2,0,1,0,1]
	[1,0,1,0,2]

Without the consideration of MPS

- 5 different platoon sizes (0, 1, 2, 3, 4) with the frequency of (30, 12, 9, 6, 3)
- Try the equation of frequency:

$$N_0 = \binom{N}{NP_{CAV}} N(1-P_{CAV}) = \binom{6}{4} 6\left(1-\frac{4}{6}\right) = 30,$$

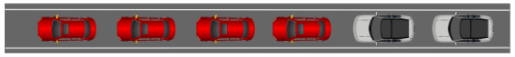
$$N_1 = (N - NP_{CAV} + 1)\binom{N-m-1}{NP_{CAV}-m} = (6-4+1)\binom{6-1-1}{4-1} = 12,$$

$$N_2 = (N - NP_{CAV} + 1)\binom{N-m-1}{NP_{CAV}-m} = (6-4+1)\binom{6-2-1}{4-2} = 9,$$

$$N_3 = (N - NP_{CAV} + 1)\binom{N-m-1}{NP_{CAV}-m} = (6-4+1)\binom{6-3-1}{4-3} = 6,$$

$$N_4 = (N - NP_{CAV} + 1)\binom{N-m-1}{NP_{CAV}-m} = (6-4+1)\binom{6-4-1}{4-4} = 3.$$

Verification for Platoon Size Frequency

Combination Type	Platoon Size
	[0,0,4]
	[0,3,0,1]
	[3,0,0,1]
	[0,2,0,2]
	[2,0,0,2]
	[0,1,0,3]
	[1,0,0,3]
	[0,4,0]
	[3,0,1,0]
	[2,0,2,0]
	[1,0,3,0]
	[4,0,0]
	[1,0,2,0,1]
	[2,0,1,0,1]
	[1,0,1,0,2]

Without the consideration of MPS

- 5 different platoon sizes (0, 1, 2, 3, 4) with the relative frequency of (0.5, 0.2, 0.15, 0.1, 0.05)
- Try the equation of relative frequency:

$$f_0 = \frac{N(1-P_{CAV})}{N(1-P_{CAV}^2)+P_{CAV}} = \frac{6(1-\frac{4}{6})}{6(1-(\frac{4}{6})^2)+\frac{4}{6}} = 0.5,$$

$$f_1 = \frac{(N-NP_{CAV}+1)(1-P_{CAV})}{N(1-P_{CAV}^2)+P_{CAV}} \prod_{i=1}^{1} \frac{NP_{CAV}-i+1}{N-i}$$

$$= \frac{(6-4+1)(1-\frac{4}{6})}{6(1-(\frac{4}{6})^2)+\frac{4}{6}} \cdot \frac{4-1+1}{6-1} = 0.2,$$

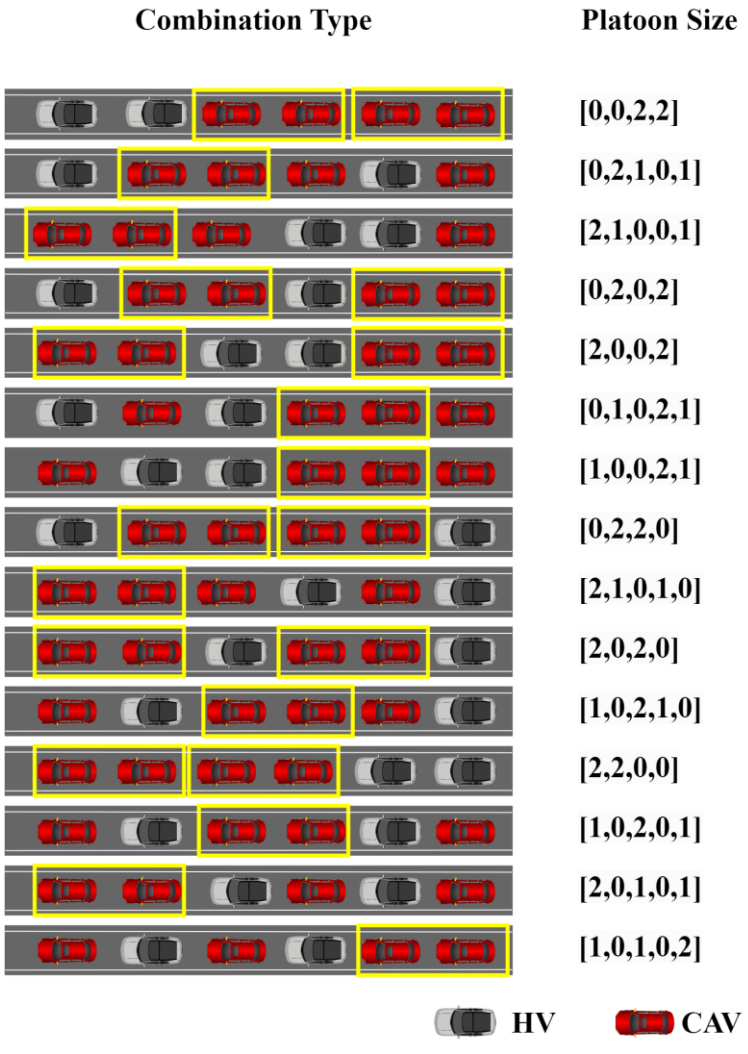
$$f_2 = \frac{(N-NP_{CAV}+1)(1-P_{CAV})}{N(1-P_{CAV}^2)+P_{CAV}} \prod_{i=1}^{2} \frac{NP_{CAV}-i+1}{N-i}$$

