INTRODUCTION TO MARKET STRUCTURES

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Introduction to Market Structures

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Lecture 1

Introduction to Industrial Organization, Preferences

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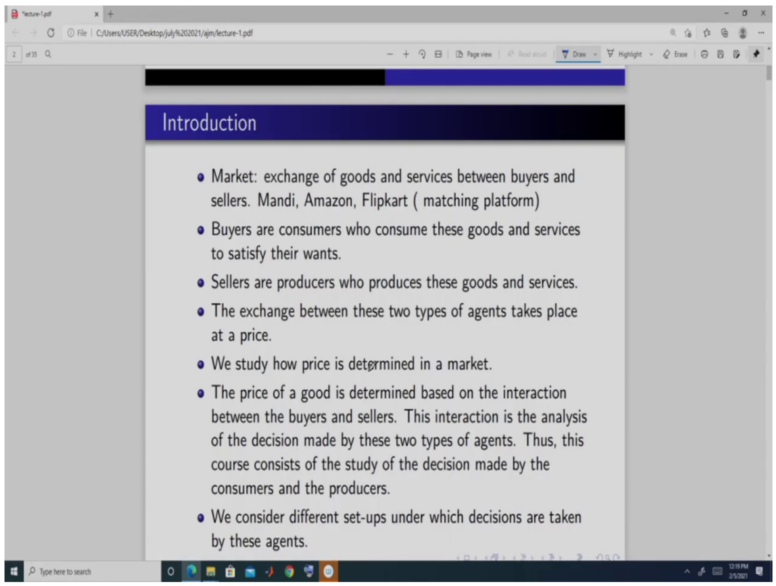

Hello, everyone. Welcome to my course Introduction to Market Structure. Mainly market means it is a place of exchange of goods and services between buyers and sellers, like Mandi, where the buyers and sellers meet to exchange goods. Amazon, Flipkart they are matching platform where the buyers put up their demand and the supplier, and or they choose the supplier from which they want to buy their good and the supplier provides them.

So, buyers are consumers who consume these goods and services to satisfy their want. Sellers are producers who produce these goods and services which are demanded by these buyers or the consumers. And the exchange between these two types of agents that is buyers and producers takes place at a price. So, we study how price is determined in a market; in this course our objective is to study this price, how it is determined.

Now price of any good is determined based on the interaction between the buyers and the sellers, right. Now this interaction is mainly analysis of the decision made by these two types of agents. So, in this course, in a way we will study the decision made by the consumers, and the decision made by the farmers or the producers. And what we will do, we will take different setups, different environments you can say and under which these decisions will be taken, okay.

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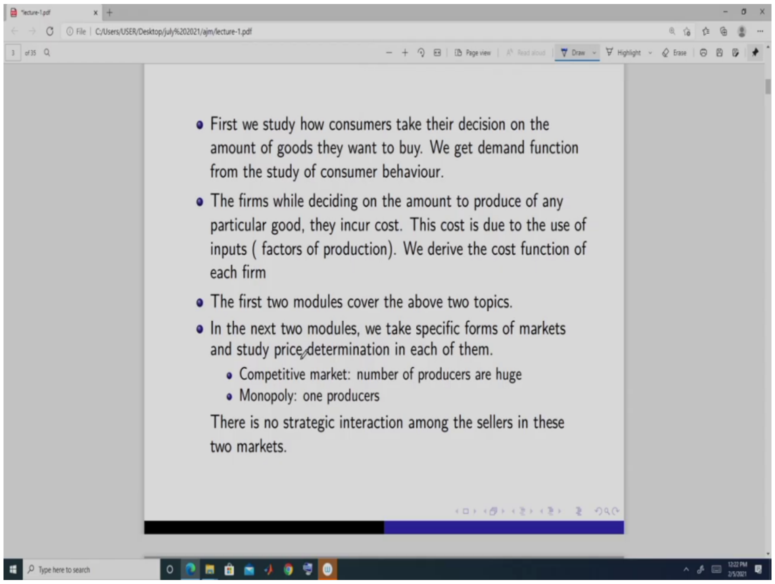

First, we study how consumers take the decision, that is how much amount of goods and services they want to buy if they face any price and if they have a fixed income, okay. So, we do that in the consumer behavior, that is our first module. And the firms while producing output, they need to employ inputs like land, labor, capital that is machine and so they incur costs in hiring these inputs. So that will give us something called the cost function.

And so, we will derive the cost function in this second module, where we will study how the firms takes decisions regarding the inputs that they are going to employ. So first these two are going to the first two modules. And the next modules are going to be on a specific form of market structure.

First, it is the competitive market and the second, is the monopoly market. In a competitive market, we assume that there are many sellers or many producers. So, none of them can determine the price. So, price is completely given independent of their decision. So, price is determined in the something called in the market, where supply is equal to demand. We will do these things in detail and then at that time you will come to know how do we get a supply curve, how do we get a demand curve, okay.

The next topic is the monopoly. In monopoly, we have only one producer. Now this one producer, so everyone who wants to buy this good or service, they have to buy it from this producer only, or this firm only. So, in that case the firm or the monopolist can determine the price. So, it can charge a very high price or it may charge a low price depending on the nature of demand or the objective of the firm. So, in these two form of market, we do not have any strategic interaction.

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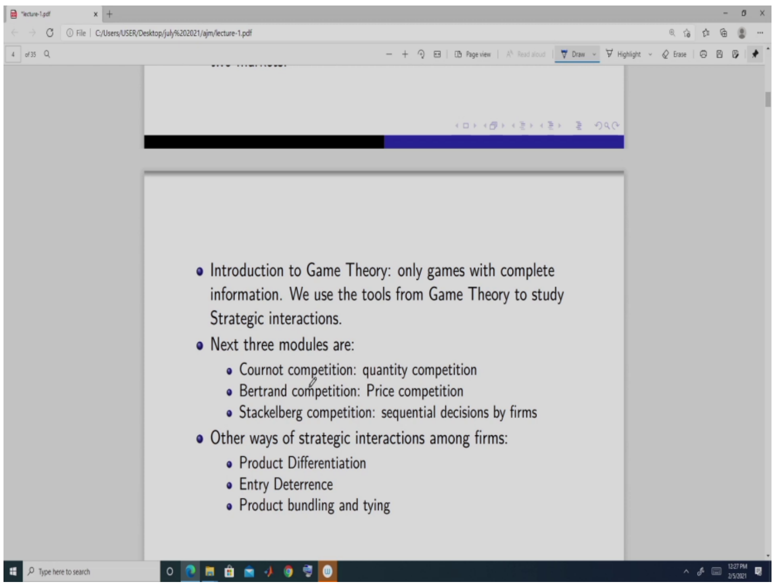

So next we study strategic interaction. Now what do we mean by strategic interaction? Strategic interaction means that if I take a decision, I have an opponent and I always consider how my opponent is going to react to my decisions. So, to study this kind of things we use Game Theory, and we use the tools from Game Theory, and specifically the non-cooperative games, okay. And we will study a little bit of Game Theory in the next module, and we will only study complete information games, both static and extensive games.

Now in games, generally what happens, we have some players, and the players behave in a specific way. They take decisions in a specific way and while taking the decision they always consider how the opponents are going to take their decision. Like for example, chess. In chess we have two players; one who is using the black game and another is the white one.

And they move alternately and while each one making a decision to make a move, they always consider how the other one is going to react to it. And so, we use mainly this kind of methodology to study the next, other forms of market. And we use some solution concept to study these games, specifically Nash equilibrium and subgame-perfect Nash equilibrium for extensive games. So, we will do this in a separate module.

And next, using these tools we will study market forms like Cournot competition, Bertrand competition and Stackelberg competition. In Cournot competition, firms decide how much amount of output to produce. While deciding that they take the output of the other firms as given. Then they see that if they react in this way how the other firm is going to react, and based on that they take the decision, okay. And here firms mainly decides on the output. And the market price is given from the market demand curve where each, when each quantity is already given.

Next form that we are going to study is Bertrand competition. In Bertrand competition firms decides the price. Now if suppose there are two firms, firm one and firm two and they produce a homogenous good. Homogenous good means that a buyer is indifferent between buying from firm one and/or firm two. It is the same type of good, okay.

Now if firm one sets a price, say firm two will decide whether to set the price little bit higher or little bit lower. And while firm one, taking this decision it will definitely see how firm two is going to react. And so, there is a going to be a competition during setting this price. So, we study this kind of market in this kind of competition in Bertrand competition.

Next in Stackelberg competition, we study, we actually change the way decisions are made by the firm. In above two markets like Cournot and Bertrand the decisions are taken simultaneously. But in this form, in this form of market that is Stackelberg, decisions are taken sequentially. Like firm one who is first, decide first on the quantity and then firm two decides on the amount of quantity it is going to produce. Or first firm one decides on the price after observing that then firm two decides how much amount of price to set. So, this is going to be Stackelberg competition, and we will study it.

Next, we will study some other form of strategic interactions among the firms. So, first is the product differentiation. What do we mean by product differentiation? Product differentiation means that the firms try to differentiate their product. They do not try to produce homogenous product. So, if there are supposed two firms, firm one and firm two, they will produce goods like firm one's output is not completely substitute of the firm two. So, you can think in terms of different attributes these goods have. They will be differentiated in that term.

Second, in this part is the entry deterrence that we are going to study. Entry deterrence is that suppose there already exists a firm. And a new firm wants to enter this firm, market. Now whether the existing firm will allow this to happen? So, they may react in such a way that there may not be any incentive for this new firm to enter this market.

So, under what condition we can get this kind of situations? So, we will study this. So, this is also very important for studying like Antitrust policies, especially a firm may be deterring entry of new firms and trying to reap more profit, okay**.** And, so that is why the competition is not there in the market. So that is why entry deterrence is quite important.

Next, we are going to study the way the firm sell their product. Like they want to bundle it, like you will get this product only in specific quantity or you can get this quantity, these goods as whatever amount you want to, like they are completely divisible. Like when you are buying suppose rice, so you buy it in half kilo, like 1 kilo or 5 kilos or 10 kilos.

Instead, if we suppose bundle it, like it is only available in 5 kilos, 10 kilos and 25 kilos. So, this, whether the firm should use this kind of strategy or method, or they should allow it whatever amount the consumer wants they should be providing, or it is pre-determined, fixed available in only specific quantities.

And another thing that we are going to study is the tying. In tying what do we do? Suppose a firm is selling, suppose a product, a good and if you buy that good from that firm then you also get some other good from it. Like if you buy suppose a trouser then you also get a shirt along, or if you buy shoes, you also get a socks along with it, or if you buy suppose laptop you get a mouse along with it. So, this is like tying.

Now if one firm does this, follows this strategy whether the other firm is also going to follow the strategy or it is going to follow some other strategy? So, we are going to study these kinds of things in this section that is the tying; and the in bundling we will see in what quantity. And one portion of the bundling we will also study in monopoly, where we will study price discrimination in monopoly and in that thing, in the first degree and second degree it is related to a form of bundling, okay. So, this is going to be the structure of this course and I hope you are going to enjoy the same.

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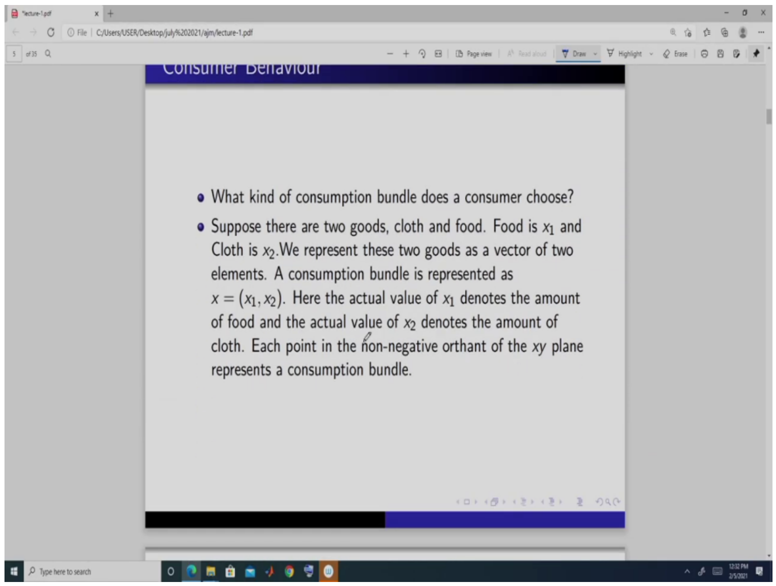

So now let us move to our first topic and that is consumer behavior. So here our objective is to derive the demand curve, okay, of a good. So, first question that we address in this section is what kind of consumption bundle does a consumer choose? Suppose you are given, you are in a world where there are only two good, cloth and food. And we represent food by x1 and cloth by x2. It can be any good but only two goods.

And these two goods are represent as a two-element vector that is x=(x1, x2). x1 is for good 1 and x2 is for good 2. And good 1 is food and good 2 is cloth, okay. So, a consumption bundle is, any consumption bundle is represented as a point in xy plane, positive orthant of that plane or you can say non-negative to be very specific.

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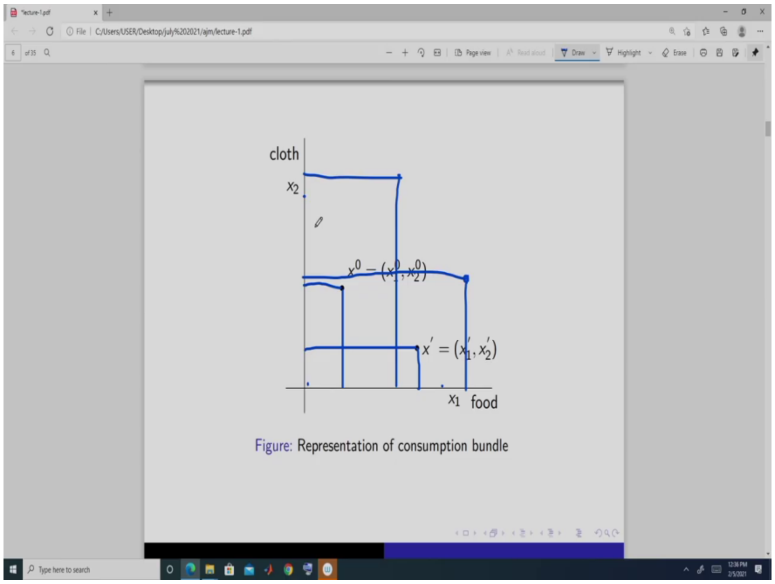

Figure: Representation of consumption bundle

So, it is like this. So, we take, in this axis, this vertical axis good 2 and in this horizontal axis we take good 1 and each point in this orthant gives us one consumption bundle. Like this point x naught, i.e x_0, it gives this much amount of, this height is this much amount of good 2 and this much amount of good 1.

Actually, if we look at this axis then each point here denotes 1 unit. So, if you are looking at this, suppose this point, suppose this point so here this much amount of good 1 is demanded or is there. Then each point in this distance, in this length, determines the amount and then each point here determines 1 unit in this line, okay.

Now suppose this, here this much amount of good 1 and this much amount of good 2, okay. So, each point, like suppose if we take this point then here this much amount of this height is giving me the good 2 and this base, this much is giving me the good 1. If we take this point, then this much amount of good 1 and this height is giving me amount of good 2. Like this each point here gives me the consumption bundle.

Now here if we take, this point then in this bundle we have 0 unit of good 1 and this much amount of good 2. If we take this point, then we have this much amount of good 1 and 0 units of good 2. But we will not take any point in this orthant, or this orthant, and this orthant because here it is negative. So, it means that we have a negative amount, so it does not make any sense, what is a negative amount of a consumption bundle okay . So that is why we will only consider this point and each point here.

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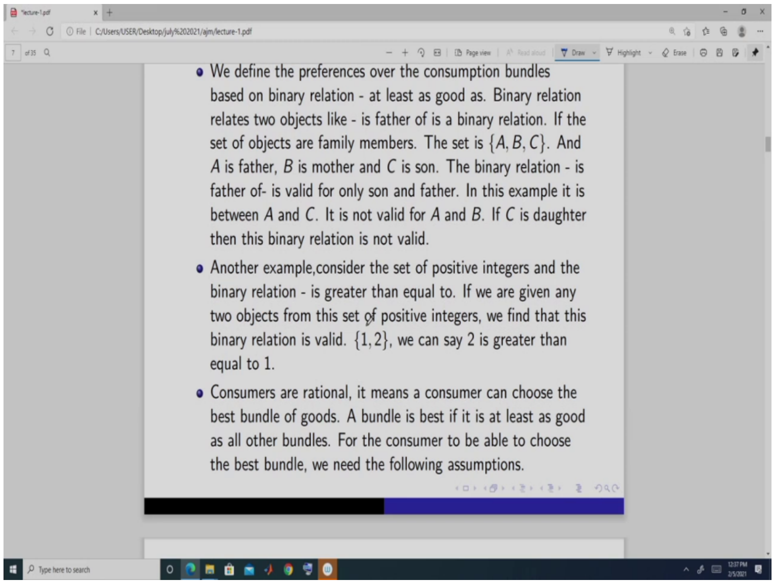

Now how do we define the preferences? Preferences means that when we are choosing then what we are doing? So, we are doing some form of a comparison, right. How do we do that? So, we introduce something called a binary relation, this. Binary relation is a way to relate two objects.

So, if we take a set, suppose like this, set {A, B, C,} it is a set of family members. And in this family, we have suppose three members, A denotes father, B denotes mother and C denotes son. And we take a binary relation like is father of, then, in this if we try to relate based on this binary then it is only valid if we take A and C because A is a father and C is the son. So, A is a father of C. But if we take A, B then it is not valid. If we take B, C then it is also not valid. If we take C, A then it is also not valid.

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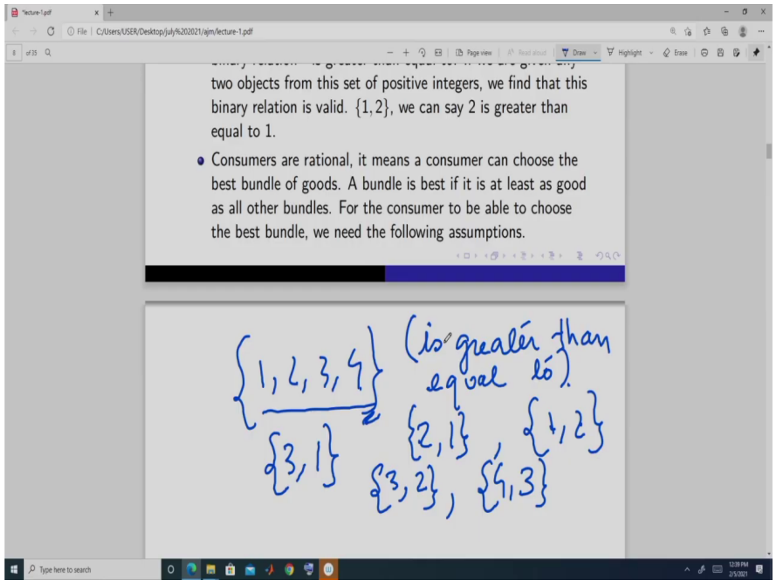

So binary relation is like a way to relate two objects. Like if we take sets of positive integer. Suppose like let us take one example, {1, 2, 3, 4} this. And suppose our binary relation is is greater than equal to, then this is valid if we write, if we take a subset from this {2, 1} of this set then this is, 2 is greater than equal to 1.

But this is not valid for this. So, what we are doing? We are relating first object with the second object. Here first object with the second object. So, if we take 1 is greater than equal to 2 it is not true. So, this is not valid. But this is valid. Then here if we take {3,1}so this is again valid. So, from here suppose if we are given this A point and we have to find out which is the greatest number.

Then we know, see 2 is greater than equal to 1; 3 is greater than equal to 1. Then 3 is greater than equal to 2, right. Then again 4 is greater than equal to 3. So, if we go on doing like this, we will find that 4 is greater than or equal to all the numbers, right. So here from this set we can find out that 4 is the greatest element in this right. So how it is possible? Using this binary relation “is greater than equal to”, okay.

So here to study the consumer behavior we will also introduce such consumer, binary relations. So, we will first assume that the consumers are rational. When consumers are rational it means that if you are given a set of bundles from which you can consume, these are consumption bundle, that is there are some amount of good 1 and some amount of good 2. Then out of this set of consumption bundle you can choose the best bundle.

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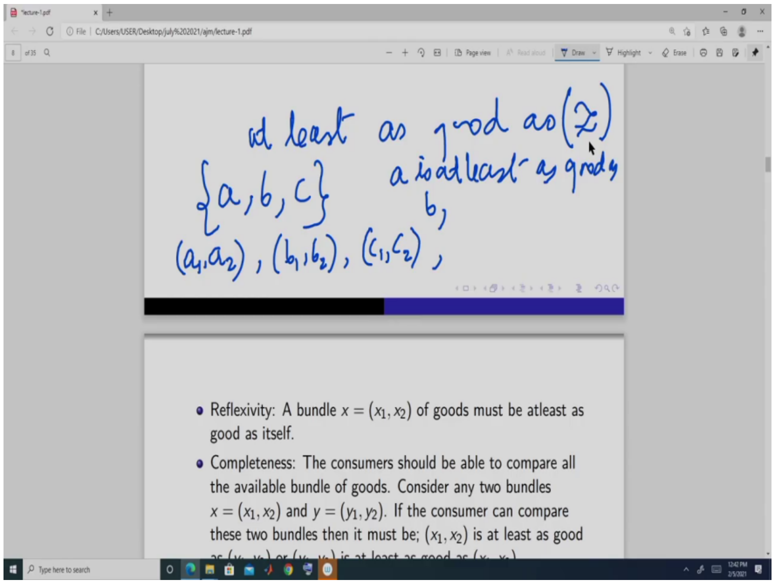

Now what is the best bundle? Best bundle is that bundle which is at least as good as all other bundles. So now here what do we do? We introduce a binary relation and that is at least as good as. So, our binary relation is “at least as good as”.

Now suppose we are given a set like {a, b and c}. So, this a is one bundle where a is something (a_1, a_2) , this b is something like this (b_1, b_2), and c is something like this (c_1, c_2), right. And then if we define this binary relation, it means a is at least as good as b or b is at least as good as c and c may be at least as good as a, like this, or c is not at least as good as a, something like this, so we define this. It is already given to us, okay. So, this is mainly binary relation that we are going to use. And what we do, this at least as good binary relation is represented in this way as a symbol, okay.

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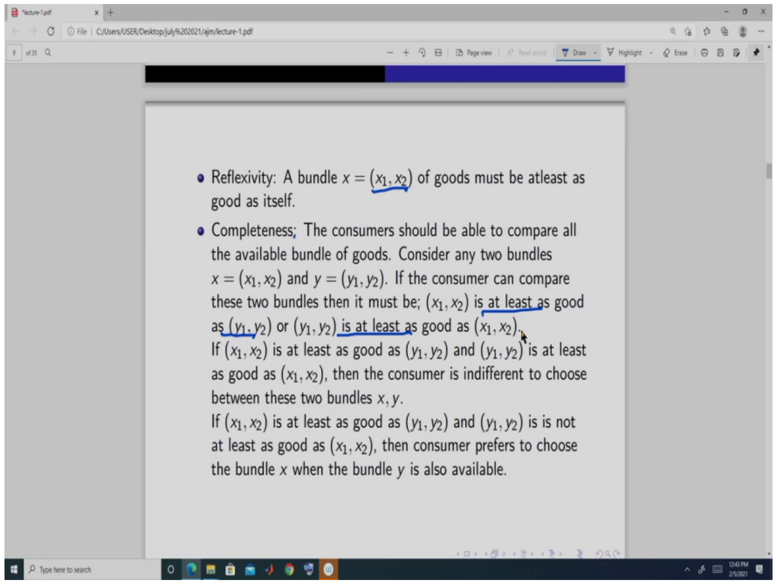

Now we specify certain assumptions on this binary relation, okay, so that we will be able to find the best bundle. First is reflexivity assumption. Reflexivity assumptions says that the bundle x, if we are given a bundlex= (x1, x2,) this x of goods then it must be at least as good as itself, okay. So, this is something like an obvious here, so if you are given a bundle then that bundle is at least as good as itself.

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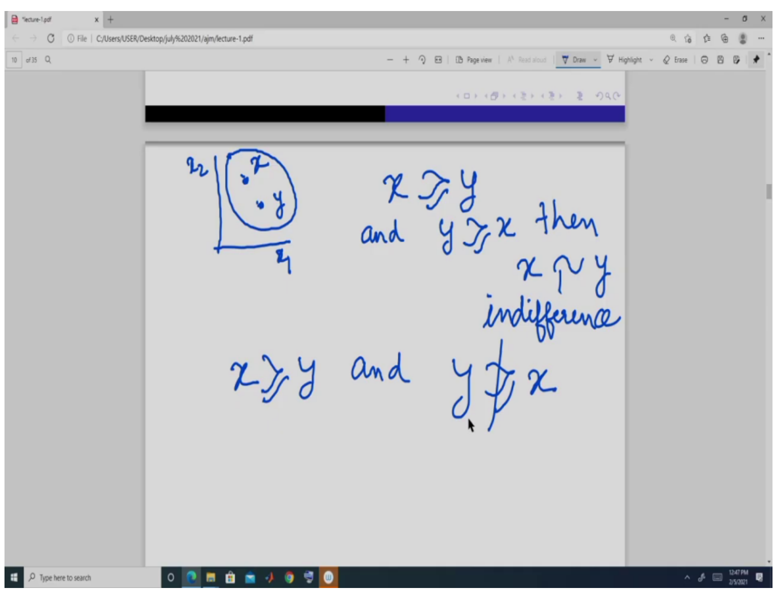

Next thing is completeness. Completeness says that we are given a set of bundle, from that pick any two bundle, okay. Suppose one is x and another is y. Now consumer should be able to compare them, that is, that x should be at least as good as y or y should be at least as good as x, right. So, suppose this is good 1, this is good 2, this point is suppose x and this point is suppose x and this point is y, okay, now, this is y.

Now, if suppose we have set, from which I have to choose a bundle is given by this set, all the points lying in this set, right. Then I should have that at least as good as y or y is at least as good as x. If it is not there then while comparing all this bundle based on this relation that is at least as good as this, then I will not be able to compare these two. right.

Then I may find out which one is best from all these things, but this when it comes to comparing these two bundles, I will be silent. I will not have anything to say. I will not, so that is why if we are given a set and from that set whatever bundle we take, it should be, it should have this relation that is "at least as good as" must be defined with all other bundles, okay. This is what completeness tells us.

Now suppose we are given any bundle that is x and another bundle is y. And if x is at least as good as y and y is at least as good as x, so in that case we say that we are indifferent between these two goods and we say, we write it in this way, that suppose x is at least as good as y and y is at least as good as x, then we write x is indifferent to y, okay. So, this is indifference i.e $x \sim y$.

Next if we have something like this, like x is at least as good as y and y is not at least as good as x, okay. Or you can write it in this form, $\sim y \geq x$, okay any form will do. No, it is better to use this because then there is no confusion. Suppose it is y is not at least as good as x, then we say that x is preferred to y. So, this is, $x > y$ is preferred to relationship, okay. So, I hope all of you are following, okay

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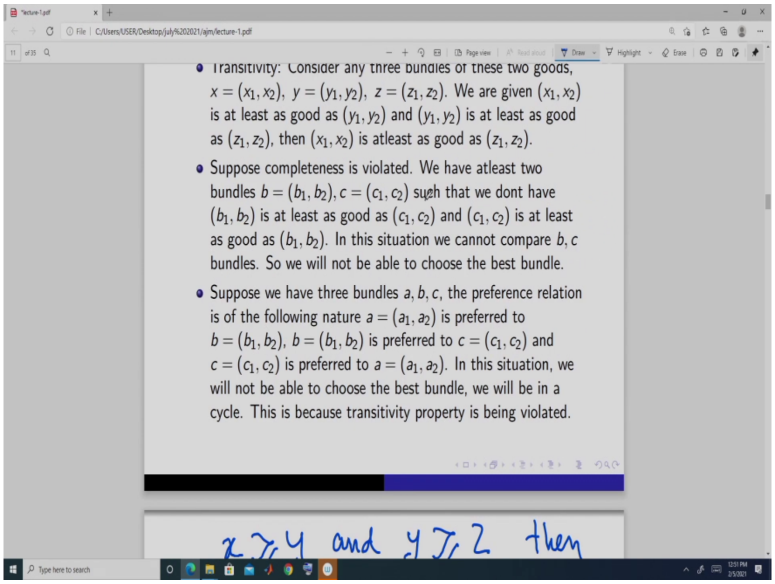

Next assumption that we or next axiom that we consider is that it is called transitivity. It is more, something some like we are taking a decision in a consistent way. So, suppose from this available set of bundles we are taking three bundles x= (x_1, x_2), y= (y_1, y_2) and z= (z_1, z_2), okay**.** Then if x is at least as good as y and y is at least as good as z, then x must be at least as good as z, okay.

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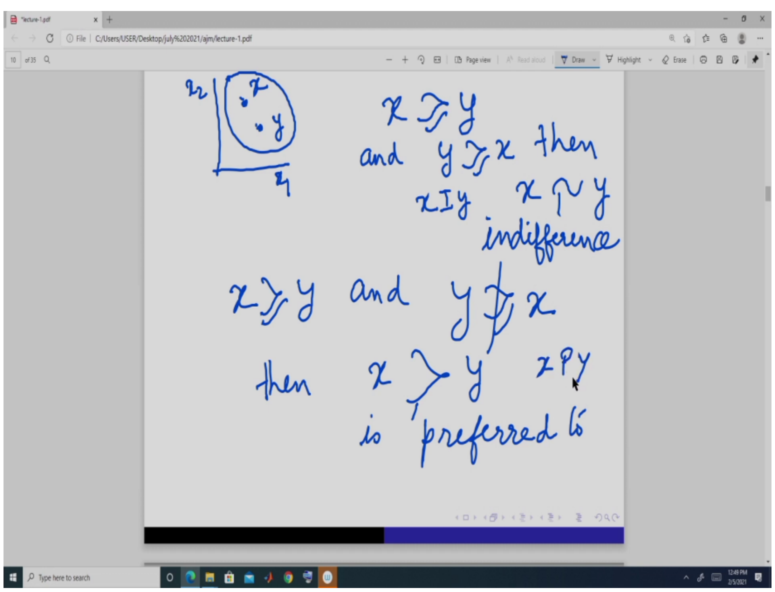

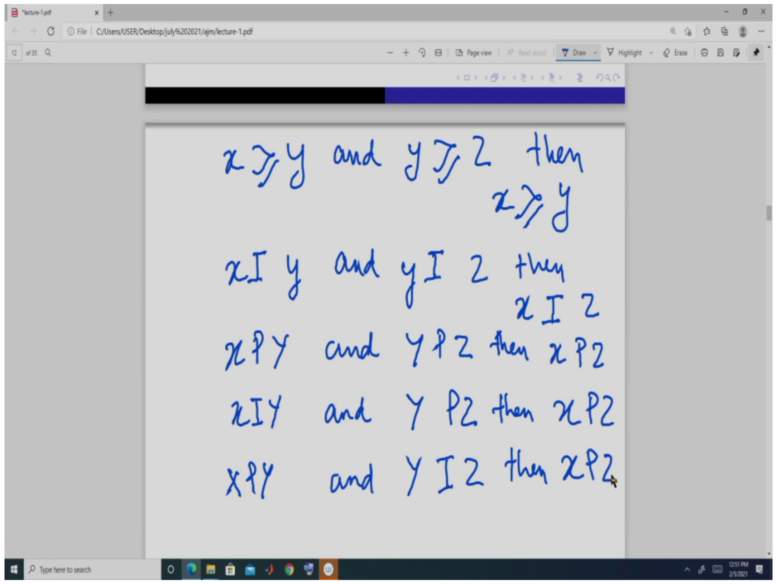

So, it is something like this, that we are given x is at least as good as y i.e x≥y and y is at least as good as z, i.e y≥z, then we must have x at least as good as y, x≥y, okay. This is transitivity. So, from here we can, see we will get, suppose x is indifferent between y and we are indifferent between y and z, then we get that we are indifferent between x and z.

Or if we take it this way, x is suppose, or you can, this indifference, if this sign is complicated we can take, write it in this way also, x indifferent to y or this we can write as x P y. This is easier, right? So, we will use that only. So, suppose we are indifferent between x, then I get this or if I prefer x to y and I prefer y to z then I prefer x to z, right.

So, or from here we can get like this. Suppose I am indifferent between x and y i.e xPy and, but I prefer y over z then x will be preferred to z. Or if I suppose x is preferred to y i.e xPy and y is; I am indifferent between y and z, then I can write that x is going to be preferred to z. Or you can just reverse this here also, it is possible, okay.

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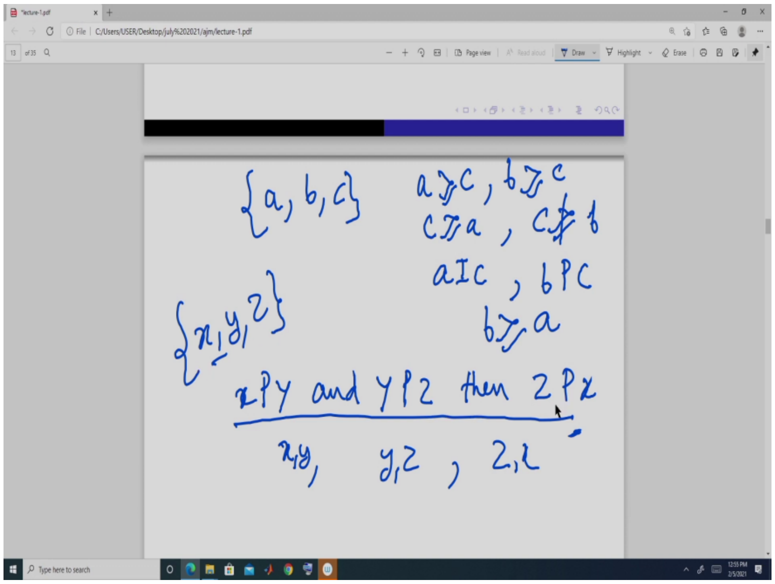

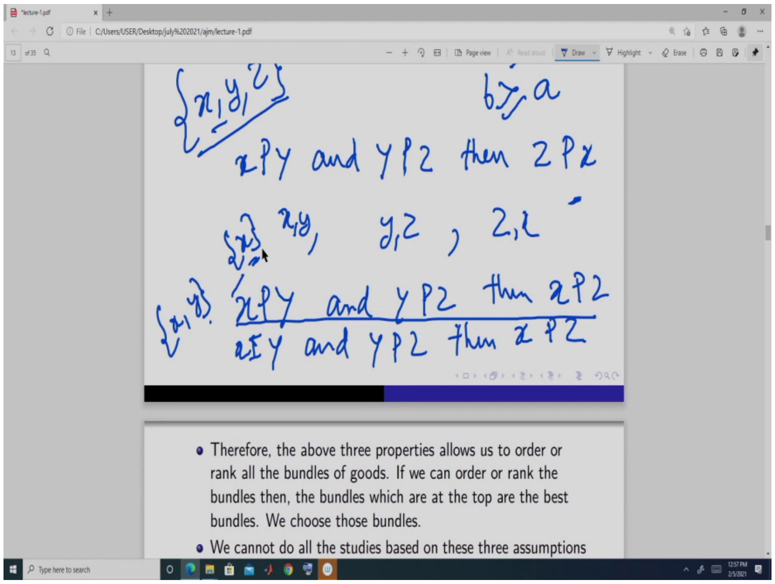

Now we will see that if suppose completeness is violated then what happens? So, suppose we are given a set of bundle like this {a, b and c}. So, we are given that a is at least as good as c, and b is at least as good as c, and we are given c is at least as good as a, and we are given that c is not at least as good as b, okay, but only this is given. We do not know what is the relation between a and b, how they are, whether a is at least as good as b or b is at least.

So, in this case from here what do we get? a is at least as good as c and c is at least as good as, so we get that a, I am indifferent between a and c. Now from here we know that b is preferred to c, right. But we can apply transitivity and then say that b is preferred to a. But we do not know, it is not specified here. And we, so, so that is why we will not be able to find or compared all the, so that is why we should be given that a is at least as good as b.

This, if we are given this then we can now apply transitivity and we can say since a is at least as good as this and from here we can, okay**,** but if we are not given this then we cannot say. So that is why completeness is a requirement, to find out how we, which one to choose from this a; because we will choose that which is at least as good as all others or which is preferred to all these bundles, all the bundles.

Now suppose transitivity is violated. So, if transitivity is violated what it means? So, it is something like this. That suppose x is preferred to y and y is preferred to z. Then from transitivity we know that x should be preferred to z. But instead suppose we are given; we are given z is preferred to x. Now you choose, which one we will choose?

From this set {x, y and z}, you prefer x to, over y. So, you should choose x if you are given these two, these two things only, x and y. And you will choose y if you are only given x and z**,** you will choose z if you are given, but then if y is also there here then you will prefer y. But since x is preferred to y you will get, so we get a cycle kind of thing. So, in this situation if we are given this we cannot conclude or we cannot come to a conclusion which one we are going to choose. So that is why transitivity is a necessary thing.

Now if these three are given then we can always choose the best element. Like from here we know if we are given this, then if transitivity is satisfied then we will have a situation like this, x is preferred to y and y is preferred to z, then from here x is preferred over z. So, from here we can choose x, right

Instead suppose we are only given that suppose x is indifferent to y and y is preferred over z then from here we know x is preferred over z and y is also preferred over z. And I am in different between x and y. So, I will be indifferent between choosing x and y bundle from this set, right. So, I will have two best bundles.

But in this case, in this situation, I will have only one best bundle and that is x, this one {x}, if we are given, if the preferences are in this way. If preferences are of this nature, then we will have two, if preferences are this nature, then we will have only one. So, these three conditions or these three assumptions allow us to find the best element or best bundle from a set of bundles, okay.

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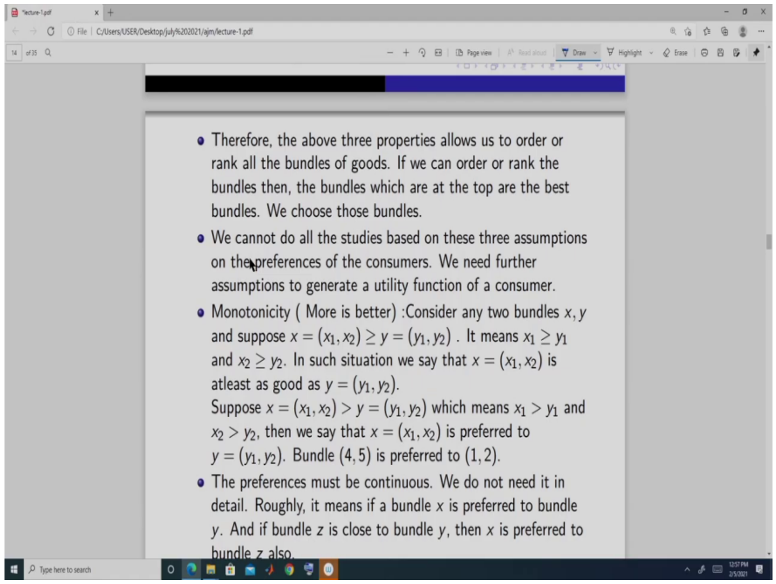

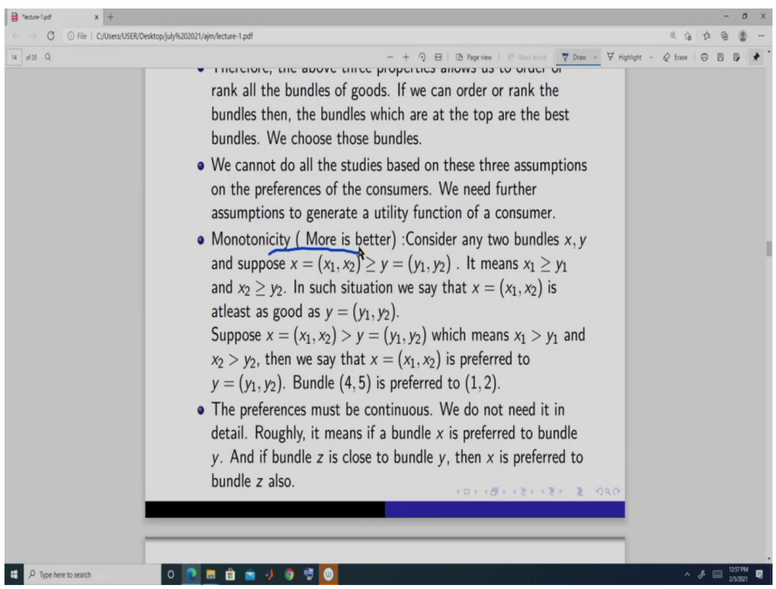

Now the problem with this is that it is too basic. So, we cannot do all the analysis or we simply cannot derive the demand curve from these three assumptions. We need some further assumptions, okay and further conditions to get, to derive something called the utility function and we will see that. So, first assumption that we make is monotonicity assumption, this. Monotonicity assumption means that more is better. What does that mean?

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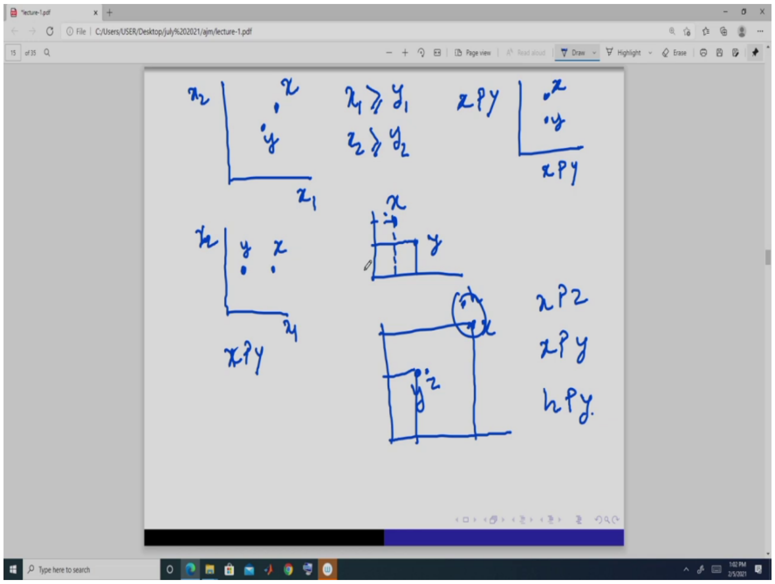

That suppose we are given a bundle x and y, okay. So, you have this bundle. This is suppose bundle x and suppose this bundle is, suppose this is x and this is y. Now from this, we know that x1 is greater than equal to y1 and x2 is greater than equal to y2. Now if the situation is something like this, then we say x is always preferred to y that is x is preferred to y, xPy, okay

Or instead of this, if the situation is something like this, suppose x is this and y is this. So here amount of good 1 is same in both x and y bundle but amount of good 2 is higher in x than here. Here again we can say that x is preferred to y. And similarly, we will also say, if suppose this is y bundle and this is the x bundle**,** then we again say x is preferred to y because amount of good 2 is same and x1 is greater than y1. So good 1 is more in this.

So, in these three situation, we can have three, these three possible ways and we know how we define the preference. But this is silent if we have a thing like this. If this is x and this is y because here this x2 is greater than y2 but x1 is less than y1. So, we do not know based on monotonicity. So, and monotonicity mainly says that more is better. If in a bundle anything more is there than another bundle then we should always prefer that bundle which has more amount or higher amount of any one of these goods, okay. So, this is the monotonicity assumption.

And next assumption is something called a continuity assumption. Preferences must be continuous. So, it means what? It means that, it is something like this. See we will not do this in detail, okay. Suppose this is y and this is x. From monotonicity we know that x is always preferred to y because both the good x1, y1; x1 is greater than y1 and x2 is greater than y2 here in this case.

Now if we have a bundle here z which is close to y then since x is preferred to y so x will be preferred to z also. So, this is mainly what continuity tells us, okay**.** Or if we have any bundle here, suppose this, which is given by suppose h then h is going to be preferred over y. Or here x is going to be preferred over z, something like this because it is near this and it is very far from, like this.

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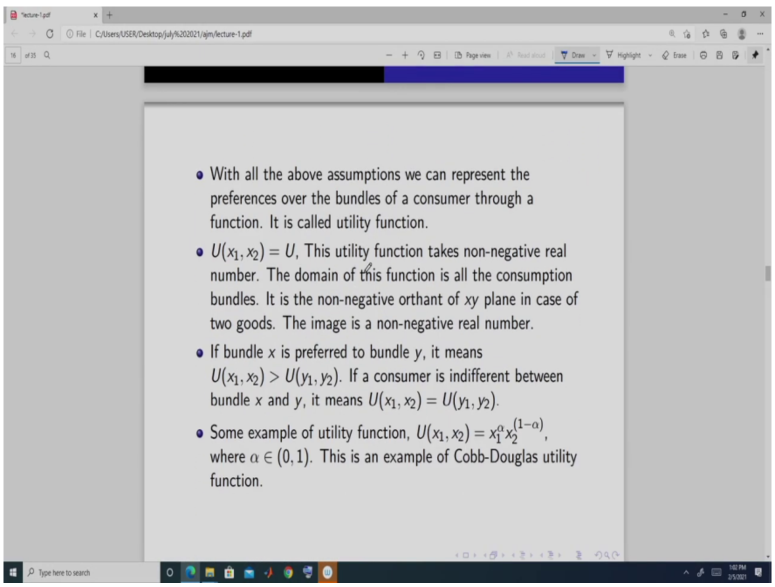

So, if we assume these five assumptions that is reflexivity, completeness, transitivity and monotonicity and further continuity then we can represent these preferences through a function. And this function is called a utility function, okay. So, it is something like this.

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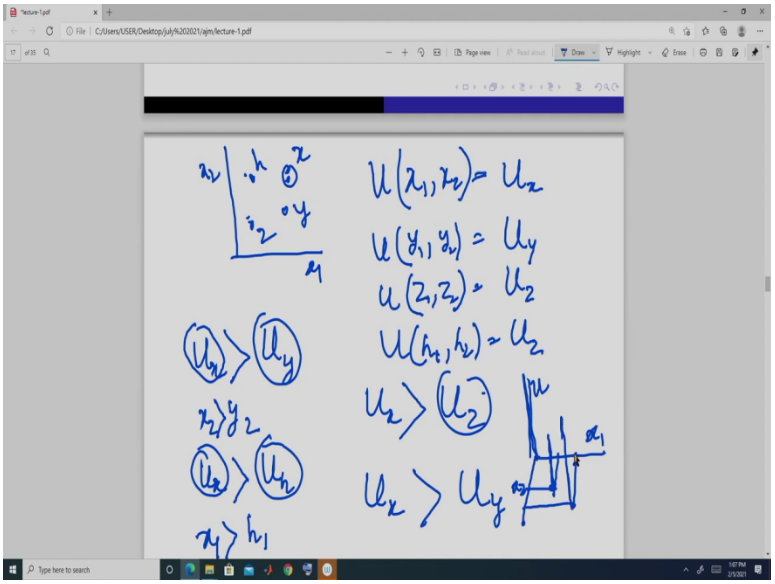

So, we have, good x1 and x2. Based on all the above five assumptions we know that if we are given, suppose this in our set, suppose these four bundles then we know which one to choose; like from monotonicity we will definitely going to choose this bundle because all goods are more in this. But here if you take this, so good 2 is same but good 1 is higher here.

If you look this here good 1 is same but more or less same but good 1 is, good 2 is higher and compare, so we will choose this bundle, right. Now what do this utility function says, that we have a function like this $U(x_1, x_2)$which is a function of this amount of good 1 and good 2 and when we plug in these bundles, we get a real number and that gives us total level of satisfaction that we get from consumption of this bundle.

So, we get a satisfaction level like this. Suppose this is, this bundle is x, this is y, this is z and this is suppose h, okay**.** Then we get some utility from x1, x2 is, suppose this is x, i.e $U\left(x_1, x_2\right) = U_x$; some utility that we get from consuming y1 y2 and suppose this is $U\left(y_1, y_2\right) = U_y$; and you get another and that is $U\left(z_1, z_2\right) = U_z$ and we get $U\left(h_1, h_2\right) = U_h$. Now here based on monotonicity we can say that utility from x1 this is greater than utility of y. Why? because x2 is greater than y2, y2 whereas x1 in both the case are same.

Again, we can say that x is greater than h i.e $U_x > U_h$, why? Because x2 is same as h2 but x1 is greater than h1. So, we have something like this. Now this U_x takes a real number, this U_y takes a real number, this U_x takes a real number and this U_z also takes a real number. So, we can compare these real numbers, you can always compare. So, from here if we know this and we know then U_x is always greater than U_z. Why, both x1 is greater than z1 and x2 is greater than z2, if you compare it.

Now this will also be a real number and we can compare the real numbers. So, since this is here, from here we get U_x is greater than U_y and now depending on the actual value that we assign to this we will, but this is, we can find out which bundle to choose from here, right out of this because we will have these four utilities and we will have these four real numbers and we can compare these real numbers and that will give us which one to choose from these four, right.

So, this is the advantage of utility function. So that is why, what we do, if we make this assumption then we can represent these bundles based on the utility that we attain from this, from the consumption of this, or the satisfaction that we get from this. So, it is something like this that if we are given like this, so this may be good 1, this may be good 2 and this is our utility.

So, any bundle here this much amount of good 2 and this much amount of good 2, the height is, this height is going to give me the level of utility. For this point again this height is going to give me the utility, right. And since these two goods are, this is preferred to this, so utility is higher. So, we get something like this.

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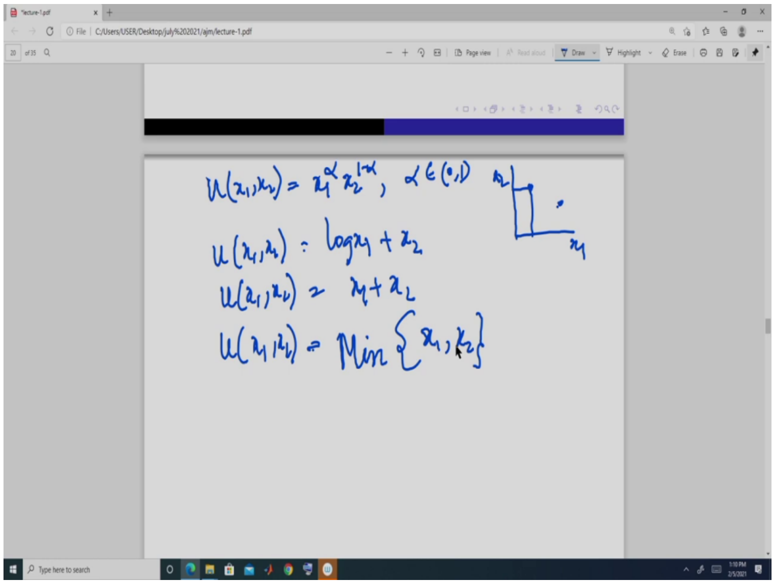

So, we will now do some examples of utility functions, okay. So, some examples of utility functions are something like this- $U\left(x_1, x_2\right) = x_1^{\alpha} x_2^{1-\alpha}$ that if we are given x1, x2 is where this alpha belongs to 0 and 1. It can be any number between 0 and 1. So this is an example of something called the very well-known utility function that is Cobb Douglas utility function, this.

And here wherein we plug in, in this a, so based on these two we will get a real number. So, if we take, this is x1 and this is x2, any point this, this much x1, this much x2 you plug in these two here, the outcome is going to give you the utility from this. Take any point, you plug in the value here. You will get the a for specific value of alpha.

Another utility function that we can talk about is this- $U\left(x_1 x_2\right) = log x_1 + x_2$.this is one, this is something called a quasi-linear utility function. Here you plug in the value of x1, x2 and you will get the all the positive value you will get the utility, right. And for here we do not have these, these values because x, in this case x will always take a value which is greater than 0, otherwise log is not defined, right.

Next example you can take is something like this, $U\left(x_1 x_2\right) = x_1 + x_2$ this, this is one. So here you can see that these goods are this one, x1 and x2, they are, so you plug in this value and the utility you get is simply the sum of these two. Or you can take another example like this. So, this is an example of perfect substitute and like Min of this- $U\left(x_1 x_2\right) = Min\left\{x_1, x_2\right\}$. this is example of perfect complements, goods are x1 and x2 are perfect complement. We will do these four examples in detail.

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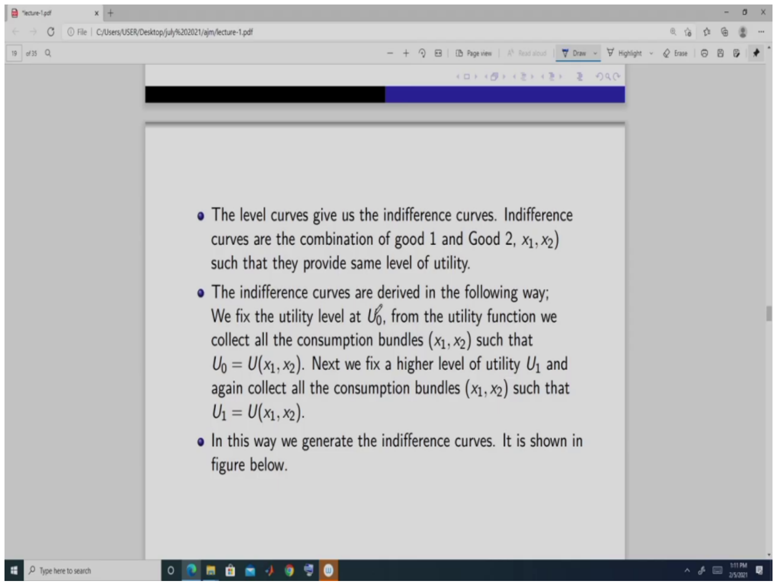

Now so what do we get? That we assign a number to each point like this based on the specific utility that the consumer has, utility function of that consumer we get the real numbers for each of this point, in this positive orthant, okay. Now from here we define something called indifference curve. So, what this indifference curve says? Indifference curves are like level curves. So, it is something like this, this.

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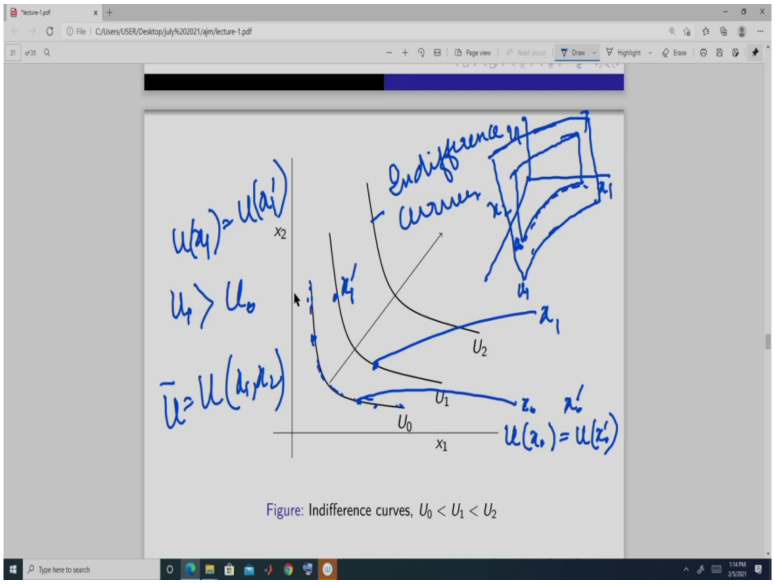

So, what we are doing here that this is a level curve which is the utility from each of these points we get is u naught, i.e U_0. So, this point is suppose x naught i.e x_0 and this is suppose x naught dash. Then utility from x naught is equal to utility from x naught dash, these two, okay. So, we take all this. So, it is something like this that if we take x in this axis x1 and y2 in this good 1 good 2 and utility in this axis then these points, if this is one indifference curve then height is going to be same for each, so this height is same height.

Now this you will get at the same height, each, if all of these are at the same level. Suppose this is u1 and this is u naught, okay, something like this. So here if we take this, suppose this is x1 and this is suppose x1 dash then utility from x1 is equal to utility from x1 dash, i.e $U(x_1) = U(x_1^{'})$. And since this is at a higher level so and we are making that assumption from the monotonicity. So, this utility from u1 is greater than u naught.

So, in this north east direction utilities are increasing. But each point is giving me same level of utility. So, these are called, something called indifference curves, okay. So, we generate indifference curve in this way. What we do? We fix suppose a level of utility at this and we have a utility function which is given like this $U = U(x_1, x_2)$and then we find out at this level of utility u bar all the combination of x1 and x2 which are giving me same level of utilities.

And so, then we join them and we get the indifference curve. Or you can think it in terms of three dimension that is all the points which have the same height and their height is giving you the utility, level of utility, okay.

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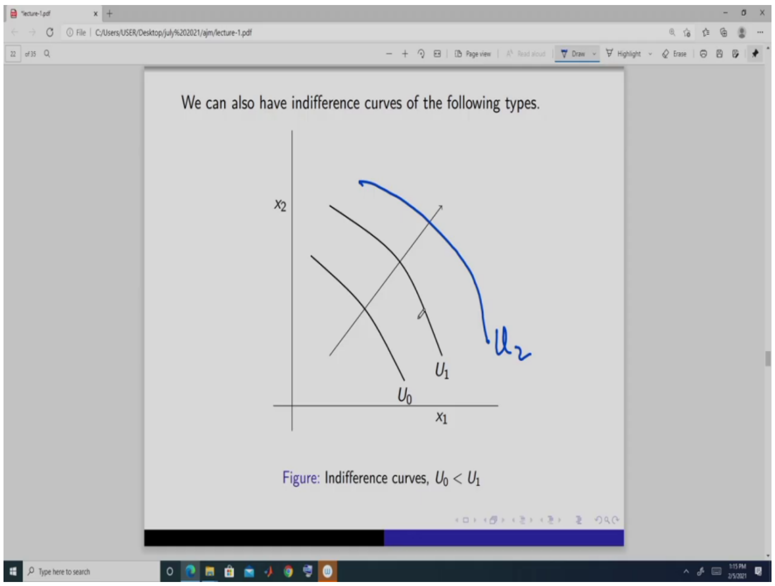

But here from till the assumptions that we have made till now we may have a indifference curve of this nature also and utility is increasing. Suppose this is again, it is something like this, like this, we may get this. Now what is the problem of this kind of utility function? We will see that when we do, when we actually derive the demand function not now.

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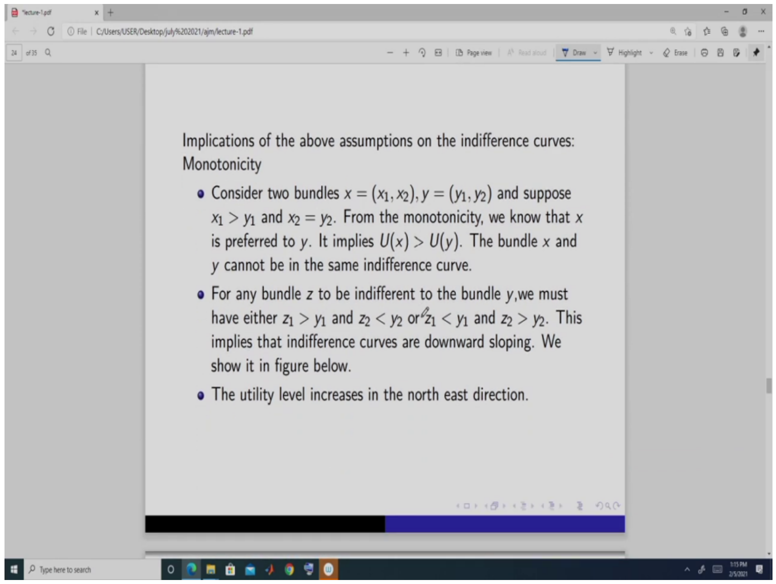

So, but these are a specific form of utility functions and we in this course we do not want this kind of indifference curves. So, what we will do now? So, we need some further assumptions. And before moving to further assumptions we will see actually the two assumptions that we have made monotonicity and transitivity, what is the implication of these assumptions.

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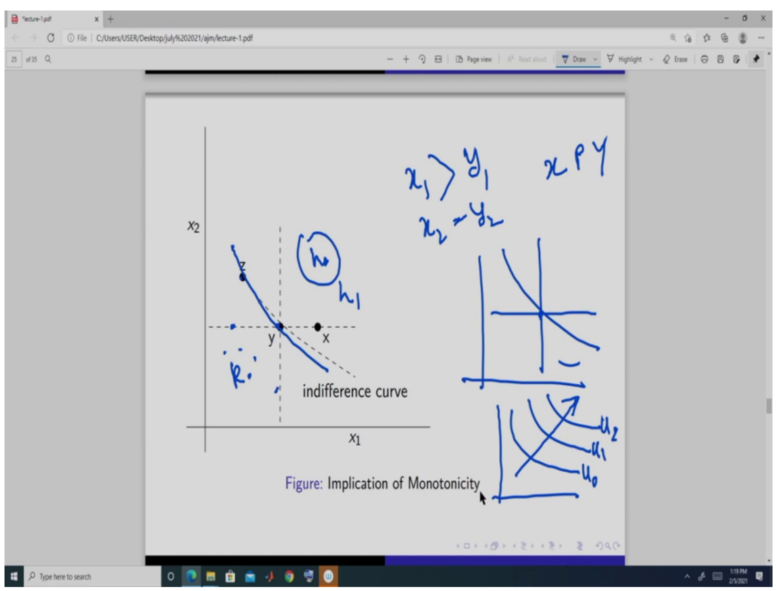

First the monotonicity. Now suppose we are given three points. This is, or three bundles x, y, z. Now if you look at these bundles you will see that in this bundle x1 is greater than y1, but x2 is equal to y2, right. So, from monotonicity we get x is preferred to y, right. And if you take any point here in this region and compare it with y, so any point suppose like h which is here or it may be suppose h naught is here, or h1 is here, then all of these are going to be preferred to y.

Now if you take any point here. Suppose these are k, okay, this is suppose k. Then y is going to be preferred to k from monotonicity, it is obvious because each of them are greater. And if you take point in here or here also y is going to be preferred. So, if you are taking this point then any point which needs to, which will be indifferent to this should be either in this portion or it should be in this portion, okay.

So, this gives us what? This gives us that the indifference curves should be downward sloping like this. So, we get the indifference curve should be like this from the monotonicity assumption. They should always be like this. So, if we specify any point here than any point which must be indifferent to this should either lie in this section of the curve or it should lie in this section. It cannot lie in this portion or in this portion**,** okay.

So, the first implication of monotonicity assumption is that the indifference curves are downward sloping, indifference curves are downward sloping, okay. And second thing is that if we take this a then the, here still, the second implication is that these level curves, the utility here, this, this, so U_2, U_1 should be greater than U_0, U_2 should be greater than U_1.

So, utility level or this level curve should be increasing in this north east direction. So, the height of this third a which gives me the utility should be increasing, okay. So, this is the implication of monotonicity.

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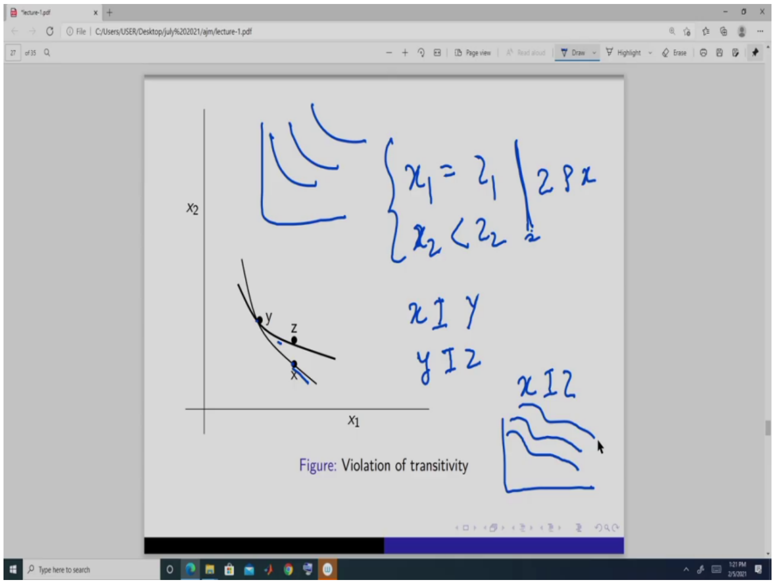

Now what is the implication of transitivity? So, the transitivity is, now take three points x, z and y like this. And suppose the indifference curves are like this nature. So, this is one indifference curve and this is another indifference curve. Now from here we know that x1 is equal to z1 and x2 is less than z2, right. It is obvious from this.

But now see x and y are in the same indifference curves, so x is, I am indifferent between x and y, right. x and z are in same indifference curves. So, I am indifferent between y and z. right. So, apply transitivity, you will get what, that I should be indifferent between z and x right. But we have this. So, from here we know that z is preferred over x. So, we get a contradiction.

So that is why if transitivity is there, then indifference curves are never going to intersect. So, these level curves are never going to intersect. So, they will be always one over another, right. So, indifference curves are always going to be of this nature, or the indifference curves depicting different levels can be of this nature like this, but they are never going to intersect, okay. So, in today's lecture we will cover only till this much and in next lecture we will continue from this, okay. Thank you very much.

Introduction to Market Structures
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Lecture - 2
Utility Maximization and Derivation of Demand Curve

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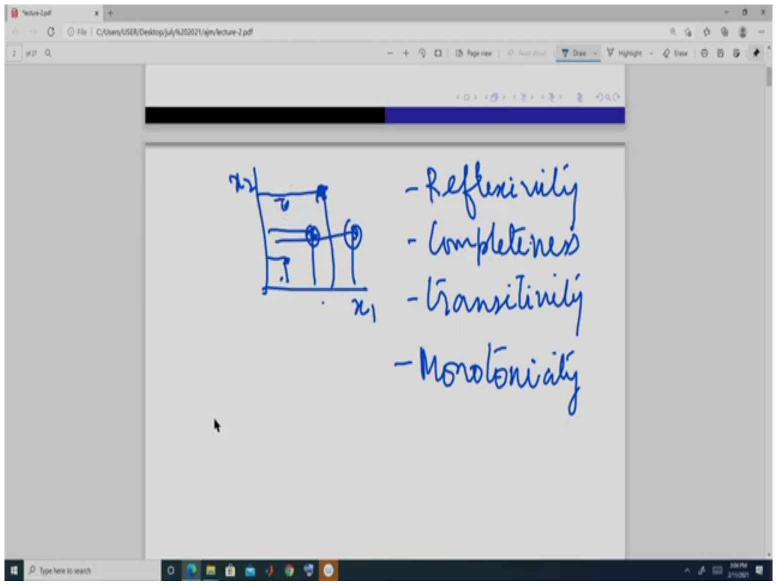

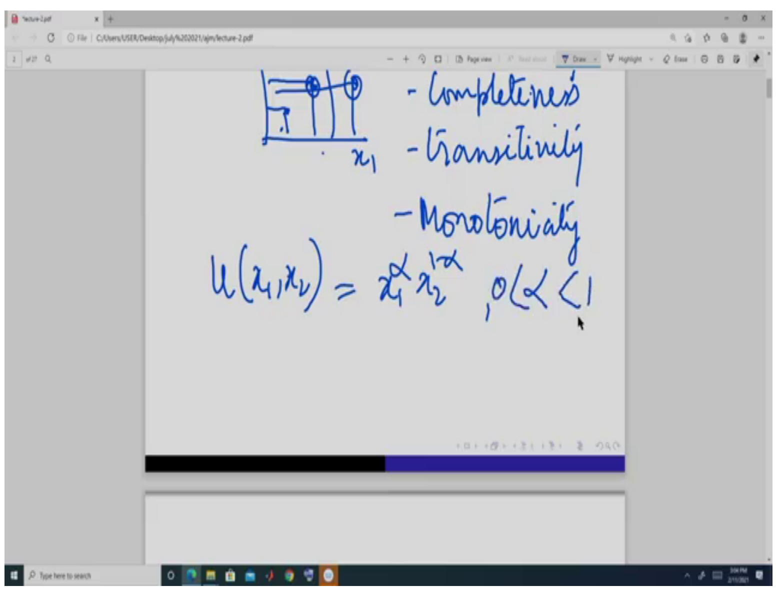

Hello, everyone, welcome to my course Introduction to Market Structure. So, in this second lecture, we will first recap what we have done in the first lecture. So, in first lecture we have defined preferences and we have done only for two goods suppose, good 1 and good 2. So, these points in the positive quadrant represents the consumption bundle and we have defined certain assumptions on these preferences a consumer has over this bundle.

So, first is like reflexivity, we have defined reflexivity that bundle should be at least as good as itself, then we have defined something like completeness which says that we should be able to compare all the bundles which are there in this positive quadrant. Then, we have like transitivity. Transitivity says that if a bundle x, bundle y and bundle z suppose we have three bundles and x is at least as good as y, y is at least as good as z, then x should be at least as good as z.

So, one implication of this is that if x is preferred to y and y is preferred to z, then it implies that x must be preferred to z. We cannot have z preferred to y; z preferred to x. So, in that case, if we have z preferred to x, then we will not be able to choose from this three bundles because we will go on cycling.

Now we have introduced another property and that is monotonicity, and monotonicity gives us that if we have a bundle and if the quantity in that bundle is greater than another bundle, then we choose that bundle in which quantity is greater. So, suppose if we take this bundle and this bundle, we will choose this bundle or prefer this because here both the goods are greater than this.

So, here compared to this and this, we will choose this one, if compared to this and this we will prefer this one. But compared to this and this, we cannot say which one we prefer based on simply under monotonicity, right. Next, we assume that the preferences are continuous, so, it means that there is no jump in it and we do not go into details.

So, based on these five assumptions, we can say that we can define utility function over these bundles. So, utility function we have defined it something like this – U(x1, x2). So, if we are given a bundle like this, then we can assign some number to this bundle. So, one example is suppose, like this- $U(x_1, x_2) = x_1^{\alpha} x_1^{1-\alpha}$ where alpha lies between 0 and 1, this is a Cobb Douglas utility function.

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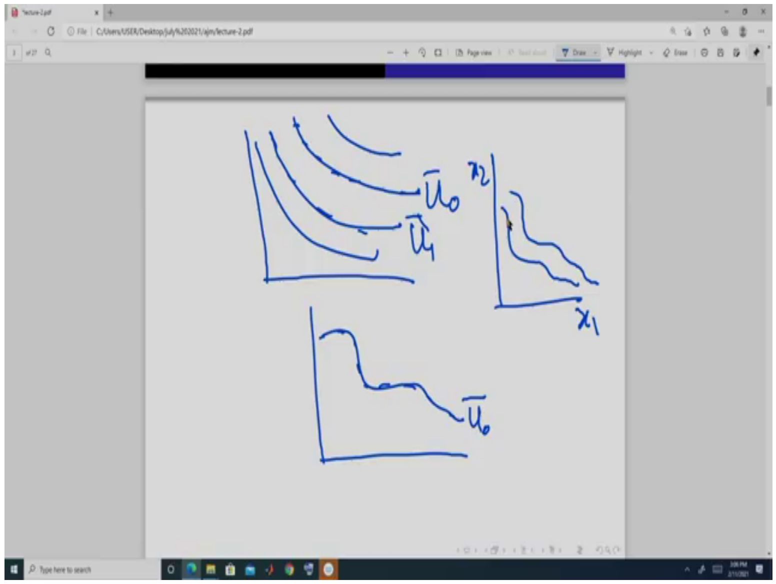

And from this utility functions what we have done, we have derived something called a set of indifference curve like this, where each bundle in this curve gives us same utility. So, this is same and then here we have got this is another a where we have got, now here the utilities are higher than this. So, these are something called indifference curves we have got this.

So, we may also get an indifference curve of this nature, right? where all these bundles give me the same utility like this, it is fixed. But from monotonicity what we have got that these utility indifference curves should be always downward sloping. It should be like this, like this, okay.

Now, if we have indifference curve like this, if our utility function is such that we generate this kind of indifference curves then we may have a problem while doing the optimization, that is maximizing utility subject to budget constraint. So, we do not want such indifference curves.

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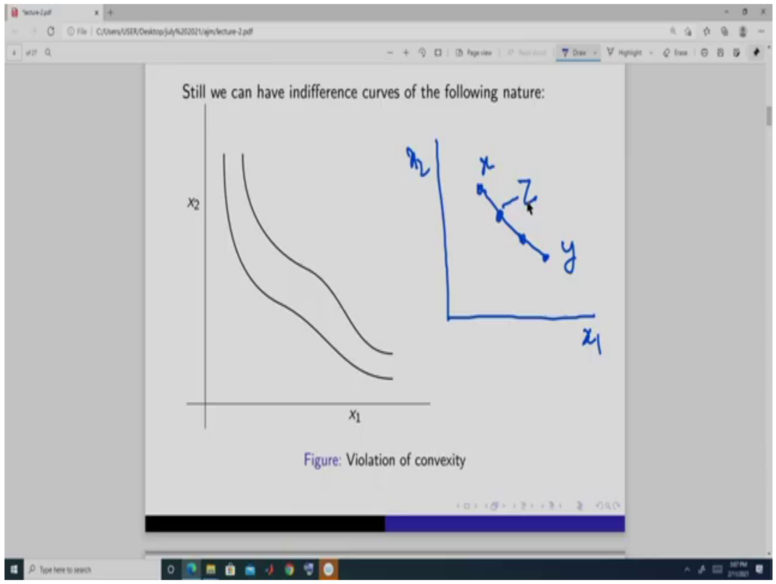

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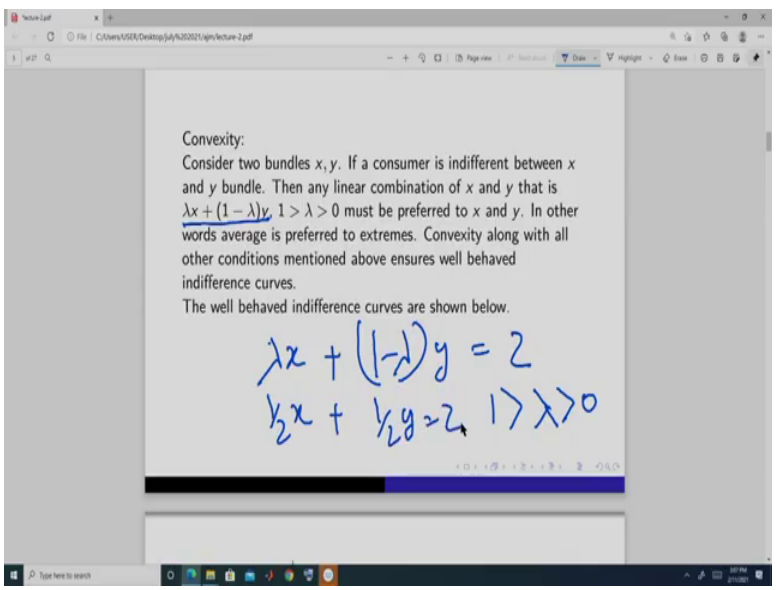

So, what we do, that we get these kind of indifference curve when we are violating another assumption and that is the assumption of convexity. What convexity says, it is something like this, that if we are in this a, given two bundle this is suppose x and this is suppose y, okay. Now, if we take any combination of this linear combination of this suppose we, so suppose this bundle or this bundle is suppose z. So, how do we have arrived at z?

We have arrived at z through this kind of linear combination- λx +(1-λ)y, 1>λ>0. What we have done, we have taken lambda fraction of x plus 1 minus lambda fraction of y and that has given me z. So, this lambda which always lies between 0 and 1, okay. Now, so, it can be like half of x plus again half of y give me z, it is something like this- 1/2 x+1/2 y= z, so, here we have got this. Now convexity says that this bundle should be at least as good as x and y. In fact, strict convexity says that this should be always preferred to x and y. So, it means that convexity implies that average is always preferred over extremes, okay.

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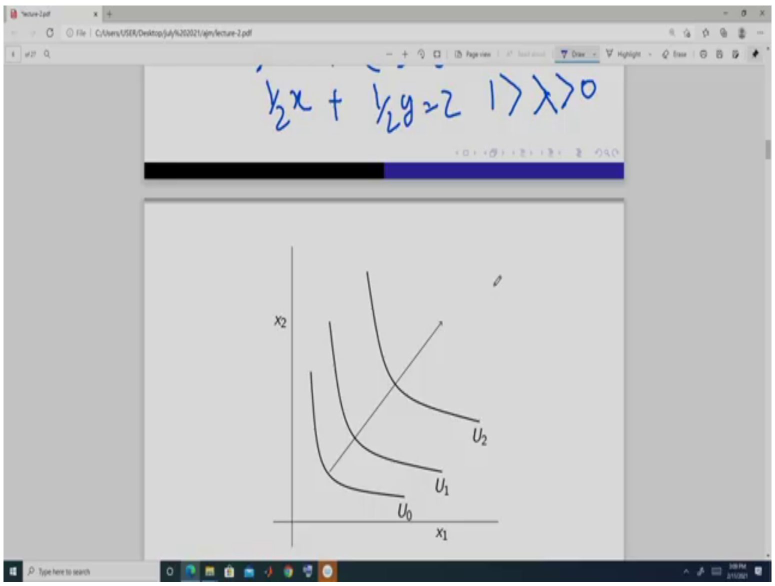

So, that is why we get this kind of indifference curves when convexity assumption is satisfied and these are something called an well-defined utility function. So, utility level is increasing from u naught to u1 to u2 and we get such kind of indifference curves, all these combinations give me same level of utility this gives me same level, but these are higher than this, right**.** So, we will assume that our preferences are well behaved, so, we will always get a well-behaved indifference set of indifference curves.

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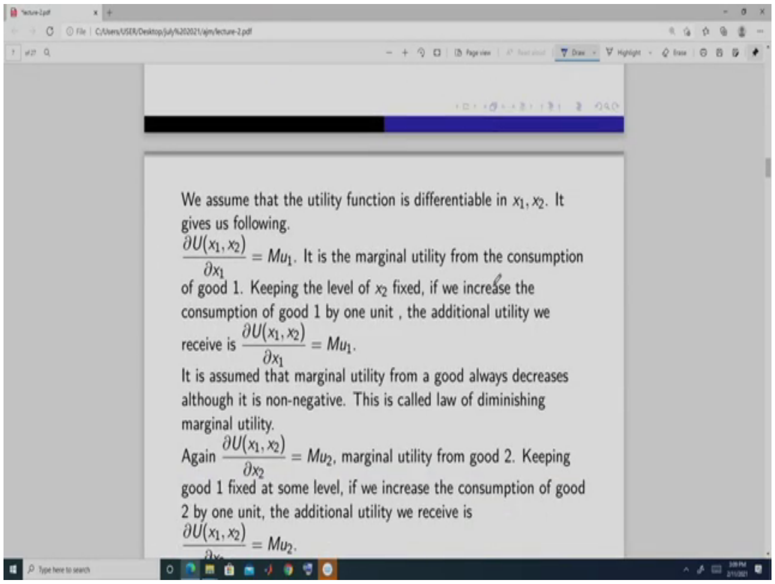

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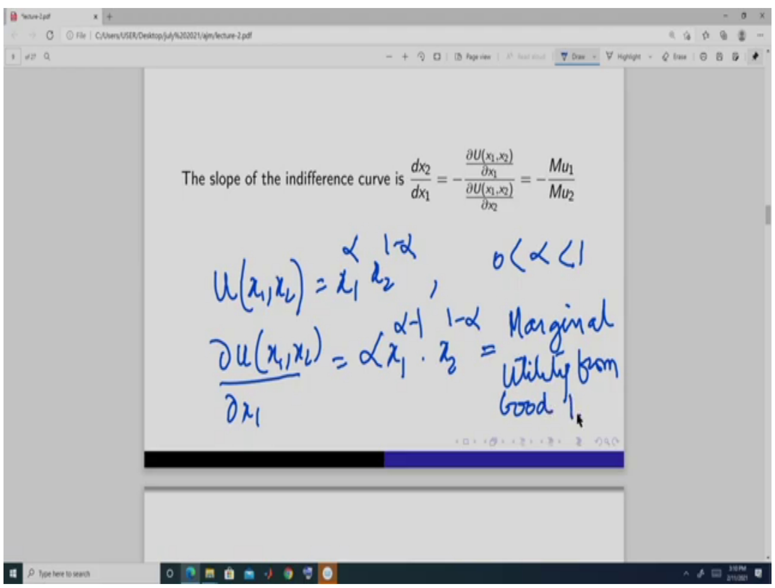

Now, further we will assume that the utility, so, because we can define our preferences through a utility function. So, if you give me a bundle, I know how much utility from, I am getting from that consumption of that bundle and that is given by a real number, positive real number, right. Now, here we further assume that these utility functions are differentiable.

So, if they are differentiable what does it means, that, if we are given a utility function like this- $U(x_1, x_2) = x_1^{\alpha} x_1^{1-\alpha}$ same as we have done earlier Cobb Douglas utility function where alpha lies between 0 and 1. Then it means we can take partial of this, we can take partial derivative of this and this is something called marginal utility from good 1, that is if we keep good 2 fix and if we increase the amount of good 1, then how much additional utility do we get? This is called marginal utility from good 1.

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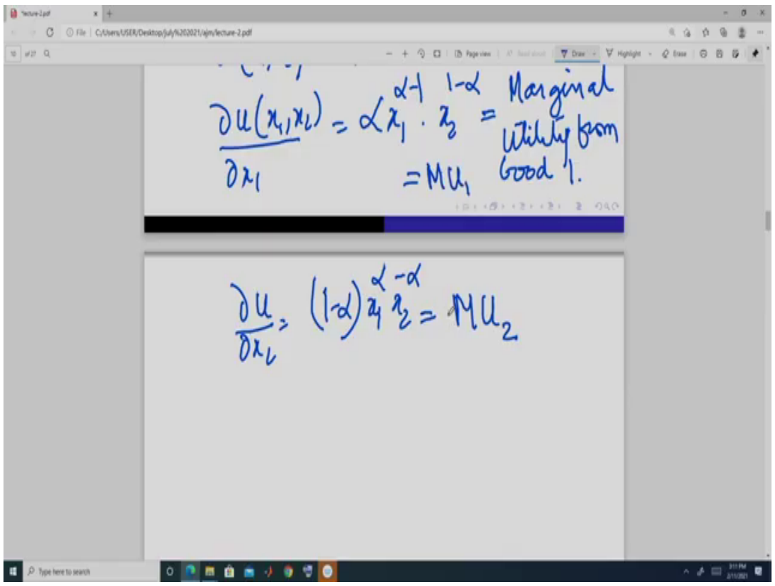

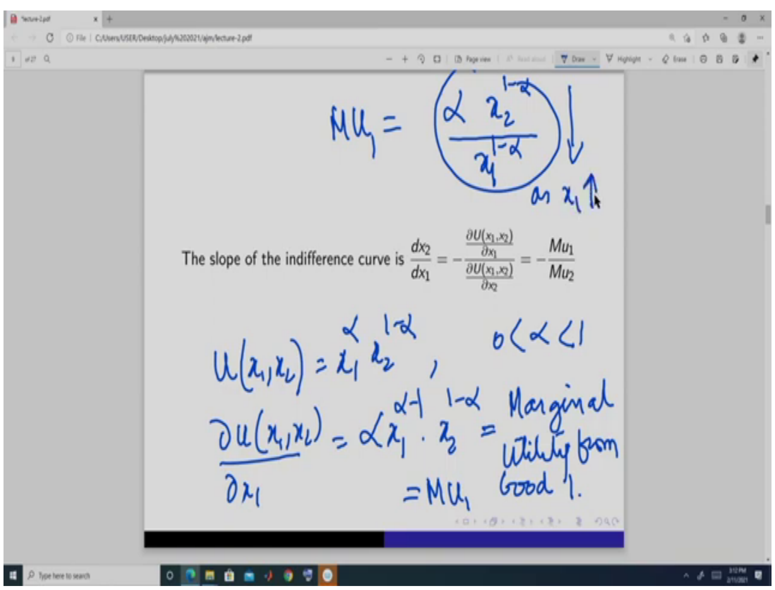

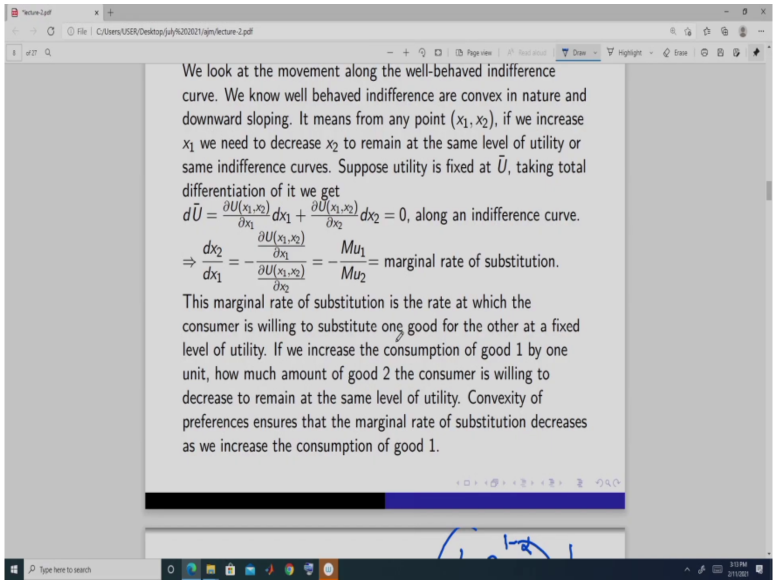

We look at the movement along the well-behaved indifference curve. We know well behaved indifference are convex in nature and downward sloping. It means from any point (x_1, x_2), if we increase x_1 we need to decrease x_2 to remain at the same level of utility or same indifference curves. Suppose utility is fixed at $\bar{U}$, taking total differentiation of it we get

$d\bar{U} = \frac{\partial U(x_1,x_2)}{\partial x_1}dx_1 + \frac{\partial U(x_1,x_2)}{\partial x_2}dx_2 = 0$, along an indifference curve.

$\Rightarrow \frac{dx_2}{dx_1} = -\frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\frac{Mu_1}{Mu_2}$ = marginal rate of substitution.

This marginal rate of substitution is the rate at which the consumer is willing to substitute one good for the other at a fixed level of utility. If we increase the consumption of good 1 by one unit, how much amount of good 2 the consumer is willing to decrease to remain at the same level of utility. Convexity of preferences ensures that the marginal rate of substitution decreases as we increase the consumption of good 1.

Similarly, the marginal utility from good 2 is δu/δx2= $(1-\alpha)x_1^{\alpha}x_1^{1-\alpha}$, *right,* and we represent it as Mu2 this is equal to Mu1, okay. So, we have something called law of diminishing marginal utility. What law of diminishing marginal utility says that, as we increase the consumption of one good keeping the amount of other good fixed, the marginal increase or the additional increase in utility is going to go down as we go on increasing good 1 or go on increasing good 2.

This marginal utility of good 1 should go on decreasing as we go on increase good 1 keeping good 2 fixed. So, it is obvious right**,** you will see what, you will see that this term is actually so, if I write it here you will see that the marginal utility of good 1 is actually this $MU_1 = \alpha\frac{x_2^{1-\alpha}}{x_1^{1-\alpha}}$, right**,** because 1 minus alpha is positive, alpha minus 1 is negative.

Now, as we go on increasing x1 this term is going to go down as x1 increases. So, this means that the marginal utility goes up and this is based on a law that is the law of diminishing marginal utility that is, if we keep the amount of good 2 fix and if we increase the amount of good 1 as we go on increasing good 1, the additional utility that we are going to get, it goes on decreasing.

Similarly, if we fix the amount of good 1, and if we go on increasing good 2 then the marginal utility or the additional utility that we get from consumption of good 2 is going to go down, it will remain positive, but it will go down. Now, since, we have assumed that our utility function is very well-behaved utility function and it is also differentiable, now, we can do some more operations on it.

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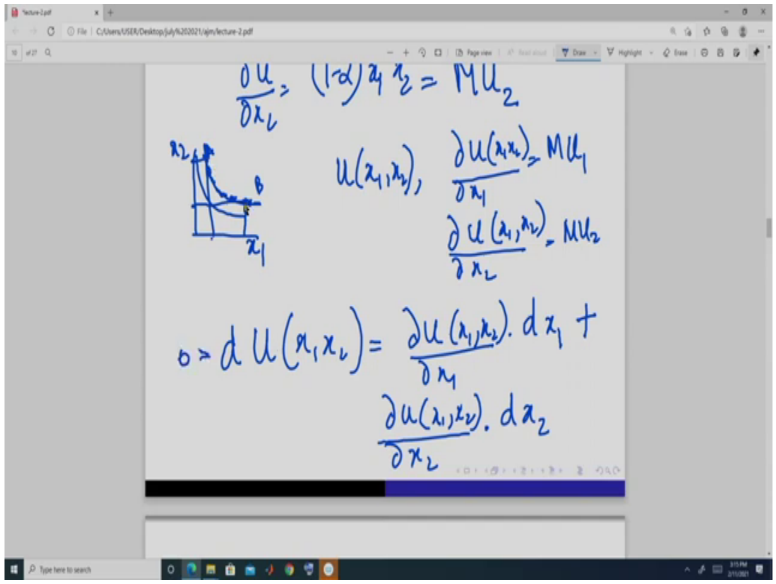

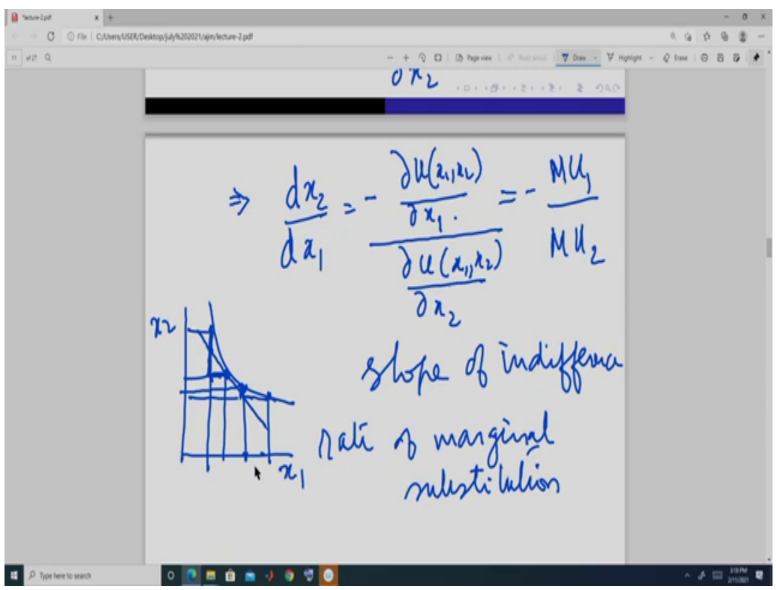

So, it is something like this that what, so, we will get a indifference like this indifference curves are going to be like this. Now, what happens if we move in this indifference from here to here. So, it means what, if we move from this point to suppose this point from A suppose to a point B our utility is not changing.

But our bundles are changing, right**,** here, it is a high amount of good 2, and low amount of good 1, here low amount of good 2, and high amount of good 1, right**.** So, what is happening in this movement? So, that is called movement along an indifference curve, and we will see that, it gives us something called a marginal rate of substitution and you derive it based on this.

So, our suppose utility function is this U(x1, x2), so, this utility function from this we will get this is what marginal utility of good 1, similarly, we will get marginal utility of good 2, right. Now, from this, if we take the total differentiation of this, what do we get? We will get this- $dU\left(x_1 x_2\right) = \frac{\delta U\left(x_1 x_2\right)}{\delta x_1}.\ dx_1 + \frac{\delta U\left(x_1 x_2\right)}{\delta x_2}.\ dx_2$. So, now, this is not changing when we are moving from point A to point B.

So, this gives us one important thing and that is, we get- $\frac{dx_2}{dx_1} = \ - \frac{\frac{\delta U\left(x_1 x_2\right)}{\delta x_1}}{\frac{\delta U\left(x_1 x_2\right)}{\delta x_2}}$, and this is equal to marginal utility of good 1 divided by marginal utility. This is called the slope of indifference curve and this gives us something called the rate of marginal substitution, the rate of marginal substitution. And this gives us that if I want to increase one unit of good 1 then how much unit of good 2, I must give up, so that I remain at the same level of utility.

So, it is something like this for this a, suppose this, so here when I move from this point to this point then if I increase x1 by this much I have to give up x1, x2 by this much to remain at the same level of utility. If I move from this, to this, then I have to give up this much only, okay. If you look at these two points you will see when I increase x1 by this much amount I have to give up only this much amount of x2.

So, what is happening? So, the amount of good 2 we are giving up that goes down as we go on increasing x1. So, this is the marginal rate of substitution goes on decreasing as we move downward in this indifference curve, okay. And this is actually what is, what we are willing to substitute, that is if I want to increase one unit of good 1, how much unit of good 2 I am willing to give up to remain at the same level of utility. This is willing what, because it is based on my preference, okay. Now, so, this much we are going to cover in a utility or the preference portion.

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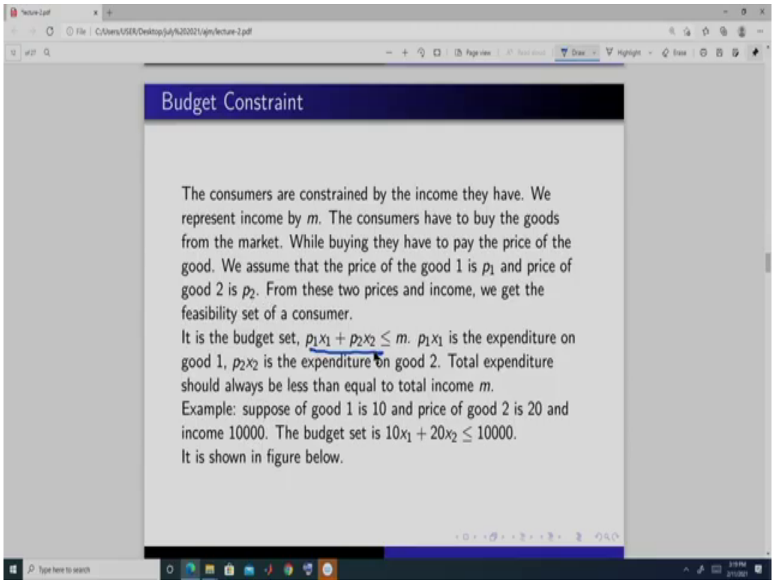

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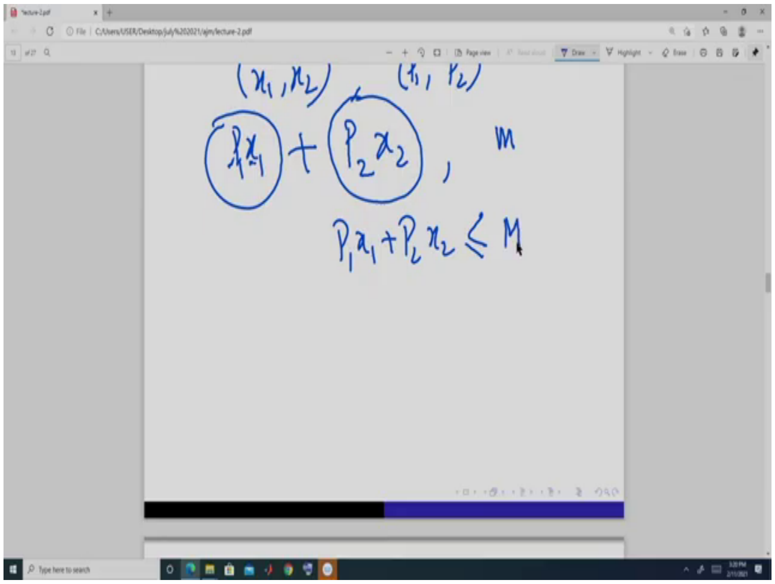

Next, we move to something called a budget constraint. What do we mean by budget constraint? Now, when we are buying goods in the market, we pay some price, we know we are in a world of two good that is good 1, good 2 and suppose price of good 1 is, p1 and price of good 2 is, p2, okay. Price of good 1 is p1 and price of good 2 is p2.

Then budget set is defined in this way- $p_1x_1 + p_2x_2 \leq m.$, where this amount p1 into x1 is the amount, I am spending on good 1, p is the price into the quantity of good 1 x1, so, this is expenditure on good 1 plus p2 into x2 this is the expenditure on good 2. These two expenditure is my total expenditure and these two should always be less than equal to my income, what is my income? My income is m. So, we get an equation and that is called a budget equation of this form - $p_1x_1 + p_2x_2 \leq m.$, should always be less than equal to m, okay, which is given in this form.

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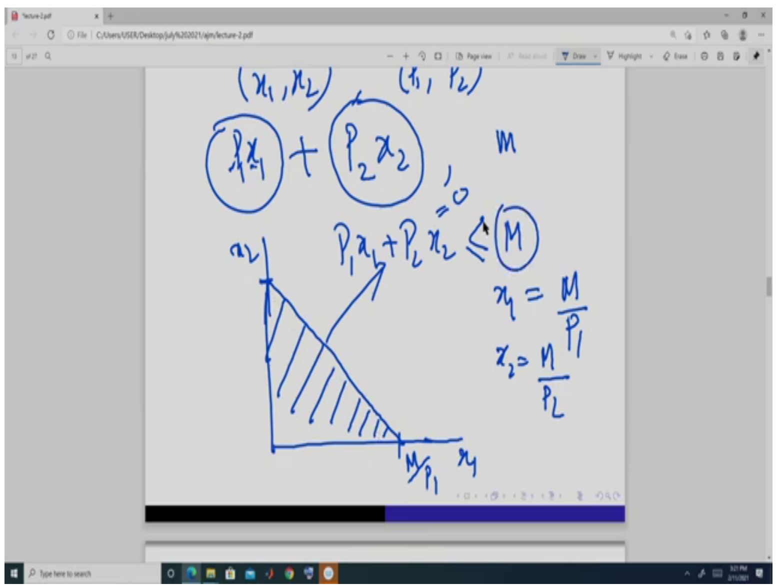

Now, how do we draw this budget line? So, from this if we spend everything on good 1, then this good 2 is 0. So, x1 is equal to m by p1. So, suppose this is m by p1 and if we spend everything on good 2, we will, how much we can buy, we can buy m by p2. And suppose this is and m by p2 then, this is our this $p_1x_1 + p_2x_2 \leq m$ equation and this whole set is the feasible set or all this bundle in this set, we can afford it, I can afford if my income is m, okay.

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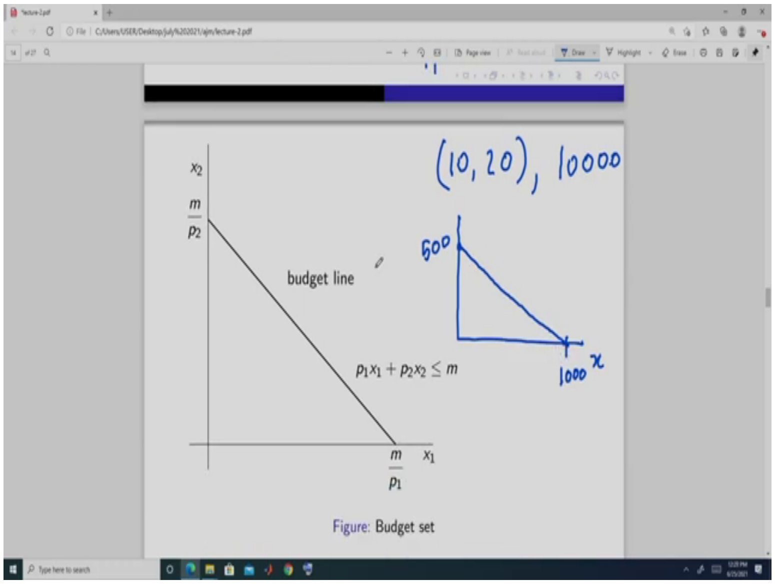

Let us take one example where price of good 1 is suppose 10 and price of good 2 is 20 and the income is suppose 10,000. So, if we try to look at the budget line, it will be like this. So, this point is m by p1. So, m is sorry this is 10,000. So, m is this. So, this is going to be 1000, this point is this divided by 20. So, it is going to be this divided by 20 which is going to be 500. Now, this, you joined it and we get the budget line. So, this is the budget line that we get, okay.

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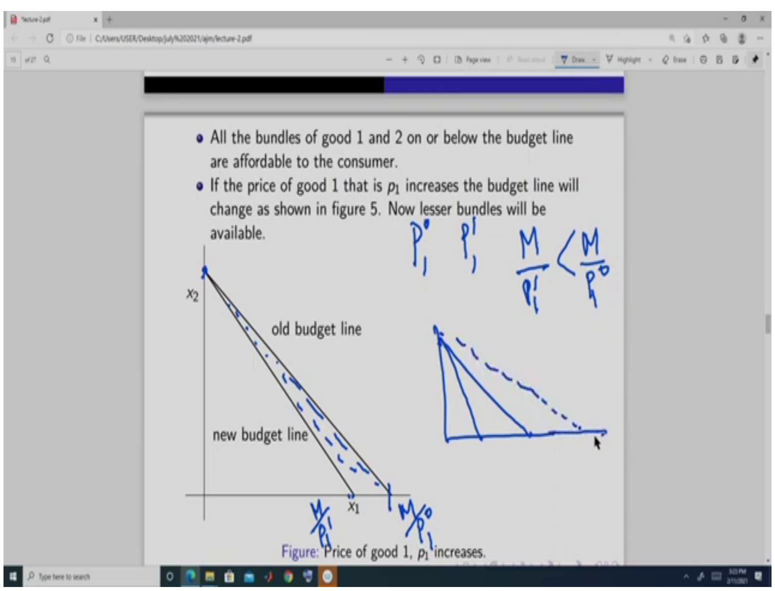

Now, suppose the price of good 1 increases from price of good 1 was p naught 1 now it increased to p11, right, then what is going to happen this term, this is going to be less $\frac{M}{P_1^1} < \frac{M}{P_1^0}$, right.So, if this point was m by p naught 1 and then this is, it will shift like this. So, these bundles are no more, I cannot afford this bundle when, right, but I can choose this bundle because if this bundle is I am only buying good 2 and zero units of good 1.

So, price of good 2 is fixed so, it is same. So, I can afford this bundle, but all these bundles which was earlier affordable it is those are not affordable anymore because price has increased. So, whenever price increases budget set contracts in this form. So, when price increases, budget line will contract in this way, and when the price decreases it will expand in this way, okay, for good 1.

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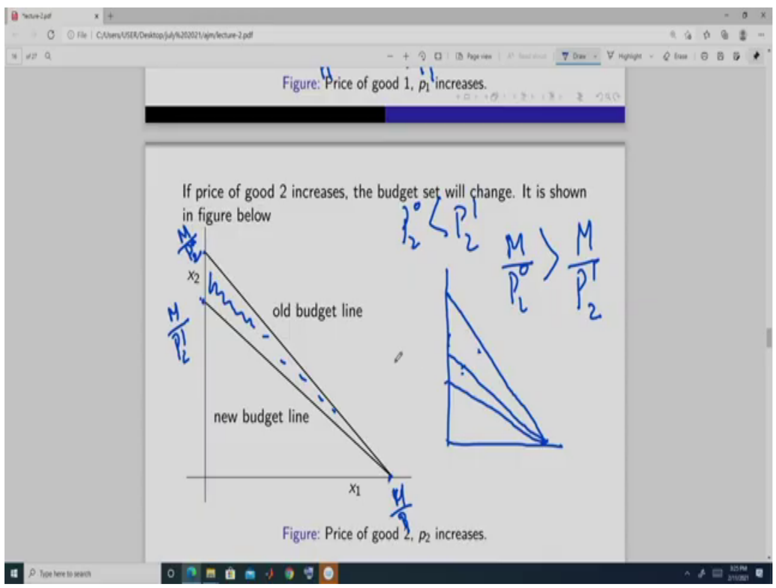

Similarly, if suppose the price of good 2 changes suppose the initial price of good 2 is p naught and it has increased to p naught, p21 or p12. So, it is like this- $P_2^0 < P_2^1$ then this is going to be like this. So, we will move from this point to this, this was for p 2 naught and this is for p21 this. So, what are we going to get?

That these bundles are no more affordable, but this bundle which is m by p1, I can still buy this because price of good 1 is fixed, only I am changing price of good 2. So, when I change the price of good 2 keeping the price of good 1 fixed and if I increase the price of good 2, budget set moves like this. If I decrease the price of good 2, keeping price of good 1 fix it will, budget set will expand like this, okay. And so, this is how the budget set behaves, changes as we change the price of each good.

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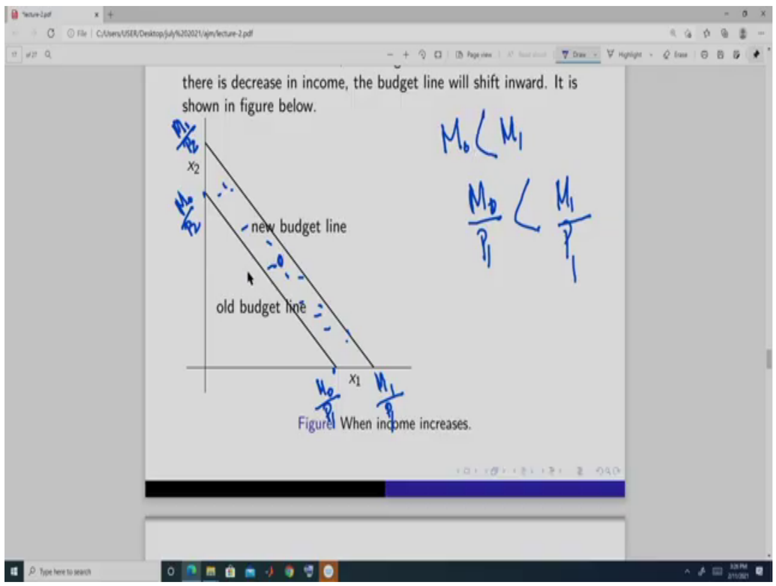

Now, suppose prices are fixed and income is changing. So, income has increased from suppose m naught to m1, then what happens? m naught by p1 this is going to be less- $\frac{M_0}{P_1} < \frac{M_1}{P_1}$. So, this which was m naught by p1 now, this is suppose m1 by p1 this is suppose m naught by p2 and this is m naught by p2 this is m1, right.

So, budget set expands as my income increases, now, more bundles are affordable to me, these bundles which was earlier not affordable now I can buy. So, more bundles are affordable to me. So, budget set expands. If suppose income decreases, suppose initial income is m1 and the new income is m naught, so this is less. So, this budget set will contract in this way, okay, it will move in this. So, this is how the budget set changes, when we change income and change prices.

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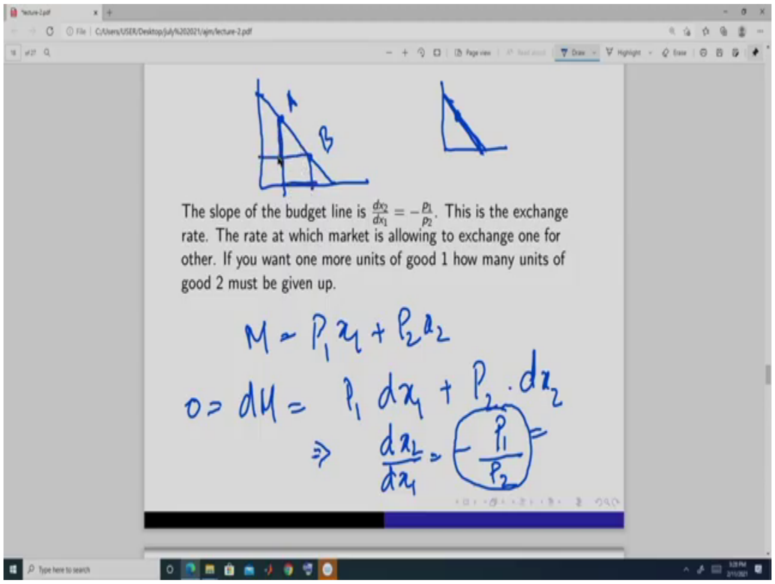

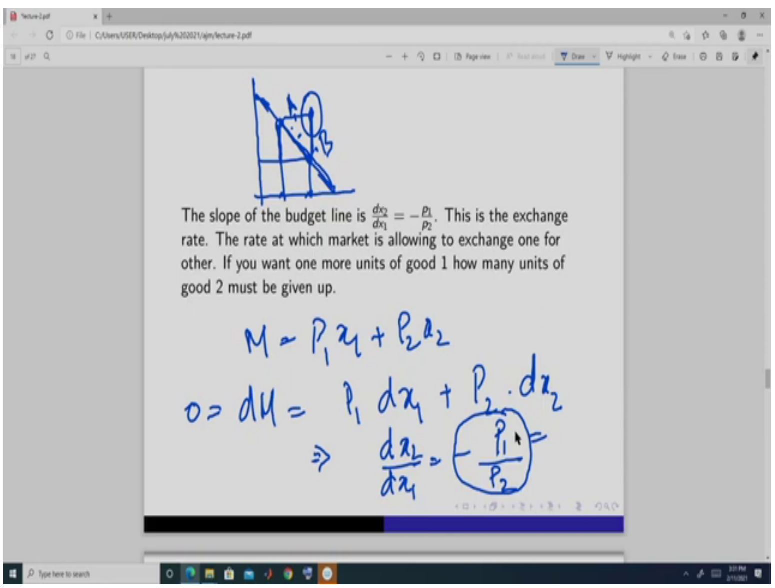

And from the budget set we will see so; our budget equation is this- $M = P_1X_1 + P_2X_2$, right. Now, if we take the total differentiation of this and what do we get? We get changes in m is equal to- $du_1 = P_1 . dx_1 + P_2 .\ dx_2$actually because price 1 and price 2 are taken as given by the consumer. So, this since income is not changing, so, if we move along our budget line like this, we get a slope this - $\frac{dx_2}{dx_1} = -\frac{P_1}{P_2}$, so this slope or budget line is giving me when we move from this to this point suppose from, okay, let us suppose move from point A to point B, if I have same amount of income, if I increase amount of good 1 from this one to this, I have to give up good 2 by this much amount.

So, the slope is giving me if I want to increase one unit of good 1 how much unit of good 2, I have to give up, so that I do not spend any extra amount or how much the market is allowing me to exchange. If I want to increase one unit of good 1, how much amount the market is allowing me to substitute the amount of good 2, that is how much amount of good 2 I have to give up, okay. So, this slope is also you can say is the exchange rate, okay.

And so, what we have done, we have defined the preferences that is how we choose the best bundle and we have got a utility function. So, we know that if we consume one bundle then how much utility we are going to get. Now, suppose we move from point A to point B, so, what does it mean that you increase the amount of good 1 from this unit to this unit. So, we increase the amount of good 1 from this much unit to this much unit.

Now, to since our income is fixed, so to remain the same budget set and so, that this bundle is affordable for us, when we give up this much amount like if we keep this and we keep increase the amount of good 2, then we get this bundle and this bundle is not affordable to me because it is outside the budget line. So, I have to give up good 1, good 2, how much I have to give up, this much amount.

So, this movement along the budget line is we can give it is defined by this slope which says that if I want to increase one unit of good 1 how much amount of good 2, I must give up, so that my expenditure remains same, okay. So, this you can say is a relative price of good 1 in terms of good 2 or you can say this is the exchange rate that is if you want to increase one unit of good 1 how much unit of good 2 you have to give up, okay.

So, now, we have done, how the budget set changes or how our sets which is defined or the bundles which are available to me or which are affordable to me how that changes when we change price of each good or when we change the income. And we also know how we choose from a given set of available bundle that is based on our utility. So, next we move to which bundle actually we choose.

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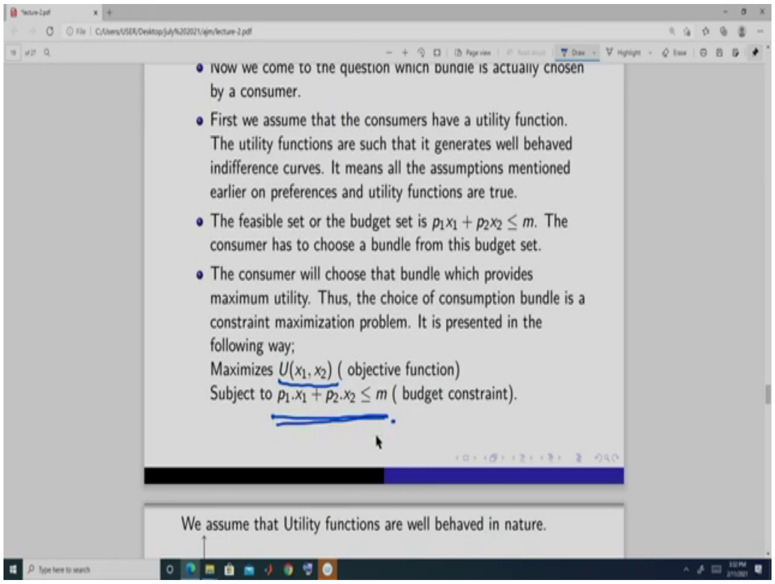

So, first here we assume that our utility function is we have a utility function. So, that means our top five assumptions are satisfied that is reflexivity, completeness, transitivity, monotonicity and continuity. Next, we further assume that our preferences are also convex. So, we get a well-defined utility function. So, the indifference curves are convex.

Next, we assume that we have a feasible or a budget set. So that is price of good 1 is p1 price of good 2 is p2 and the income is m and then we move to this problem that is we always want to maximize our utility function U(x1, x2) subject to this budget constraint- $p_1x_1 + p_2x_2 \leq m$. . What does this mean?

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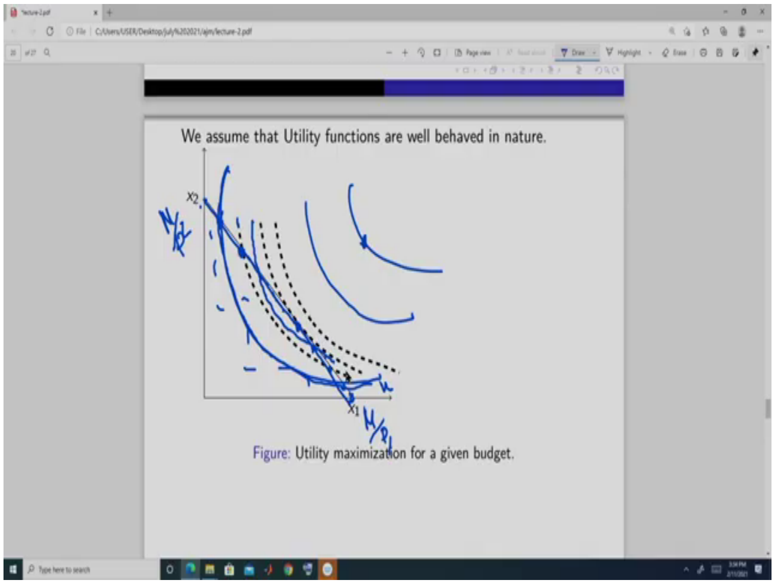

It means that we have a budget set like this, this all these points are feasible points, this point is m by p1, this point is m by p2 and we have indifference curve like this. Now, this point is not affordable to me. So, it I will not check how much utility I get from this, I will check only those bundles, which are in this set. And I will choose that one, which is giving me the maximum utility.

So, how do we arrived at that point, so suppose I choose this bundle, then I will get this much level of I am in this indifference curve. So, I will get this much utility some effect. Instead of this if I choose this one, then I will be at a higher indifference curve, right. So, that means I will get more utility. So, instead of this I should choose this bundle. So, in this way, I should move and I should, I will get this point.

Again, if I choose this, here, I get more utility than this, so, I should move in this. Again, in this a I get more utility in from this bundle than this because this indifference curve lies below these indifference curves and we know indifference curve cannot intersect because otherwise it will violate transitivity.

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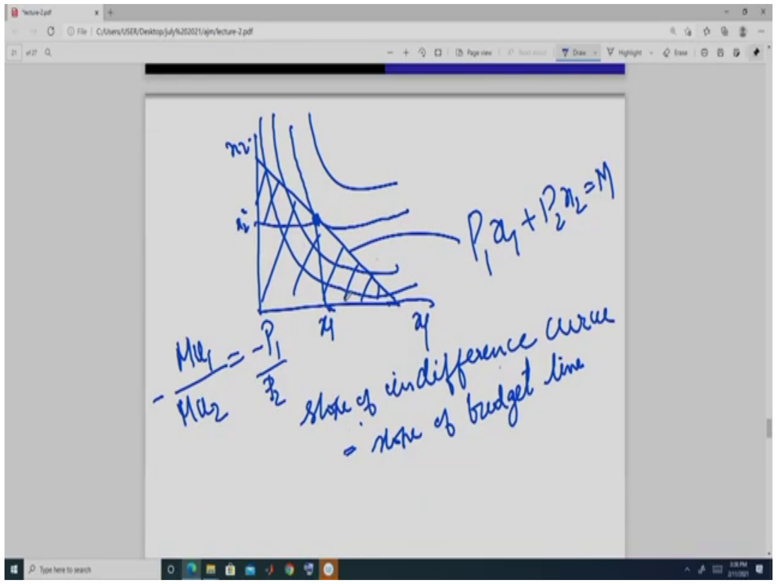

So, we get a situation like this, so this bundle, suppose this is a unique bundle(x_1^*, x_2^*) star is a bundle that maximizes my utility. So, utility subject to this budget set or budget constraint, okay. So, this is what? so, this means that when I my utility function is such that I get this sets of indifference curves and my budget bundle available bundle is given like this feasible set, this is $P_1x_1 + P_2x_2 = M$ this, then I choose this bundle, when I am maximizing my utility subject to this budget.

And at this point you will see one slope of indifference curve or you can say slope of indifference curve is actually is equal to slope of budget line or you can say this budget line is tangent to the indifference curve. So, at this point what do we get that marginal utility from good 1 and good 2. So, this which is slope of indifference curve should be equal to slope of budget line- $-\frac{Mu_1}{Mu_2} = -\frac{P_1}{P_2}$, okay.

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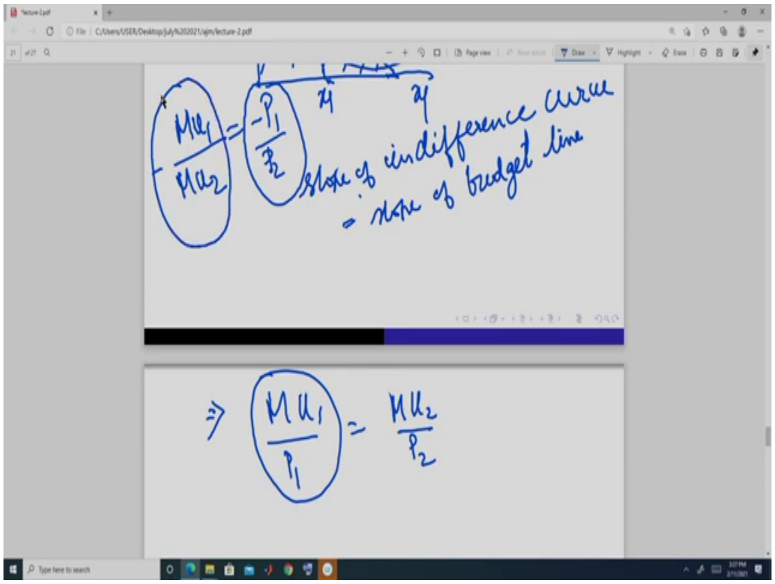

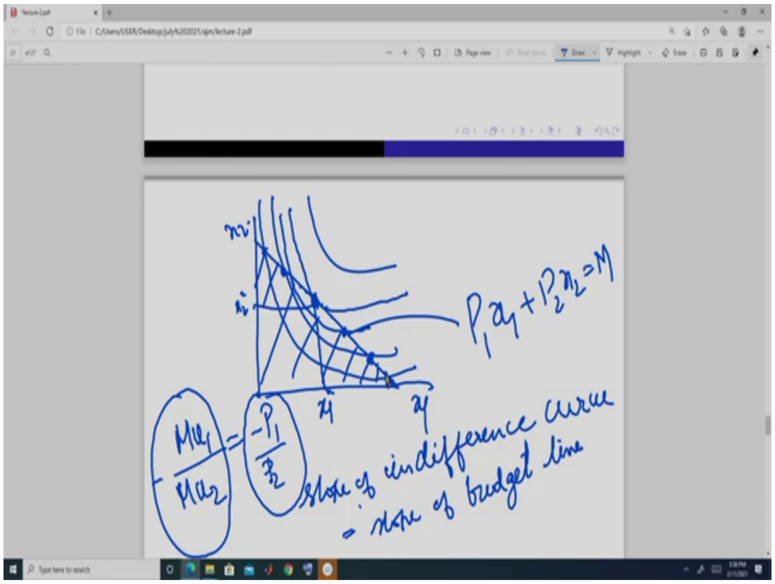

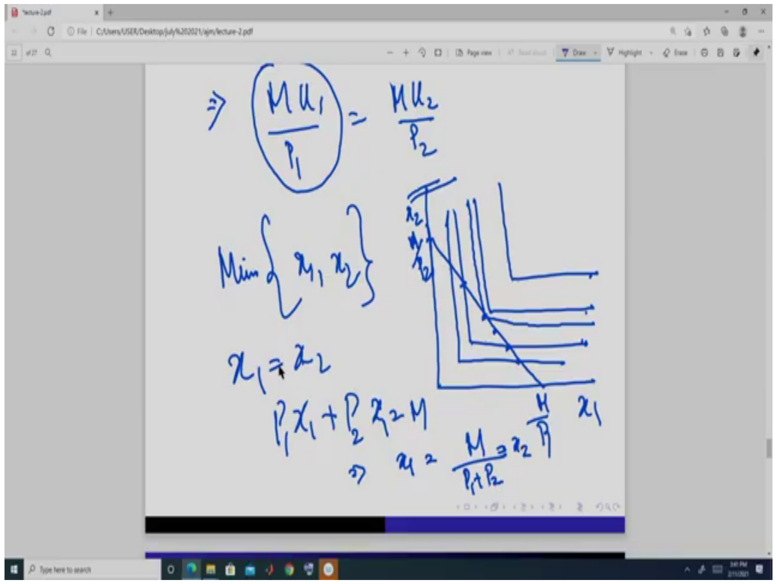

So, this actually gives us a interesting interpretation and this is like this- $\frac{Mu_1}{P_1} = \frac{Mu_2}{P_2}$. So, this means the amount of money that I, if I spent the one unit of money, the utility that I get from good 1 must be equal to the utility that I get from good 2 by spending that amount of money, right?. So, this means the exchange rate should be always equal to the marginal rate of substitution, that is how much I am willing to substitute good 2 for one unit of good 1 must be equal to what market is allowing me to exchange the amount of good 2 that I have to give give up to get one more unit of good 1, okay.

So, this should be and this point, we have understood why this is a maximum point because any point inside this curve is going to give us less utility. So, points should always be at this budget line, right. Now, if I choose this point, I get higher utility then here then this so, I will choose this so, I will move in this line and I will reach this point. Now, if I choose any point here, this point will give me higher utility than this, so, I will choose this rather than this.

This point will give me more utility than this and like this, I will hit this point where at this point the indifference curve is tangent to the budget line, okay. So, this is the optimal point or utility maximizing point. But, so, this is our tangency criteria. So, if we are given a utility function and if we are given a budget set, then we can find what is the optimal bundle or what is the utility maximizing bundle given that budget set by simply looking at the point at which the indifference curve is tangent to the budget line, right.

But can we do this for all things. Now, if utility function is suppose of this nature, min (x1, x2) like the case when the good 1 and good 2 are perfect complement. In this case this function is not differentiable and the indifference curves are of this nature, right and this utility is increasing, right. Now here how do we choose in this case. So, we will again draw the indifference budget line and this is m by suppose p1, here m by p2 and this bundle this side.

So, I will get a point like this and this point is going to give my utility maximizing point because if I choose any point here, this will is at a higher indifference curve. So, utility will be higher so, I will choose not, I will not choose this, instead I will choose this. In here I will choose this and not this one, so I like this. But here I will choose this and not this. Again, if we go further, it is not affordable. So, I will stop here.

So, this is my utility maximizing bundle in this case. And here we will, you can see how to solve this. So, in this kind of way, what do we do, we will equate these two x1 and x2 and see this will be equal and this bundle should be affordable. So, we will get x1, so this it should be in the budget set, so we get this. So, x1 is actually equal to M/P1+P2, like this and x2 is same from this, right. So, this is how we proceed in this.

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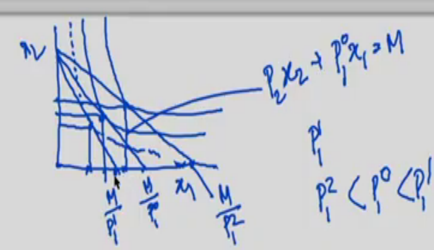
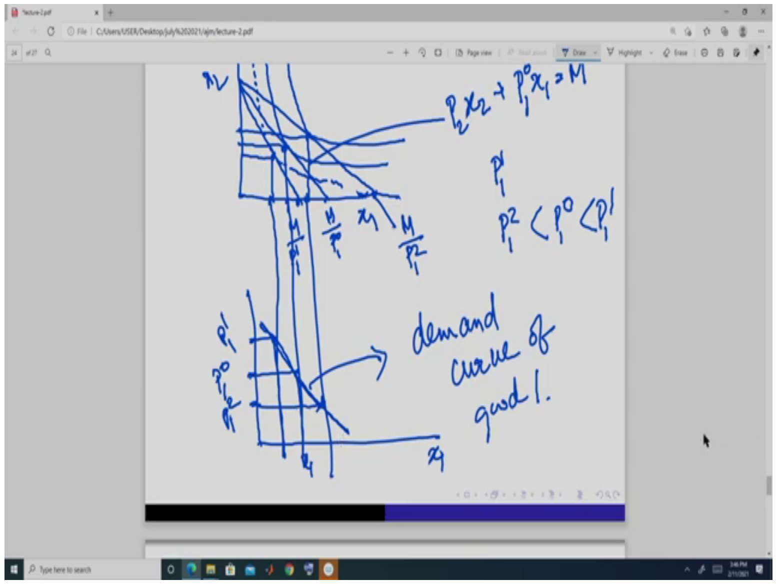
- At the point where utility is maximized subject to a budget constraint, we the following condition is satisfied; The slope of indifference curves= slope of budget constraint

$$-\frac{Mu_1}{Mu_2} = -\frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

- Based on the maximization of utility subject to a budget constraint, we derive the demand curve for each good.





Now, we know, this is our tangency condition- $\frac{-Mu_1}{Mu_2} = -\frac{\frac{\delta U(x_1x_2)}{\delta x_1}}{\frac{\delta U(x_1x_2)}{\delta x_2}} =- \frac{p_1}{p_2}$ that we get when we are maximizing our utility. Now, we will see how to derive the demand curve of a function from this utility maximum. What do we do like this? How do we derive? Good 1 and good 2 and we want to derive the demand curve of good 1, okay.

Suppose this is our budget constraint- $P_2x_2 + P_1^0x_1 = M$, this is we have fixed price of good 2 at p2 and suppose price of good 1 is some p naught, income is fixed that is m. Now here our, this is our optimal bundle suppose, okay. Now, we increase the price of good 1 from p naught 1 to p11, okay. So, what is going to happen? Our budget line is going to shift like this.

So, earlier this was this- M/P_1^0, this point is now this- M/P_1^1, so, suppose this is our optimal bundle. So, now, all these bundles are not affordable. So, our new feasible set is this and, in this set, this is suppose our optimal bundle, right. So, mark this. Now, suppose we take a price P_1^2 which is less than P_1^0, P_1^1.

So, our budget line shifts something like this. So, this point is like this $M/P_1^2$and here so, all these bundles are affordable and some more bundles are affordable given by this region because our price of good 1 has fallen. So, now, suppose this is the optimal point and this is the optimal demand of good 1. This is the optimal demand of good 1 when price is p naught. This is the optimal demand when price is p1, this is the optimal demand when price is p2.

So, in this, what do we do, we plot this point and we know p1 is greater than suppose this is p_1, p_0 is this and p2 is this. So, then we get this much demand of good 1 when the price is p11, we get this much demand of good 1 when the price is p1 naught and when price is p12 we get this much**,** right**,** and by joining these point, this is actually the demand curve of good 1, okay. And this demand curve is always downward sloping for a normal good, we will come to what is normal good later on not now.

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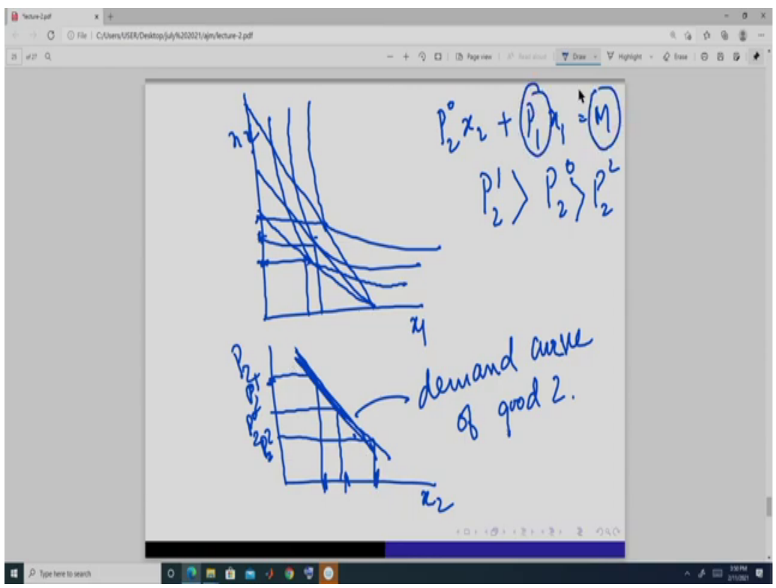

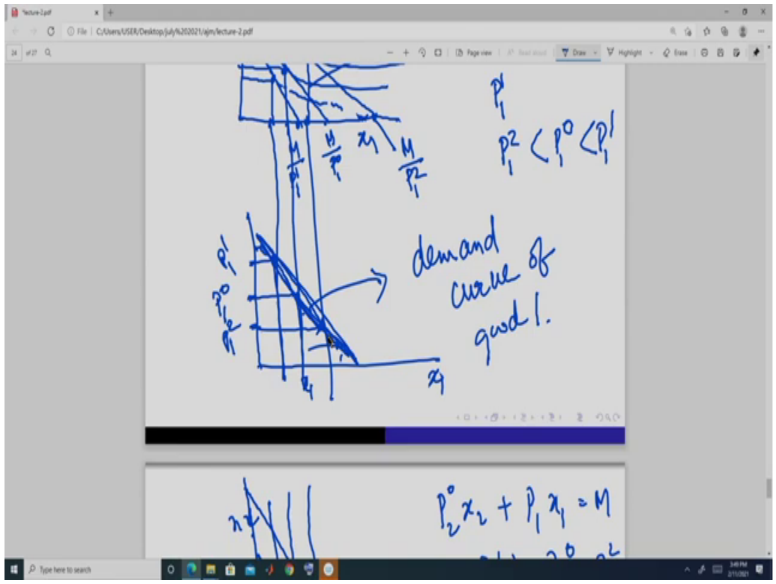

Let us do one more example and suppose we do it for good 2 now. So, this is good 1 and this is good 2. Suppose price of good 2 is p naught 2 and price of good 1 is fixed at p1 this- $P_2^0 x_2 + P_1 x_1 = M$. So, now price of good 2 is suppose increased and so what will happen? It will like this and suppose optimal bundle is here, okay. Now suppose it is so P_2^2 is suppose less than p naught 2 i.e $P_2^2 < P_2^0$. So, it is this by this, then optimal bundle is suppose this or these are the optimal bundles.

So, here in this axis we take p2, px2 and p2 this bundle is suppose this much, this bundle is suppose this much, and this bundle is for this much. This much when price is this much when price is P_2^1it is the greatest one, suppose this P_2^1. So, this next, P_2^0 , this is the bundle of good 2, this is P_2^0 . Now P_2^2 is this bundle highest, this bundle suppose. So, this is P_2^2 , so the demand curve for good 2 is this. This is the demand curve of good 2 and we get it in this way.

Now what do we see from here, so this individual is going to demand this much amount when price is P_1^1. This much amount when the price is P_1^0 , this much amount when price is P_1^2 of good 1. This much amount when price is something else, and this much amount like this so we get a curve function like this. We will derive this function algebraically also in the next class, okay.

So, and for good 2, similarly we get a demand curve like this and we see that it is downward sloping, that is when the price is high quantity demanded is low, when price is low quantity demanded is high. We have got this, when we have kept the income of this person this is fixed and the price of good 2 is fixed. We have got similar kind of demand for good 2 of this downward sloping demand curve of good 2 when price of good 1 is fixed and income is fixed, okay.

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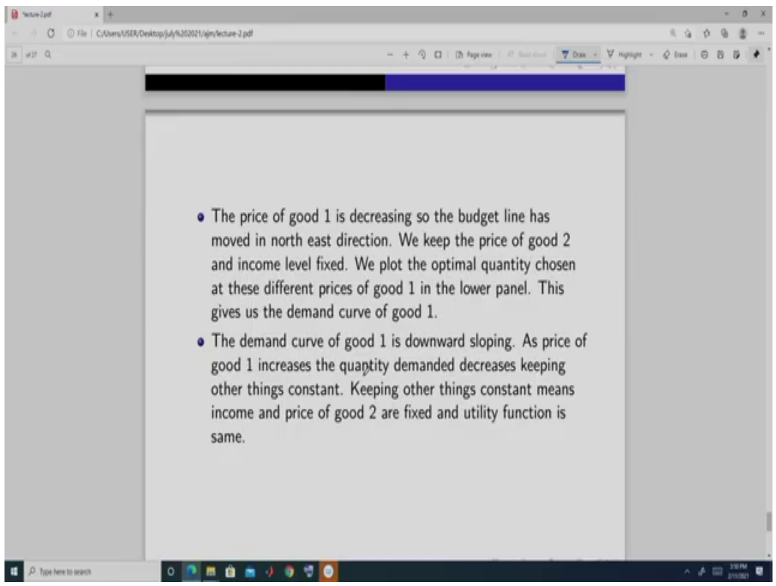

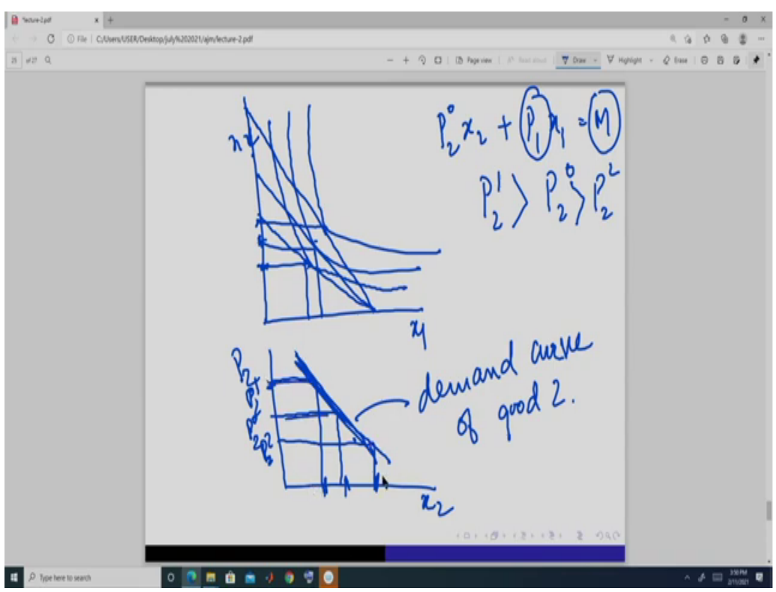

So, we have derived the demand curve. So, demand curve is actually is a function of prices and income. It says that as we change the price of a good, suppose as we change the price of good 1 how the quantity demanded of good 1 is going to change, okay. So, it is giving me this. So, at each price of good 1 how much quantity demanded when getting.

So, this is for one person and this information actually is giving me, is defining my demand curve. And what we will, do we will get the similar demand curve for all the individuals and then that will allow us to derive the market demand curve and that market demand curve is actually used by firm to determine how much amount of output to produce, okay.

So, in this way this consumer behaviour is playing an important role in the market. How the prices are going to be determined, okay. Because we get the demand curve from the preferences and the budget constraints, when the individuals are maximizing their utility subject to a budget constraints. So, I am stopping today at this level, at this point and next class we will solve this demand curve algebraically, okay. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
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Lecture - 3
Examples of Utility Maximization, Demand Curve and Market Demand Curve
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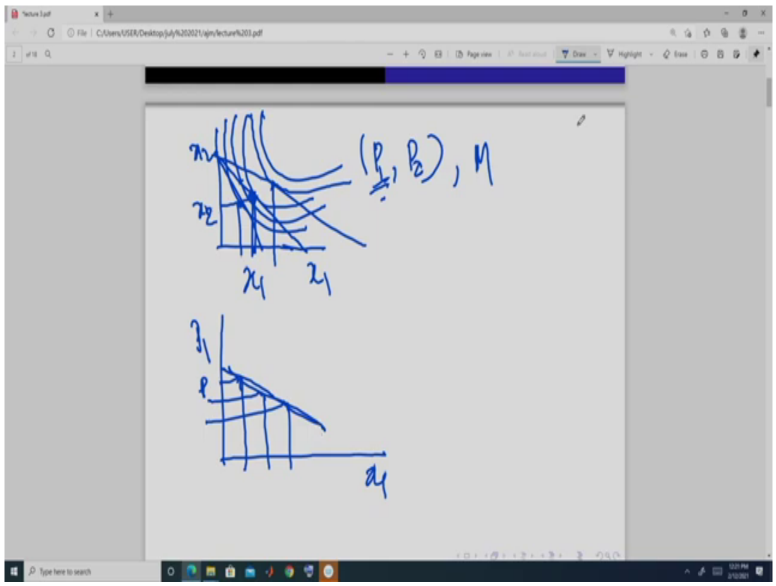

Hello, everyone. Welcome to my course Introduction to Market Structure. Before starting the lecture three, let me recap what we have done in lecture two. So, in lecture two we have derived demand curves. So, suppose we have two good, good1, good 2 and this is our budget line. So, all these points are feasible points we have to choose out of these points, and these are our indifference curves which we have got from the utility function and this is our optimal point.

So, at p_1 and p_2 price, we have demanded this much amount of good 2, good 1 and this much amount of good 2. So, this is our optimal point. Why this is an optimal point? Because if we choose any point here, then utility at this curve is higher than this. If we choose any point inside the set, then this curve point which is in this manner point, which is tangent to the budget curve is giving me higher utility.

So, that is why this is an optimal point which maximizes my utility subject to the budget constraint and that is why we have chosen that bundle and we have arrived at this for a given set of price p_1 and p_2 and an income M. Now, if we change the price of good 1, suppose like this, then we will get suppose this is our optimal point if we reduce the price this is our, this is our optimal point.

So, like this, we will get different optimal point as we change the price of good 1 and so, we will get the demand curve. So, this is the demand for good 1 when price is this, this is the demand for good 1 and this is the demand so, like this. So, this is, this is at this price, at this price, at this price, okay. So, like this we get a demand curve. And today, we will derive this algebraically.

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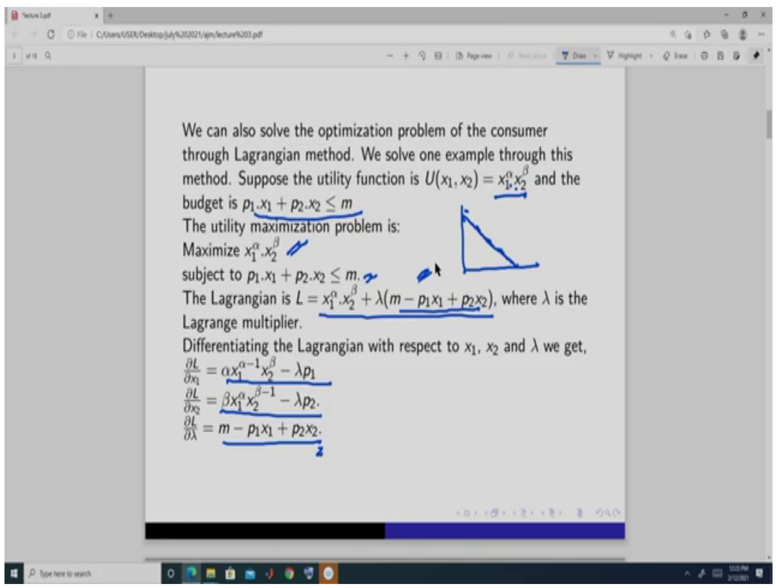

So, or what do we do? Suppose, for convenience, we have assumed a utility function like this- $U(x_1, x_2) = x_1^\alpha x_2^\beta$, this is a Cobb Douglas utility function, where alpha takes a value which is between 0 and 1 and beta takes a value which is between 0 and 1 and the budget constraint is like this- $p_1x_1 + p_2x_2 \leq m$. So, the problem is we need to maximize this utility subject to this budget constraint.

So, to do this what we do, we use something called a Lagrangian method. And here you will see that since our utility functions are of this nature, since we have assumed that they are convex, so, the utility will always be at the boundary, optimal point is always going to be at the boundary, okay. So, we can take the Lagrangian in this form, okay.

So, we do not have to use something called a Kuhn-Tucker method in this case, okay, we use the simple Lagrangian method. So, this is our objective - $x_1^{\alpha} x_2^{\beta}$ and this is our constraint- $m - (p_1 x_1 + p_2 x_2)$, okay. And this lambda (λ) is something called the Lagrangian multiplier. So, what we are doing in a way while solving this maximization problem that we are maximizing this part i.e. $x_1^{\alpha} x_2^{\beta}$ and we are actually trying to minimize this part, i.e. $m - (p_1 x_1 + p_2 x_2)$. Because when we choose a bundle, we want to choose it in such a way that it gives you maximum bundle and at the same time the expenditure should be minimum, okay, then only this is satisfied.

So, we set up the Lagrangian in this form, and then since it is differentiable, so we take, we optimize this with respect to x_1, x_2 and lambda (λ). So, here when we take the first derivative of this expression, this Lagrangian,- $L = x_1^{\alpha} . x_2^{\beta} + \lambda \left(m - p_1 x_1 + p_2 x_2\right)$ we get this term- $\frac{\delta L}{\delta x_1} = \alpha x_1^{\alpha - 1} x_2^{\beta} - \lambda p_1$, second a with x2, we will get this term- $\frac{\delta L}{\delta x_2} = \beta x_1^{\alpha} x_2^{\beta - 1} - \lambda p_2$ and when we differentiate with respect to lambda we get this term- $\frac{\delta L}{\delta \lambda} = m - p_1 x_1 + p_2 x_2 , \, okay.$ So, this concept.

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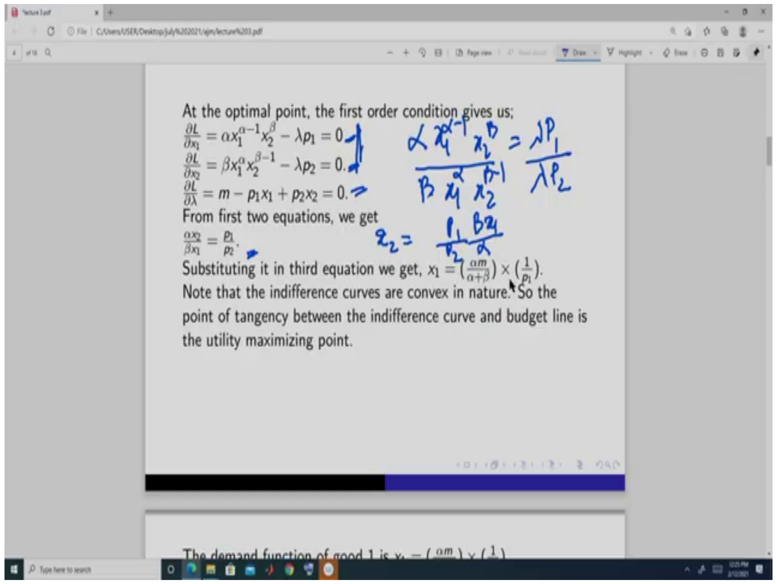

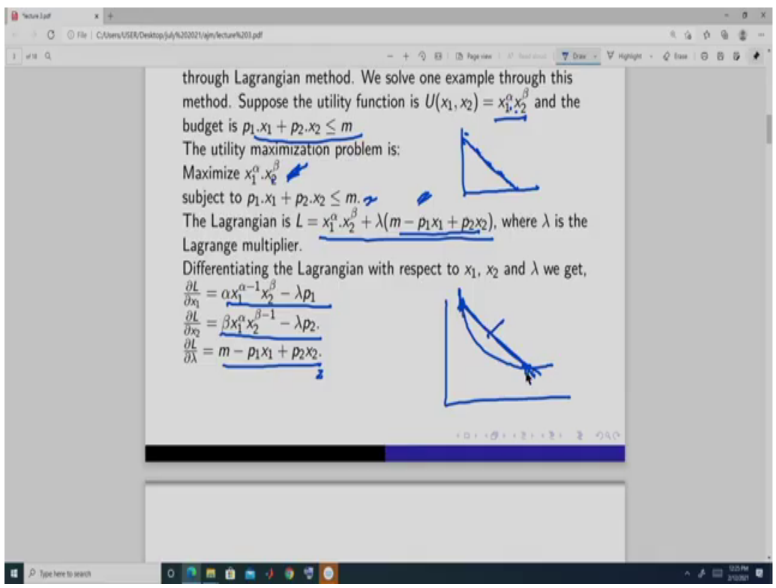

Now, the first order condition of this optimization gives us that this should be always equal to 0, i.e $\frac{\delta L}{\delta x_1} = \alpha x_1^{\alpha-1} x_2^{\beta} - \lambda p_1 = 0$, this should always be equal to 0, i.e $\frac{\delta L}{\delta x_2} = \beta x_1^{\alpha} x_2^{\beta-1} - \lambda p_2 = 0$, and this should be always equal to 0, i.e $\frac{\delta L}{\delta \lambda} = m - p_1 x_1 + p_2 x_2 = 0$. So, from this first order condition, what do we get if we equate these two, we get this thing, i.e $\frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$, right, because it is a simple calculation, right, we take this to this side and then divide it by this and we will get it like simply if we, okay, and this you can see it will give me this- $\frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$.

Now, from here what we do, we write it in terms of suppose x2 so, we can write x2 as p1 divided by p2 beta x1 divided by alpha i.e $x_2 = \frac{P_1}{P_2} . \frac{Bx_1}{\alpha}$ and we substitute this in this equation in place of x2 and we get x1 in this form- $x_1 = \left(\frac{\alpha m}{\alpha + \beta}\right) . (\frac{1}{p_1})$. So, here note that this is the first order condition and if we are doing, solving any optimization, we need to look at the second order condition also, right. But since this, i.e $x_1^{\alpha} x_2^{\beta}$ is a well-behaved utility function, if you look at the indifference curves of this a, it is something like this.

So, it satisfies something called the convexity property right, if we take this point and this point, then any point which is the linear combination of these two, it is preferred to these two points, right. So, that is why we do not need to check the optimal second order condition here. And, and if you want to pursue it, you can look at any graduate textbook you will find it, but in this course, we do not need that much, okay.

So, we will generally assume that the utility functions are such that the indifference curves are convex to the origin, okay. And so, we will get, we do not need to take the second order condition, first order condition is sufficient to give us the optimal point. okay

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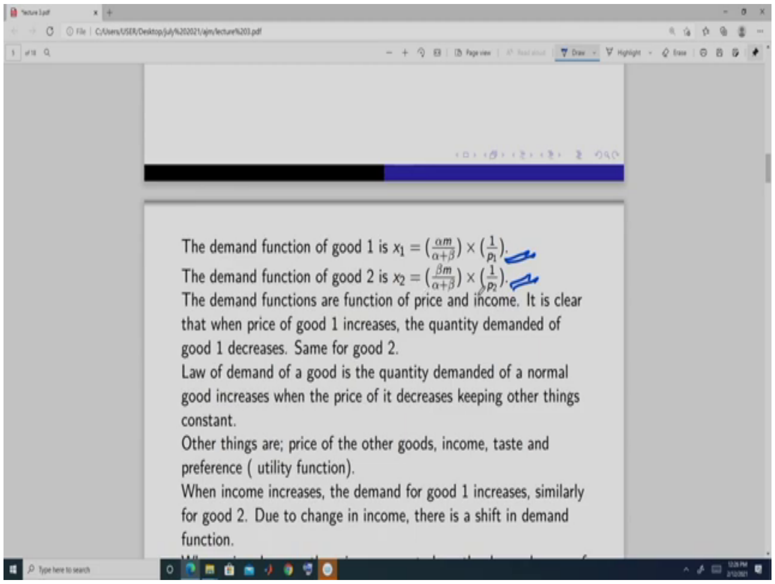

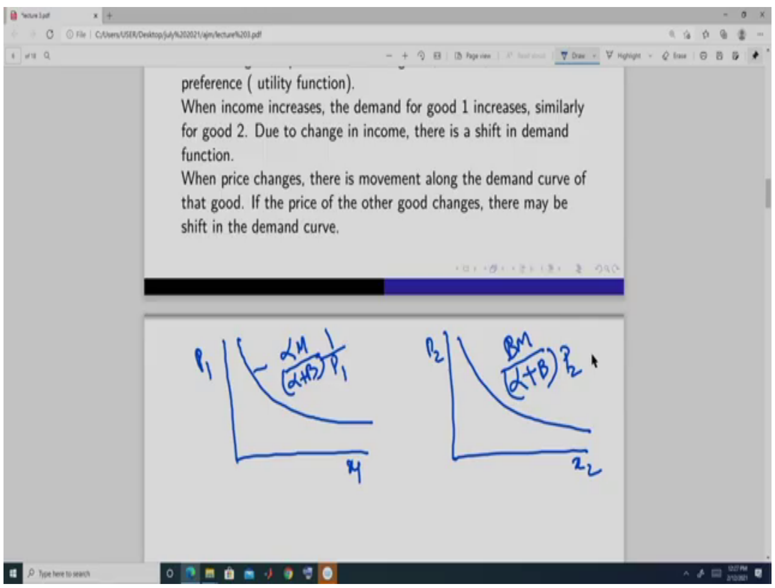

So, now, based on this, we get this - $x_1 = \left(\frac{\alpha m}{\alpha+\beta}\right).\ (\frac{1}{p_1})$ as a demand curve of a demand function of good 1 and this is demand function of good 2- $x_2 = \left(\frac{\beta m}{\alpha+\beta}\right).\ (\frac{1}{p_2})$. And if you look at them carefully, you will see that when it is something like this, that if we plot x1 here, p1here, it is curve line like this and it is something like this and this is, you can say this curve is like this- $\left(\frac{\alpha m}{\alpha+\beta}\right).\ (\frac{1}{p_1})$ this curve is beta m is like this-$\left(\frac{\beta m}{\alpha+\beta}\right).\ (\frac{1}{p_2})$. , so they are downward sloping.

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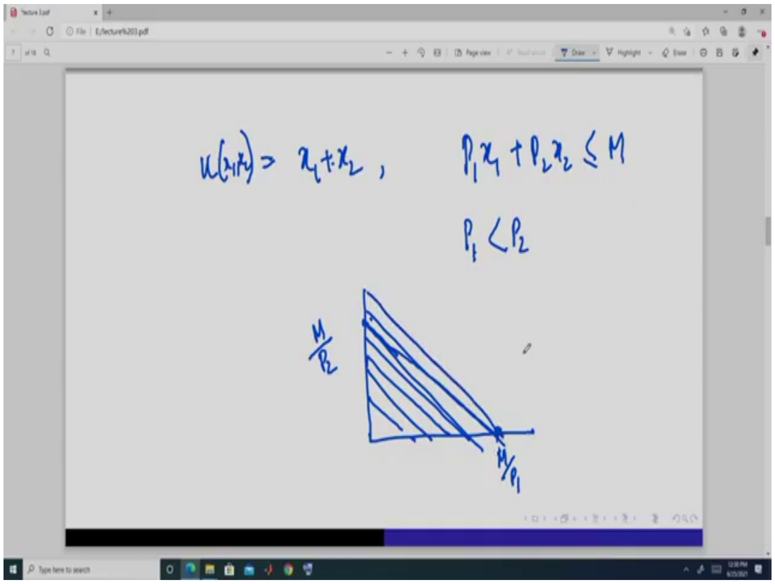

Now, let us solve another problem and that is suppose good 1 and good 2 are perfect substitute. So, this is your utility function, utility function is given like this- $U\left(x_1, x_2\right) = x_1 + x_2$ and our budget set is our budget line is supposed this- $P_1x_1 + P_2x_2 \leq M$. Now, we again further assume, we assume that p1 is less than p2.

Now, here if we take this then this is going to be m by p1and this is going to be m by p2 and it will be like this. So, this height is going to be less than this base, but if we plot these indifference curves, these are straight line and these heights and base are going to be seen. So, if we do this, let us do this, we will get like this and this is going to be our optimal point.

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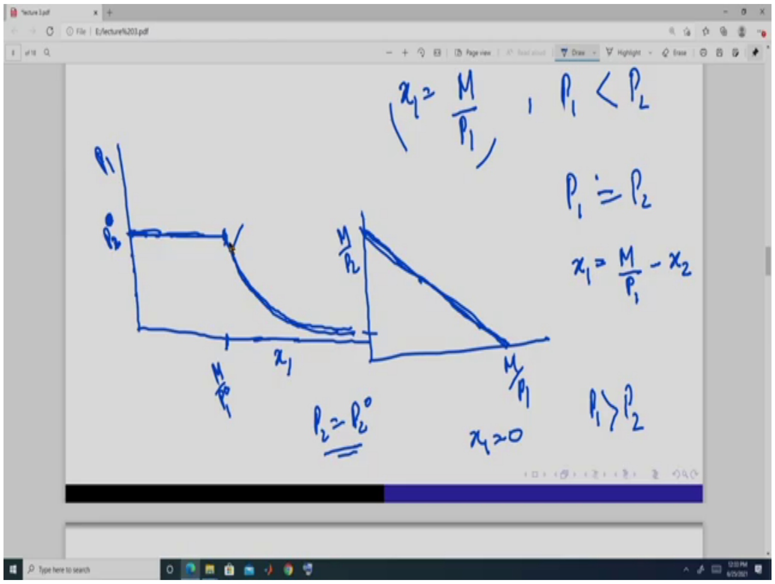

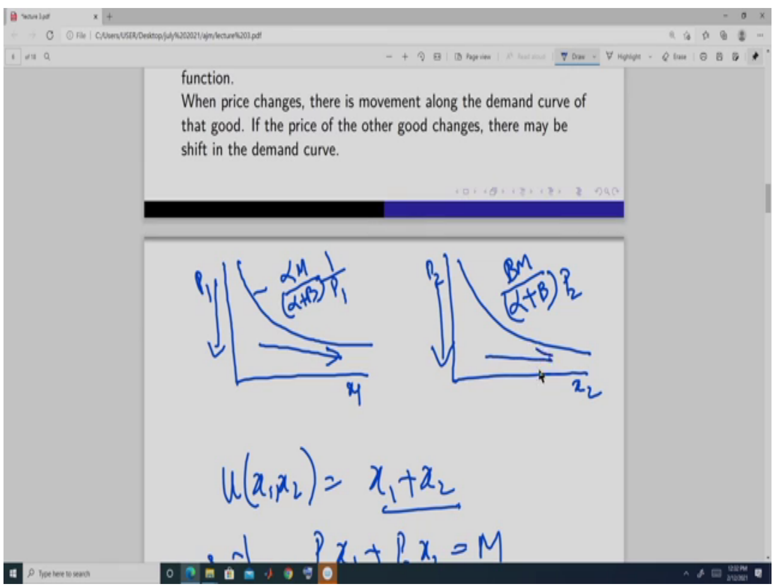

So, we get that x1 is actually this- $x_1 = \frac{M}{p_1}$ when p1 is less than p2. Now, if p1 is equal to p2 in this case you draw the budget line, this height and base are same ,m by p1 is same as m by p2. So, all these points are any points are going to be an optimal point. So, the budget so, the demand curve of firm one is going to be this- $x_1 = \frac{M}{p_1} - x_2$ because p1 is equal to p2 we can cancel it so, you whatever you have bought amount of x2 and what is left, you are going to buy x1.

So, any point like this so, if we and whenever p1 is greater than p2, we get that from same argument we get that x1 is going to be 0. So, if we plot the demand curve, we will get something like this, that suppose this is the p2, okay. So, if p1, p2 is fixed at this level and here in this we are taking p1 and in this axis we are taking x1. So, suppose this amount is m by this when p1 and here this is p2 some p2 dot so this is p1 dot suppose, okay. So, this is the amount it is demanded when or this is the amount demanded and x2 is 0, okay.

Now, when p1 falls below this a at this a when there is a same demand is going to be you can demand anything. So, and when it is less, demand curve is this. So, the demand curve is going to be a it is as the price p1 of good 1 falls, it is going to be something like this. So, this is the demand curve where we have perfect substitutes of good1 keeping price of good 2 fixed at p2 naught, p2 is equal to suppose p2 when it is naught this and we get a demand curve of this nature done.

So, now, here if you look at this demand curves, you will see that these demand curves are mainly downward sloping in prices, it means what, that as the price decreases, quantity demanded increases, same here as the price decreases quantity demanded increases and this is called the law of demand.

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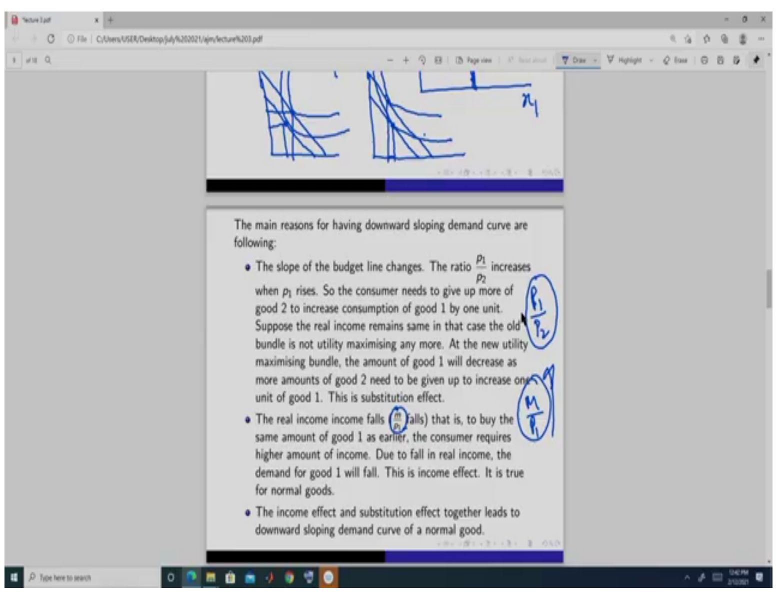

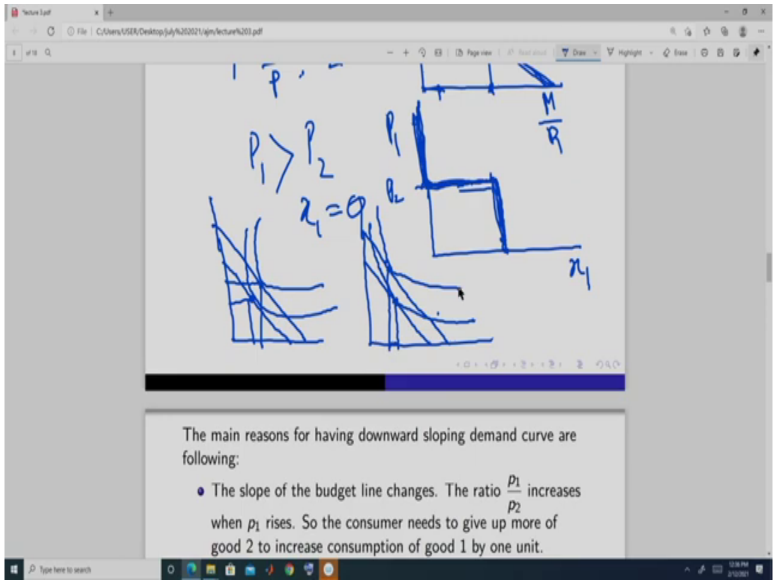

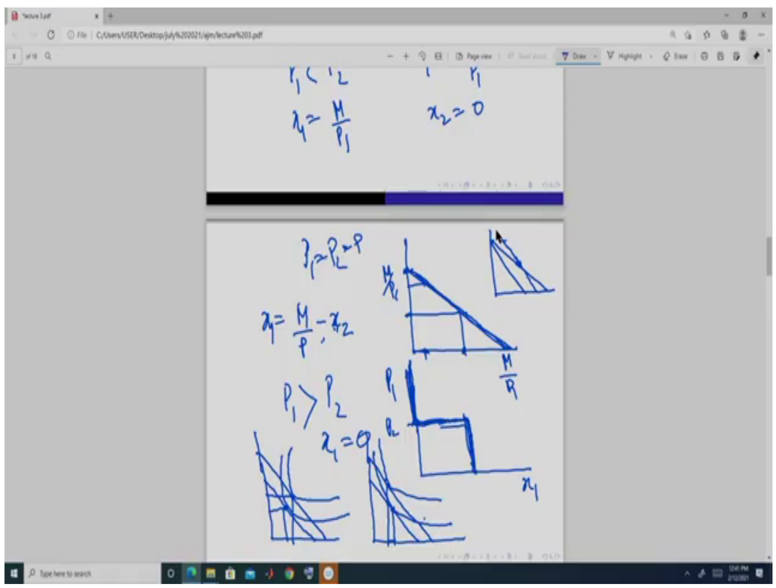

So, the law of demand says that if the good is a normal good, we will come to what is a normal good now just assume that it is a normal good, then if the price increases the quantity demanded should always goes down, given other things constant. What are other things? The other things are price of the other good, income and the taste and preferences here means the utility function is fixed that means the indifference curves are fixed.

Now, when I say normal good, it mainly means that suppose, I have this budget line and suppose this is my utility function and I have got this. Now suppose income increases, I am at this point, okay. So, this is your new optimal point, when income is increasing in this way budget line has parallelly shifted upward, demand for both good has increased, right.

So, this you can say is a normal good, but suppose the situation is something like this. So, what is happening to the demand for good 1. As the income has increased, demand for good 1 has gone down. So, in this case we say that good 1 is an inferior good when income increases demand for that goes down. So, when we are seeing that it is a normal good then it means that whenever income increases demand for that is going up or it will at least not good down, okay.

So, whenever our consumption bundle is such that both the goods are normal then the law of demand says that if the price of good 1 suppose goes up, then quantity demanded of good 1 is going to go down because price has increased. Keeping other things constant it means that the price of good 2, p2 is fixed, m is fixed and the utility function is also fixed in this case, which means that our tastes and preferences are same, okay.

Now, why do we get such a downward sloping demand curve? One, reason is it is mainly because of two effects and these are called income effect and substitution effect. Income effect says that, suppose your a price is fixed p1and p2 and whenever the price of p1increases, then this ratio that is m by p1, i.e $\frac{m}{p_1}$it falls.

So, it means what? That now, whatever earlier amount of maximum amount of good 1 you could have bought, now, you cannot. So, in real terms, we say that your real income has fallen. So, this has fallen, but you can buy what the maximum amount of good 2 what was earlier, what you were buying, then this we know the way the budget line shifts whenever price of good 1 changes, right.

So, whenever price increases, what happened budget line will shift inward, right and it will shift in such a way that now some of these bundle of good 1 which was earlier affordable now it is not affordable. So, that is why we will reduce our demand for this good 1, okay**.** Because there is an effect of actually in real terms our income has gone down because this- $\frac{m}{p_1}$has fallen, okay, this is income effect.

Another thing is the moment p1 and, p1suppose increases this ratio p1 by p2 what happened, this is what, this is your exchange rate we have done it, it is the slope of the budget line that changes. So, in this case suppose here what is happening? We have like this, now, price of good 1 has suppose in case so, it will be like this.

So, earlier suppose the optimal point was, is at this point now, here, if we look at this point only, this point, the slope is now changing at this point. So, this bundle, whatever I am willing to substitute, if I want to consume one more unit of good 1, how much amount of good 2 I am willing to substitute that is given by the slope of indifference curve. And what I am supposed to substitute that is the exchange rate is given by the slope of the budget line that is this has changed.

Now, what it means that the market is asking you to give up more of good 2, if you want to consume one more unit of good 1. So, that is why what you will do, you will reduce your consumption of good 1. So, this is something called a substitution effect, that moment, the price of good 1 increases, what happened?

The p2 by p1this ratio, that ratio is the exchange rate, that is what if you want to increase one more unit of good 1 how much market is allowing you to reduce the amount of good 2 asking you to reduce the amount of good 2 that is the exchange rate or slope of the budget line. Now, that changes, it actually increases this ratio and once it increases so, it means what you have to now give up more unit of good 2.

So, and you are getting the utilities in the same way. So, you will reduce the consumption of good 1 rather, okay. So, that is why the demand for good 1 decreases when price of good 1 increases and similarly, when the price of good 1 decreases the demand for good 1 is going to go up because the moment the price decreases, it means what? this ratio- $\frac{M}{P_1}$is going to go up. So, your real income is actually increasing.

So, you will demand more of this good and when the price of p1 falls it means what this ratio is going to go down. So, now, your market is asking you to reduce less amount of good 2 to increase one unit of good 1, okay. So, you will consume more of good 1, okay. So, this is the main reason for why a demand curve should always be downward sloping if the goods are normal goods, okay.

Now, we will move from so, what we have done? We have done that the demand for any good is derived, we have derived the demand for any good from a utility maximization problem. Now, we move to the market demand curve. So, we know that suppose these many peoples are there in this market and we know that demand curve of each individuals then what is going to be the market demand curve. Market demand curve means that at each price how much is the total quantity going to be demanded in that market.

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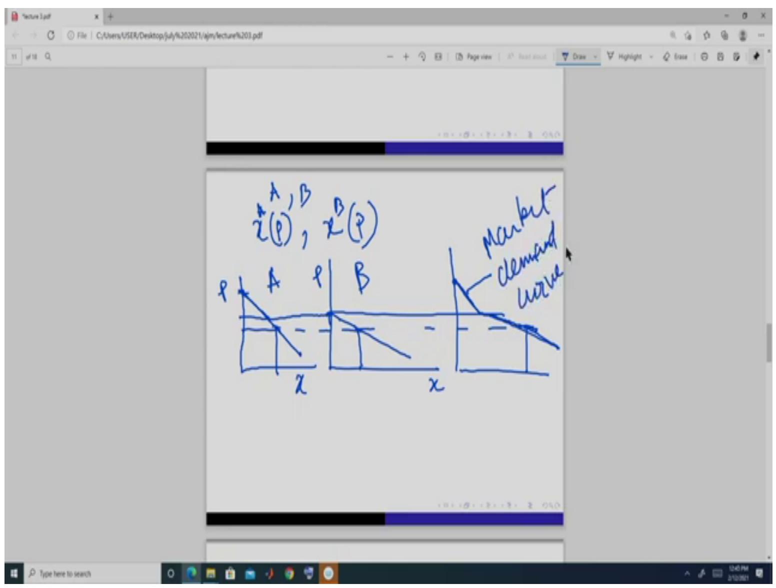

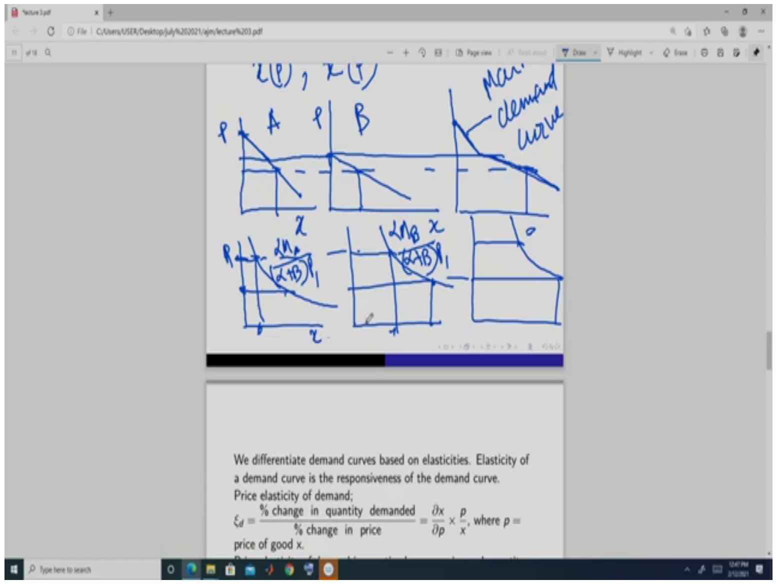

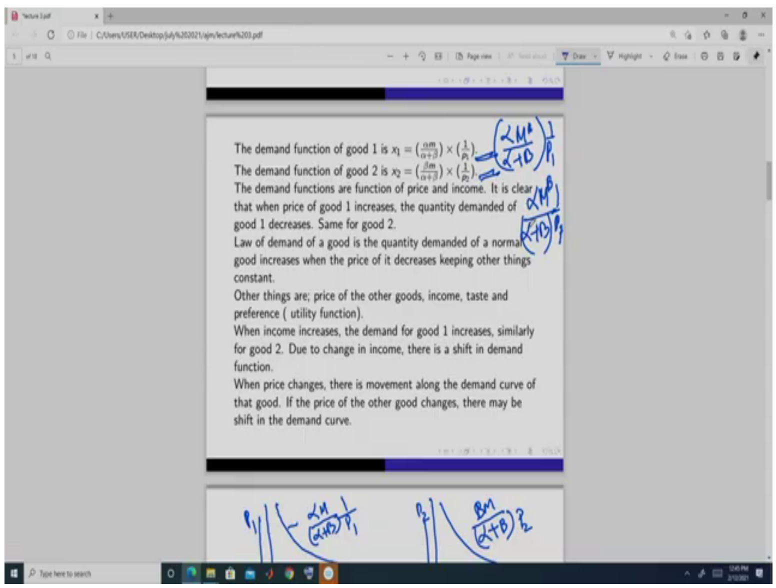

So, what do we do to get the market demand curve? We do something called horizontal summation, and what is this, suppose we have two individual, individual A and individual B. Demand curve demand function of individual A is suppose this- $x^A(P)$, it is some function of income and price and demand curve function of B is this-$x^B(P)$, okay, something like this for suppose this is good A, person A and this is for person B, okay

Now, the market demand curve is at this price what is the demand, only person A is demanding, B is not demanding anything. So, it is like this, so till this price, our demand curve is only this, that is same as this, but after this price we have to add both. So, at this price quantity is this much and this much**,** right. So, this together gives me some amount like this. So, we will get a horizontal curve some like this, this is going to be the market demand curve, okay.

Now, in our example, where we have derived this demand curve, suppose this is the demand curve- $x_1 = \left(\frac{\alpha m}{\alpha+\beta}\right).\ (\frac{1}{p_1})$ and suppose for person A it is mA alpha plus beta 1 by p1, i.e $\left(\frac{\alpha M^A}{\alpha+\beta}\right). \frac{1}{P_1}$ and for person B it is alpha by B alpha plus beta 1 by p1, i.e $\left(\frac{\alpha M^B}{\alpha+\beta}\right). \frac{1}{P_1}$. So, if we plot them, we will get something like this- p1 will be like this, where we can write it as alpha mA alpha plus beta p1, i.e $\left(\frac{\alpha M_A}{\alpha+\beta}\right). \frac{1}{P_1}$and suppose it is something like this for alpha B alpha plus beta p1, i.e $\left(\frac{\alpha M_B}{\alpha+\beta}\right). \frac{1}{P_1}$ this is for person B and this is for person A.

And if we, when we do the sum, we will get a curve like this and this is going to be the market demand curve. So, what we do is for each price we add the quantities at this price this much is the quantity, at this same price this much is the quantity so, market demand is this plus this. So, we get this much, here at this price we get this much quantity this, this much quantity. So, market demand is something here like this. So, this horizontal summation of each individual demand curve will give us the market demand curve, okay. And we will need the market demand because the firms are going to face the market demand, okay.

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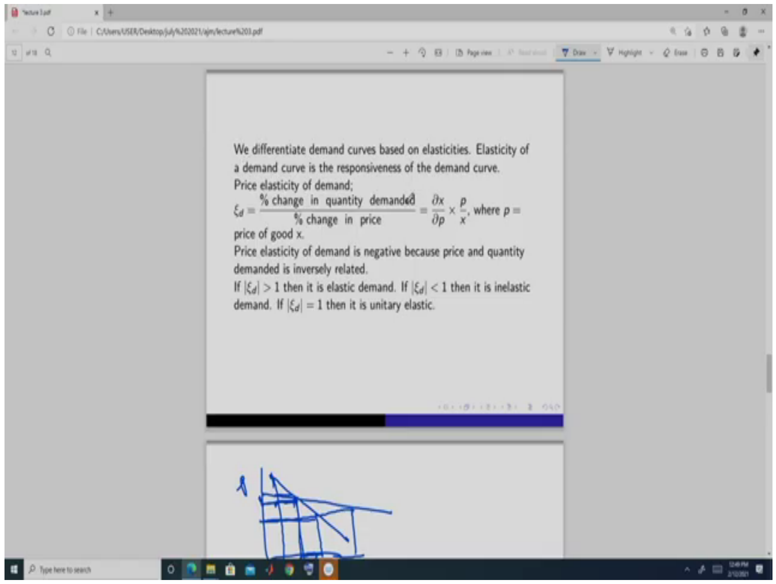
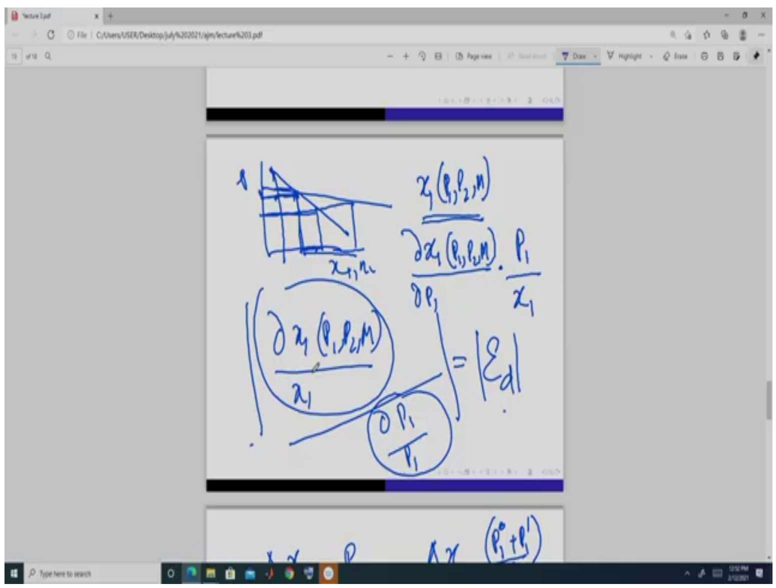
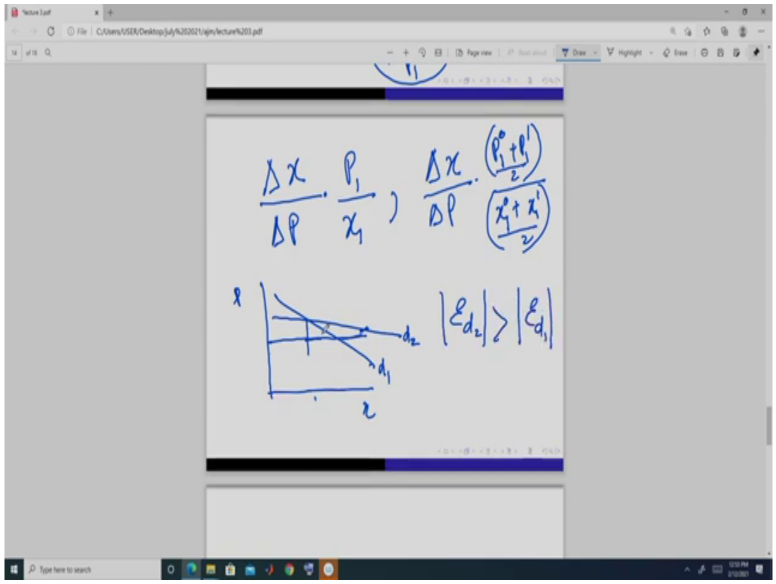


We differentiate demand curves based on elasticities. Elasticity of a demand curve is the responsiveness of the demand curve.
Price elasticity of demand;

$$\xi_d = \frac{\text{\% change in quantity demanded}}{\text{\% change in price}} = \frac{\partial x}{\partial p} \times \frac{p}{x}, \text{ where } p =$$

price of good x.
Price elasticity of demand is negative because price and quantity demanded is inversely related.
If $|\xi_d| > 1$ then it is elastic demand. If $|\xi_d| < 1$ then it is inelastic demand. If $|\xi_d| = 1$ then it is unitary elastic.





Next, we come to another topic and that is the elasticity. Elasticity actually gives you how responsive the demand curve is to, to changes in price. Suppose, you have a demand curve of this is good and this is the price, suppose good 1 and this is the demand curve of one individual, okay or this is the demand curve in this market you can say or you can say suppose here we are plotting two types of good good 1and good 2, this is 1 and another demand is suppose something like this, right.

So, now, when we are changing price suppose the price is here and if we change the price so, at this price quantity demanded is same, but when we decrease the price here quantity demanded has increased by this amount, in this case it has. So, in this case responsiveness is much higher than this for a change in price, right. So, here if we have, say in this case, we have to reduce only this much price to get this much change in a.

But here we have to reduce this much a huge amount of price to get same amount of, so, these things are defined based on something called elasticity, how responsive the demand curve is to change in price. So, this is called the demand elasticity, okay, mainly the price elasticity of demand. Okay. So, when we calculate it as percentage change in quantity demanded divided by percentage change in price.

So, if we are given a demand curve like this- x(p), p like this, suppose this is demand curve there are some other variables also. So, this is suppose p1 and it is something like this- $x_1(P_1, P_2, M)$, then if we want to find the price elasticity of good 1 then what we do, we take, this is what?, i.e $\frac{\delta x_1(P_1, P_2, M)}{\delta P_1}$ this is the slope of demand curve. If we change price slightly how much quantity demanded is going to change, multiplied like this- $\frac{\delta x_1(P_1, P_2, M)}{\delta P_1} . \frac{P_1}{x_1}$.

So, here this term- $\frac{\delta x_1(P_1, P_2, M)}{x_1}$ this is percentage change in quantity demanded divided by this term- $\frac{\delta P_1}{P_1}$ this is percentage change in price or if you are looking at suppose change from this unit of price to this unit of price you can do it in this way. That is changes in quantity demanded divided by changes in price p1, this one- $\frac{\delta x}{\delta P} . \frac{P_1}{x_1}$ and this you can take the initial price or you can take average of these two price, both are possible.

So, this is suppose initial price plus like this. So, this here this is again percentage change, this also you can say a percentage change, okay. Now, here we can categorize demand function in terms of elasticities. Now, suppose the elasticity now this elasticity see here since the demand curve is always downward sloping, so this slope is always going to take a negative value for a normal good.

So, what do we do? We generally take the absolute value of this and we would call this as elasticity of demand is generally represented in this form- $\frac{\frac{\delta x_1(P_1, P_2, M)}{x_1}}{\frac{\delta P_1}{P_1}}$, okay. So, because what do we want to see that we know that since we are looking at only the normal goods, so, it is always going to be downward sloping. So, all of them will going to take negative value. So, that is why we make them positive by taking only the absolute path.

Now, if the absolute value of this elasticity is greater than 1, we say it is elastic, if it is less than 1we call it inelastic. And if it is equal to 1, we call it unitary elastic. And you can look at these things like suppose your, something like this, this is your demand curve and your demand curve is this. This is more elastic than this, because when you change a price, the same amount. quantity demanded is changing in this case much more. So, if this demand curve is suppose d2 and this is d1, then elasticity of d2 is definitely going to be greater than elasticity of d1, right.

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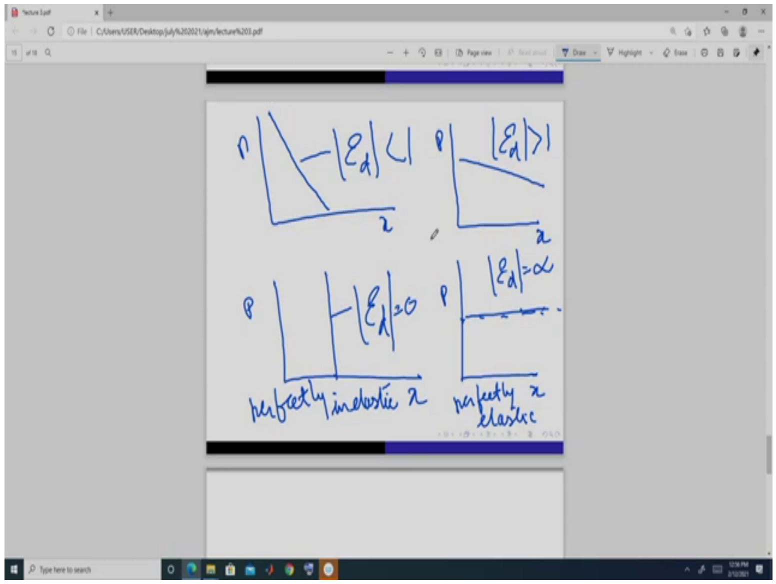

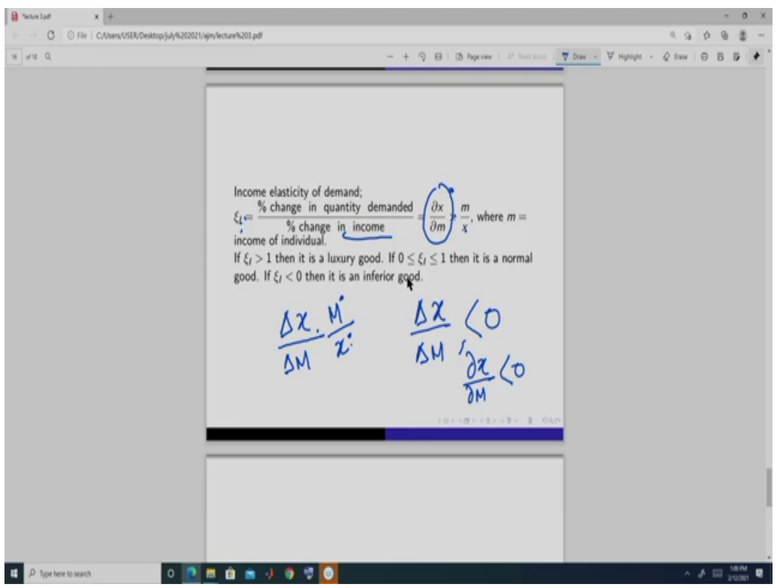

And if our demand curves are something like this, then these are generally inelastic demand curve where elasticity is generally less than 1. And if we take a demand curve of this nature then the elasticity is generally elastic and it is greater than 1, okay. And from here actually, you can see that if we take the demand curve is of this nature suppose whatever may be the price, quantity demanded is fixed, okay, this is when elasticity is actually 0, it is perfectly inelastic.

And so, this is and if we get something like this, demand curve is suppose, of this nature then elasticity is actually infinity. So, it is perfectly elastic than we demand any amount at this price, this amount or this amount or this amount, right. So, if we slightly change increase the price the quantity demanded becomes 0 and here even if we reduce, it does not make any difference because at this price anything can be demanded, okay. So, this is called perfectly elastic.

And so, if a demand curve is more closer to this vertical line, then it means that the elasticity is less so, it is inelastic, so, less than 1 and if it is more closer to the horizontal line, then it is it means that elasticity is more elastic it is elasticity is greater than 1, okay. So, these things we may require in the course, because specially when we do the monopoly, we will see the how the price is determined in a market it depends a lot on the price elasticity of demand. Okay.

Next, we do something called an income elasticity. Income elasticity is given by this formula that is- percentage change in quantity demanded divided by percentage change in income. So, given your demand curve, we know it is always going to be a function of income. So, if you change the income, so, how the demand is going to respond and then we convert it into percentage change. So, it is you can look at it is this changes in quantity demanded by changes in income and then income divided by - $\frac{\delta x}{\delta m} \cdot \frac{m}{x}$.

So, this is something called a at a point if we slightly change the income how slightly the quantity demanded is good, or we can convert it into this- $\frac{\delta x}{\delta m} \cdot \frac{m}{x}$, okay. So, this is changes in quantity demanded due to changes in income and this is the initial income and the initial quantity demanded, okay. So, this is now, if this takes a positive value, then it is not an inferior good and when it can take a negative value it is when this $\frac{\delta x}{\delta m}$ is negative, or this term is negative, this is what it means.

It means if you increase income by how much amount quantity is going to increase this. Now, if this is negative then it means what if you increase income this much decrease, if you decrease this then this must increase, then only this is a, so, this is only when we have an inferior good **or** then when our income increases, we reduce the consumption of that good and we get also in marginal terms, we get this to be negative.

And we would call those goods whenever, if we have a demand curve and that demand curve is such that whenever we increase the income and if the quantity demanded decreases, then we say that it is an inferior good. And if this income elasticity is greater than 1 that is, if the income changes by 1 percentage unit, then the amount of quantity demanded, if that increases by more than 1 percent then we say that it is a luxury good.

And when there is 1 percent increase in income, the quantity demanded it increases by more than 1 percent then we say it is a luxury good. And if it lies between 0 and 1we say it is a normal good that is whenever if there is a 1 percent increase in income and the quantity demanded that increases, but it lies between less than 1 percent or less than equal to 1 percent and if it is greater than 0, then we say it is normal good, okay.

So, this is how we classify goods in these three terms if we look at normal, inferior and luxury. Another is elastic, the good is elastic in demand the price elasticity is when it is greater than 1, we say it is inelastic when price elasticity of demand is less than 1, okay.

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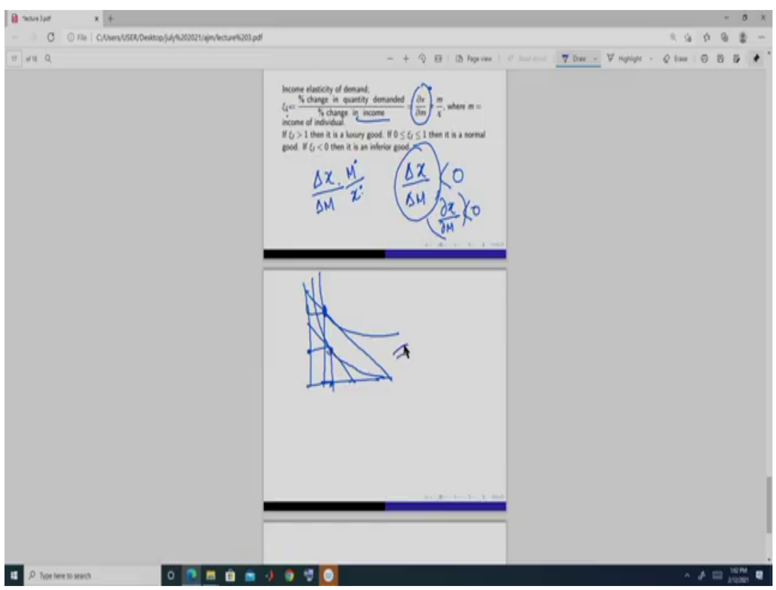

And now here in this case, I just simply I should give you one example simply to see that what actually it means. I have already shown it suppose we have this, now suppose income increases by this much amount and for some reason our optimal point is this. This is the optimal demand for good 1 and this is for good 2 when income increases from such that the budget curve, budget lines shift from this line to this line, demand for good 1 optimal bundle is this, so demand for good 1 has gone down. So, this is negative or you can say this is negative. So, this is an example of inferior good, okay.

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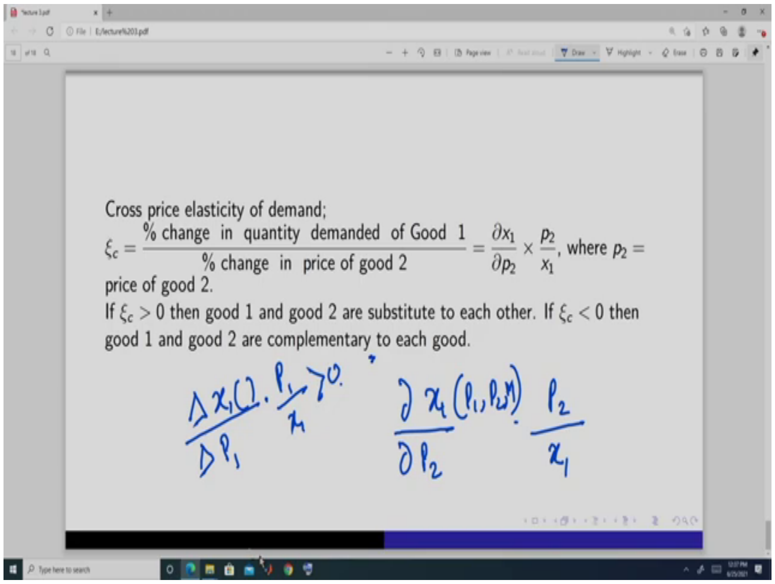

Now, we will look at cross elasticity of demand. Cross elasticity of demand means that if suppose there are two good, good 1 and good 2, if I vary the price of good 2, how much there is change is going to be in the demand of good 1. So, that is how responsive is the demand of good 1. So, if suppose these good 1 and good 2 are substitutes.

So, if price of good 2 increases, then the demand for good 1 should also rise because when the price of good 2 increases, then the demand for good 2 is going to go down and so, people will substitute to good 1 and so, the demand for good 1 is going to go up. So, whenever this goods are substitute cross elasticity is positive, price of good 2 has gone up. So, demand for good 1 has gone up.

But if the goods are complimentary, then when the price of good 2 increases, then the demand for good 2 is going to go down. So, demand for good 1 is also going to go down because they are complimentary in nature. So, it is negative and we calculate this based on this- $\frac{\delta x_1}{\delta p_2} \cdot \frac{p_2}{x_1}$. So, we will have the demand curve. So, demand curve is this- $\frac{\delta x_1(P_1, P_2, M)}{\delta P_2} \cdot \frac{P_2}{x_1}$ which is a function of p1, which is a function of p1, p2, and it should be a function of m also, then we take this derivative partial with respect to p2 and this is p2 divided.

And or if we look at in terms of difference, it will be difference in x1 divided by difference in price of x divided by this or we can take the average of these prices- $\frac{\Delta x_1}{\Delta P_1} . \frac{P_1}{x_1}$. So, this is going to be positive when we have substitute goods and this is going to be negative when we have complimentary goods, okay.

