Introduction to Market Structures

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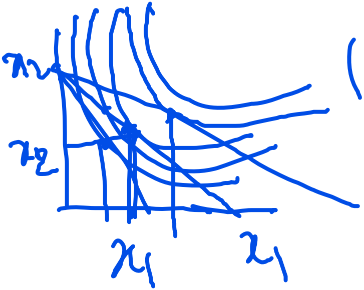
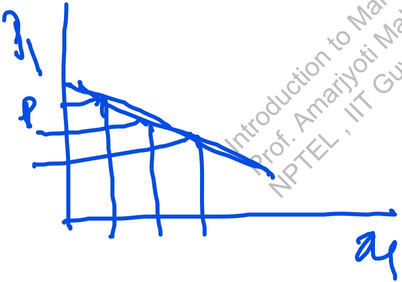
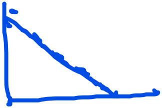


(P_1, P_2), M

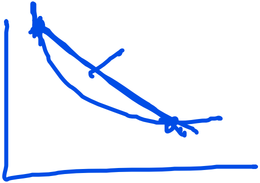


We can also solve the optimization problem of the consumer through Lagrangian method. We solve one example through this method. Suppose the utility function is $U(x_1, x_2) = x_1^{\alpha}x_2^{\beta}$ and the budget is $p_1.x_1 + p_2.x_2 \leq m$



The utility maximization problem is:
Maximize $x_1^{\alpha}.x_2^{\beta}$
subject to $p_1.x_1 + p_2.x_2 \leq m$.
The Lagrangian is $L = x_1^{\alpha}.x_2^{\beta} + \lambda(m - p_1x_1 + p_2x_2)$, where λ is the Lagrange multiplier.
Differentiating the Lagrangian with respect to x_1, x_2 and λ we get,

$$\frac{\partial L}{\partial x_1} = \alpha x_1^{\alpha-1}x_2^{\beta} - \lambda p_1$$
$$\frac{\partial L}{\partial x_2} = \beta x_1^{\alpha}x_2^{\beta-1} - \lambda p_2.$$
$$\frac{\partial L}{\partial \lambda} = m - p_1x_1 + p_2x_2.$$



At the optimal point, the first order condition gives us;

$\frac{\partial L}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{\beta} - \lambda p_1 = 0$

$\frac{\partial L}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta-1} - \lambda p_2 = 0.$

$\frac{\partial L}{\partial \lambda} = m - p_1 x_1 + p_2 x_2 = 0.$

$$\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}} = \frac{\lambda p_1}{\lambda p_2}$$

From first two equations, we get

$\frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}.$

$$x_2 = \frac{p_1}{p_2}\frac{\beta x_1}{\alpha}$$

Substituting it in third equation we get, $x_1 = (\frac{\alpha m}{\alpha+\beta}) \times (\frac{1}{p_1})$.

Note that the indifference curves are convex in nature. So the point of tangency between the indifference curve and budget line is the utility maximizing point.

The demand function of good 1 is $x_1 = (\frac{\alpha m}{\alpha+\beta}) \times (\frac{1}{p_1})$. $= \left(\frac{\alpha M^{\alpha}}{\alpha+\beta}\right)\frac{1}{P_1}$

The demand function of good 2 is $x_2 = (\frac{\beta m}{\alpha+\beta}) \times (\frac{1}{p_2})$. $= \left(\frac{\alpha M^{\beta}}{\alpha+\beta}\right)\frac{1}{P_2}$

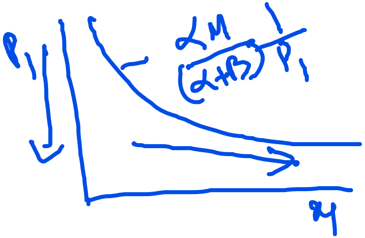
The demand functions are function of price and income. It is clear that when price of good 1 increases, the quantity demanded of good 1 decreases. Same for good 2.

Law of demand of a good is the quantity demanded of a normal good increases when the price of it decreases keeping other things constant.

Other things are; price of the other goods, income, taste and preference (utility function).

When income increases, the demand for good 1 increases, similarly for good 2. Due to change in income, there is a shift in demand function.