$$= \frac{(6-4+1)(1-\frac{4}{6})}{6(1-(\frac{4}{6})^2)+\frac{4}{6}} \cdot \frac{4-1+1}{6-1} \cdot \frac{4-2+1}{6-2} = 0.15$$

$$f_3 = f_2 \cdot \frac{4-3+1}{6-3} = 0.1,$$

$$f_4 = f_3 \cdot \frac{4-4+1}{6-4} = 0.05$$

Verification for Platoon Size Frequency



When MPS = 2

- 3 different platoon sizes (0,1,2) with the frequency of (30,18,21)
- Try the equation of frequency:

$$N_0' = \binom{N}{NP_{CAV}} N(1-P_{CAV}) = \binom{6}{4}6(1-\frac{4}{6}) = 30.$$

$$N_1' = \sum_{i=0}^{C} N_{m'+iL}, \quad C = \lfloor \frac{NP_{CAV}-m'}{L} \rfloor$$

$$= \sum_{i=0}^{C} N_{1+2i}, \quad C = \lfloor \frac{4-1}{2} \rfloor = 1$$

$$= N_1 + N_3 = 12 + 6 = 18.$$

$$N_2' = \sum_{j=1}^{D-1}\sum_{i=0}^{L-1} jN_{jL+i} + \sum_{i=0}^{NP_{CAV}-DL} DN_{DL+i}, \quad D = \lfloor \frac{NP_{CAV}}{L} \rfloor$$

$$= \sum_{j=1}^{D-1}\sum_{i=0}^{2-1} jN_{2j+i} + \sum_{i=0}^{4-2D} DN_{2D+i}, \quad D = \lfloor \frac{4}{2} \rfloor = 2$$

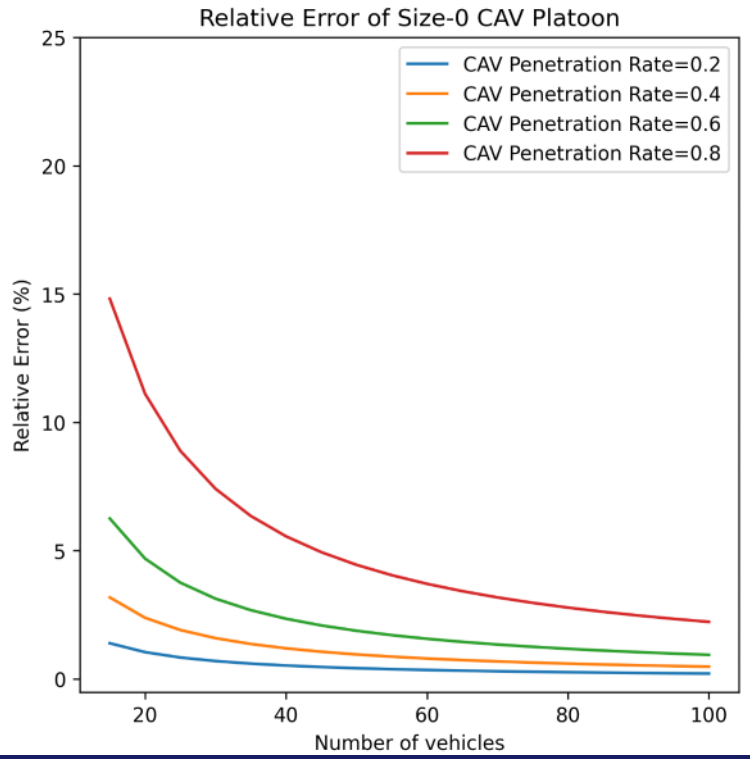
$$= \sum_{j=1}^{1} (jN_{2j} + jN_{2j+1}) + \sum_{i=0}^{0} 2N_{4+i}$$