Now, whenever this is positive, it means that when price of good 1 increases, demand for good 2 also increases. So, when price of good 1 increases, it means what, demand for good 1 is going to go down and cross elasticity suppose that the demand it is positive. So, the demand for good 2 is increasing. So, it means, that good 1 and good 2 are substitutes because, whenever the price of one good increases, we try to substitute it with some other goods which are now whose price has not gone up.

Similarly, when the cross elasticity is negative, that is when the price of good 1 has suppose increases and increased and the demand for good 2 has also gone down it means what, it means that price of good 1 has increased. So, the demand for good 1 is going down. So, at the same time a good, which is complimentary to good 1, it is demand is also going to go down. So, that is why whenever cross price elasticity is negative, we say that these 2 goods are complimentary in nature.

It is something like this, then when the price of suppose ink increases, then at the same time the demand for pen is also fountain pen is also going to go down. So, this is what we are going to cover in consumer behavior. So, what we have done, we have done that we know how to get a demand curve from a utility maximization and once we know the demand for a good from there, we can find out the market demand curve by doing a horizontal submission.

And then we also know how to classify the demand different demand curves like in terms of price elasticity, in terms of income elasticity and also in terms of cross price elasticity. So, in this course, we will only need this much of consumer behavior. So, from next class we will start the second module and that is the production and cost, okay. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture 4
Tutorial

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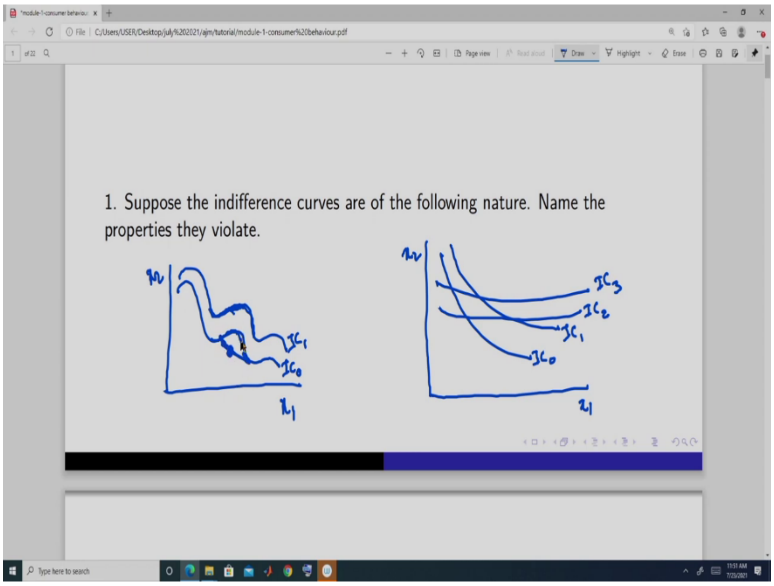

So, let us discuss few problems related to consumer behaviour and demand curve. We have already discussed this topic. So, now let us do some problems. Now, the first problem is suppose the indifference curves are of the following nature and name the properties they violate. So, there are two good x1, x2 and suppose the indifference curve is of this nature, this is IC_0, IC_1this is 1.

Another is it was again two good, good 1, good 2, and it is like this, this is IC_0, IC_1, $IC_{2,}$ IC_3.

So, these are the indifference curves. So, we have to say or name the properties they violate. So, in this case what do we see? We see that the indifference curves is like this it is upward sloping, here also it is upward sloping and here it is like this. So, if you take suppose take this point and this point take any linear combination, we do get what this point is less preferred then this point or this point because it lies in a lower indifference. So, this violates 2 properties.

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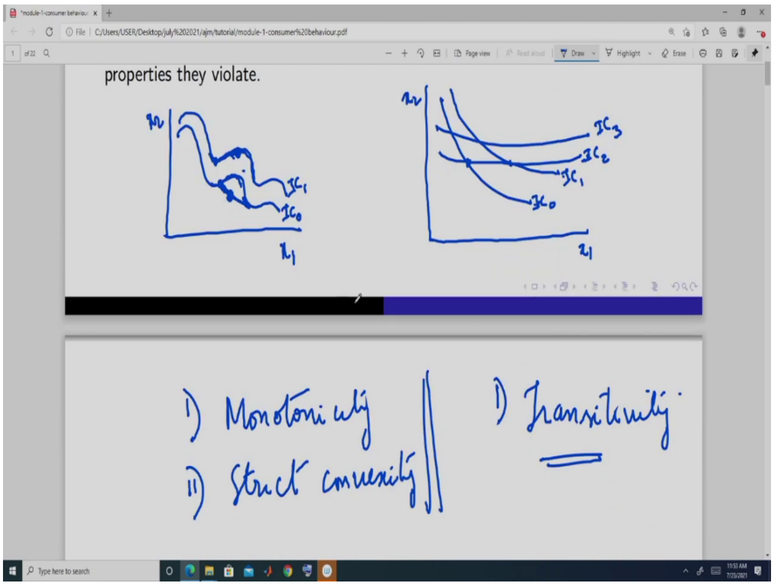

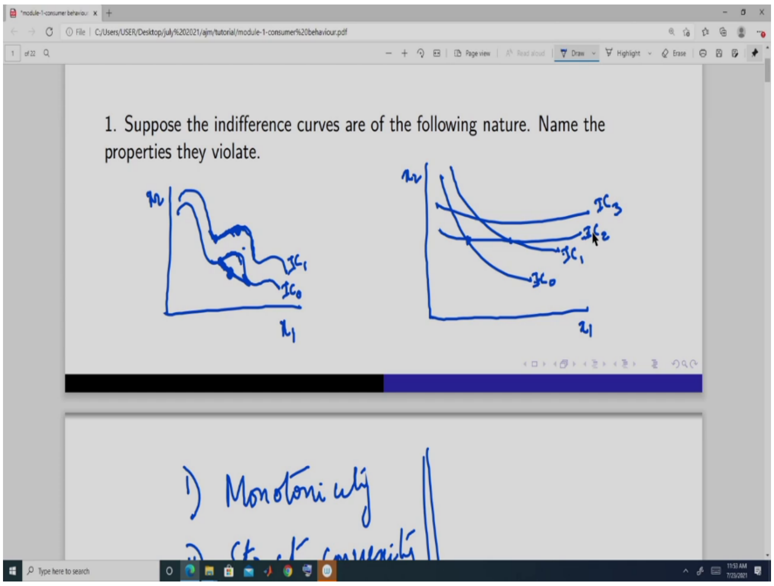

One, it is first is monotonicity, monotonicity means if X and Y if x1 and x2 both are higher than the utility should be higher, here in this point, both are higher than this point, but we are getting same utility. So that is why it violates monotonicity. Second here, it violates strict convexity of the preference, how? Here when we are taking a linear combination of this point and this point, we get any point in the straight line.

And those points are less preferred than these 2 extremes. So, it is just opposite of extremes are preferred than the average. So, that is why it violates strict convexity. So, these 2 are violated by this set of indifference curves. Now, here, if you look at these are the Ic curves and they gives different levels of utility and the utility is increasing in this direction, we know that.

So, this point, so these indifference curves intersects and we have seen, we have discussed this if they intersect, then they violate what is called transitivity. So, the second type of indifference curves violates transitivity.

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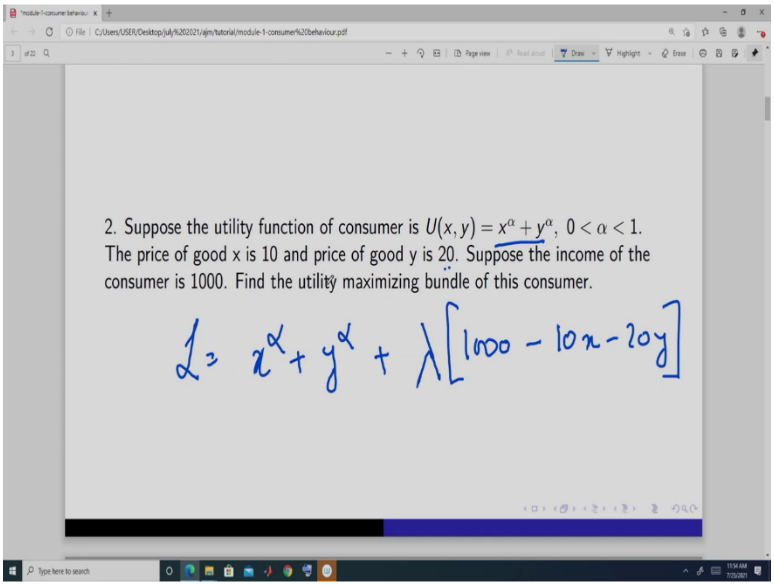

Next, let us look at this problem. This suppose we have a consumer and it consumes 2 good x and y and the utility function is this - $U(x, y) = x^{\alpha} + y^{\alpha}$. Price of good x is 10. Price of good y is 20. And income is 1000. We have to find the utility maximizing bundle of this consumer. So, let us since all of these are differentiable, so let us say the Lagrange and the Lagrange is this- $L = x^{\alpha} + y^{\alpha} + \lambda[1000 - 10x - 20y]$. This is the Lagrange multiplier. And we can write it, this is the budget constraint and this is the utility function. Now we maximize this.

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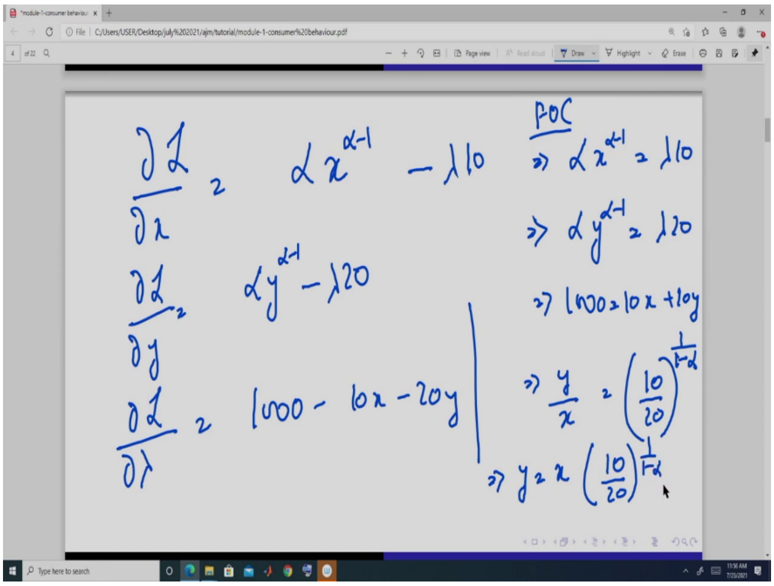

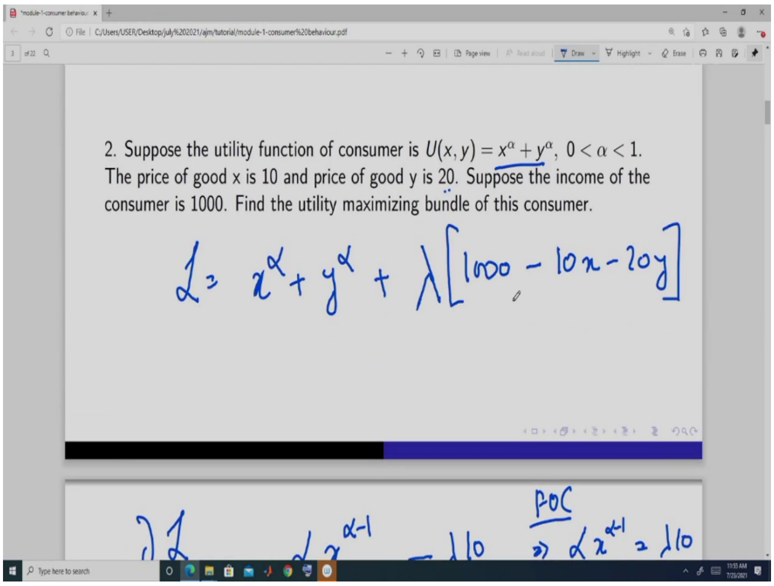

So, what do we do? We take the derivative with respect to x and what do we get? We get this, what do we get? It is 20 and now first order condition, first order condition will imply that this is equal to, i.e $\alpha x^{\alpha-1} = \lambda 10$, this will be equal to - $\alpha y^{\alpha-1} = \lambda 20$and this is equal to- 1000=10x+ 20y , get this. Now, from these two equations we get x, because here this alpha lies between 0 and 1. So, alpha minus 1 it is less than 1. So, I can write it in this way and this portion will be, sorry, this will be y by x actually, okay. So, now, here from this we get to y is equal to x , i.e $y = x\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$this.

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$$1000 = 10x + 20 \cdot x \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$= x\left[10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\frac{1000}{\left[10 + 20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]} = x$$

$$y = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}} \left[\frac{1000}{10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}\right]$$

2. Suppose the utility function of consumer is $U(x, y) = x^{\alpha} + y^{\alpha}$, $0 < \alpha < 1$. The price of good x is 10 and price of good y is 20. Suppose the income of the consumer is 1000. Find the utility maximizing bundle of this consumer.

$$\mathcal{L} = x^{\alpha} + y^{\alpha} + \lambda\left[1000 - 10x - 20y\right]$$

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1} - \lambda 10 \Rightarrow \alpha x^{\alpha-1} = \lambda 10$$

$$\frac{\partial \mathcal{L}}{\partial y} = \alpha y^{\alpha-1} - \lambda 20 \qquad \Rightarrow \alpha y^{\alpha-1} = \lambda 20$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 1000 - 10x - 20y \qquad \Rightarrow 1000 = 10x + 10y$$

$$\Rightarrow \frac{y}{x} = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow y = x\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

Plug in this in this budget constraint and you will get first order condition gives, it will be this x, and the next one, so, x is and this. So, this is the demand for x and demand for y you simply put the value we know this is 10 by this, this into this. You can simplify I am not simplifying this any further. So, this is the demand for y, okay. So, we can find out the utility maximizing bundle is given by these 2, x is equal to this - $\frac{1000}{10+20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}$and y is equal to this.- $\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\frac{1000}{10+20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}$

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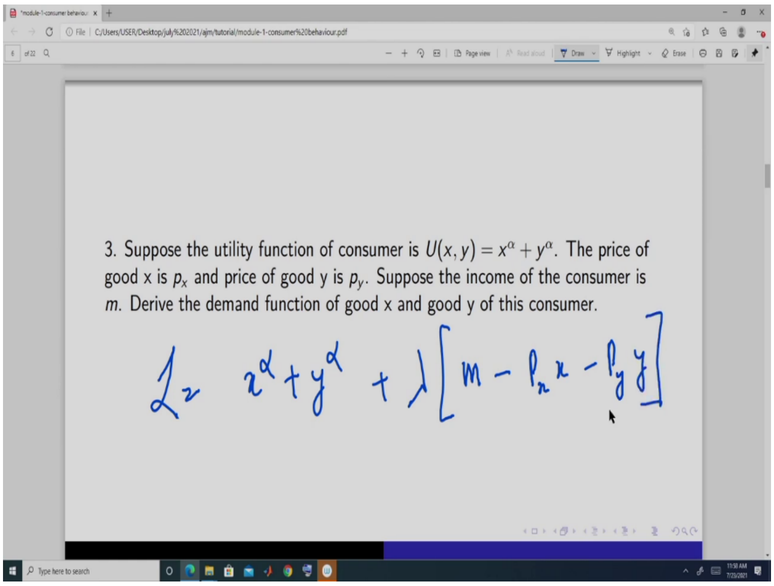

Now, let us take another example here. So, this we have kept the same utility function only we have now take, we have not taken any specific number for this prices and income, so, that we can derive the demand function. So, in this case our, we will follow the same thing Lagrange is going to be same. Here it will be- $L = x^{\alpha} + y^{\alpha} + \lambda[M - P_x x - P_y y]$

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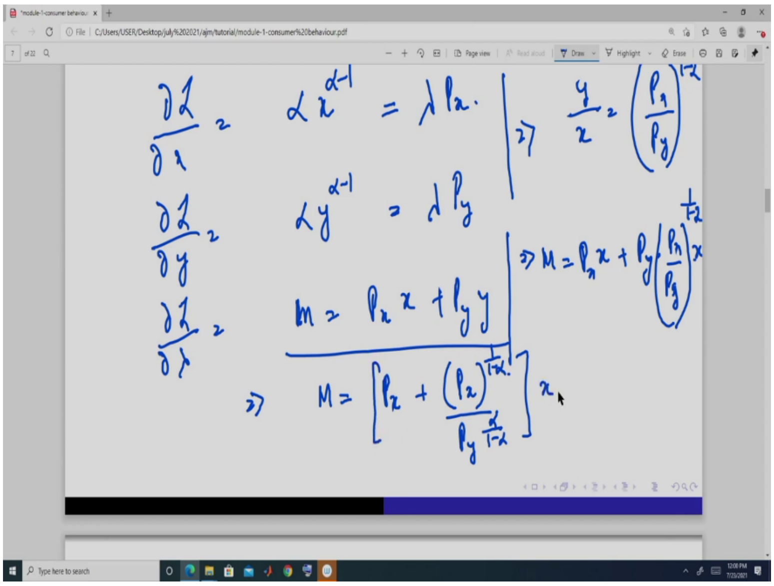

It is going to be this and the first order condition. This will give me- $\frac{\delta L}{\delta x} = \alpha x^{\alpha-1} = \lambda P_x$, $\frac{\delta L}{\delta y} = \alpha y^{\alpha-1} = \lambda P_y$, $\frac{\delta L}{\delta\lambda} = m = P_x x + P_y y$, , from these two I get this- $\frac{y}{x} = \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}}$. Next plug in this here, here, we will get m is equal to px plus py, i.e $M = P_x x + P_y y$.so, from here we will get, into x, i.e $M = P_x x + P_y \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}} . x$. So, this will give me m is equal to Px. So, Px here. So, it will be the Py. So, here Py is to the power 1 by 1 minus alpha and here it is power is 1. So, it will be we can write this alpha and this Py is this x- $M = \left[P_x + \frac{(P_x)^{\frac{1}{1-\alpha}}}{P_y^{\cdot\frac{\alpha}{1-\alpha}}}\right]. x$

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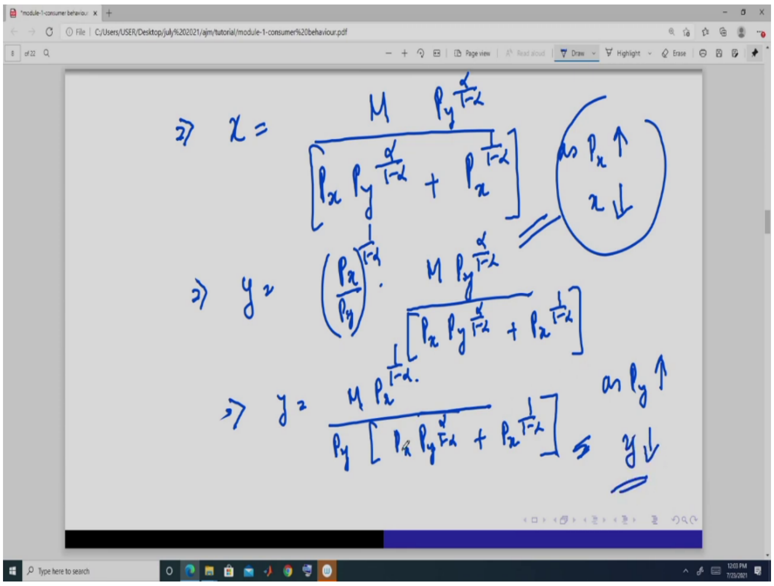

So, the demand function of x is m divided by Px to the power Py plus Py, Px to the power 1 minus alpha and Py,. So, this is the demand function of good x - $x = \frac{M P_y^{\frac{\alpha}{1-\alpha}}}{P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}}}$ and we see as and it is downward sloping in Px. So, as Px increases, x falls because it is at the denominator of this expression that is why. Now, y we know is Px into x, so, x is this, it is this. Now, here from this we can get y, because here the power is 1 by 1 minus alpha and this is Py alpha to the power 1 by alpha.

So, this we can write m Px to the power 1 minus alpha. So, this will be Py. So, it is going to be this one, is the demand function of good y, i.e $y = \frac{M P_x^{\frac{\alpha}{1-\alpha}}}{P_y[P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}}]}$, and you can see that the y's are in the denominator. Here Px are in the denominator, so, that is why it is this. Here again Py is in the denominator. So, that is why as Py increases demand also, it is again a downward sloping demand curve. So, we have got this.

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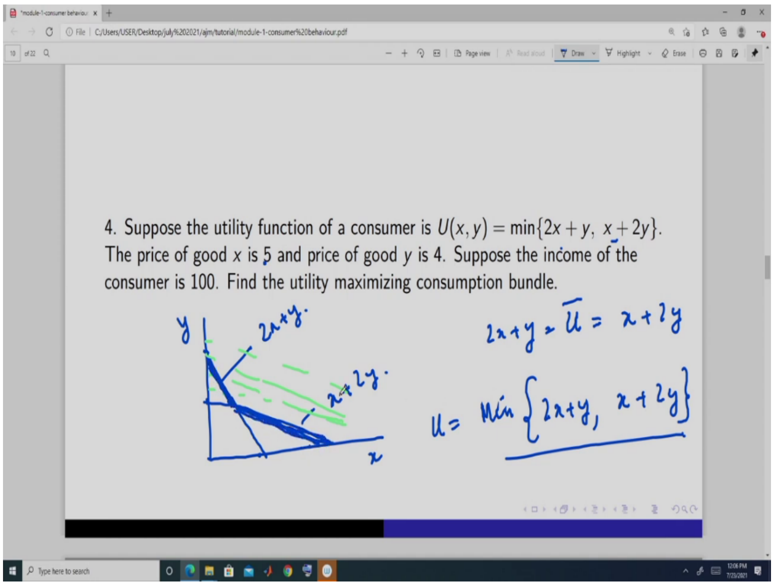

Now, let us do a slightly complicated utility maximizing problem. So, suppose the utility function is given in this form, price of good x is 5, price of good y is 4. If we are given this- $U(x, y) = min\{2x + y,\ x + 2y\}$, now this is mean of this expression. Now, if we take x, here y here, this suppose is equal to some u, and this is equal to some utility level fix. So, this is what? This is going to be this y is going to be u then this is going to be u bar by 2, this is if it is this is u.

So, it is this like this. Here, this is going to be u bar and this is going to be u bar by half. So, this line, this, so, this is 2x plus y and this is x plus 2y and we are given min of 2x plus y, utility function is this. Now, here if you look at these curves, these curves, so, what is happening? Level curves are increasing. So, if we are moving above this, this line, then min is given by this point, this line.

So, it is going to be this, because otherwise it is increasing this side is higher. Now, if we look at these curves like this its level curves are increasing, but this level curves is same as this point. So, that is why it is going to be like this. So, this utility function is can be graphically represented in this form.

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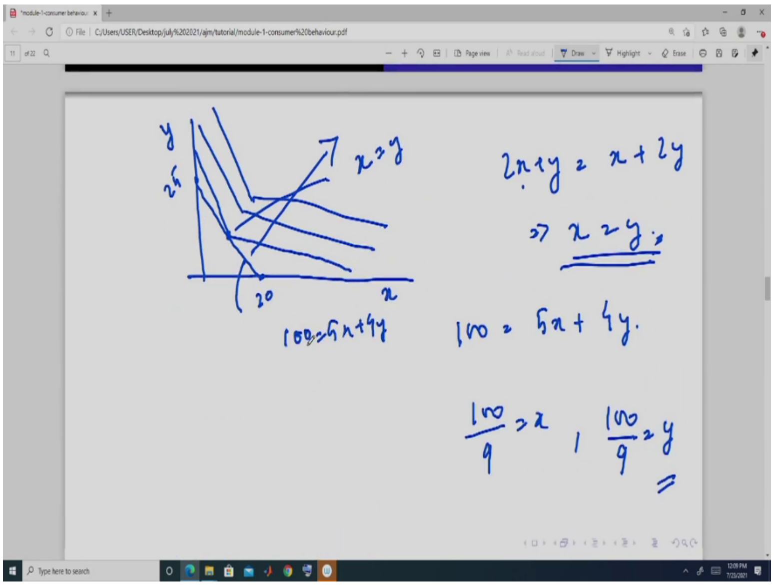

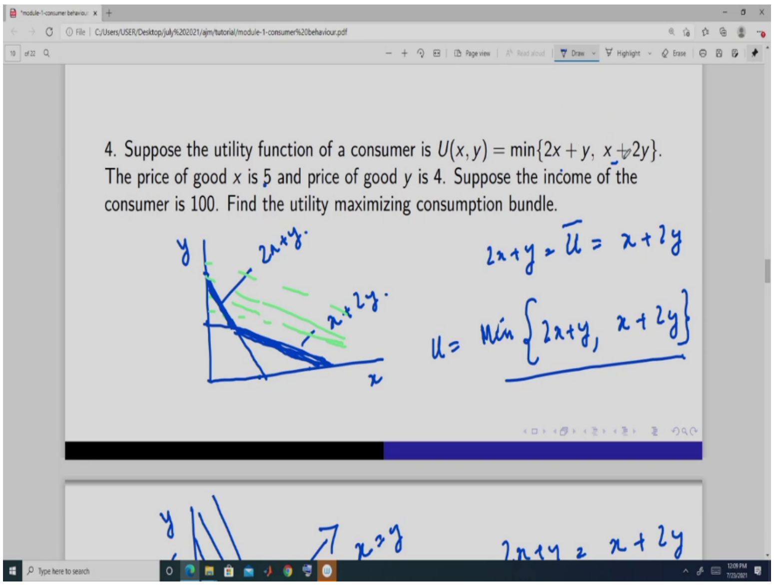

So, it is something like this. If it is x is here, y is here, it is this and it is this, this point is x is equal to y because 2x plus y is equal to x plus 2y and this implies x is equal to y, like this. So, utilities are increasing these are the Ic curves indifference curves. So, good is this 4, budget constraint is 100= 5x plus 4y. So, this point is 20 and this point is 25. So, we get a, if this is 20, then this is 25 this is, so, we will get a curve like this.

This is here, this is the and here, because if this budget line this slope is same as the slope of this then it would have match this, but this slope is less. If this slope is same as this then it would have match this, but this slope is higher than this slope. So, this point is only going to intersect here, not intersect it is going to be tangent to this point. But if this matches to any one of this then we will get different here, but here it is not matching.

So, in this case the optimal point is always going to be x is equal to y and here what we are going to get 10 by this and this is going to be the, this is 9, this is 9, okay. But here the main problem is or the difficult part is this how to draw the indifference curves, okay.

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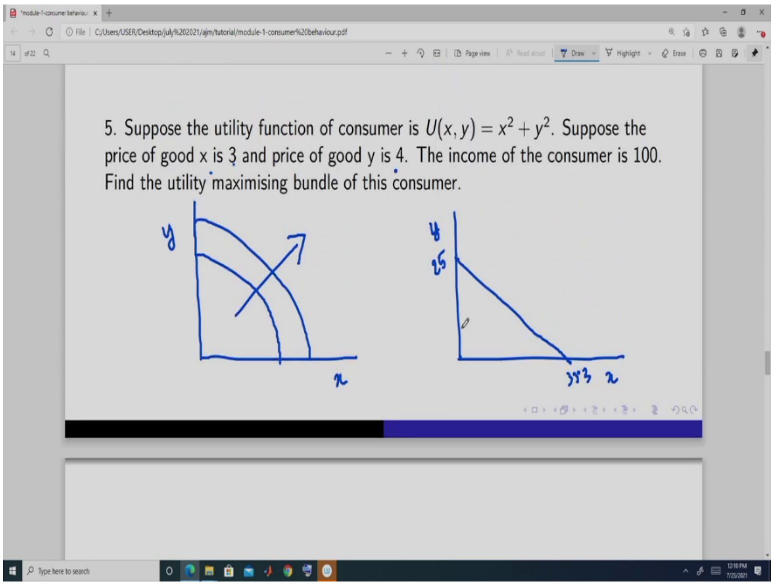

Once you can draw the indifference curve you will be able to find the a. Now, if we are given a utility function of this nature- $U(x, y) = x^2 + y^2$, x is here. If we are given this it is going to be somewhere here, utilities are increasing in this way. Price of good x is 3, price of good x is 4, income is 100. So, our budget it is 4, so, it is 25 and here it is 3, so, it is going to be 33 point something 33, this. Now, how to find the optimal point we see.

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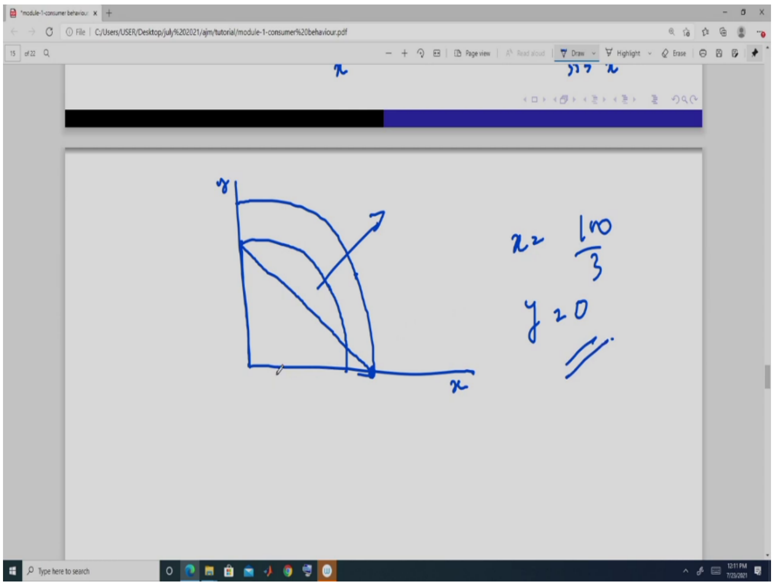

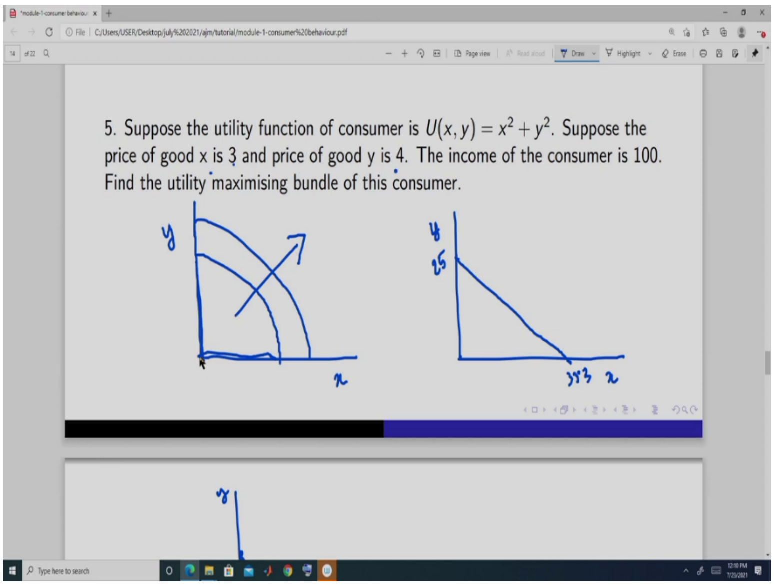

Here, so, the intercept this is less than the intercept here of the budget line, okay. And these indifference curves are such that these intercepts are seen. So, if I draw a, it will be somewhere here and if we draw here it will be here. So, this is at a higher level than this, since utility increases in this way, so, the optimal point is this. So, x is equal to 100 by 3 and y equal to 0, this is the optimal bundle in this case.

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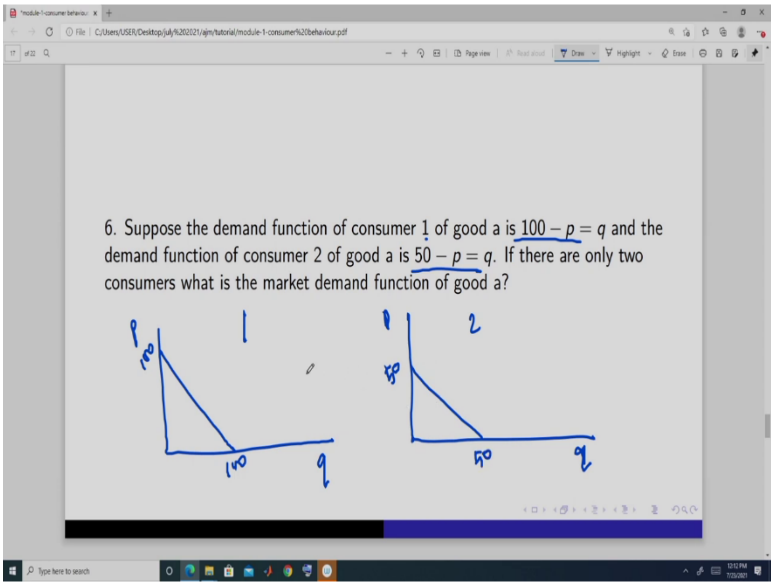

Now, suppose we are given 2 demand curves. This is for consumer – 100-p=q and this is for consumer 2- 50-p= q. Consumer one’s demand curve is you can say if you plot q here, p here it is 100, 100. Consumers 2, this is 1, consumer two’s is q here, it is 50 and 50, right. Now, what is going to be the market demand curve is there is only these 2 consumers?

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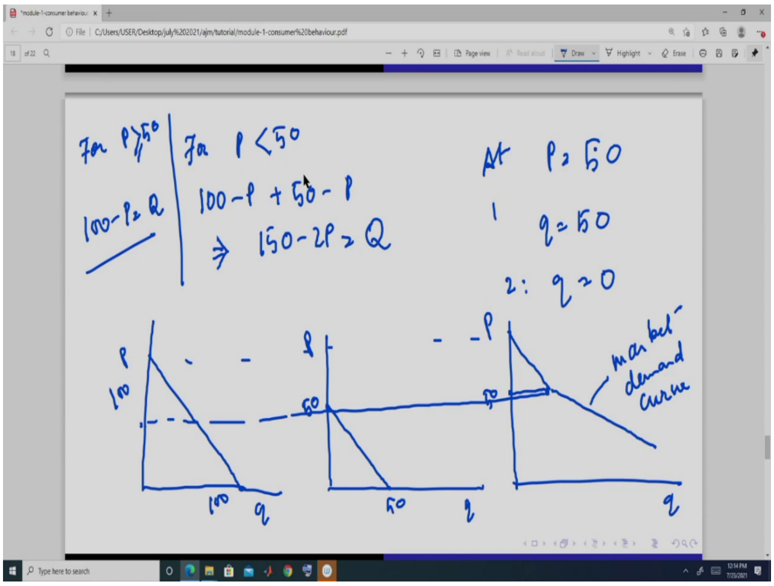

So, at P is equal to 50, what is the demand for consumer 1? Consumer one’s demand is 50 and what is the demand for consumer 2? It is 0. So, we are going to do simply a horizontal summation of the demand curves, 50. So, it is going to be 50. So, it is till this point it is this. Okay, I should have drawn this slightly, okay, like this and after this I am going to add this too. So, it is going to be so, for P less than 50 it is going to be 100 this plus 50, this is which is equal to 150.

This is going to be the total demand and if this we will get something like this. So, this is going to be the market demand curve. So, if we write it in a compact way, it is this. So, for this and for P greater than or equal to 50 it is, this is the market- 100-P= Q and for P less than this we are going to get this- 150-2P=Q, okay, this is the market demand.

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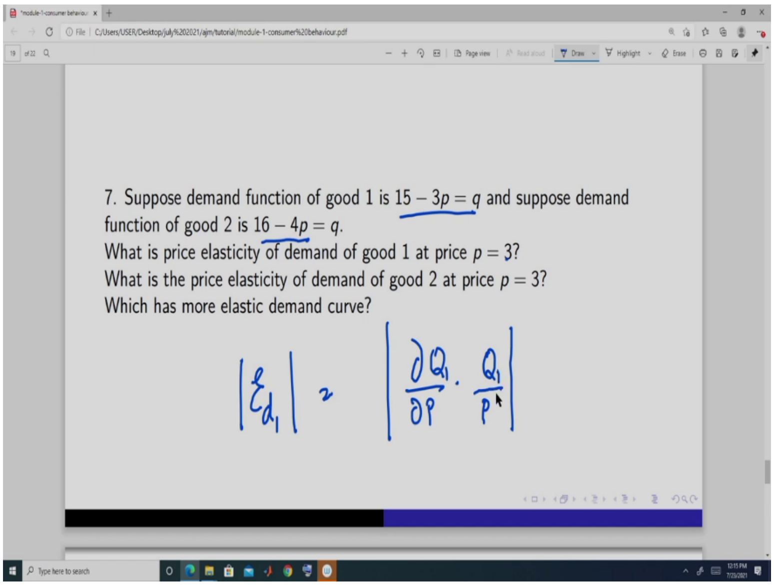

Now, suppose we are given a demand function this- 15-3p=q, this is of good 1. And suppose the demand function of good 2 is this- 16-4p=q. We have to find the price elasticity of demand of good 1 and good 2 and price and price history. So, we know the elasticity of demand is-$|E_{d_1}| = |\frac{\delta Q}{\delta P} \cdot \frac{Q}{P}|$, get this, right. So, this if we simply take derivative of this, the demand function 1.

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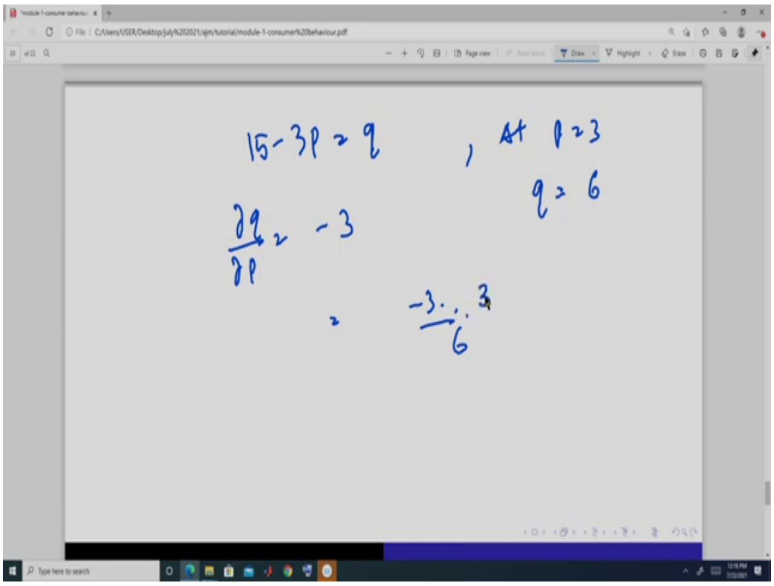
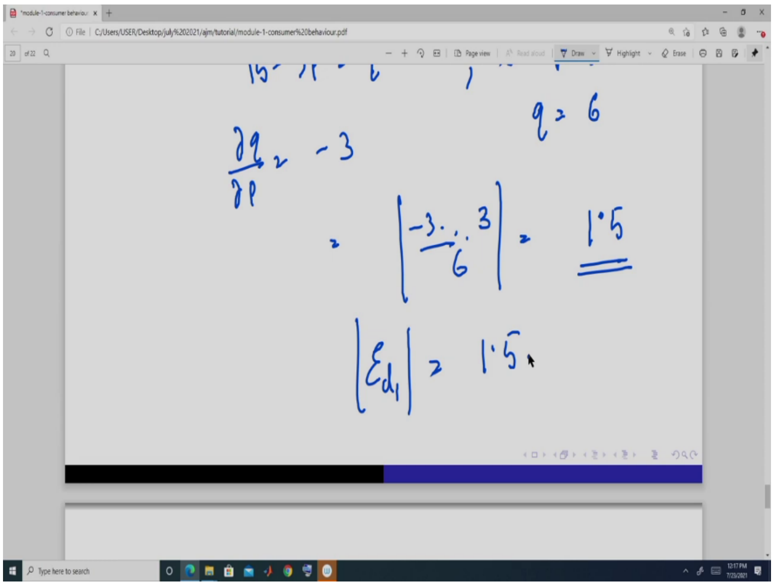
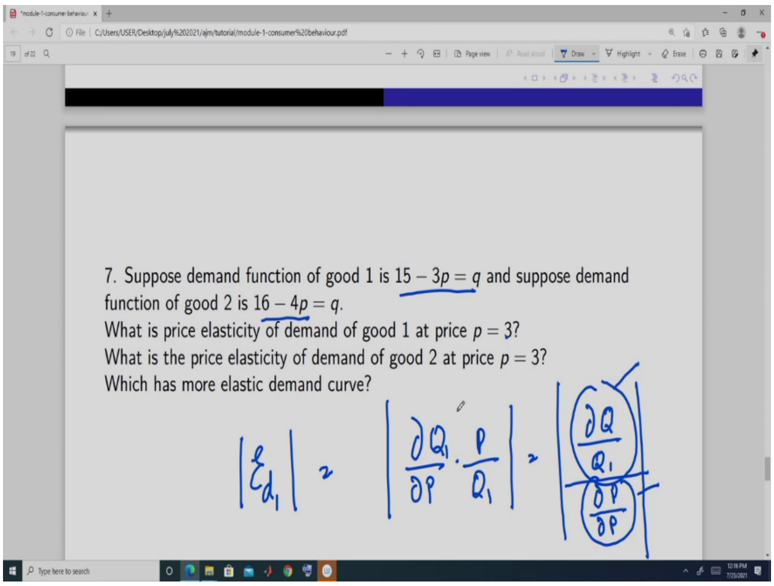

$15 - 3p = q$, At $p = 3$
$q = 6$
$\frac{\partial q}{\partial p} = -3$
$= -3 \cdot \frac{3}{6}$


$q = 6$
$\frac{\partial q}{\partial p} = -3$
$= \left| -3 \cdot \frac{3}{6} \right| = 1.5$
$|\varepsilon_{d_1}| = 1.5$


7. Suppose demand function of good 1 is $15 - 3p = q$ and suppose demand function of good 2 is $16 - 4p = q$.
What is price elasticity of demand of good 1 at price $p = 3$?
What is the price elasticity of demand of good 2 at price $p = 3$?
Which has more elastic demand curve?
$|\varepsilon_{d_1}| = \left| \frac{\partial Q_1}{\partial P} \cdot \frac{P}{Q_1} \right| = \left| \frac{\partial Q / Q_1}{\partial P / \partial P} \right|$

So, what do we get, demand function is this, right and we have to find that P is equal to 3. So, at P is equal to 3, q is, is equal to 6 and derivative of this q is equal to minus 3. So, plug in minus 3 at 3, 6. So, here it will be 3 and this will be, sorry, I have made a mistake in this is P divided by Q, because it is, this is percentage change in quantity and this is the percentage change in price, okay.

So, that is why we get this. So, this is equal to and when we take the modulus it will be equal to 3. So, one minute, it will be 3 by 2, so, it is 1.5. This is the elasticity of good 1 is equal to 1.5.

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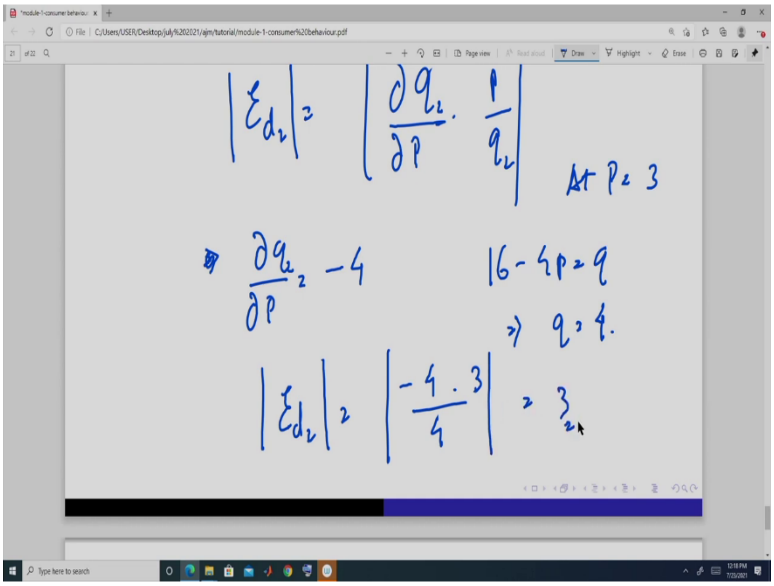

Similarly, elasticity of good 2 at this is, this- $|E_{d_2}| = |\frac{\delta Q_2}{\delta P}.\frac{P}{Q_2}|$, at price is equal to 3, demand function is, so, it is q is equal to 4. So, and the slope is minus 4. So, this is.

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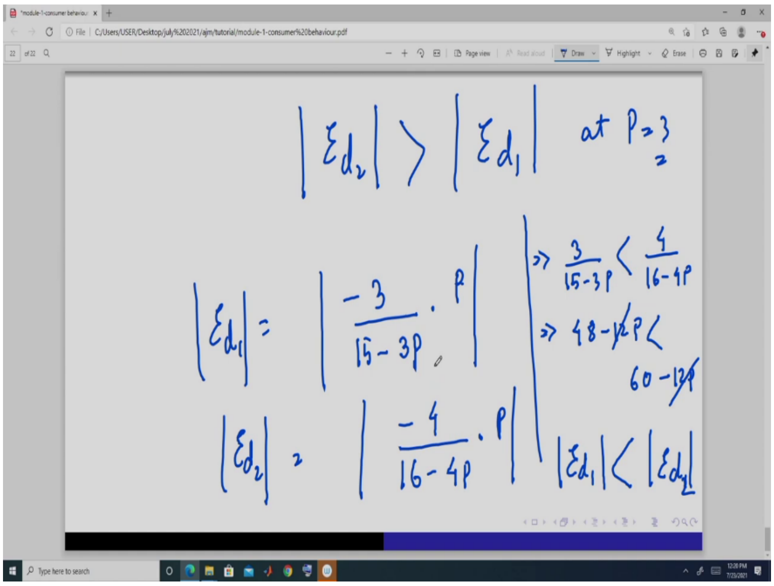

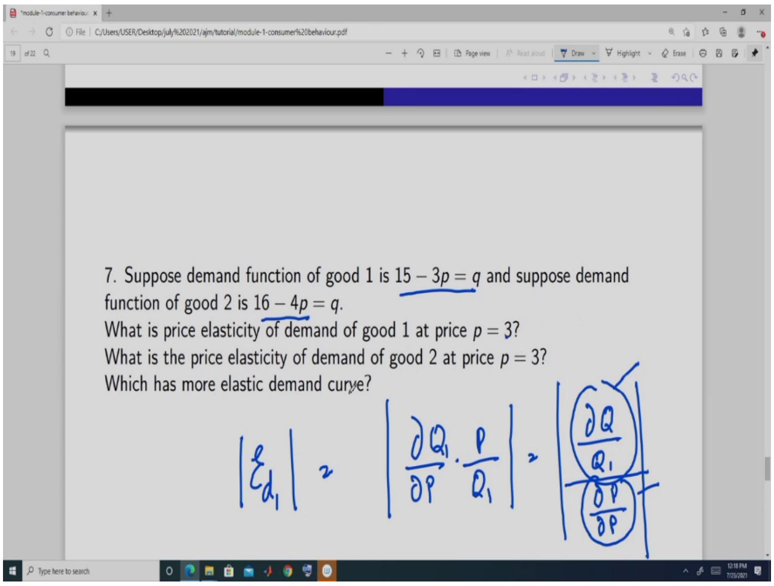

So, what do we get? We get elasticity of good 2 is greater than elasticity of good 1 and price is equal to 3. Now, can you say about the whole demand curve which one is more elastic? So, we have find out the elasticity. So, for this it is 1.5 for this it is 3 so, it is greater and which is more elastic. Now, these we can simply use the formula. So, slope of this is so, we can write this for a general price is this, which is this- $|E_{d_1}| = |\frac{-3}{15-3p}.P|$, it is this- $|E_{d_2}| = |\frac{-4}{16-4p}.P|$.

Now, we compare this, if we compare this what do we get? So, this removed the, so, it is 3 by, suppose, it is of this nature, okay, this, so, we get this is 48 minus 12P, 60 minus 12P this cancels and 60 is greater than 40, so, we get this is greater than this. So, we get that for each price elasticity of good 1 is less than the demand elasticity of good 2, we get this. So, these are few problems. So, you will get similar kind of problems in the assessment and question in the exam, **okay.** Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Science
Indian Institute of Technology, Guwahati
Module two: Production and Cost Curves
Lecture Production Function

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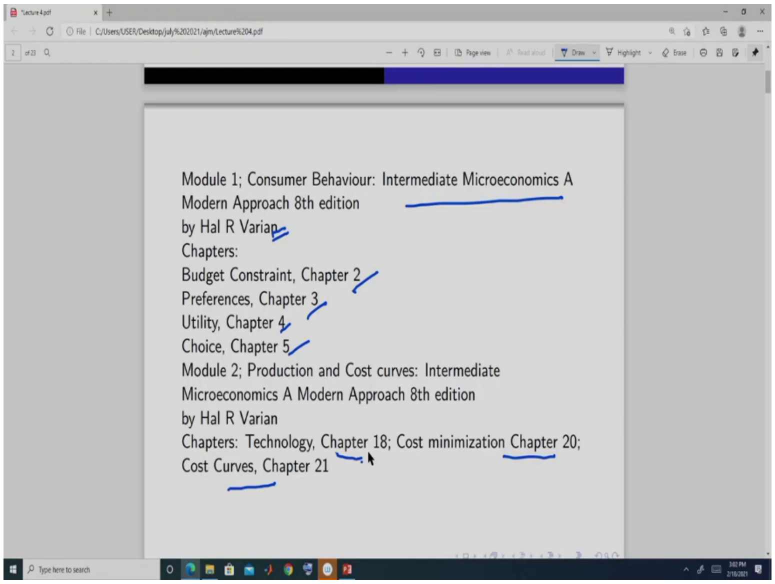

Welcome to the course Introduction to Market Structures and today we are going to start module two, that is cost and production cost function in that. So, before that, let us first give you, I will give you some references for consumer behavior that is module one, you follow this book if you want to follow a book that is Intermediate Microeconomics A Modern Approach by Hal Varian eighth edition and the specific chapters that you can go through from this book is like chapter two, chapter three, chapter four and chapter five from this book.

Or you can alternatively the power point slides that you will get it is sufficient for this portion. And today, we are going to start module two that is production and cost curve. So, for this also, you can follow the same book intermediate microeconomics a modern approach, eighth edition by Hal R, Varian and the chapters are, chapters are like technology, it is chapter *eighteen*, cost minimization chapter *twenty* and cost curves chapter *twenty one*. So, these three chapters, so, today we will do chapter *eighteen* that is technology.

So, when we say production, what do we have in mind? So, it is like this, suppose, think of a farmer who owns a plot of land and in that plot of land he uses his own labor along with some machine to cultivate this land and he produces a food grain or some vegetables or some other crops. So, this whole process is a production, like you plow the land using your labor and some machine you sow seeds and then you harvest once these plants are grown up and from that you get if it is food grain, then you get food grains, if it is vegetables you get the vegetables like that.

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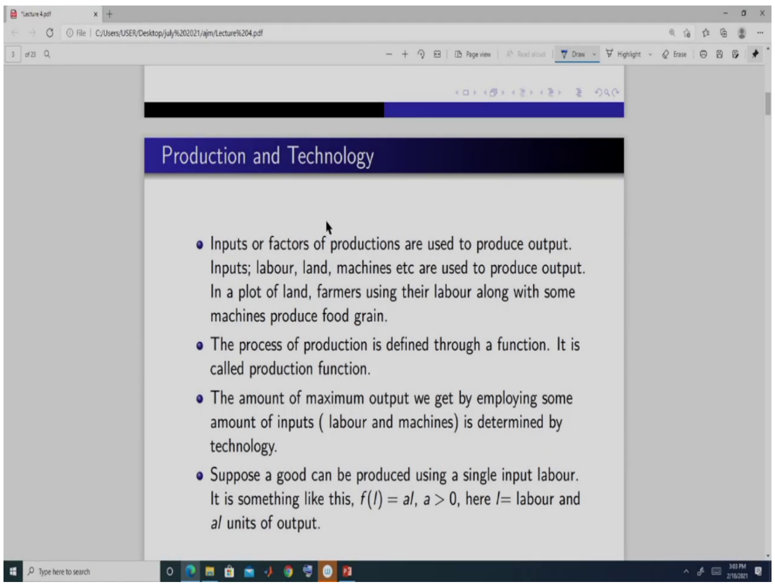

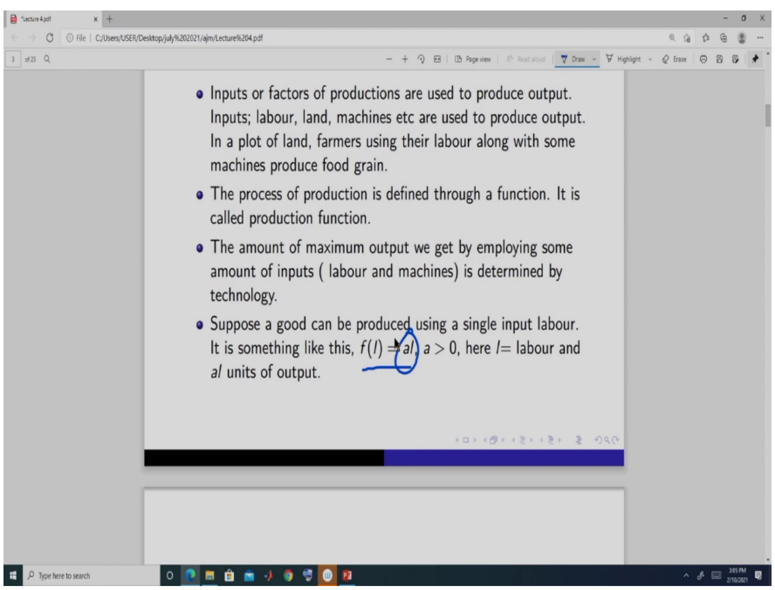

So, this whole process has some inputs, these are factors of production. So, in this example inputs are like land, labor, the machines like the tractor or the harvester or etc or Bullock and output is the food grain the crops those are outputs or alternatively you can think of suppose you are producing automobile. So, here the inputs are like iron and steel, some machines which you along with the workers, you use these machines to give different molds, different forms to the structure of the automobile.

And again using some steel and machine you use the machines and then you together assemble them and then you get a automobile. So, this whole process is determined or whole process is not determined it is represented as a function. It is like you provide input and you get some output and here output is our the goods that we want and inputs in the most general case are like labor, land, capital means machines, this three and the raw materials like the intermediate was like to produce machines, we need iron, iron and steel.

So, that is a raw material, we convert this into machines, right. So, those are raw materials. So, what do we do, we represent it as a function. So, for simplest case suppose we take the output which can be produced by a single input and that is suppose labor and then we can write it in as this way- $f(l)=al, a>0$. So, here a into l, l is denoting the labor, okay.

So, a is some positive number. So, you plug in labor and the amount of output you are going to get is this much, okay. And when you plug in some specific amount of input how much amount of maximum possible output that we are going to get that is determined by what is called a technology, okay.

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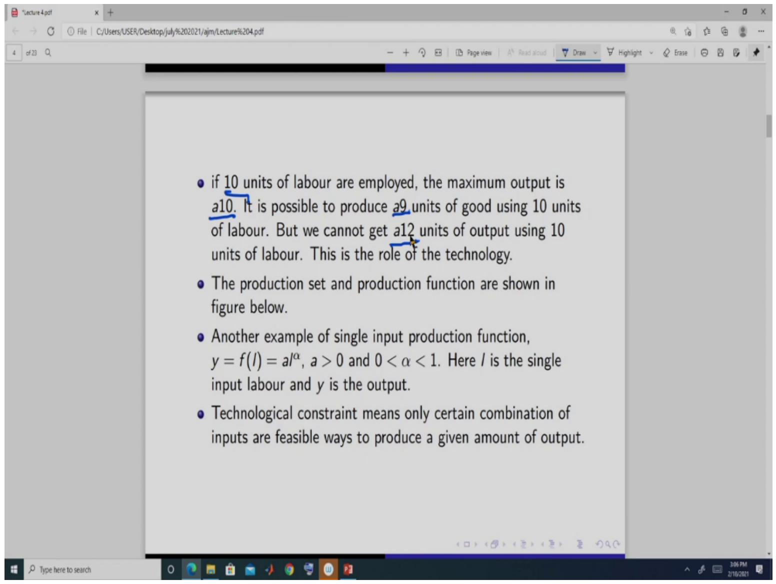

So, it is something like this that suppose we employ 10 units of labor. And if we employ, employ 10 units of labor then we will get using that above production function our output is going to be a into 10. So, now, it is possible to produce a into 9 that much units of output because it means we are not whatever maximum it is possible is a into 10. But still we can produce this a into 9.

But we will never be able to produce a into 12 units of output by employing 10 units of labor. So, this is the role of the technology. So, it defines or the determines the maximum possible output given input.

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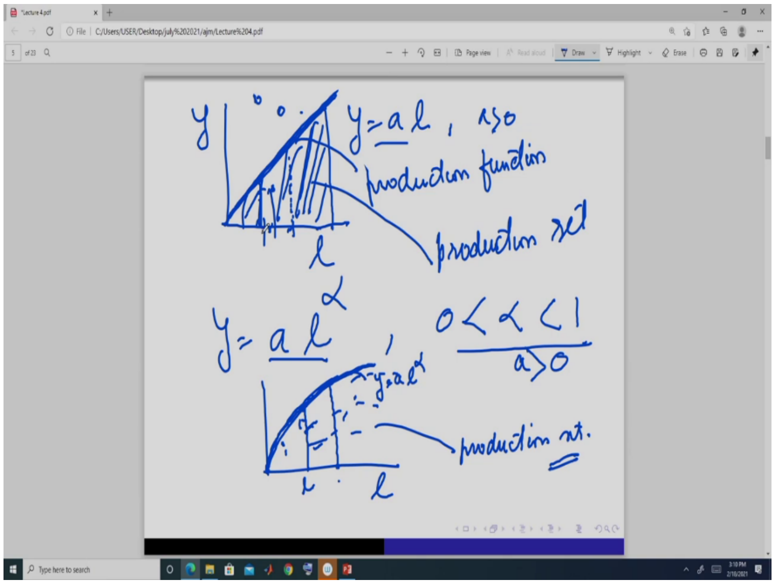

So, like here if we take in this a labor suppose, and here we take output and our production function is a into l where a is some positive number, then we get a line like this. So, we plug in this much amount of labor the output, maximum output is this much. But we can produce each of these units like if we employ this much amount of labor, the maximum output is this much. But we can produce any one of these possible levels of output.

But we will not be able to produce this much amount of output if we employ this, to produce this much amount of output we need to employ this much amount of labor okay. So, this is the production actually function okay. And this whole set is called the production, oh sorry, this is called the production function and this whole set this is called a production. So, anything here this point it is feasible because, if we employ this much amount of labor we can produce this.

But this is not feasible by employing this much amount of a. Another example of production function you can take like here-$y = al^{\alpha}$, where alpha takes a value between 0 and 1 this. So, if we plot labor here and alpha is some lies between a number between 0 and 1 and a is a positive number. So, we get a curve like this. So, this is like this, here if we employ this much amount of labor the maximum output possible is this much, if we employ this much amount of labor, maximum possible output is this much.

So, this curve is the production function and this whole set this is the production set in this case that is all these points are feasible okay. I hope it is clear. So, what we are doing we are plugging in input that is factor of production and we are getting output and we define this thing through a function and that is production function. So, we get when we are using only one way input, so, we are getting our production functions, these are some examples of production. Now, let us complicate this.

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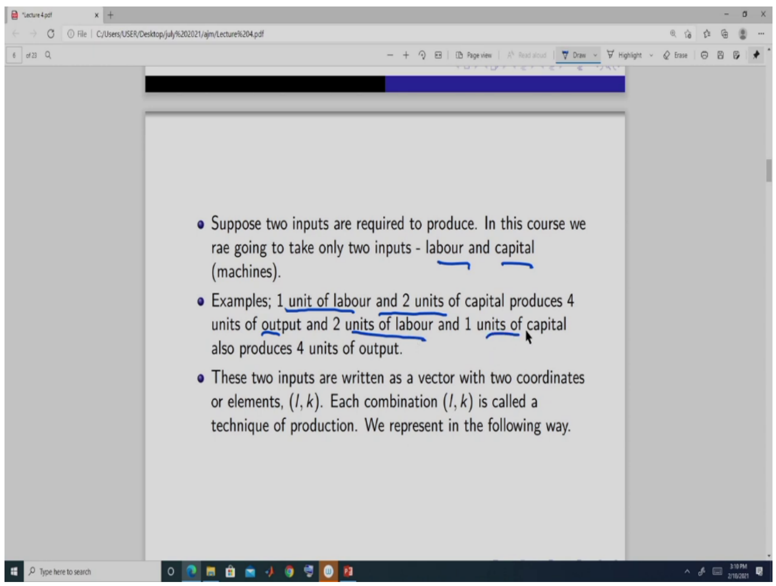

Suppose we have two inputs and these two inputs are labor and capital, capital means some machines. So, labor along with some machines produces output okay. For example, suppose we can do and now here since we have two inputs. So, now, we can combine them in different ways. For example, 1 unit of labor and 2 units of labor can produce 4 units of output okay.

So, this is one technology. So, the technology is this now, what we can do? We can also produce four units of output using 2 units of labor and 1 unit of capital. So, what we are getting? We are getting different combinations of these two inputs may give us same output.

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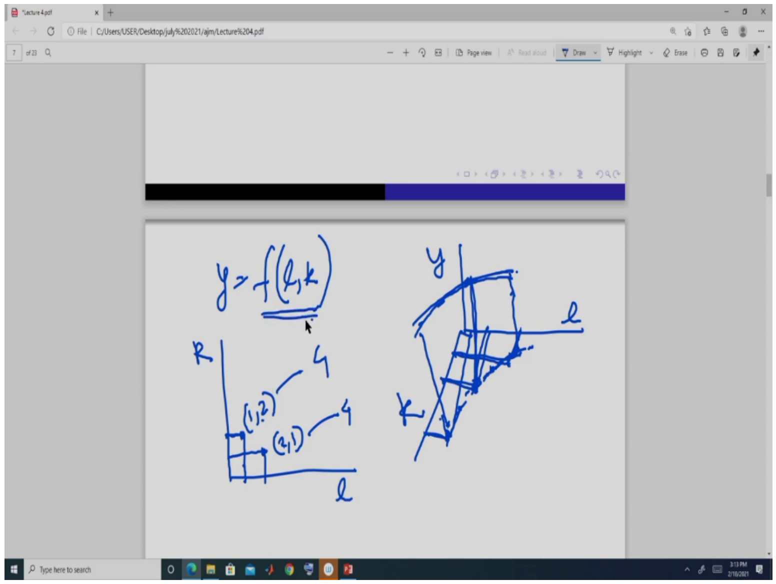

So, our production function is now actually something like this-y=f(l, k), okay. And what we do in this axis we plot labor in this axis we plot capital and suppose we are 1 unit of labor and 1 unit of labor and 2 units of capital give me 4 units. So, this point gives me, so this (1, 2)gives me 4 units of output. Same this point which is (2, 1) also gives me 4 units of output. So, these two if we take if we use the height of this a height to represent the output, then this two point should be at the same level, same height, okay.

So, this is our now technology. So, technology here means that this is giving me 4 units. This is giving me again 4 units. So, and this is a specific technique of production and this is also a specific technique of production okay. Now, here what we can do, if we plot labor here and capital here and output here. So, then suppose this point, this much amount of capital, this much amount of labor and height is giving me the amount of output.

So, if we have a combination of points like this and all of them are at suppose same height like this, then we are getting same output from these different combinations. So, each of this is a separate technique of production okay. So, here we are employing this much amount of capital and this much unit of labor. Here we are using this much unit of capital and this much unit of labor. Here we are using this much unit of capital and this unit of labor. But each of this technique is giving me same output. So, here we have more capital less labor, here we have less capital more labor like this. So, same thing like this.

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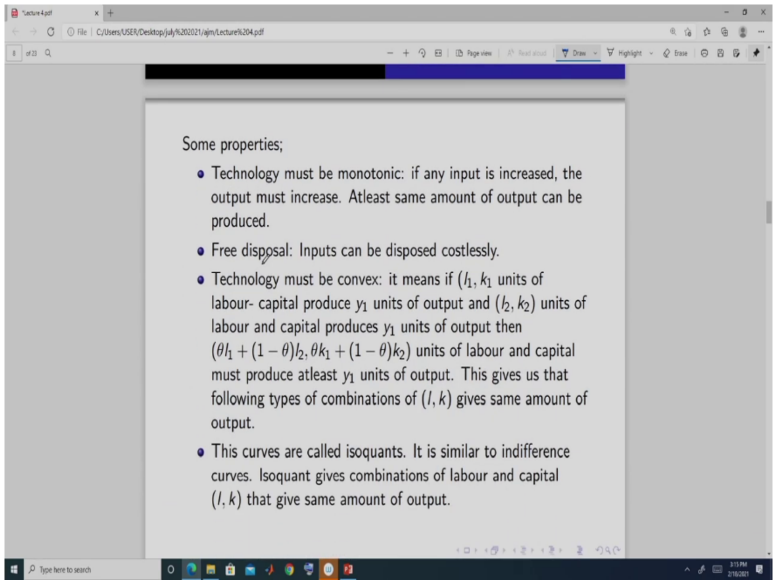

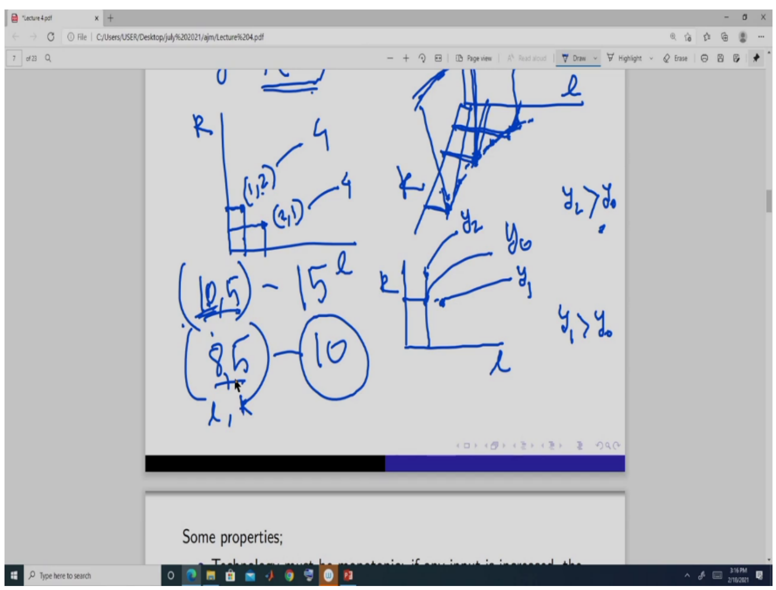

Now, in this production function, we impose certain conditions and these conditions are first that the technology must be monotonic, what it means that if we increase any input then the output must increase. So, it is something like this, if we take like this labor and capital and if we are at this point suppose, and at this point gives me suppose, y naught unit of output. If we combine this much amount of capital and this much amount of labor, then if I increase capital, so, this must give me some y 1 unit of output.

And y 1 must be greater than y naught, or if I move to any point like this, where we have kept the amount of labor same and increase the amount of capital and suppose, this point is y 2, then again y 2 must be greater than y naught because I am employing more of capital here than at this point. Here, I am employing more of labor than capital. So, again this output should be more than this okay. So, technology must be monotonic.

And second that we should have something called a free disposal. Free disposal means, that the input can be disposed costlessly. So, it means suppose, I have suppose 10 units of labor and suppose 5 units of capital. And I can combine suppose 5 units of capital with 8 units of labor to produce 10 units of output, okay.

And 10 units and 10 units suppose give always be more, it gives me suppose 15 units of output okay. Now, suppose I want to produce only 10 units and I already have 5 units of labor, then I will not cost anything extra, if I want to dispose 2 units of labor okay. So, I will not. So, I can easily switch from this combination to this combination. And I will not bear any extra cost for that okay. So, this is what free disposal means. And the second, the technology must be convex. What does it mean?

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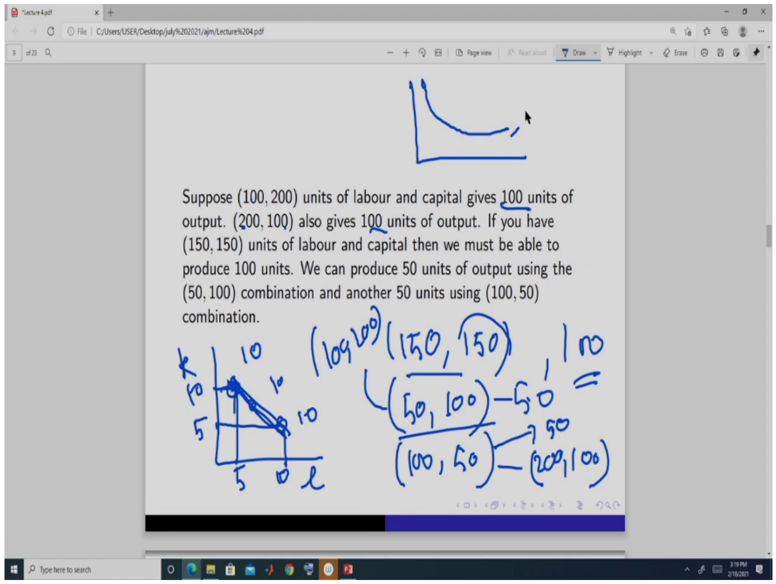

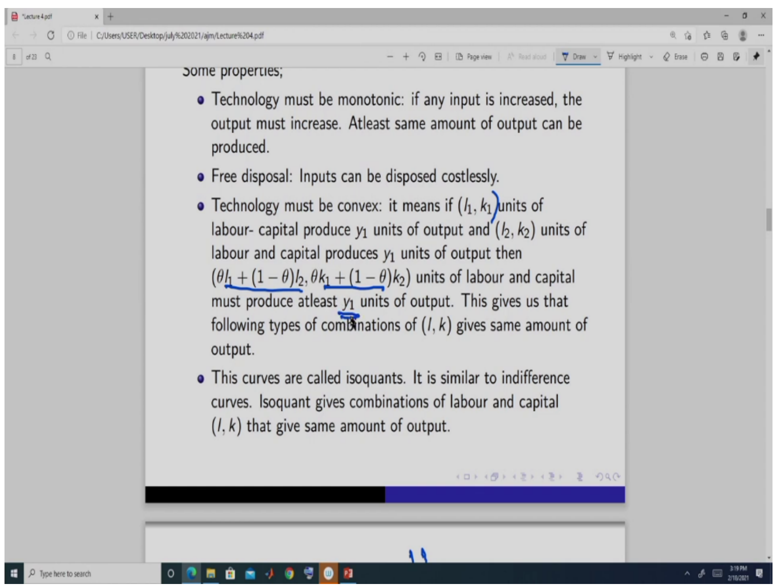

Convex means that if suppose, you take labor here capital here and suppose this point, this is suppose 5 units and 10 units. And suppose this is 5 and 10. And both of them give me suppose 10 units and 10 ten units. Then, if I take a combination of these points, then I should be able to produce at least 10. So, this is something like this suppose, I am given with 100 units of labor and 200 units of capital. I can produce 100 units of output.

And same with 200 units of labor and 100 units of capital, I can produce 100 units of output okay. Now, if I want to use 150 and 150 units of labor and units of capital, I should be able to produce 100 units of output, how? By taking 50 and 100 units of labor sorry, 50 units of labor and 100 units of capital, and 100 units of labor, 50 units of capital. So, this will allow me to use the first technique that is (100, 20).

So, this will allow me to produce 50 and this here, I am using 200 and 100. So, this will allow me to produce (100, 50) again. So, (50 50) we can produce 100 units, right, this is, is it clear? I think this, this is a simple example. So, graphically it means that if we take this point, this is one technique and this another technique.

And if we take a linear combination of these two techniques suppose, any point then I should be able to produce whatever I am being able to produce using this technique or this technique okay. So, this assumption, if we assume same we will give us level curves of this form. And this is something called isoquant, we will come to it now.

So, formally it is like this, that if we use l1 and k 1 units of labor to produce suppose y 1 units of output. And again we can use l 2 and k 2 units of labor and capital to produce y 1 units of output. Then any linear combination, that is theta l1 plus 1 minus theta l2 this linear combination of labor- $\Theta l_1 + (1 - \Theta) l_2, \Theta k_1 + (1 - \Theta) k_2$). And again linear combination of capital, that is theta k 1 plus 1 minus theta k 2 will give us at least y1 units of output, okay. So, this is what is means that the technology is convex.

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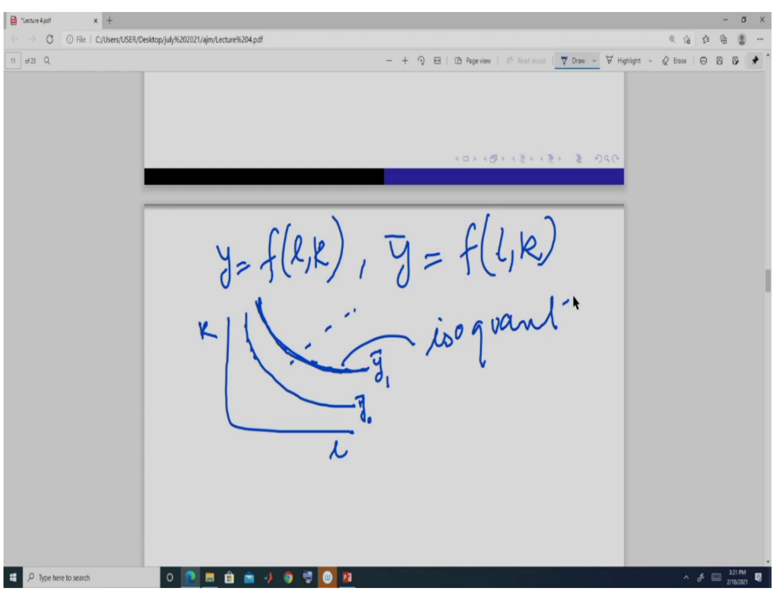

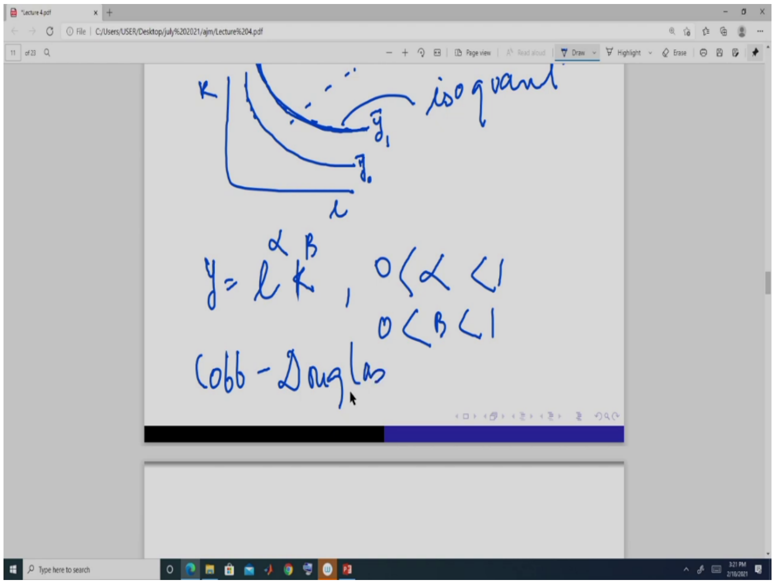

And what we can do since we have this production function like this- y=f(l,k)now if we fix the level of output k bar and we find out all the combination of l and k, which is giving me this here. Suppose, if we assume that the technology is convex we get a combination like this. So, each point here gives me y bar units of, if I increase this level curve, suppose this is y bar naught and this is y bar 1, here output is more and all these combinations of capital and labor give me y bar 1 units of output okay.

And as we move in this northeast direction, output is increasing. This is straight from monotonicity and convexity gives us this okay. Now, so, these are called isoquant, something similar to indifference curve. And we will require them while solving a optimization problem that the firm solve, okay.

Now, let us do some example. Suppose our production function is this- $y = l^{\alpha} k^{\beta}$. So, l to the power alpha, k to the power beta, where alpha takes a value (0, 1) beta takes a value (0 and 1), okay. This is a very famous production function and it is called a Cobb Douglas production function, one variation we have done in Cobb Douglas utility function okay.

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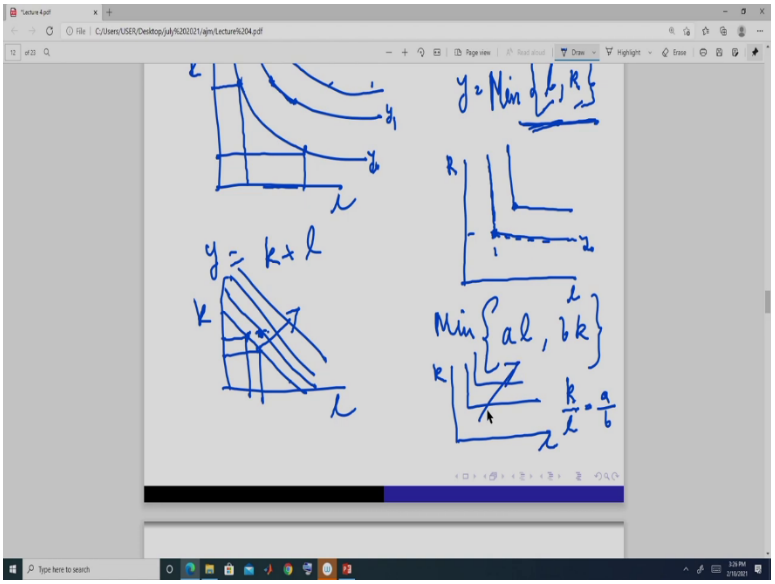

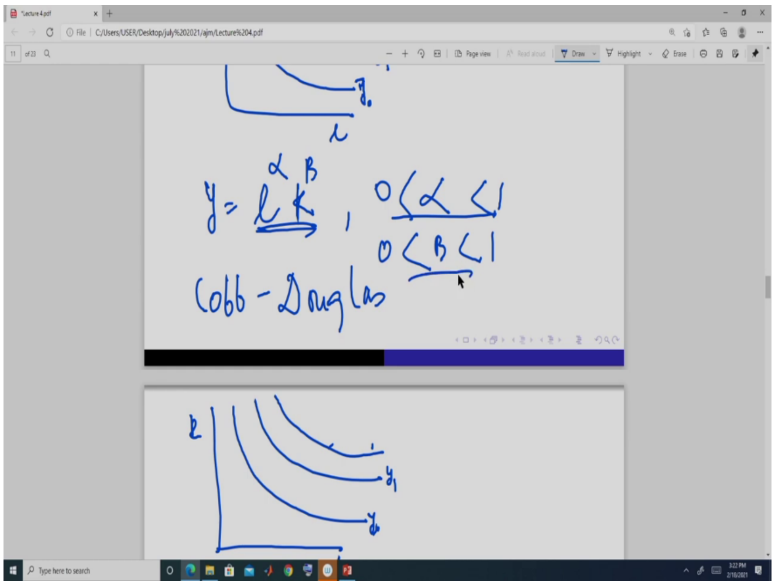

Now this function isoquant of these functions will be something like this. And here this is output is y naught here y, y 1 so output is increasing in this, okay, we will get curves. So, these are level curves or isoquant when our production function is this. And this alpha and beta takes the value in this range. So, here what do we see that we can substitute 1 capital and labor at some degree. So, this we are using this much capital and this much labor.

Here we are using this much capital and this much labor. So, if we move from this point to this point, what we are doing? We are substituting, we are reducing the use of capital and increasing the amount of labor. Same here, if we move from this to this, what we are doing? We are increasing the use of capital and decreasing the use of labor okay. So, there is some form of substitutability allowed in this production function.

So, this technology is allows us to substitute a bit of capital and labor. But suppose, our production function is this y=min{l,k} this. Now, if we plot 1 here and k here, if we look at this a, it will this, it will be suppose for. So, this production function are generally called a fixed proportion. So, we produce output using a fixed proportion at this. So, even if we keep on increasing the amount of capital, our output is not going to increase given a fixed amount of labor.

So, you can see that here this is violating monotonicity and also this is not a differentiable function okay. Here, if we fix this amount of capital and if we keep on increasing more labor, output is not going to increase it is going to remain fixed. So, that is why to produce a level of output we require labor and capital in fixed proportion and that is here (l k), okay. So, this would be equal proportion.

So, here this does not allow us to do any substitute. So, factors are not substitutable in this production function, we require both the inputs in fixed proportion. Here, it should be always equal. But instead suppose, our production function **is** suppose a l and suppose b k. Then here also it will be again, something like this. But, they will be required in the ratio suppose if we look at k sorry, we generally take k by l. If we look at this k by l ratio here, it should be always in the **a** by b ratios.

So, in this production function the inputs capital and labor should always be used in this ratio to get output okay. Now, suppose let us take another production function and this is suppose simply k plus l- y=k+l. So, here if this is labor and this is capital, it is going to be a straight line. So, these two inputs are perfectly substitutable. So, if I move from here to here, I produce the same level of output. So, to produce but if I increase the keeping same a I will increase the output.

So, I will be at a higher isoquant okay. So, isoquants here are increasing in this, production are increasing in this, direction, here production is increasing in this direction okay. So, these are some examples of production function okay. Now, here to keep our life simple, we generally assume that the production function is differentiable okay.

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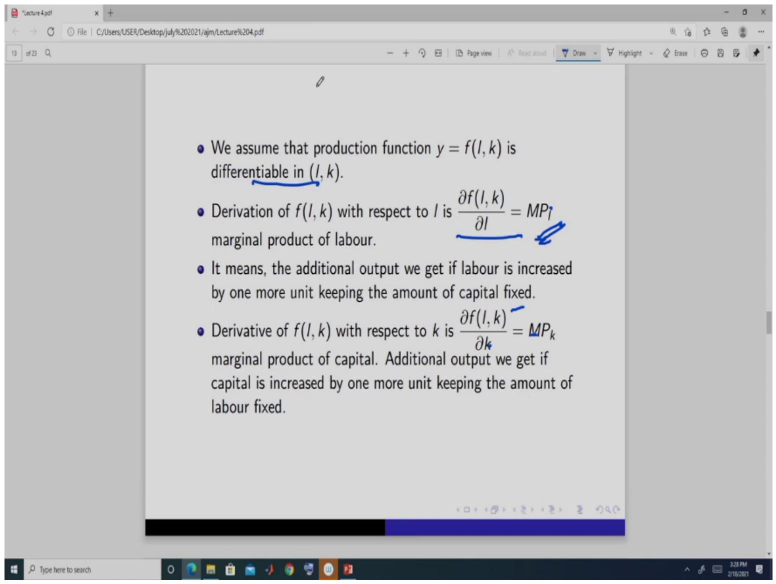

If they are differentiable in both labor and capital then these partial derivatives are possible. And this partial derivative that is our total output with respect to labor is called marginal product of labor, that is if we keep the amount of capital fixed and if we increase labor, how much amount of output, additional output we are going to get it is given by this. So, it is something like this. So, at the margin, if we increase one more unit of labor, how much additional output do we get, if we keep the level of capital fixed.

Here this- $\frac{\delta f(l,k)}{\delta k} = MP_k$, the partial of the production function with respect to capital is the marginal product of capital. And we write it MPk. So, it means that if we fix labor and if we increase capital then how much amount of additional output we are going to get? So, at the margin if we increase capital by 1 more unit, by how much unit the output is going to increase? okay.

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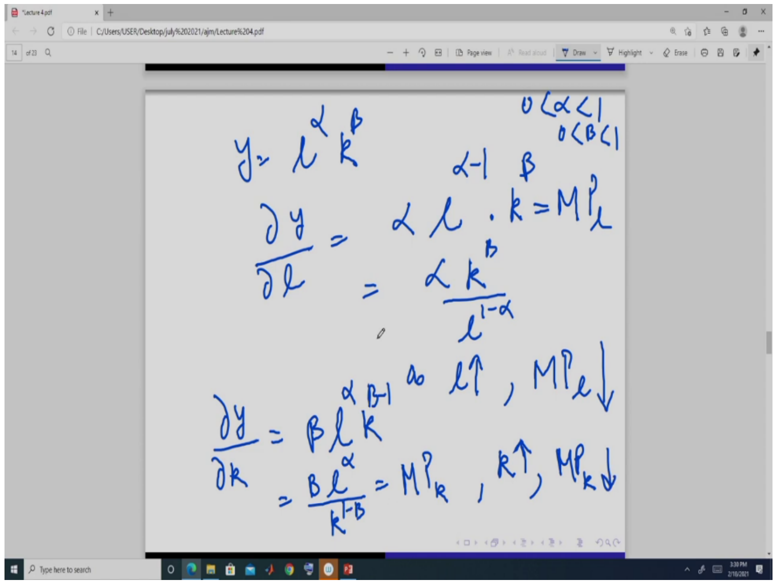

So, it is suppose our production function is the Cobb Douglas production function. So, this, like this- $y = l^{\alpha}k^{\beta}$, then this- $\frac{\delta y}{\delta l} = \alpha l^{\alpha-1}.k^{\beta} = MP_l$ is, right, and we know alpha lies between. So, this is actually like this- $\alpha k^{\beta}/l^{1-\alpha}$. So, it means what? That as labor increases, this is you can call marginal product of labor, marginal product of labor decreases, *okay*. Same, if we take this- $\frac{\delta y}{\delta k} = \beta l^{\alpha}.k^{\beta-1} = MP_k$, why? Because beta lies between again 0 and 1, right. So, this is marginal product of capital. So, as capitals increases, marginal product of capital decreases from here, right.

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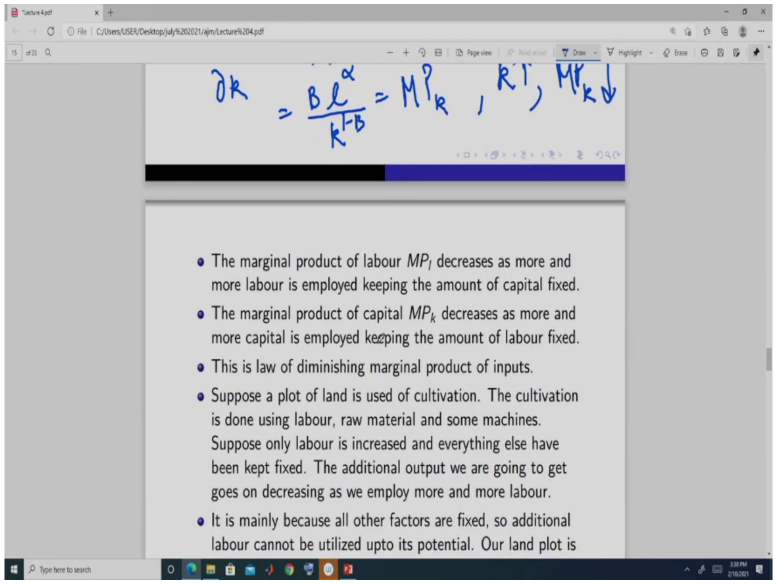

So, this thing is called the law of diminishing marginal product. So, it is something like this, that if we fixed one input and keep on increasing the other input, then the additional output that we are going to go, get it is going to increase. But it is going to go up, it is going to be positive. But it is the additional outputs are going to be go on decreasing.

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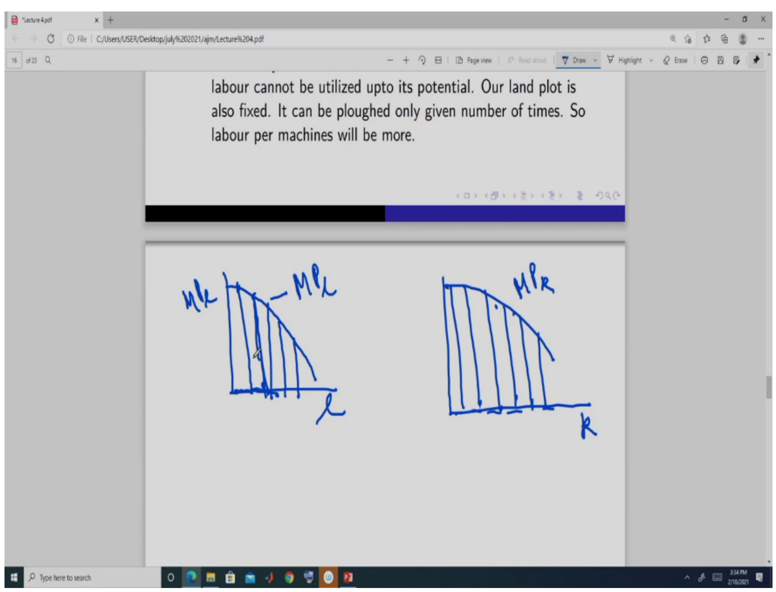

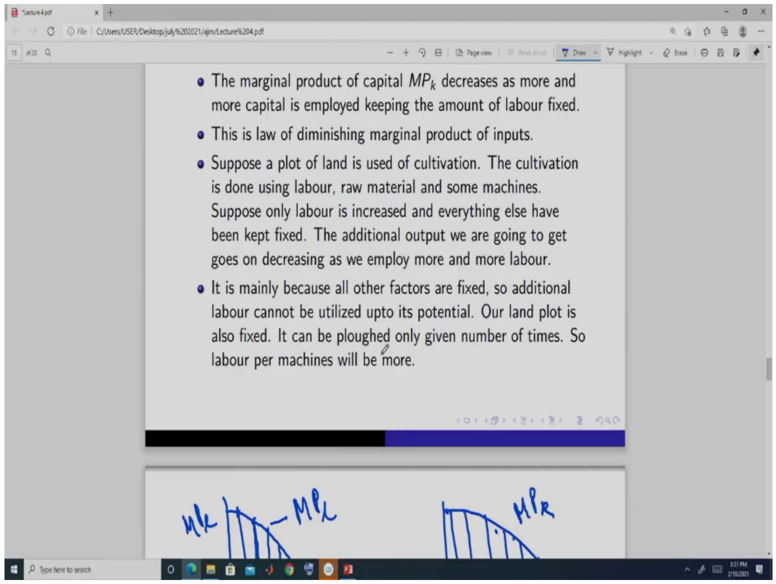

So, it is something like this, that if we plot labor here and marginal product of labor is going to be something like this. If we plot marginal product of labor here. So, that means, keeping capital fixed as we keep on increasing labor, marginal product is positive, but it is going down. So, this marginal product means the additional output that we are going to get. So, at this level of labor marginal product is this much.

If we increase further, marginal product is gone down, but it is still positive, okay. Similarly, marginal product of capital is going to be, going down as we go on increasing capital. So, here as we go on increasing capital marginal product is decreasing, it is positive but it is going down, like this. And this is what the law of diminishing marginal product says. The main idea is something like this. So, suppose, you fixed a plot of land and you are cultivating that land.

So, your land plot is fixed, you are only using labor. Now, if you keep on increasing the labor what is happening? And suppose your machines are also fixed. So, you can only plow that field only for a fixed number of times. You cannot go on plowing that land. And further machines can also be used only for plowing. Suppose only a few number of times. So, then the additional labor is not adding or you cannot use the additional labor to its potential.

So, if each labor can work for eight hours. So, you now you what you do each one is doing any at least fully for two hours or three hours. So, the additional output that you are going to get is less or you can think of suppose you have a place, you have a restaurant which uses one stove to produce some dish suppose but now, if you keep on increasing your staff and if you keep the number of gas stove same, then you will not be able to use increase output by use thing, additional output will, you will have some additional output.

But it will be going down as you keep on because this since you have your stove is fixed only now, what they can do? They can only reduce their labor, can reduce the amount of time they are working at to utilize that stove, and if you want to utilize all the workers together. Otherwise, they cannot use it because one person is already using and there is only one stove. So, other person will lie idle.

So, what he can do when this person can get tired so, other will substitute then the output is not going to go down. So, now the when the person gets tired so, he produces very at a lower rate. Now, since a new person will substitute him. So, that lower rate is not going to go down. But now if you keep on increasing the labor. So, this if you are using it for suppose you are opening that restaurant for eight hours, then you cannot employ all the people and produce at the same rate because your number of stove is same.

So, that is the idea that the factors get congested and that is why we marginal additional output is although it is positive, but it is less, it goes on decreasing okay. So, this is the idea. So, as we increase labor the additional is going down here, as we increase capital the additional output is going down okay.

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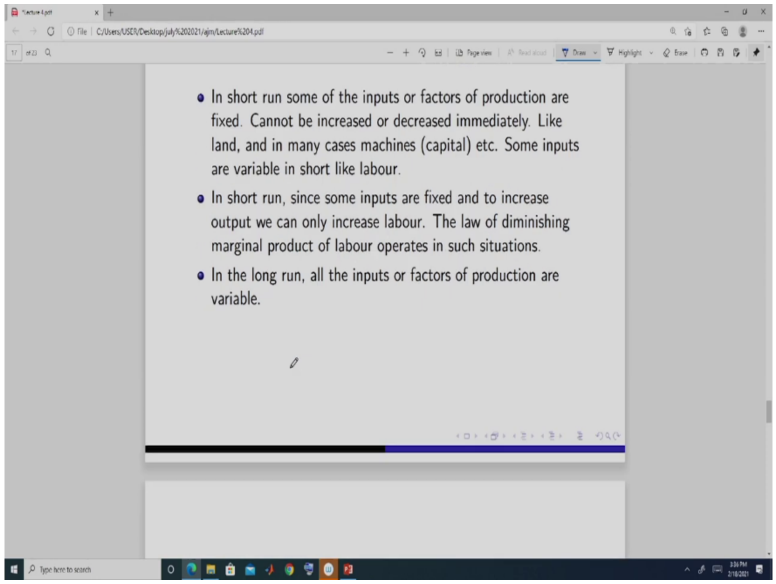

So, now, what do we see that in the short run some of these factors are going to be fixed, like machines or land. We cannot change them immediately, it takes some time. Suppose we want to use more machines. So, these machines need to be procured and again this needs to be installed and all those things requires a lot of time. So, immediately we cannot do this, cannot change the our level of machines that we are using.

So, some factors are fixed in the short run. But in the long run, all the factors of productions or inputs are variable, we can change as many machines we want to use we can do it. Because it is long run. So, here there is no distinct demarcation that this many time period denotes short run. And this many times, if it is more than this many time periods then it is long run. But the idea is something like this, that if some of these inputs cannot be varied, cannot be increased or cannot be decreased immediately then we say we are in a short run.

But if all the factors can be varied, can be changes, then we say we are in the long run okay. So, now since in a short run, some of these factors or inputs are going to be fixed. So, that is why we see the law of diminishing marginal product may operate, that is why it plays some important roles okay.

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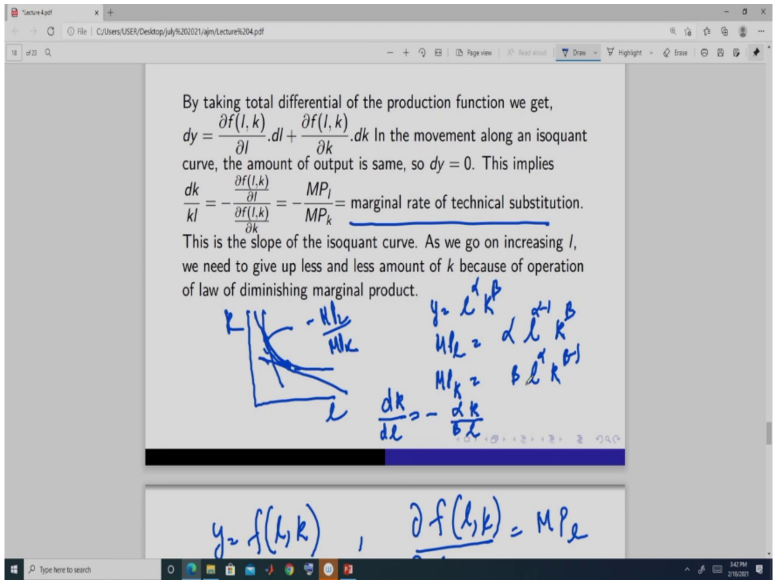

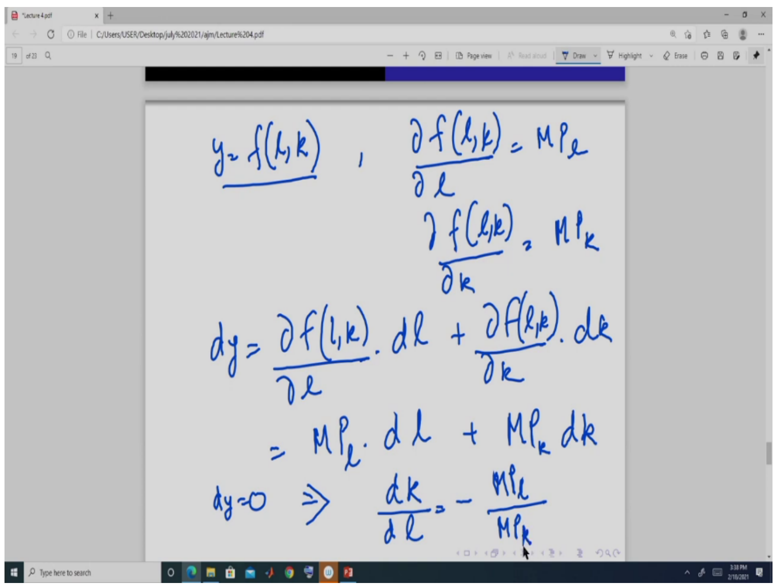

Now, we will study what happens when we move along a isoquant okay. So, suppose our production function is something like this- y=f(l,k), right. Now, it is differentiable. So, we have this thing- $\frac{\delta f(l,k)}{\delta l} = MP_l$. So, this is marginal product of labor, again, this is - $\frac{\delta f(l,k)}{\delta k} = MP_k$. marginal product of capital, right. Now, if we do what, take the total differentiation of this. So, this is going to be what, This- $dy = \frac{\delta f(l,k)}{\delta l}.\, dl + \frac{\delta f(l,k)}{\delta k}.\, dk$ now, this is you can write marginal product of labor- $dy = MP_l.\, dl + MP_k.\, dk$, this. Now, here suppose take our isoquants are something like this.

If we are moving from this point to suppose this point, what we are doing? We are increasing labor and decreasing capital, right, it is movement along an isoquant. right, So, what is happening? This changes in output is 0. So, this gives us this- $\frac{dk}{dl} = -\frac{MP_l}{MPk}$. So, this is what? This is the slope of isoquant. This, this is actually equal to minus of marginal product of labor by marginal product of capital. And if, this, this is the isoquant, slope of isoquant at this point.

And you will see that as we move in this way, that as we are going employing more and more labor to keep the output same, we have to give up less and less amount of capital. Why? Because of especially the law of diminishing marginal product. As we keep on increasing labor, the additional output that we are going to get, it is going to go down. If we keep the amount of capital fixed. So, now, so, this additional we do not want to increase output, we are moving along an isoquant. So, the amount of capital that I have to give up is going to be less.

Because the additional output is also less right. So, that is why what happens? The slope goes on decreasing as we go on increasing labor. And this is something called this ratio, slope is something called the marginal rate of technical substitution. So, it is marginal rate of technical substitution that is, as you go on increasing one input, how much amount of other input has to be decreased to stay at the same level of output.

So, it is that if you increase one unit of labor, how much amount of capital should be reduced to produce the same amount of output, okay. So, this is, this ratio is given by the actually the technology or the production function that you were using. So, in our case suppose, here we take the let us take the production function to be of this nature, that is Cobb Douglas. So, here we know marginal product of labor is this- $MP_l = \alpha l^{\alpha-1} k^{\beta}$, marginal product of capital is- $MP_k = \beta l^{\alpha} k^{\beta-1}$.

So, the slope of this isoquant is a ratio of these two things. So, it is going to be like this- $\frac{dk}{dl} = -\frac{\alpha k}{\beta l}$. So, what happens? So, as a goes up, and this needs to will going down, as we move here. So, this ratio is going to be smaller and smaller, since it is a negative a. So, it is going to be a bigger one, but we ignore the negative a, if we simply look at the absolute value, what do we get? We get that because the negative sign is mainly implying here, that as we increase labor capital has to be decreased. So, the negative relationship, but if we simply look at the absolute value, that then we see that it is it will go one decreasing, okay this is the main idea here.

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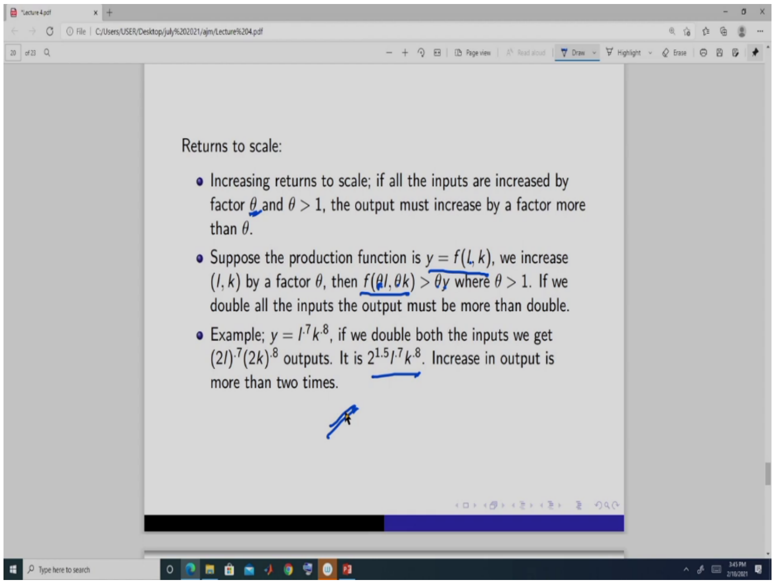

Now, we do another concept and that is returns to scale. Return to scale means, means mainly see in the long run all the inputs can be varied. And this plays an important role in the long run. So, now we differentiate the production function in three things and that is in terms of scale, returns to scale. So, that is increasing returns to scale, constant returns to scale and decreasing returns to scale.

What increasing returns to scale says? That suppose we are given a production function is like this- y=f(l,k), okay. Now, if we increase all the inputs by a factor theta (ϴ) and this theta must be greater than 1. So, if we suppose double all the inputs, then the output should be more than theta times the present output, okay. So, it is like this. So, the output must increase by a factor more than theta. So, if we increase all the inputs by a factor this theta and multiply all the inputs by factor theta, then the output, this output- f(ϴl, ϴk) should be greater than theta times y, where y is this.

For example, suppose production function is like this- $y = l^{.7}k^{.8}$ l to the power 0.7 k to the power 0.8. Now, here if we double labor and double capital, what do we get? So, it is this- $2^{1.5}l^{.7}k^{.8}$ so, it is 2 to the power 1.5, l to the power 0.7 k to the power 0.8. So, this means that the output has increased by a factor more than two times. So, when we have this kind of production function we say it exhibits increasing returns to scale. That is as we increase input and the output is going to be more than the multiple by which we have increased the inputs, okay. So, this is an example of increasing returns to scale.

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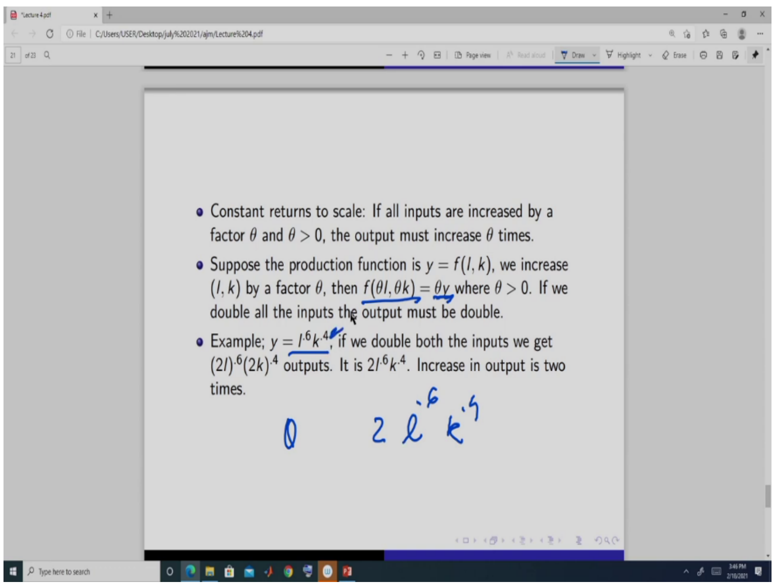

Next we have constant returns to scale, constant returns to scale says that if we increase, if we change the inputs by a multiple theta. Here theta can be any positive number then output should also change by only that theta factor. That is, if we increase all the inputs by a multiple of theta here this- - f(ϴl, ϴk)= ϴy, then the output should also increase by only this multiple, theta multiple. So, for example, suppose the production function is like this- $y = l^{.6}k^{.4}$ l to the power 0.6 and k to the power 0.4.

Here if we double all the inputs, that is 2 l and 2 k then we get the output is 2 l to the power 0.6 k to the power 0.4. So, output is exactly doubled now, because we have doubled the inputs. So, this is something called an constant returns to scale, okay. And this is one example of constant returns to scale production function.

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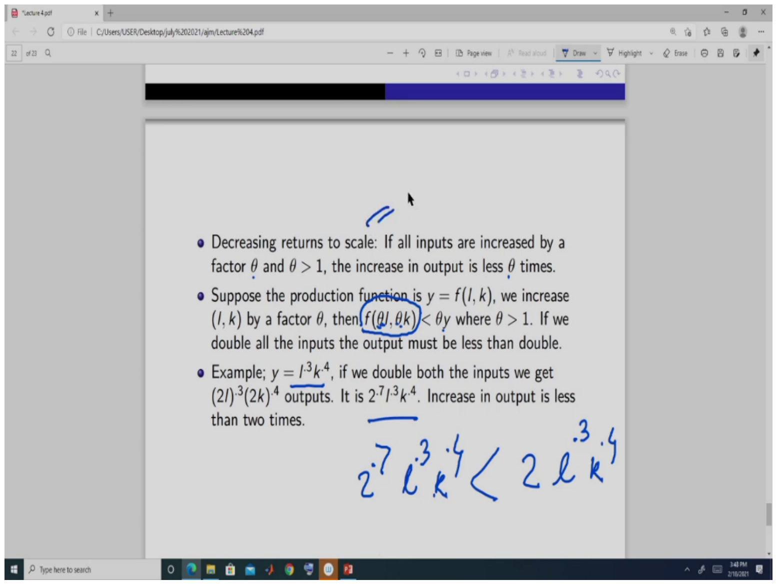

Next, something called decreasing returns to scale. What is the meaning of decreasing? It says that if all the inputs are increased by multiple theta. Here theta is a number which is greater than 1, then the input must, then the output must be increased by a factor which is less than theta. So, if we double all the inputs then the output should be less than double or if we triple all the inputs, the outputs should be less than triple.

So, it is like this, if all the inputs are increased by a multiple theta, that is theta greater than 1, then the output is this much, because this is the production function and it should be less than theta times y, where y is this- y=f (l,k), or it is this if we take this production function where l is to the power 0.3 and k is to the power 0.4, i.e $y = l^{.3}k^{.4}$. Now, if we double both the inputs labor and capital then the output is only 2 to the power 0.7 into l to the power 0.3 k to the power 0.4.

So, this 2 to the power 0.7, l to the power 0.3, k to the power 0.4. This is actually less than 2 to the power this-$2^{.7}l^{.3}k^{.4} < 2l^{.3}k^{.4}$,okay. So, output is now has not increased by, has not doubled it is something less than double, it is definitely it has increased but it is less than double. So, this is decreasing returns to scale, okay. So, today we will end at this only and next class we will see how firms decide on the amount of inputs, this l and k how much they are going to demand or how much they are going to use to produce a given level of output.

So, that decision of the firm we are going to study in the next module, in the next class. Today we have defined the technology that is a production function. So, firm when it is deciding how much amount of input of labor and capital to use, it is first given constant by a technology and that technology is given by this production function, okay. So, thank you very much.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 2: Production and Cost Curves
Lecture – 6 Cost Minimization Problem

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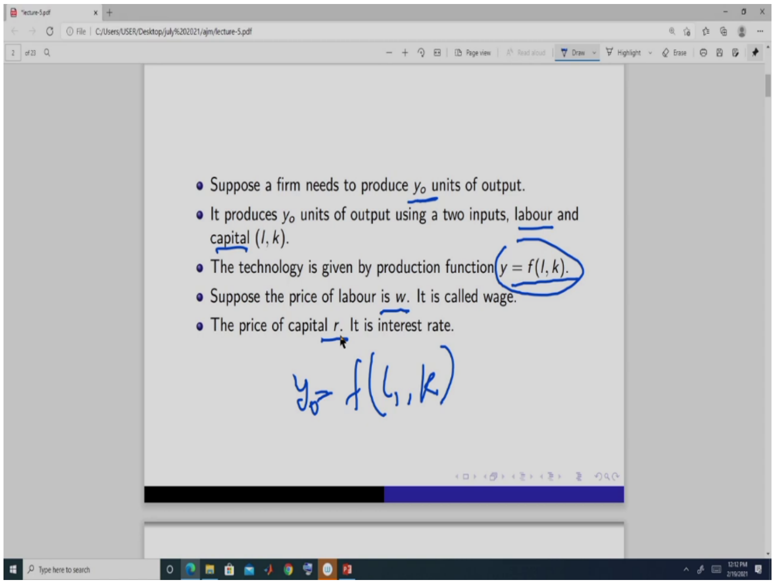

Hello, welcome to my course Introduction to Market Structures and today we are going to do cost minimization. So, now, suppose we have a firm and firm needs to produce this much y naught units of output. Now, we know to produce output it requires inputs and in our simple case we assume that it requires two inputs that is labor and capital, okay. And we have a technology which is given by the, or represented through a function this is production function.

So, we plug in the value of labor and capital and we get output here- y=f(l,k). So, to get y naught amount of output, we will have to plug in some specific amount of L and some specific amount of K to get this okay. Now, to get this input the firm needs to hire this from the market like labor market, from labor market it will hire labor, from the capital market it will hire capital. So, it will have to pay some price, it will pay wages that is 'w' for hiring labor and it will pay 'r' which is interest for hiring capital.

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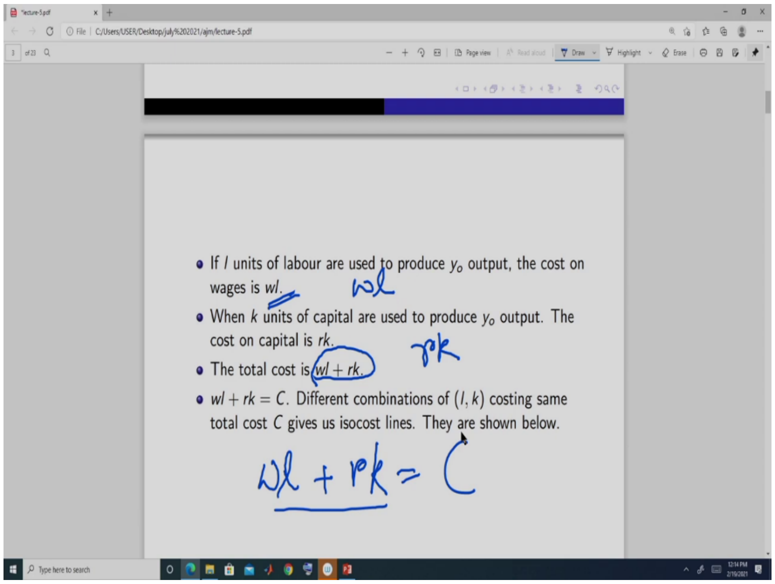

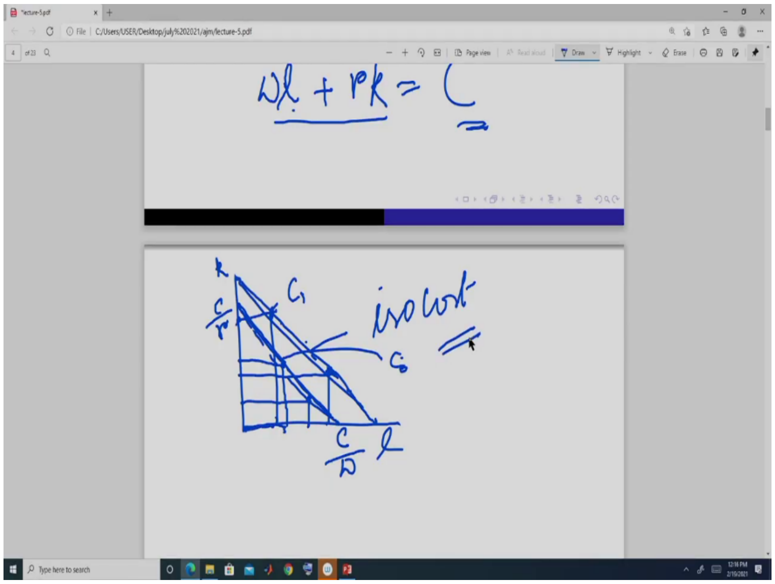

Now, suppose it so, the expenditure that it is going to incur on labor is this much- wl, w that is the wage rate price of labor into total units of labor it has bought that is wl and the expenditure it is going to incur on the cost it is going to incur on the capital is r into k. So, this is r is the rate of interest or the price of capital and k is the amount of capital. So, together this- wl+rk is the total cost that a firm is going to incur to produce output.

Now, what we call this we get a relation like this wl and this you can think something similar as a budget constraint and this is suppose equal to some number that is C, i.e wl+rk=C. This if we look at different combinations of l and k such that the C is same. These lines are called isocost lines and they look something like this if we plot l here and K here capital, this is you can say an isocost line where this is C divided by r and this is C divided by w and this is C divided by r okay.

Now, if we change the amount of total cost, if we increase it, this curve is going to shift like this. So, these are called isocost lines or you can say these lines if we are at any point in this line, then it gives the total expenditure that a firm going to incur is this much for this different combination. So, at this point, we are having this much amount of capital and this much amount of capital and this much amount of labor, but the total cost is this C suppose 1.

Here the total cost remains same, but our combination of capital and labor is different, here we have less capital and more labor here in this a, total cost is less than C one and suppose this is C naught here the combination of capital is this and labor is this much or here we have increased the amount of labor and we have decreased the amount of capital but our cost is same as C naught like this. So, these lines are, gives me the total expenditure that we are going to incur. So, different combinations of labor and capital that their total cost is same okay.

So, this is these are the isocost lines and if we are moving along an isocost like this from this point to this point what we are doing, we are decreasing capital and we are increasing labor but total cost is same and like this we can get what is called the slope of this a, isocost line and that we derive in this way- $dC = \frac{\delta(wl+rk)}{\delta l} Dl + \frac{\delta(wl+rk)}{\delta k} dk.$

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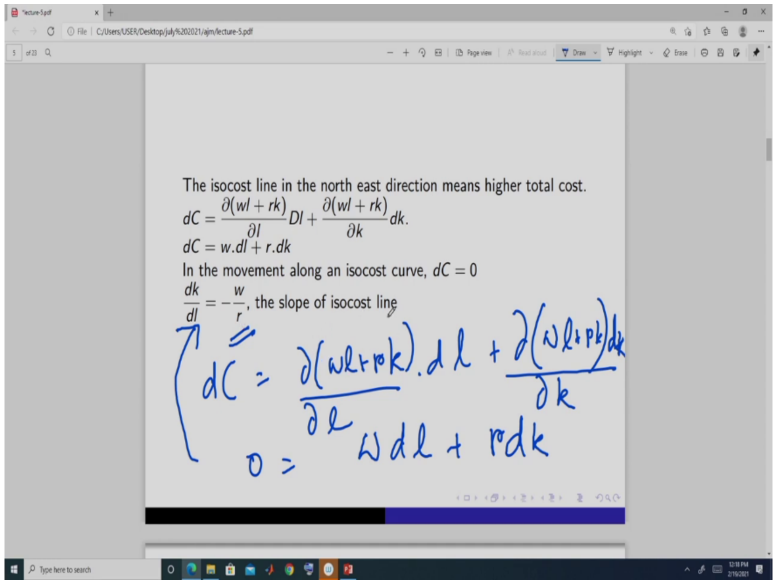

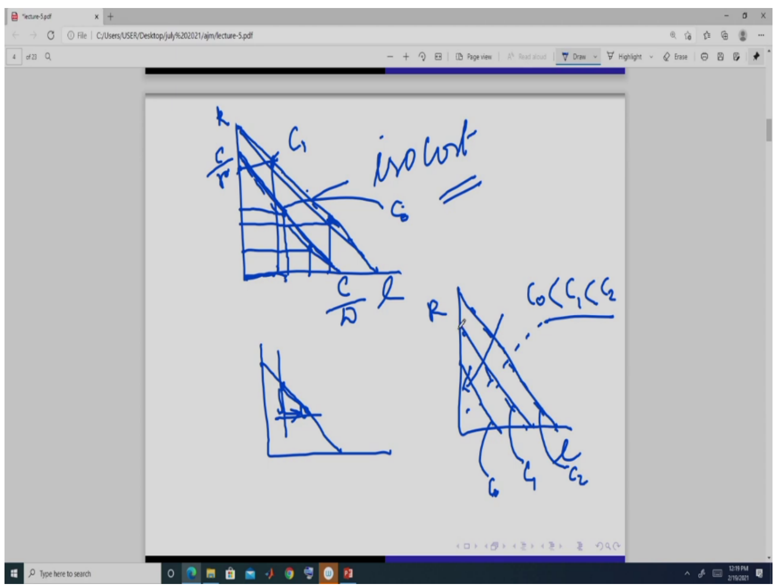

So, if we take the total differentiation of the isocost curve, so, it is what, it is this $\frac{\delta(wl+rk)}{\delta l} dl$ plus $\frac{\delta(wl+rk)}{\delta k} dk$. So, this is simply what? This- dC= wdl+ rdk. And now the changes along an isocost curve is 0. So, this is **a** and so, we get the slope as this. So, this means that if we want to increase one more unit of labor, then if we want to use one more unit of labor, then how much unit of capital the market is allowing us to substitute so, that our cost remains same. So, it is given by this ratio- $\frac{dk}{dl} = -\frac{w}{r}$. So, if I increase one unit of labor, so, I have to decrease some amount of capital.

So, how much the market is allowing us to do so, that our total cost remains the same. So, this is the idea of the slope of the isocost line, okay. So, what we have got that if we look at this capital here and suppose this is the isocost line, now, if we look at this isocost line here, all the total cost is higher in for all these combinations of labor and capital and compared to this and if we take this, this even lower.

So, here if this is C naught, this is C1 and this is C2 then total cost, the rank of this total costs are like this or the position order, okay. So, in this direction in the northeast direction, total cost is increasing okay, and if we have to reduce the total cost we have to move in this direction okay. And we will use this in solving the cost minimization problem okay.

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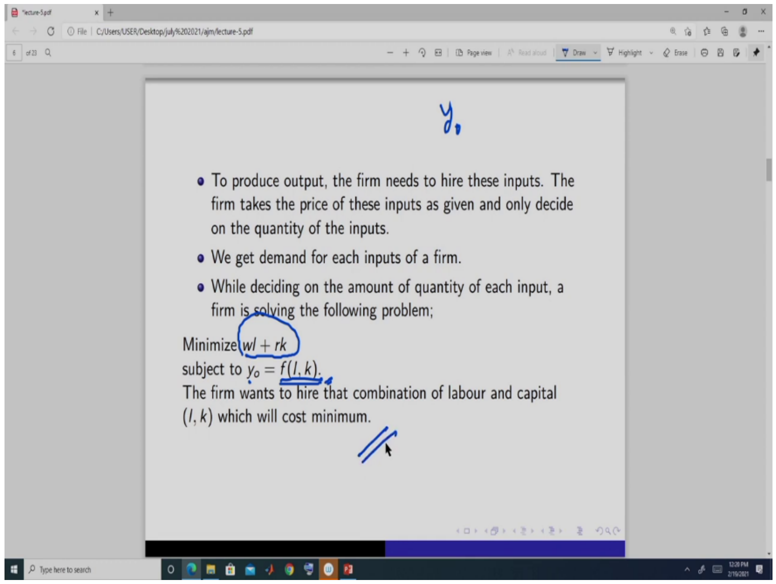

Now, what happens, so the firm wants to produce an output of suppose, y naught unit so, for that it needs to hire labor and capital. Now, while deciding the a, the amount of labor it is going to hire and the amount of capital it is going to hire, it takes the price of labor that is w and the price of capital that is r as given, the firm cannot determine the price of labor and capital, it is given.

So, now, the firm only decides how much amount of this each of these labor, they are going to hire how many units okay. So, the firm is actually going to solve this problem that is its wants to minimize this cost it is w into l plus r into k such that it is subject to it wants to produce this given the production function or technology in this way.

So, now, suppose firm wants to produce 100 units of output then to produce 100 units of output, it can use several combinations of labor and capital. Now, it will choose that combination of labor and capital such that the cost is minimum. So, this is the idea of this problem okay.

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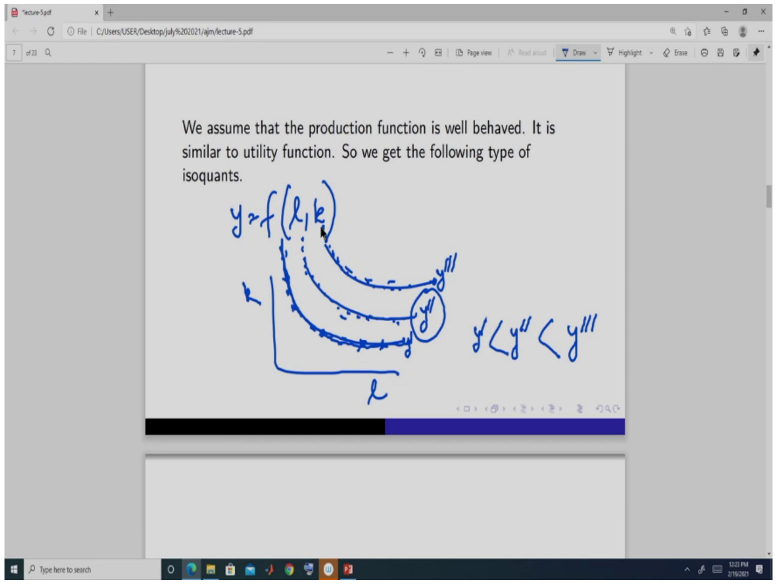

Now, so here to solve this problem, we will be given a production function of some nature this- y=f(l,k). We assume that the production functions are well behaved. So, we get following types of iso-quants. Iso-quants are level curves like this. So, this is for suppose I have no output y dash this is for output y double dash and this is for output y triple dash and here output in y dash is less than output in double dash and here even, i.e $y^{'} < y^{''} < y^{'''}$.

So, outputs are increasing in this way so, we will get this now, suppose we want to produce y dash units of output then we this is fixed so, we can choose from these combinations of capital and labor either we can choose this, we will choose this, we can choose this any one of this, okay. Now, out of these points or these combinations, we want to choose that one which is costing us minimum or suppose now, we want to produce **y** double dash amount of output.

Then all these combinations of labor and capital are allowing us to produce y double dash units of output out of these combinations we want to choose that one which is going to give us or which is going to cost us minimum. Similarly, when we want to produce suppose y tripled dash units of output then these combinations of labor and capital is actually allowing us to produce y triple dash units of output and out of these combinations of labor and capital, we want to choose that combination which is giving us or which will cost us minimum, okay. So, now, how to proceed?

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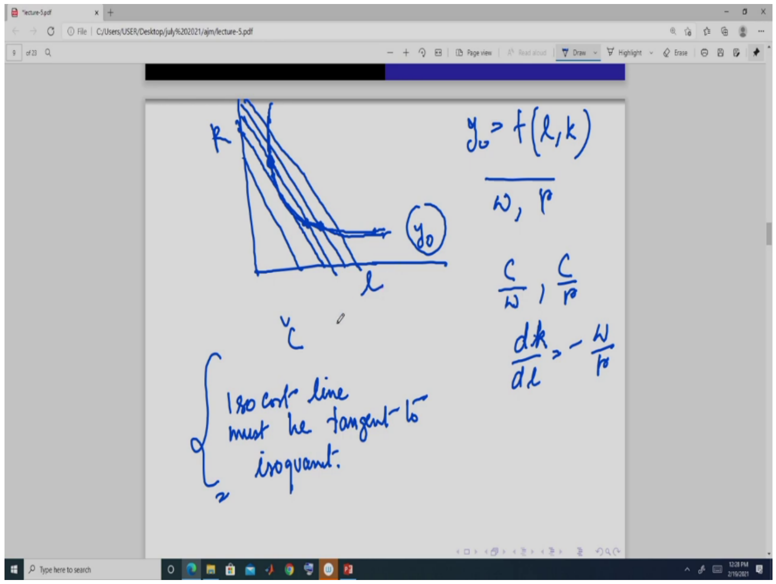

So, first we will solve this graphically and then we will solve these through Lagrange multiplier, okay. So, suppose labor is measured in this axis, capital is measured in this axis and we want to produce y naught unit of output and our production function is suppose something like this- $y_o = f(l, k)$, now we have fixed. So, we will get isoquant and suppose this is the isoquant and this is y naught, okay. Now, we have assumed that this is well behaved so it is convex to the origin and it is also continuous all these properties are eligible.

Now, suppose the price of labor is w and price of capital is some r, okay. So, we have isocost line like this and isocost lines are going to be parallel, why? Because isocost lines are given by joining these two points C and R since w and R are constant, it is taken as fixed or given by this firm. So, these are going to be parallel and the or you can say this since the slope this is equal to w by R and it is going to be same. So, the angles are going to be same. Now, here we can have all these different isocost lines and this is our isoquant, right? we want to produce y naught units of output.

So, we can produce using this combination of labor and capital, this, this, this, this, like this, now if we choose this combination suppose, then, we are in this isoquant, instead, if we choose a point here it we will be at a lower level of isocost line. So, it means our cost can be reduced. So, it is better to choose a combination here rather than this point. Now, suppose we are choosing a combination suppose this if we choose this combination of labor and capital, then we are at this level of isocost line.

But if we switch from this point to this we increase some amount of capital and reduce some amount of labor. Then we are at a lower level isocost curve. So, our total cost is less now, so this is costing us less than this. So, we move in this a, and finally, we reach that isocost curve which is tangent to the isoquant. So, isocost line must be tangent to the isoquant, okay. So, this ensures that the cost is minimum.

Because if we are at any other point compared to this the point of tangency then there is always a possibility that the cost can be reduced provided our production functions are well behaved so, isocost, isoquants are like this, okay.

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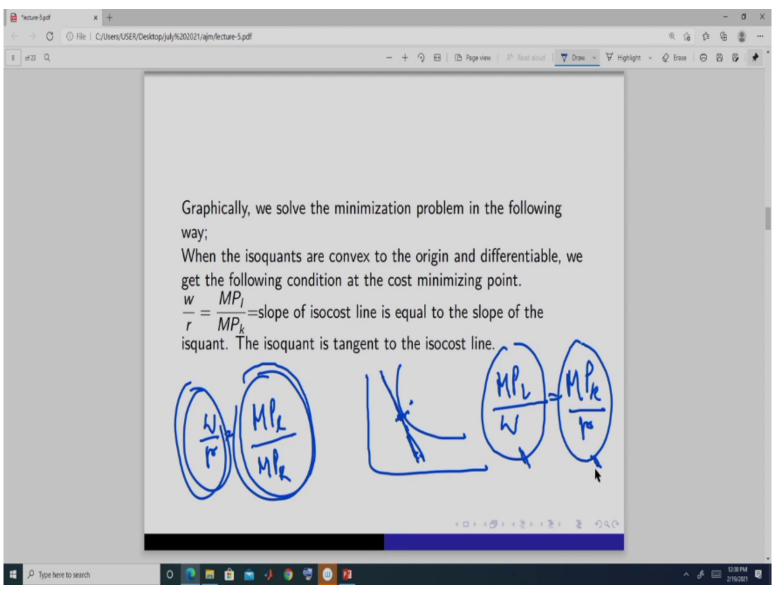

So, this condition you can say is something like this- $\frac{w}{r} = \frac{MP_l}{MP_k}$. So, when they are tangent when the isocost line is tangent to the isoquant it means what? that the slope of isocost and the slope of isoquant are same, right. At that point slope of isoquant at that point, so, it is something like this. So, suppose this is the slope of at this point, right? is given by this, so, again this is a tangent to this, so, the slope must be same of this line and the isoquant at this point, so we get this condition-$\frac{w}{r} = \frac{MP_l}{MP_k}$.

So, this we have got from the slope of isoquant and this we have got from the slope of isocost. So, this should match that means, how if we want to increase one unit of labor, how much unit of capital we should decrease, how much it is possible given the technology that is given by this isoquant and how much the market is allowing us to do given the prices, it is given by this-$\frac{w}{r}$. So, when we our cost is minimum these two things should match or from here you can say that we can also write it in this way-$\frac{MP_l}{w} = \frac{MP_k}{r}$., this condition is similar to what we have done in the consumer behavior.

So, this much is to increase one additional unit of output, the amount of expenditure we have to do on labor it is this-$\frac{MP_l}{w}$ and to increase one unit of output the amount of expenditure we have to do in the capital it is this-$\frac{MP_k}{r}$. So, these two should match. So, if we want to increase one unit of output, then the amount of expenditure done in the labor should be equal to amount of expenditure done on capital when we are at a optimal point and here optimal point is the point at which cost is minimum, okay. So, this is the important condition that the isocost line should be tangent to the isoquant at the optimal point okay.

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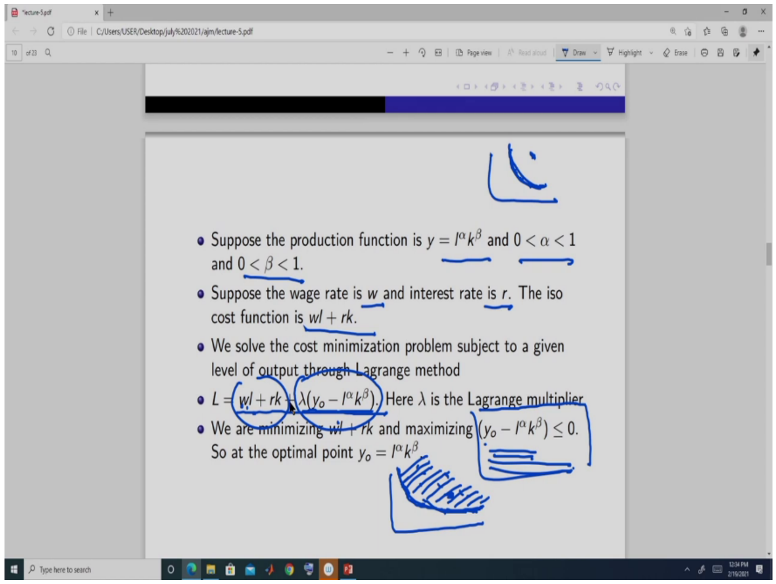

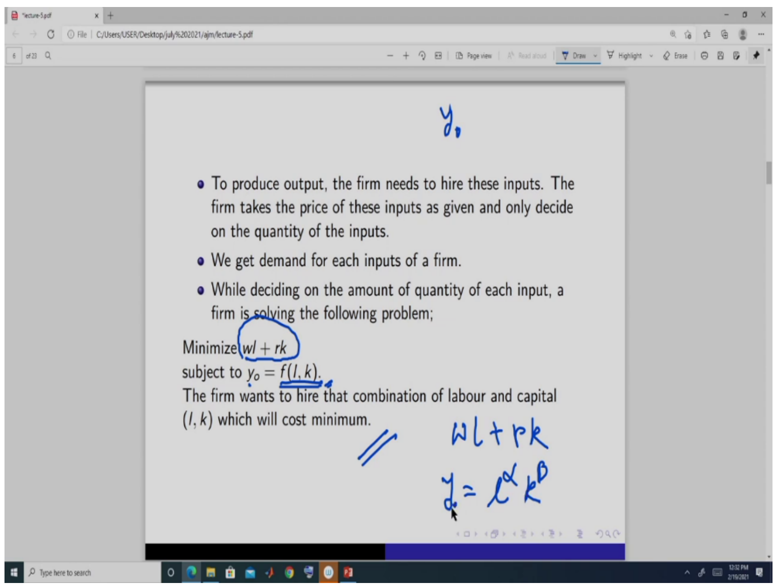

Now, let us solve this problem algebraically, how do we do? So, for this we assumed a very specific form of production function it is this- $y = l^{\alpha}k^{\beta}$ l to the power alpha and k to the power beta, l labor, k capital and alpha takes a value any value between 0 and 1 and beta takes a value between 0 and 1, okay. So, here again we assume that the wage rate is w and the interest rate is r some positive numbers and so, the isocost line is given like this w into r, w into l plus r into k.

Now, we solve this problem this, we want to minimize this- wl+rk, w subject to our production function like this- $y = l^{\alpha}k^{\beta}$, suppose we specify some amount of output and that is suppose, y naught, okay. So, we write the Lagrange in this form- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. So, this Lagrange it is saying it is a function and you can think it to be something like it is a saddle point also you can say we want to find the saddle point of this a. So, what we are doing, we want to minimize this component, this portion- wl+rk and we want to maximize this portion- $\lambda(y_o - l^{\alpha}k^{\beta})$. and this portion is written in this form- $(y_o - l^{\alpha}k^{\beta}) \leq 0$.

So, this is maximized whenever it is equal to 0 because otherwise it is because if we fix the isoquant, right, what we are doing this whole set we take any combination of this when we can produce. Now this is the most efficient way in the sense employing minimum labor and capital we can produce the output. Now here you bring this combination also we can produce this much, but here we are employing more labor and capital, right.

So, in this sense, we want to be always at the boundary. So, that is what that while minimizing the cost, we always want to be at the boundary or we always want to be at that isoquant which specifies that level of output, okay, this. So, since we want to maximize so, when we want to find the optimal combination of labor and capital, so we will always be at this point we will not use any combination of like this we will always use like this.

So, this is now here this lambda is the Lagrange multiplier. So, this thing- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. is what? this thing is in terms of output and this is in terms of value because it is price into quantity, price of labor into quantity of labor, so it is in value terms. So, this is actually converting this output into this. So, this you can say as some price of this good, of this output or you can say like shadow price, I will not discuss that or you can think it is simply the price of this, okay.

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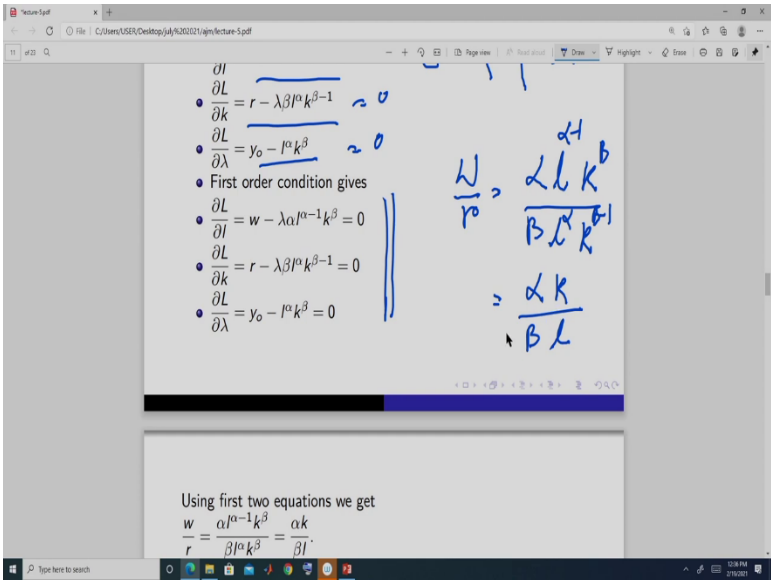
- Suppose the wage rate is w and interest rate is r. The iso cost function is $wl + rk$.
- We solve the cost minimization problem subject to a given level of output through Lagrange method
- $L = wl + rk + \lambda(y_o - l^\alpha k^\beta)$. Here λ is the Lagrange multiplier
- We are minimizing $wl + rk$ and maximizing $(y_o - l^\alpha k^\beta) \leq 0$. So at the optimal point $y_o = l^\alpha k^\beta$

- Since the production function is differentiable in (l, k). So we take the following derivatives;
- $\frac{\partial L}{\partial l} = w - \lambda \alpha l^{\alpha-1} k^\beta$ $= 0$
- $\frac{\partial L}{\partial k} = r - \lambda \beta l^\alpha k^{\beta-1}$ $= 0$
- $\frac{\partial L}{\partial \lambda} = y_o - l^\alpha k^\beta$ $= 0$

$L = wl + rk + \lambda(y_o - l^\alpha k^\beta)$

- First order condition gives
- $\frac{\partial L}{\partial l} = w - \lambda \alpha l^{\alpha-1} k^\beta = 0$
- $\frac{\partial L}{\partial k} = r - \lambda \beta l^\alpha k^{\beta-1} = 0$
- $\frac{\partial L}{\partial \lambda} = y_o - l^\alpha k^\beta = 0$



Now, what we are going to do we have set this Lagrange and Lagrange in our case is this L is equal to w L, w into L plus r into k plus lambda which is Lagrange multiplier y naught minus L to the power alpha k to the power beta like this- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. Now, we take the first derivative of this with respect to three variables that is L labor, k capital and also lambda. Lambda is also a variable in this case, okay.

Now, and the first order condition gives us this or you can say the derivative this- $\frac{\delta L}{\delta l} = w - \lambda\alpha l^{\alpha-1} k^{\beta}$ is w minus this portion, okay and this is-$\frac{\delta L}{\delta k} = r - \lambda\beta l^{\alpha} k^{\beta-1}$ r minus this first derivative of this and if take the derivative of this expression l Lagrange with respect to lambda we will get this- $\frac{\delta L}{\delta\lambda} = y_o - l^{\alpha} k^{\beta}$because, this portion is going to . Now, the first order condition says that we always we should equate this to 0, this to 0 and this to 0.

So, these- $\frac{\delta L}{\delta l} = w - \lambda\alpha l^{\alpha-1} k^{\beta} = 0$, $\frac{\delta L}{\delta k} = r - \lambda\beta l^{\alpha} k^{\beta-1} = 0$, $\frac{\delta L}{\delta\lambda} = y_o - l^{\alpha} k^{\beta} = 0$, are the first order conditions. Now, from here what we get if we use these two conditions, first two equation we can write it in this form- $\frac{w}{p} = \frac{\alpha l^{\alpha-1} k^{\beta}}{\beta l^{\alpha} k^{\beta-1}}$this, right. And this expression we can reduce it into alpha into k divided by beta into l this- $\frac{\alpha k}{\beta l}$.

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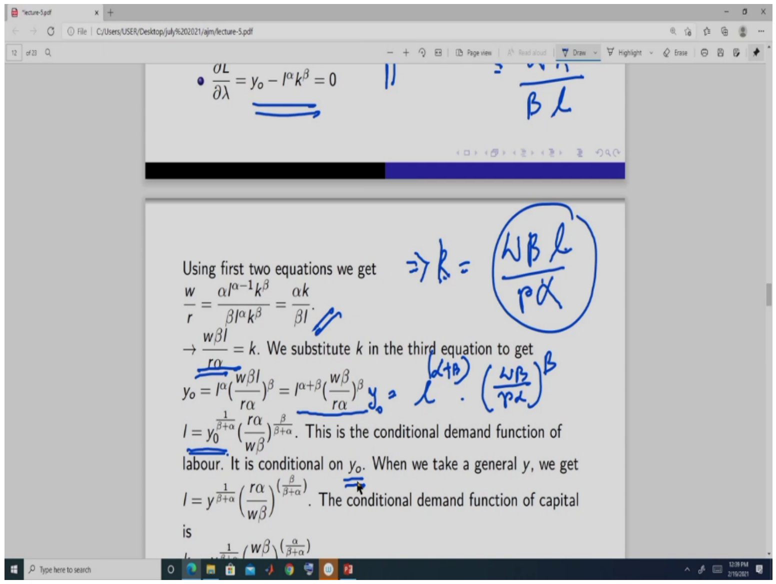

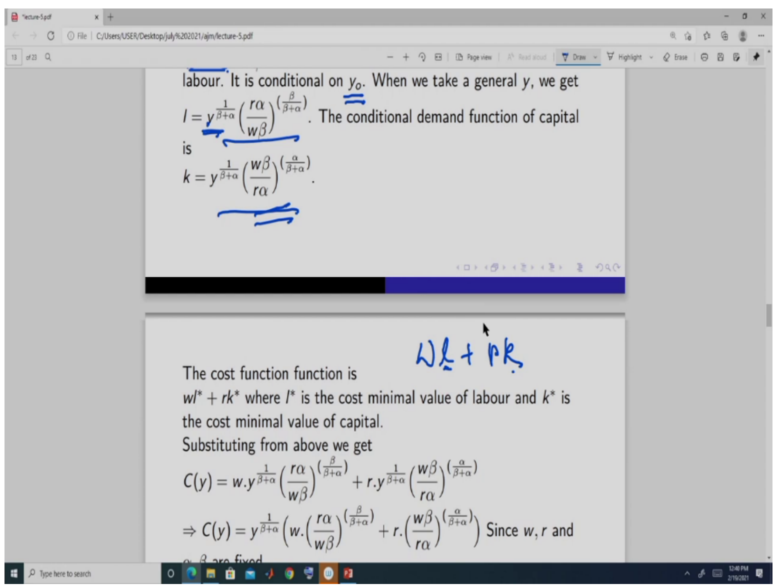

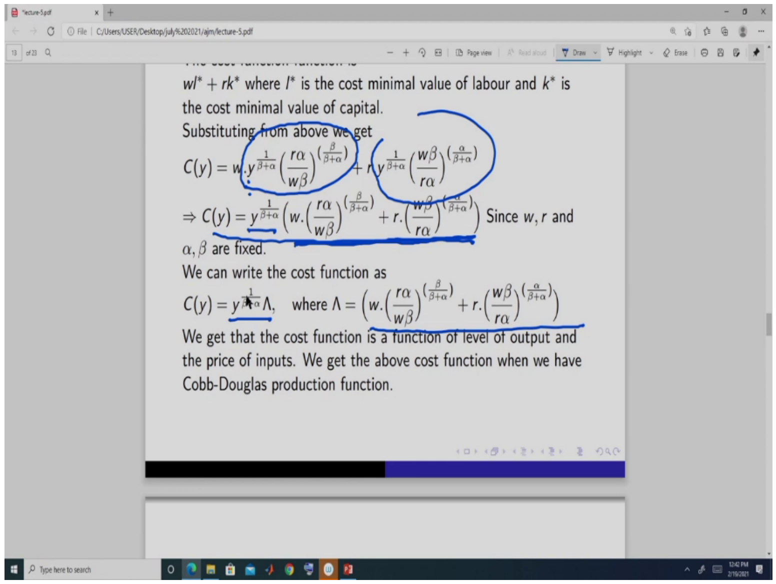

Now, from here we can write labor in terms of capital or capital in terms of labor. So, we what we do we write now capital is equal to w beta L divided by r alpha, i.e $k = w\beta l/r\alpha$, okay.So, this and now, so we know that if we specify any value of l we know how much amount of k to we should buy this, right. Now, again what we do, we substitute this in the production function here because this is also another first order condition.

So, if we do that what do we get this L to the power alpha where k to the power beta. So, in this place of K we plug in this and we get this whole expression this- $y_o = l^{\alpha}\left(\frac{w\beta l}{r\alpha}\right)^{\beta} = l^{\alpha+\beta}\left(\frac{w\beta}{r\alpha}\right)^{\beta}$ where l is to the power alpha plus beta and then we have an expression where this is, to the power beta, okay. So, this now, from here, we can find out the demand for labor and that is if we take this portion to the, because y naught is equal to this we know how much amount of output we want to produce.

So, when we want to produce y naught? So, if we want to produce y naught then what is going to be the demand for l? So, y naught now, if we take this to this side and if you take the power to be 1 by alpha plus beta then we know the demand of labor. So, this is the demand for labor, i.e $l = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}}$. So, now, plug in the amount of output you want to produce you will start getting the demand for labor it is this now, from here what you do you plug in this, here you will get the demand for capital.

Now, this is something called a conditional demand why? because it is conditional on y naught it is like conditional on the amount of output we want to produce if we know the amount of output we want to produce we will know how much amount of labor we want it is this given the price of the labor and capital. Similarly, we can find this demand for capital and it is this again you can see it is conditional on y, i.e. $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$

Now, from y naught we make it a general y because you plug in any y you will get the this function only, okay. So, that is why we take instead of y naught we make it y the same here okay. So, this gives us what this gives us that when we are want to choose a combination of labor and capital subject to a production function and the prices of the labor and capital we choose, we get a demand for labor and demand for capital okay.

Now, when we plug in these optimal values or this demand curve in this so, our total cost is this- wl +rk, right?. And if we want to produce some specific unit of output, our demand for capital is this- $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ and demand for labor is this- e $l = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}}$ now plug in this demand for labor and demand for capital here optimal demand less and that gives us something called the total cost. So, total cost, this total cost at the optimal point is called the cost function.

Because cost function is actually the minimum cost of producing some huge amount of output given the price of labor and capital. So, when we plug in this, this here- $C(y) = w.y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$, so this is the demand for labor and this is the demand for capital. So, we plug in here, we get y to the power 1 by beta plus alpha and this term and here we get r into y to the power 1, 1 by beta plus alpha into this term. Now, from here you can take this- $y^{\frac{1}{\beta+\alpha}}$ common this and what is left this term- $w.\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$.

So, here if you look at this term- $C(y) = w.\,y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\; y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ a firm takes the price of labor that is w as given it cannot determine w. It takes the price of labor that is a price of capital that is r as given it cannot determine it, its production function is given. So, this beta and alpha is given. So, this whole portion, this portion, this is already given to this firm. So, this is fixed. Now, whenever a firm has to decide, that it is going to produce some amount of output that is y then its cost function is given like this in this form- $C(y) = w.\,y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\; y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ provided that the production function is a Cobb Douglas production function of this nature.

So, we can write it in this one in a more compact way - - $C(y) = y^{\frac{1}{\beta+\alpha}}\,\wedge$ where this big lambda $\wedge$ is this- $\wedge = w.\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\;\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ which is always given as fixed a firm so it cannot determine this, right. So, this is the cost function of a firm and this is the total cost function, okay. So, we see that the total cost function is a function of output, okay. Now, let us do some another example.

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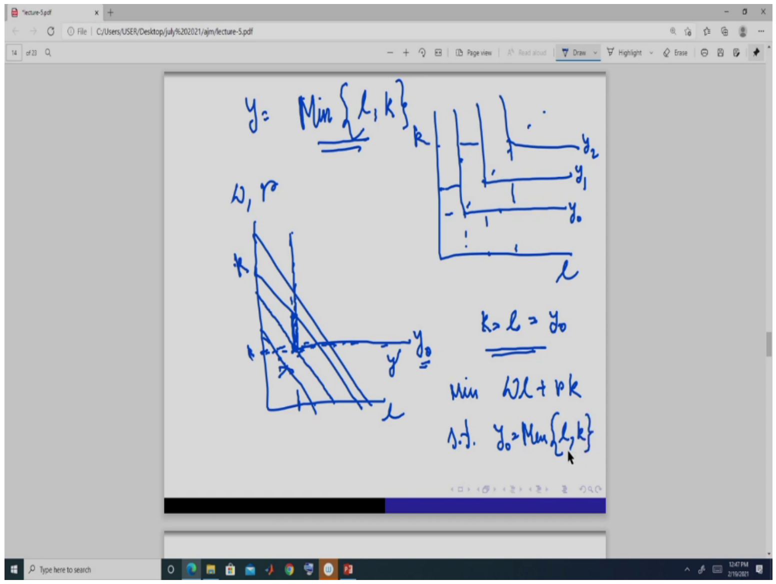

Suppose we take the example of fixed proportion, production function is suppose like this- y=Min{l,k} okay. So, the firm what it does if the production function is this then the isoquants are of this nature. This is suppose y naught, this is y 1, this is y 2 and this is the point of equality, equality of labor and capital like this. So, this production function is not differentiable okay. But it is, you can see convex to the origin if you take any combination of this and this it will at least you can produce this much amount of output, right.

Now, you see what we are, suppose the wage rates are also and the price of the capital is fixed. So, isocost lines are like this and suppose, we want to produce y naught units of output and this is here. Now, this point is going to be the optimal point, why? See, now, because we can produce by combining any point here this y naught unit, but here we have already employed this much k amount of capital. Now, if we go on increasing labor more than this, then what is happening we are not getting any additional output, output is remaining same.

So, there is no point in it. So, we should employ this much only if we have employed this much amount of labor and if we keep on increasing capital we are not getting any extra output, right. So, it is better to say or you can think in this term isocost lines are something like this and like this. Now, this isocost line is lying below this right. So, any point here it is not going to give us y naught unit of output. So, this is not possible. So, isocost line must cross this isoquant, right. if we use this combination, then compare this and this, this isocost line lies below this isocost line.

So, the cost is less here. So, this is better, because this much extra unit of capital is not giving us any extra output, right and we only want to produce y naught units of output. So, we should produce this. So, the optimal point is given in this way, where k is equal to l in this case and that is equal to y naught. Now here if we know the output, we should employ that much amount of labor and that much amount of capital. So, in this problem minimizing min wl+rk this subject to in this- y= Min{l,k}what do we get?

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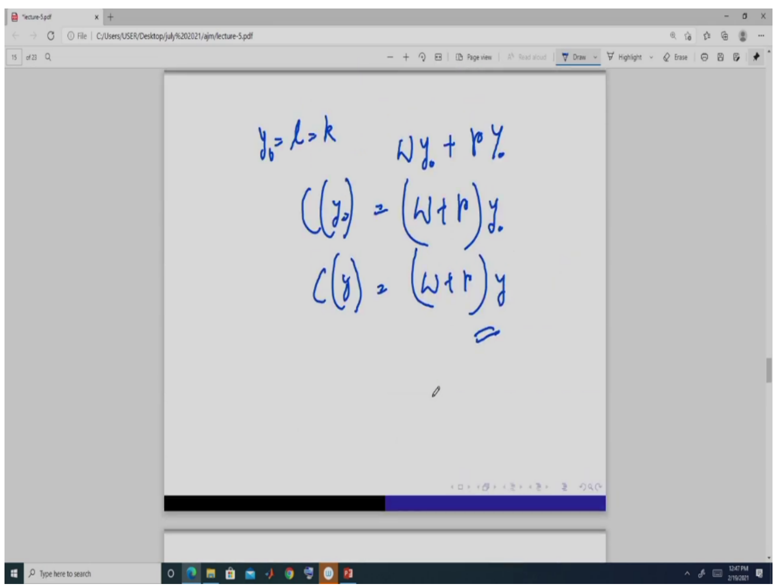

We get that the optimal point is when y is equal to l is equal to k. So, our cost is going to be w. So, this- $Wy_o + ry_o$ is this$C(y) = (W + r)y_o$, now here you plug in different values of output, you will get the cost function of this nature. So, the general cost function is this- $C(y) = (W + r)y$,right. Now, we will see that, how it looks, how this cost function looks when we take different when the production function different exhibits different types of returns to scale, okay.

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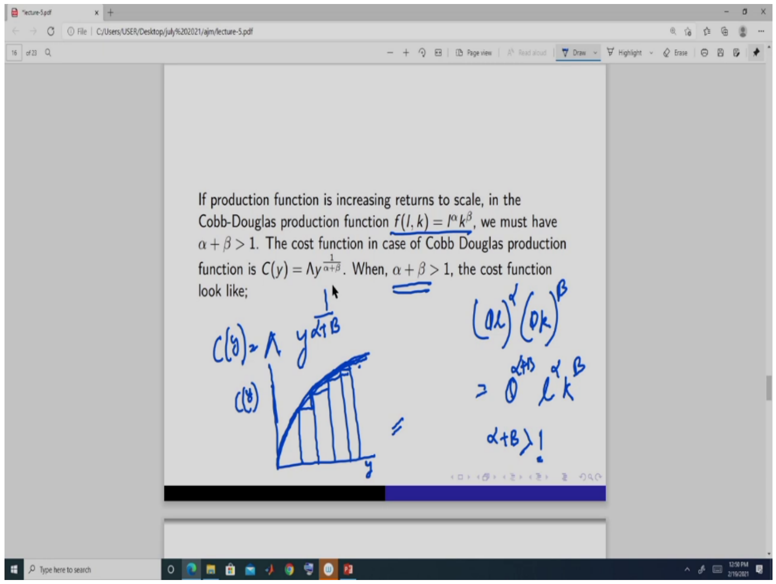

Now, suppose we have increasing returns to scale and the production function is Cobb Douglas. If it increasing returns to scale and Cobb Douglas we have seen in the last class what happens so, this if we take here, this a, what do we get? that if we take a multiple of, theta multiple of both the inputs then this we get theta to the power- $\theta^{\alpha+\beta} l^{\alpha} k^{\beta}$. And if it is increasing returns to scale then this output should be more than theta times this and this is possible when alpha plus beta takes a value greater than 1, right.

So, whenever we are using a Cobb Douglas production function and we want it to exhibit increasing returns to scale, then we always we must have this- $\alpha+\beta>1$, right. Now, we must have this condition that alpha plus beta should be greater than 1. Now, our cost function here when we have a Cobb Douglas production function is this-$C(y) = \wedge .\ y^{\frac{1}{\alpha+\beta}}$. So, alpha plus beta is taking a value greater than 1 then and this is you can take this big lambda to be some constant, some positive number.

So, if we try to plot the cost function here you take output and here you take the this. Now, plug in 0 amount of output cost is 0 here, but this alpha plus beta is greater than 1. So, the cost function is going to look something like this nature. So, what is happening here as we go on increasing output the additional cost is going down, okay. So, we will do this in detail in the next class, okay. So, this is what that total cost curve looks like when we have increasing returns to scale and the production function is Cobb Douglas okay.

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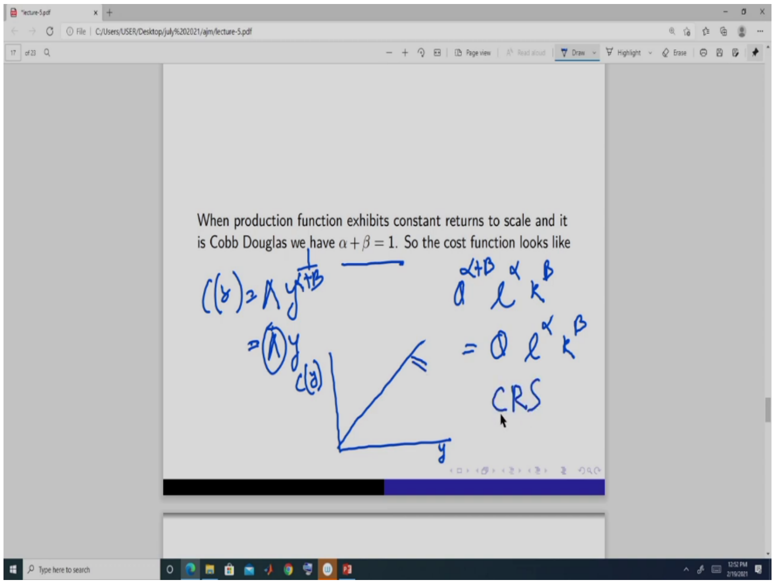

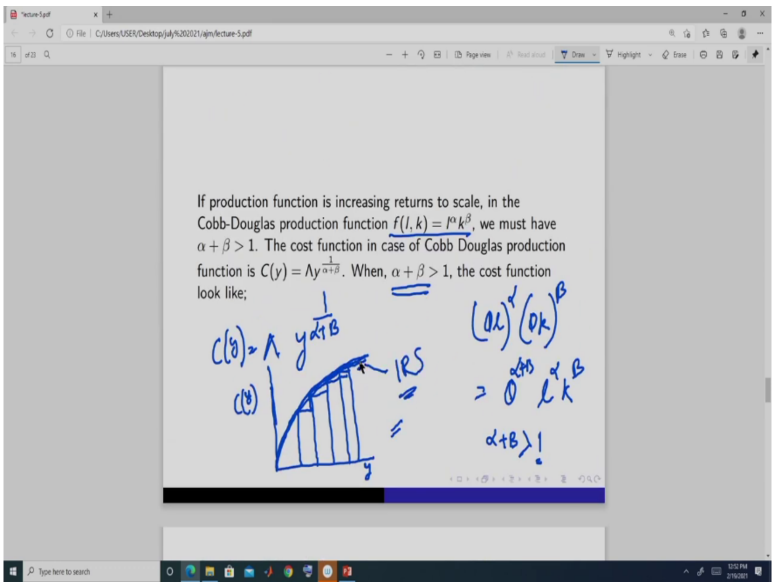

Now, suppose we have constant returns to scale. So, in case of constant returns to scale what we will have, we will have this -$\theta^{\alpha+\beta}\, l^{\alpha}k^{\beta}$should be equal to this-$\theta\, l^{\alpha}k^{\beta}$. So, this means what? Alpha plus beta should always be equal to 1, right because if we take a theta multiple of labor and capital so output should also increase by the same multiple right. Now, our cost function for the Cobb Douglas is big capital lambda y to the power alpha plus beta this- $C(y) = \wedge.\ y^{\frac{1}{\alpha+\beta}}$.

Now, here this is equal to 1. So, alpha is big lambda. So, our total cost function if we take plot y here and cost here, it is going to be a straight line where the slope is given by this big lambda, okay, capital lambda. So, this is our total cost function, when we have CRS that is Constant Returns to Scale and when we have IRS Increasing Returns to Scale, our cost function is like this.

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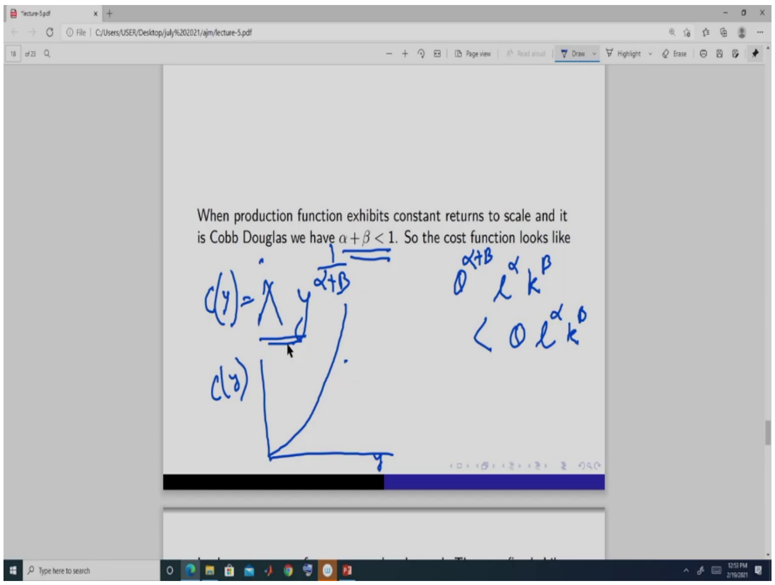

Now, suppose we have decreasing returns to scale. Decreasing returns to scale means, when we have Cobb Douglas production function if we take theta multiple of both the inputs, then this- $\theta^{\alpha+\beta} l^{\alpha} k^{\beta}$ should be less than theta where theta takes a value greater than 1, okay. So, this means that alpha plus beta should always be less than 1 if it is less than 1 then the our cost function in the case of Cobb Douglas, it is this- $C(y) = \wedge.\ y^{\frac{1}{\alpha+\beta}}$, right. So, this alpha plus beta is less than 1.

So, it will be of this nature this function, right is it I hope you are following. So, this is 1 because Cobb Douglas is mostly used in the literature. So, we have concerned, we have only, we have stick to Cobb Douglas, you can try any other form also but technique is going to be same. And next we come to a very important thing like we have discussed at the beginning that in the short run, not today in the last lecture, some of these factors can be fixed. And that is why we see something called Law of Diminishing Marginal products.

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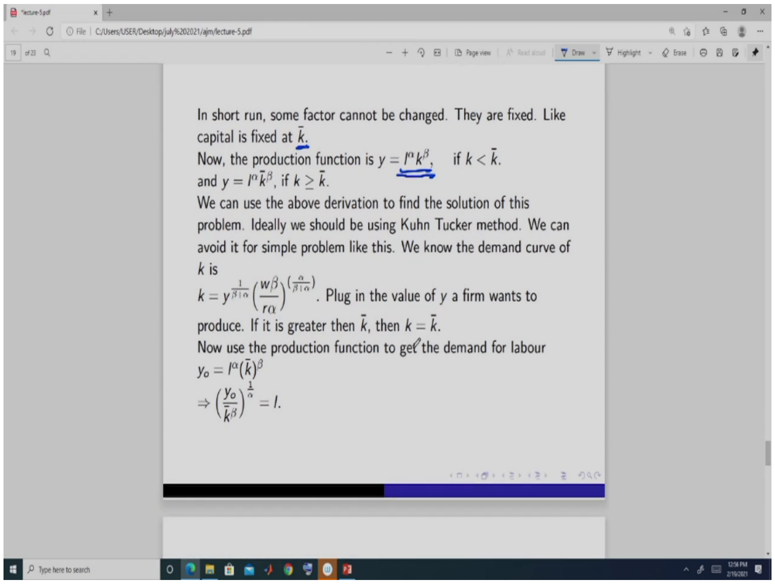

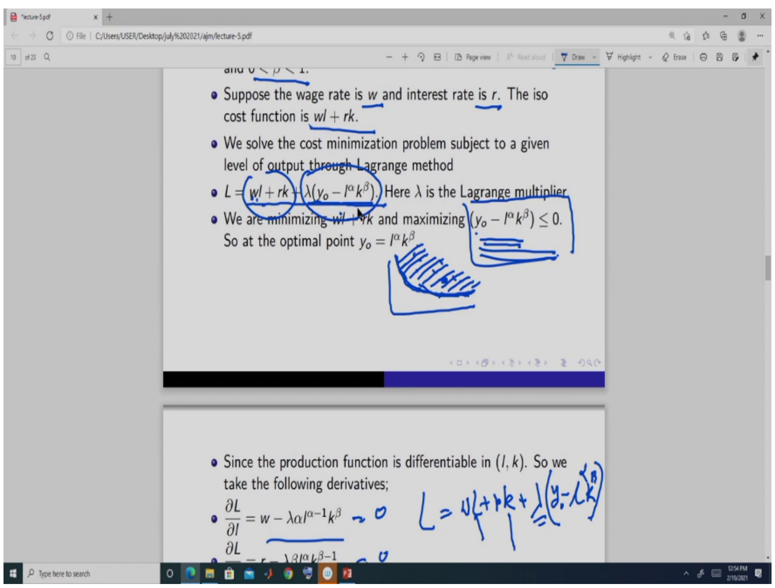

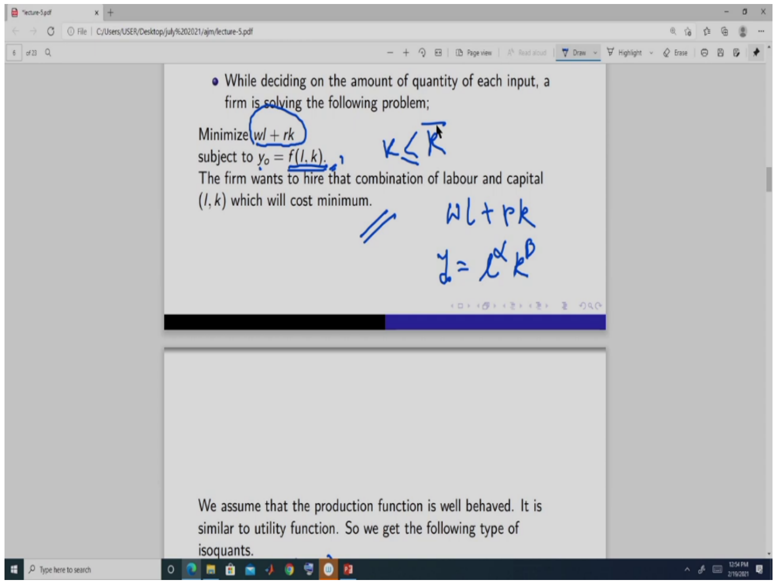

And we assume that suppose k is fixed, okay. Now let us do this problem this what we have done here we have solved this Lagrange thing- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$, we have solve this problem using a Lagrange method and this problem was minimize this- wl+rk subject to this- $y_o = l^{\alpha}k^{\beta}$. Now we add one more thing that the k has to be less than equal to k bar, okay. We cannot use more than k bar.

So, here if we are provided some kind of constrain like this then we should actually use a method that is Kuhn-Tucker method, okay, but we will not do that method, what we will do we will use a more simpler method in this case because this problem is a very simple problem because we have only two inputs labor and capital, but if we have multiple than Kuhn-Tucker would have been a better option. Because here we can do it graphically so that is why it is easier to do without using Kuhn-Tucker.

Now, suppose we keep the production function same as the Cobb Douglas. Now here the production function is like this if k is less than a then we will get production function is of this nature- $y = l^{\alpha}k^{\beta}$ l to the power alpha, k to the power beta. Now, if we want to have more k then k bar it is not possible. So, our production function is something like this l to the power alpha into k bar to the power beta. Now, if we want to increase more output, we can only change labor, capital is fixed we cannot change.

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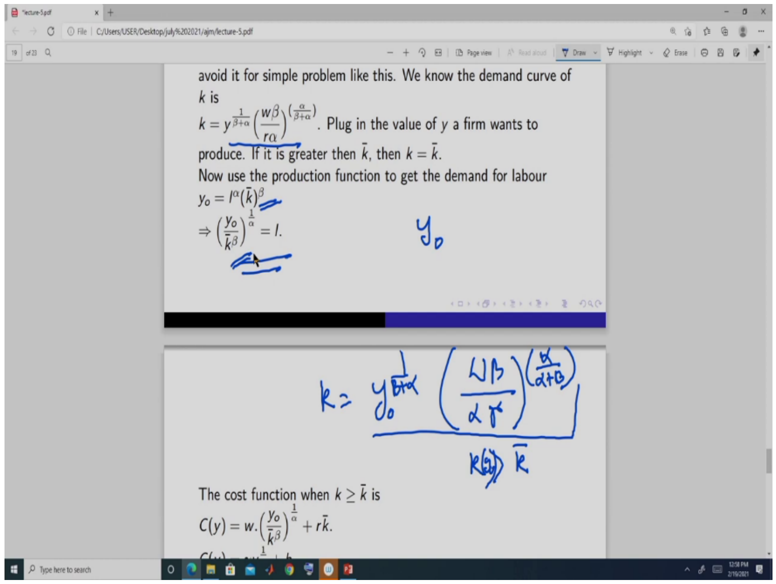

So, what we will do? So, we know from the Cobb Douglas thing the demand curve of labor is a capital is this- $k = y_o^{\frac{1}{\beta+\alpha}} \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$. Now, if we want to find out how much is going to be the demand if we have some fixed amount of output that is y naught we know it is going to be given like this where k is equal to 1, sorry k is equal to output alpha into this divided by this is power alpha divided by alpha plus beta okay. given this,

Now, we know the output y naught now plugin y naught here. Now, if so we get a specific value of k if this k is suppose greater than k naught when we have y naught output then we cannot use it, right. So, here what we can do the maximum we can use is k bar. So, plug in k bar, right. So, k is fixed now, once k bar is, k is fixed it is at k bar then we know how much amount of labor we need to employ to produce y naught amount of output it is given by the production function it is this- $y_o = l^{\alpha}\left(k^{-}\right)^{\beta}$ you plug in k equal to k bar and then you will get what? this is the demand for labor.

Now, the question is whether this is an optimal point or not, optimal point in the sense whether this is a cost minimizing point or not it may. Now, from the, since the wages and the price of the capital is fixed, and the production function is also given in this way. So, we know from the

Cobb Douglas production function that the demand for capital is going to be like this- $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}, right$. Now, since we cannot employ the optimal a, because this optimal capital because this is more than what is available to us. So, the minimum that is we can employ is this- $k = \frac{y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}}{k}$.

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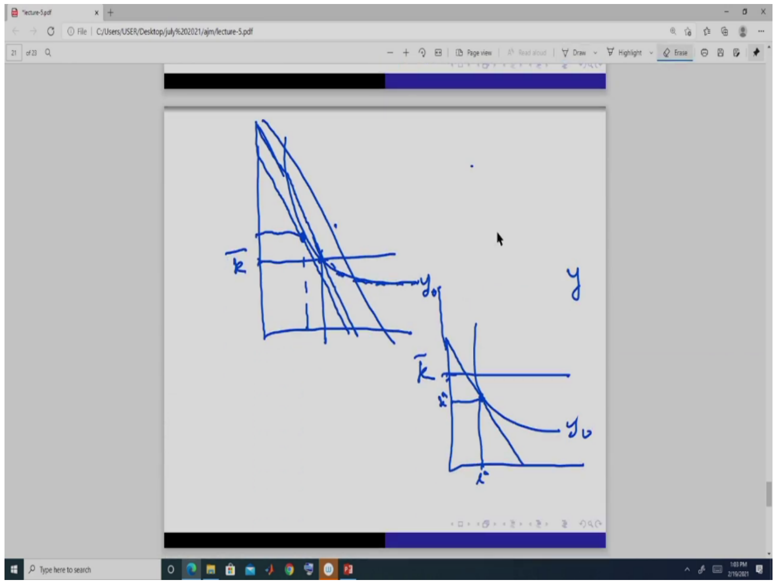

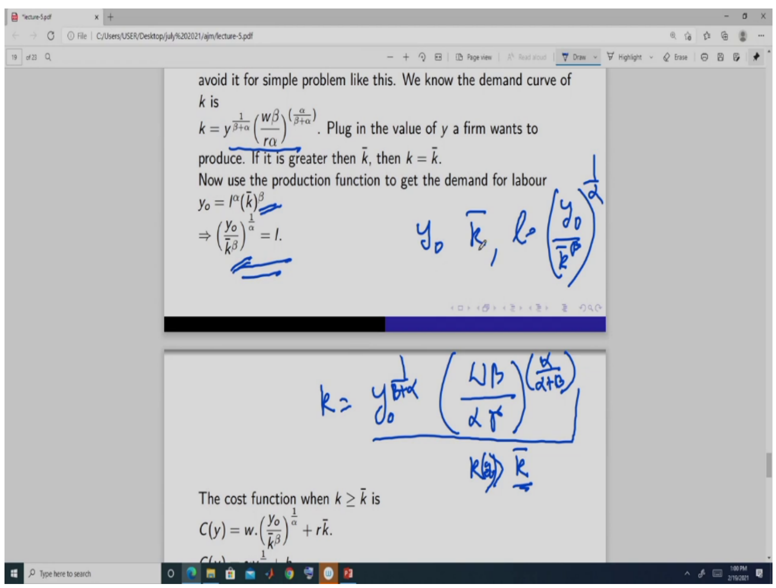

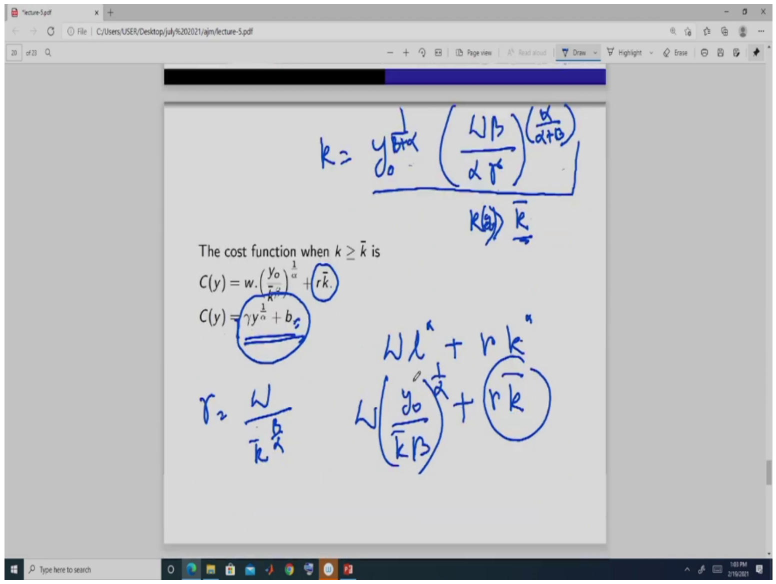

So, it is going to be something like this suppose this is the a, and suppose this is the optimal and suppose k is fixed here k bar. So, our optimal points would have been here, but we cannot have more capital than this k bar. So, we say that this is going to be our optimal point how do we find this we fix k bar and then we find the corresponding demand for labor from the production function fixing the production function at this y naught amount of output, right, this.

Now, what is possible we can choose any point from these points, right. Now, this point is at this isocost line, any point which is right to this point, only those points we can choose, right. So, it will be at isocost which is higher than this. So, that is why this point is going to be the point which will give us the minimum cost. So, that is why this k bar and l is equal to y naught divided by k bar to the power beta then whole to the power 1 by alpha, this is the demand for labor- $l = \left(\frac{y_o}{k}\right)^{\frac{1}{\alpha}}$ and based on this we will get the optimal combination of labor and capital, capital is going to be fixed.

But here in the same thing, suppose the situation is something like this, suppose, this and in this case, suppose this is the optimal point cost minimizing point when and suppose k bar is this. Now, this is going to be the demand for capital and this is going to be the demand for labor star. Now, here we can always employ this much amount of capital because our capacity is this. We already have this much amount of capital and we can take use this. But here this optimal point is not getting binded by this constraint. But here it is binding, optimal point is higher than what is possible for us to employ. So, that is why in this case we can go ahead with the previous thing.

Now, in this case what we will have to do when this is binding or not what how do we find? we in the demand curve we can plug in those specific output and from there we can find this specific demand for capital and then we compare it with our constraint amount and then if it is constraining, then we will fix it at that level and if we fixed it at that level then after that we will find the optimal amount of labor rate at this.

So, if we do that, now, our cost function is going to be what, cost function is this as the optimal point, optimal point, this optimal point is what? k bar so, this is already fixed and this is like this . So, this you can write it in this form- $C(y) = \gamma y^{\frac{1}{\alpha}} + b$ gamma into y to the power 1 by alpha plus 1 plus b, b is this portion and gamma is you can think gamma as this- $\gamma = W/k^{\frac{\beta}{\alpha}}$, right. So, our in the Cobb Douglas cost function now, become looks like this, okay.

And since alpha takes a value which is always less than 1 and greater than 0, so if we plot the cost function like this, then you will have a think if your output is here and your a, this is the cost function, okay, let us draw it in a different page.

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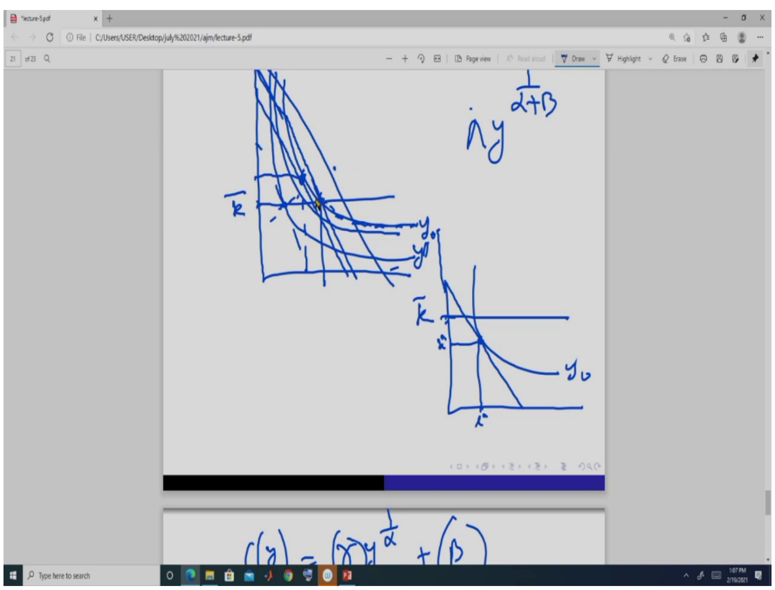

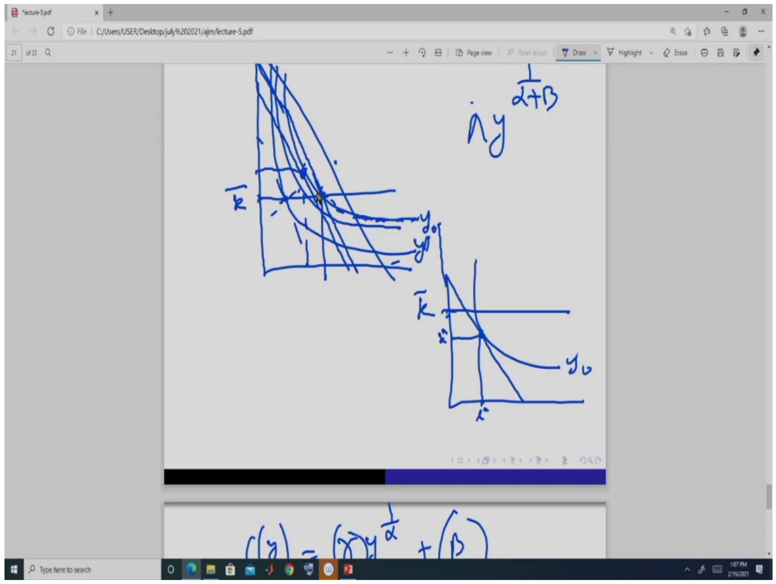

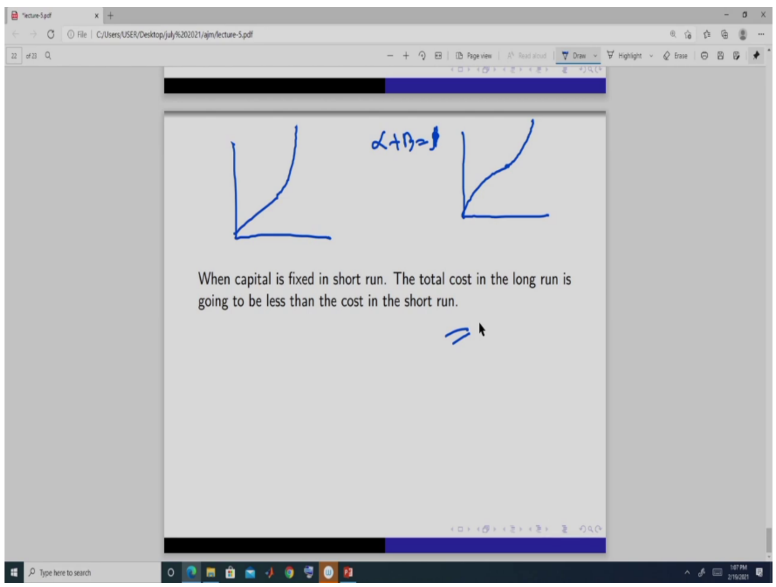

So, our cost function is now this- $C(y) = (\gamma)y + \beta$. So, this is a positive number, this is a positive number and if we take put here, so, when it is 0 it is this and as it goes on a. So, since alpha is less than 1, so, we will get a curve like this, this is our cost, total cost function. see how it got changed when we are employing this, okay. Now, here actually this cost function there is a see what is happening if we just take why I have taken see.

This cost function is only valid when our output is more than see in this case, when this is the binding point, right? if we are producing output which is less than this then what do we get? it is so, any point output which is less than this then only, right? because otherwise we will get the same cost function like this right? alpha plus beta till this y double dash level of output level.

So, actually our cost function should be if we do it properly should be like this it will be this nature right? and then it will be this nature depending on whether alpha plus beta is equal to if alpha plus beta is less than 1 and then after this it will be since it will be power is a, it will be further like this or if this is if one it will be like this and from this point it will be like this or it can be like this and after a point it will be like this.

So, our total production total cost function can be of this nature. But we will see encounter some form of costs function in this way, this form also when we do the actual industrial organization and there you can think that actually some problem like this we are facing some kind of problem like this, okay. So, from this what do we get? We get that in the short run, if we have a problem that is that the capital is fixed then we cannot go beyond it, so, we cannot use capital beyond it.

So, what may happen our optimal point which should have been this instead of that it becomes this which is fixed at this, but in the long run if we can vary the capital then it will be this, but this lies at a lower isocost line than this. So, in the long run the total cost will never exceed the short run cost. So, this is what we will, we get okay. So, the long run cost is always going to be less than or same as the short run cost, the short run cost it is possible to be greater than long run, but it will never be less than long run, okay. Thank you very much.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture 7
Derivation of Cost Curves

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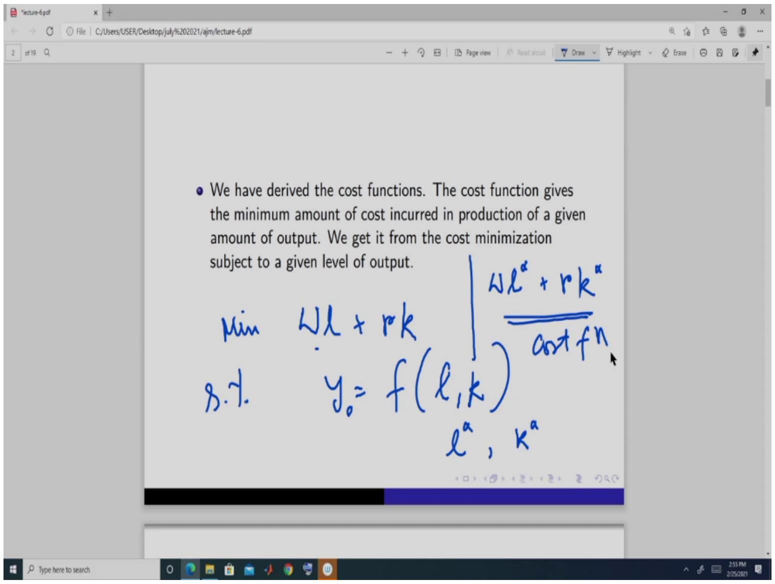

Welcome to my course, Introduction to Market Structures. So, today we are going to do cost curves. Now, we have already derived the cost function, how we have done the cost function? Cost function, we have got in the following way. We have minimize this- wl+rk, this expenditure on labor, expenditure on capital. This is wage into labor plus price capital that is interest rate into the amount of capital subject to some production function this- $y_o = f(l, k)$.

And we have specified that suppose, we want to produce y naught amount of output. Then, when we solve this, we get a cost minimum amount of labor and cost minimum amount of capital. And when we plug in this, here in the objective function-$wl^{\alpha} + rk^{\alpha}$ we got this with a cost function. We have done that in the last class. So, cost function gives me the minimum amount of cost incurred to produce a fixed amount of output, okay, or given level of output.

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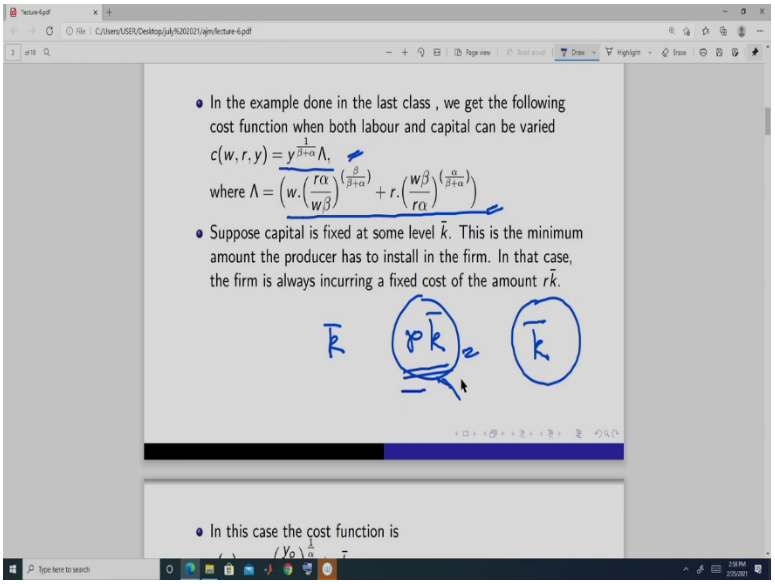

Now, if we take this as a Cobb Douglas production function, then we get a cost function of this nature- $c(w, r, y) = y^{\frac{1}{\beta+\alpha}}\Lambda$, where this capital lambda is actually given by this expression- $\Lambda = w.\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$, we have derived this in the last class. So, what do we get? We get that our cost function is actually is a function of the output that we want to produce and the price of labor and the price of capital. But since this firm always takes the price of capital and labor as given, this firm cannot determine these prices.

So, we take this cost function to be only a function of output, okay. Now, so, we get a cost function of this firm. When both these inputs, labor and capital are divisible and you can employ any amount you want to, want to, to produce that output, right? So, none of these labor input is fixed. But in short run, we know that capital can be a fixed input. We cannot use more than a given amount of capital. So, in that case, what happens this suppose capital is fixed at k bar, then price of capital is R.

So, this become a fixed cost, whatever amount of output you produce, you have to bear this cost. Then this component, because you need some amount of capital and you know that capital is only available in suppose chunk, capital is not divisible continuously. So, in that case, suppose the minimum capital you want to have is this, you need to have is k bar. So, in that case, you will always incur this much amount of costs, whatever amount of output you want to produce.

Now, suppose you want to employ more capital. So, you will further increase your amount of capital, but that will take time. So, in the short run, only this amount is available. So, now, if you want to produce more and more amount of output, what you can do? You have to employ more and more labor. So, in this case, we have seen that we get a cost function of this nature-

$$c(y) = w.\left(\frac{y_o}{\bar{k}^{\beta}}\right)^{\frac{1}{\alpha}} + r\bar{k}$$

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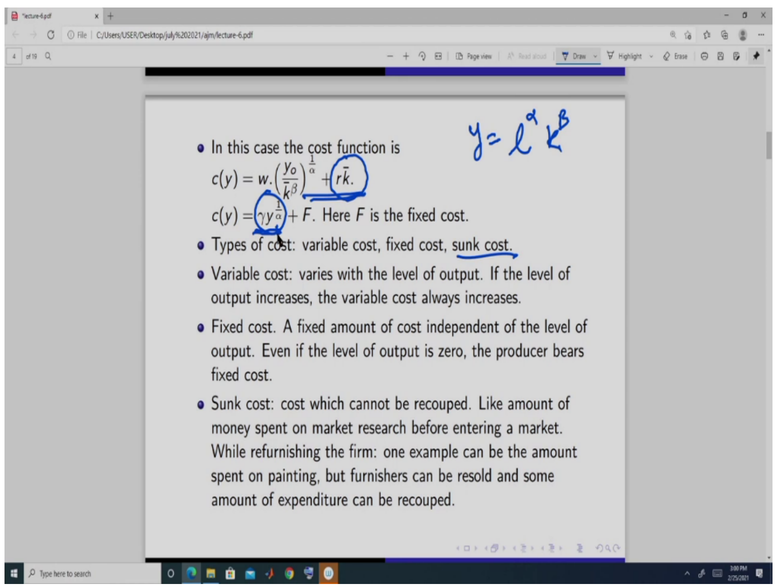

If our production function is a Cobb Douglas production function that is of this nature- $y = l^{\alpha}k^{\beta}$. Right? We have seen that, now here you can say that this portion is a fixed cost, and this is a variable cost, okay. So, our cost function may have two component, one is the variable cost, another is the fixed cost. And further we may have another type of cost and that is a sunk cost okay. So, first what is a so, what is a variable cost?

Variable cost is that cost, when you want to change your output that cost also changes. That means, that if you want to increase your output, you will require more input, at least one of the inputs needs to be employed in more quantity. So, the cost in that related to that input is going to go up. So, that component is giving you the variable cost. So, here it is this part- $\gamma y^{\frac{1}{\alpha}}$, okay. And if you want to reduce your output, then you are not going to employ that much amount of input.

So, in that case you will reduce your employment of that input. So, you cost, you are going to incur in that factor or input is going to go down. So, this portion is going to change. So, it can change when you increase your output. So, the cost will go up, when you decrease your output, the cost is going to go down and that is going to happen in this portion. Fixed cost is this given-F, you cannot, the moment you keep on increasing output, this cost is going to stay same, it is going to be fixed.

If you reduce your output, even if you are producing zero units of output then also you will incur that cost and that is this f fixed costs. Sunk cost is another type of, it is a fixed cost only it is a cost which you cannot recoup. Like in fixed cost if you have seen, that we have this capital these machines, now you can resell this machine if you are just going out of the business, or if you think that you are not going to produce this much amount of output, you are going to reduce your capacity.

So, you do not require that much amount of machine, then you can sell off some of your machines and you will get some price. So, you will get some money from that. But suppose, you have entered this market and before that you have done some market research. And that has cost you some amount of money. Then that cost you cannot resell that thing, anything. So, that is a sunk cost or suppose, you have refurnished your plant.

So, you have put new paints in this say. So, this paint you cannot sell to anyone, because you have painted your firm. So, this is going to be a kind of sunk cost, you cannot get any return on it. If you have incurred it, it is gone. So, you have incurred that cost. But if you think of furnitures or if you think of machines, then you can resell them. And in that case, you while reselling you will get some money from them, while reselling.

So, you will be selling them at some price. So, at least there is some way of generating some amount of money from by reselling. But if you incur the cost of this nature like painting your firm or market research, those kind of thing, then you cannot resell those things. So, that is why, these are kind of sunk cost, okay. But we will not explicitly require sunk cost in this chapter. We may introduce sunk cost when we do entry deterrence later in this course, okay. So, we have seen this different type of cost.

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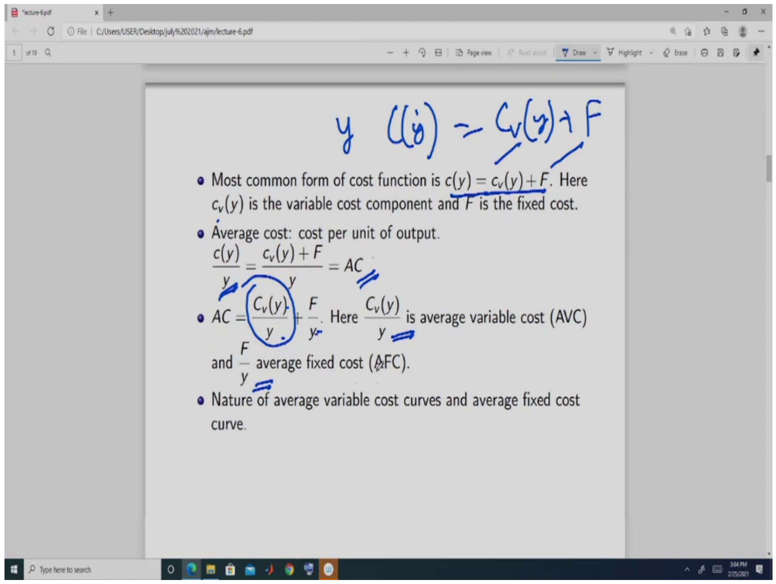

And most commonly, we are going to use this type of cost function- $c(y) = c_v(y) + F$, where this part is the variable cost and this part is the fixed cost, okay. So, since this is variable, so we have put a subscript v and this fixed, so, we have put it v, this F. Now, average cost is so, this function $c(y) = c_v(y) + F$ is giving me the total cost. If I want to produce y unit of output. So, total cost is given by this function. And it is this- $c(y) = c_v(y) + F$, it has a variable component and a fixed component.

