Cheat Sheet

June 20, 2023 2:55 PM

FPNS

$\{\beta, t, L, U\} \Rightarrow \pm\ 0.d_1 d_2 \ldots d_t \times \beta^p$ for $L \le p \le U$ & $d_1 \neq 0$.
or 0.

Rounding $(\beta = 10, t = 3)$: $f(0.1234) = 0.123$ (if bit is < half of β)
$f(0.1235) = 0.124$

Machine Epsilon: $E = \frac{1}{2}\beta^{1-t}$ (bound of relative error $\frac{|f(x) - x|}{|x|}$)

Absolute error bound : $|f(x) - x| \le \frac{1}{2}\beta^{1-t}\beta^{j-1}$

$$\text{RelError}(ab+c) = \frac{|(a \otimes b) \oplus c - (ab+c)|}{|ab+c|} = \frac{|(ab(1+\delta_1)+c)(1+\delta_2) - (ab+c)|}{ab+c}$$
$$\le \frac{|ab\delta_1(1+\delta_2)| + |\delta_2(ab+c)|}{\ldots}$$
$$\le \frac{|ab|E(1+E) + |ab+c|E}{ab+c}$$

Page Rank

$M = \alpha P' + (1-\alpha)\frac{1}{R}ee^T$ $e = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ $P' = P + \frac{1}{R}ed^T$

↳ add random teleport

↳ 1 if node is terminal

Eigenvector problem ($p = Mp$ steady state).

Linear Algebra

Back subs: $x_i = \frac{z_i - \sum_{j=i+1}^{N} u_{ij} x_j}{u_{ii}}$ (For upper △) $(O(N^2))$

Forward subs: $x_i = \frac{z_i - \sum_{j=1}^{i-1} l_{ij} x_j}{l_{ii}}$ (For lower △) $(O(N^2))$

Gaussian Elim → Aug matrix, upper △, back sub.

LU Factorization → $LU = PA$ ($Ax = b \Rightarrow L\underbrace{Ux}_{z} = Pb \Rightarrow Lz = Pb$) $(O(N^3))$

↳ swap rows to get largest pivot → SWAP mults in L too!

↳ row reduce ($(i) - \frac{r_i}{r_1}(1)$)

↳ store multipliers in L ($\frac{r_i}{r_1}$)

$A = \begin{bmatrix} 12 & 12 & -24 & -48 \\ 4 & 0 & 2 & 8 \\ 6 & 4 & -11 & -16 \\ -4 & -12 & 20 & 40 \end{bmatrix}$

↓

$P = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$ $\begin{array}{l} \\ (2) - \frac{4}{12}(1) \\ (3) - \frac{6}{12}(1) \\ (4) - \frac{-4}{12}(1) \end{array} \begin{bmatrix} 12 & 12 & -24 & -48 \\ 0 & -4 & 10 & 24 \\ 0 & -2 & 1 & 8 \\ 0 & -8 & 12 & 24 \end{bmatrix}$ $L = \begin{bmatrix} 1 & & & 0 \\ 1/3 & 1 & & \\ 1/2 & & 1 & \\ -1/3 & & & 1 \end{bmatrix}$

↓ swap (2)&(4)

$\begin{bmatrix} 12 & 12 & -24 & -48 \\ 0 & -8 & 12 & 24 \\ 0 & -2 & 1 & 8 \\ 0 & -4 & 10 & 24 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

IVP

(1) Dynamics Eq
(2) Initial State

Ex: $\frac{dx(t)}{dt} = V_x$ $\frac{d^2y(t)}{dt^2} = -g$.

$z_1 = x$, $z_2 = y$, $z_3 = \frac{dy}{dt}$

Eulers : $y_{k+1} = y_k + h_k f(t_k, y_k)$

event func
↳ neg if over.

$l_{n+1} = \| \hat{y}_n(x_{n+1}) - y_{n+1} \|$ $O(h^2)$

$\varepsilon_{n+1} = \| \hat{y}_0(x_{n+1}) - y_{n+1} \|$ $O(h)$

Mod Euler : euler step average w/ slope of next point.
↳ $O(h^3)$
↳ 2nd order Runge-Kutta.

↳ diff to estimate error

Stability → $y(0) = 1$, $y' = -\lambda y$ $(\lambda > 0)$

$y(x) = e^{-\lambda t} \Rightarrow \lim_{t \to \infty} = 0$.

$y_{n+1} = (1 - \lambda h) y_n \Rightarrow y_{n-1} = (1 - \lambda h)^n y_0 \ \ldots \ h < \frac{2}{\lambda}$