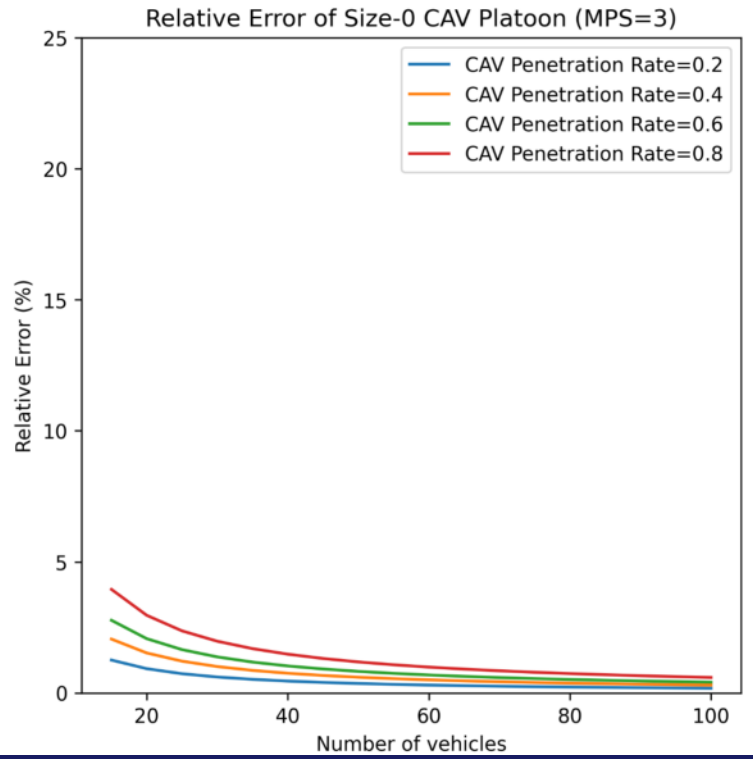
$$= N_2 + N_3 + 2N_4 = 9 + 6 + 2 \cdot 3 = 21.$$

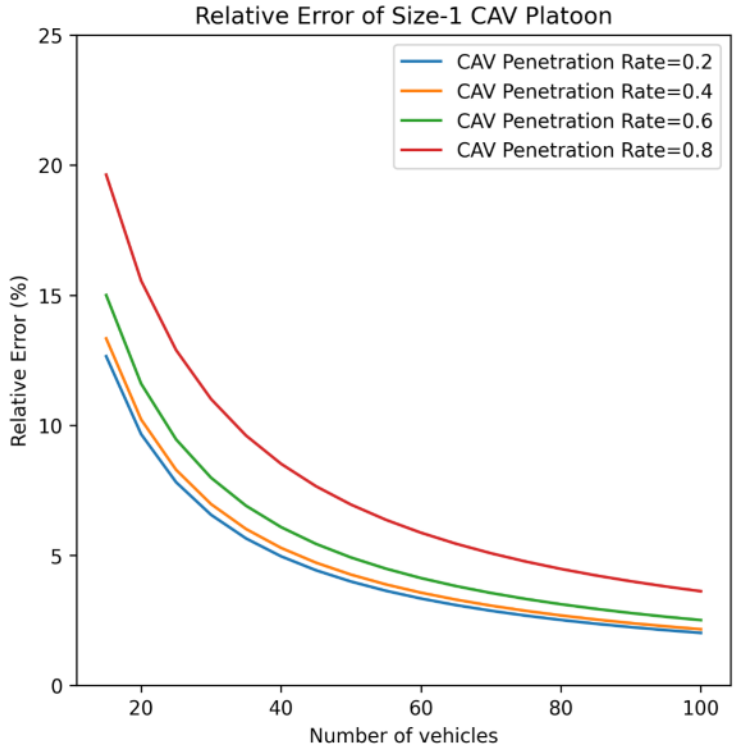
Verification for Platoon Size Probability Distribution

- We calculate the relative error between proposed probability distribution and actual relative frequency.