Now, average cost is the cost per unit of output. So, it is total cost divided by the amount of output I am producing, this- $\frac{c(y)}{y} = \frac{c_v(y)+F}{y} = AC$. So, this when you open it up this total cost is variable cost plus fixed cost divided by the total unit of output. This is called the average cost and we write it as AC, denoted it as AC that is the average cost. Now, AC you can see, it is this - $\frac{c_v(y)}{y}$ plus this- $\frac{F}{y}$. So, this part is called the average variable cost, and this part is called the average fixed cost, okay.

So, average cost is sum of two type of costs, that is average variable cost plus average fixed cost. Average variable cost gives you the average cost coming from the variable component. And this is coming from the fixed cost component, okay. Now, we look at the nature of these two functions okay.

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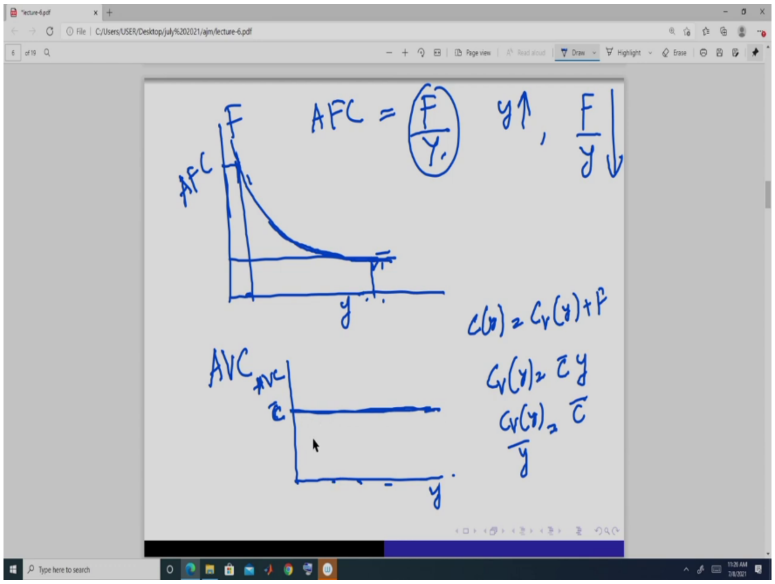

So, fixed cost is fixed, whatever be the amount of output, it is fixed. So, average fixed cost, AFC is this divided by units of output. Now, if we plot output in this axis, and average fixed cost in this axis. Then you will see that this curve is going to be something like this, it will go down asymptotically hit y at infinity. And at output equal to 0 this will take a very high value. So, average fixed cost is always going to go down.

Because as output increases, because as y increases this f is fixed. So, this portion is going to go up so, this is going to go down. So, it is going to be something like this, right. So, this is the average fixed cost. So, fixed cost goes on decreasing. So, it means that more amount of output gives you less amount of average fixed cost, less amount of output gives you higher unit of average fixed cost, okay. Why? Because the fixed cost is same, so, the more you are producing, so less per unit it is costing, okay.

Now, we looked at the average variable cost, okay. Suppose, the average variable is of this nature. When we can have this kind of average variable cost? This is the variable component- $c_v(y)$ and there is a fixed cost-F. And suppose, the variable component is of this nature- $c_v(y) = cy$. So, it is a constant. Then this is the average variable cost and this is equal to this, i.e $\frac{c_v(y)}{y} = c$. So, it is constant. So, if this is independent of the output of the firm, whatever may be the output it is producing, its average variable cost, that is cost per unit of output on the variable input it is remaining constant.

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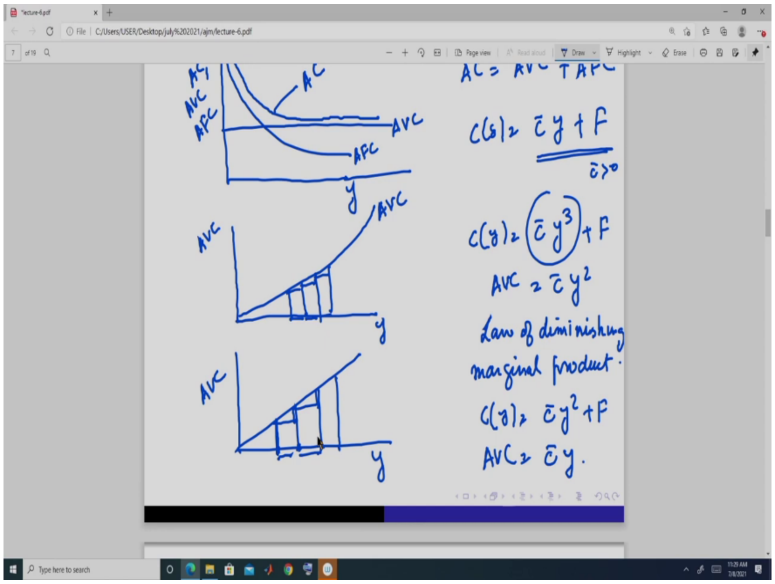

So, here if we have that now, what do we have? Our average fixed cost is this, average variable cost is this. So, this is our average cost, because average cost is sum of average variable cost plus average fixed cost in this axis, we have taken output and all the cost, average cost, average variable cost, average fixed costs are in this axis. So, we get a curve and this average cost is going to be asymptomatic to this here.

So, we get this kind of average cost curve, average variable cost curve. For example, if we have the cost function is of this nature- c(y)= cy+F, where c bar is some positive number. Now, we may have a different set. Now, average fixed cost is always going to be of this nature. Our average variable component of this cost coming from the variable component, that can be of this nature also, this is can also average variable cost.

When we can have suppose for example, our is this- $c(y) = cy^3 + F$. So, this is the variable component. So, the average variable cost here is a c bar y square. So, it is this, it is increasing. And when do we get such kind of one, when one factor is already fixed. So, due to law of diminishing marginal product. So, what is happening? As we are increasing the output, per unit cost is increasing , right? here. So, this means, what? This means, we are actually employing more of this a, to get this get same increase in output.

And this is why because the marginal product is decreasing, so we need to employ more and more of input to get the same additional amount of output. So, that is why our average variable cost is increasing here. One another example of this kind of a can be this- $c(y) = cy^2 + F$, where AVC if you see, it is this and it is a straight line, upward sloping straight line.

Here also you see that as we go on increasing the output, the average cost or average variable cost per unit of that is here, average variable cost is higher, here it is higher than this, this is higher than this. So, actually what is happening? Because the marginal products are decreasing, so we are employing more and more input. So, that is why the cost is increasing.

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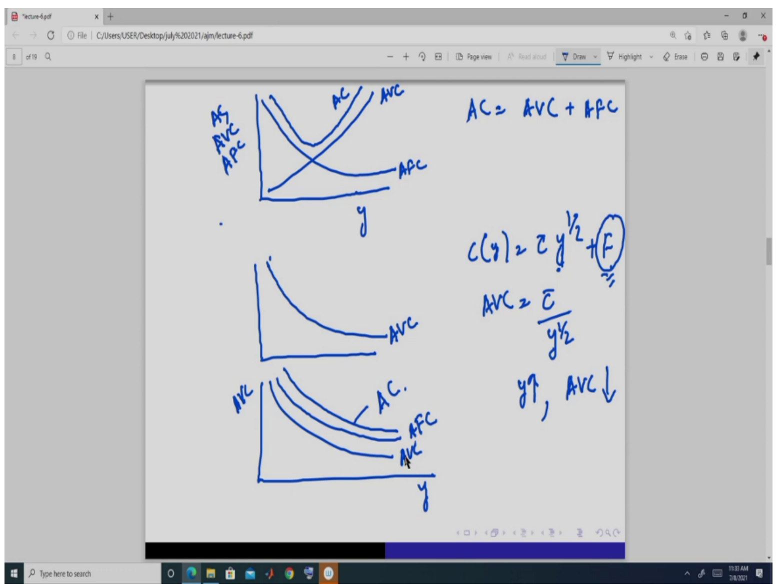

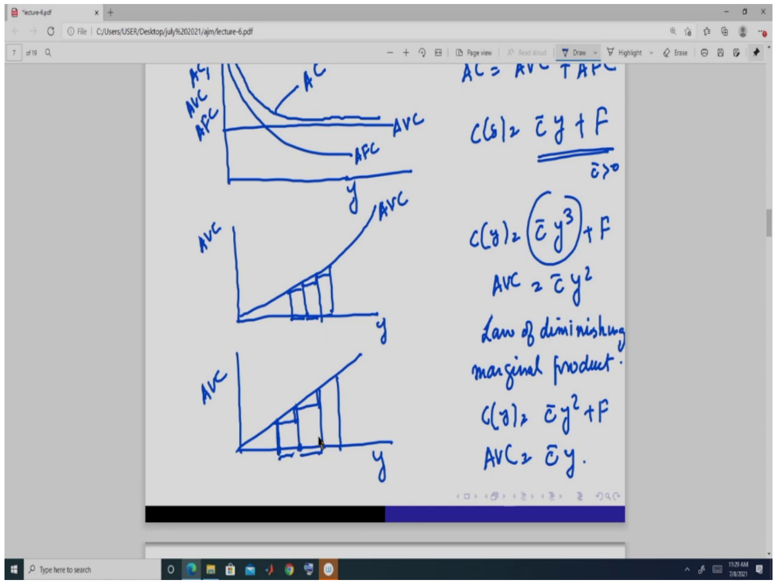

Now, if we have average variable cost of this nature which are always upward sloping, then the average cost is going to be of this nature, average variable cost is suppose this, we know average fixed cost is always this. So, the average cost is of this nature. Because average cost is sum of average variable cost plus average fixed cost. So, we get a u shaped average cost when, when do we get a u shaped average cost?

One possibility is, when the average variable cost is always increasing and since average fixed cost is always decreasing. So, this sum of these two will give us a u shaped average cost, okay. Now, another form of this is going to be of this nature. Suppose, the average variable cost is of this nature. When do we get this? One example, can be of this- $c(y) = cy^{\frac{1}{2}} + F$. Here average variable cost is this-$\frac{c}{y^{\frac{1}{2}}}$. So, this means what? That as y increases AVC goes down.

Now, these when do we get such? Now so, the problem here we have a fixed component also, fixed cost. Now, here if we, we can obviously say that if we have increasing returns to scale, then we may have a this kind of variable cost component. But since we have fixed cost, so, here we can say that in few, suppose, this is the plant size, the fixed cost is coming from the plant size and it is fixed given, and or you can say think of like the building, the building is fixed.

But you can vary the labor and the machine. So, the cost of the rent that has incurred in the hiring that building or that plot of land it is fixed, but, you can get different combination or technology or different technique by combining different amount of labor and machine. And that is coming from here. And you are having suppose increasing returns to scale in this return. So, this can be one possibility, when we have a situation like this.

And since average variable cost is of this nature, and we know so, in this situation, our average variable cost is this. And suppose, the average fixed cost is this, then the average cost is this. So, average cost is always downward sloping, when the average variable cost is also always downward sloping.

Now, we have got this are the nature of average variable cost, an average fixed cost and, and some of these two giving us the average cost of this nature, when we have only one form of average variable, means either it is strictly it is constant like this or it is strictly increasing like this or it is strictly decreasing like this.

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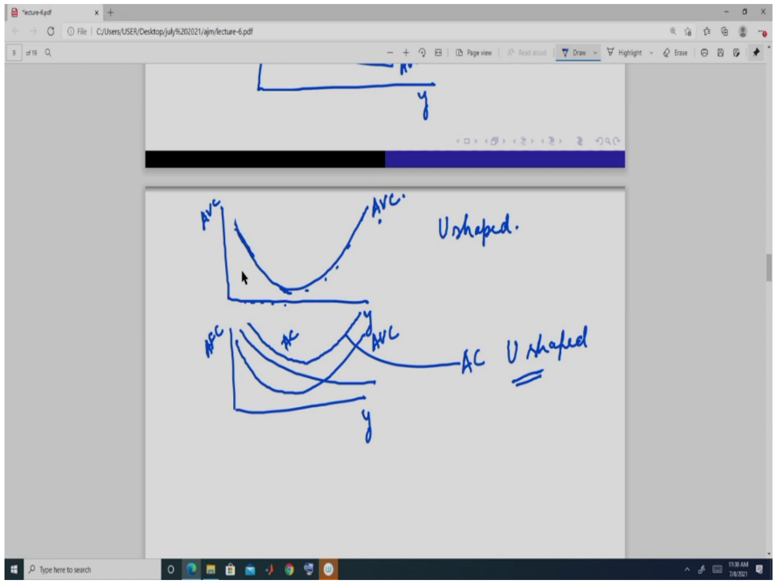

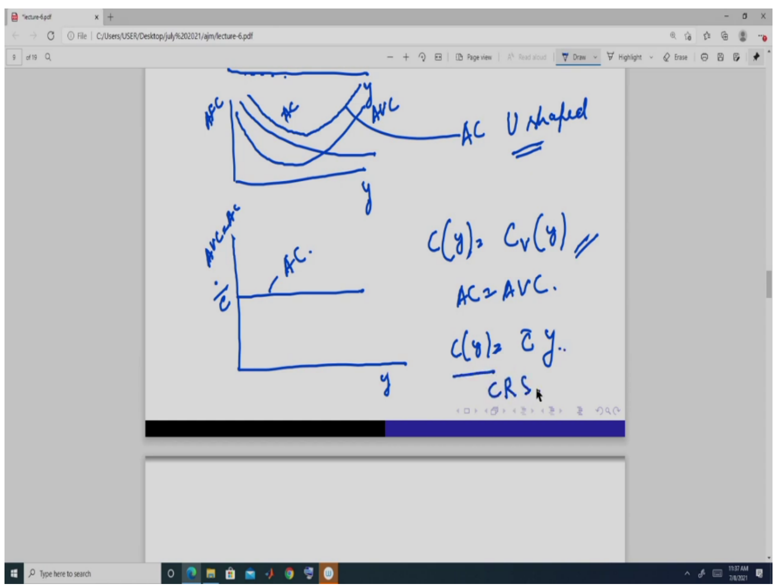

But we may have a situation where, our average variable cost is u shaped, it is like this. So, in this kind of situation again, what we do? suppose this is our, our building is fixed, our plot is fixed. And we are paying the same fixed amount of rent for that. Now, initially when we are combining the labor and capital in some combinations, we the production function is such that it is giving us in some form of an increasing returns to scale.

And so, that is why it is, it is so, here when we are seeing increasing returns to scale we are fixing the land or the building and only we are wearing two inputs, that is machines, that is capital and labor, labor, okay. And then it is giving me. So, what is happening? Average variable cost is going down, as we go on increasing output the average cost. So, the cost per unit of output that is going down.

Because we are employing less and less amount of labor and capital to get the same additional amount of output, okay. And that is because we are having some form of an economies of scale. And then, after a point it starts increasing and it is of this nature. And it is here mainly, because now the land size is or the building is fixed. And if we keep on increasing the machines and the labor, then it becomes congested.

So, it is, we are going to get some form of a diseconomies of scale. And that is why it is average variable cost is increasing. So, if we have a average variable cost of this nature, that is u shaped, it is this nature. And since, the average fixed cost is always like this. And suppose our average variable cost is this, then the average cost is going to be of this nature. So, this is, so again average cost is u shaped.

So, we get u shaped average cost curve, because of two reasons. One, when the average variable cost is always increasing, and when the average variable cost is u shaped, that is it has, it exhibits some form of economies of scale and then it has some form of diseconomies of scale. So, we get this. Now, suppose we can have another thing like, suppose our cost function is simply this- $c(y) = c_v(y)$. We do not have any fixed component.

That means none of the factors are now fixed, you can vary all of them. Then we may have a situation like this, this is the average cost. Now, here if you look at this average cost is same as average variable cost, because there is no fixed component. And if we have this, then this is something like, and this is c bar. When do we have a situation like this? So, this is when we have CRS, constant returns to scale.

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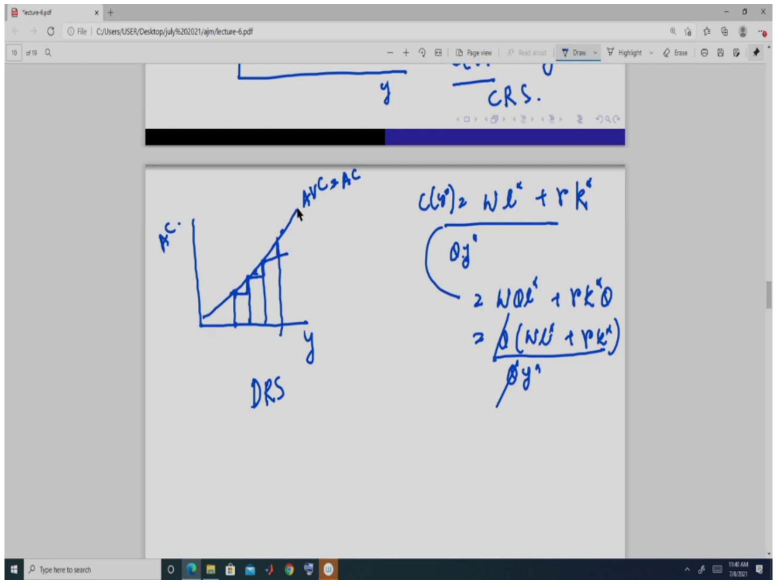

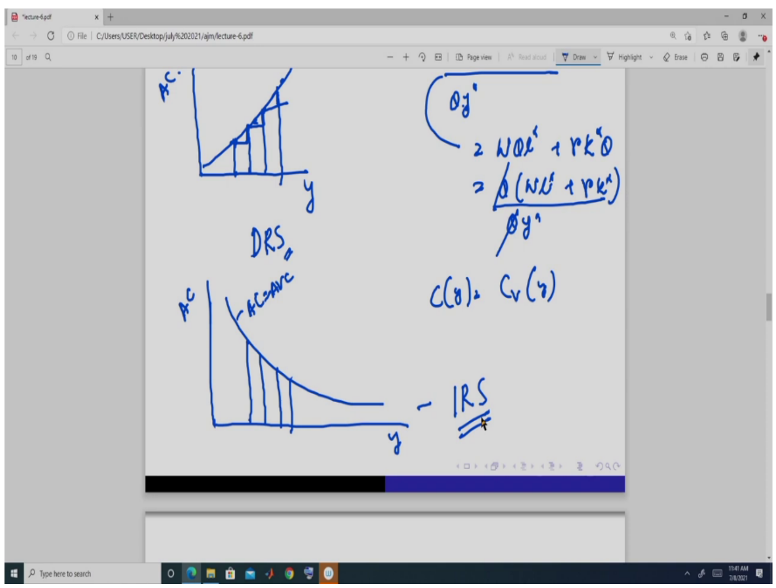

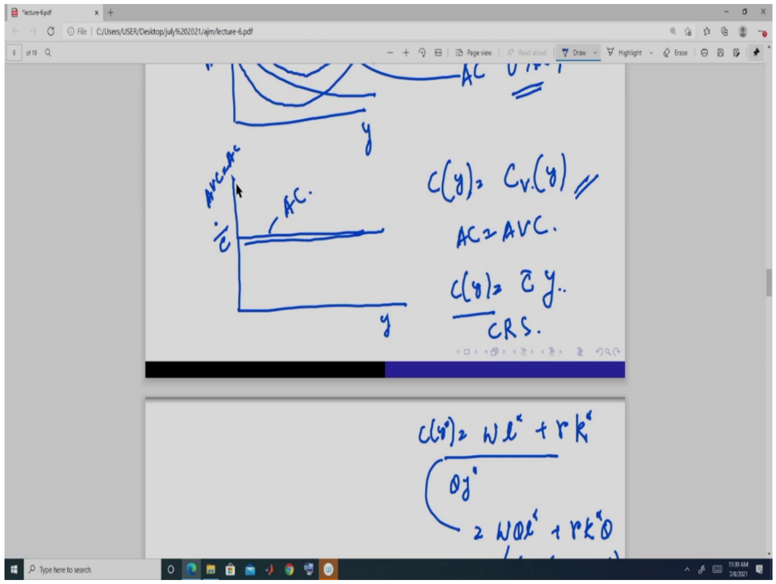

So, that means that as we are increasing, so, your cost function, this is actually what? L star, this- $c(y) = wl^{\alpha} + rk^{\alpha}$. Now, for a given fixed level. Now, you increase the a. Suppose, we want to now increase it to theta this- Θy^{α}. So, this will be what? From CRS we know, it is going to be because that is why it is CRS. So, it is this- $c(y) = \Theta(wl^{\alpha} + rk^{\alpha})$. and then this- $\Theta(wl^{\alpha} + rk^{\alpha})$ divided by this- Θy^{α} , so this cancel so it remains same. So, that is why it is a CRS.

So, this if we have a cost function of this nature only, then the, and when average variable cost is same as average cost. So, it is a situation where there is no fixed cost. Then this shows that there is a constant returns to scale. Another thing is, suppose we have a situation like this. And our average variable cost is suppose equal to average cost and it is like this. So, here what you see, as you are increasing the output per unit cost, average cost is increasing.

It is here, now it is this much. So, this much increase in addition per unit cost, this much increase in per unit additional increase. So, that is why it is increasing, average variable cost or average cost it is increasing. So, we get this, when suppose we want to increase the output by theta unit. Now, if we have decreasing returns to scale, what is going to happen? We, to get suppose you want to increase output 2 by 2 times.

Then you have to employ the labor and capital more than two times. So, that means, what? Your cost is going to be more, if you want to increase your output by 3 times then you have to employ labor and capital by more than 3 times. So, if you look at this situation, so, this denominator, numerator- $\Theta(wl^{\alpha} + rk^{\alpha})$ is increasing by more than the theta times, then the in a. So, that is why, you will get a increasing average cost curve.

So, when our production exhibits decreasing returns to scale, okay. Another is this, AC is this, when we have only this- $c(y) = c_v(y)$, we do not have any fixed cost. In this situation, what is happening? As the output is increasing, our average cost that is cost per unit of output it is going down. So, when do we have this situation? When we have IRS, increasing returns to scale. When we have increasing returns to scale, it means that if we increase the output suppose now two times, then we do not need to employ the labor and capital two times.

We can get two times output, twice the output with less than two times labor and two times capital, or when you want to increase the output by some theta times, we do not require to employ labor and capital as theta times of labor and theta times of capital. We can get theta times of output less than the theta times of labor and theta times of capital. So, that is why the cost is going down.

So, because we are getting increasing returns to scale so that is why, our average cost is going down. So, we get a situation like this, when we have increasing returns to scale. So, these are the different forms of average cost curve and average variable cost and a average fixed cost.

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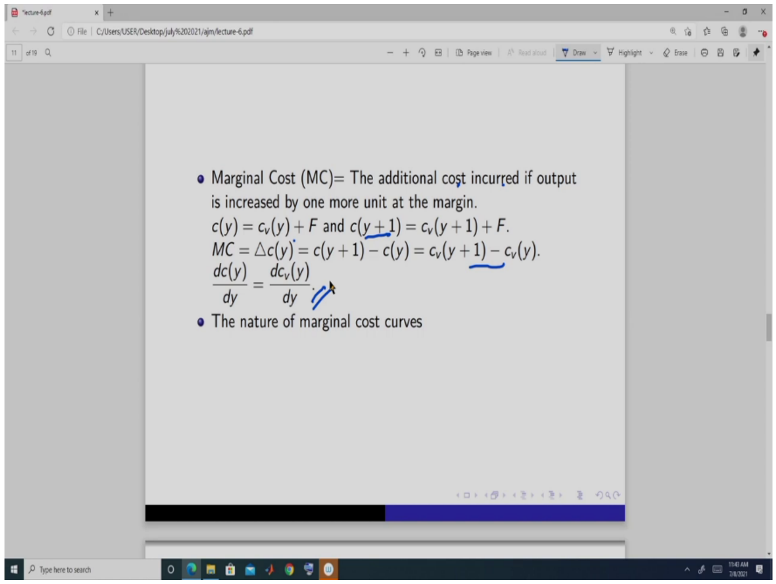

Now, we move to another concept and that is the marginal cost, what is the marginal cost? Marginal cost is the additional cost incurred if output is increased by one more unit at the margin. Average cost is, cost per unit of output. Marginal cost is, if we increase one more unit of output at the margin suppose, we are already producing 10 units of output. Now, you want to produce the 11th unit, then how much additional cost we are going to incur?

So, it is supposed like this- $c(y) = c_v(y) + F$, and then we increase- - $c(y + 1) = c_v(y + 1) + F$. So, the difference between these two is going to be this- $c_v(y + 1) - c_v(y)$. So, actually this comes from the variable component, fixed cost is to fixed. If we have a cost curve function of this nature, or if we simply take the derivative of this- $\frac{dc(y)}{dy} = \frac{dc_v(y)}{dy}$we get the cost at the margin, okay. So, at the margin, we simply taking the derivative and plugging in that output, we will get the.

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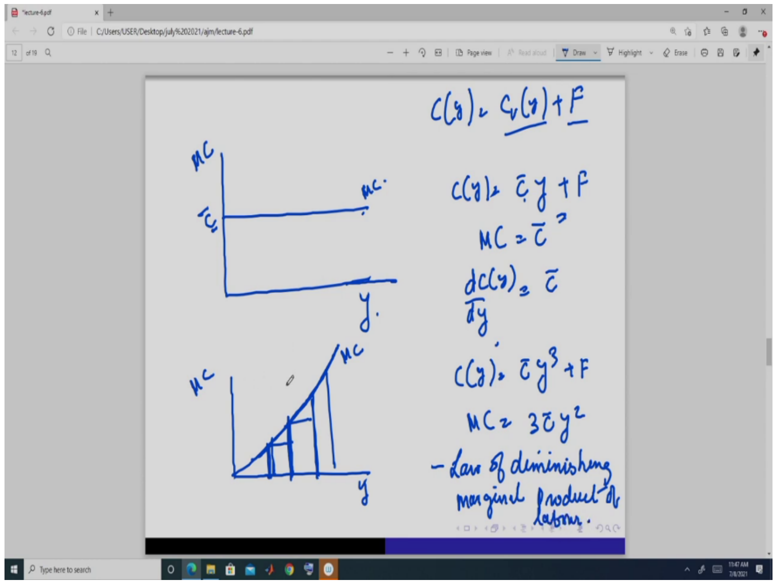

Now we will look at the nature of marginal cost. Now suppose, our cost function is of this nature- $c(y) = c_v(y) + F$. So, we have a variable component and we have a fixed component, okay. So, this is coming from this suppose fixed plant size, and this from differing different combination of labor and capital, or suppose the capital is fixed and you are simply this is the cost you are incurring in the fixed capital and you can only vary the labor and that is coming from this.

Now, if our, is this nature- $c(y) = c_v(y) + F$ then marginal cost is simply this- MC=c. Because you take the derivative of this, we get this-$\frac{dc(y)}{dy} = c$. Now, this is when we have a marginal cost is constant, fixed. Whatever may be the input you are employing, you are getting the same cost, you are incurring the same cost at the margin, it is not changing, okay. This can be one form. Another form here, can be of, if we take this.

Because, suppose this is this- $c(y) = c\ y^3 + F$, then marginal cost is how much? So, it is upward sloping. This is the marginal cost- $MC = 3cy^2$, . So, as we go on increasing the output, our additional cost that we are incurring at the margin that is increasing, right. From here to here, we are increasing this much. From here to here, we are, output increase. So, and we get this when we have a cost function of this nature, it is mainly because of law of diminishing marginal product.

Easiest way to explain, because we have capital is fixed and we require more and more labor to produce the same amount of output, one more additional unit of output. Because marginal product is decreasing. That means, if I increase one more unit of labor, the additional output we are getting, that is going down. So, I have to employ more labor to produce one unit, as we go on increasing more and more output. So, that is why, we get a this kind of thing, and it is mainly because of law of diminishing marginal product of labour, you can say of labor.

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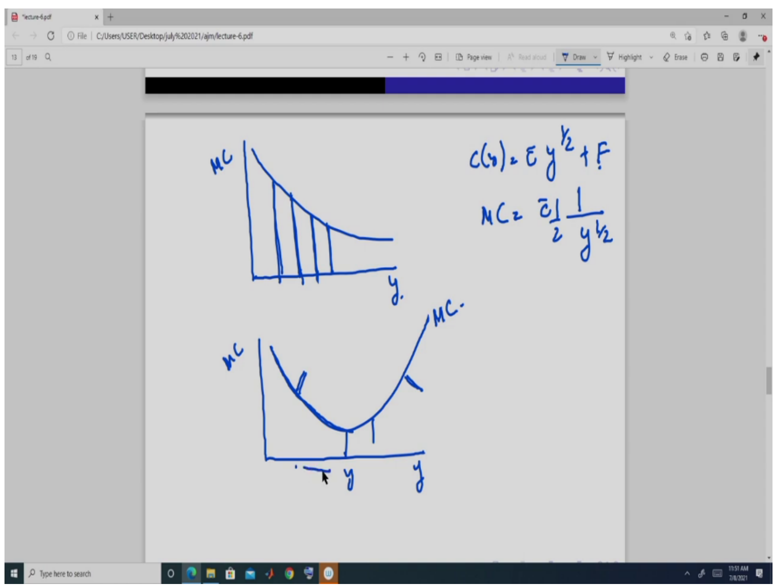

Another form of marginal cost, can be this nature, when suppose our is this- $c(y) = cy^{\frac{1}{2}} + F$. So, marginal cost here is- $MC = \frac{c1}{2} 1/y^{\frac{1}{2}}$. So, what is happening? As we are increasing the output, at the margin the additional cost we are going to bear, that is going down. This, as we have explained earlier, when we take this to be a component, fixed component coming from the plant or from the building or from the rent that we pay for the land or for the building.

And suppose we can vary both labor and capital and our production is and so that exhibit suppose some form of economies of scale here. So, what do we get here? As we go on increasing the output so additional cost we are going to incur it is going down, okay. But since we have this fixed component, so we do not say exactly that this is mainly because of increasing returns to scale because we cannot vary all the factors.

But we get a idea, that suppose the plant size is fixed or the land is fixed and all other inputs are variable like labor and machines or labor and capital, these are variable. Then, we get, we may get a situation like this, okay. So, this is simply take the derivative of this you will get this. So, we get a marginal cost of this nature. Another form of marginal cost, that we may get is of u shaped. So, the argument is same.

So, till this level of output, our additional cost incurred to produce one more unit of output at the margin, that is going down. So, here we are getting some form of economies of scale and then here we are getting some form of diseconomies. So, the marginal cost is starting, why? Because suppose your plant size is fixed, so as you go on employing more and more labor and capital, more machines, then what is happening?

It is becoming more congested and then your law of diminishing marginal product may operate. But, when I say law of diminishing marginal product, it has a specific meaning. It means, keeping all the other factor fixed if we vary one input. So, we cannot directly use that argument. So, what we are saying, the idea is something like this that since, it is more congested means, subsequently if we can increase the plant size also or if we are looking at suppose agriculture cultivation, we are employing more machine, more labor, but if our fixed, land size is fixed, then after a point we do not get any additional benefit.

And our additional cost is go on increasing. But, if we can vary the land size also, then we may not have that situation, but as of now, we are keeping that land size is fixed. So, we are getting a u shaped. So, for this reason our costs are going down, marginal costs are going down as we go on increasing the output. And after this our marginal costs so that is the additional cost is going up. This is one.

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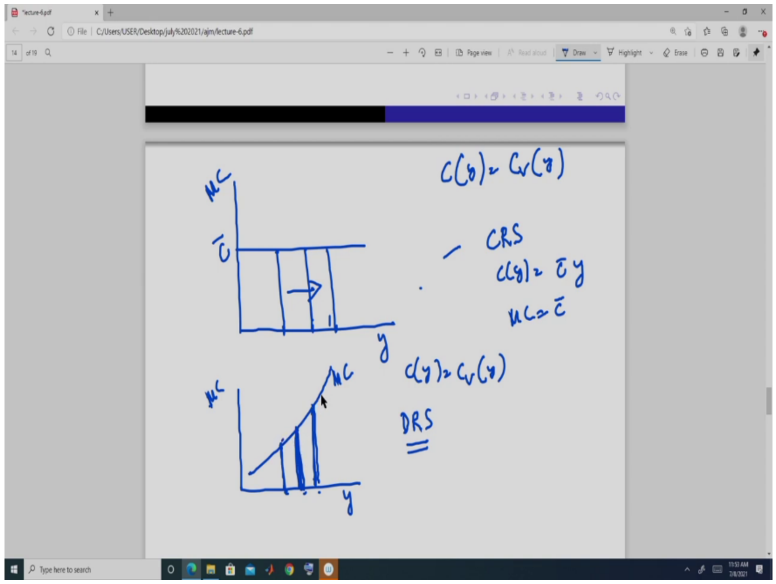

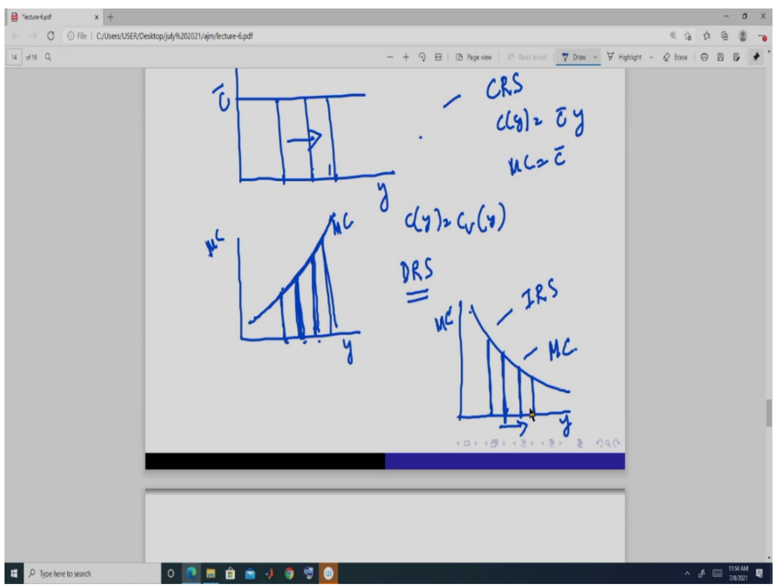

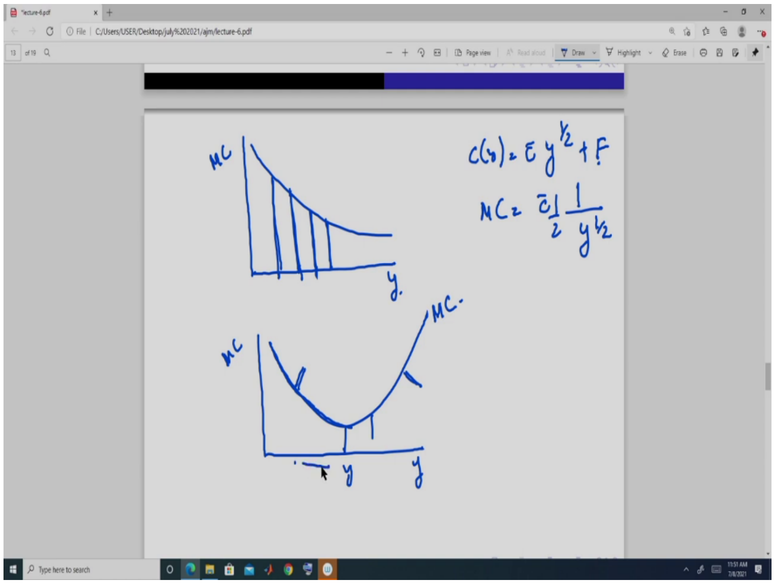

Now, when we have a cost function of this nature- $c(y) = c_v(y)$. So, we do not have any fixed cost. So, all the factors are variable. Then, if we get a situation like this, then this is CRS. We can directly say it is CRS. And we get a situation, CRS, this. And you can do, if you get it from the simply cost minimization problem and you will get that it is a CRS kind of thing. So, what is happening? Whatever may be the output you go on increasing the output, your marginal cost is same.

So, the idea is same, that is you employ labor and capital in the fixed ratio, and you go on doing that. If you want to increase your output two times, you have to increase your labor and capital also two times, okay, whatever be the level of output. So, that is why the marginal cost remains same. So, this is CRS things. Now, if our marginal cost is this, and our cost function is of this nature- $c(y) = c_v(y)$. , so we do not have any fixed cost.

So, then we have a decreasing returns to scale and that is DRS. Why? Because as we go on increasing the output, the additional cost that we are getting, that is higher this. So, because as we go on increasing, the output, what do we need? We need to employ more labor and more capital, if we want to double the output, we need to employ labor and capital more than double. So, that is why, marginal cost also goes on increasing, okay. So, we get a curve like this.

And another form that we may get, it is of this nature, it is this marginal cost, this is the marginal cost curve, the output. This we get, when we have increasing returns to scale. Because as we go on increasing the output, here our additional cost is going down. Because, as we are increasing more and more output, suppose we are increasing output two times, three times now, we are employing less than three times labor and capital. So, that is why the marginal cost is going down, okay. So, these are the nature of marginal cost. And another marginal cost is of this nature u shaped.

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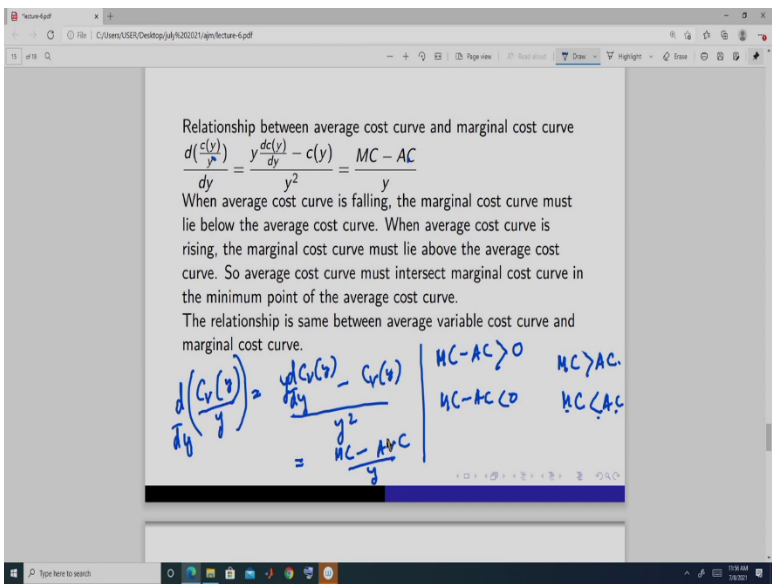

Now, we look at the relationship between marginal cost and the average cost. Now, if you look at this- $\frac{d\left(\frac{c(y)}{y}\right)}{dy} = \frac{y\left(\frac{dc(y)}{dy} - c(y)\right)}{y^2} = \frac{MC - AC}{y}$, it is what? This is the derivative of the average cost. So, derivative of average cost with respect to output is giving me this-$\frac{MC-AC}{y}$. So, this means what? So, this portion, this is positive when marginal cost is greater than average cost. So, that means whenever average cost, this is what? Slope of the average cost, average cost is decreasing out, this has to be increasing.

Then marginal cost is greater than average cost. And when we have this, this means marginal cost is less than average cost. So, whenever slope of average cost is negative, then is marginal cost should be less than average cost. Now, here if you look at this situation-

$\frac{d\left(\frac{c(y)}{y}\right)}{dy} = \frac{y\left(\frac{dc(y)}{dy} - c(y)\right)}{y^2} = \frac{MC - AC}{y}$, instead of this if you take this- $\frac{d}{dy}\left(\frac{c_v(y)}{y}\right)$ you will get the same thing. So, you will, this- $\frac{\frac{ydc_v(y)}{dy} - c_v(y)}{y^2}$ which is equal to marginal cost minus average cost divided by y. So, it is same, right, here it is AC because we have taken this the average cost, this we have taken average variable cost so that is why it is AVC, average variable cost, okay.

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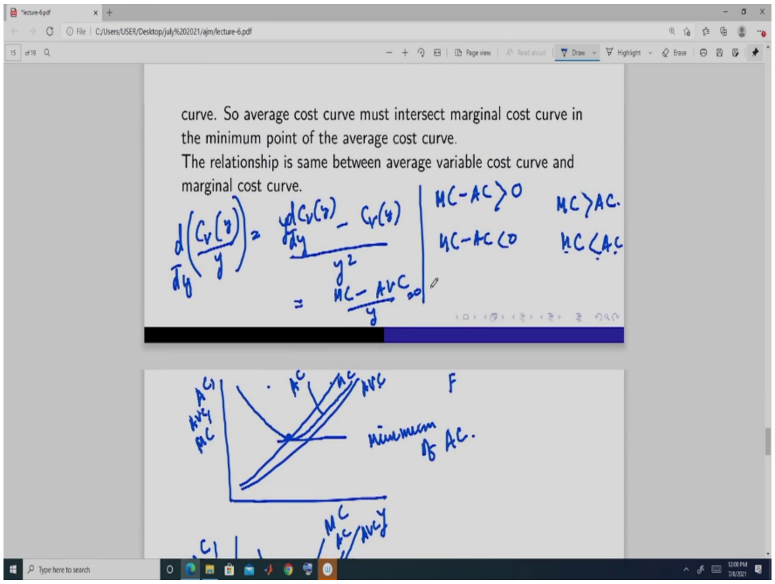

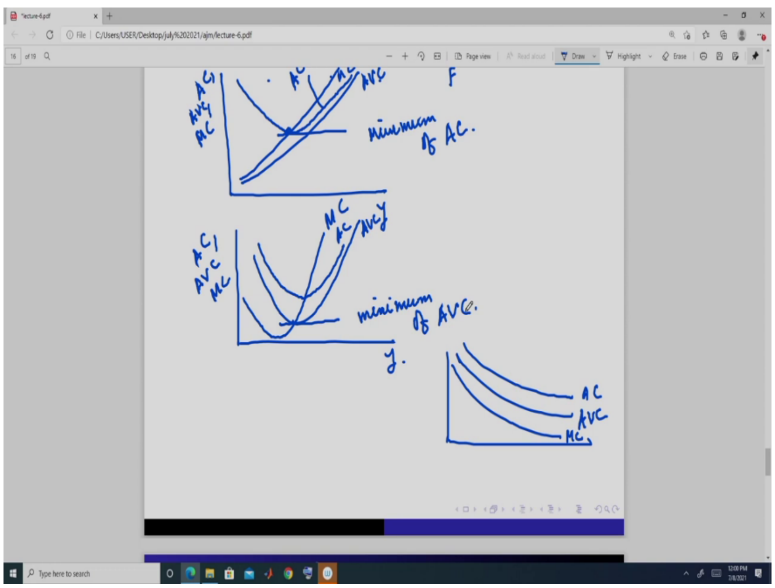

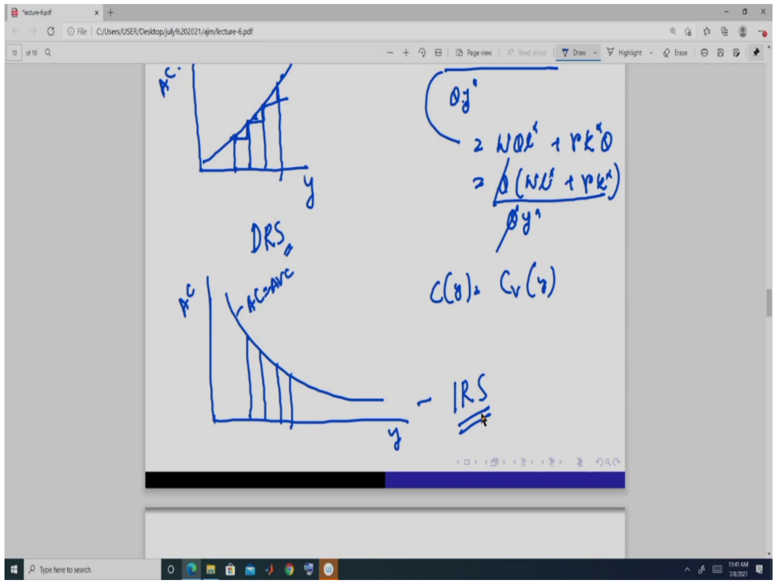

So, from this we get that, so all the cost AC, AVC, marginal cost like this. Now, suppose our average variable cost is of this nature. It is always increasing, then if this is the AVC, then marginal cost will always lie above it, right. And the average cost in this situation, average cost is of this nature right. If we have a fixed component, if suppose average cost is not same as average variable cost, then this will hit at the minimum of AC.

Because, average variable cost is this and then we will get a u shaped, this curve is going to be the AC because of the fixed component part. So, this will intersect at the minimum. So, when we have increasing portion of AC, AC will be less than the marginal cost. When we have decreasing portion of AC, AC will be higher than the marginal cost, like this, okay. Now, if we have a situation like this suppose, the average variable cost is of this nature.

Then the marginal cost is going to be like this, again this is going to be the minimum of AVC. So, this is clear from actually this condition. Now, here this is equal to 0. So, that is why at this point they should intersect minimum, slope is 0. Here, slope is 0 when these two are equal. So, when they intersect, it should intersect at minimum of AC. So, that is why, and in this situation, average cost curve is also going to be something like this, right.

So, here in this axis, we have taken all the cost, and in this axis we have taken all the output. So we get, so mainly this is the relationship between the average cost and the marginal cost, okay.

And if they are decreasing, suppose continuously. Average variable cost is also of this nature, average fixed cost to we know, so average cost is of also this nature, then the marginal cost is going to be always below this, okay, from this -MC-AC<0and this condition- $\frac{MC-AVC}{y} = 0$, okay.

So, next, so all these things are for the, we have done it for the short run. So, at least we have our plant size was fixed or when we talk about increasing or decreasing returns to scale like in this situation or in these situations, we have ignored that the plant sizes can also vary. But, so, this was mainly the analysis was for short run.

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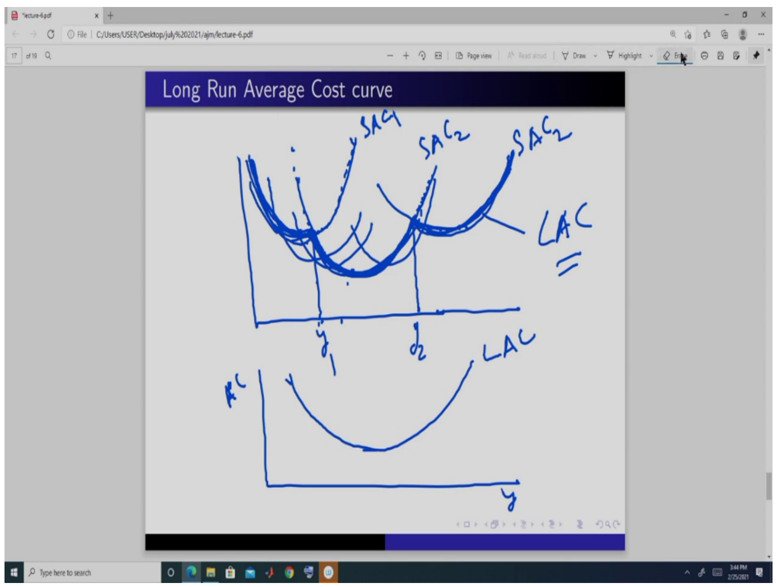

Next, we move to long run average cost curve. Now, in the long run what may happen, we may have a situation like this, output and suppose this is one average, short run average cost curve. This is suppose for one plant. So, when we say long run, here we can change the plant size or you can think that we may have one size of plot where we have one plant and we can keep on increasing the land plot and we can set up a bigger and bigger industry.

So, this is suppose another short run average cost curve and we have further another short run average cost curve of this nature, okay. So, the plant size are increasing, as we move in this. So, this is like this, this is like this and this is like this. Till this level of output, we should always choose this plant size. Because if we choose this plant size, the average cost is higher, right? So, till this output, we will choose the plant one.

But if we move to produce more than this, and if we continue with this plant our cost, average cost is this per unit cost, but average per unit cost is can be lower, if we shift to this plant two. So, we will do this, we will continue this in plant two. But now, suppose we want to produce further and if we continue with this plant, our average cost is here, right. But if we shift to plant three, our cost is going to be like this. So, we can go on.

So, this outer envelope that is the lower envelope of all the short run average cost gives me the long run average cost curve. So, in the long run, the average cost will always be the lower envelope of the average short run average cost curve, okay, it is something like this. Because, when we are producing this much, we can use the plant one and since it is long run we can change the plants, plant sizes are also variable.

So, here now, if we can continuously change the plant size, then we will get a this kind of long run average cost curve, from here like instead of this we may have another here like this, we may have another like this, we may have like this. So, if we do go on like this, then we can get a smooth curve like this. And it is, it will be of this nature, okay. So, this is the long run average cost. Now, what is long run marginal cost?

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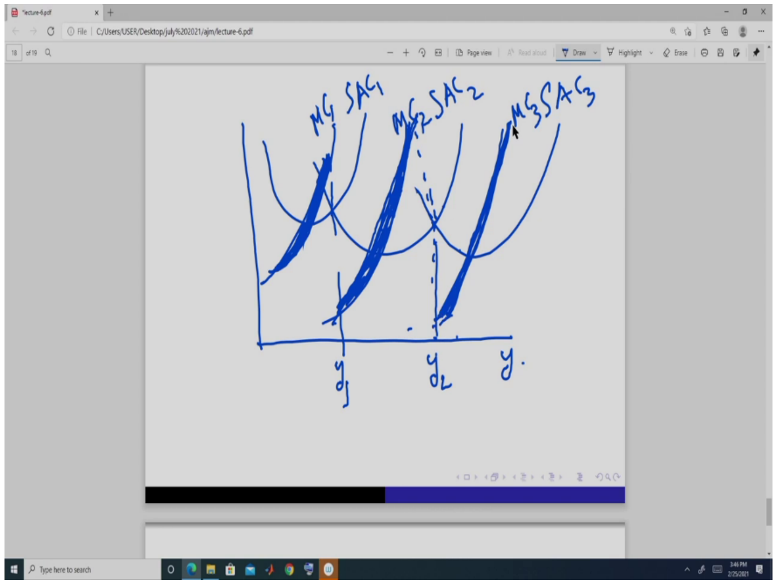

See, so in the long run, we have seen that the plant size are variable. So, we have done, taken three plants. This is short run average cost of plant one, this is short run average cost of plant two, this is short run average cost of plant three. This plant, we suppose, it has a marginal cost curve of this nature, this has a marginal cost of this nature and this has a marginal cost of this nature. Now, we know we will produce till this output in plant one.

So, our marginal cost is going to be of this marginal cost. Then we are going to shift to this plant and here the marginal cost are this. And we are going to continue to produce till this level of output. So, this is the marginal cost, right, plant two. And then plan three we will shift after this much level of output. Because, average cost is less here then continue using plant two, if you continue using plan two.

So, long run marginal cost is like this is, so we will get a discontinuous marginal cost curve in the long run, why? Because we are changing the plants, okay. And so, our cost curves are also changing, okay. So, this is what we require in this module. And if you want to read it, because these notes are sufficient, this class a.

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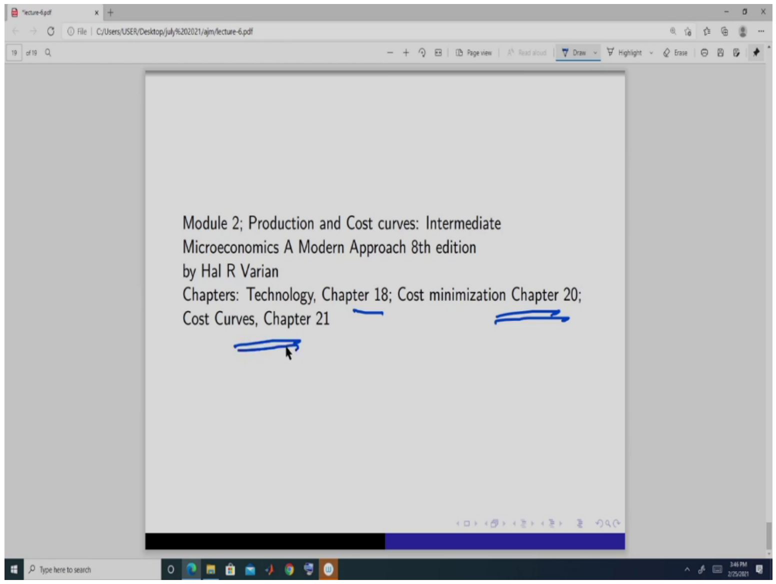

Or if you want to read it, so you can read chapter 18 from Hal Varian, Intermediate Microeconomics, A Modern Approach, chapter 18 for technology part that is production function for cost minimization, you can read chapter 20 and for cost curves, this kind of curve you can read chapter 21, okay. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture - 8
Tutorial on Production and Cost Curves

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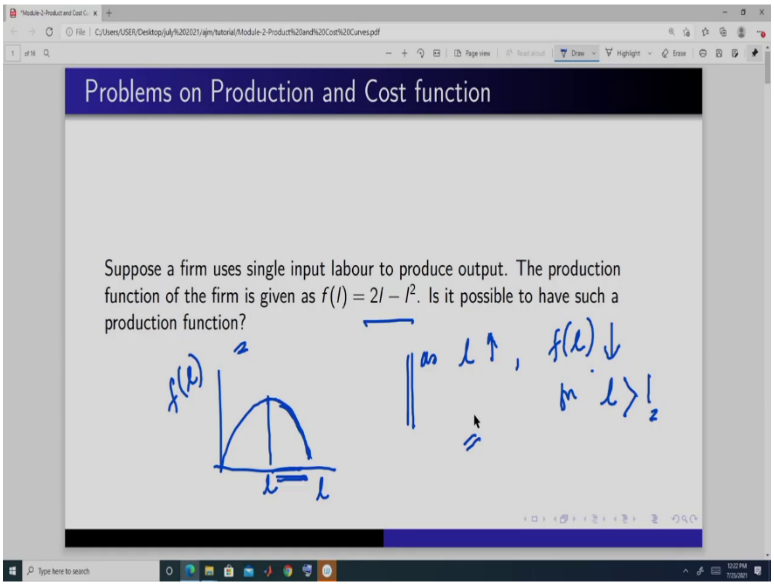

Okay, let us solve some problem for the topic that we have done in module two, that is production and cost. So, here suppose a firm uses a single input that is labor to produce output and the production function of the firm is given by this function- $f(l) = 2l - l^2$, okay. Now, is it possible to have such a production function? Now, if we plot this, if we look at labor here and if we look at this, this function is something like this output here.

Now, if we look at this, what is happening? This portion, it is decreasing but labor is increasing. So, what we are getting? As labor increases, it falls after for l greater than 1 this point is actually because if you solve this the maximum point is at point 1, okay. So, it cannot be a production function, why? Because we know that if we increase input the output should always increase, but we are not getting this here. If we increase labor further more than 1 then output is decreasing. So, that is why this is not a production function, okay.

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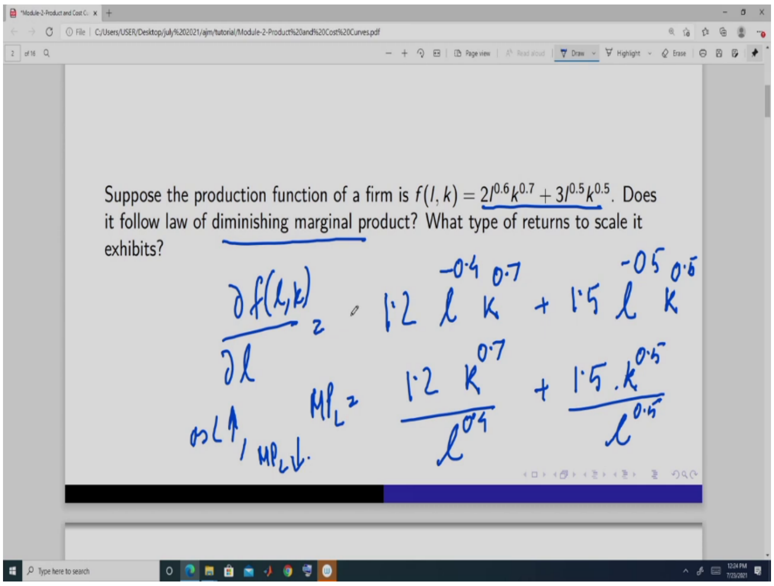

Now, let us take another example and suppose this is the production function of the firm- $f(l,k) = 2l^{0.6}k^{0.7} + 3\,l^{0.5}k^{0.5}$. Does it follow law of diminishing marginal product? So, what do, do we know that this is a differentiable production function and it has two inputs labor l and capital k. So, if we take the derivative of this with respect to l, we are going to get 1.2 $l^{-0.4}k^{0.7} + 1.5\,l^{-0.5}k^{0.5}$ or you can write this, this- $1.2\frac{k^{0.7}}{l^{0.4}} + 1.5\frac{k^{0.5}}{l^{0.5}}$. Now here, you will see this expression as we increase labor this is going to go down, as we increase labor this is going to go down. So, that is why marginal as l increases marginal product of labor it falls. So, that is why it follows law of diminishing marginal product.

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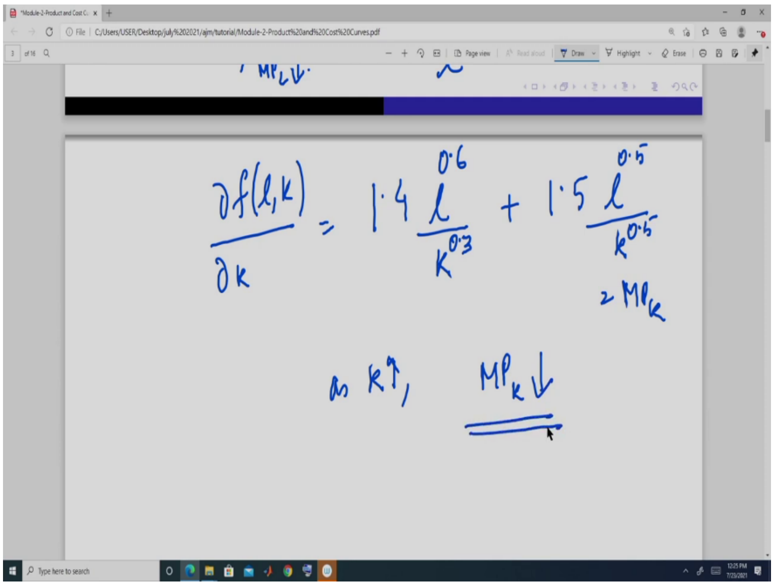

Similarly, if you do the, take the derivative with respect to k, you are going to get this is 0.7. So, it is 1.4. This- $\frac{\delta f(k,l)}{\delta k} = 1.4\frac{l^{0.6}}{k^{0.3}} + 1.5\frac{l^{0.5}}{k^{0.5}}$, now here this is the is equal to marginal product of capital. And if you look at this, what do we get? As k increases, this portion goes down and this portion also goes down because ks are in the denominator. So, that is why marginal product of capital goes down. We get this, so that is why law of diminishing marginal product is satisfied. Now, what type of returns to scale it exhibits. So, let us increase both the labor and capital by a factor theta, okay.

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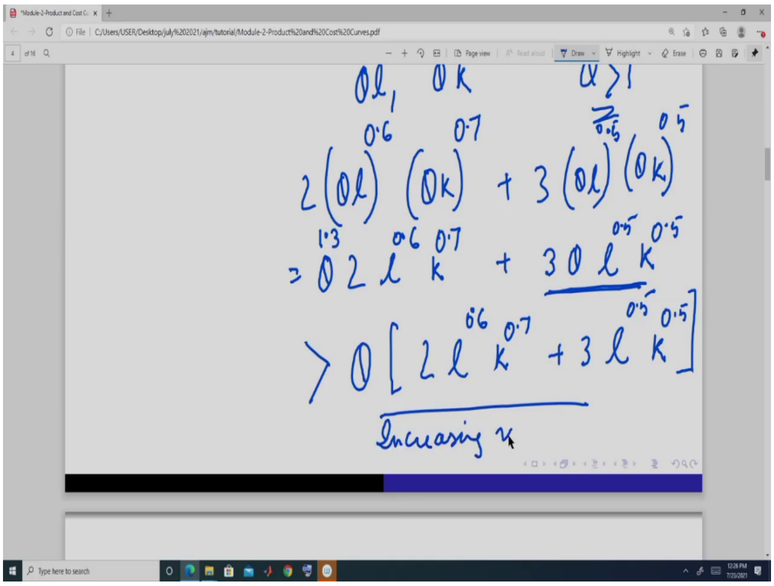

And theta is greater than 1. So, we will get this- $2(\Theta l)^{0.6}(\Theta k)^{0.7}$ plus this-$3(\Theta l)^{0.5}(\Theta k)^{0.5}$. So, this is equal to this-$(\Theta)^{1.3}2l^{0.6}k^{0.7} + 3\Theta l^{0.5}k^{0.5}$, now if you look at this here this portion $3\Theta l^{0.5}k^{0.5}$is theta times this. But here it is theta to the power 1.3. So, it is more. So, this term you can write it is more than theta terms, this-$> \Theta\ [2l^{0.6}k^{0.7} + 3l^{0.5}k^{0.5}]$. So, that is why we see that there is increasing returns to scale, okay, we get this.

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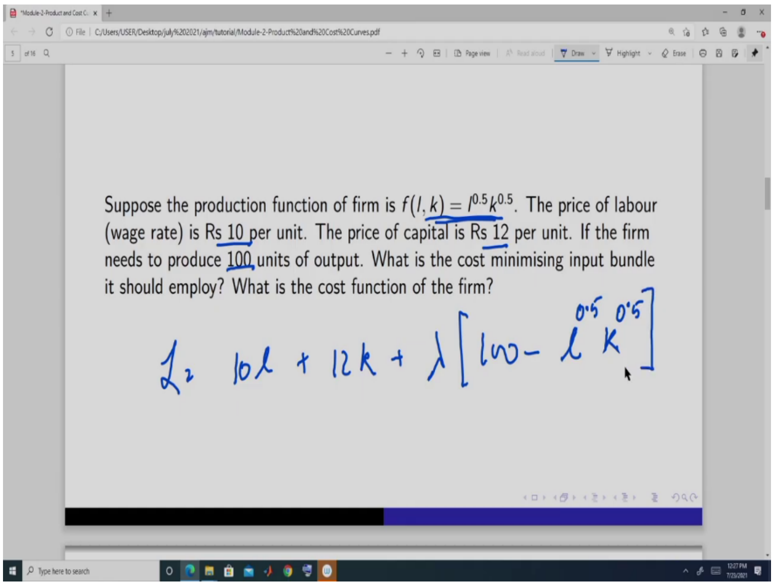

Now, let us do another problem and this is cost minimizing problem. So, here is suppose the production function is this- $f(l,k) = l^{0.5}k^{0.5}$ and the price of capital is 12, price of labor is 10. And the firm wants to produce 100 units of output. So, we see that the production function is a differentiable production function. So, we set the Lagrange on in this form this- $L = 10l + 12k + \lambda[100 - l^{0.5}k^{0.5}]$, this and what do we get?

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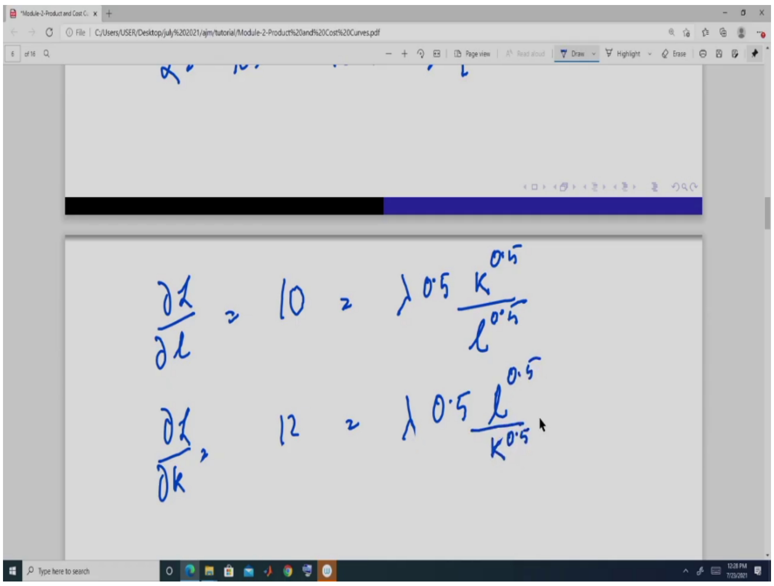

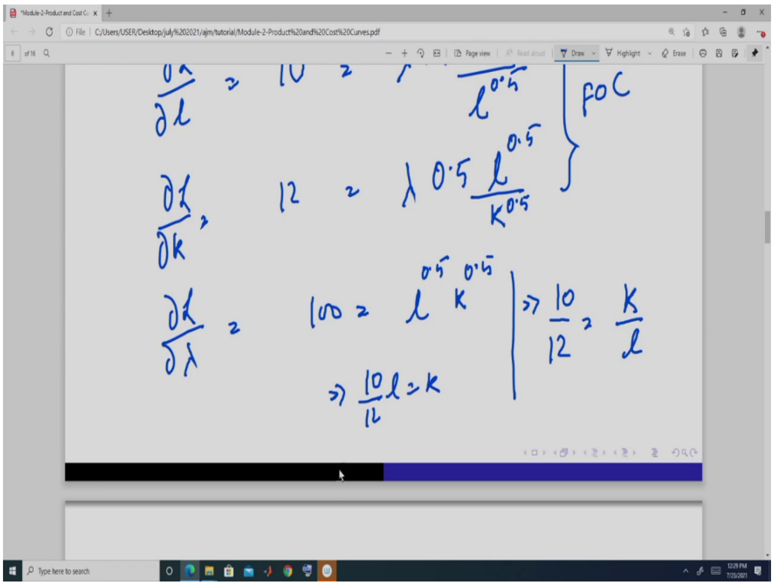

We get the first order condition. I am not, I am simply writing the first order conditions and we are going to get this as- $\frac{\delta L}{\delta l} = 10 = \lambda^{0.5} \frac{k^{0.5}}{l^{0.5}}$, $\frac{\delta L}{\delta k} = 12 = \lambda^{0.5} \frac{l^{0.5}}{k^{0.5}}$, $\frac{\delta L}{\delta \lambda} = 100 = l^{0.5} k^{0.5}$, okay. These are the first order condition. So, from these two, first two equations, we get 10 by 12 is equal to, if I take this divided by this, then I will get this is equal to k by l, okay, sorry, this- $\frac{10}{12} = \frac{k}{l}$. Now, from here, what do we get that 10 is equal to k, i.e$\frac{10}{12} l = k$. So, you plug in here, you will get- $100 = l^{0.5} \left(\frac{10}{12}\right) l^{0.5} => 100 = l \left[\frac{10}{12}\right]^{0.5}$, this.

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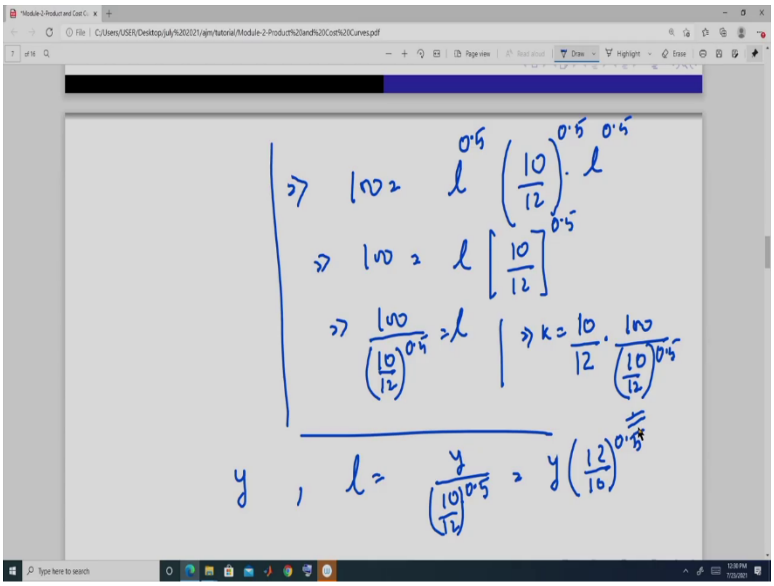

So, this is l. So, k is equal to 10 by 12 into l- $k = \frac{10}{12} \cdot \frac{100}{(\frac{10}{12})^{0.5}}$ you can simplify this. Now, we are again asked, what is the cost function of this? Cost function is a function of output. So, instead of taking 100 if we take some output a suppose y is, output is suppose y, then here what do we get? Labor is equal to y by this- $l = \frac{y}{\frac{10}{12}^{0.5}}$, or you can write this.

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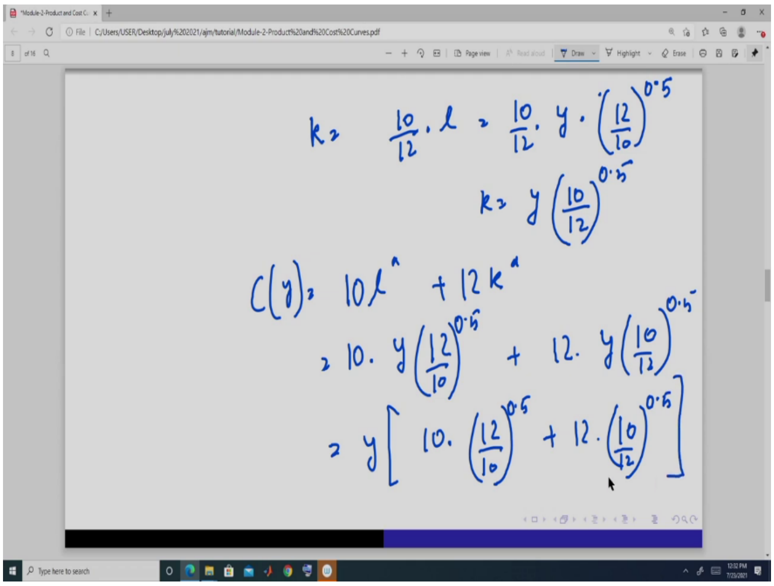

And capital k is equal to y, this 10 by 12 into l, which is 10 by 12 into y, this- $k = \frac{10}{12}. l = \frac{10}{12}. y. \left(\frac{12}{10}\right)^{0.5}$. So, k is equal to y this- $k = y\left(\frac{10}{12}\right)^{0.5}$. So, the cost function is 10 l star plus 12 k star, so 10 into this plus 12 into this. So, this is what? y so this is the cost function in this case, i.e $c(y) = y[10. \left(\frac{12}{10}\right)^{0.5} + 12\left(\frac{10}{12}\right)^{0.5}$

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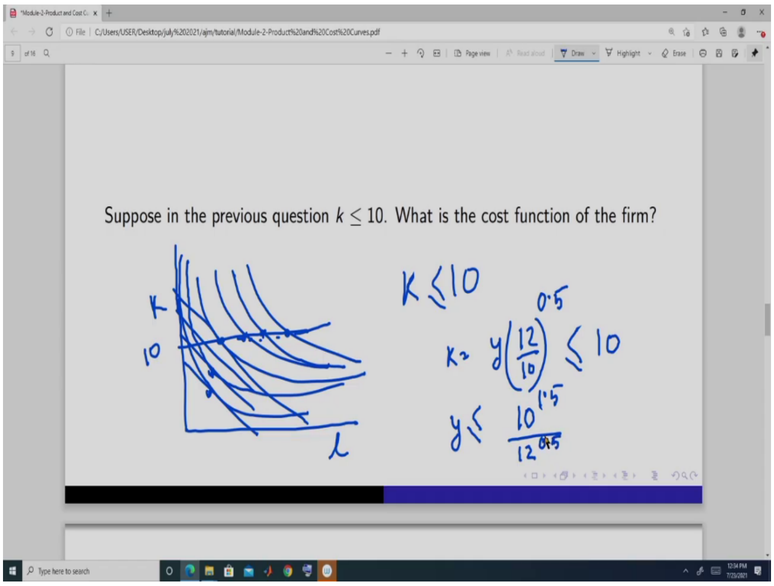

Now, suppose we are fixing the k, k cannot be greater than this 10. Now, what is going to be the cost function? Now, so, this means that if we take labor and capital given the isoquant it is something like this. And we are fixing this k 10 price are such that we will, it is 10, 12. So, it is going to be somewhere here like this. Now, we may get like this, we may get like this. But once we are here we our outputs are going to be at this point, optimal points are this.

We have done this while doing this in the class, okay. Why these are the optimal points? Okay. So, what do we find? Now, k is should be less than or equal to 10, what is the demand for a k? K is always equal to y 12 this- $k = y\left(\frac{10}{12}\right)^{0.5}$. So, this should be less than equal to 10. So, y should be equal to 10 to the power 1.5 and 12 to the power 0.5, 0.5 this- $y \leq \frac{10^{1.5}}{12^{0.5}}$. Whenever, y is less than this we will get the cost function to be of this nature.

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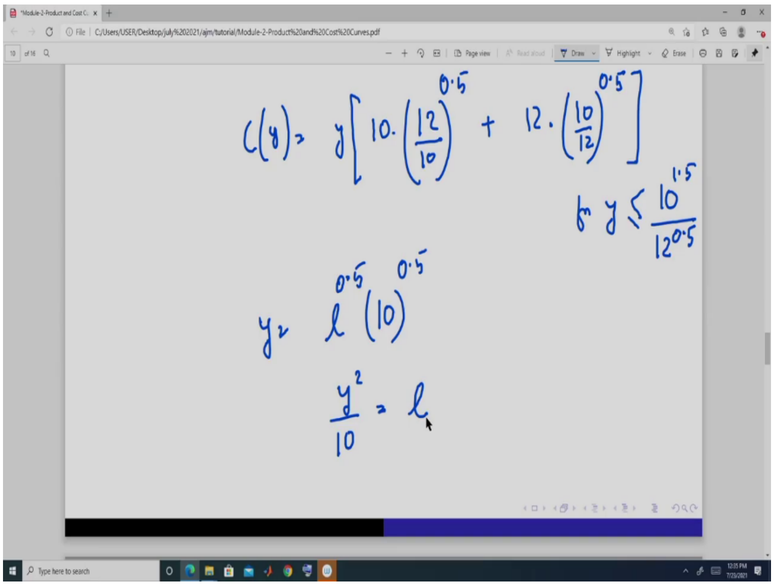

So, we get the cost function is this- $c(y) = y[10.\left(\frac{12}{10}\right)^{0.5} + 12\left(\frac{10}{12}\right)^{0.5}$ for y less than equal to this- $y \leq \frac{10^{1.5}}{12^{0.5}}$. And whenever y is greater than this, then we know how the output is produced. This from the production function it is this, and capital is fixed so it is 10. So, the demand for labor is, it is this, right. So, what is going to be the cost function?

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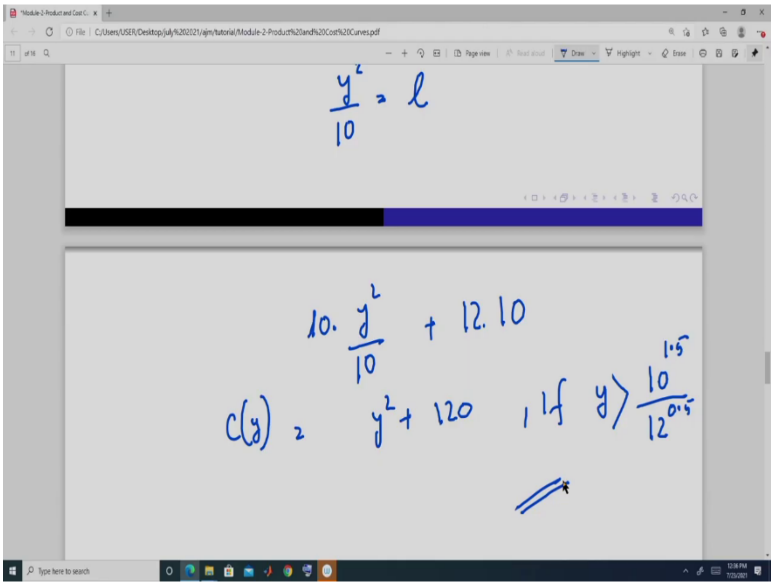

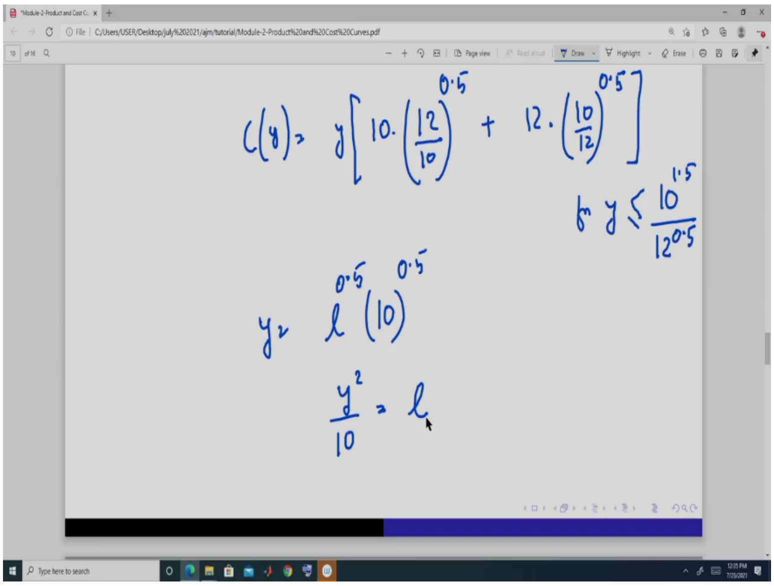

Cost function, now is 10 into this y to the power plus 12 into 10- $c(y) = y^2 + 120$. Because 10 is the a, so, this is square plus 120, if y is greater than 10 to the power 1.5 12 to the power this. So, this is the cost function, okay. And for this, it is this, okay. Thank you.

Introduction to Market Structure
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Module 3: Perfectly Competitive Markets
Lecture 9
Optimal output and supply curve of a firm

Hello, welcome to my course Introduction to Market Structures. So, today we are going to start Perfectly Competitive Markets. So, from today actually we are starting the market structure. Before that we have done some preliminary things that is, those are required to do this analysis.

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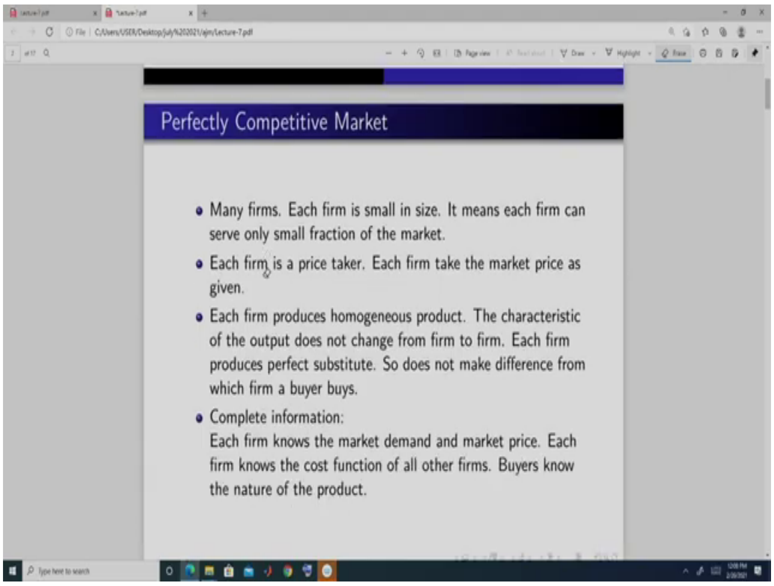

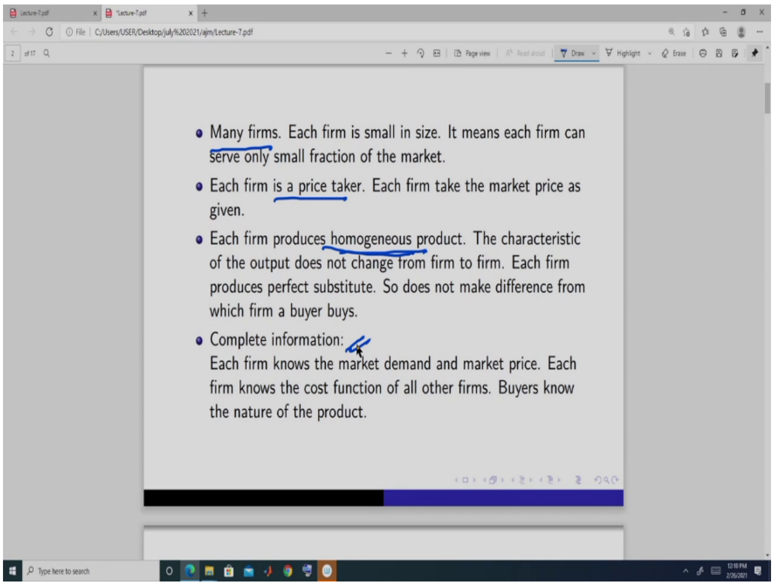

So first thing that we are going to do is what is Perfectly Competitive Market? In a Perfectly Competitive Market there are many firms. So, first characteristics is the many firms and each firm is small in size, it means that each firm can only serve a small fraction of the market, each firm cannot provide to a big chunk of the market.

Next each firm is a price taker. What do we mean by price taker? That the market price or the price at which a consumer is buying that good. that price is not determined by the firm, by an individual firm. Each firm takes that price as given and then it and only decides the output that is going to produce, okay. How that price is determined, we will do that later, okay.

So, first we assume that each firm takes the market price as given; that is the price at which consumers are buying that product. Next, the goods that are being produced by these firms are homogenous; that means that they are; suppose there are three firms, firm 1, firm 2, firm 3, output produced by each of these firms are going to be similar, 100 percent similar.

So, it means that whether I buy from a firm 1 or buy from firm 2 it does not matter. So this is the homogeneous product, okay. This assumption is very important. So, then what happens? So, actual identity of the firm does not matter because the products are homogeneous and next assumption is the complete information assumption.

What do we mean by complete information? It means that each firm knows the market price completely. If there is no uncertainty in the market price, so firm knows what is going to be the market price. Second, firm knows what is going to be its cost function, so if there is no uncertainty with respect to the cost function, like the input, price of the inputs are completely given and they are certain, okay.

So, the firm while deciding how much amount of output to produce, so it will decide how much amount of inputs to produce and doing that, in that decision there is no uncertainty involved, okay and the buyers also know the exact nature of the product, also know the price of the product, so there is also no uncertainty in that respect.

And each firm knows what are the types of firms they are there in the market; that is, the different production technology or the different product firms with different production function that are present in that firm market, okay if there are, so it may happen there are some firms which are using a different technology and some firm may be using different technology, so that these two types have different production functions.

So, firm knows that there are these many of these types of real firm and there are these many of these many firms, okay. So, there is no uncertainty involved in this.

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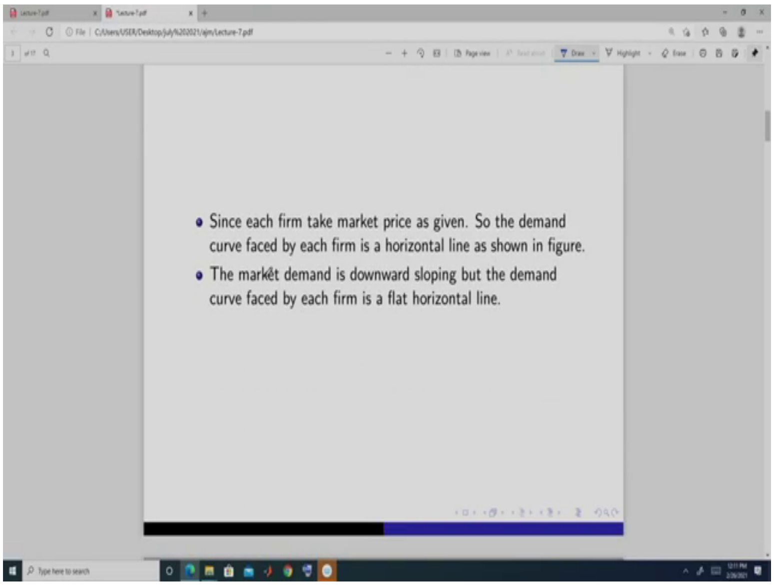

Now here, first the assumption that we have made, so these are the main important assumption , okay and there is another assumption, but we will specify that later, not now. It is regarding free entry and exit of firm and since we are not dealing with that right now, we are not specifying it, okay. Now the next is, that from the first assumption that the firms are always price takers, that means they cannot determine the market price. It means that the demand curve faced by each firm is a flat horizontal line, okay. So, you will see what do we mean by this.

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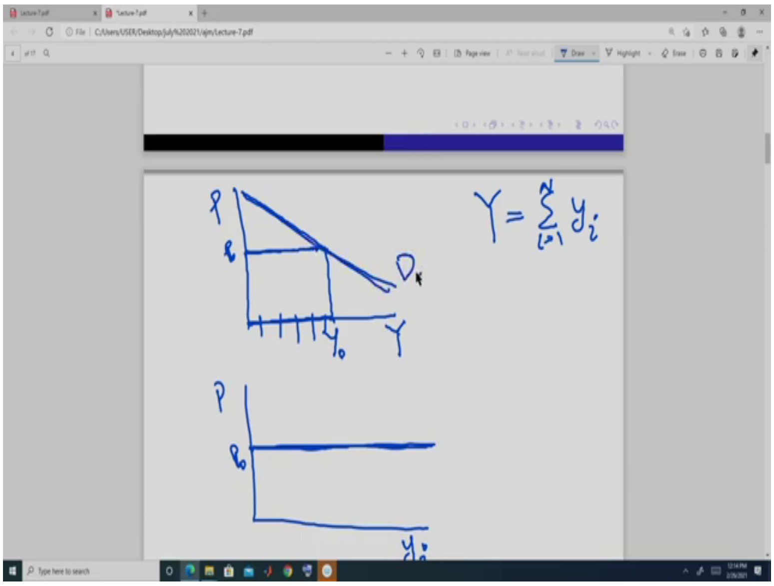

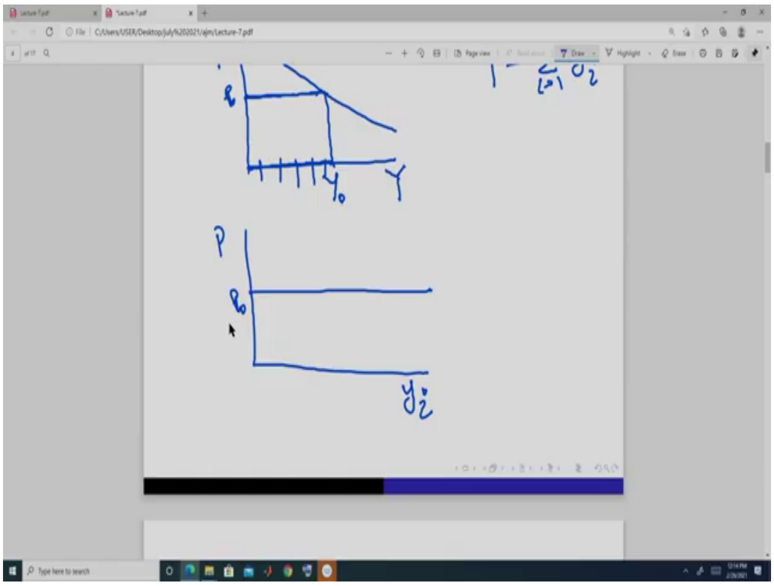

Suppose, this is the market output, okay and this is the price, so market output is actually sum of the output of each firm and we have not yet specified the number of firms present in this market but suppose there are n firms and this is, each firm is producing Y_i units of output, okay and this is the market output. And suppose this is the market demand curve. We know how to derive a market demand curve, it is horizontal summation of the individual demand curve, right?

And suppose it is like this. Now suppose market price is this p, okay, p naught suppose, then the total amount demanded in this market at this price is this much-Y_0, right, so each firm is going to produce some amount, suppose like this, some like this, some like this and then the sum is going to be this much. So, the market demand curve faced by each firm is simply this horizontal line, so we take, suppose any firm Y_i and price here the market demand curve is like this at this price p naught, so it is flat.

Whatever amount of output it wants to produce it can produce and sell, that is the, because it cannot determine the price. And actually, their capacity is going to be so low or so that is why they will not be producing that much amount of output, okay, we will see that. So, the demand curve faced by each firm is this a horizontal line.

Although the market demand curve is this, this is the demand curve for that commodity, okay. Suppose, like take for example, the agricultural goods, so agricultural markets are in many times assumed to be perfectly competitive in the sense that the farmers when they are selling they cannot determine the price at which they are selling. So, they take the price as given, okay. So, whatever amount they want to sell they can sell at that price. So, the idea is this, okay.

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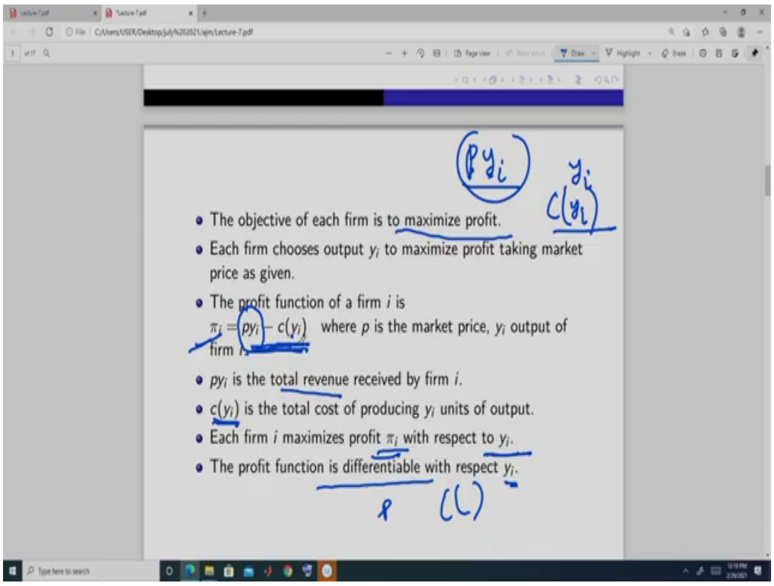

Now, we move to the objective of this firm. So, each firm wants to maximize profit that is their objective. Now, what is profit? Profit is while selling your output you will be selling at some price, so this price into the total amount of output you are selling will give you the total revenue minus the cost that you are going to incur while producing that much amount of output is your cost.

So, this is your profit- $\pi_i = py_i - c(y_i)$, where this term p into yi, this is the total revenue, this is the total revenue because this is price firm cannot decide the price, it takes it as a given, it only deciding the output, suppose it is a firm i is producing yi units of output, so this is the total revenue and there is a cost function, we have derived the cost function in the last class and this is the total cost suppose.

To produce yi units of output, the cost a firm incurs is suppose this- $c(y_i)$. Now, we are not going to specify what kind of cost function this firm has, we will come to it later on, okay. So, the profit function is actually total revenue minus total cost, okay. So, firm i or in fact any firm i wants to maximize this by choosing an appropriate y_i, okay. So, for this firm the p is given, the cost function is given and based on this, it has got a profit function which is price into quantity, it wants to sell that is the total revenue minus the total cost, okay and this is the total cost it is incurring for producing yi units of output.

Now, here firm wants to maximize this function, this is profit function with respect to yi- $\pi_i = py_i - c(y_i)$. Now, here we assume that this profit function is differentiable with respect to yi, okay. This is not that strong assumption because we know that the cost function are differentiable with respect to yi, right. We have done that in the last class. Why we have got it in that firm?

Because the inputs, that is labor and capital are continuously divisible and they can be employed at any level, so that is why or they can be changed continuously, so the cost function is actually a differentiable cost function, okay.

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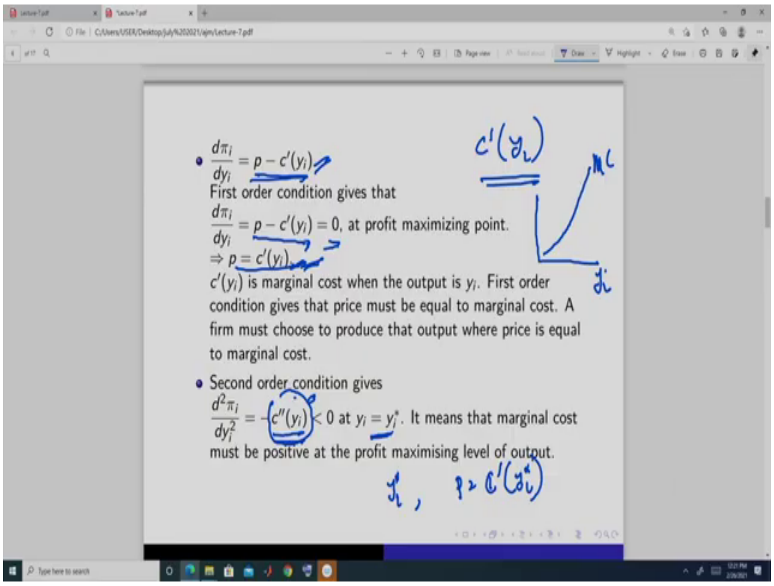

Now, what we are going to do? We are going to maximize this. Now, since this is a differentiable function so we take the derivative with respect to yi. So, each firm while choosing their output what they are going to do, they are going to take this derivative- $\frac{d\pi_i}{dy_i} = p - c^{'}(y_i)$ and the first order condition gives that this should be equal to 0- $\frac{d\pi_i}{dy_i} = p - c^{'}\left(y_i\right) = 0$, right? So, then this means that the price should be equal to this term. i.e $p = c^{'}\left(y_i\right)$.

Now here, this $\{c^{'}\left(y_i\right)\}$ is what?, this is the first derivative of the total cost, this is marginal cost, this is what, this is the changes in the total cost, right? at yi units of output. So, this first order condition gives us that when the profit is at a maximum point, then the price should be equal to marginal cost, so a firm while choosing is profit maximizing output, it should ensure that the price that is taking as given must be equal to the marginal cost of producing that much amount of output, okay.

So, this is, okay, so this is the first order condition that the price should always be equal to marginal cost- $p = c^{'}\left(y_i\right)$, okay. Now, we move to something called the second order condition, so we take the second derivative of this- $\frac{d\pi_i}{dy_i} = p - c^{'}\left(y_i\right)$ with respect to yi that is the output of firm i and we get, since in this a or in this thing p is not a function of yi, it is given, it is fixed for this firm. So we get this term- $-c^{''}(y_i)$ and this term should be negative at this, i.e $\frac{d^2\pi_i}{dy_i^2} =- c^{''}\left(y_i\right) < 0\ at\ y_i = y_i^*$.

So, we get this y star by solving this equation- $\frac{d\pi_i}{dy_i} = p - c^{'}\left(y_i\right) = 0$. So, y star, yi star is such that p is equal to this- $P = c^{'}\left(y_i^*\right)$, so when this is here, we get this y star and once we get this y star, what we do we take the second derivative and plug in y star here, then this, expression which is the changes in the marginal cost or the slope of the marginal cost function should be negative at that output.

So, this means that this term- $-c^{''}(y_i)$, since there is a negative term, so this term should always be positive, right. Now, for this to be positive that means the marginal cost curve should be increasing like this, right.? We have done this in the last class when the marginal cost function is something like this, then it means that the production function is exhibiting decreasing returns to scale.

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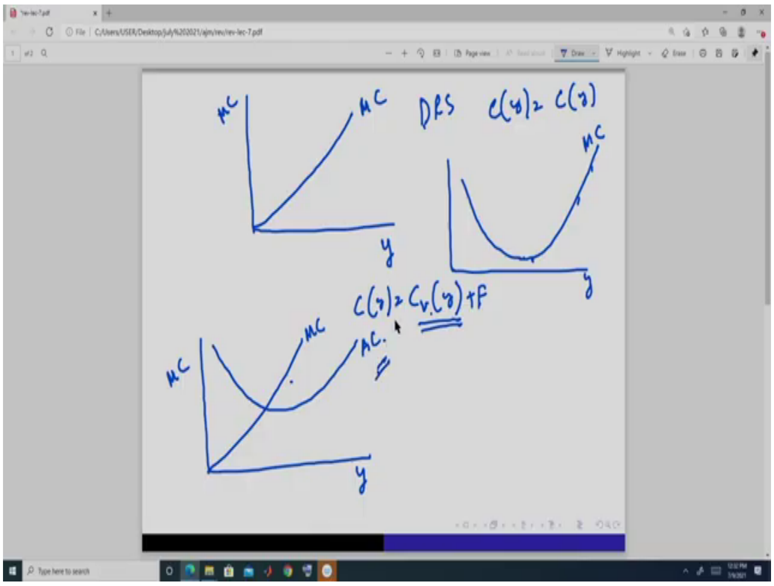

So, we have seen that when we have decreasing returns to scale, marginal cost is of this nature, this is y and this is mc, okay. So in this case our cost is, cost function is this- c(y)= c(y), so we do not have any fixed cost, okay. Or we may have a combination of increasing and decreasing returns and then also marginal cost can be of this. So, here we have economies of scale and then we have diseconomies of scale.

So, we have explained this in the cost function while doing the cost function. Now we may have another type of here also when, and we have discussed this in detail when we have done the costing. So, if we have a cost function like this- $C(y) = C_v(y) + F$ and this is a variable component it is giving you this kind of upward sloping marginal cost. So, here, this cost marginal cost is mainly due to law of diminishing marginal product, this.

So, we have discussed that if we have DRS, so in a competitive market we require decreasing returns to scale, but further even if we have, we do not have decreasing returns to scale, but we may have a situation like this, then we will get a marginal cost like this and in this situation we know the average cost is going to be of this nature, okay and you will see that we will use this case mostly in the perfectly competitive market.

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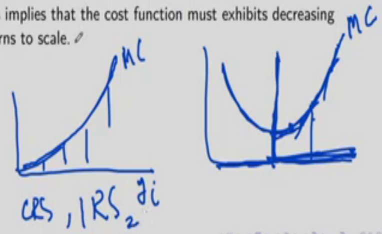
- When the profit function is differentiable, we get that for the profit maximising output to exist, the cost function must be such that marginal cost is increasing in output.
- This implies that the cost function must exhibits decreasing returns to scale.



- $\frac{d\pi_i}{dy_i} = p - c'(y_i)$

  First order condition gives that

  $\frac{d\pi_i}{dy_i} = p - c'(y_i) = 0$, at profit maximizing point.

  $\Rightarrow p = c'(y_i)$

  $c'(y_i)$ is marginal cost when the output is y_i. First order condition gives that price must be equal to marginal cost. A firm must choose to produce that output where price is equal to marginal cost.

$c'(y_i)$

- Second order condition gives

  $\frac{d^2\pi_i}{dy_i^2} = -c''(y_i) < 0$ at $y_i = y_i^*$. It means that marginal cost must be positive at the profit maximising level of output.

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$y_i, \quad p = c'(y_i^*)$

So, what do we get? So, we get that the for profit maximizing output to exist, so that this point exists- $p - c'(y_i)$ and this is a profit maximizing point we require this point condition, i.e $-c''(y_i)$ and this ensures that the marginal cost function is of this nature, right? or the marginal cost function can be of this nature. But if marginal cost is of this nature then the profit maximizing output should be in this region, right? it cannot be in this region.

Here it will not, because then the marginal cost is downward sloping, it is going down as output, so then this violates this condition- $-c''\left(y_i\right) < 0$. So, if marginal cost is of this nature then the profit maximizing output should be in this, output should be greater than here. Because here if you take this point here, marginal cost is positive, here it is also positive, here it is, everywhere marginal cost is, the slope of marginal cost function is positive, right?

For this kind of marginal cost, we will always have any point, any output is going to be a profit maximum, can be depending on the price, okay. So, this portion here, we know it is exhibiting decreasing returns to scale and here the whole, it is exhibiting decreasing returns to scale throughout, so here till this output it is increasing returns to scale and after that it is decreasing.

So, this means that we will never have something called a CRS or IRS production function in this type of market. We will show later why we should not have such type of production function, okay or if we have such kind of production function then what is going to be the outcome, okay.

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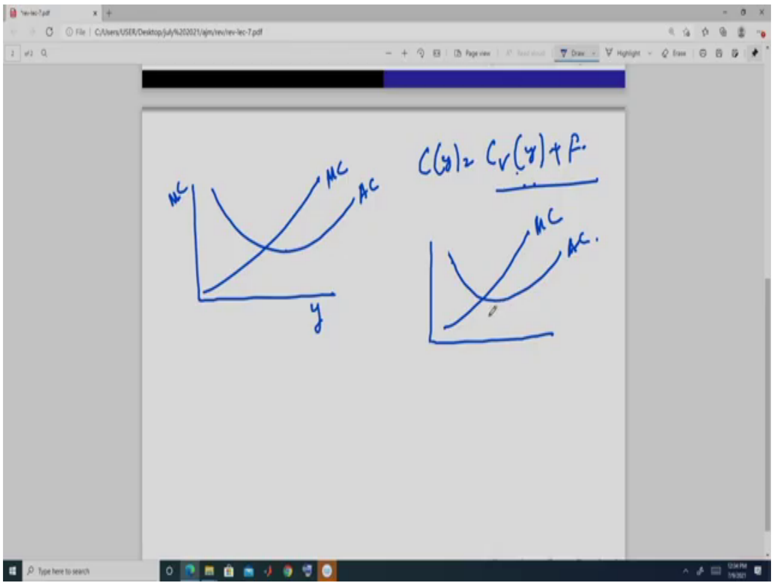

Now, let us do this profit maximizing thing which we have solved algebraically in a graphical way. So, our cost function can again be of this nature- $C(y) = C_v(y) + F$, right? and when it is this, we know that one factor is fixed and the another factor is varying, so we have law of diminishing marginal product it is operating and so that is why our marginal cost is of this nature, right?, and the average cost you will get is of this nature.

So, this should be at the minimum point, okay, intersect at the minimum point. But further we may have a situation when we have, we can vary both machine and the labor but our plant size is fixed or the land is fixed, so in that case we fix, get the fixed cost from that land or that rent is giving us a fixed cost but main inputs like capital and machine, labor those are varying, then we may have a situation of decreasing returns to scale where we have got decreasing returns to scale by varying both labor and capital but our land or the plant size is fixed.

So, then we again may get, this kind of can thing, right? so these two are, it maybe although they look same, but maybe different. In the sense here only law of diminishing, one factor is fixed machine is fixed, only labor is varying, here both labor and machine are varying, labor and capital are varying but the land is fixed, okay and the moment we have this kind of marginal cost function, we will get the optimal point in a perfectly competitive outcome.

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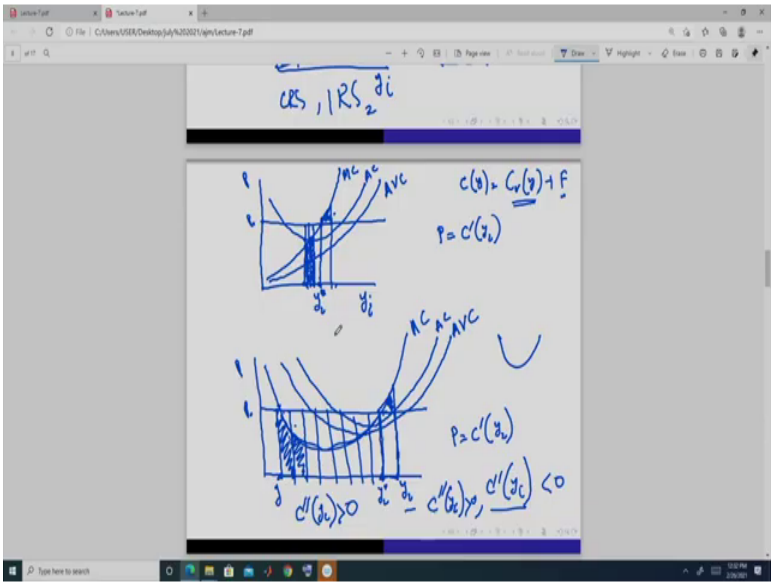

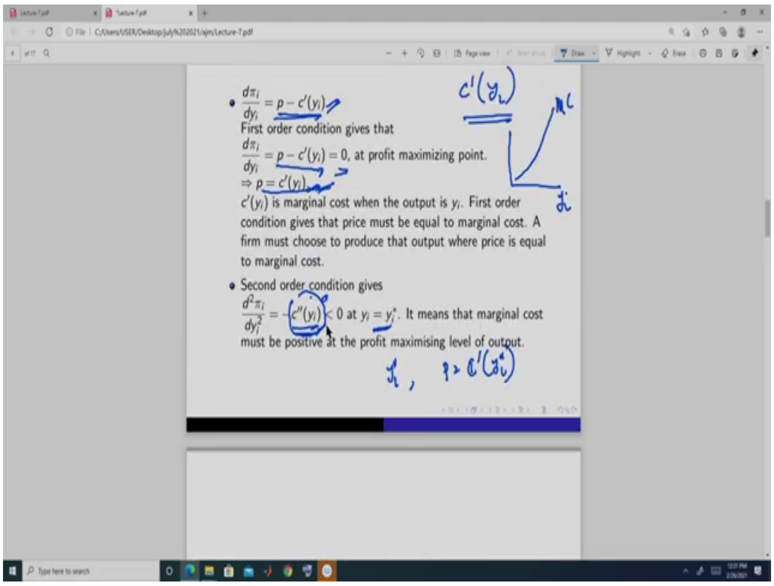

So, let us take any firm yi, output of that firm is given in this axis and the price is here and suppose the market price is this, okay. Initial market price is this p naught, okay. And the marginal cost function is something like this, okay if the marginal cost is like this it is throughout increasing, then we know that the average variable cost is going to be like this and the variable cost function, sorry, the average cost function is suppose something like this.

So, if we have taken this firm, then it means that our cost function is of this nature- $C(y) = C_v(y) + F$, right? and this is such that the marginal cost is always increasing, right?

If we did not have this part- F, then average cost would have been same as average variable cost and it will be something like this, okay. Now, here what do we see? We get that this is the profit maximizing output. Why? Because at this output, price is equal to marginal cost because these two curves intersect at this point.

Now, suppose take any other output here, if we take this output then it means what? Marginal cost at this point and revenue or the price is there. Now, if we increase the output slightly from here to here, here, what we are doing? We are total cost; that is the region under the marginal cost is this much extra cost we are bearing, but this whole region we are adding as the total revenue because this into this.

This is the difference in the output and this is the price, this whole region, so we are adding some additional part to the total profit, this is the profit, right. So, what is happening? So, I have a tendency to increase output. Now, from here if I do further like this, what I am doing? Marginal cost curve is this, so this region is the total increase in the cost but total additional revenue profit is given by this region.

So, I will again further increase. So like this I will go on like this y star unit of output, but if I move beyond this, like if I move to this portion, what I am doing? total revenue that I am getting, additional total revenue is this, right? this rectangle, but the total cost is this portion, so here this much is the additional cost we are earning, which is more than the revenue, so this much amount of loss I am making.

So, I will not have any tendency to produce output more than y star. So, that is why we get a condition that the price should always be equal to marginal cost, okay. Now, here when this is the price and this is the marginal cost so, and suppose the average cost is like this and the average variable cost is like this optimal point is like this, okay and here you see that since the marginal cost is throughout increasing, so the second order condition is always true.

Now, when second order condition is going to be binding, we will see that now. Suppose let us take this a price in this axis and suppose this is the market price, okay at this level and suppose the marginal cost is of this nature, okay. Okay, marginal cost and this is suppose average variable cost and suppose this is the AC, average cost, okay. In this case we have, in two points we have this condition, p is equal to marginal cost.

At this point and another point is this, out of this point we will always choose this y because now if we are at this point price is equal to marginal cost, but if I increase output a little bit like this, what is happening, my additional cost is given by this region and my additional revenue is this rectangle, so I am making a profit of this region. So, that is why I will produce this much not this, I will not stop at this point, so this is not going to be an optimal point.

Now, if I produce further, here the additional cost is this much, additional revenue is this rectangle, so this much is the addition to the profit, so I will go on producing output till this point and then I will continue like this because this region is giving me the additional profits, right? and further I will this way reach this point. If I produce more than this, suppose I produce here, then what is happening?

My additional cost is given by this region, but my revenue is only this so I am making a loss of this portion. So, that is why I will not produce beyond this y star unit. So, in this situation when marginal cost curve is u-shaped that is, it has both increasing returns to scale component and also decreasing returns to scale component, then we see that this second order condition this is a very important. Why?

Because if you look at the marginal cost here you will see that its slope is downward sloping that means this second order condition this is negative, so then it means if we take negative of this, if we take negative it becomes positive, so that is why it is not a and also from the argument that I have given you it is clear.

But at this point it is clear that the slope is positive, so slope of marginal cost is positive. So, this term is negative at this point, not at this point, okay. Now, let us look at one more thing that suppose we, so we now know what is going to be the optimal point here and what is going to be the optimal point in this case of marginal cost, okay, so only this type of marginal cost we get an optimal point, okay.

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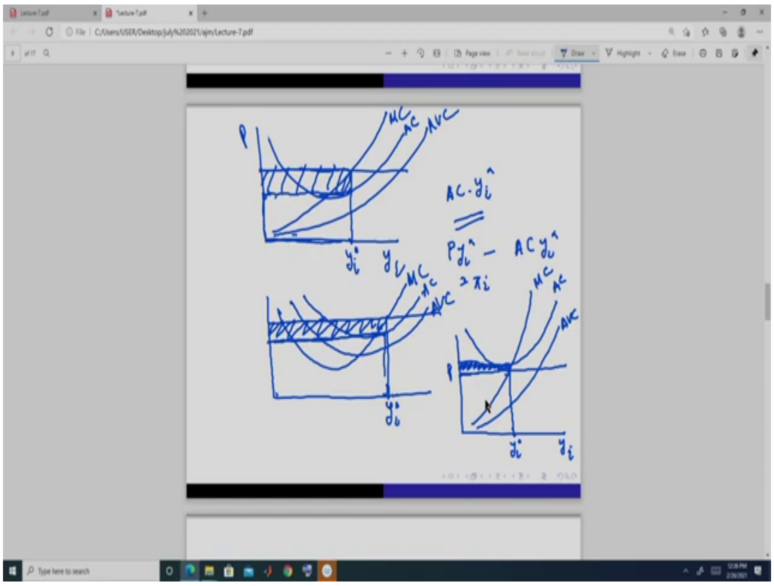

Now, suppose again take this is the price and suppose the marginal cost curve is this, then the average variable cost is this and suppose the average cost is this, okay. So, the optimal output is given by this point yi star (y_i^*). Now, here what is the profit that the firm is going, is making? So, this is the, when I produce this much amount of output, average cost is this much.

So, this region average cost, this rectangle gives me I can say average cost into height is the average cost into yi, yi star so this region is the total cost, this is the total cost. Total revenue, this height is the price into this quantity, so this gives me the total revenue, right? So, price minus you can say AC is going to give me the profit. So, profit is this rectangle, total profit, okay.

We get this. And when we are in this situation, suppose marginal cost is like this, so average variable cost is this, average cost is this and suppose the price is this. So, we know the optimal output is given by this much level of output. Now here total profit the firm is making is given by this rectangle because at this level of output average cost is this much and when the average cost is this, total cost is this rectangle.

And total revenue is this rectangle so this shaded portion is the profit, okay. So, in this case we see that there is a profit there and a positive profit. Now, suppose the situation is something like this, okay. So, optimal output is this, right? at this profit price is equal to marginal cost, right? suppose the price is this. But at this a output, this output total cost is this much. So, here in this case firm is making a loss of this much amount because total cost is given by this bigger rectangle.

And the total revenue is given by this smaller rectangle. So, it is possible that the firm may make some loss in the short run, okay. We will define what do we mean by short run, now just take it as a term. Here, then this question comes that these two condition that or you can say if we look at this graph and suppose the price is somewhere here, then from this marginal cost and this is intersecting here so optimal output is this much.

But the average cost is this much, so the firm is making a loss of this much amount in this case, so this gives rise to another question, whether a firm is going to produce when they are making loss or they are not going to? That means whether they are going to produce any positive amount of output when price is quite low. So, from this what do we get, that when the price is low then the firm makes a loss and when the price is high the firms are making positive profit, okay.

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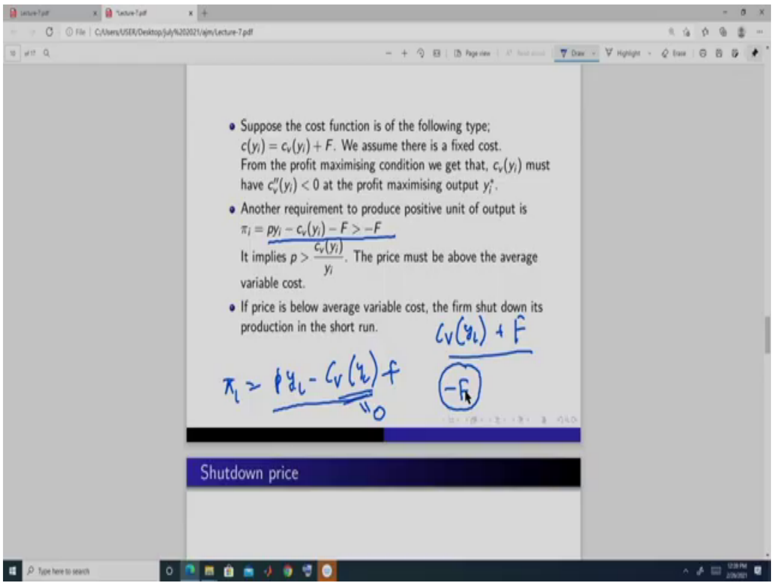

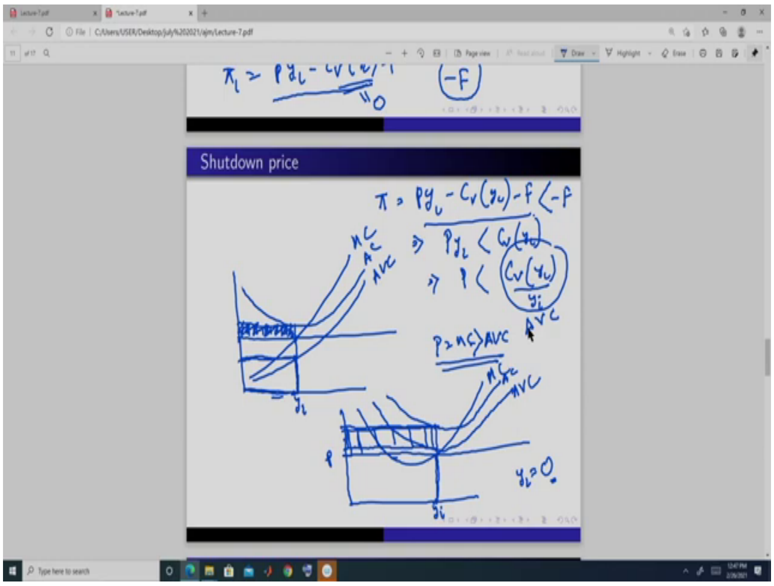

So, we defined something called a shutdown condition, okay. How do we define that? We define it based on this criteria- $\pi_i = py_i - c_v(y_i) - F >- F$. So, we have got that our cost function is of this, going to be of this nature-$c_v(y_i) + F$, right? Now, profit is this as given, so this profit can be negative when we are making loss, now it should be that much, now what is happening? It can be negative. Now see firm is, if it produces zero amount of output then this part is going to be equal to 0, i.e$c_v(y_i) = 0$.

But this part is there, fixed cost it is always, it is independent of the level of output, right? so if it produces 0 output, it is also equal to 0, right. So, this portion is 0, i.e

$py_i - c_v(y_i) - F = 0$, right? So, the profit is minus F, so firm is making a loss of F amount, okay. So, the whole fixed cost is going as a loss. So, if a firm makes a profit which is negative that is if a firm makes a loss and that amount of loss is the whole fixed cost that is F, then a firm should, it is better to produce 0 amount of output. Because if firm produce any positive amount of output, okay.

So, here suppose firm makes a positive amount of output and this is this- $\pi_i = py_i - c_v(y_i) - F$ and they are making a loss and this loss is suppose less than this loss, i.e $\pi_i = py_i - c_v(y_i) - F < -F$, okay, then this implies what, this implies, this implies what, this is what?, this is average variable cost- $c_v(y_i)/y_i$, when the price is below average variable cost we get that the loss is going to be more than the fixed cost, okay, and so it is better not to produce any output, because if we produce any output and we sell it at a price which is less than the average variable cost then our profit is going to be negative.

So, that means we are going to make loss and that loss amount is going to be more than the fixed cost. So, it means by producing we are increasing our loss, so it is better not to produce. So, that is why we get the shutdown condition as this- $\pi_i = py_i - c_v(y_i) - F > -F$, okay. So, the shutdown condition it means that the firms are not going to produce any output whenever price is below the average variable cost. So, for a firm to stay in the market or to produce some positive amount of output, the price should always be greater than the average variable cost.

What does this mean now? So that means that if we are given like this, suppose the marginal cost is like this then the average variable cost is like this and suppose this is the average cost and suppose the price is here, okay, then this is going to be the optimal output. So, firm is making a loss of this much amount, right. Now, if firm suppose does not produce, because see if you look at this curve, this much is the average variable cost.

Average variable cost into output is going to give you the total variable cost, so this rectangle is the total fixed cost, right. Now, if it does not produce then the loss it is going to make is this, because it has already incurred the fixed cost because it is in the market right? so it has already bought the machines. So, now if it does not produce then the machines are going to be lying idle, so it means that he has to pay for this machines and they are not going to and this entrepreneur is not going to get any return on this payment.

Why? Because it is not producing and so then the loss it is going to make is this whole rectangle. But if it produces then this loss, it is still making a loss but this loss is this much only, so it is better to produce, so because the loss is now less amount, if it produces, so that is why a firm is always going to produce in this situation, right? So, when the marginal cost is always increasing, we know that the average variable cost is always going to lie below it.

And the moment average variable cost lies below it, then it means what? from this a the optimal point or the profit maximizing output is given by price is equal to marginal cost and since marginal cost is always greater than average variable cost, here, so this condition is satisfied, this shutdown condition automatically gets satisfied. So, for this kind of marginal cost curve if the technology is such that we get a marginal cost curve of this nature then this problem of shutdown will never arise.

So, firms are always going to produce some amount of positive amount of output because if it does not produce then the loss it is going to make is more than when it produces some positive amount of output and so it will always produce the optimal amount of output, okay, which is given by this point. Now, suppose consider this case, okay, marginal cost is this, average variable cost is this and average cost is this and suppose price is here, okay.

So, the optimal output from the first order condition if it is maximizing, when it is maximizing profit is this, at this output, total loss it is making is this much, right. This whole rectangle because this is the average cost when it produces this much amount of output, so the total cost is this big rectangle. Total revenue is, because this is the price is this, so the total loss is this rectangle.

Now, average variable cost is this much, is this, so if it does not produce any output then its loss is going to be only this smaller rectangle, right, but if it produces then it sells at this price and this price is less than the average variable cost, so it is making a higher loss. So, in this situation a firm is never going to produce output.

So, it is going to produce y optimal output is 0 and because the shutdown condition is not being satisfied that is the price is below the average. So, we get this that this shutdown condition is binding when we have a u-shaped marginal cost curve. When the marginal cost curve is always increasing then this is not going to bind, we will always, shut down condition is always going to be satisfied.

So price is always going to be higher than the average variable cost, but when we have a marginal cost which is u-shaped then it is possible that the price may go below the average variable cost and so the optimal output is such that at that price marginal cost is less than the average variable cost, okay and when the marginal cost is less than the average variable cost then what we will get?

We will get that when it produces it makes a more loss then when it is not producing anything so the optimal output is, optimal decision is going to be not to produce any output, okay. So, from here what do we get? We get that when a firm is going to produce, firm is always going to produce when the price is greater than average variable cost, right. When price is always greater than average variable cost?

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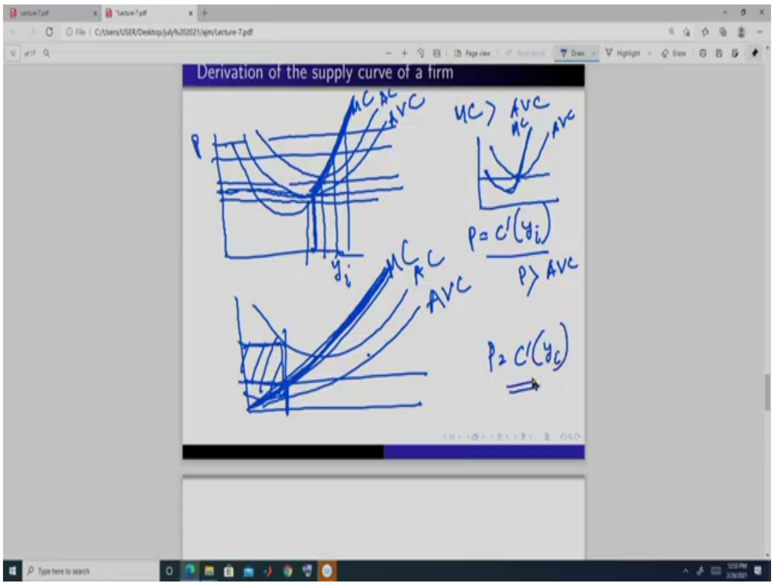

We know that a condition that, we have a condition that whenever marginal cost is greater than average variable cost, whenever average variable cost is increasing, right? so we know this thing. So, this intersection, this is the marginal cost and this is, this curve is suppose marginal cost and this curve is suppose average variable cost then this intersection takes place at the minimum of average variable cost, right?

So, whenever price is above this minimum, then firm is always going to produce. If price is below this minimum, then it is not going to produce. So, from this we can derive something called a supply curve of a function, supply curve of a firm, so if the output is like this, okay, so price is like this, marginal cost is suppose this, this is average variable cost, this is average cost and suppose price is here.

In that case this is the optimal approach, so firm is not going to produce. So, price should always be above this level, right, this much output. So, a firm is always going to produce this much amount of output and then as the price increases like this, here, optimal output is this, if price is here, the optimal output is like this, if price is here the optimal output is this, so from this position this, this is the supply curve of a firm.

So, price here and the amount of output it is going to produce is this, **this** much, it is not going to produce anything below this because if the price is below this then it is possible that they may produce, but then while producing that it makes more losses then while not producing any output. So, that is why it is not going to produce. So, the supply curve is this. So, its output is going to start like that.

So, it is that portion of marginal cost curve which is lying above the average variable cost, so the supply function of a firm can be written like this. This condition is giving me the supply curve, where the price is greater than average variable cost or you can say it is when the price is greater than minimum of the average variable cost, okay.

So, in this situation we will get a like this, but suppose the marginal cost curve is of this nature, then we know the average variable cost is always going to lie below it and suppose the average cost is this, then this whole marginal cost is the supply curve, because if we produce, suppose the price is here, then optimal amount of output is this and it is lying above the average variable cost, so if it does not produce its loss is this much.

And if it produces its loss is this, so it is better to produce, right? So, this whole the marginal cost is the supply curve, in this situation the supply curve is actually this of any firm. Now, why do we need this supply curve? Because from this supply curve we will get the market supply curve because it is going to be the horizontal summation of each supply curve of its firm and then that will give us something. It will allow us to derive the market demand, okay. We will do that later.

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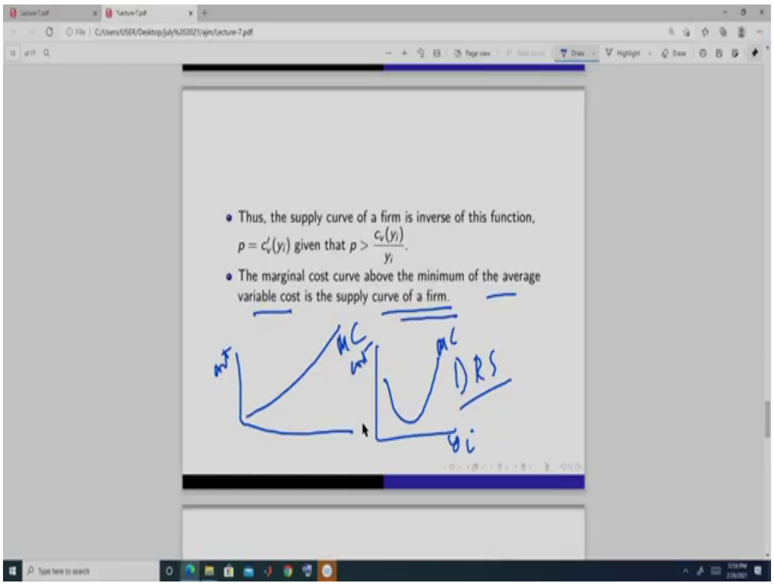

So, marginal cost curve which lies above the average variable cost is the supply curve of a firm, so we have got that. Now the thing is in this whole whatever we have done till now, we require the cost function to satisfy decreasing returns to scale at least partially, so the cost function can be either like this, marginal cost function can be like this or the marginal cost function can be of this nature, right?

So, this is when we have decreasing returns to scale we know marginal cost is always going to be like this, okay and if we have one fixed component of cost and suppose all, suppose labor and capital is variable, but the fixed component is coming from the rent that we pay from for the building that we have hired or for the land, then also we can have a decreasing returns to scale and we may get a this kind of u-shaped marginal cost.

Another reason when we can have this kind of marginal cost is when we have law of diminishing marginal product, which is operating, that means suppose land is, capital is fixed, it is fixed at some a and it is lumpy, you have to buy this much and a is variable, labor is variable then we have seen that we will get a marginal cost curve, which is always increasing of this nature and but fixed cost is coming, will be there. So, together we will get this. So, due to law of diminishing marginal product, we will also get this kind of marginal cost and so the average cost curve will be of this nature, okay.

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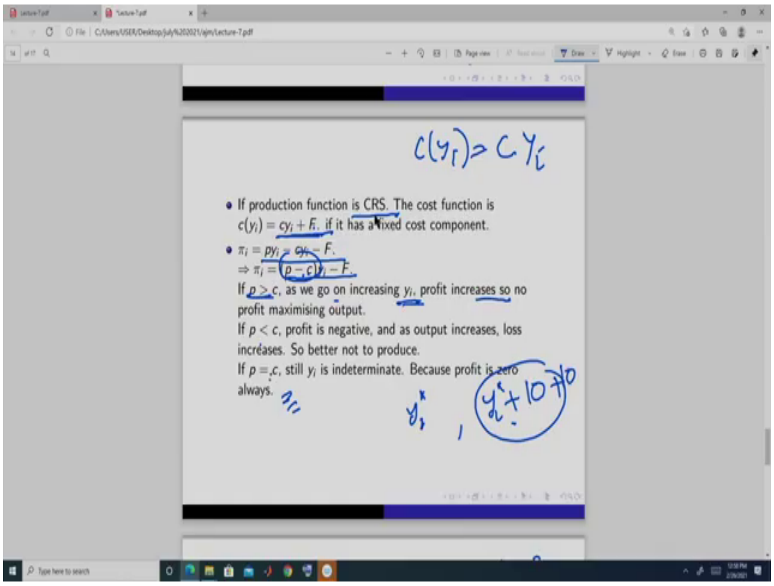

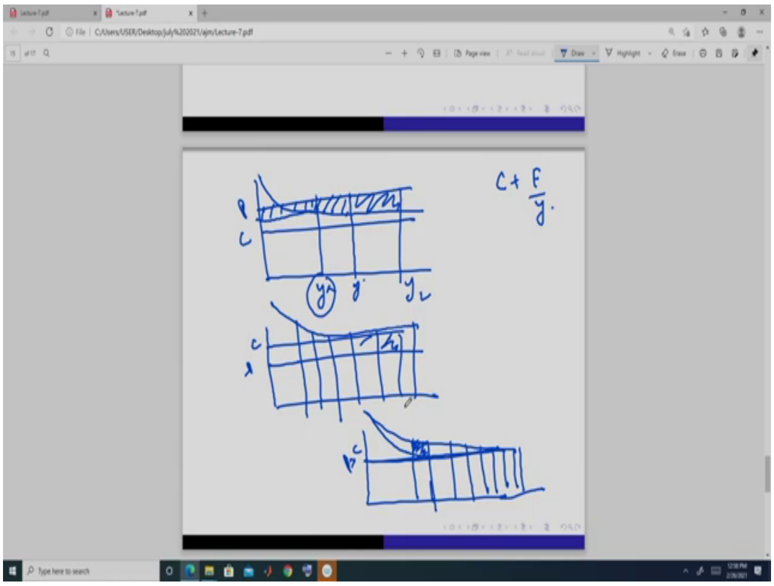

Now, what happens if we have suppose Constant Returns to Scale – CRS? We know one form of a CRS can be this kind of cost function- $c(y_i) = cy_i + F$ where we are multiplying the output with a constant number, fixed number, real number plus a fixed cost component, it is like this. So, if it is like this or the another form it can take is going to be only this- $c(y_i) = cy_i$, whatever form it takes here we get the profit function in this form- $\pi_i = py_i - cy_i - F$.

So, here we can take it in this a- $\pi_i = (p - c)y_i - F$., so it means what? Now, here you can see the price minus this marginal cost into output. Now, see we will use this kind of cost function a lot in later in this course, okay, as of now we are not using it. Now, here if suppose price is greater than c, if in this case, so what is happening, this is taking a positive number. Now, if we go on increasing output this profit is going on increasing.

So, if you produce suppose y star units of output and then instead of that if you produce y star plus some more suppose 10 units, then this will give you a more profit. So, if you produce this then plus suppose 10 more, it will give you more unit profit because it is something, diagrammatically we can do it like suppose the price is this fixed and c is fixed, so if I produce output of this much, then the profit that I get and suppose this is the average.

Suppose this is the average cost, we know the average cost curve is going to be like this, right, so it is like this. So, if we produce this much amount of output, then I am making this much amount of profit, right. If I produce this much amount of output, I am adding this much amount of profit, so I will not produce this instead this, but if I produce this I am adding more, so what is going to happen, so I will not be able to determine what is the optimal amount of output, right?

As we increase output profit goes on increasing so we do not have any profit maximizing output. In this case suppose the price is less than the marginal cost, okay, so this is suppose c and this is p and this is our average cost, right, so what is happening? We are always going to make losses, if we go on increasing, so these are losses, right, output our losses are increasing. So, we will not produce any output here in this case, okay.

Now, in this situation if price is equal to the marginal cost, so if we are at this, price is also equal to, now profit is always, so it will be like this, average cost is going to intersect, right? so we will go on producing output, right. If we produce here loss is, this much is going to be the loss, right. So, in this situation also we do not get because as we, a output is indeterminate. So, what do we get? If we take CRS, then actually we cannot determine the optimal amount of output, okay.

So, that is why we generally do not take CRS in competitive outcome. Now, when we do the market demand curve and when we do the derivation of the market price then I will again discuss this CRS, okay but as of now for each firm they cannot determine their optimal amount of output okay. Now, what happens we when we have an increasing returns to scale?

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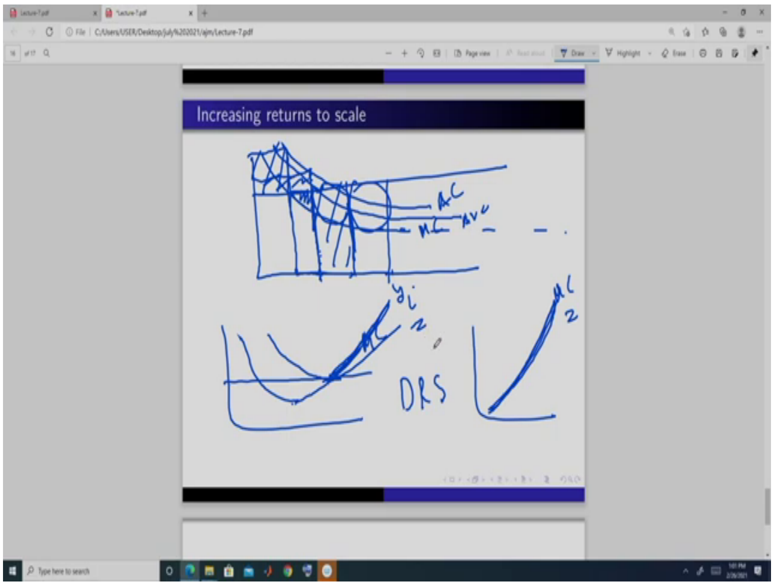

When we have increasing returns to scale we know marginal cost curve is something like this, right? if marginal cost is something like this, average cost is going to be like this, okay and suppose average variable cost is also like this. So, at this price we get price is equal to marginal cost, but if I increase output, this much is the revenue and this much is the additional cost. So, I make some additional profit.

Although I am making a loss in aggregate amount, but that loss has gone down. Earlier the loss is of this whole amount, now the loss is only of this amount, so instead of this if I produce, here my profit increases by this region, if I produce here profit increases by this region, so I will go on producing, so here again when we have increasing returns to scale, we find that the, it is not, we cannot determine the optimal amount of output of each firm.

So, that is why for in a perfectly competitive market we always assume that the technology or production is always decreasing returns to scale or it may have initially some increasing returns and then again decreasing return. So, marginal cost curve should always be of this nature or the marginal cost curve should be of this nature, okay.

And based on these two type of marginal cost curve what we can do we can find out the optimal amount of output a firm is going to produce at any price and we know in this case, the whole marginal cost curve is going to be the supply curve and in this case, only suppose this is average variable cost is like this, then the supply curve is going to be this for the firm, okay.

So, we end today's lecture at this point and in the next class what we will do from this supply curve of each firm, we will move to market supply curve and then at market we will determine the price and then we will see what happens, what is the dynamics here, we will not do explicit dynamics, but we will simply describe the function here, how, what happens when the price is below average cost and when the price is above the average cost, okay.

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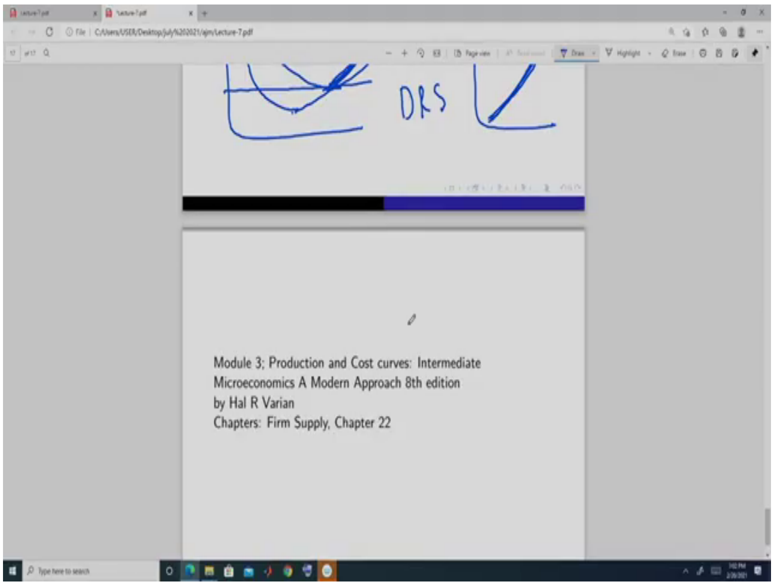

And you can start reading chapter 22 of this book Hal R Varian Intermediate Microeconomics A Modern Approach, that is Firm Supply or this class notes is going sufficient, I have covered exhaustively, okay. Thank you very much!

Introduction to Market Structure
Professor Amarjyoti Mahanta
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Module 3: Perfectly Competitive Markets
Lecture 10
Derivation of Market price in Competitive market

Hello, welcome to my course Introduction to Market Structures. So, let us first do a little bit of recap. We have started Perfectly Competitive Market and the main assumption of Perfectly Competitive Markets are that the price, each firm takes the market price as given and they decide only on the amount of output they are going to produce and there is complete information and firms, there are many firms.

And also another a is that, there, it is not an assumption but is an outcome that we have got from the profit maximizing property, from the property the profit maximizing outcome that there should be decreasing returns to scale.

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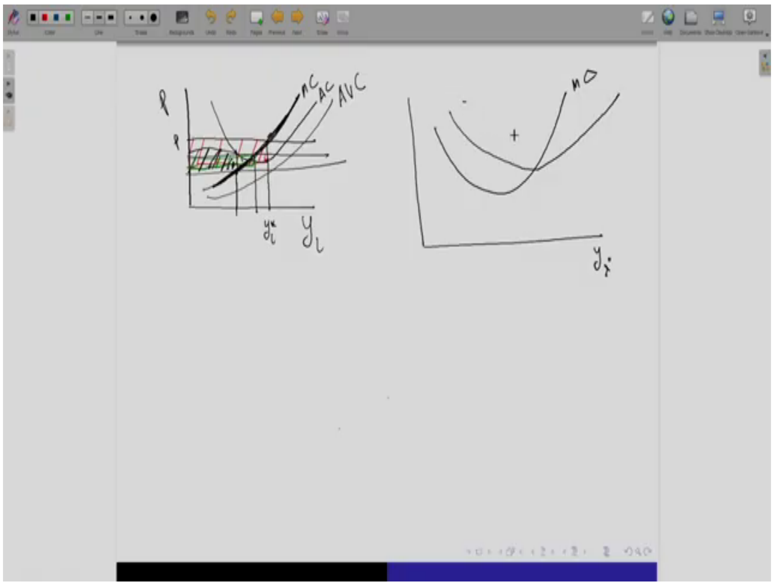

So here and suppose the output of firm i is given in this axis, price is given in this axis, we know that the, suppose this is the market price and the marginal cost is of this nature, MC and if marginal cost is like this we know average cost, average variable cost is going to be of this nature and the average cost is this. Now here, from the last class we have got this is going to be the optimal amount of output produced by a firm I, okay.

Now here, suppose instead of this price, price is this then the optimal output is this, if the price is this, then the optimal output is this. Now, here you see when the price is this, profit made by the firm is this rectangle, right? when price is this, marginal optimal output is this, average cost is here, so this green box is the profit, right? When price is this optimal output is this much and at this output average cost is this, so this box is the loss, right?

So, we get what, this curve, actually the optimal output is always in this curve and this is the marginal cost curve and this is also the supply curve of a firm, okay. This we have done in the last class. Now, so we get a supply curve is the marginal cost curve of the firm, but is it the whole marginal cost or some portion of the marginal cost?

From the shutdown condition that we have done in the last class, we get that the marginal cost curve which lies above the average variable cost is only the supply curve. Suppose the marginal cost curve is of this nature, then the average variable cost is of this nature and suppose the average, okay, let me draw again.

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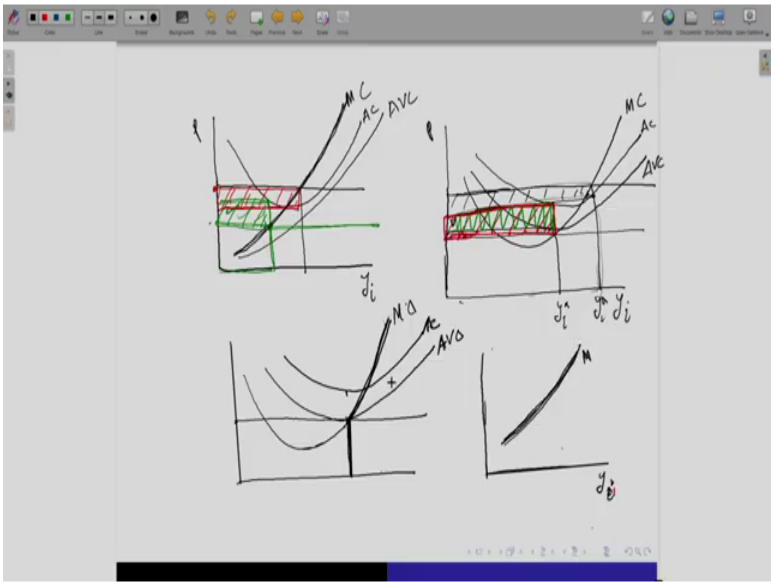

Profit is this when the price is at this level and suppose the price is this, market demand curve is this for the firm, then the optimal output is this, so cost is here, this green a is the loss because this average cost, average cost heist into the output total cost, price is this much, so price into total quantity, height is given, so total revenue, so this box is the loss. Here this box, this red box is the profit.

So, in this case we get that if the marginal cost is of this nature always increasing, then the whole marginal cost curve is the supply curve of each firm. Now, let us take another example. When, so this is suppose a and this is the price, this is again the output, this is the price. Suppose the marginal cost curve is of this nature and the average variable cost is of this nature and average cost is of this nature.

Suppose the price is here, so from the optimality condition we know this is the optimal amount of output at this price, so average cost is here, this rectangle is giving me the total cost, this rectangle and this rectangle is giving me the total revenue, so the profit is this box, right? Now, suppose price is here, then we know price is always equal to marginal cost, this is the optimal amount of output of firm, average cost is this at this output.

So, this box is the loss, right? average variable cost is here at this output. It is this, so this green box which is the green rectangle is the difference between average variable cost into output and the average fixed, average cost into output, so this box is the fixed cost green box and the loss is given by this red, whole red box. So, here loss is more than the, than the total fixed cost. So, the shutdown condition is violated.

So, the firms are not going to produce at this output. So, if we follow this argument, then we get, suppose the marginal cost is of this nature, this is average variable cost, this is average cost and price should always be above this level because this is the minimum of the average variable cost, this. If price lies below this, then firms are not going to produce because shutdown condition is violated.

So, if the price is above this then the firms are going to produce even if they are going to make loss. So, this portion is the supply curve of this type of marginal cost, so we get that the supply curve of each firm is this upward sloping portion of a marginal cost curve but not whole of the upward sloping portion of the marginal cost curve because in this case this portion is also upper sloping but this is not part of the supply curve, okay. So, it is always above the minimum of average variable cost, okay, if this is the output so supply curve of a firm is this.

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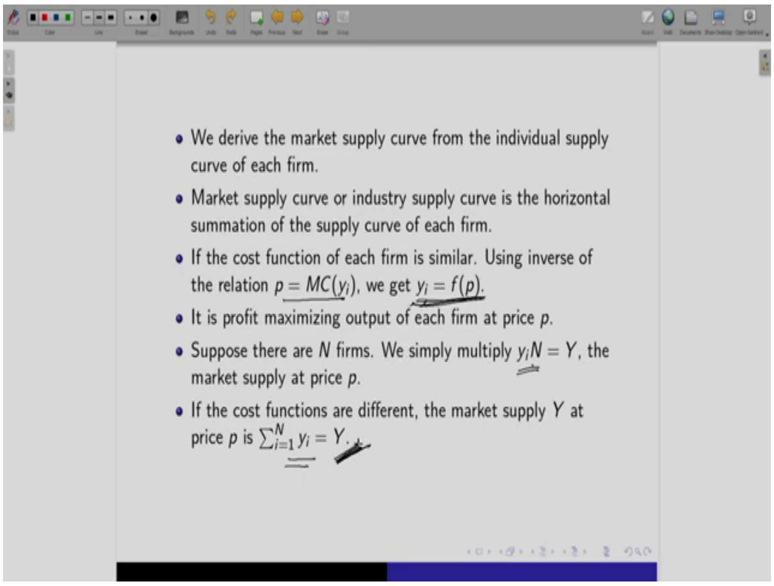

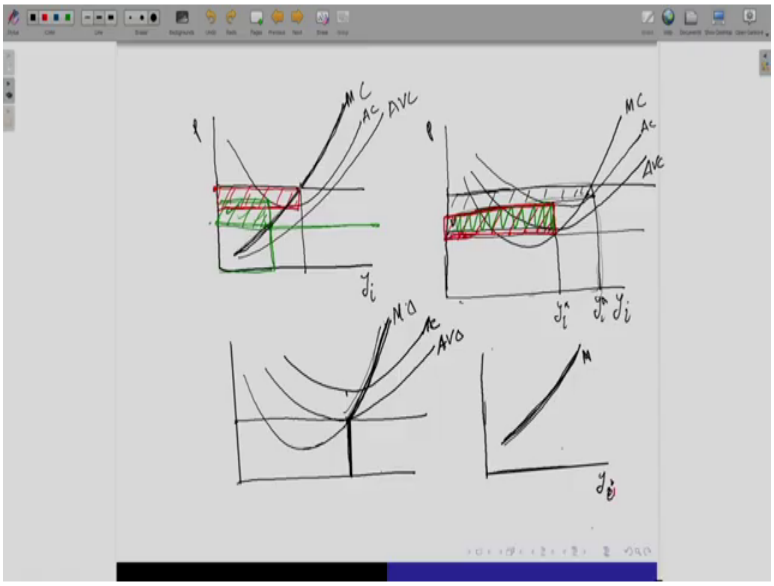

Now today what do we do, we start or derive the market supply curve. How do we get the market supply curve? So, remember in deriving a market demand curve what we have done, from individual demand curve we have done a horizontal summation and that horizontal summation has given us the market demand curve.

It is like at each price what each individual is demanding and if we sum this, the demand of each individual, we get the market demand. Similarly, here at each price how much each firm is supplying, so each firm from this condition- $p = MC(y_i)$ we can get like this- $y_i = f(p)$ which is if we take the inverse of this- $p = MC(y_i)$, okay. Since this marginal cost curve is always increasing in this portion, is always increasing like this or here like this, so we can always have the inverse and from that inverse we will get this- $y_i = f(p)$, okay.

Now, what we do, so this we get for each price if this marginal cost is above the minimum of the average variable cost. Now, so what we do? We sum this, so we get a like this - $\Sigma_{i=1}^{N} y_i = Y$and this gives me the capital y is the market supply at the price p. If suppose this firms are similar, then this y is going to be same if the cost function are same. So, we simply do yi into n the market output- $y_i N = Y$, N is the total number of firms present in the market and this is common knowledge, okay, everyone knows about this, all the firms. So, we get this is the market.

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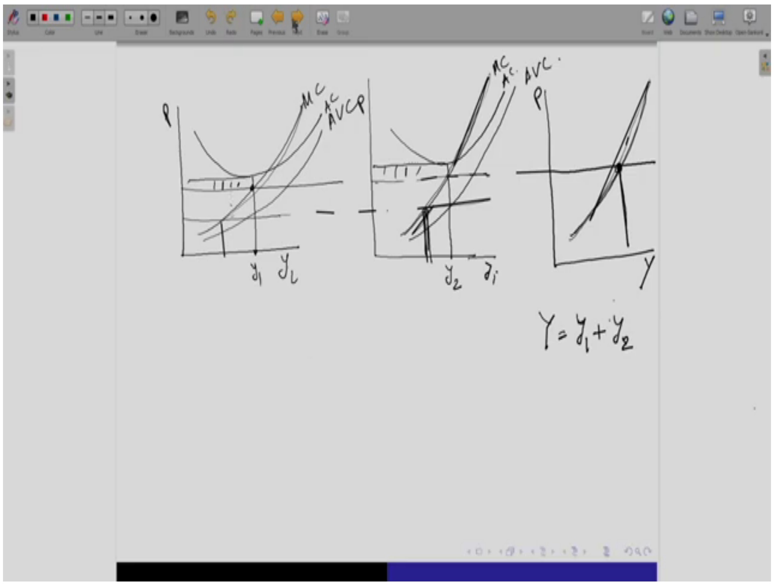

So, here what do we do? How do we get this? This is suppose, there are only two firms suppose, marginal cost is something like this, average variable cost is something like this, average cost is something like this and suppose this is the market price, okay. So, this is the loss they are making, but this is the output they are going to sell.

Similarly, we have another firm, this is MC, this is the AVC and suppose this is the AC of that firm and it produces this much amount of output, suppose y2 and it is also making a loss of this amount, it is also making a loss of this much amount and this is price, price and this… so the market supply at this price and this is capital y is equal to $y_1 + y_2$.

So at this price it will be sum of suppose this much amount of output, here this amount of output, this is what, this is y1 plus y2. So for each price we will get this much, this much, so it will be sum of this curve plus this curve, so this curve plus this curve, so the supply curve is going to be of something like this, this is the market supply curve, okay.

Now, when we, so we will get the supply curve of this and if there are suppose three firms then we will take this and horizontal summation of this three marginal cost, if there are n firms so it will be or in this way horizontal summation of the marginal cost curve of n firms and we will get a curve like this if the marginal cost are curves are of this nature.

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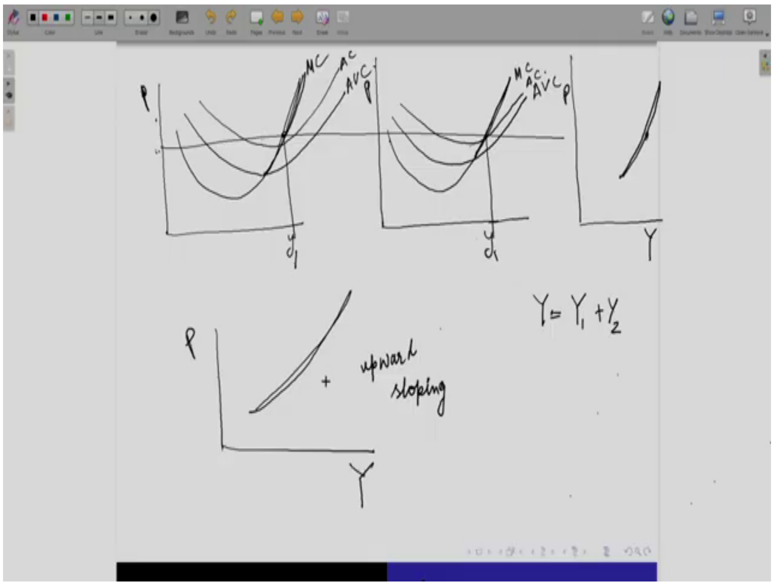

Next, suppose this is output of firm one, suppose marginal cost curve is of this nature and I hope you remember that this point is the minimum of AVC and again this point is the minimum of AC, okay. Here you just take, I have, this diagram is not that correct because this curve should have gone pass through the minimum of these two curves but remember this when you draw and suppose the market price is this, right?

So optimal output for this firm is here, optimal output is here and here this is the y2, this is the capital output, capital Y is, okay. so this is given by this point where this is sum of this output and this output, okay. Now here remember this is, supply curve is only this portion and the supply curve here is this portion, so the market supply is going to be sum of these two curve at each price is here, okay, so it is going to be a curve like this, okay, not the whole of the marginal cost.

So from this what do we get that the market supply curve, this price here is always going to be upward sloping, okay, that means as the price increases quantity supplied is always going to go up. Now we have to derive the market equilibrium price that this price, which each price, each firm takes as given we have to determine that. How do we do that?

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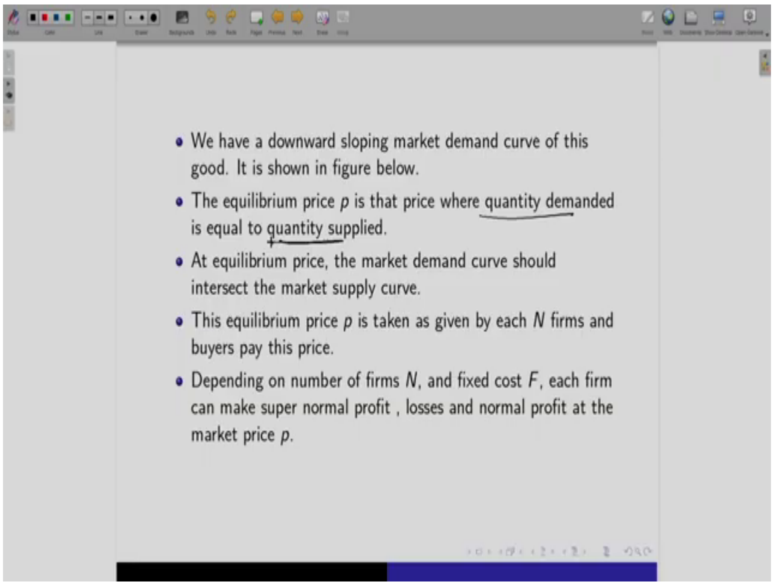

So, we know that the market demand curve is a downward sloping demand curve; we have assumed that at the very beginning. Now, we also have derived the market supply curve for a given number of firm and that is suppose n, so in that case it will be the horizontal summation of the marginal cost curve of each firm and that portion of marginal cost curve which lies above the average variable cost, minimum of the average variable cost, okay.

Now, market price equilibrium price is determined where quantity demanded is equal to quantity supply, this, this is the most important condition, it is that price at which market is cleared you can say, whatever quantity is being demanded that amount is being supplied, okay and that price is taken as given by each firm, okay so we will derive that.

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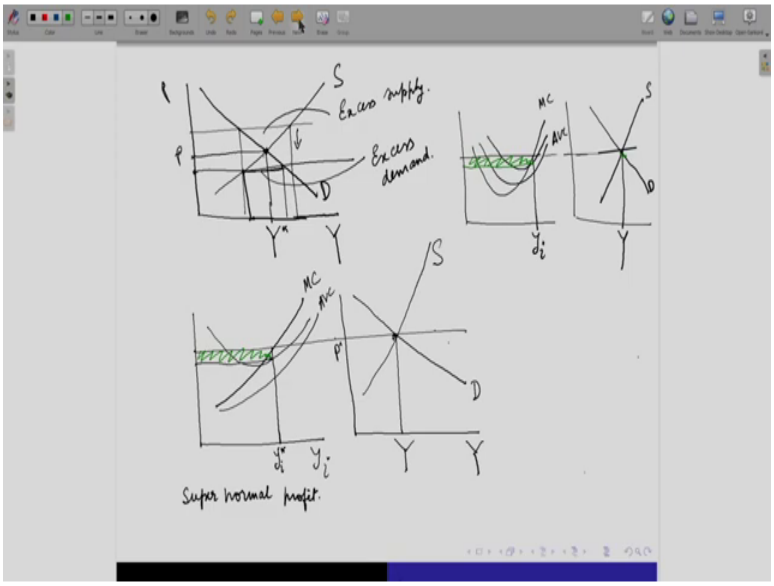

See we have the demand curve like this which is the D, price here, quantity y, market demand. Market supply is this which is the horizontal summation of the marginal cost curve. You can write this as, okay, it is going to create unnecessary confusion, simply take it like that, at this price what is happening at this price, quantity demanded is this, quantity supplied is this it is equal, but if we take this price what is going to happen?

At this price quantity demanded is this much, quantity supplied is this much, so quantity supplied is more than quantity demanded, so what is going to happen, there is going to be excess supply in the market and so price will fall and it will move here, so it will move like this and it will come down to this. If the price is here, at this price quantity demanded is this much and quantity supplied is this much.

So, there is excess demand which is this much, right? and this is the excess supply, whenever there is excess supply the price will fall and whenever there is excess demand price is going to rise and this whole thing is going to happen instantaneously, instantly. So, there are firms which are going to, which will get a signal that this is the market price, okay.

And if at that market price there is going to be excess supply, then price is going to immediately fall and it will come to this price and if at that signal the price is this, is the signal and there is excess demand, price is going to rise and it is going to rise to this point. So, this whole adjustment takes place instantaneously.

And this is an assumption and this is a very big assumption in this or you can say it is a very strong assumption which you made in the Perfectly Competitive Market that the price adjustment takes place instantaneously , okay and as this price adjustment takes place, so firm always take the equilibrium price as the given price, because if there is excess supply then the price is going to go down and immediately there is going to be equilibrium.

And if the price is such that it is low and there is excess, demand price is going to rise and equilibrium price will be attained. Now here, this whole story we can explain it in this way, okay. see, so I hope you have understood this. So, this is suppose, take any firm and firms are, any firm i and all the firms are identical suppose, this is the market, and this is the market demand curve, okay. Suppose the marginal cost curve is of this nature, okay.

Average variable cost is of this nature, average cost is of this nature and market price is suppose this. What does this mean? That the supply curve is like this and this is the market and this is the price which is prevailing at the market at that time, and each firm is making this much amount of, producing this much amount of output, right. Here this is the average cost at this output, so firms are making this much amount of profit.

This profit is positive amount and this situation is called firms are making something called super normal profit, okay. At this equilibrium price which has, which we have attained by where market demand is equal to market supply, amount demanded is equal to amount supplied in the market, okay this or in case suppose our market are of this nature.

Suppose the marginal cost curve is this, average variable cost is this, average cost is this and suppose the market price is this, then suppose this is the market supply and this is the market demand and this is the market output, this is the output of each firm. And here in this case they are making a super normal profit of this much amount, right. So, in this situation we say that the firms are making super normal profit because the profit is more than the, means it is positive amount okay and this price is determined in this way in the market.

Now, there may be a situation that the price is such that the firms are making only normal profit. When do we have normal profit when the profit is equal to 0, that is total revenue is equal to total cost and that situation is something like this.

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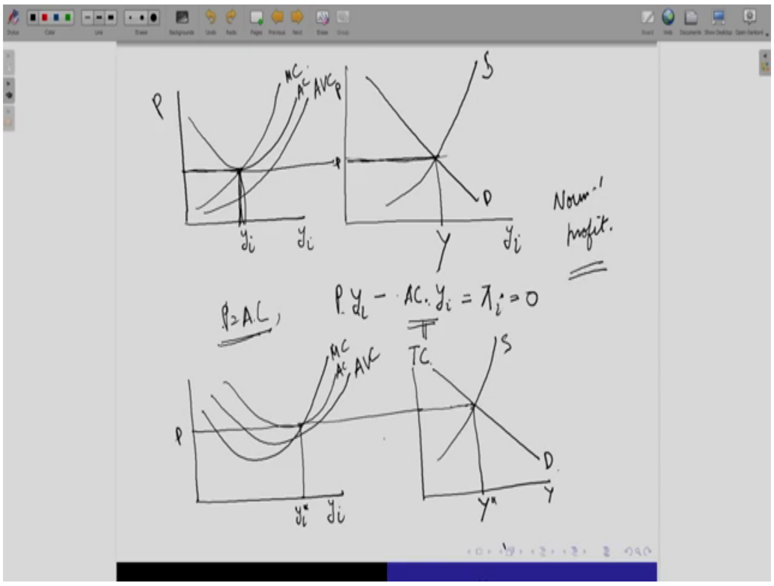

Marginal cost is of this nature, average variable cost is of this nature, average cost is this and suppose here at this price if this is the quantity and this is the price, this is the price and this is market demand is suppose this, this is suppose the market supply and this is the market equilibrium output and this is the output, optimal output of each firm, here if this is the equilibrium price suppose then what is happening.

At this price average cost is this, price is also this, so price is equal to average cost, so you can, is the profit, right? average cost into the output, this is equal to what, total cost, this is total revenue, so this is equal to 0 because when price is equal to average cost we get this, so this is situation is called the firms are earning, what is called normal profit- $P.y_i - AC.y_i = \pi_i = 0$. So, normal profit here it means that the remuneration that the entrepreneurs or the owner gets is already part of the cost, okay, otherwise why entrepreneur is going to produce.

Now, in case marginal cost curve is of this nature and the average variable cost is this, average cost is suppose this, this is the optimal amount of output and suppose this is the market price which we have got and this is the market demand, market supply, this is the market output, right? so this price has been attained in the market where quantity demanded is equal to quantity supplied and this price is taken as given by each firm.

And the optimal output produces this, at this optimal output price is equal to average cost, so firms are earning zero profit and this is a situation where we call the firms are making normal profit, okay.

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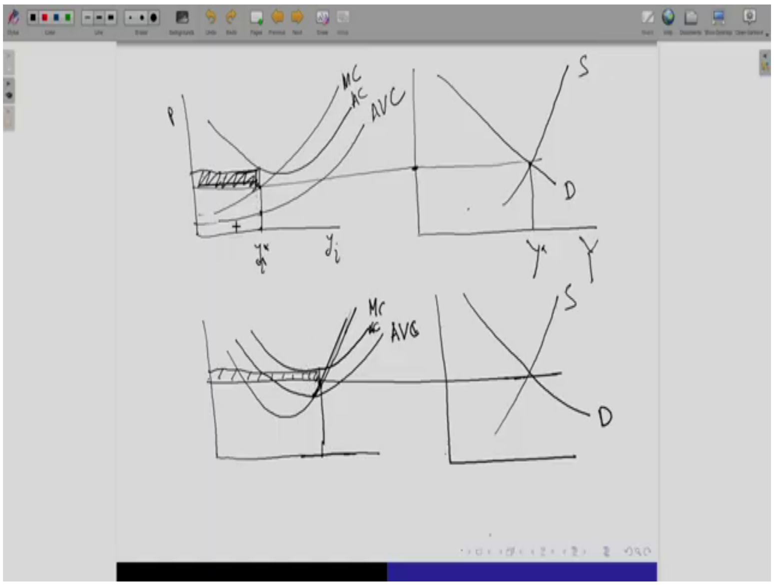

Next we will look at a situation where firms are making losses. Suppose, this is the output of firm i, this is the price, marginal cost is of this nature, average variable cost is of this and average cost is this and suppose market price is this, this is the market demand and this is the market supply. So, market price is determined at this level, and here each firm is producing because at this price, marginal cost, price is equal to marginal cost is at this point.

So optimal output of each firm is this so average cost is this much, so this triangle, this rectangle is the loss they are making and in the short run we know, since this loss is less than the fixed cost this amount, because fixed cost here is this whole big drag rectangle, because it is average variable cost is this much, so this quantity into average variable...

This rectangle is going to give you the variable cost and this quantity into this height is going to give you the total cost, so minus, total cost minus total variable cost gives you the fixed cost so which is this rectangle and this small rectangle is the loss they are making, so the loss is less than the fixed cost, so firms are going to produce some positive amount of output because only when price is below the average variable cost and that is the minimum of the average variable cost firms are not going to produce in the short run.

Now, let us take the case where the marginal cost is u-shaped, it is something like this, this is the marginal, okay and this is average variable cost, okay. This is suppose average cost and market price is somewhere here, which, here this is the supply curve, right? So suppose this is the market supply curve which we get as the sum of the horizontal summation of the marginal cost curve, here this is the optimal output produced by each firm.

And the loss they are making this is them, is this much amount, this loss is again less than the fixed cost, so firms are going to produce, so here firms are making losses. Why we have discussed this? Because now we will move to long run and in long run there is free entry and exit of firms and once there is free entry and exit of firms, then we will see how this super normal profit, normal profit and losses, how they are going to change, okay.

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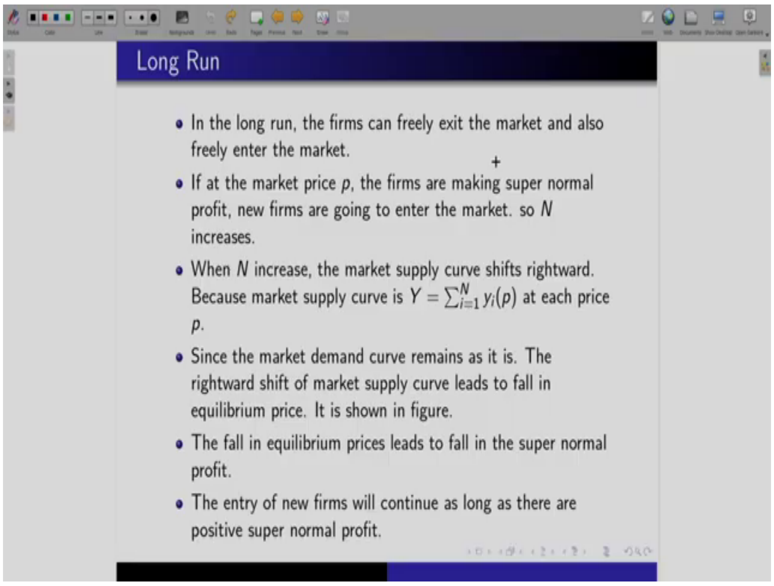

Now, let us do the long run situation. In the long run what happened, firms can freely exit the market or firms can freely enter the market. What freely, why freely is important? Because here firms do not have to make any expenditure or have to bear any cost for entering the market or for leaving the market. Now suppose firms are making some super normal profit at any price today and what happens…

So they are making super normal profit, so other firms which are want to enter this market they will observe this and they will see okay these firms are earning super normal profit that is some positive amount of profit, so they are going to enter and as the firms enter what is going to happen, so there is at that price there is going to be, because they will enter and they will take that price as given and what will happen, there will be more entry of firms.

So, the total output is going to go up, so this means what, that the supply curve is going to shift, the moment supply curve shift what happens, the price is going to go down and as the price goes down the profit, super normal profit also goes down and this process will continue, as long as there are some super amount of super normal profit that the firms can earn there is always going to be an entry of firm.

So, firms are going to keep on entering as long as there are some positive amount of super normal profit. Once, as these firms are going to enter and there is not always, this does not take place in a very coordinated way, so what may happen that some of the firm, then some of the firms which should not have entered they have also entered, so what will happen that the firms may, price has gone down so much that the price, firms start making some losses.

In the short run we have seen that if that loss is not more than the fixed cost then the firms are going to produce, so since they are going to produce so they are going to stay in the market in the short run, but in the long run, so if they stay and since they are making loss, so no new entry, so what, it is going to happen, so they are going to make the same losses continuously.

So, since they are going to make losses continuously in each period what will happen? Some firms are going to leave this market and moment some firms going to leave this market so what is happening, at that price the quantity supplied in the market is going to be less, so supply curve is going to shift upward or you can say is going to shift leftward.

The moment supply curve shift leftward the market price is going to go up and as it goes up firms will start making some positive profit or they are going to earn some super normal profit and as there are super normal profits some more firms are going to enter and this process will go on, entry and exit as long as there are some super normal profit and there are some losses.

And so this will come to an halt when price is always equal to the average cost and it is equal to the marginal cost. So, this is the additional condition that we get in the long run because of free entry and exit of market, of firms and what is this additional condition, and this additional condition is that that the price should be equal to the average cost and the average and it should be equal to the marginal cost, okay.

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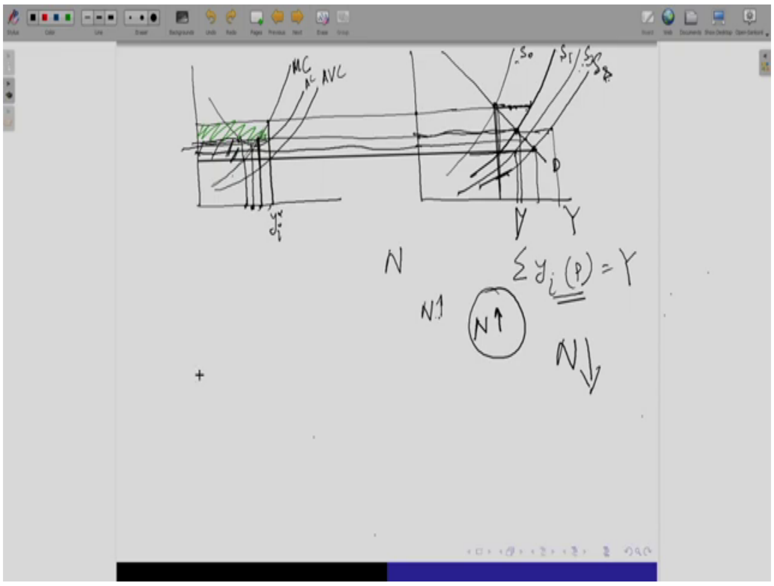

So, we will now describe this through diagram. Suppose this is the marginal cost, this is the average variable cost and this is the average cost, okay and suppose the market price is this, so in this case this is the whole of this is, so supply curve is something like this, so this supply curve is what, this is simply sum of yi at each price, right. So, each firm is making this much amount of output. So, they are making a this much amount of super normal profit.

So firms are going to observe this, new firms, so there is going to be an entry of this firm, moment there is entry of this firm so this summation was initially, suppose there are only n firm, now what is going to happen? This n firm now increases, so this sum is going to go up, so at this price we will have more output, so this new supply curve is something like this, okay. So, this is suppose initially S naught and this is S1 okay.

So at this price as there are more firm has entered what is going to happen, there is quantity demanded is this, but quantity supplied is this, so there is excess supply. Now we know this cannot be sustained because there is excess supply, so price is going to fall and what is going to be the new equilibrium, when this is the supply curve and this is the demand curve new equilibrium is this, at this price, this price the quantity demanded is equal to quantity supply.

So at this price there is going to be entry of firms, moment they enter the equilibrium price is going to change from this price to this price, this and if this is the price, what is going to happen, each firm if they are similar then their output is going to go down, from this they are now producing this, but since the total number of firms are now more so the aggregate output is more this compared to this level okay.

So, but still if you look at this curve you see that the margin, this a is above the average variable cost, so firms are making some still some positive profit or that is the super normal profit so there is going to be further entry of firm and as there are further entry of firms, suppose the supply curve shifts like this, so some more firms have entered, so at this price market supply is this much amount, so there is excess supply.

So, the price is going to down because there is excess supply and the equilibrium price is going to be this much and at this equilibrium price if this price is taken as given, firms our optimal output is this which is again gone down, but now there are more firms, it has gone up because N has gone up further, right? so this, when we have shift this, it means that the N, N has gone up further and as N has gone further so this supply curve has shift rightward again.

So, the price has further fallen and so each firm is producing this much, which is less than earlier, initially it was this much, more entry of more firms, this much further entry of firms this much, now here at this price average cost is this, so it is average cost is more than the market price, so the firms are making losses. So, if the same number of firms stays in the market, at least in the short run they are going to stay then each firm is going to make some losses, so of this amount, this rectangle amount.

So, it means that in the long run over time some of these firms are going to leave, so as these firms are going to leave then what is happening? This n is again now going down, as this n goes down, so what is happening from this supply curve, supply curve will shift upward, that is leftward, at and each price now it is supplying less, because each firm is producing less and number of firms has also gone down because N has fallen because as all the firms are making losses, some of them have decided to leave the market.

And so supply curve shifts like this and suppose this is S3. So, this was S naught, this is S1, then S2 and this again S3. So, here what is happening? At this price it is this, the market supply, demand is this much, so there is an excess demand in the market and so the supply is, price is going to go up at this, so market price is going to be this much, at this price is this, so margin, firms are going to produce this much, it has increased from this to this. But it is still below the average cost, so it is making some amount of losses.

So, in that period some firms are going to produce this much amount of output, but over time some of them will leave, so this N is going to go down, so what will happen, this supply curve is going to shift right and leftward further and so the equilibrium price will again rise and then finally it will be such that it will be at this level, where price is equal to the marginal cost and it is equal to average cost, okay. So, this is the whole mechanism of price, how the price adjustments are going to take place in the long run, okay. Thank you, we will continue in the next class.

Introduction to Market Structure
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Indian Institute of Technology, Guwahati
Module 3: Perfectly Competitive Markets
Lecture 11
Long run market price and Pareto optimality

Hello, welcome to this course Introduction to Market Structures.

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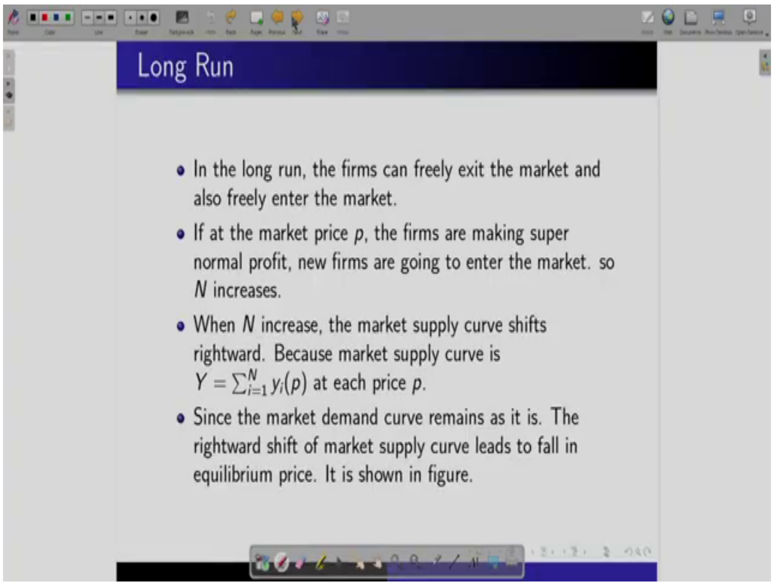

Today we will first do the long run thing in a Perfectly Competitive Market. So in the long run the firms can freely enter and, freely exit the market and freely enter the market. What does this mean? It means that if I want to enter the market then I do not have to pay anything or I do not have to bear any extra cost and if I want to exit this market I do not bear any extra cost, okay. Now when the firms are making super normal profit, super normal profit that means the profit is a positive profit then the firms are going to enter, new firms are going to enter.

And the moment new firms are going to enter what is going to happen? This N that is the total number of firms present in the market it is going to increase and we know that this N determines the location of the market supply curve. So, now as this increases what it means? This is- $Y = \Sigma_{i=1}^{N} y_i(p)$ going to increase because it is sum over N, now we are having more N, so market supply curve is going to shift rightward, okay. Now what happens? So market supply is shifting rightward and the demand curve is same, so it is going to, so the equilibrium price is going to fall and we will show this in a diagram.

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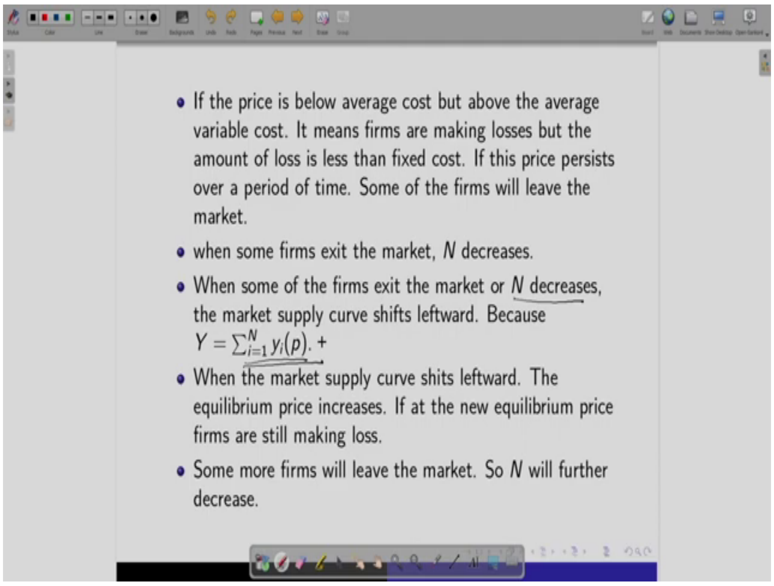

Next, so as the equilibrium price falls what happens, so super normal profit goes down, but still if the firms are still earning some super normal profit then what is going to happen? Then some more firms are going to enter and this will continue. So, it may happen that the number of firms has increased so many that the price has fallen by a very big margin and so the firms have started making losses and once they start making loss, so what it means?

So there are now huge number of firm and they are making losses. So, if the same firm, number of firms stays in the market then they are going to continuously make loss, so some of these firms are going to leave the market and once the firms start leaving the market what is going to happen? This supply curve will shift leftward and given the same demand curve, if supply curve shifts leftward then what is going to happen?

The market equilibrium price is going to rise and the moment the equilibrium price rises what happens, the firms are now, either they are going to make less loss or they are again going to make some profit. And if there is a and when they are going to make some positive profit that is super normal profit then again there is going to be an entry of firms and so again the N increases and again the supply curve shifts rightward.

And moment if the entry of the new firm is so much that the price has fallen by such a margin that again the firms have started making losses, then what will happen? After sometimes some of these firms will leave this market and moment they leave what happen, N decreases and as N decreases this supply curve, market supply curve shifts again leftward and then again equilibrium price may rise, will always rise.

And this process will continue and it will halt or it will stop at that point where the price is such that the firms are making normal profit or that is 0 profit and at that price the price is always equal to average cost.

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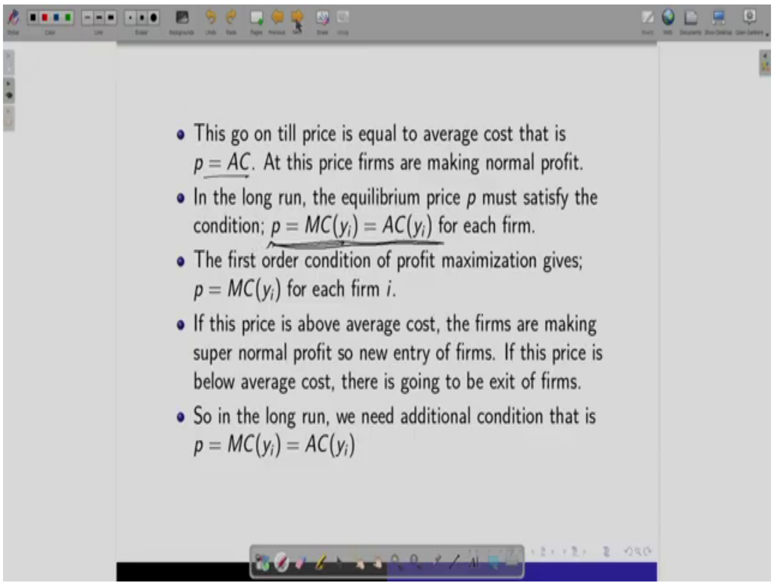

So, for this, in this market we see for the long run equilibrium price should always be equal to average cost and from the profit maximizing condition we know price is equal to marginal cost, so we get this condition- $p = MC(y_i) = AC(y_i)$ for the equilibrium point, okay. So, price should always be equal to marginal cost and marginal cost should be equal to average cost, for in that situation there is not going to be any entry or any exit of firms and so this is the long run equilibrium point, okay and now we show it through the diagrams.

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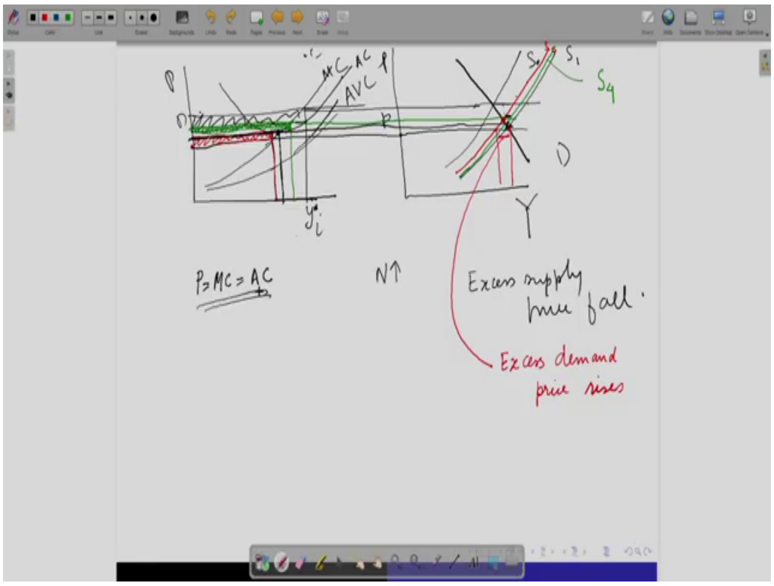

So, what happen this is the output of any firm I and we assume that the firms are suppose similar. This is the marginal cost, so average variable cost and so this is the average cost, okay. This is market outcome, price, suppose the, this is the market demand, this is the market supply, okay and this is the present equilibrium, this is the optimal output of each firm and this is the margin, oh sorry, okay, there is a problem in this diagram.

I have done the average cost curve in the wrong way, this is going to be the average cost curve and this is the optimal, so this is the AC, okay because it intersects the MC at the minimum average cost, so this amount is the, so this amount is the optimal output of each firm and this is the average cost, so this rectangle is the profit, right? because this average cost into quantity, average cost height into quantity, total cost this is the total quantity into price, total revenue.

So, the profit is this, so there is a positive profit, so there is going to be entry of firm, so it means n is going to increase, so moment that happens, what happens? the supply curve shifts like this, suppose this was the initial supply curve and this is the next period. So, at this market price since there is a entry of new firm, what do we see that there is going to be an excess supply.

And we have argued that moment there is excess supply, what happens? price falls, so it will fall till this point means this price will fall till this point, okay, this level and next equilibrium point is at this price. Now, at this and this whole process takes place instantaneously. So, again this price is taken as given by the firm, moment this price, so this is the optimal amount of output and so firms are, now see average cost is this much, price is this.

So, firms are making a loss of this red amount, this red rectangle, so what is going to happen? Now, some firms are going to leave the market, the moment they leave the market what is going to happen, supply curve will shift like this and suppose this is S2. Now, this was the equilibrium at S, when the supply curve was S1, now supply curve has shift leftward like this, so what is this?

So there is, this much amount is the excess demand, so price rises and the new equilibrium price is going to be this price. Now, at this price we get this marginal cost intersects the price at this so this is the new optimal output for each firm, average cost is here, so firms are making some positive profit or super normal profit which is given by this green rectangle, so there is going to be entry of firm.

Moment there is going to be entry of firm, supply curve is going to shift rightward and this is going to be the next supply curve, this is S4, S4 okay. Now, here what is happening, I mean there is a S4, so we get this much amount of excess supply. Moment we have some amount of excess supply what is going to happen? Price is going to fall, right? and as the price falls what will happen, this point is going to be the new equilibrium price.

So, it will lie somewhere here and like this it will continue and suppose this is the price where we get this point where price is equal to MC is equal to AC- $p = MC\left(y_i\right) = AC\left(y_i\right)$, okay, at this point and so this is going to be the long run optimal amount of output for each firm and the long run price is going to be this price where this condition is satisfied, okay.

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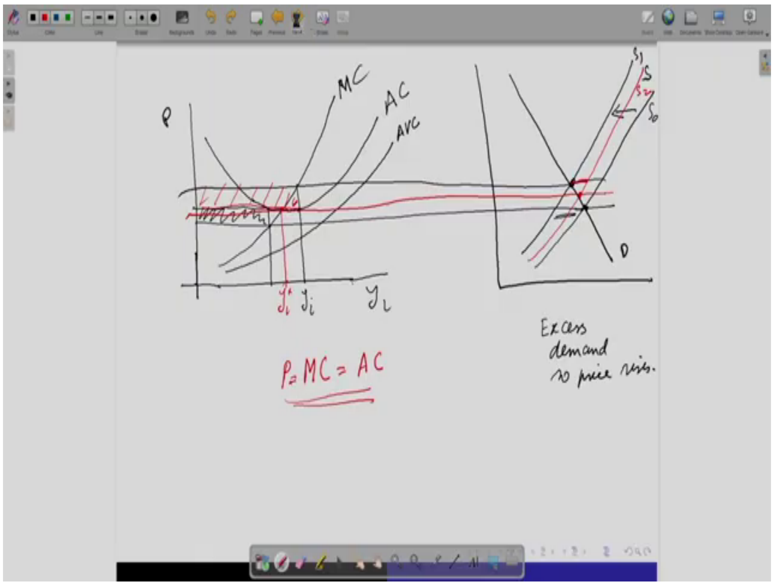

Now, let us take another example, let us start from the, supposed initially the firms are making losses, okay, this is the marginal cost, this is okay. Okay? and suppose this is the output of any firm, this is the supply curve, this is the demand curve, okay. Now here optimal output is this, so firms are making a loss of this amount, right? The moment they are making this loss what is going to happen? In the long run if the same number of firms persist in this economy some of them will leave because they are making losses.

The moment they leave what is going to happen, this supply curve is going to shift like this, so at this price what do we get here? we have, at this price this is the, we have something called excess demand, so price rises and that happens instantly and instantly price move from this level to this level and at this level when the equilibrium price is this each firm take that, price has given and the output they produce is this much.

And here see average cost is this so they are making a profit of this much amount, so moment they are making some profit there is going to be entry of firm and they are going to be of this much suppose and here you see that there is an excess supply and instantly price will move from this level to this level because moment there is excess supply it means that there is less demand than the quantity being sold in the market so the price will immediately fall and it will fall to this level and the firms are going to take this price as given.

And finally this is going to be equilibrium price and the optimal output is going to be this much and at this level of output what happens, price is equal to marginal cost and marginal cost is equal to average cost at this price and we get this as the, so if there is firms initially they are making losses, then some firms are going to leave the market and then market price is going to rise.

And due to rise in the market price the firms may make some profit, super normal profit and if that happens then again some new firms are going to enter and this process will come to a halt or it will stop when the price is equal to marginal cost and marginal cost is equal to average cost, okay. So, this is the, how the we get, this is the outcome or the equilibrium outcome in a Perfectly Competitive Market in the long run.

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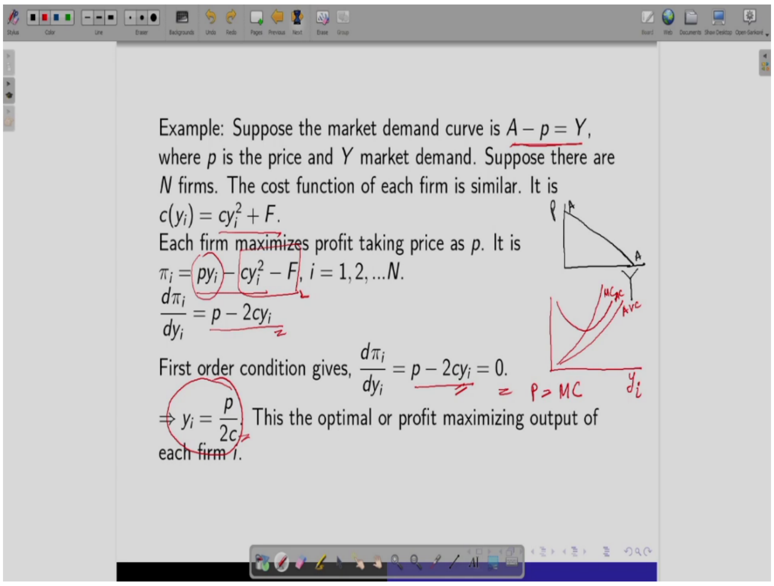

Now let us solve an example numerically. Suppose this- A-p=Y is the market demand curve, okay so you plug in prices the quantity demanded you are going to get is this, so if you look at this demand curve it is market demand curve it is something like this, okay this is A and this is, sorry. Suppose the cost function of each firm is this- $c(y_i) = cy_i^2 + F$, so this means that if we take output of this firm here in this case marginal cost curve is this.

Average variable cost is going to be something like this and average cost, so this is average variable cost, average cost is like this, right? so remember this we will be using these diagrams, means in while doing numerically we do not need to use them directly, but while, previously we have used this diagram so you can look at the relationship between this, numerical thing and the diagrammatic representation.

So, profit function is this- $\pi_i = py_i - cy_i^2 - F,\ i = 1, 2, ...N$, so this- py_i is the total revenue received by each firm and this amount is the total cost- $cy_i^2 + F$, okay. Now each firm wants to maximize profit taking price as given, so we maximize this profit with respect to yi and we get this- $\frac{d\pi_i}{dy_i} = p - 2cy_i$, first order condition gives that this should be always equal to 0, i.e $\frac{d\pi_i}{dy_i} = p - 2cy_i = 0,$, so this condition is actually, this first order condition is actually giving you price is equal to marginal cost so this is the marginal cost of each firm. From here we get this- $y_i = \frac{p}{2c}$, so if the price is p what is the optimal amount of output a firm is going to produce is p by 2c, okay, is the optimal amount of output of each firm.

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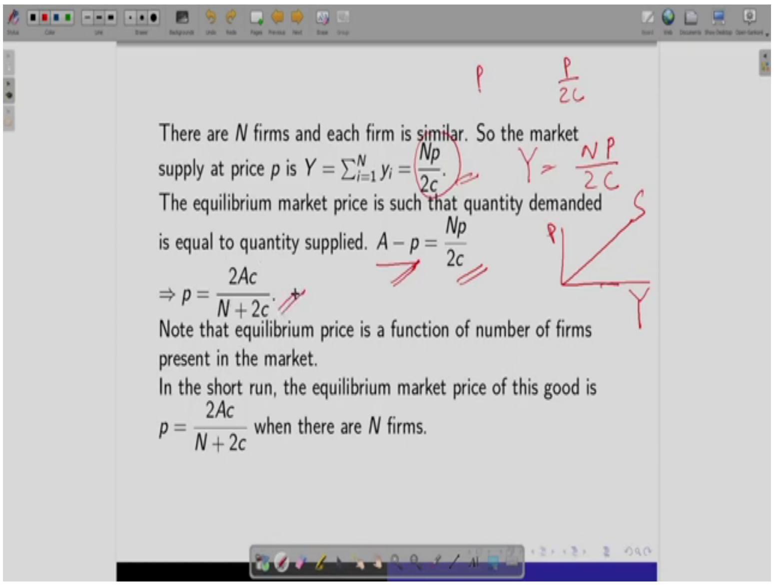

Now, there are N firms and we have assumed that they are similar, each firm at p produces p by 2c and since there are N firm, so the market supply at p is this- $\frac{Np}{2c}$, it is some horizontal summation of each output at that price, so we get this-$Y = \Sigma_{i=1}^{N} y_i = \frac{Np}{2c}$, right? Now, so this if I plug in the price then what is going to be the market supply, it is going to be this much- $\frac{Np}{2c}$.

Now, market demand curve is this-$A - p = \frac{Np}{2c}$ and market supply is this-$\frac{Np}{2c}$ because we can write this as this equal to- $Y = \frac{Np}{2c}$, okay. Now, so this supply curve is something like this, okay. So, at equilibrium this market demand should always equal to market supply, so we have to find that price where quantity demanded is equal to quantity supply.

So, we just solve this equation in this form- $A - p = \frac{Np}{2c}$ and we get this- $p = \frac{2Ac}{N+2c}$, so the market price when there are N firms is this- $p = \frac{2Ac}{N+2c}$. Now, here you note that this price is always a function of number of firms present in and the marginal cost, okay. So, in the short run suppose there are N firms then we know that the equilibrium market price is this.

