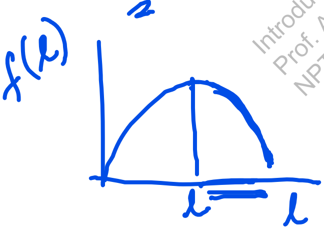
Problems on Production and Cost function

Suppose a firm uses single input labour to produce output. The production function of the firm is given as $f(l) = 2l - l^2$. Is it possible to have such a production function?



as $l \uparrow$, $f(l) \downarrow$ for $l > \frac{1}{2}$

Suppose the production function of a firm is $f(l,k) = 2l^{0.6}k^{0.7} + 3l^{0.5}k^{0.5}$. Does it follow law of diminishing marginal product? What type of returns to scale it exhibits?

$$\frac{\partial f(l,k)}{\partial l} = 1.2\, l^{-0.4} k^{0.7} + 1.5\, l^{-0.5} k^{0.5}$$

$$MP_L = \frac{1.2\, k^{0.7}}{l^{0.4}} + \frac{1.5\, k^{0.5}}{l^{0.5}}$$

as $L\uparrow$, $MP_L\downarrow$.

$$\frac{\partial f(l,k)}{\partial k} = 1.4\,\frac{l^{0.6}}{k^{0.3}} + 1.5\,\frac{l^{0.5}}{k^{0.5}}$$

$$= MP_k$$

As $k \uparrow$, $\quad \underline{\underline{MP_k \downarrow}}$

$\theta l, \quad \theta k \qquad \theta > 1$

$$2(\theta l)^{0.6}(\theta k)^{0.7} + 3(\theta l)^{0.5}(\theta k)^{0.5}$$

$$= \theta^{1.3}\, 2\, l^{0.6} k^{0.7} + \underline{3\theta\, l^{0.5} k^{0.5}}$$

$$> \theta\left[2 l^{0.6} k^{0.7} + 3 l^{0.5} k^{0.5}\right]$$

Increasing returns to scale.

Suppose the production function of firm is $f(l,k) = l^{0.5}k^{0.5}$. The price of labour (wage rate) is Rs 10 per unit. The price of capital is Rs 12 per unit. If the firm needs to produce 100 units of output. What is the cost minimising input bundle it should employ? What is the cost function of the firm?

$$\mathcal{L} = 10l + 12k + \lambda\left[100 - l^{0.5}k^{0.5}\right]$$

$$\frac{\partial \mathcal{L}}{\partial l} = 10 = \lambda\, 0.5 \frac{K^{0.5}}{l^{0.5}}$$

$$\frac{\partial \mathcal{L}}{\partial K} = 12 = \lambda\, 0.5 \frac{l^{0.5}}{K^{0.5}}$$

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$$\frac{\partial \mathcal{L}}{\partial \lambda} = 100 = l^{0.5} K^{0.5}$$

$$\Rightarrow \frac{10}{12} = \frac{K}{l}$$

$$\Rightarrow \frac{10}{12} l = K$$

$$\Rightarrow \quad 100 = l^{0.5} \left(\frac{10}{12}\right)^{0.5} \cdot l^{0.5}$$

$$\Rightarrow \quad 100 = l \left[\frac{10}{12}\right]^{0.5}$$

$$\Rightarrow \quad \frac{100}{\left(\frac{10}{12}\right)^{0.5}} = l \qquad \Big| \quad \Rightarrow k = \frac{10}{12} \cdot \frac{100}{\left(\frac{10}{12}\right)^{0.5}}$$

$$y \quad , \quad l = \frac{y}{\left(\frac{10}{12}\right)^{0.5}} = y \left(\frac{12}{10}\right)^{0.5}$$

$$k = \frac{10}{12} \cdot l = \frac{10}{12} \cdot y \cdot \left(\frac{12}{10}\right)^{0.5}$$

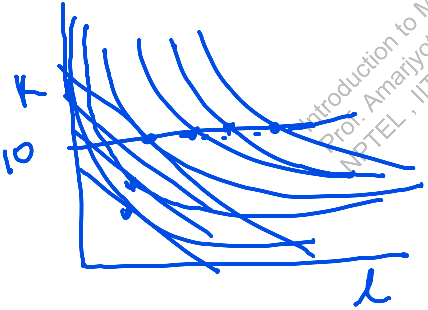
$$k = y \left(\frac{10}{12}\right)^{0.5}$$

$$C(y) = 10 l^{a} + 12 k^{a}$$

$$= 10 \cdot y \left(\frac{12}{10}\right)^{0.5} + 12 \cdot y \left(\frac{10}{12}\right)^{0.5}$$

$$= y \left[10 \cdot \left(\frac{12}{10}\right)^{0.5} + 12 \cdot \left(\frac{10}{12}\right)^{0.5} \right]$$

Suppose in the previous question $k \leq 10$. What is the cost function of the firm?



$$K \leq 10$$

$$K = y\left(\frac{12}{10}\right)^{0.5} \leq 10$$

$$y \leq \frac{10^{1.5}}{12^{0.5}}$$

$$C(y) = y\left[10\cdot\left(\frac{12}{10}\right)^{0.5} + 12\cdot\left(\frac{10}{12}\right)^{0.5}\right]$$

for $y \leq \dfrac{10^{1.5}}{12^{0.5}}$

$$y = \ell^{0.5}(10)^{0.5}$$

$$\frac{y^2}{10} = \ell$$

$$10 \cdot \frac{y^2}{10} + 12 \cdot 10$$

$$C(y) = y^2 + 120 \quad , \text{ if } y > \frac{10^{1.5}}{12^{0.5}}$$