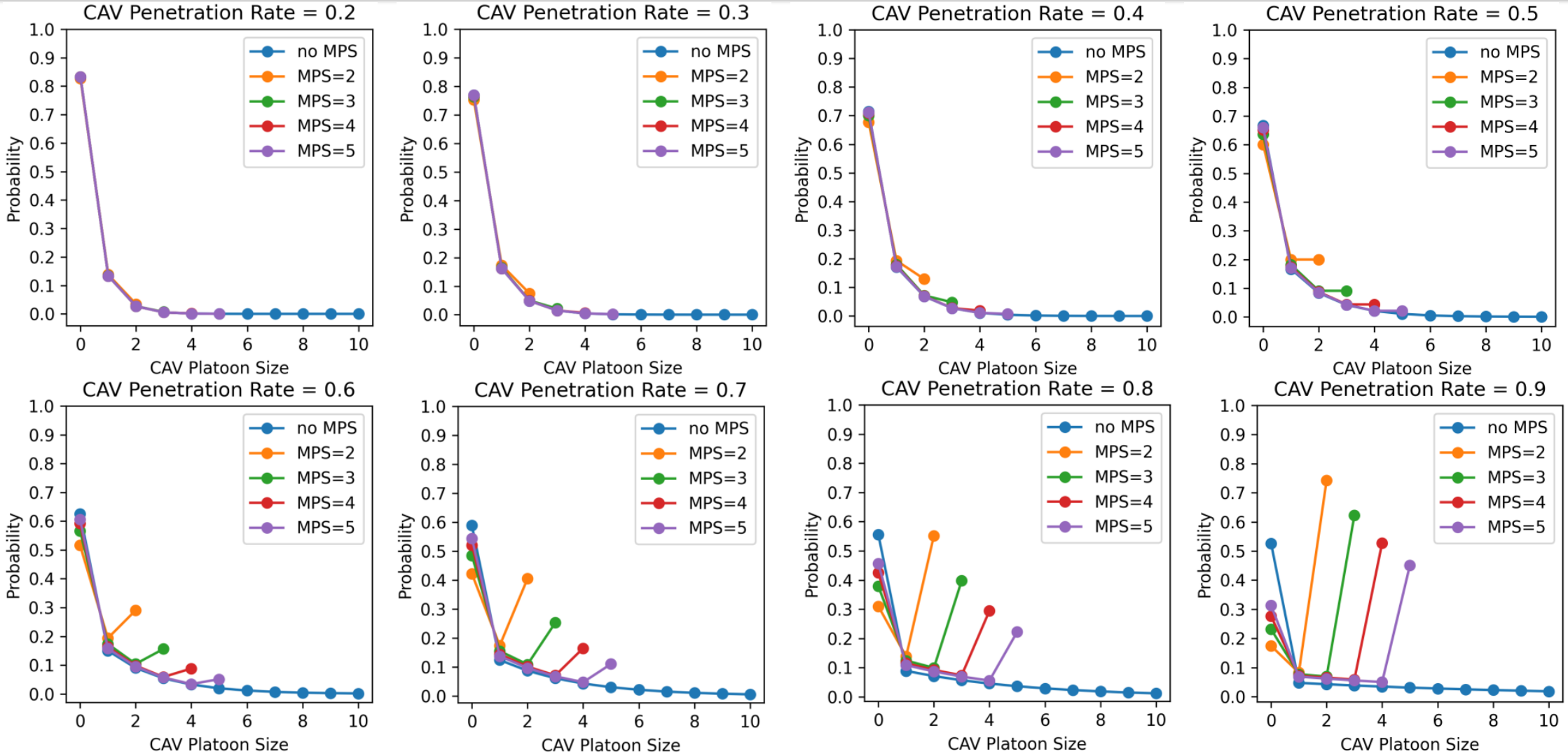
$$\epsilon = \frac{|\text{Theoretical Probability} - \text{Actual Relative Frequency}|}{\text{Actual Relative Frequency}}.$$







Platoon Size Probability Distribution Visualization



Conclusions and Contributions

Conclusions:

1. An accurate computing method for the frequency of different CAV platoon sizes is proposed with or without the consideration of MPS.
2. A CAV platoon size probability distribution is derived from corresponding frequency, which can accurately and effectively reflect the distribution of different CAV platoon size.
3. Proposed probability distribution contributes to provide a basis for subsequent traffic characteristics modelling, such as capacity.

Future Work:

1. Proposed probability distribution would be applied in traffic capacity modeling to explore how platoon size influences traffic capacity. Meanwhile, the influence of platoon size on other traffic characteristics could be further explored.
2. This research formulates the probability distribution based on different combinations of HVs and CAVs and the maximum platoon size. More metrics of CAVs platooning behavior could be taken into considered.
3. Real world data could be used for further calibrating the proposed theoretical model.

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Thank you

Welcome the feedback to make us work better.

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