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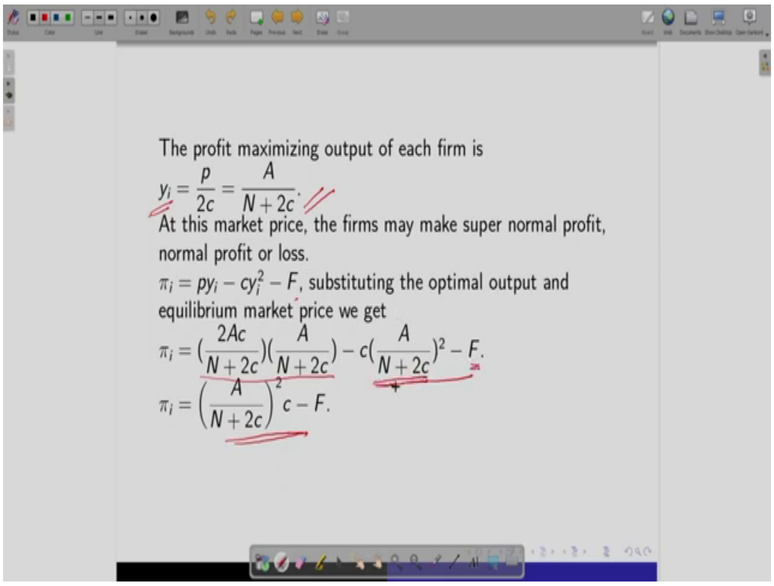

And the moment we know the equilibrium market price from the optimal conditions that we have got that the output of each firm is this so we plug in this value we get this is the output of each firm- $y_i = \frac{P}{2c} = \frac{A}{N+2c}$. And then we find out whether firms are making super normal profit or normal profit or losses at this price. So profit is this- $\pi_i = py_i - cy_i^2 - F$, so we plug in this quantity and the price we get, the profit function in this form- $\pi_i = \left(\frac{2Ac}{N+2c}\right)\left(\frac{A}{N+2c}\right) - c\left(\frac{A}{N+2c}\right)^2 - F$ and we get it after simplification we get it this- $\pi_i = c\left(\frac{A}{N+2c}\right)^2 - F$.

So, this is the price into quantity of each firm, so this is the total revenue price and this is the quantity square, so this is the total variable cost minus the fixed cost, so this is total revenue minus total cost. And after simplification we get this.

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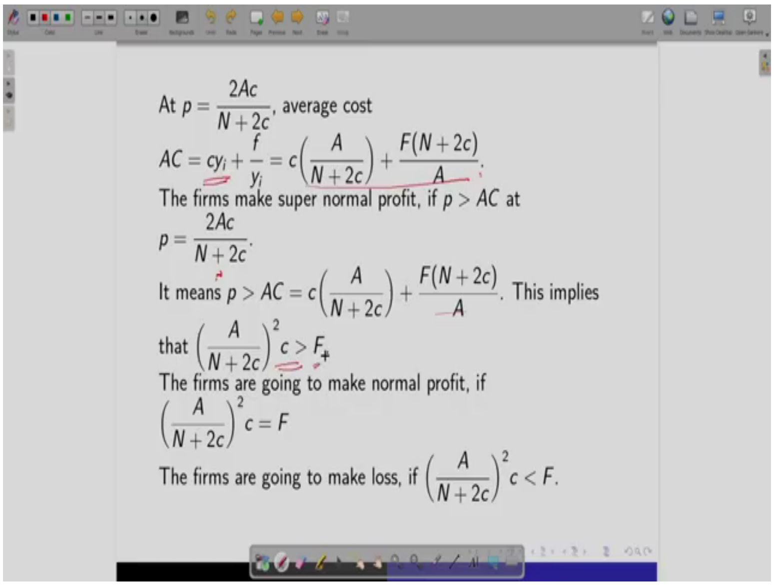

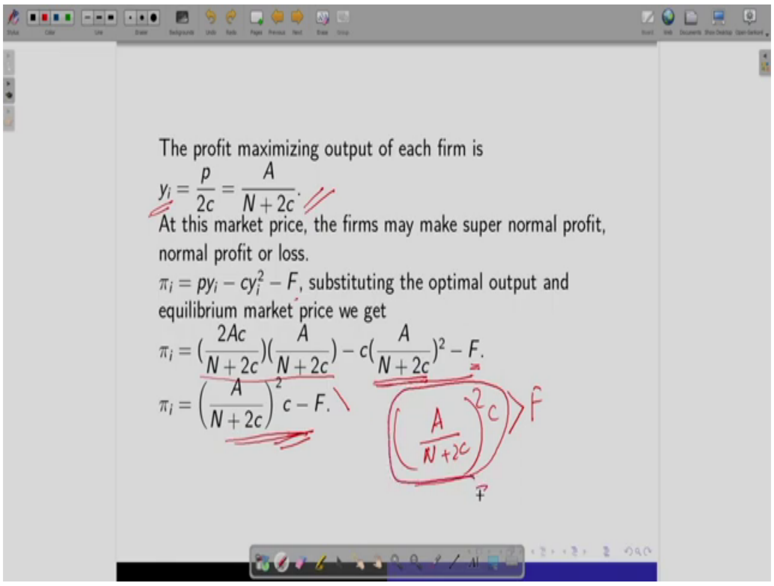

Here we can see when a firm is making a super normal profit when the price is greater than average cost. Now average cost in this case is given by this a form- $AC = cy_i + \frac{f}{y_i} = c\left(\frac{A}{N+2c}\right) + \frac{F(N+2c)}{A}$, okay, and when we plug in this y1 we get the average cost to be this- $c\left(\frac{A}{N+2c}\right) + \frac{F(N+2c)}{A}$ and we know the price is this- $p = \frac{2Ac}{N+2c}$, equilibrium price, so it means whenever this price is greater than this average cost then the firms are making super normal profit. So, we simply look at this- $p > AC = c\left(\frac{A}{N+2c}\right) + \frac{F(N+2c)}{A}$ and we get this- $c\left(\frac{A}{N+2c}\right)^2 > F$, right?

So, the fixed cost has to be less than this amount- $c\left(\frac{A}{N+2c}\right)^2$ or we can simply say that this should be, fixed cost should always be less than this amount, okay. Now, we have got this, when they are going to make normal profit when fixed cost is exactly equal to this amount, when they are going to make losses when the price is less than the average cost, when this fixed cost is greater than this amount- $c\left(\frac{A}{N+2c}\right)^2 < F$, right?

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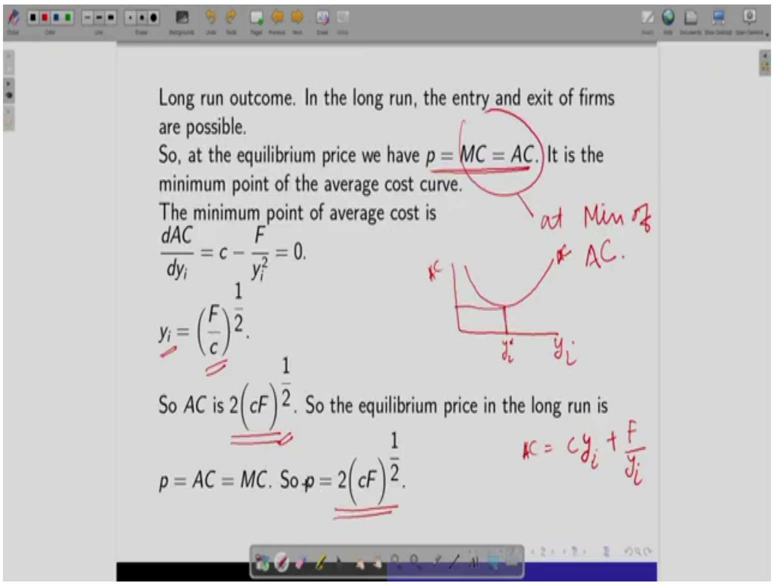

So, now depending on the actual value of F, A, C, N, we can get whether there is a super normal profit or there is a loss or there is a normal profit. Now, we see what is going to be the outcome in the long run? Okay. So, in the long run we know the equilibrium is at this- p=MC=AC, condition is this, so we find the minimum of the average cost because this is, this happens only at the at mean of AC, average cost.

So we take the average cost function and thus take the, minimize it with respect to output of each firm, so we get this- $\frac{dAC}{dy_i} = c - \frac{F}{y_i^2} = 0$and from here we get this output- $y_i = \left(\frac{F}{c}\right)^{\frac{1}{2}}$.

So, in this case when the average cost is something like this so this is the minimum point and this is the, and so output is this. So, we know that in the long run each firm is going to produce at this, this much level of output, okay. So, now plug in this in the average cost, so this height is giving you the average cost.

And so this is the output, you plug in it in the average cost we get this, right? because the average cost function is this- AC=cq, right?, plug in here we get it, this amount and price is equal to, average cost is equal to marginal cost, so price is equal to this-$p = 2(cF)^{\frac{1}{2}}$, so price is we have got this in the long run. So, in the long run the equilibrium market price is this that is the minimum of the average cost, this much amount-$p = 2(cF)^{\frac{1}{2}}$.

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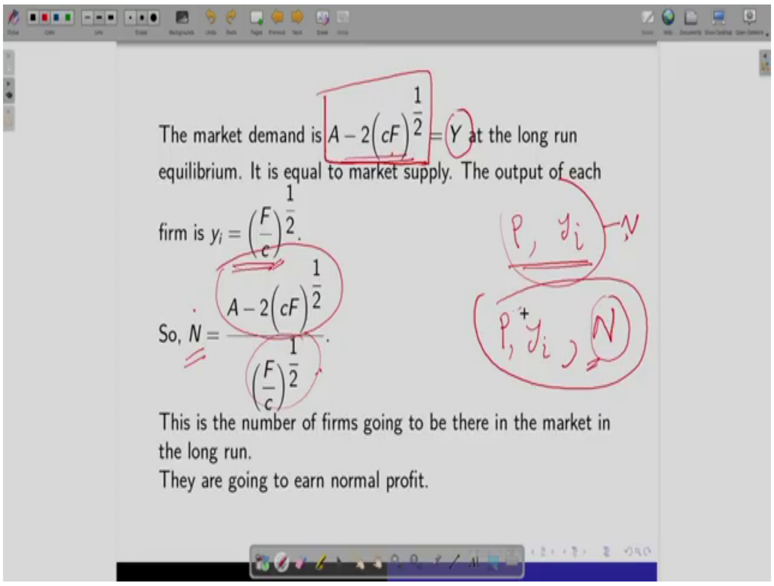

Now, you plug in this market price in the demand curve, we get the, this is the market demand which is given by this, this whole expression, this is the market demand- $A - 2(cF)^{\frac{1}{2}}$. Now in equilibrium market demand should always be equal to market supply, right? and we know each firm is going to produce this much because the price is always equal to the minimum of the average cost and the minimum of average cost is attained at this- $y_i = \left(\frac{F}{c}\right)^{\frac{1}{2}}$.

So, how do we find the number of firms that are going to be present in this market in the long run? So this total demand in the market, quantity demanded divided by the output of each firm, this at the equilibrium point and this, we get this N, i.e $N = (A - 2(cF)^{\frac{1}{2}})/\left(\frac{F}{c}\right)^{\frac{1}{2}}$, so this n is the number of firms that are going to be present in the long run, okay. So, in perfectly competitive what do we get? In the short run we know the equilibrium price and we know the optimal output of each firm.

So, we know these things in the short run. In the long run, so we know this, assuming that there are some given number of firm that is N, in the long run we find this equilibrium price, we find the optimal amount of output of each firm and also we can find the total number of firms that can be present, so this is also determined.

Here this is taken as given in the short run, in the long run this is also determined, so these three things are determined in the long run. So this is mainly Perfectly Competitive Market, where we have determined the price, we have determined the output that each firm is going to produce and we also know what is the total amount of demand that the consumers are going to demand. And from there what happens depending on the demand function of each individual we know the optimal amount of output that they are going to consume, okay.

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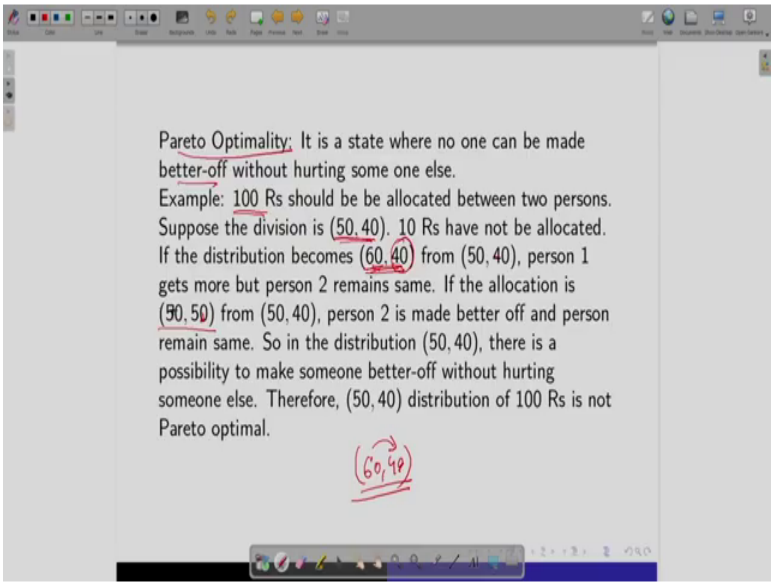

Next we define few concepts and then we, so a very important result that is a perfectly competitive outcome is always Pareto optimal and also social welfare maximizing, okay. How do we show this? So, first let us define what is Pareto Optimality. So Pareto Optimality it is a state where we cannot make anyone better off without hurting someone else.

So, if there are two persons and if I have to make someone better, then I have to hurt the other person if we are not in a Pareto optimal situation, but if we are in a Pareto optimal situation then I cannot make anyone better off by without hurting someone. So, this is, so suppose take a 100 rupees, now you want to divide it between two person, person one and person two. Now, you have done this division suppose fifty and forty, okay.

So, ten rupees left with you. Now, if you give 10 rupees to this person 1 then the allocation is (60, 40), right? Now, from here if you compare this and this what you have done, you have kept the level of person two same, but you have improved that person one, right? but from here you cannot move to any other allocation, like from 60 and 40, since 100 is your total amount if you want to improve this person's share you have to reduce this person's share.

Or if you want to improve this person's share further you have to reduce this person's share, so then what it means, if you want to improve this person you have to hurt this person and if you want to improve this person, then you have to hurt this person, right? so this is a Pareto optimal allocation, but this is not a Pareto optimal allocation when you have 100 rupees because you can make one person better off without hurting another.

Now instead of this we could have done this division also (50, 50). So, this is also another Pareto optimal allocation because from this allocation if 100 is our total amount, we cannot make anyone better off without hurting someone else, so if I want to make person 1 better off i have to reduce some amount of person 2 and give it to person 1 and or if I want to make person 2 better off I have to take some amount from person 1 and give it to person 2. So, this will, this has to be done, so this is a Pareto optimal allocation, okay.

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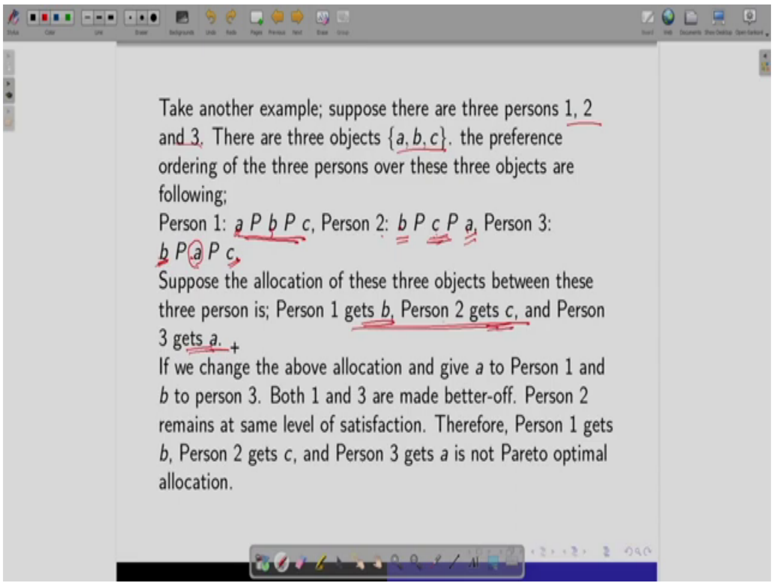

So, let us now do another example, suppose we have three person, person 1, 2 and 3, and there are three objects, these are a, b, c, okay, this can be anything like this is bat, ball and this is suppose wickets, okay or you can say this is suppose some chocolate, this is ice cream and this is suppose some biscuits or cookies, okay. Now, you have one person and we know the preference of this person is that person 1 prefers a over b and prefers b over c.

So, person will be the happiest person if he gets a will be slightly less happy if he gets b and it will be worse off if he gets c. Now, the preference of person 2 is, person 2 prefers b over c and c over a, so it means the person 2 will be will be very happy when it gets b, the object b, will be slightly less happy if it gets the object c and he will be worse off if he gets object a.

And person c preference is like this, like person c prefers b over a and a over c, so person 3 will be happiest and will be most satisfied if he gets the object b, he will be slightly happy or not that satisfied if he is, if he gets a and he will be worse off if he gets c. Now, suppose I have to allocate this three object among this three person a, b, c. And if I have done, suppose I have done the allocation in this way.

Person a, person 1 gets b, person 2 gets c and person 3 gets a, okay? now here we have to say whether this outcome, this allocation is Pareto optimal or not, that is from this allocation whether we can move to a better allocation such that, that I do not have to make anyone happy or better off I do not have to hurt anyone. If I cannot do such movement then it means that I am, in a Pareto optimal state.

Or if I cannot move to that situation from this where I can make someone better off without hurting someone else then I am in a Pareto optimal state. Now, here if you see, if we change the allocation of person 1 and person 3, that is if I give in this a, a to person 1 and this from, I take the a from person 3 and give it to person 1 and take the b from person 1 and give it to person 3 and leave person 2 same.

So, then what happening, see earlier person 1 was getting b and person 3 was getting a and when we have done this interchange what has happened, person 1 has got a, is more satisfied, and person b has got, has got b, is now more satisfied than a and person 2 is more or less in the same state because he earlier he was also getting b, now also he is getting b, okay so this is you can say is a this allocation is not a Pareto optimal allocation.

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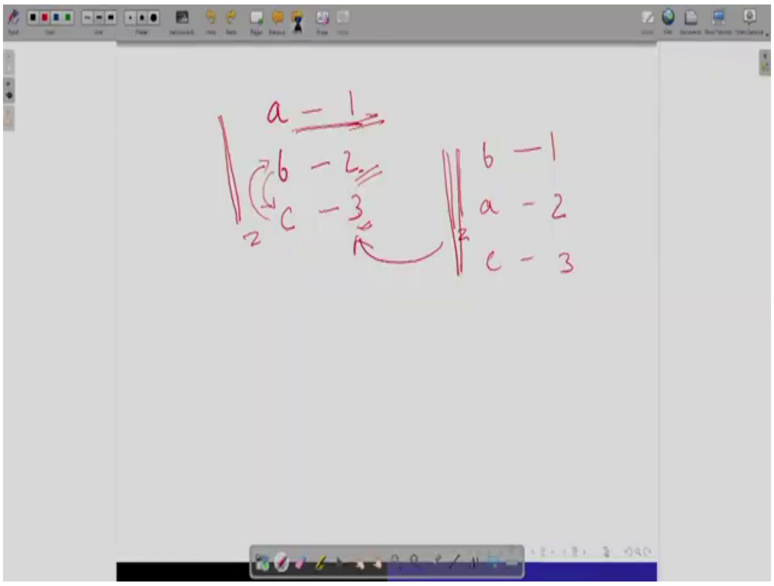

So, we can do some lot of combinations like this, suppose we take instead of this a to person 1, we give b to person 2 and we give c to person 3. This, now is this a Pareto optimal, yes, this is a Pareto optimal allocation. Why? Because now a is in the best position, he has got his most, that bundle which gives him the maximum utility or he has the most preferred one, b has also got that, he has got that object which gives him the maximum utility or the most preferred one, c has got this, he has got that object which is second best, not the first one.

Now, if I have to make suppose person 3 happier, how to do that? I have to do some interchange here, I have to either give this 2 and this, like this interchange I have to do but in that case what is happening, to improve the situation of person 3 I have to hurt person 2. So, that is why, and since he is always best, if I try to do any change here he will get only hurt, so that is why this is a Pareto optimal.

Now, here you can see that instead of this if we do like this to 1, a to 2 and this is actually not a Pareto optimal allocation this, because we can move from this to this and we can make both 1 and 2 happier, more satisfied or we give most preferred bundle to this person, okay. So, this is what Pareto Optimality is. Pareto optimality means that it is a state where you cannot make any person better off without hurting someone else, okay.

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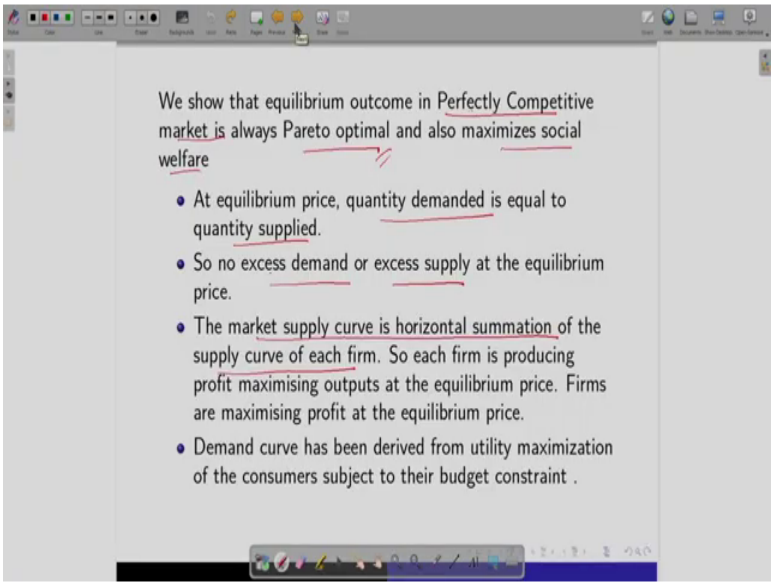

Now, let us do some more a, so now we have to, so the main result is that Perfectly Competitive Market is always Pareto optimal, so the total amount of output that is being produced in the competitive market, so it is produced at that level where quantity demanded is equal to quantity supplied and we have got a equilibrium price and that price determines the total amount of output, right?

So that output is Pareto optimal and it also maximizes social welfare, we will come to it what do we mean by social welfare, but we have defined Pareto optimality. Now, let us first do what do we mean? How do we show this? So, at equilibrium price in a competitive market all quantity demanded is always equal to quantity supply. So, there is no excess demand or no excess supply, okay equilibrium price is such that the market always clears that is quantity demanded is equal to quantity supply.

And market supply is what, it is the horizontal summation of the supply curve of each firm, so if we are in a market supply, so it means it is also taking some point in the supply curve of each firm, right? so in the supply, how we have derived the supply curve of each firm, we have derived the supply curve of each firm from maximizing the profit taking the price as given, so firms are actually maximizing profit when we are talking about this equilibrium price.

So, since it is a price in the supply curve also, supply curve, so and each point in the supply curve is, has been derived from some profit maximization, so it is maximizing the profit. And how we have derived the demand curve, the derived demand curve is the horizontal summation of the individual demand curve and how we have got the individual demand curve, it is from a utility maximization of the consumer subject to a constraint that the consumer face in terms of the market price and the income.

So, so we have these two thing, so the moment we have this, what is happening, at that price both the utility of the consumers are being maximized and also the profits are being maximized, so that is why it is a Pareto optimal solution. Now, it may happen that we will discuss that slightly later, okay or just wait a minute, okay.

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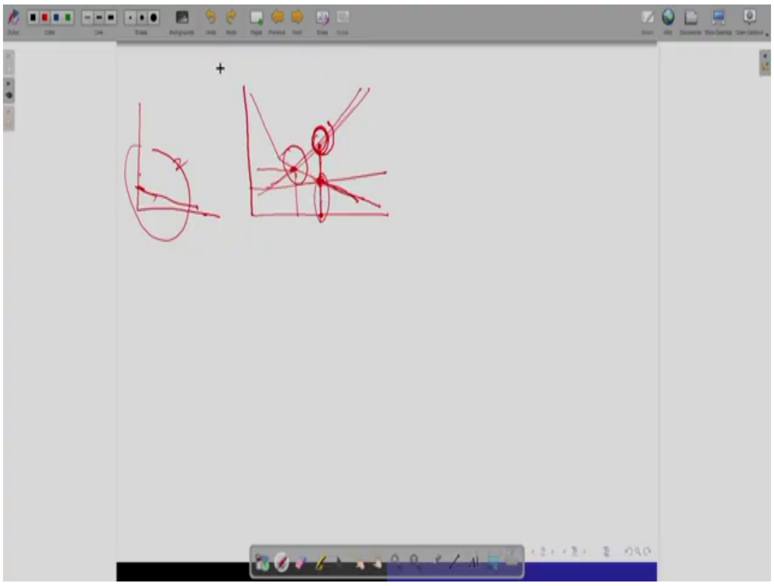

See suppose the market demand curve is something like this okay and because it is a horizontal summation or it is sum of a two demand curve, so and suppose the supply curve is like this, so equilibrium price is here, right? and we are saying that this point is an Pareto optimal thing. Now, why, so this point lies in this market supply, so each point here, firms are optimizing their profit, so or maximizing their profit, taking the market prices given.

So, profits are being maximized. Now, it is also in the demand curve, market demand curve, but it may happen that in this demand curves, in this portion we may have some individual whose demand curve is like this and when we add this add all the demand curves we get like this, so this person, if this is the market price cannot buy any quantity of this, okay but there are many other individuals whose demand curves are like this, so we have derived the market even they can buy at this.

Now, why this is a Pareto optimal? Now, see this is in the market demand, so that means it is lying in the some demand curve of some individuals, but it may not lie, so for this person he or she cannot afford this price, so he is not buying, he or she is not buying any amount, but there are many other individuals which who can buy. Now, what happen? If I reduce this price suppose, suppose I make it like this, then at this price what happens this person can buy.

And the other persons are also buying and they are, since the price is less they can now buy more, and so the, what is happening total utility of all the consumers have increased you can say, but if I keep this price what is going to happen? You can say at this price, so but sellers are, so if we keep this price the demand is this much, at this demand, since this is the supply curve seller wants the price to be of this but the market price has been kept at this.

So, sellers are not in their supply curve, so this means what, so it is firms are not maximizing their profit, so when we are making some individual better off, this individual we are making the firms worse off, so that is why only this point is the Pareto optimal point, any other points are not Pareto optimal points, okay.

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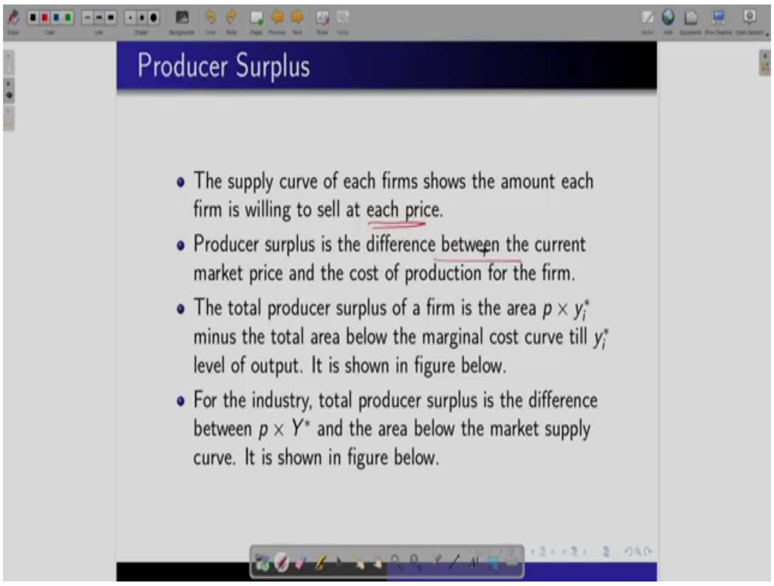

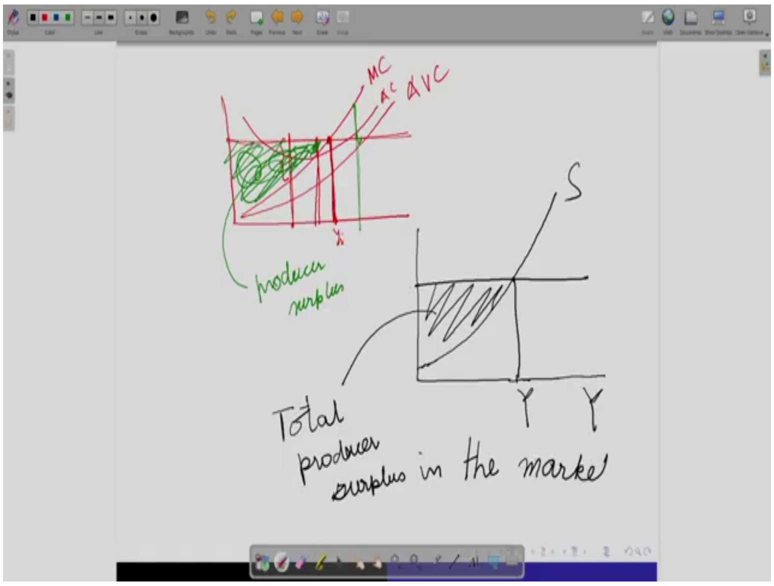

Now, let us define quickly define few terms and then we will define the social welfare. What do we mean by producer surplus? So, these are required to define the social welfare. Producer surplus is actually the difference between the current market price and the cost of production of the firm. So, you can think that suppose this is the marginal cost, this is the average variable cost and this is the average cost.

And we know the amount of output a firm is going to produce does not depend on the fixed cost, since we are doing optimization with respect to yi amount of output, so it depends only on the variable cost. So, here suppose the market price is this, okay. If the market price is this firms are producing this much amount of output, at the margin market price is, sorry, market price is this, it is taken as given by the firm.

And marginal cost at this output is this much, but marginal cost at this much level of output is only this much, so this much surplus it is getting, when it is producing this much suppose, so it is getting this much amount of surplus, so this is actually producer's surplus and this whole reason is the total producer surplus. So, the firm produces till the producer surplus is 0. Now, here if you look at this, the price is this much but the marginal cost is this, so at the margin firms are making a negative surplus, okay.

So, this region is the producer surplus. And what do we get from here? From here if you take the market thing this is going to be the market supply and suppose the market price, equilibrium price is this then firms are, market price is this so it is going to be total output produce, this is going to be this. So, this whole region is the total market surplus or the total producer surplus in the market okay, so this is the surplus.

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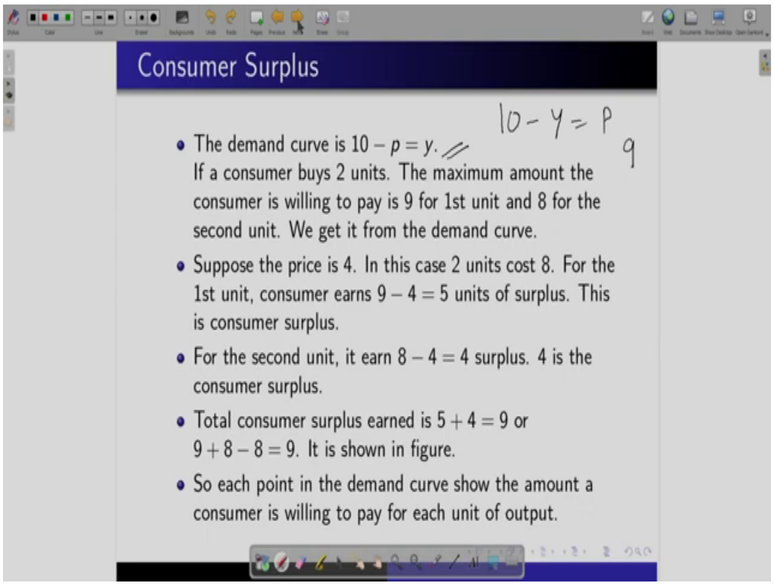

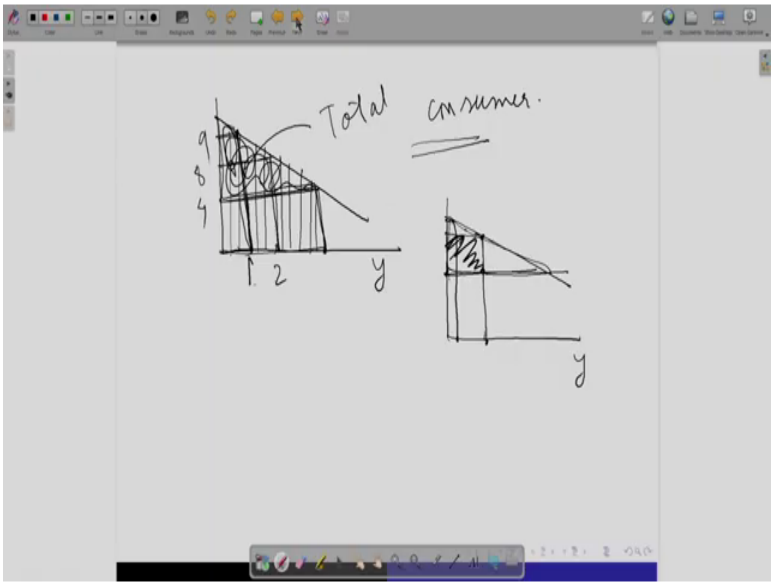

Now, let us define another concept that is consumer surplus. So, when the demand curve is like this-10-p=y and suppose a consumer wants to buy two units, so now here you see, so the inverse demand curve is this, right? So, you buy one unit, you are willing to pay, you can say 9, you two unit, the second unit, you plug it two, you are willing to pay again 8, so this thing you can represent in this way.

So first unit you are willing to pay this much unit, first, second unit you are willing to this is suppose 9, this is 8 so what is happening here, see when you are buying two two units, you are willing to pay this much amount, when you are buying one unit you are willing to pay this whole amount, right? 9, so when you actually pay 8, you are getting this much surplus, right? but suppose the price is at 4 and you have bought this much amount of output, okay.

So, you have paid this much this 4 into total output, but you are willing to pay for each dot this much amount the height of the demand curve, so this much amount is the surplus that you are getting, so this total amount is the total consumer surplus or you can say if you look at only, this is the total consumer surplus, but if this is the case and you are willing to pay, for this amount you are willing to pay this much this whole and you pay actually this is the price.

So, then this is the surplus you are getting, so like this for this unit you are willing to pay this much, so like this we can go on like this, the whole this triangle is going to be your surplus if we make it very smooth, each dot as one, okay. So, the consumer surplus means that the maximum you are willing to pay minus what the amount you pay, so that surplus. So, social welfare is actually the sum of producer surplus plus sum of consumer surplus, okay.

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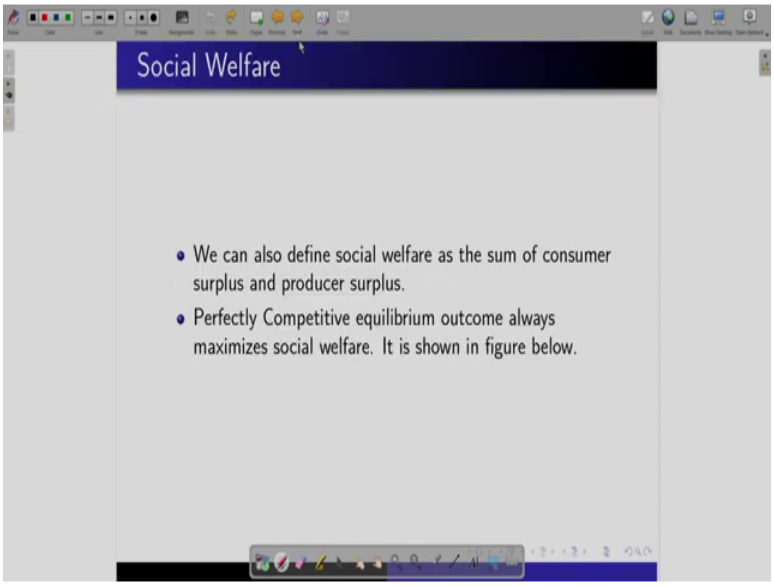

So, we have done this. So, social welfare is the sum of consumer surplus and the producer surplus and we have to show that this sum which is the social welfare is always maximized and when we are in a Perfectly Competitive Market equilibrium outcome, okay. When the outcome is perfectly competitive then this social welfare is maximized. We will do it in the next class. Thank you.

Introduction to Market Structure
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 3: Perfectly Competitive Markets
Lecture 12
Tutorial

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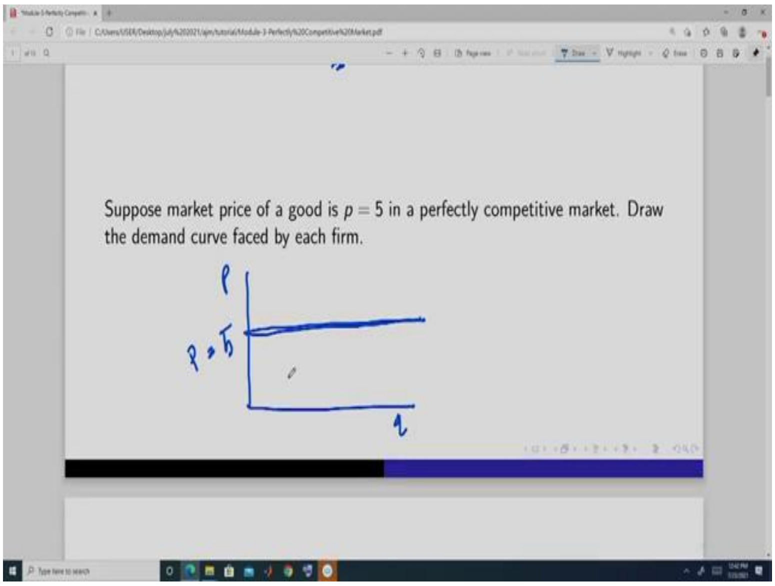

So, let us discuss some problems on module 3 that is Perfectly Competitive Market, okay. Suppose the market price is p is equal to 5. Now, what is the demand curve faced by each firm? So, the demand curve faced by each firm is this horizontal line at price is equal to 5, this is going to be the demand curve faced by each firm because firms are price takers, whatever be the price it cannot determine it, it will take that price as given. So, that is why the demand curve is like this.

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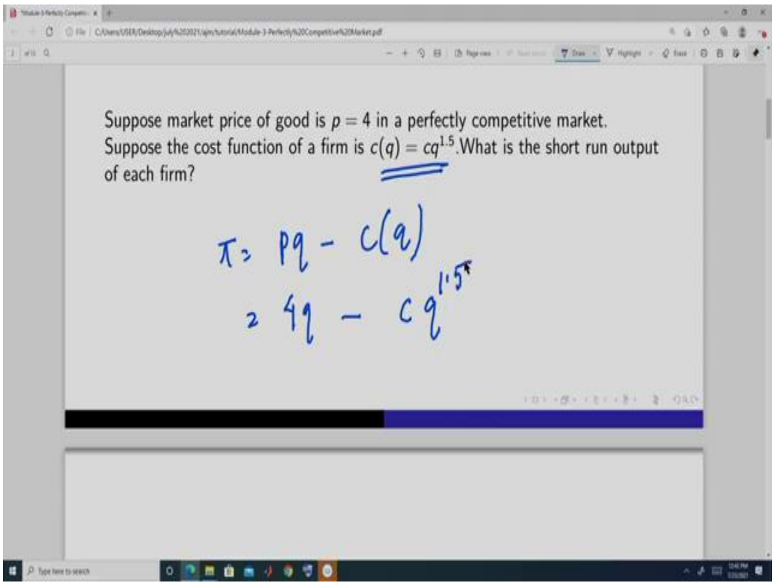

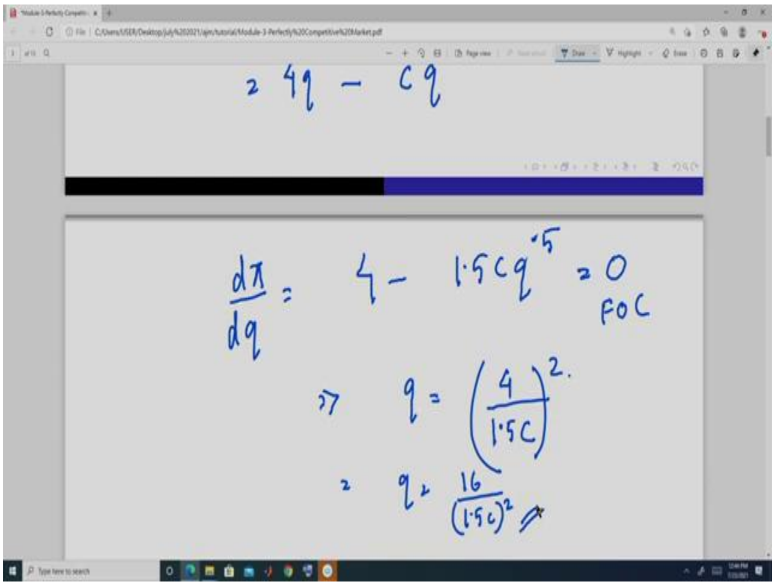

Now suppose the price is this, p=4 and the cost function of a firm is this- $c(q) = cq^{1.5}$and there are supposed many firms what is the short run, and they are similar in terms of the cost and what is the short run output of each firm? So the profit of any firm is this price into q into this cost function- $\pi = pq - c(q)$, which we can write price is fixed at 4, so this into q cost is this- $\pi = 4q - cq^{1.5}$.

Now we take the derivative of this with respect to output because we want to maximize this, so we get this- $\frac{d\pi}{dq} = 4 - 1.5$ cq, first order condition gives this is equal to 0, so we get equal to this, sorry, this is the output of each firm c takes, some positive number, okay, here, so we get this- $q = \frac{16}{(1.5c)^2}$.

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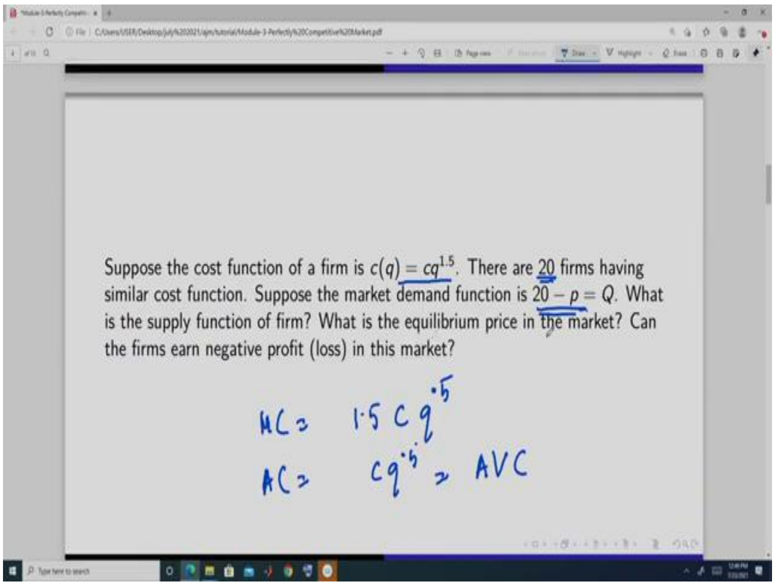

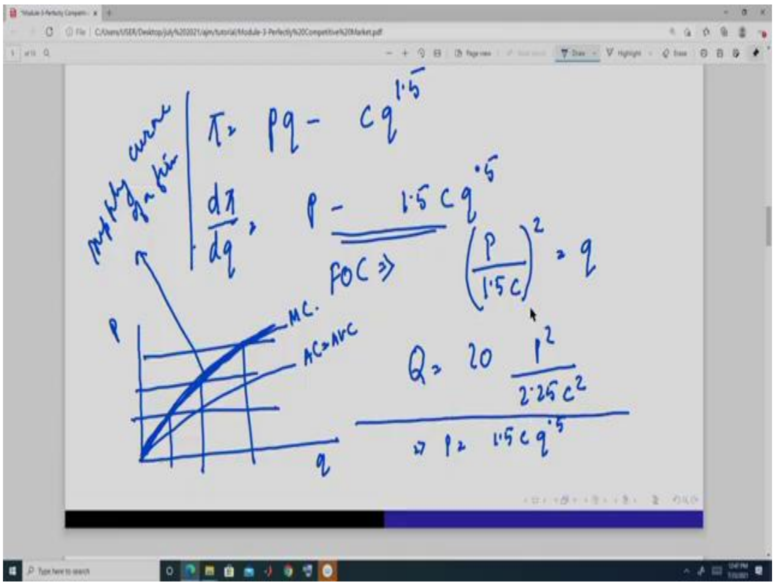

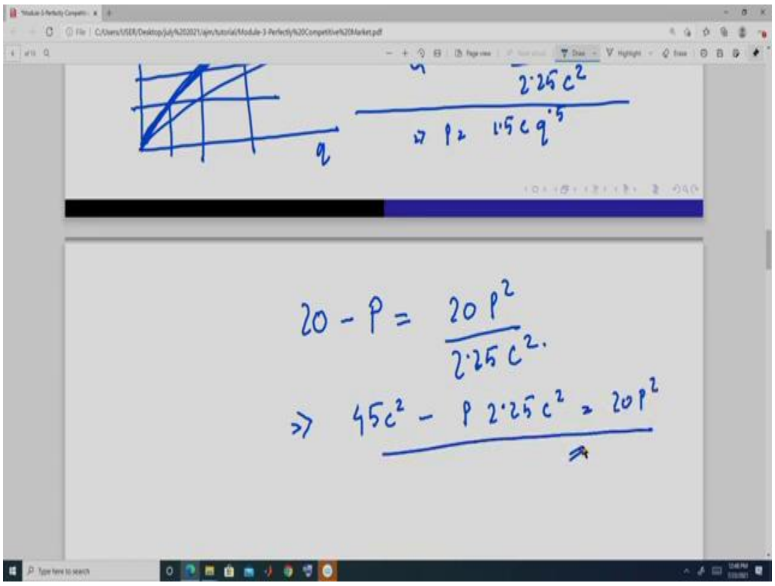

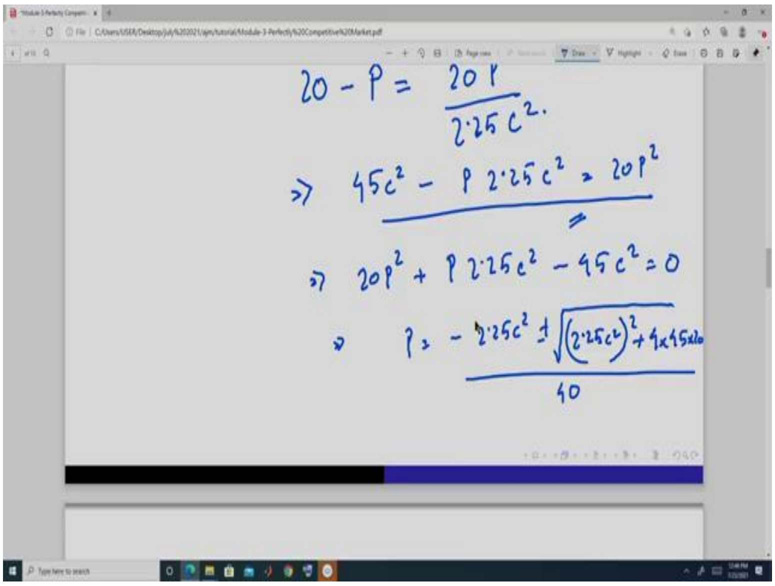

Next suppose, suppose the cost function of a firm is this- $c(q) = cq^{1.5}$, same as the previous one and now we have 20 firms, okay, having similar cost function, 20 firms, earlier we have not specified the number of firms, now we have specified and suppose the market demand function is this- 20-p=Q, what is the supply function of the firm and what is the equilibrium price in the market and can the firms earn negative profit or loss in this market.

If you look at this cost function, what is the marginal cost? Marginal cost is-.$MC = 1.5\ cq^{.5}$, is this, what is the average cost, average cost is c- AC= $cq^{.5}$, which is same as average variable cost because, so this is the case of strict decreasing returns to scale kind of thing, okay. Now, what is the profit of the firm**s**, of any firm, this is the profit function- $\pi = cq - cq^{1.5}$., first order condition implies it is this- $\left(\frac{P}{1.5c}\right)^2 = q$, right? Now, we have 20 firms, so the total supply at price p is 20 into p square.

This you can write this- $Q = 20\ P^2/2.25c^2$. What is the market, so this is the supply, supply of each firm is this- $P - 1.5\ cq^{.5}$, price is equal to this, if we take this output ,price here, marginal cost is this, so it is something like this average cost and average, it is this, so this whole you take any price here, this is going to be the optimal price, if the price is here, it is going to be this, if the price is this, so this whole this line is the supply curve of a firm, right?.

So, supply curve of a form is p is equal to 1.5 c, this- $P = 1.5\ cq^{.5}$. So, but the market supply is this, when there are 20 firms. Now, market demand is 20 minus p is, this is the market demand- 20-P and this is equal to this 20, i.e $20 - P = \frac{20P^2}{2.25c^2}$, this. So this should be equal to this- $45c^2 - \mathrm{P}\ 2.25c^2 = 20\mathrm{P}^2$, we solve this quadratic equation and we will get a price and with that price is going to be the market equilibrium price.

So, this is going to be this one- $20c^2 + \mathrm{P}\ 2.25c^2 - 45c^2 = 0$, this, so this term is going to be definitely greater than this, so we will get a positive price, okay. Now, here question is what is the market equilibrium price? So, by one will be the negative price, another will be a positive, negative price is not possible so the positive one is going to there.

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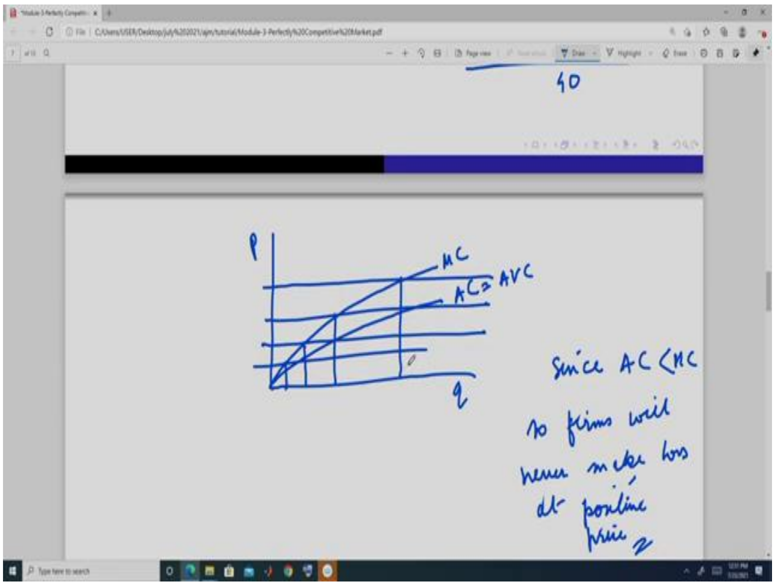

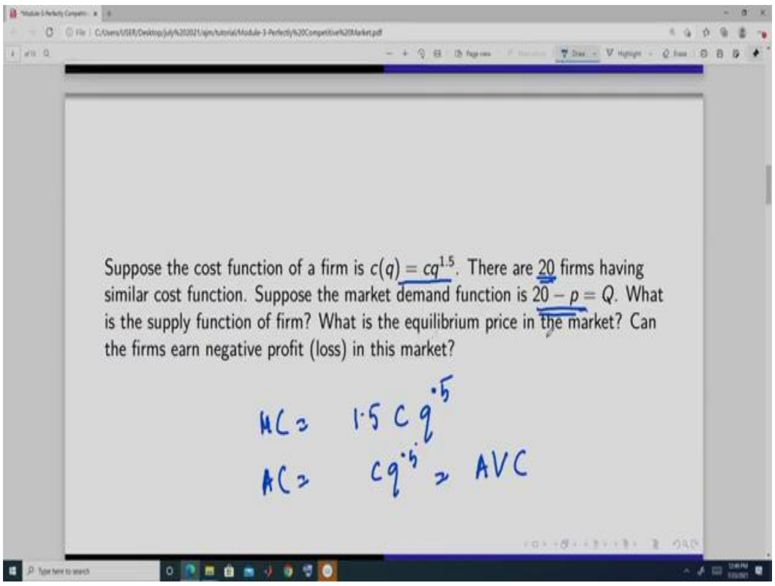

Next, the question is whether can the firms earn negative profit in this market? Now, see this is an interesting part. Marginal cost is this, average cost is this which is equal to average variable cost, you take any price, market price, this is the output, this is the optimal output, this is, so since average cost, since AC is always less than MC so firms will never make loss at positive price, okay, we get this.

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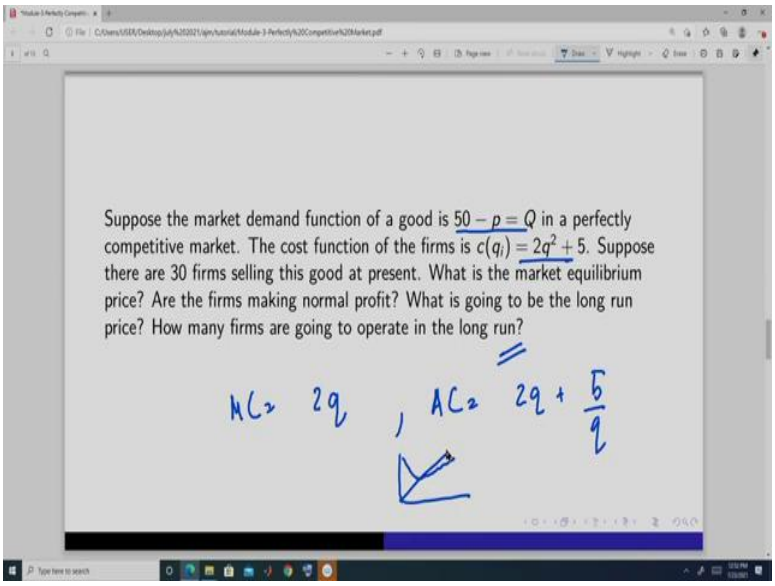
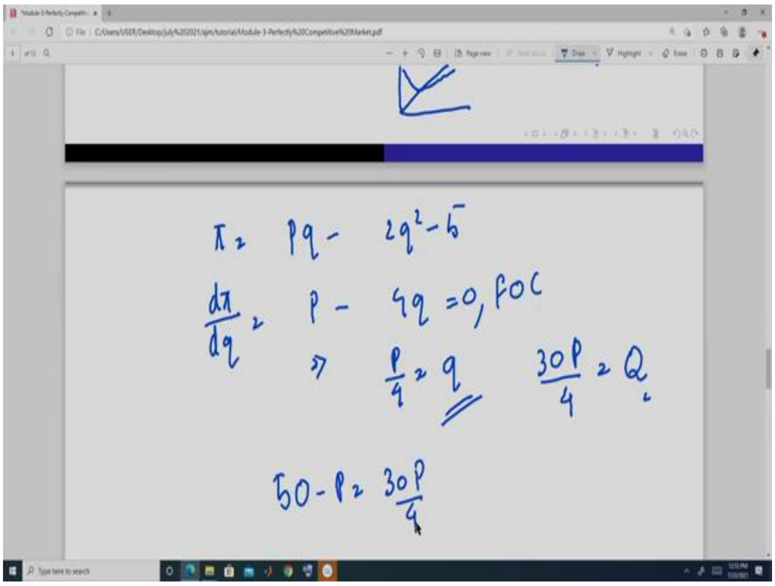
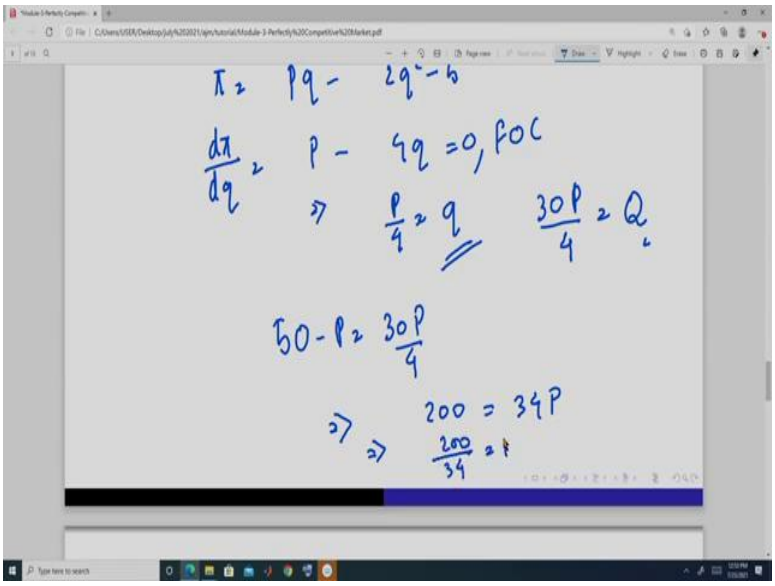






$$\Rightarrow \frac{100}{17} = p$$

$$\frac{p}{4} = q \Rightarrow \frac{100}{4 \cdot 17} = q \Rightarrow \frac{25}{17} = q$$

$$\pi = \frac{100}{17}\cdot\frac{25}{17} - 2\left(\frac{25}{17}\right)^2 - 5$$

$$= 4\cdot\left(\frac{25}{17}\right)^2 - 2\left(\frac{25}{17}\right)^2 - 5$$

$$= 2\left(\frac{25}{17}\right)^2 - 5 < 0$$

Firms are making loss.

Firms are making loss.

Some firms are going to leave or exit.

$$AC = 2q + \frac{F}{q}$$

$$\frac{dAC}{dq} = 2 - \frac{F}{q^2} \Rightarrow q = \sqrt{\frac{F}{2}}$$

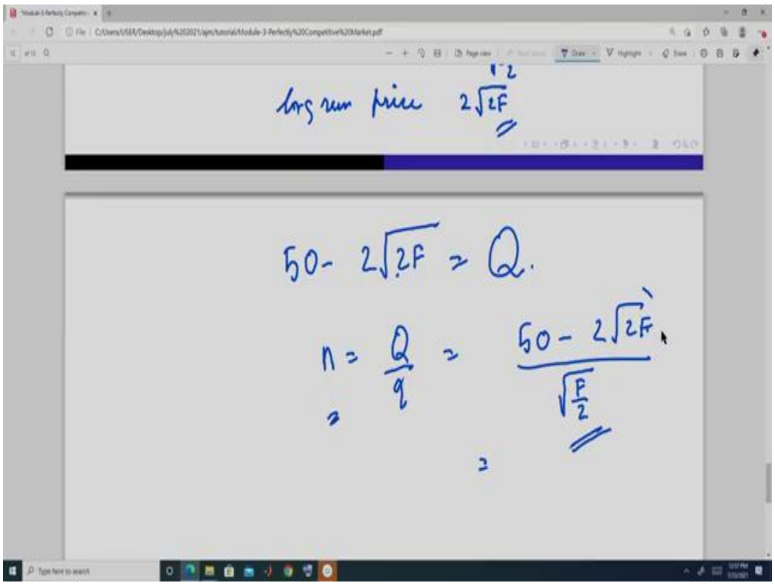
or exit.

$$AC = 2q + \frac{F}{q}$$

$$\frac{dAC}{dq} = 2 - \frac{F}{q^2} \Rightarrow q = \sqrt{\frac{F}{2}}$$

$$AC = 2\cdot\frac{\sqrt{F}}{\sqrt{2}} + \frac{F}{\sqrt{\frac{F}{2}}} = 2\sqrt{2F}$$

long run price $2\sqrt{2F}$



Now, let us solve another problem that is suppose the market demand function is this- 50-p=Q and the cost function is of this nature- $c(q_i) = 2q^2 + 5$. Now, we have a fixed component, fixed cost component and suppose there are 30 firms selling this good. What is the market equilibrium price? Are the firms making normal profit? What is going to be the long run price and how many firms are going to operate in the long run?

Now, here if you look at this marginal cost is 2q, okay, average cost is this- $AC = 2q + \frac{5}{q}$, okay? so marginal cost is this, average cost is something, it is something like this, okay. Now, the profit of any firm is this, it takes the price as given, so we get this profit function- $\pi = pq - 2q^2 - 5$, so this is equal to first order condition- $\frac{d\pi}{dq} = p - 4q = 0$, cost function is this so p by 4 is equal to, this- p/4=q is the output of each firm, there are 30 firms, so 30, i.e- 30p/4=Q.

This is the, market demand curve is this- 50–p=Q, get this- 50–P= 30P/4, this- 200=34P, so 100 by 17 is the price, right? Now, are the firms making normal profit? So, this is the price- 100/17=P, so profit is this, output of each firm p by 4, so this is the profit- $\pi = \frac{100}{17}.\frac{25}{17} - 2\left(\frac{25}{17}\right)^2 - 5$, this- $\pi = 4.\left(\frac{25}{17}\right)^2 - 2\left(\frac{25}{17}\right)^2 - 5$so this you can write it is something like this- $\pi = 2\left(\frac{25}{17}\right)^2 - 5$. Now, this number is less than 2, so this is- $\pi = 2\left(\frac{25}{17}\right)^2 - 5 < 0$. So, firms are making loss. So, some firms are going to leave or exit. So, in the long run what is the average cost?

Average cost is going to be the minimum of this which is we get this by taking derivative of this with respect to output, which is equal to this- $\frac{dAC}{dq} = 2 - \frac{F}{q^2}$ so minimum point is this. So, if this is the a, so what is the AC? So, this is equal to two times… So, the long run price is this-$2\sqrt{2F}$. Now, what is going to be the number of firms? So, this- $50 - 2\sqrt{2F} = Q$, is going to be the aggregate output when we plug in the price, right? What is the output of each firm?

Output of each firm is this-$\sqrt{F}/2$, so n is this divided by this-$n = Q/q$, so this is 50 this-$\frac{50 - 2\sqrt{2F}}{\frac{\sqrt{F}}{2}}$

, so this is the number of, so we take the closest integer of this and we get the number of firms which are going to be there in the market in the long run. This is the number, okay. So, because the number of firms are always going to be integer, so we take the integer which is lower than this real number and this is definitely a positive number if F is not that big, okay. So, these are some of the problems that we discuss in module 3. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 04
Monopoly Price

Welcome to my course Introduction to Market Structures. Today we will do, first do the last portion of perfectly competitive market, and then we will do monopoly.

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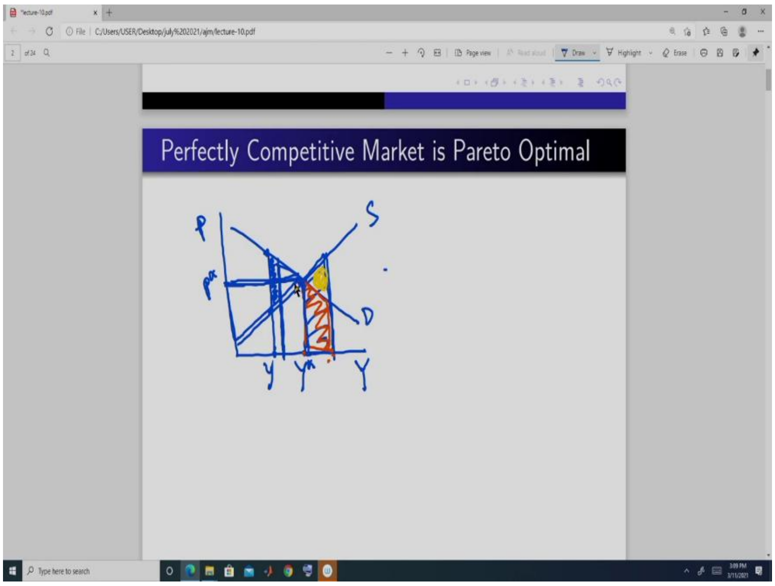

In a competitive market we know how the price, market price is determined. If this is the market quantity, this is the price, this is the market demand and this is the market supply, then the equilibrium price in the market is this and the aggregate output produced and sold in the market is this y star.

Now, we know that this point is in the demand curve and also in the supply curve. So since it is in the supply curve it means it is a point in the marginal cost of each firm. And at this price marginal cost is equal to the price. So it is maximizing the profit of firms. And also at the same time, since it is in the demand curve so it is also maximizing the utility of the individuals.

But suppose we take an output like this, this output. This output, this, at this output people willing to pay is this much and the cost is this much, okay. Now if they can produce a slightly more, here, so what is happening? This much addition is coming. Like this is the, we know this amount is the producer surplus and this is the consumer surplus. So this much is the addition to the surplus. So that is why this output is not Pareto-optimal because we can make both the individual better off.

Similarly, instead of this if we are producing here then the amount of additional cost is this, and the amount people are willing to pay is only this portion. So this yellow portion is the additional cost which cannot be borne by the seller, because the producers, buyers are only willing to pay this much amount and seller wants this whole amount. So this is the loss that they are going to make.

So, that is why to produce more than y star, and suppose it is this it is not Pareto-optimal. You cannot make anyone better off without hurting anyone, okay. So that is why since we cannot move to any other point, then from this, so that is why this point is a Pareto-optimal point, okay. This we had done.

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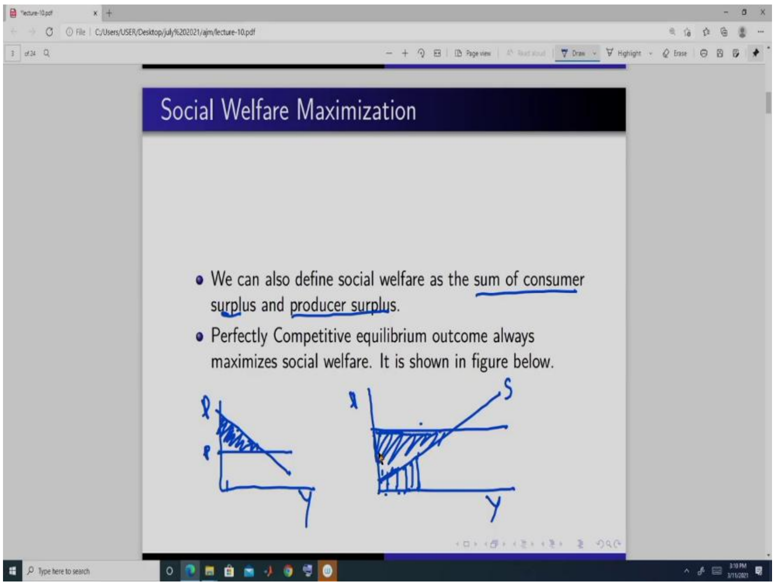

Now, we will do what do we mean by social welfare. This we also have, we have just defined it in the last class. So social welfare is the sum of consumer surplus and the producer surplus. And suppose this is the demand curve, market demand curve. And this is the price. And suppose this is the market price.

Then this triangle is the consumer surplus, because for each unit consumers are willing to pay this point which are in the demand curve. But they are actually paying this. So this much is the surplus that it they are getting. And if we take this price and suppose this is the supply curve, market supply curve and the market price is this, suppose, then this region is the producer surplus.

Why? Because their cost is only this much for each of these units, marginal cost, right? Sum of the marginal cost, horizontal sum of the marginal cost gives me the supply curve. So these heights are the marginal costs for producing this much amount of output.

Now, but they are getting this much prices for each of these units. So this whole region is the total producer surplus. So competitive outcome, the outcome in the competitive market, what it does? Actually maximizes this sum of consumer surplus and the producer surplus.

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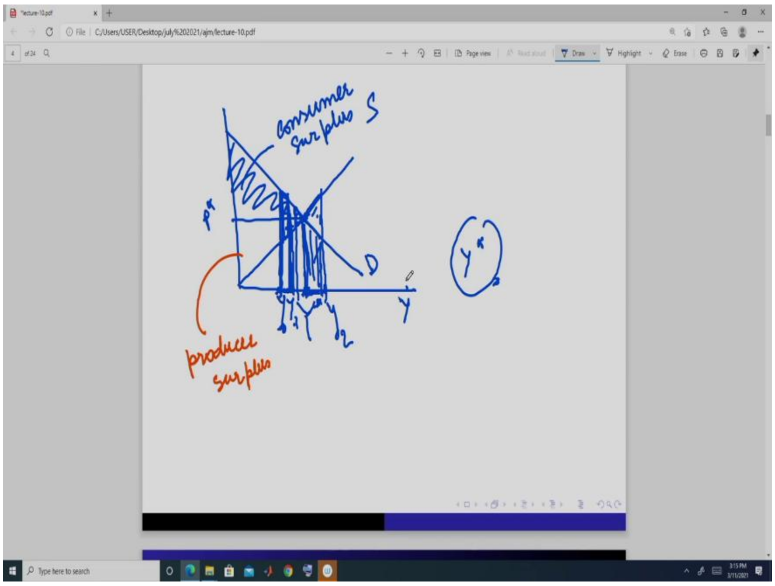

How that happens? Suppose this is the demand curve and for simple exposition we are taking them to be the straight line. So this is going to be our market price. And this is going to be the market output, optimal market output or the equilibrium output. Here this region is consumer surplus. And this region, this orange color is the producer surplus, right?

Here instead of producing this, suppose we produce this much. Then if we increase the output by little bit amount, this much, the market output then what we are doing? We are adding this portion, this whole portion. Now here cost is only this much which is given by, suppose this. So initially this is the equilibrium output, market output. And suppose instead of this we produce this, y naught.

At y naught what we are doing? We are producing this much. Now suppose instead of this we produce y1. Then we are adding this portion. Here this much is the cost, and this much is the total revenue, because consumers are willing to pay this much amount. And so this into this, so this whole the producer can take because they are willing to pay this whole amount and this is the cost. So this much is the surplus.

So, if we stick to this then we are not adding this much. So if we switch from this y naught to y1 we are adding some additional surplus which is sum of producer; and the sum of producer and the consumer surplus. So this is not social welfare maximizing. So we will move to this. So like this we can go on arguing till we reach this point, right? Because if we are here again we can add this triangle as a surplus which is sum of consumer and producer surplus. So we will continue till this point.

Now, suppose we produce this much amount of output which is y naught. Now here instead of producing y star that is the equilibrium output if we produce this y2 then additional cost borne by all the firms is this much. An additional amount the consumers are willing to pay is only this. So this triangle is the loss, right? That is the additional cost, part of that extra cost which cannot be borne by the amount consumers are willing to pay. So that is why this is going to be something like a loss.

So, if we move right to this y star then there is a possibility that we may make some loss. So that is why it is not optimal to move in the right of this y star. So that is y star, this amount that is being produced in a perfectly competitive market is the social welfare output, okay. So this is the perfectly competitive market.

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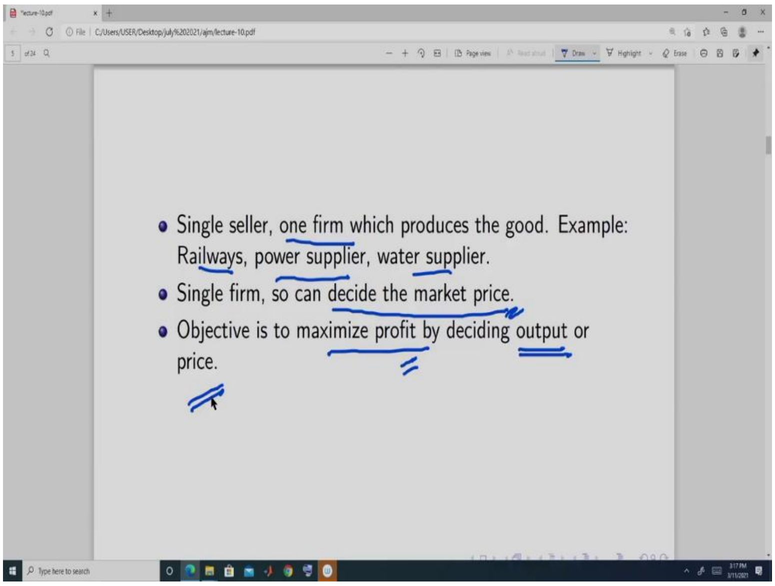

Now, we will do another topic and that is monopoly market. And monopoly market is where, as a special form of a, in a monopoly there is only one firm okay, one firm or there is a single seller. Or you can say that there is only one producer.

Example like Railways it is a single firm or it is a only one, we have only one seller that is the Indian Railways. It may be owned by a government but it is only one provider, okay. Power supply in our house it is mainly by, we do not have the option to choose from multiple sources or from multiple firms. We have only one power supply. Water supplier, all these are monopolies. Water supplier in most of the places, it is done by the municipal corporations. So all these are form of a monopoly.

So, in a monopoly there is only one firm. And since there is only one firm they can decide the market price, okay. Now what the objective of this firm is, which is a monopoly firm, is to maximize profit. And it can maximize its profit either by deciding the amount of output it is going to sell in the market or the price that it is going to set in the market. So either it can decide the profit maximizing price or it can decide the profit maximizing output. These both will give the same outcome, okay.

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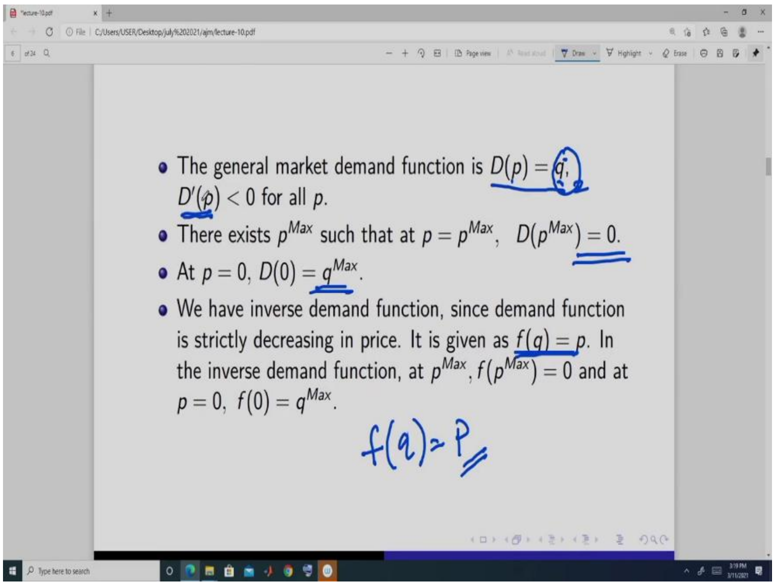

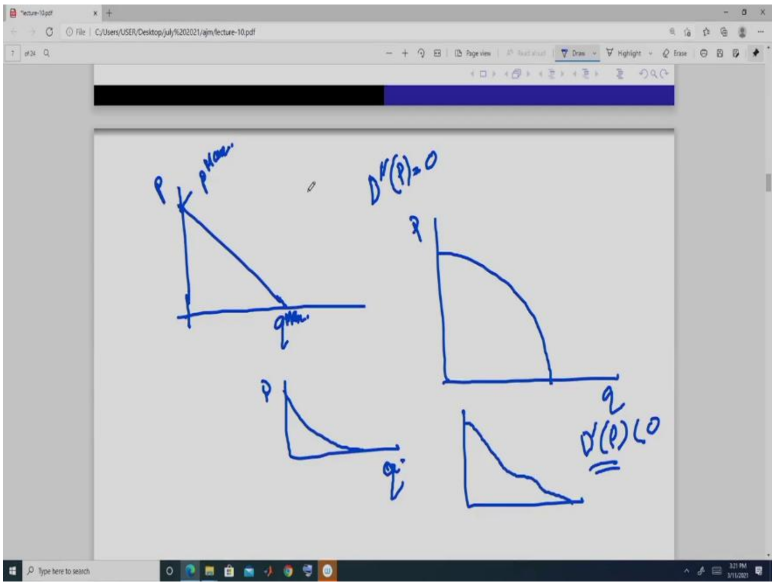

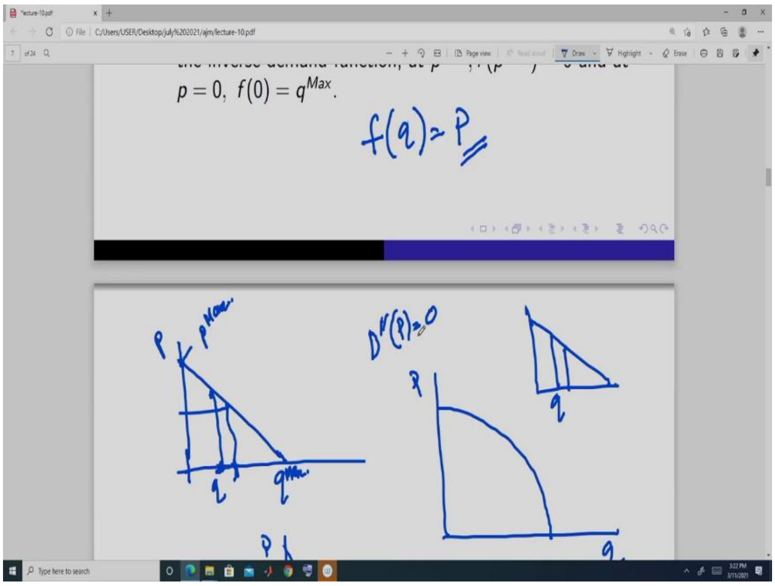

And next we will specify the market that is the nature of the firm in this market. So first we will specify the market demand. So this is our demand curve, demand function you can say which is given by this function D- D(p)=q. So you plug in the prices and you know the market output is this q, market demand.

So, at each price p, how much is demanded in the market of this good, of this particular good is q. So if this is suppose electricity you can say, you plug in the price of electricity, how much amount of electricity is been demanded with this.

We have got this from the individual demand curve which we have got from the consumer optimization. There are each consumer does, that is the utility maximizing, maximization subject to a budget constraint. And to this function see we will keep a general function here and then we will solve.

Later on we will solve one example and we will also give a diagrammatic explanation of this thing. So then things will be more clear, if you are getting a bit confused or if it is a bit vague for you, right now.

So, we specify this demand function in such a way that it is downward sloping. So that is why the slope is negative. And this demand curve has a price that is P Max. If you plug in P Max it is so high that there is no demand at this price. So this is demand is 0, i.e $\mathrm{D}(\mathrm{p}^{\mathrm{Max}}) = 0$. And at a price, zero price, so the maximum demand that is generated is this q Max, i.e $\mathrm{D}(0) = \mathrm{q}^{\mathrm{Max}}$, okay.

Now, since this is a strictly decreasing because function because of this slope is negative so we can have an inverse of this, and the inverse function is this- f(q)=P. So what this inverse does? Inverse allows us to make this function. Now it is a function of q. So we plug in quantity. We know what price the consumers are willing to pay. If we plug in this much output in the market in this function then how much maximum the consumers are willing to pay is this price, okay.

Now, given this specification of demand curve we will get this if we take price here and quantity here, this price is the p max. So at this price the quantity demanded is 0. And this price is q Max. So, when the price is 0 this is. So this demand curve is downward sloping if it is like this. It is a straight line, if it, second derivative is 0. It can take a form like this also, right? Or it can take a form like this also, okay. We can, any one of this will do.

In fact, it can take a form, not like this so much curl is not possible but at least like this it is possible. As long as the derivative is negative, it is, it can take any form, right, so all these variations are possible. Out of all these variations we will again assume only few and that we will come to later. We will specify later. But all these demand functions are possible.

Next we specify, so the demand situation scenario is clear. So it is a downward sloping demand curve. So if you want to sell more you have to reduce the price. So if you are selling this much amount of output, now if you want to increase the market output then you have to reduce the price. At this price you cannot sell more than this, this amount. So this is, this idea is very important.

Like when we are saying downward-sloping it means that if you are selling this much amount of output at this price and if you want to sell more output you have to reduce the price, the market price, okay. Otherwise it is not possible.

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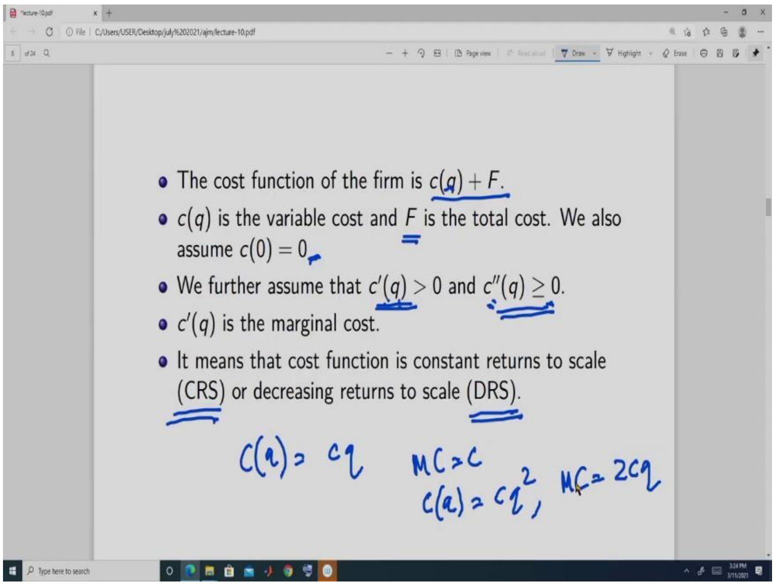

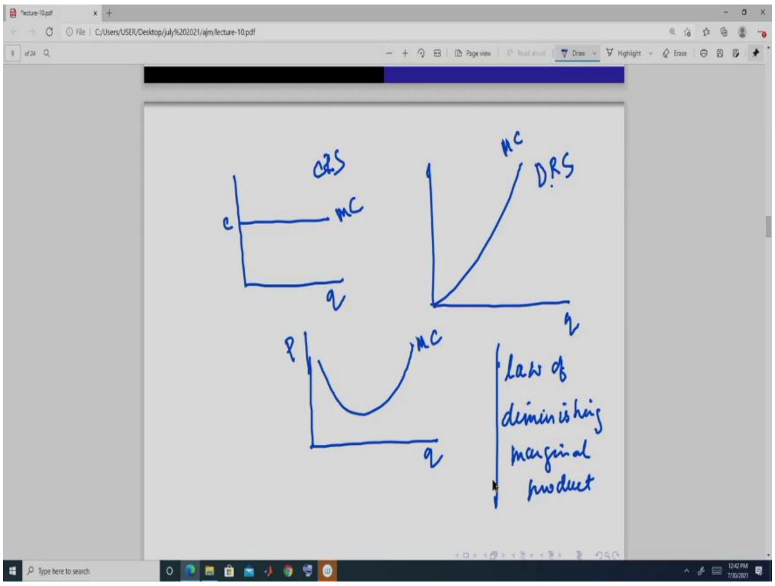

Now, cost function is of this nature- c(q)+F. So this portion is the variable cost and this F is the total cost. And we assume that if output is 0 this c is equal to 0. So that means variable cost is only incurred whenever there is a positive amount of output, okay.

Now, we assume that it is, this is the marginal cost- $c'(q) > 0$. It is always positive, this is marginal, and the second derivative of this cost is always non-negative, this- $c''(q) \geq 0$. So what does this mean? It means that either this can take a 0 value or it can take some positive value. It means that either it the technology, cost function is CRS or it is like DRS, decreasing returns to scale.

So, this portion you can specify either like this- c(q)=cq, when this is positive because marginal cost here is c and the derivative of marginal cost is 0. So this is also satisfied. Or you can take a form like this- $c(q) = cq^2$. Now here marginal cost is like this- MC= 2cq, okay. These are the examples of marginal cost. And this we get when we have CRS or DRS and also when one of the factor is fixed and the other is variable.

Like a capital is fixed and labor is variable and due to operation of law of diminishing marginal product we get a upward-sloping marginal cost function. And here when we take this F to be a fixed component it is coming from, suppose the rent that we pay for the building, that. Or when we have only one factor is fixed that is giving us this component. Together that with rent is giving us this component. And the second derivative is also positive, right? and it is 2c.

So, this means that our, if we plot output here then marginal cost can be of this nature when it is CRS, or it can be of this nature when there is DRS. This is the marginal cost and this is the marginal cost. So we consider only these two forms.

But there is a possibility that we have a marginal cost like this. So we do not introduce that the optimization that we are going to do but we will consider this case separately. Later on you will see. So this is one example of DRS we have, marginal cost.

But this kind of marginal cost function may be due to law of, due to law of diminishing Marginal product, marginal product. When one input is fixed, another input is variable. Like capital is fixed and labor is variable. Then also we will get this. So we have, there is or we may have here this kind of situation, okay.

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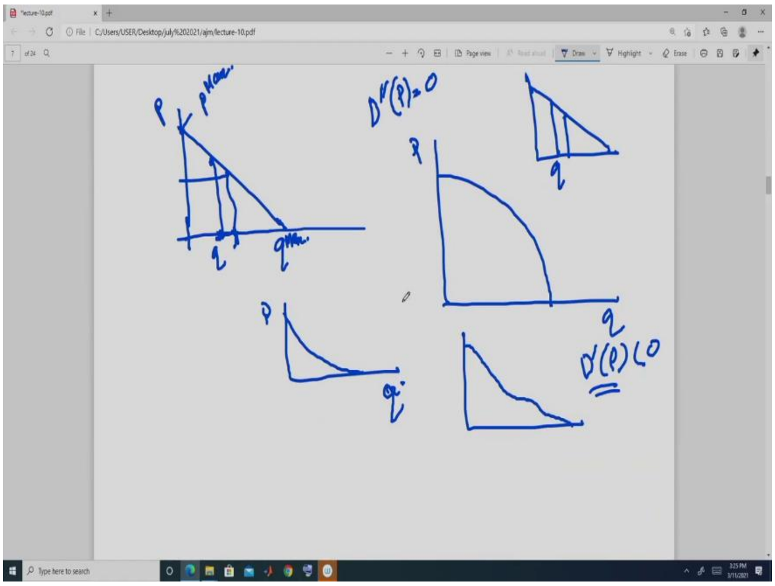

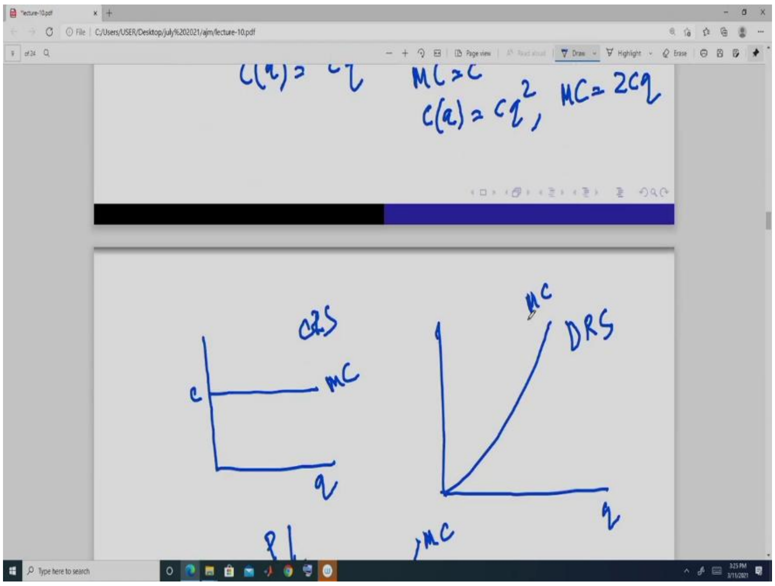

So, what we have? We know the demand curve in the market is of any one of this form. And the cost curves are; marginal cost is either this or this, okay.

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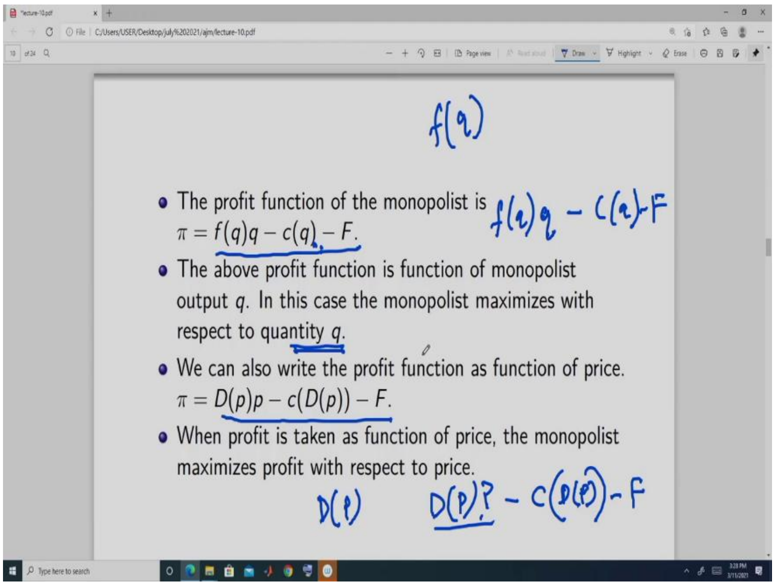

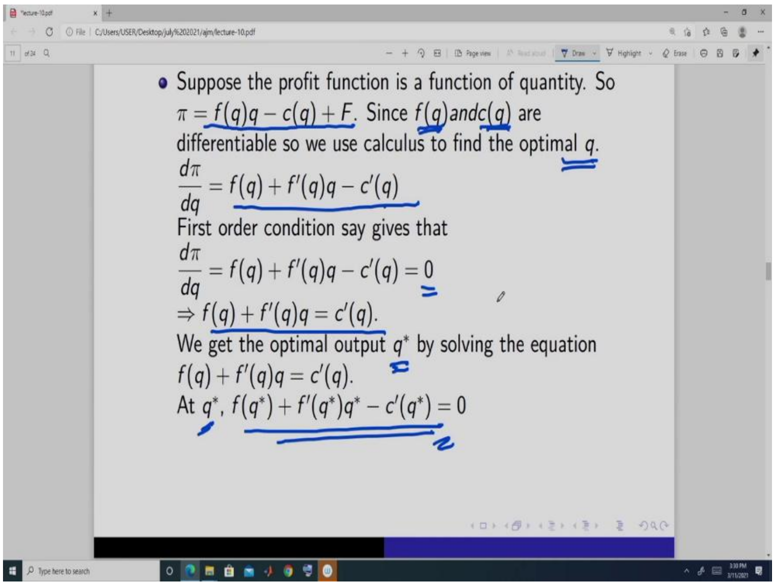

Now, we specify the objective of the firm. So objective of the firm, we know is to maximize profit. Now we can specify the objective in this form- $\pi = f(q)q - c(q) - F$. So this is, here this portion is, so this is price into q, amount of output produced by the monopolist. So this is the total revenue- f(q)q. Cost is c minus f, this is the total cost- $c(q) - F$, okay. So this is the profit of this firm, total revenue minus total cost.

Here you notice that this profit function is a function of output, so output that firms are producing. So if we want to maximize this produce, this, maximize the profit, since the profit function is a function of output so we will maximize it with respect to quantity that is q. So the firm will decide the amount of output it wants to produce. And from that we can derive the market demand. And that is the monopoly price by plugging in the demand curve that is this, inverse demand curve.

Or we can do, write the profit in this form- $\pi = D(p)p - c\big(D(p)\big) - F$. Here this amount, this DP is the, you plug in the price, you get the quantity. So this is the quantity into price. So this is the total revenue- $D(p)p$. Here cost is always a function of quantity. So it is this- $c\big(D(p)\big)$. So this is the variable cost minus, so this portion is the total cost and this is the total revenue. So this is the total profit- $\pi = D(p)p - c\big(D(p)\big) - F$.

But here this profit function is a function of price. So if monopoly considered this, its profit function in this form then the monopolist will decide the price, and given the market demand, if you know the price you know the amount that the consumers will buy from here. So in this objective function the price will be determined and then also, corresponding output will be, we will get the corresponding output from this demand curve.

In this here what will happen? The output will be determined monopoly output and corresponding monopoly price will be determined from this, okay. So this is the objective of the monopoly's, objective function of the monopolist, okay.

Now monopolist is going to maximize this. And it is a conventional in the whole, in the literature to take this function- $\pi = f(q)q - c(q) - F$, okay. We generally do not consider this function- $\pi = D(p)p - c\big(D(p)\big) - F$. So all the monopolist decides the or the monopolist decides the output. And the corresponding monopolist price is determined based on the demand curve, okay.

So, suppose this is the profit function- $\pi = f(q)q - c(q) - F$, okay. And we know this function, this inverse demand curve and this cost, variable cost function they are differentiable. We have assumed that. So we now use calculus to find the optimal q.

How do we do? We take the derivative of t***he*** profit function with respect to q. If we take the derivative we will get this- $\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q)$. So derivative of this is this f q plus f dash q into q. And this is the. Now here first order condition will give that this should be equal to 0. So this is the first order condition-$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q) = 0$. From here we can write this- $f(q) + f'(q)q = c'(q)$

Now here see, so if we are given this first order condition, now what we can simply do? We can solve this first order condition. So we will get an equation. If you plug in specific form of this here we will do that later on when we do an example.

But generally also you can do. You can solve this equation and you will get a q star. And this q star is, should be a profit maximizing output, right? So we will get this. So at q star, it should be, first order condition should be satisfied- $f(q^{*}) + f'(q^{*})q^{*} - c'(q^{*})$., okay, by solving this equation, okay.

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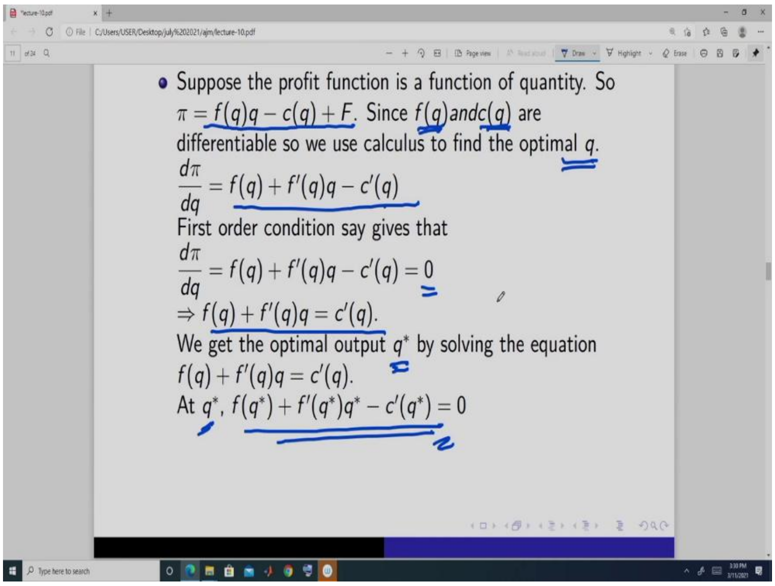

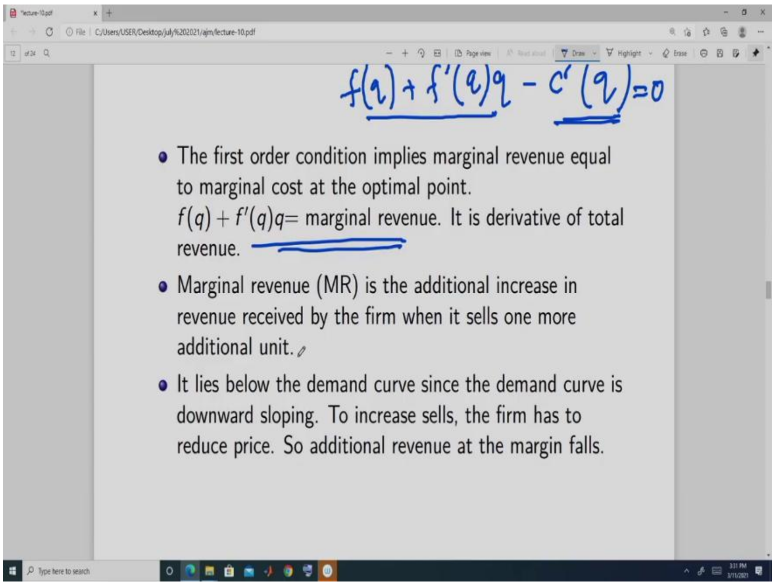

Now, here you will see what is this portion. If you look at this, this is actually something called the marginal revenue- $f(q) + f'(q)q$. So the first order condition says that marginal revenue, first order condition is what? Is this, is this- $f(q) + f'(q)q - c'(q)$. So this says that this portion is the marginal revenue should be equal to, because it is equal to 0, should be equal to, this is the marginal cost because it is the derivative of the cost function.

And what is the marginal revenue? Marginal revenue is the additional increase in revenue received by the firm when it sells one more additional unit. If you want to sell one more unit at the margin then how much additional revenue you are going to get.

Now, marginal revenue curve is always going to lie below the demand curve. Why? Because we know from the demand curve that if you want to sell more you have to reduce the price. So that means if you want to sell one more unit you have to reduce the price. So the price is falling. So price into quantity is going to give you the revenue. So the revenue is going to go down.

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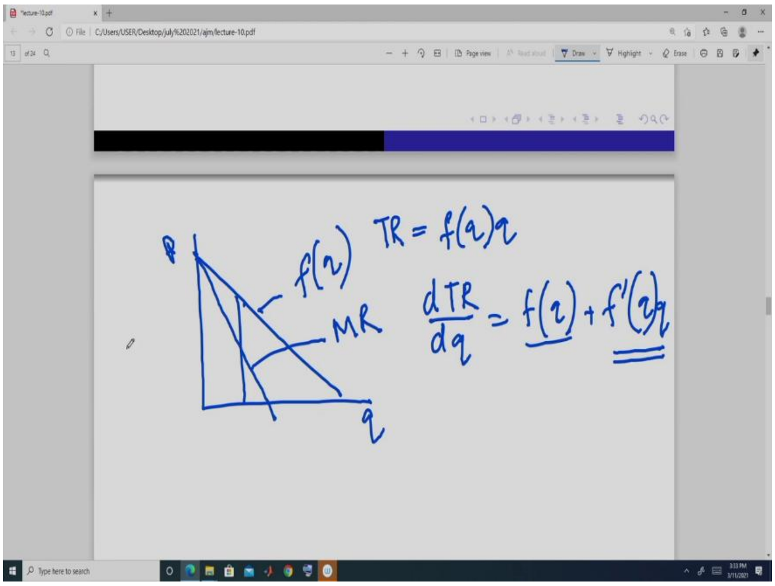

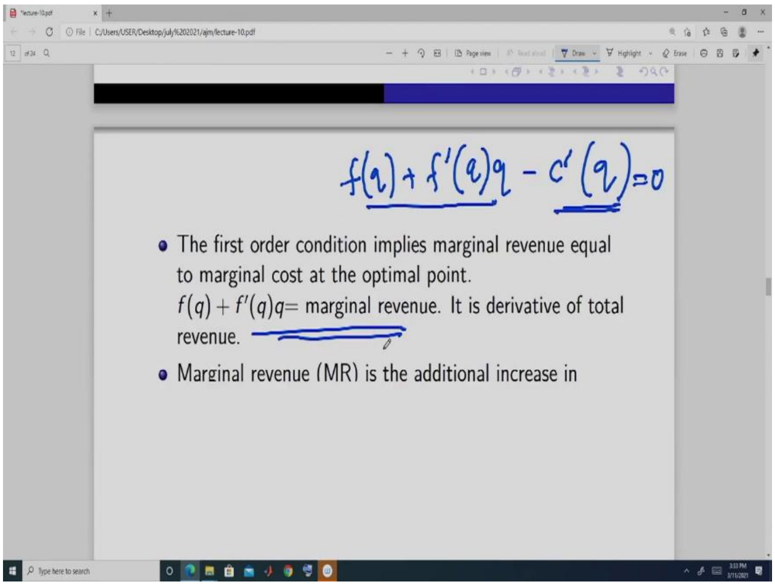

So which you can see from here, so if this is the price and this is a and this is the, supposed inverse demand curve like this, marginal revenue curve is. How do we get marginal revenue curve? So total revenue is this when it take it to be a function of output of a. Total revenue, this is the price into quantity, right? Total revenue.

Now if you take derivative of this with respect to output you will get this- $\frac{dTR}{dq} = f(q) + f'(q)q$. We know demand curve is downward sloping. So this portion- $f'(q)q$ is going to take a negative value. Now plug in outputs. This is the price plus some negative here. So that is why it is going to be something like this. And this is the marginal revenue curve, okay.

So, the idea is something like this. Since the demand curve is downward sloping, so from here if you want to increase some more amount of output so you have to reduce the price. So the additional revenue you are going to get, that is going to go down, okay. So that is the idea. So this is the marginal revenue and this is the, this portion is the marginal cost.

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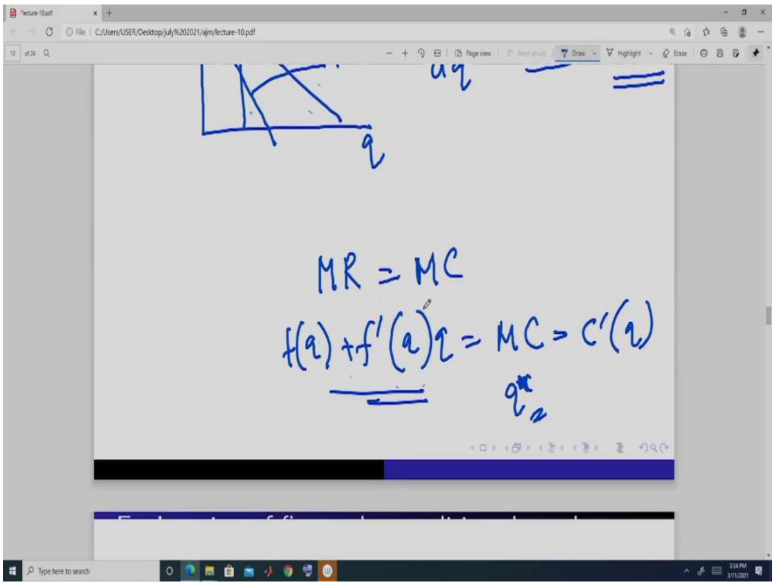

So, first order condition says that the profit maximizing output will be such that the marginal revenue should always be equal to marginal cost, okay. And we can solve this function, sorry this equation which we get like this- $f(q) + f'(q)q = MC = c'(q)$. And solving this we will get the optimal output q star, okay.

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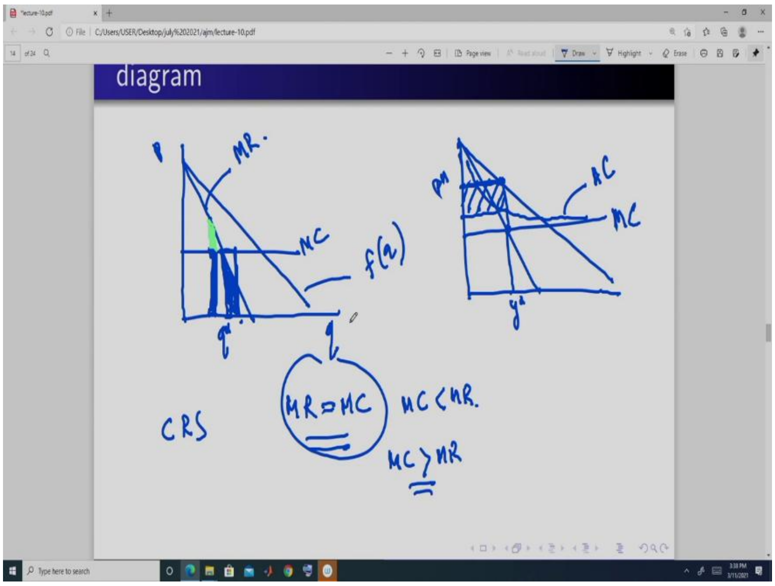

Now, let us diagrammatically explain this how the optimal output is determined. Suppose output here, price here and this is the demand curve. So we will get a corresponding marginal revenue curve. So this is suppose the marginal revenue curve, okay.

Now, take the case of CRS. So the marginal cost is this. So the optimal output is always determined here, this. This is the optimal output q star, so or the profit maximization output. And this output, why this is a profit maximum? Because suppose instead of this monopolist produced this much output. At this much output its marginal cost is this.

Marginal revenue is this height. If it slightly increases then output, then this much is the additional cost. It is here. Additional revenue is the total region it is going to get. So this green part is the additional revenue. So the monopolist will be at better off by increasing his output. So it will go on doing that. So it will be, so the marginal revenue should always be equal to, sorry marginal revenue should be always equal to marginal cost.

But if it produces the output more than, where, here, so here marginal cost is this and the marginal revenue is at this height. So if it produces this much amount of output extra then this is the additional cost, right, because this much amount of output and this is the cost. So this whole region is the total extra additional cost. But the revenue is this total area which is below the marginal revenue curve. So this triangle is the loss that they are making if it increases output more than q star.

So that is why it is not optimal or profit maximizing to produce output which is more than q star where, because at q star this condition is satisfied- MR=MC. If you produce more output more than q star then what is happening? MC is greater than MR. So at this, profit is not maximizing.

If you produce less than this, then what is happening? MC is less than MR. Here also profit is not maximizing. In this case the monopoly, so profit is maximized when we have this condition. So the actual monopoly profit in this situation is something. So this is the marginal cost and suppose the average cost here it is going to be something like this. And this is the monopoly output y star.

So, this is the monopoly price. This is AC. So this price is the monopoly price pm. Because if it produces this much amount of output the market people are willing to pay this much from the demand curve we get. And so this is the monopoly price. And this is the average cost. So this rectangle is the monopoly profit, okay. So this is how the monopolist decides the output. Moment it decides the output it gets the monopoly price from the market demand curve.

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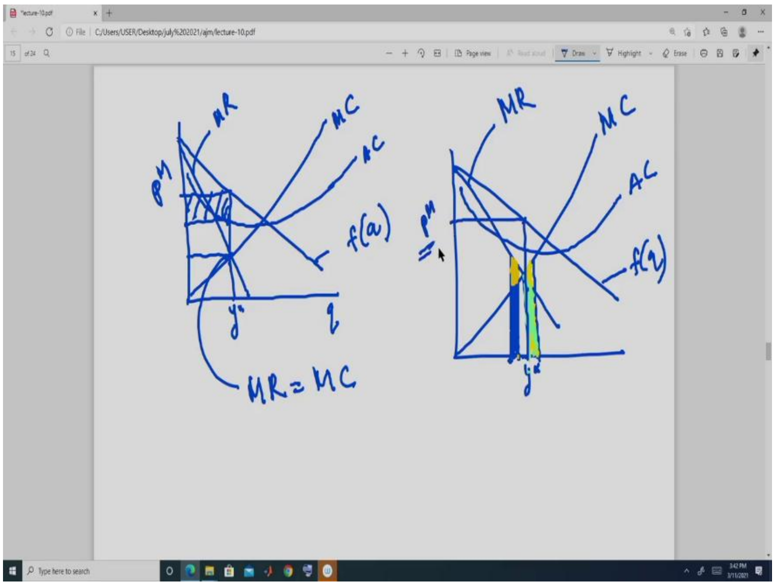

Now, let us take the case when we have decreasing returns to scale. So suppose again this is output. This is the market demand curve. This and suppose this is the marginal revenue curve. And suppose this is the marginal cost because it is decreasing cost, decreasing returns to scale and this is the average cost. This is again I have made mistake, okay. This is the AC.

Now, here you notice that at this point, now this output, at this output marginal revenue is equal to marginal cost. So firm should always produce y star units of output and the monopoly price is going to be this much, this pm and the monopoly profit is going to be, this is the average cost. If it produces this much amount of output so it is going to be this rectangle, okay.

Now, why this is the profit maximizing output? Because if we take the same case here, this is market demand, and this curve is marginal revenue, and this is marginal cost, and this is average cost. So this is the profit maximizing output of the monopolist.

If it produces something less than this, here, this much amount of output then instead of producing here if it increases slightly till this then additional cost is this region below the marginal cost because it has already incurred the fixed cost, so additional cost is only this much. Additional revenue is this total region below the marginal revenue curve. So this whole, so this portion, this is the additional profit that the surplus, that the monopolist is going to get by producing this instead of this.

So, the monopoly should not produce this much. So it should continue like this until this point y star. If it produces more than this y star here then what is happening? Total additional cost is given by this portion, this whole yellow portion. But the total additional revenue is only given by this, this region. So this yellow extra portion is left as the additional loss that the firm is going to, the monopolist is going to make if it produces more than y star. So that is why the optimal output it is going to be y star. At y star price is this. So this is the monopoly price. So this is how the monopoly determines its optimal or profit-maximizing output and the monopoly price, okay.

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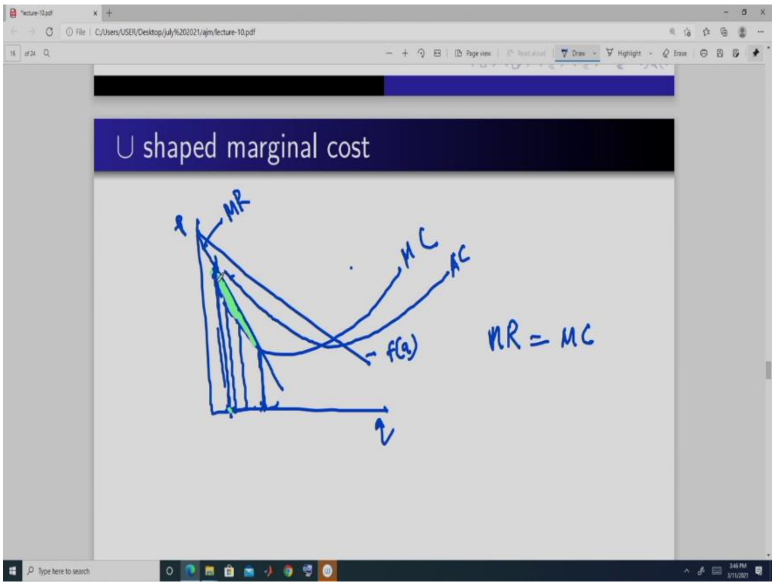

Next we will take the case of U shaped marginal cost, okay. So let us take the same demand curve like this and this is the marginal revenue. Let me make it slightly flatter so that it is easier to, okay suppose this is marginal cost. And this is the average cost. Now here notice we have two points at which marginal revenue is equal to marginal cost. One point is this. Another point is this. This is the demand curve and this curve is the marginal revenue curve. Here we are plotting output and the cost also.

But if we, the monopolist produce this output then what is going to happen? If it increases further this much, so this much is the additional surplus it gets. If it produces this much then additional cost is this region below the marginal cost and additional revenue is this region below the marginal revenue. So this portion is the additional surplus.

So, that is why the monopolist will not choose this output but instead it will choose this output. Because at this output we know if it increases more than this then it is going to make a additional loss. If it produces less than this then it is, it can by producing a little bit more, it can generate some additional surplus. So that is why this is, we know, it is a monopoly output.

Now, whether this is a monopoly output or not? But this is not. Because if it produces more than this then it can generate some more surplus given by this a. And it will go on this, this whole region. This whole region is the additional surplus it can generate. So that is why this output is not a.

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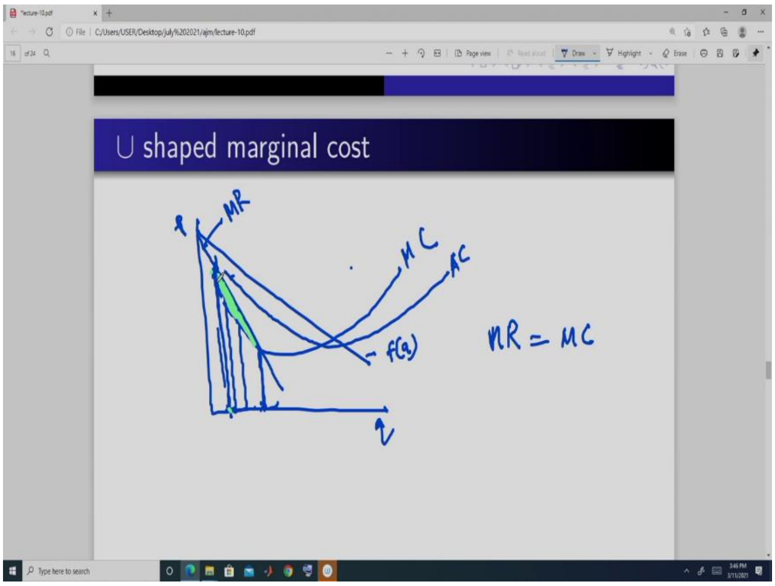

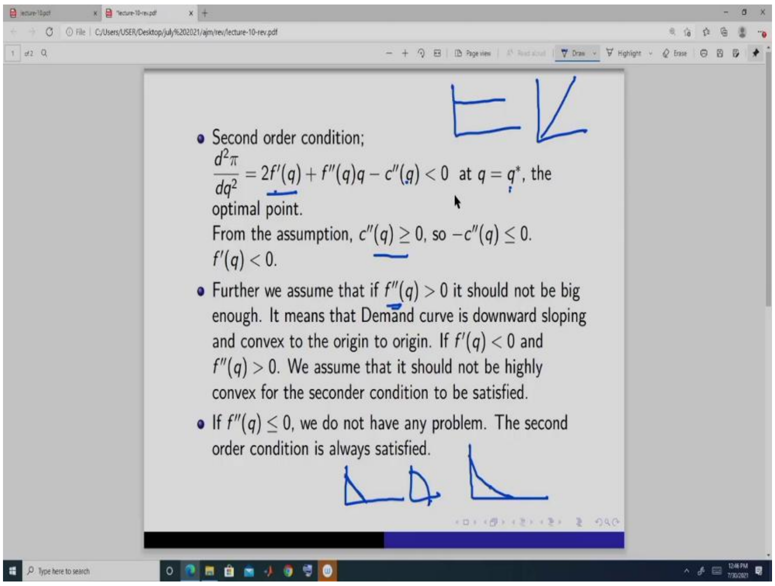

Now here, so this gives rise to our second order condition. And second order condition says that the, when the marginal cost intersects the marginal revenue curve it should be upper sloping, or it should not have a negative slope, okay. Here it is the slope is negative, right? So second order condition is given in this form- $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0$, so which we have again taking the derivative of the first order condition.

Now, here we know this portion, this portion- $2f'(q)$ is going to be negative. Why? Because downward sloping demand curve. This- $f''(q)q$ we are not very sure. We have not yet specified, okay. So let us be silent. This we have already specified that our marginal costs are of this nature or marginal cost are of this nature. So this 2 kinds satisfies this, okay.

So, from the assumption this, we get this portion is this. So this portion-$c''(q)$ is negative because they are either 0 or it is positive. This kind of marginal cost curve when we have CRS, this when we have either DRS or one factor is fixed and we have law of diminishing marginal product operating, okay.

Now, this portion-$f''(q)q$, so this- $2f'(q)$ is negative already and this- $c''(q)$is negative. So this is negative. But what happens to this-$f''(q)q$? Now this we have also assumed that our demand function can be of this. So in this case it is this. It is convex to the origin.

So, what happens in this case? In this case what do we get that, for this overall to be a negative at the optimal point, i.e $\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0 \; at \; q = q^*$we need this curvature to be less convex. So even if it is convex should not be that convex. So then we will get this whole to be a negative number, okay. But in case of a concave demand function which is like this or like this, this is always going to be negative, so second order condition is always satisfied, okay.

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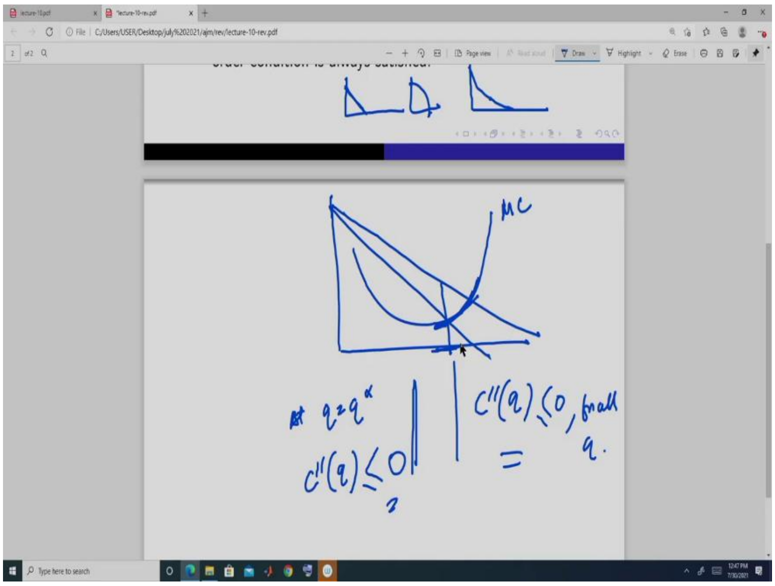

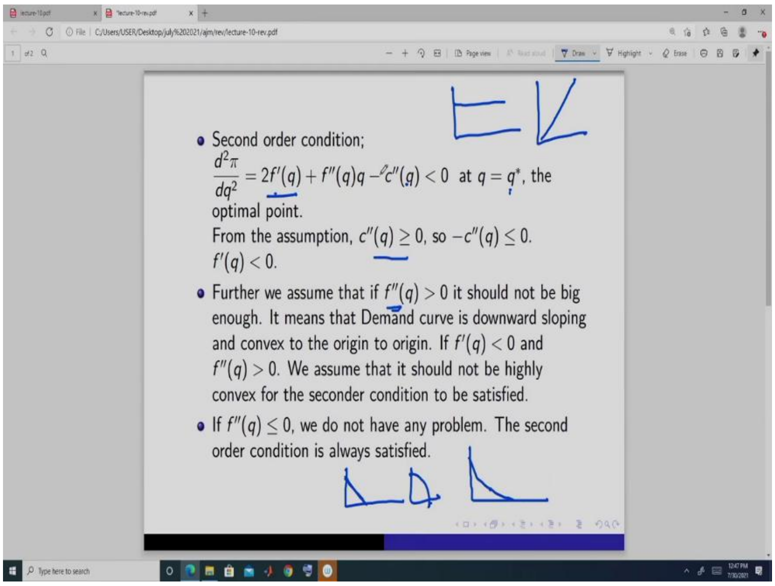

Now, we have taken another example and that is when our marginal cost curve is U shaped. It is like this. So in this situation what do we get in the monopoly outcome? Monopoly outcome is somewhere here and it is like this. So marginal cost is increasing at this. So in this situation, even if this condition- $c''(q) \leq 0$ for all q is not satisfied, at least for this it has to be satisfied, okay.

So, if we take this, when it is satisfied for this so then again when, for at q is equal to q star should be this-$c''(q) \leq 0$. So this is a local assumption that is within in this range. So this ensures again that the second order condition is satisfied. So when we have a U shaped marginal cost then also the second order condition is satisfied, okay. So this is the importance of the second order condition.

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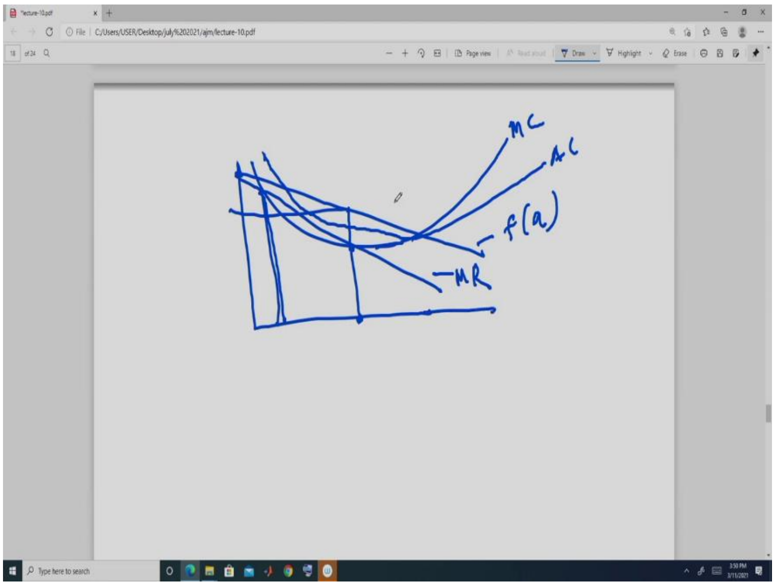

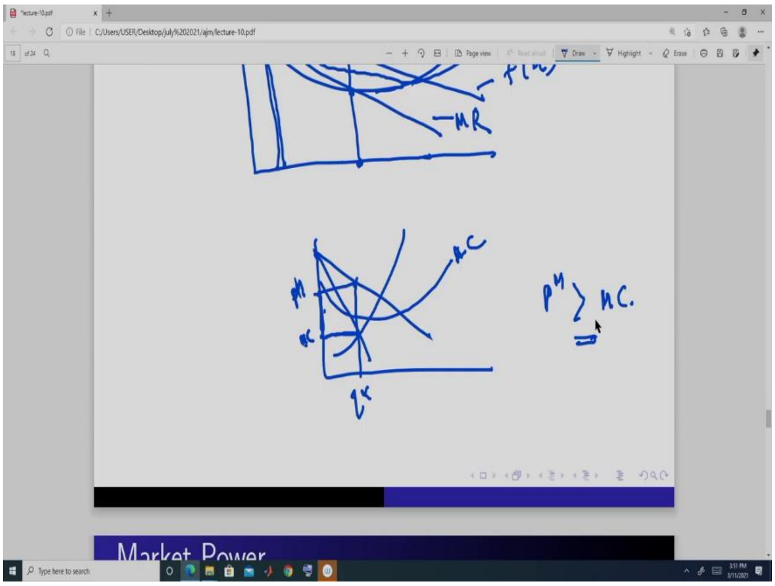

So, mainly the second order condition will allow us to, if we, our marginal cost curve is like this, demand curve is like this, marginal revenue, this is demand, this is marginal revenue, this is the, this is the average cost, then this is never going to be the optimum output. This is going to be the optimum output. From the second order condition we get that. And also we can argue logically why this is not going to be that.

So, we know how the market price is determined. If we are given a demand curve like this we will get a marginal cost like this and the cost function is, marginal cost is like this and the average cost is like this then the monopoly output is this y star and the monopoly price is this pm, right? We know this.

Now, the question is, suppose the cost function is similar. Then our cost functions are different. When a monopolist can charge a higher price and when it can, it has to charge a lower price? Because here if you see that this price is always greater than the marginal cost, because this quantity marginal cost is this and the monopoly price is this. So the price, monopoly price is greater than the marginal cost. How big it is going to be, what determines that.

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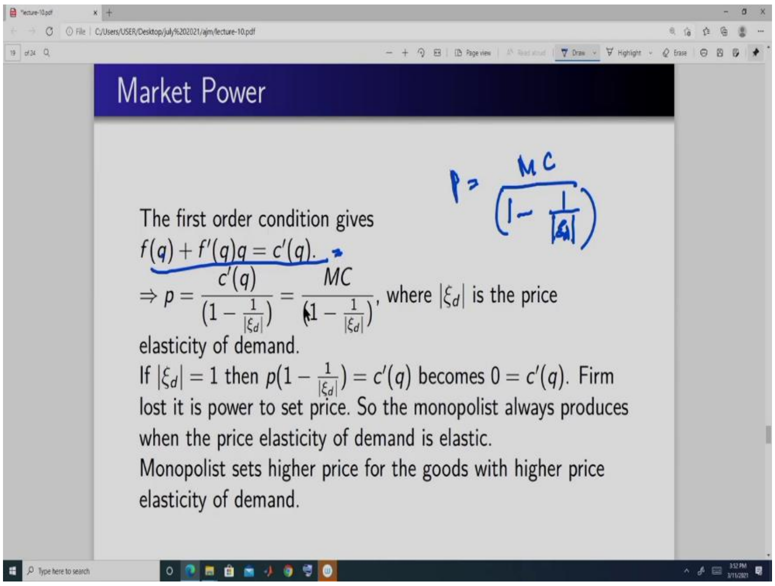

So, that is based on the market power. And how this market power is given? We will do this. So first order condition gives us this condition- $f(q) + f'(q)q = c'(q)$, and if we do a little bit manipulation we can derive this- $p = \frac{c'(q)}{1 - \frac{1}{|\xi_d|}} = \frac{MC}{1 - \frac{1}{|\xi_d|}}$.

So, we can say that the monopoly price is always equal to marginal cost divided by 1 minus 1 by price elasticity at that a. So we know this output. So you plug in this output here. So you know the elasticity at that point. So this will give you the price, how much price. And this should always take a value which is less than 1. So the monopoly price will be always greater than the marginal cost.

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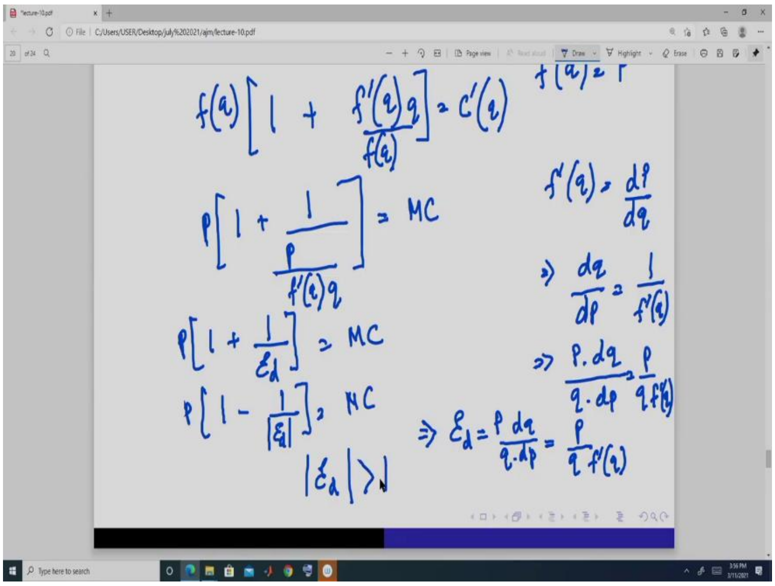

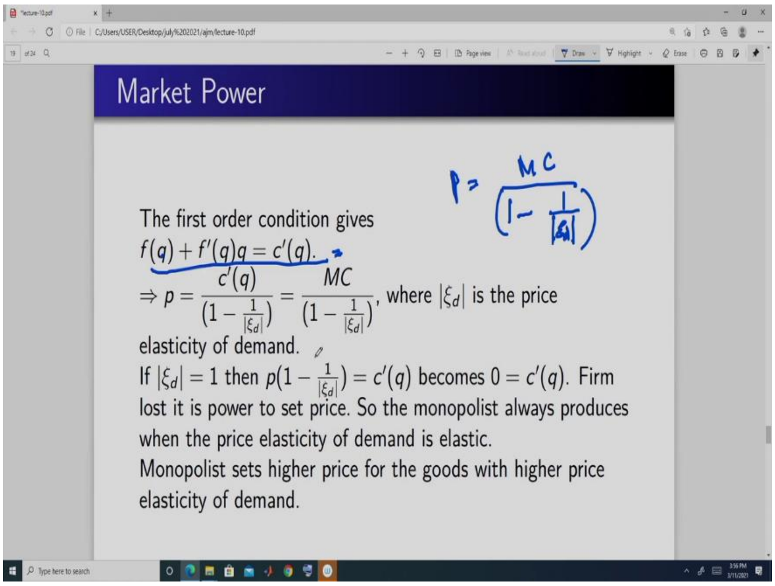

How do we derive this? We derive, first order condition gives us this- $f(q) + f'(q)q = c'(q)$. This portion-$c'(q)$ is equal to marginal cost, okay. Now we know this is equal to price. So if we take that common we get this- $f(q)[1 + \frac{f'(q)q}{f(q)}] = c'(q)$. yes. Now here, so using this- f(q)=P we can write, we can write it in this form-$P[1 + \frac{1}{\frac{P}{f'(q)q}}] = MC$. You can say marginal cost

Now, this is what? Suppose we can take the reciprocal of this-$f'(q) = \frac{dP}{dq}$. So we can write this-$\frac{1}{f'(q)} = \frac{dq}{dp}$ and to this if we do this we can write it in this form- $\frac{P.dq}{q.dP} = \frac{P}{qf'(q)}$. So this is what? This is the point elasticity which we by big epsilon, capital epsilon. It is like this- $\xi_d = \frac{\mathrm{P.dq}}{\mathrm{q.dP}} = \frac{\mathrm{P}}{\mathrm{qf'(q)}}$.

So, here we plug in this, this- $\mathrm{P}\left[1 + \frac{1}{\xi_d}\right] = \mathrm{MC}$ and elasticity of demand, since this is always negative so we can write it in this form- $\mathrm{P}\left[1 + \frac{1}{|\xi_d|}\right] = \mathrm{MC}$. We have done this, right? So price you can say now we have got this is equal to marginal cost that is this. So here this is the elasticity.

Now, if suppose at that quantity, price elasticity of that demand is unitary elastic. So if this is equal to this unitary elastic this takes 1, so this 0. So 0 is equal to this. i.e $0 = c'(q)$. So a monopolist will not have any power to set the price. So the monopolist will always produce when the price elasticity of demand is elastic that is this price elasticity is greater than 1.

The moment it is greater than 1, so this is a less than 1 so it is, so this price is always going to be greater than marginal cost. So this price is always, since it is going to be greater than marginal cost, so this price is going to be greater than the perfectly competitive market price because at the perfectly competitive market the price is always equal to marginal cost. Here this is due to this here.

So, what do we get? That the monopolist, means it only has only a high power, it has a higher market power that is high power to set a higher price when the elasticity of demand, or price elasticity of demand is more elastic. The moment the good has a more elastic demand at that price, so it can, at that quantity it can charge more prices. If the elasticity is less then it will charge or set a less price, okay. So this is actually, this condition is called the Marshall Lerner index, okay.

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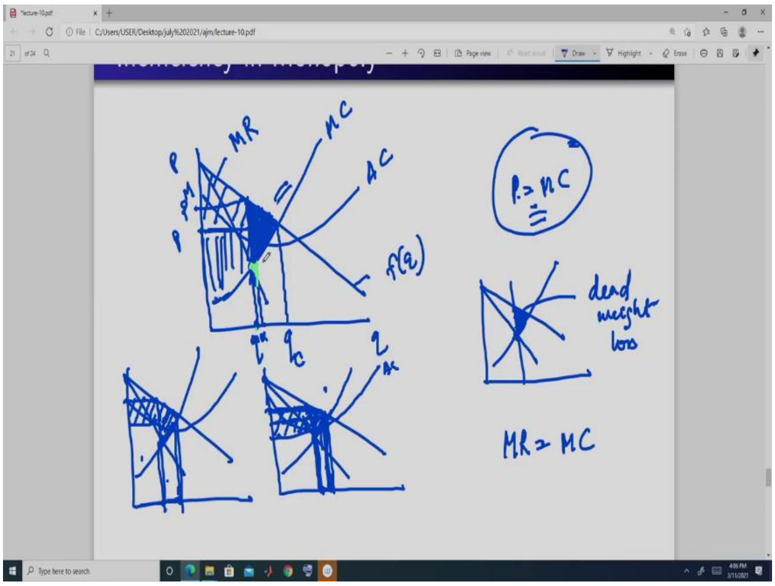

Now, we will show the inefficiency that is generated in the monopoly, okay. So this is the output. This is price. This is the market demand. This is the marginal revenue. Suppose this is marginal revenue. This is marginal cost. And this is the average cost. Optimal output is this much q star, monopoly optimal output or profit maximizing. Monopoly price is this much, right.

Now, suppose in this case there is one firm and that and that firm has to produce at that output where price is equal to marginal cost, suppose, okay, because had it been a competitive market the outcome would always be this, price is equal to the marginal cost, right?

So at that point output would have been this. So this is suppose q c, right? At this, market price is this. Now here this is the monopoly price. And if we know that this is the optimal, or profit maximizing price. But suppose firm is producing this much, then see the revenue it is, extra revenue it is getting is this much, this region. And this, this is the cost. So this because the cost, there is a net cost or net loss. So that is why the profit maximizing is this.

But if you look at it slightly differently you will see that consumers are willing to pay this much here. So if this is the a, then total producer surplus is, consumer surplus is this much and the producer surplus is this whole region. And from here we minus the fixed cost, okay.

But if we leave apart the fixed cost this is the producer surplus. And this is the consumer surplus. So this additional surplus is, we are not taking the benefit of that. We are not generating this a. So this is the amount of surplus we are foregoing.

So, here if we take this case, so this triangle is something called the dead weight loss. And because the monopoly price is, marginal revenue equal to marginal cost instead of price equal to marginal cost there will always be a dead weight loss, okay. Because we get the optimization optimal output at this, and not at this point. So this additional amount, sum of consumer plus producer surplus is, monopoly is foregoing that much amount of surplus, okay. If it uses this pricing then this benefit, this additional surplus could be borne, okay.

If we take the sum of the surplus that is producer and consumer surplus, because here you will notice that the moment price. So this is the monopoly price, okay. And suppose this is the AC, so this is the monopoly profit. Now if the price is this suppose for some reason. Then the monopolist is going to get, this is the average cost, so this rectangle is going to be the monopolist profit. So this rectangle is less than actually this rectangle. So that is why this is not a profit maximizing condition. So here profit is maximized.

But if you look at in terms of social welfare you will see that here this extra surplus is not, no one is getting, neither the producer nor the consumer. But there is a possibility we can get this, right? So that is why and if we get this much amount here, if we take the sum of this a, so we can make the producer. So what we are doing?

So if we take this, so monopoly is charging this price. So consumers surplus is this triangle, right. Now what happens? You ask the monopolist to produce at this much level of output. Suppose. Then and consumers are, and you set the price to be at this level, okay.

Now, what you can do here? So what I have to show that this is an inefficient outcome. Inefficient outcome means this is not Pareto-optimal. So if it is not Pareto-optimal that means I can make at least someone better off if I move from this monopoly to this outcome, right.

Now that is, how that can be done? So here if the price is here then there may be some person whose demand, who cannot buy this much amount, who cannot buy at this price, so, but who are willing to pay this much prices. So what we do? We charge, we give this much amount to those people in aggregate and charge them this whole amount. So cost is here. So these people are going to pay this whole amount. And the monopolist cost is this much. And if we pay, ask them to pay this whole thing then this will be received by the monopolist. So this is received by the monopolist. So we are making the monopolist better off, right?

Or instead we can do like this also. If we take this and take this and take this here, so what we do? Monopoly market price is there. And we take some individual who cannot afford this price. And what we do? We give them this much amount, and we charge them this much amount of price.

So monopolist is making this amount of profit. But it is producing this much amount of output and also this much amount. This much is going to someone else. And those people who are willing to pay this much, they are getting a benefit of this. So these people who were earlier not able to afford it, now they are getting this much amount and they are getting a surplus of this much. So we are making at least someone better off without hurting the monopolist, some consumers.

So, in this situation we are benefiting the consumer. In this situation we are benefiting the monopolist without hurting the consumer, here without hurting the monopolist. So in both the cases we can move to a Pareto-optimal situation. So that is why monopoly outcome is not a Pareto-optimal. So that is why it is an inefficient outcome. And there is this problem of dead weight loss, okay.

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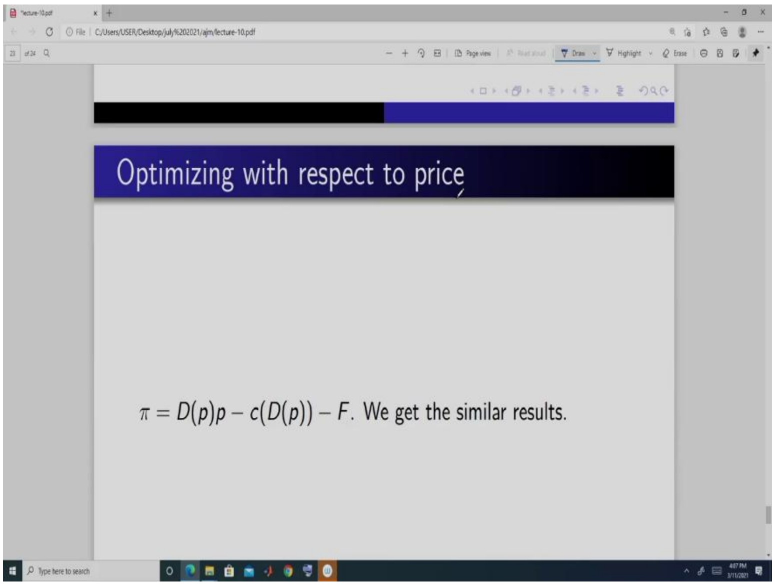

So, next we will do natural monopoly and the optimization with respect to price. You will see it is more or less same and that we will do in next class. Thank you.

Introduction to Market Structures

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Module 4: Monopoly
Lecture 14
Price Discrimination I

Hello. Welcome to my course: Introduction to Market Structures. Today we will do the remaining portion of Monopoly and we will start price discrimination. So, we were doing inefficiency in monopoly.

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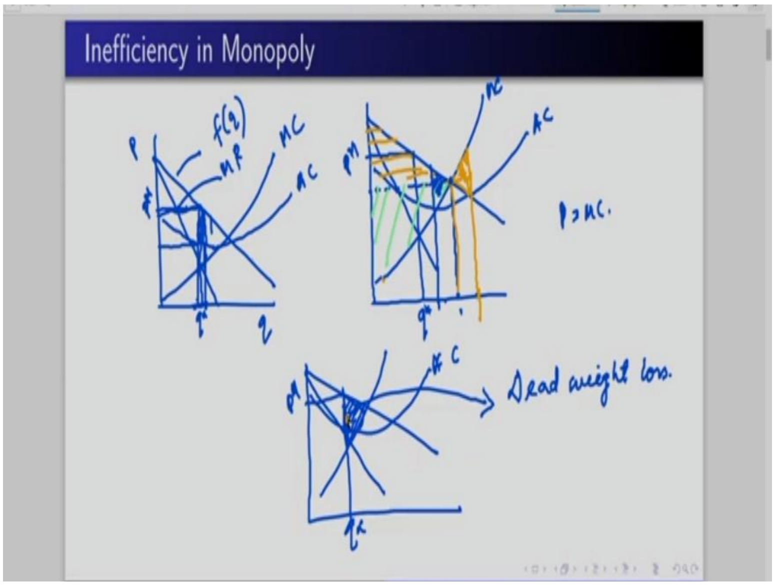

So first, we will see that suppose, this is the output in this axis and this is the price and this is the demand curve and this is suppose the marginal revenue curve and this is the marginal cost curve. So, monopoly output is this and the corresponding monopoly price is this and suppose the average cost is this, okay.

So, the monopoly profit is this rectangle, okay. Now, suppose a firm instead of producing this, if it produces here, what happens? We know it is not a profit maximizing output. We have done that but, what we can see is this that if it produces this much, then and suppose, it knows a consumer who is not able to buy at this price, but who can buy if the price is less.

So, it takes that consumer and it charges this whole amount and to get this much amount, so this amount is charged. So now and the price for or the cost that the monopolist is bearing is only this much because fixed cost is already borne and this is the total variable cost, this region. So this much is amount charged by the monopolist. So, what is happening? So, this consumer who is not able to buy at this price now, can buy because this is the market price.

Those who can buy this good at this price they are buying and they are buying some amount, okay.

So, the total amount is this q star. Now, the monopolist knows few consumer who cannot afford this good at this price and for them and suppose the monopolist can distinguish between who can afford this price and who cannot. So, there are many assumptions and then monopolist can charge this region as the price, so this, whole amount for this amount of output, so this consumer will pay this and this consumer will get the good and this much extra monopolist is getting and this much is the cost that the monopolist is bearing. So, this must surplus.

So, what we can do? So already those consumers who were buying at this price, they are getting it and there are some additional consumers are getting it by paying this much amount, okay, but they are not earning any consumer surplus because they are this whole amount is being paid, but the monopolist can earn surplus like this.

So, what is happening? Now here, through this construction, we can show that the monopolist outcome is not a pareto optimal. Why? Because here, we have kept the level of satisfaction of the consumers same because already those consumers who were buying this good were buying at the same price. Some additional consumers are getting this good and they are getting it in such a way that they are not earning any consumer surplus, and the monopolist is earning some extra surplus.

So, we can keep the satisfaction level of all the consumers at the same level and as the monopoly, but we can make the monopolist better off. So that is why the monopoly outcome is not bad at all at the optimal. Okay. So here you can see this whole region so we can go on doing like this.

So, like take this. This is the monopoly price and suppose this is the prices AC, and now suppose the price is this, okay. So, the monopolist is selling at this price. If this is the case, then what is going to happen? Total surplus that the monopolist will get, is this region, right? and total surplus that the consumers are going to get because they are going to pay this much is this, okay. Now, if you produce any output more than this here, we know from the argument that we have done in the case of perfectly competitive here what is happening, this much is the additional cost and this whole triangle cannot be this amount is the losses that they are A is going to make.

So, at this outcome, this output we can see that the surplus total surplus that is a consumer surplus, and the producer surplus is getting maximum, because if it is less than this, if it is less here, then we can move in this direction and make this surplus, which is sum of consumer and producer but if you produce more than this, then we make a loss of this amount. So, at this amount, the social welfare is maximum. This, we have done. Now here, if we take the case of monopoly, price is this, output is this.

So, this amount is not received by either the consumer or the producer. So, this is the lost surplus. So, this amount is called dead weight loss. So, in a monopoly, there is always going to be inefficient outcome because there is always some dead weight loss and in through this argument, we can also argue that the monopoly outcome is not a pareto optimal outcome, okay, and here we can from this, we can show that it is not social welfare maximizing.

So, there is a problem of inefficiency here because this dead weight loss, this triangle or this region is this surplus is not going to the consumer and the producer which they can, if the pricing is the price is equal to marginal cost, then this is going to be the price and this is going and the output is this. So, this whole amount is going to be this yellow amount is going to be the consumer surplus and this green amount is going to be the producer surplus, but which is not the case if there is a monopoly. So that is why there is a problem of inefficiency in a monopoly.

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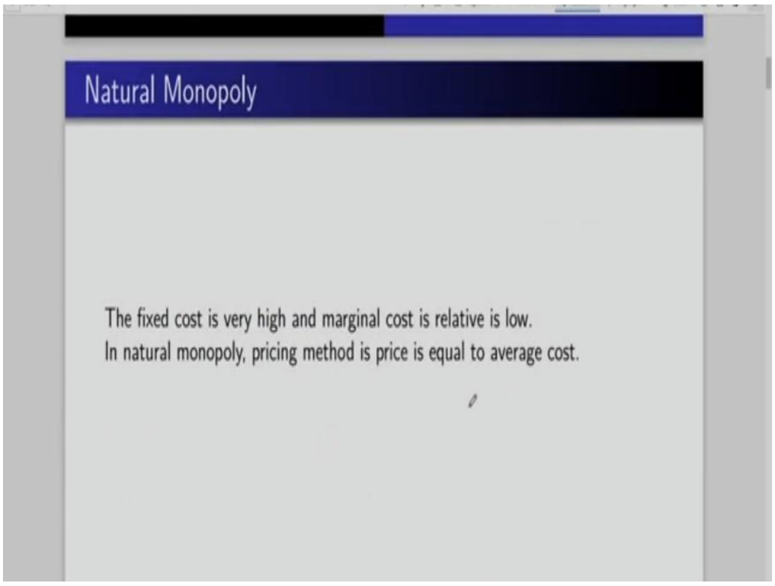

Next, we will do a special type of monopoly that is called a natural monopoly. A natural monopoly and the monopolist whose fixed cost is very high. The moment fixed cost is very high. It means that the average cost will take a different shape and the marginal cost is relatively low, not that high. So, we will say see. This is the situation.

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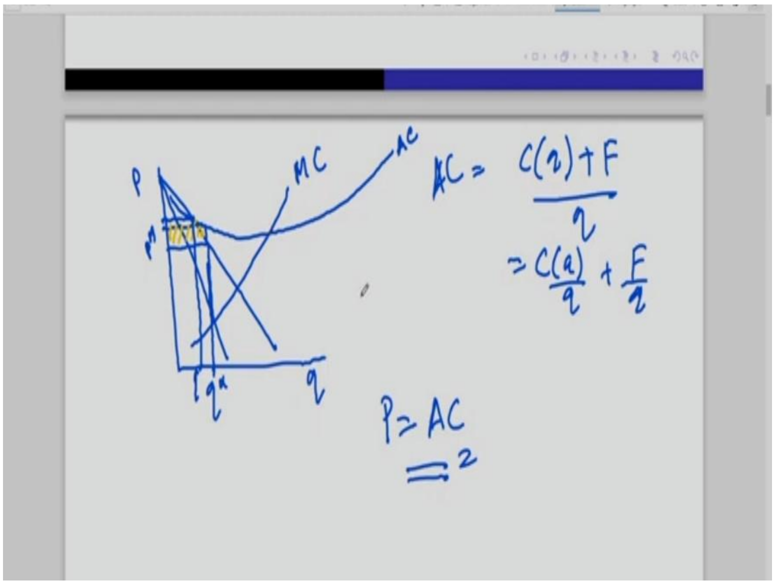

This is the output, price, okay, this is the market demand and this is the marginal revenue and suppose this is the marginal cost. So, monopoly output is this and the monopoly price is this. Now, suppose the fixed cost, average cost is what? Average cost is this- $\mathrm{AC} = \frac{\mathrm{c(q)+F}}{\mathrm{q}}$. Now if this F is very high, then we may have a situation like this. Average cost is very high.

So, at this monopoly price, if the monopoly uses the same pricing method that is price, the output is marginal revenue equal to marginal cost, and the marginal cost should be upward sloping at that output, and to that output corresponding price from the demand curve gives us the margin monopoly price. Now here monopoly if uses this produces this output and say this price is going to make a loss of this rectangle, this yellow coloured rectangle.

So, this then the monopolist should not produce and this generally arises in the case, when the fixed cost is very high. That means the set-up cost is very high or like the case of electricity or the case of water treatment plant, or like those power production plant. All these things has very high fixed cost.

So, they may so even a monopoly may not be profitable. So here, what they do, they use the average pricing method. So, the price is equal to AC. So, at this point so firms produce this much amount of output in this case and the so it is they are making normal profit. That is profit is equal to 0 in this case.

So, this is the pricing mechanism for natural monopoly and natural monopoly is common in certain sectors like railways you can say it is a natural monopoly or you can say electricity power supply or the water treatment thing. Those kinds of things, or even a huge big iron and steel plant, all these have a characteristic of natural monopoly that is the marginal cost is relatively low sticking less here, but average cost is very high. Why? Because this fixed cost is very high. So, in this case, the monopoly pricing may not be profitable. So, the firm uses this method that is price is equal to average cost- P=AC.

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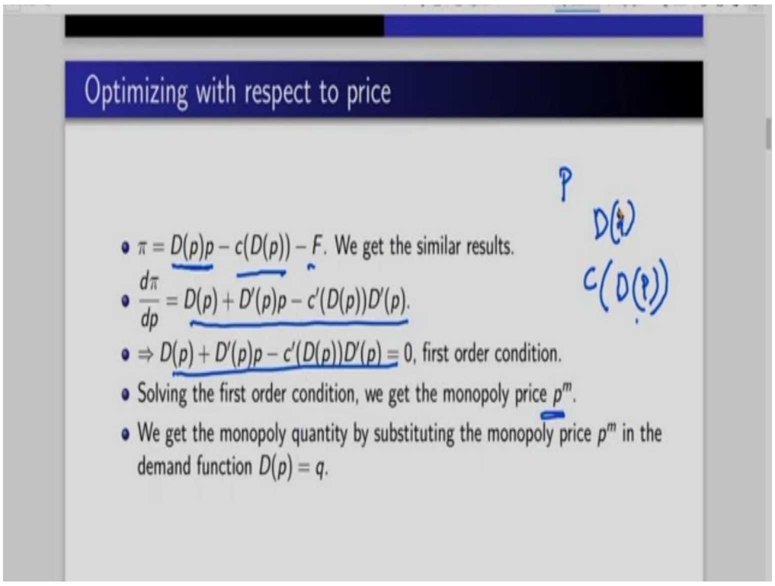

Till now we have done all the optimization or what we have done, we have determined the monopoly output and given that output, we have found the monopoly price by from the demand curve. Inverse demand curve. Now, let us do this optimization in terms of price, that is suppose the monopoly first determines the price and then from that price using the demand curve or demand function we get the monopoly output.

So, in that case, we know the demand curve function is this- $\pi = D(p)p - c\big(D(p)\big) - F$ so this portion is quantity into price. So, this is the total revenue- $D(p)p$. So, this amount at price p quantity demanded at price p, quantity demanded is this- $D(p)$. We have specified it in the last class. So, the total quantity produced should be this.

So, this is the variable cost- $c\big(D(p)\big)$and this is the fixed cost- F . So, this is the profit function- $\pi = D(p)p - c\big(D(p)\big) - F$. This is the total revenue- $D(p)p$ and this portion is the total cost- $c\big(D(p)\big) + F$. Now, we optimize since we know these functions are differentiable. So, we find the optimal price by differentiating it with respect to P. That is the price like this and if we do this, we will get this expression- $\frac{d\pi}{dp} = D(p) + D'(p)p - c'\big(D(p)\big)D'(p)$, right?

And from here, the first order condition, this should be equal to 0 and we get this equation- $D(p) + D'(p)p - c'\big(D(p)\big)D'(p)$=0, and we solve this for this p and we get the monopoly price that is by solving this equation this and now, we substitute this monopoly price in this difference curve, and that will give us the monopoly quantity, okay. So, this is how and we will get the same thing.

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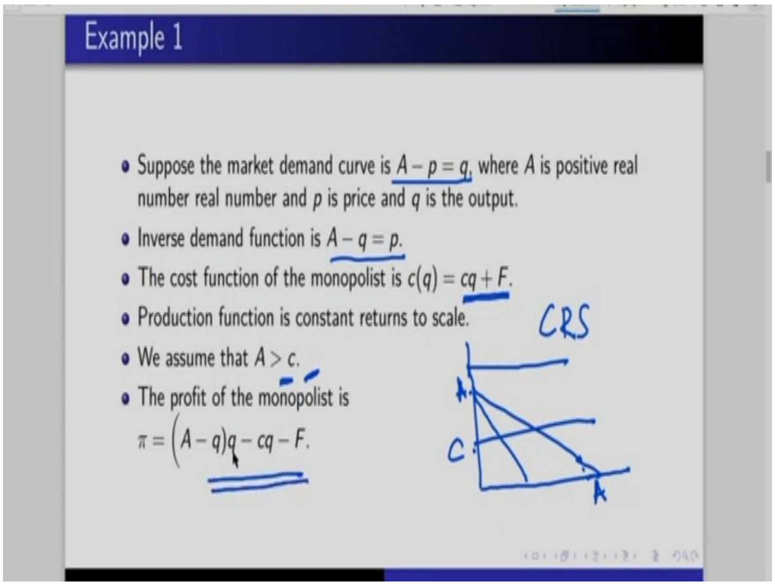

So, let us now do some example and then things will be more clear. Suppose, the demand curve is this A minus p is equal to q- A–p=q. So here this is A is real number positive real number and P is the price, q is the output, market output and if we take this demand curve, then the inverse demand curve is this- A–q=p and the cost function is of this- $c(q) = cq + F$. So that means that the firms have CRS the constant returns to scale and we assume this- A>c.

Here, this F that fixed cost, we are getting, it is from the rent that we pay in the building and here we have assumed this portion, i.e $cq + F$ is like CRS, so you can think that the labour and capital are variable and so it is giving you a constant returns to scale from these two factors and the building or that plot of land is fixed and that is giving you a fixed cost, okay.

Why we have made this assumption? It will be clear from the diagram. Like if we take demand curve is like this, this is A and this is A, then the marginal revenue curve is going to be something like this, marginal cost curve is this. So that is why C has to be less than A. If C is greater than it will be like this.

So, the cost is always greater than the maximum price that the consumer is willing to pay. That is the A. So that is why we require this assumption, okay. So, in this case the profit function is this- $\pi = (A - q)q - cq - F$ A minus q, this is the total revenue- $(A - q)q$ and output it is producing is q and this is the variable cost- cq and this is the fixed cost- F, we get this. Now, we solve this.

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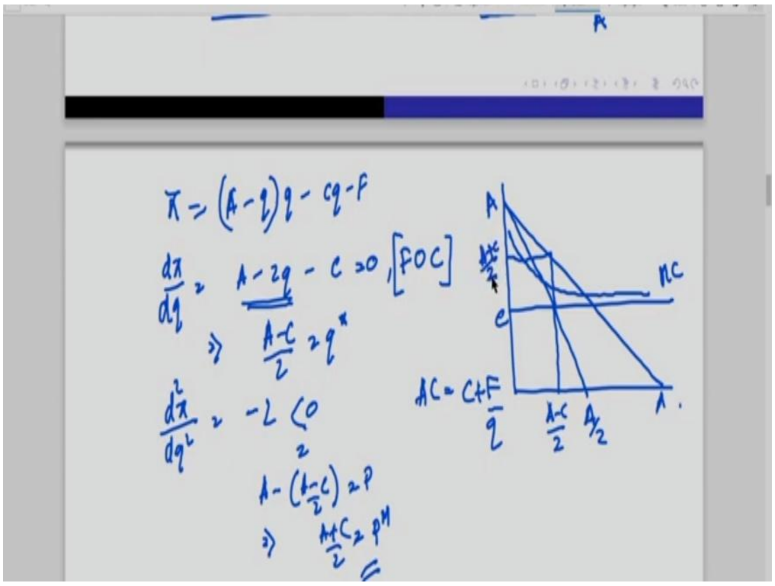

So, profit is A minus, i.e this- $\pi = (A - q)q - cq - F$, first order condition will give this- $\frac{d\pi}{dq} = A - 2q - c = 0$, first order condition will give this is equal to, so this means that this is the monopoly output- $\frac{A-c}{2} = q$, optimal output and if you take the second derivative you will see this is equal to 2, which is always less than equal to 0, i.e $\frac{d^2\pi}{dq^2} = -2 < 0$, okay. Now here, what you do monopoly price is this- $A - \frac{A-c}{2}$. This is the monopoly price-$\frac{A+c}{2}$. and how do we diagrammatically in this case.

So, this is the, so this point is A by 2. This is the marginal revenue curve, which is given by this portion. Marginal cost, which is this AC. Here AC is C by F by Q. So, it is something like this. This output is A minus C divided by 2. This monopoly price is A plus C divided by 2 this. So, this is the monopoly outcome in this case when we have constant returns to scale.

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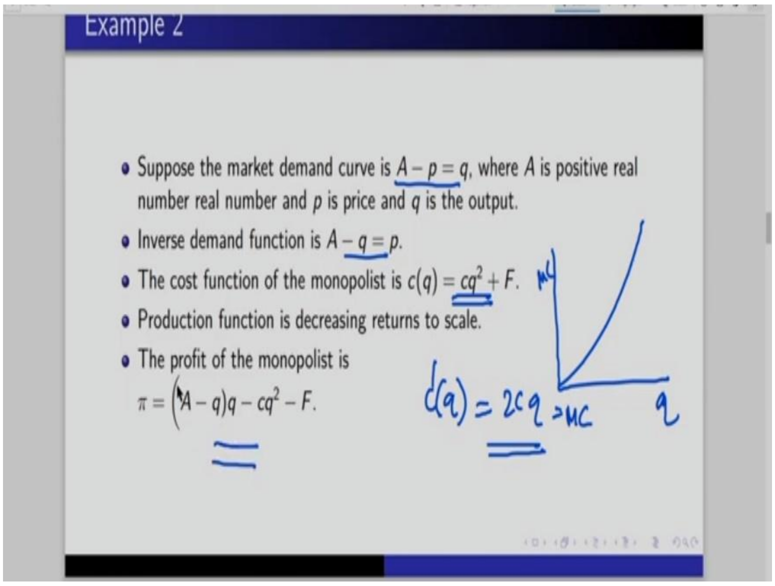

Now, let us do another example, we take the same demand curve that is the linear demand curve. So same inverse demand curve and suppose the cost function is of this nature- $c(q) = cq^2 + F$. So, if it is this nature, then the marginal cost is of this, right? because if the cost function is this, then the derivative of this is- 2cq, so this is the marginal cost. So now the profit function is of this nature- $\pi = (A - q)q - cq^2 - F$ because this is the total revenue- $(A - q)q$ and this is the total cost-$cq^2 + F$. So, what do we do?

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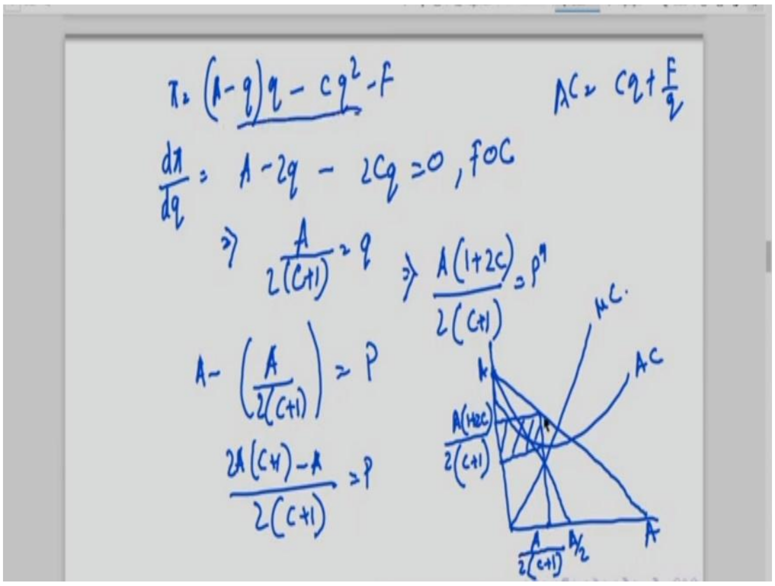

So, profit is A minus q, q this is the total revenue, q square minus f- $\pi = (A - q)q - cq^2 - F$. So, since every all the functions are differentiable, we take the first derivative and we get this- $\frac{d\pi}{dq} = A - 2q - 2cq$. This is the marginal revenue-. This is the marginal cost equal to 0 at first order condition or the optimal point, i.e $\frac{d\pi}{dq} = A - 2q - 2cq$=0. So, we get this is equal to- $\frac{A}{2(c+1)} = q$, right? and this is the monopoly output and monopoly price what we do, we substitute this in the inverse demand function and we get the monopoly price- $A - \frac{A}{2(c+1)} = P$ and it is, so this is the monopoly price- $\frac{A(1+2c)}{2(c+1)} = P^*$. So diagrammatically this is something.

Suppose, this is A, this is A, marginal revenue curve, demand curve, inverse demand curve it is this, marginal cost curve it is this, average cost curve, average cost curve is here it is this so it will be some nature like this. This output is A by 2 C plus 1, this and this monopoly price is this and the profit is this by plugging in this output. In this function, we get the profit, which is given by this rectangle. So, this is the second example when we have decreasing returns to scale.

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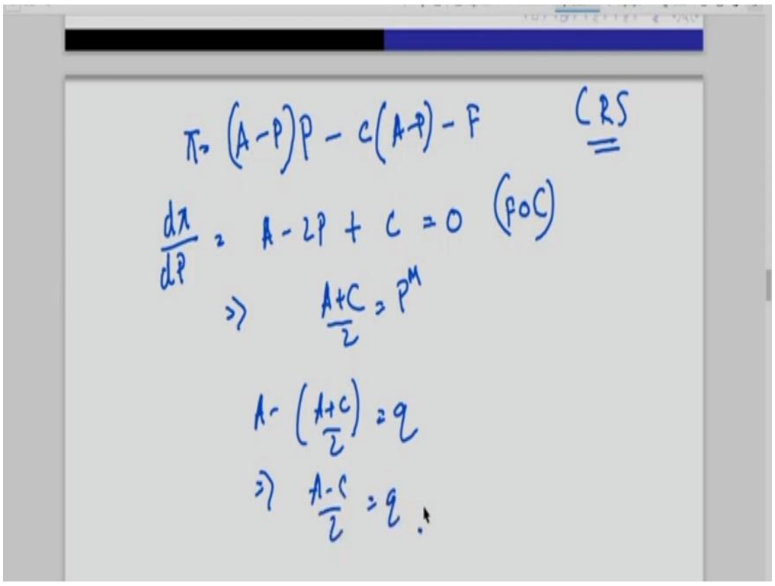

Now, we can do the same thing here by if we first determine the price and then the output. So here in this case, this is going to be the demand curve- π= (A–P). So, we are now going to use demand function, not the inverse demand function and this it is going to be this one- $\pi = (A - p)p - c(A - p) - F$. So, when we have CRS case, that is the example one. So now we get this. This is the first order condition- $\frac{d\pi}{dp} = A - 2p + c$. Now, this is the monopoly price- $\frac{A+c}{2} = p^*$and you substitute here, you will get the monopoly output- $A - \frac{A+c}{2} = q$ and that is and if you compare this and this you will see, they are same. So, the outcome is same.

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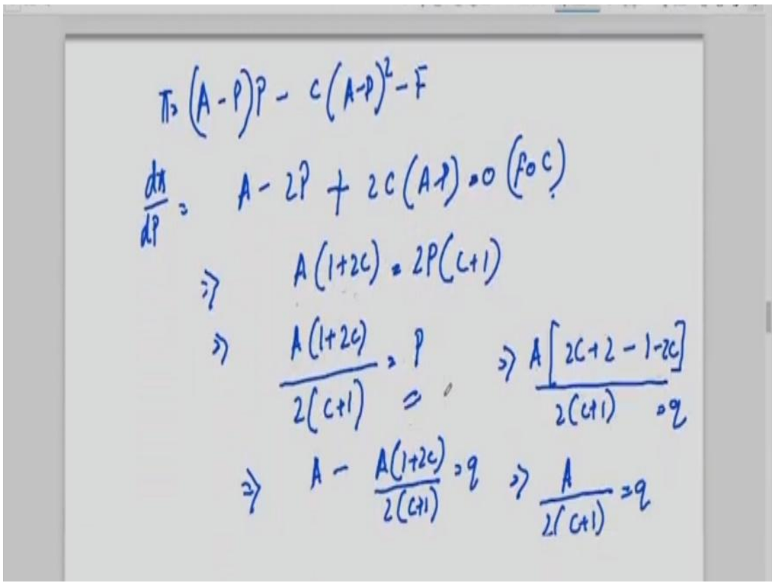

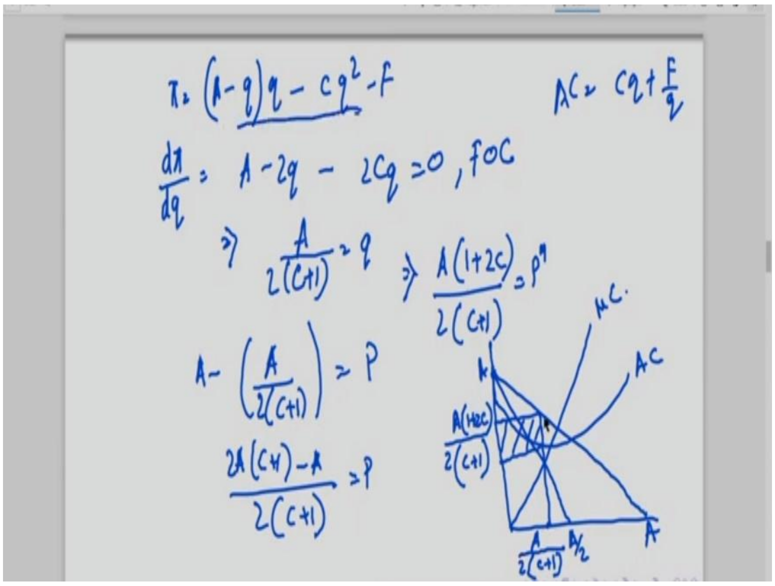

Now, let us take the second example. So, profit is A minus P- $\pi = (A-P)P - c(A-P)^2 - F$. So, this is the output into price total revenue. Total variable cost, first order condition and this- $\frac{d\pi}{dP} = A - 2P + 2c(A-P) = 0 \Rightarrow A(1+2c) = 2P(c+1)$. So, this is going to be the monopoly price, i.e $\frac{A(1+2c)}{2(c+1)} = P$ and the monopoly profit, monopoly output is going to be this much- $A - \frac{A(1+2c)}{2(c+1)} = q$ and from here, you will get this- $(A[2c+2-1-2c])/2(c+1) = q \Rightarrow A/2(c+1) = q$ and if you compare this with the outcome in the second example, you will see that they are same. So, it does not matter whether you decide the output or you decide the price. You will get the same profit.

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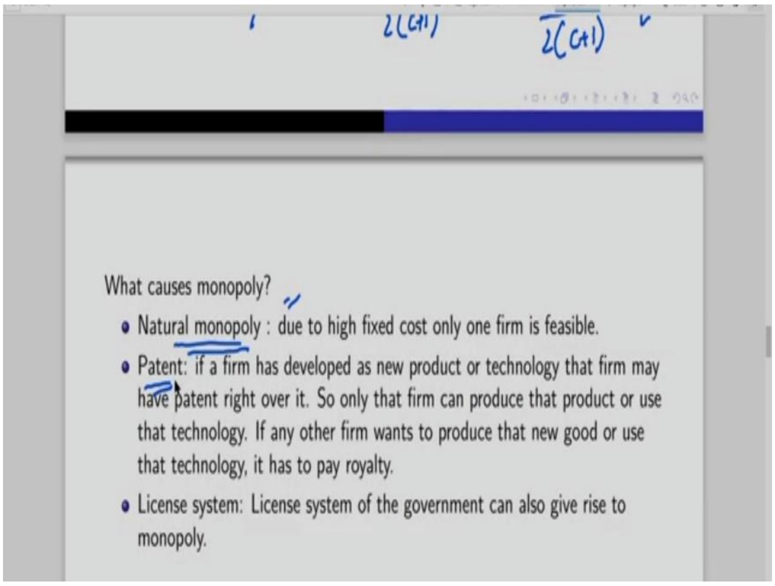

Now, the next question is, what causes monopoly? So, monopoly means that there is only 1 firm or 1 seller in a market. So, 1 may be the natural monopoly. If a industry has characteristics of that industry such that the fixed cost is so high that only 1 firm is feasible that if there is only 1 firm, then only it makes profitable. So natural monopoly, will always lead to a monopoly kind of thing. That is why we see in case of like electricity or that is the power supply or power transmission or like water treatment, all these are a form of a natural monopoly. Next is the patent right.

That is, if a firm develops a new product or a new technology and based on that if it when it develops a new product and suppose it wants to produce it and sell it, then it can have a patent over it. So only that firm now can produce that output. If any other firm produce that output, then that firm has to pay some royalty to this firm, which has developed this product or suppose a firm has developed a new technology. So, the firm while producing any output, it will have lot of advantage or it will produce a completely different product.

So, if it has a patent over it, then this firm can only, only this firm can produce this output and in that case what happens, all other firms, if they want to produce this output, or if they want to use this technology they will have to pay royalty to this firm. So, this firm has a monopoly power. Here it means that it has a lot of power to determine the price. Next is the license system in like prior to 1991 in India, we have a license system where the to set up a firm or to produce output, you require a license from the government.

So, in that case, sometimes happen that government only issue license only 1 firm. So that will automatically generate a monopoly or if suppose a firm is producing using some hazardous substance. So, then government generally tend to give very few or 1 big firm is allowed to produce such things. So, in that case also there is a tendency to have a monopoly in the market, okay. So, these are some natural ways through which the monopoly is generated or come into existence in the market.

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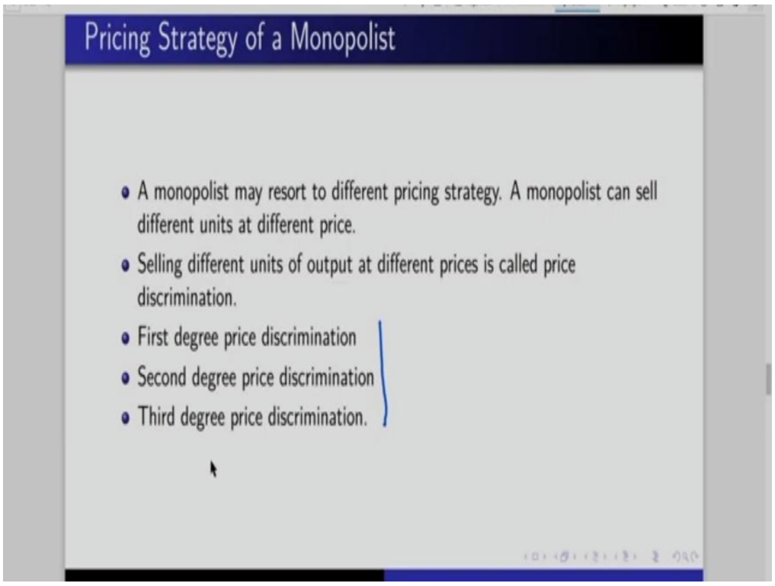

Next, we will study the pricing strategy of a monopolist. A monopoly can so we have seen before, how a monopoly save the market price and the optimal output its producing. Now, the next question is, is it the only way because the monopolist is the only 1 firm and if anyone wants to buy this product that person or that consumer has to buy from that firm only or that producer only. It has no other option. So, the monopolist can resort to different ways or you can use different strategy of pricing. So, one way is that you sell each item or each quantity at different prices.

So, this way of selling different prices or setting different prices for different quantities. This is called a price discrimination and the monopolies generally or they can do something called price discrimination. So, there are three forms of price discrimination. First, is the first-degree price discrimination. Second is the second-degree price discrimination and third is the third-degree price discrimination. So today we will only do the first-degree price discrimination.

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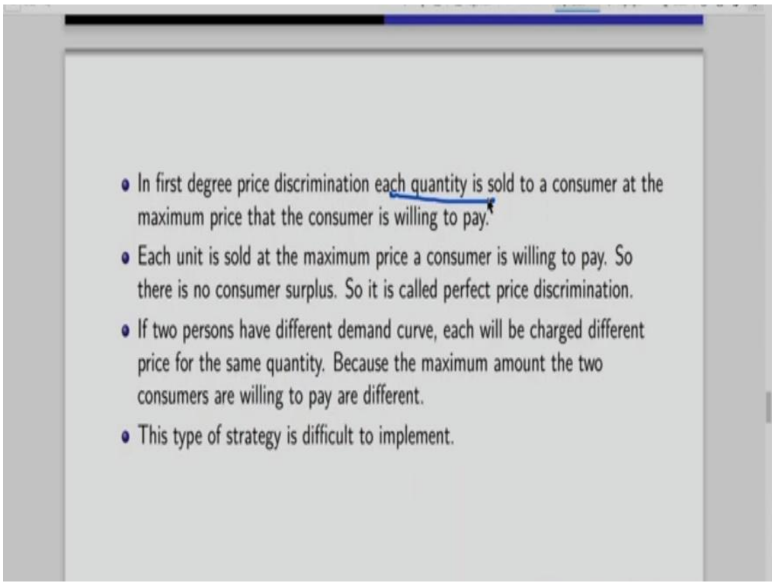

So, in first degree price discrimination, what happens, each quantity is sold to a consumer at the maximum price that the consumer is willing to pay, okay. So, what is the meaning of this?

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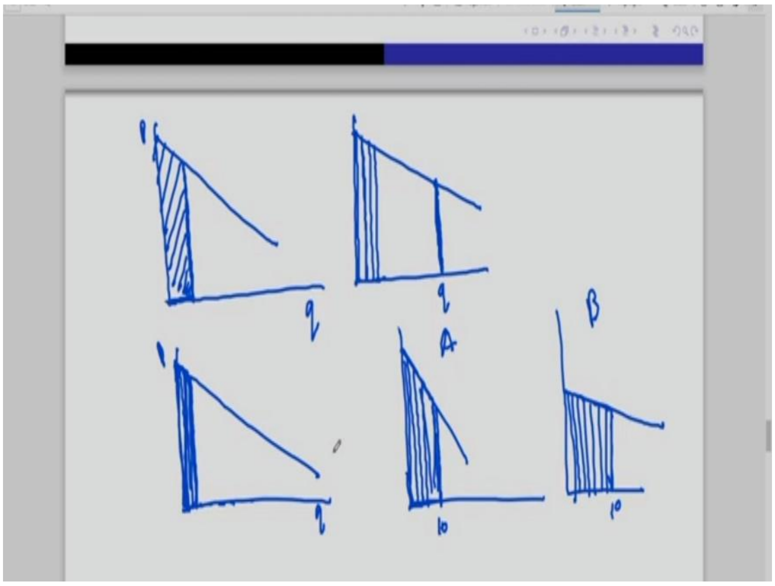

Meaning of this is this. Suppose, this is the demand curve of that person. So, if a consumer wants to buy this much amount of output, this whole amount so this amount. Then the maximum consumer is willing to pay is this whole amount, right? or for that, or if we take this amount and if consumer is willing to pay this amount for the first unit and suppose consumer is maximum consumer wants to buy this much. So, the maximum for the last unit consumer is willing to pay this much and we know why the demand curve is like this because as the price falls then only the demand increases. So, consumer has already bought this much these many units.

So, for this last unit, he is willing to pay less or you can understand it through marginal utility. So, the utility from each unit goes down. So, the maximum it wants to buy, wants to pay is only this much. So, what this monopolist can do, they can for each quantity. So, each dot the monopolist can charge what the maximum amount the consumer is willing to pay like this. So, in case, when suppose demand curve is like this and each dot here represents 1 unit or 1 quantity and for this each dot, we have a maximum that the consumer is willing to pay is given by this point in this demand curve, right?

So, the monopolist can charge this price if he is buying this much quantity then the monopoly will charge this whole amount. So, for each unit, it will charge the different prices. So, this is called a perfect degree of price discrimination. You can perfectly discriminate for each quantity you are charging the different price to a individual.

So, an individual suppose the demand curves are different. Suppose, there are 2 individuals. One's demand curve is this A and person B's is demand curve is suppose this. Then for the first suppose, if this is the 10 unit. For the 10-unit, consumer A will be charged this whole unit. This whole portion because and it will be charged each quantity wise.

First unit, it will be charged this, for second unit it will be charged this, for second unit it will be charged this, second like this it will go on. So, this whole 10 units will cost this area consumer A and suppose this is 10 here and B also buys 10 unit. Then B will be charged this for each units. So, for 10 units, the B will end up being this whole reason. So, this is actually first degree price discrimination where you can discriminate across individuals that to each individual, you can charge different price and to each quantity, you can charge different price.

So, it is discrimination in both terms, consumer wise and also quantity wise, okay. Now for this if you want to do then use the simplest thing that suppose there are 3 individuals and suppose they buy different quantities, then you will have to, a monopolist will have to do lot of calculations, right? It will have to charge different prices to each individual and for each quantity. So, it is very difficult to implement this method. So, what in practice a monopoly can do.

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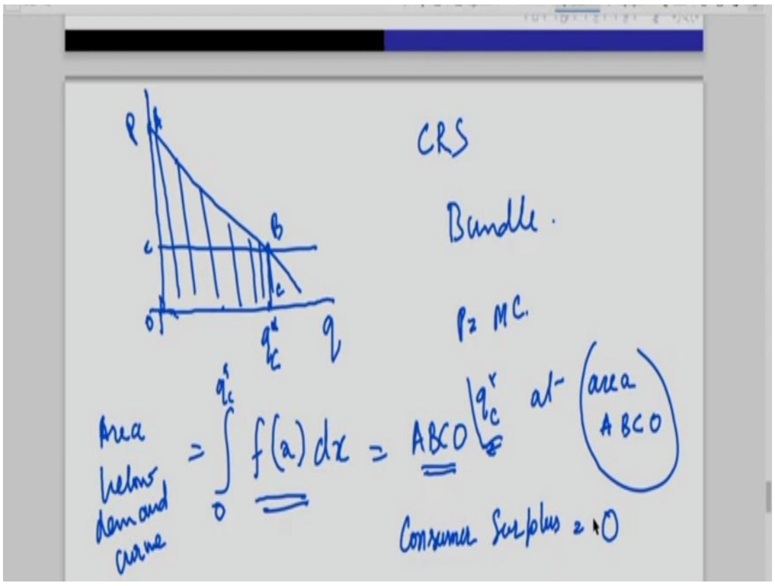

Monopoly can resort to this kind of and monopolist here suppose this is the quantity and this is the price, okay and this is the demand curve, okay. Suppose the marginal cost is this so it is the case of CRS, right? Now here monopolist can practice something called it can bundle the good. Suppose this is for one consumer and in this situation, what we do, we make a very strong assumption that all the consumers are similar. So, this is the demand curve for each individual and so all the individuals have the similar demand curve. So, market demand curve is simply n times this, okay.

Now, for an individual if we consider the demand curve of one individual and this is suppose the marginal cost and this at this quantity, which we can call competitive because the price is equal to marginal cost because this is the price. Price is equal to marginal cost at this point. Suppose the monopolist produces this quantity, okay and this suppose this point is A, this point is B, this point is C and this point is O and monopoly says that you bundle it in such a way that you can buy q star c at the price given by the area ABCO.

So, you cannot buy any intermediate good or any other goods than this, other quantity than this. So, it is available only in one quantity that is Q star and the amount one has to pay, the buyer has to pay is this area ABCO, okay or you can say this whole region is this is q star c demand curve is and we integrate over this one. So, this whole region is the area below the demand curve, right? So, this is the area below the demand curve and which is equal to area of ABCO, okay. So, this is one way. So, suppose we are at n buyers.

So, n buyer each one will buy this much quantity by paying an amount of this amount and monopolist will charge this amount, right? when the cost function is CRS and the demand curve is of this nature, okay. So, this is one way. So here what is happening? So, consumer, the monopolist is extracting all the consumer surplus, this whole. So, the consumer surplus is 0. The buyers do not earn any consumer surplus is equal to 0, okay. Everything is extracted by the seller or the monopolist.

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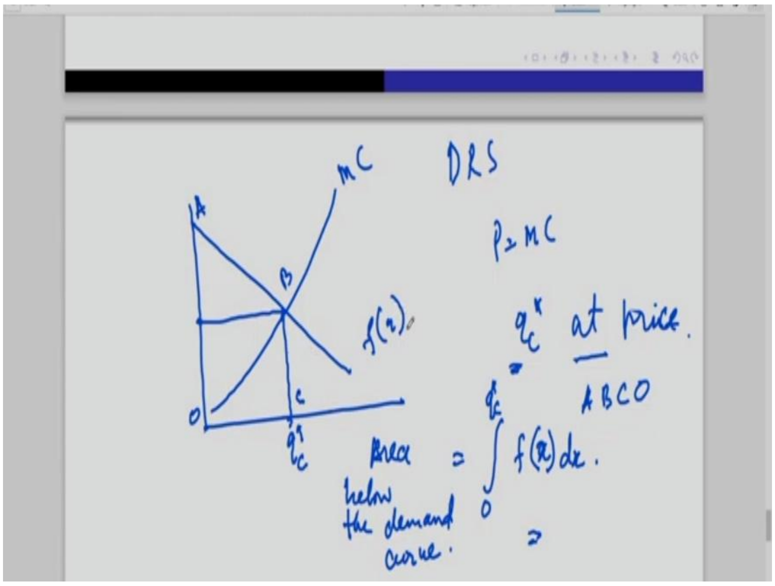

In case of decreasing returns to scale, we also get the same thing. This is the demand curve and the cost function is, marginal cost is this. So, this is q star c, why, because it is a competitive way because the price is equal to marginal cost at this output. So, this is the price. This price should be the competitive price.

But here monopolist will set this quantity at price for this quantity is this whole region, A B C and O. So, it is at price ABCO or the price is from 0 to q star c area under the demand curve, which is this. So, this area below the demand curve, so this is the case in case of decreasing returns to scale. So here the monopolist can charge that I can extract the whole surplus from the consumer, okay. Now this outcome has a very interesting property. If the monopolist can do first degree price discrimination, then you will see that it is a efficient outcome.

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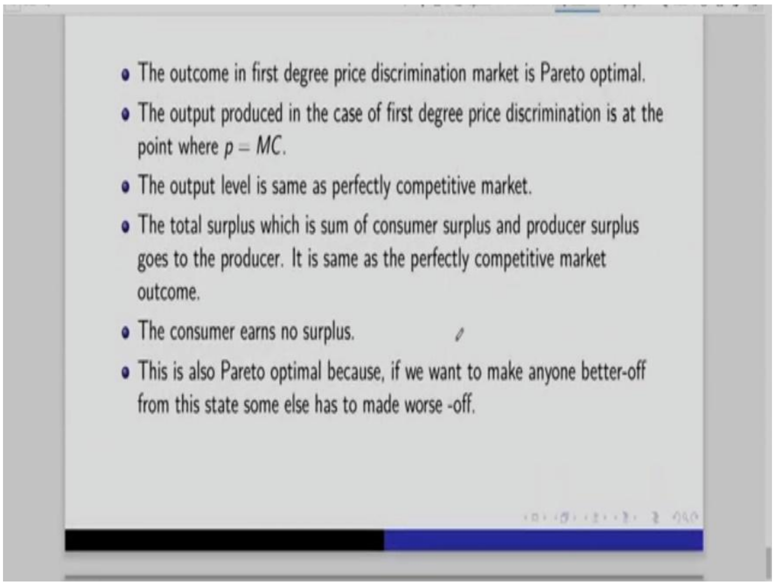

Why it is an efficient outcome? See or we can say efficiency in terms that it is a pareto optimal outcome, in the sense that consumer, we cannot make either the consumer or the monopolist better off without hurting someone else, or we can say the total surplus, sum of consumer plus producer surplus is maximum.

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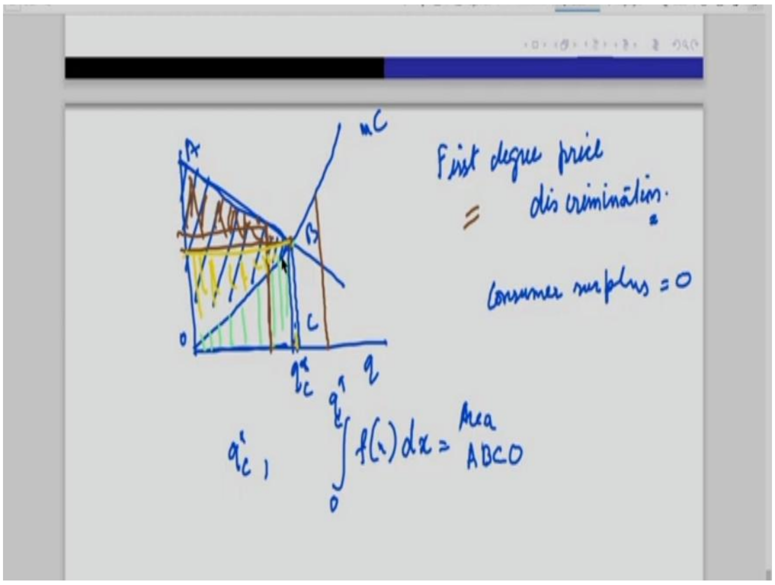

Here marginal cost is this and the demand curve is this. This is the output produce when there is first degree price discrimination, first degree, okay and the price charged so quantity sold to each individual is this and the price charged is, this whole area AB, right? This whole region. So, a consumer, the maximum a consumer is willing to pay for this much amount of output is this whole region. For each unit, it is willing to pay the point in the demand curve.

So, the whole region is a maximum for this much amount of output. So, the price is also that. So, price is A0 to q star c. This is the demand curve and we integrate over this demand curve- $\int_0^{q_c^*} f(x)dx$.

So, this whole region is giving me so the price is this or you can say area of ABCO, okay. So here consumer surplus is 0. So those who are buying they are not getting any surplus but what is the producer surplus? This whole region is the producer surplus from each buyer because if it is selling this much amount of output, the cost it is bearing is this for production of this. Fixed cost is already there. So, we are not bothered about the fixed cost. It will always be there. Now, or now, but this whole surplus because if it once produced this amount or this amount or this amount, it does not matter.

Fixed cost will be there right? So, but this whole amount is going to the monopolist. So, this whole surplus is received by the monopolist. Now, but the output produces this, right? where price is equal to marginal cost. Now, if you produce less output, suppose, okay, less output here. Then what is happening?

This much amount of surplus we are foregoing. It does not matter who is getting consumer or producer but this much sum of the surplus is from that sum of the surplus is being lost or we are forgoing that much amount. So, it is not better to produce amount of output less than this and if we produce more than this, then we are adding this much loss, okay. So this much negative amount.

So, it is going to be less than this total sample. So social welfare, which is sum of consumer plus the producer welfare is going to be maximum at this point. So, if the social welfare which is sum of consumer and producer, so that is maximum when we have the first-degree price discrimination.

But because the social welfare is silent about the distribution of surplus between the consumer and the producer but the distribution is very skewed, all the surplus is being extracted by the monopolist and the consumers are not getting any surplus but still the total surplus is maximum at this point, okay. So that is why social welfare is maximum.

So, this outcome is same as perfectly competitive outcome. Social welfare is being maximized and also this is a pareto optimal outcome. Why? Because if we now produce anything less here, then this surplus we are forgoing, right? and if we are producing more, then we are adding some loss and further what is happening? If we do a kind of discrimination, like changes in the distribution. The distribution is such, 0 surplus to consumer and all the surplus to the producer. Then, what is happening? The producer is, if we fix any price here, then what is happening?

This much surplus is being received by the consumer and or if we take this A and fix this price, okay. So output is this and price. So now, what is happening? Consumer is getting this much surplus and the producer is getting this much surplus, right? but earlier the whole amount was received by the producer and 0 was received by the consumer. Now, what is happening here? We are making consumers better off at the cost of the producer. So that is why the initial distribution that is consumers are getting 0 surplus and the producers are getting all the whole surplus. It is a pareto optimal outcome.

So, even if the market price is fixed here, then the output going to be produced is this much and producer surplus is going to be this yellow region and the consumer surplus is this brown region, but what is happening compared to the first-degree price discrimination outcome here when the firm does the bundling.

Here, all the surplus is received by the producer and consumers were not receiving any surplus. So, there is a change in the distribution of the surplus but the total surplus is same. So, that is why the first-degree price discrimination is a pareto optimal because here, if we move to any other outcome where the total surplus is same, what you have to do? You have to change the distribution.

So, it means that if you want to make suppose the consumers better off, you will have to make the producers worse off at the cost of the producers. So that is why first-degree price discrimination is pareto optimal and it is also we have seen it is a social welfare maximising. So, this is actually from the chapter 25 of Hal Varian and the monopoly is from chapter 24 and in the next class we will do second degree price discrimination and the third-degree price discrimination. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
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Module 4: Monopoly
Lecture 15
Price Discrimination II

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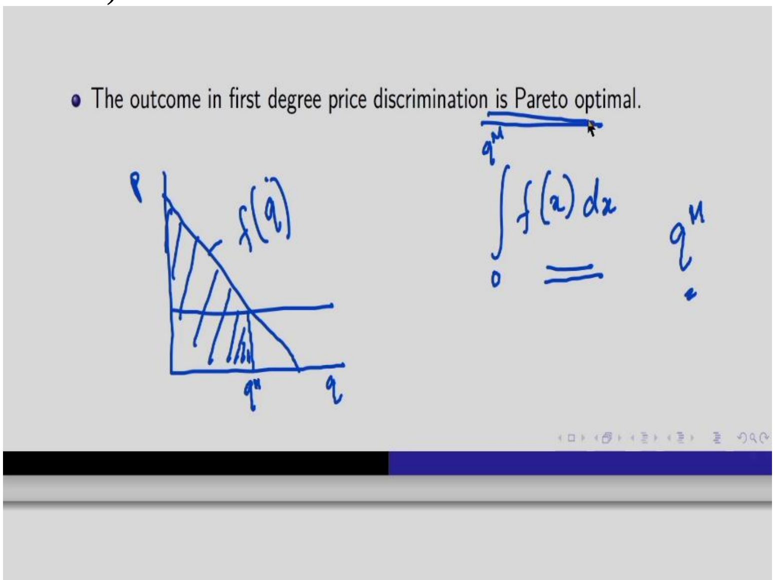

Hello, welcome to my course, Introduction to Market Structures. So, we were doing first degree price discrimination in a monopoly and we have seen that suppose here is the price, here is the quantity, and this is the demand curve. Suppose there are individuals are similar in terms of the demand curve, so the aggregate market demand curve is this and suppose the marginal cost is this, it is CRS then the monopoly is going to sell this much amount to each consumer and the price each consumer has to pay is this, right? this whole.

So, if this is the demand curve which is given, in this way then the price is from 0 to q m whole area under the demand curve. So, this is the amount- $\int_0^{q^M} f(x)dx$, they have to pay and each consumer will get this amount-q^M. And if there are "n" individuals so it will be "n" into this, okay? Now we will show that this is actually a pareto optimal situation, why?

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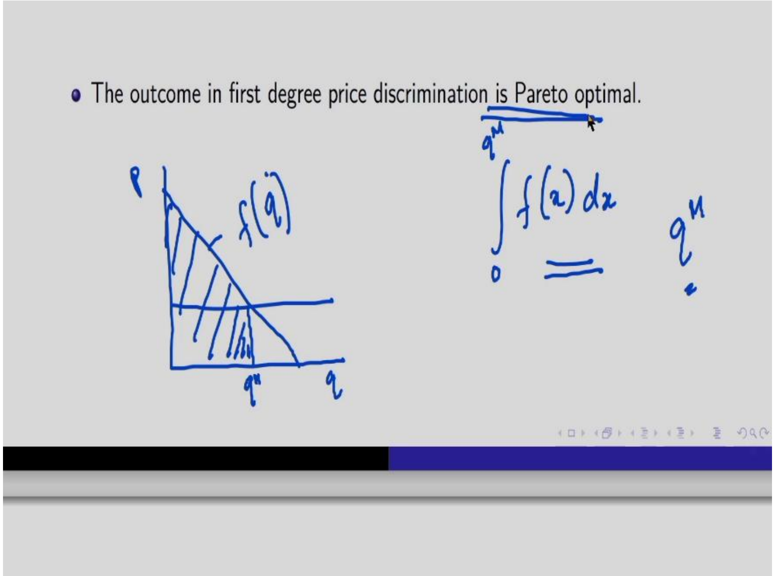

Say, take the case of a competitive outcome, in competitive market we know that, price is equal to marginal cost. So, the output produced is such that you have price is equal to marginal cost. So, suppose the demand curve is this and the marginal cost is this, then competitive outcome would be this, this much quantity because at this quantity price is equal to marginal cost. Although in a competitive market we have seen that we generally do not take CRS because the output is indeterminate in that.

But suppose it is a monopoly market and suppose the government regulates the price and they said that price should be equal to marginal cost so it is like this- P=MC. So, the output is this and at the same time this condition is also being satisfied. So, here, this is the output, monopolist is producing but now suppose the monopolist is not under any regulation and it can discriminate the price so it will charge the whole area under the demand curve as the price and the price is going to be 0 to q M, this- $\int_0^{q^M} f(x)dx$, right?

So, here it means if you look at this marginal, this is the marginal cost so this area under the marginal cost, so this green region, this is the total variable cost, fixed cost does not matter whatever be the level of the output so that is it. So, total surplus in this economy, in this market is this orange area.

And this whole surplus is now is going to the monopolist when they can do the first-degree price discrimination, okay. But the output produced is the competitive output so we know if we are here, if the output is this, then if we produce output more than this here, then we are making a net loss of this amount.

If we are producing less than the this amount here, then we are forgoing this much grey amount of surplus that is why it is not pareto optimal, we can produce a little bit more and we can generate this much surplus and if we are producing more than this, then actually we are making a loss of this pink amount, right?

So, that is why this output where price is equal to marginal cost is the pareto optimal thing, that output. And here we are not talking about the distribution of the surplus, in the competitive market this whole surplus would be going to the consumer but in a first-degree price discrimination this will be going to the monopolist so here monopolist is getting all the surplus, but if we want to make anyone better off here then what we will have to do, we have to give some surplus from the monopolist to the consumer so if we want to make the consumer better off then we have to hurt the monopolist.

So, that is why this situation where all the surplus is accruing to the monopolist is a pareto optimal situation because if we want to move from that to any situation where we want to make anyone better off, we will have to hurt someone else. So, if we want to make any consumer better off, we have to hurt the monopolist and if we want to make the monopolist better off, then we do not have any other option, okay. So, in this sense, this is a first-degree price discrimination is a pareto optimal situation.