When price changes, there is movement along the demand curve of that good. If the price of the other good changes, there may be shift in the demand curve.



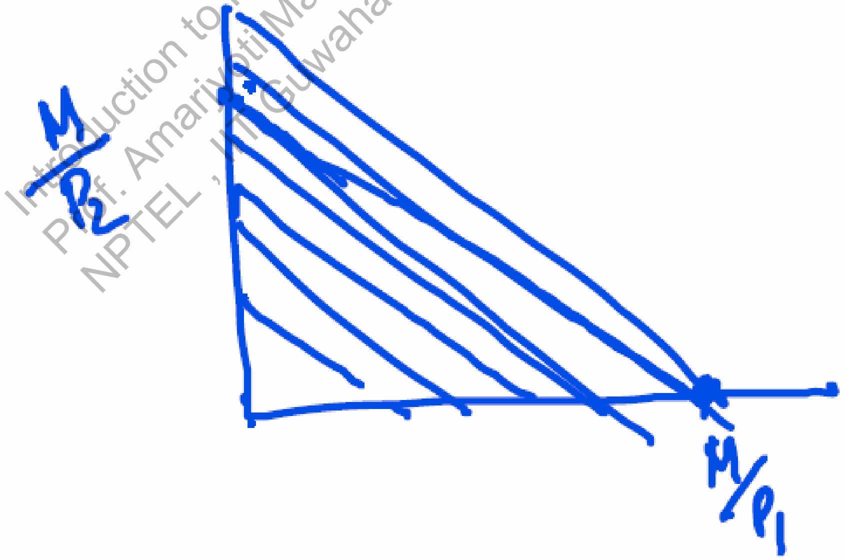
$$U(x_1, x_2) = \underline{x_1 + x_2}$$

s.t. $P_1 x_1 + P_2 x_2 = M$

$$P_1 < P_2$$

$$u(x_1,x_2) = x_1 + x_2\,, \qquad p_1x_1 + p_2x_2 \leq M$$

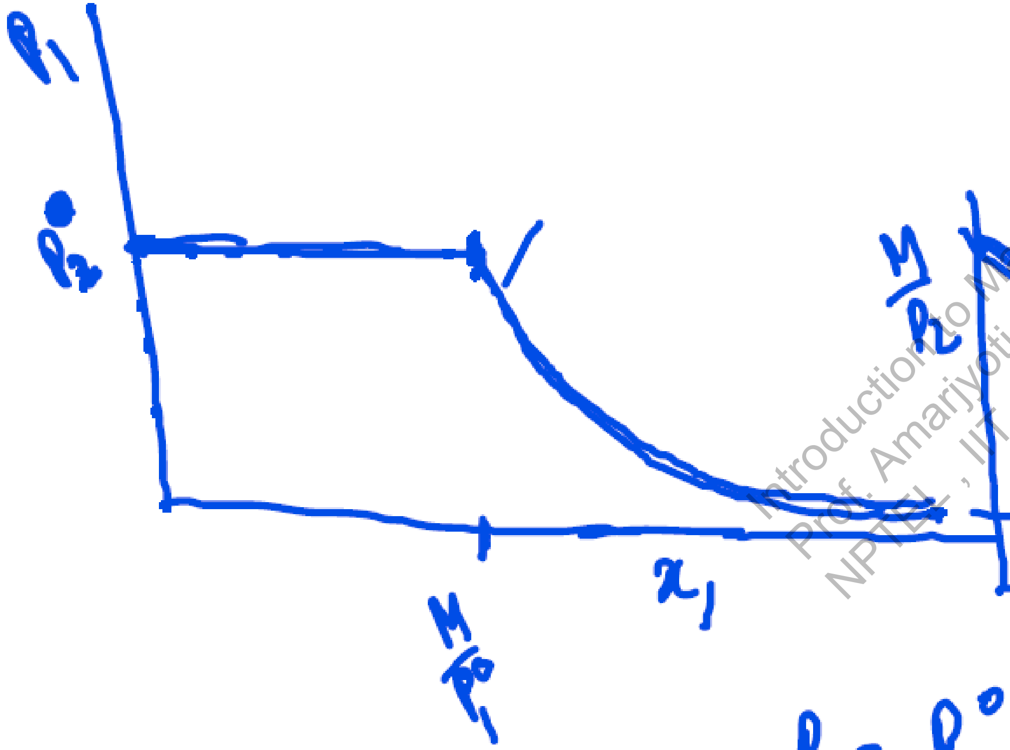
$$p_1 < p_2$$



$$\left(x_1 = \frac{M}{p_1} \right), \quad p_1 < p_2$$

$$p_1 = p_2$$

$$x_1 = \frac{M}{p_1} - x_2$$



$$p_2 = p_2^0$$

$$x_1 = 0$$

$$p_1 > p_2$$

The main reasons for having downward sloping demand curve are following:

- The slope of the budget line changes. The ratio $\frac{p_1}{p_2}$ increases when p_1 rises. So the consumer needs to give up more of good 2 to increase consumption of good 1 by one unit. Suppose the real income remains same in that case the old bundle is not utility maximising any more. At the new utility maximising bundle, the amount of good 1 will decrease as more amounts of good 2 need to be given up to increase one unit of good 1. This is substitution effect.

$\frac{P_1}{P_2}$

$\frac{M}{P_1} \uparrow$

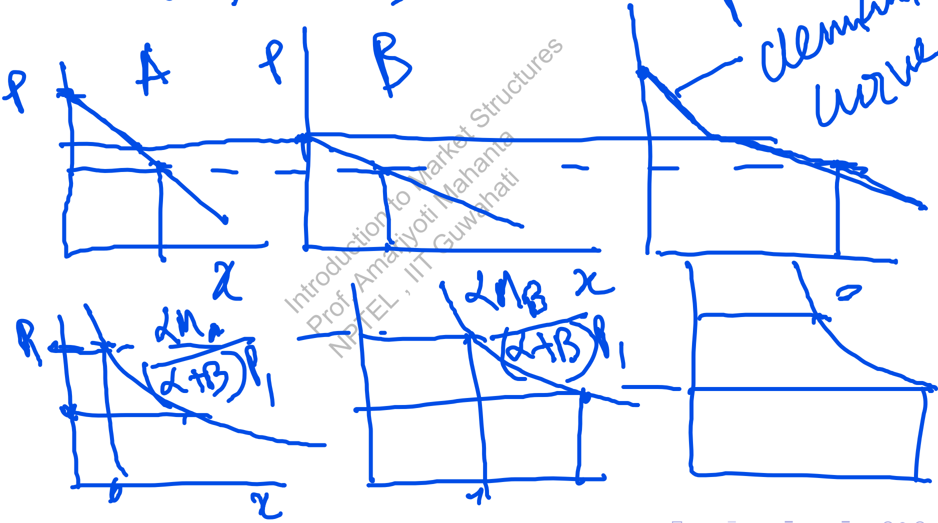