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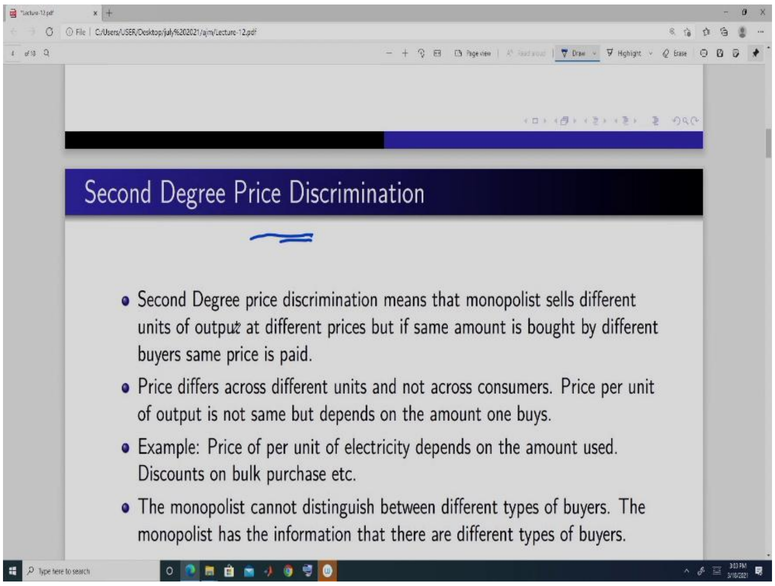

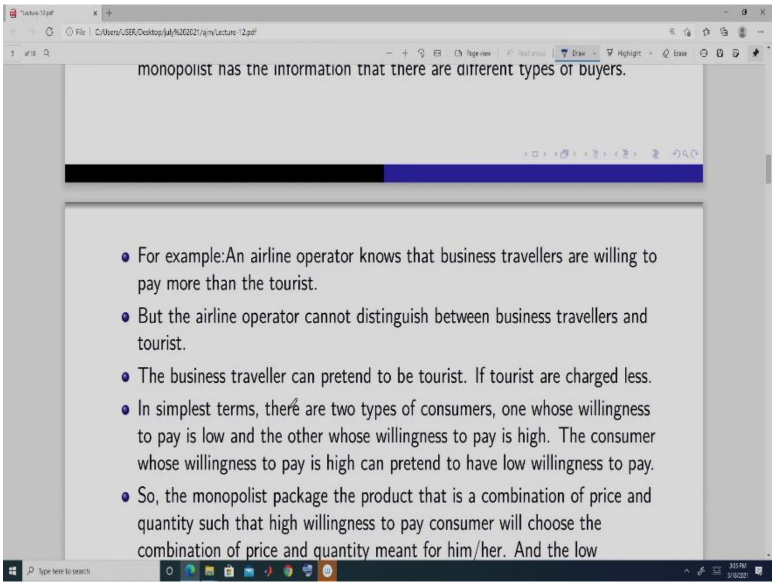

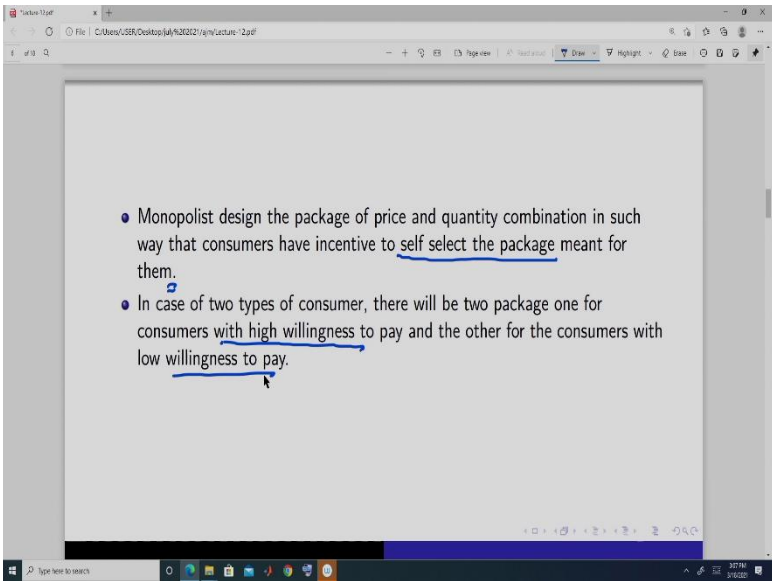

Next, we move to something called second degree price discrimination, in the first-degree price discrimination what we have seen is that the monopolist can distinguish between the consumers and also, they know the monopolist knows that who is whose consumer is of what type and so the monopolist is going to able to extract all the surplus from the consumers and it is very difficult to implement so it requires us a lot of information.

Now, in a second-degree price discrimination what we assume that the monopolist sells different units of output at different prices but if the same output is bought by different buyers' same price is paid.

So, it means the monopolist cannot distinguish between the consumers, it knows that there are different types of consumers or there are different varieties of consumers and when we say different types of consumers or different varieties of consumers it means that the demand curve of these consumers are different.

It means that their willingness to pay is different from each unit of output and so we get different demand curve, so we have different types of consumers and monopolist knows that there are these two types or three types or four types of consumers but if a consumer comes to buy a product it cannot say or it cannot distinguish, what is the types of this a. So, what the monopolist is going to do?

The monopolist is going to package these goods or you can say bundle so it will give a combination of price and output in such a way that, it is for, this bundle is for this type, another bundle is for another type and each type is going to select their respective bundle which is designed by the monopolist. So, it is something like this, that airline provider, it knows that it has two types of travelers, one is the business travelers and another is the tourist.

Business travelers are willing to, they have more or they have high willingness to pay why because it is binding on them, for this business things, they have to make this travel and the tourist, they are not, their willingness to pay is not that high because they can wait if the price is very high, they may postpone their travel or they may reschedule their whole schedule.

So, in that sense the willingness to pay is different for the business travelers and for the tourist traveler but the airline operator or the person who is selling the ticket he or she cannot distinguish between a business traveler and a tourist traveler, so but they know that there are these two types of travelers.

So, they will package these tickets in such a way that the business travelers are going to choose the package which is meant for them and the tourist travelers are going to choose that package which is meant for them. So, this criteria is called self-selection method, okay. So, monopolies design the package of price and quantity combination in such a way that consumers have incentive to select the package meant for them.

So, in case there are only two types of consumers then one is, has a high willingness to pay and other consumer has a low willingness to pay then there will be possibility of having two package and one is meant for this high willingness to pay and another one is for this low willingness to pay.

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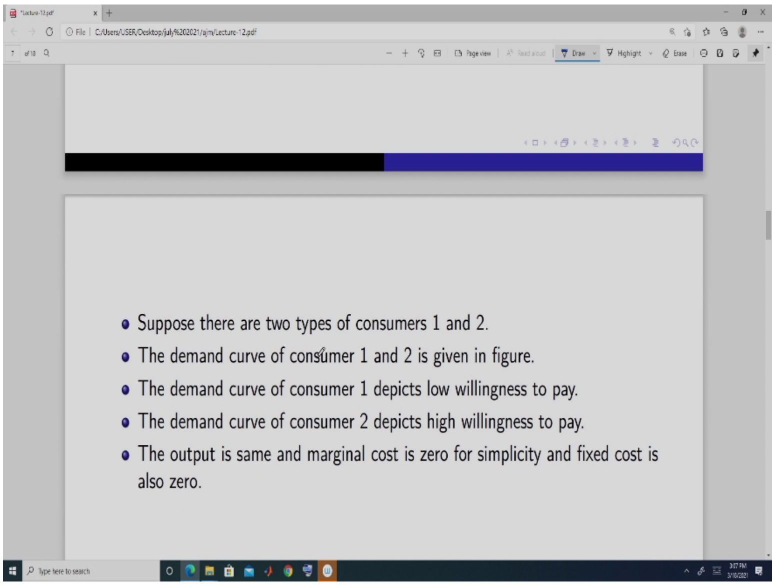

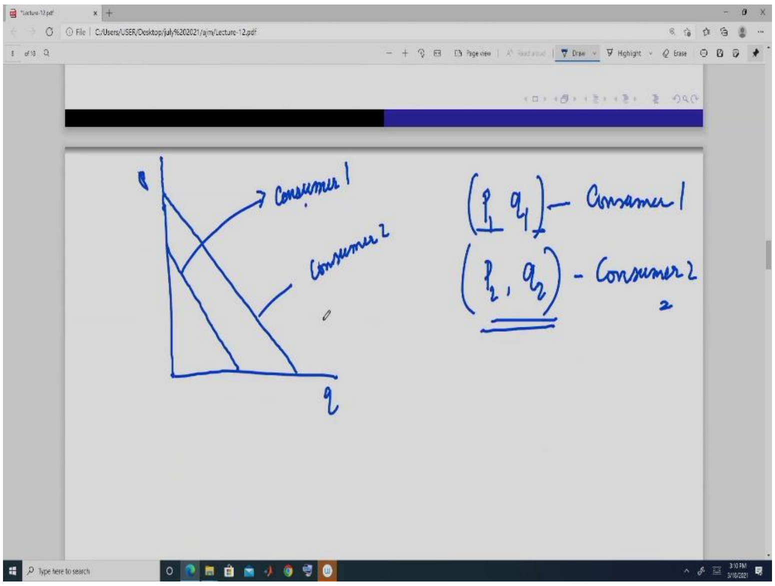

So, now we do a model on this. Suppose there are two types of consumers, consumer one and consumer two. Consumer one and two's demand curve is given here, okay. Consumer one has this, okay? This is for consumer 1 and this is suppose consumer 2, so this is the quantity and this is the price so the consumer 1 has low willingness to pay for each quantity and consumer 2 has high willingness to pay or at each price consumer 2 demands a higher quantity than consumer 1, okay.

So, monopolists know that there are two types of consumers and their demand curve is of this nature, okay? Monopolist know this demand. But monopolists further, if it wants to discriminate price perfectly, it has to know which consumer is of which type but monopolist does not have that information okay.

So, what monopolist can do, it can design one price bundle that is p1 and q1, another price bundle p2 and q2. And this (p_1, q_1) is for consumer 1 and this (p_2, q_2) is for consumer 2, okay. And when the consumers are buying, the consumer 1 will always buy this bundle and consumer 2 is always going to buy this bundle. So, this means that the bundles are designed in such a way that the consumer 1 is always going to buy the bundle meant for him or her and the consumer 2 is going to buy that bundle which is meant for him or her, okay.

Now, how to do this? Here we make a further assumption that the marginal cost is zero and the fixed cost is zero. And, So, the cost is completely zero so that is simplifying assumption because it will give us an easier way to analyze this simple case.

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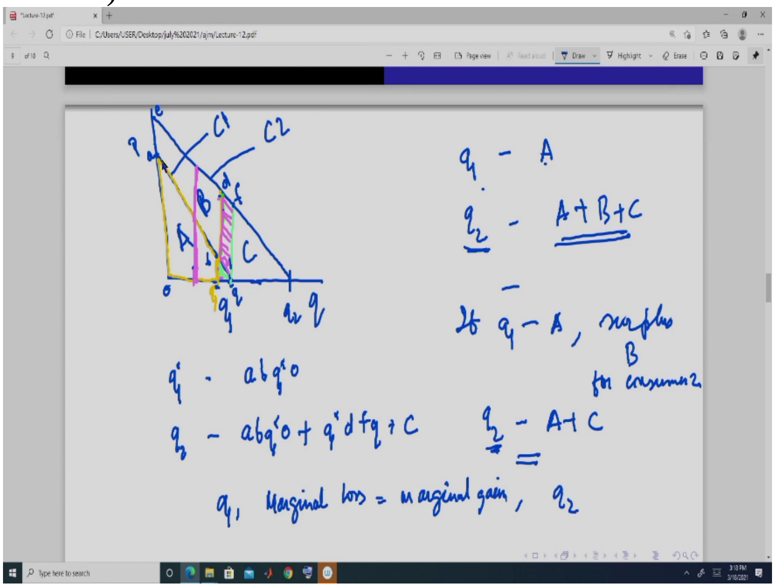

So, suppose price here and this is for consumer 1 and this is for, so this is for C1 and this is for C2, price. So, monopolists can sell this q1 unit to consumer 1 and q2 unit to consumer 2. Now suppose this whole triangle is this A, okay and this region is suppose B and this triangle is suppose C.

Now, if consumer, if the monopolist, sells this at price A, this whole area of this A, okay. And sells quantity 2 at a price A plus B plus C then this bundle which is meant for consumer type 2, they are not going to buy this bundle, they are going to buy this bundle because if they buy this bundle, they are going to get this much and they are willing to pay this whole area and they are actually paying this so they get a surplus of amount B, if they buy q1 which is sold at A then the surplus is B for consumer 2.

So, monopolist should not set this bundle instead monopolist can set q should those who are willing to buy bundle 2, this much amount they should pay A plus C. So, if the consumer is charged this amount for this bundle and the same A amount for bundle A, then those who are buying this bundle which is will be bought by consumer 2, they are going to get a surplus of amount B and they are also going to get a surplus of amount B if they buy this bundle A so they are indifferent. So, once they are indifferent, then we assume to break that indifference that consumer 2 buys this bundle okay.

So, this is one way of bundling. Now the question is, whether that is a profit maximizing bundle or not? Okay. So, here you can see that now suppose instead of this, if I make bundle for consumer 1 this is suppose q1 then what is happening? Consumer 1 is going to pay this much amount, this much amount, so what I am losing?

I am losing this small triangle, I am losing this amount, okay and what I can charge consumer of type 2, now I can charge them this additional amount, this whole amount to consumer B so suppose now I make this amount suppose this is O so then, and this point is suppose small a, o, this is q1, q1 star, and this is not O, this is suppose small b, okay this amount is suppose small c, this is d, this is b, this is q and this is suppose e, okay**,** e I have already given, this is suppose f so then for q1 which is this yellow, which is now q1 star.

Then the amount they have paid is given by the region a, b, q1 star and o, this region, okay. And q2 which is same, this amount, now the monopolist can charge this a, same amount this a, b, q1 star o, this whole region plus q1 star, d, d, f, q plus c. So, this pink area is the net gain if the monopolist follows this strategy rather than following this strategy and so the profit is higher here by this amount so the monopolist will go on.

So, like this it will go on as long as this at the margin, if it increases from here to here, this much is the loss and the gain is this much, right? At the margin. So, till at that point where marginal loss is equal is marginal gain. So, the bundle this q1 will be determined in such a way that marginal loss is equal to marginal gain, okay. And q2 is going to remain same and this whole region is going to be charged to for the second consumer and the consumer 1 will be charged only this amount, okay? So, this is second degree price discrimination, okay.

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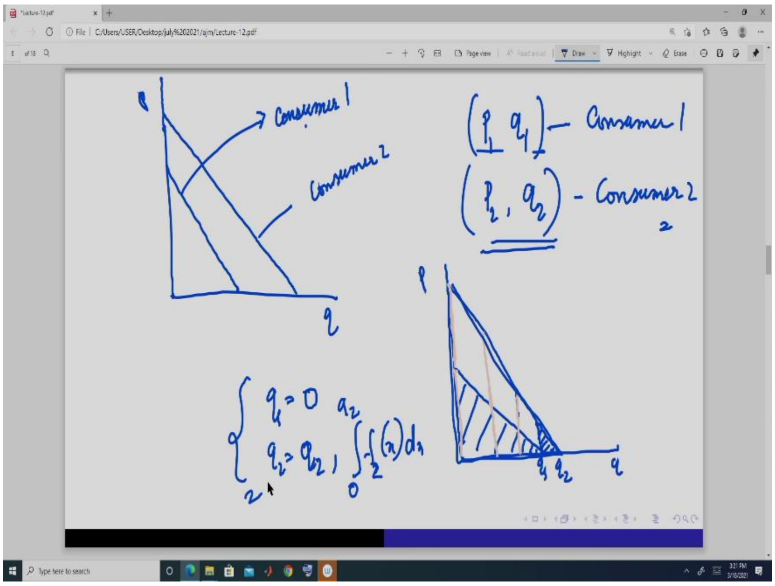

But here we may have a problem, the problem is that now think of a situation like this, let me use this portion a little bit. Suppose the demand curve is like this and demand curve is like this, okay? this is q1, this is q2, okay. q1 it can charge this amount and q2 it can charge this amount but if we use that marginal principle what is happening, if we do here like instead of this, if we shift this q1 here so this much is the loss and this much is the net additional unit, so we are going to reduce this.

If you do this, here this is the, at the margin this is the loss and this is the gain, right? So, we are gaining more so we will go on doing like this and then here you will see the gain is more than this, it is possible to do. So, in this case, what will happen or in this case, what we will get that q1 is always equal to 0 and it will have a package of this and it will charge this whole triangle.

So, q2 is A, q for the consumer 2 it is going to be this q2 and the price is going to be this whole, this is the demand curve of consumer 2, this whole amount. So, here consumers which are of type 1 they are not going to buy any bundle. So, this can be the optimal bundle in this situation, okay.

So, this is one case where the consumers the monopolist is only selling the product to those consumers whose willingness to buy is very high and it is not selling the product to those consumers whose willingness to pay is low, okay. So, this is, this can be an outcome when the monopolist cannot distinguish between the types of consumers and it wants to discriminate the consumers because then it can earn more profit. So, then in that case it is only going to sell to the consumers who are willing to pay more, okay.

Next type of market price discrimination is third degree price discrimination. So, what we have done? In first degree we know that the monopolies can distinguish between the type of consumers and it can charge each quantity at a different price, okay. So, a same consumer if it is buying different quantities it will be charged different prices or if the consumers nature are different then for the same quantity, they may be charged different amount, okay? Because their willingness to pay is different.

But that is possible and what we have done? If the consumers are of same nature suppose they are similar, we have assumed that and then we have devised a strategy where a fixed bundle is fixed amount of quantity is sold in the market and that is where price is equal to marginal cost, okay in the first-degree price discrimination.

In the second degree we have assumed that the monopolist cannot distinguish between the type of consumers and monopolist will package the goods into a combination of price and quantity in such a way that it will be meant for a specific type of consumer. So, for consumer type 1, it will have a price and quantity and for consumer type 2 it will have a different price and quantity, okay.

And it will be such that the profit is maximum, and when the profit is maximum for some specific type of demand curve, we see that when there is the marginal loss, if you charge slightly less to the low paying, low willingness to pay consumer and the marginal gain, that you get by charging a little bit higher to the high willingness to pay consumers, if they are equal then you are at an optimal point, okay.

Now, this condition can also lead to an outcome were depending on the nature of the demand curve where the monopolist is not selling any quantity to the low, to the consumers whose willingness to pay is low and it is only selling to the consumers whose willingness to pay is higher amount or who is willing to pay higher amount, okay. And we have given one example, diagrammatic example.

Next is the third degree, in the third degree what it assumes that the consumer knows that there are two types of market and these two types of market is because of there are two types of consumers but for each it charges same price per unit of quantity for whatever amount you are willing to buy.

So, it is something like this, that if you go to a movie hall you will get some student discount, if you go to suppose some, if you use some services, some public utilities or services, you get senior citizenship discounts so those are. You know that this is a market for the senior citizen and there are another type of consumer who are not senior citizen so you can charge different prices to them, so you can distinguish these two types of a.

Or say for example the students who has I card and if you only show the I card you get the discount so you can distinguish between the consumers and you know their demand curves are different, okay. So, depending on this type of consumers you charge the same price for whatever amount they want to buy.

So, here you are not discriminating in terms of quantity, you are discriminating only in terms of types of consumer, in second degree price discrimination you cannot distinguish between the consumers so you bundle or you package the goods in a combination of price and quantity in such a way that you charge different prices for different quantities but here you charge same prices but you charge same prices that price may differ according to the type of consumers because you can distinguish between them so that is why this is different from the first degree because in first degree you do the both.

So, if a consumer is, you can charge according to, you can charge different prices to different consumers and also you can charge different prices for each or different quantities to each consumer. But here you charge same prices for any amount, the same price per unit for any amount they are buying but you can charge, you can differentiate in terms of the types of consumers, okay so that is.

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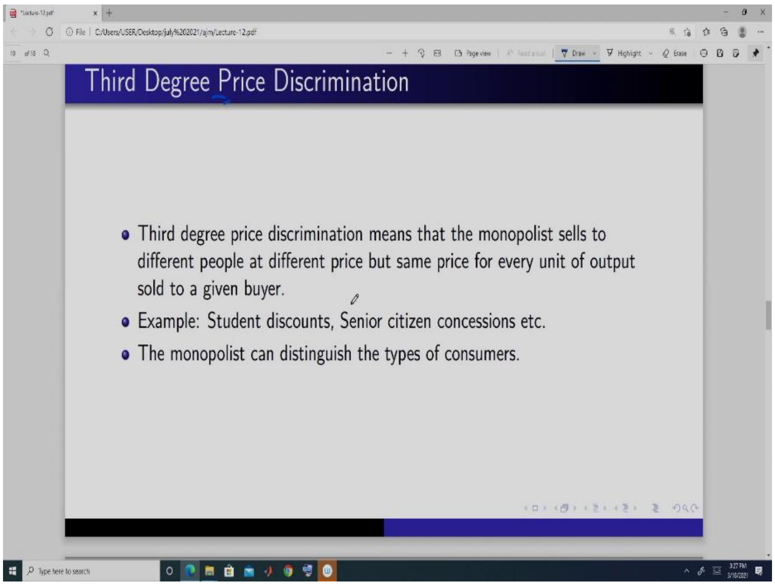

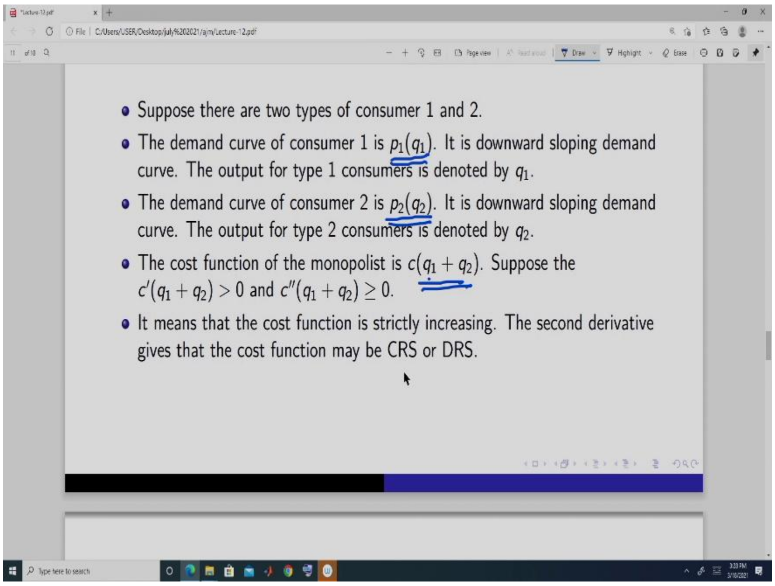

So, the informational requirement is much less compared to the first-degree price discrimination, okay. So, here we will again, suppose there are two types of consumers, consumer 1 and consumer 2. Consumer 1's demand curve is this- $p_1(q_1)$ and consumer 2's demand curve is this-$p_2(q_2)$. And the quantity you are selling to consumer 1, so you take a monopoly it is given by q1 and consumer quantity you are selling to consumer 2, is given by q2, okay.

So, these demand curves are downward sloping and they are not same, they are different , okay and you have to assume that the cost function is for this nature-$c(q_1 + q_2)$, we have assumed a general cost function so this is strictly increasing because marginal cost is always positive and marginal cost can either be constant or it can be increasing so the possibility of

CRS or constant returns to scale or DRS that is decreasing returns to scale, we consider only these two situations.

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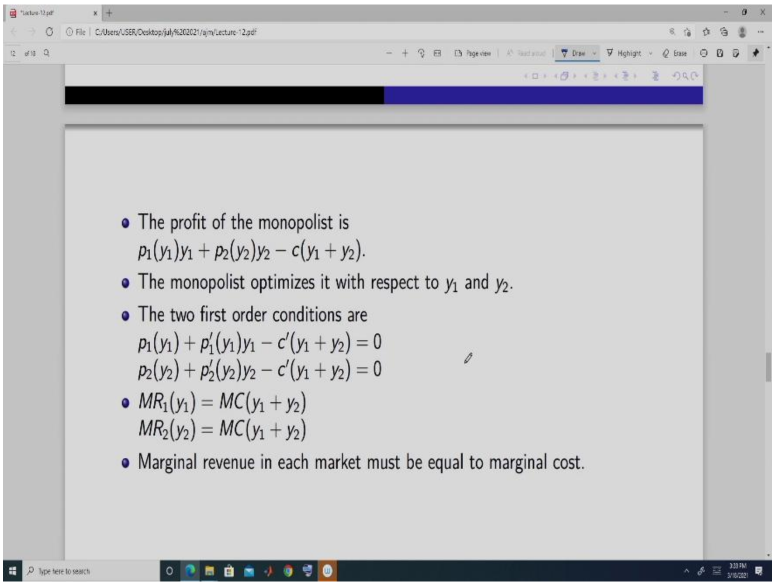

So, aggregate profit of the monopolist is given by this amount- $p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2)$ because, this amount- $p_1(y_1)y_1$ is the revenue from type 1 consumers or market 1, that is you are selling y1 unit of output, okay I have messed up here.

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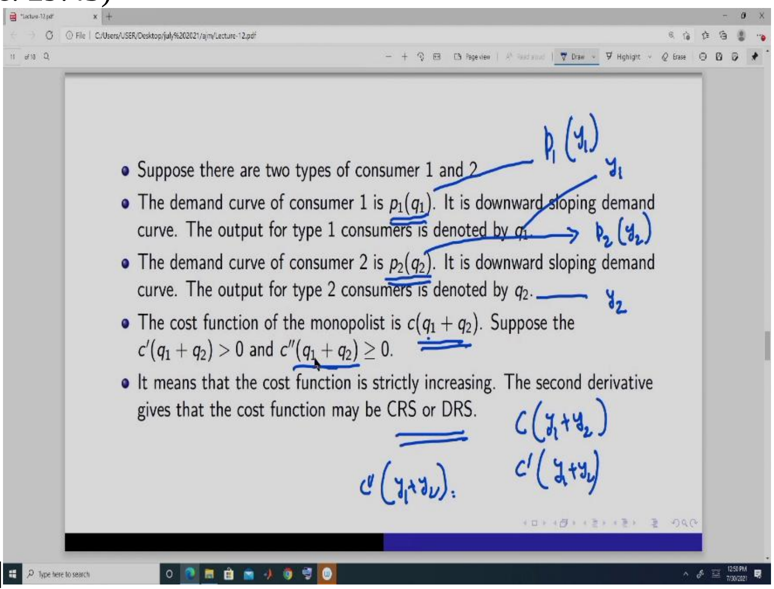

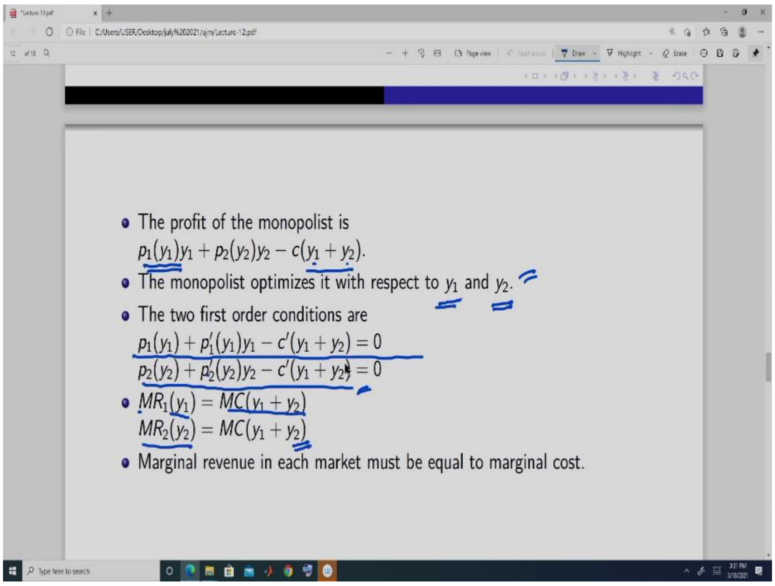

So, let us take this thing this to be instead of q1 let us take y1, instead of q2, let us take y2, okay? Output here is not q1, it is y1, output is y2, okay. And this here cost function is c y1 plus y2, i.e $c(y_1 + y_2)$ and marginal cost is this- $c'(y_1 + y_2)$ and the derivative of the marginal cost is this- $c''(y_1 + y_2)$, okay. I just messed up the symbols.

So, CRS or DRS will give satisfy this- $c''(y_1 + y_2) \geq 0$, but this condition is also satisfied when one of the factor is fixed and the other is variable and the law of diminishing marginal product is operating. In that case we will get this, that is the marginal cost is upward sloping and so we will get the second derivative of cost function to be a positive number.

So, you take this-$p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2)$, so this portion-$p_1(y_1)y_1$ is the revenue from selling to type 1 consumers, this is the revenue from selling to the type 2 consumers-$p_2(y_2)y_2$and this is the total cost of producing y1 and y2- $c(y_1 + y_2)$, this is the total output it is producing and it is selling the goods are identical in nature so the cost is same, okay.

Now monopolist is going to maximize this profit with respect to y1 and with respect to y2 because these two are for two types of market, for consumer type 1 and for consumer type 2. And so, since all of these are differentiable so we take the derivative and the first order condition when we optimize with respect to y1 is this- $p_1(y_1) + p_1'(y_1)y_1 - c'(y_1 + y_2)$=0.

So, this is marginal revenue- $p_1(y_1) + p_1'(y_1)y_1$ and this is the marginal cost-$c'(y_1 + y_2)$. And this- $p_2(y_2) + p_2'(y_2)y_2 - c'(y_1 + y_2)$=0 is the first order condition from when we optimize with respect to y2 and since the demand curve is downward sloping and, it is not convex, strictly convex and this is always positive or this is second derivative is positive one, equal to 0 so we will always have an optimal point which maximize x with respect to y1 and y2,okay.

So, this first order condition is marginal revenue from consumer or type or market 1 should be equal to marginal cost and marginal revenue in type 2 market should always equal to marginal cost, so from this first order condition, okay. So, that means marginal revenue should always be equal in this both the market.

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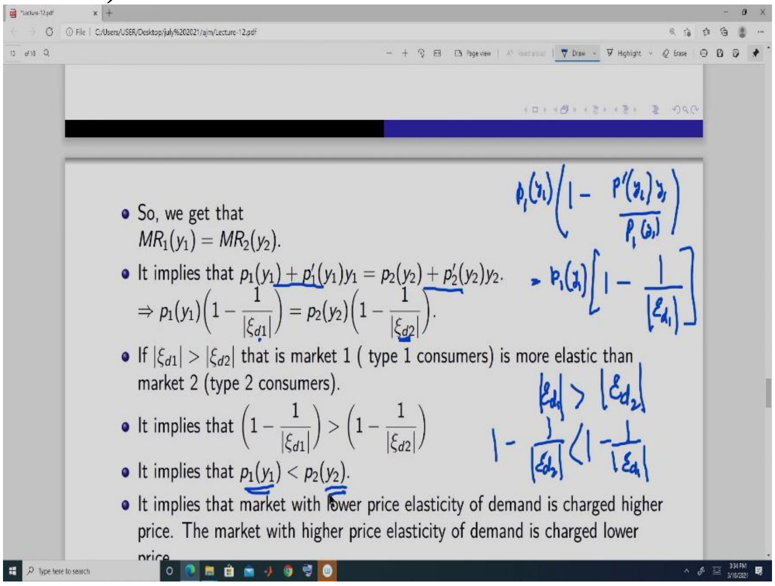

So, from there we get this marginal revenue- $MR_1(y_1) - MR(y_2)$, these two should always be equal, if that is the case then it implies that this portion which is marginal revenue in market 1 should be equal to marginal revenue in market 2, i.e $p_1(y_1) + p_1'(y_1)y_1 = p_2(y_2) + p_2'(y_2)y_2$. So, we get this, this should be equal to this.

Now here we know, if we take this common, we can write it this- $p_1(y_1)(1 - \frac{p_1'(y_1)y_1}{p_1(y_1)}$ and this we have shown, can be written as elasticity of the demand curve of market 1- $p_1(y_1)[1 - 1/|\xi_{d_1}|$] and this is also same, is the elasticity or price elasticity of the market 2- $p_2(y_2)[1 - 1/|\xi_{d_2}|$] and this is from market 1. Now these two things should be equal and suppose the elasticity in market 1 is more than elasticity into market 2, then what do we get?

We get that, if we get this- $|\xi_{d_1}|$, we can show that this why? Because this is the case if we take the negative then here, okay, this is the demand price elasticity in market 1 and price elasticity in market 2. Price elasticity of market 1 is more than price elasticity of market 2, i.e

$\left|\xi_{d_1}\right| > \left|\xi_{d_2}\right|$, and from there we know that one this, then if we take the negative sign this, will be, we can write this so we get here.

So, this means, since they are equal so price in market one should be less than price in market two- $p_1(y_1) = p_2(y_2)$. So, this means what? That if price elasticity of demand is less then you charge a higher price and if the price elasticity is high, price elasticity of demand is high you charge a lower price.

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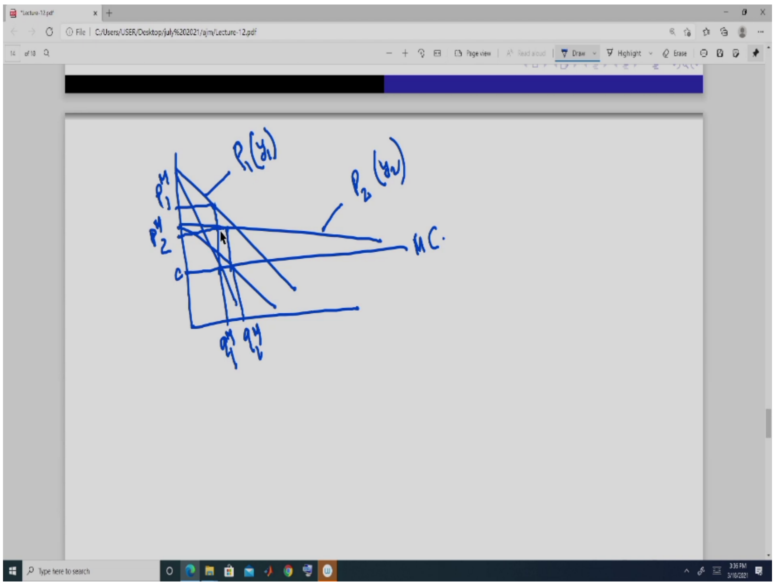

So, diagrammatically what we are doing is suppose this is, okay, so this is the marginal revenue, this is suppose p2 and it is, this is the marginal revenue, okay? And suppose this is the marginal cost, say, okay? so for market one this is the, and for market two this is the revenue so price here is this, price here is this, this is the monopoly price in market 2, this is the monopoly in market 1. So, this is the price, this demand curve you know, is more flat so it is more elastic. This demand curve is more steeper so it is less in elastic.

So, you are charging or the monopolist is charging a higher price in inelastic demand curve and it is charging a lower price in elastic demand curve, okay, this is happening. So, this is mainly the third-degree price discrimination, okay.

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$$P_1(y_1) = P_1 = 100 - y_1$$

$$P_2(y_2) = P_2 = 50 - y_2$$

$$c(y_1+y_2) = c(y_1+y_2), \quad c>0, \quad c < \frac{50}{2}$$

$$\pi = (100-y_1)y_1 + (50-y_2)y_2 - c(y_1+y_2) - F$$

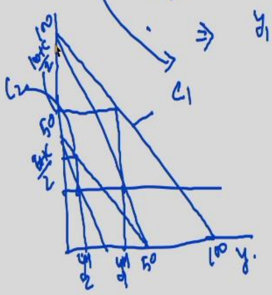
$$\frac{\partial \pi}{\partial y_1} = 100 - 2y_1 - c = 0$$

$$\frac{\partial \pi}{\partial y_2} = 50 - 2y_2 - c = 0 \Rightarrow y_2 = \frac{50-c}{2}$$

$$\Rightarrow y_1 = \frac{100-c}{2}$$

$$\frac{\partial \pi}{\partial y_2} = 50 - 2y_2 - c = 0 \Rightarrow y_2 = \frac{50-c}{2}$$

$$\Rightarrow y_1 = \frac{100-c}{2}$$



$$\Rightarrow P_1 = 100 - \left(\frac{100-c}{2}\right) = \frac{100+c}{2}$$

$$\Rightarrow P_2 = 50 - \left(\frac{50-c}{2}\right) = \frac{50+c}{2}$$

Now, we will solve one problem or one example. Suppose demand curve which is this or you can say price inverse demand curve is this, let us take some examples, this- $P_1(y_1) = P_1 = 100 - y_1$ okay, this is type 2- $P_2(y_2) = P_2 = 50 - y_2$ and this is type 1. Just the opposite of what we have done earlier, okay.

And the marginal cost is equal to suppose, C, sorry not marginal, variable cost is this- $C(y_1 + y_2) = c(y_1 + y_2)$ where C takes a positive number and C is less than suppose 50, okay. So, the profit of the monopolist is from market one that is type one consumers, total revenue.

Total revenue from type 2 consumers, here the willingness to pay for the type 2 is less than the willingness to pay of type 1, okay. So, this is a constant and suppose some fixed cost, okay? So, here the monopolist is going to do what? Y1, so this is first order condition, it should be equal to 0, i.e $\frac{d\pi}{dy_1} = 100 - 2y_1 - c = 0$

Second, this first order condition, the second first order condition-$\frac{d\pi}{dy_2} = 50 - 2y_2 - c = 0$, from this we will get that y1 is equal to 100 minus C divided by 2- $y_1 = \frac{100-c}{2}$, and this will give y2 is equal to 50 minus C divided by 2- $y_2 = \frac{50-c}{2}$, then what we are going to do, so monopolist price is, so it is going to be equal to 100 plus C- $P_1 = 100 - \frac{100-c}{2} = \frac{100+c}{2}$

And this in the market 2 and it is going to be this- $P_2 = 50 - \frac{50-c}{2} = 50 + c$. So, here, if you try to do it diagrammatically so this is the output so this is suppose 100 and this is 100, this is 50, this is 50. And So, these are the two-demand curves, this is for consumer type 1, this is for consumer type 2.

Marginal cost is something like this, marginal revenue here, marginal revenue here, so this is the monopoly price in, which is 50 plus C divided by and this is in market 2, this is in market 1 and the price is this much, okay. And this price is 100 plus C divided by 2, okay. So, this is the third-degree price discrimination, okay.

So, this is actually the end of this module, module 4 where we have done the monopoly thing and in the monopoly first we have, what we have done? We have defined the monopoly market and then we have derived the optimal output and the monopoly price. then we have looked at monopoly behavior that is since there is only one firm in the monopolies and all the buyers have to buy from that firm only, so it can vary according to different types of buyers and also the amount of information it has.

So, it can involve in or engaged in price discrimination so we have done three types of price discrimination. First degree price discrimination, where the monopolies can charge different prices for different units of output and it charges also it can vary the price according to the types of consumers.

In second degree price discrimination, what we have seen is that the monopolist cannot distinguish between the consumers, so if a consumer, so it can only bundle the price and quantity combination in such or it packages the price and quantity in such a way that it is specific to its type of consumers, for consumer type 1 it will have a specific bundle of good and price or specific combination of price and quantity.

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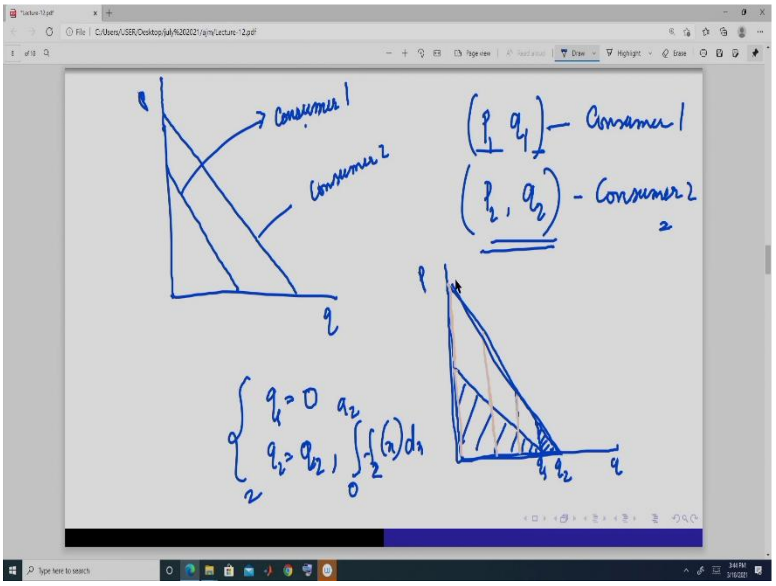

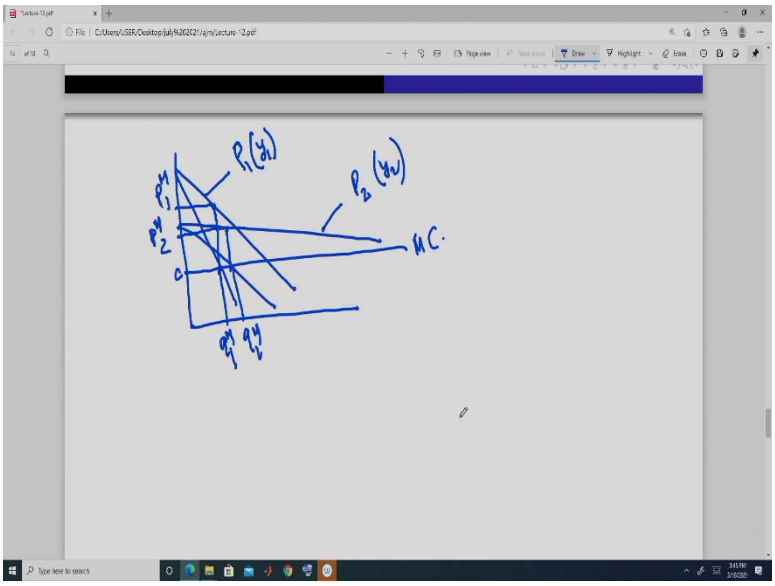

For consumer type 2, it will have a specific combination of price and quantity, and if the demand curves are of certain type as we have seen in this situation then it may happen in second degree price discrimination the monopolist is not selling any output to the consumer whose willingness to pay is low, for this consumer whose demand curve is this. It is only selling to the that consumer whose willingness to pay, willingness to pay is high.

And then we have done the third-degree price discrimination and in the third-degree price discrimination it is assumed that the monopolist same price for whatever amount you are going to buy so that is per unit price remains same but it only distinguishes in nature among the types of consumers or among the types of market, okay.

So, we have seen that a market whose price elasticity of demand is high in that market the less price will be charged and if the price elasticity of demand is low then a higher price will be charged, okay. So, this is what we have completed, covered in monopoly and the next topic that we are going to do is the game theory.

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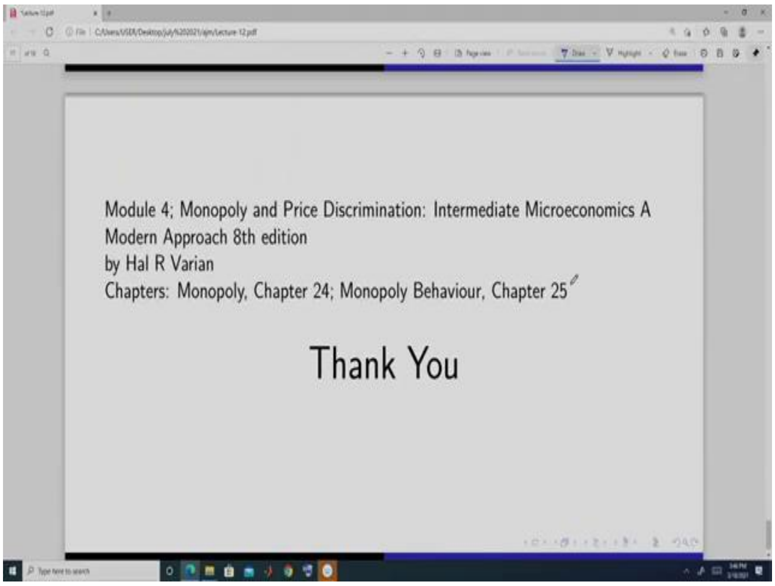

So, for this portion what you can do is you can read from this chapter 24 and chapter 25 of this book, Intermediate Microeconomics – A modern Approach 8th edition by Hal Varian. So, these two chapters, this is the monopoly and the price discrimination is from this chapter and the monopoly is from this chapter, okay. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture 16
Tutorial on Monopoly

We will discuss few problems on monopoly. So, let us look at this first problem.

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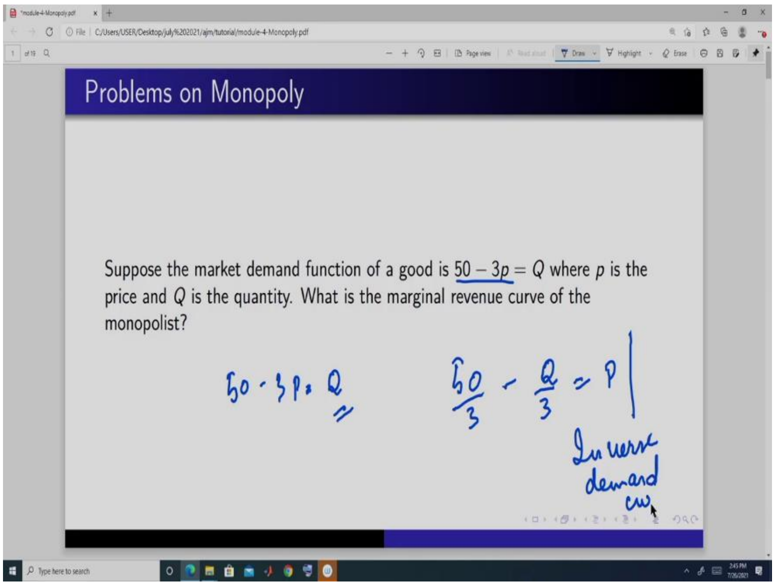

Suppose the market demand function of a good is this- 50–3p=Q where p is the price and this Q is the quantity demanded, okay. What is the marginal revenue curve of the monopolist? Now, from this, if this is the demand curve, then this is the inverse demand curve- $\frac{50}{3} - \frac{Q}{3} = \text{P}$, okay.

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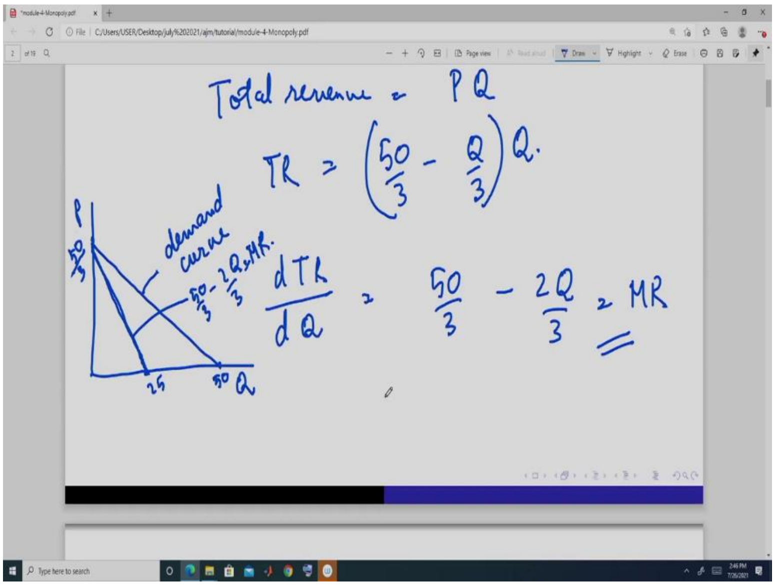

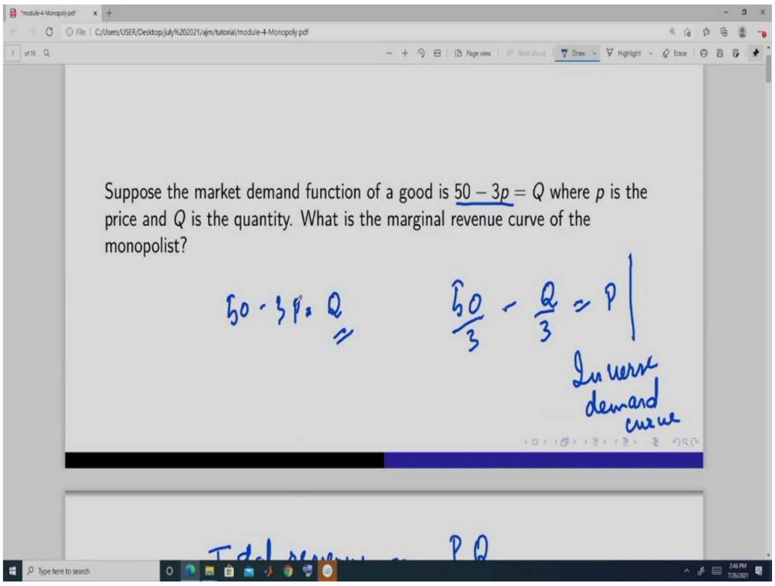

Now, total revenue is equal to price into quantity. So, here you apply the, this is the price- $\frac{50}{3} - \frac{Q}{3}$ and quantity is this- Q because total revenue is always a function of the quantity. We get this, this is the total revenue- TR= $(\frac{50}{3} - \frac{Q}{3})Q$ and since it is differentiable take the derivative of this with respect output that is q we get, this, this is MR=>$\frac{50}{3} - \frac{2Q}{3}$ that is the marginal revenue and if we want to plot this we will get something like this if it is price and this is quantity.

This point is 50, it is this, it is this divided by 3 this point is 50. This is the demand curve and this is the marginal revenue curve this curve and this point is going to be 25 from here, okay, this is how we get the marginal revenue curve of a firm our monopolist.

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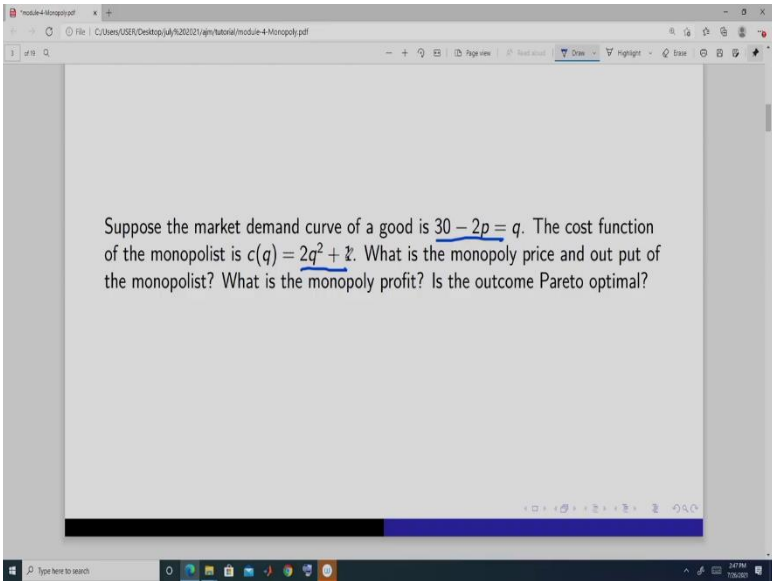

Next suppose next problem suppose the market demand curve of a good is this- 30–2p=q, cost function of the monopolist is this- $c(q) = 2q^2 + 1$. So, marginal cost is strictly increasing here, this, what is the monopoly price and output of the monopolist. So, this and we have to find the monopoly profit also.

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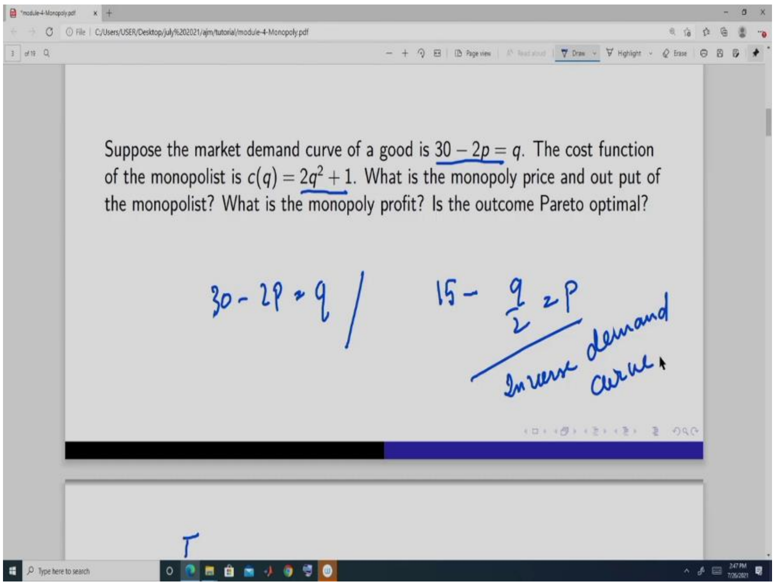

So, the profit function so, from here let us first derive the. So, if this is the demand curve-30–2p=q, then the inverse demand curve is, is this- $15 - \frac{q}{2} = P$, this is the inverse demand curve, okay.

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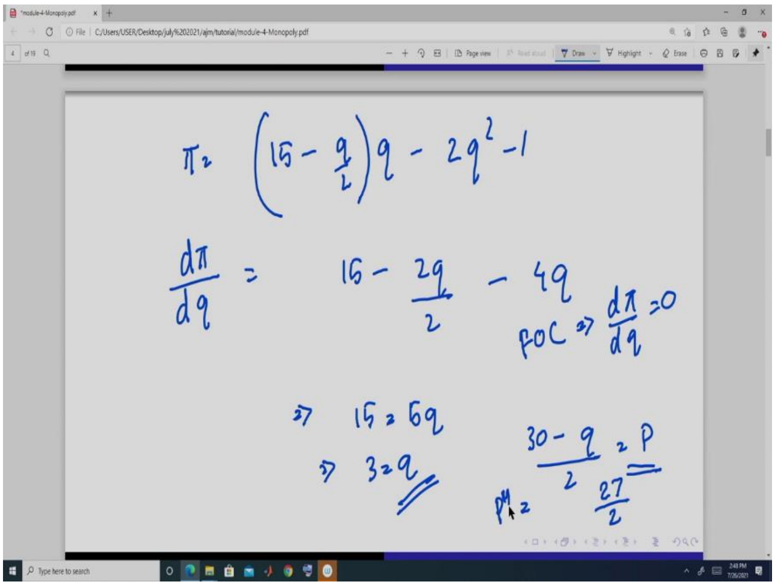

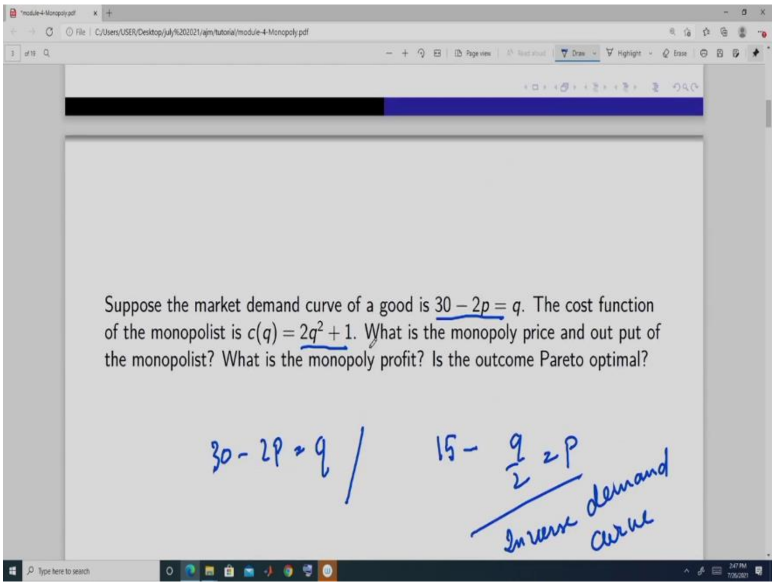

Now, so, the profit function is 15, cost function is this 2 output is q and this is the fixed costs. So, this is the profit function- $\pi = \left(15 - \frac{q}{2}\right) q - 2q^2 - 1$ it is a differentiable function. So, when the monopolist will optimize this or maximize this with respect to output and we will get this first order condition will give that this equal to 0, i. e $\frac{d\pi}{dq} = 15 - \frac{2q}{2} - 4q = 0$. So, we get this is the monopoly output= 3=q and what is going to be the monopoly price, monopoly price is going to be this- $p = \frac{30-q}{2}$. So, plug in here it is 27 by 2. So, this is the monopoly price- $p = \frac{27}{2}$ and this is the monopoly quantity- 3=q.

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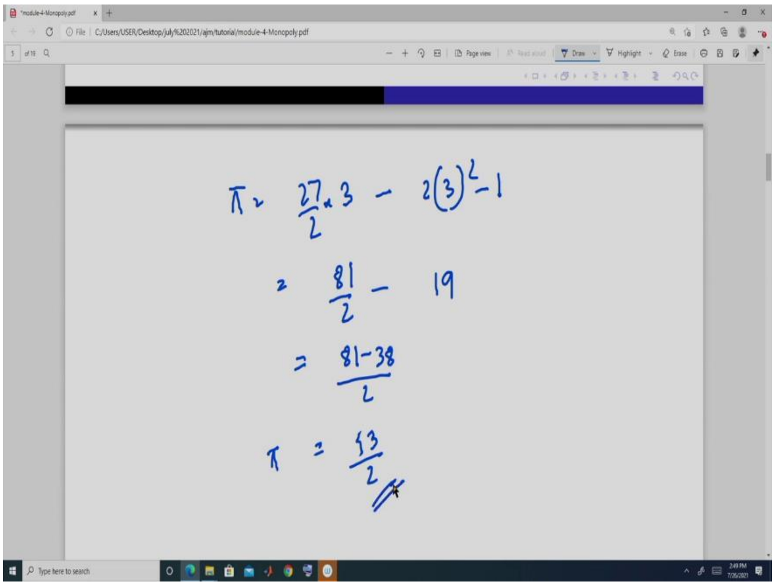

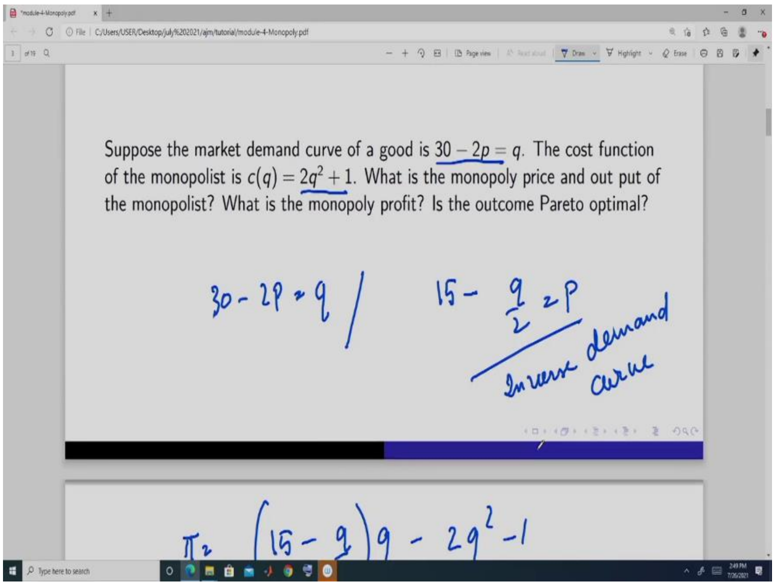

Now, what is going to be the monopoly profit? Monopoly profit is price is this, output is this and fixed cost is 1, so it is going to be this- $\pi = \frac{27}{2} * 3 - 2(3)^2 - 1$. So, this is 19. So, it is, so, it is going to be what? 43 by 2, this is going to be the monopolist profit- $\pi = \frac{43}{2}$.

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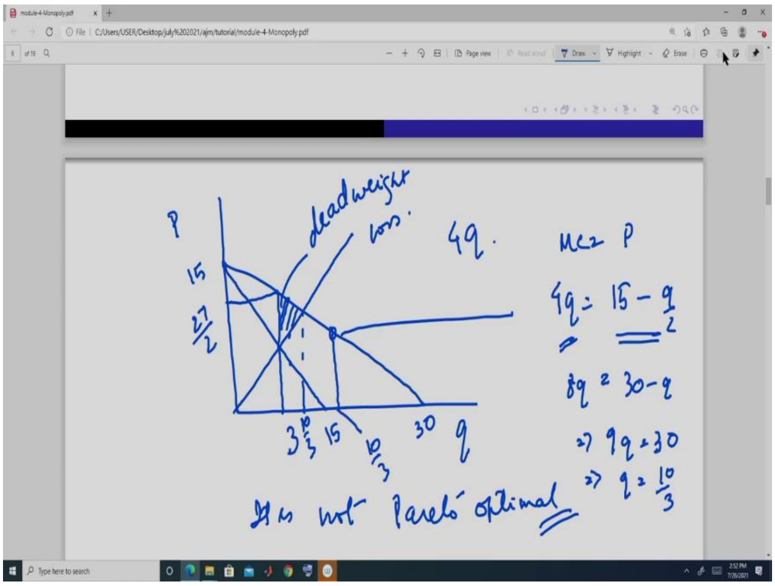

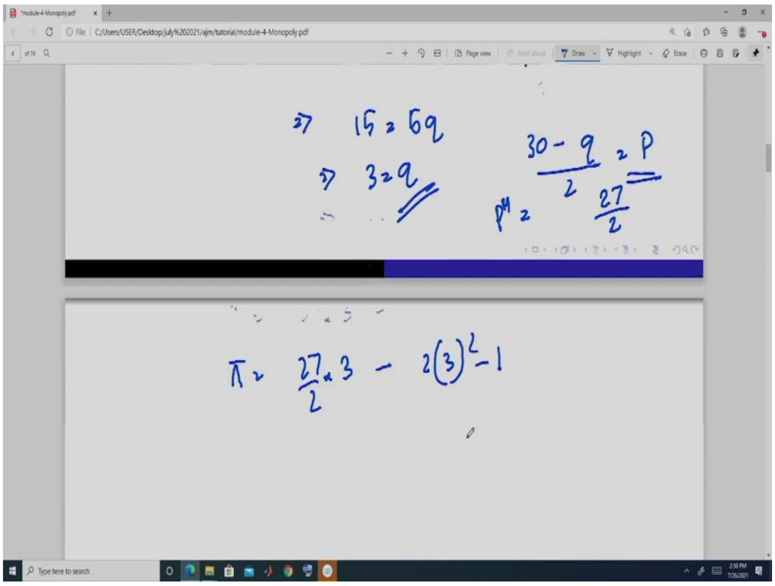

Now, whether it is a Pareto optimal point or not how do we decide that? See, this point is if you look at this is 15, this is 30, this is 15, marginal cost this is 4q, it is this this is 3, this point is 27 by 2, this point is what this is 4q equal to 15, right? this where marginal cost intersects the demand curve. So, it is MC is equal to price at this. So, this if you solve this you will get this a and this is what sorry, it is this- $4q = 15 - \frac{q}{2} => 8q = 30 - q => 9q = 30 => q = \frac{10}{3}$.

So, this point is 10 by 3. So, this diagram will be slightly like this, suppose this is the point 3 and this point is 27 by 2 and this is 10 by 3. So, this reason is the deadweight loss. So, that is why it is, it is not Pareto optimal, we get this, okay. Now, let us solve another problem.

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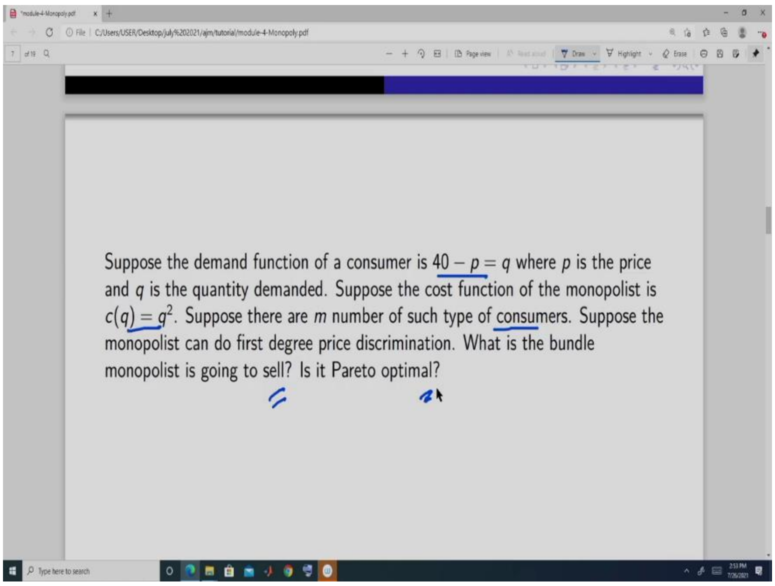

Suppose the demand function of a consumer is this- 40-p=q, it is a downward sloping, where p is the price and q is the quantity demanded and suppose the cost function of the monopolist is this- $c(q) = q^2$, okay. And suppose there are m number of such type of consumers and suppose the monopolists can-do first-degree price discrimination. First degree price discrimination means that it can charge different prices for each unit of good and it can charge different prices to each person, okay.

Now, here all the persons are similar. So, it can charge different unit price to different amount of unit that it is selling. So, in this case, we have seen that the monopolist is actually doing a kind of bundling. What is the bundle monopolist is going to sell and is it a Pareto optimal? okay?

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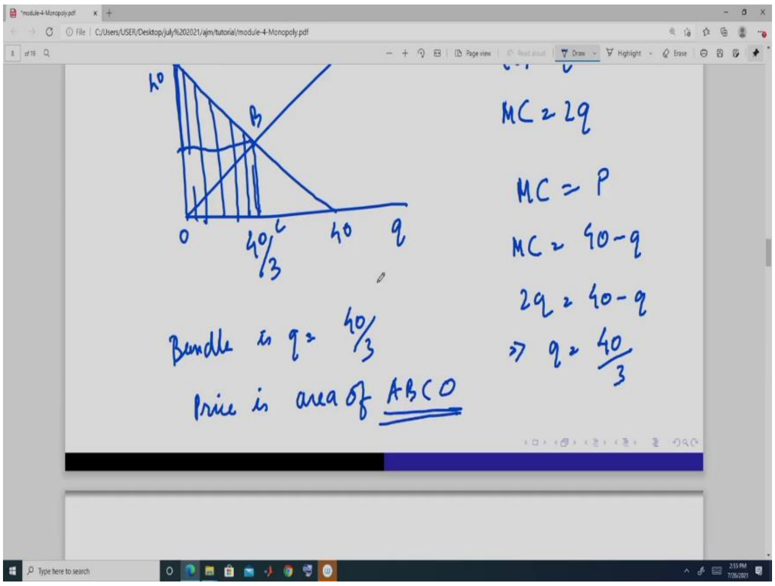

So, this is we will see, this is suppose quantity, price this point is 40. This point is 40. This is the demand curve and the marginal cost is because cost function is q square So, marginal cost is 2q. So, this is MC, it is this, this point is where MC is equal to price. So, it is equal to 40 minus q, MC is 2q, so this is equal to. So, this point is 40 divided by 3, okay. Now, monopolist we know first degree will charge this whole amount.

So, monopolist will do it is the bundle is like this you get so, the bundle is q is equal to 40 by 3 and you get a price which is given by this whole area this is suppose A, B, C and O. So, price is area of A, B, C, O this, right?

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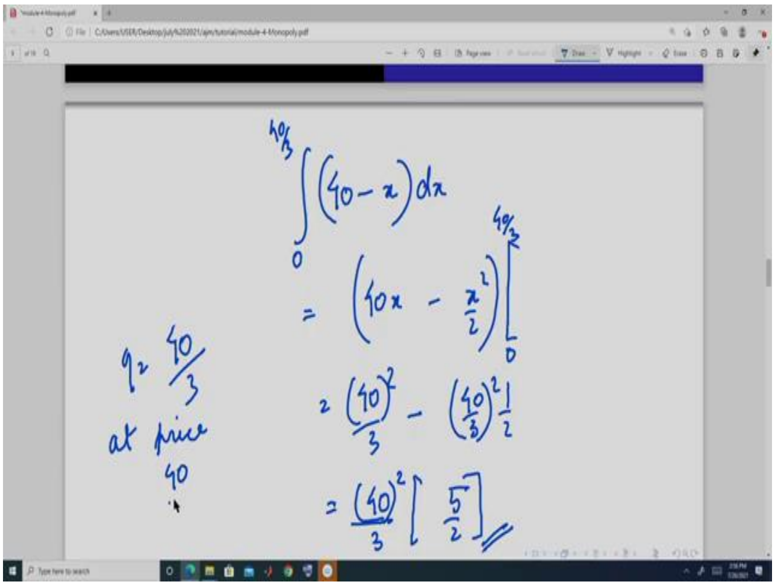

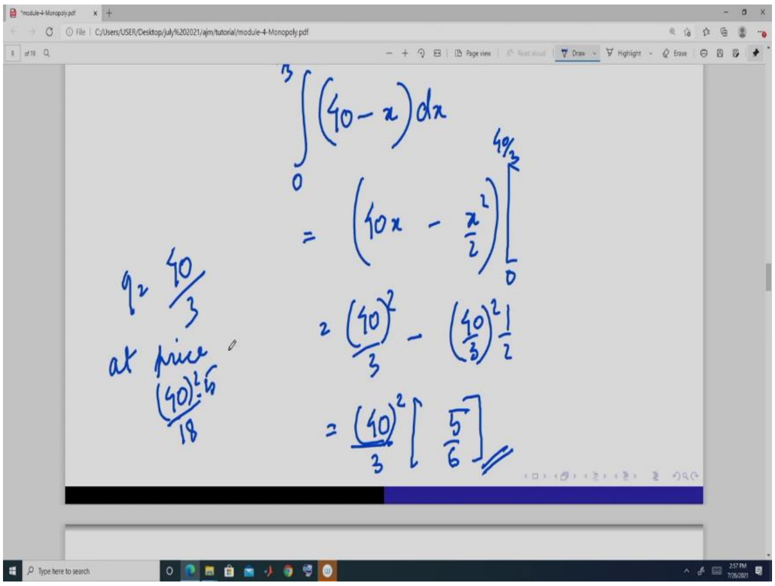

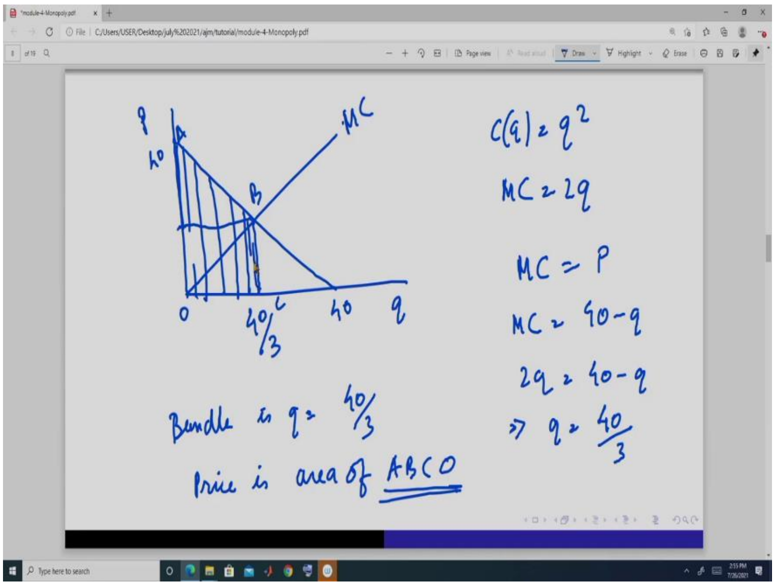

So, this area you can find out it simply 0, 40 by 3 area under the demand curve and demand curve function you can write integrating over x here x is the quantity- $\int_0^{\frac{40}{3}}(40 - x)dx$. So, it will give me this whole region and this is what? this is going to be so, this is you will get this and this amount is going to be this much amount. So, the price is bundle is that quantity 40 divided by 3 at price 40 square divided by sorry it is going to be 6, you can say it is divided by 18 into 5 this this is the price of this good, i.e $P = \frac{(40)^2}{18}.5$.

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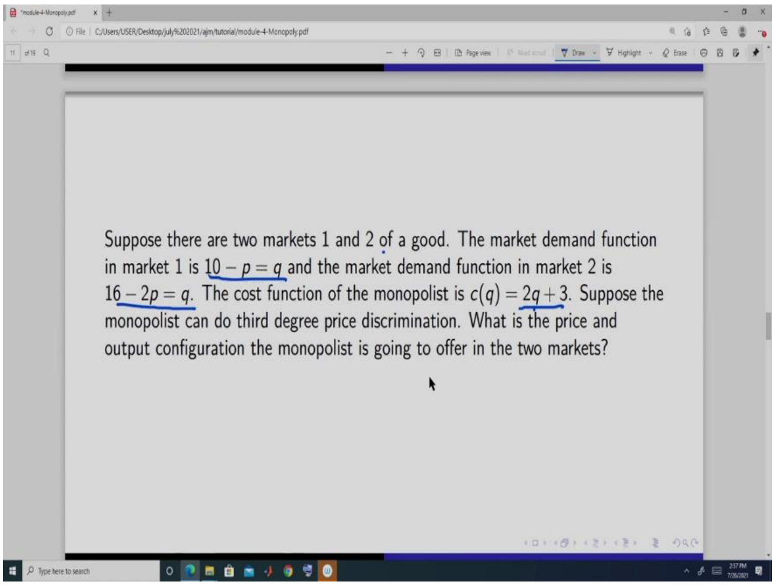

Now, let us do another problem related to price discrimination and suppose this is this problem is related to third degree price discrimination. So, suppose there are two markets, market 1 and market 2 and demand function of market 1 is this- $10 - p = q$ and the demand function of market 2 is this- $16 - 2p = q$, cost function of the monopolist is this- $c(q) = 2q + 3$, okay. So, it is a constant marginal cost and suppose the monopolists can-do third-degree price discrimination it means what?

That the buyer of market 1 cannot go to buyer of market 2 and buyer of market 2 cannot go to buyer of, means buyer market 1 consumers which are there in market 1 cannot go to market 2 and the consumers who are there in market 2 cannot move to market 1, okay. This is like because of specific characteristics of the buyers like old person and the young person or the students or non-students or like geographically different location and they are very far apart, okay.

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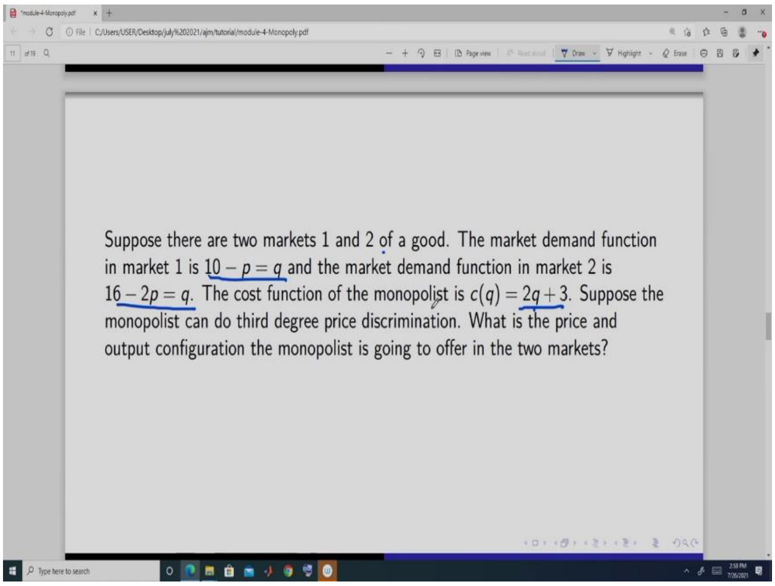

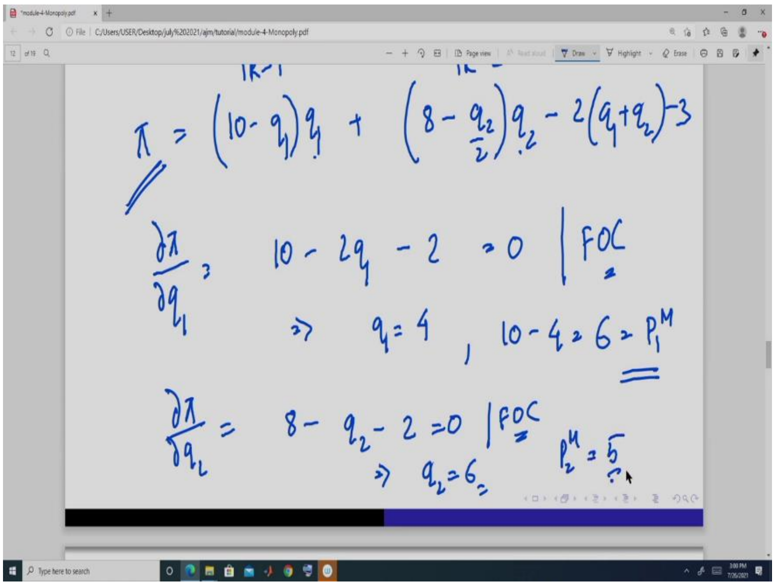

Now, we will get this here. Market 1's demand function is this- $10 - p = q$ market 2's demand function is this- $16 - 2p = q$. So, market 1's output is q1, this is the total revenue TR from market 1- $(10 - q_1)q_1$ plus this is, this is total revenue from market 2- $\left(8 - \frac{q_2}{2}\right)q_2$, minus 2q. So, this is the total mono profit earned by the monopolist- $\pi = (10 - q_1)q_1 + \left(8 - \frac{q_2}{2}\right)q_2 - 2(q_1 + q_2) - 3$. So, monopolist will choose q1 and q2 such that this profit is maximum. So, it will be this.

So, we will get this is first order condition- $\frac{d\pi}{dq_1} = 10 - 2q_1 - 2 = 0$ and then this will give me q1 is equal to 4. So, the monopoly price is going to be this in market 1- $10 - 4 = 6 = P_1^M$.

Then it is again this is the first order condition will give me this- $\frac{d\pi}{dq_2} = 8 - q_2 = 0$. So, q2 is equal to 6. So, plug in q2 here. So, p 2 M is actually 5, i. e $P_2^M = 5$.

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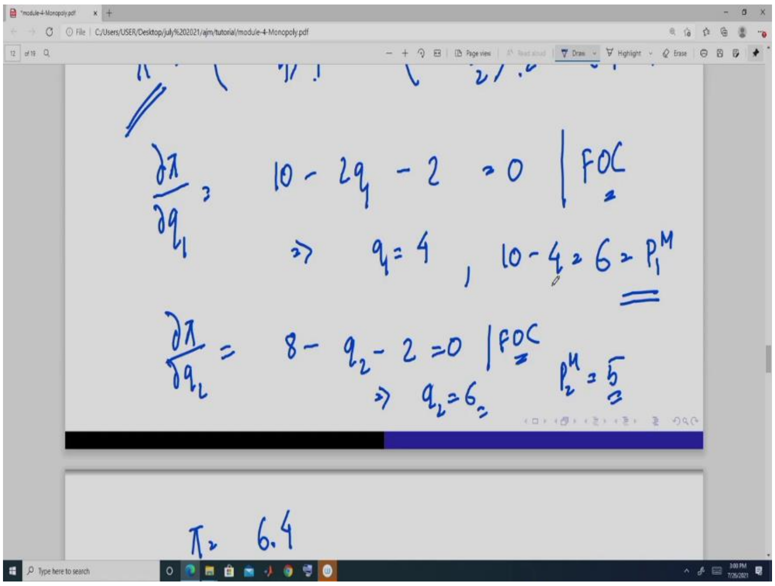

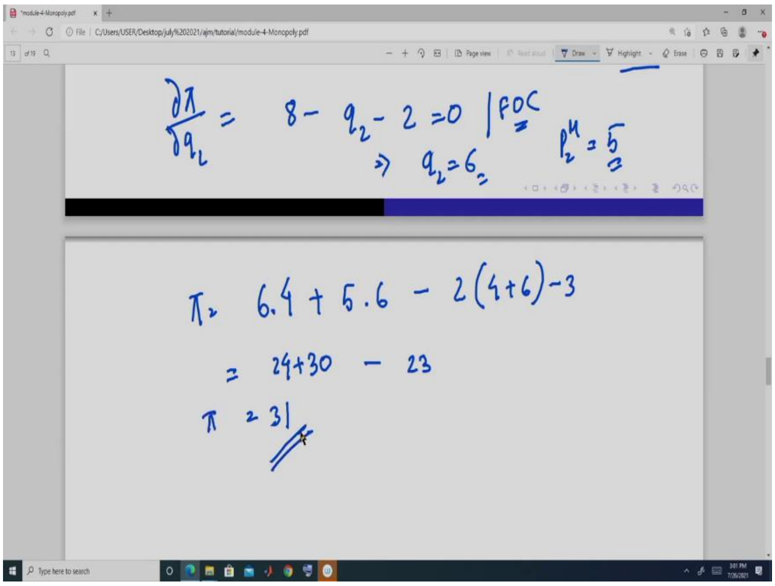

So, what is the monopoly profit here now? Monopoly profit is price is 6 here and output is 4, right? 6, 4 plus price is 5 output is 6 minus 2, 4 plus 6. So, this is 24, 30, 24 plus 30 minus this is 23 So, this is 31. This is the profit if it does third degree price discrimination- $\pi = 6.4 + 5.6 - 2(4 + 6) - 3 => 31$, okay.

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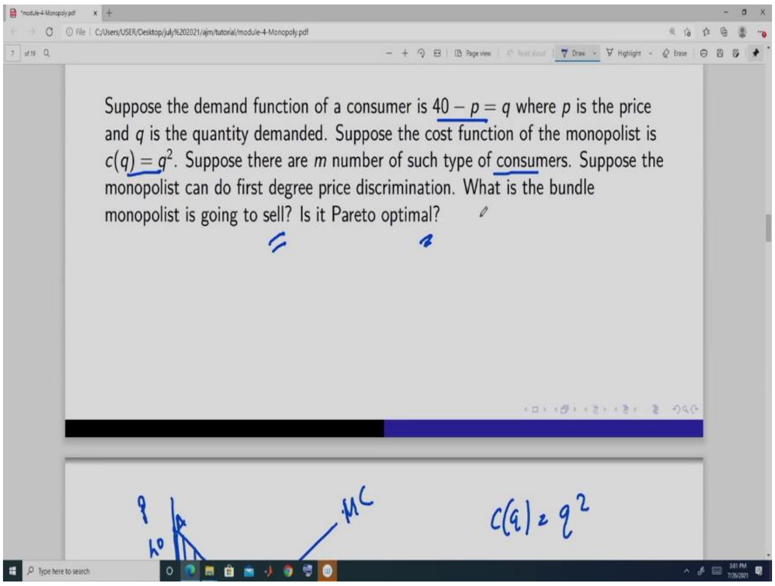

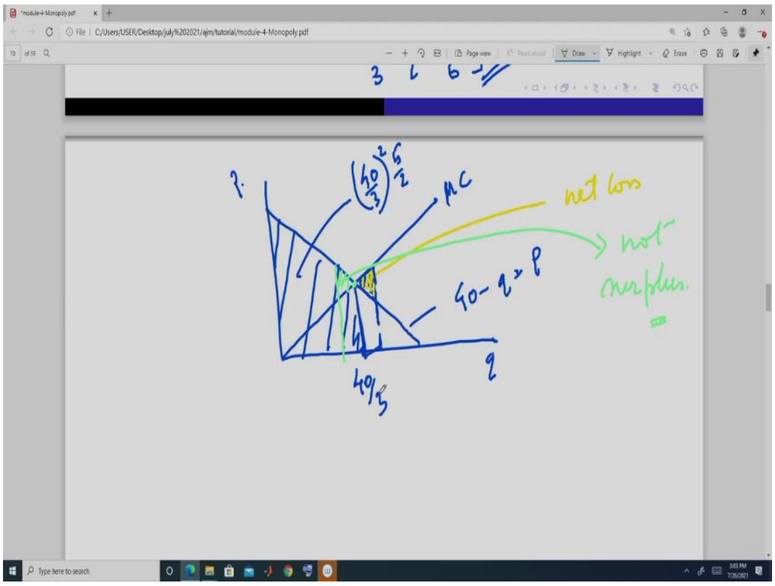

I have left one thing in the previous question and that is this question that is, is it Pareto optimal? That we have already discussed. So, what is the outcome we have got? This is the marginal cost, this is demand and this is the price, okay. This whole shaded area is the price and that price is 40 by 3 square 5 by 2, this- $\left(\frac{40}{3}\right)^2 \cdot \frac{5}{2}$. Now, if it produces here then net loss it makes is this amount, this yellow amount is the net loss if it produces here is the net loss.

Because revenue it is getting his additional revenue is this, but additional cost is this much so, this is the net loss. If it is producing here, then it is foregoing this much revenue and the cost is this much so this much is the net surplus it is forgoing. So, that is why this point here it is not forgoing any loss for it is not forgoing any surplus neither it is making any losses. So, that is why this point is the Pareto optimal in the sense that it is making the monopolist getting all the surplus and the consumers are not getting any surplus.

But if you want to move anywhere, if, then there are some amount of surplus that they can make, but no one is making or they make some losses and the monopolist is making losses here. So that is why this is the Pareto optimal position, okay.

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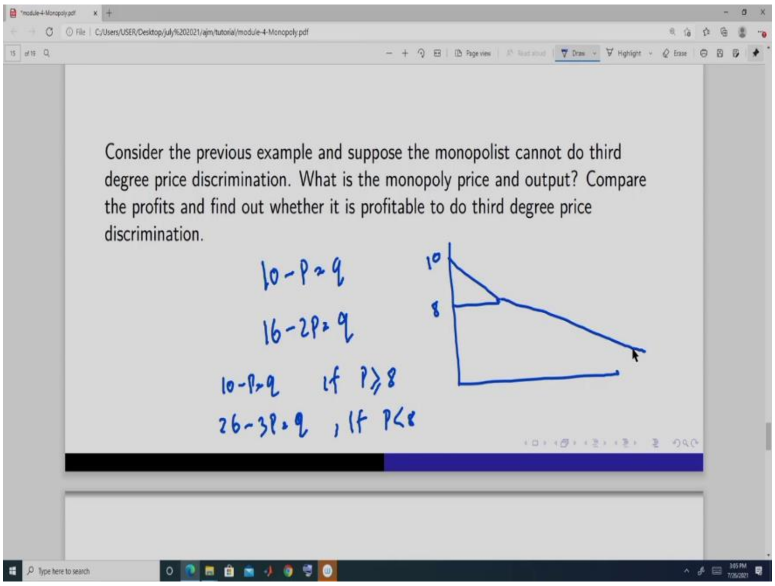

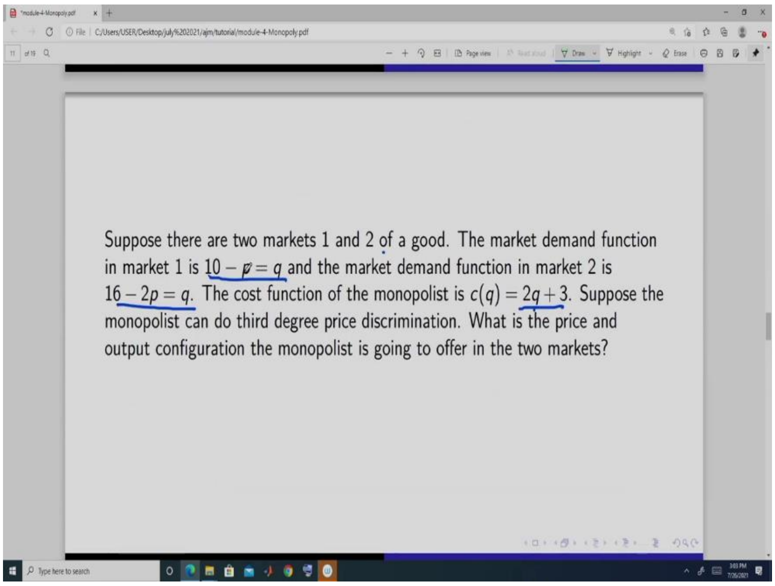

Next, let us solve another problem. That is this consider the previous module where this this problem these demand curves, there are two demand curve this- $10 - p = q$ and this- $16 - 2p = q$ and suppose here the firm cannot do third-degree monopoly, third-degree price discrimination then what is going to be the market? So, the if you look at this, it is this, okay. So, market demand curve is 8 and it is like this, this is the market demand curve- 26-3P=q, if P<8.

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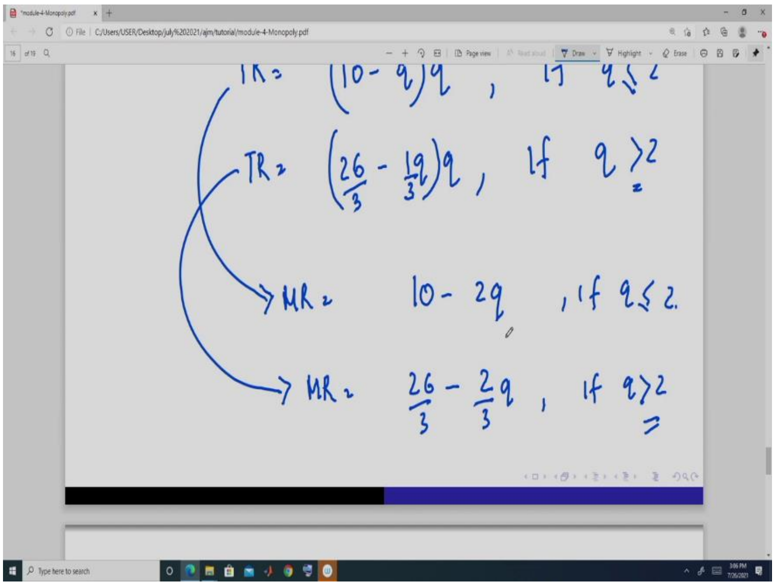

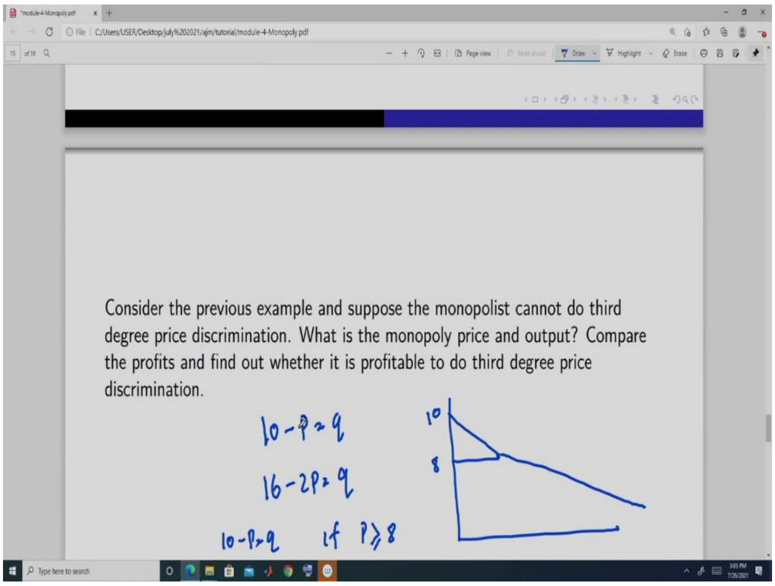

So, from here what do we get? We get the total revenue curve is- $TR = (10 - q)q, \text{if } q \leq 2$ if you plug in this equal to 8 here quantity is, if q is less than equal to 2 and the total revenue is this- $TR = (26 - 3q)q, \text{if } q > 2$. Now, let us look at this marginal revenue curve, this marginal revenue curve is this- $\mathrm{MR} = 10 - 2\mathrm{q}$, if $\mathrm{q} \leq 2$ if q is this marginal revenue curve is it is this, right? So, it is this now, if we look at this, if you plug in 2 here two this is equal to 5.

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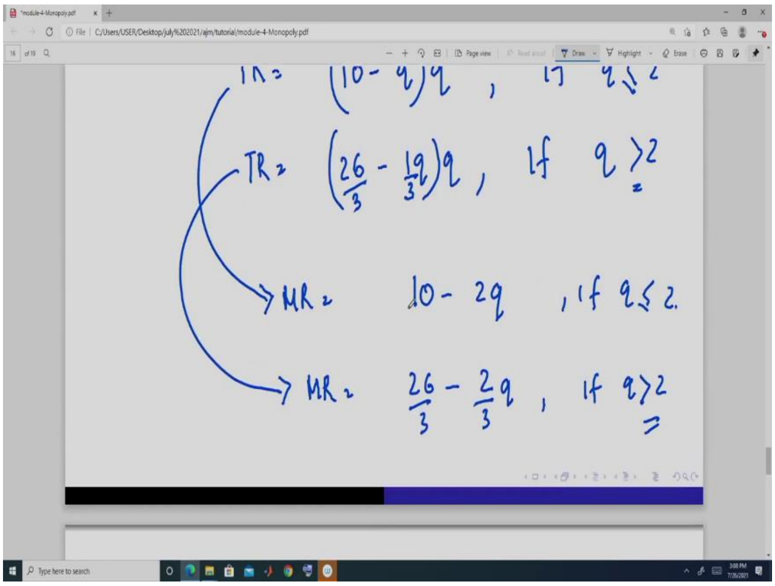

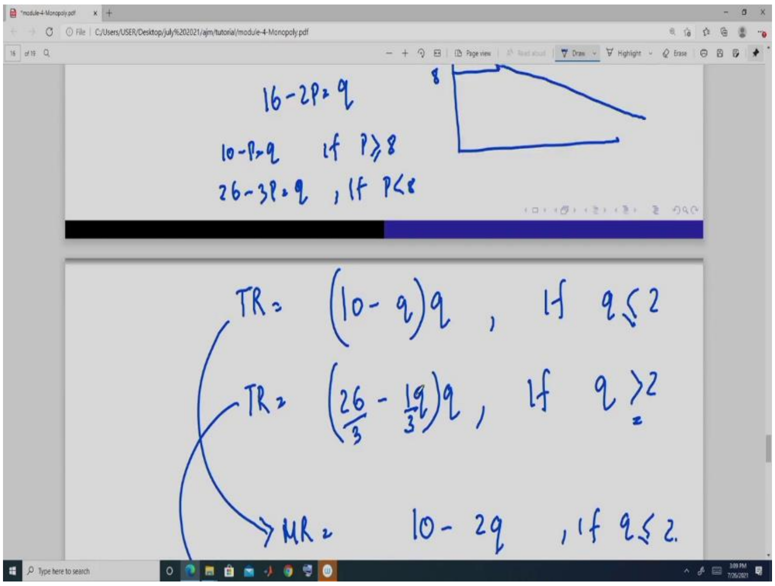

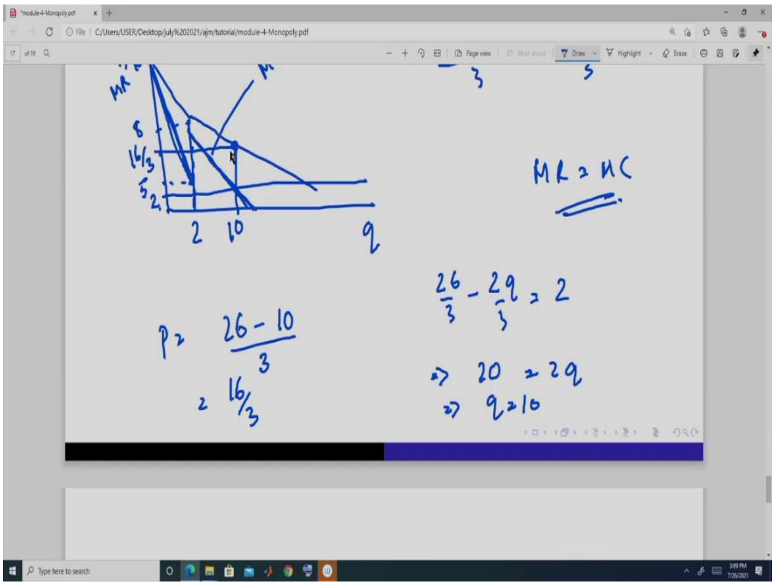

And if you plug in 2 here this is equal to 26 minus, so, this is 22 by 3- MR $= \frac{26}{3} - \frac{2}{3}$q, if q $>$ 2 $=> \frac{26-4}{3} = \frac{22}{3}$, right? and if you plug in 2 here it is 5. So, if you look at price here, total revenue, marginal revenue here this point is suppose, this is 2, then this is somewhere here, this is suppose 5 and this is 7 point something, so, it is this. So, this is the marginal revenue curve. This for here and this till this much and marginal cost is this is 2.

So, the monopoly profit is, monopoly output is this because marginal revenue is equal to marginal cost it is here. So, it is this would be equal to 2. So, this is q is equal to 10 this point is 10 and this price is plugin price in this portion 10, so, it is price is 26 minus 10 divided by 3. So, it is 16 by 3. So, this point is actually 16 by 3, okay.

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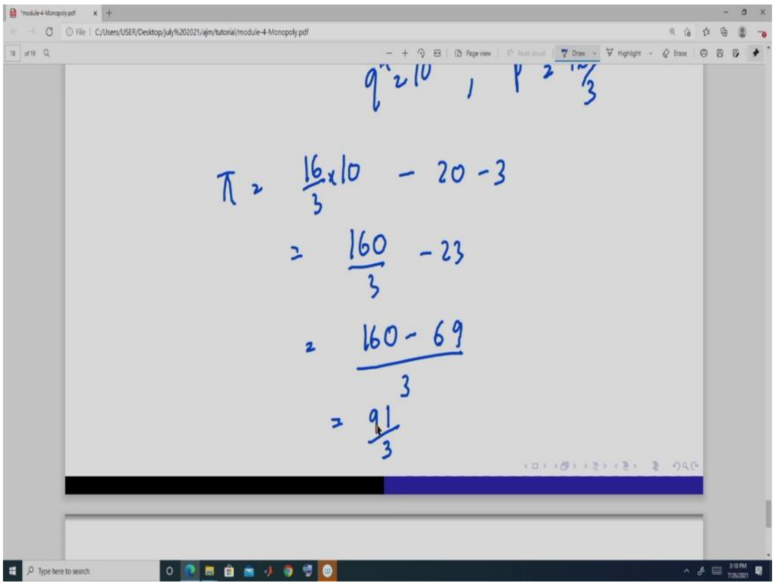

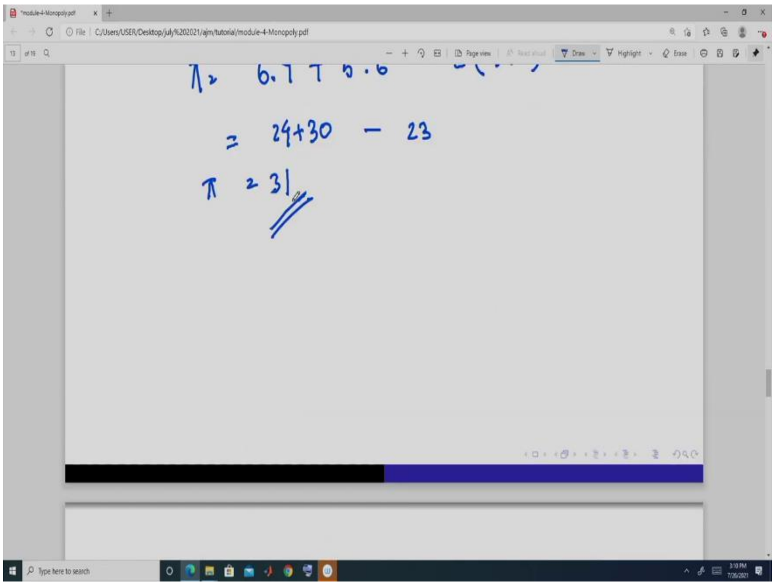

So, here monopoly output is 10 and monopoly price is 16 by 3. What is the profit here? Profit here is this- $\pi = \frac{16}{3} * 10 - 20 - 3 => \frac{160}{3} - 23 => \frac{160-69}{3} = \frac{71}{3}$ price 13 this revenue cost is 20 fixed cost is 3. So, this is, so this is, 71 by 3. If we do this because to be 10 and it will be this, what is so, this is 30 point something 30.33 and the profit here is 31. So, if we compare the a what do we get?

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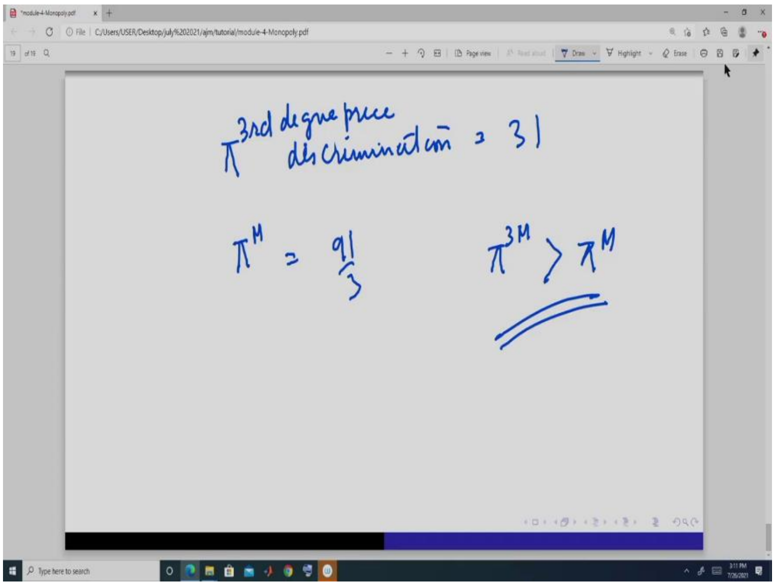

We get that the profit in third degree price discrimination is equal to 31 and simply monopoly price is 91 by 3. So, third degree M is greater than by simply M. So, there is always going to be a tendency among the monopolist to do third degree price discrimination if it is possible for them, okay. Thank you.

Introduction to Market Structure
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture 17
Introduction to Game Theory, Iterated Elimination of Dominated Strategies

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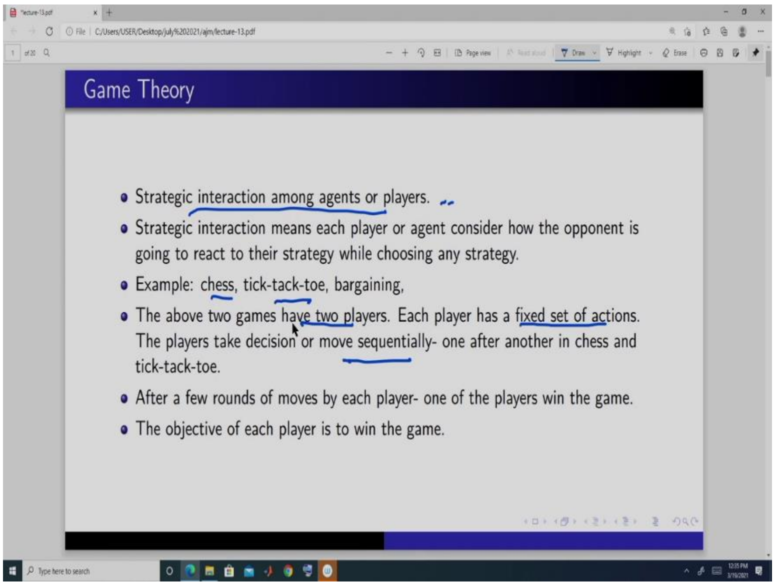

Today we are going to do game theory and this is module 5. So, in Game Theory it is mainly strategic interaction among players. So, what do you mean by strategic interaction it means that if we have suppose a number of players then while each of them taking any decision, they will always think how their opponent is going to react to the action they are going to choose.

So, for example, suppose in chess, what do we do? While making any move, I always try to think how if I moved in this way, how the other, my opponent is going to move in reaction or like in a tick-tack-toe. In tick-tack-toe what happens the players move sequentially and in each move they make a decision, right? and then after a set of moves you, we can find whether there is a winner or there is not a winner.

So, what happens in all these games we have a set of players. So, in these two examples, we have two players and each of them has set of actions that they can choose from and further we have to define whether they are taking the decision simultaneously or move sequentially or move one at a time, but in case what happened they move one after another in tick-tack-toe what happens they move one after another and after a few rounds, we know who is the winner and the objective here is to win the game.

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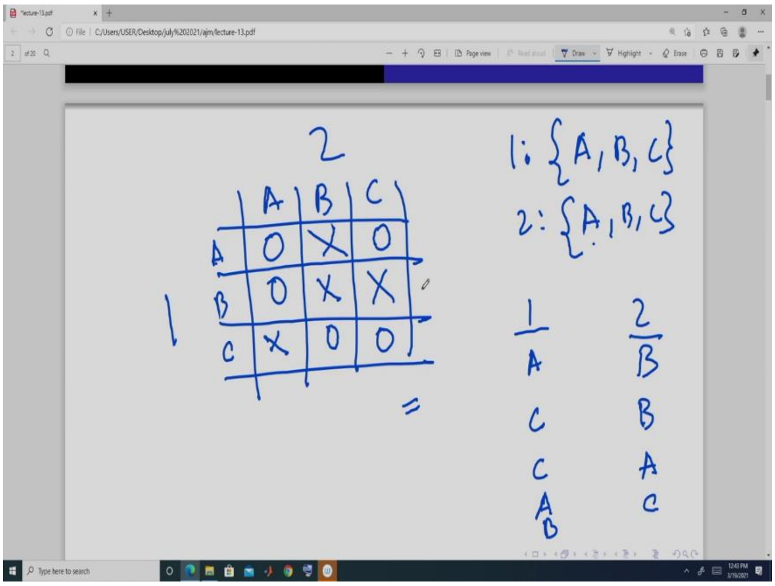

So, let us play one Tick-tack-toe game. So, here this is supposed be player 2 and this is player 1. So, this, okay? it can choose any place from A and player 2 can choose from these places A, B, C these columns and player 1 will choose from these row A, B, C. Suppose player 1 moves first as chosen and player 2 has moved second and it has chosen this place and next again player.

So, we can say that the action that the player can choose from these three places and action of player B is from these 3 places. And the moves are like this, suppose player 1 moves first A, player 2 has moved B then suppose player 1 is again moving and it has suppose chosen this place. So, it has chosen this. Now, player 2 will always choose this position. If he chooses something else, then player 1 will place here and it will win the game.

So, this is their strategic interaction. So, while choosing this position player 2 is deciding is thinking that if he if it does not do place its mark here, then player 1 will do and player 1 will win. So, this is what we mean by strategic interaction. Now, see, player 1 will always choose this position. This because if player 1 leaves this then player 2 will place here and it will make.

Next player 2 is going to move. So, player 2 if it leaves this then what is going to happen player 1 will choose this and it will win the game. So, player 2 will always choose this so it is A, next if player 1 leaves it, then what is going to happen. Player 2 will choose this and it will win it so win the game. So, player 1 is going to choose here.

So, it is again going to be A then if player 2 leaves this part, this block that is C for player 2, then player 1 will place here that is B for player 1 and it will win the game. So, so, there is only one position left and that is this it will be here. So, this is C and then again it will place at B. So, in this game we cannot find any winner, okay. But I hope you have understood what do we mean by strategic interaction that while making my move or making decision regarding my placement where I am going to place my mark, I have to think about what my opponent is going to place, where my opponent is going to place if I place my any specific place, okay.

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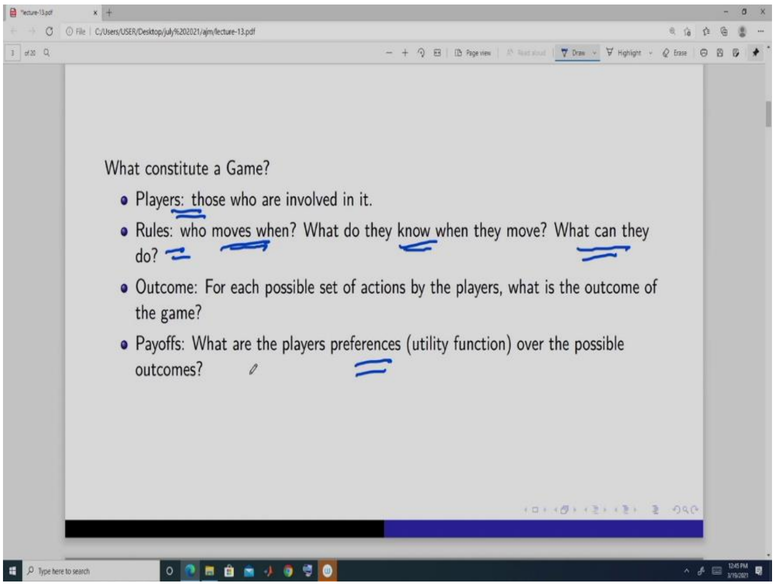

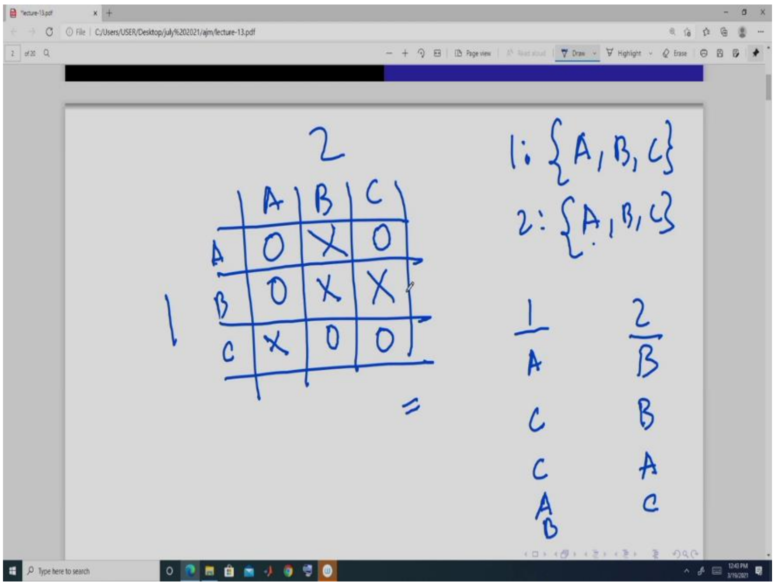

So, what mainly constitutes a game? It will have players who are going to play the game, we will have to define the rules like in tick-tack-toe, what are the rules? Rules are one player will move first and then the second player will move and then again the first player will move and then again the second player will move. So, this is one rule that is the sequence. So, in how the game is going to be played.

So, who moves when? So that has to be specified, then what do they know when they move. So, here when you play the tick-tack-toe, when player 1 was making this a player 2 while moving second, while player 2 was choosing this place, player 2 has observed where player 1 has placed his mark otherwise player 2 would have a means a different information if it does not observe this, but if it observes then its observation or the information set is going to be different.

So, this the role of the information set while making any decision plays a very important role and we will do this specifically when we do the extensivity, okay. And next we have to specify what they can do like here they have to choose from this player 1 has to choose from this A, B, C all these positions, okay. In A it has 3 positions in B it has 3 positions, in C it has 3 positions or places, okay.

So, these are the actions that they can take and we have to define the outcome. So, outcome means that when they are playing here, when they are playing So, if I suppose player 2 while player 1 has moved this player 2 has been moved this then player 1 has moved this this position, then if player 2 suppose placed somewhere else not here, then player 1 will win by placing it here and then it will win the game.

So, what is going to be the outcome here. So, if all these three matches in any line like this way or this way, this way or this way, then we say a player wins and if it does not match, then we say no one wins. So, in that way, we have to define the outcomes, okay. So, those are the outcomes and then we have to define the payoff, that if I win, what do I get? I get if I win, I get lots of satisfaction. So, we have to define the utility function, okay.

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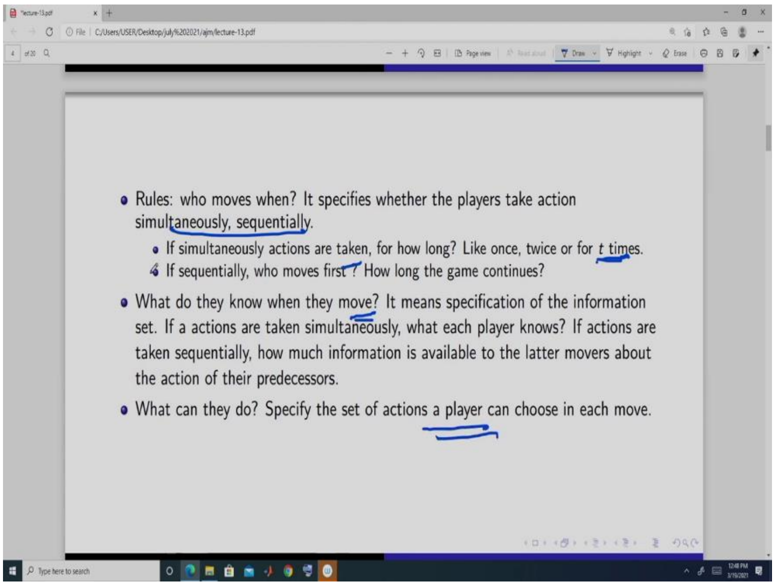

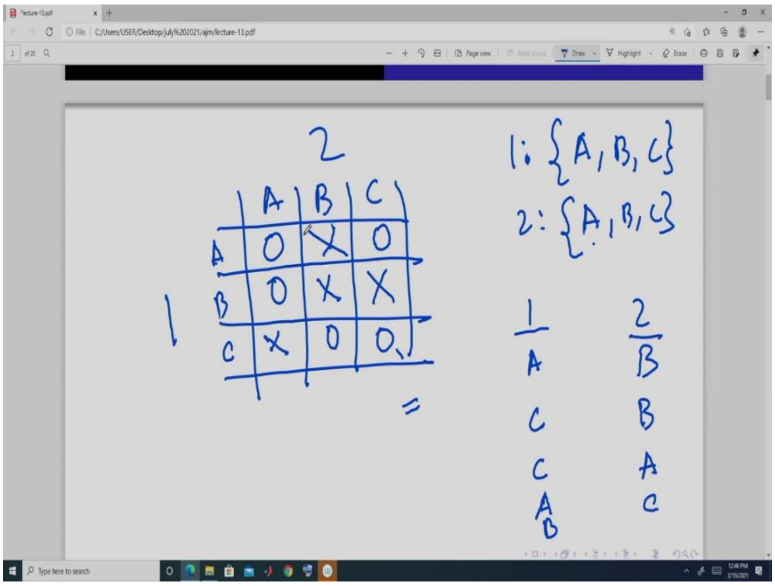

So, mainly a game constitutes this. So, when we say rule, when we say that who moves when it specifies whether the player takes action simultaneously or sequentially a little bit later we will see what do we mean by decisions are taken simultaneously. It means that both the player are choosing their action or their strategy at the same time. So, if they are simultaneously choosing then we have to specify whether they are going to play this game for one time or they are going to choose it for 2 times or they are going to play it for suppose some t times, okay.

So, it is like this that in this tick-tack-toe we know we cannot say beforehand how much but we say that the play will continue as long as all the places are not if any of the places is vacant or there is a winner if you get a winner then the player the game will end and if we have not got any winner, then the game will continue. So here we have specified that the game is going to go on each player is going to choose as long as there is no winner or there is any vacant place to be marked.

So, this is like a in and in sequential game we have to define who moves first and who moves second and how long the game is going to continue or how long the players are going to choose their actions. Now, what do they know when they move it means the specification of the information say like when I am choosing my action, I should know what are the possible actions that the player 2 can choose?

Or if I am playing after player 2 has played then I should know what are the actions that has been already taken by player 2 or I have to specify means if I do not know some of these actions or some of the payoffs then I will have to define some probability distribution over them. And that part we are not going to do in this course that is incomplete information game. Next, when we say what can they do we specify the set of actions for at each move, okay.

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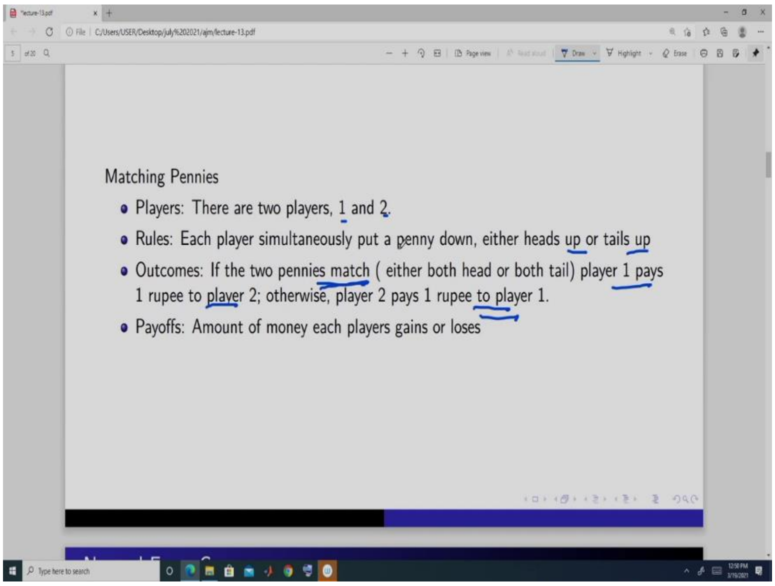

Now, let us look at the game that we are generally going to study in this course like one is these matching pennies. In the matching pennies what happens there are 2 players, player 1 and player 2 and they simultaneously what they do they toss a coin, okay or they will place a coin in a table, okay they will put a penny down.

So, what happens if both of them do that thing. So, either they will have both head or they will have tail, okay. So, what will happen if both the pennies matches that if either both are head and or both are tail then player 1 pays 1 rupee to player 2. If the pennies are not matching like 1 is head and another is a tail, then player 2 pays 1 rupee to player 1. So, this is the outcome and the payoffs are the amount of money each player gains or each player loses and what are the rules?

Rules here is the simultaneously place a coin in a table, okay. So, while making this action, both of them are taking their decisions at the same time or they are taking the decision together, but they know that what is going to be the outcome. So, they know all the possible outcomes. So, based on that they will take a decision, okay. So, this is one form of matching pennies, one form of a game which we call as matching pennies. It is a simultaneous move game and it is also single shot. So, generally we defined this kind of simultaneous move which is played only once as a normal form game.

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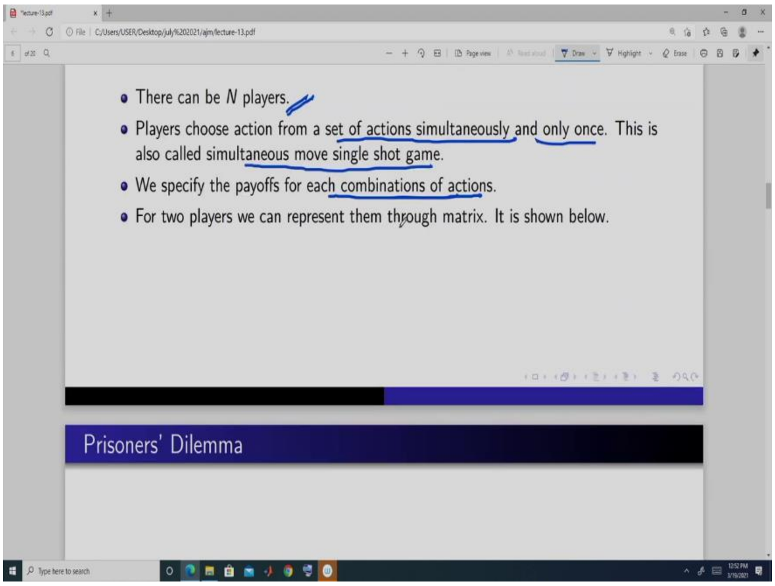

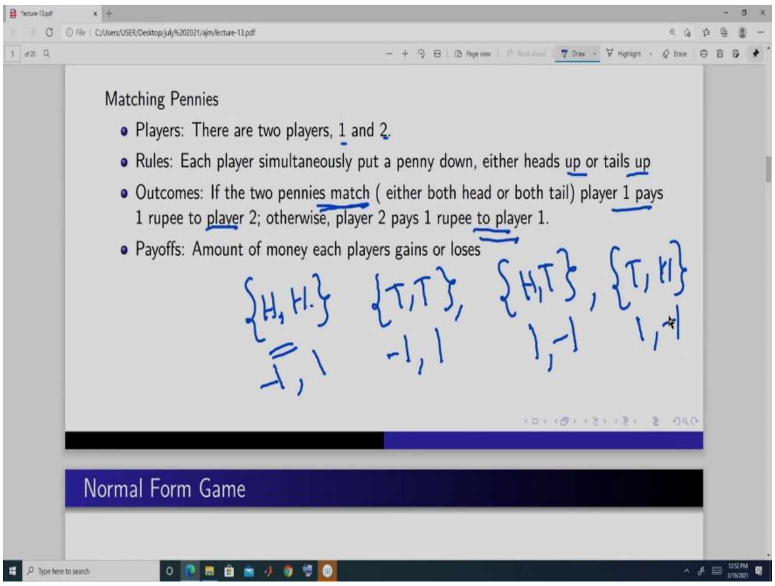

And it can be played between among N players. So, the number of players is not a restriction and player can choose a action from a set of actions simultaneously and they only choose only once. So, the game is played only once and it is played simultaneously. So, this is also called a simultaneous move single shot game, okay. And here we specify the payoff for each combinations of actions, okay like in this.

So, what are the possible outcomes? So, one possible outcome is head and head. So, we define the payoff here another is tail and tail, another is head and tail and the another is tail and head. So, if it is head and head, player 1 pays 1 rupee to player 2, so, it is minus 1 to player 1 and plus 1 to player 2. If it is tail and tail, so, it is minus 1 for player 1 and 1 for player 2 and if it is head and tail, then it is 1 for player 1 and minus 1 for player 2. And if it is tail and head again you will. So, these are the 4 possible outcomes and these are the payoffs they get in these outcomes, okay. So, this is a normal form game. So, normal form game can be represented through matrix and it is very convenient way.

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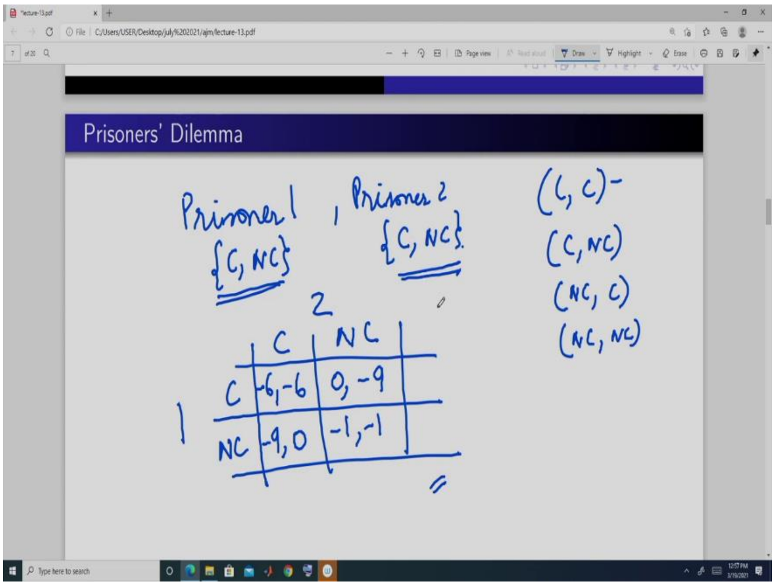

So, we will do a now I will specify a very famous game that is called a prisoner's dilemma. So, there are 2 prisoner, prisoner 1 and prisoner 2, okay and they are interrogated simultaneously in two separate, okay and they have committed some crime each one can either confess C or D or it may not confess So, it has 2 possible actions. Similarly, prisoner 2 has 2 possible actions either can confess or it may not confess.

So, how many combinations we have? Combinations of outcomes, so, this is C, C both of them confess, okay or it is let us define the combinations not in through this, **so** combination is C, C this is C not C then we have not C, C and not C not C, okay. So, there are 4 possible combinations from these 2 actions. So, these are called the action space, i.e (C,C), (C, NC), (NC, C) and (NC, NC) or strategy space, and these are the combinations of each element from this strategy or action space, okay.

And now we define some payoff from here. So, if this is the outcome what is going to be the payoff or what is going to be the utility for each of these individuals, okay what they are going to get if this is. So, this whole thing can be represented through a matrix in this form. Suppose here this column represents player 2, so, it has 2 actions, one is C and other is not C. Player 1 again it has 2 actions that is C and not C, and here this block gives me the outcome when both of them are choosing C and C.

This is when player 1 is choosing C and player 2 is choosing not C. This when player 1 is choosing not C and player 2 is choosing C. This outcome is when player 1 is choosing not C and C and the player 2 is also choosing and C that is not confess, non-confess, or this is confess, and if both of them confesses, then they get a little suppose 6 years of jail and so jail is a you get a disutility from jail.

So, that is why I have written it as minus 6. But if player 1 confesses and player 2 does not confess, then player 2 is not given any punishment, but player 2 is given a stricter punishment and that is 9 years of jail, okay. So, it is so if player 2 1 does not confess, but player 2 confesses, then player 1 gets 9 years of jail and player 2 is not given any punishment. If both of them does not confess.

But since the police know that they have committed a crime, but they are not very sure whether they are sure they will get some amount of punishment and that is 1 year of jail. So, if we look at this here, then each of them should prefer this outcome, right? because it is 1 year, 1 year. So, this first element is for player 1 that is for the player whose actions are specified in the row and the second element is for the player 2 whose actions are represented in the columns, okay. So, this is a representation of the prisoner's dilemma game. And this is called matrix representation of normal from.

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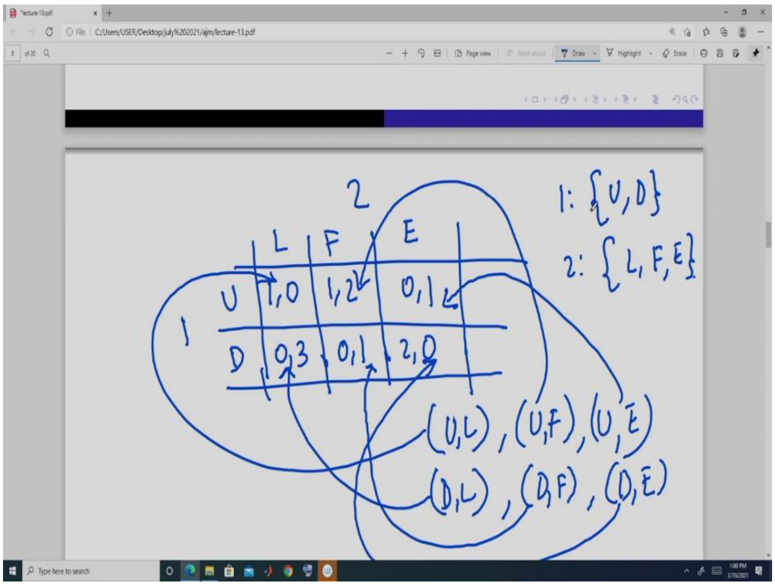

Another type we can think suppose again we have 2 player, player 2 and so, this is suppose U and D for player 1 for player 2, it is L F and E, okay. So, now this is a type of asymmetric game where action or strategy space of player 1 is U and D and for player 2 it is L, F and E, okay. And now suppose we define the payoffs. Suppose the payoffs are like this, okay. So, here so when player 1 place of the different total combinations that is possible combinations of outcome is this- (U, L), (U, F), (U, E), (D, L), (D, F), (D, E)

So, this outcome is represented here and these are the payoffs. This is given here and these are the payoffs. This is for player 1 who plays who have played **a** chosen action U and this is for player 2 who has chosen F. This is for this for this player 1 has chosen U and player 2 has chosen E, this outcome is this. 0 for player 1 and 3 for player 2, player 1 has chosen D. So, it has got 0 when player 2 has chosen L it has got 3.

This combination gives you this payoff this outcome. So, it is 0, 1; 0 for player 1 when it chooses D given that player 2 has chosen F and player 2 gets 1 when it chooses F and player 1 has chosen D. This is for player this combination is D, E when player 1 has played or choosing D and player 2 has played E, okay. So, this 2 is the payoff for player 1 and 0 is player 2. So, this is also another game where the action state or the strategy space are asymmetric, okay.

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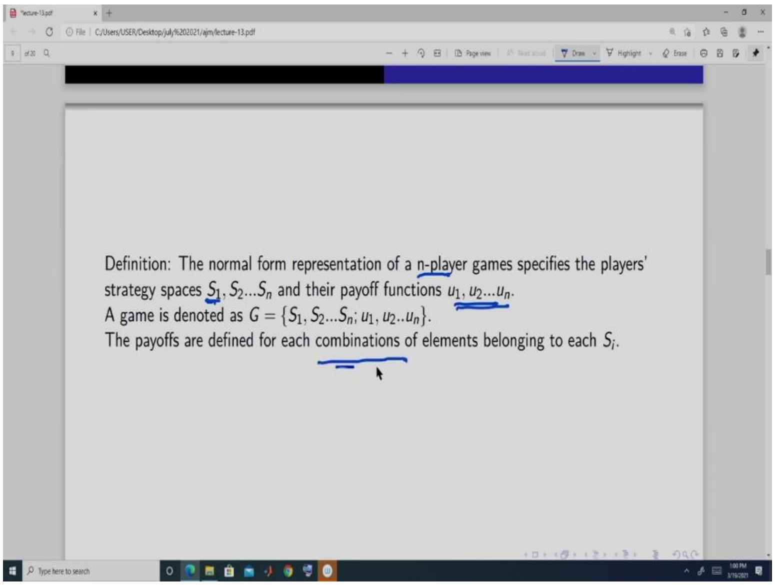

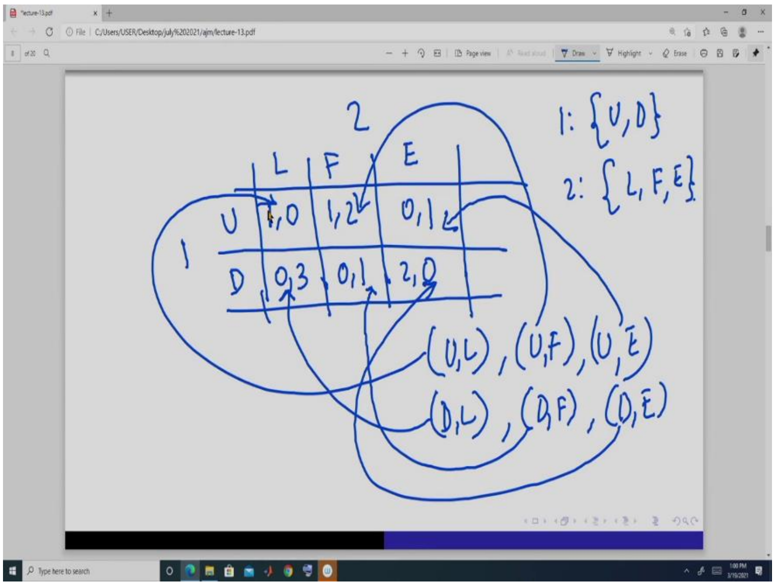

So, we define in a compact form the normal form game in this way that suppose there are n players and each player has a strategy space like this, this is capital S1 for player 1 capital S2 for player 2 and like this for gap n. For player n it is Sn and they have a payoff function which is which gives you this the amount you get when you play any when there is a specific combination of outcome like this.

So, there is a utility function defined for each player and payoffs are defined for each combination of elements belonging to. So, each combination means, so, here all these combinations all possible combinations. So, for this we have to define the utility how much the sorry, we have to define the payoff how much they get, okay. Now, we have to play this game. So, we have to define a solution concept.

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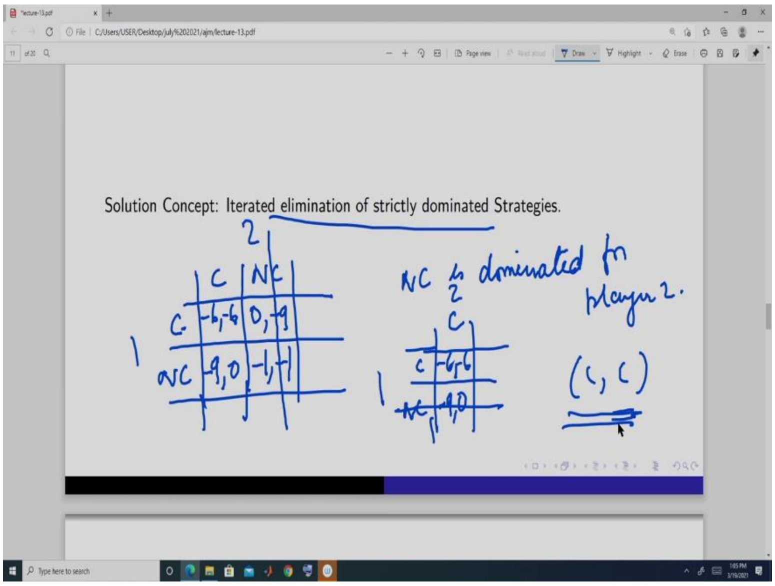

So, first solution concept that we are going to do is iterated elimination of dominated strategies. So, that is, so first let us consider the prisoner's dilemma game. So, player 1 it can confess or not confessed player to confess or not confess. If both of them confess confess payoffs and minus 6, minus 6, if player 1 confesses player 2 does not confess 0 comma minus 9, if player 1 does not confess player 2 confess, then it is minus 9, 0.

Both the players do not confess this. Now, what iterated elimination of dominated strategy says that you eliminate the dominated strategy. What do we mean by dominated strategy? So, for player 2, suppose it plays so in this case, minus 9 here minus 6, so this is less than this, right? Here, for player 2, this is 0 and this is minus 1. So again, this is less. So, this strategy is actually dominated.

So, NC is you can say dominated for player 2, so, if we start with player 2, then this can be removed. So, we have eliminated the dominant. So, that means when you specify a action and or a strategy of player 2, and you look at all the possible actions that the player 1 can do. It can choose C or it can choose this if it choose C then it will get this if it chooses this it will get you have to compare this.

So, in all these comparisons, what will happen minus 9 is greater, is smaller than minus 6, minus 1 is smaller than 0. So, this if it plays this if player 2 plays this strategy or this action, then it is going to get less than whatever player 1 is going to choose. If it chooses C it will get minus minus 9 it will get minus 9 which is again less than minus 6 if it chooses player 1 to NC it will get minus 1 which is again less than 0.

So, that is why this is dominated. So, this will remove then the game is so, player 2 has no option now, right? but player 1 has options. So, it will compare minus 6 and minus 9 and it know minus 6 is less than a greater than minus 9. So, it is so, what is the outcome? Outcome is going to be C, C always, player 1 will choose C after this elimination.

Because suppose player 2 starts this process of elimination then we remove this then we arrive at this reduced form and from here player 2 does not have any actions or strategies to remove it has only 1 A, player 2 has player 1 has to from it, it removes the NC and we get end up having only this outcome. So, this is the outcome of this game, okay. If there is iterated elimination of dominated strategies.

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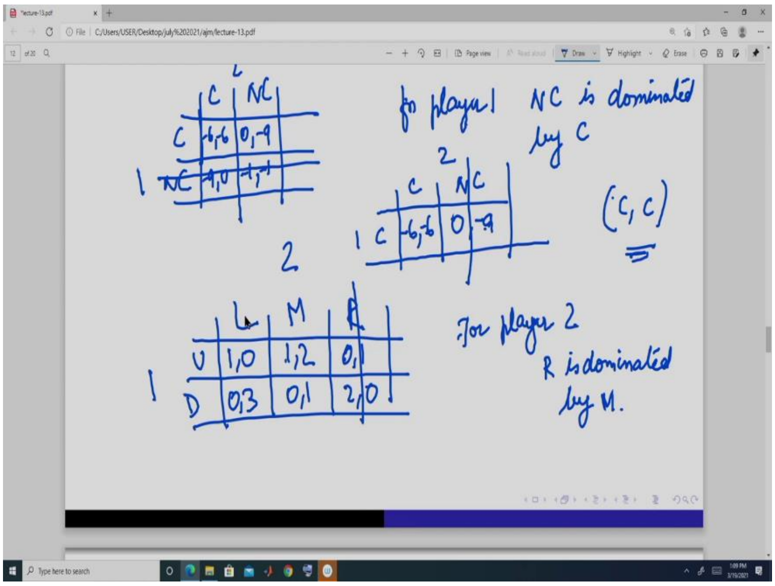

Now, here the same game if it is suppose the elimination is started by player 1, okay. So, player 1 it will player 1 suppose, chooses this NC. So, if it chooses this you will see that compare it with C minus 6, minus 9, 0, minus 1. So, if it plays this for each outcome, it will get less compared to this. So, that is why for player 1 and C is dominated by C. So, we eliminate this. So, next the reduced form is, is this.

Now, player 1 has only one action of strategy so it cannot eliminate further. Player 2 has 2, it has C and NC. So, if it plays C it gets minus 6 if it plays NC it gets minus 1. So, this can be eliminated. So, again the outcome is C and C. So, in this case we see that whether we start from player 2 we will start the process of elimination from player 2 or from player 1 we get the same outcome that is C and C.

Now, let us take another example, okay. So, this was this is for player 1 and, okay this is for player 2, okay. So here so, if we take player 1’s this U and compare it with D, so (1, 0) (1, 0,)(0, 2). So, for this 2, this dominates this or this is dominated by this but for this U gives more than D. So, we cannot eliminate any action for them, but if we compare this M and R, we will see that 2 is greater than 1 or 1 is less than 2, 0 is less than 1. So, for player 2, R is dominated by M. So, we remove this, so, this is our game left.

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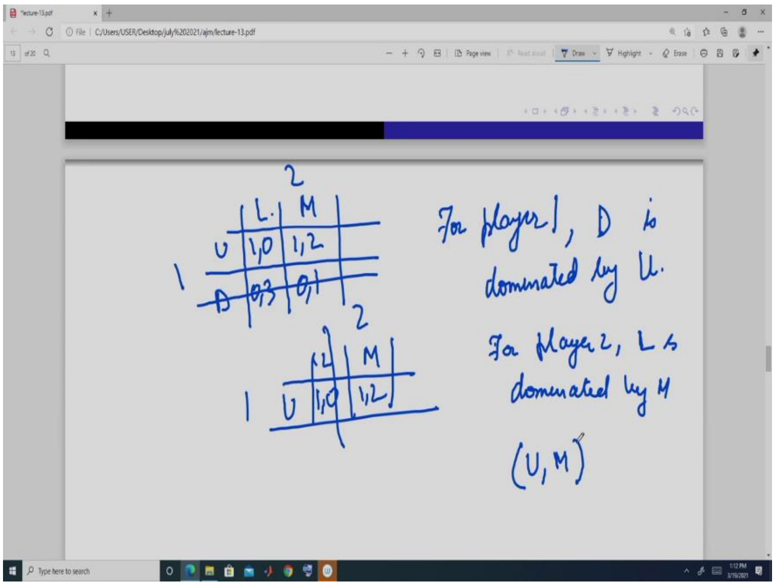

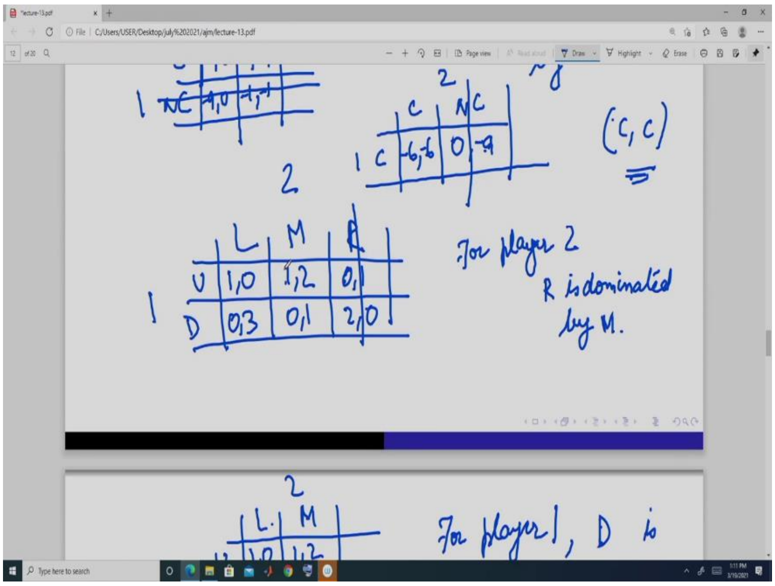

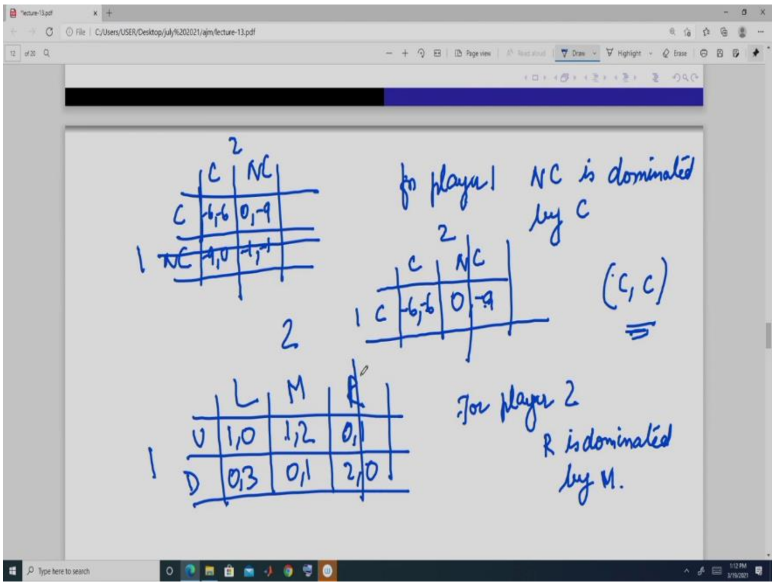

So, now, if you compare for player 2 again L 0, M 2, so, this is greater than, but here 3, 1 So, this is greater. So, 2 cannot decide, 2 cannot say whether this is dominating this or this is dominating this, and for player 1 it knows U 1, 1, D 0, 0. So, this is dominated for player 1, D is dominated by U. So, next our next is this. So, for player 1 there is only 1 action and that is U for player 2 there is L and M and from this we can remove L for now, for player 2 L is dominated by M. So, the outcome in this game is (U, M).

In this game it is going to be this. So, when players are playing once and they are playing simultaneously, all these eliminations will be done by on their own they will not interact, but they will know. So, here we are assuming that the player 1 knows that player 2 is also going to behave in this way, and player 2 knows that player 1 is going to behave in this way.

So, there is player 1 knows that player 2 first is going to remove this and then player 2 is also going to remove this and then player 2 again going to remove this and it is going to come end up having this outcome. Player 2 also knows that player 1 is will after looking at this game will know that player 2 since player 2 is the rational is going to is never going to pay this strategy.

So, this is important. So, this so, then it again knows that since player 1 is itself a rational player, so it will remove this so it is a left with this game. So, then it again knows that player 2 is also a rational player. So, it is not going to play this action and that is so they end up having this. So, since both the player behaves rationally and also both the player thinks that the other player is also rational and other player also thinks that he itself.

So, player 1 thinks or player 1 knows that player 2 is rational and player 1 knows that player 2 knows that player 1 is also rational and so using that argument, we can eliminate all the dominated strategies and we will end up having one outcome one this.

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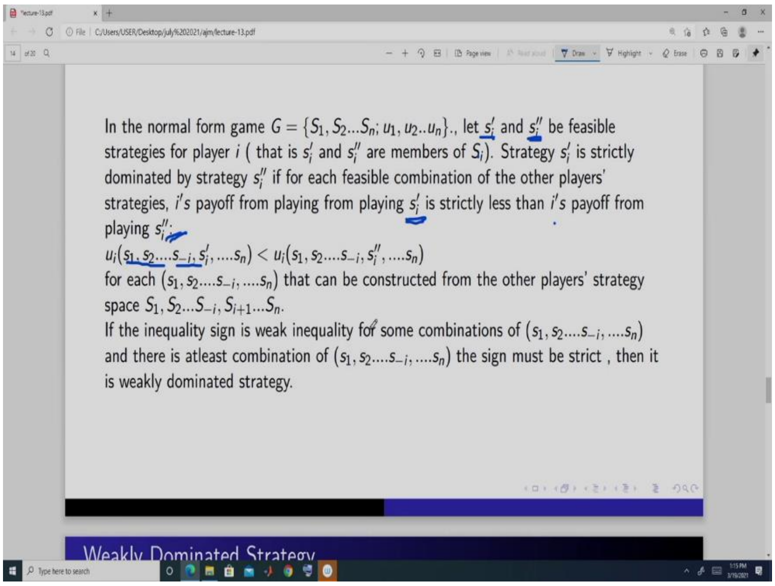

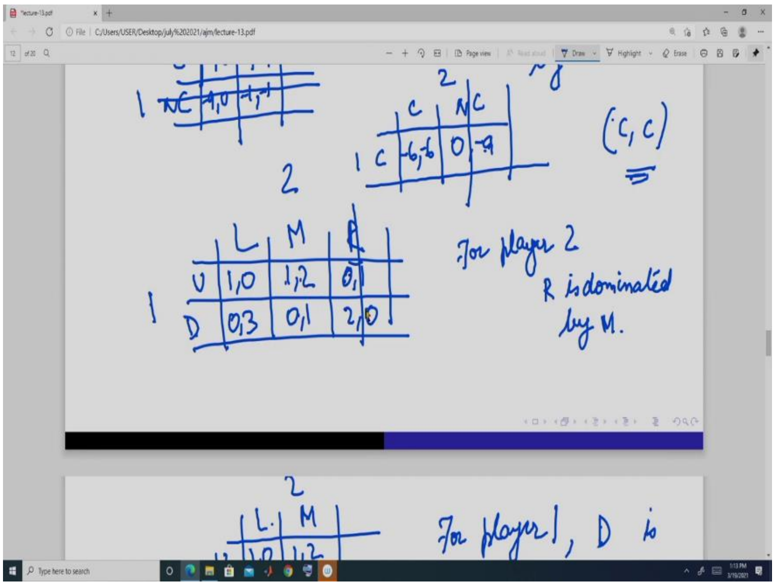

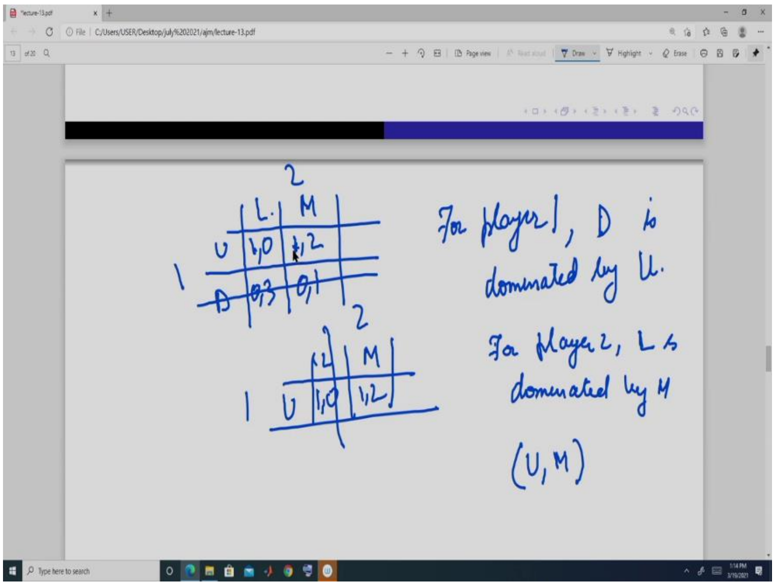

So, more formally, we can define a dominated strategy in this way suppose take any player i and let this Si dash and Si double state double dash be 2 strategy for player i, okay so they are 2 member of this strategy space i okay. Then strategy Si dash, single dash is strictly dominated by a strategy of player i double dash if for each feasible combination of other player strategies. So, it is like this.

So, here if this is dominated by this then all the possible combinations of player 1 that is this and this so, you have to compare this with this and this with this, right? So, that is what it says that for each feasible combination of other player strategies Si's payoff from playing this Si dash is strictly less than a Si's payoff from playing double dash. So, this utility from playing single dash strategy is less than the playing double dash.

So, single dash a player i is dominated by the strategy or action as double dash, okay. And so, here when we look at this, so, this is one combination, so for each of these combinations, okay for each Si that can be constructed from the other player this here. So, when we are eliminating this then what we are doing player 1 is eliminating. So, it is looking at all the possible combinations, okay. So, it for this this has to be less than this and this has to be less than this, okay. And when we say it is weakly dominated, then when some of them are equal and some of them are at least 1 must be unequal.

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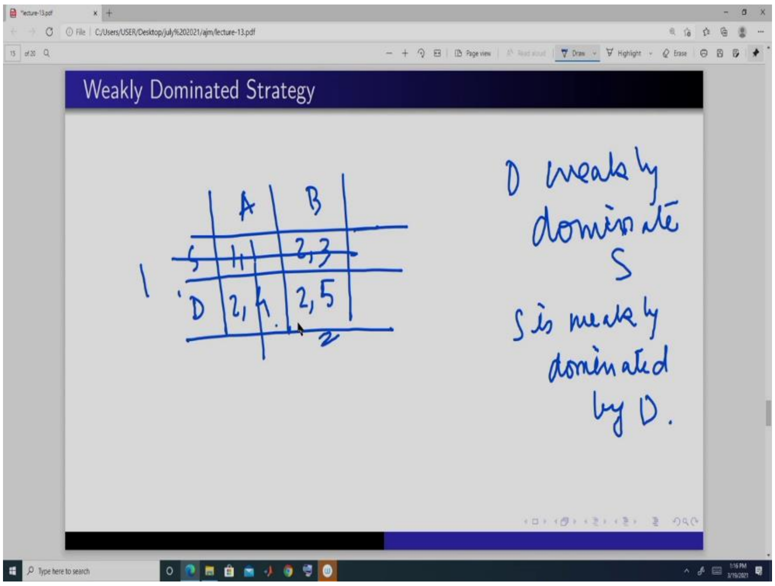

So, this can be 1 this is see now here so, if you compare the payoff for player 1 then you will see that this S and D so, this is greater but here it is same. So, D weakly dominates or weakly dominates S or S is weakly dominated by D in this case, okay. And then here again you can remove this so, you can remove here and so, this this will be removed. So, outcome is this and if you look at so, we have to do only this way, okay.

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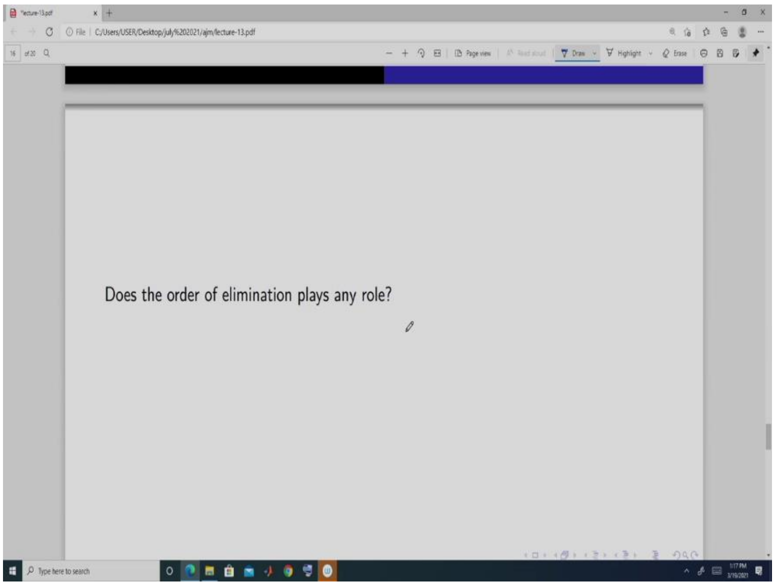

The next thing that we are going to do, and it is going to be in the next class, and that is whether the role or the order in which we eliminate the strategies or actions, whether that has any important role to play that if we change the order of elimination, then do we get a different outcome? Okay.

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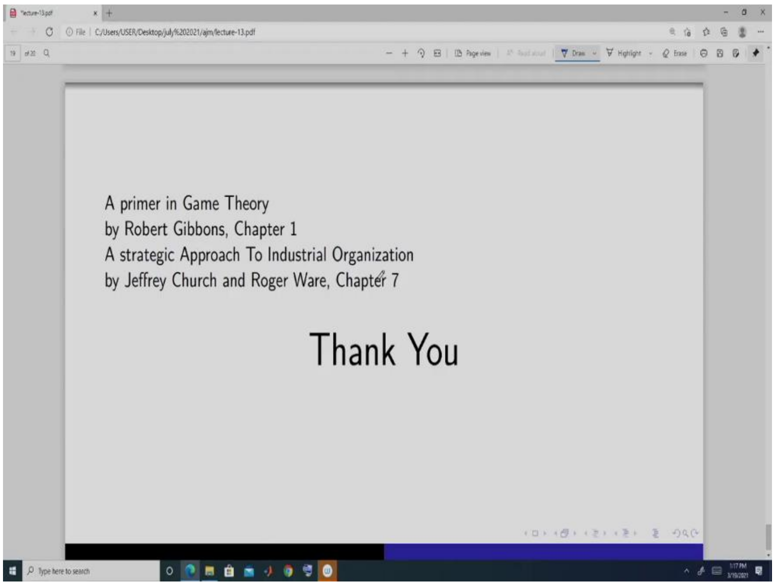

For this person, you can read chapter 1 of the book by Robert Gibbons, A Primer in Game Theory, or you can read chapter 7, from this book, A Strategic Approach to Industrial Organization by Jeffrey Church and Roger ware. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 05: Game Theory
Lecture 18
Pure Strategy Nash Equilibrium

Hello. Welcome to my course Introduction to Market Structures, and we were doing Game Theory and we have done the first solution concept that is iterated elimination of dominated strategies.

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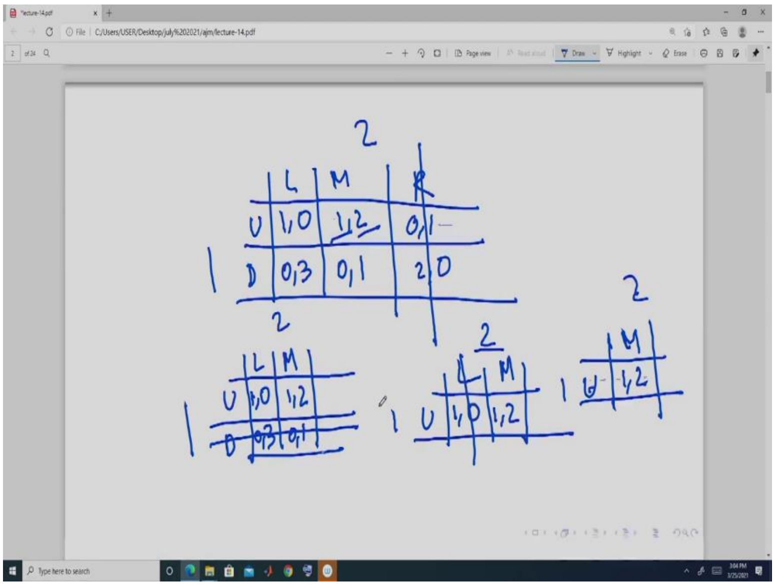

And let us recap through one example and then we will start the new thing today. So, suppose we have a two-player game. Player 1 is in the row. It has two actions or two strategies U and D. Player 2 is the column player and it has 3 strategies or action that is L, M, R. And payoffs are like this, okay. Now, what do we do in a iterated elimination of dominated strategies? From here we see that M dominates R or R is dominated by M for player 2. Why? 2 is greater than 1.1 is greater than 0. So, we can remove this, because player 2 will never going to choose R when M is available, okay. So, our game is now of this form.

Now, if look at this 0, 2, 3, 1. So, neither M dominates L nor L dominates M. But if we look at U and D; (1, 0); (1, 0). So, U dominates D or D is dominated by U. So, we can remove this. So, we are eliminating this action of player 1 that is D. When U and D are available player 1 is always going to choose U because for each action of player 2 U gives greater payoff than D.

So, the game now is, is this. And if we look at this here L is dominated by M because 0 is less than 2. So, we leave this. So, we… so through this process of elimination we come to the conclusion that player 1 is always going to choose U and player 2 is always going to choose M. So, this is going to be the outcome of this game. And this whole process of elimination is going to be taking place in the minds of this player. And when the game is played only once and simultaneously each of them are going to choose.

Player 1 is going to choose U. Player 2 is going to choose M, because player 2 will do this elimination of dominated strategies, and similarly player 1 is also going to do this, okay. Now, here and we have done weakly dominated strategy also in the last class.

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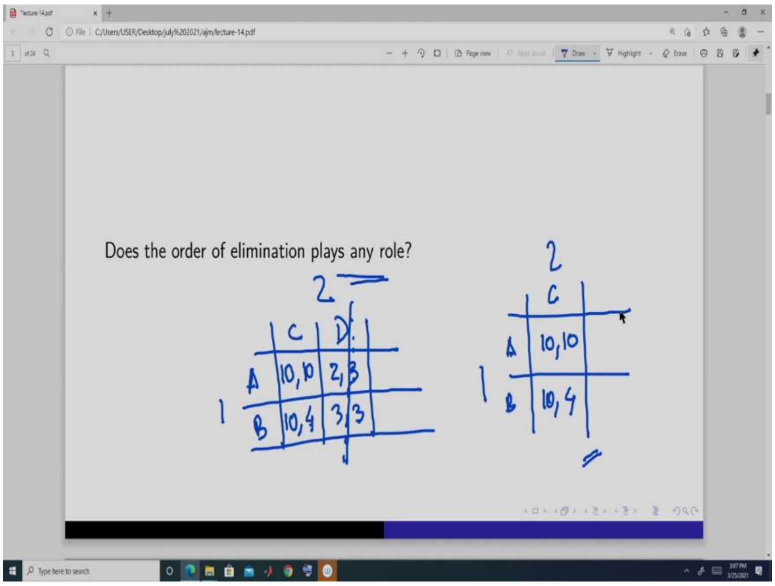

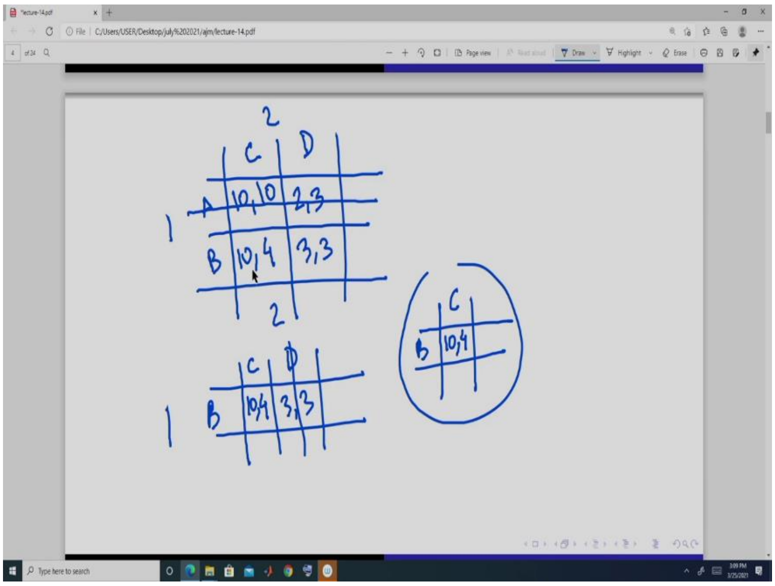

Now, the question is does the order of elimination plays any role? Means if we start the elimination from the strategies of player 1 then do we get the same outcome as when we start the process of elimination from the strategy set of player 2. So, let us do one example and then see what happens. Suppose we take a game of this form. Player 2 is again in the column and its action is C and D. And player 1 is in the row and its actions are A and B. Or you can say these are the strategies, okay. And payoffs are suppose (10, 10); (2, 3); (10, 4); (3, 3). Now, here if you look at this A. Now, let us start the game from, in this way. Suppose player 2 we eliminate, so we know that C; D is dominated by C because 10 is greater than 3 and 4 is greater than 3. So, we can eliminate this, this action or strategies of player 2.

So, the game is now; if we eliminate this… so now only one strategy for player 2 but there are two actions and we cannot eliminate because it is weakly dominated. They are same. A is not dominated by B and neither B is dominated. So, we end here. So, we do not get any precise result, okay. So, the outcome is that if we start from the player 2, if we start the elimination process from the actions of the player 2 then we end up here. Now, let us take the same example and let us start the process of elimination from the action set of player 1, okay. Now, here if you look at this player 2 and player 1. So, for player 1 strategy B weakly dominates A or A is weakly dominated by B. So, we can remove this. So, we are left with this.

Now, we can see from here for player 2, D is dominated by C so we can remove this and we end up in this outcome, okay. So, if we start from the elimination of the strategies of player 1 then we get a precise outcome and that is player 1 chooses B and player 2 chooses C. But if we start the elimination process from the strategy sets or action sets of player 2 we remove this first and then we end up in this situation. And then we cannot remove. So, we, this is an imprecise here.

So, from here we can say that order of elimination from how we start the elimination process, from whose strategy set or action set; it actually determines the outcome that we are going to end up. And it is true only in the case of weakly dominated strategies. So, if we have weakly dominated strategies then we will face this problem, otherwise we will not face this problem, okay.

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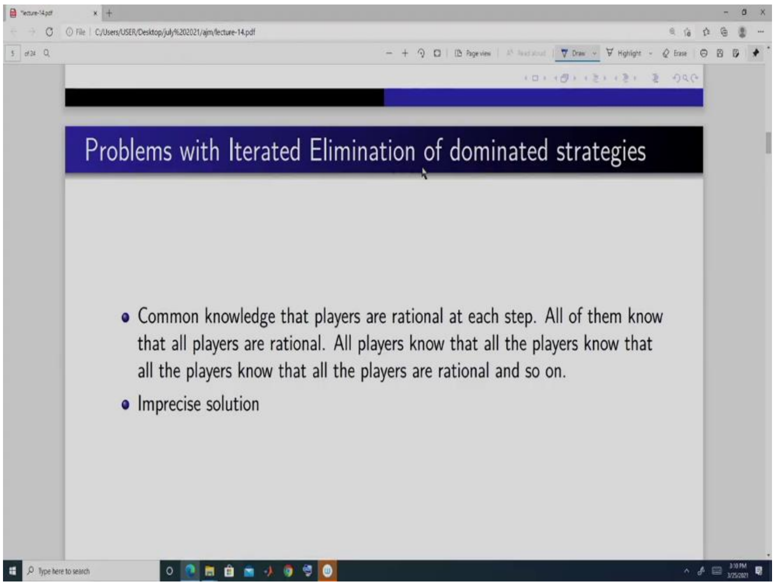

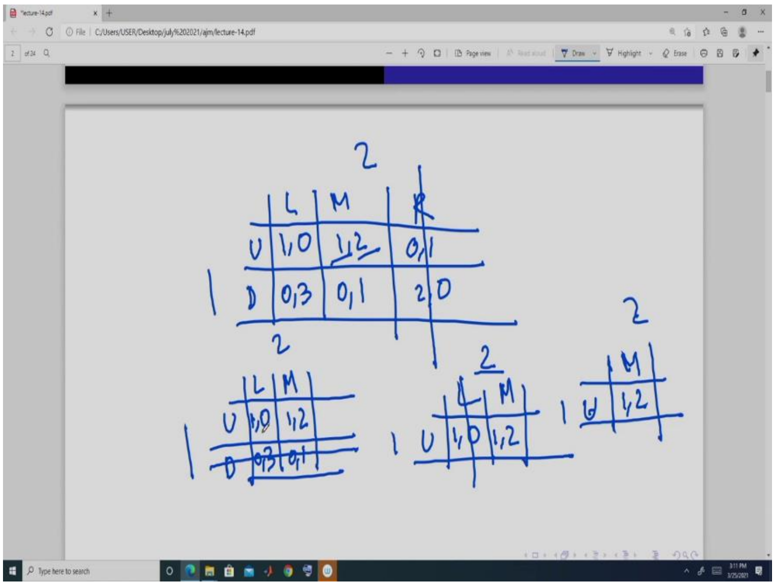

Now, the next important thing here is what is the problem in the, this solution concept that is iterated elimination of dominated strategies? So, first while doing this elimination, when we have done this example, so what we have done here? We have done that suppose player 2 eliminates this because R is dominated by M. Then player 2 eliminates… if suppose player 2 while thinking about which strategy to play, first it will eliminate this. Then it will eliminate this and then it will eliminate this. And then we will be at this.

Similarly player 1 while choosing U will first eliminate this, then eliminate this and then eliminate this and then end up in this sequence. So, what is happening? So, first player 2 is assuming that player 1 knows that player 2 is going to remove this. Then player 2 is again making an assumption that player 1 is rational and that it knows that player 1 knows that 2 is rational, so that is why it is removing this.

And again, player 2 again in the next step it is assuming that the player 1 again going to behaving in a rational way and so player 2 again knows that player 1 knows that it is going to behave in a rational way, because player 1 again knows that player 2 is rational, so player 2 is also going to behave in a rational way so player 2 is going to remove this. So, in this way we have to assume rationality in a sequence of rationality, okay.

So, this is a very strong assumption. And so this is, and also we call it as a common knowledge that; common knowledge is that the all players behave in a rational way, and in each step they are behaving in a rational way. So, this is actually a very strong assumption which we have implicitly made in this.

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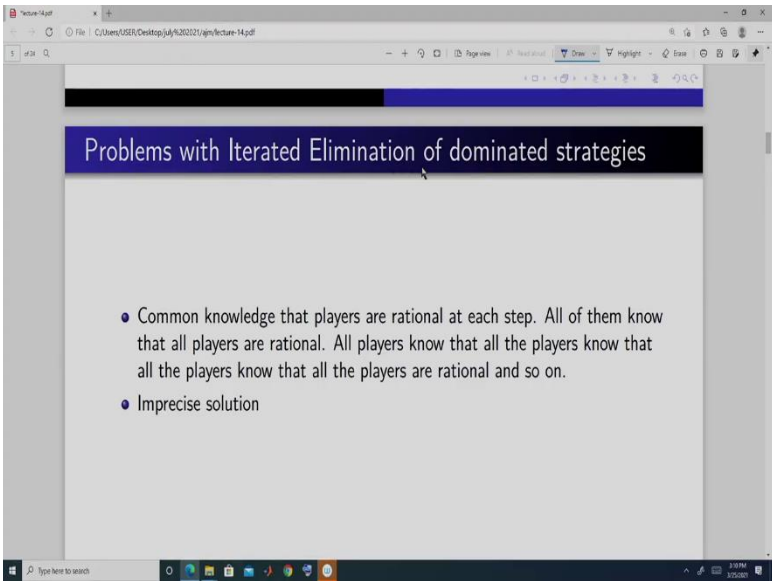

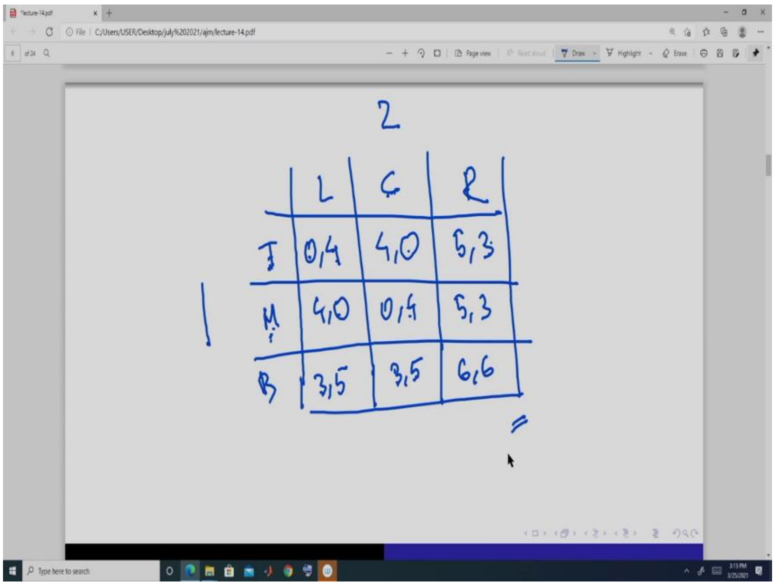

Another problem is that we always, we may get an imprecise result, that we may not be able to eliminate any strategies. Now, let us take an example of this. And the example is this. Suppose there are two players; player 2 which plays the column, player 1 which is a row player, okay. And suppose the payoffs are, for each combination of… So, these are the payoffs for each combinations of outcome. So, in T, L 0 to player 1, 4 to player 2 like this we will have all these 8 outcome, 9 outcomes and all these payoffs are defined.

Now, if we look at this game player 1 has 3 strategy or 3 action T, M, B. Player 2 has 3 strategies or action L, R, N. Now, if you compare this R and C, (0, 3); (4, 3); (5, 6). This does not dominate this neither this dominates this, okay, because 4 is greater than 3. 5 is less than 6 and 0 is less than 3. Again if you compare this (4, 0); (0, 4). So, we cannot say L is dominated by C or C is dominated by C. Now, compare L and R. 4, 3 greater than this. This is less than this. So, we cannot eliminate any of the strategies of player 2 in this here.

Now, look at the player (1; 0), 4; (4, 0). So, we cannot say T is dominated by M or M is dominated by M. Now, compare this (4, 0; 3). So, neither M dominates by B nor B dominates M. Now, compare T, B; (0, 3); (4, 3). So, neither B dominates T nor T dominates B. So, here if we use this solution concept that is iterated elimination of nominated strategies we cannot choose any action or any strategy. So, player 1 and player 2 is going to stuck here. They cannot eliminate any strategies. So, they will not be able to choose any actions. So, we need to improve our solution concept.

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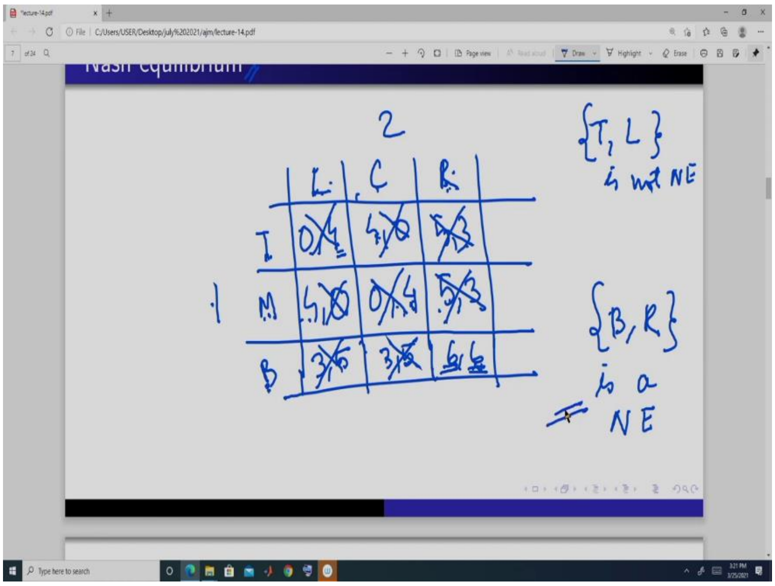

So, here comes this solution concept, and that is Nash equilibrium, okay. And we will find the Nash equilibrium of this game, okay. So, let us take this game again. L,C, R for player 2; this is T, M and this is B. (0,4); (4, 0), (5,3); (4, 0), (0,4); (5, 3); (3, 5); (3, 5); (6, 6), okay. Now, here suppose player; how do we play the game if we are using Nash equilibrium concept? So, player 1 thinks that suppose I choose T. Then what player 2 is going to do? So, player 2 will compare; if he plays L it will get 4; if he plays C it will get 0; if he plays R it will get 3. So, the best response for this T is to play this, that is L because 4 is greater than 0 and 3.

Now, suppose player 2; player 1 things that if I played T then player 2 is going to choose L. Now, if player 2 is going to choose L then what is my best response? So, best response is not T but it is M. So, therefore this T and L is not Nash equilibrium, okay because if player 2 plays

L I am not going to choose T; instead I am going to choose M. So, this cannot be a Nash equilibrium, okay.

Now, so we can also strike off this and this outcome because whenever player 1 plays T player 2 chooses L not C nor M. So, these are not an outcome, right. Now, here if you look at this, if player 2 plays this L, player 1 chooses M. So, we can also remove this. So, this is not a Nash equilibrium. This outcome is also not a Nash equilibrium. This is not a Nash equilibrium. This is not a Nash equilibrium, right.

Now, suppose player 1 chooses M, suppose. Then player 2, the best response, what is the action that gives player 2 the maximum payoff? It is 0, 4, 3; out of this it will always choose C because 4 is highest. So, player 1 when it chooses M, best response for player 2 is to choose C.

Now, when player 2 chooses C best response for player 1 is to choose T because T gives him maximum; 4 is greater than 0 and 4 is greater than 3, this. So, this (0,4) is not a Nash equilibrium, okay. And we know this is already not a Nash equilibrium; because when player 1 chooses T, the best response of player 2 is to choose L. What is the best response? The strategy or action which gives him or her the maximum payoff.

Now, suppose player 1 chooses B. If he chooses B; 5 from playing L, 5 from playing C and 6 from playing R. So, player 2 chooses R. And if player 2 chooses R; player 1 if it plays T it will get 5, if it plays M it will get 5, if it plays B it will get 6. So, this is maximum. So, it is this. So, when player 1 chooses B, player 2 chooses R. And when player 2 chooses R player 1 chooses B. So, this outcome, this is Nash equilibrium, okay.

Now, here you know when player 2 is playing R player 1 is choosing B not M. So, this is not a Nash equilibrium. When player 1 is choosing B player 2 is choosing R not C. So, this is not a Nash equilibrium. And we also know that this is not a Nash equilibrium because when player 1 chooses M best response is to play C for player 2 because …

So, in all these outcomes we see that there is a tendency for any one of the player to deviate. So, if… but here we see that there is no tendency to deviate. So, if player 1 chooses by B, player 1 is going to choose R because it gives him the maximum payoff. And if player 2 chooses R player 1 is going to choose B because it gives … So, that is why this is a Nash equilibrium. So, this is the idea of Nash equilibrium, okay.

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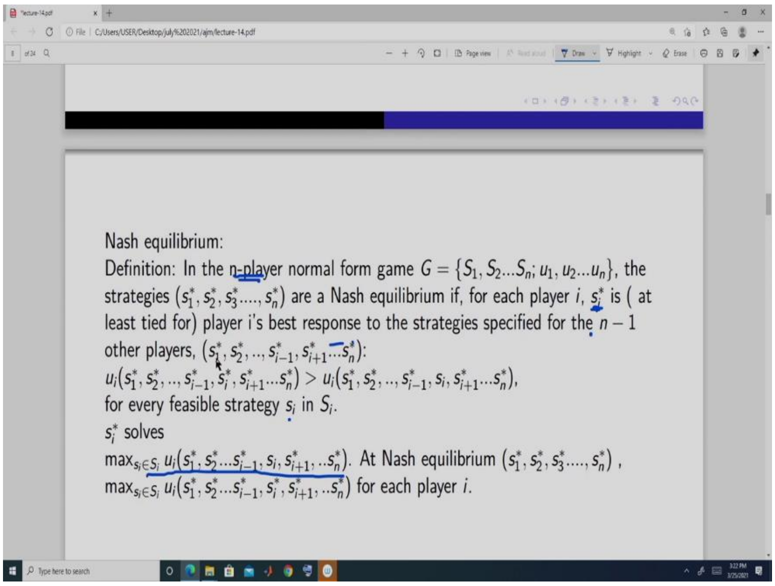

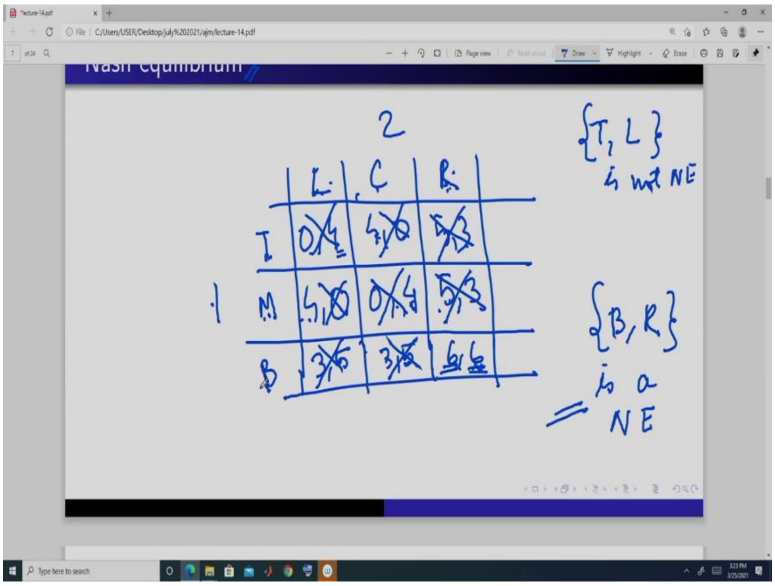

So, formally we define Nash equilibrium in this way. In a normal form game where there are n players, so the game is represented in this way- $G = (S_1, S_2, ... S_n . u_1, u_2 ... u_n)$; so there are n players, each player has a strategy space or action space given by S, capital S, and there is a payoff defined for each of this outcomes u1 for player 1, u2 for player 2 like this.

So, the strategy this- $(S_1^*, S_2^*, ... S_n^*)$, so there are many strategies like each for one player, so there are strategies; are a Nash equilibrium, it is Nash equilibrium if for each player i, si that is here is at least tied for, is player i's best response to the strategy specified for the n minus 1 other players. So, here if you look at these strategies, so these are the strategies of all other players except for the i.

Now, here if all others are playing this then player i should play this strategy. So, it means that the payoff from paying this si star is more than any other element that is there in S other than star, okay. If we plug here si star then it should be equal. But for any other it should be greater, okay for every feasible strategy si in this. So, it means that si solves this problem, that it maximizes the utility given all others are playing this strategy, okay. And it should be true for all players.

So, when player 1 is choosing so si star, si star should maximize the payoff of player 1 when all other players like player 2, player 3, player 4 they are choosing… like player 2 is choosing s2 star, player 3 is choosing s3 star, player 4 is choosing s4 star, like this, okay. So, it is actually a maximization problem for each player, okay.

So, as we have done here like when this is played by player 1 what is the action of player 2 that gives him the maximum payoff? It is R. And when player 2 chooses R it is B which is… So, we have fixed this and we have found the best strategy of player 2 and it is R. Now, we have fixed R and then we have seen whether what is the best strategy for player 1 and it is B. So, that is why this is a Nash equilibrium, okay. So, in a Nash equilibrium we see that there should not be any tendency to deviate, okay.

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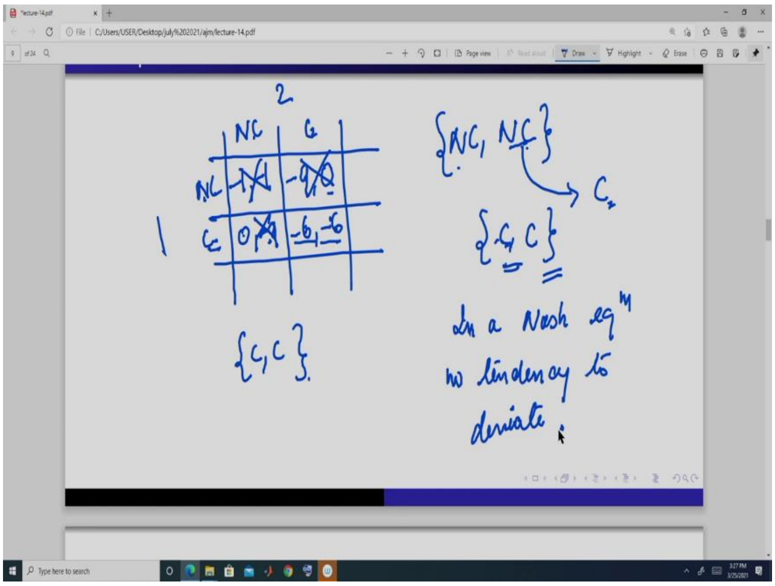

So, now let us do some example. So, let us do the Prisoner's Dilemma example. So, Prisoner's Dilemma we have already done. This is for player 1 and this is for player 2. This, now here, if player 1 chooses NC that is not confess then player 2 will compare minus 1 and 0. So, 0 is greater. So, it is going to choose this C. When player 2 chooses C player 1 will compare minus 9 and minus 6, because if he chooses NC it will get minus 1 and if it chooses C it will get minus 6. So, it will choose this.

So, this is a Nash equilibrium because if we choose, player 1 chooses this player 2 is going to choose minus, compare minus 9 and minus 6 and it will get this. So, one Nash equilibrium is this. Now, from here it is obvious that this is not a Nash equilibrium and here it is obvious because if player 2 chooses C it deviates here. This is also not a Nash equilibrium because if player 1 chooses C player 2 chooses here, C. And this (-1, -1) is not a Nash equilibrium. Why? Because when player 1 chooses NC player 2 chooses C, so not this.

So, what do we see? That if we take this outcome {NC, NC}, suppose. So, player 1 is choosing NC player 2 has a tendency to deviate from NC. And it will deviate to C. But here if you look at this, when player, and when player 2 is choosing NC, is choosing C, player 2 is suppose choosing C here then player 1 is not going to choose this but it is going to choose this {C, C}, okay. So, we get this {C, C} outcome. So, this is a Nash equilibrium.

But if you look at this here when player 1 is choosing C, this; player 2 if it is choosing C it has no tendency to switch from C to NC. And when player 2 is choosing C and player 1 is choosing C, player 1 has no tendency to switch from C to NC. So, here we see that in a Nash equilibrium no tendency to deviate, okay. So, none of the player has any tendency to deviate from it, okay. So, that is one another criteria to check whether outcome is Nash equilibrium or …

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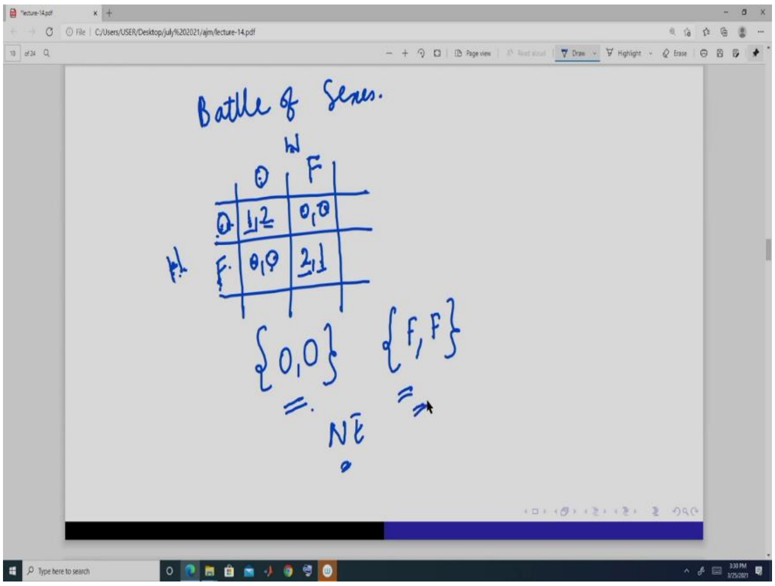

Let us do another game and that is, this is very which is called Battle of Sexes. Suppose there are two, there is a couple, husband and wife. So, this is husband and this is wife. They have to choose from two actions that is either they can go to watch an Opera or they can go and watch a Fight, Opera and Fight, okay. So, husband prefers Fight and woman prefers Opera. So, the payoffs are of this nature.

So, if both of them are in Opera Opera, woman the wife gets 2 and husband gets 1. If husband is in Opera and wife is in Fight since they want, prefer to be together so they get (0, 0) payoff. If the husband is in Fight and women is in Opera they again R not in the same place together. So, they get (0, 0). But if both of them are in Fight husband gets more payoff than wife.

Now, here you will see that if player 1 plays, if husband plays Opera best response for the wife is to choose Opera, and if wife, because 2 is greater than 1. If wife chooses Opera best response for husband is to choose Opera because 1 is greater than 0. So, one Nash equilibrium is this, this {0,0}, okay.

Next, so this is one Nash equilibrium. Now, if husband wife, or husband chooses F, best response for the wife is to choose Fight because 1 is greater than 0. If wife chooses Fight best response for husband is to choose Fight because 0 is less than 2. So, again this is an… So, we have two Nash equilibrium in this game, in this Battle of Sexes game, okay.

And here you can see that if we are in this outcome player 1 choosing Opera and wife also choosing Opera then there is none of the player has any tendency to deviate because player 1, if wife has chosen Opera and husband has also chosen Opera. Husband does not have any incentive to choose, shift from Opera to Fight because 1 is greater than 0. If you are looking at this, this outcome, if player, if husband chooses Fight; player 2, that is wife has is also choosing Fight so the outcome is this, and it has no tendency to shift to O because 1 is greater than 0. So, this, so we can see that at Nash equilibrium, so we will see no tendency to deviation, of deviation.

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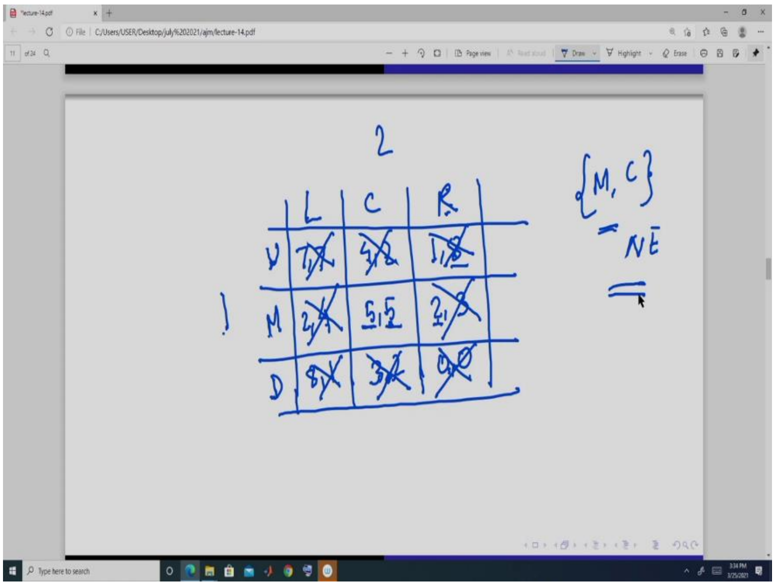

So, let us do another example. So, let us take a slightly bigger game. So, there is player 2 and its actions are L, C and R. Player 1, its action is U, M and D, okay. Now, we have to find the Nash equilibrium. How do we do? Suppose player 1 is playing U, okay. So, this is again a simultaneous move single shot game. So, each player is going to choose only once and they are going to choose their actions simultaneously and only once, okay.

So, player 1 while choosing which action to choose, it will do this mental calculation. So, player 1, if chooses U, best response for player 2 is to play R because 7, 2, 8; 8 gives the maximum. So, it is choosing R. Now, if player 2 chooses R best response for 1 is to choose M not U. So, this is not a Nash equilibrium. So, it is going to choose this, okay. Now, if player 1 chooses M, player 2 best response is to choose C because 4, 5, 3; out of that 5, C gives the maximum. So, it will choose 5.

And if player 1; so if player 2 chooses C, best response for player 1 it will compare 4, 5 and 2. So, that is, U gives 4, M gives 5, D gives 3. So, player 1 is going to choose M. So, this is a Nash equilibrium; {M, C}. Because when player 1 chooses M best response for player 2 is to choose C. When player 2 chooses C best response for player 1 is to choose M. So, this is a Nash equilibrium.

Now, we have to check whether there exist any other Nash equilibrium or not, okay. So, from here we know that this {4,2}is not Nash and this {3,2} is not Nash, okay. Again, we know already this is not; because if player 2 chooses M it chooses this. So, this {2,4} is also not a Nash equilibrium. If player 1 chooses D, best response is to choose this {8,1} for player 2. So, this is not a Nash equilibrium. If player 1 chooses U we know already it is choosing this. So, this is not a Nash equilibrium because U, L there is a tendency for player 1 to deviate, for player 2 to deviate from L to R if player 1 chooses U.

Here this is again not a Nash equilibrium. Why, because if player 1 chooses D player 2 has a tendency to deviate from this outcome and it will choose C. So, there is unique Nash equilibrium and that is M, C. Player 1 is always going to choose M and player 2 is going to choose C, okay. So, these are some of the examples that we are doing try to understand he Nash equilibrium, okay.

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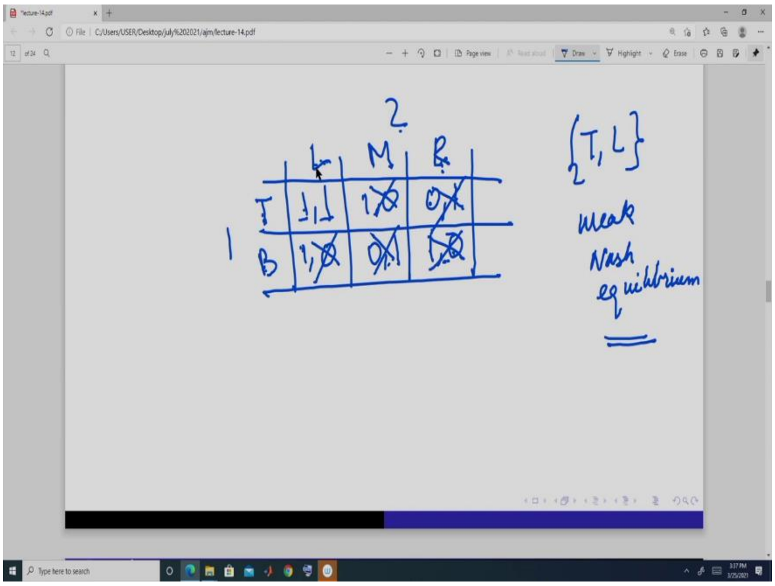

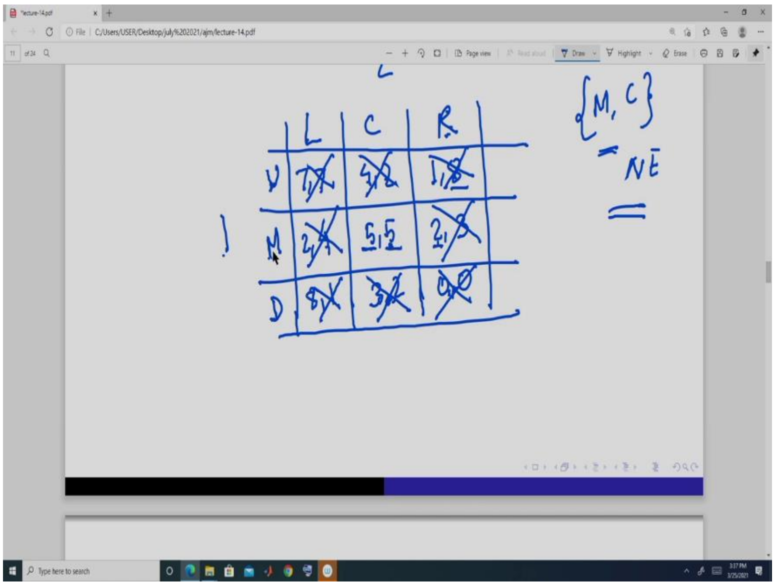

Now, let us do one more example and then we will … So, this is for player 1. This is for player 2. L, M, R and this T and B. Now, here notice if player 1 plays T best response for player 2 is to L 1, R 1. So, player 2 is indifferent between L and R. It is greater than M but it is same, okay. Now, suppose player 2 chooses L, then player 1 is again indifferent between T and B because {1, 1}, okay.

So, this {T, L} you can say is a weak Nash equilibrium, okay; because when player 1 plays T, player 2 is indifferent between L and R. When player 2 plays L, player 1 is indifferent between T and B. But it is in same payoff. But if player 1 plays B player 2 does not play L, it deviates to M. So, this is not a Nash equilibrium, okay. And player 2 plays M player 1 switches to T not … So, this is not a Nash equilibrium. If player 1 plays B this, player 2 is going to switch to

M because 1 is greater than, so this {1,0} is again not a Nash equilibrium. If player 1 is choosing, so this we know this is not a Nash equilibrium.

Again here if player 1 is choosing R, player 1 is going to choose B. So, this is not a Nash equilibrium. So, again the unique Nash equilibrium here is T, L and it is a weak Nash equilibrium, okay. So, we can have both strict Nash equilibrium where this here it is weak because T and B is giving same payoff. And again L and R is giving the same payoff. So, that's why in that sense it is a weak.

But here if you look at this game here M, C here it is strict because if it plays for M; 4, 5, 3, so 5 is strictly greater than 4 and 3. So, that is why it is a strict here. And if player 2 plays C; 4, 5 and 3, again this 5 is a strictly greater than 4 and 3. So, that is why it is a strict Nash equilibrium, okay. So, we can have both strict Nash equilibrium and also we can have weak Nash equilibrium, okay.

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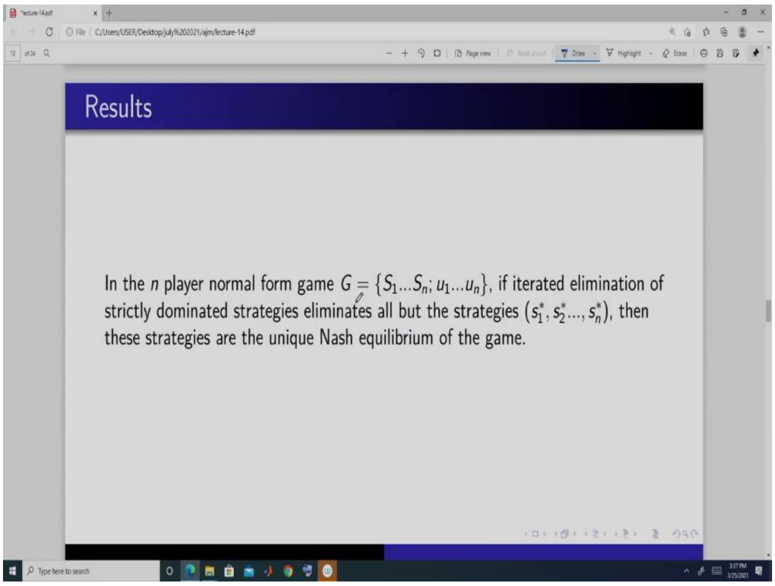

Now, we discuss one result. That is in n player game which is a normal form game where it has this strategy sets S1, S2, Sn, i.e $G = (S_1, S_2, ... S_n. u_1, u_2 ... u_n)$; and the payoff function u1, u2, un. If iterated elimination of strictly dominated strategies eliminates all but the strategies this, so we are left with only one outcome this then these strategies are the unique Nash equilibrium of the game, okay. So, we will not prove this result because this course is an introductory course. So, but we will do a more or less general example to show this result, okay.

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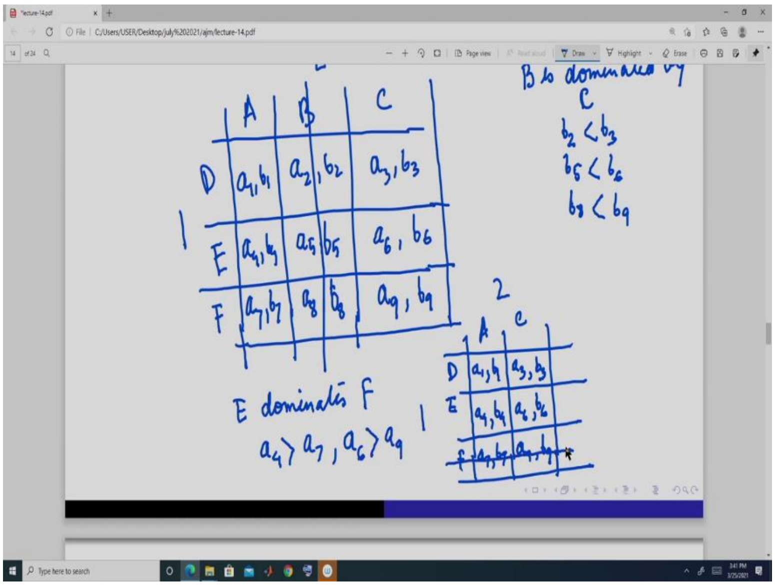

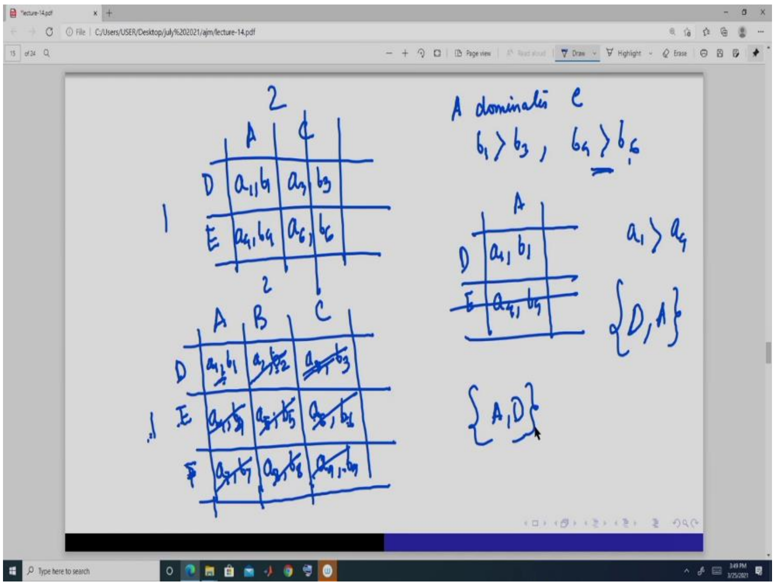

Now, let us take a game where there are two players; player 2 and player 1, and suppose the strategy is A, B and C. And here strategies of player 2 is D, E and F. And the payoffs are (a, b1); (a2, b2); (a3, b3); (a4, b4); (a5, b5); (a6, b6); (a7, b7); (a8, b8); (a9, b9), okay. Suppose C is dominated, or suppose for A, B is dominated by C. So, if B is strictly suppose dominated by C then it means what? b2 is less than b3; b5 is less than b6; b8 is less than b9. So, that is why we remove this, okay.

And next the reduced form is A and C; D; (a1, b1); (a, b3), (a4, b4); (a6,b6); (a7, b7); (a9, b9), okay. And suppose this E dominates, dominates and suppose here E dominates F. So, this means that a4 is greater than a7 and a6 is greater than a9. So, we remove this. So, we end up having this, okay, and player two and suppose A dominates C. So, this means b1 is greater than B3 and b4 is greater than b6. So, we are now…

So, A, D and E; a1, b1; and a4…. (a1, b1); (a4, b4). And suppose a1 is greater than a4. So, this B dominates E. So, we are left with this. So, the outcome of iterated elimination of dominated strategy is D and A and we have eliminated them in this process. That is first we have eliminated B, then we have eliminated F, and then we have eliminated C, and after that we have eliminated E and we end up here. And in this elimination process we have… why it is possible? Because of this X, because of this condition- $\mathrm{b}_2 < \mathrm{b}_3, \mathrm{b}_5 < \mathrm{b}_6, \mathrm{b}_8 < \mathrm{b}_9$; this condition- $\mathrm{a}_4 > \mathrm{a}_7, \mathrm{a}_6 > \mathrm{a}_9$, this condition- $\mathrm{b}_1 < \mathrm{b}_3, b_4 > b_6$ and this condition- $\mathrm{a}_1 > \mathrm{a}_4$, okay.

Now, we find the Nash equilibrium of this game using these conditions, okay. We are in this. Now, we know from this first A that b3 is greater than b2. So, if player 1 plays D, b3 is greater than b2. So, it is not going to choose this. So, this is not. B3 is greater than b2. So, it is not going to choose this. So, we are here. Again we know that b1 is greater than b3 so it is going to choose this if player 1 chooses D.

Now, if player 2 chooses A, from this we know a4 is greater than a7; a4 is greater than a7. So, this is not a Nash equilibrium. And again we know a1 is greater than a4. So, this is not again… so this is a Nash equilibrium from here because if player 2 chooses A best response is to choose D because a1 is greater than a4 and a7. If player 1 chooses D it is best response is to choose A because b1 is greater than b2 and b3, okay. Now, we can remove so that these are also not a Nash equilibrium. Why? Because if player 1 is choosing E since this is A, so b4, b5 is less than b7 so this is not a…. Now, here it is choosing this.

Now, if player 1 is choosing E and the player 2 is choosing C, this because, suppose it is choosing this. Then we know if we compare this; a6 is greater than a, so this is greater than this, okay. So, it is going to choose this. Now, here we have to compare from this a6 and a3, okay. Now, if player 1 chooses E, player 2 is not going to stick with this. So, this, it will shift to this. Again if we compare b4 and b6, from here we know that b4 is more than b6, so it will choose this. And from here we know that this is, a1 is greater than or not. So, this is not a Nash equilibrium.

Now, here if player 1 chooses F we know b9 is greater than b8 so this is not here. So, if it chooses this, from here we know that a6 is greater than a9. So, this it will be deviate to this. And again from here we know this is not a Nash equilibrium. So, this has a unique Nash equilibrium and this outcome is {A, D}.

So, these two things matches if we look at… So, this result actually holds for a general two player and three strategies. And if you look, if you do, we can actually prove it for a general case for taking k Strategies and n player but that is going to be more involved so we are not doing it here. But this is a simple proof.

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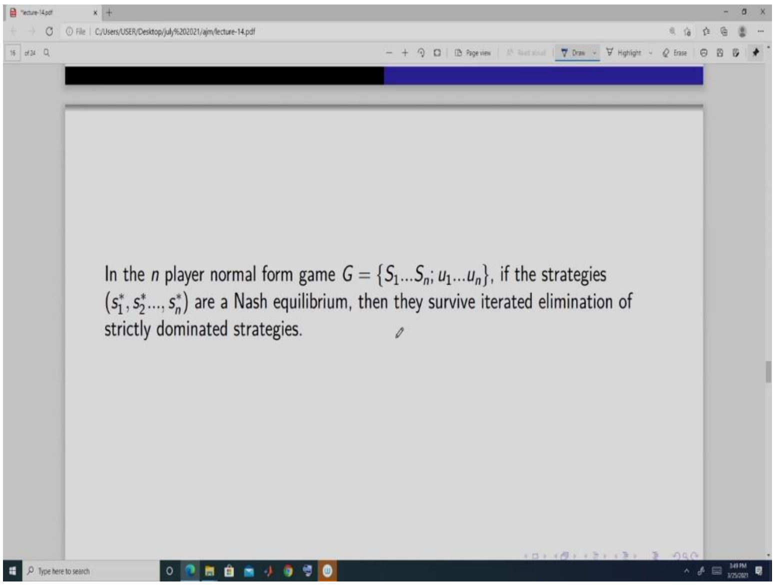

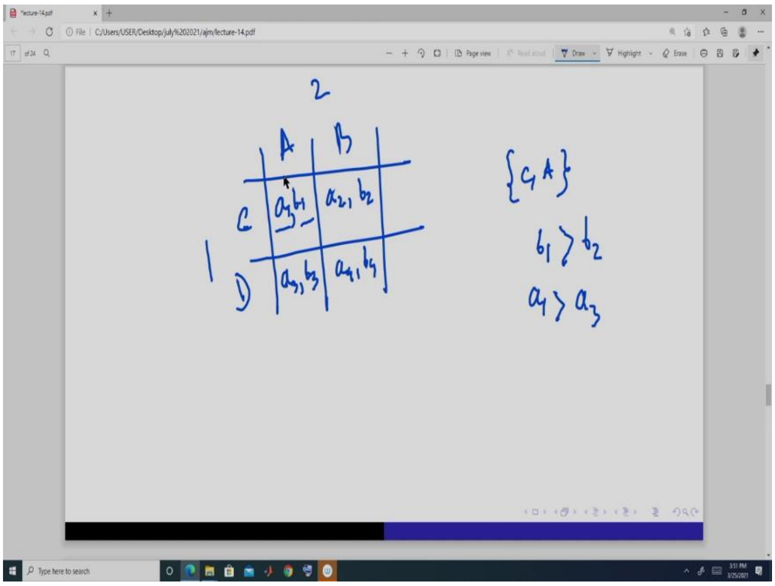

Now, we can have a converse of this also. So, if we have a Nash equilibrium, then of a game, if we have a n player game and this is the game and if there is a Nash equilibrium this, then this will always survive the iterated elimination of strictly dominated strategies. So, one example, simply take this example suppose A, B, C, D; this is for player 1; this is for player 2; take like this.

Now, suppose this {C, A} is a Nash equilibrium. So, if this is in Nash equilibrium it means what? b is greater than b2, and a1 is greater than a3. Now, if b1 is greater than b2, then this in this A this is greater than this. So, we cannot remove, eliminate A because b1 is greater than b2. And you again cannot eliminate C because a1 is greater than a3. So, therefore this survives the iterative elimination of strictly dominated strategies. So, that is why this is an example of this result, okay.

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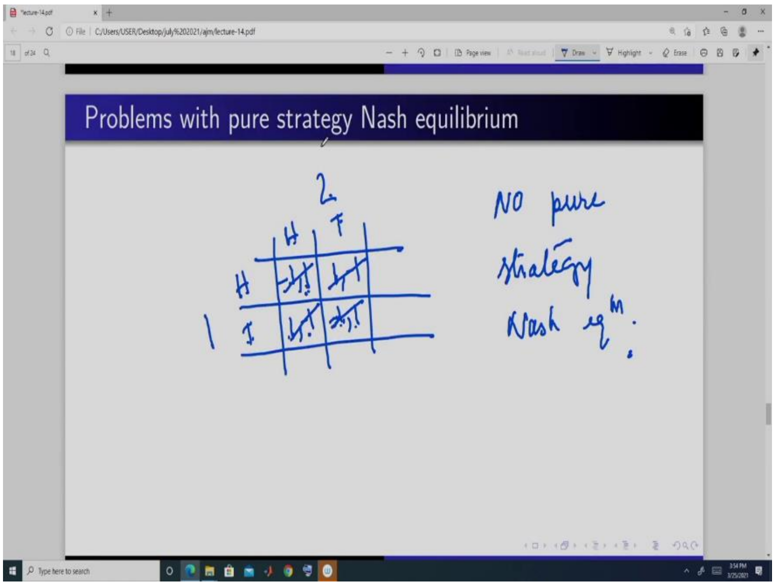

Now, we want to see what is the… So, can we find the Nash equilibrium in all the situations? So, let us give you an example. Let me give you an example where we do not find a pure strategy Nash equilibrium, okay. And this is the famous Matching Penny example that we have done in the first class on the Game Theory.

Suppose this is player 1 this is player 2; and they have two actions that is head and tail, okay. So, these are the players. So, if player 1 plays head and player 2 plays head; player 1 gets minus 1, player 2 gets 1. Head and tail player 1 gets 1, player 2 gets minus 1. If player 1 plays tail and player 2 plays head player 1 gets 1 and player 2 gets minus 1. If player 1 plays tail and player 2 also plays tail, player 1 gets minus 1 and player 2 gets 1, okay. So, this is the game.

Now, here we will see that there is no pure strategy Nash equilibrium So, if player 1 plays H then the best response for player 2 is to play H, 1 and minus 1. When player 2 play H, best response for player 1 is to play tail because minus 1 and 1, okay. So, it will deviate. So, this H, H is not a Nash equilibrium.

Now, if player 1 plays tail best response for player 2 is to choose, compare between these two head and tail and it is tail rather than head, so this (1, -1) is not a Nash equilibrium. And if player 2 play tail best response for player 1 is to choose head; 1 is greater than minus 1. So, this (-1, 1) is not a Nash equilibrium. And if player 1 plays head best response for player 2 is to choose head; because 1 is greater than this, so this is… So, we do not have pure strategy. No pure strategy Nash equilibrium. Pure strategy here means choosing its action like head or tail or again head or tail, okay.

So, we will now define strategies and we will attach probabilities to it. And if the probabilities are 1 and 0, 0 then it is a pure strategy. And if there is some probability to some of the strategies then it is a something called a mixed strategy. So, here in this game we are going to find the mixed strategy Nash equilibrium.

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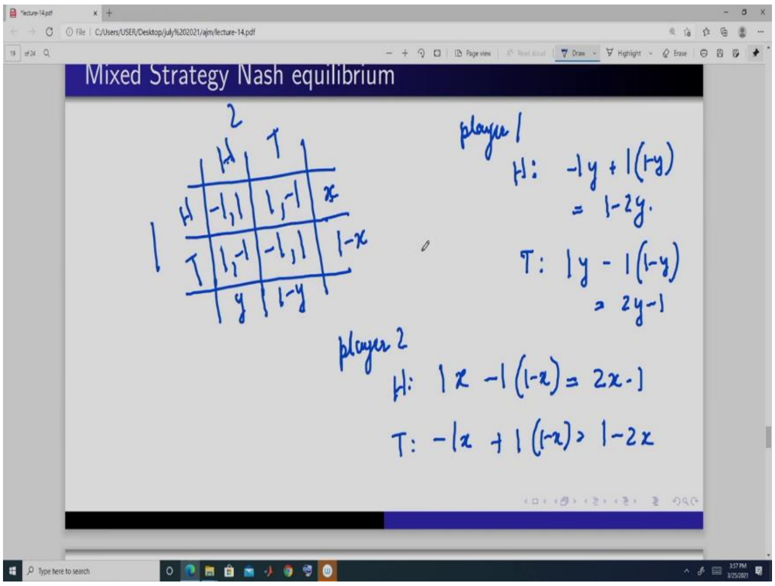

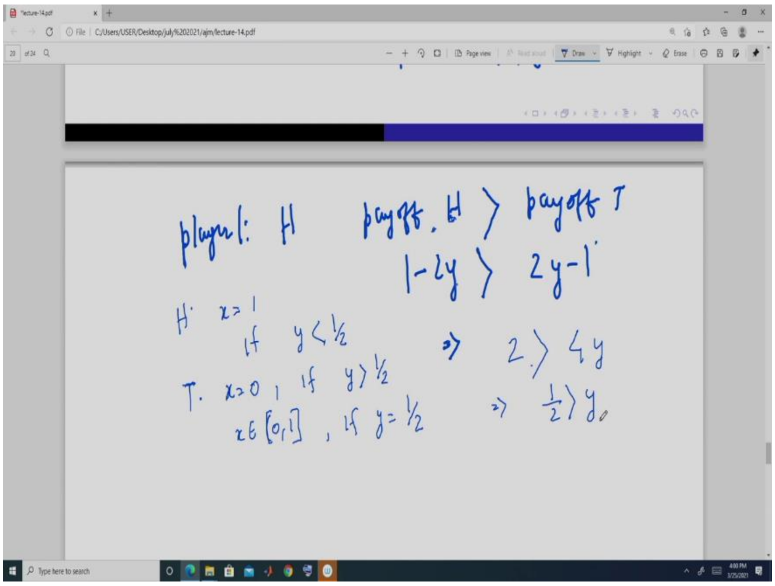

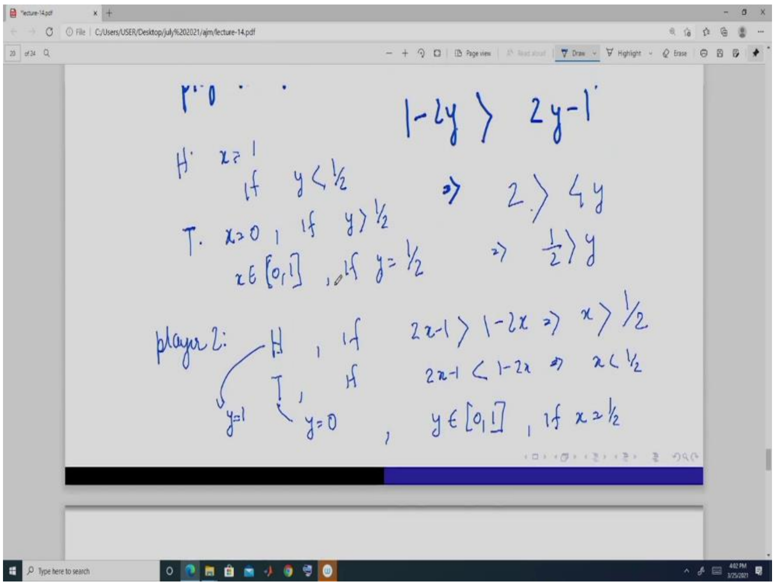

So, in mixed strategy what happens? Each player attaches some probability to their actions or their strategies. Suppose player 1 attaches probability x to this action each and it attaches 1 minus x to this. And player 2 attaches y to this strategy H and 1 minus y to the strategy, okay. Now, if 1 player 1 plays head; for player 1, okay so head, if it plays head then it will get minus 1 with probability y and it will get 1 with probability 1 minus x. So, the payoff is 1 minus 2y if it gets head. And if it plays tail it will get 1 with probability y and minus 1 with probability 1 minus y. So, the payoff is 2y minus 1, okay. So, this is the payoff for player 1.

Now, let us calculate the payoff of player 2. Player 2 if it plays head, if player 2 plays head it will get 1 with probability x with probability 1 probability and minus 1 if with probability 1 minus x. So, this payoff is 2x minus 1, i.e H: 1x-1(1-x)= 2x-1. And if it plays tail it will get minus 1 with probability x and plus 1 with probability 1 minus x. So, the payoff is 1 minus 2x, i.e T: $-1x+1(1-x)= 1-2x$.

Now, when a player is going to attach probabilities? Now, if, for let us look here if player 1 is going to play head; player 1 is going to play head if payoff from head is greater than payoff from tail; that means if 1 minus 2 Y is greater than 2y minus 1. This means that whenever, whenever 2 is greater than 4 y. So, that means whenever y is less than half, player 2 is going to choose each for sure because his payoff is greater than… And whenever…

So, this is going to play, player 1 is going to play head always. So, it is going to play head with probability x is equal to 1, right, if y is less than half. And it is going to; from here it is going to play tail that is x is equal to 0 if y is greater than half. And it is going to attach some probability that is x is going to lie between 0 and 1 if y is going to be equal to half, okay.

So, we can derive this because whenever this is greater than this so it is going to always play head. So, x is equal to 1. x equals to 1 means it is always going to be a head. If y is greater than half then this is just opposite sign, right. So, tail is giving the player 1 more. So, that is why player 1 is always going to play tail. So, x, means it is going to attach 0 probability to this. So, it is this.

Now, if y is equal to half so these two are equal. So, it will attach any probability here, okay. So, we get this, okay. So, in this situation we get that whenever y is equal to half then only x takes some positive probabilities here. Otherwise x will either take 1 or it will take 0, okay. Next we look at the player 2. Player 2 plays head if this- $2x-1$ is greater than this- $1-2x$. So, player 2 plays head if 2x minus 1 is greater than 1 minus 2x.

So, this means x is greater than half. And it plays tail if x is less than half. So, it means it will always play head. So, in this case what is happening? Here y is always equal to 1 and here it means y is always equal to 0. And y will be taking value, any value between, any value between 0 and 1 if x is equal to half, okay. So, if you look at these two this- [H: $x=1$, if $y<1/2$, T: $x=0$, if $y>1/2$, $x \varepsilon [0,1]$, if $y= ½$] is something called the reaction function of firm 1. And this- [H: $y=1$, if $2x-1>1-2x$, T: $y=0$, if $2x-1<1-2x$] is called the reaction function of firm 2, okay.

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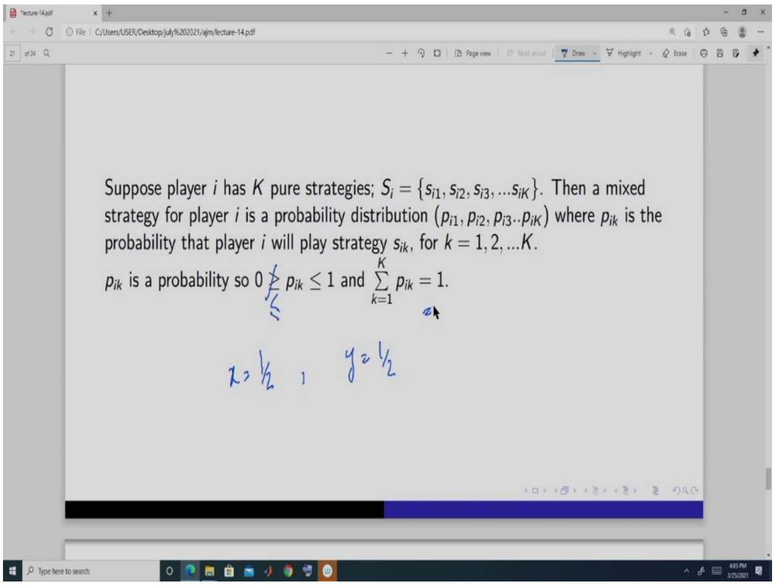

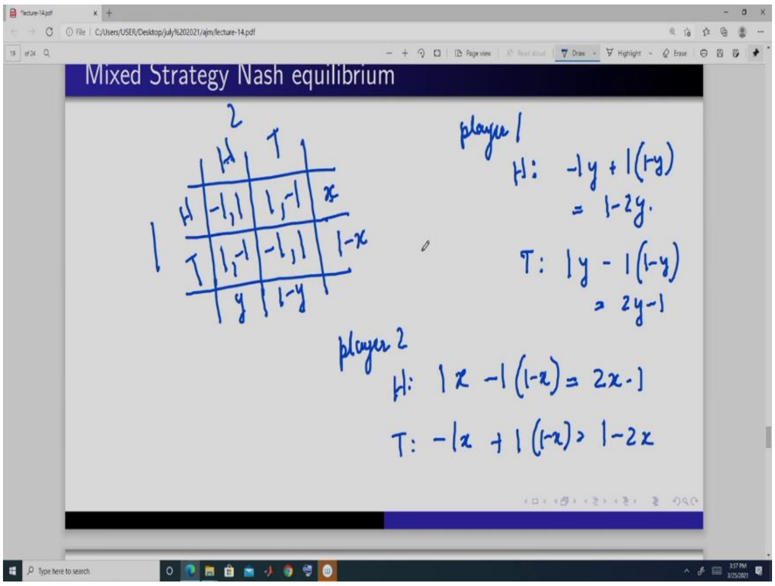

So, we see that whenever x, in this situation, we see that x is equal to half and y is equal to half is a Nash equilibrium and that is a mixed strategy Nash equilibrium, okay. So, how do we arrived at the mixed strategy Nash equilibrium? So, it is that what we do? We find that value of y where we are indifferent between head and tail. Player 1 is indifferent between head and tail. And we find that value of x where player 2 is indifferent between head and tail. And that value gives us the mixed strategy; because if we are not indifferent then we will attach the full probability to this A.

So, if H is greater than T in payoff so that is 1 minus 2y is greater than 2y minus 1 then, I will always, player 1 will always play head with full certainty. And if 2 minus, 2y minus 1 is greater than 1 minus 2y then player 1 is going to play tail with full certainty. So, when it is indifferent? When y is equal to 1 by 2. And when same for player 2 when x is equal to 1 by 2 player 2 is indifferent between head and tail. So, it is attaching some probability to this and this. Here also it is attaching some probability to this and this. So, at this point when x is equal to half and y is equal to half we get a mixed strategy Nash equilibrium.

So, formally suppose player i has K strategies so strategies set of player I is this- $S_i = \{s_{i1,} s_{i2} \ldots s_{ik}$ then a mixed strategy for player i is a probability distribution this where pi is the probability that player i will play strategy this for k is equal to 1 to this, okay. And here this should always lie between this….oh; oh this inequality should be … this is wrong, okay. And this…. So, we are attaching some probability to each action, and when it is a best response compared to the other players strategies then it is…

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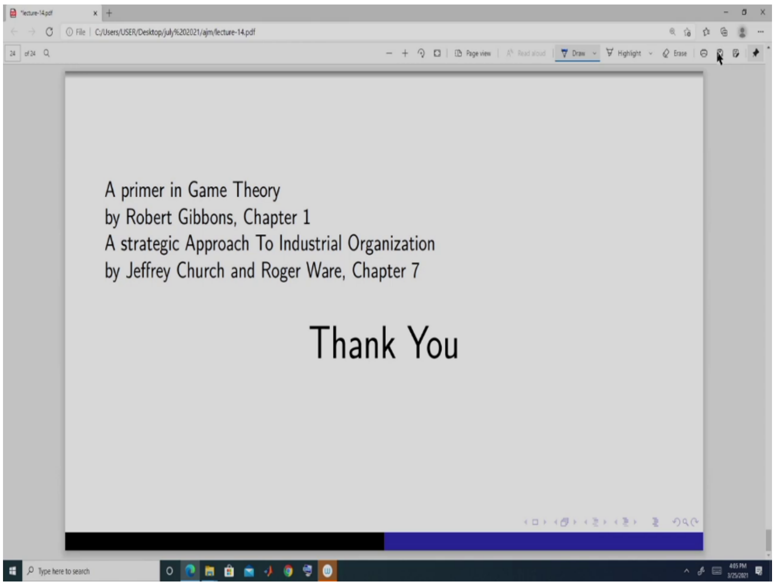

So, we will do one example in the next class, okay and for this you can read chapter 1 from Gibbons and chapter 7 from Church and Ware, okay thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 05: Game Theory
Lecture 19
Mixed Strategy Nash Equilibrium

Hello. Welcome to my course Introduction to Market Structures. So, we were doing mixed strategy. And let us do one example.

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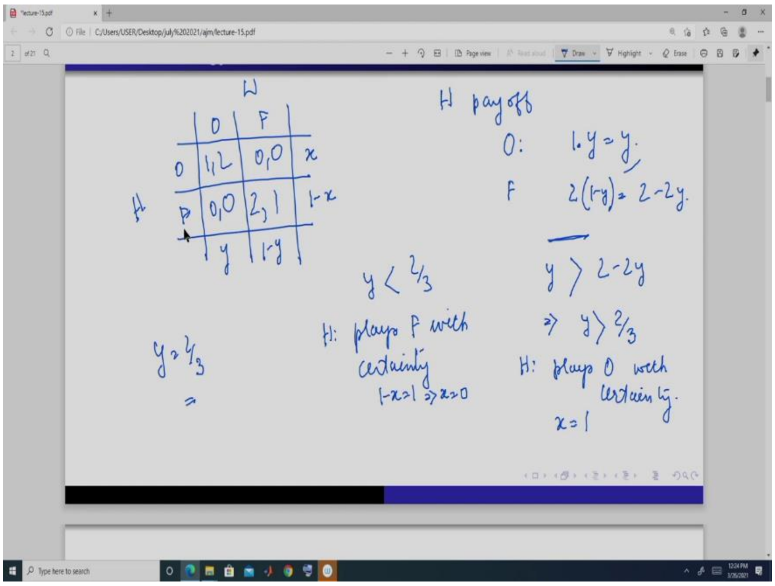

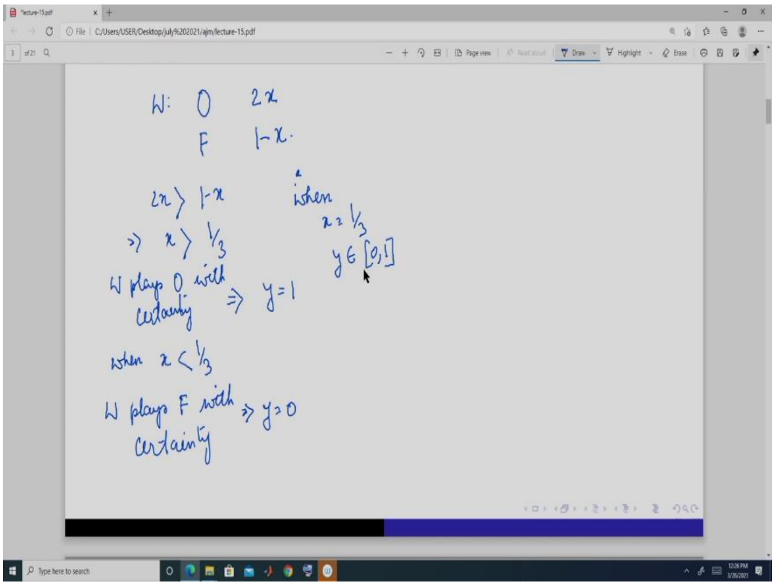

So, let us consider a game, and suppose this is a game of Battle of Sexes. We have done one version of it in the last class, right. And now we will try to find the mixed strategy in this case. So, these (1,2), (0,0), (0,0), (2,1) are the payoffs, right. Now, here suppose player 1 that is the husband attaches probability x to strategy O and attaches probability 1 minus x to the strategy F. And wife attaches y to the strategy O or action O, and 1 minus y.

Now, if you look at the, suppose husband's H payoff, okay so payoff from O is, if it plays O, if there is a chance that it will get 1 with probability y and 0 with probability 1 minus y. So, the A is payoff from playing O is 1 into y which is y. Payoff from playing F for player 1 is; it will get 0 with probability y and 2 with probability 1 minus y. So, 2, 1 minus y which is equal to, i.e $2(1–y)= 2–2y$, okay….

So, this is the payoff for the husband. And when husband is always going to play O? When y is greater than 2 minus y. And this implies, since this is greater than this, so then I am not going to attach any probability to O and F. I am always going to play O. So, this is when y is greater than 2 by 3.

So, then in this case it will; H plays O with certainty, right? And if y is less than 2 by 3 that means F is, so this- $1–y=y$ is less than this-$2–2y$. So, then H plays F with certainty. What does this mean? This means that if O is played with certainty, that means x is equal to 1. And if F is played with certainty that means 1 minus x is equal to 1. So, this means x is equal to 0, right. And when it attaches some probability to it? It attaches some probability when y is equal to 2 by 3.

Whenever player wife plays O and F such that probability of playing O is 2 third and probability of playing F is one third then player 1 that is husband attaches some probability to O and F. What? We will come to it later, okay. But if y is greater than 2 by 3 then player husband is always going to play O. And if y is less than 2 by 3 then husband is always going to play F with full certainty. So, these are, okay.

Now, let us do it for wife. Wife if it plays O, if it plays O, it gets 2 with probability x and 0 with probability 1 minus x. So, payoff is 2x. And if it plays F it gets 0 with probability x and 1 with probability 1 minus x. So, it is 1 minus x. And when wife is always going to play A? When 2x is greater than this- $1–x$, so this means when x is greater than one third W plays O with certainty, okay. And when x is less than 1 by 3, W plays F with certainty. So, with certainty here it implies that y is equal to 1. Here it implies that y is equal to 0. And when it will going to attach some probability that y takes some value between 0 and 1? When and, when x is equal to one third y will take some value between 0 and 1.

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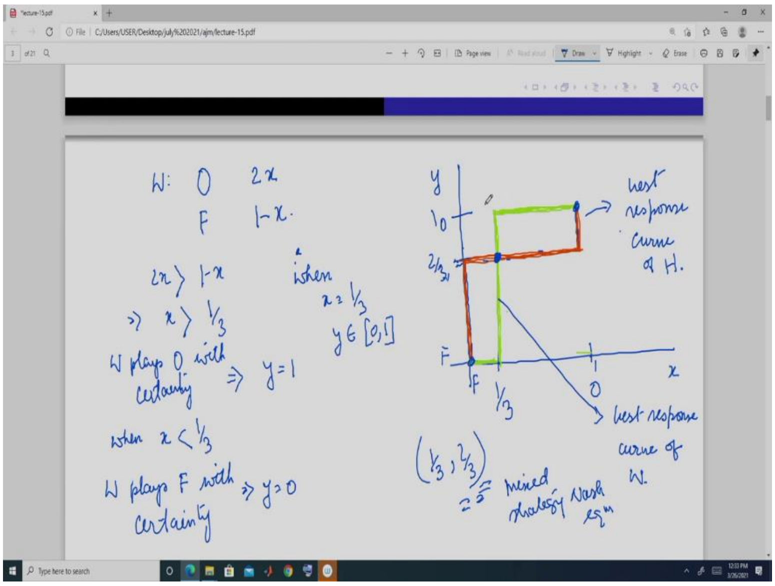

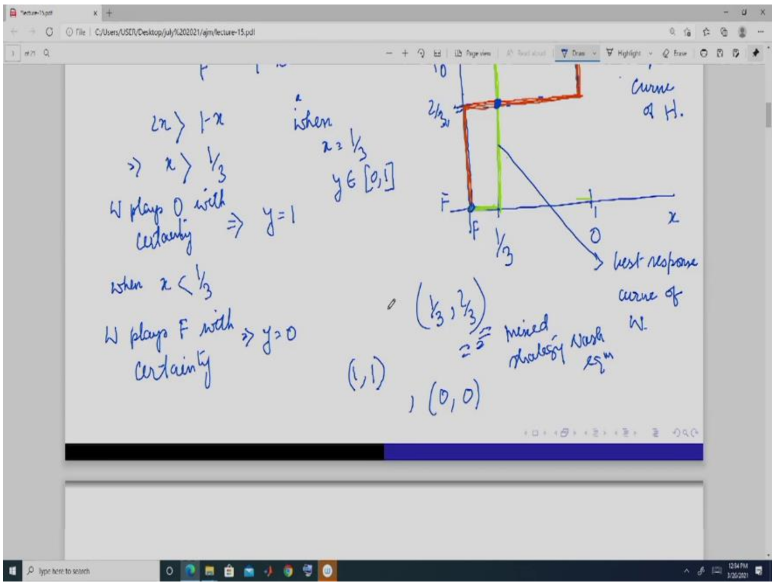

Now, we represent this whole information through a graph, okay. So, in this axis we plot x and in this axis we plot y. So, if x, so suppose this is 1, this is 1. If x takes value 1 it means what? That x is taking value 1. So, that means H is playing O with full certainty. So, this means it is playing O. So, this is O. And when x takes value 0 it means player H, husband is playing F with full certainty. So, this is F. This is 0.

Similarly for player wife; when y takes value 0 it means that it is playing F with full certainty when F, y takes value 1 then it is playing O with full certainty. And suppose this point is two third and this point is one third, okay. From this argument if y is greater than two third, when y is lying here then x takes value 1 because it plays O with full certainty. And when y takes value less than two third it plays F with certainty. So, x takes value 0. So, it is this, this region. And when y takes value two third; it, x can take any value between 0 and 1 because it is indifferent between playing O and F, right. So, this is what we can call the, this is the best response curve of H given any y how it is going to attach probability to its actions or strategies. So, it is given by this red curve, this red line, okay this.

So, when y take value greater than two third x is 1. It is 1. When x takes, y takes value less than two third, x is 0. So, it is here, any point, y any point below two third, it is y, x is 0. And when y takes value two third x can take any value in this. So, it is this. Now, let us draw the, this best response curve of wife, okay. So, when x is less than one third, so when x is here, then when x is less than one third then this, when x is less then… wife plays F with certainty.

So, that means y is equal to 0. So, y is equal to 0 means here, whenever x is less than one third, x is less than one third it is going to play F. So, it is here, y taking the value 0, here. And when x is greater than one third so it is, x is here then y takes the value 1. So, it is here. So, it is this curve. And when x takes value one third y can take any value between 0 and 1. So, it is this.

So, this green line is the best response curve of player 2, of wife, okay. And here you will see that these two curves intersect at three points; at this point, at this point and at this point, okay. This point is (F, F); this point is (O, O) and this point is when x takes the value of one third, y take the value two third. So, this is the mixed strategy Nash equilibrium.

So, this, so that means when x take the value one third player 2 is indifferent between playing O and F. So, it is attaching some probability, okay. And when player 2 is playing, attaching probability two third then player 1 is going to attach some probability to F and O. And what is that probability? Out of all these probabilities it is going to attach only this, okay when player 2 is doing this.

Now, when player 1 y, sorry, when player 1 takes one third, player 2 is going to choose any of these, right? But if it chooses anything higher; then it is, player 1 is going to choose here. If it chooses anything lower player 2 is not going to attach any probability. It is going to play this. So, that is why player 2 when it is attaching any probability from this range it is going to attached two third, so this.

So, that is why this is the point of intersection of two best response curves and so this is the mixed strategy Nash equilibrium, okay. And these 2 are degenerated Nash equilibrium because here x is taking value 1 and y is taking value 1. So, if you represent it in terms of probabilities x taking value 1 y taking value 1; this is also a Nash equilibrium. And x taking 0 y taking 0 is also a Nash equilibrium because these two curves are intersects, okay. So, this is how we find the mixed strategy Nash equilibrium.

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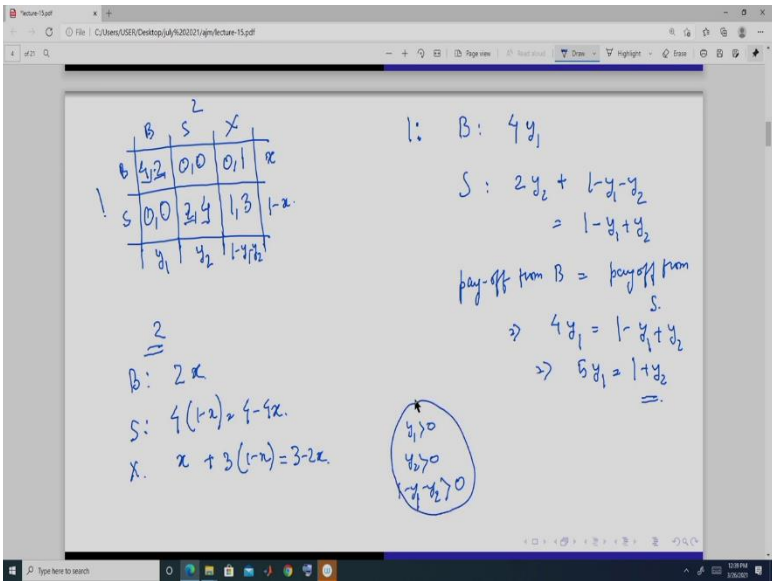

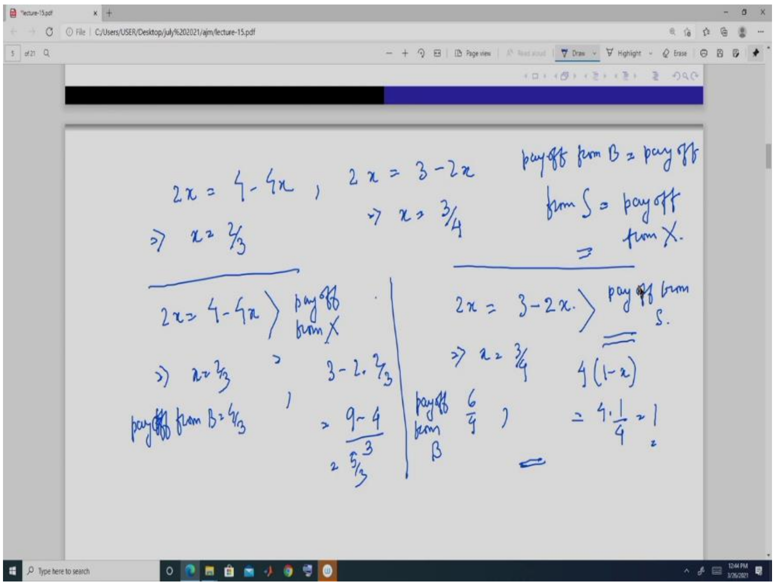

And we have shown in one case in the last class where we have, again where we do not have any pure strategy but we have a mixed strategy Nash equilibrium, okay. Now, let us do another example. So, there are 2 player. This is player 1. It is column player. And player 2 is the, sorry this is the row player and this is the column player, okay. So, this is the game. if you look at this game then this B, if player 1 plays B then the best response for player 2 is to choose B because 2 is greater than 0 and 1. And if it chooses B then it is this. So, this is one Nash equilibrium. If player 2 chooses S player 1 is going to choose S because 2 is greater than 0. And if it chooses S then it is going to choose S. So, this (3,4) is one Nash equilibrium.

If player 1 chooses this we know, so this is not a Nash equilibrium. But player 2 choose x then it is going to choose this S. And when it chooses S, player 2 is going to choose… So, there are only two pure strategy Nash equilibrium. And here we are going to check whether there is any mixed strategy Nash equilibrium or not, okay. So, this, suppose player 1 attaches probability x to this action or strategy B, and to action strategy S it at attaches 1 minus x. And player 2 attaches suppose y1 to B, y2 to S and 1 minus y1 minus y2 to x.

So, now let us write the payoff for player 1. Player 1's payoff, if it plays B it is going to be 4, it is simply going to be 4 y1, because rest are (0, 0). If it plays S then it is going to be 2 y2 plus this, i.e $\mathrm{S} = 2\mathrm{y}_2 + 1 - \mathrm{y}_1 - \mathrm{y}_2$. So, this is equal to- $1 - \mathrm{y}_1 + \mathrm{y}_2$, okay. And from here we know, when know when player 1 is going to attach some probability; x taking some positive value and 1 minus x also taking some positive value when payoff from B is equal to payoff from S.

Otherwise if suppose payoff from B is greater than payoff from S then it is always going to choose B. If payoff from S is greater than B then it is always going to… So, we will attach some probability only in this sequence. So, this means that 4 y1 is equal to 1 minus, i.e- $4\mathrm{y}_1 = 1 - \mathrm{y}_1 + \mathrm{y}_2$ So, this means 5 y1 is equal to 1 plus y2, okay. So, remember this- $5y_1 = 1 + y_2$

Now, let us find the payoff of player 2, player 2. If player 2 plays B its payoff is 2 with probability x so it is 2x. And it is 0 with 1 minus x so it is 2x. If it plays S its payoff is 0 into x and 4 into 1 minus x, i.e $\mathrm{S} = 4(1 - x)$. So, this is 4 minus 4x, okay. If it plays X, capital X then it is 1 with x; plus 3, 1 minus x. So, it is 3 minus 2x, i.e $X = x + 3(1 - x) = 3 - 2x$, okay.

Now, first here we have to identify. Suppose player 2 attaches some probability to all these three strategies or all the three actions. When, when it will attach? Some positive, so some number which is y1 is positive, y2 is positive and y minus, this- $1 - y_1 - y_2$ is also positive, fine, when the payoffs are same in all these three strategies. Because if suppose a situation is that if payoff from B is greater than these two then player 2 is going to play B with full certainty. That means y1 is taking value 1 and the rest (0, 0).

So, we have to now check for this situation- $y_1 > 0, y_2 > 0, 1 - y_1 - y_2 > 0$. First we will check this. So, it is 2x must be equal to 4 minus 4x, i.e $2x = 4 - 4x$. So, then this means x is equal 2 by 3, i.e $x = \frac{2}{3}$. Again we need x is equal to 3 minus 2x. So, this means x is equal to 3 by 4, i.e $2x = 3 - 2x => x = \frac{3}{4}$. So, this is not equal. So, that means we cannot have a situation where payoff from B is equal to payoff from… payoff from S is equal to payoff from X. So, this payoff from all the three actions or strategies cannot be equal. This is for player 2. This is clear from this thing.

But now suppose we may have a situation that this is- $2x = 4 - 4x$, we attach some probability to these two. And this is greater than payoff from X, i.e$2x = 4 - 4x > \text{pay} - \text{off from X}$. So, this means that x is equal to 2 by 3. If x is equal to two third then payoff from from A, from B is equal to 4 by 3. And payoff from X is 3 minus 2, i.e $3 - 2.\frac{2}{3} = \frac{9-4}{3} = \frac{5}{3}$ It is 5 by 3. So, again this is not possible. When payoff from B and payoff from S is equal but it is greater than payoff from X it is not possible. So, we ruled out this also, okay. Next, look for a situation where payoff from B is equal to payoff from X and this is greater than payoff from S, right. So, if we plug in here this to the a so it will be what? It will be 6 by 4, payoff from B. But what is going to be the payoff from S? So, it is 4(1-x), this, so it is $4.\frac{1}{4} = 1$

So, this is possible because 6 by 4 is greater than 1. So, this is one possibility. So, that is why player 2 attaches some probability to B and some probability to X, okay. And when it attaches some probability to B and X? When X player 1 plays B with this, with probability 3 by 4; and S with probability 1 by 4, okay.

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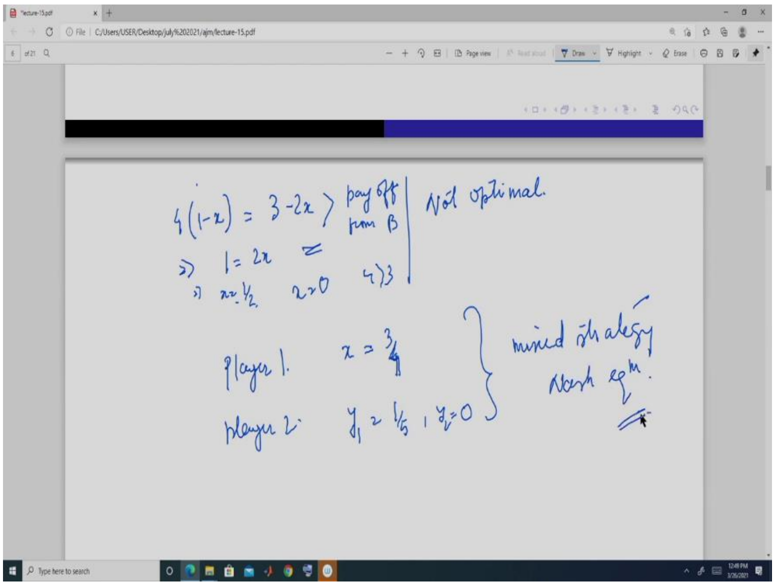

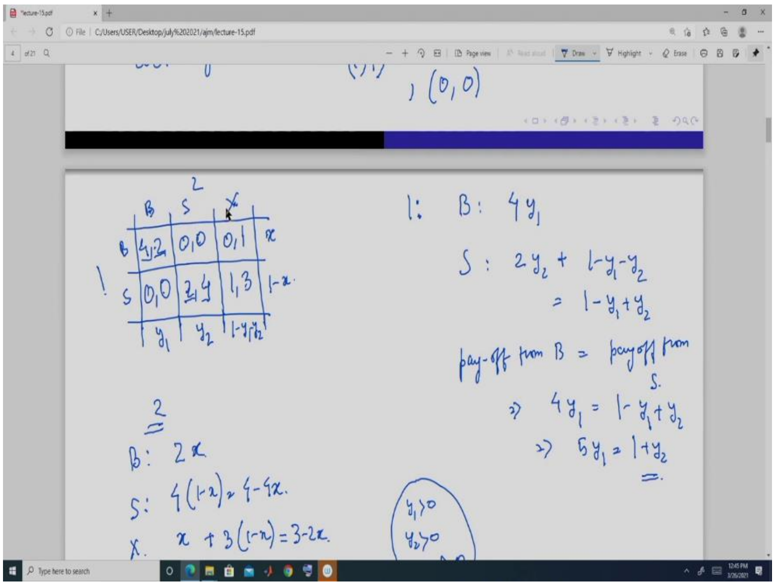

Now, let us look at another possibility that is when payoff from S is equal to payoff from X and it is greater than the payoff from B, i.e $4(1-x) = 3-2x >$ pay off from B. So, this is equal to $1 = 2x \Rightarrow x = \frac{1}{2}$when x takes value half. But here now remember this. Look at the game. If player 2 plays S and Y with some probability and plays B with 0 probabilities right? then player 1 knows that if it attaches some probability to B it will get 0; and it will get positive always when it attaches some probability to S. So, it will always play S. So, if it always play S in this situation then the best response for player 2 is to play 4. So, that is why this situation is not going to happen. So, this is not optimal, okay.

This is not optimal because if player 2 attaches some probability to S and X then player 1 is always going to play S. So, x is always going to be 0. The moment x is always going to be 0, here you plug in x is equal to 0, so 4 is greater than 3. So, that is why player 2 is always going to play S and not X and not going to attach any probability to X.

So, from these what do we get? So, we get that only this- $2x = 3-2x >$ payoff from S, is one possible outcome, okay. So, so if this is one possible outcome that is when player 1 attaches probability 3 by 4 to this strategy and 1 by 3 to this strategy then player 2 plays this with some probability and this with some probability and this with 0 probability.

So, here plug-in y is equal to 0. So, it means that x y1 is equal to 1 by 5, i.e $y_1 = \frac{1}{5}$. So, it means that player, so what is going to be our mixed strategy? Mixed strategy for player 1, that is x is going to be 3 by 4 and for player 2 y1 is going to be 1 by 5, y2 is going to be 0, okay. So, this is the optimal outcome, because if player 1attaches probability 3 by 4 to this and 1 by 3 to this then player 2 is indifferent in playing B and X. And the payoff it gets from playing B with some Probability and X some probability is greater than S. So, it is going to attach 0 probability to y is equal to, y. So, y2, 0 probability to y, 0 value to y is equal to 2, sorry y2.

So, since payoff from S is less than payoff from B and X so it is attaching 0 probability here. So, y2 is equal to 0. And y1 and this is going to take some value. What is that some value? Now, for player 1 to be indifferent so that it is going to attach some probability to B and S x y1 should take this when y2 is taking value 0. So, so that is why this- Player1: $x = \frac{3}{4}$, player 2: $y_1 = \frac{1}{5}, y_2 = 0$ is a mixed strategy Nash equilibrium, okay.

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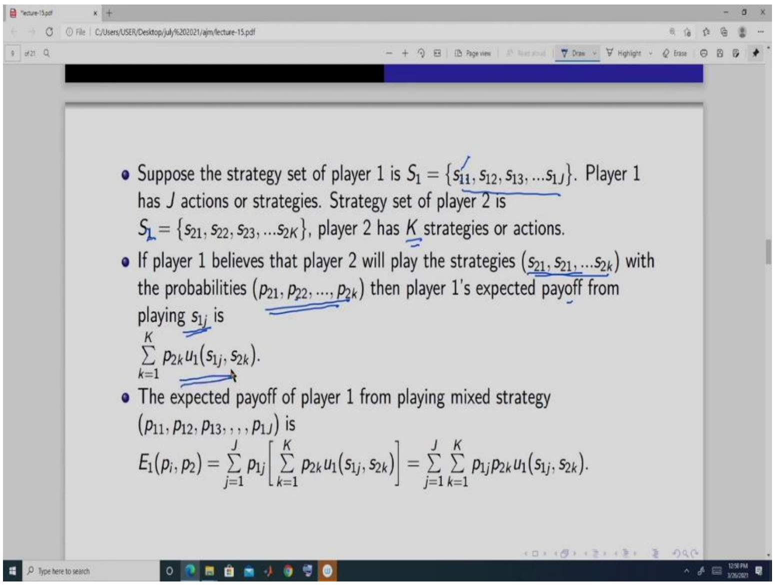

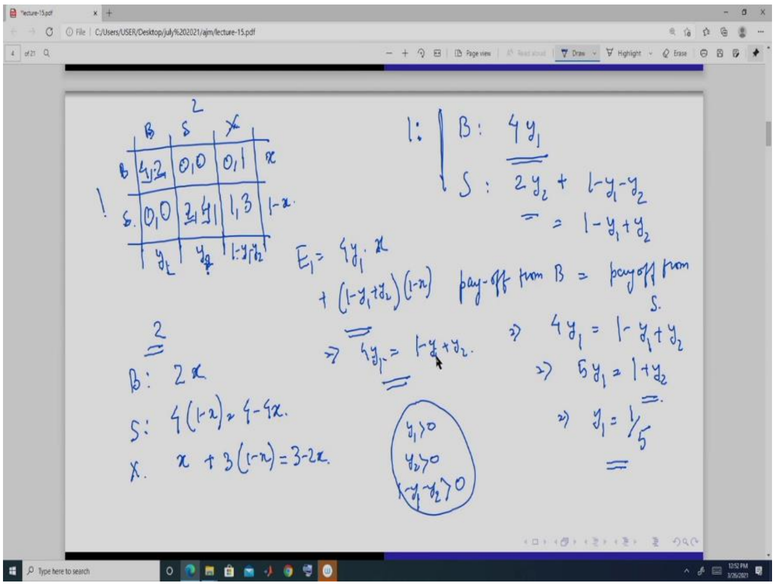

Now, more formally what do we define? That suppose there are 2 player and the strategy set of player 1 is this- $S_1 = \{s_{11,}s_{12} \ldots .s_{1j}$. So, player 1 has J strategies from 1 to J. So, all this small, so this is for, signifies player 1 and this is the strategy 1 or action 1. So, there are J actions, capital J. And for player 2 this should be S2= $s_{21}, s_{22} \ldots \ldots s_{2K}$. First subscript is player and the second is the strategy.

So, s21, s22 s23, s2k. So, these are the strategies of player 2. And it has k strategies, okay. And player 1 believes that player 2 will play strategy this- $\{s_{21}, s_{22} \ldots \ldots s_{2K}\}$with probabilities this- $\{p_{21,}p_{22} \ldots . p_{2k}\}$, okay. So, it is attaching some probability. So, this can be any number between 0 and 1, okay each of these. Then player 1's expected payoff from playing any strategy j from this set is this- $\Sigma_{k=1}^{k} = \mathrm{p_{2k}u_1(s_{1j}, s_{2k})}$. So, this is actually you can say here; when I say that player 1 is playing B, player 1 is playing B, so it is summing over all the possible outcomes from this action that is B with probability y1, S with probability y2, X with probability 1 minus x1 minus x2. So, it is this- $B = 4y_1$. And when it plays S it is 0 with probability y1, 2 with probability y2, 1 with probability 1 minus x1, y1 minus y2. So, it is this. So, this is what this definition, this says.

Now, the expected payoff of player 1 from playing mixed strategy, this. Now, player 1 is playing the mixed strategy, okay. So, it is attaching this. So, this, if it attaches this probability- $\{p_{11}, p_{12}, \ldots . p_{1j}\}$ to this-$E_1(p_i, p_2) = \Sigma_{j=1}^{j} p_{1j}[\Sigma_{k=1}^{k} p_{2k} u_1(s_{1j}, s_{2k})$, it is going to be this- $\Sigma_{j=1}^{j}\Sigma_{k=1}^{k} p_{1j} p_{2k} u_1(s_{1j}, s_{2k})$. So, where, in this situation, so the expected payoff of 1 is 4 y1 into x plus 1 minus y1 plus y2. So, this is the expected payoff- $E_1 = 4y_1.x + (1 - y_1 + y_2)(1 - x)$. And player 1 is going to choose the expected payoff in such a way that it should be maximum with respect to x.

So, it will choose such that… so if you simply maximize this with respect to x you will get that 4y should always be equal to this- $4y_1 = 1 - y_1 + y_2$. So, it will attach probability x in such a way that these two are equal, okay. Or you can say it is only going to attach some probability when they are equal, okay. So, the expected payoff of player 1 when it is playing this mixed strategy is this-$E_1(p_i, p_2) = \Sigma_{j=1}^{j} p_{1j}[\Sigma_{k=1}^{k} p_{2k} u_1(s_{1j}, s_{2k})$. And when we… because it has J strategies so we have J probabilities. These are numbers lying between 0 and 1, and we get this, and we have this as the expected value which is this- $E_1(p_i, p_2)$.

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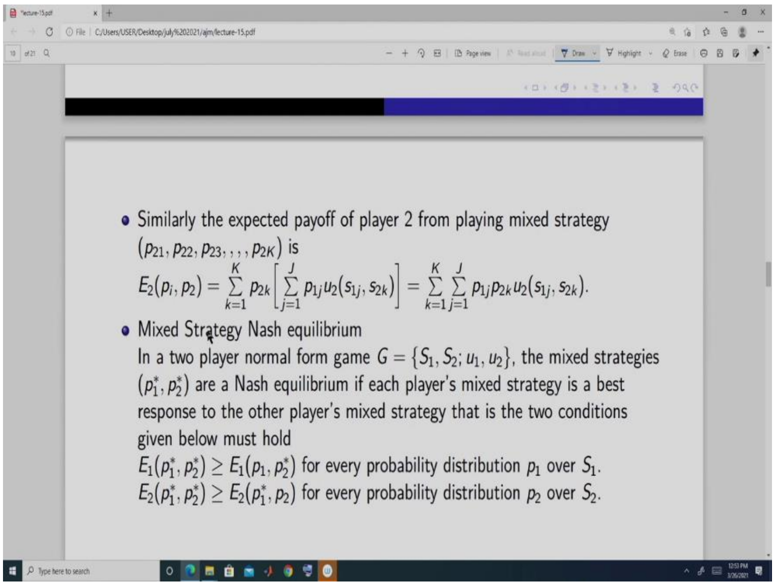

Similarly for player 2 we can write the expected, now it will be sum over first, the player 1 when player 1 is attaching some probability, and then it will be sum over its own probabilities it is this-$E_2(p_i, p_2) = \Sigma_{k=1}^{k} p_{2k}[\Sigma_{j=1}^{j} p_{2j} u_2(s_{1j}, s_{2k}) = \Sigma_{j=1}^{j} \Sigma_{k=1}^{k} p_{1j} p_{2k} u_1(s_{1j}, s_{2k})$, okay. And how do we define the mixed strategy? We define the mixed strategy that the mixed, in the 2 player game this, where S1 as we have defined above, S2 as we have define above, and these are the payoff functions; the mixed strategy, this p1 some probability attached to each strategies, p2 some probability attached by player 2 to each strategies are a Nash equilibrium if each player's mixed strategy, if each players, so like player 1 the p1 is the best response to the other players mixed strategy to p2 star. That is this, expected value of this should be greater than any other probabilities that we are assigning here- $E_1(p_1^*, p_2^*) \geq E_1(p_1, p_2^*)$. And this should also be greater- $E_2(p_1^*, p_2^*) \geq E_1(p_1^*, p_2)$..

So, together, these two defines the mixed strategy. And one way to find out the mixed strategy from these two conditions you will see that whenever the players are indifferent between their actions or strategies then only it is going to attach some probabilities. And so that gives me the probability of the other player. And for the player 1 it will attach probability in such a way so that player 2 is indifferent among its strategies. So, that gives me the mixed strategy Nash equilibrium. Thank you very much.

Introduction to Market Strategies
Professor Amarjyoti Mahanta
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Module 5: Game Theory
Lecture 20
Existence of Nash Equilibrium in Games with 2 Players and 2 Strategies

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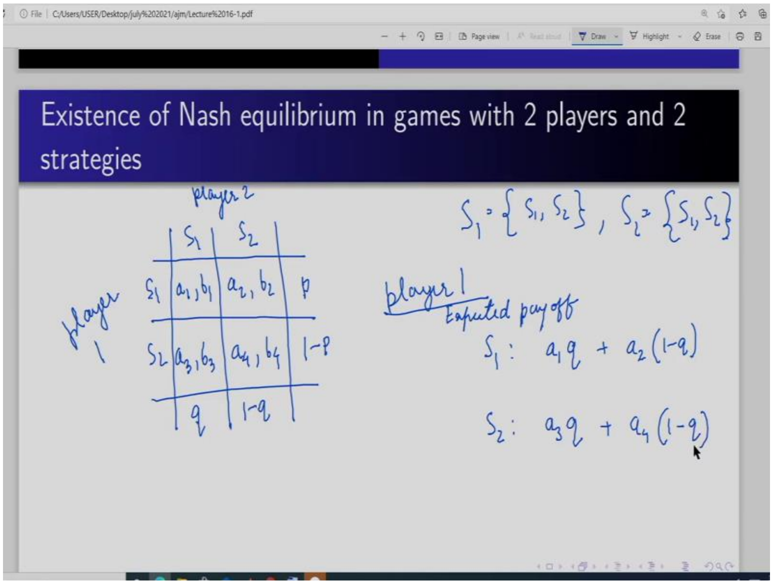

Hello, welcome to the course - Introduction to Market Structures. We were doing game theory so today we will show the existence of Nash Equilibrium in a game which has 2 players and 2 strategies and we will do it for a general case. So, the general game is this, okay, player 1 is the row player, player 2 is the column player.

The strategies are they have 2 strategies or 2 actions S1 S2, okay? So, you can take Si which is big A, it consists of $S_1 = \{S_1, S_2\}$ and S2 consists of 2 elements S1 and S2, i.e $\{S_1, S_2\}$, okay. And the payoffs are a for player 1, b for player 2, okay. So, this is the general game which has 2 players, player 1 and player 2. And each player has same strategies but they have 2 strategies S1 and S2 or 2 actions, okay.

We will show that in this kind of game there always exist a Nash Equilibrium, it can be a pure strategy Nash Equilibrium or it can be a mixed strategy Nash Equilibrium. So, here and the probability attached by each player is P to this strategy by player 1, 1 minus P to this strategy by player 1 to strategy 2, Q by player 2 on strategy 1 and 1 minus Q to strategy 2 by player 2, okay. So, I have specified the game, I have also specified the probabilities.

Now, let us look at the payoffs, okay expected payoffs. So, here let us look at the reaction function of player 1 first, okay. So, we will find all sorts all possible best response function or the reaction functions, okay. So, the expected payoff from S1 is a1 into q plus a2 into 1 minus q, i.e $S_1: a_1q + a_2(1-q)$ and the expected payoff from S2 so this is for player 1, okay, player 1 is a3 q so this they get if player 2 attaches q probability to it and it gets a4 with 1 minus q4 probability, okay if it plays S2, i.e $S_2 = a_3q + a\_4(1-q)$, okay.

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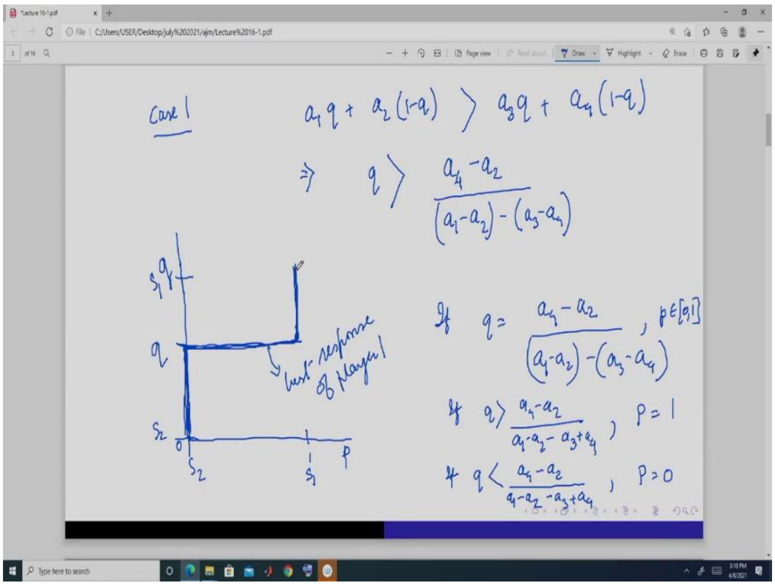

So, one possibility and let us that is case one is that, that a1 q plus a2 1 minus q is suppose greater than a3 q plus a4 1 minus q, i.e $a - 1\ q + a_2(1-q) > a_3q + a_4(1-q)$. So, from here we get that $q > \frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$, okay, this. So, here we now plot the best response curve, so in this axis we plot the probability attached by player 1 and in this axis we plot the probabilities attached by player 2. If p takes value 0, that means if p takes value 0 then it is playing S2 for certainty and if p is equal to 1 then it playing S1 with certainty. So, this is suppose 1 so this is S1, this is 0, so this is S2. Similarly, for player 2 this is S1 because this value is 1 and this is S2, okay.

And suppose this is q and this q is this value, q is equal to a4 minus a2, a1 minus a2 minus a3 minus a4, okay this is q- $\frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$. So, if q is greater than this, then we have this situation, so this means that expected payoff from S1 is greater than expected payoff from S2 for player 1. So, if q is greater than this- q> $\frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$., then p is equal to 1. So, for here p is equal to 1 it means this, okay and if q is less than a4 minus a2 divided by a1 minus a2 minus a3 plus 4 this- $q < \frac{a_4-a_2}{a_1-a_2-a_3+a_4}$ then p is equal to 0, so it is here, right? And it is indifferent when q is this- $\frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$, so when q is….if q is this then p belongs to this range any value in this range so we get this as the…this is the best response of player 1 when we assume this that suppose this is greater than this one so that means whenever q is greater than this, that is this takes a positive value and this is lying between 0 and 1 then we get this, okay, this situation.

So, this is case one this is one possibility of best response function based on the payoffs. Suppose the payoffs are because this q is derived from the payoffs of player 1 and if q takes a value like this, suppose it takes a value like this if q is greater than this then p1 takes value here if q is less than this then p1 takes value 0 and p can takes any value lying between 0 and 1, if q is equal to this-$\frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$, if it is this, okay. So, this is case one.

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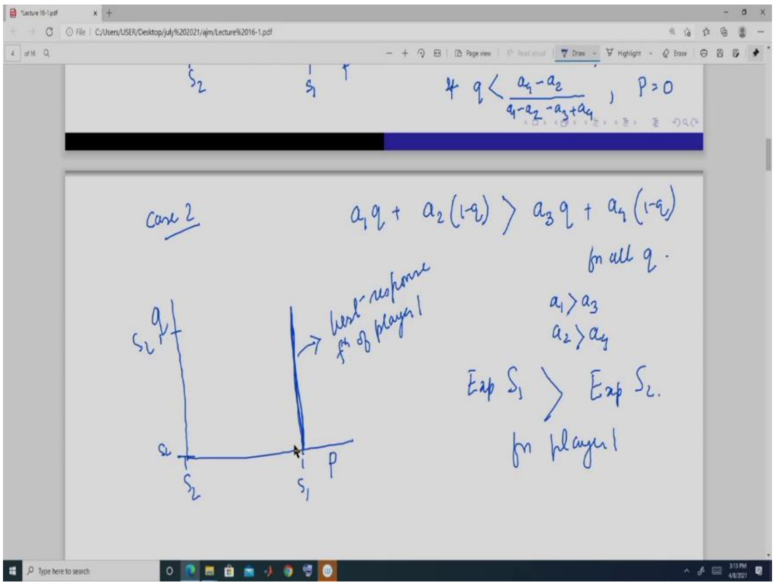

Now case 2, case 2 is, suppose, this- $a_1q + a_2(1-q) > a_3q + a_2(1-q)$ is true for all q, for all q this is true, whatever q takes value this is true this is possible when a is greater than a3 and a2 is greater than a4. So, a1 is greater than a3, a2 is greater than a4, right. So, we get this condition, so here again let us plot the best response function.

So, here whatever value q takes this is, so this is what expected payoff from S1, expected S1 is always greater than expected S2 for player 1. So, whatever q takes p is always going to be expected value of q and we know when p is 1 it is S1 when p is 0 it is S2. So, q any value it is this, so this is the best response function of player 1. Plug in any value of q you will get what is p, p is always 1 in this situation- $a_2 > a_4$. So, here you can say that S1 is a dominant strategy so S1 always dominates over S2, so that is why it always choose S1.

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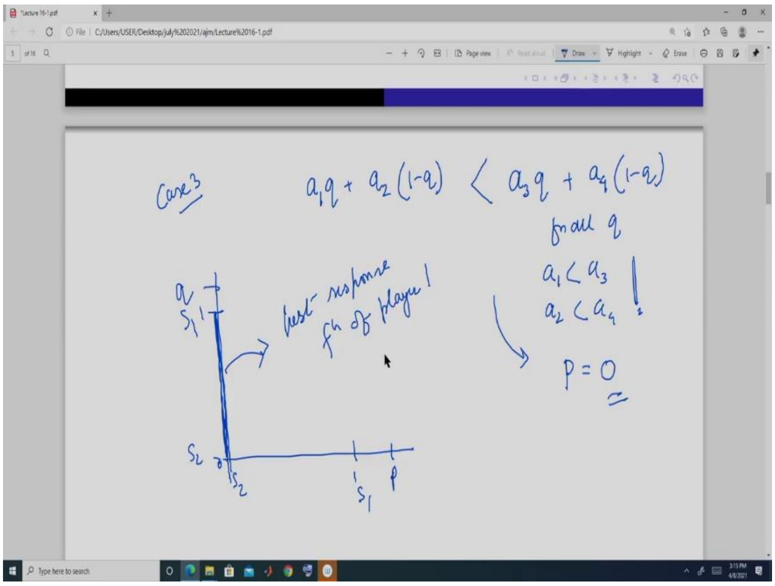

Next case, is this is always less than- $a_1q + a_2(1 - q) < a_3q + a_4(1 - q)$, okay, for all q, so this is possible when a1 is less than a3 and a2 is less than a4, so this is one situation where we will have such situation. So, in this case what is going to happen? Since the payoff from S2 is always greater than the payoff from S1, so player 1 is always going to choose S2, so here it is p, here it is q, this is p is equal to 1 and so this is S1, this is p is equal to 0, so this is S2, this is S2 for player 2, this is S1 for player 1 because this is 1, okay. Then whatever value q takes p is… so here p is always equal to 0. So, it is this… so this is the best response function of player 1, so this is the case.

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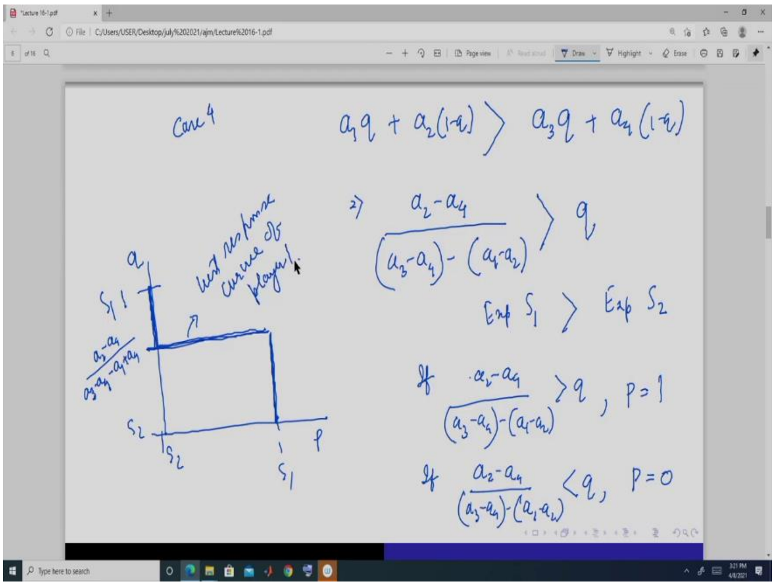

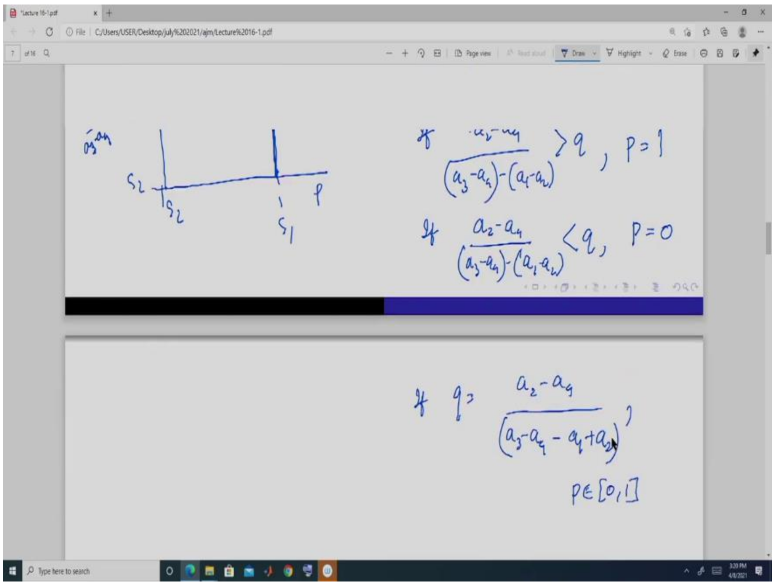

And now we do the last case, so again the expected payoff from S1 is this- $a_1q + a_2(1-q)$, expected payoff from S2 is this- $a_3q + a_4(1-q)$. So, if q is less than this number- $q < \frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$ and suppose this is a positive number lying between 0 and 0, so this number lies between 0 and 1, then if this is the case then if q is less than $\frac{a_4-a_2}{(a_1-a_2)-(a_3-a_4)}$ so we want the situation to be the opposite of… so if we have got this from here, we can write, suppose this is the case-$\frac{a_2-a_4}{(a_3-a_4)-(a_1-a_2)}$, okay, so if q is less than this number then the expected S1 payoff from S1 is greater than expected payoff from S2.

So, here if q takes a value like this- $\frac{a_2-a_4}{(a_3-a_4)-(a_1-a_2)}$, which is that q is less than this number then p is equal to 1, okay. And if this- $\frac{a_2-a_4}{(a_3-a_4)-(a_1-a_2)}$ is greater than q, then p takes a value equal to 0. So, here best response function, this is S1, this is S1, this is S2 so when q and this is suppose this value this is a2 minus a4 divided by a3 minus a4 minus a1 plus a4 this value so if q is less here then p is equal to 1. P is equal to 1 if q is less than here and q p is equal to 0 if q is greater than here and if q is exactly equal to this number that if q is equal to a2 minus a4, a3 minus a4 minus a1 plus a2, if this- $q = \frac{a_2-a_4}{a_3-a_4-a_1+a_2}$ then it is p, if this then p takes any value between 0 and 1. So, it is this, so this is the best response curve of player 1, okay.

So, this is not a function because this is a correspondence because for this point q taking a… this point we get a set and that set is this p= [0,1] so it is mapping from point to set. So, that is why it is not a function it is a correspondence, okay. But we do not need this all these things in detail but in case if you have any query regarding this, we get that it is a correspondence for each value of q we get this is the reaction of player 1. So, if q is less than this value, p is equal to 0, if q is greater than this value p is always equal to 0, if q is less than this value then p is always equal to one and if q is equal to this value, then p can take any value in this, so it is a set.

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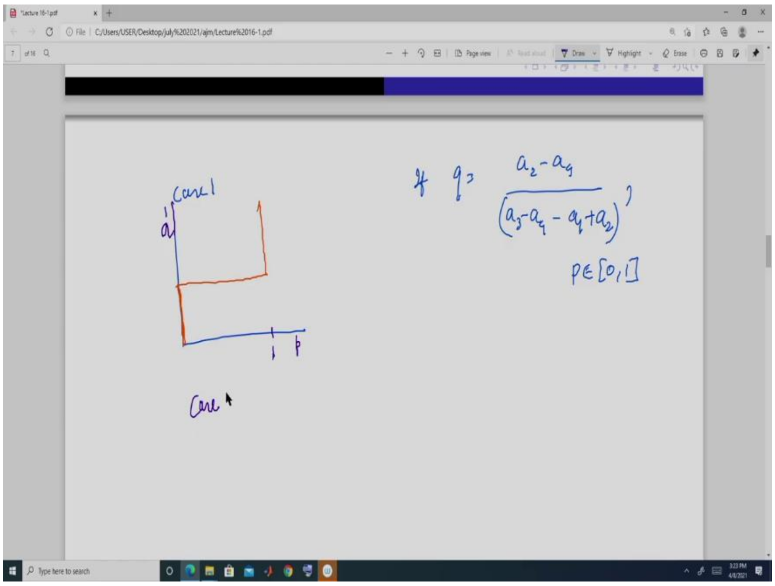

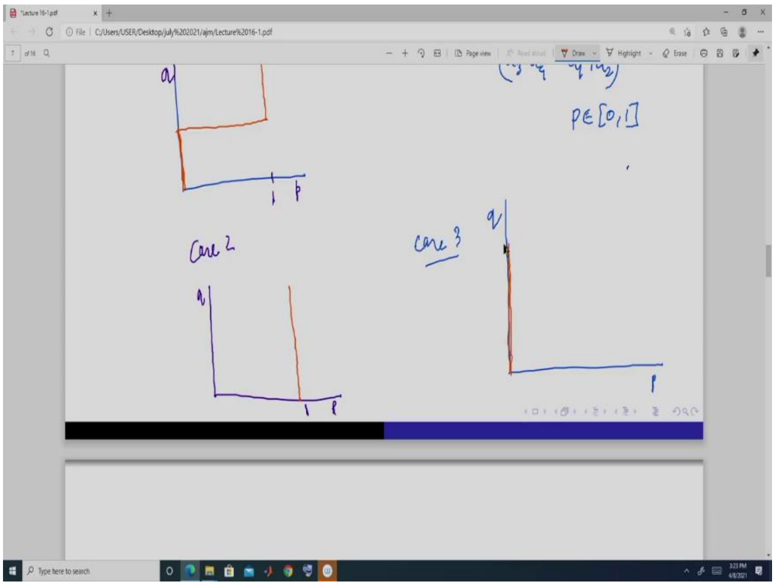

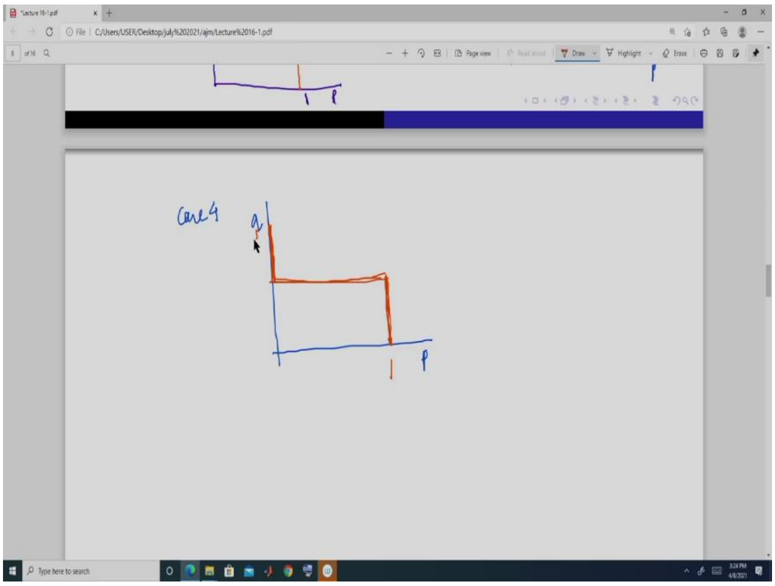

So, we get 4 possible cases what are these 4 possible cases? Case 1, we have got this ok let me draw it by using a different colour. So, this is the, right? case 2, so this is p and this is q, p is 1, q is 1 here so we have got this. Case 2, plug any value of q p is always 1, okay. Case 3, p and q… so any value of q, p is always equal to 0, this is the case. And case 4 we get this, is the best response, okay. So, these are the all-possible situations depending on the different payoffs, okay.

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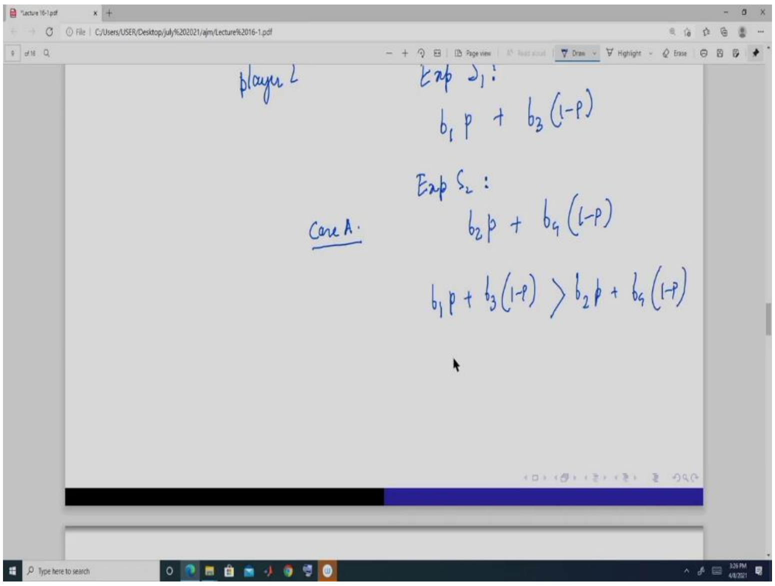

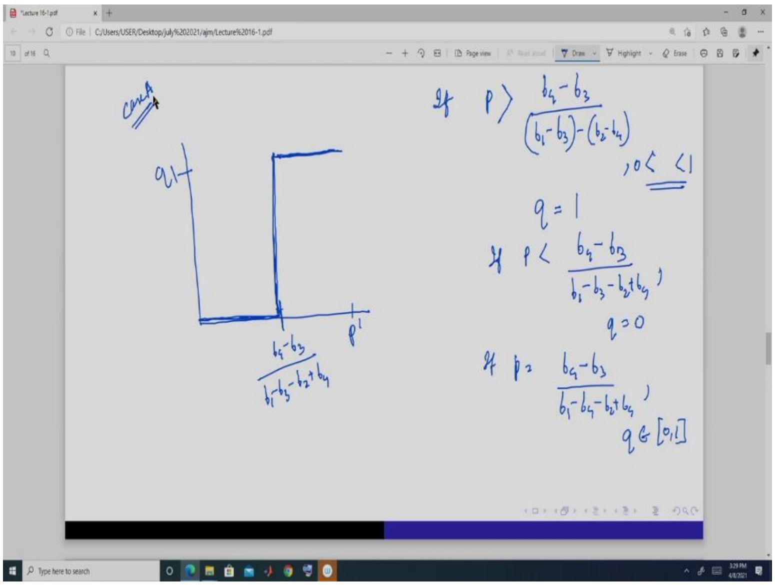

Now we look at the case of player 2, so if you look at this game and the payoffs of player 2 the expected payoff, so now we are looking at player 2, so expected from S1 is b, 1, p plus b3, 1 minus p, i.e Exp $S_1: b_1p + b_3(1 - p)$ and expected payoff from S2 is b2 p plus b4 1 minus p, i.e Exp $S_2: b_2p + b_4(1-p)$ and we may have a situation where b1 p plus is so this is suppose case A- $b_1p + b_3(1-p) > b_2p + b_4(1-p)$, okay, this and we have got,

We have got- $p > \frac{b_4-b_3}{(b_1-b_3)-(b_2-b_4)}$, so here in this situation if p is greater than this value and suppose this lies between 0 and 1. So, the payoffs b1, b2, b3, b4 are such that this lies between, this is greater than 0 and this is less than 1, if it is like this. If this value lies in this range- $0 < \frac{b_4-b_3}{(b_1-b_3)-(b_2-b_4)} < 1$, then q is always equal to 1, right? And if p is less than this- $p < \frac{b_4-b_3}{b_1-b_3-b_2+b_4}$ then q is equal to 0 and if p is equal to this- $\frac{b_4-b_3}{b_1-b_3-b_2+b_4}$ then q lies between 0 and 1.

So, here in case 1, if we take plot p here, q here, 1 so p, suppose this is the p this is b4 minus b3, b1 minus b3 minus b2 plus b4, okay, this value is this. Then if p is greater than this, then q takes value 1, q is 1, okay. If p is less than this q is 0, it is this, okay. So, this is one possibility of which is case A.

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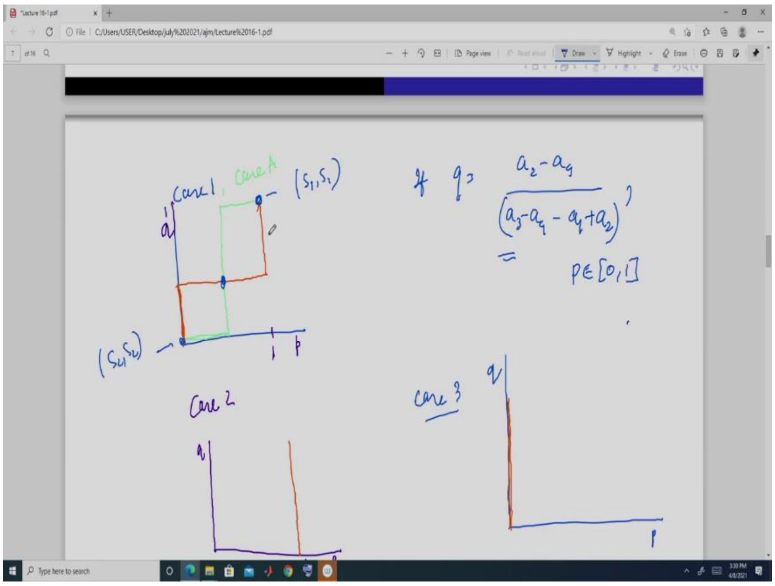

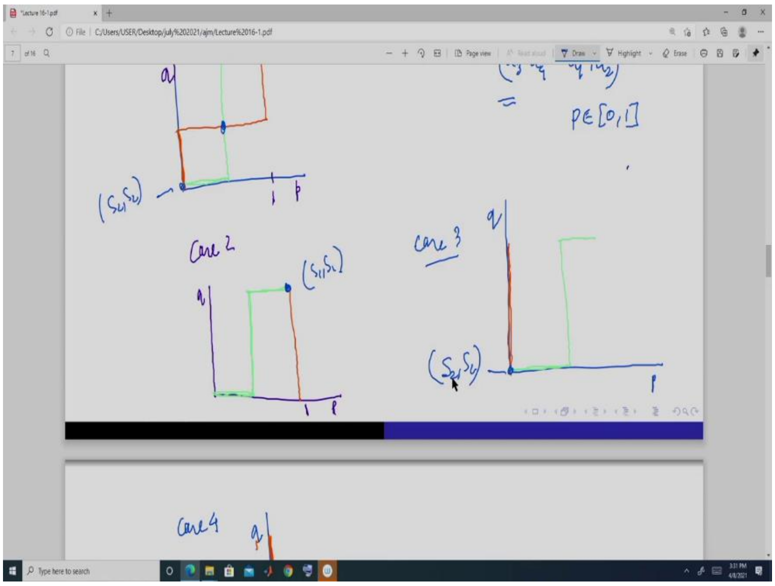

Now here just look at this so we have this case and we can have any one of these 4 cases of player, here player 1. So, now just in this graph just look at this here so if case 1 and so we can have a combination like case 1 and case A, so in that case this so we have 3 Nash Equilibrium, this is 1, where it is (S1 S1), this is another which is (S2 S2), and this is the mixed strategy where q takes this value- $\frac{a_2-a_4}{a_3-a_4-a_1+a_2}$ and p takes this value-$\frac{b_4-b_3}{b_1-b_3-b_2+b_4}$, okay. So, this is the situation so we have 3 Nash Equilibrium in this case.

And suppose instead of case 1, we have suppose case 2, then this is the reaction function of player, best response function of player 2, so we have only 1 Nash Equilibrium in this situation and that is this and this is S1, and this one S1, okay. In this situation, if suppose we have case 3 of player 1 and case A of player 2, so this is the point of intersection of best response function, so we have 1 Nash Equilibrium it is (S2, S2).

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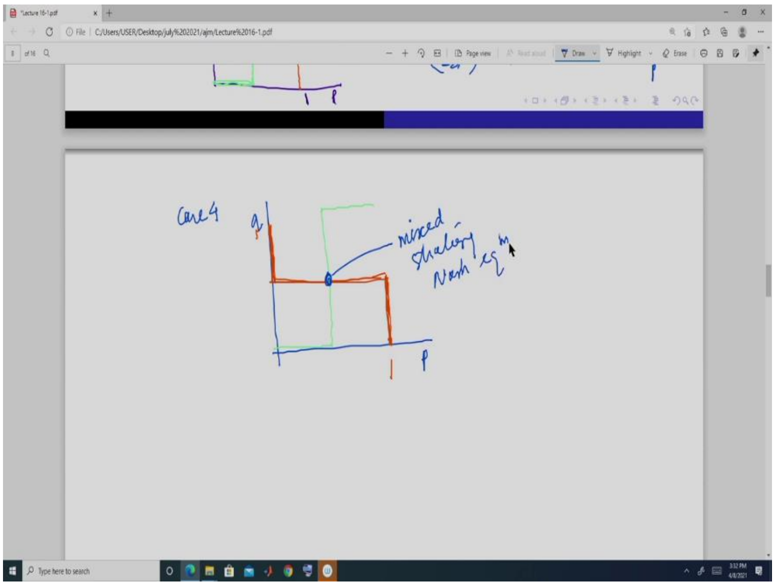

In this situation, we have this we have only 1 Nash Equilibrium in this situation and it is the mixed strategy, q so this is the mix strategy Nash Equilibrium, where p, this p takes this value sorry p takes this value and q takes this value, okay. So, what we have got if we if the payoffs are such that best response of player 2 is case A and take any case of player 1 either case 1 or case 2 or case 3 or case 4, we have at least 1 Nash Equilibrium, okay. So, this is one possible outcome.

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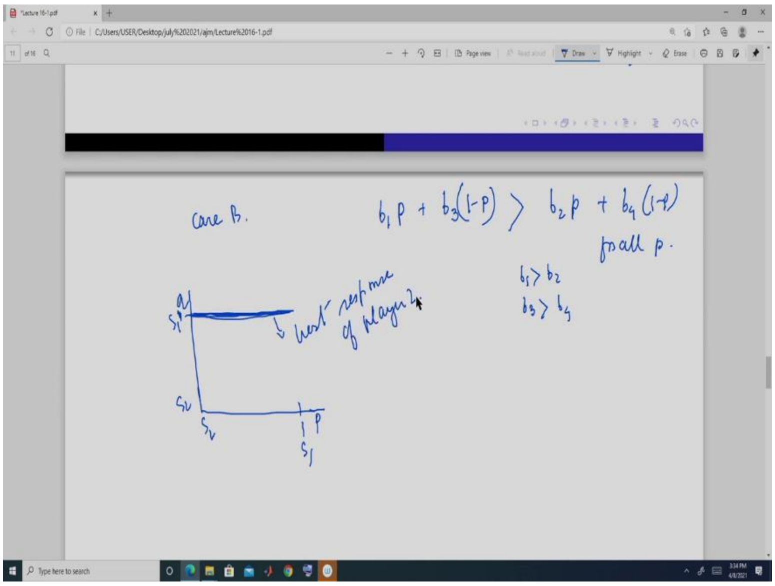

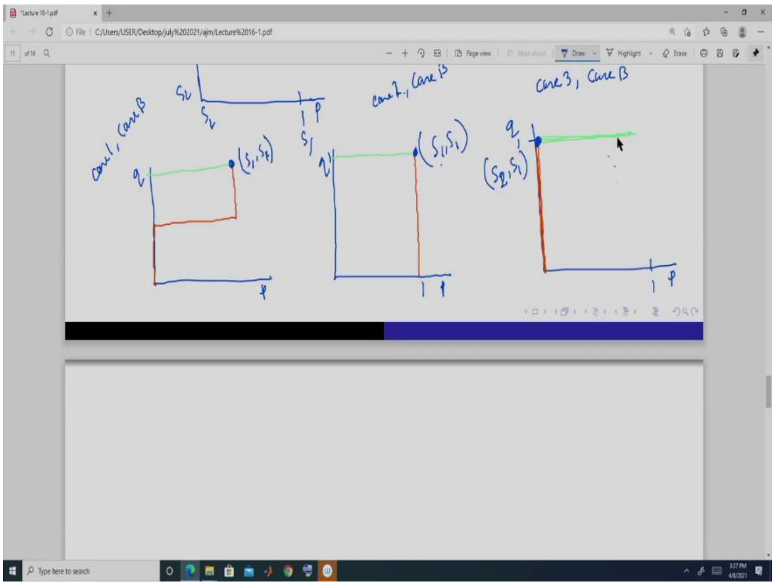

Now, let us look at case B of player 2. Case B is, suppose this is always, i.e $b_1p + b_3(1-p) > b_2p + b_4(1-p)$ for all p, so then it means one possible way is b1 is greater than b2 and b3 is greater than b4. So, it always chooses S1. So, in this whatever be the value of p player 2 is always going to choose S1 in this case. So, here it is, this is suppose 1, is S1, S2, this is S2, so whatever be the value of p player 2 chooses S1. So, this is the best response this is the player 2, best response curve or you can say best response function, here it is a function, right?

So, now if you look at all the possible, so this is case B, okay. So, suppose this is case 1 and case B, okay, so then this is the reaction function of player 1 and the reaction function of player 2 is this. So, this is the Nash Equilibrium and Nash Equilibrium is (S1, S1), okay in this case. So, this is for case 1 and case B. Now, we have may have case 1, case 2 and case B. Case 2 is this, this right? and case B is this, so the Nash Equilibrium is this and it is (S1, S1) okay. Next is suppose case 3 and case B, so here this is the reaction function of player or the best response of player 1 and the best response of player 2 is this case B. So, the Nash Equilibrium is this point and this is (S2, S1). S2 of player 1 and S1 of player 2, okay.

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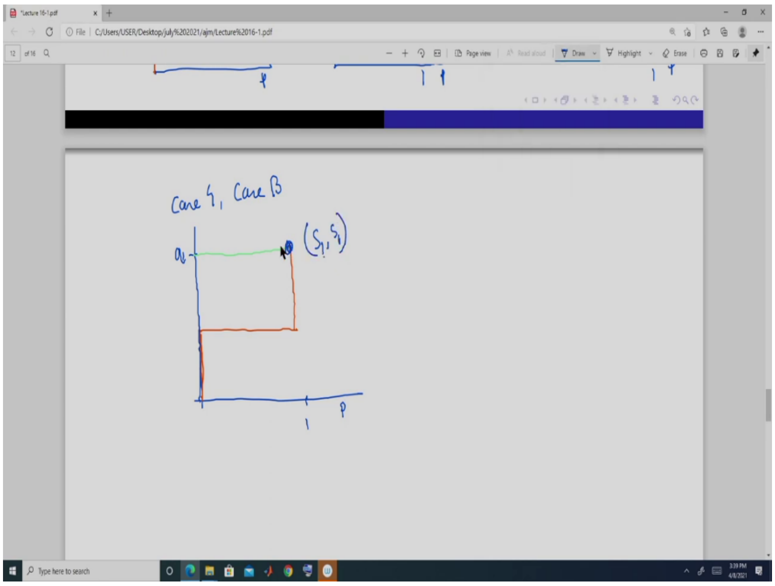

Now, let us look at this case that is case 4 and case B. So, in this situation we have best response of player is this and best response player 2 is, sorry, okay and best response of player 2 is this. So, the Nash Equilibrium here is again this, so this is S1 of player 1 and S1 of player 2, this. So, here we have shown that in this case also we always have a Nash Equilibrium.

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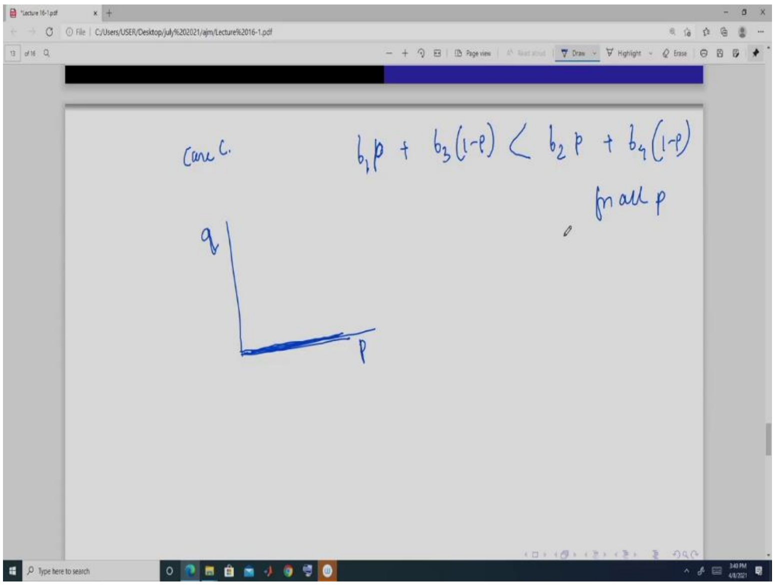

So, we can go on doing this so we will get 2 more cases and this are case C where we have b1 p plus b3 1 minus this is always less than b2 p plus b4 1 minus p for all p, i.e $b_1 p + b_3(1-p) < b_2 p + b_4(1-p)$, okay. So, in this case the whatever be the value of p q is always this, this is the reaction. Now, here you compare this with all the force case of player 1 and you will see that there exists at least one intersection point. So, we always have at least one Nash Equilibrium in this case.

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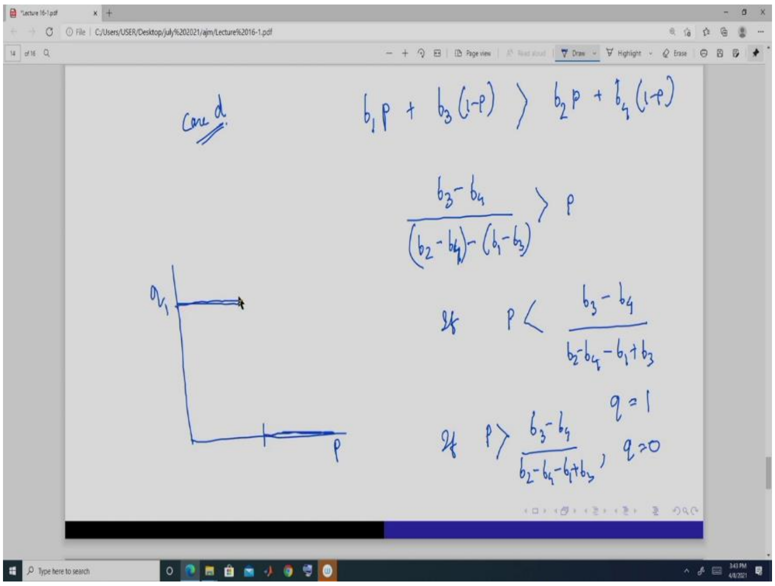

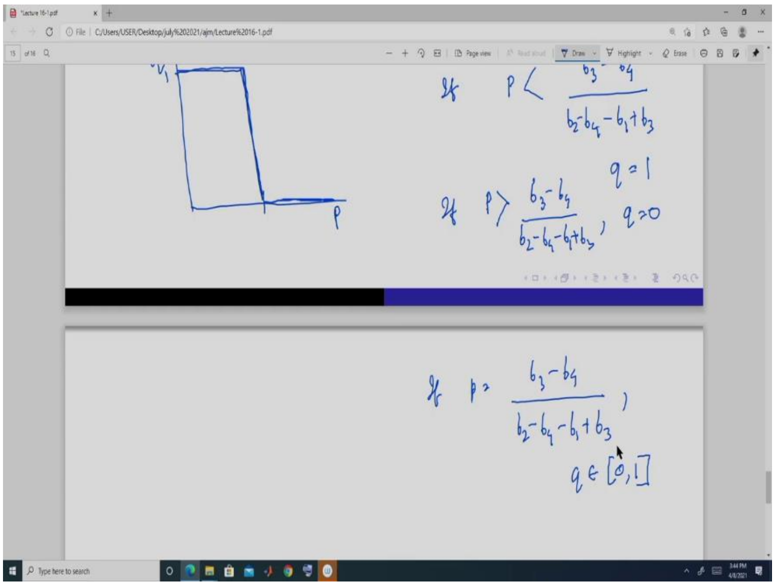

Again, the case D is suppose this- $b_1 p + b_3(1-p) > b_2 p + b_4(1-p)$, okay, so this is, so if p is less than this number, i.e $p < \frac{b_3-b_4}{(b_2-b_4)-(b_1-b_3)}$ then q is equal to 0 and if p is greater than this number, was this the case A that we have done, this is what we have done here is this, if p is like this if p is greater than this number-$\frac{b_4 - b_3}{b_1 - b_3 - b_2 + b_4}$, p is greater than this number then we have this situation.

So, here if p is less than this number then we have this situation, so here it is q is not equal to 0 but q is equal to, q is equal to 1 and in this situation if q is this, then q is equal to 0. So, here if we plot p and if we plot q, if p is less than, then q is 1 and if p is greater than then q is 0, this- $p > \frac{b_3 - b_4}{(b_2 - b_4) - (b_1 - b_3)}, q = 0$. And if p is equal to this number-$\frac{b_3 - b_4}{(b_2 - b_4) - (b_1 - b_3)}$, then q can take value, then q takes any value between 0 and 1, so this is the case 4, case D of player 2.

So, now here again in this case we can look at all the 4 possibilities of payoffs of player 1 and then we will see at least in one point the reaction functions are going to intersect. If we take this case and we take all the 4 possible cases of player 2, player 1 then again, we will see that at least in 1 point they are going to intersect the best response functions are going to intersect. So, this means that there always exist a Nash Equilibrium when we have 2 player and 2 strategy game.

So, actually Nash prove this for n player, any arbitrary number of player and any arbitrary number of strategies and he has proved it using both Brouwer's Fixed Point Theorem and Kakutani Fixed Point Theorem, okay. So, we do not require those things so it is beyond the scope of this course, so we will skip this portion.

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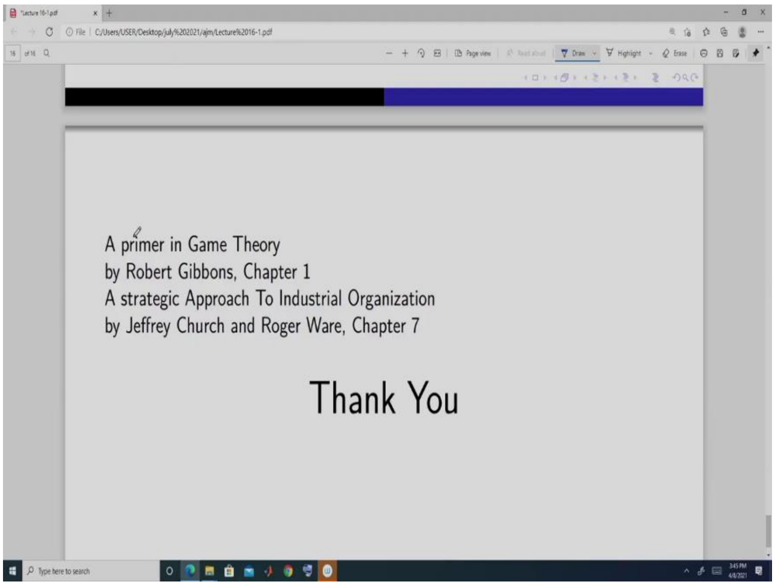

So, this is the end of Static Game Theory and you can read it from chapter 1 of, A Primer in Game Theory by Gibbons or you can read it from chapter 7 of a Strategic Approach to Industrial Organization by Jeffrey Church and Roger Ware. Thank you!

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 5
Lecture 21
Tutorial on Normal Form Games

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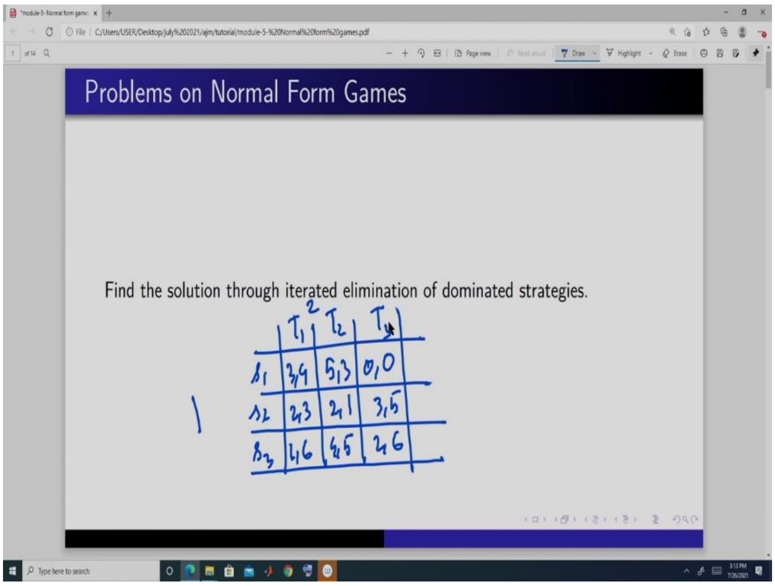

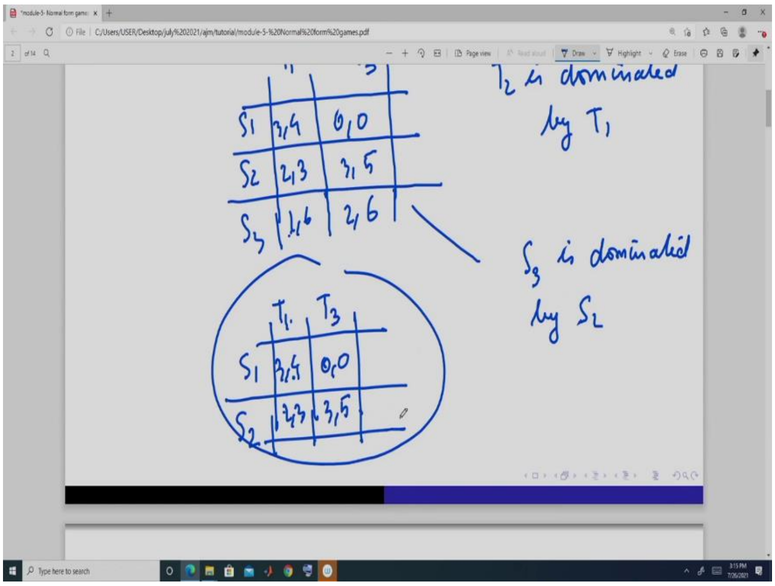

Week 5, module 5. It is Normal Form Game. So, let us solve some problem on Normal Form Game, okay. So, let us take this example S1, S2, S3, T1, T2, T3, this is player 1, this is player 2, payoffs are like this- (3,4), (5,3), (0,0), (2,3), (2,1), (3,5), (1,6), (4.5), (2,6), okay. So, this is a Normal Form Game, okay, player 1 it has 3 action that is S1, S2, S3 and player 2 has 3 action T1, T2, T3, okay. Now, we want to solve this, find the solution through iterated elimination of dominated strategies.

So, if we look at this game you will see that this 2, 2, 3, okay. 3, 4, 3, 5, 0, 3 but here it is this… in this case (2, 1), (2, 4) so that is… but if you compare T1, T2 we will say 4, 3, 3 so this can be eliminated. So, T2 is dominated by T1 so we remove T2 and the game is now like this. So, this is (3, 4), (0, 0), (2, 3), (3, 5), (1, 6), (2, 6), okay. So, here….so again here S2 is dominated by S2, okay. So, S2 actually dominates S3, S3 is dominated by S2.

So, we are left with T1, T3, S1, S2, (3, 4), (0, 0), (2, 3), (3, 5), okay. Now, here 4 is greater than 0 but 5 is greater than 3, so neither T1 dominates T3 nor T3 dominates T1 as it is (3, 2), it is (3, 2) so as here and it is (0, 3). So neither S1 dominates S2 nor S2 dominates S1. So, we are left with this, so if we use iterated elimination of dominated strategy, we will end up here only, we cannot move further, okay. So, that is why we use something like a pure strategy Nash Equilibrium to find out. And let us solve one problem related to Nash Equilibrium, okay.

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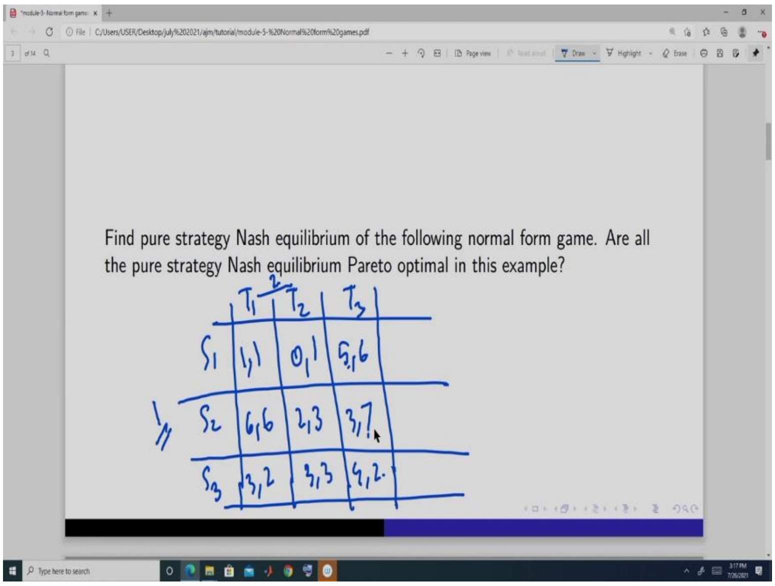

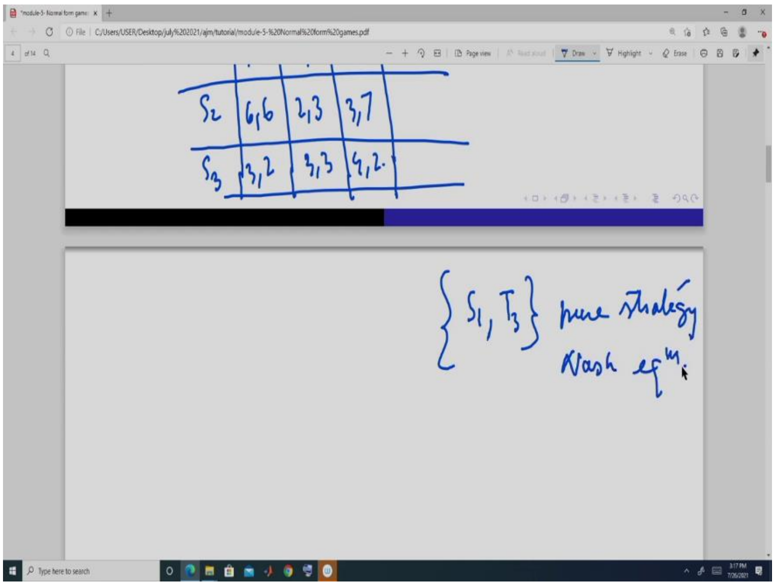

So, suppose the game is of this form T1, T2, T3, S1, S2, S3, okay? Payoffs are of this nature- (1,1), (0,1), (5,6), (6,6), (2,3), (3,7), (3,2), (3,3), (4,4), okay now if we look at this game let us S1, suppose player 1 plays S1, then you will see T3 gives the highest, right? So, if T3 is played then S1 is again best response because 4, 3, 5 if it plays S2 it is 3 and if it plays S3 it is 4. So, {S1, T3} is a pure strategy Nash Equilibrium, okay.

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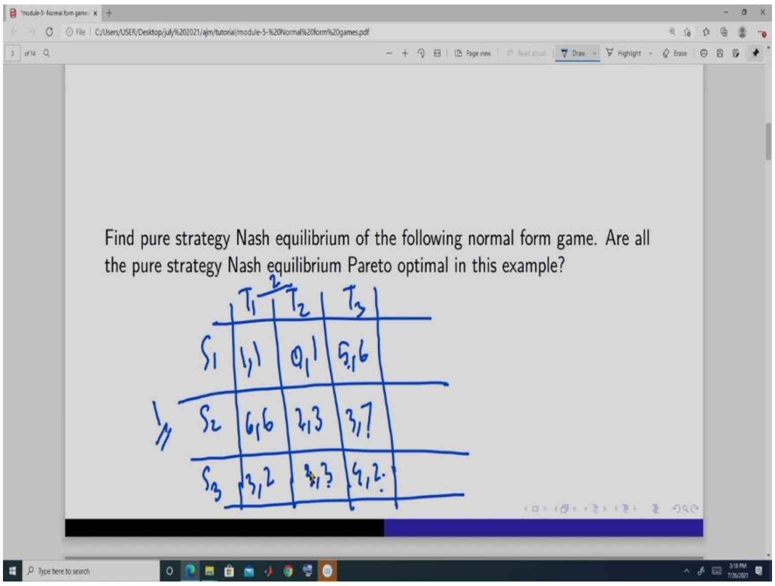

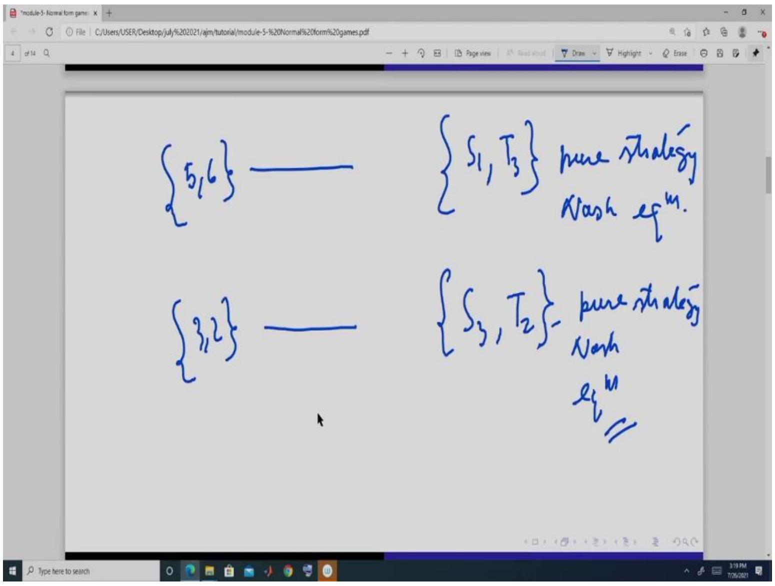

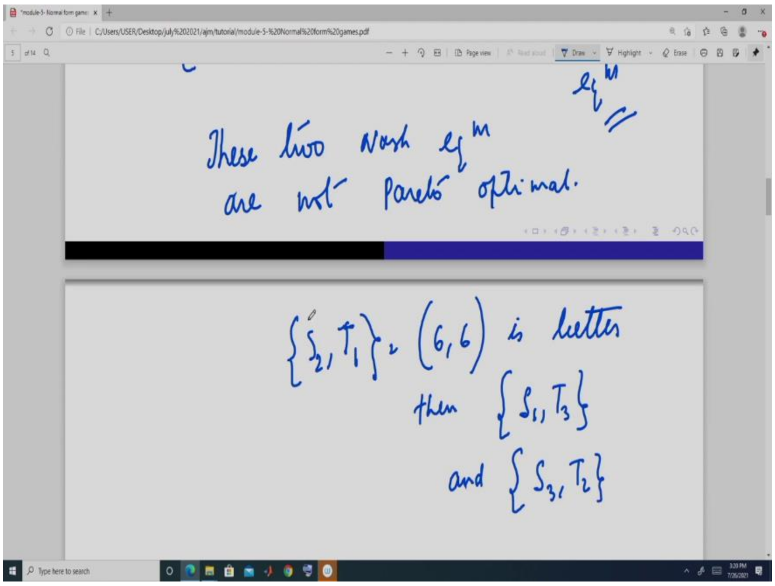

Now, let us play S2, if S2 is played then player 2 because player 2 is player 1, okay? Player 2 is going to choose this T3, if T3 is played we know it is going to choose S1 so there is no Nash Equilibrium where S2 is a strategy or action of player 1. Now, let us play S3, if S3 is played then the best response is T2 because 2, 3, 2 and if T2 is played 0, 2, 3, so S3 is going to… So, again another is {S3, T2}, this is again a pure strategy Nash Equilibrium, okay.

So, here we have two pure strategy Nash Equilibrium. Now, the next question is are all the pure strategy Nash Equilibrium Pareto Optimal in this case? So, what are the Nash Equilibrium out payoffs? One is (5, 6) when we have this $\{S_1, T_3\}$as a Nash Equilibrium another is (3, 2) when we have this $\{S_3, T_2\}$ as a Nash equilibrium. But we have a payoff like this (6, 6) we also have a payoff like (3, 7) but here this is not…. but this (6,6) is a Pareto optimal. If we move from here to here from (5, 6) to (6, 6) what we are?

We are keeping the payoff of player 2 same but we can improve the payoff of player 1. Here this is 1 so we can improve the payoff of both the player if we move from this (3,3) to this (6,6). So, that is why these two, these, these two Nash Equilibrium are not Pareto Optimal, why? Because there exists another payoff that is this (6,6) combination, combination of payoff which gives which is better than both these two Nash Equilibrium outcome. Because this S1, sorry {S2, T1} which is equal to (6,6) is better than {S1, T3}. And because these two are the Nash Equilibrium {S3, T2} and {S1, T3}, this, so that is why it is…

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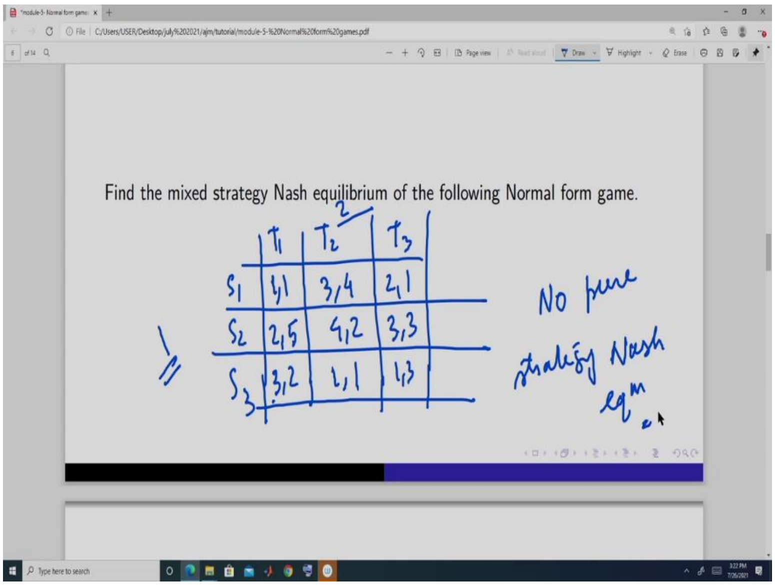

Now, let us solve one, find the mixed strategy Nash Equilibrium. Suppose the game matrix form or the normal form game is of this nature (1, 1), (3, 4), (2, 1), (2, 5), (4, 2), (3, 3), (3, 2), (1, 1), (1, 3), okay. Now, if you look at this if S1 is played then this is best response T2, T2 is played S2 is the best response, if S2 is played T1 is the best response, if T1 is played S3 is the best response, if S3 is played T3 is the best response, if T3 is played S2 is the best response, if again S2 is played then T1 is the… so we see there is no pure strategy, this is for player 2, this is for player 1, okay. No pure strategy maximum, so let us find the mixed strategy now.

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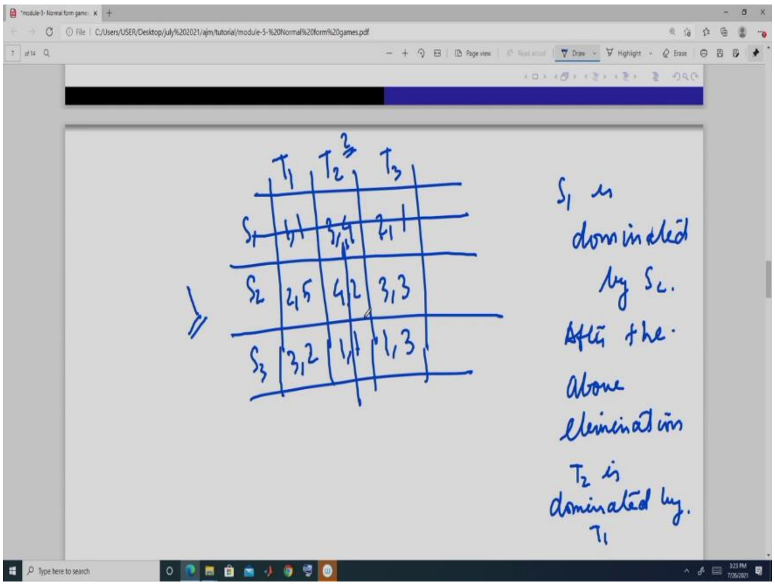

Now, here so we have found that there is no pure strategy Nash Equilibrium, okay. Now, here if you compare 1, 2, 3, 4, 2, 3 so this dominates… so S1 is dominated by S2 so remove this eliminate this, now we are left with this here you will see 5, 2, 2, 1. Again this is (3, 2), (1, 3), so again T2 is dominated by T1 we are left with this.

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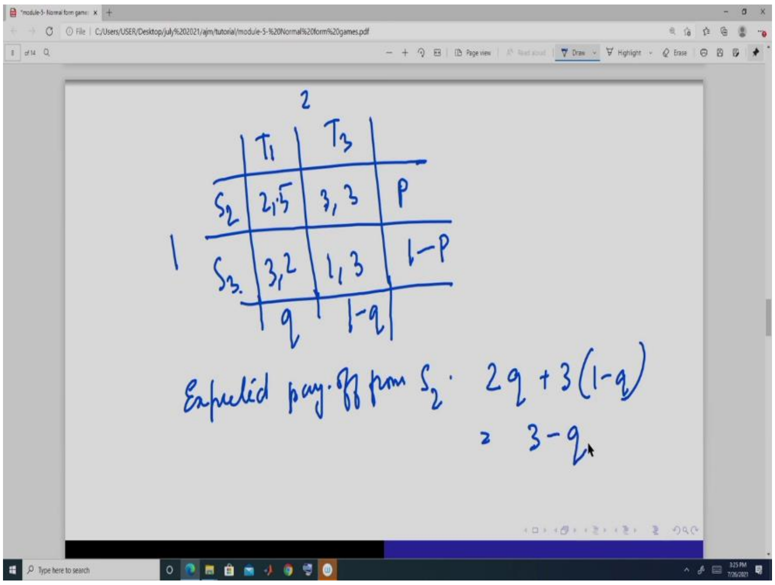

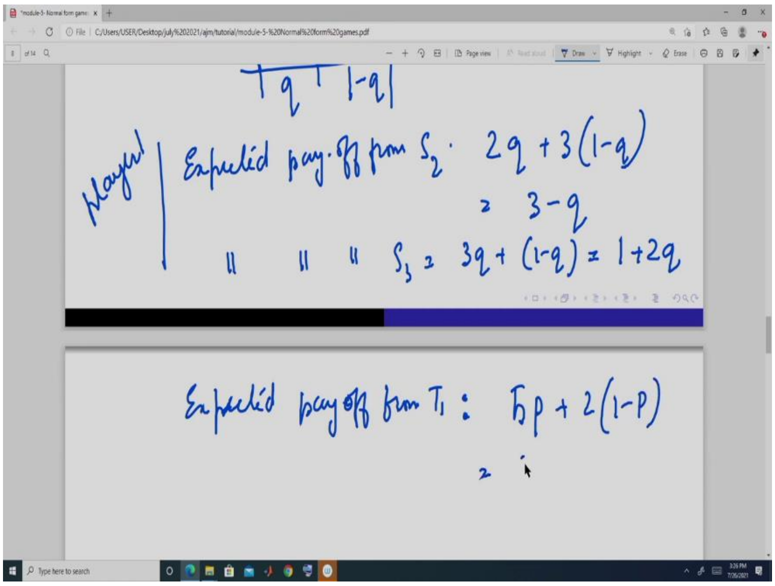

So, what is the game we are left with, we have T1… now here if S2 is played then T1, if T1 is played the best response is to play S3, if S3 is played best response is to play T3, if T3 is played best response is S2, so again no pure strategy. So, suppose player 1 attaches probability P to this S2, action and 1 minus P to this action- S3, and player 2 attaches Q to T1 and 1 minus Q2 T3, okay. So, the expected payoff from S2 is 2 Q plus, so this is equal to 3 this and again expected payoff from S3 is 3Q plus this- $S_3 = 3q + (1 - q)$. So, this is- 1+2q this…

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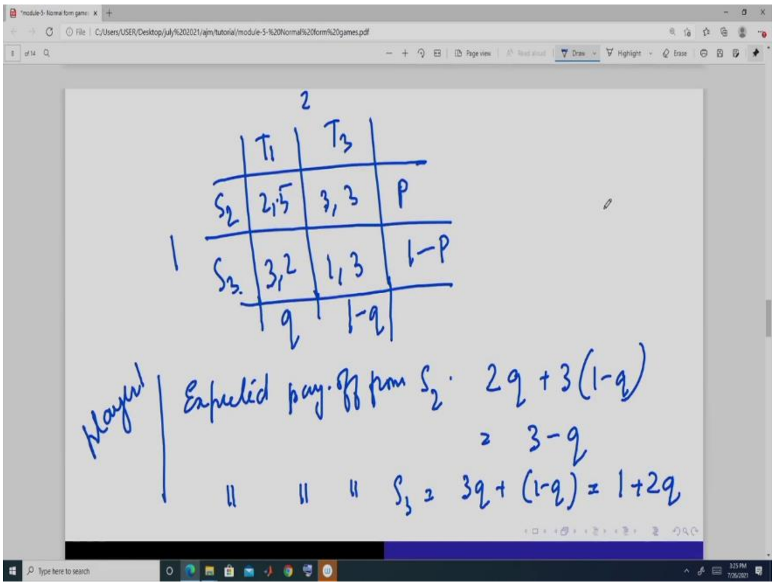

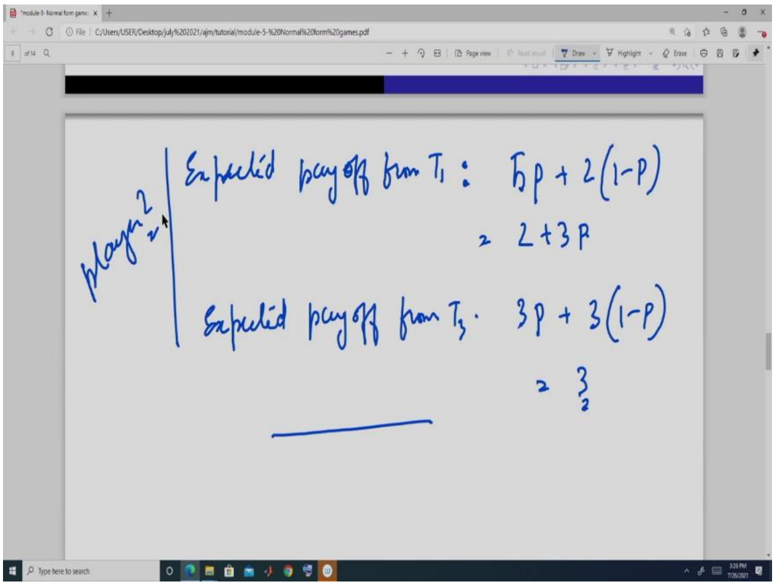

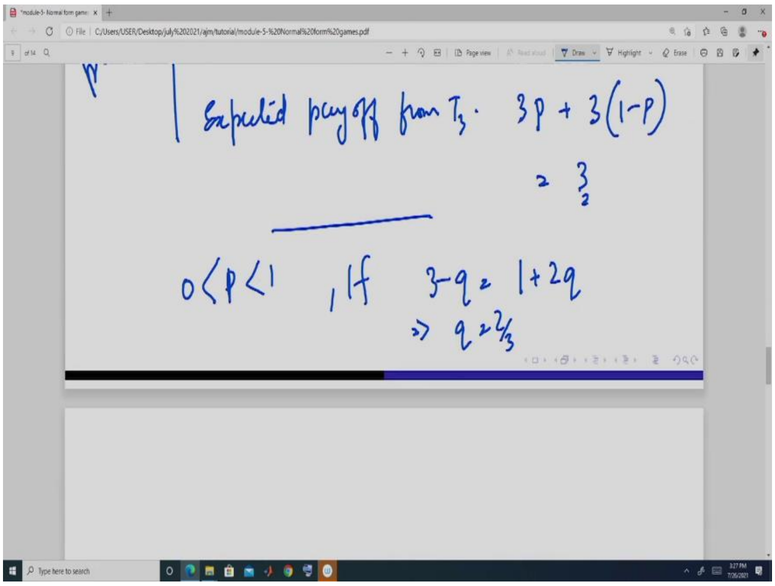

Now, if you look at this expected payoff this is for player 1, expected payoff from T1 is, T1 is, this-$T_1 = 5p + 2(1-p) = 2 + 3p$. Again, expected payoff from T3 is T3 is (3,3) it is same 3P plus T1 minus so it is 3- $T_3 = 3p + 3(1-p) = 3$. Now, we know player 1, this is for player 2. Now, we know player 1 will attach some positive probability that is P will be some positive value that is P will lie between 0 and 1, if these two are equal, i.e $3 - q = 1 + 2q$ only when… it is this… so this is equal to Q is equal. When Q is equal to 2 by 3 then P takes a value lying between 0 and 1, okay.

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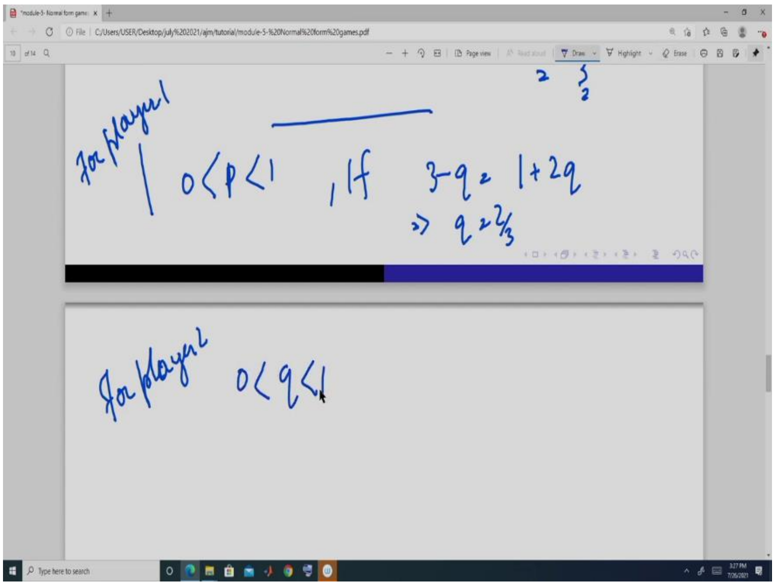

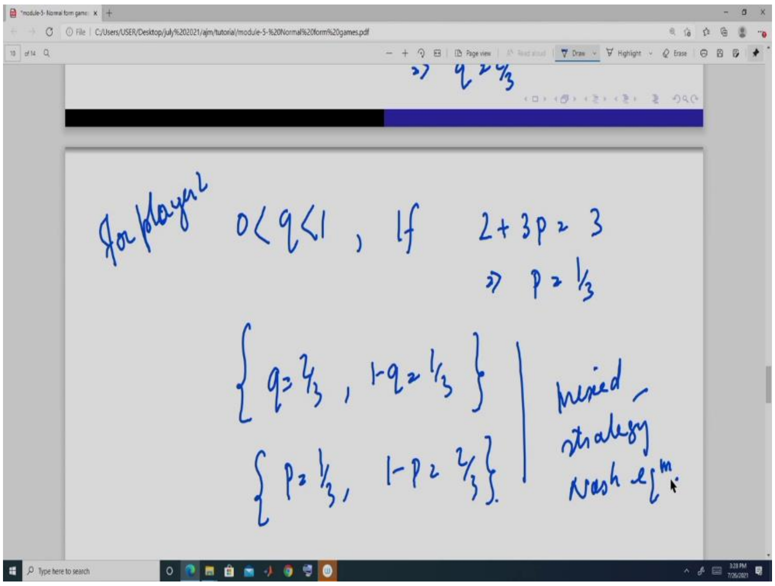

Now, when Q, this is for player 1, for player 2, Q takes a value this- 0<q<1 if these 2 are equal- 2+3p=3, so that means 2 plus 3 P is equal to 3. So, this means P is equal to 1 by 3. So, Q lies between here, so this implies that Q is equal to 2 by 3, 1 minus Q is 1 by 3 this- $\{q = \frac{2}{3}, 1 - q = \frac{1}{3}\}$ when P is equal to 1 by 3 and 1 by P, i.e $\{p = \frac{1}{3}, 1 - p = \frac{2}{3}\}$, okay. And this is the mix strategy Nash Equilibrium, okay. Thank you!

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities & Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 22
Dynamic Games, Backward Induction

Welcome to my course Introduction to Market Structures. So, we were doing game theory and we have completed the static game of complete information and now today we are going to do dynamic games of complete information.

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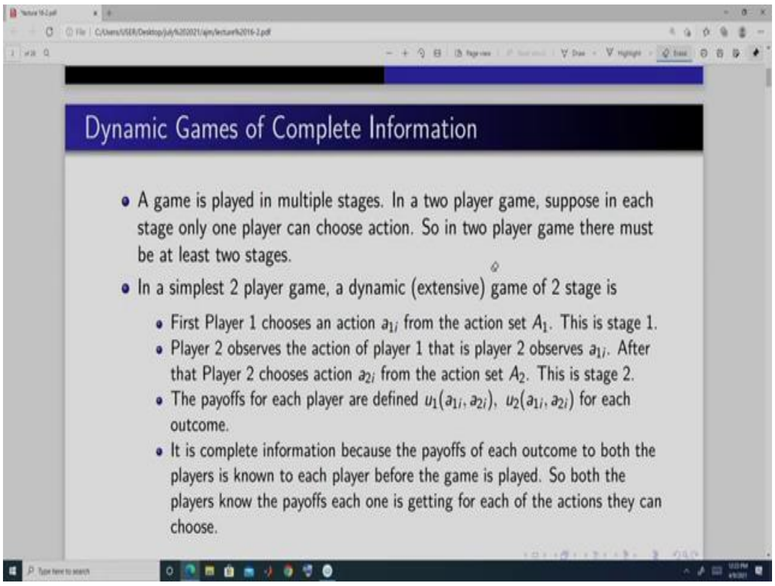

So, in a dynamic game actually game is played in multiple stages. So, in a static game, game is played only once and it is played simultaneously. Here the players may play the game sequentially one after another.

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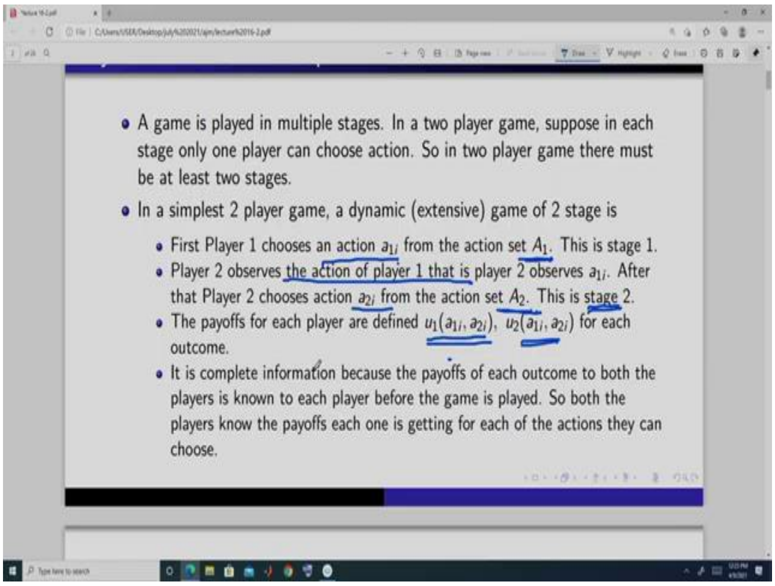

So, in a simplest possible game, if we have two player, so it can be something like this. So, Player 1 first chooses an action from, chooses an action ai from a set of action that is **A**i. So, that is stage 1. And Player 2 observes the action chosen by Player 1 and then Player 2 chooses an action a2i from its set of action that is capital A2. So, this is simplest, you can say, two stage game, where this is, in second stage Player 2 makes the decision.

And we define the utility function or a payoff function for this for each player and for each outcome, okay. And now here what I have said, Player 1 moves first, Player 2 moves second and Player 2 while making decision knows what is the action chosen by Player 1. So, this is a complete information and perfect information game. Why complete information, because the payoff of each player for each outcome is known by both the players.

So, Player 1 knows all the possible outcomes of Player 2 and its own outcome payoffs and also Player 2 knows what are the possible payoffs in each of the outcomes in, of Player 2, Player 1 and also of itself, okay. So, they know what each other's payoff. So, that is why it is a complete information and perfect because when Player 2 is choosing it knows the history, so it knows what is the action of Player 1, okay because it is moving second. So, because of that it is a perfect information.

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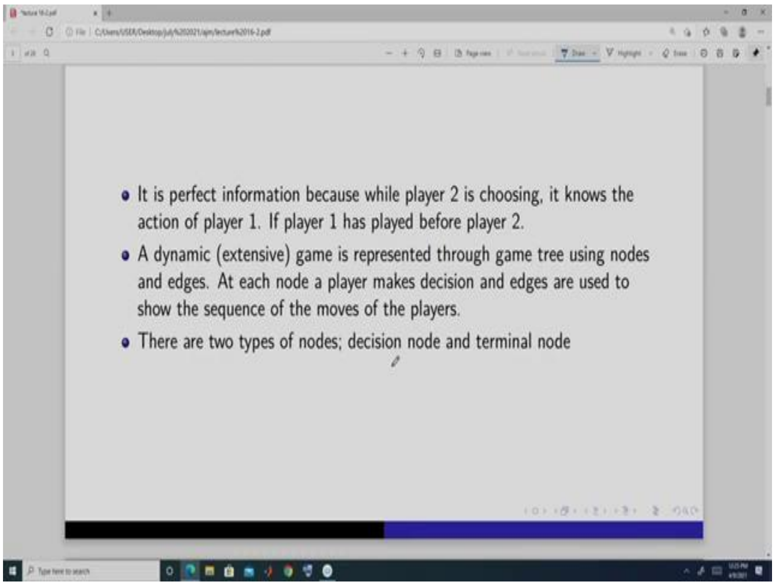

So, again dynamic games are generally represented through a game tree and in the game tree we denote it through edges and nodes. Nodes are where each player is making some decision or taking, choosing their action and edges are to denote the movement of the game, okay.

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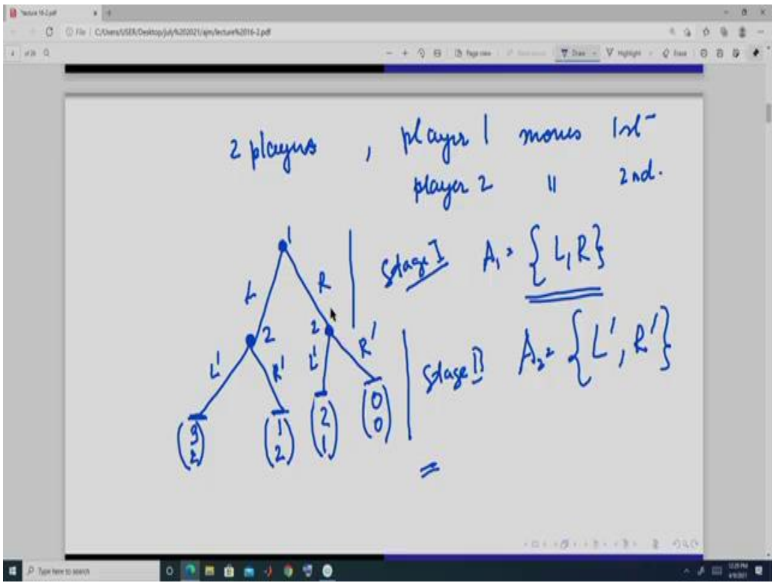

So, you will see one example here. So, this is suppose there are two players, okay. And player 1 moves first, player 2 moves second. So, we represent the game in this way player 1. Suppose the action set of player 1 is L and R, okay, so this is you can say a two player game, where player 1, action set of player 1 is this L and R. So, player 1 moves first, so it is this node. It is the starting point of the game and so this is the stage 1 you can say and this nodes from here is stage 2.

And here in stage 1 player can choose any action from this set, either it can choose L or it can choose R. And in stage 2 player 2 knows either it can be in this node or it can be in this node. If L is chosen, it is in this node, if R is chosen it is in, player 2 is in this node. And here, player 2's set of action is less L dash and R dash. It is same for, in each situation or in contingency, okay and these are the payoffs. So, this tree is, the first element is the payoff of player 1 and the second element is the playoff of player 2. So, like this. So, this is a terminal node.

In the terminal node, we specify the payoffs. So, these are the end of this game. And these nodes are the decision nodes and these are the edges. And based on these edges we specify the movement or how the sequence in which the game is played like first player 1 will choose and player 2 is going to observe that and then player 2 is going to choose, okay. So, this is an example of two stage game which is played between two players and it is a perfect information. Why it is perfect, because it is, player 2 knows what is, in which node it is, when it is choosing and making a decision. Here again player 2 knows in which node it is lying when it is making a decision, okay.

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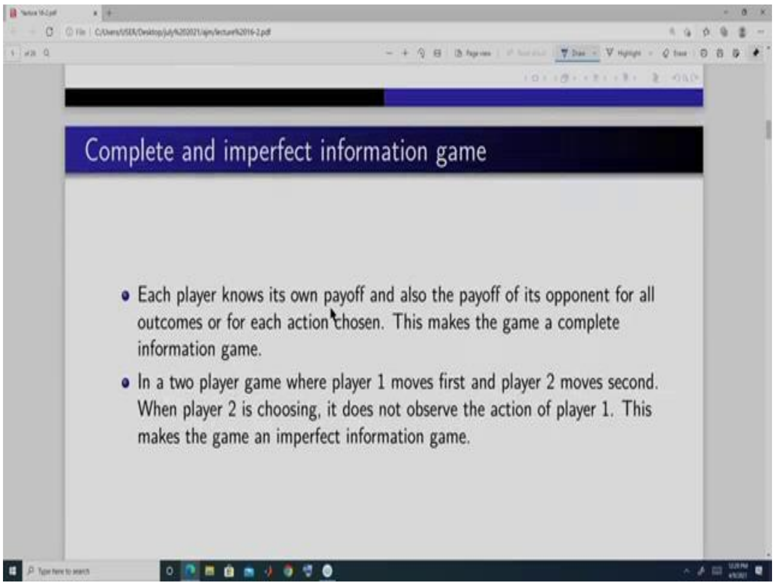

Next there can be another type of game and that is complete information and imperfect information, okay. So, it is a complete information. Why, because player 1 knows the payoffs of each outcome of itself and also the payoff of player 2. Similarly, player 2 knows what its payoff for each outcome and also the payoffs of player 1 for each outcome, okay. So, that is why it is a complete information.

But it may be imperfect, because while player 2, suppose the game is played between player 1 and player 2 and it is a two stage game, where player 1 moves first and player 2 moves second, but while player 2 is moving or choosing its action, then player 2 does not observe the action chosen by player 1.

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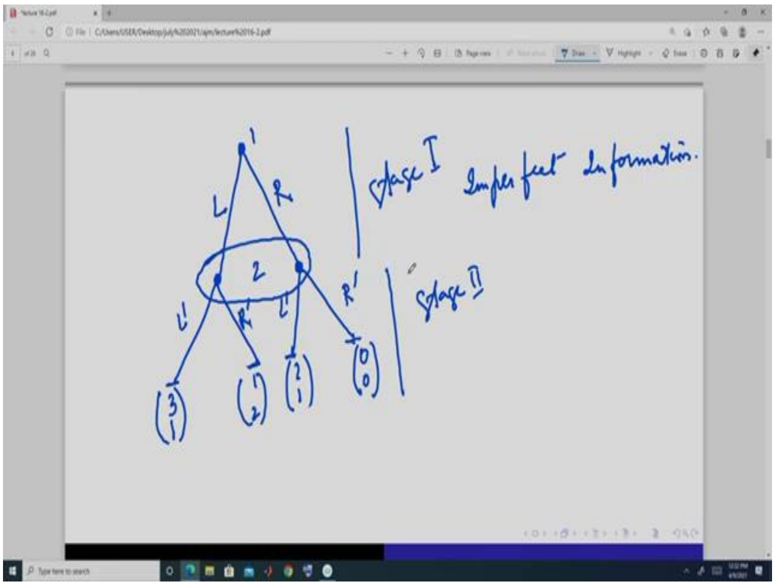

So, if we take the similar game as we have discussed, so it will look something like this. This is player 1, this is player 2, L, R, L dash, R dash, okay. So, here player 1 has chosen and player 2 is supposed to choose here and then we represent it through a, like this and we write here. So, player 2 knows that player 1 can choose either L or it knows or it can choose R. It knows all these outcomes, all these payoffs. But he does not know what is the exact choice of player 1 either L or R. So, player 2 knows that it can be either in this node or it can be in this node, okay.

So, in this sense it is imperfect information, because it does not know in which node it is lying, okay or what is the action chosen by player 1 or by the previous player. So, here also it is, this is stage 1 and this is stage 2, okay. And these are the decision nodes and this is the terminal nodes. The game ends in this way, okay. So, this is the way to represent games in a complete information dynamic games that is it can be of perfect information and it can be of imperfect information, okay. So, first we will discuss perfect information and then later on we will do imperfect information, okay.

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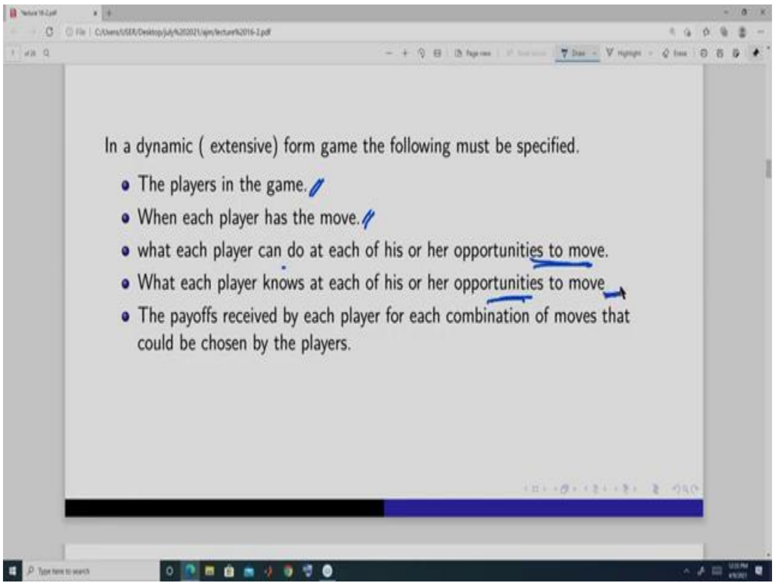

So, from here what do we get that in, while specifying a dynamic game or an extensive game, we should first specify the players, we should specify when each player has the move that is in this game we know that or in this game we know that player 1 moves first and player 2 moves second, similarly, here also we know player 1 moves first and then player 2 moves. And then we have to specify the action set so each player can do at, what each player can do at each of his or her opportunities to move when it is making a decision what it can choose from the set of actions, okay that has to be specified. And also we have to specify what each player knows at each of his or her opportunities.

So, here this is required, because it can be of this nature, because here player 2 does not know whether it is in this node or in this node because player 2 has not observed the action taken by player 1. But in this game player 2 has observed the action of player 1 in stage 1. So, player 2 knows whether it is in this node or it is in this node while making a decision in stage 2. So, we have to specify that this thing and we call this as the information set of each player. We will specify the, specifically define the information set later on, okay. And we have to also specify the payoffs. If we do not specify the payoffs, then we will not be able to play the game, okay.

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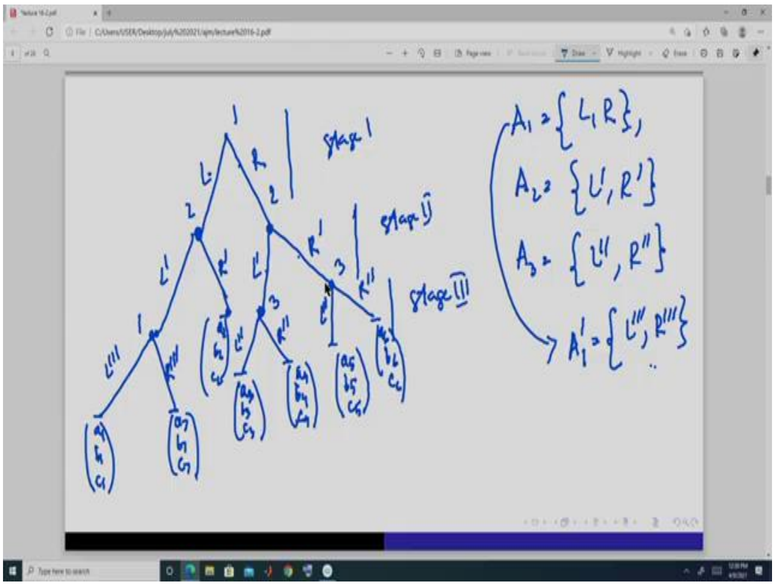

So, it is something like this. So, we have to specify that, for example, if we take another game like this player 1, player 2, player 2, player 1's action set is L, R. So, game ends here. Suppose this is a game between three players, player 1, player 2, player 3. So, the sequence is, this is stage 1, this is stage 2 and this is stage 3. So, stage 3 may be played or may not be played, okay depending on what is the action chosen by player 1, okay. So, this is stage 1, this is stage 2, and this is stage 3, okay.

So, we have specified the players player 1, 2, 3. We have specified their action set. So, player 1's action set is L, R, i.e $A_1 = \{L_1, R_1\}$ player 2's action set is L dash, R dash, i.e$A_2 = \{L', R'\}$ again player 3's action set is L double dash, R double dash, i.e $A_2 = \{L'', R''\}$in this game. And now we specify and again we have specified the moves, who moves first, who moves second, who moves third. We have also specified who knows what, like when player 2 is choosing, making a decision here it knows the action of player 1.

When player 2 is making a decision here, it knows the action of player 1. Again, player 3 knows the action of player 1 and also knows the action of player 2. Again, player 3 knows the action of player 2 and also player 1, so all this, and we specify the payoffs here suppose (a1, b1, c1), (a2, b2, c2), and this is (a3, b3, c3) and this is (a4, b4, c4) and this is suppose (a5, b5, c5) and this is (a6, b6, c6), okay, so this.

And here interestingly we make, can make this game further complicated. So, we can do like this. Suppose here we again, so if player 1 chooses L dash, then player 1 can again move and suppose it is and the game ends here. So, it is (a1, b1, c1) and this is (a7, b7, c7), okay. So, then the action set of player 1 is a1 and we have another action set and that is A dash that is L. This action set $A_1' = \{L''', R'''\}$ is here in stage 3. So, in stage 3 we may have either this game being played or this game or this game, okay. So, we can have different ways in which we can represent a game depending on what is our objective, okay.

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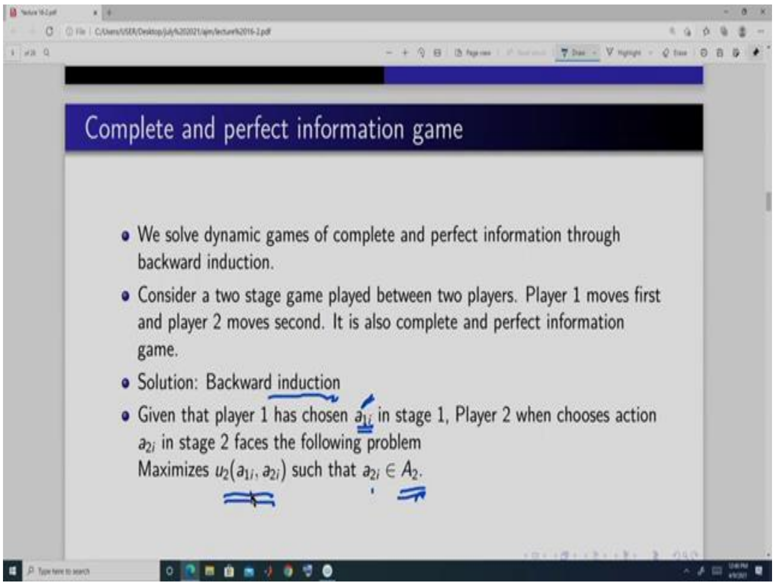

Now, the question is how to solve this problem? What is the solution concept? So, we use something called a backward induction method, okay. What do we do in a backward induction? So, let us take the simplest case that is we have two players, player 1 and player 2. Player 1 moves first and player 2 moves second. While player 2 moving or choosing, it knows the action of player 1. So, this is a simple game like this. Player 1 has moved first, player 2 has moved second in stage 2. It knows the action chosen by player 1, right? So, it knows that it is either in this node or it is in this node, okay.

So, here what player 1 will do? So, player 2 first, suppose player 1 has chosen an action from its set that is a1 then, in stage 2, so when we say we are using backward induction, then it means that we are moving from backwards. So, we will first go to the last stage that is in this case stage 2. So, we know suppose player 1 has chosen an action this in stage 1, then player 2 is going to do what, it is going to maximize its payoff function such that by choosing an action which belongs to his set of action, okay. So, given a action of player 1 that is a1i in stage 2 player 2 is going to maximize this, okay with, such that this action belongs to this set of action.

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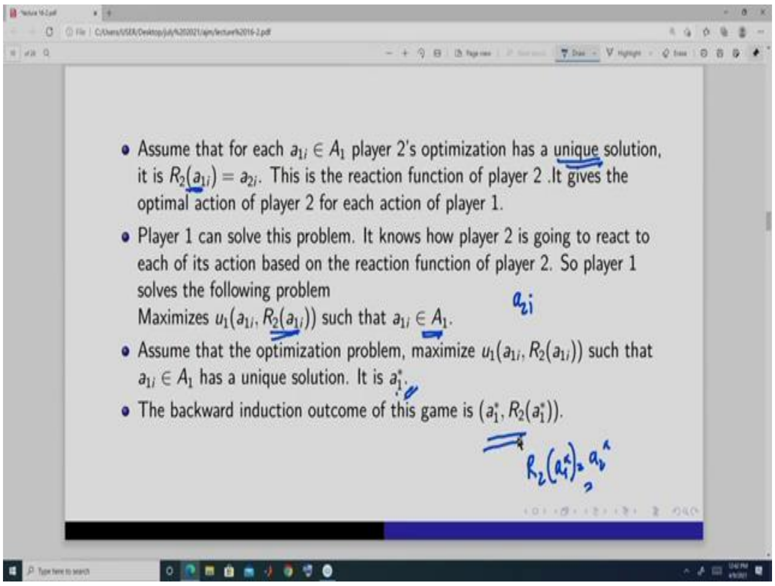

So, then it means what, then we will have a solution to this optimal problem and we assume that there is a unique solution, okay. So, then what do we get, we get a function like this- $R_2(a_{1i}) = a_{2i}$ and we call this a reaction function. So, we plug in the value of a1i that is a choice of action or the action of player 1 and then we get the optimal choice of player 2. So, this is a reaction function of player 2. So, it is a function of the action of player 1, okay.

Now, since it is a complete information game, so player 1 will know this reaction function. So, player 1 knows that if I choose a1i then what is going to be the decision of or what is going to be the choice of the player 2 in stage 2 based on this reaction function. So, in stage 1, what player 1 will do, will plug in this- $R_2(a_{1i})$ in case of a2i, okay. And then it will choose a1i such that it maximizes its payoff function and also it should belong to the set of actions, okay. And we assume that it has a unique solution, otherwise, it is a problem, and then we get a like this- a_1^*, okay an optimal solution.

So, the backward induction outcome is that a1 is always, a is always going to choose a1 star, and player 2 is going to choose a2 star which is always equal to a1 star like this. So, this is the backward induction outcome, okay.

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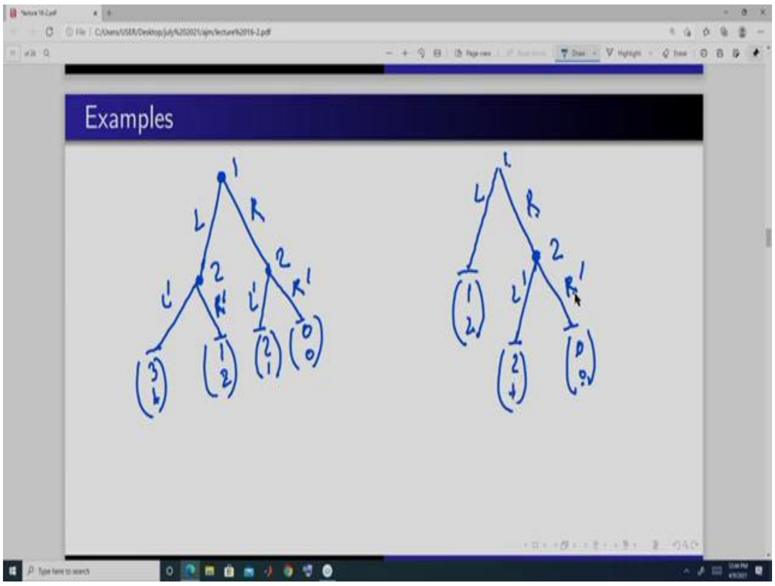

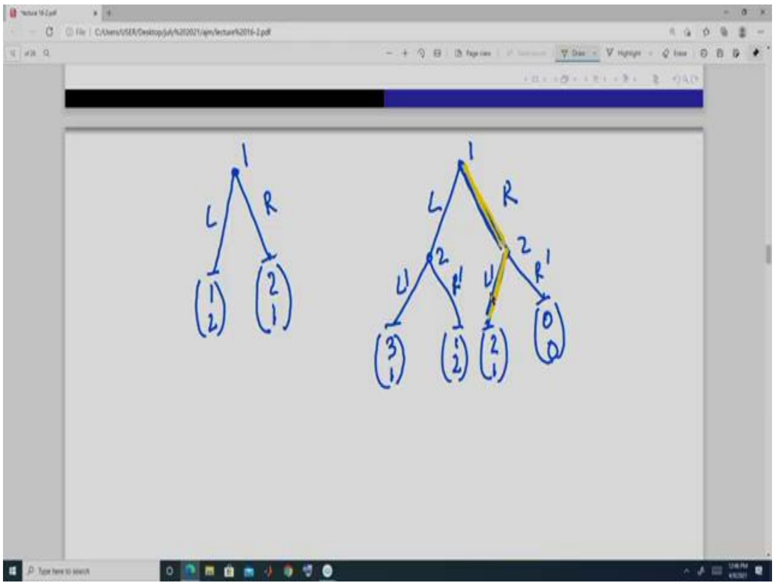

So, now, let us solve some examples. So, we have already specified one type of game. Let us take that game. Player 1 moves first, L, R, player 2 moves second L dash, R dash, okay. So, this is the game. So, this is the, in stage 1 player 1 makes a decision and in stage 2 after observing the action of player 1 player 2 makes a decision, okay. So, let us start with the second stage here or here. So, in second stage we can have decision can be taken in this node or decision can be taken in this node, okay. So, suppose decision is taken in this. So, player 2 knows that player 1 has chosen L. So, it is in this node.

So, here player 2 is always going to choose R dash, because it is compared to 2 and 1. So, R dash is this. So, what we do in this portion we represent that the optimal choice of player 2 given that player 1 has chosen L is to choose R dash, so the outcome is this. So, we delete this portion and we write it is going to be this, okay. So, it is this. Now, suppose player 1 has chosen R, then player 2 is here. If it is here, it will compare between 1 and 0. If it plays L dash it gets 1, if it plays R dash it gets 0. So, player 2 is going to choose L dash.

So, so player 1 while it is choosing it knows how player 2 is going to behave. If it chooses L player 2 is going to choose R. So, this is the outcome. If it chooses R player 2 is here, player 2 is going to choose L dash. So, it is outcome is this. So, player 1 will choose based on this. So, it will choose R, this. So, backward induction outcome in this game is actually, okay so backward induction outcome is this and this. This path is the backward induction outcome, okay.

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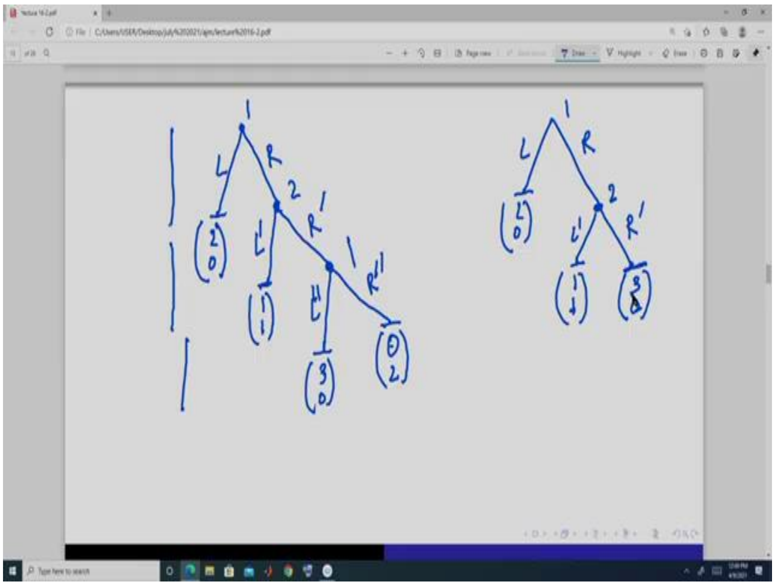

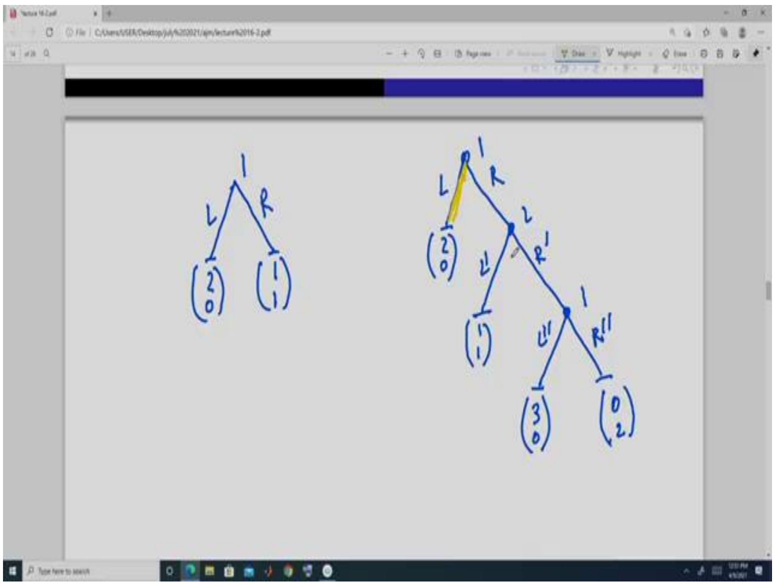

Now, let us do another example. So, suppose the game is played between two player, player 1 moves first and its action is L and R. Suppose, so this is a game played between two players player 1 and player 2. Player 1 moves first. It is, this is stage 1, this is stage 2 and this is stage 3, okay. So, if player 1 chooses L, then the game ends. If player 1 chooses R then player 2 can move. It can choose either L dash or R dash.

If player two chooses L dash, then again the game ends. If he chooses R dash, then again player 1 get a chance to move and it can choose from L dash, L double dash and R double dash, okay and these are the payoffs- (3,0), (0,2), okay. So, the first element is the payoff of player 1 and the second element is the payoff of player 2.

So, let us, since we are using now backward induction, let us start from this stage, so that means we have to start from this-L'', R'', okay. Here, so when player 1 is here suppose, it is making a decision from here, it is always going to choose this (3,0), because 3 is greater than 0. So, here we can write this. So, here it is going to get, player 2 is going to get, because player 1 is going to choose L double dash, so it is going.

Now, here player 2 is making a decision. Player 2, it will compare, if it chooses L dash, it is going to get 1. If he chooses R dash, it is going to choose, going to get 0. So, here player 2 will choose L dash, because 1 is greater than 0. So, it is, this becomes this- L= (2,0), R= (1,1). So, player 1 is always going to choose L, this rather than this, right? because 2 is greater than 1.

So in this game, the backward induction outcome is that the game is only played by player 1 and that is also only once. So, these edges represents its action, okay and movement who moves what, okay. So, the backward induction outcome here it is this. So, player 1 is always going to choose R and the game ends. Instead the game could have been played for three stages, but actually if we use backward induction as a solution concept, the game is played only once and that is in only one, first stage, okay. So, this is one example.

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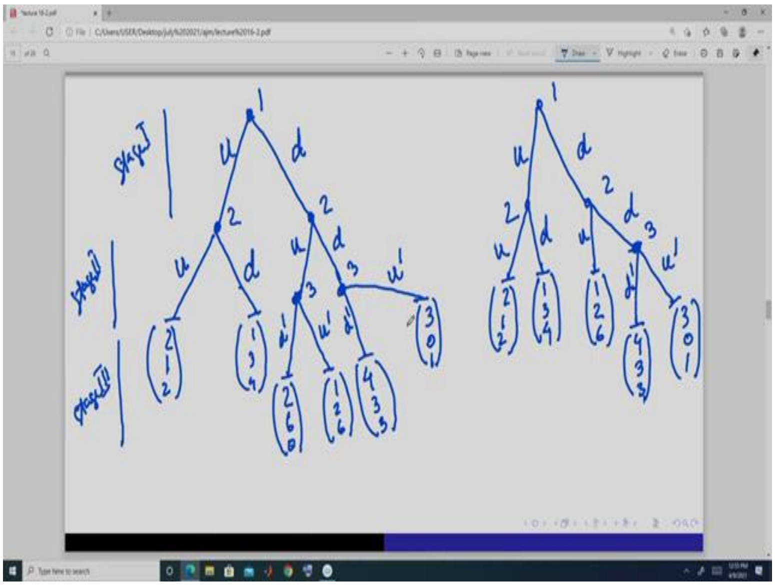

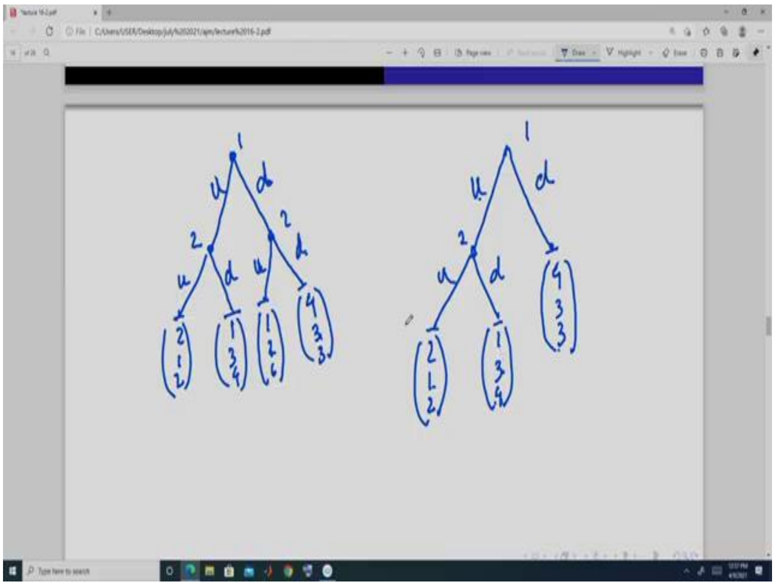

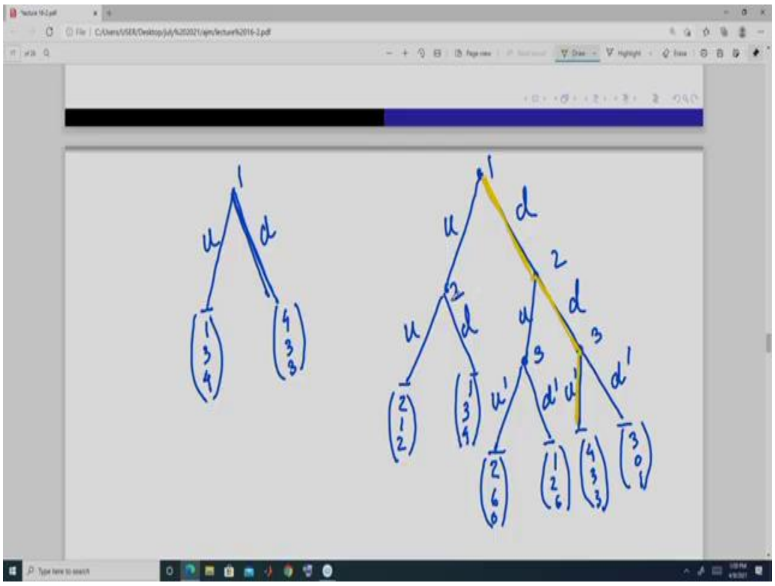

Now, let us do another example and this is a slightly bigger game. Let me use blue color, okay. So, this is the game. And this is a three stage game way. So, in the first stage, player 1 makes a decision, so this is stage 1. In stage 2, player 2 makes a decision either in this node or in this node. So, this is stage 2. So, the game may end at stage 2 if player 1 chooses u and player 2 is choosing either u or d. But here we can have another stage and that is from here for this. In stage 3 player 2 chooses or moves if player 1 chooses d and so then they will be in this path and player 2 will get the chance to make the decision. So, then we will have third stage, okay.

So, since we are using backward induction, let us start from the third stage. So, suppose here, suppose player 1 has chosen d and player 2 has chosen u and so it is here. So, player 3, so first element is the payoff of player 1, second element is the payoff of player 2 and third element is the payoff of player 3, so 0 to 6. So, player 3 is always going to choose u dash, because 6 is greater than 0. So, this game becomes. So, here if player 1 chooses d and player 2 chooses u, player 3 is always going to chose u dash, because 6 is greater than 0. So, here it is (1, 2, 6), okay.

Now, we have this portion. Here, player 3 if it chooses u dash it gets 1, if it chooses d dash it gets 3. So, player 1 is going to choose d dash, okay. So, the game is now like this. We have only two stages now. We have played the third stage. So, in stage 2, if player 2 is, if player 1 has moved, chosen d if it is here, it knows if it plays u, it is going to get 2, if it plays d it is going to get 3. So, player 2 is going to choose d. Here it is going to choose d. So, it is this. And here if player 1 chooses u, player 2 is going to choose d, because 3 is greater than 1, it is this and player 2 is going to choose d here, because 4 is greater than 1.

So, the backward induction outcome of this game is. So, the backward induction outcome here it is like this. Player 1 chooses d, player 2 chooses d and player 3 chooses u dash, okay. So, this path soon by these yellow colored edges gives me the backward induction outcome in this game, okay. So, these are some of the examples of, to solve dynamic game with complete information and also perfect information using backward induction, okay.

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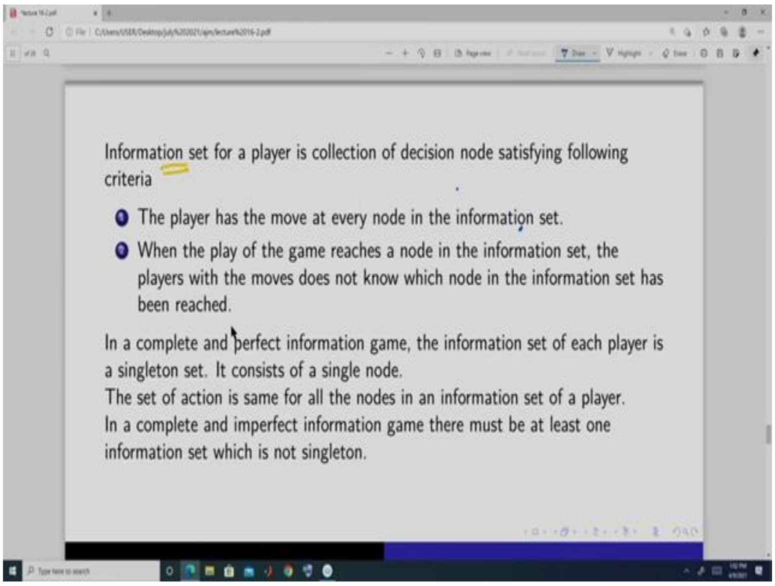

Now, we will specify the complete information game of perfect and imperfect information in a more precise way by using a concept called information set. And what is an information set? Information set is actually a collection of nodes for each player, okay. So, each player will have some information node. So, for each, so you can understand information set as that for each stage we will have one information set and for one player, okay depending on the player who is moving or making decision in that stage. So, it is a collection of nodes.

So, it gives the player has the move at every node in the information. So, since it is a collection of nodes, so in each node which belongs to an information set a specific player has to move or has to make a decision in each of these nodes. And if there are many nodes in a information set, the player, so information set is specific to a player. So, the player will never know in which node it is lying or because player 1 will not observe the history if there are multiple nodes in an information set, all the previous histories.

So, when the play of the game reaches a node in the information set, so you have an information set and the game while playing it has reached a node then the players with the moves does not know which node in the information set has been reached, okay. So, these definitions have been taken from Gibbons, A Primer in Game Theory, you can follow from there also, okay.

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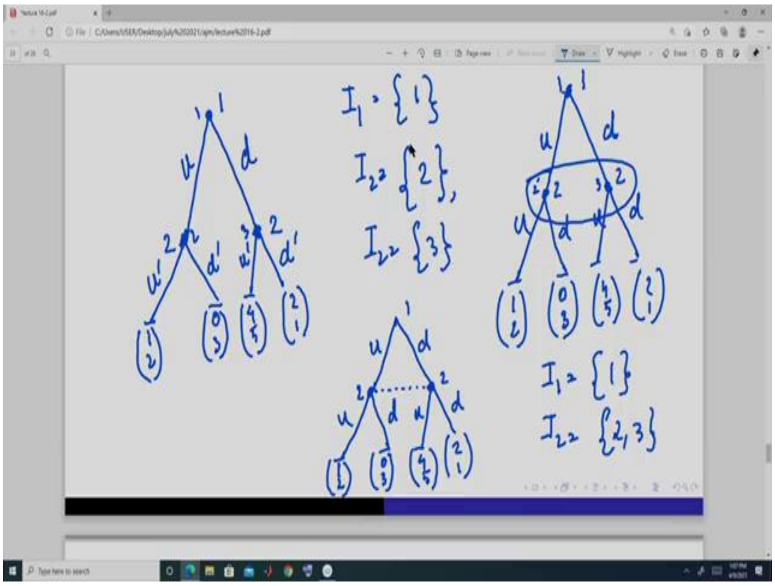

So, if we take this game, suppose, okay and suppose the payoffs are like this-(1,2), (0,3), (4,5), (2,1) okay. So, this is the two stage game and it is a game between two players, player 1 and player 2. Player 1 moves first, player 2 moves second and player 2 observe this. So, what we can say that the information set of player 1 is only this node. So, here we have to number the nods also. Suppose nodes are numbered in again in a 1, 2, 3, this way, okay. So, player 2 it is only node 1 information set. Information set of player 2 it has 2 information set, this and this.

So, why player 2 has 2 information set, because see player 2 knows that it is in this node. Again, here player 2 knows that it is in this node. But if we place all these two nodes, because player 2 has two nodes in which it is, it can make decision in a same information set, then it means that player 2 will not know whether it is in this node as the game is being played, whether it is, it has reached this node or it has reached is node, okay. But in this game player 2 knows that it is in this, whether it is in this node or in this node. So, that is why we have to separate this and these are singleton sets.

So, if all the information sets are singleton, then it is a game of perfect information. And if it is not singleton, then it is a game of imperfect information. So, it is like this suppose, same game, but it is like this. So, here player 1's information set is only this node that is node 1, this node is node 2 and this node is node 3, okay. So, but here player 2, player 1 can choose u or it can choose d. If it has chosen u, player 2 is in this node. If it has chosen d it is in this node. The game has played will move in this path, again game move in this path.

But player 2 does not observe the action of player 1, but it knows player 1 can choose either d or it can choose u. It can be either in this node or in this node, but it is not sure whether it is in this node or it is in this node, so that is why we mark a like this, okay. We mark it in this form.

Another way to mark it is in this form is to make a dotted line like this. Connect these nodes by a dotted line. Then it means that player 2 does not know whether it is in this node or it in this node. That means player 2 has not observed the action of player 1.

So, here, information set of player 1 is node 1, this- $I_1 = \{1\}$ because player 1 moves first, okay. Then player 2 information set is this node 2- $I_2 = \{2,3\}$. So, there are two nodes. It does not know whether it is in, whether, while making a decision, whether it is in this node or in this node. But here player 2 knows that whether it is in this node or it is in this node. So, that is why we have to make two separate information sets for player 2. But here we do not need. So, then in this way we can define the complete information, imperfect information game and information, perfect information game.

How, if all the information set of each player is singleton then it is perfect information. If there exist at least one information set which is not singleton then of any player, then it is an imperfect information game, okay.

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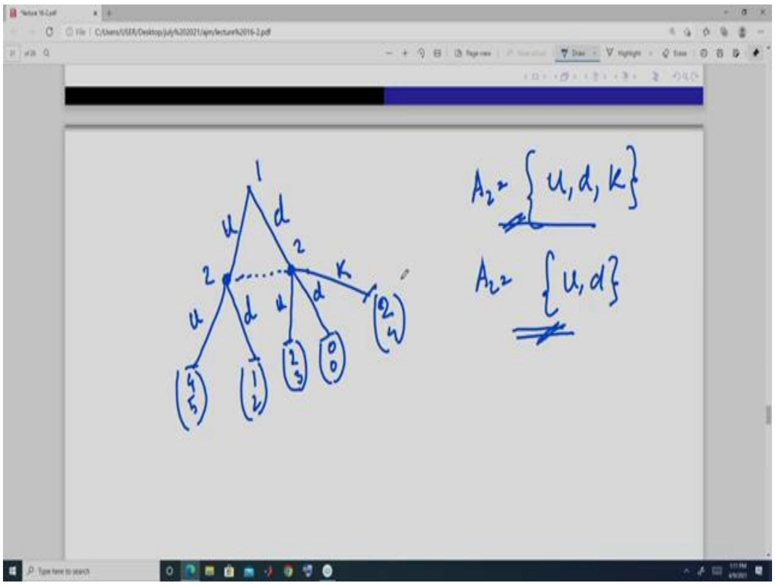

And further we require one more thing. Suppose if there are two nodes in one information set and in each node the number of actions are different then also player will be able to distinguish. So, the action set should be same for each nodes in a information set, okay. So, it is something like this. And suppose, so this is an imperfect information game, because player 2 does not know whether it is in this set, node or this node.

But suppose player 1 knows, player 1 plays this u, then ideally since it is connected by the dotted line these two nodes, so player 2 should not be able to distinguish between this node and this node. But here if it is here, then the action set is this- u= (4,5), d= (1,2). So, that is player 2 will know that, okay. But if it is here, the action set is u, d, k. So, it has two different action sets, right? So, if player 1 plays this d, then the action set that the player 1 has to choose from is d, k u, d, k, i.e. $A_2 = \{u, d, k\}$. And here this action set is only u and d. So, it does not matter whether player 2 observes the action of player 1 or not.

If it knows that whether it has to choose from this-$A_2 = \{u, d, k\}$. or it has to choose from this-$A_2 = \{u, d\}$, then because it will come to know, right? moment it is here, its action set is this-$A_2 = \{u, d, k\}$ or you can say action space is this. If it is here, its action space this- $A_2 = \{u, d\}$. So, it can distinguish between these two. So, here, even if player 2 has not observed the action of player 1, it can make this distinction and it will, from here it will know whether it is in this node or it is in this node.

So, that is why if we are in a information set, there are multiple nodes or nodes more than one, then the action space or the set of actions available should be same. So, here we should remove this k, okay. So, the action space in each node should be same if these nodes belongs to the same information set, okay.

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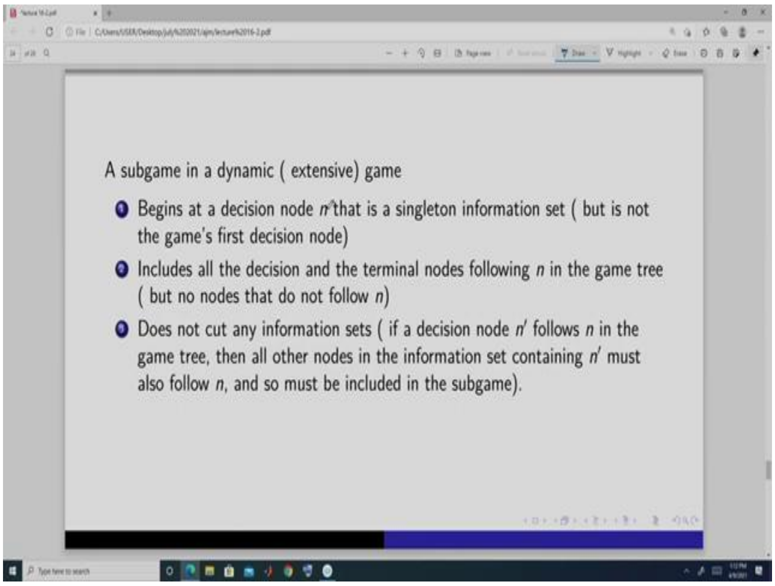

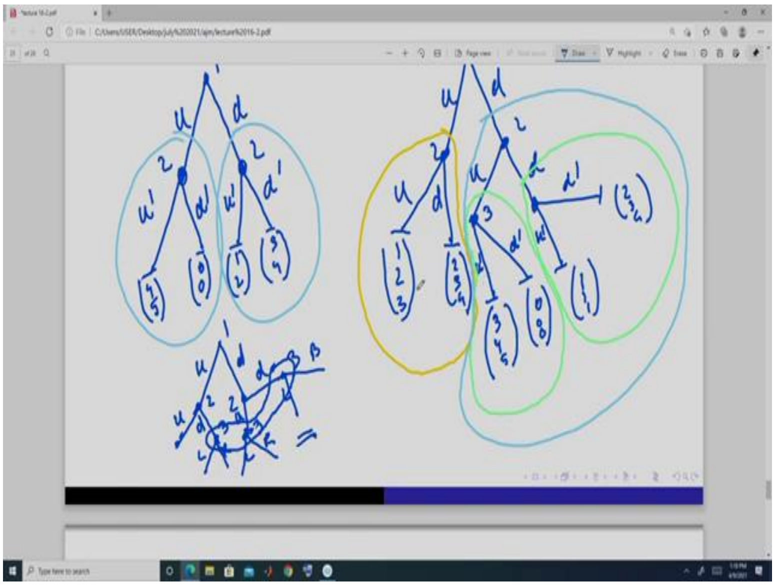

So, in this game, in this complete or dynamic games, we defined it through a game tree and we have multiple stages. Now, here we bring in another concept that is sub-game. And sub-game here, if we take this game, okay sub-game here it is defined in this way that the sub-game should begin at a decision node, node numbered n, any number that is a singleton information set, okay. So, a sub-game should start from a game here, either it can start from here or it can start from here. But it should not be the topmost that is the first node, okay. So, it should not be the first decision node, okay.

And it should include all the decision node following from it. So, if we have a game like this, suppose it ends here, suppose this is a game, okay. So, it is a three stage game. So, here, a game, this is a sub-game, because it starts from a decision node which not the first one and again this is a sub-game, again this is a sub-game and this is also a sub-game, because from here all these nodes which follows it belongs to this game only. So, it will have this. This is one sub-game- u= (1,2,3), d=(2,3,4), this is another sub-game- $u' = (3,4,5), \mathrm{d}' = (0,0,0)$, this is another sub-game-$u' = (1,1,1), \mathrm{d}' = (2,3,4)$, and we will have another sub-game like this- $u' = (3,4,5), \mathrm{d}' = (0,0,0), u' = (1,1,1), \mathrm{d}' = (2,3,4)$, okay.

So, this game has 1, 2, 3, 4, sub-game. This has only two sub-game. This is one and this is one, okay. So, sub-games are that it includes all decisions and terminal, decision node and terminal node following n in the game tree. So, if you start from any node then all the nodes that follows it, it should belong to that game, okay and it should not cut any information set. So, this means that if the game is suppose of this nature, now suppose player 3 also moves here, okay player 2 always moves u and d, player 1 moves at 3, L, R, L, R, L, R, so the game is something like this, okay.

So, here player 1 moves, either it can choose u or d. Then player 2 moves, it can choose u, d. If it is here, player 2 if it chooses d then player 3 can make a choice. Again, player 2 here it can choose either u and d and then player 3 can choose from here. So, in this game what you can see the sub-games are only one sub-game that is, actually it has no sub-game, because if you start from here what happens, this and this. But this game ends here. But if you start from there, it belongs to this information set. And this information, here we do not have any information set.

But this, if we start the game from here, then this node also belongs to this information set, but this node is not in this path of the game played. This is, this node is from this path, right? So, that is why this is, there is no sub-game in this game, okay because if you start from this, then we move here or we move here. But here this node it belongs to this information set and these nodes are not in the path that follows from here.

But if you look at this, if the game starts here, then all the paths, you can go to all the nodes that follows u. From here it, but from here you cannot go to this which are in the information set, this information set. So, that is why here is no sub-game in this game, okay. So, this is way we define the sub-game.

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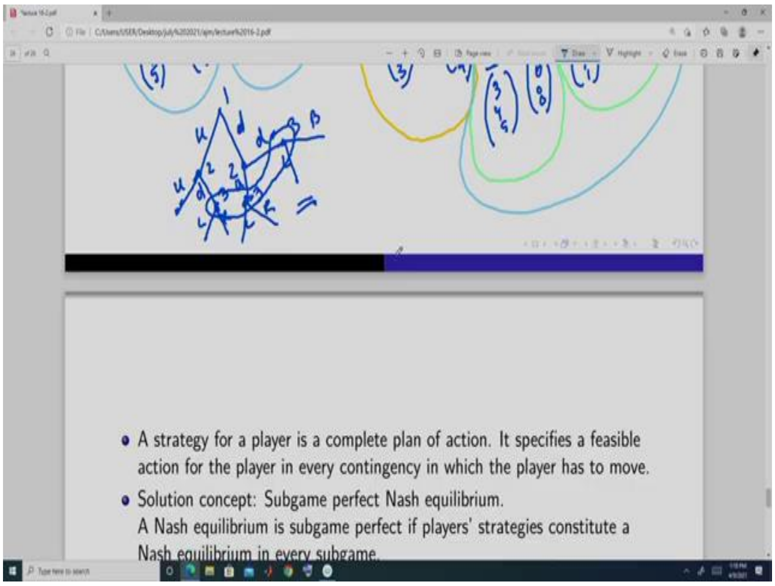

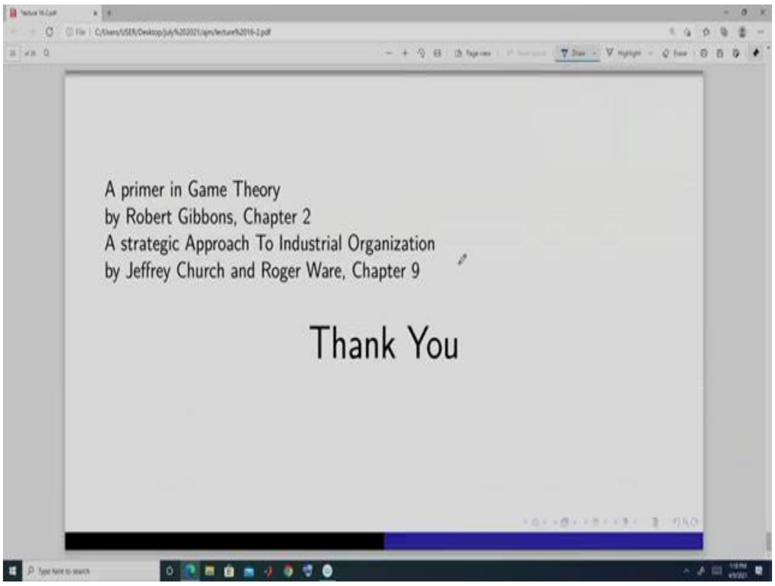

And then we will use a concept that is sub-game perfect Nash equilibrium and that we will do in the next class, okay. So, thank you. And for this you can read chapter 2 or chapter 9 from this book, okay.