- The real income income falls ($\frac{m}{p_1}$ falls) that is, to buy the same amount of good 1 as earlier, the consumer requires higher amount of income. Due to fall in real income, the demand for good 1 will fall. This is income effect. It is true for normal goods.
- The income effect and substitution effect together leads to downward sloping demand curve of a normal good.

From the individual demand curves, we derive the market demand curve. The market demand curve is the horizontal summation of the individual demand curves.

Suppose there are two consumers 1 and 2. The demand for good 1 of consumer 1 at price p is $x_1^1(p)$ and the demand of consumer 2 is $x_1^2(p)$ at the same p. The market demand at p is $x_1^1(p) + x_1^2(p)$.

A, B

$x^A(p)$, $x^B(p)$

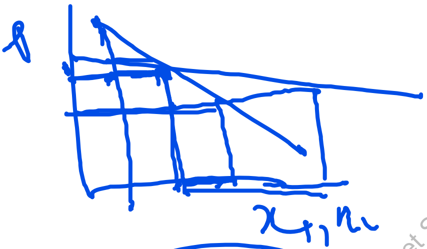


P
A
P
B
Market demand curve
x
P
λm_A
$(\alpha+\beta)P_1$
λm_B
x
$(\alpha+\beta)P_1$
x

We differentiate demand curves based on elasticities. Elasticity of a demand curve is the responsiveness of the demand curve.
Price elasticity of demand;

$$\xi_d = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}} = \frac{\partial x}{\partial p} \times \frac{p}{x},$$ where $p =$ price of good x.