Introduction to Market Structures
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Lecture No. 23
Subgame Perfect Nash Equilibrium

Hello, everyone. Welcome to my course Introduction to Market Structures. So, we were doing dynamic games that is extensive games. So, today we will conclude that portion.

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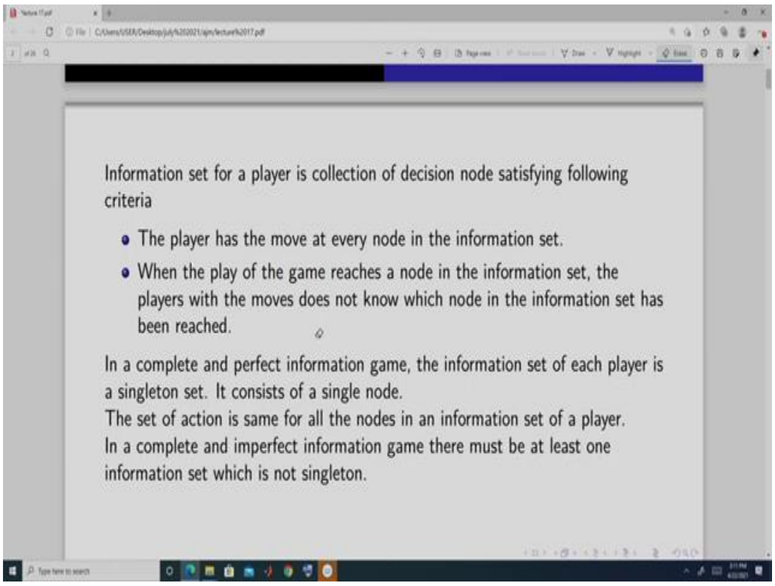

So, we have already done what is information set. Information set is a collection of decision nodes of a player and it can be singleton set or it may have many decision nodes. If it has many decision nodes that is more than one decision node, then the player does not know in which decision node he is taking the decision. So, he only knows that in which node, in which information set it belongs to, but it does not know exactly which node it is in, okay.

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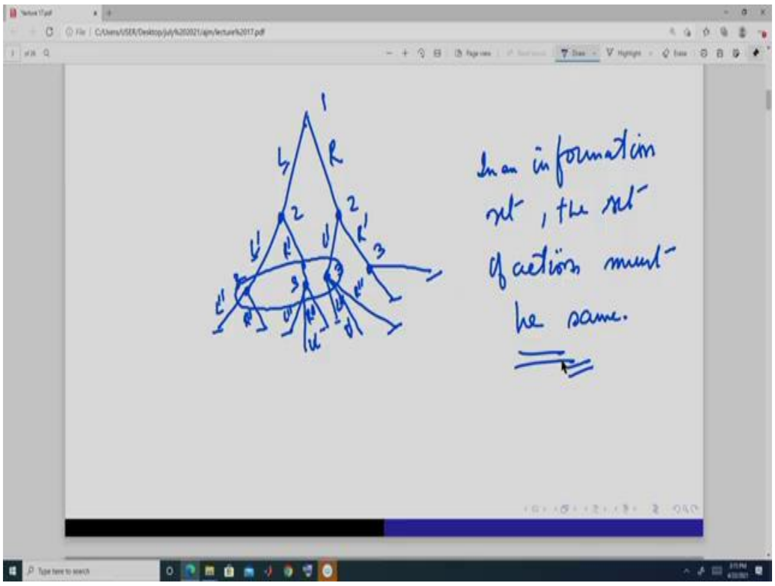

So, to make, so if suppose a player is already they have played a game like this suppose. Suppose they have already played a game of this nature. This is player 1, L, R and this is player 2, this is L dash, R dash, L dash, R dash, and then suppose again player 3, okay. So, this is player 3, player 3, 3, 3. Now, suppose I have a situation like this, okay. And here I have some payoffs. Payoffs I will not specify now.

Now, here player 3, in this situation player 3 is it is moving it does not know whether it is in this node or in this node, or this node, because player 3 has not observed these movements of or these actions of player 1 and player 2. So, that is why it is imperfect information. But it is complete information because all the payoffs are known, okay. So, here what is happening, in this situation player 3 does not know which node it is.

But suppose we have a situation where it is L double dash, R double dash, L double dash, R double dash, L double dash, R double dash, L double dash, R double dash, but it has one more strategy and that is suppose u, one more action that is u and this has one more action and that is suppose d dash. So, then see each node has a different sets of action. In this node it has L double dash, R double dash. In this node it has L double dash, R double dash and u dash. In this node it has L double dash, R double dash and d dash.

So, if player 1 has chosen this action and player 2 has chosen this action then the game is here and player 3 has not observed this action. But if player 3 only can choose from these two actions, from these two, only from these two actions it has to choose then it knows it is in this node. So, for this reason what happens that the all the action set must be seen in a information set. So, in an information set the set of actions must be same, okay.

Otherwise, the player 1, it will no more be an imperfect information, because player 3 even though it has not observed actions, but from the set from which it has to choose an action, it will, it can decipher or it can infer what are the past actions, okay. So, that is why the action must be, set of action must be same, okay.

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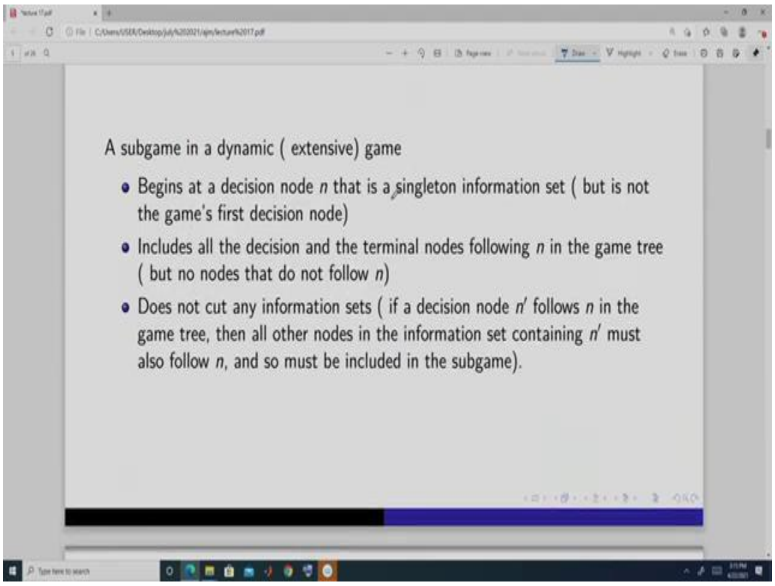

And then we have also defined what is subgame. Subgame, it begins from any node which is other than the first node and subgame, it follows all the decision nodes from any specific decision node that is includes all decision and terminal nodes following n in the game tree, and does not cut any information set. So, if, a whole information set should belong to only one subgame. Information set cannot be partitioned into two different subgames, okay. So, this is.

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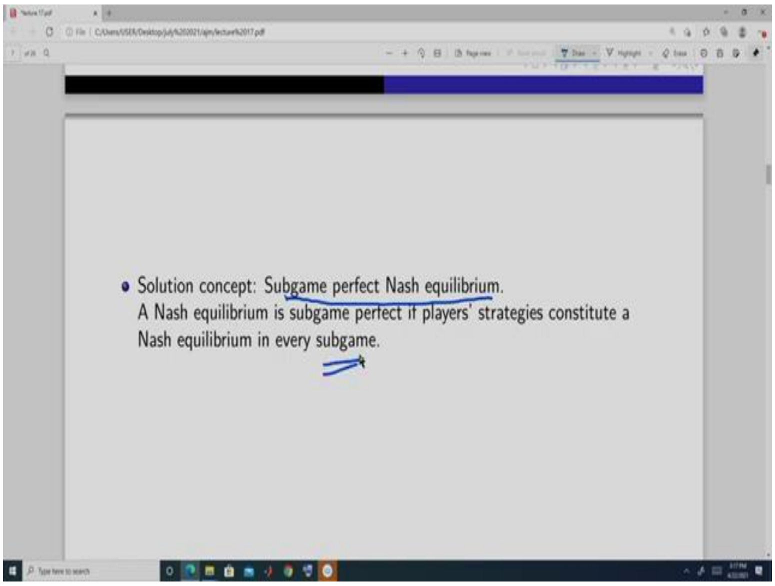

Now, we will discuss some solution concept. So, we have done backward induction. Backward induction is a method of solving the game. But what solution, while using backward induction what we were doing? We were moving from the last subgame to and then we have moved backwards and we have tried to find the Nash equilibrium at each stage, okay. So, a specific solution concept for this type of dynamic game is subgame perfect Nash equilibrium and this was proposed by Selten, okay.

So, it says a Nash equilibrium, it is a Nash equilibrium is subgame perfect if players' strategies constitute a Nash equilibrium in every subgame. So, let us take, in every subgame.

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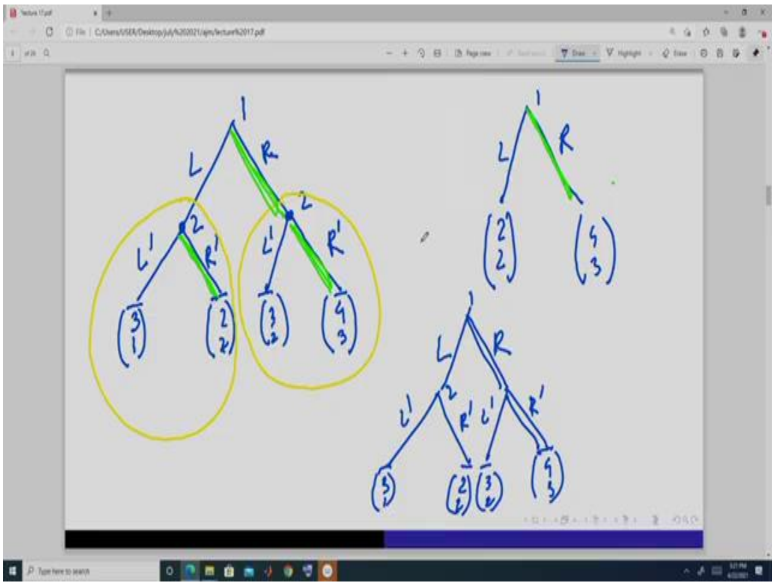

So, let us take an example, okay. Player 1, its action is L and R, player 2 its action is L dash, R dash, L dash, R dash, player 2, so it is a complete information game, okay complete information and also perfect, okay. This is the subgame. Now, here, it has two subgames. This is starting from this and this is a node. We have this- $\mathrm{L}' = (3, 2), \mathrm{R} = (4, 3)$ as one subgame and this- $\mathrm{L}' = (3, 1), \mathrm{R}' = (2, 2)$ is another subgame, okay. So, in this subgame, so, how do we solve? We use the same the backward induction only, but here we specify that it has to be a Nash equilibrium in each subgame. So, it is a Nash equilibrium in this subgame, it has to be a Nash equilibrium in this subgame and also in the whole game, okay.

So, this, if player 2 is here, then we know it is optimal to choose this R dash- $\mathrm{R}' = (2, 2)$, because 2 is greater. So, in this game, this-$\mathrm{R}' = (2, 2)$ is the action that player 2 is going to choose. In this game, when player 2 is choosing it is again going to choose this- $\mathrm{R} = (4, 3)$ 3 is greater than 2, here 2 is greater, right? And then, so this game, now, it is what. So, player 1, since we are using backward induction, so it knows that if it chooses L player 2 is in this node and player 2 is going to choose R. And if player 1 chooses R, player 2 is going to be in this node and it is going to choose R. So, player 1 knows this by, means player 1 can infer this based on this game, because it is a, information is complete.

So, player 1, what it will do? It will compare these two payoffs. So, for player 1, it is going to be like- L= (2, 2), R= (4, 3), so player 1 is going to choose this action- R= (4, 3). So, it is going to be this here. So, in this game subgame perfect Nash equilibrium is, in this (L) game it is this- $\mathrm{R}' = (2, 2)$, and in this (R) game it is this- R= (4, 3), and then it is chooses this one. So, this, but while we were doing backward induction outcome and we are trying to find the optimal outcome in the backward induction in the same game, it is here, it is this, here it is this and then from this it is here.

So, we do not need to, because we do not need to specify this. So, backward induction outcome is only this-$\mathrm{R}' = (4, 3)$, but subgame perfect Nash equilibrium outcome is this. We have to specify the Nash equilibrium in each subgame. In backward induction outcome we do not need to. So, this is the difference. So, it is a complete profile of this, all this has to be provided, okay.

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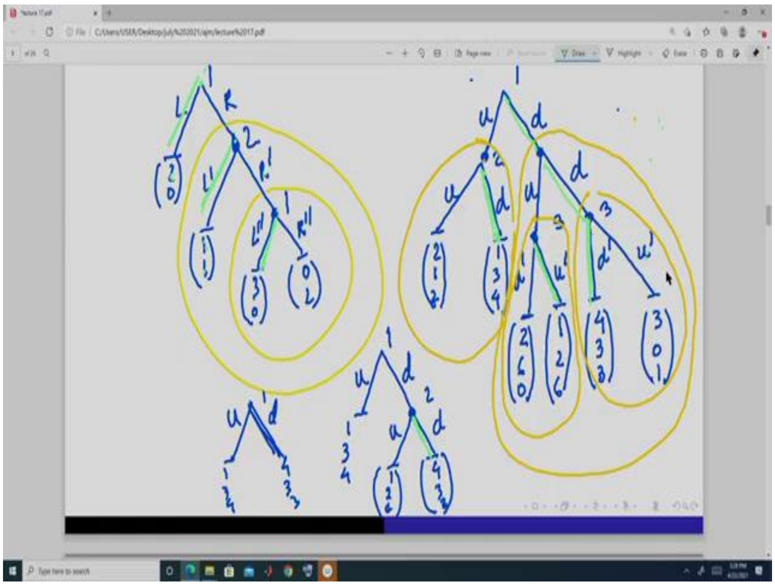

So, let us do another example. Suppose player 1, player 2, again, player 1, okay this is L, R, L dash, R dash, L double dash, R double dash, payoffs are- (2,0), (1,1). (3,0), (0,2). So, the subgames are, this is one subgame- $\mathrm{L}'', \mathrm{R}''$, this is another subgame- $\mathrm{L}'', \mathrm{R}'', \mathrm{L}', \mathrm{R}'$, okay. Now, here we have to find the Nash equilibrium. In this subgame-$\mathrm{L}'', \mathrm{R}''$ player 1 is always going to choose this-$\mathrm{L}''(3,0)$. So, player 2, it has this if it chooses L dash 1, and if it chooses R dash it will, player 1 is going to choose L dash, so it is going to get 0.

So, player 2 chooses this-$\mathrm{L}'(1,1)$. And player 1 if it chooses L it is going to get 2, if it chooses R then it is going, player 2 is going to choose L dash, so it is going to get 1. So, it is going to choose L. So, this is the subgame perfect Nash equilibrium outcome. So, in this starting the games this is the initial node, the action is this- L (1, 1). In this, action is this. In this L'', action is this one- (3, 0), okay.

Let us take another example. So, this example is slightly big game. It is played between three players. It is u, d, this is again u, d, it is again u, d, okay. Suppose this is the game tree. There are three players and game tree is of this nature. So, the subgames are, it has many subgames.

This is one subgame-$d'(4,3,3), u'(3,0,1)$, this is another subgame-$d'(2,6,0), u'(1,2,6)$, this is another subgame- u(2, 1, 2), d(1, 3, 4) and this whole is also a subgame-$d'(4,3,3), u'(3,0,1), d'(2,6,0), u'(1,2,6)$. So, it has 1, 2, 3, 4, 4 subgames in this whole game, okay.

So, let us start from this. In this game, player 2, it will compare 1 and 3. So, player 1 is always going to choose d here in this subgame we know. Then let us start from this subgame. This subgame, player 3, d dash, u dash, if d dash gives 0, u dash gives 6 so player 3 is going to choose this. In this subgame, player 3 is choosing d dash, u dash, d dash 3, u dash 1. So, it is this. And here this now you can see that this game has become something like this. So, player 2 is compared 2 and 3. So, player 2 is going to choose this and here this is going to be, and this is (4, 3, 3), so player 1 is going to choose this.

So, in the whole game you can look at it in this way, it is this and it is this. So, in this subgame, the subgames are, Nash equilibrium are this, this and here it is this, in this it is this, so then finally subgames are represented by this green lines in this game tree, okay. Here again by the green lines in this game tree, okay.

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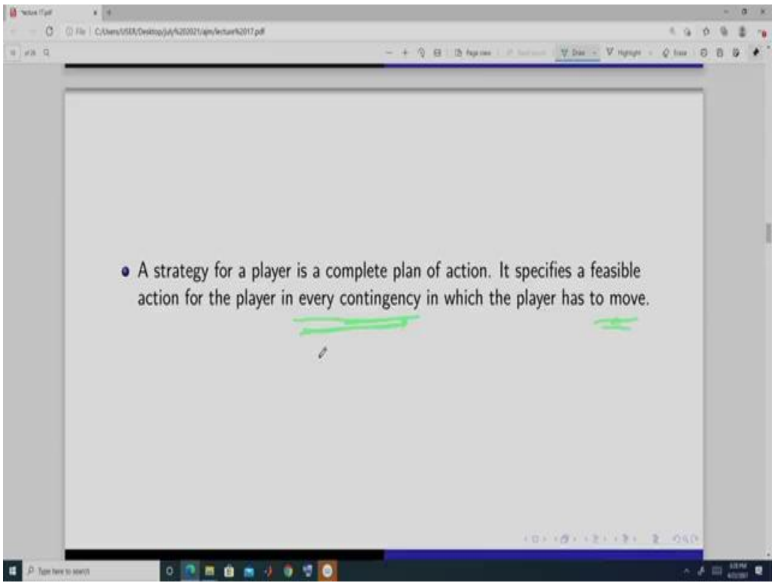

Now, we are going to define what is the strategy. We have talked the actions here. In this game, what is the action, in this game, subgame what is the action, in this subgame what are the actions like that. Now, we will specify strategies. What is a strategy of a player? Strategy of for a player is a complete plan of action. It specifies a feasible action, feasible action for the player in every contingency in which the player has to move, okay. So, keep this in mind, in every contingency in which the player has to move, okay.

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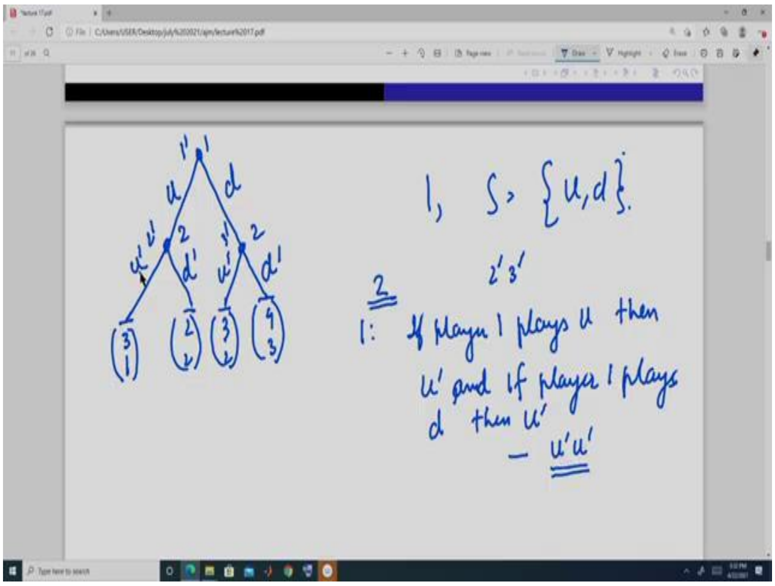

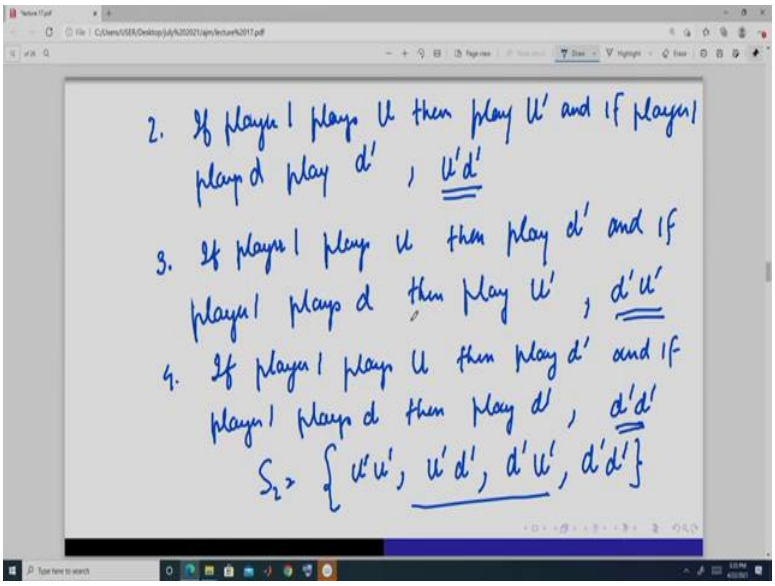

So, now let us define it for a very simple game, okay. So, let us take this initial game u, d, u dash, d dash, u dash, d dash, okay. Let us take this game. We have solved this game. We have no, we know what is the subgame perfect Nash equilibrium of this game, right? Now, player 2 is taking decision in two contingency. It has two contingency, in this node and this node.

So, contingency means mainly in how many nodes it is taking a decision. It is not or you can say in how many information set, okay. Best is going to be one information set implies one contingency. For player 1 it is one only, it is one decision node. So, the set of actions, so for player 1 set of action or the strategy sets are same and it is u and d, okay. But for player 2, it has two. So, let us take, name this. This is 1 dash and this is 2 dash. These are the name of or you can say, sorry, this is 3 dash, okay. So, these are the name of the nodes. So, its contingency is 2 dash and 3 dash, okay. So, how do we, we have to define the complete plan of action. So, we will suppose strategy 1 of, these are for player 2, okay strategy 1.

So, if player 1 plays u then u specify u dash and if player 1 plays d, then u dash. So, this is u dash, u dash. This you can say it is one strategy of player 1, because it has two contingencies or it has two information set on in which it is going to take the decision. So, it is u dash, here also it is taking u dash, and here also it is taking u dash. So, this is suppose strategy 1.

Then we can specify strategy 2, second strategy. Second strategy is, if player 1 plays u then play u dash and if player 1 plays d play d dash. So, this is u dash, d dash strategy. This is strategy 2. This is for node 1, node 2 dash, this is for node 3 dash. Third strategy, if player 1 plays u then play d dash and if player 1 plays d, then play u dash. So, this is d dash, u dash strategy. This is the third strategy.

And we have a fourth strategy and that is if player 1 plays u then play d dash and if player 1 plays d then play d dash. So, it is d dash, d dash strategy. So, the whole strategy set of player 2 you can say it is u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash, i.e $S_2 = \{u'u', u'd', d'u', d'd'\}$. So, we have four strategies. So, it is, here it will play this (3,1), this (3,2) one strategy, this (3,1), this (4,3) the second strategy, this (2,2) and this (3,2) third strategy, this (2,2) and this (4,3) it is going to be the fourth strategy, okay. So, these are the strategies. But for player 1, it is only this- S={u,d}.

Now, so this is the way we are going to define the strategies. Now, once we know the strategies, so then we can specify this dynamic game as a normal form or a static game. And there how do we do, I will show you just now.

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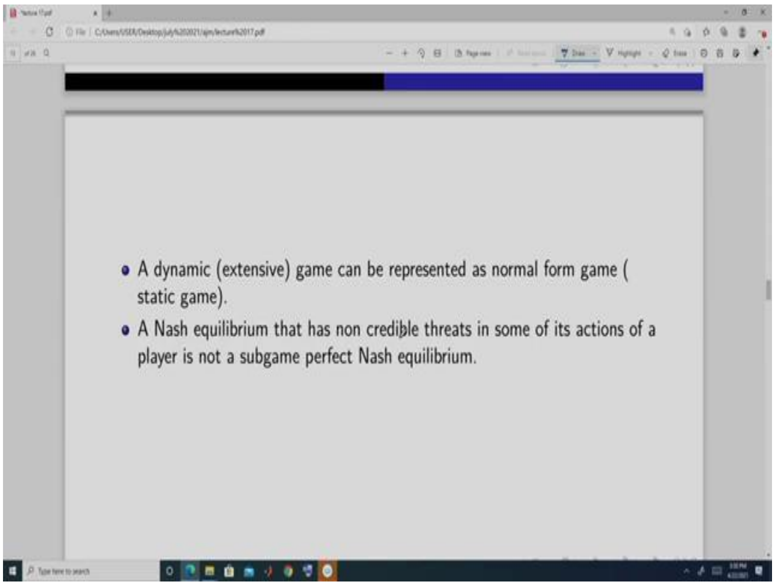

And then we know if we have a static game or a normal form game, then we know the how to find the Nash equilibrium and then from all the set of Nash equilibrium of that game how to find the sub game perfect Nash equilibrium only those which do not have any tradeable threat. If there are some tradable threat, then it is subgame perfect Nash equilibrium and if the Nash equilibrium and there are some actions which are not credible enough, then we say it is not subject subgame perfect Nash equilibrium. So, it is a Nash equilibrium, but it is not a subgame perfect. So, I will discuss that now, okay.

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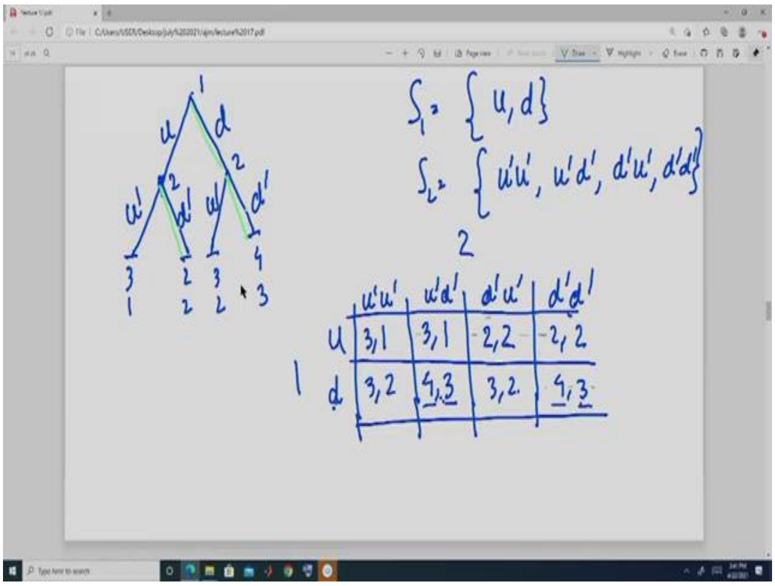

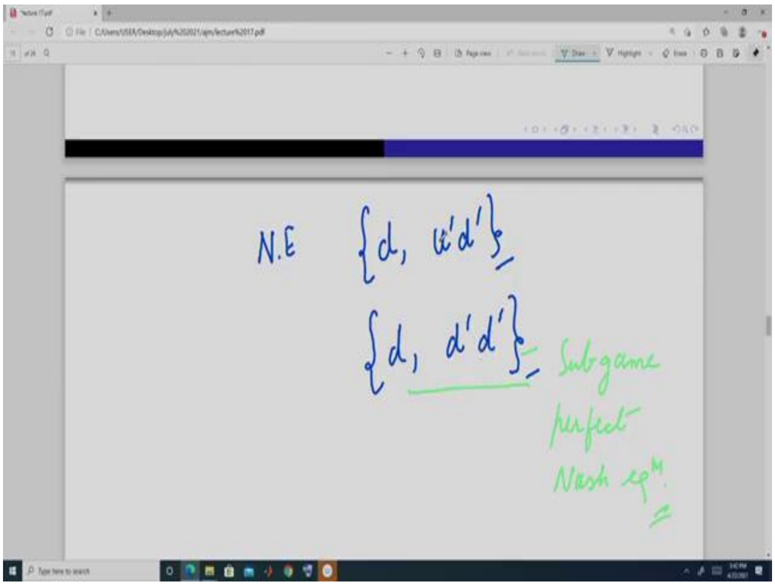

So, let us take this game. So, this game, if you look at this game, what are the, player 1, u, d, player 2, u dash, d dash, u dash, d dash (3, 1), (2, 2), (3, 2), (4, 3), set of strategies for player 1 it is u and d, i.e. $S_1 = \{u, d\}$, set of strategies for player 2 it is u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash- $S_2 = \{u'u', u'd', d'u', d'd'\}$, right? Now, normal form game can be u dash, u dash, u dash, d dash, d dash, u dash, d dash, d dash. This is for player 2. U, d and this is for player 1, okay.

Now, if it is, player 1 has suppose played u so here and it is, so this, payoff in this block it is going to be (3, 1). And here it is going to play this and this, but player 1 has played this, so it is going to be (3, 2), okay. Player 1 is playing u and it is choosing u dash, okay. And here, and player 2 is choosing d, it is doing d dash, so it is. And player 1 is playing u it is choosing d. When player 1 is choosing u it is choosing d dash, so it is again. Here, player 1 is choosing d and it is choosing u dash. Here it is choosing d dash. So, this is how we will get the payoff metrics of a normal form game.

And in this normal form game, if you want to find the pure strategy Nash equilibrium, what are these pure strategy, if it plays u, so player 2 is indifferent between these two strategies- $d'u'$and $d'd'$ and if it plays this player 1 is going to follow this- $d'd' = (2,2)$. So, this is not a Nash equilibrium, right? But if it plays again this player 1 is going to choose this. So, again this- $d'u' = (2,2)$ is not a Nash equilibrium. But if player 1 chooses d, player 2 is indifferent between this- $u'd' = (4,3)$and this- $d'd' = (4,3)$.

And here if player 2 chooses this, so this (4,3) is one Nash equilibrium. It is a weak Nash equilibrium, pure strategy, weak pure strategy, but it is a Nash. Here again if player 2 chooses this (3,1), player 1 is going to choose d and if it chooses d, it is indifferent. So, this (4,3) is again a Nash equilibrium. So, the pure strategy Nash equilibrium, are d and u dash, d dash and again d and d dash, d dash, right? Now, we have two pure strategy Nash equilibrium.

Now, we have to find out which is subgame perfect. If you look at this here, we know here it is going to choose this- $d' = (2,2)$ and here it is going to choose this- $d' = (4,3)$ and player 1 finally chooses this, so d, d dash, d dash,- $\{d, d'd'\}$. So, this is subgame perfect Nash equilibrium. This- $\{d, u'd'\}$ is not a subgame perfect. Why, because see, what it is saying. If player 1 plays u, this is u dash, d dash. If player 1 plays u, if it is in this node, then player 2 is going to choose this. And because of that, because it is giving a threat that if you are, if you choose this, I will choose u dash.

So, better choose this-d, then I am going to choose this- d'. But then player 1 knows this that if player 1 chooses this, when player 2 is here, it is never going to choose this-u', instead it is always going to choose d dash because this is 2 is greater than 1. So, that is why this strategy u dash in node 1, node 2 dash and first node here and d dash the second node of player 2, so this and this it is not credible. This is credible, but this is not.

So, that is why this is not a subgame perfect Nash equilibrium. But this, it is always going to choose this- d' here in this subgame and this- d' in this subgame so that is why this is a subgame. So, we have a Nash equilibrium in each subgame here in this strategies-$\{d, d'd'\}$. But here this u dash is not a subgame if we take this, okay.

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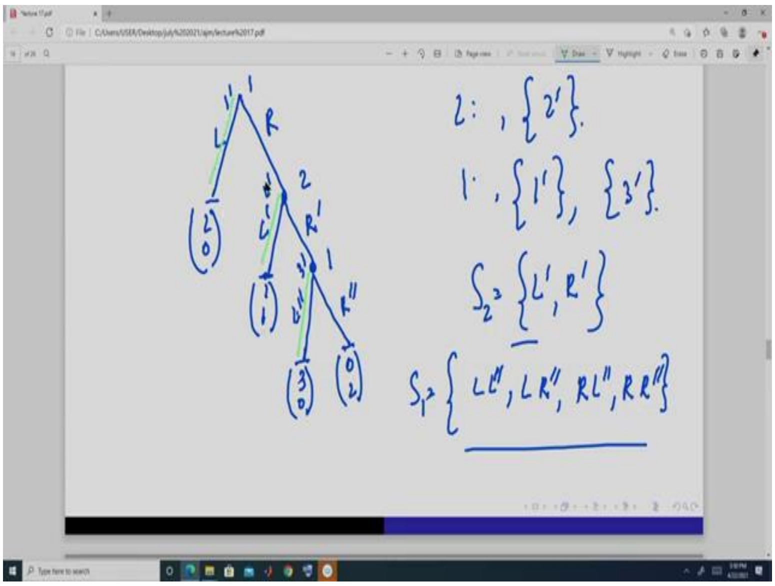

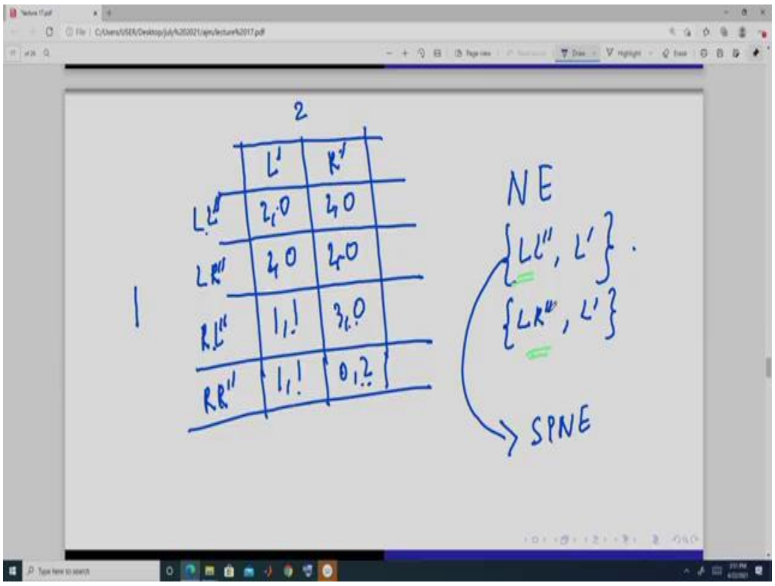

So, now let us do another example, okay, okay. So, this game has this as one subgame, this is another subgame, okay. Now, here, player 2 has only one information set. So, suppose these nodes are 1 dash, 2 dash, 3 dash and player 2's information set is only this 2 dash, but player 1's information set it has two information set that is 1 dash and 3 dash. So, it has two contingencies, player 2 has only one contingency. So, the action strategy set of player 2 is L dash and R dash- $S_2 = \{L', R'\}$. But for player 1 it is this L and next it is L dash, then again L and R dash this is for the second contingency.

We have to discuss all the possible plan, right? So, we have to specify the actions for each nodes or each information set, R, L dash, R, R dash- $S_1 = \{LL', LR', RL', RR'\}$. But here if player 1 plays L, then it is never going to choose this here, because the game ends here, but still we have to specify it. So, these are the action strategies of player 1 and these are the strategies of player 2.

So, the normal form game, it is for player 1 this is, sorry, player 2 L dash, R dash. This is L, L double dash, L R double dash, R double dash, it is this. Now, if player 1 plays L, player 2 does not get any chance to play. The game ends here. So, this for this thing if player 1, so this is for player 1, if it plays L it is going to be 2, 0, it is going to be 2, 0, because since it has played L so player 2 it is same. So, for all these contingencies we have get the same payoffs, okay.

But suppose player 1 has played R and player 2 has played L dash, L dash then it is going to be this one, 1, 1, and here player 2 is not getting any chance, because player 1 is playing R so the game has moved to the second node and player 2 is making a decision that is L dash. So, the game ends here. So, that is why the payoff, in this case, it is also going to be (1, 1) and then we will get this, okay.

Now, here if you look at this game and you have to find out suppose the pure strategy Nash equilibrium, then if player 2 plays this strategy L dash, player 1 is indifferent between these two. If it plays this he is indifferent. So, this is one Nash equilibrium- $\{LL'', L'\}$. And here if it plays this- $\{L\,R''\}$ he is indifferent and if it plays this he is indifferent between these two. So, this is also a Nash equilibrium- $\{LR'', L'\}$. These are weak Nash equilibrium.

If it plays this-R' , player 1 moves here- (3,0). If it plays this- $\{RL''\}$, it will, player 2 is going to switch here. So, this (3,0) is not a Nash. Again this is not a Nash equilibrium. If it plays this- RR'' , player 2 is going to choose this-(0,2). But if it choose R dash, player 1 is going to choose R L double dash, so this is. So, these are the two pure strategy Nash equilibrium. Out of this, we will, we have to find the subgame, which is subgame. So, this is double dash, okay. Otherwise, it will be confusing, right? Make it double dash.

So, this, in this, it is this. Here it is, if we plug in this value here it is 3, 0. So, player 2 is going to choose here. It is 1, 1. So, player 1 is, so these are the, we have already shown the subgame, Nash equilibrium of each subgames. So, here it is going to choose. In this here it is going to choose this L double dash, not L, R double dash. So, that is why this $\{L\,R''\}$ is not a Nash equilibrium. This $LL''$is a Nash equilibrium. This is a Nash equilibrium, but this is not a subgame perfect Nash equilibrium. So, this is you can say SPNE, subgame perfect Nash equilibrium, this is not, because here L double dash is not credible enough.

If you play this strategy and player 2 plays this strategy, sorry if you play this action, player 2 plays this action I will be here, player 1 is going to be again here, then it is not going to play this. This is not credible. This there, it is going to be this, because 3 is always greater than 0. So, that is why in this strategy L, L double dash, it is credible. But L, R double dash, it is not credible. So, in that sense it is a subgame perfect Nash equilibrium.

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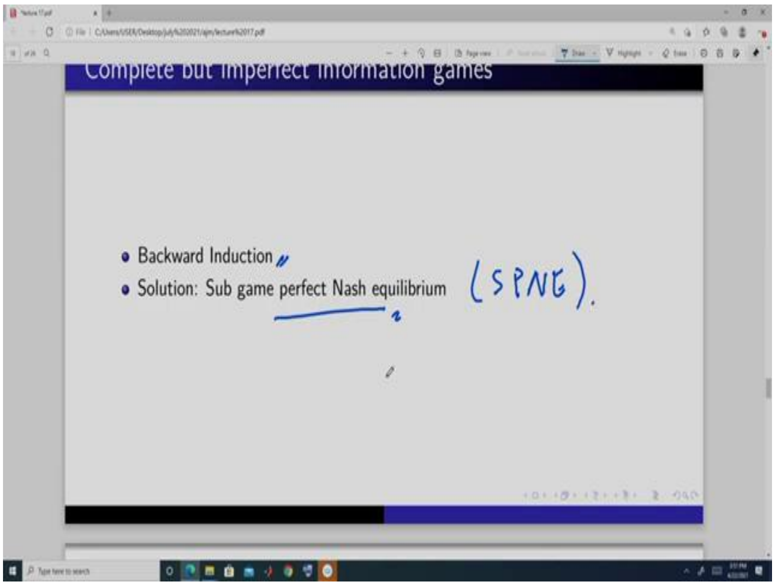

Now, let us, we have already done or talk, discuss about perfect information case, now we will do imperfect information. Now, the solution concept, we will again use the backward induction that is we will start from the lowest or the last subgame and then we will move backward and the solution concept is same as subgame perfect Nash equilibrium or SPNE. Let us do some example to understand this.

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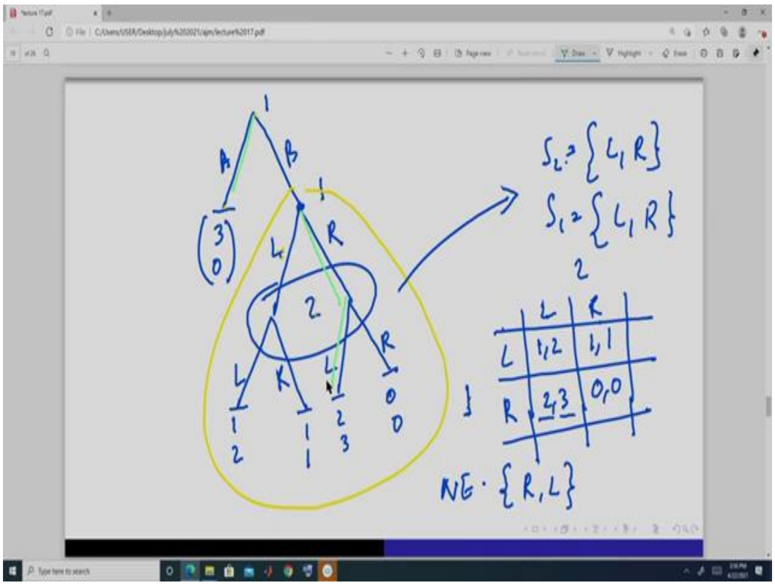

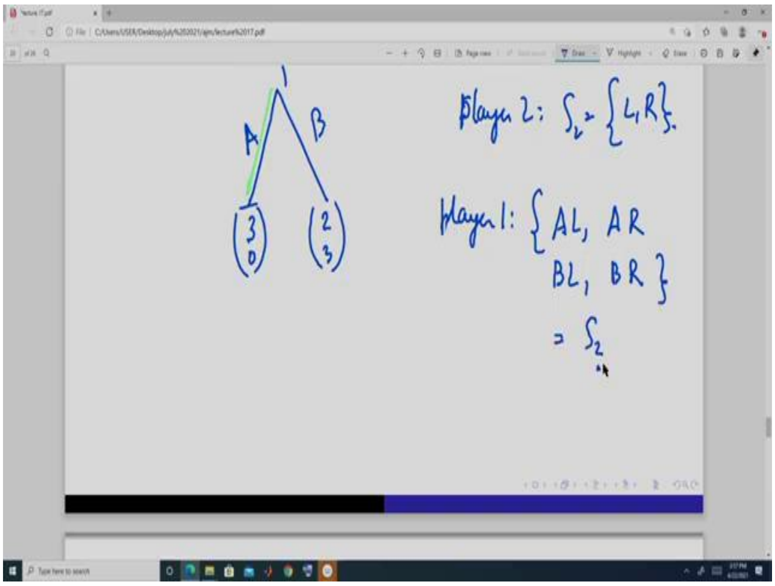

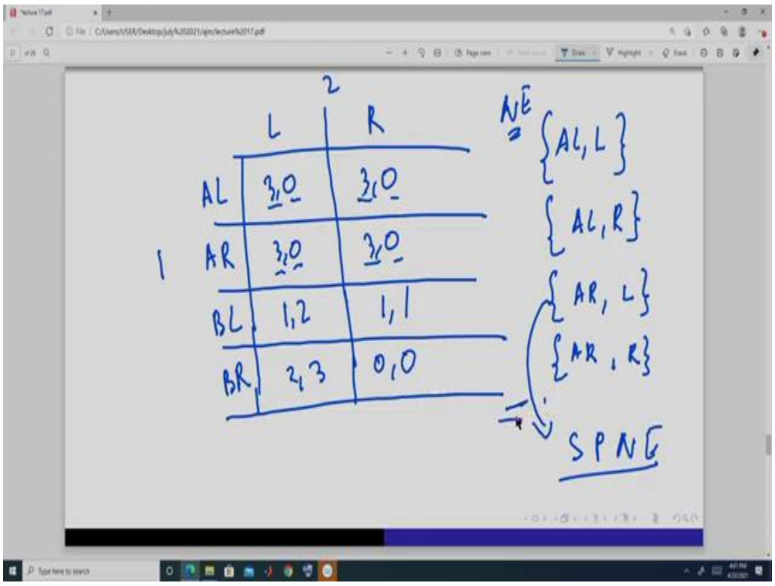

So, it is again a two player game and suppose it is player 1, it is A, it is B, again player 1, okay L, R. So, this game, this player 2 is not going to observe the action of player this one and also it is not going to observe this, okay on this is not known. Player 2 when it is moving it will know that player 1 has chosen B, otherwise it will not get the chance to move at all, okay. So, in that sense player 2 knows that player 1 has chosen this. But player 2 is not going to observe the action of, this action L R. And similarly player 1 is not going to observe the action of R, okay.

So, now, how do we solve this? So, we will start with this subgame, okay. So, this subgame is you can think this subgame as a simultaneous move game or you can say a normal game, where since this is the information set of player 2 and it has two nodes, so it is not sure. So, action set is same here. So, the here strategy set of player 2 is simply L, R, strategy set of player 1 in this subgame is again L and R. So, this game can be represented as a normal form game and it is in this way. This is for player 1, this is for player 2 actions L and R, so player 1 L, player 2 L, L, L this combination 1, 2. L, that is L and then R, this is R, 1, 1, this R, R then player 2 L, R, L, R, R, R, R. So, this is the normal form game, okay.

Now, so we play this game and we find the Nash equilibrium of this game. So, in this the Nash equilibrium is, so it is obvious that for player 2 L is a dominant strategy or R is dominated by L. So, it is going to be choosing this and so player 1 will choose this (2,3), so this is a Nash equilibrium. So, here in this game the player 1's action or a, so the Nash equilibrium is R, L, okay. Then what do we do? Again for player 1 A, B, so it is (3, 0) and this is (2, 3), so player 1 is going to choose this- A= (3,0).

So, here you can represent the Nash R and L, this, okay. But how do we specify them strategies of players. This is strategies are for only this subgame. For the whole game, the strategies are like this. For player 2 it is, a strategy set is same. It is, it has only one information set here and then it has two nodes in it and they have same. So, it is L and R. But for player 1 it is taking decision at two nodes here and here, right? So, it is going to be AL, AR, BL, BR. This is the strategy set of player 2, okay whole game.

Now, we can also represent this game as a normal form game and because we know the strategies, so you can say this is L and this is R, AL, AR, BL, BR, right? AL, AR, if it plays A then it is game ends here. So, in all these A so the payoff is same. It does not matter. Player 2 does not get any chance or player 1 does not get the second chance or second move. Here it is B and then player is choosing L and then again L, so this- (1,2). So, now here we will place this game here, payoffs here. So, it is 1, 2, okay.

Now, we can find all the Nash equilibrium on this game. What are the Nash equilibrium If it plays L, it can choose this- AL= (3,0) or this- AR= (3,0) indifferent between, but it will be here it is going to be indifferent between this, this, so this is a Nash equilibrium, again these are the Nash equilibrium. So, you can say, all these- {AL, L}, {AL, R}, {AR, L}, {AR, R} are Nash equilibrium. But are all of them subgame perfect, because here if you look at this game it is L, R, so it is L, R, only this, because L, L, L, L, L, L it is not, it is dominated by, if you play this, by this, right? this, so R.

Now, so that is why in this subgame, the Nash equilibrium of this subgame is not part of this strategy profile. Here again, this is L, R, this is also not A. So, this is all, that is why this is not a subgame, but this is R, L, R, L, R, L. So, in this subgame, it is a Nash equilibrium is L R, L. So, this is subgame. This is R, R, this R, R, it is not a Nash equilibrium in this subgame. So, that is why only this is SPNE. But in this normal form game we have four pure strategy Nash equilibrium this- {AR, L}, {AR, R}.

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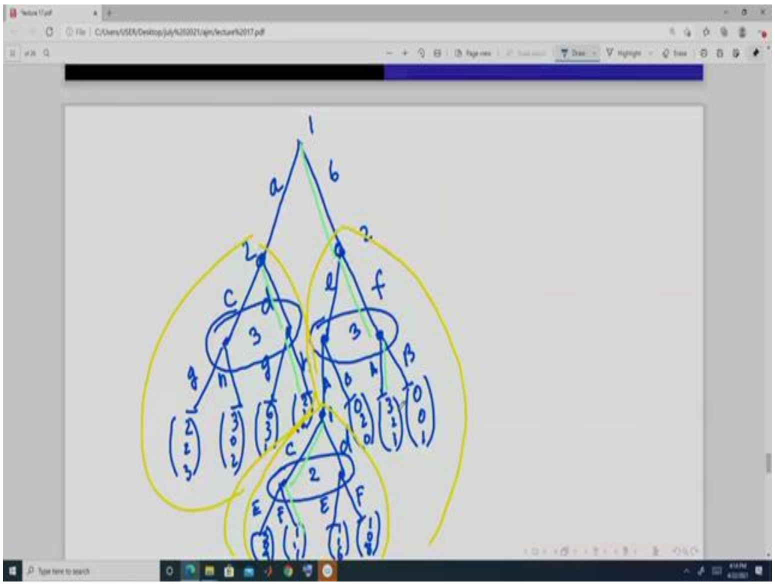

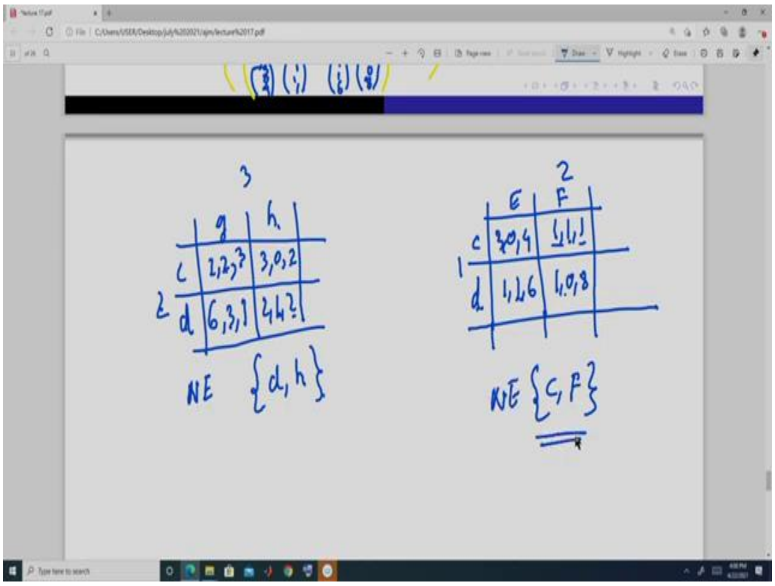

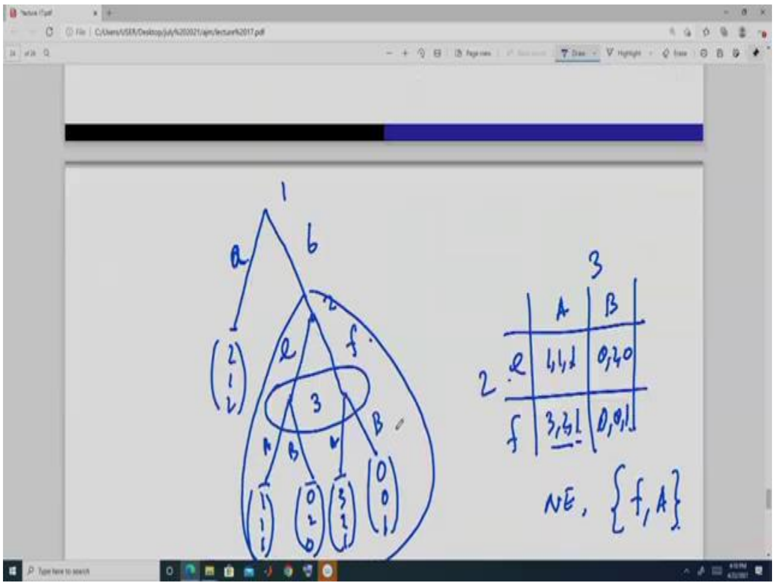

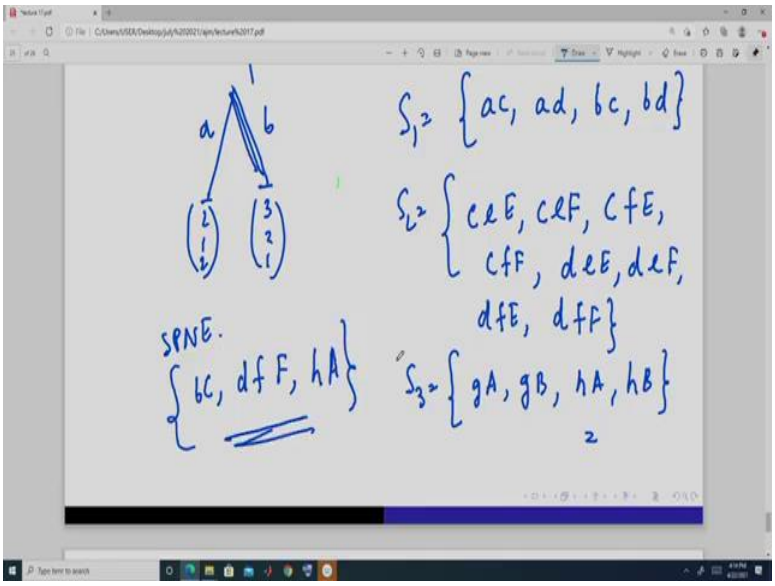

Let us do another example and this is going to be our last example and with this we will end the game theory portion and start industrial organization, okay. Suppose is this, okay so imperfect game between player 2 and 3, A, B, A, B, C, D, E, F, E, F, (3, 0, 4), (1, 1, 1), (3, 0, 4), okay this is (1, 1, 6), this is (1, 0, 8), this is (0, 2, 0), this is (3, 2, 1), this is (0, 0, 1), again this, suppose this here, this payoff here is (2, 1, 2), payoff here is (6, 3, 1), payoff here is (3, 0, 2), here is (2, 2, 3). So, in this game, if you look at the subgame, one subgame is starting from here, another subgame is starting from here and this whole, another subgame is this, okay. So, it has three subgames.

So, let us first solve this subgame. This is between player 2 and player 3. Now, this is an imperfect, because player 3 does not know that it is, what is the action chosen by the player 2, okay. So, we can, so it is c, d and g, h, payoffs are c, g, so it is (2, 2, 3), then it is (3, 0, 2), (6, 3, 1), (2, 1, 2), okay. So, it is played between player 3 and player 2. And the payoffs are, we have to compare 2, 3, this 0, 2, 3, 1, 1, 2.

So, here a player 2 plays c, player 3 is going to play h, 3 is greater than this and if player 3 plays g it is going to play d because 3 is greater than 2 and if it plays this- h(2,1,2) it is going to choose this one-2, because 2 is greater than 1 and if it plays h it is going to choose d, because 1 is greater than a, because d is actually a dominant strategy for player 2 here if you look at this. C is dominated by d. So, in this the Nash equilibrium is d, h. So, in this subgame Nash equilibrium is d, h.

Now, let us look at this subgame. This is played between player 1 and player 2. Player 2 is E, F, player 1 it is c, d, okay and the payoffs are, you can look at the payoffs from here. It will be like this. And here if you play, player 1 plays c, player 2 is going to play F, because 1 is greater than 0. If it plays F player 1 is going to be indifferent between these two. But if it plays d, player 2 is going to play E and when it plays E it is going to, so this- F (1,1,1) is a Nash equilibrium. It is a weak Nash, sorry. This (C, F) is a pure strategy Nash equilibrium.

Now, from this we can get, so the first this game becomes here for this payoff is (2, 1, 2), right? d and h and then here for player 2 if it plays, so here the outcome is c, f. So, this is (1, 1, 1) and rest we know. So, we now play this subgame, this. Play this subgame. So, here again we have two player, player 2 and player 3, player 3 A, B, player 2 it is e and f. So, if it plays e and A so it is (1, 1, 1), e and B, so it is (0, 2, 0), e, f and it is A (3, 2, 1), (0, 0, 1).

So, we have to compare the last two digits. So, for player 2, if it plays e best response for player 3 is A, because 1 is greater than 0. If it plays with A, player 2 is going to choose f, because 2 is greater than 1. If it plays here f, it is indifferent okay between 1 and 1. But if it plays this, player 2 is going to choose e, not this. So, this is a Nash equilibrium. So, here Nash equilibrium is f and A, okay.

So, this game now, a, b, a is (2, 1, 2), and this is (3, 2, 1). So, player 1 is going to choose this. So, in this game, if we want to find the path which shows the game this and then it is c and f and here it is d and h, d and h, it is d and h and then finally here it is f and A, it is f and A. So, we get the subgames in this way. In this subgame, this is the Nash equilibrium. In this subgame this is the Nash equilibrium. Then in this subgame, this is and these are the Nash equilibrium. So, finally this.

So, but here if we try to present this game as a normal form game then it will take a huge space, so we will not do that. Instead we will simply specify the strategies of each player, suppose strategy says of the whole game of each player. Player 1 has two nodes, right? this and this. So, its action set is or its ac, ad, bc, bd, i.e. $S_1 = \{\mathrm{ac}, \mathrm{ad}, \mathrm{bc}, \mathrm{bd}\}$, right? two contingencies. For player 2 it is taking a decision here, it is taking a decision here and it is taking a decision here. Here each information set is singleton, this is again singleton, but this is not a singleton. So, its strategy profile it has three nodes or three different information sets, so three contingencies.

So, these- $\mathrm{S}_2 = \{\mathrm{ceE}, \mathrm{ceF}, \mathrm{cfE}, \mathrm{cfF}, \mathrm{deE}, \mathrm{deF}, \mathrm{dfE}, \mathrm{dfF}\}$ are the complete set of action plan that player 2 can have. For player 3 we have 2a, one is here and another is here. So, for player 3 it is the gA, gB, hA, hB, i.e. $\mathrm{S}_3 = \{\mathrm{gA}, \mathrm{gB}, \mathrm{hA}, \mathrm{hB}\}$. So, from this we know what is going to be the SPNE from this A, it is bc for player 1, dfF, hA so this- {bc, dfF, hA} is actually subgame perfect. Why bc, because player 1 when it is choosing it is choosing here b and here it is choosing c. For player 2 you see in this it is choosing d here, it is choosing e here and it is choosing F here. So, it is df and F, dfF. So, this is this one, dfF. Then here it is hA for player 3. Player 3 here it is choosing h and here it is choosing A. So, this is a subgame perfect Nash equilibrium, okay.

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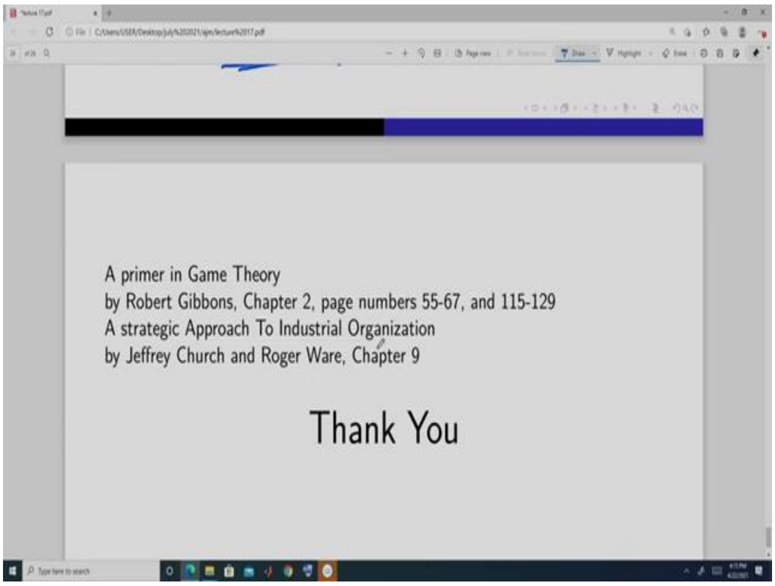

So, thank you very much and this is the, from where you can read this portion, okay Chapter 2 of Gibbons and Chapter 9 of Church and Ware, okay. So, these are the specific page numbers you can look at. Thank you again.

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities & Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 24
Tutorial on Dynamic Games

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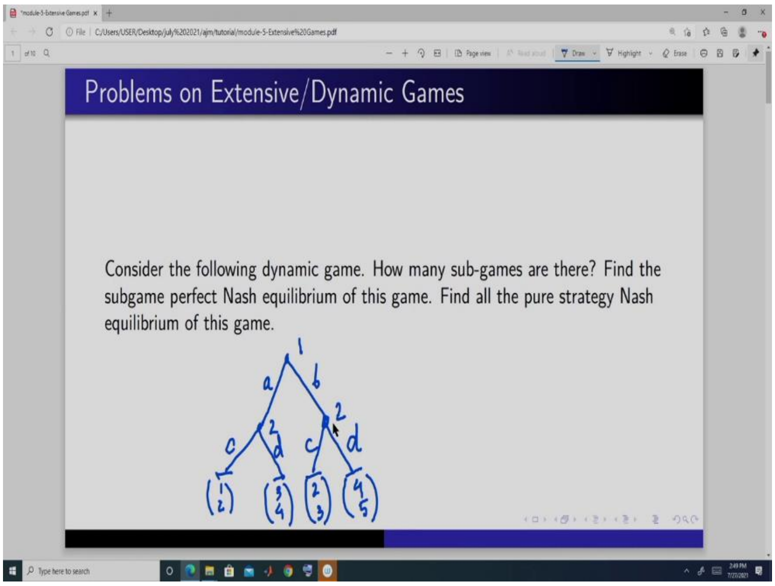

So, let us solve some problems on dynamic games, okay. We have already solved few problems, but let us do some more. Suppose let me first draw the game tree, layer one a, b, c, d, c, d, (3, 4), (2, 3), and this (4, 5), okay. So, this is a two stage game. First player 1 moves. It can choose two, it has two actions a and b, then player 2 moves and the player 2 has two actions c and d. So, we have to find out how many sub-games are there and we also have to find out the sub-game perfect Nash equilibrium, also the pure strategy Nash equilibrium of this game, okay. Now, let us do this.

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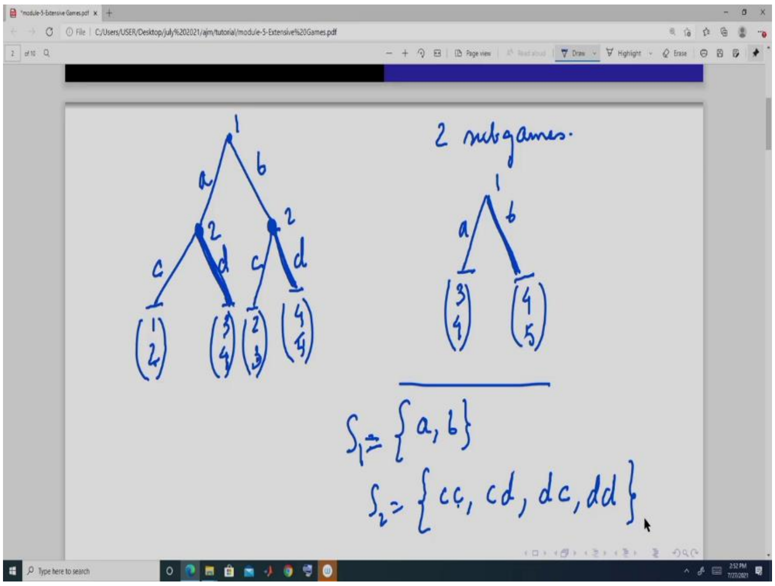

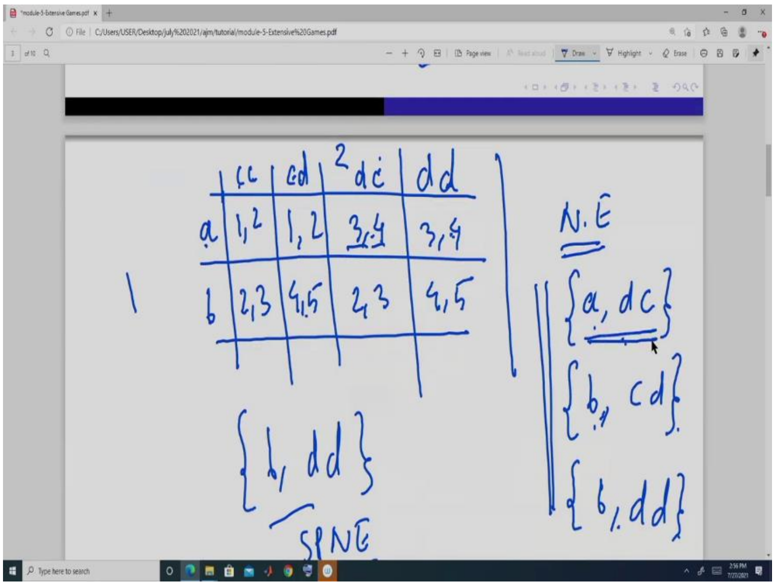

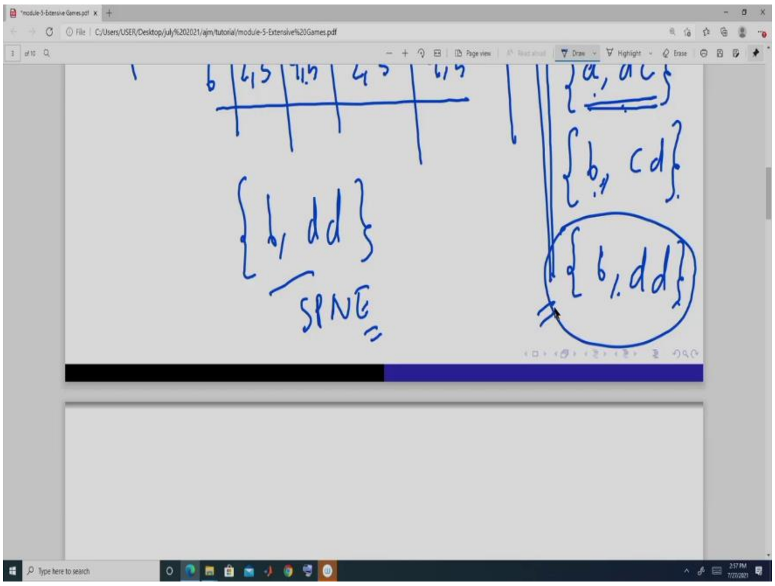

So, our game is this a, b, then this 2, c, d, c, d, okay. Now, we know we have two, how many sub-games, one sub-game from here, another sub-game from here. So, we have two sub-games. And in the dynamic games, we solved it using backward induction. So, suppose player 1 has played this action, then player 2 will have to choose between c and d. If it chooses c it gets two, if it chooses d it gets 4. So, it will choose this d.

Now, suppose player 1 has chosen two, sorry, chosen b, then it will be here. 2 will be in this nod. So, when 2 is in this node, if it chooses c it gets 3, if it chooses d it gets 5. So, it will choose this d. So, this game is now reduced form is (3, 4) and (4, 5). So, player 1 will choose b, because 4 is greater than 3. So, this is the sub-game perfect Nash equilibrium. So, player 1 chooses b and player 2 when it is in this node it chooses d and when player 2 is here it chooses d, okay.

Now, here, this is the sub-game. Now, how do we find the pure strategy Nash equilibrium? Pure strategy Nash equilibrium, then we have to convert this dynamic game into normal form game. And for that we have to specify their strategies. How many, so player 1 has only one nod from which it makes a decision. So, its actions or strategy sets are same. So, S1 is suppose this- $S_1 = \{a, b\}$, the strategy same or the action set, sorry it is same.

But for player 2 it has two node, this and this. So, we have to specify an action for each node then only it will be a complete profile of action, set of actions. So, it is possible that cc, c in nod 1, c in nod 2, then it is cd, c in nod 1, d in nod 2, then it is dc, d in nod 1 and c in nod 2, then dd that is d in nod 1 and d in nod 2. So, this is the strategy shape of player 2.

Now, we specify the payoffs of this. We represent this as a normal game. So, for player 2 is in this column and player 1 is in this row. This is a, this is b, cc, cd, dc, dd, okay. So, and this here if you look at the game, you will get 1, 2, this is playing b and player 1 it is c in nod 1, c in nod 2, so it is 2, 3, it is again a and it is playing c in, then it is (4, 5), (3, 4), (2, 3), (3, 4), (4, 5), okay. So, here, we have to find that in this normal form game we have to find the Nash equilibrium. So, if player 1 plays a, player 2 is indifferent between this (dc) and this (dd), because 4, 4. But if player 2 plays this, player 1 plays this. So, this (3, 4) is one Nash equilibrium.

So, Nash equilibrium was the $\{a, d, c\}$, this is one. Now, if he plays this, player 2 plays this (dd) strategy, player 1 plays this (3, 4). So, this is not there. These two are not an outcome we have got. If player 1 plays b, player 2 is indifferent between this (cd) and this (dd), but if this is played he moves this b (4, 5). So, immediately b and cd is a Nash equilibrium- $\{b, cd\}$. Again here if d is played he plays b and if b is plays he is indifferent between cd and dd. So, again this {b, dd} is a weak Nash equilibrium.

So, we get three pure strategy Nash equilibrium. And which one is sub-game from this. Sub-game we have got, sub-game is this, this and here this. So, it means when player 1, player 2 in its first node that is, it will play d and in second nod it will again play d and player 1 will play b. So, it is this. So, b, dd this is SPNE, sub-game perfect equilibrium. And these two- {a, dc}, {b, cd} are Nash equilibrium, but they are not sub-game perfect because there is no credible trait in it.

It is like this that player 1 here, if you look at this, player 1 here, it is playing a, because player 2 is playing that if you play b, I will threaten you by playing c, so better play a. So, its outcome is this, a, d, so it is a, d, because player 2 is threatening that if you play b, I will play c. But then it is not credible, because if player 1 plays b, player 2 if it is in this node, it will always play d and not c. So, that is why this is not credible.

Again look at this, here it is c and d. So, here player 2 is saying that if you play a here, then I will play c. So, you play b so that I will play d. But then here again this is not credible. Why, because if player 1 plays a, then player 2 will not play c, but instead it is going to play d, because 4 is greater than 2. So, that is why only this {b, dd} has that credibility. So, that is why this is a sub-game perfect Nash equilibrium and other two Nash equilibrium, pure strategy Nash equilibrium are not sub-game perfect Nash equilibrium, okay.

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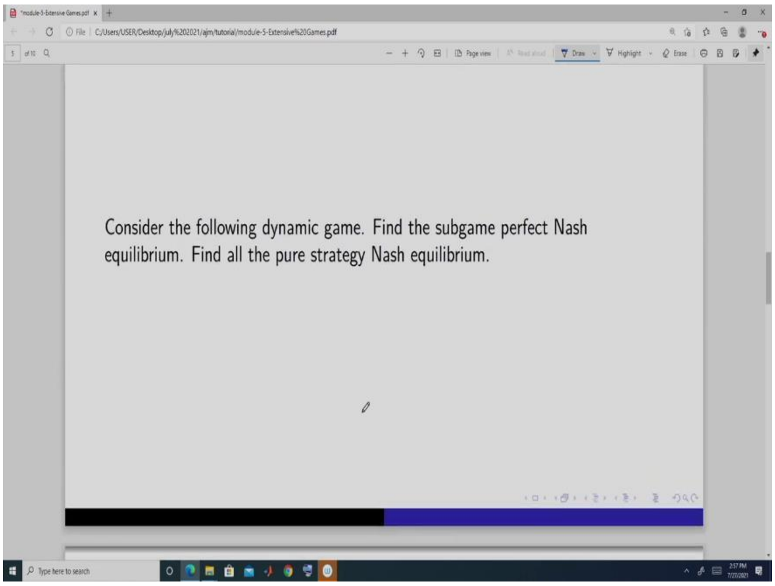

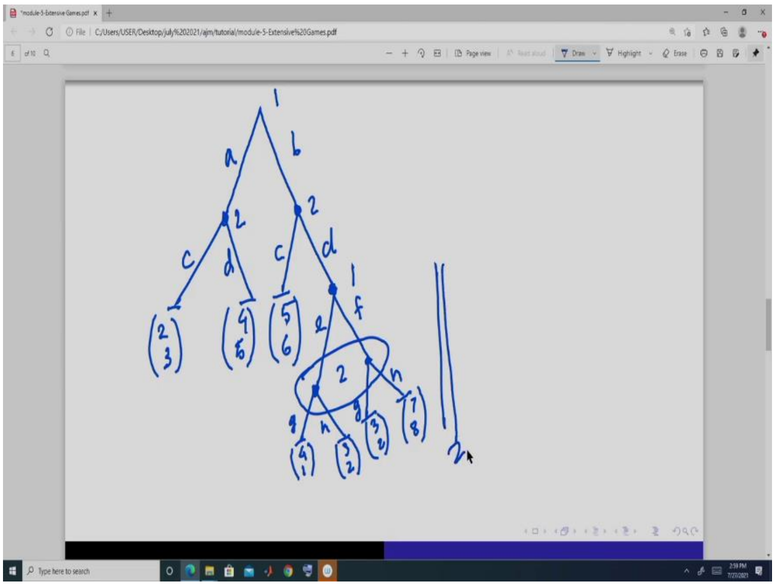

Now, let us solve another problem and this is an imperfect game, okay. Let me first draw the game. So, this game tree is slightly, okay. So, this game is an information but imperfect information, because here if this node is rich then the player 1 and player 2 simultaneously choose their action, okay or even if they do it sequentially but the actions are not observable. So, that is why the information set of player 2 here it has two nodes, okay. So, this we will first specify this as a normal game.

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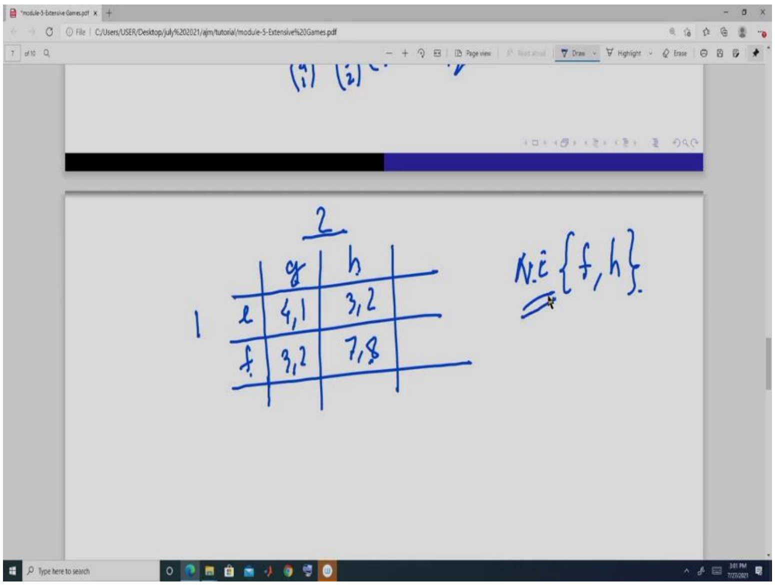
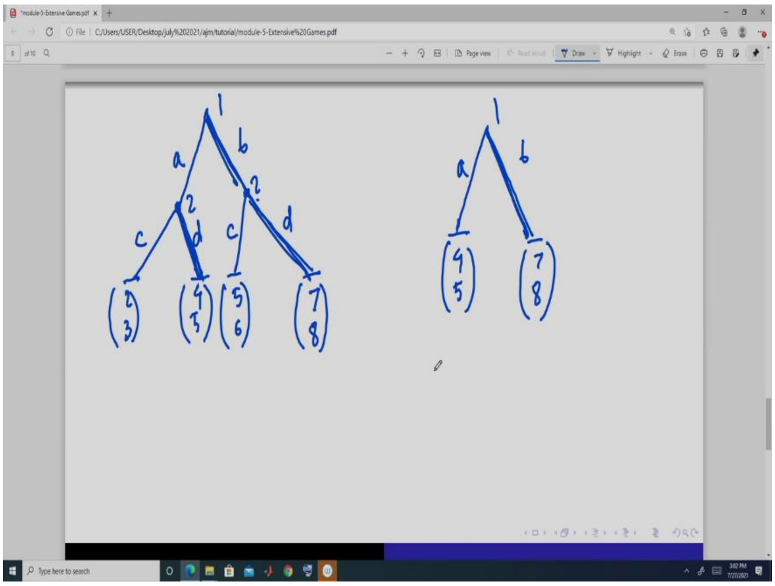
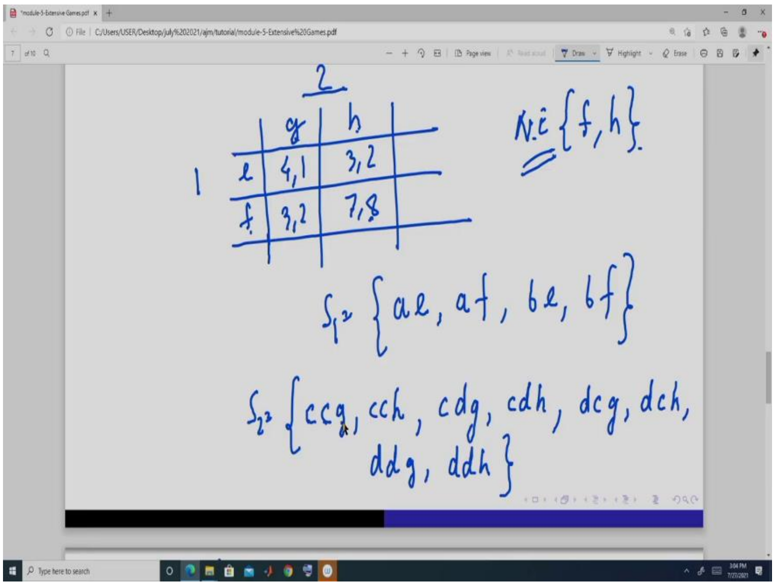


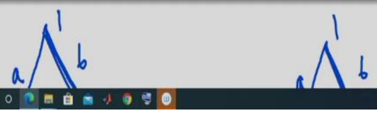




1	e	4,1	3,2
	f	3,2	7,8

$S_1 = \{ae, af, be, bf\}$

$S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$



		2							
		ccg	cch	cdg	cdh	dcg	dch	ddg	ddh
1	ae	2,3	2,3	2,3	2,3	4,5	4,5	4,5	4,5
	af	2,3	2,3	2,3	2,3	4,5	4,5	4,5	4,5
	be	5,6	5,6	4,1	3,2	5,6	5,6	4,1	3,2
	bf	5,6	5,6	3,2	7,8	5,6	5,6	3,2	7,8

N.E: $\{be, dcg\}$, $\{be, dch\}$, $\{af, ddg\}$

N.E: $\{be, dcg\}$, $\{be, dch\}$, $\{af, ddg\}$

$\{bf, ddh\}$, $\{bf, cdh\}$

SPNE

And so this game becomes like if you take player 2 in the column and player 1 in the row then player 1 has two action and those are e and f, player 2 has two actions g and h, and so, this is the game that they play when it is starting from this node, right? Now, this is the simultaneous move game, single shot simultaneous move game.

So, if we look at this game, player 1 if it plays e, best response for player 2 is to play h. If player 2 plays h best response for player 1 is to play f. And if player 1 plays f, best response is to play h. So, here Nash equilibrium is {f, h}, okay. This is the node. We plug in this outcome here. So, the ever game now it is, the outcome here is 7, 8, this. So, suppose again player 1 plays a, then player 2 starting here is going to play d, because 4 is greater than 3. So, this is the outcome.

If player 1 place b, then player 2 starting from this node, it will compare 6 and 8 it will play d (7, 8). So, the game reduced form is now (4, 5), (7, 8). So, player 1 will choose this- b(7, 8). It will be this. So, this is the outcome and this is the sub-game perfect Nash equilibrium. Now, we have to present this as a normal form game. So, if we look at this as a normal form game, then we have to specify the actions.

Now, specify, the actions are already being specified. So, we have to specify the strategies. Player 1 is making decision two nodes in this node and in this node. Player 2 is making decision in three positions, you can say this, this and in this information set. It has two nodes, but this is single information set. This is one information set, this is one information set, this is one information set. So, it is taking at three positions.

So, action set, strategy set of player 1 say it is S1= {ae, af, be, bf}, because here in this node it takes, it can take only two actions e and f and here it has two a and b. So, that is why combinations we are getting this. So, these are each point, each element here is a strategy of player 1. For player 2, we have to specify like c in node one or information set one, c in information set two and then you choose g and h. So, it is g in the third information set and then again you can have this, then again we can have this. So, these are the strategies of player 2- $S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$.

So, this is one strategy where information set one it play c, information say two it plays c, again it plays g in the third information set when it is making a decision here in this, right? So, like that we will, can construct the, these are the strategy of player 1 and player 2, i.e. S1= {ae, af, be, bf}and $S_2 = \{ccg, cch, cdg, cdh, dcg, dch, ddg, ddh\}$. Now, let us represent this as a normal form. Now, normal form of game it will be very big game actually. So, it will be ae, af, be, bf, it will be like this and we write the payoffs. So, these are the payoffs, right? So, this is the normal form game where this is the player 2 which is in the column, player 1 which is in the row.

If player 1 plays this al, player 2 is indifferent between these four strategies- dcg, dch, ddg and ddh, okay. And if it is this is played player 2, 1 is indifferent between these two be, bf. If this is played, is again indifferent between this, this, this. So, this is one Nash equilibrium- dcg and dch. So, here if I write the Nash equilibrium set, then I get one be and dcg. If this- dch is played again he is going to indifferent between this- be (5,6) and this- bf (5, 6). If this is played, he is indifferent between this, this, this, this.

So, again this is be, dch. And if this is played he is, player 1 is indifferent between this, this and this. And if this is played he shifts here. So, this is not an outcome. If this is player 1 plays here this is indifferent between this, this. So, this is one Nash equilibrium, i.e ddg (4, 5). If this (ddh) is played then player 1 is going to play this (2, 8) and if this bf is played he is going to play this (2, 8). So, another Nash equilibrium is bf and it is ddh, ddh. And if suppose this bf is played, because he is going to be indifferent between this and this player 2 and if this ddh is played, he is going, so this is one. If this is played by player 2, then he is going to, if this is he is going to choose this, if this is so this (7,8) is also another Nash equilibrium that is bf and it is cdh.

Now, if this is played, I know we have already discussed, he is going to be indifferent between these and we have found out all the Nash equilibrium in this region. If this is played, then he is indifferent between these four, player 2 is indifferent between these four strategies and we have find out. So, these are the 1, 2, 3, 4, 5 are the pure strategy Nash equilibrium in this game, i.e {bl, dcg}, {bl, dch}, {af, ddg} {bf, ddh}, {bf, cdh}.

And what to do with the sub-game perfect Nash equilibrium. Sub-game perfect Nash equilibrium we have seen b and player is f. So, here bf, so these have bf, then ddh and here cdh. Ddh, so it here d, d and in this game it is h. So, this is the sub-game SPNE and rest are not sub-game perfect Nash equilibrium. These are the pure strategy Nash equilibrium, but they are not sub-game perfect, because in one of their nod**es**, we will always find that it is not credible enough, okay. So, that is why it is not a sub-game perfect Nash equilibrium and only we have a unique sub-game perfect. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
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Indian Institute of Technology Guwahati
Module 7: Cournot Competition
Lecture 25: Cournot Duopoly

Hello everyone, welcome to my course, Introduction to Market Structures. We have completed the game theory portion. Now, we are going to apply that game theory tools to study the market behavior and how the firms decide, okay.

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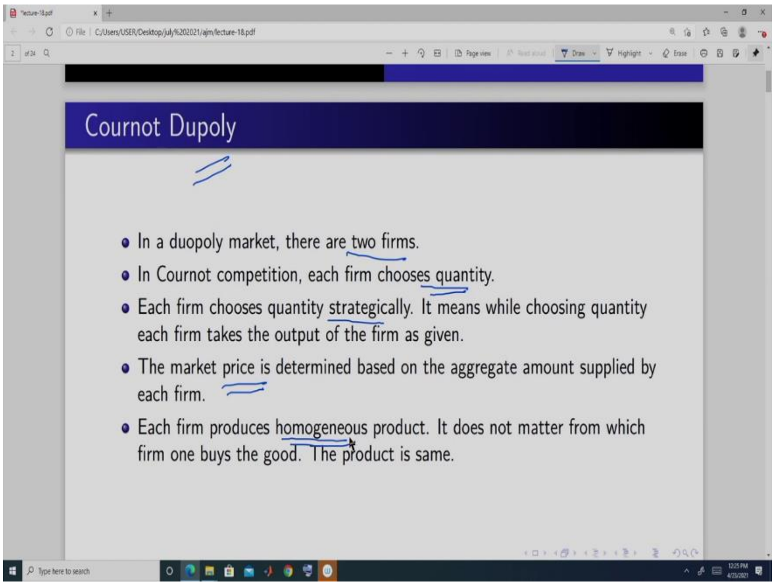

So first model that we are going to do is Cournot duopoly. In Cournot duopoly, there are two firms, okay. And in Cournot competition, each firm chooses quantity, okay. So they will decide how much amount of output they want to produce or they want to sell in the market, okay.

And now while deciding these outputs, they are going to decide it strategically. What do we mean by strategically? That means when firm 1 decides its output, it will take the output of firm 2 as given, that some amount of output is going to be produced by firm 2. Similarly, when firm 2 decides its output that is q2 it will take the output of firm 1 as given some amount. So in this way, they behave strategically.

But in the previous models of markets that we have done in a competitive market, they take the market price as given. So how other firms are behaving it does not matter in that model or in monopoly there is only one firm, so it does not matter how other firms are because there does not exist any other firm but here there are two firms and so while deciding output, they will take the output of other firm as given. So this information is known, okay.

Now again based on this price output the aggregate we will get the aggregate output and that aggregate output is going to determine the market price, okay. And here we make one more assumption and that assumption is that each firm produces homogenous product. It means that whether a consumer buys from a firm 1 or from firm 2, it does not matter. They are going to **get** the same product or same good, okay.

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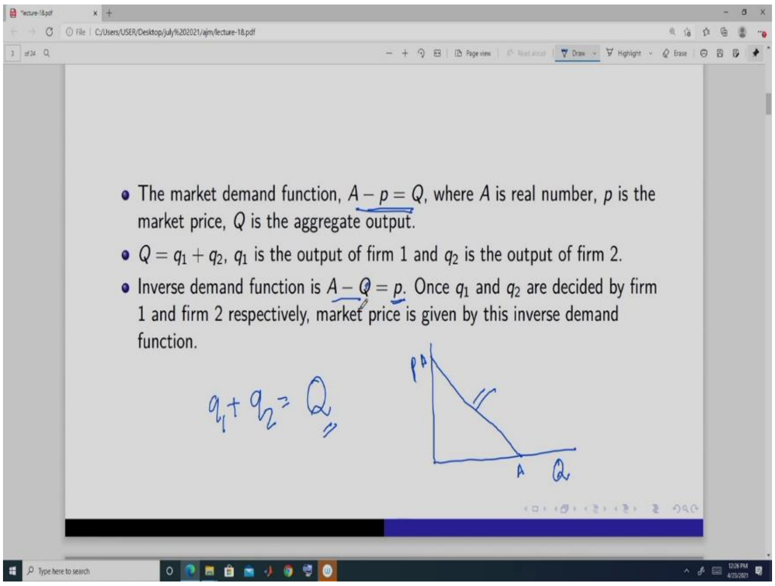

And first we will assume that the market demand is this-$A - p = Q$. So it is a downward sloping demand curve. So if we take output, aggregate output here, price here then this is something like this where this is A and this point is A. So this is our market demand curve, okay. And this Q is the aggregate output. So it is sum of output of firm 1 that is q1 and output of firm 2 that is q2 that gives us the aggregate output that is capital output**,** Q, i.e $Q = q_1 + q_2$, okay.

Now here once this q1 and q2 are decided, q1 by firm 1 and q2 by firm 2 then we plug in this we get this - $A - Q = p$ and from this inverse demand curve, we get the market price. So market price is determined based on the aggregate market demand curve, okay. So each firm decide the output it is going to sell and then that determines the aggregate output and that aggregate output determines the market price. So the market price is not decided by the firm. Market price is an outcome of the output that is chosen by each firm, okay.

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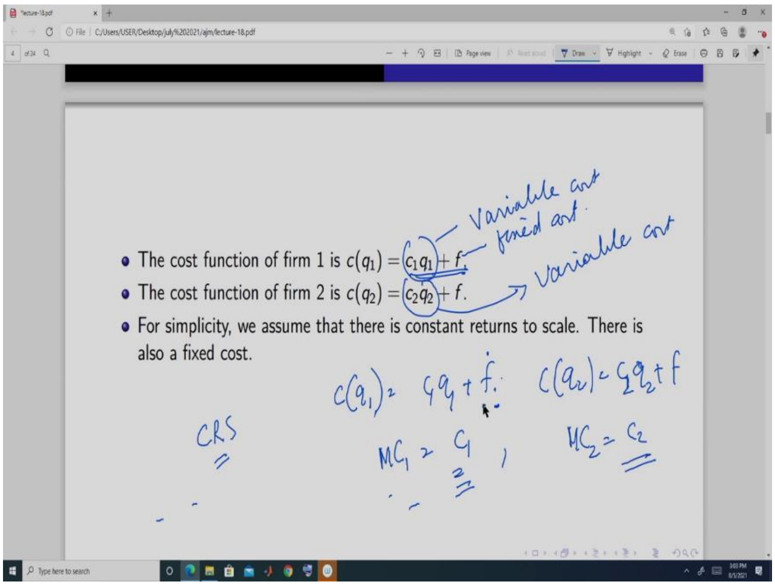

Now we specify the cost function because there are firms and to produce output they will require input and so they have a cost and we have done this already. So we assume that the cost function is of this form for firm 1- $c(q_1) = c_1q_1 + f$ so this is c1 q1 plus a. q1 is the output so this portion-c_1q_1 is the variable cost and this- f is the fixed cost.

And same here this portion- c_2q_2 is the variable cost and this- f is the fixed cost. And we assume that the variable costs are not same for each firm. Why, because they may have some differences in technology or they may say different wages or many reasons, okay or they may be using a different combination, so that is why, okay.

So here if we take this cost function of firm 1 in this then the marginal cost of firm 1 is c1. And marginal cost of firm 2 we will get in this form-c2. So it is constant, so we are assuming that there is CRS production function, okay. And this fixed cost may arise from some factor which is fixed like you can take **a** rent that we pay for the land.

So we are mainly varying machines and labor but land is supposed fixed. We do not vary. We suppose this land size is sufficiently big and we can keep on expanding our capacity so that in that case we can, the rent that we pay it can be a form of fixed cost. Another fixed cost can be like the license fee that we pay to set up a firm so those kind of things will constitute this fixed cost.

Now here we are making I should have specified. Here we are making that firm 1 knows the cost function, okay. We will do this later, not now. So we have got this specification of cost of each firm, okay.

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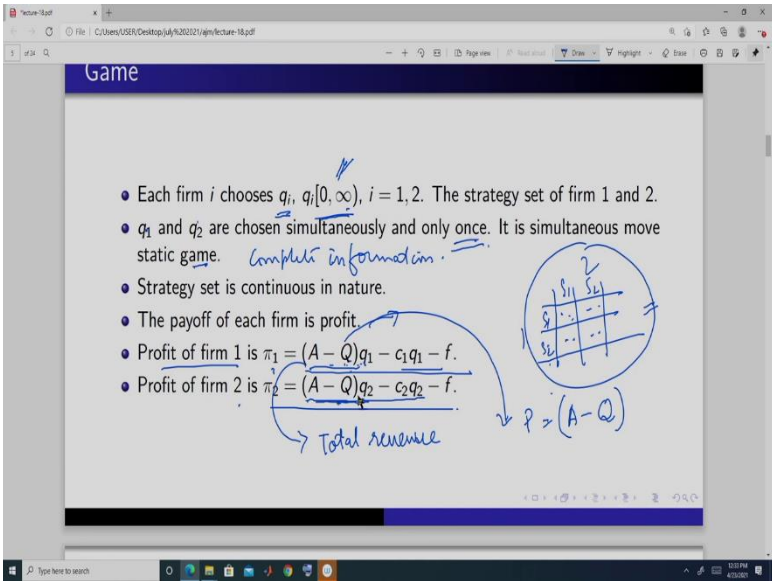

Now let us talk about the game. Now here what is happening, each firm choose qi firm i, i 1 and 2 so that is 2 firm, firm 1 and firm 2. And they can choose q from this range from 0 to infinity, okay. Infinity is not included, okay. So this you can say is the strategy set, okay of firm 1 and firm 2. And the strategies are to choose q1 of firm 1 and q2 of firm 2. They simultaneously choose this and they choose it only once. So you can think of this as a simultaneous move static game, okay.

So both these firms are taking the decision to produce output simultaneously at the same point in time or at the same time and they are choosing it only once, okay. Right? now here we have so this is a part of complete information. Now this is a simultaneous move static. Now this is also a complete information game, complete information. Complete information in the sense that firm 1 knows the payoff function of firm 2. So this- $\pi_2 = (A - Q)q_2 - c_2q_2 - f$ is known to firm 1 and firm 2 also knows this- $\pi_1 = (A - Q)q_1 - c_1q_1 - f$. What are these, I will come to it, okay.

Now here we have done the normal form game in the game theory portion. There we have assumed like this kind of games, right? So this is player 1, this is player 2, their strategy 1, strategy 2. So they have 2 strategies or two actions and these are the payoffs. We have not specified here. So this is kind of normal form or strategic form game that we have done.

Here this strategy space or the strategy set is a continuous thing. It lies here, okay. Instead of this discrete action or discrete strategies, we have now continuous strategies, okay. This is the difference from the thing that we have done earlier and the payoffs of each firm we have to specify so we have specified the strategies or the actions that strategies set.

Now we have to specify the pay-off so payoff is a profit, okay and for firm 1 it is this-$\pi_1 = (A - Q)q_1 - c_1q_1 - f$. This portion-$(A - Q)$ is what, this portion is price because price is equal to A minus aggregate output so this is price then this price into q1 so this is a total revenue. This portion is total revenue-$(A - Q)q_1$. This the total cost- $c_1q_1 + f$, we have specified the cost so total revenue minus total cost it gives me the profit. So this is the profit of firm 1.

Now this is the price because market price is going to be the same for each firm. So this is the price into output of firm 2 q2 minus this total cost so this is the profit of firm 2. So firm 1 maximizes this-$\pi_1 = (A - Q)q_1 - c_1q_1 - f$, firm 2 maximizes this-$\pi_2 = (A - Q)q_2 - c_2q_2 - f$, okay.

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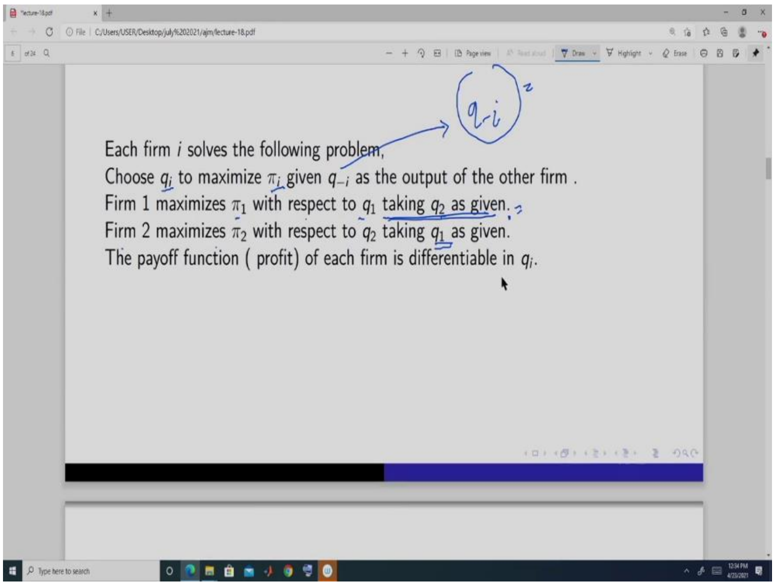

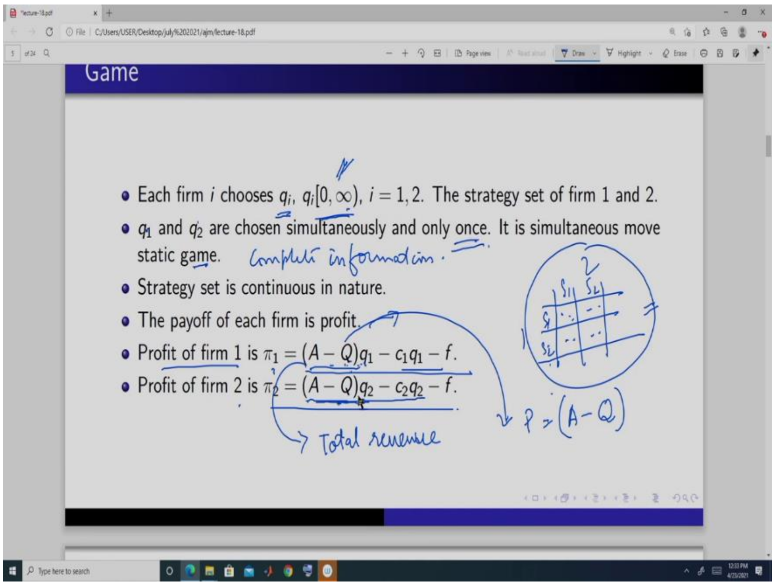

Now what is firms solve? They solve this problem. So firm i will choose qi to maximize this profit assuming that the there is a some given level of output of the other firm and this we represent it in this way- q_i. This is the output of other firm, okay.

So firm 1 maximizes profit pi one, i.e π_1 with respect to q1, taking q2 as given firm 2 so this portion is bringing in the strategic aspect, okay. And firm 2 maximizes profit that is pi 2 with respect to q2 taking q1 as given, okay and here if you look at this profit function it is obvious that they are differentiable, okay.

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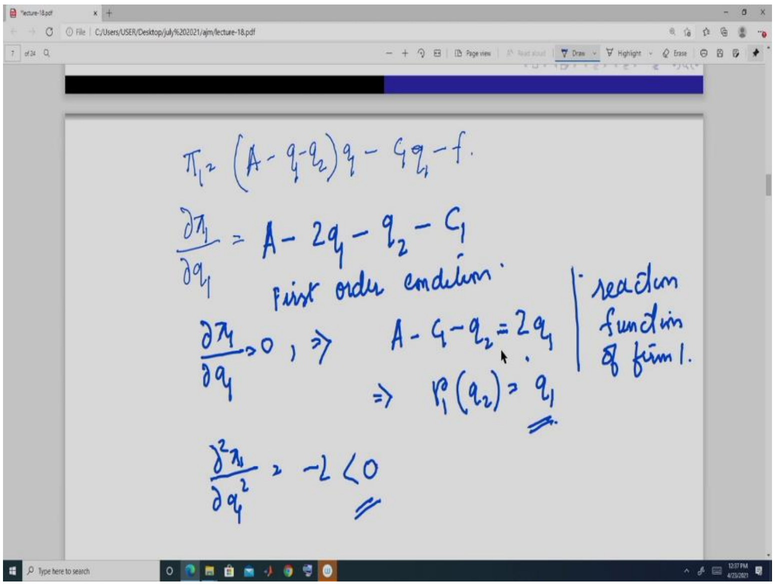

So now we solve this problem. So profit of firm 1 is this- $\pi_1 = (A - Q)q_1 - c_1q_1 - f$ so we maximize this, so this will give me if we differentiate with respect to take the partial with respect to q1 this will be this- $\frac{d\pi_1}{dq_1} = A - 2q_1 - q_2 - c_1$ now simply first order condition of maximization this gives me this- $\frac{d\pi_1}{dq_1} = 0 => A - c_1 - q_2 = 2q_1$ and this we called as reaction function of firm 1 or we can write this as it is a function of q2 in this way-$p_1(q_2) = q_1$.

So it means whatever if firm 1 takes some level of output of firm 2 as given then what is the optimal output it should produce, it is given by this function- $A - c_1 - q_2 = 2q_1$. So that is why it is called the reaction function. What it is? That if firm 1 believes that firm 2 is going to produce q2 amount then what is the optimal for firm 1 it is given by this firm. This function so this is the reaction function.

And here if you take the second derivative of this you get the second order condition of this, and it is minus 2 which is always negative so that is why this a is always, so this is always going to be the, is going to maximize the profit, okay. Second order condition is negative at this level. So we have got the reaction function of firm 1.

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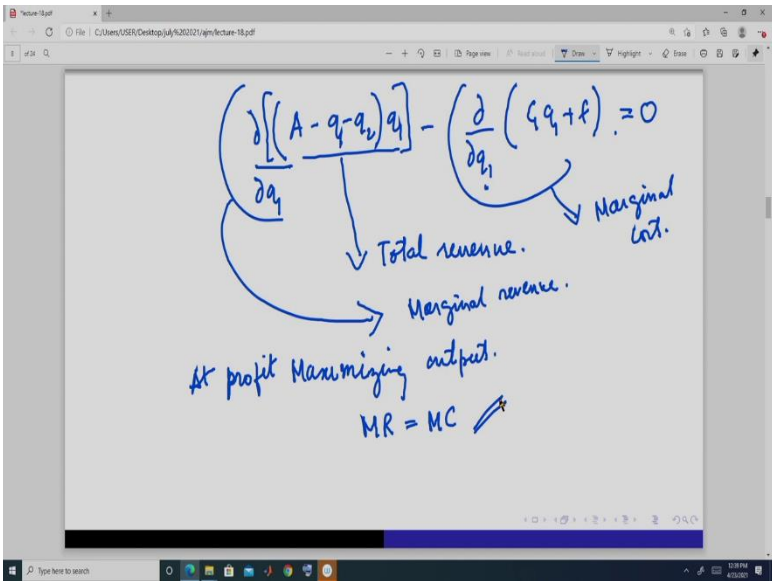

If you look at this function carefully what do we get? So this is- $\frac{\delta(A-q_1-q_2)}{\delta q_1} - \frac{\delta(c_1q_1+f)}{\delta q_1}$, right? So this is what? This portion is total revenue, so this taking this is the marginal revenue- $\frac{\delta(A-q_1-q_2)}{\delta q_1}$. This is the total cost, so this is marginal cost-$\frac{\delta(c_1q_1+f)}{\delta q_1}$. So at the optimal point or at profit maximizing output, we get marginal revenue should be equal to marginal cost because this is equal to 0 at the optimal point right and this condition is same as what we get in the monopoly thing.

So in the monopoly also the optimal output is decided where the marginal revenue is marginal cost so this optimization is similar to what we have done in the monopoly, only difference is here is, that we have considered some given level of output of firm 2, okay this portion- $A - 2q_1 - q_2 - c_1$.

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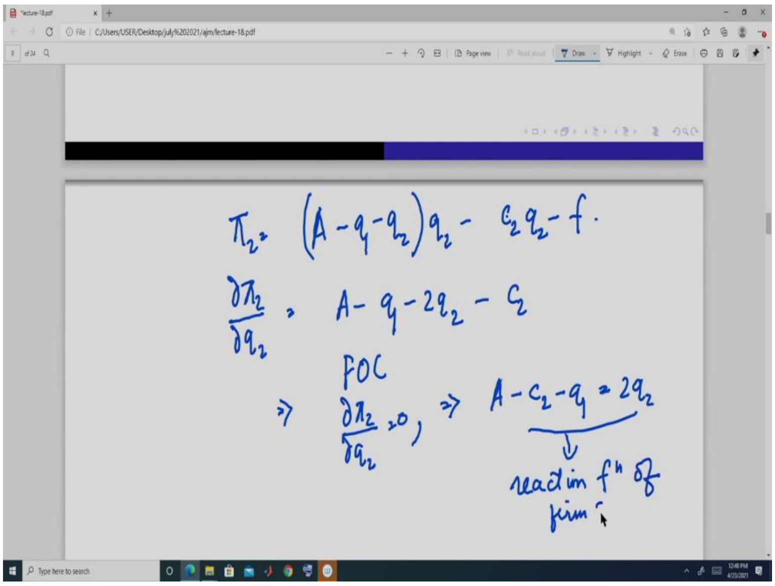

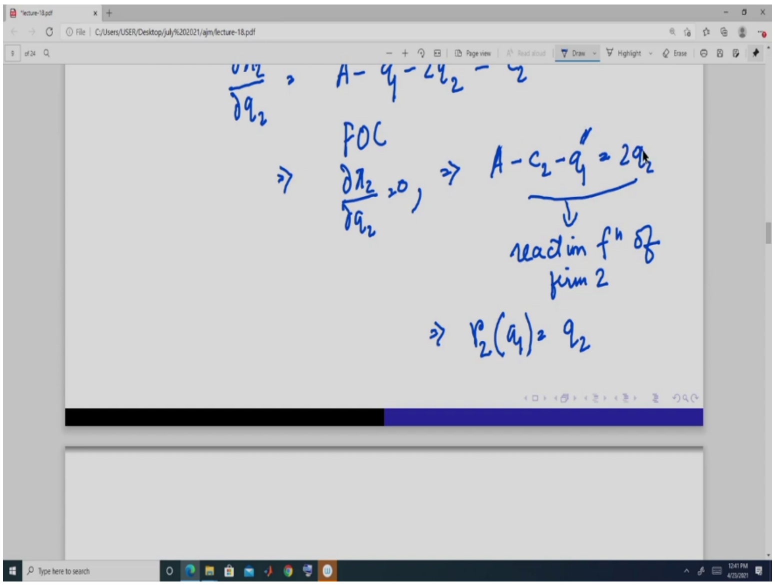

Similarly, the profit of firm 2 is this, this is again first order condition implies-$\frac{d\pi_2}{dq_2} = A - q_1 - 2q_2 - c_2$, so this implies so this is the reaction function of firm 2-$A - c_2 - q_1 = 2q_2$, which we write as function of output of firm 1. So if firm 2 believes that the output of firm 1 is this much then what is the optimal output it should produce, it should produce based on this function and this much, okay. It is given by this.

So now we know the reaction function, okay. So we know that if firm 1 believes output of firm 2 is this much then how much it is going to produce. And similarly if firm 2 believes that our firm 1 is going to produce this much then how much output it should produce.

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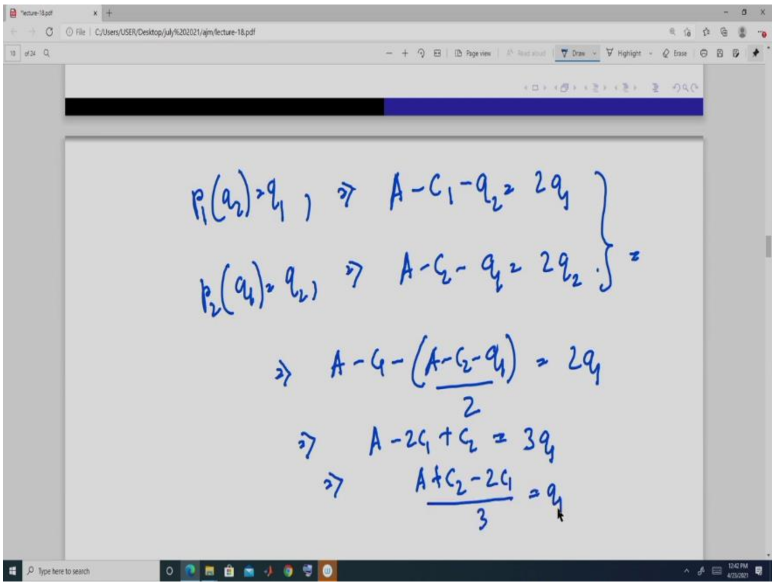

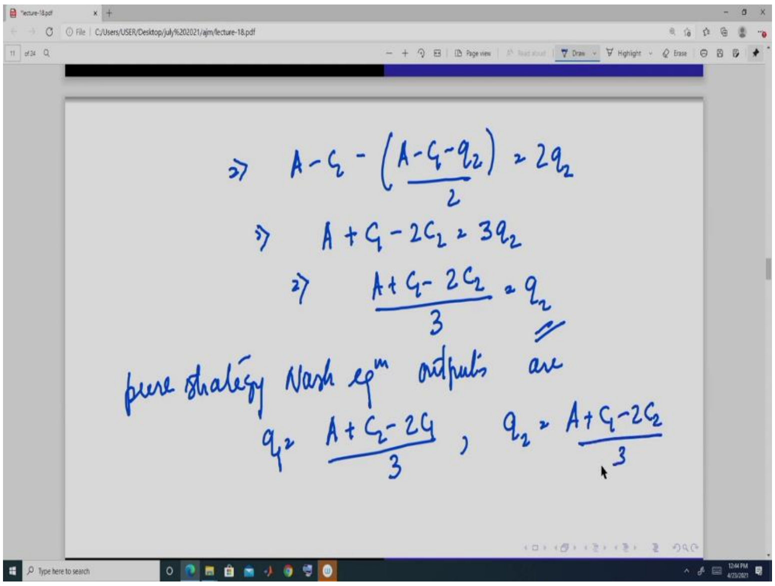

So based on these 2 reaction function so that is this- $p_1(q_2) = q_1$, we get the, this one- $A - c_1 - q_2 = 2q_1$, this- $p_2(q_1) = q_2 => A - c_2 - q_1 = 2q_2$ Now we solve these two equations. These two are linear equation and we can solve them. This is the output of firm 1- $\frac{A+c_2-2c_1}{3} =$

q_1 and for output of firm 2, we can write simply this- $\frac{A+c_1-2c_2}{3} = q_2$, so these two points is going to solve this.

So that means when firm 1 believes that the output of firm 2 is this-$\frac{A+c_1-2c_2}{3} = q_2$, then its optimal output is this- $\frac{A+c_2-2c_1}{3} = q_1$and simultaneously, firm 2 believes the output of firm 1 is this much- $\frac{A+c_2-2c_1}{3} = q_1$ so it produces this-$\frac{A+c_1-2c_2}{3} = q_2$. So that is why the Nash equilibrium here, the Nash equilibrium or you can say pure strategy Nash equilibrium outputs, are q1 is equal to this- $\frac{A+c_2-2c_1}{3}$.

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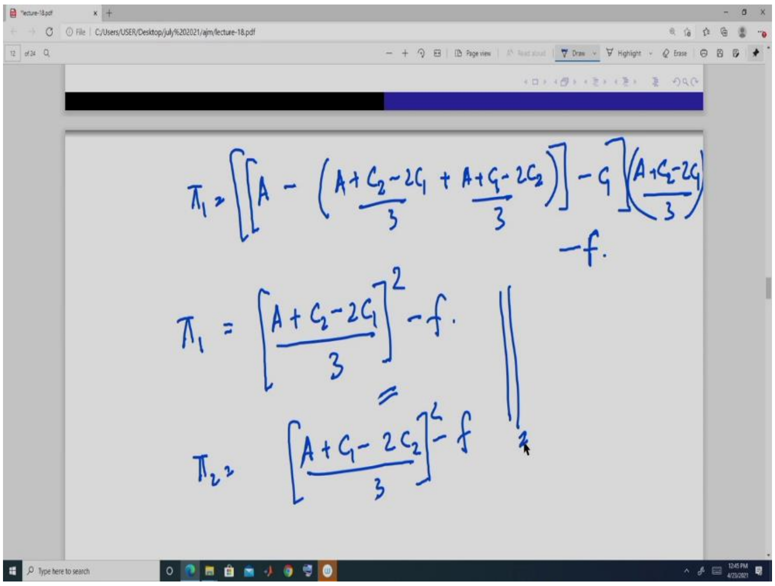

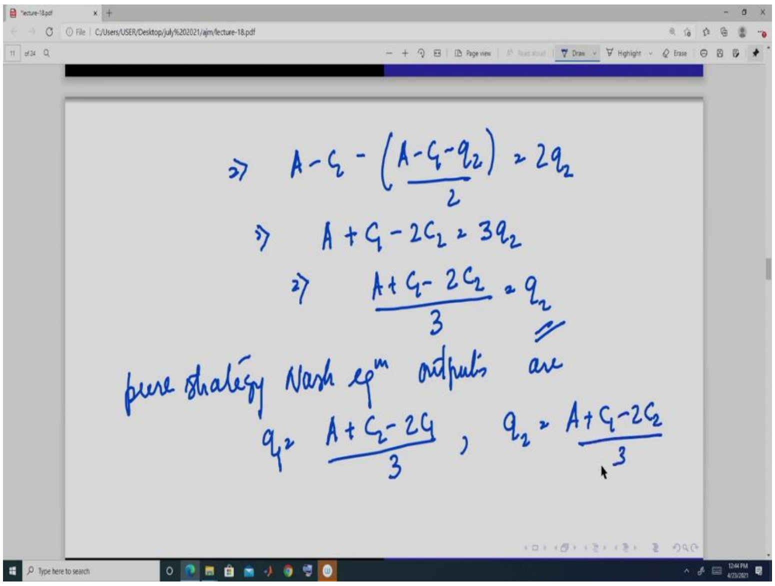

And when we plug these outputs in the profit function we get the Nash equilibrium profit. So while after solving this we get it. So this is the profit of firm 1-$\left[\frac{A+c_2-2c_1}{3}\right]^2 - f = \pi$ and similarly profit of firm 2 if you solved it, is this-$\pi_2 = \left[\frac{A+c_1-2c_2}{3}\right]^2 - f$. So this is a Nash equilibrium outcome, okay. Now here we have got this outcome.

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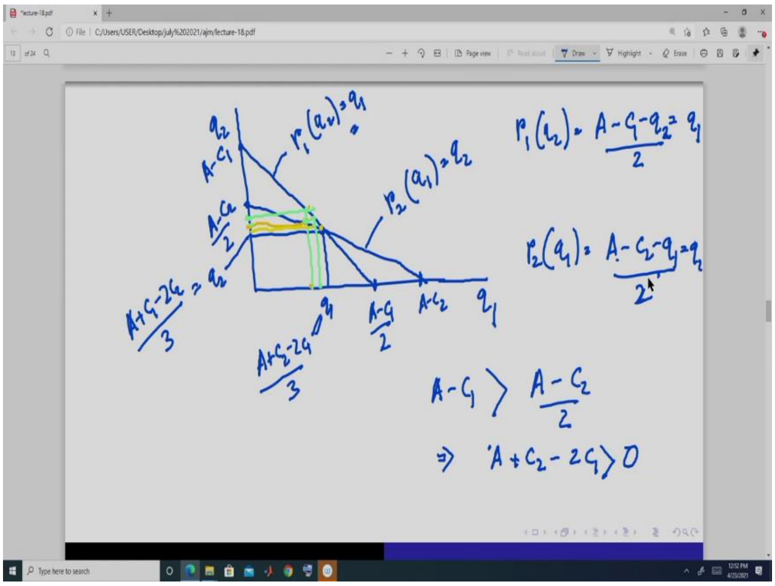

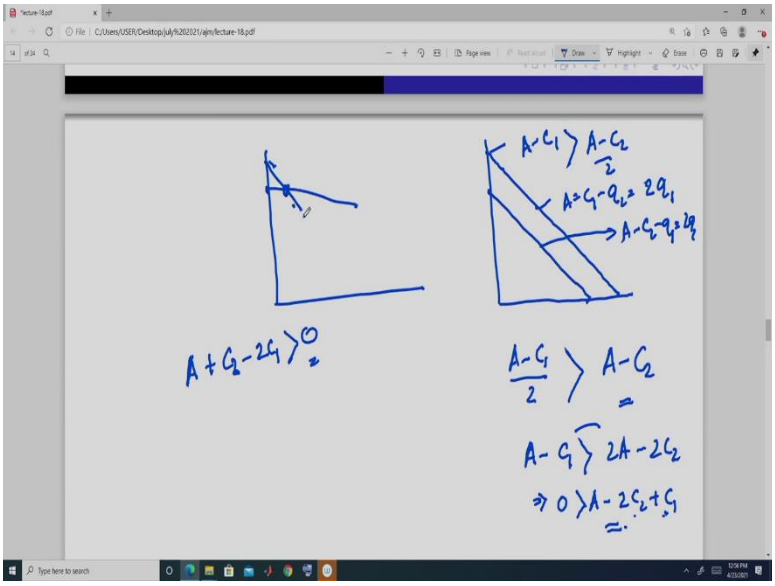

Now actually let us solve this through diagram to understand it better way. So we will use the reaction function. So reaction function of firm 1 is this-$p_1(q_2) = A - c_1 - q_2 = 2q_1$ so when output is 0 this is 0 then q2 is here it is this, right. And when q2 is 0 q1 is here this is and this is a straight line. It is this. So this is the reaction function of firm 1, okay or you take it like this, okay.

Now here reaction function of firm 2 is this-$p_2(q_1) = \frac{A-c_2-q_1}{2} = q_2$ so if q1 is 0 it is q1 **is** 0, it is A minus c2 is equal to, divided by 2 is equal to q2. So suppose it is this. And when q2 is equal to 0 it is A minus c2 is equal to q suppose it is here, A minus C2, okay.

And this is a reaction function of firm 2. This is the point of intersection that means at this point these two equations is the, this is the solution of these two equations. So this q1 is A plus c2 minus c1 divided by 3. This q2 is equal to A plus c1 minus 2 c2 divided by 3. We have found that.

Now here this is the point where two reaction functions are intersecting and that point gives us the pure strategy Nash equilibrium. Why? Because see suppose firm 1 this is the reaction function of firm 1. If firm 1 thinks that the output of firm 2 is this, then it is going to produce, suppose firm 1 thinks the output of firm 2 is this, then the optimal output it should produce is this much from the reaction function, right?

But firm, here when firm 2 thinks that the output of firm 1 is this much, it should produce this much amount. It should produce this much amount, not this. So there is a mismatch in the belief. So that is why it is not a Nash equilibrium. Then what will happen if it thinks in this way, then firm 2, since firm 2 has thought that output is this, so it has produced this much.

Now firm 1, it will come to know that output of firm 2 is this much, so it will assume that output is this, so it will produce this so its output is going to be this much. When firm 2 thinks that the output of firm 1 is this, it is going to produce this much. So there you will see that they will slowly come to this point and this is the point Nash equilibrium point.

Now here in this situation we get this solution this under what assumption? So we have to, we have not yet specified that assumption but we are, we have implicitly assumed it. So that assumptions are that this A minus c1 should be greater than a minus c2 divided by 2, i.e $A - c_1 > \frac{A-c_2}{2}$. So this should be greater so then we will get so this implies that A minus A should be positive- $A + c_2 - 2c_1 > 0$ or so if we have only this situation so that means this is going to be lower than this.

Then since they are downward sloping from this a equation we know these two equations. So that is why what we are going to get we are going to get that actually it will be since we are assuming it is like this and it is like this, this point so it will be there. So we have a point of intersection, right? But we may can we have a situation like this. This is A is this is greater than a minus, right? like this can we have a situation like this?

So there is no intersection point so there is no solution. Can we have this situation because from here we have got this right? if we have this situation then only we will get, right. We have assumed this but then we can have this situation, if we have this situation that means what from here what do we get a minus c1 divided by 2 is actually greater than a minus 2, i.e. $\frac{A-c_1}{2} > A - c_2$, okay.

So this we can have, this will be, this is assuming what so this will imply what. This will give us this-$0 > A - 2c_2 + c_1$. Now we have assumed this- $A - c_1 > \frac{A-c_2}{2}$so we have got this- $A + c_2 - 2c_1 > 0$. Now we may have this situation so if we have this situation if this is true, i.e $A + c_2 - 2c_1 > 0$ then it does not imply that this- $0 > A - 2c_2 + c_1$is not true. This can be true, okay. If c2 is sufficiently small, right?

If c2 is sufficiently big, okay or c1 is sufficiently small, okay we can have these two situations and, in that case, we will have no intersection point in the positive orthant. So, we will require this. So, this may not happen. So, we require this condition also that 1 minus c2 is greater than, i.e $A - c_2 > \frac{A-c_1}{2}$, so this implies a minus 2 c2 plus c1 is positive- $A - 2c_2 + c_1 > 0$.

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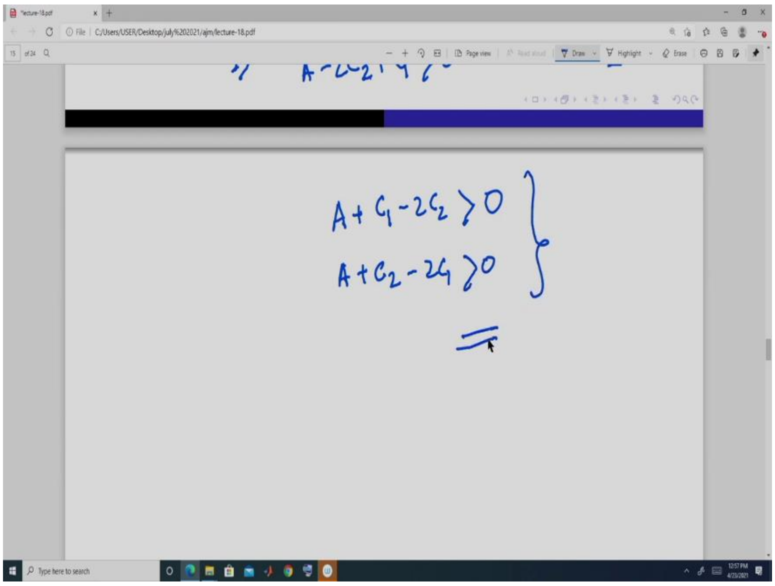

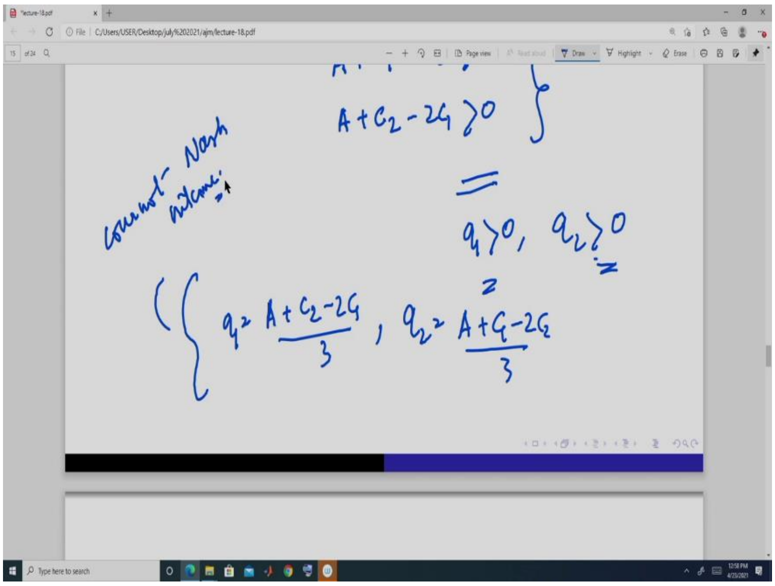

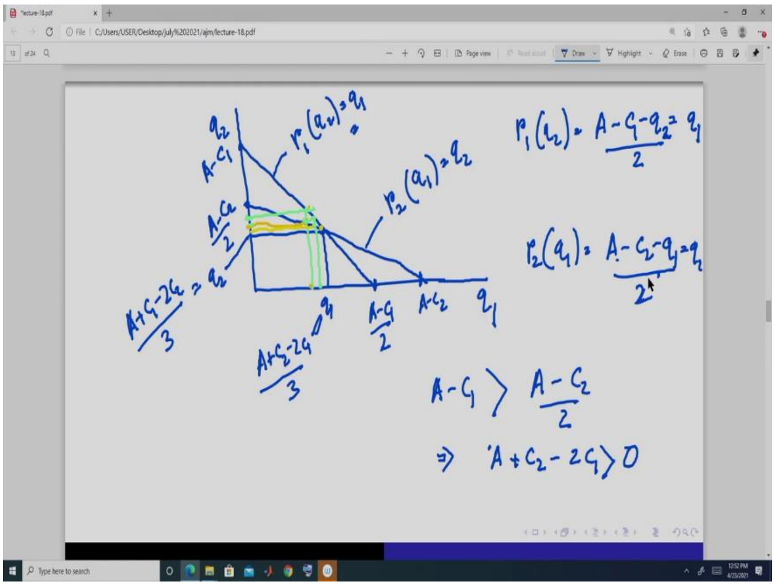

So for a solution, to exist in this case, we require these two conditions- $A + c_1 - 2c_2 > 0, A + c_2 - 2c_1 > 0$. Then only we will have a pure strategy Nash equilibrium and the situation will be such that it will be positive that is, these two conditions will ensure that Q1 is positive and Q2 is positive. If we do not have this situation, then we will not have a pure strategy Nash equilibrium where output of each firm is positive.

And this is also this outcome that is q1 is equal to $\frac{A+c_2-2c_1}{3}$ and q2 is- $\frac{A+c_1-2c_2}{3}$, this is also called Cournot Nash outcome, Why? because when Cournot proposed this model, at that time game theory was not developed. So Nash later on developed this non-cooperative game theory. And then combining these two we get this as the outcome, okay.

Now here while we are drawing this, you see at this point only believes are also match. That is when I assume that the firm 2 is going to behave in this way and in response I behave choose this, similarly firm 2 thinks my output is this and then chooses this as output. So if that is not there then we will not be in a Nash equilibrium.

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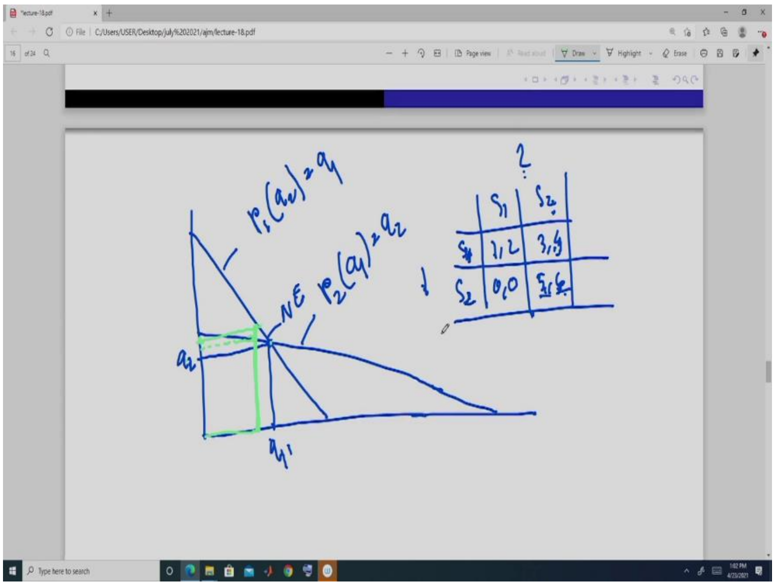

So that it is something like this that when we are, suppose consider this game how we have found the Nash equilibrium, okay. If player 1 thinks – if player 1 is choosing this, okay player 1 is choosing this then so player 2 thinks that suppose player 1 is playing this what is going to be my best is? My best is to choose this, okay.

So here you can say when firm 2 is choosing its output it should think that what is the firm 1 is going to produce, okay. And then based on that it is going to choose the optimal output so if firm 2 thinks that firm 1 is going to choose this, its best response is this, okay. And then when firm 2 is choosing this best response is to choose this. So that is why they are deviating.

It is firm 2 assume that firm 1 is producing this and then it has decided to produce this, choose this strategy. Then when firm 1 is assuming that this is the strategy of firm 2 then it is not choosing this but it is this. So that is why this (3,5) is not a Nash equilibrium so here firm 1 is choosing S2 when firm 1 is choosing S2 this firm 2 is going to choose S2, so that is why this (5,6) is a Nash equilibrium, pure strategy Nash equilibrium.

Same thing is happening here. If this is the reaction function of firm 1 and this is the reaction function of firm 2. At this point what is happening? Firm 1 believes that firm 2 is going to produce this much, so it produces this okay and firm 2 assumes that firm 1 is going to produce this and so it produces this, so they are match, so that is why this is a Nash equilibrium.

But consider any other point like this here suppose firm 1 thinks the output of firm 2 is this much then it is going to produce this much level of output. If it produces this much output so it is given by this amount then firm 2 its reaction function is this, it should produce this output. This much, not this, so that is why they are not match.

So only the solution of this reaction function is going to give you the Nash equilibrium. Because here it is deviating, right? from this output firm 1 thought that output of firm 2 is this much and then based on that it produce this but firm 2 when it assumes that the firm 1 is going to produce this much, it is not producing this but it is producing less than that. So it is deviating so that is why this is not a Nash equilibrium only the point intersection point is the Nash equilibrium, okay. So this is the solution of Cournot outcome in a 2 firm case, okay.

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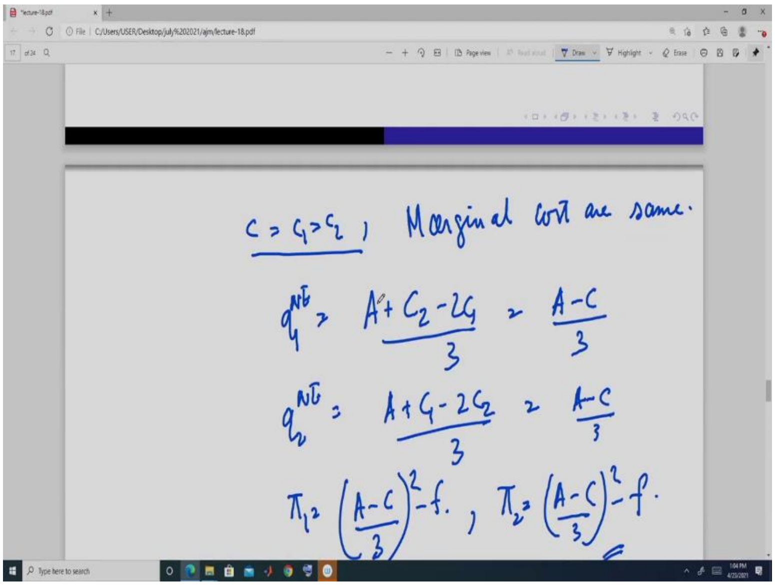

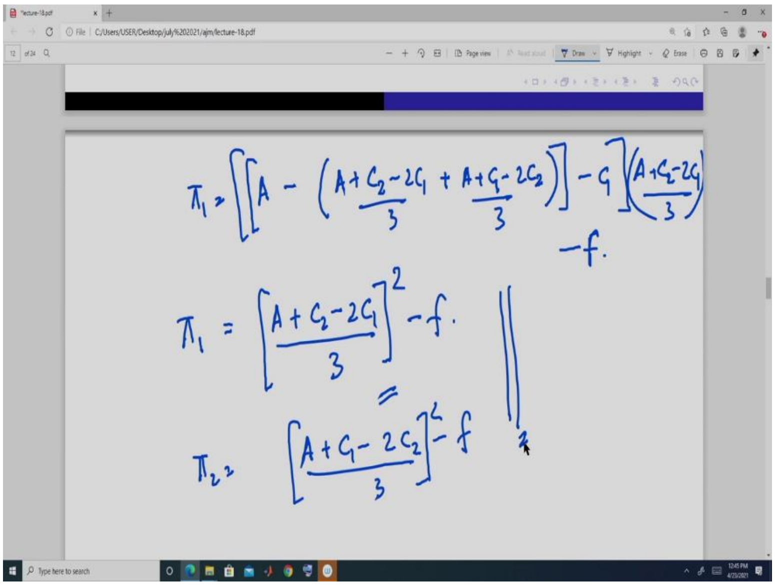

Now we are going to, we will show that this Cournot outcome is similar to something like we have done that the Prisoner's dilemma kind of thing. So to do that what we will assume for our simplicity that suppose c1 is equal to c2 that is marginal costs are same. So this Nash equilibrium output, which was this- $q_1^{NE} = \frac{A+c_2-2c_1}{3}$ it becomes- $\frac{A-c}{3}$, right?

And if you plug in these outputs in the profit, profit is going to be this $\pi_1 = \left(\frac{A-c}{3}\right)^2 - f$ and, is going to be this- $\pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, right? simply in this function, you do the substitution, you will get this – this thing so the Nash equilibrium profit of firm 1 is this and it is this firm 2.

Now instead of this playing strategically choosing output simultaneously without any coordination or without any discussion among them suppose the firm 1 and firm 2 make a collusion, collusion in the sense that what they do now from here this assumption makes that these two firms are similar, right?

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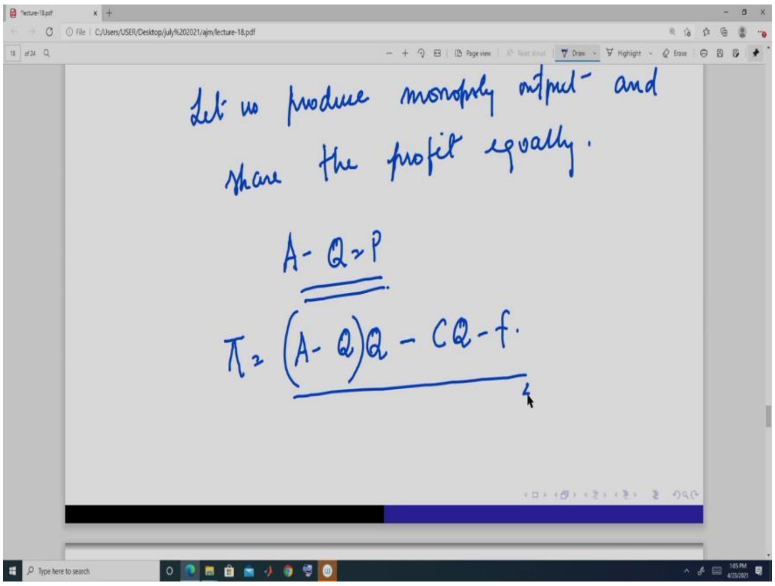

Since these two firms are similar so they decide that let us produce, let us produce monopoly output, monopoly output and share the profit equally. What does this mean? So we know the monopoly thing. So these firms are similar so it does not matter whether firm 1 produces or firm 2 produces, okay.

So market demand inverse market demand is this-A-Q=P now assume that they are going to act as a monopoly so only one firm produces. So it is you can think one firm is going to sell only. So this is the profit- $\pi = (A - Q)Q - CQ - f$.

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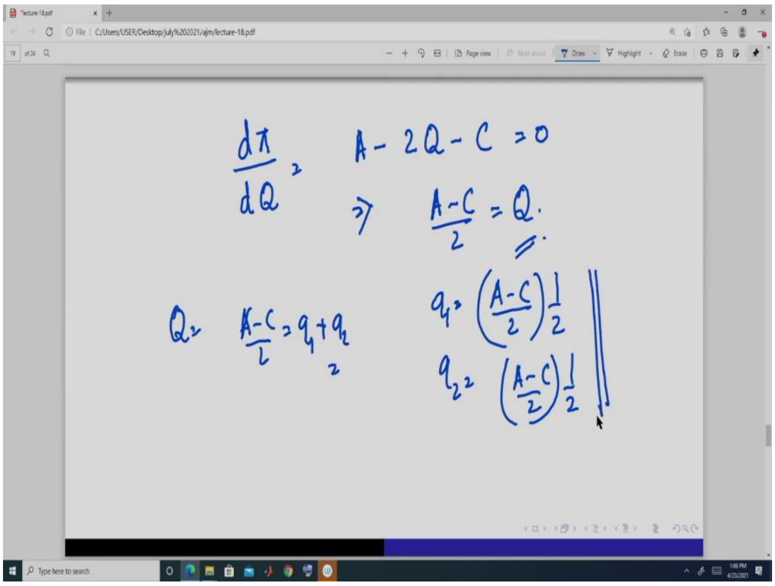

Now we, this is the monopoly output- $\frac{\mathrm{A-C}}{2} = \mathrm{Q}$. Now what it can do since the firms are similar, so this they can share. They say that you produce half of this and I produce half of this. So this is suppose $q_1 = \left(\frac{\mathrm{A-C}}{2}\right).\frac{1}{2}, q_2 = \left(\frac{\mathrm{A-C}}{2}\right).\frac{1}{2}$, okay, they are selling this and you can think is this, which is equal to q1 plus q2. Now if this is the case then what is the profit of these firms?

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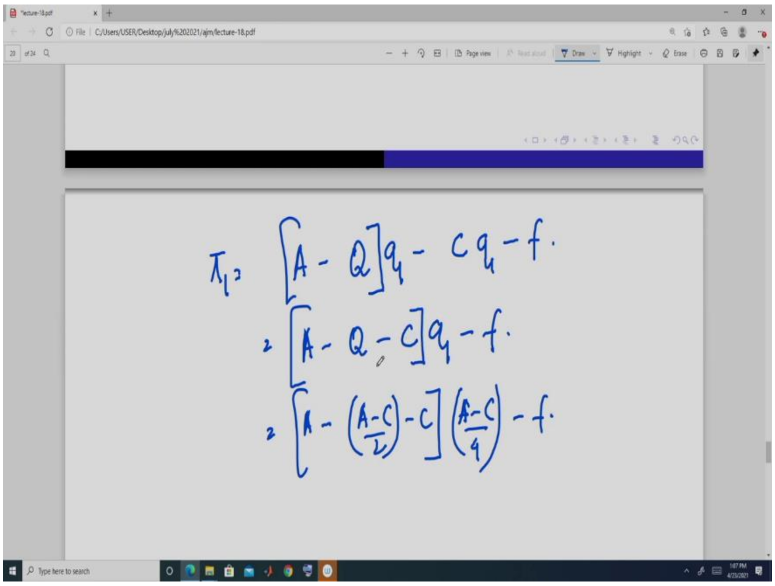

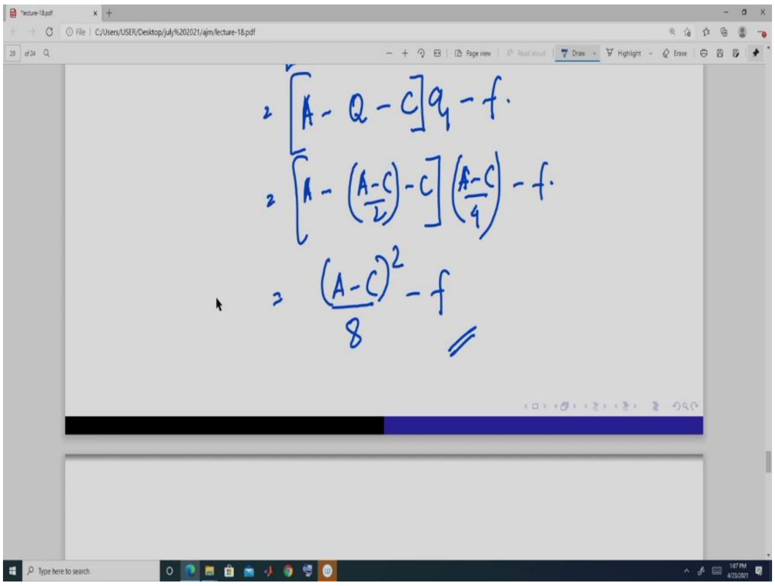

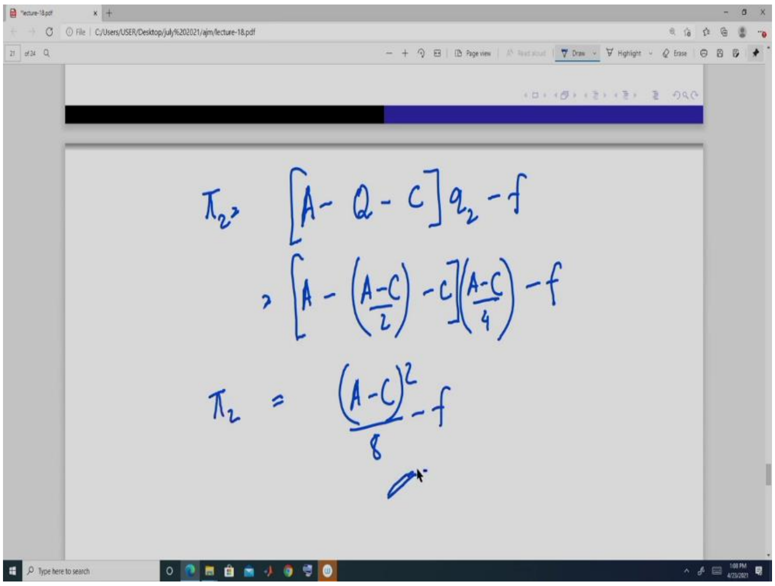

Profit of these firms are; profit of firm 1, we know this-$\pi_1 = [A - Q - C]q_1 - f$, we know this-$\pi_1 = [A - \frac{A-c}{2} - C].\left(\frac{A-c}{4}\right) - f$. It is this- $\frac{(A-C)^2}{8} - f$ and if we follow the same method, going to be this, is this-$\pi_2 = \frac{(A-C)^2}{8} - f$. Now the question is whether they are going to play this Cournot kind of game or they are going to share the market equally.

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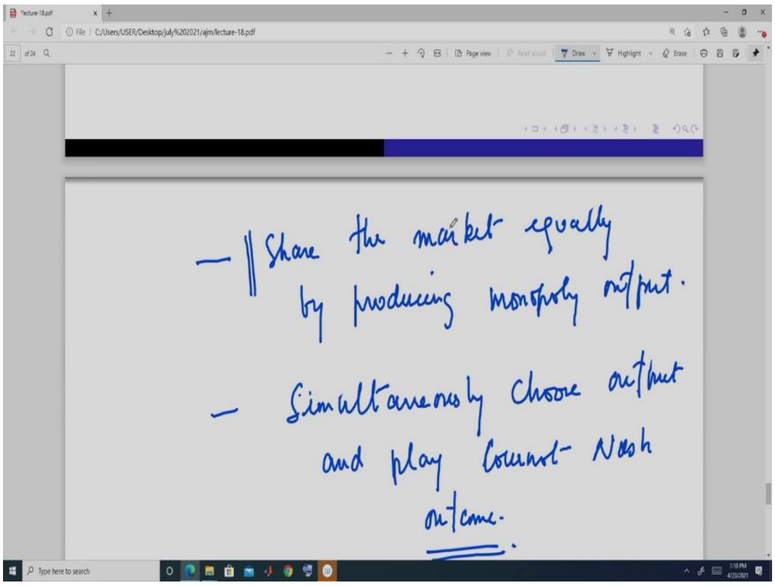

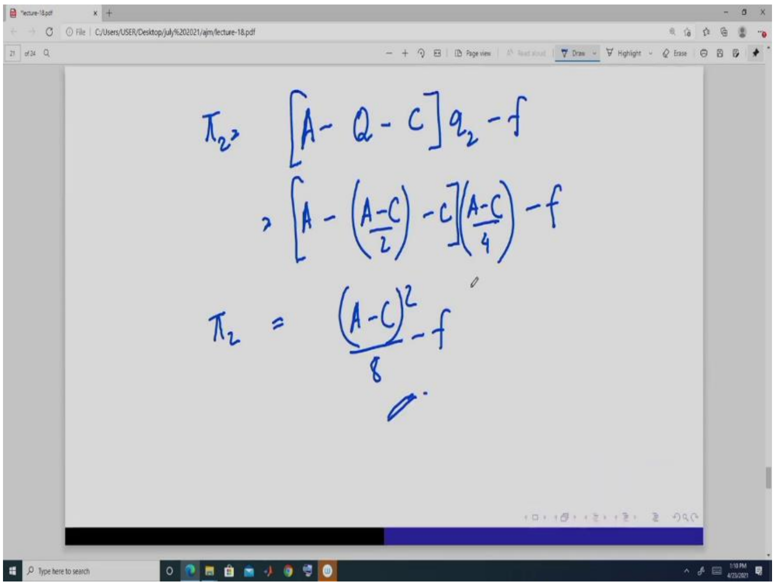

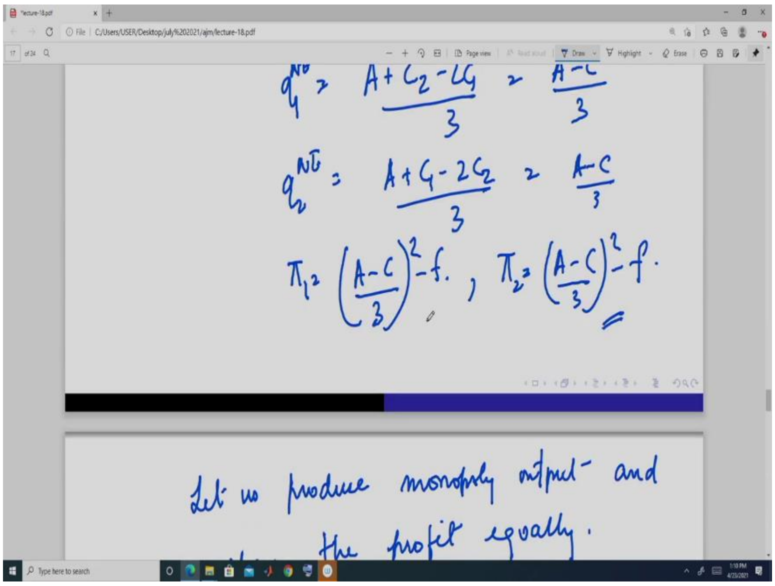

So the question is here that whether to share the market equally by producing monopoly output so this is one you can think as this is one strategy or do not bother, simultaneously choose output and play Cournot Nash outcome, okay. Suppose this.

Now here in this situation, in this situation, what are the profits, in this situation profits are this- $\frac{(A-C)^2}{8} - f$ so, when they share this, so the profit is this and in this case, the profit is what is the profit, the profit is this much-$\left(\frac{A-c}{3}\right)^2 - f$, okay. Now suppose firm 1 whether this is going to be implemented automatically.

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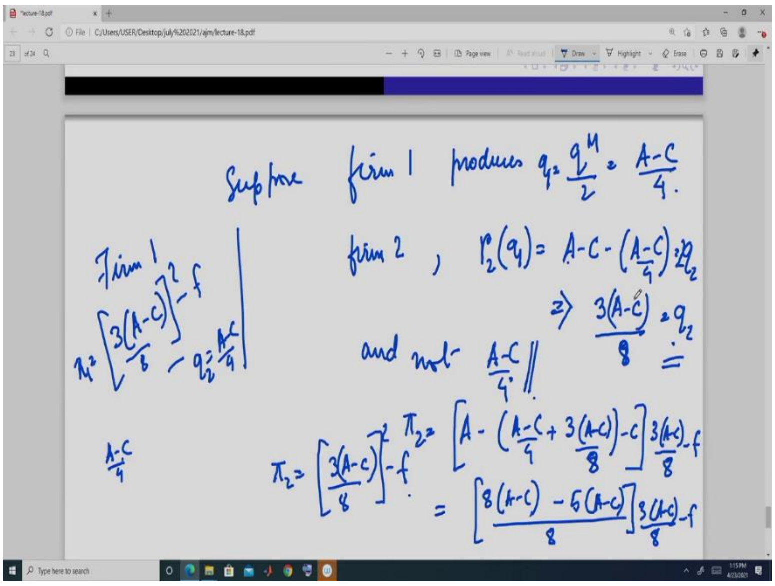

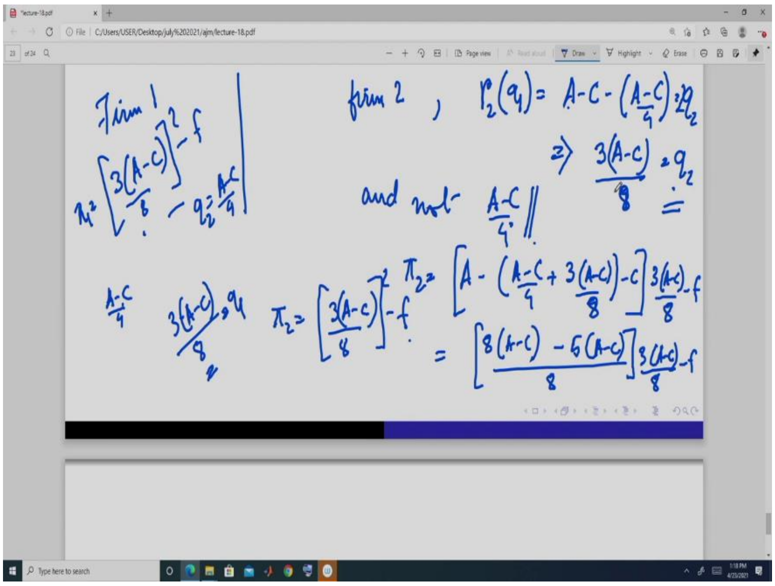

$$\pi_2 = [A - Q - C]q_2 - f$$

$$= \left[A - \left(\frac{A-C}{2}\right) - C\right]\left(\frac{A-C}{4}\right) - f$$

$$\pi_2 = \frac{(A-C)^2}{8} - f \qquad \frac{(A-C)^2}{8} - \cancel{f} < \left(\frac{3(A-C)}{8}\right)^2 - \cancel{f}$$

$$\Rightarrow \quad 1 < \frac{9}{8}$$

— Share the market equally by producing monopoly output.

— Simultaneously choose output and play Cournot-Nash outcome.

$$\pi_1 = [A - Q]q_1 - Cq_1 - f.$$

$$= [A - Q - C]q_1 - f.$$

$$= \left[A - \left(\frac{A-C}{2}\right) - C\right]\left(\frac{A-C}{4}\right) - f.$$

$$\pi_1 = \frac{(A-C)^2}{8} - f$$

Suppose firm 1 produces monopoly half of this, this- $\frac{q^M}{2} = \frac{A-C}{4}$ then firm 2 how much it should produce if we use the Nash optimization, so we get a reaction function of like this. So this is going to be output of firm 2 is this-$\frac{3(A-c)}{4} = q_2$ and not this- $\frac{A-c}{4}$, right? because this is the half of monopoly. Suppose firm 1 produces this then from the reaction function of firm 2 we get firm 2's best response is this. It is optimal and we have got this from optimizing the output of firm 2 given a output of firm 1, okay.

So if we take this combination so then the profit of firm 2 is what, this is, so the reaction function it will have are 2 here, right. So this will be 3 by 8 this will be 3 by 8 like this, so this is going to be, so it is again 8 A minus C 5 A minus C. So profit of firm 2 is going to be this- $\pi_2 = \left[\frac{3(A-c)}{8}\right]^2 - f$, right?

Similarly, if we do the same thing so we get the profit of firm 1, what it is going to be? It is going to be 3- $\left[\frac{3(A-c)}{8}\right]^2 - f$. It is going to be this one when firm 1 produces, firm 2 produces this half a monopoly. Now what you are doing? So firm 2 suppose firm 1 is producing this much amount of output- $\frac{A-c}{4}$, so firm 1 suppose produce this much amount of output$\frac{A-c}{4}$, okay.

Then if it produces this much firm 2 produces this much-$\frac{A-c}{4}$, its profit is going to be this much- $\frac{(A-C)^2}{8} - f$ right but instead if it produces this much- $\frac{3(A-c)}{8}$based on the reaction function, its profit is going to be this much$\left[\frac{3(A-c)}{8}\right]^2 - f$ so now let us compare the profit here.

This and you will see that this $\frac{(A-c)^2}{8} - f$ is always less than $\left[\frac{3(A-c)}{8}\right]^2 - f$, so this is going to be, so for firm 2 it is optimal to deviate, it is optimal to deviate instead of producing half a monopoly, it should produce more output and this. So that is why what is happening firm 2 is deviating and similarly firm 1 if we compare this profit and this we get that it is also optimal for firm 1 to deviate if firm 2 produces half of the monopoly output.

So that is why what do we get they are never going to decide that let us produce half of the monopoly. Although if you look at this profit, this profit is greater than this profit. If you simply compare, this and this you will see that this is greater than this, this $\left(\frac{A-c}{3}\right)^2 - f$ is greater than this$\frac{(A-c)^2}{8} - f$ right? because this is divided by 8 and this is going to be divided by 9, So even though the Cournot outcome is suboptimal compared to the monopoly. Why it is optimum because if they decide that let us produce half of the monopoly then the profit can be at a higher level. But what happen, they cannot decide on that, they cannot come to a conclusion because if firm 1 decided that let us produce half of the monopoly profit, firm 2 based on its reaction function is going to produce this much level of output and its profit is going to be this much, which is higher than the half of the monopoly profit.

Similarly, if firm 2 produces half of the monopoly output then firm 1 is not going to choose half of monopoly instead it is going to produce this much amount of output and so the profit is going to be this much which is greater than the half of the monopoly profit. So that is why they will not they cannot come to outcome which is better than a Cournot outcome because Cournot outcome is going to give this much profit to each but they end up having this much profit but they could have got better profit. This is half of the monopoly profit.

So we will stop at this today. We have done the Cournot duopoly thing and we have also shown that the Cournot outcome is actually similar to the Prisoner's dilemma that is there is an another outcome, which is better than these 2 outcomes but still since the firms behave in a non-cooperative way, in the sense that they behave what is best for them given the output of other firm and there is no possibility of any negotiations between them so that is why we get the outcome as same as the Prisoner's dilemma outcome, okay. And in the next class we will extend this model to n firm, okay. Thank you very much.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
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Module 7: Cournot Competition
Lecture 26: Cournot Oligopoly

Hello everyone. Welcome to my course Introduction to Market Structures. So we will initially do what we were doing in the last class that is Cournot duopoly and so we assume that there are two firms and the market demand is this- A-p=Q.

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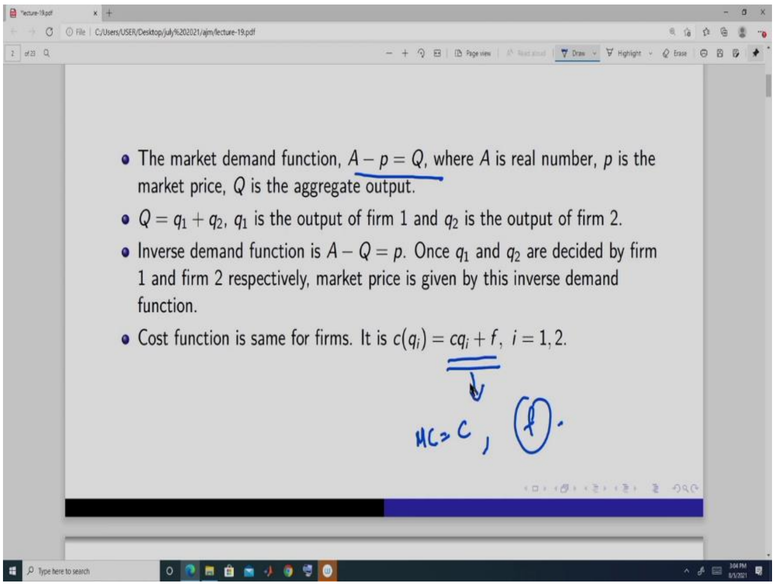

So cost function is of this form- $c(q_i) = cq_i + f, i = 1,2$. So we have assumed that the marginal cost is constant and so here marginal cost is c and we have a fixed cost which comes from here. So this kind of cost function we can see when we have either one factor is fixed and another factor is varying. Then we can have or we can vary most of the factors and suppose we have all the factors and we have paid some license fee so that is giving us this f fixed cost or suppose the rent that is we have paid in land so that may give this kind of fixed cost, okay.

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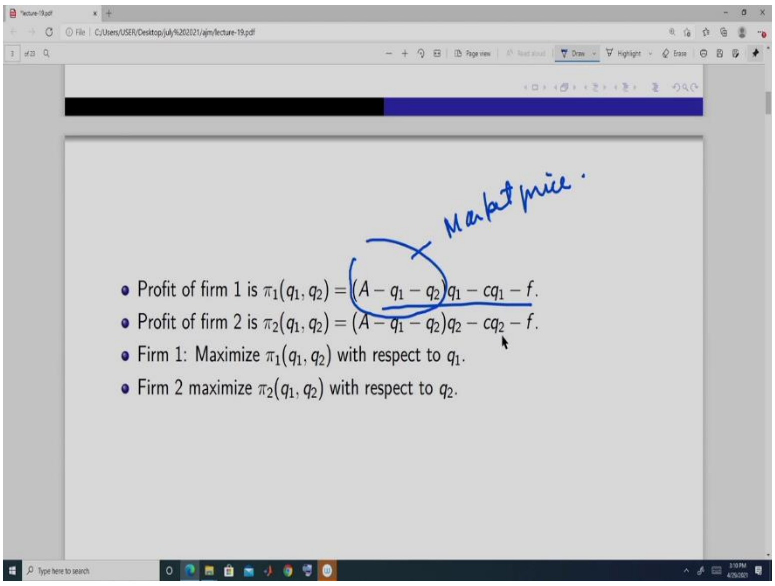

So profit of firm 1 is given by this function. It is this- $\pi_1(q_1, q_2) = (A - q_1 - q_2)q_1 - cq_1 - f$ where this portion$(A - q_1 - q_2)$ is the market price and market price into output of firm 1 is giving its the revenue of firm 1 and this is the total cost. So this is the profit function of firm 1 and similarly this is the profit function of firm 2- $\pi_2(q_1, q_2) = (A - q_1 - q_2)q_2 - cq_2 - f$. And firm 1 what it will do, it will maximize this profit with respect to q1 taking q2 as given and firm 2 will maximize this with respect to q2 taking q1 as given, okay.

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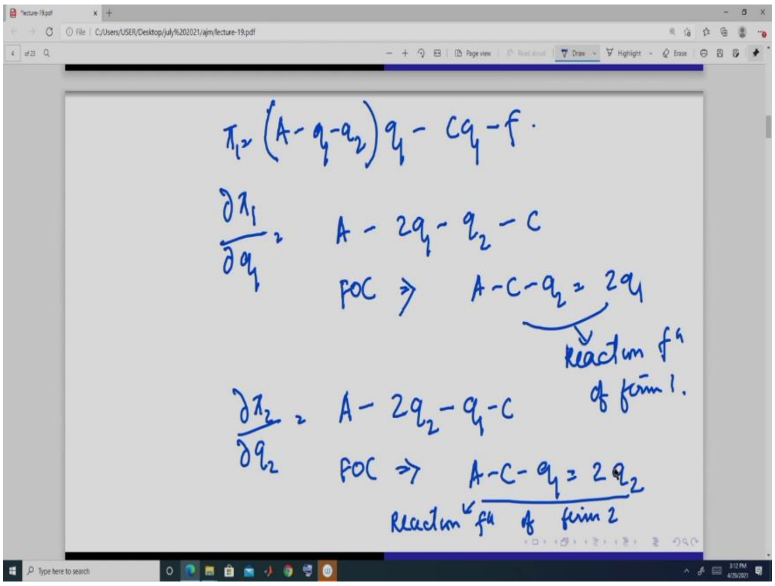

So for firm 1 we see since it is differentiable since the profit function if you look at the profit function it is this, right? and we have assumed that the cost function is same for 2 firms. So we get this is and First Order Condition implies- $\frac{d\pi_1}{dq_1} = A - 2q_1 - q_2 - c$. So this- $\mathrm{A} - \mathrm{q}_2 - \mathrm{c} = 2\mathrm{q}_1$ is the reaction function of firm one.

Similarly, we will find the reaction function of firm 2 and this will be equal to again the First Order Condition will imply- $\mathrm{A} - \mathrm{q}_1 - \mathrm{c} = 2\mathrm{q}_2$. So this is what reaction function of firm 2. So we have got these two reaction functions and we know that the pure-strategy Nash equilibrium is the point given by the intersection point of the reaction functions.

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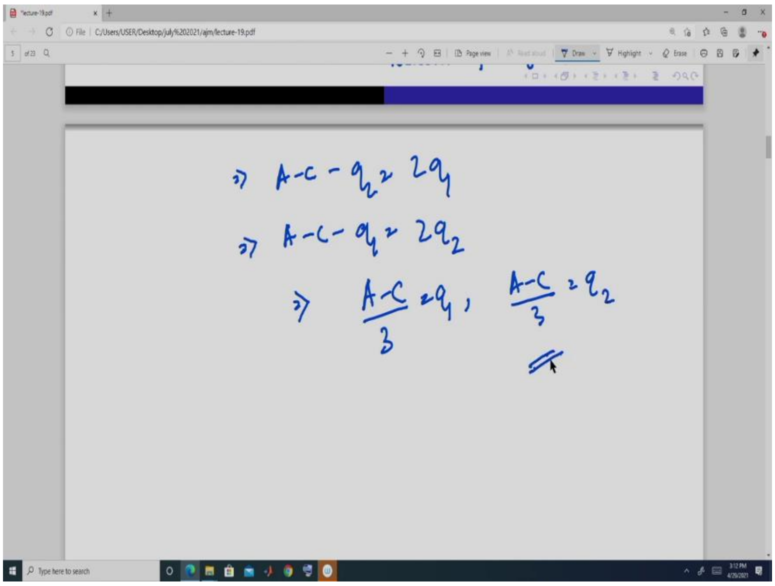

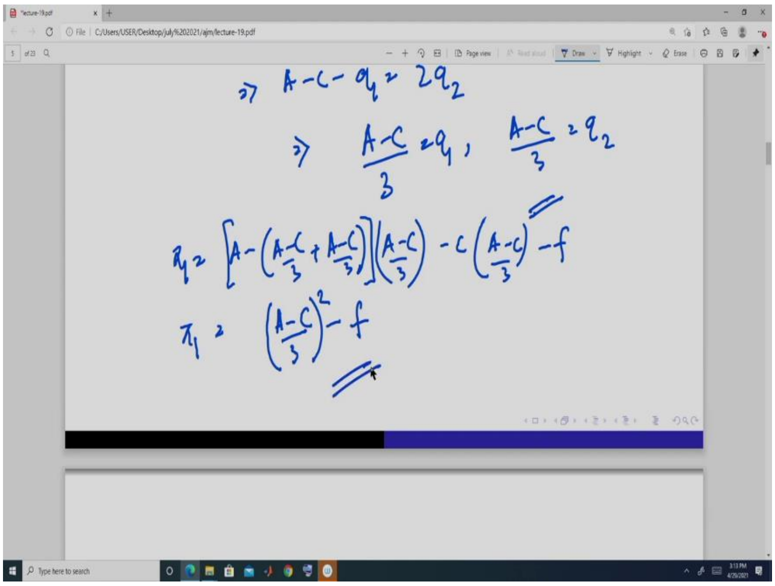

So here in this case the Nash equilibrium is given by solving these two equations- $\mathrm{A} - \mathrm{q}_2 - \mathrm{c} = 2\mathrm{q}_1, \mathrm{A} - \mathrm{q}_1 - \mathrm{c} = 2\mathrm{q}_2$, right? This and we get after solve this- $\frac{\mathrm{A-c}}{3} = \mathrm{q}_1, \frac{\mathrm{A-c}}{3} = \mathrm{q}_2$. So this is the Cournot outcome and when we plug in this in the profit function of firm 1, it is this pi 1 that is the profit of firm 1. It is going to be this- $\pi_1 = \left[A - \left(\frac{A-c}{3} + \frac{A-c}{3}\right)\right] . \left(\frac{A-c}{3}\right) - c\left(\frac{A-c}{3}\right) - f$ and if we, when we solve this we get. So this is the profit of firm 1 in the Cournot competition- $\pi_1 = \left(\frac{A-c}{3}\right)^2 - f$

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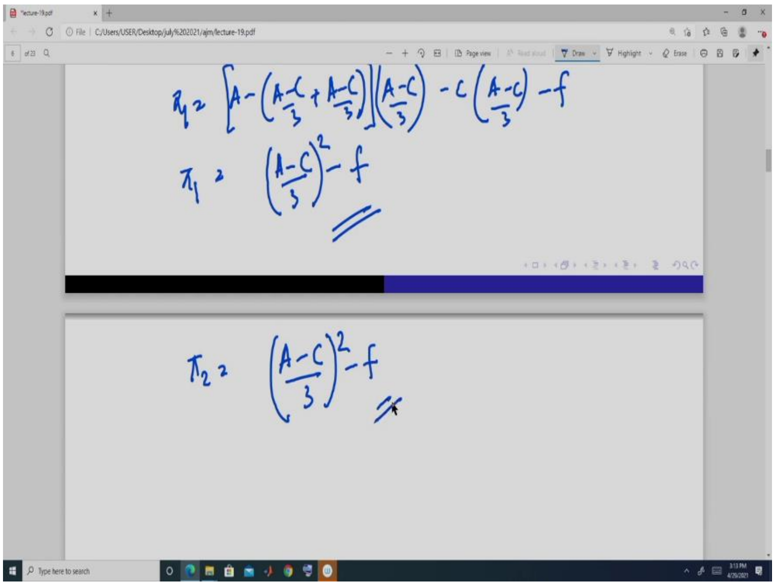

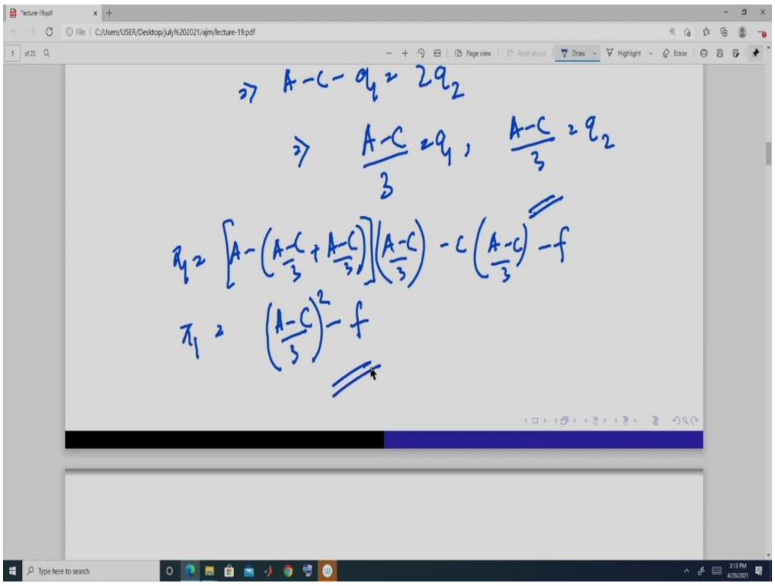

And similarly profit of firm 2 is-$\pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, so whenever both the firms are deciding the output simultaneously and only once and then we get the Cournot outcome and the Cournot outcome outputs are this for firm 1, this is for firm 2. Profit for firm 1 is this and profit for firm 2 is this. So this is the Cournot model.

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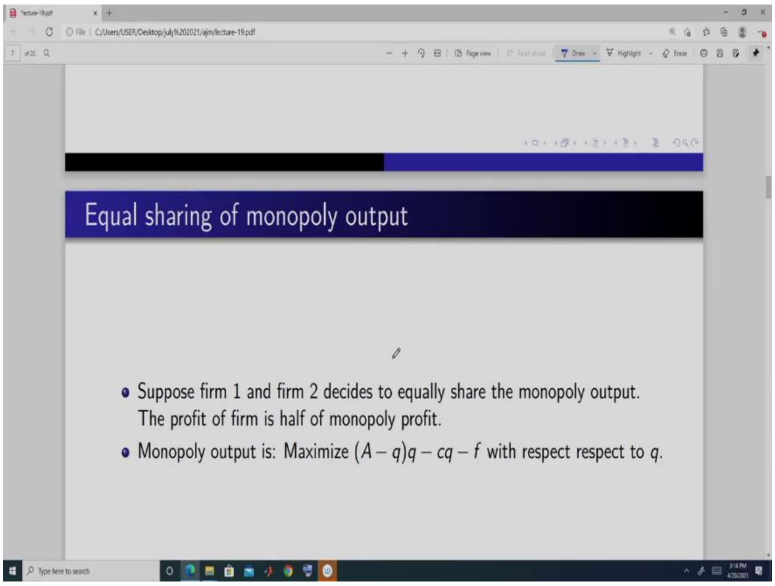

Now we ask the question can we improve upon this? So suppose instead of playing the Cournot thing what happens firm 1 and firm 2 they have a discussion or a negotiation and they say that let us produce half of monopoly output each and then we will sell that half of monopoly output, okay. Now the question is whether that is a Nash equilibrium outcome or not. To show that we will now do the following steps.

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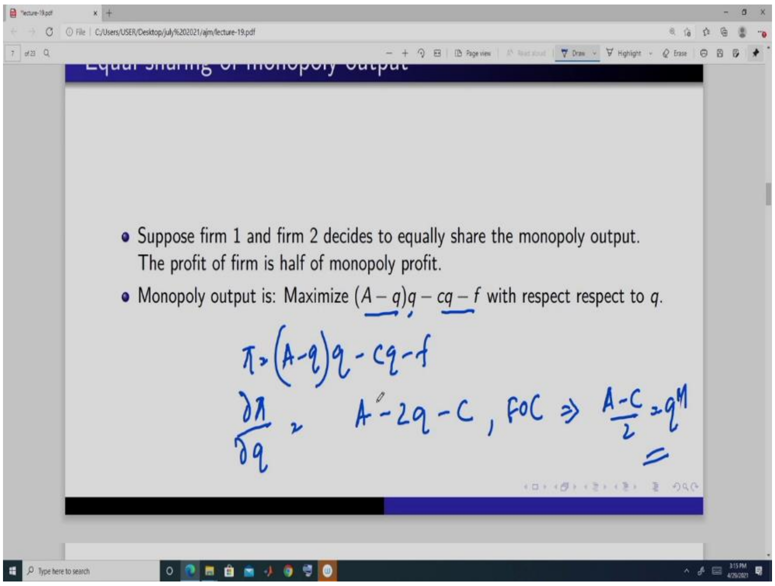

So first suppose each firm decides that we are going to produce half of monopoly. So how to find the monopoly output in this case. So this is market demand and this is the cost function of each firm- $(A - q)q - cq - f$ and this is the output of each firm. Since if there is a monopoly then that means only one firm so this output of one firm can be firm 1 or firm 2 that is giving you the market price A minus q is giving you the market price into q output of that single firm this. So profit is of a monopolist is simply this- $\pi = (A - q)q - cq - f$ and we maximize this with respect to so we get, so then first order condition gives us so this is the monopoly output- $\frac{A-c}{2} = q$

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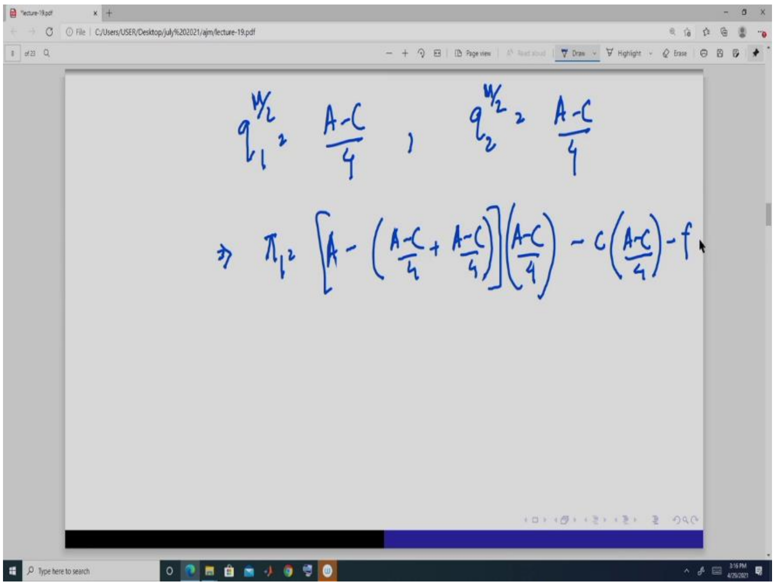

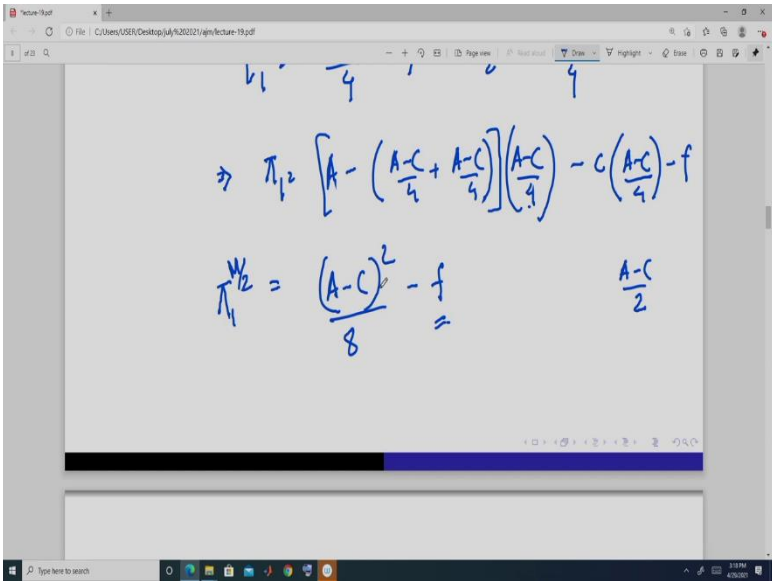

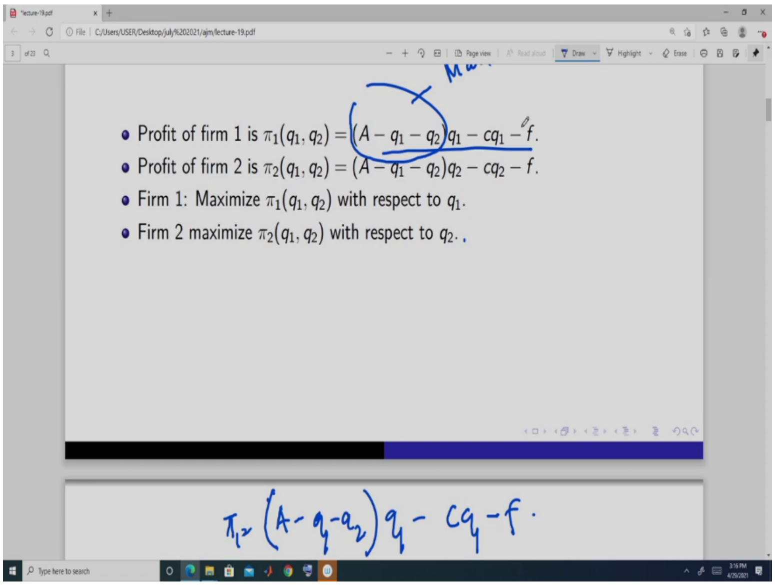

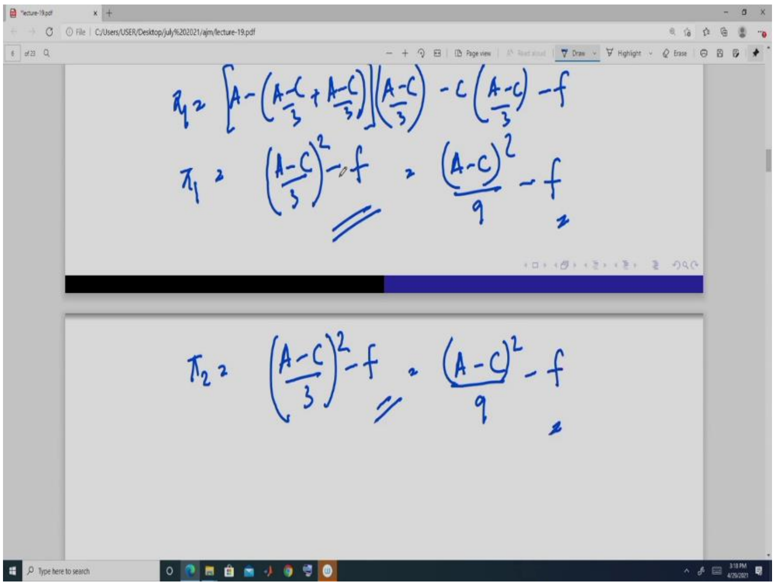

And so if each firm decides to produce half of monopoly output so firm 1 is going to produce this-$\frac{A-c}{4}$, firm 2 is going to produce this-$\frac{A-c}{4}$, right? Now let us look at the profit. So profit of firm 1 so this is f the fixed cost. This is the profit-$\pi_1 = \left[A - \left(\frac{A-c}{4} + \frac{A-c}{4}\right)\right].\left(\frac{A-c}{4}\right) - c\left(\frac{A-c}{4}\right) -$ f. We know the profit function in the duopoly thing is given by this so we got similar kind of profit thing.

Now if we simply it, we will get, it will be this-$\left(\frac{A-c}{4}\right)^2 - \text{f}$. So each firm producing this, so the total output is going to be a monopoly output, this much- $\frac{A-c}{2}$. Now if you plug in here you will get, so this portion it will be this-$\frac{(A-c)^2}{8} - \text{f}$, okay. So this profit of firm 2 have this, it is easy to see that this is greater than this thing-$\left(\frac{A-c}{3}\right)^2 - f.$, because this is, right? so now the question is whether this- $\frac{(A-c)^2}{8} - f$ can be a possible outcome of some game and we will define the game slightly later on, okay. But we know that this is giving a higher profit, okay.

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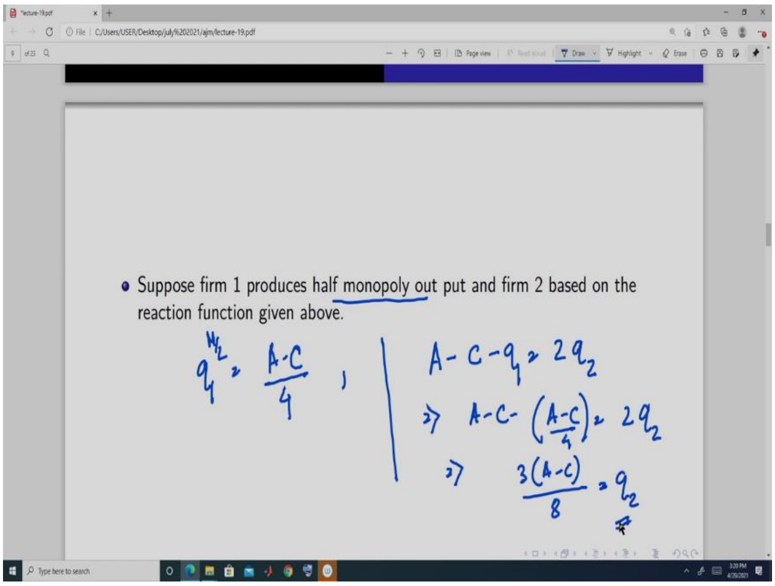

Now suppose we take another one. Firm 1 produces half monopoly output that is q1 is this-$\frac{A-c}{4}$ and firm 2 produces based on the reaction function of that we have got in the Cournot thing. So the reaction function of firm 2 is this- $A - q_1 - c = 2q_2$, so you plug in the output of firm 1 you will get the optimal output of firm 2 so here firm 1 produces this so the optimal output for firm 2 is this, this is the optimal output of firm 2- $\frac{3(A-c)}{8} = q_2$.

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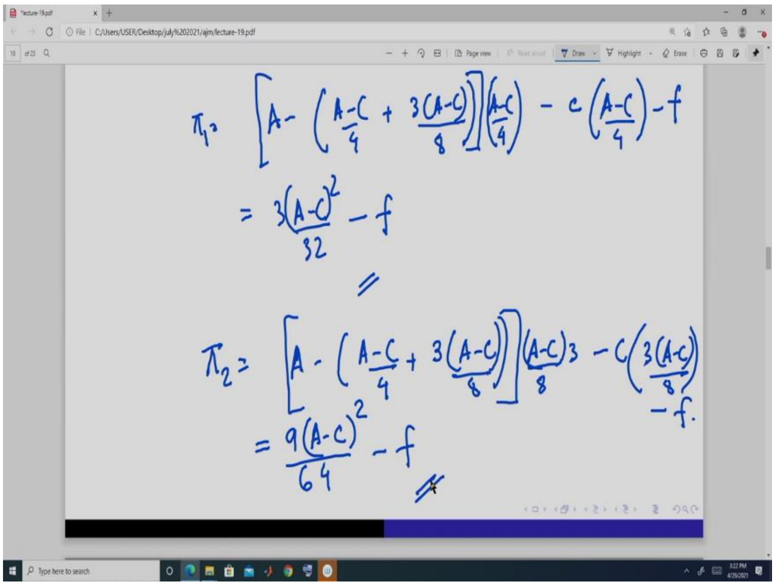

Now let us find out the profit in this case. So profit of firm 1 is again this, so this is going to be 5 by 8 and so this is so again this is going to be 32 minus f. This is the profit of firm 1-$\frac{3(A-c)^2}{32} - f$. And profit of firm 2, it is this, so this is equal to this-$\pi_2 = \frac{9(A-c)^2}{64} - f$. So this is the profit of firm 2, when firm 2 decides its output based on the Cournot reaction function. This is the Cournot reaction function and firm 1 decides that it is going to produce half of monopoly output.

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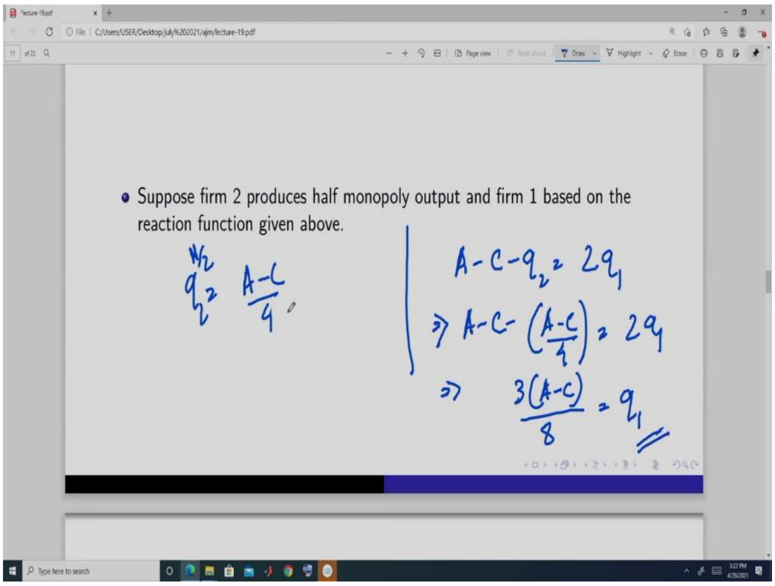

Now suppose firm 2 decides that it is going to produce half of monopoly output. So it is this-$\frac{A-c}{4}$ and firm 1 produces based on the Cournot reaction function. So firm 1's Cournot reaction function is this- $A - c - q_2 = 2q_1$, so you will get this as this- $A - c - \left(\frac{A-c}{4}\right) = 2q_1 => \frac{3(A-c)}{8} - q_1$.

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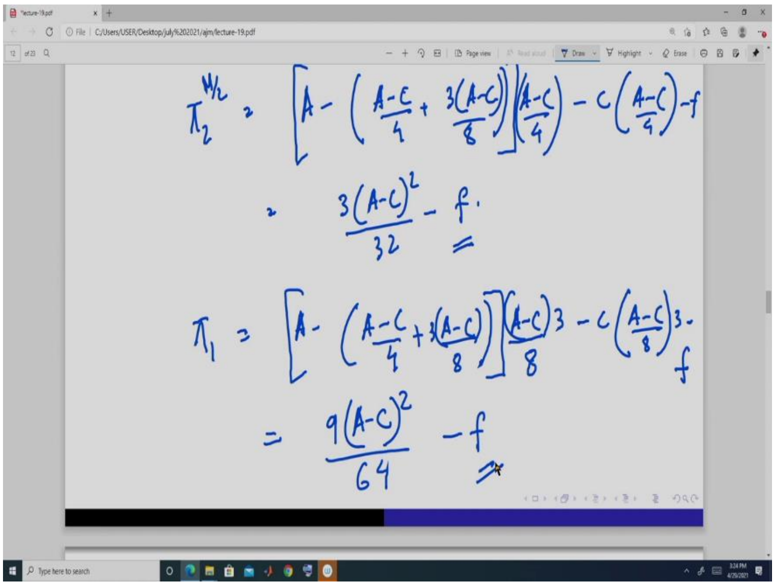

Now let us find out the profits. Profit of firm 2 when it produces monopoly output. Half of monopoly output is so market price is determined by the aggregate output so it is this. You get this- $\frac{3(A-c)^2}{32} - f$ and the profit of firm 1 when it produces based on the Cournot reaction function is, and this will give profit to be this- $\frac{9(A-c)^2}{64} - f$, right?

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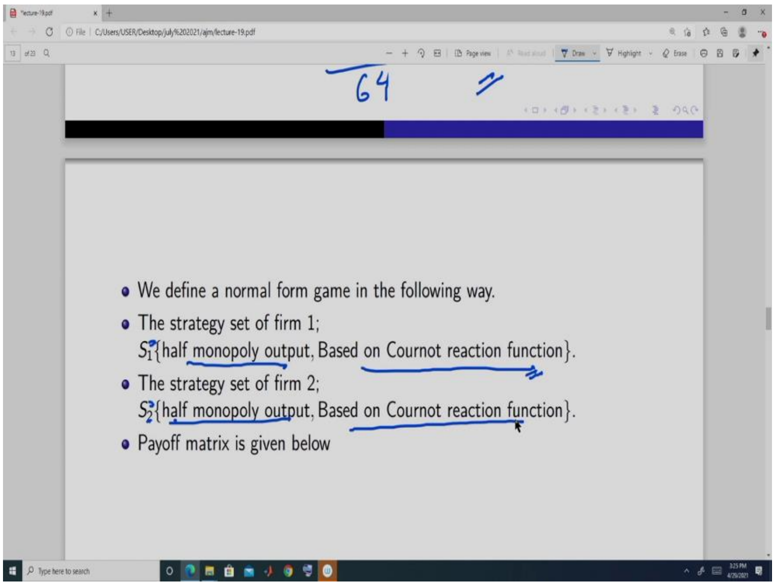

Now let us define a game. Game is a normal form game where it is played only once, and, okay. So this should be S21, S2 played once and it is played simultaneously between firm 1 and firm 2. Strategy of firm 1 it is either choose half monopoly output or it can choose based on Cournot reaction function. If both the firms chooses Cournot reaction function, we get the Cournot outcome. If both the firm chooses half monopoly then we get the half monopoly outcome. And reaction strategy set of firm 2 is half monopoly or based on reaction Cournot reaction function, okay.

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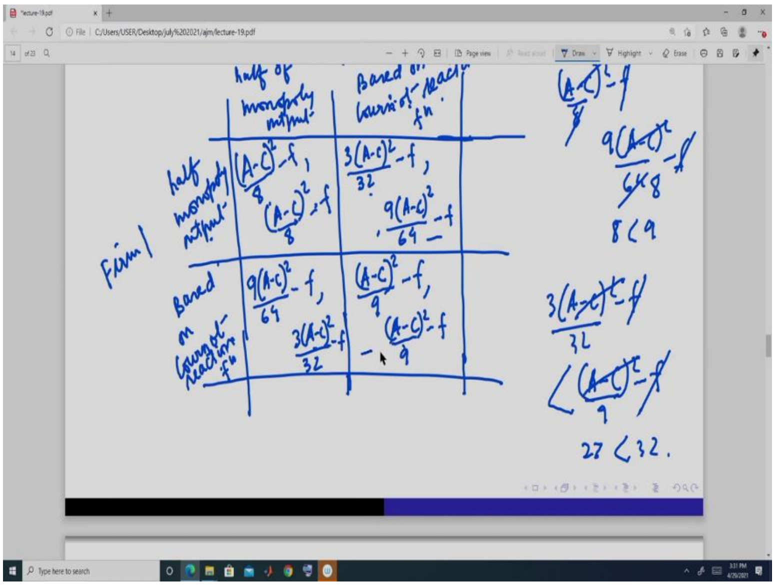

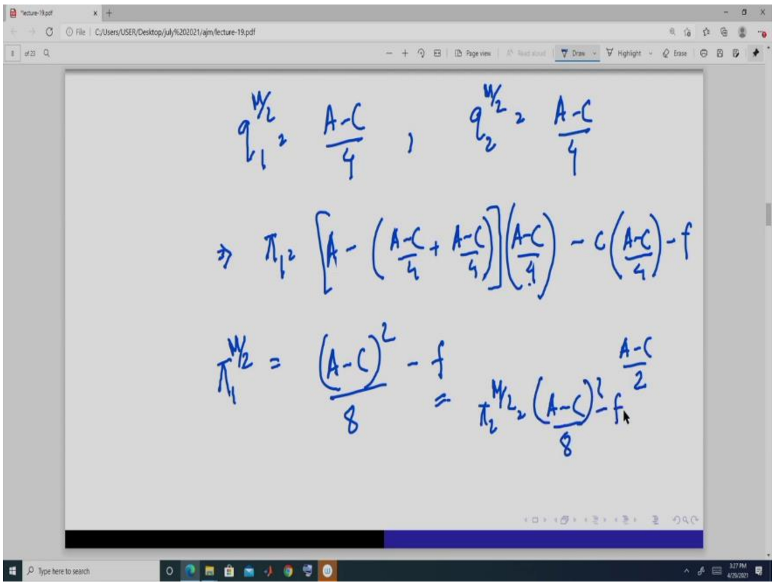

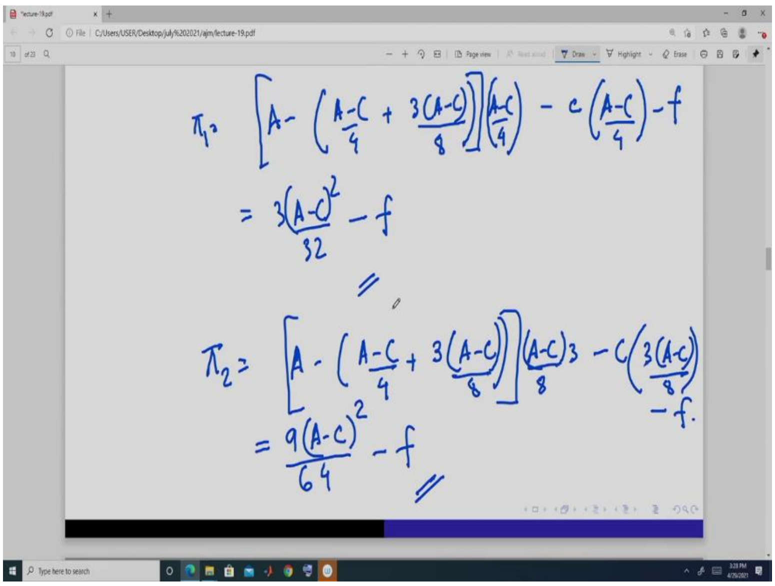

Now how do we define the payoff matrix, payoff matrix is given in this way. So let us make this block a slightly bigger one so this is for firm 2. Firm 2 is a column player; firm 1 is a row player and this is half monopoly output and this is based on Cournot reaction function. Similarly, this is half of monopoly output, so these are the strategies, okay. And this is based on Cournot reaction function. If no half monopoly half monopoly this is both this payoff in this block is both the firms are playing half monopoly output.