Price elasticity of demand is negative because price and quantity demanded is inversely related.
If $|\xi_d| > 1$ then it is elastic demand. If $|\xi_d| < 1$ then it is inelastic demand. If $|\xi_d| = 1$ then it is unitary elastic.

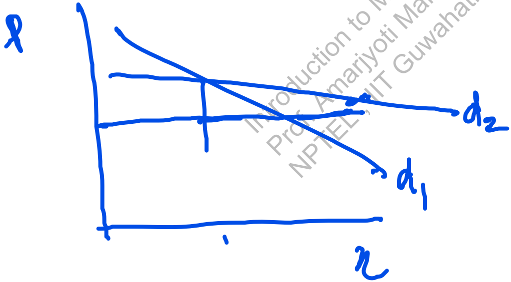


$$\underline{\underline{x_1(p_1,p_2,M)}}$$

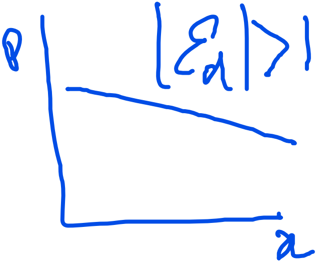
$$\frac{\partial x_1(p_1,p_2,M)}{\partial p_1}\cdot\frac{p_1}{x_1}$$

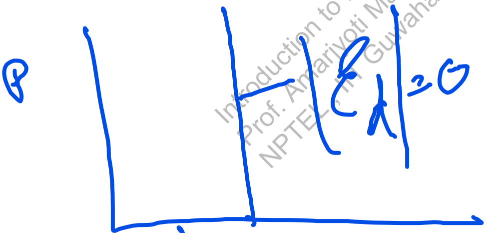
$$\left|\frac{\dfrac{\partial x_1(p_1,p_2,M)}{x_1}}{\dfrac{\partial p_1}{p_1}}\right| = |\varepsilon_d|$$

$$\frac{\Delta x}{\Delta p} \cdot \frac{p_1}{x_1}, \qquad \frac{\Delta x}{\Delta p} \cdot \frac{\left(\frac{p_1^0 + p_1^1}{2}\right)}{\left(\frac{x_1^0 + x_1^1}{2}\right)}$$



$$|\varepsilon_{d_2}| > |\varepsilon_{d_1}|$$





perfectly inelastic x

perfectly elastic

Income elasticity of demand;

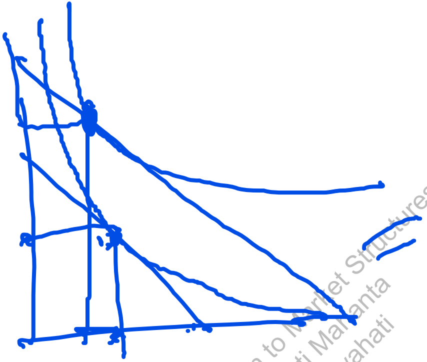
$$\xi_I = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in income}} = \frac{\partial x}{\partial m} \times \frac{m}{x},$$ where $m =$ income of individual.

If $\xi_I > 1$ then it is a luxury good. If $0 \leq \xi_I \leq 1$ then it is a normal good. If $\xi_I < 0$ then it is an inferior good.

$$\frac{\Delta x}{\Delta M} \cdot \frac{M}{x}$$

$$\frac{\Delta x}{\Delta M} < 0$$

$$\frac{\partial x}{\partial M} < 0$$



Cross price elasticity of demand;

$$\xi_c = \frac{\%\ \text{change in quantity demanded of Good 1}}{\%\ \text{change in price of good 2}} = \frac{\partial x_1}{\partial p_2} \times \frac{p_2}{x_1},$$

where p_2 = price of good 2.

If $\xi_c > 0$ then good 1 and good 2 are substitute to each other. If $\xi_c < 0$ then good 1 and good 2 are complementary to each good.

$$\frac{\Delta x_1}{\Delta P_2} \cdot \frac{P_2}{x_1}$$