So if both are playing half monopoly we know the profits are this, this-$\frac{(A-c)^2}{8} - f$ for both the firms right. So since it is profit of A is this so profit of firm 2 is also going to be same because the cost function is same, so payoff are this. So in this normal firm game we write the payoffs. This is the payoff of firm 1-$\frac{(A-c)^2}{8} - f$, okay, this is the payoff of firm 2-$\frac{(A-c)^2}{8} - f$. Then firm 1 produces half monopoly output and firm 2 produces based on the Cournot reaction function. This case, payoff of firm 1 is this, and payoff of firm 2 is this.

And payoff of this similarly this is firm 2 produces, firm 1 produces based on Cournot reaction function and firm 2 produces half monopoly profit so this is this-$\frac{9(A-c)^2}{64} - f$ and this is the Cournot outcome and the Cournot outcome is we know firm 1, firm 2-$\frac{(A-c)^2}{9} - f$. Now we have to find the pure-strategy Nash equilibrium of this game.

Suppose this firm 1 plays half monopoly output, then firm 2 will decide this and this. So from here we know if we compare this, this cancels 8, this cancels, so 8 is less than 9 so we get that for firm 2 this is greater than this. So if firm 1 produces half monopoly output, firm 2 is going to produce based on the Cournot reaction function, not this-$\frac{(A-c)^2}{8} - f$, okay**.**

Now suppose firm 2 produces based on Cournot reaction function. So whether firm one is going to choose this, half monopoly output or based on Cournot reaction function. So we have to compare this payoff- $\frac{3(A-c)^2}{32} - f$ with this payoff-$\frac{(A-c)^2}{9} - f$, right? Now here it is easy to see that this is greater than this because this cancels out this, cancels out and 27 is less than 32.

So when firm 2 plays best, chooses output based on Cournot reaction function, this. Firm 1 also chooses output based on Cournot reaction function. Now we have to choose whether when firm 2, firm 1 chooses based on Cournot reaction function, whether firm 2 chooses half monopoly or this. Now if we compare this and this, we know this is so it will choose this.

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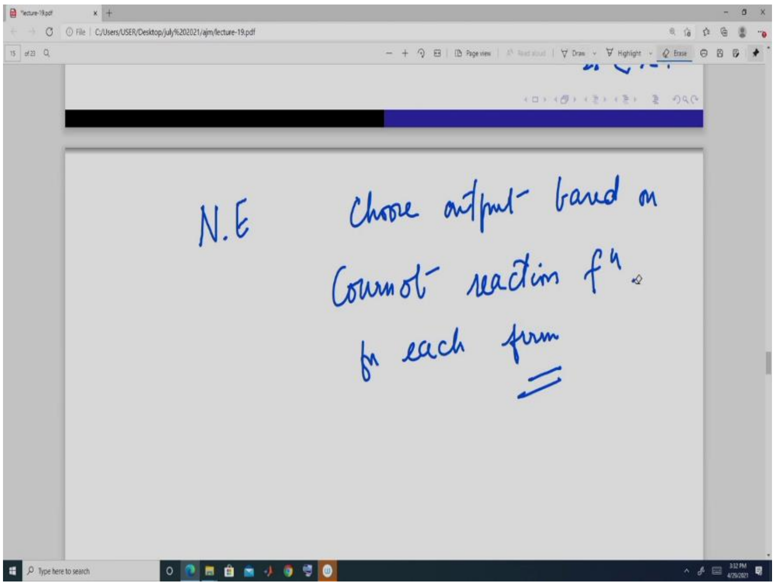

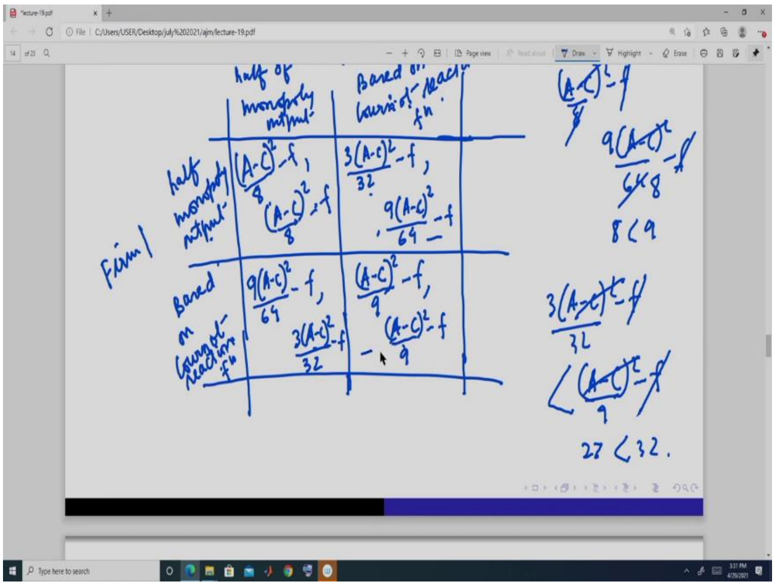

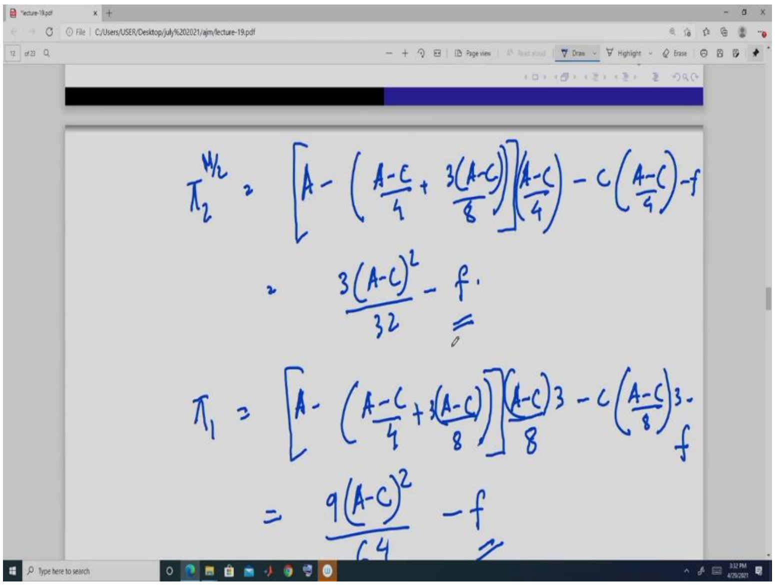

So the outcome is, so the Nash equilibrium, Nash equilibrium is to choose output based on Cournot reaction function for each firm. So this if you compare here this, this- $\frac{(A-c)^2}{8} - f$ is a better outcome for each firm than this outcome-$\frac{(A-c)^2}{9} - f$. But still firms choose this. Why? Because choosing output based on Cournot reaction here it is a Nash equilibrium and also it is a dominant strategy if you look at it, okay. So because of this reason we do not get the Cournot outcome. We do not get the sharing of monopoly market between the two firms. Instead they are going to play the Cournot game, okay.

So this is an important result. In a Cournot competition we will see that the firms if they collude or if they form any cartel then what they are going to do, the maximum profit is the monopoly profit in that market. So they are going to share the monopoly profit but that is not possible in this case if the game is played only once between 2 firms, okay.

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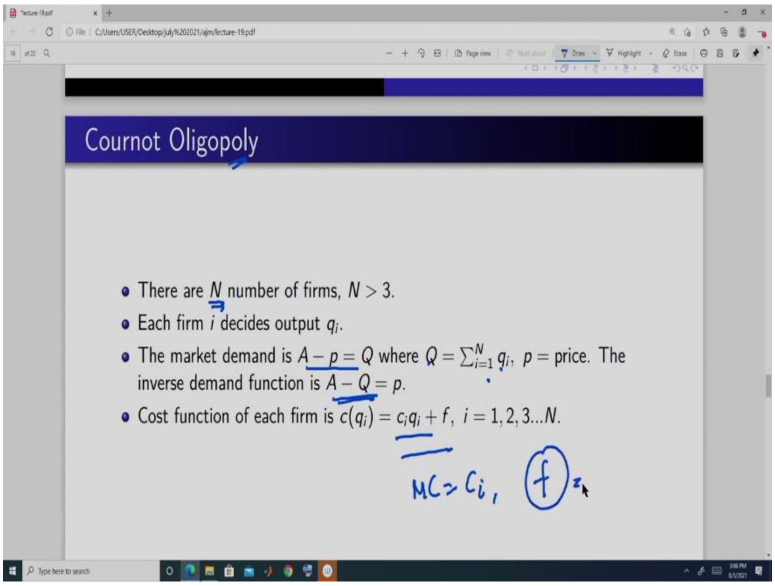

Now we will extend this model into n firms and that is why it is oligopoly. Oligopoly means that there are many firms. So there are n firms. So n is some positive integer, which is greater than 3, okay. Market demand we have kept same it is this-A-p=Q and so this capital Q is sum of all the outputs of each n firm each firm and there are n firms this p is the market price and the inverse demand function is this- A-Q=p, so cost function is of this firm is of this form- $c(q_i) = c_i q_i + f, i = 1,2,3..N$.

So it means that the marginal cost is Ci, i denote for each firm and it is constant for each firm and we have a fixed cost and fixed cost is given by this f. So we can see this kind of cost function when one of the factor is fixed and other factor is variable.

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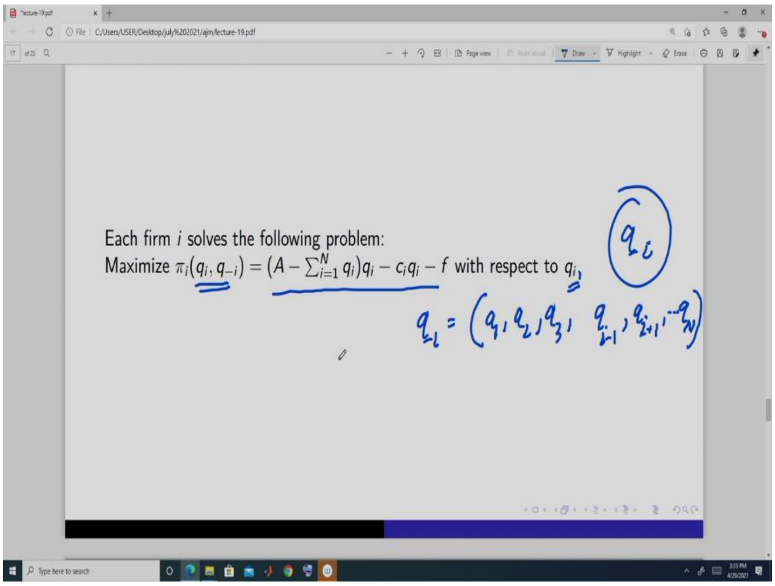

So now what each firm is going to do. The profit here when we write the profit function as a function of firm i as output of firm i and this. This q minus i this means actually this-$q_{-i} = (q_1, q_2, q_3, q_{i-1}, q_{i+1} \dots)$, i plus 1 this whole vector is denoted by this q. So this is a short firm notation that is used in game theory, okay. So the profit of firm A is this- $\pi_i(q_i, q_{-i}) = \left(A - \Sigma_{i=1}^{N} q_i\right)q_i - c_i q_i - f$ and firm i is going to maximize this profit with respect to its output taking output of all the firms as given, okay, taking q this as given. So since it is a differentiable profit function, so we can solve it in this way.

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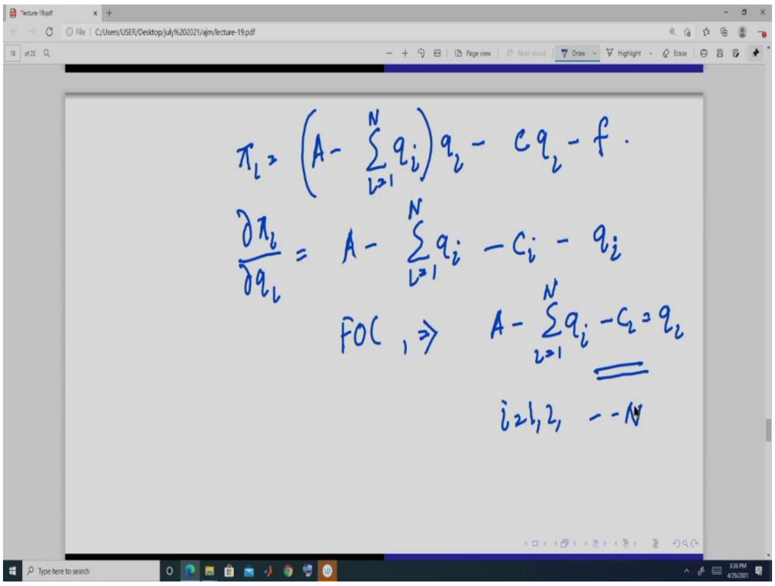

Then here if we differentiate this with respect to qI, we will this is so we will get qi so this is qi then the first order condition this will imply- $A - \Sigma_{i=1}^{N} q_i - c_2 = q_2$. Now we will have such for each i, i is equal to 1 to 2 and so we will get this.

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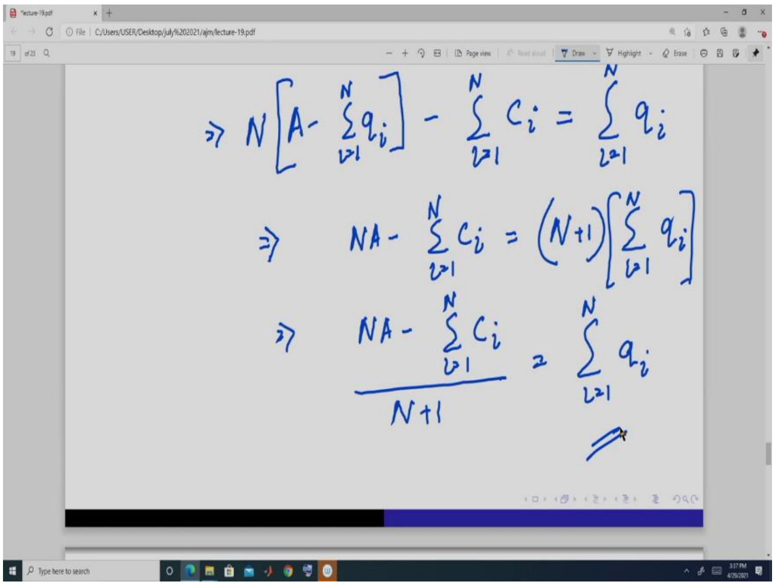

So if we sum this, so the summation, we can write so since there are n equations so this part is going to be same for each equation. So this marginal cost is going to be different for each firm and we will take the sum of this and here RHS is also going to be the sum of all the outputs like this. So from this we get-$N\left[A - \Sigma_{i=1}^{N} q_i\right] - \Sigma_{i=1}^{N} c_i = \Sigma_{i=1}^{N} q_i \Rightarrow NA - \Sigma_{i=1}^{N} c_i = (N + 1)[\Sigma_{i=1}^{N} q_i]$, so the aggregate output in the market is going to be, aggregate output is this in the market- $\frac{NA - \Sigma_{i=1}^{N} c_i}{N+1} = \Sigma_{i=1}^{N} q_i$.

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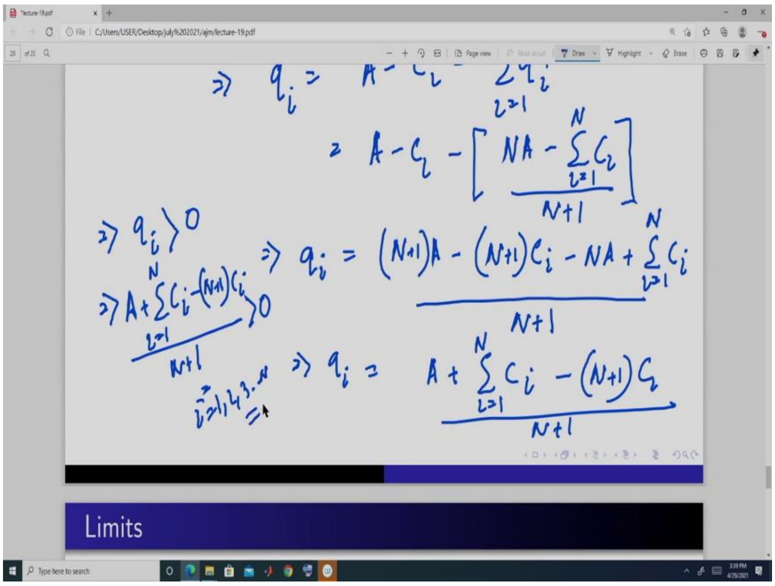

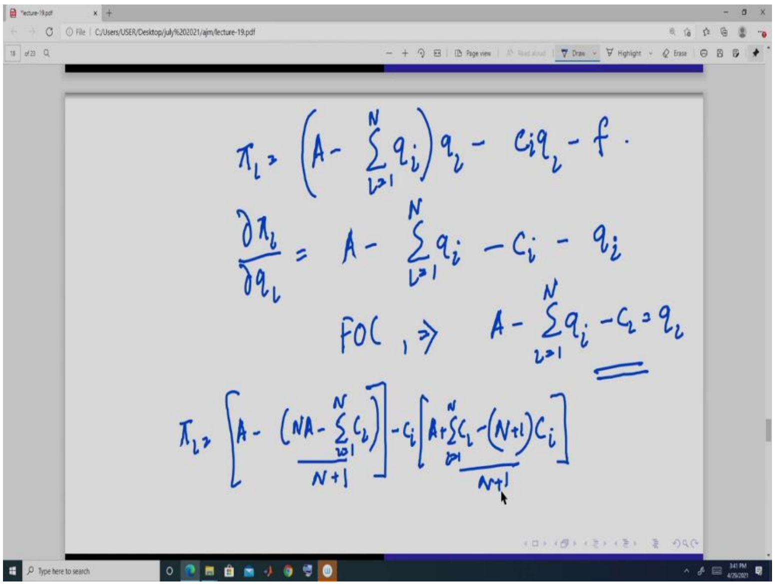

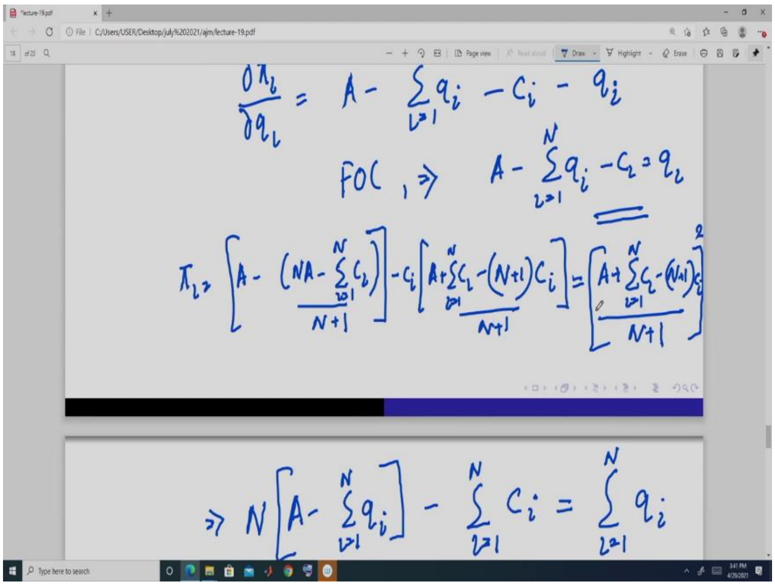

Now so the output of each firm qi you can simply get it, it is this- $q_i = A - c_i - \Sigma_{i=1}^{N} q_i$ from the reaction function of each firm and we get. It is this- $A - c_i - \left[\frac{NA - \Sigma_{i=1}^{N} c_i}{N+1}\right]$ so this is going to be, it is going to be this one-$q_i = \frac{(N+1)A - (N+1)c_i - NA + \Sigma_{i=1}^{N} c_i}{N+1}$. So q1 is actually A plus sum of all the marginal cost of all the firms and plus 1 into Ci marginal cost of firm i divided by, so this is the output of each firm- $q_i = \frac{A + \Sigma_{i=1}^{N} c_i - (N+1)c_i}{N+1}$, right? We get this as the output. For this to be a solution, we put the condition that this should be greater than A, so this means that should be greater than 0, okay. So for i equal to 1 for each firm i.

Now we plug in this in this profit function. This profit, this profit function- $\pi_1 = \left(A - \Sigma_{i=1}^{N} q_i\right) q_2 - c q_2 - f$ and then we will give the Cournot profit and if we do it here you will see this is and if you do the simple calculation, this is going to be. it is going to be this one $\left[\frac{A + \Sigma_{i=1}^{N} c_i - (N+1)c_i}{N+1}\right]^2$, okay.

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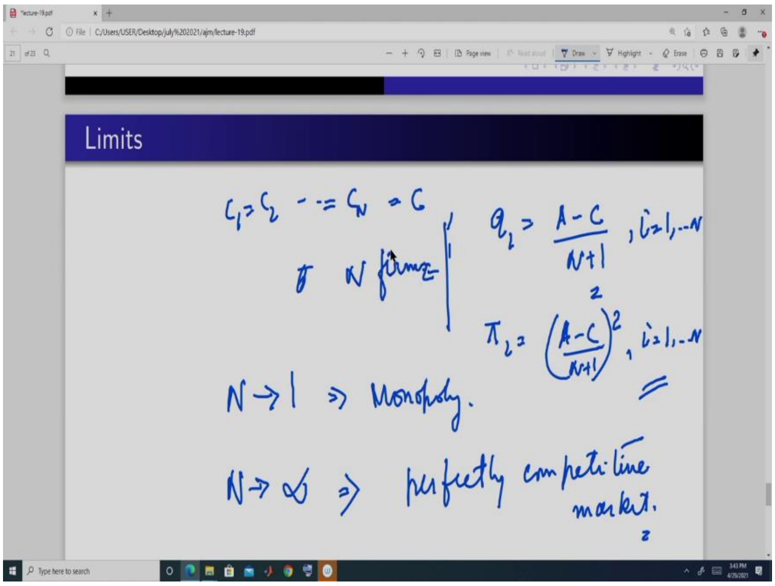

So and now here if you take the C to be common, same for each firm marginal cost is same that is C n is equal to this then qi is this- $\frac{\mathrm{A-c}}{\mathrm{N+1}}$, okay. And profit of so i equal to 1 to N, profit is this- $\pi_2 = \left(\frac{\mathrm{A-c}}{\mathrm{N+1}}\right)^2$, right? Now here we look at the limit now. What do we mean by limit? So when this n tends to 1 then it is implies monopoly and when n tends to infinity then it implies that we have perfectly competitive market. You can see this. So this is the, so we have assumed that the cost is same and our output of each firm is this when there are n firms. This is when we have n firms, right?

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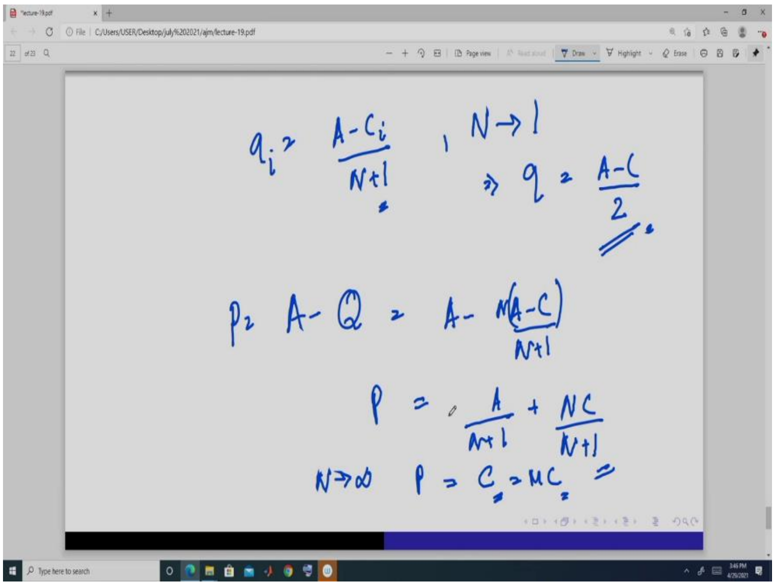

Equal sharing of monopoly output

- Suppose firm 1 and firm 2 decides to equally share the monopoly output. The profit of firm is half of monopoly profit.
- Monopoly output is: Maximize $(A-q)q - cq - f$ with respect respect to q.

$$\pi = (A-q)q - cq - f$$

$$\frac{\partial \pi}{\partial q} = A - 2q - c, \quad FOC \Rightarrow \frac{A-c}{2} = q^M$$

$$\Rightarrow q_i = A - c_i - \sum_{i=1} q_i$$

$$= A - c_i - \left[\frac{NA - \sum_{i=1}^{N} c_i}{N+1}\right]$$

$$\Rightarrow q_i = \frac{(N+1)A - (N+1)c_i - NA + \sum_{i=1}^{N} c_i}{N+1}$$

$$\Rightarrow q_i = \frac{A + \sum_{i=1}^{N} c_i - (N+1)c_i}{N+1}$$

$$\Rightarrow q_i > 0$$

$$\Rightarrow \frac{A + \sum_{i=1}^{N} c_i - (N+1)c_i}{N+1} > 0, \quad i = 1, 2, 3 \ldots N$$

Limits

Limits

$c_1 = c_2 = \cdots = c_N = c$, # N firms

$$q_i = \frac{A-c}{N+1}, \quad i = 1, \ldots, N$$

$$\pi_i = \left(\frac{A-c}{N+1}\right)^2, \quad i = 1, \ldots, N$$

$N \to 1 \Rightarrow$ Monopoly.

$N \to \infty \Rightarrow$ perfectly competitive market.

So it is what? this N tends to 1, so this q is and this we know we have found the monopoly. We have already calculated the monopoly profit and the monopoly profit is given like this and it is this monopoly output is this. So here again we have got the monopoly profit and that is this- $q = \frac{A-c}{2}$.

So we know the Cournot, from Cournot we can derive the monopoly output so in a Cournot market there are n firms, Cournot oligopoly market and there we have found the Cournot outcome. Cournot outcome is this. Each output of each firm is this. Profit we have calculated it and then when we take the limit on n and the limit is that suppose from n firm we reduce it to one so they are only one firm then the outcome is same as the monopoly output and the profit is also going to be the same of monopoly profit.

Next what is the price in this Cournot thing, this, it is going to be this one- $P = A - \frac{N(A-c)}{N+1}$ because output of each firm is this- $\frac{A-c}{N+1}$ and there are n firms so it is n into this. Cost is same. So it is going to be this. So this is what, this is, this price is this in the Cournot oligopoly when the cost is same we get this as the market price- $P = \frac{A}{N+1} + \frac{NC}{N+1}$.

Now here when you take N tends to infinity this price you see is going to be equal to this which is equal to marginal cost and we know the when moment of price is equal to marginal cost then the pricing is same as the perfectly competitive pricing.

So in this case when the firms decide its output but there are infinite number of firms then it is same as perfectly competitive. So you can say the Cournot thing is in between monopoly and a perfectly competitive and the main strength of this result is this that at the limit we get monopoly that is when there is one firm and when there are many firms, infinitely many then the price is same as the marginal cost. So it is outcome is same as competitive market.

So when each firm is deciding its output taking the output of other firm as given we know in that case it is the Cournot market. In that case when the number of firms are very large then this same outcome tends to competitive pricing or the outcome is same as the competitive market. So this is the importance of Cournot market. So Cournot market is you can say from there it is lying in between monopoly and the competitive market.

And we know that in reality competitive market hardly we see. In the competitive market mainly the firms are price taker so and there is a homogenous output and there is free entry and exit of firms. So these characteristics are perfectly competitive market, it is very difficult to see in the real life.

Monopoly and 100 percent monopoly is also very difficult to see but what do we see in between these two and that is Cournot kind of thing where each firm if they are producing homogenous output they are choosing their own output given taking the output of other firm as given, okay.

So this is the end of Cournot competition, Cournot model, so we have done Cournot duopoly where we have taken 2 firms and then we have form the pure-strategy Nash equilibrium. Then we have shown that there is a possibility of a better outcome where they can share the equally share the monopoly market but monopoly output but instead of that firm chooses to play the Cournot kind of competition, okay. That we have shown.

We have shown in the last class also and today also we have shown it in detail and then we have derived the Cournot oligopoly result taking CRS production function so that the cost function has a variable component and a fixed component.

And we have assumed a linear demand curve and based on that we have got the market outcome and market outcome is given by this and this is the pure-strategy Nash equilibrium of this game. Here we have to assume this condition so that the output is positive for each firm, okay. And from this Cournot oligopoly outcome we have derived the monopoly thing taking the limit when n tends to 1 and the perfectly competitive outcome when n tends to infinity, okay. So this is what we require in a oligopoly, it is not oligopoly in a Cournot oligopoly, okay.

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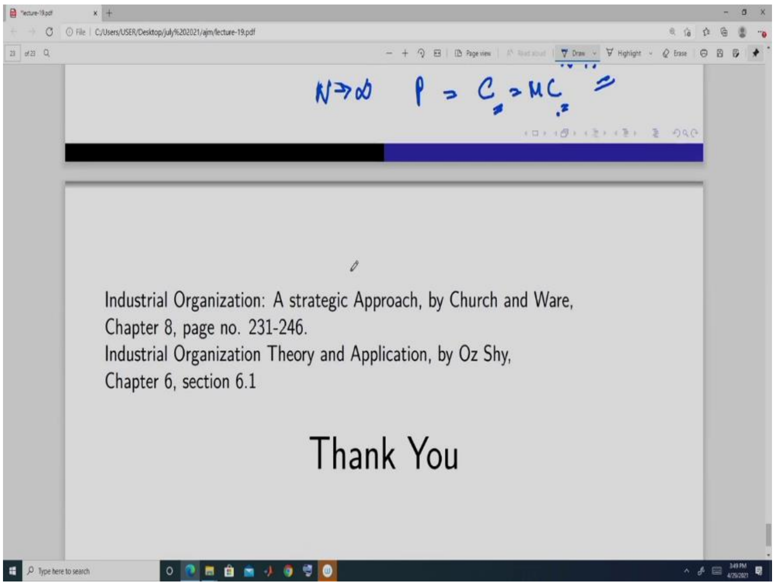

So you can read this portion from chapter 8 of Church and Ware Industrial Organization: A Strategic Approach. These are the page numbers. Also you can read from another book Industrial Organization Theory and Application by Oz Shy and this is chapter 6, section 6.1, okay. Thank you very much.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology Guwahati
Module: 7
Lecture 27: Tutorial

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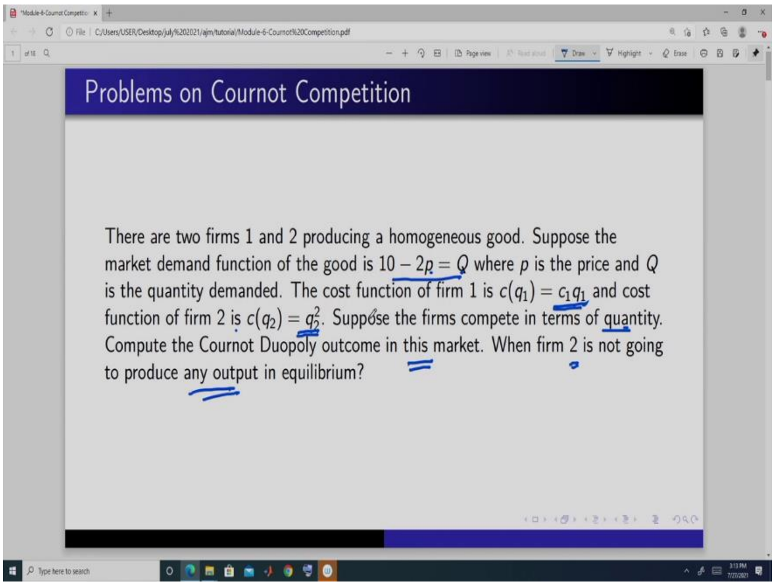

So let us discuss some problems on Cournot competition. So in this example suppose there are 2 firms producing a homogenous product and the market demand function is this- 10–2p=Q where p is the price and Q is the quantity demanded. Cost function of firm 1 is this- $c(q_1) = c_1q_1$, cost function of firm 2 is this- $c(q_2) = q_2^2$.

So it is CRS and it is decreasing returns to scale and suppose the firms compete in terms of quantity so it is a Cournot competition. Compute the Cournot duopoly outcome in this market and when firm 2 is not going to produce any output in equilibrium. So we have to find out this also, right?

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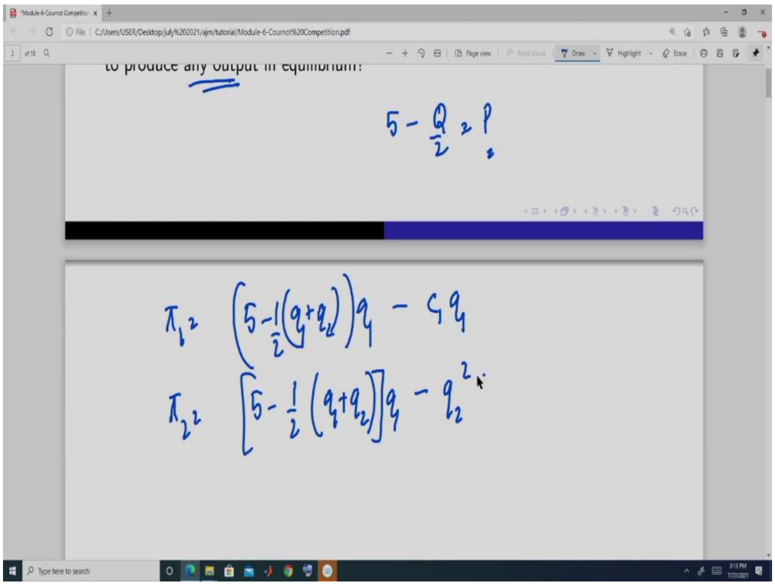

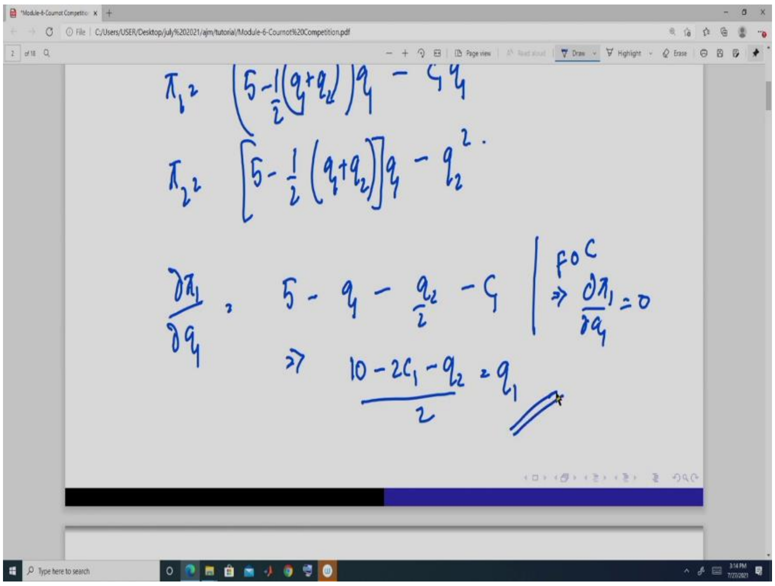

Now so let us, so from this, we get the inverse demand function is, it is this- $5 - \frac{Q}{2} = P$. So the profit function of firm 1 is this- $\pi_1 = \left(5 - \frac{1}{2}\,(q_1 + q_2)\right).q_1 - c_1q_1$ profit function of firm 2 is-$\pi_2 = \left(5 - \frac{1}{2}\,(q_1 + q_2)\right).q_1 - q_2^2$ where q1 and q2 are output of firm 1 and firm 2. It is q square I think. It is this, they are differentiable, so we optimize this with respect to their outputs. So this we get- $\frac{d\pi_1}{dq_1} = 5 - q_1 - \frac{q_2}{2} - c_1$, First Order Condition implies. So this, is the reaction function of firm 1- $\frac{10-2c_1-q_2}{2} = q_1$, okay.

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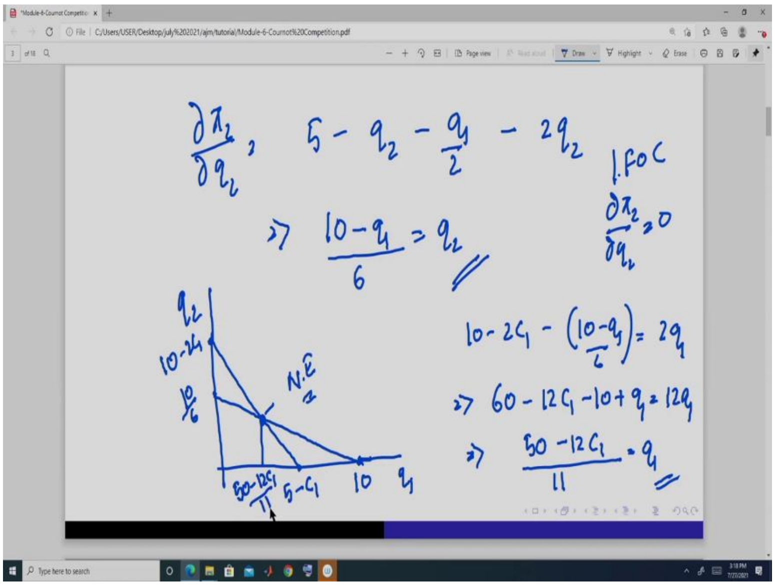

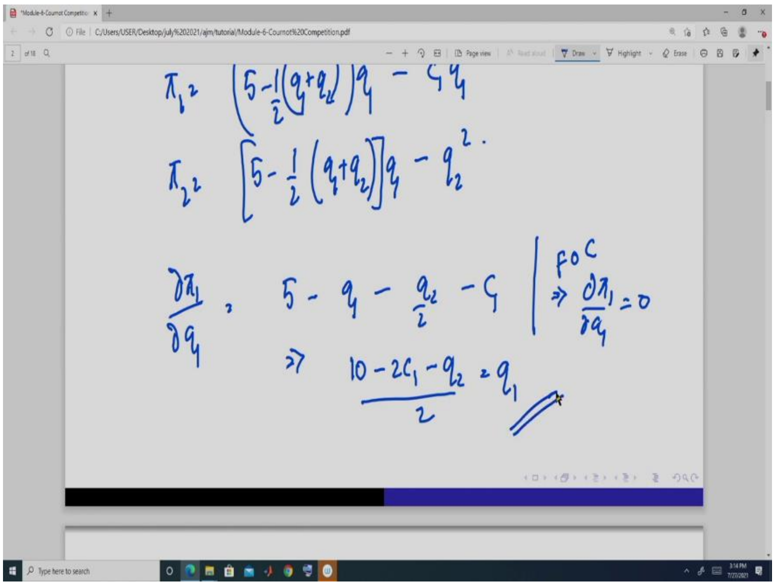

And so here again, you get this first order condition implies is equal to this. So we get this is the reaction function of firm 2- $\frac{10-q_1}{6} = q_2$. So if we plot these reaction functions, okay. If we are plotting these 2 reaction functions, this, so this is and this is equal to 0. This is 5 and when this is equal to 0 then because when q1 is equal to 0, it is going to be this point is 5 minus c1 and when q2 is going to be 0 then it is, this is, so we take common because this is equal to 0. So this is 5, see this 10 minus 2c1 equal to q2 so this point is 10 minus 2c1 and this point is 5 minus c1.

It is this and this reaction is q is equal to, this is 10 and this point is suppose 10 by 6. And these two intersect here. So this is the Nash equilibrium. And we get it by solving these two reaction functions. So it is 10 minus 2c1. If we do this, we get this, this is output of firm 1- $\frac{50-12c_1}{11} = q_1$ so this point is 50 minus 12c1 divided by 11.

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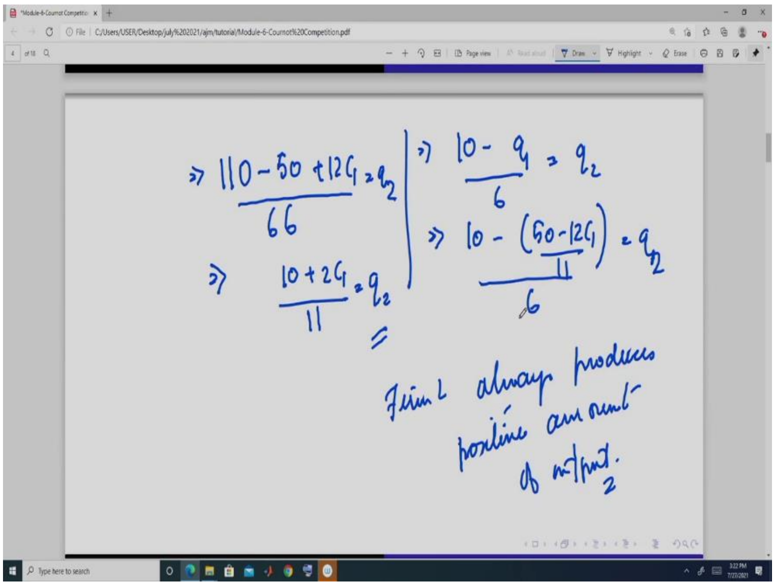

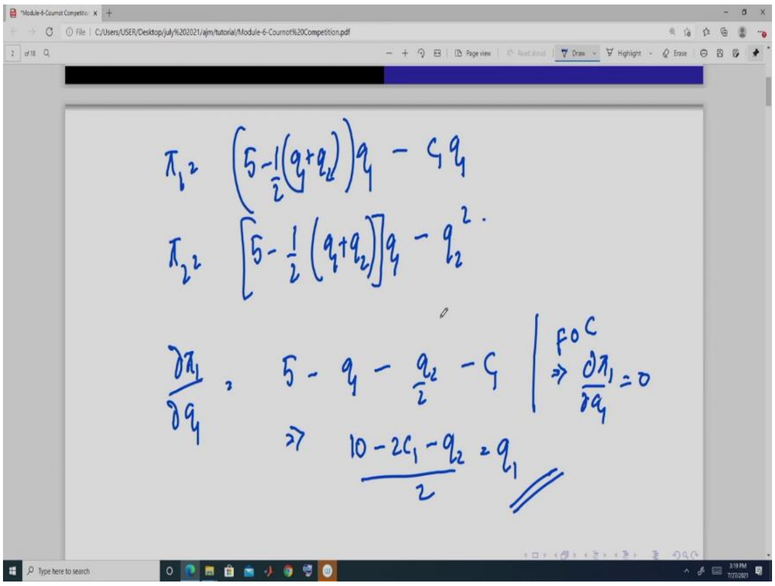

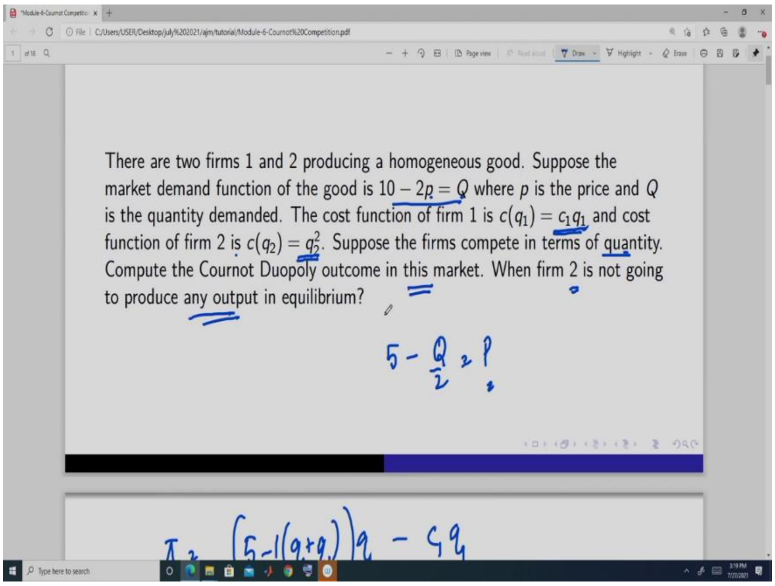

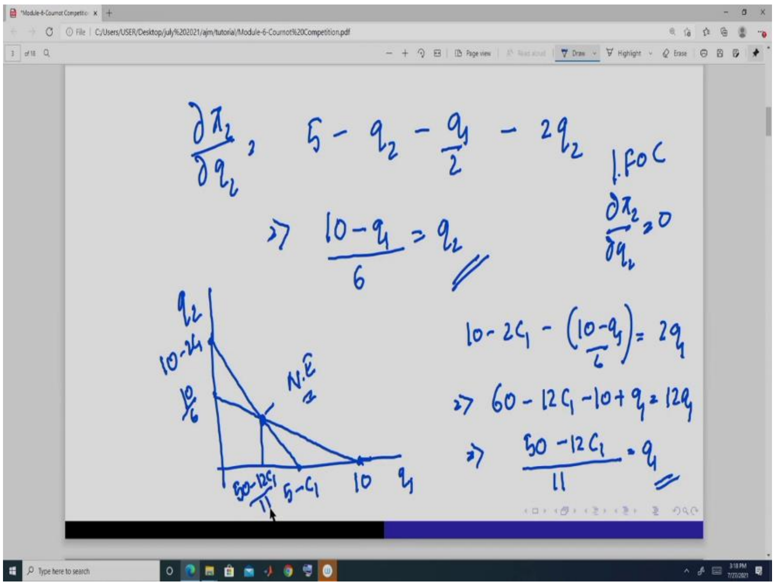

And this height is, it is 10 minus q1, this. So it is 10 minus 50 minus 12 c1 C2. So this is we can again write this, our 10 plus 2c1 divided by 11, this- $\frac{10+2c_1}{11} = q_1$. So these- $\frac{10+2c_1}{11} = q_1, \frac{10-q_1}{6} = q_2$ are the two pure strategy Nash equilibrium or Cournot Nash equilibrium outcome of this game and we can plug in them in the profit function here and we will get the profit.

Now here again we have to find when firm 2 is not going to produce any output in equilibrium. So it is, so this is the reaction function of firm 1, right? Now it is not going to produce any output, when? So when this, this and it is this, suppose this is the reaction function of firm 1 which is given by this point, this point- $\frac{10-2c_1-q_2}{2} = q_1$ or okay, the question is this. When firm 2 is not going to produce any output in equilibrium, when firm 2 is not going to produce.

So see here it is this is the Nash equilibrium outcome. When do we get this? When this is less than this and this is less than this, right? Now this is the reaction function of this. So this has to be so much here that it has to go below it. Then only it is not going to produce anything but and it is not this diagram, okay.

So an output for this equal to 0, c has to take some negative here, which is not possible. So that is why firm 2 always produces positive amount of output, always. But firm 1 may produce 0 when c1 is sufficiently high that is when this is or this line is less like this.

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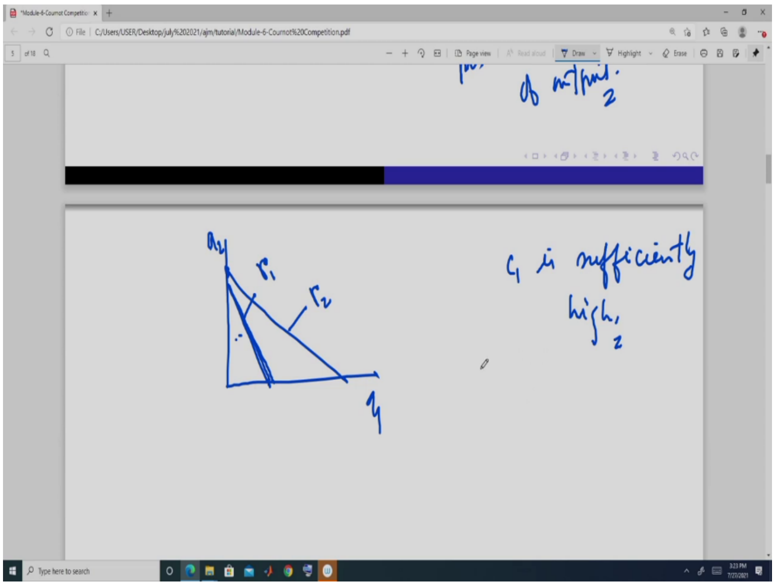

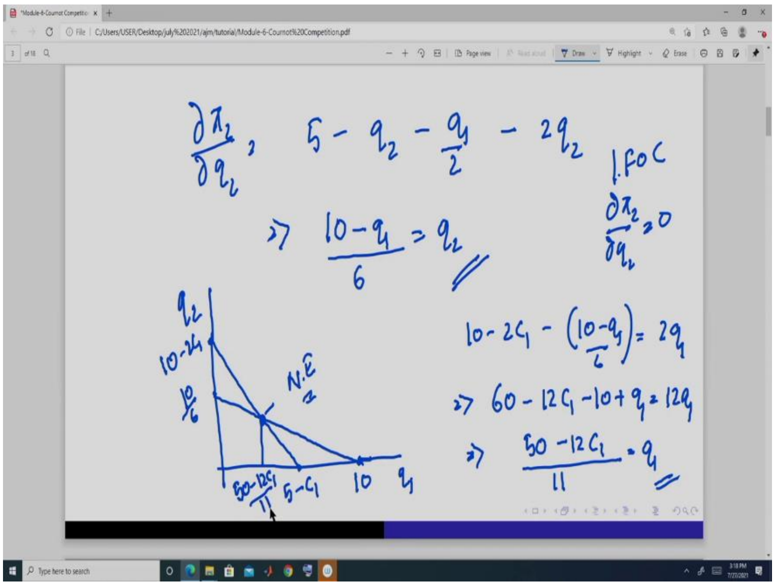

So if we have a reaction function of firm 1 and this is the reaction function of firm 2. So firm 1 will not produce anything, firm 2 is going to produce this much amount of output. But since we cannot reduce this below put this curve below this or we cannot have this curve greater than this, why?

Because if you look at this point, this is 5 minus c1. So it cannot be greater than 10. So that is why we cannot have this curve line above this curve, okay. Reaction function of firm 1 lying above the reaction function of firm 2. So that is why firm 2 is always going to produce some positive amount but firm 1 may not in this situation when c1 is sufficiently high, okay. So we get this.

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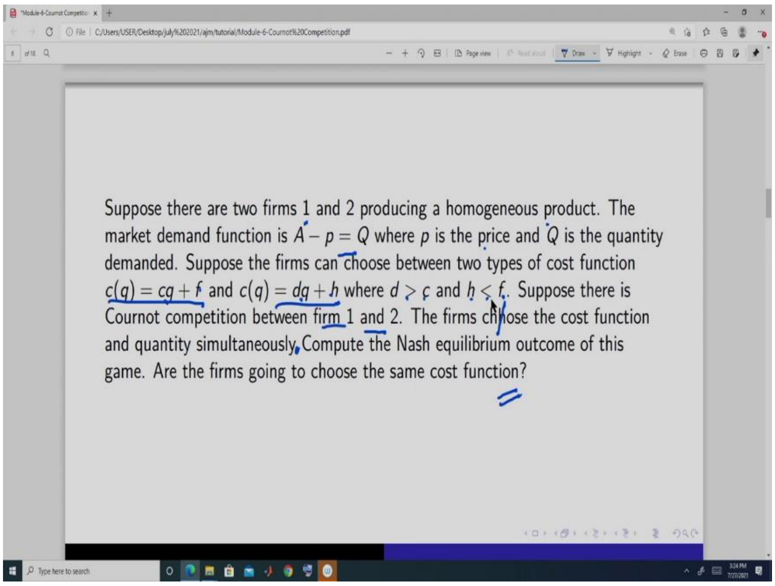

Now next, solve another problem. This is slightly involved problem. Suppose there are two firms; firm 1 and firm 2 and they produce a homogenous product and market demand function is this- A–p=Q. A is a positive number and Q is the quantity demanded, p is the market price. Suppose the firm can choose between 2 types of cost functions. Its cost function can be of this nature- $c(q) = cq + f$ and cost function can be of this nature- $c(q) = dq + h$. Both has this option where d is greater than c and h is less than f.

So here in this cost function marginal cost here is less than the marginal cost this. But here fixed cost is greater in this case than this. And suppose there is Cournot competition between firm 1 and firm 2. And the firms choose the cost function and quantity simultaneously, okay. So these two decisions are taken simultaneously and compute the Nash equilibrium outcome of this game. Are the firms going to choose the same cost function? okay So these are the questions.

Now we have this specification. So choosing between 2 different types of cost functions, it means choosing different type of technique or different combinations of input bundle to produce the same amount of output or to choose a different technology or to choose in such way that the setup cost is also here different because f and d, okay.

So there are many possibilities which may give rise to this situation. It is mainly because of 2 different production functions or it may be two different types of price of inputs or it may be different plant sizes that we have discussed that is one land plot is big, another land plot is small and they have same – they can choose between machines and labor. Both of them are variable. All these are possibilities are there.

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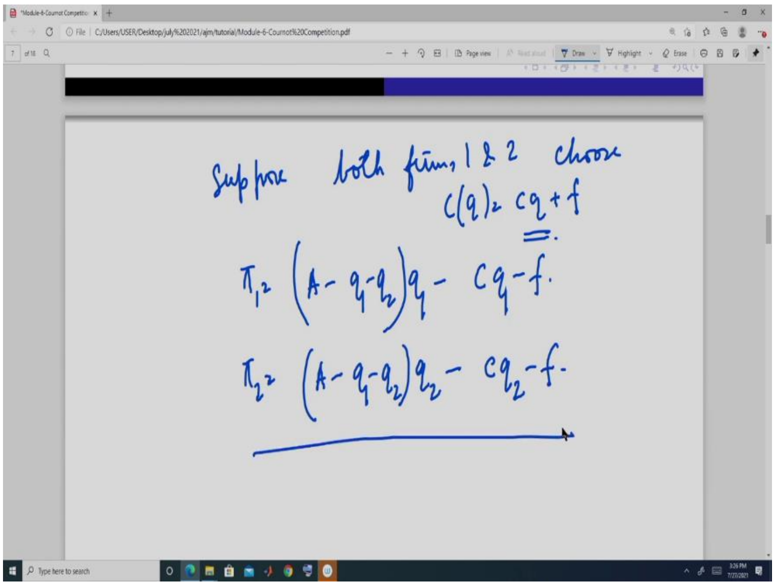

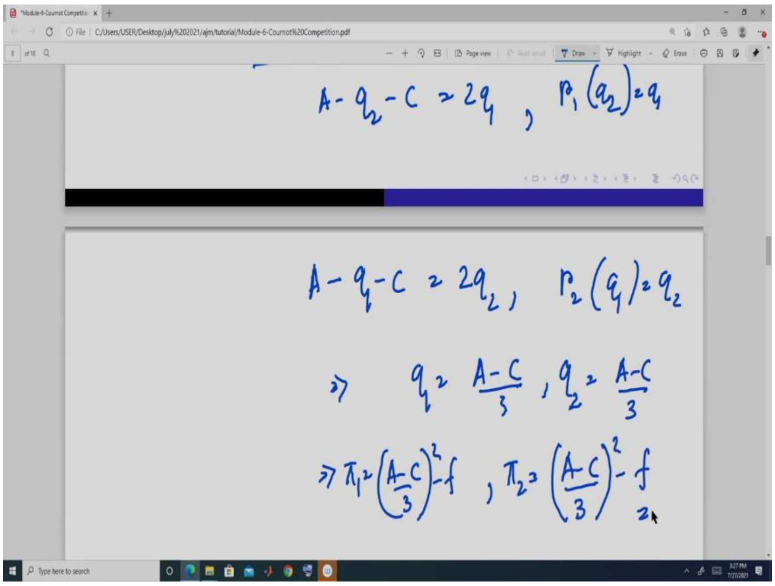

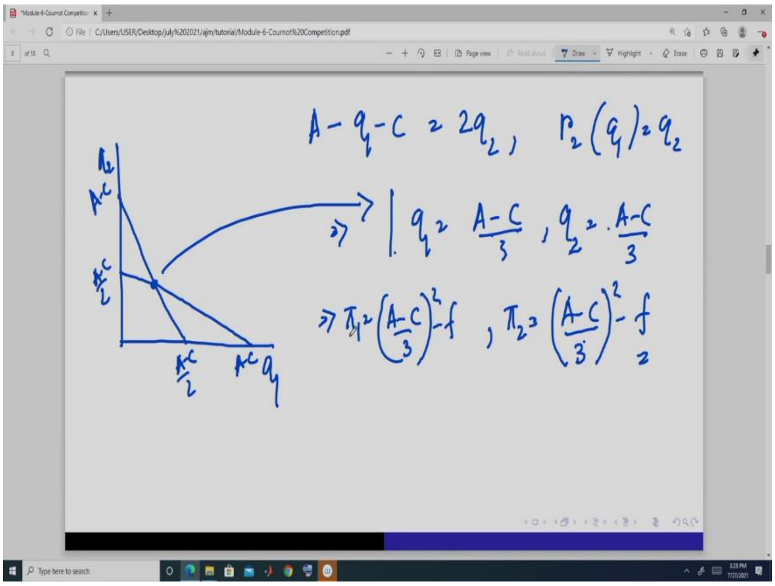

Now in this case, suppose, both firms 1 and 2 choose this production function- c(q)= cq+f. Then profit of firm 1 is this- $\pi_1 = (A - q_1 - q_2)q_1 - cq_1 - f$, profit of firm 2 is this-$\pi_2 = (A - q_1 - q_2)q_2 - cq_2 - f$. So, we do the usual process so the reaction function of firm 1 from this we get A minus this- $A - q_2 - c = 2q_1, p_1(q_2) = q_1$. This is the reaction function and the reaction function of firm 2 is- $A - q_1 - c = 2q_2, p_2(q_1) = q_2$ so if we solve this we get the Cournot outcome as we have already got this- $q_1 = \frac{A-c}{3}, q_2 = \frac{A-c}{3}$ and in this situation we know profit of firm 1 is, it is this-$\pi_1 = \left(\frac{A-c}{3}\right)^2 - f, \pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, right? Now if we look at the reaction functions, and this is the point, this point. This point is this, right?

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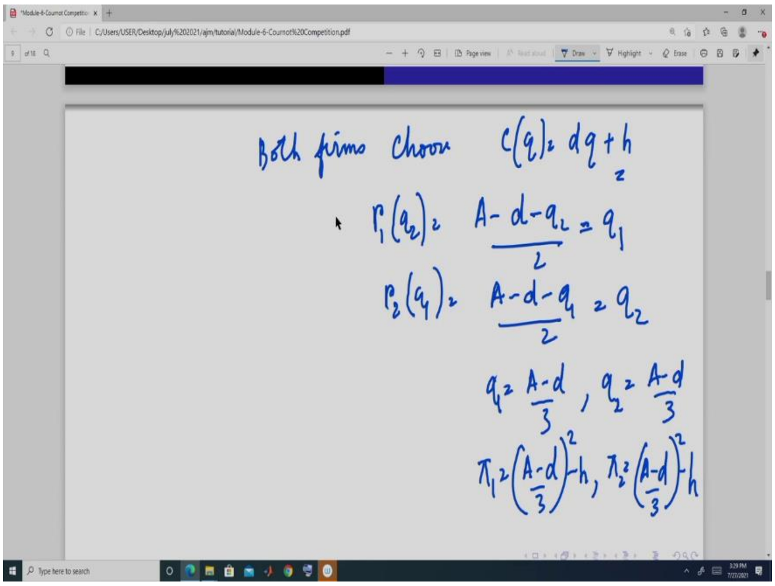

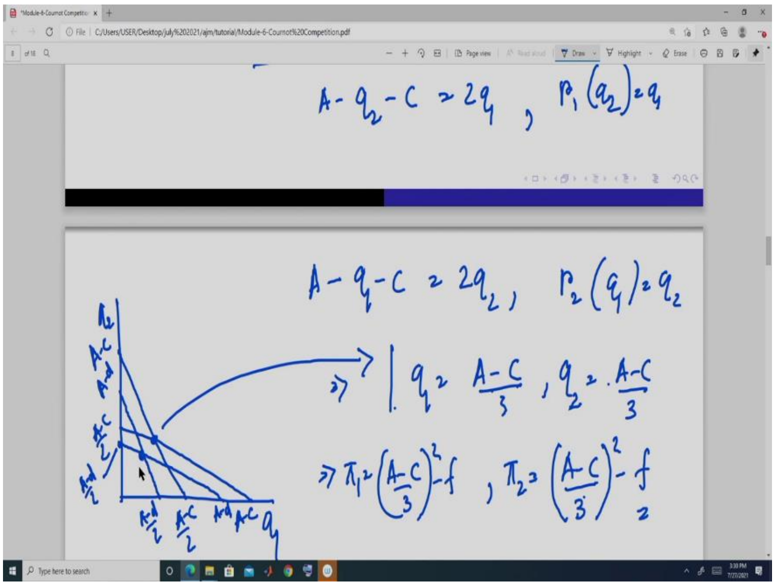

And now suppose both the firms, both firms choose this-c(q)= dq+h then again we know what is going to be the reaction function. It is this-$p_1(q_2) = \frac{A-d-q_2}{2} = q_1, p_2(q_1) = \frac{A-d-q_1}{2} = q_2$ and so the outcome is, so profit is- $\pi_1 = \left(\frac{A-d}{3}\right)^2 - h, \pi_2 = \left(\frac{A-d}{3}\right)^2 - h$. And in this reaction function we will get d is greater than c so we will get line like this A minus d this is A minus and this is this A minus d this point is A minus d by 2 so this is the outcome, right?

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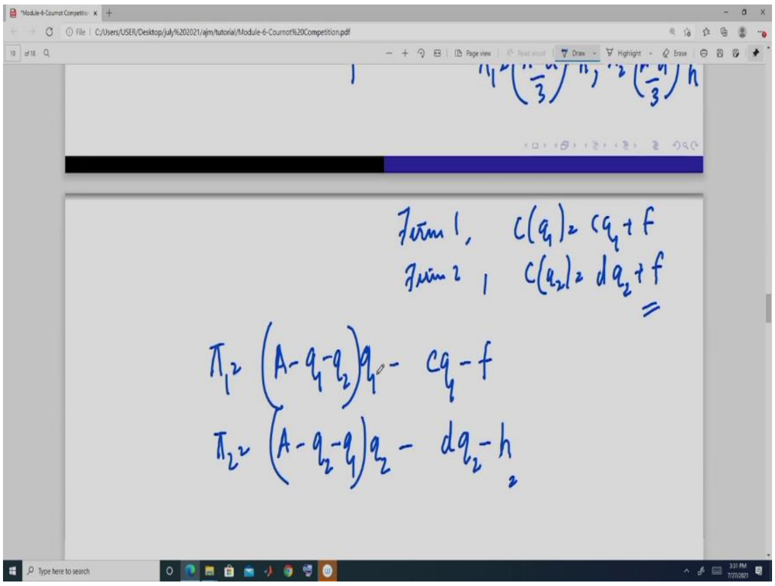

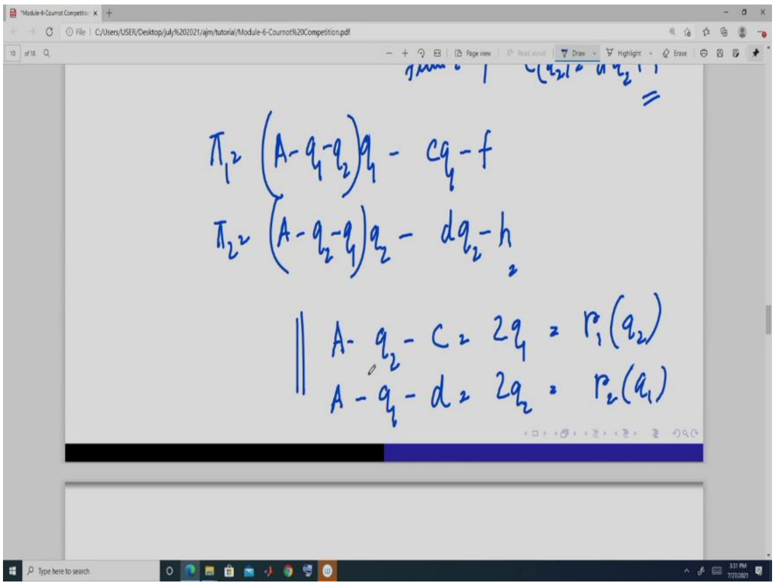

Now if firm 1 if any one of them chooses. Suppose firm 1 chooses this- $c(q_1) = cq_1 + F$ and firm 2 chooses this- $c(q_2) = dq_2 + F$, then what is going to happen? Profit of firm 1 is this- $\pi_1 = (A - q_1 - q_2)q_1 - cq_1 - f$. Profit of firm 2 is this-$\pi_2 = (A - q_2 - q_1)q_2 - dq_2 - h$. And the reaction functions of firm 1 is going to be n reaction function of firm 2.

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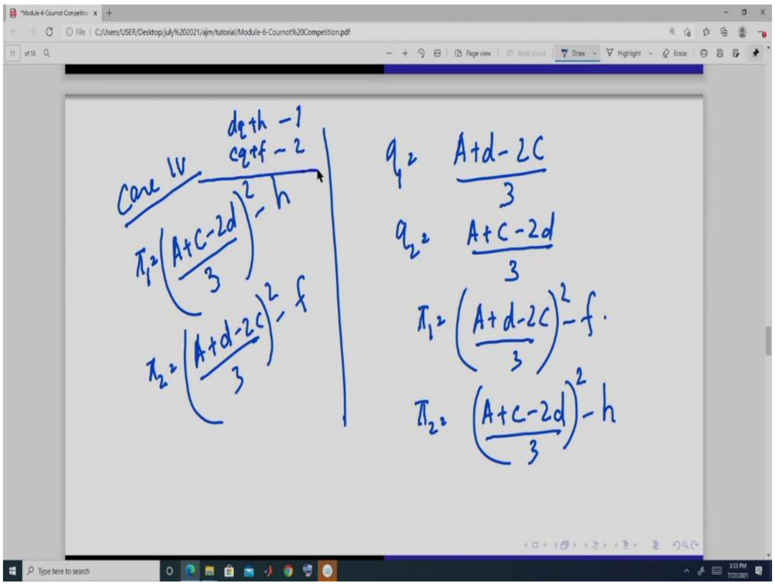

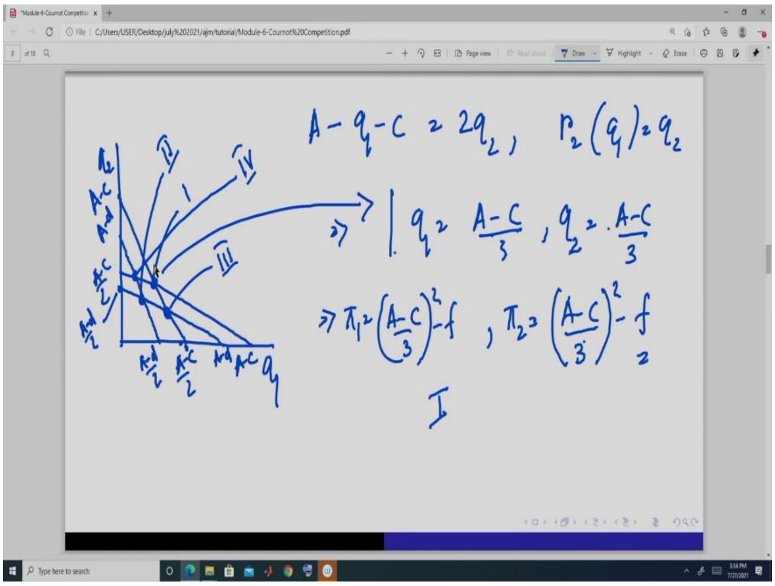

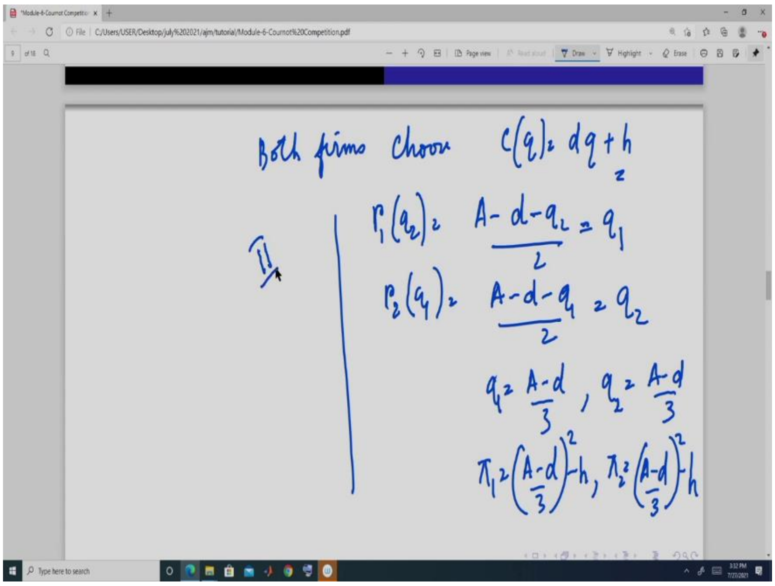

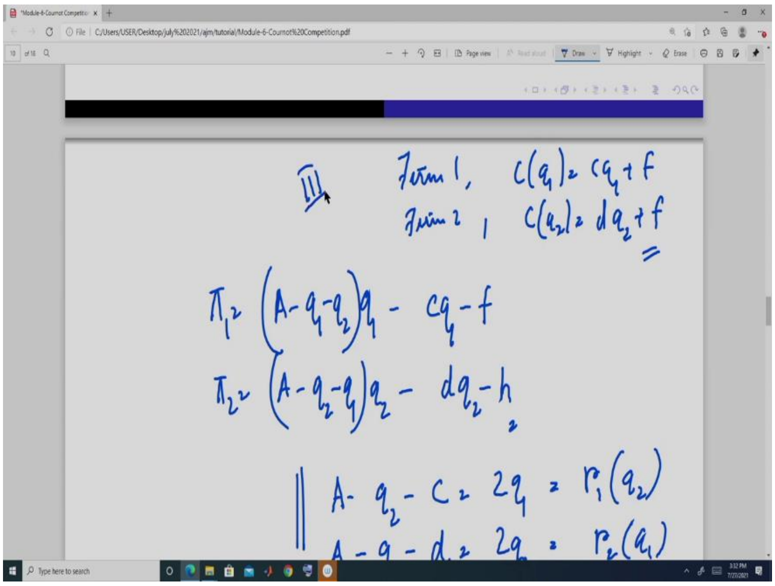

Solving these two, solving these two we get that q1 is A plus d, i.e. $q_1 = \frac{A+d-2C}{3}$ and q2 is A this and this-$q_2 = \frac{A+c-2d}{3}$. So and you can look at profit, profit is also. This is for firm 1- $\pi_1 = \left(\frac{A+d-2c}{3}\right)^2 - f$ and for firm 2-$\pi_2 = \left(\frac{A+c-2d}{3}\right)^2 - h$. It is this and if we look at this diagram, reaction function of firm 1 is this, reaction function of firm 2 is this. So this is the outcome. So this is when we are in case 1. This is case 2 so this is case 1. This is case 2. This is case 3. And case 4 is so we will just get the opposite. It is going to be h because in case 1 firm 1 is choosing the h. This is firm 1 and this is firm 2, okay. So we get this- $\pi_1 = \left(\frac{A+c-2d}{3}\right)^2 - h, \pi_2 = \left(\frac{A+d-2c}{3}\right)^2 - f$. Now we have to compare this profit and these decisions are being taken simultaneously. So it is something like. So this is case 3 and this is case 4.

So now when we can have this case or this case or this case or this case. So when we have this case that means firm 1 has chosen this. Its reaction function is this because its cost is, marginal cost is c and firm 2 has also chosen marginal cost such that it is c and they have got this here. So then firm 2 has 2 options either to choose c or it can choose D these two. If I fix the near firm 1 suppose this is the A, it has 2 choices, either to choose this or to choose this. Now how we get this? Which one is chosen?

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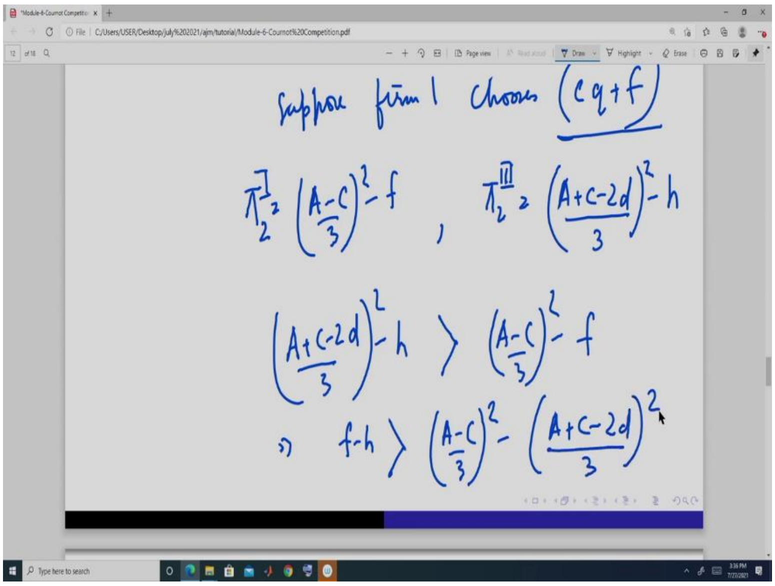

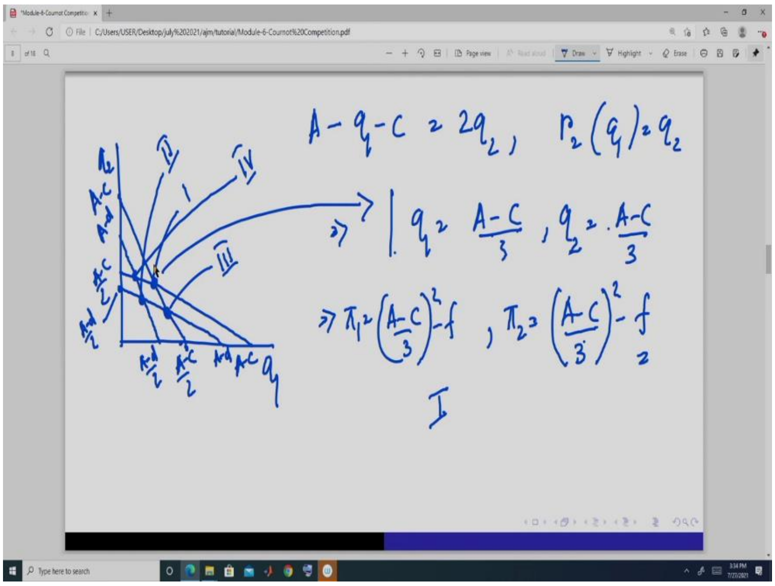

So suppose firm 1 chooses this- cq+f, okay. Then firm 2 profit it has to compare between these 2 cases. Case 3 and case 1, case 3 and case 1. So the profit of firm 1 in case 1 is, of firm 2 is- $\pi_2^1 = \left(\frac{A-c}{3}\right)^2 - f$ and profit of firm 2 in case 3 is it is this-$\pi_2^{iii} = \left(\frac{A+c-2d}{3}\right)^2 - h$, when it is going to choose this. It has to compare between these two when it is possible. When this$\left(\frac{A+c-2d}{3}\right)^2 - h$, is greater than this- $\left(\frac{A-c}{3}\right)^2 - f$ and this implies f minus h should be this.

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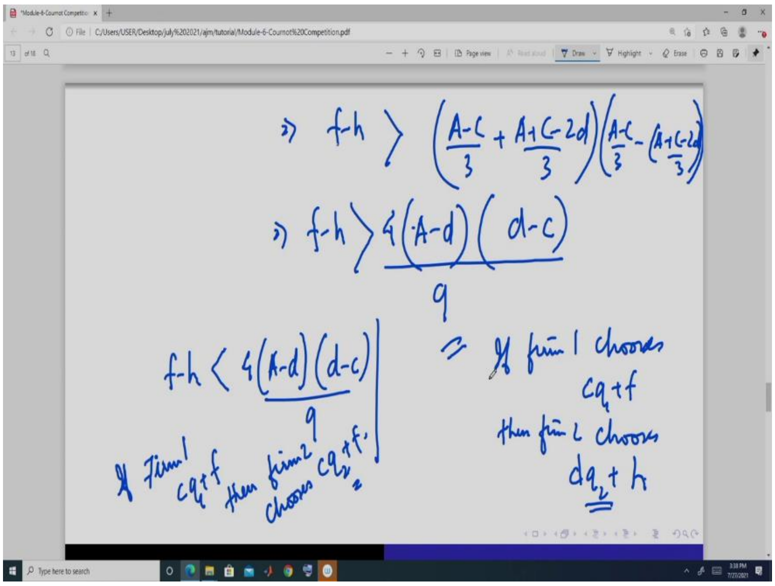

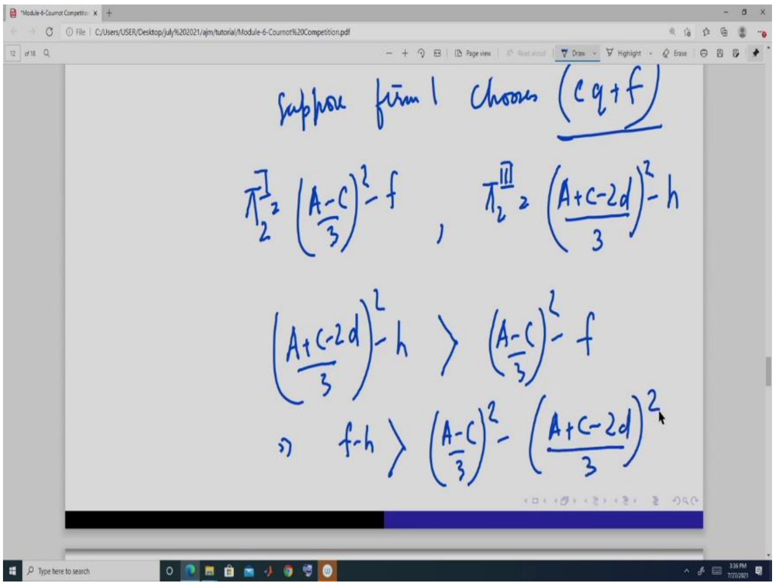

So this implies that f minus h, so this implies f minus h should be greater than this- $\left(\frac{A-c}{3}-\frac{A-c+2d}{3}\right).\left(\frac{A-c}{3}-\left(\frac{A+c-2d}{3}\right)\right)$. So this we know f is greater than h but that difference should be greater than this if this is the case then when firm 1 choses this firm 2 chooses. So if this is the case when then if firm 1 chooses cq this then firm 2 chooses this as the cost function, right? or if this is not satisfied if then firm 2 both will, if firm 1 chooses this then firm 2 chooses this only. If this, then firm, if firm 1 chooses cq plus f then firm 2 chooses cq2 plus f. It should be h.

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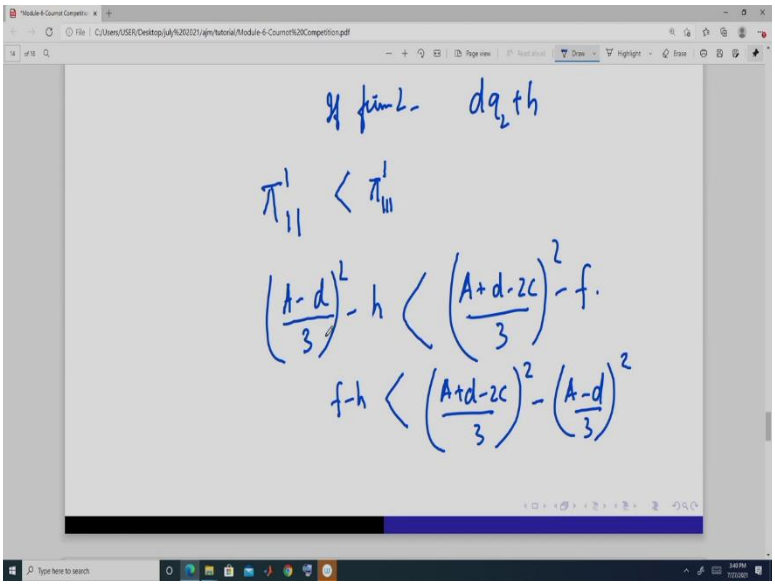

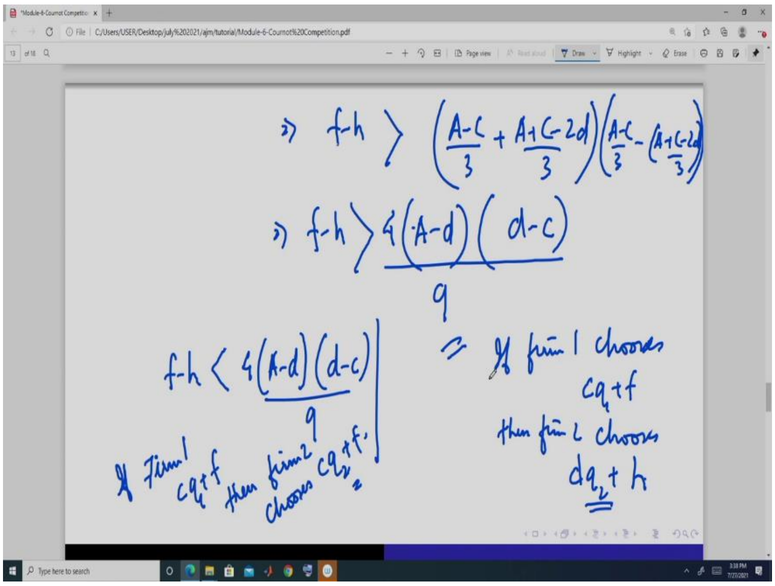

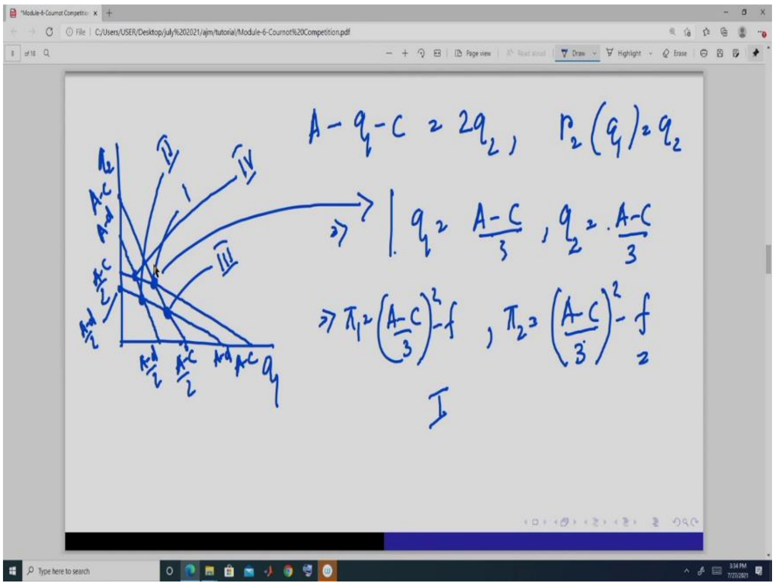

Now if firm 2 is suppose choosing this. If firm 2 is choosing this-$dq_2 + h$ given firm 1 has chosen this so is it optimal for firm 1 to find out whether that it is a Nash equilibrium. So we have to compare this with firm 1 this point this should be. So suppose firm 2 has chosen this then we have to compare firm 1 whether it will choose this point or this point. This point or this point oh not 4, it is 2. It has to be 2. So then this means. It is this-$\left(\frac{A-d}{3}\right)^2 - h < \left(\frac{A+d-2c}{3}\right)^2 - f$, so then it means it is this.

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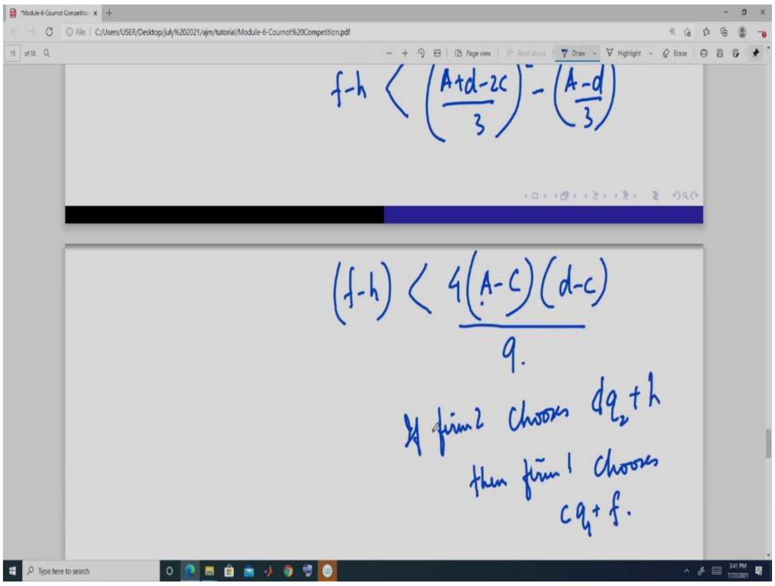

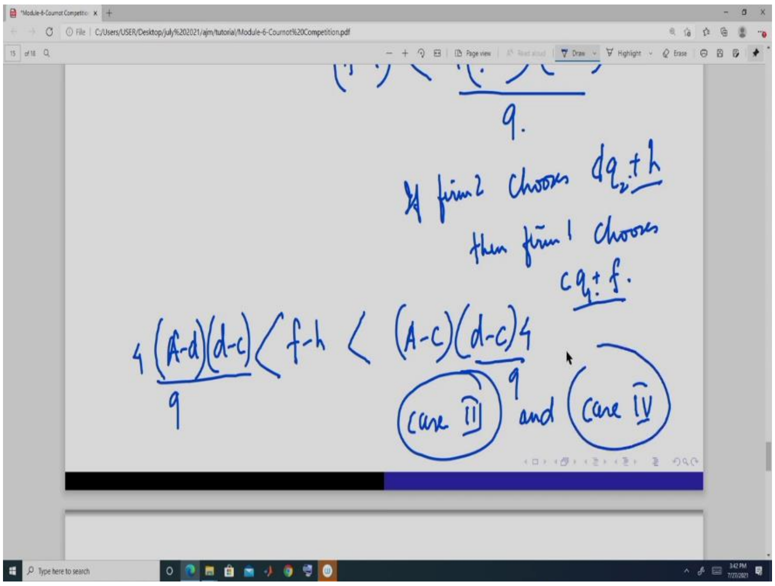

So again this means it should lie here. So if this is the case then if firm 2 chooses this cost function-$dq_2 + h$ then firm 1 chooses- $cq_1 + f$ it has to be h chooses this cost function. So we get that if this is less than A minus c d minus c 4 by 9 and if this is greater than A minus d into d minus c. If it lies here then the outcome is either case 3 and case 4, both can happen. Either firm 1 chooses this cost function-$cq_1 + f$ and firm 2 chooses this cost function- $dq_2 + h$. So this is the case- case iii, or firm 1 chooses this cost function- $dq_2 + h$ and firm 2 chooses this cost function- $cq_1 + f$. So this is the case, i.e case iv.

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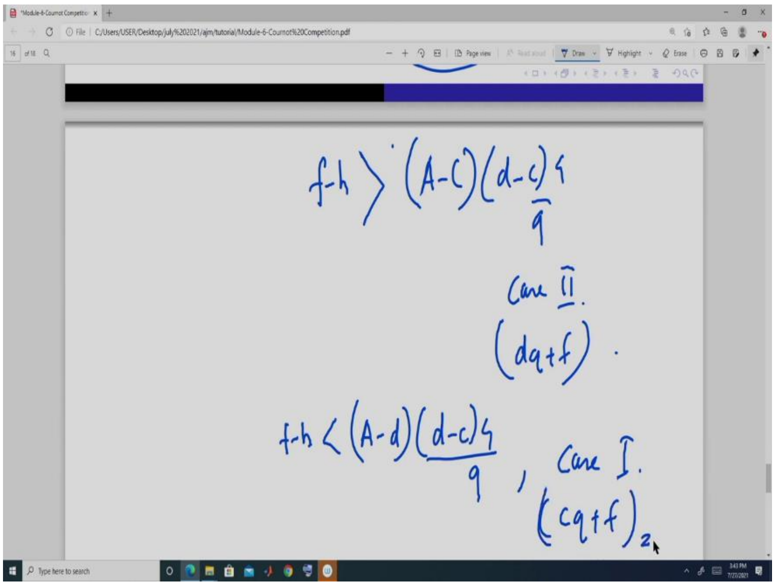

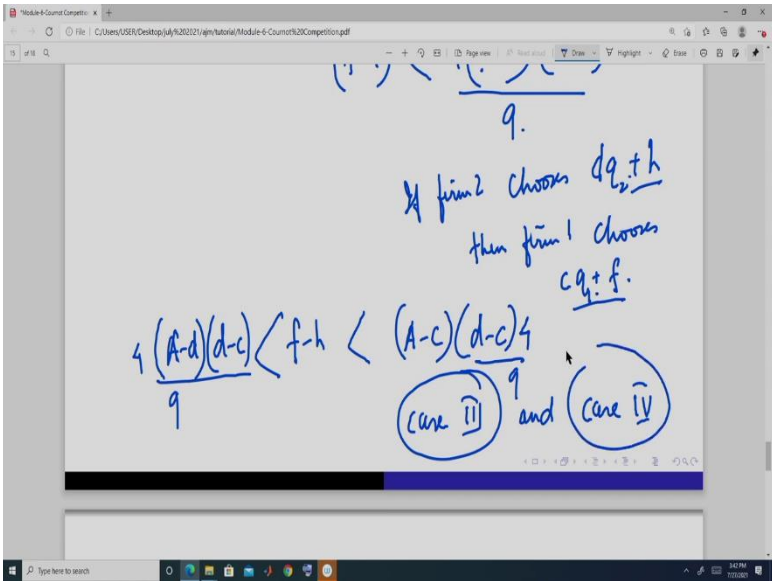

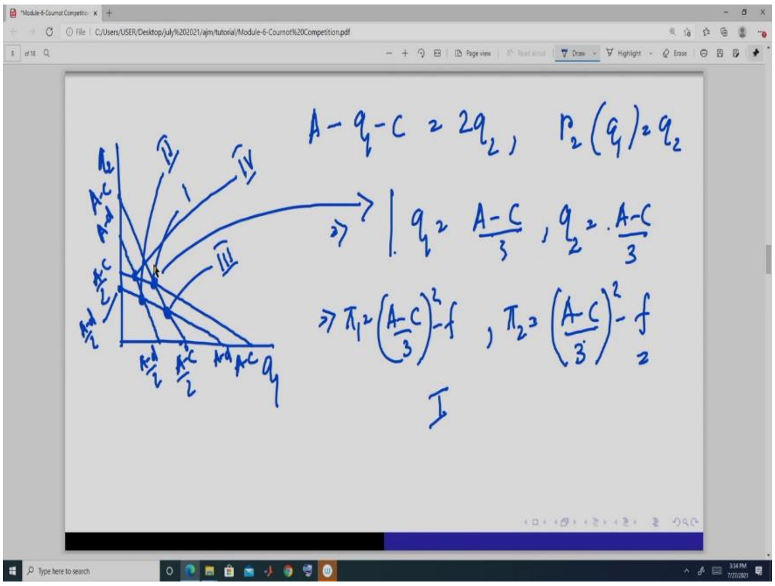

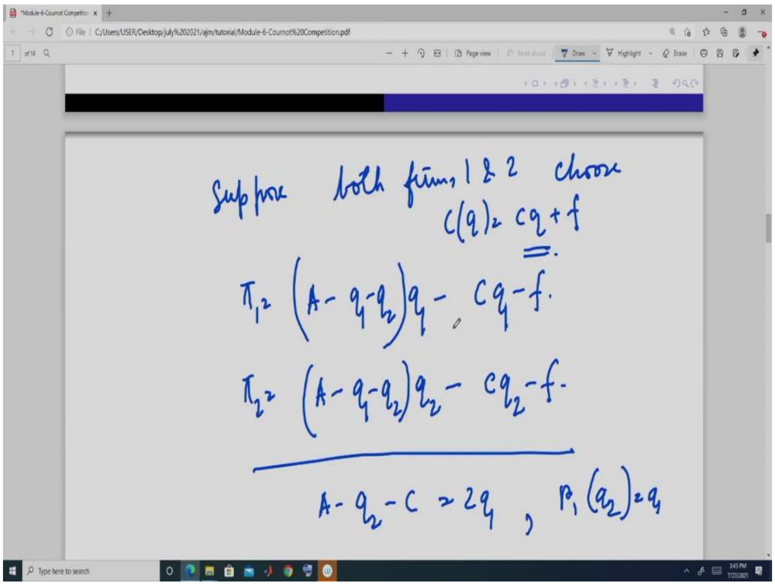

And if this is greater than, this, i.e. f-h> (A-c)(d-c)4/q then we have case 2. Both firms choose this cost function-(dq+f) and if we have this,- f-h< (A-c)(d-c)4/q . This is less than this then we have case 1 and both firms choose this as the cost function. So what do we get? We get many possibilities. So we have one situation where we have a symmetric situation that is case 2 and case 1. We have this when, the difference between the fixed cost is either sufficiently high or the difference is sufficiently low. Case 2 when it is sufficiently high, case 1 when it is sufficiently low.

When it lies between a range then we have an asymmetric situation and it is this. Firm 1 chooses first type of cost function and firm 2 chooses second type of cost function. This is 1 that is case 3 and we may have a this case also. So we have multiple Nash equilibrium here in this equation. See which simply introduction of a choice over the cost function that is if you have choice over cost function that means a choice over different types of technology or different prices of the input then it will, break that unique equilibrium that we get.

Because if you look at this case, so we have these four possible outcomes now, right? depending on different and either we can have this as a unique outcome, either we have this as a unique outcome or we can have these two as a multiple outcome, right? Because when this happens we will also have this, right? But either we can have 1 or we can have this 2, okay this case. So depending on the variety of the cost function and which we get from either production function or from the prices of inputs, okay. Thank you.

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 28
Bertrand Competition with and without fixed cost

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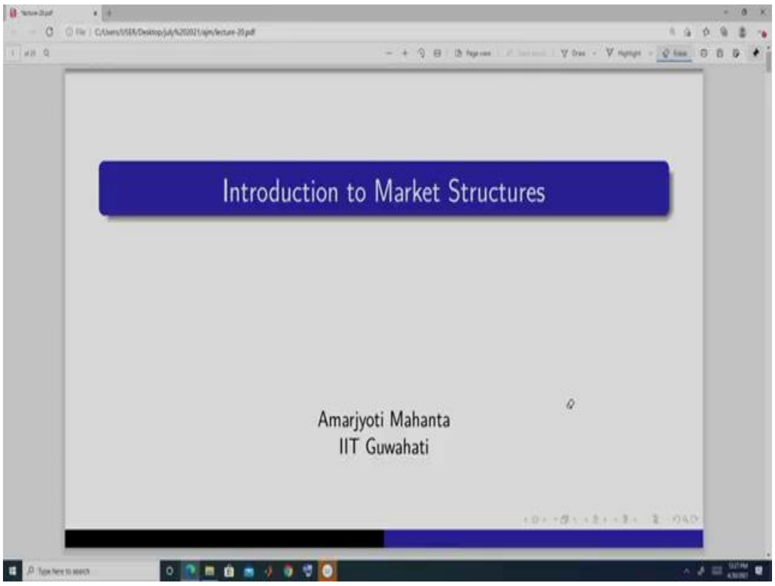

Hello, everyone. Welcome to my course Introduction to Market Structures. We have completed the Cournot model. We have done both Cournot duopoly market, Cournot oligopoly market. In Cournot competition what we have seen that the firms decide the output. And the aggregate, and based on the aggregate output the market price is determined, okay. And while deciding the output they play a strategy game.

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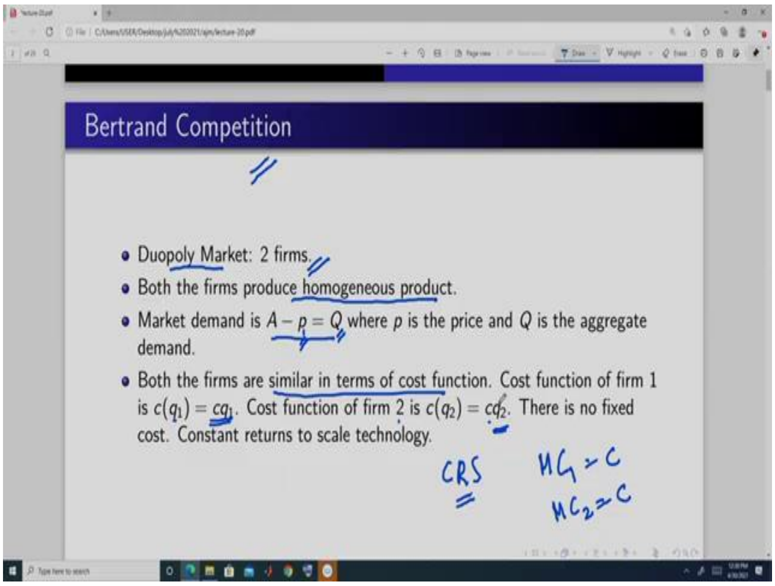

Today, we are going to do Bertrand competition. In Bertrand competition what happened instead of quantity they will choose price. So, how, now let us specify the market. So, for simplicity we will assume that there are two firms. So, it is a duopoly market, okay. Next, both firms produces homogeneous product. What do we mean by a homogeneous product? Homogeneous product means that whether I buy from firm 1 or buy from firm 2 buy being the same price it does not matter. The type of good or the nature of the good is going to be same. So, they are perfectly substitutable, okay.

So, market demand is this-A-p=Q. It is same as what we have considered in the Cournot. So, this p is the market price. We will come to this. But now in a market we know there is only one price and it is this price and this is the aggregate quantity demanded at that price, okay. Again, for simplicity we assume that the, we will see in this Bertrand competition we will take different types of costs.

So, first both the firms are similar in terms of cost function. So, cost of firm 1 is c into q1, q1 is the output of firm 1. And cost function of firm 2 is c into q2. So, this is a CRS technology, constant returns to scale and there is no fixed cost, okay. And both the firms are same. So, here marginal cost of firm 1 is c, marginal cost of firm 2 is again c, okay. So, they are same.

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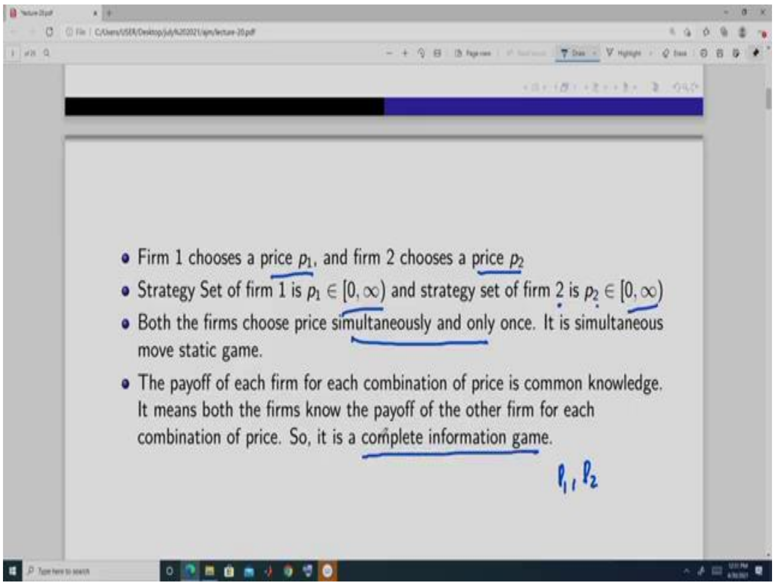

Now, let us specify the game, the strategic interaction in this. So, firm 1 chooses a price that is p1 and firm 2 chooses a price p2, okay. Now, here just previously I have said that in market share is only one price. So, the market price is actually the minimum of these two price, okay. So, okay so strategy set of firm 1 is this p1 and which lies between 0 to infinity and strategy set of firm 2 is again this which lies between 0 to infinity, okay. And both firm, both the firms choose price simultaneously and only once. So, this is a static game of, static game which is played simultaneously, okay.

Next, firm 1 knows the payoff of firm 2 for all the combinations of price p1 and p2. Firm 2 knows all the payoffs of, payoffs for all the combinations of p1 and p2 of firm 1. So, therefore, it is a complete information static game okay and it is played only once.

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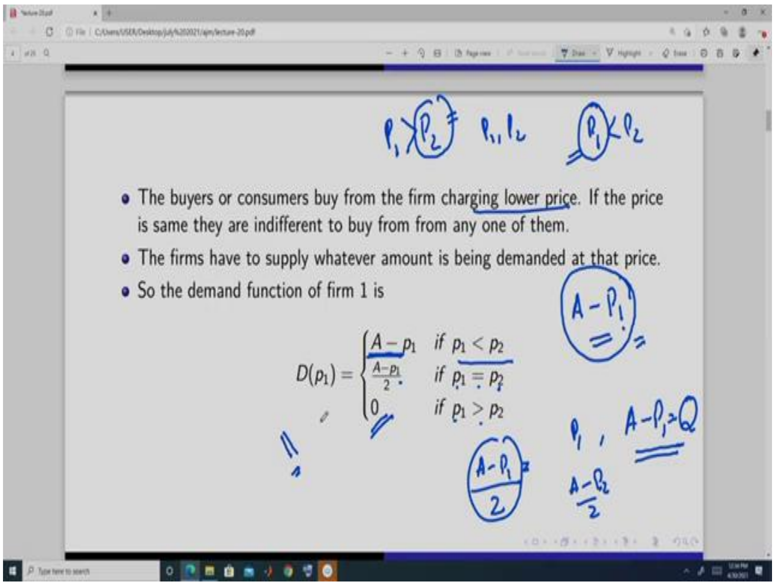

Now, we specify some further details, okay. Like buyers, they will buy from the firm that is charging the lower prices. So, if we have two price p1 and p2 and if p1 is less than p2, then everyone is going to buy from firm 1. And if p1 is greater than p2, then everyone is going to buy from firm 2, okay. So, this will change the demand curve faced by each firm, okay. We will come to it. And further if firm 1 sets a price p1 and suppose p1 is less than p2, then there must be a demand and that demand is going to be this much- $A - P_1$. So, this demand has to be satisfied by firm 1, whatever be the quantity, okay.

So, this firm has to supply this much amount of quantity, okay. So, based on these two assumptions, we specify the demand curve of firm 1. So, firm 1 if suppose sets a price p1 and it is less than p2, then it has to serve the whole market. And at p1 market demand is A minus p1. So, this whole amount is to be satisfied or is to be supplied by firm 1. So, demand curve of firm 1 is this- $A - P_1$.

And if firm 1's price is same as the price of firm 2, p1 is equal to price p2, then the market is equally shared. It is this-$\frac{A-P_1}{2}$. So, at this, this is the amount of quantity demanded and this is the demand faced by firm 1. Similarly, firm 2 will face this if the prices are same. And if price of, set by firm 1, p1 is greater than p2, then the amount is 0. It does not, nobody buys from that firm, okay. So, based on this demand function, we will get the payoff function.

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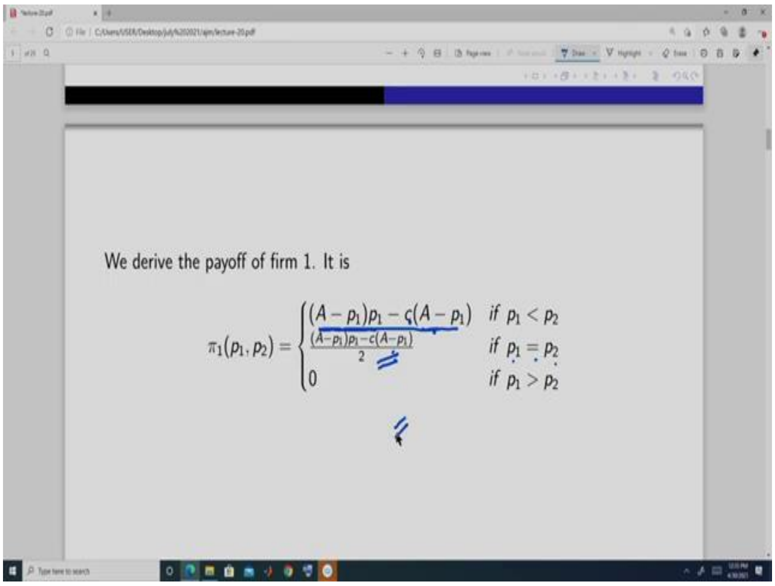

So, the payoff of firm 1, if price of firm 1 is less than the price of firm 2, then it has to serve the whole market. So, the demand is this much, so demand into price. So, this is the total revenue and this is the marginal cost c into the amount it is going to produce A minus p1. So, this is the total cost. So, this is total revenue minus total cost. So, this is the profit-$(A - p_1)p_1 - c(A - p_1)$.

So, profit of firm 1 when it sets a price p1 and firm 2 sets a price p2 and suppose p1 is less than p2 it is this- $(A - p_1)p_1 - c(A - p_1)$. And suppose price are such that p1 is equal to p2, then the profit is this-$\frac{(A-p_1)p_1-c(A-p_1)}{2}$, because this is the total revenue minus total cost divided by 2, because the this is going to be shared half, this is again going to be shared half. So, this is the payoff. So, this is the payoff function of firm 1.

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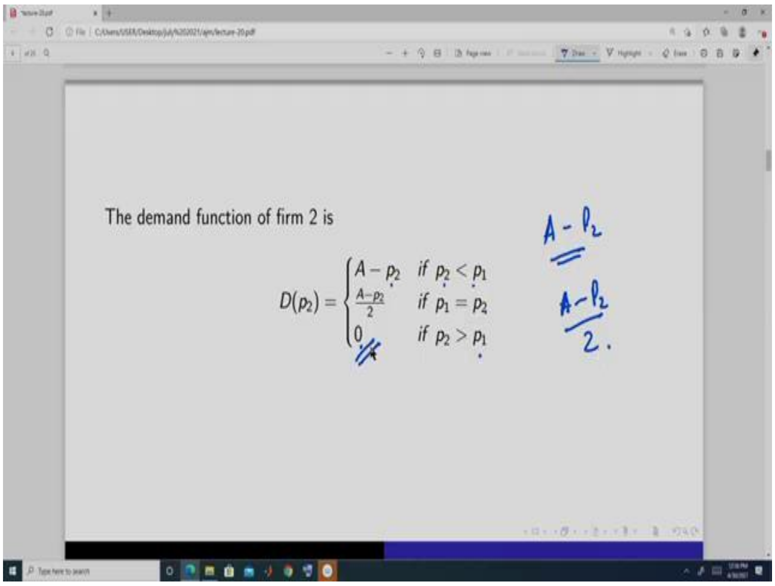

Similarly, demand function of firm 2 is A minus p2, if p2 is less than p1 that is if firm 2 sets a price less than the price of firm 1, it has to serve the whole market and the demand at p2 is this- $A - p_2$. And if it sets a price which is same as the price set by firm that is p1 is equal to p2, then the market is shared equally. So, the demand is this much- $\frac{A-p_2}{2}$. And if firm 2 sets a price which is greater than the price of firm 1 they need, gets 0 demand that is nobody buys from it. So, its demand is 0.

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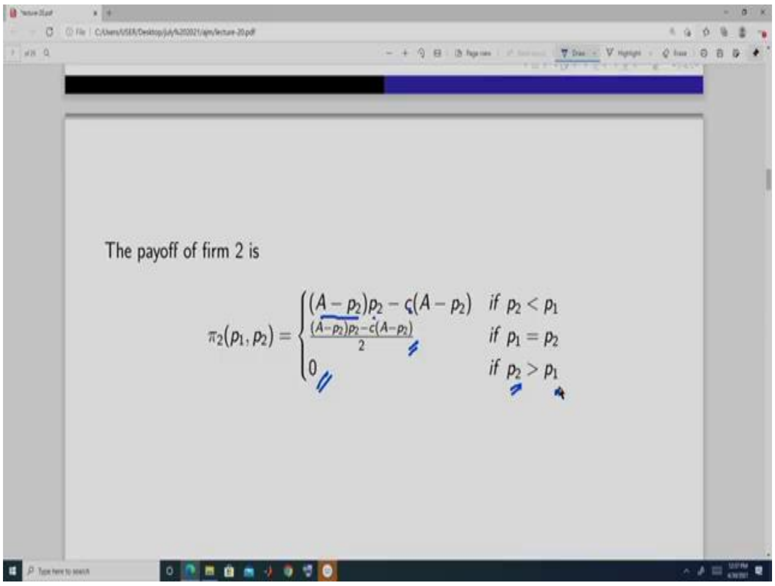

So, from here, from this demand function, we get the payoff of firm 2. Payoff of firm 2 is this- $(A - p_2)p_2 - c(A - p_2)$. So, this is the demand that the firm 1, firm 2 faces when its price is less than the price of firm 1. So, demand into price. So, this is total revenue, marginal cost and it is same as the average cost. So, this is into the amount it has produced, so total cost. So, this is the profit. And when prices are same p1 is equal to p2, then this is A minus p2 into p2 divided by 2.

Again, c into A minus p2 divided by p2. This is the total cost and this is the total revenue. So, the profit is this-$\frac{(A-p_2)p_2-c(A-p_2)}{2}$. So, it is simply half of this if the prices are same, but if they match and if in one case it does not match, okay. And profit is 0 if price is, of firm 2 is more than the price of firm 1. Now, given this specification we have to find the pure strategy Nash equilibrium of this game, okay.

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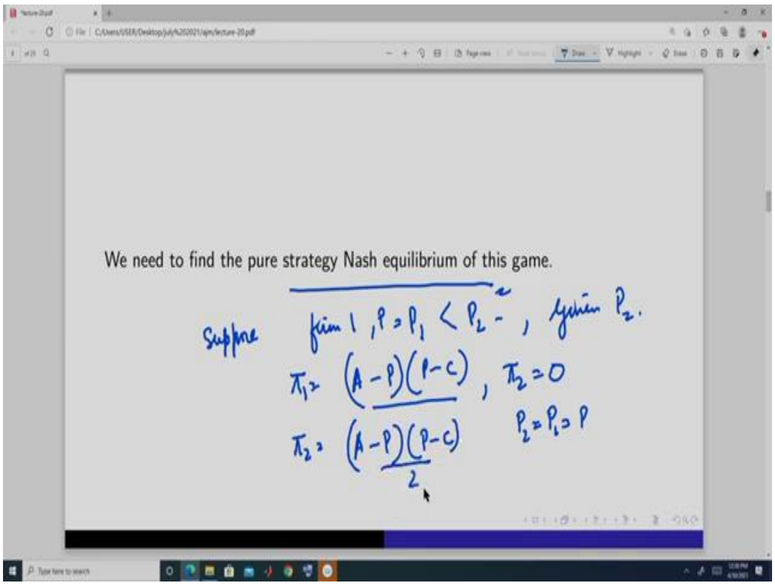

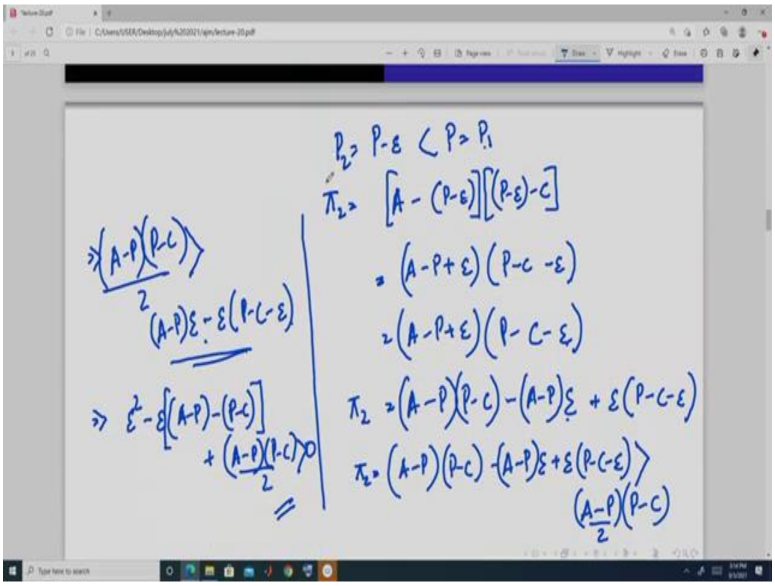

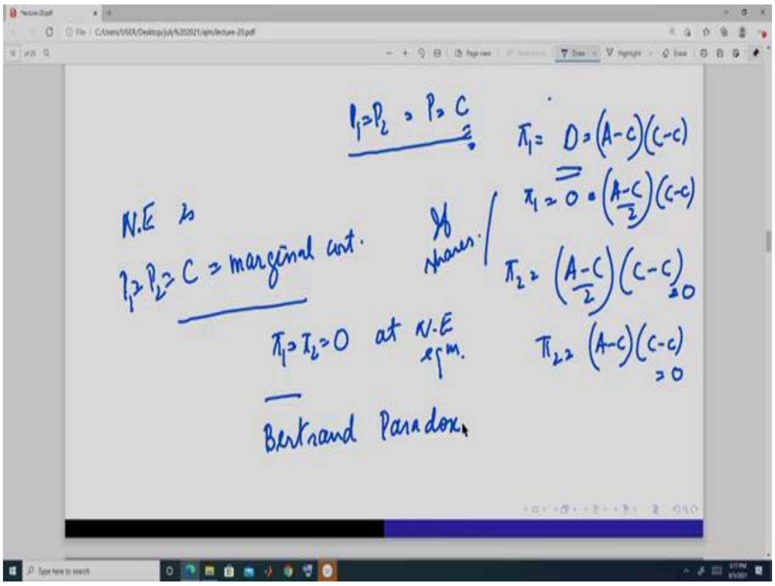

So, first I will show you algebraically and then we will do it geometrically that is through diagram and through diagram it will become more clear. Suppose firm 1 sets a price p1, okay and p1 is suppose less than p2, already there is a p2, given p2, So, the profit of, and suppose this is equal to p, i.e $P = P_1$ is A minus p, p minus c. This is the profit- $\pi_1 = (A - P)(P - c)$. And profit of firm 2 is 0. So, firm 2 if it sets a price p2 is equal to this, p1 is equal to p1, then the profit of firm 2 is, it is this-$\pi_{1=2} = \frac{(A-P)(P-c)}{2}$, right? And instead if firm 2 sets a price which is p2 suppose is equal to p minus some epsilon which is this- $P_2 = P - \varepsilon < P = P_1$, then the profit of firm 2 is, it is this-$\pi_2 = [A - (P - \varepsilon)][(P - \varepsilon) - \varepsilon]$, right?

Now, if we compare this profit and this profit, we see that, this is c, right? So, we have, we can get this- $(A - P + \varepsilon)(P - c - \varepsilon)$. And then if we, okay solve this we have got this. This can be written this way- $(A - P)(P - c) - (A - P)\varepsilon$. This part is multiplied with this part and then we are left with e p this-$(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$, okay. So, if firm 1 sets a price p1 and firm 2 sets a price epsilon less than that price, then the profit of firm 2 is going to be this-$(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$. Now, if this epsilon is very small, if it is very small, then this part we can show that this which is, this- $(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$ is actually greater than this- $\frac{(A-P)(P-c)}{2}$when firm 1 sets this and it shared a market.

So, when the firm 1, firm 2 set the price same as the price of firm 2 and shared the market equally, because from here we get that, so if epsilon is sufficiently small we can construct an epsilon like this which is, which satisfies this- $\frac{(A-P)(P-c)}{2} > (A - P)\varepsilon - \varepsilon(P - c - \varepsilon)$. So, if this is because then actually we get a quadratic form and this quadratic form is of this nature that is, I should have taken it in this form, this, so we can have epsilon which satisfies this-

$\varepsilon^2 - \varepsilon[(A - P) - (P - c)] + \frac{(A-P)(P-c)}{2} > 0$. And this will give us that since there exists an epsilon, so we will, it is optimal for firm 2 to reduce the price or undercut the price if firm 1 sets a price p, then the firm 2 will set a price p minus epsilon.

So, like this it will go on, because we have taken this price to be some arbitrary price. So, like this, this will continue, and then finally, we will get that the price is equal to c. And when the price is equal to c, so profit of firm 1 is 0, because it is A minus c if firm 1 sets this price. And this is, will also be equal to 0 if it shares the market at that price and profit of firm 2 it is going to be the same and it is going to be shares and it is going to be, so this is equal to 0- $\pi_2 = \frac{A-c}{2}.(c - c) = 0$, this is equal to 0-$\pi_2 = (A - c)(c - c) = 0$. So, price of firm 1 and firm 2 is going to be such that it is going to be equal to marginal cost.

And in this case, we have only one pure strategy Nash equilibrium. So, Nash equilibrium is p1 is equal to p2 is equal to c which is equal to marginal cost. We get this. So, when the cost function are same and marginal cost function are such that the marginal cost is constant and they do not have any fixed cost in that case in Bertrand competition or when the firm set the price or compete in terms of price, we get that the pure strategy Nash equilibrium is to set a price is equal to marginal cost. And this will lead to the profit of firm 1 and profit of firm 2 is 0 at Nash equilibrium. So, this is called something called a Bertrand paradox.

And why it is a paradox, because there are only two firms. And two firms are sufficient to generate 0 profit. Generate 0 profit here means that the through the price competition, the price is reduced to marginal cost. And when the price is equal to marginal cost, the firms are not earning any supernormal profit. So, they are earning same as what they get in a competitive market. But in competitive market, we need many firms. But here with only two firms, we can generate this outcome. So, that is why it is called a Bertrand paradox.

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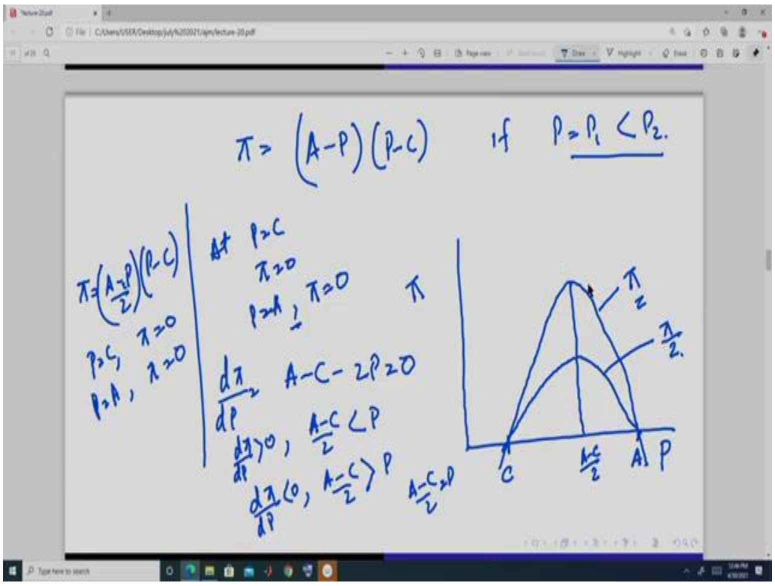

Now, let us do it diagrammatically. So, profit function, we can write it this way. If suppose p is equal, if it is this-$\pi = (A - P)(P - c)$, if $P = P_1 < P_2$. Now, if we try to plot this function here, in this axis we take price and here we take profit. If we take this, let us, this is c and this is A, okay. At p is equal to c, profit is equal to 0. At p is equal to A, profit is equal to 0. And when we differentiate with respect A, we get A minus c, this. So, this is, slope is increasing as long as p is less than this, i.e $P < \frac{A-c}{2}$ and this is this, right. So, it is maximum at price is equal to A minus c divided by 2.

So, we get this curve something like this. So, this is the profit, right. Now, if we take this curve, so again here when p is equal to c, profit is equal to 0 and p is equal to A, profit is equal to 0. And this is same as this curve only it is at the half distance. So, this point is the, you can say the monopoly price, this curve, okay, half the market share. Now, so we get the function, the plot of the payoff function in this form, okay.

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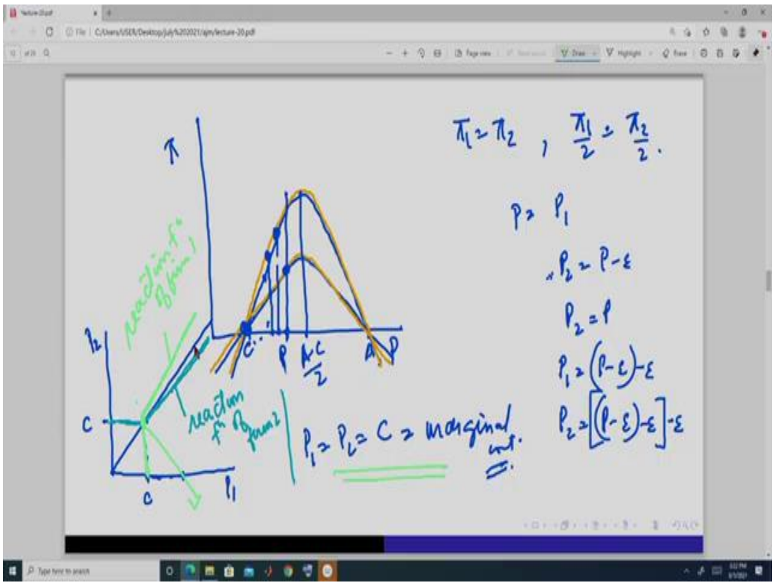

Now, let us look here, right. And since the payoff functions are same, so suppose this is for firm 2, because if we look at the payoff function it is same, right? Now, this is, now set the price like this. So, this is suppose p. P is equal to, firm 1 sets a surprise p. If firm 2 sets a price slightly less than this that is p2 is p minus epsilon something close to it, its market share is this. But if it sets a price which is p2 is equal to p, then it is here. Both the firm gets here.

So, firm 2 is going to set this price and moment it sets this price, its payoffs from here to here, right? So, it is like this. Now, if firm 2 sets this price, now firm 1 is going to set a price which is p slightly less than this, slightly less than here. So, if it sets here, its market is this. Now, firm 2 will know that if it sets a slightly less than this, then if it sets the same pA it will get this profit, but it is still higher. So, p2 is going to be this- $P_2 = [(P - \varepsilon) - \varepsilon] - \varepsilon$. Like this it will go on. So, this undercutting will go on till this point, because for any price which is greater than c this curve lies below this curve.

So, there is the moment you undercut the price, you get a bigger market share and your profit is also higher. So, that is why price is should be always equal to this that is equal to marginal cost-$P_1 = P_2 = C = MC$. So, the reaction functions of firm 1 and firm 2 in this can be represented in this form. Suppose this is the 45 degree line, and this is the marginal cost, so marginal costs are constant. And it is same.

Now, if firm 1 set a price suppose it is this, firm 2 will set a price which is less than this, we have seen this from here, from this diagram. So, any price of firm 2 greater than this. So, how it is going to respond, what it is A. So, it will be less than A. So, it will be lying here, slightly less than. And when it is c, it will be c. So, this is you can say c and then this. And if firm, price of firm 1 goes below this, it is not going to reduce the price, so it is this. So, this is the reaction function of firm 2. And the reaction function of firm 1 we will get in this way.

Firm 2 if it charges a price like this, it will set the price such that it will be slightly less than the price of firm 2. From this diagram we know this. So, it will be like this. So, it will above lie above the 45 degree line. And at price, when firm 2 sets the price c, it is going to set the price c like this. And if it moves below this, it will not charge any price below this, so it will be like this. So, this green line is the reaction function of, firm 1 and these two reaction function intersects at this point. So, that is why this is the Nash equilibrium point, okay. So, we get this as the outcome based on this reaction function.

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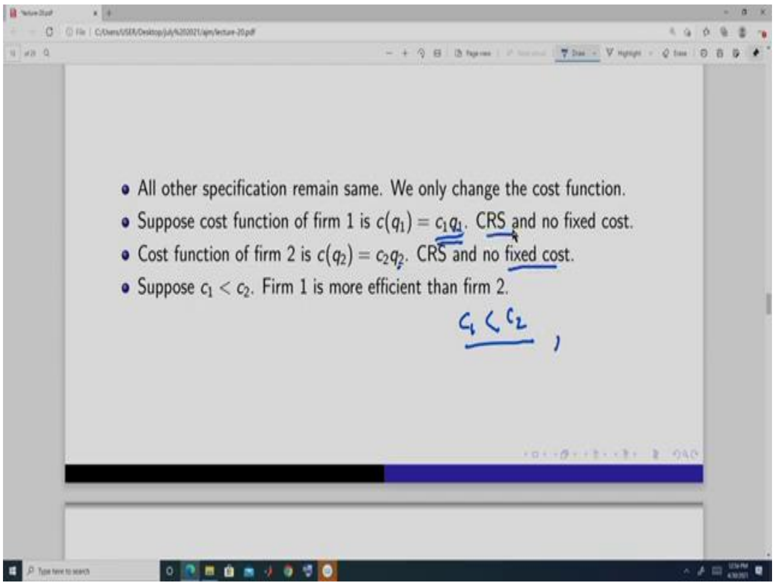

Another case is suppose we now take different function, different cost function. So, what do we take? We take the cost function of firm 1 to be this- $c(q_1) = c_1q_1$, okay. So, it is c1q1. Cost function of firm 2 is c2q2, q2 is the output, here c1 is the output. And further we assume c1 is less than c2. So, this means that firm 1 is more efficient than firm 2 and there is no fixed cost and there is CRS, okay. So, this is the specification of the cost. So, we know that firm 1 can produce output at a much lower cost than the firm 2 and because of this thing and they do not have any fixed cost. All the cost is only the variable cost and the demand curve function is same as earlier.

So, firm 1 if it sets a price lower than the price of firm 2 it gets the whole demand. This-$A - p_1$ if it sets the price same as the price of firm 2 it has to share the market equally. And if it sets a price higher than the firm 2 it gets 0 demand, okay. Similarly, for firm 2 if it sets a price less than the price of firm 1, then it gets the whole market demand. If it sets a price which is same as the price of firm 1, then it gates or it has to share the market equally and it gets 0 demand if the price is higher than the price of firm 2, okay firm 1.

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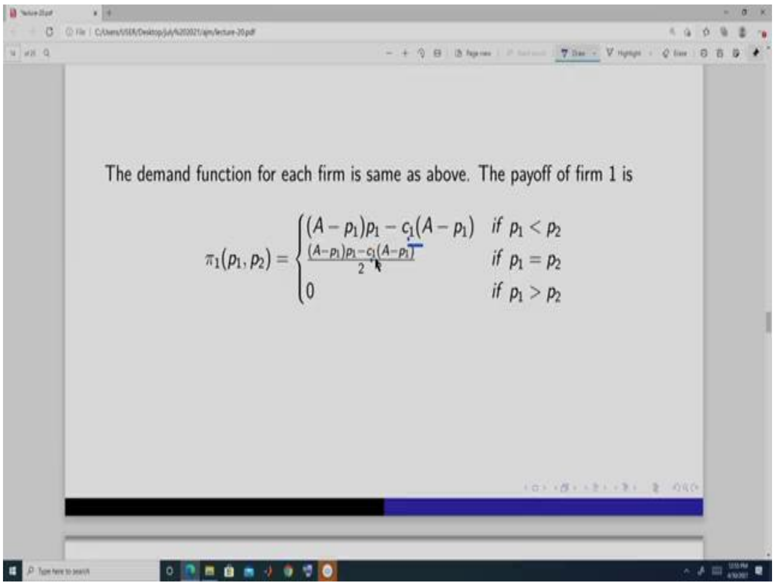

Now, given this we know the payoff function of firm 1 is going to be like this- $(A - p_1)p_1 - c_1(A - p_1)$. Here instead of c earlier it is going to be c1 and here it will be c1. Here in this case firm 1 is getting the whole market, in this case the firm 1 is getting half of the market or half of the total quantity demanded at that price, okay.

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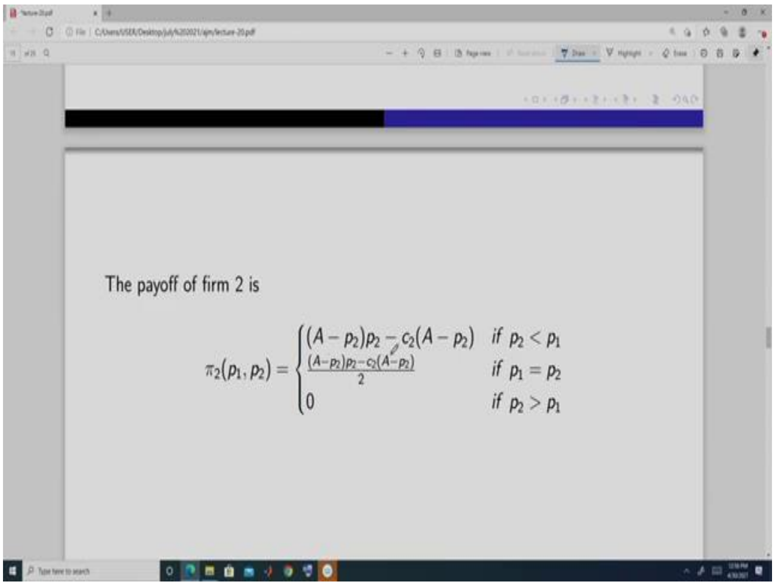

And for firm 2 at p2 when p2 is less than p1 it is getting the whole market that is at p2 whatever is being demanded firm 2 is serving or selling. It is getting this payoff- $(A - p_2)p_2 - c_2(A - p_2)$. This is the quantity and this is the price, this is the revenue, this is the total cost. So, this is total revenue minus total cost gives you the total, gives you the profit. And if the price is same as the price set by firm 1, then it shares the market equally and profit is this- $\frac{(A-p_2)p_2-c_2(A-p_2)}{2}$, okay. Now, let us solve this. And in this case we will do it through diagram, okay.

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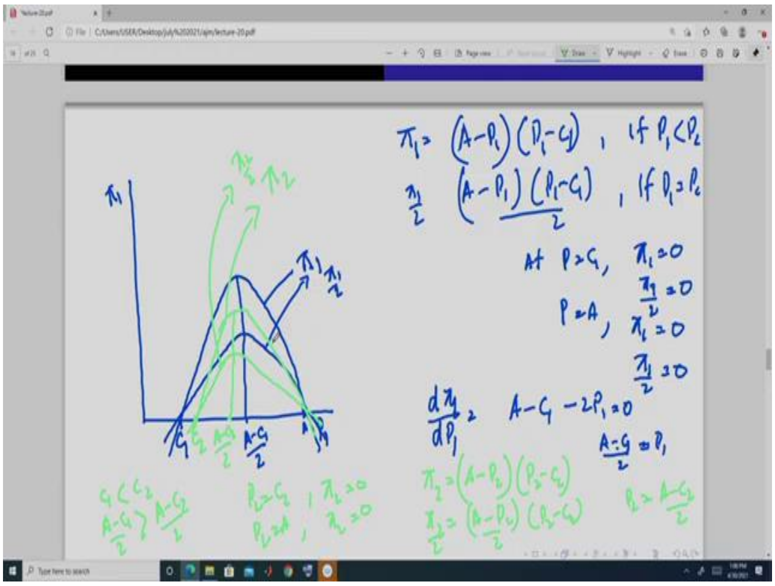

So, first let us look at the profit function of firm 1. Firm 1, profit function is this- $\pi_1 = (A - p_1)(p_1 - c)$, right? if p1 is less than p2 and this- $\pi_1 = \frac{(A-p_1)(p_1-c)}{2}$ if p1 is equal to p2, right.

So, I plug here c1. It is 0. At p is equal to c1 profit is 0, p is equal to A profit is 0, half of this is also, because this is simply half of this. And this is if we differentiate it with respect to p1, we get it first order condition- $\frac{d\pi_1}{dP_1} = A - c_1 - 2P_1 = 0$. So, this gives if, so this is the optimal point. If p1 is less than this, then it is increasing. If p1 is greater than this, then it is decreasing. So, it is something like this. So, this is the, and if market is shared, is you have got half of the market then this, this is for firm 1, okay.

Now, here for firm 2 it is this-$\pi_2 = (A - p_2)(p_2 - c)$, right? So, it is same except at p2 is equal to c2, profit is 0 and p2 is equal to A, c2 is 0. And if we differentiate, we get it to be if p2 is equal to A minus c2 and from here we know since we are given that c1 is less than c2, so A minus c1 divided by 2 this is greater than A minus c2- $\frac{A-c_1}{2} > \frac{A-c_2}{2}$. So, this curve is going to be somewhere some, so this is the profit of firm 2 and this curve is half of profit, okay and this point is A minus c2, this and this point is c2. So, now, I hope this payoff functions, the diagram of the payoff function is clear. Now, we will derive the Nash equilibrium here, okay.

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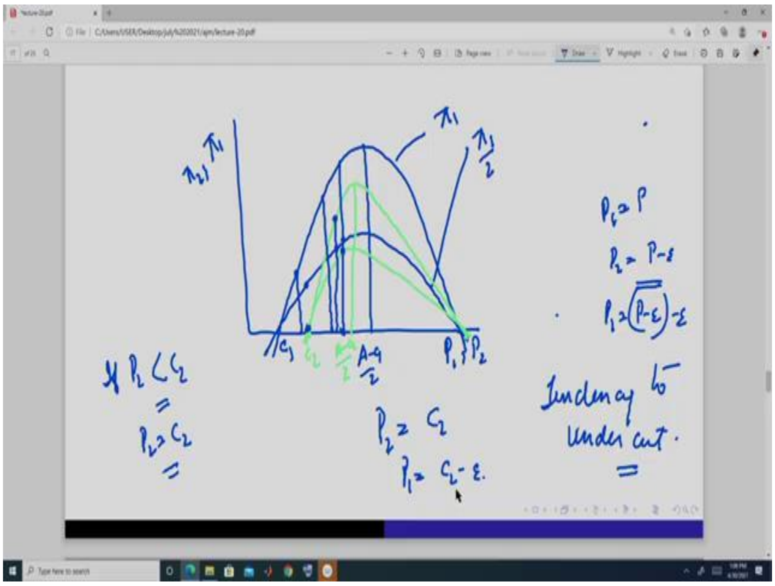

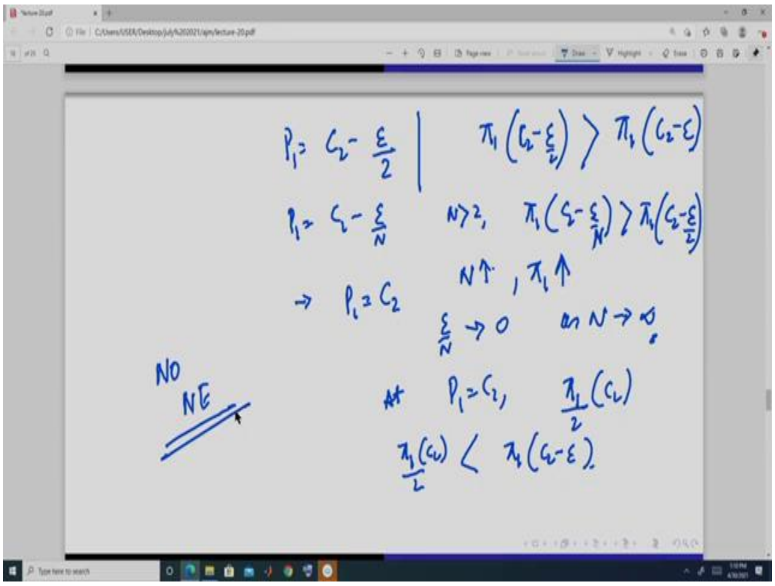

So, you will see that we may not have Nash equilibrium in this. Both are present. okay A minus c2 divided by 2 and this is c2. Now, suppose firm 1 sets a price somewhere here this price. This is p1 equal to p, this. If firm 2 sets a price which is same as this, it is going to get this as a profit and firm 1 is going to get this as a profit. But firm 2 if it sets a price slightly less, its profit is going to be here. So, firm 2 is going to set this price. So, we get this.

Now, if firm 1 sets the price same as this is profit is going to be here, but if it sets the price slightly less than this, so if p1 is this, then it moves profit to here. So, it is going to be like this. So, we see that there is a tendency to undercut and it will go on. So, it will reach like this. So, suppose p1 is equal to c2 firm 2 sets a price, firm 1 set a price which is equal to c2, okay. If it is here then firm 1, see firm 1 has set this. Firm 2 if it sets a price, firm 2, wait, let us, so like this it is going on. And suppose firm 2 sets a price p2 is equal to c as it is going goes on decreasing, it reaches a price c2 and suppose firm 2 that is p2 is equal to c2.

Then if firm 1 matches this price, then its profit is going to be here. But if it sets a price slightly less than this is, its profit is going to be here, right? So, p1 is c2. The moment it sets this price firm 1, firm 2 is not going to set any price, because if it sets a price less than this, if p2 is less than c2 its profit is going to be negative. So, it will set the price is going to be like this only.

Now, here see instead of this price, if firm 1 had set a price which is p1 c2, this- $P_1 = C_2 - \frac{\varepsilon}{2}$, half of this, profit here is going to be greater than, when it is this- $\pi_1\left(C_2 - \frac{\varepsilon}{2}\right) > \pi_2(C_2 - \varepsilon)$, because as the price increases profit increases, because it is less than this price- $\frac{A-c}{2}$. This is the monopoly price, the price at which this we have derived it now. This is maximum. So, firm 1 is going to increase again further if profit is this and N greater than 2, profit is going to be greater, epsilon, like this- $\pi_1\left(C_2-\frac{\varepsilon}{N}\right)>\ \pi_2(C_2-\frac{\varepsilon}{2})$, and will go on increasing. So, profit of 1 is going to go on increasing.

So, finally, it will hit p1 is equal to c2, because epsilon 2 tends to 0 as N tends to infinity, right. So, and when the moment p1 is equal to this c2, firm 2 sets a price which is also c2, its profit is going to be here. So, it was going like this. So, it was going, increasing like this. The moment it hits, it comes from here to here. So, again when it is here, so at p1 is equal to c2 is this which is c, so this is less than epsilon which is this amount here. So, like this it will continue.

So, we see that firm 1 will undercut and then it will go on increasing till the price is c2. And moment it reaches c2, it will again undercut and then it will go on increasing till it reaches c2. So, we see that there is no pure strategy, because when it is firm 2's price is c2 optimal response of firm 1 is to set a price which is slightly less than c2.

Now, when it sets a price which is slightly less than c2 that is epsilon less than c2, then it is better off by again taking half of that A. If we take this c2, but then if we take this half of that distance this is epsilon then it is better off. Again it is further better off if we reduce that distance further. So, it will continue like this and as this portion is going on reducing and reducing, it will hit, the price will hit c2. The moment price again hits C2, profit goes down. So, goes down to this level. So, market is shared equally.

Although firm 2 is earning 0 profit, but it is market is shared equally so that is why firm 1 will again reduce the price by some epsilon. Now, if you keep on reducing that epsilon, your profit is again going on increasing. So, you will go on reducing the epsilon and finally again hit c2. So, this process will go on and so there is no pure strategy Nash equilibrium. So, no pure strategy Nash equilibrium in this equation.

So, the moment we have a CRS production function and so our cost function is like this, but they are different. If suppose one of the firm is efficient, another farm is relatively less efficient then so in a Bertrand competition, we do not have any pure strategy Nash equilibrium, okay. So, it is based on this diagram.

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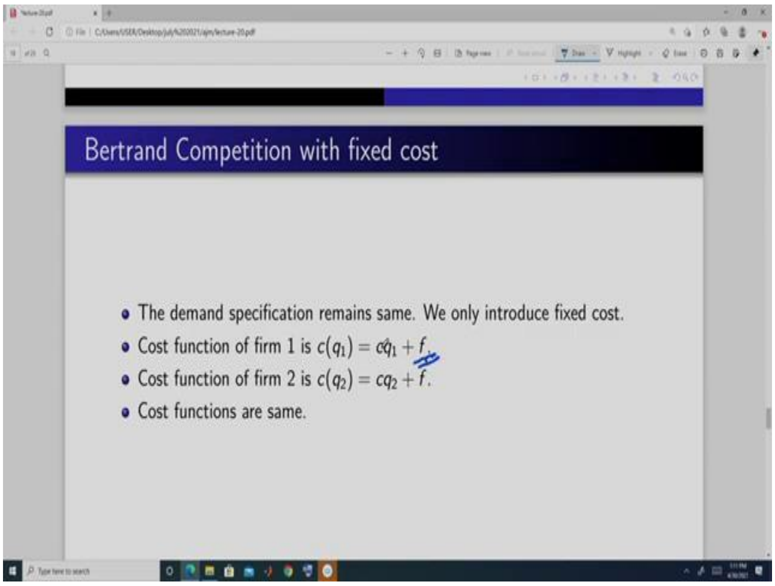

Now, let us introduce fixed cost, okay. So, cost demand specification remains the same as earlier. And the cost function it is also same, but there is a fixed cost component and it is this f.

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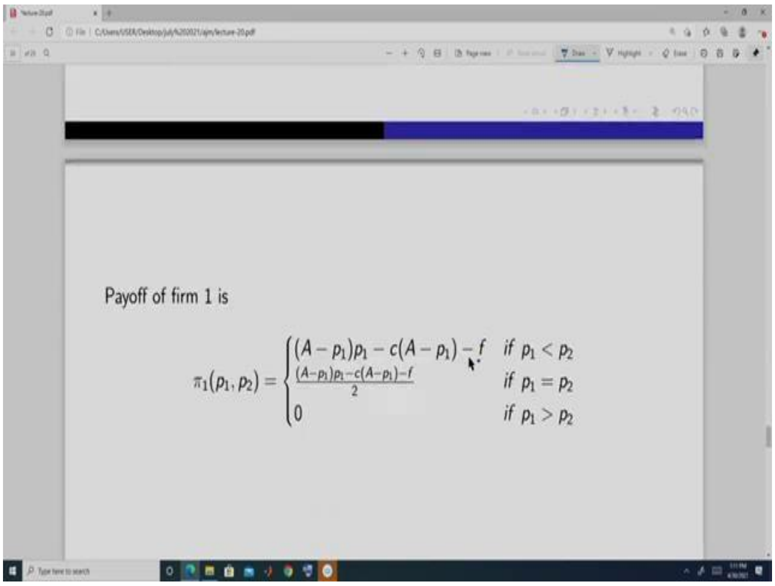

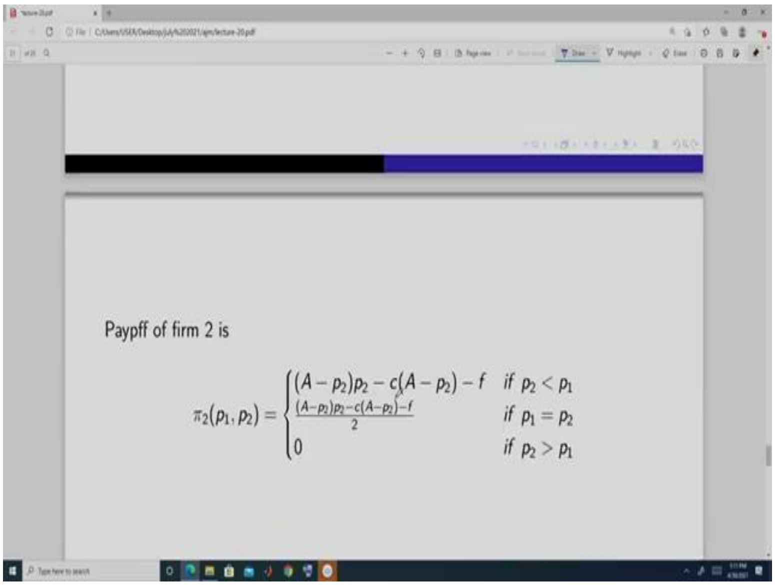

So, we have a component like this - $\pi_1(p_1, p_2) = (A - p_1)p_1 - c(A - p_1) - f$, if $p_1 < p_2$ and we have a component like this here. So, this is the payoff of firm 1 and this is the payoff of firm 2-$(A - p_2)p_2 - c(A - p_2) - f$.

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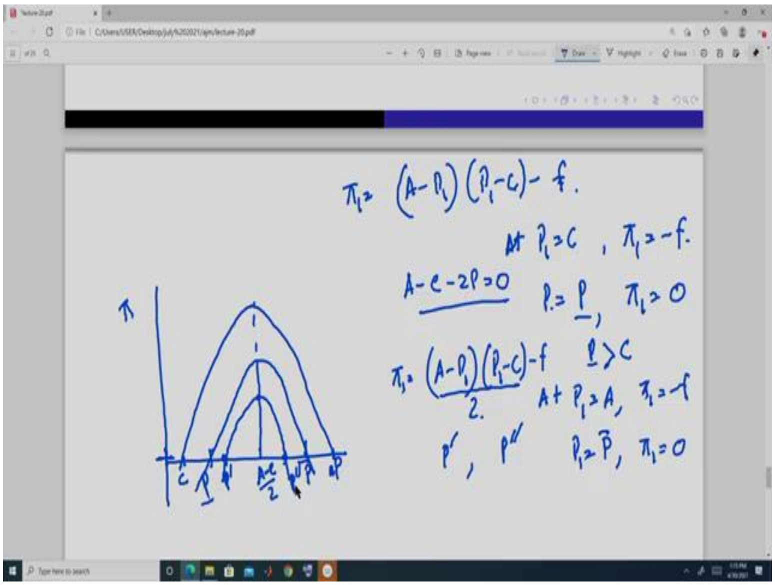
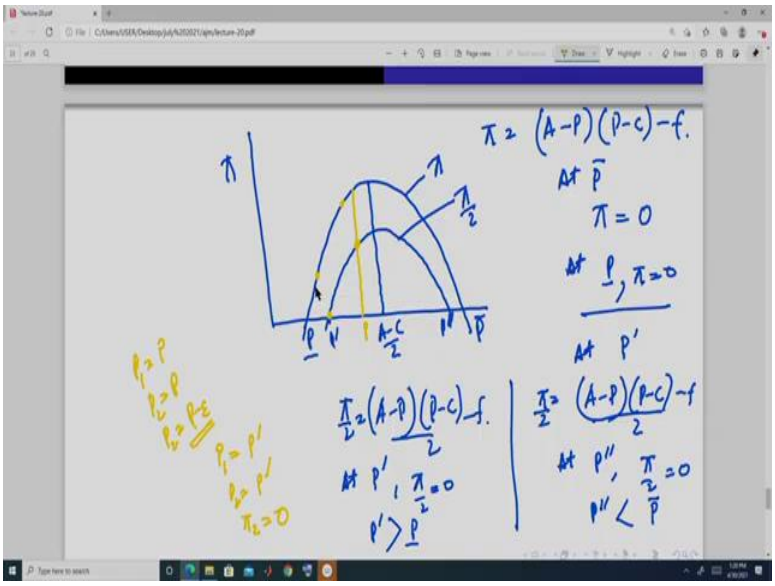
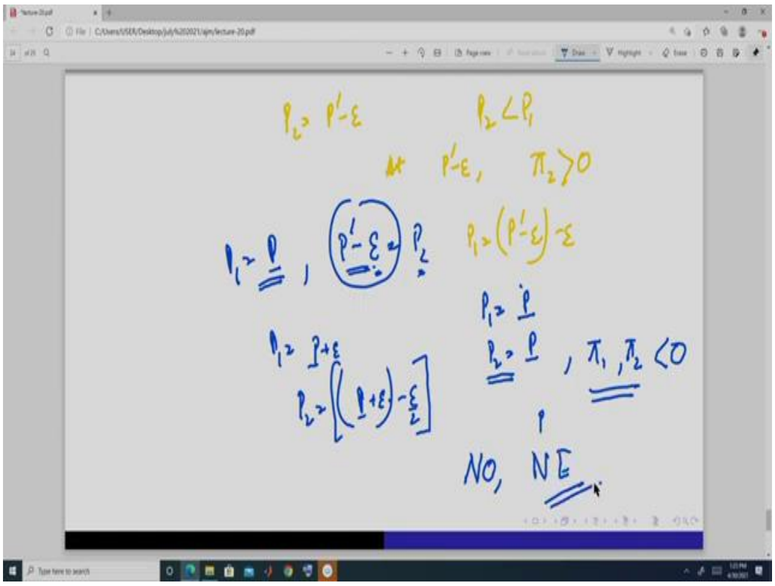
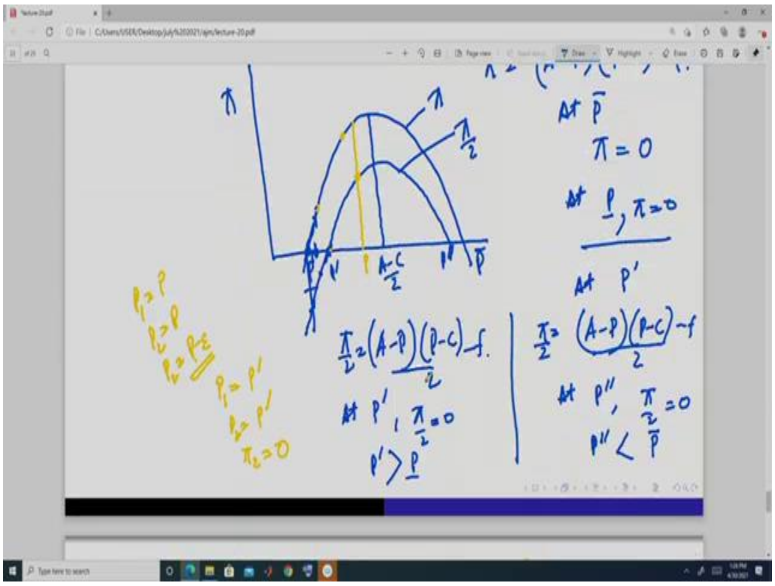



$\pi = (A-P)(P-c)-f.$
At $\bar{P}$
$\pi = 0$
At $\underline{P}$, $\pi = 0$
At P'
π
$\frac{\pi}{2}$
$\underline{P}$ P' $\frac{A+C}{2}$ P'' $\bar{P}$
$P_1 = P$
$P_2 = P$
$P_2 = P-\varepsilon$
$P_1 = P'$
$P_2 = P'$
$\pi_2 = 0$
$\frac{\pi}{2} = \frac{(A-P)(P-c)}{2} - f.$
At P', $\frac{\pi}{2} = 0$
$P' > \underline{P}$
$\frac{\pi}{2} = \frac{(A-P)(P-c)}{2} - f$
At P'', $\frac{\pi}{2} = 0$
$P'' < \bar{P}$


$P_2 = P'-\varepsilon$
$P_2 < P_1$
At $P'-\varepsilon$, $\pi_2 > 0$
$P_1 = \underline{P}$, $(P'-\varepsilon = P_2)$
$P_1 = (P'-\varepsilon)-\varepsilon$
$P_1 = \underline{P}$
$P_1 = \underline{P}+\varepsilon$
$P_2 = \left[(\underline{P}+\varepsilon)-\frac{\varepsilon}{2}\right]$
$P_2 = \underline{P}$, $\pi_1, \pi_2 < 0$
NO, NE


At $\bar{P}$
$\pi = 0$
At $\underline{P}$, $\pi = 0$
At P'
π
$\frac{\pi}{2}$
$\frac{A+C}{2}$ P'' $\bar{P}$
$P_1 = P$
$P_2 = P$
$P_2 = P-\varepsilon$
$P_1 = P'$
$P_2 = P'$
$\pi_2 = 0$
$\frac{\pi}{2} = \frac{(A-P)(P-c)}{2} - f.$
At P', $\frac{\pi}{2} = 0$
$P' > \underline{P}$
$\frac{\pi}{2} = \frac{(A-P)(P-c)}{2} - f$
At P'', $\frac{\pi}{2} = 0$
$P'' < \bar{P}$

Now, again we explain it, find the pure strategy, whether we have a pure strategy Nash equilibrium or not through the diagrams, okay. So, it is this- $\pi_1 = (A - p_1)(p_1 - c) - f$, right? Now, here you will see that c is same, okay but profit is 0 if you take this, at p1 is equal to c profit is, this portion is 0, so profit is minus f. So, we have a price which is p is equal to lower bar at which p1 is equal to 0, okay. The moment, and p bar is greater than c, marginal cost, okay. And further at p1 is equal to A this profit is, this function is 0, this function is again this is 0, so this is 0. So, we will have negative profit, i.e $\pi_1 = -f$.

So, we will have p is equal to some upper bar at which this is equal to 0. And if we differentiate this, again we get the same thing that is first order conditions this is this- A-c-2P=0. So, if we plot p here, profit here, this is p lower bar and this is p, this is p upper bar and, okay earlier it was, so this was c and this was A and fixed cost was not there, cost profit function as like this as a function of p. But now when we have a fixed costs, so it has become something like this, okay.

So, remember this thing. And this, this thing- $\pi_1 = \frac{(A-P_1)(P_1-c)-f}{2}$, right? So, this will be again further lower. It will be something like this. This portion, but rest it is, this is going to be the maximum. So, this is suppose p dash and this is supposed p double dash. This is p dash, this is p double dash, okay. So, what we have now it is like this. This is p upper bar. At p upper bar this is 0. This is p lower bar. At p lower bar is 0. This is equal to A minus p, p minus c, f, okay.

Now, we have again at p dash suppose we take the and the market is shared between is this. So, when we, earlier this point where they are equal they, it was same, this point, but here it is not like this, because at p upper bar, at p upper bar this is equal to this, but there is half it is, so it has to be lower than p. So, that if it is lower this will take a higher value. So, overall it is. So, that is why it should be at p double dash this is equal to 0 and p double dash is less than p upper bar.

Similarly, earlier again when we take this, at this, this is equal to 0-$\frac{\pi}{2} = 0$ and this p dash is greater than p lower bar. Why, because when we, what is happening, this price is less than, so the maximum for this and this it is same and it is A minus c divided by 2. Now, since this p lower bar is less than this, so as we increase the p here profit increases, right? It is same in this case also. So, at this price only it will make profit, here it will. So, it will be something like this. So, this is the profit, this is when it is shared. Now, these two payoff functions are same for both the firms, right? Now, we have to find the pure strategy Nash equilibrium in this game.

I hope it is clear how we have got this, okay. This is p double dash and this is p dash. This is p lower bar. This is p upper bar, okay. How do we get the Nash equilibrium here? Suppose firm 1 sets a price which is this, firm 1, p1 is equal to p. Now, if firm 2 sets the same, its profit, both firm is going to get this. But if it sets a price which is this epsilon, it will get here. So, this is the best response. So, **so** like this there is a tendency to undercut. And finally price will be here.

Now, see if firm 1 has supposed to reach this price, suppose p1 is equal to p dash. Now, if firm 2 sets the same price, if p2 is equal to p dash, then profit of firm 2 is also 0. It is this. But if instead, if it sets a price which is p2 is epsilon, then what is happening, p2 is less than p1. So, it shifts to this level, which is positive. So, at this is positive. So, firm 2 will not set p2 is equal to p dash, but it will slightly reduce it. So, if it reduces then it is getting some positive amount. So, it will like this. So, firm 2 will again reduce. So, firm 1 is going to like this, right? So, firm 2 is going to go like this. It will continue reducing the prices. So, it will finally reach this point, okay. So, at this point, what is happening? So, it will go on.

So, finally, suppose p1 is equal to p lower bar. Now, at this, if firm 2 sets p is equal to lower bar, then this in this A it is negative, right? So, both the firm p1 and p2 both are getting negative. So, firm 2 will not set a price like this. So, firm 2 is going to set, is not going to set a price which is in this case below p dash. Because what is happening, if we reduce go on you will continue. But moment you are here, if it hits, then you get a negative A. So, here if you want to share the market it is better to share here, right?

Now, what is going to happen? Firm 1, since firm 2 is not reducing the price anymore, because it is less here, but if firm 2, firm 1 sets a price slightly higher than this, it will, firm 2 will set a price less than it and then again it will go here. Moment it is here firm 1, is suppose, suppose firm 1 price is at p lower bar, okay and firm 2 is still it is at some price which is this minus some epsilon, okay. Now, here instead if firm 1 is slightly go on increasing but less than this, then what is happening, is profit is increasing, because you look at this graph as it moves here is profit increases.

So, suppose firm 2 stops here. So, it will increase like this. Moment it sets a price higher than this, then moment it sets a price which is suppose this plus small amount some epsilon, then best response for firm 2 is to set something like this- $P_2 = [(P + \varepsilon) - \frac{\varepsilon}{2}]$, slightly less than this. So, again firm 1 will reduce and then they will go on doing that and it will reach this price. And then the other firm is not going to reduce any further. So, it will stop at some price like this. Then firm 1 is again going to go on increasing. It will, the moment it increases more than p lower bar then firm 2 is going to react and it is. So, in this case also we have no pure strategy, Nash equilibrium.

So, if we introduced fixed cost in Bertrand competition with CRS production function or CRS cost function then we do not have any pure strategy Nash equilibrium. So, what do we get? So, in Bertrand competition where the firm set prices, if the marginal costs are seen and it is constant, that is CRS, and there is no fixed cost, then we have a pure strategy Nash equilibrium and it is such that the prices are equal to marginal cost. So, they do not make any profit and this is called the Bertrand paradox.

Now, if the marginal costs are constant, and but they are different, so one firm is more efficient than the other firm. So, we have shown that in that case also if market is shared equally when the prices are same, in that case, we do not have any pure strategy Nash equilibrium and it was given here. Next, we keep the marginal cost same and constant and also we introduced fixed cost. So, in this case also we see that there is no pure strategy Nash equilibrium in the market, okay.

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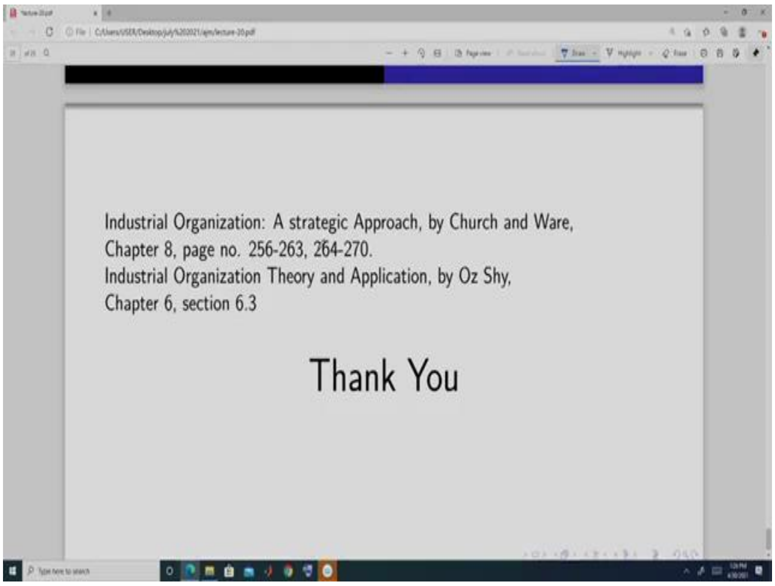

So, with this, we end this portion of Bertrand competition and you can read it from this portion. From Church and Ware you can read from these pages and from Industrial Organization Theory and Application by Oz Shy you can read section 6.3. Thank you.

Introduction to Market Structures

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Lecture No. 29

Bertrand Competition with Capacity Constraints

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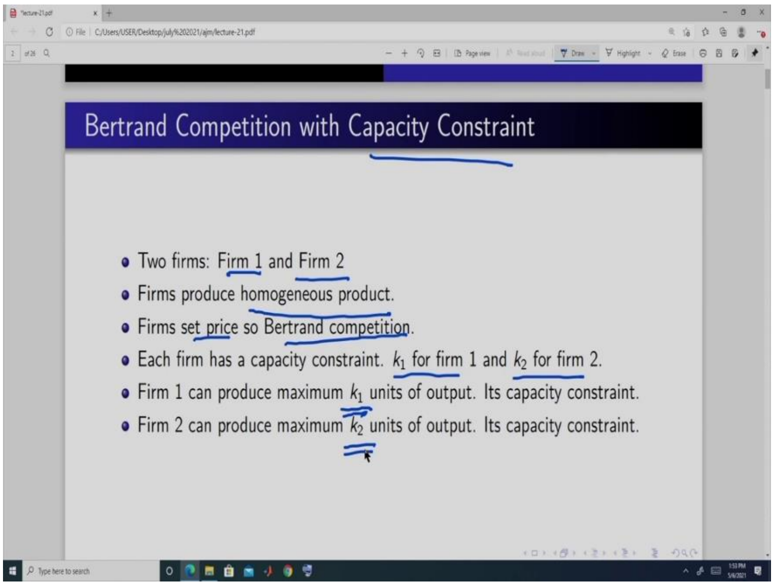

Hello everyone, welcome to my course introduction to market structures. So, today we are going to do Bertrand competition with capacity constraint. Now, we have already done Bertrand competition what is generally done in Bertrand competition. In Bertrand competition we have 2 firms and each firm chooses a price and the firm which sets the lowest price everyone or all the consumer buys from that firm.

So, there is a competition to set the price and we have seen that when there is no fixed cost and the marginal cost are same the both the firm sets the prices at price is equal to marginal cost and that is Bertrand paradox. Next when there are fixed cost we have seen that there is no pure strategy Nash equilibrium and when we have different marginal cost and 0 fixed cost at that in this situation also we see that there is no pure strategy Nash equilibrium.

Now, we introduce a new thing in this Bertrand competition and that is capacity constraint. What do we mean by capacity constraint? Capacity constraint means that the firms cannot produce whatever amount of output it wants to produce. It has some constraints on the output it can produce. So, to study this we have taken a duopoly market that is there are two firms firm 1 and firm 2. Both the firm produces homogeneous product that means the output whether

I buy from a consumer buys from a firm 1 or firm 2, it does not matter what the firm produces same type of good, okay.

And since each firm sets price so it is a Bertrand competition both the firms set a price. And now we specify the capacity. So, each firm has a capacity constraint that is k1 for firm 1 and k2 for firm 2. What do we mean by this capacity constraint? That means that firm 1 can produce maximum k1 units of output. So, it is given by its capacity and firm 2 can only produce maximum k2 units of output it cannot produce more than firm 1 cannot produce more than k1 units of output and firm 2 cannot produce more than k2 units of output. So, that is, this is given us capacity. Now, we specify the cost.

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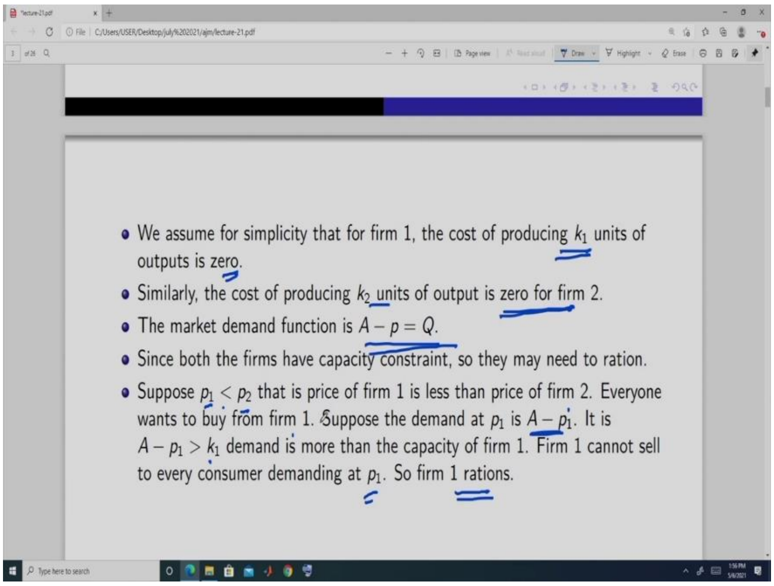

So, here we assume for simplicity that the cost is 0 for firm 1 for producing k1 units of output, its cost is 0. So, it does not incur any cost to produce this output. This is actually a simplest to simplify the game a computation that we are going to do later on to find Nash equilibrium we make this assumption. Similarly firm 2 to produce k2 units of output it does not incur any cost, okay. And the market demand is a linear downward sloping demand curve and that is this- A-p=Q and more or less we have assumed this demand curve till now.

So, this is our demand curve. Now here since each firm has a capacity constraint. So, they may have to do something called rationing and what do we mean by that? So, it is suppose price of firm 1 is p1 and price of firm 2 is p2. And price of firm 1 is less than the price of firm 2, okay so this. So, what will happen everyone will want to buy from firm 1 because the price of firm 1 is less than the price of firm 2.

Suppose the aggregate demand a market demand at price p1 is this- $A - p_1^*$ and it is such that the demand is more then the capacity of firm 1. So, firm 1 cannot satisfy the demand that is generated in the market. So, firm 1 cannot sell to every consumer that is demanding at this price. So, it can only satisfy the demand of only a fraction of that or some portion of that some will be left untouched or we will not be served by firm 1. So, firm 1 has to do something called rationing. So, it is something like this suppose you have 4 chocolates and you have to distribute it among suppose 7 children.

Now, you cannot divide this chocolate. So, these 5 chocolates how you have to you have to choose 5 out of this 7 children how are you going to do it? You will use some rule. So, here also we will specify certain rules to ration how to select which consumers are going to get this product from firm 1, okay.

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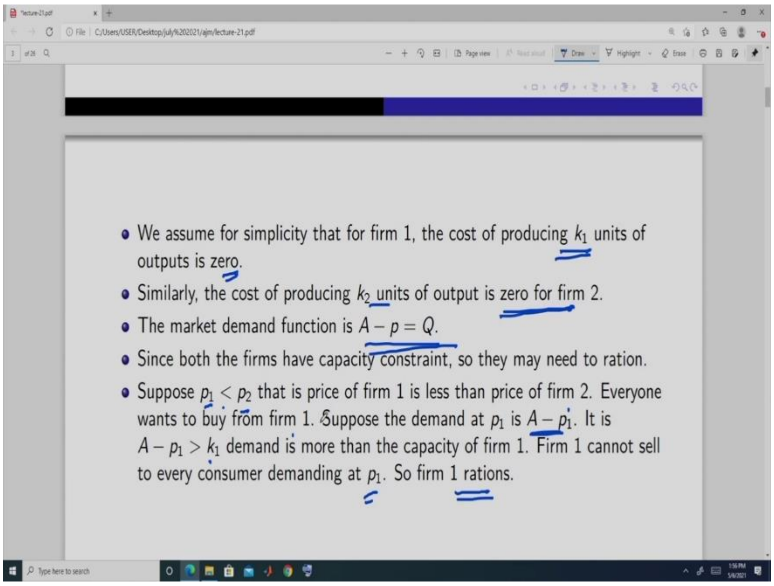

So, we have to specify a rule and one rule is this efficient rationing. So, efficient rationing says that those who value the good most they are going to get the that good from the low price firm. So, in this case firm 1 is going to serve or going to serve the demand of those consumers whose demand or whose valuation is highest or maximum and what do we mean by evaluation it means the willingness to pay. So, it means those whose willingness to pay is maximum or is highest they are going to be served first.

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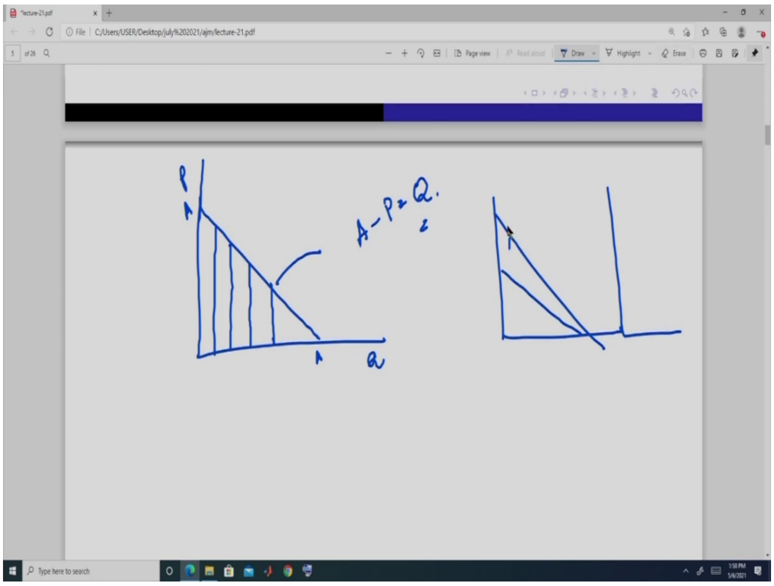

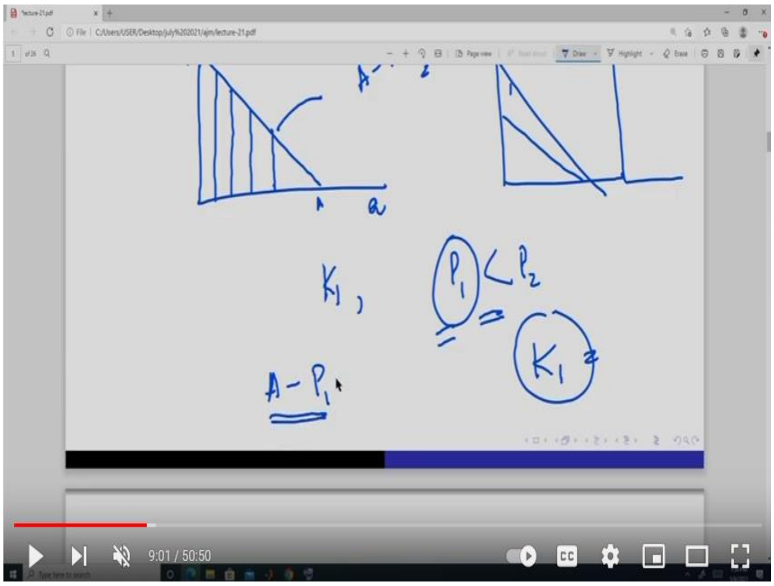

So, in terms of suppose in this case, our demand curve is this. Suppose this is q this is p, in this axis p this is A this is A. So, demand curve is this, right? This heights are giving the maximum willingness to pay by a consumer, okay. Now, this is a market demand curve. So, this demand curve we have got from horizon till summation of individual demand curves, okay. So, at so if you want this quantity what is the maximum it is willing to pay it is generally if we take individual demand curve of individual person then it is given by this height.

But here for simplicity we will assume that suppose this only gives you that willingness to pay. So, there may be some consumers who are willing to pay so, it may be something like this there is this one demand curve this is another demand curve and we have got the market demand curve firm horizontal summation of these two demand curves. So, that these demand curves everyone so, each individual whose demand curve is this their willingness to pay is higher than this something like this we will get, okay.

So, here so, capacity of firm one is this and we have assumed that this is the situation suppose- $P_1 < P_2$. So, everyone wants to buy from firm 1 since the price is less. But firm 1 can only sell this much amount only satisfy this much quantity demanded-K_1. And what is quantity demanded at this it is total demand is this-$A - P_1$.

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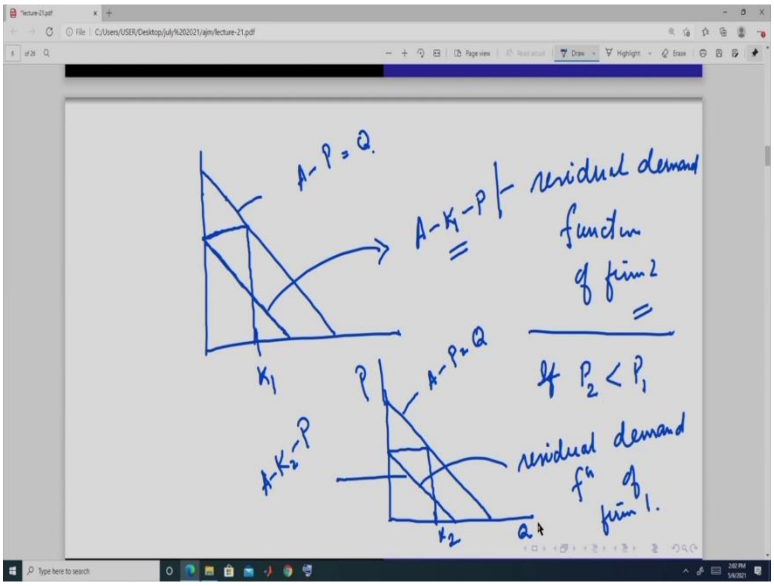

So, from this demand curve, what do we get is? So, first K units are going to be sold to those who value the goods maximum. So, this demand curve is something like this- $A - K_1 - P_1$.. This is the residual demand curve of which is faced by firm 2. So, firm 1 is always going to sell this much at each prices. So, if the price is below this price then only it can sell its full capacity that is k1 otherwise if the price is here then it will be able to sell only this much amount of quantity if here it is only this much.

So, for prices less than this it is going to first sell this much so, firm 2 whose price is higher than the price of firm 1 is going to get this demand curve. So, from this demand curve we remove this much quantity and what is left is this that is being served by firm 2, okay. So, this is how the effect of rationing. So, what we do that when price of suppose price of firm 2 is less than the price of firm 1. So, in that case here it will be something like this. So, if price of firm 2 is less than price of firm 1, so in that case so firm 2 is going to sell the first k2 units to those who value it maximum.

This is the residual demand function of firm 1 in this situation this. So, this is A minus k2 minus p- $A - K_2 - P_1$, okay. So, P is here Q is here, okay. So, now I hope you have understood what do we mean by the rationing So, rationing here when the moment a firm rations it means it will only serve those consumers whose valuation is high. Now how they are going to identify that we are not bothered about it. So, we are not modeling.

So, that is a very valid question and it is a very difficult question also to set to address. But we are not addressing it here we have address such kind of thing when we were doing price discrimination in the case of Monopoly market. But here we are assuming that suppose firm 1, if it sets a price lower than the price of firm 2, it can identify the buyers who is valuing more and who is valuing it less, okay.

And if firm 2 sets a price lower than the price of firm 1, then it can identify the buyers in terms of their valuations, okay. We assumed that. So, now we come to the demand function of each firm.

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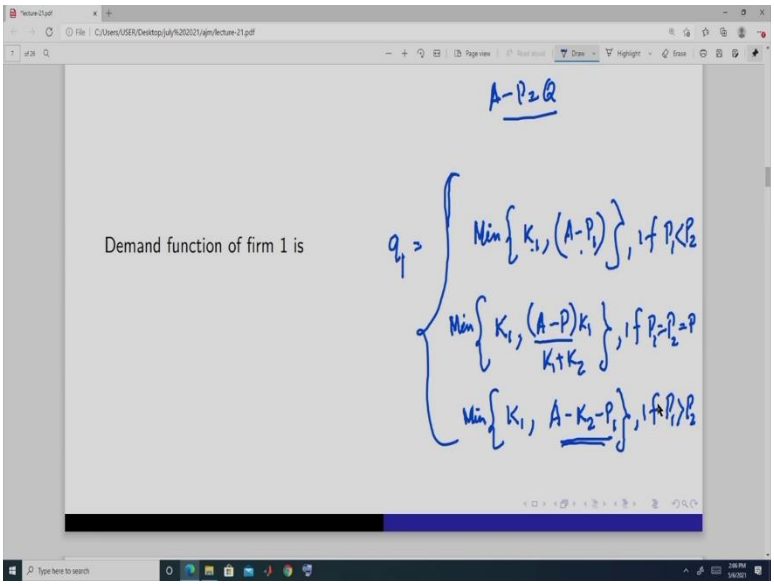

So, demand function of firm 1 that it faces is so its quantity demanded is q1 it is equal to minimum of k1 if p1 is less than p2- $q_1 = \text{Min}\,\{K_1, (A - P_1)\}, \text{if } P_1 < P_2$. If firm 1 sets a price less than the price of firm 2 can either. So, total demand is A minus p1 since the demand curve is A minus is this, right? now its capacity is k1 so it is can satisfy the demand of k1 or whichever is less than this, okay. And it is mean k1 A minus P k1 K1 plus k2 if p1 is equal to p2 is equal to p- $q_1 = \text{Min}\left\{K_1, \frac{(A-P_1)K_1}{K_1+K_2}\right\}, \text{if } P_1 = P_2 = P$.

So, if both the firms set a same price that is p then either its firm sell its capacity given if the total demand is less than its capacity, then it is shared proportionally based on their capacity it is this. So, this is the demand at price this and if so, this-$\frac{(A-P_1)K_1}{K_1+K_2}$ is proportionally shared. So, out of these k1 fraction of this whole capacity is going to be served by firm 1. And the next is k1 if firm sets a price which is greater than the price of firm 2, then either its sales is full capacity that is k1 or it serves only the residual whichever is less- $\text{Min}\,\{K_1,\ A - K_2 - P_1\}, \text{if } P_1 > P_2$.

If the residual demand is less than its capacity it is only sells is residual. So, this is the quantity demand that the firm faces in this kind of Bertrand competition with capacity constraint, okay. So, we have consider all the possible cases here.

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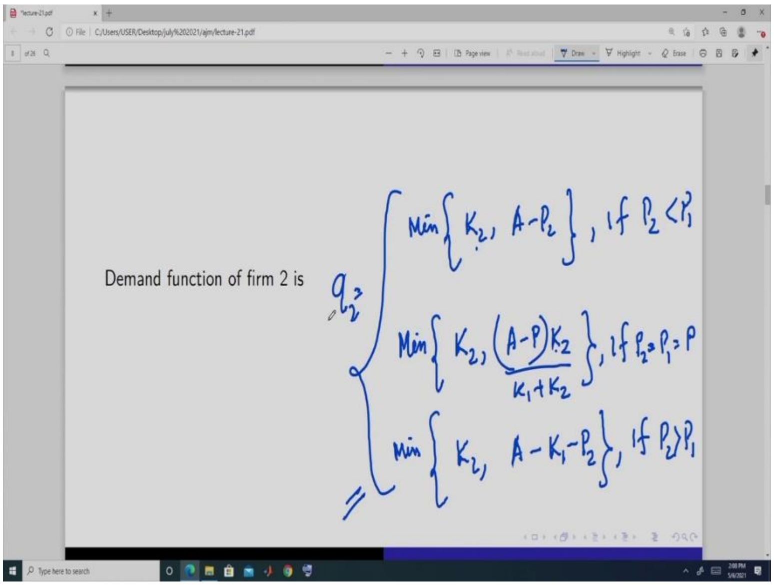

Next, similarly, for firm 2, we get q2 is equal to min k2 or A minus p2 if p2 is less than p1- $\text{Min}\,\{K_2,\ A - P_2\}, \text{if } P_1 > P_2$. So, if p2 is less than p1 then it serves if the whole demand at p2 is this, so, either it can sell this much amount or its capacity that is k2. Or it sells k2 if price of firm 1 and firm 2 are same and that is suppose p then either firm 2 sales up to its capacity or if the aggregate demand at that price is less than the aggregate capacity then its capacity then it sells this fraction- $\text{Min}\,\left\{K_2, \frac{(A-P)K_2}{K_1+K_2}\right\}, \text{if } P_1 = P_2 = P$, okay. And when it sets the higher price either it sells its capacity or it sells up to sells its residual demand if p2 is greater than p1, okay. So, this is the demand curve faced by firm 2- $\text{Min}\,\{K_2,\ A - K_1 - P_2\}, \text{if } P_1 < P_2$.

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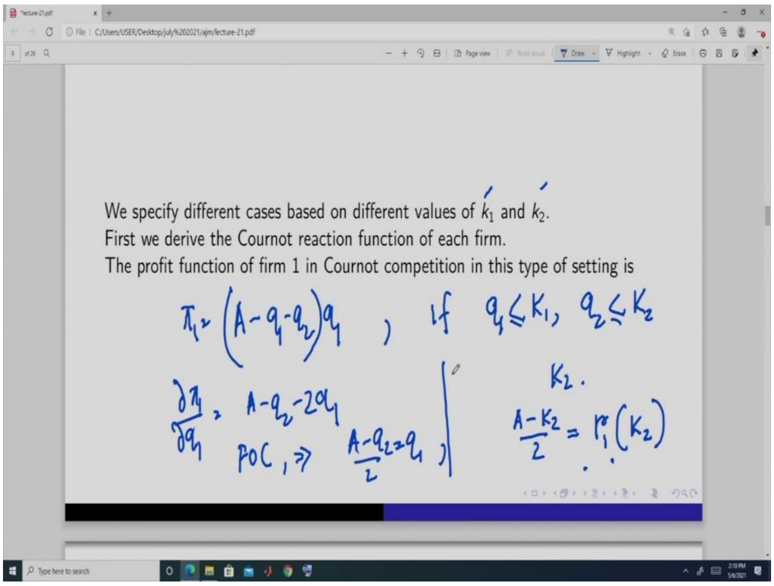

Now, we will specify different values of k1 and k2 different way and based on that we will specify different combinations of k1 and k2 to solve the, or to find the pure strategy Nash equilibrium. So, before doing that let us first look at the Cournot reaction function of this in this case. So, profit of firm 1 can be written in this way if k1 is like this- $\pi_1 = (A - q_1 - q_2)q_1, \text{if } q_1 \leq K_1, q_2 \leq K_2$. Now, so, this is the Cournot profit function and provided this are satisfied then optimizing with respect to q1 we get.

So, first order condition gives us. So, this is the Cournot reaction function- $\frac{A-q_2}{2} = q_1$. From this reaction function we can derive another reaction function and it is of this nature. Suppose capacity of firm 2 is k2, okay. Then what is the best response suppose firm 2 is producing at its capacity k2, what is the best response of firm 1? Best response of firm 1 based on this is this- $\frac{A-K_2}{2}$. So, this we can write in this way as the reaction function of firm 1 when firm 2 is producing at its capacity.

So, now plug in different values of k2 different capacity of firm 2 and you will get the best response of firm 1 what is the best capacity it wants this or it wants to produce it is this from the Cournot thing, right? So, we will require this remember.

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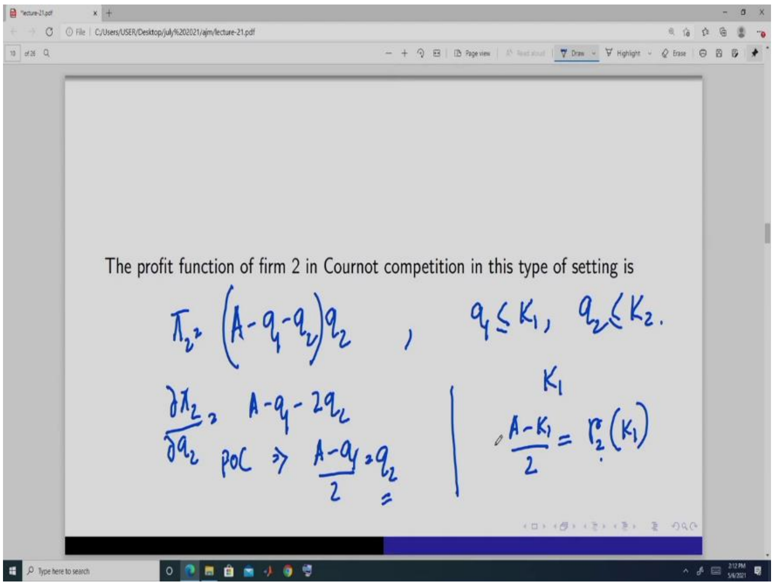

Next profit of function of firm 2 in this Cournot if we take this is the aggregate price this is the market price and this is the output given this since 0 cost so, cost portion is not there, i.e. $\pi_2 = (A - q_1 - q_2)q_2$. So, first order condition gives us this- $\frac{A-q_1}{2} = q_2$. So, this is the reaction function of firm 2 given output of firm 1. So, now suppose the capacity of firm 1 is k1 you plug in the capacity here of firm 1 suppose it is producing at its capacity then the firm 2 the best output a base response of firm 2 is given by this function.

And this is so, plug in values of capacity of firm 1 and you will get the best response of firm 2 this- $\frac{A-K_1}{2} = P_2(K_1)$, right? So, these are the so, now we will define the capacities based on these two reaction functions, okay.

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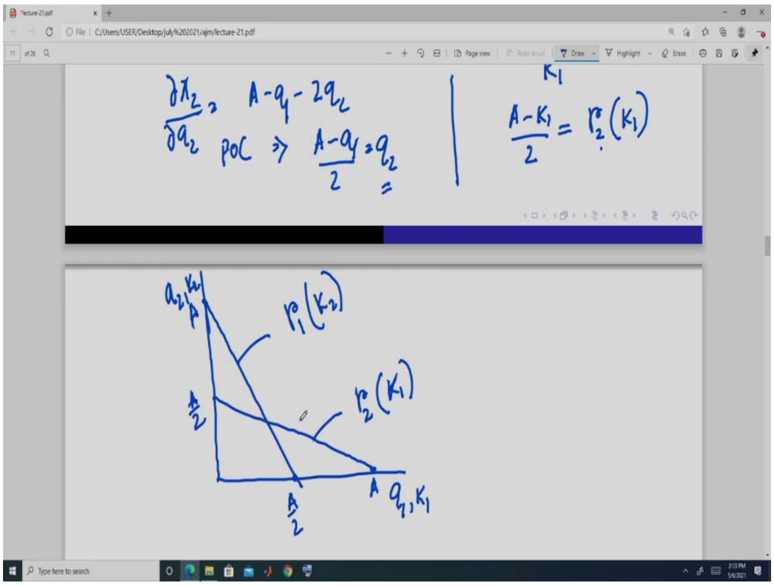

So, if we simply plot these reaction functions this. So q1, q1 comma k1, q2 comma k2 this is A and this point is A by 2 and this you can say this from this here when this- $P_2(K_1)$, is equal to 0 that means k2 is A k2 is here for each k2 we get and when k2 is 0 this is A by 2. So, we get this; similarly, the reaction function of this is A and this is A by 2 and this is react, plug in the value of k1 we get the best response of firm 2 when k1 is 0 this k1 is 0 this is equal to A by 2 this is this and when this is equal to 0, so, it is A, right?

So, when k1 is A this is equal to 0 you can say. Now, we so, this hope you are following what I have done. Now, we specify the case.

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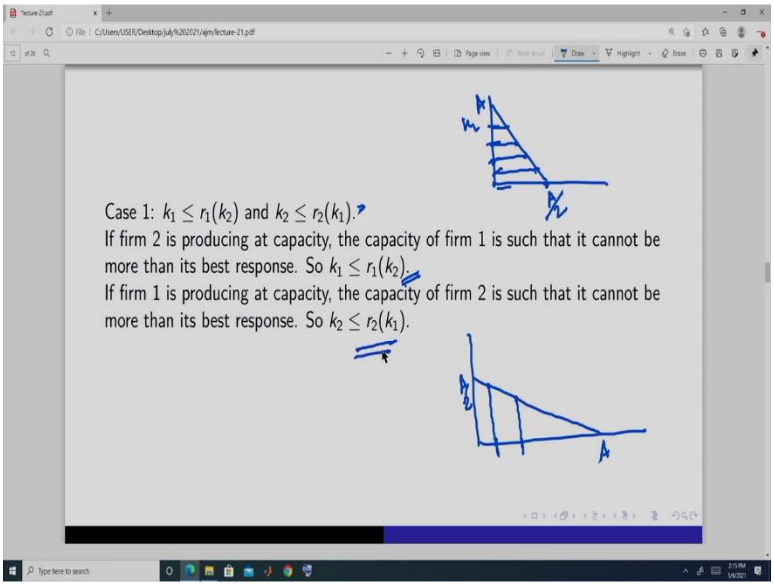

So, first case is this that capacity of firm 1 is always less than equal to this function this reaction function of firm 1. And capacity of firm 2 is always less than equal to this reaction function. That means, here if firm 2 is producing at its capacity that is k2 the capacity of firm 1 is such that it cannot be more than its best response. So, that is why it is this. So, in this situation if this is A and this is A by 2 you plug in any value here k2 you will the k1 is here if it is k2 is this k1 is this if k2 is this k1 is this k2 is this k1 is this if k2 is 0 k1 is this A by 2, okay.

Similarly this if firm 1 is producing at capacity, then capacity of firm 2 is such that it cannot be more than its best response. So, k2 is less than k1, i.e $k_2 \leq r_2(k_1)$ so, it is something like this. So, plug in here suppose this is the capacity of firm 1 if it is producing here firm 2 is capacity is should not be greater than this, if capacity of firm 1 is this capacity of firm 2 should not be greater than this. So, it is from here, okay.

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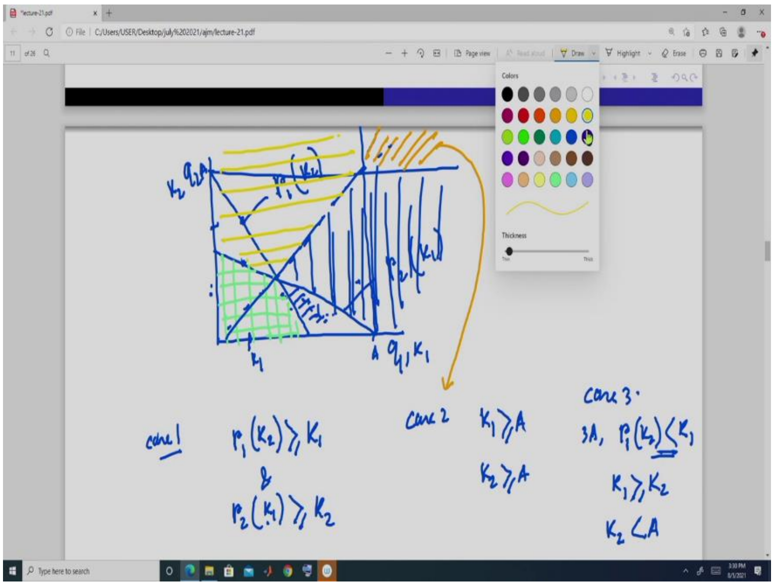

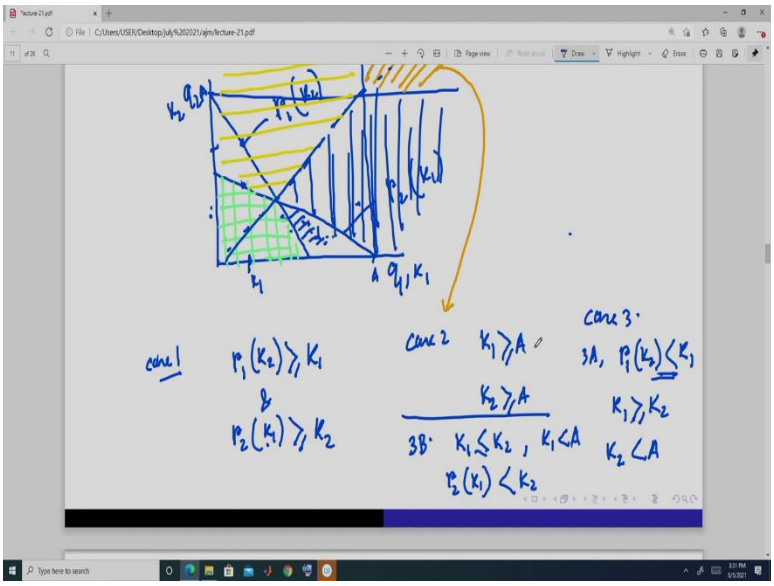

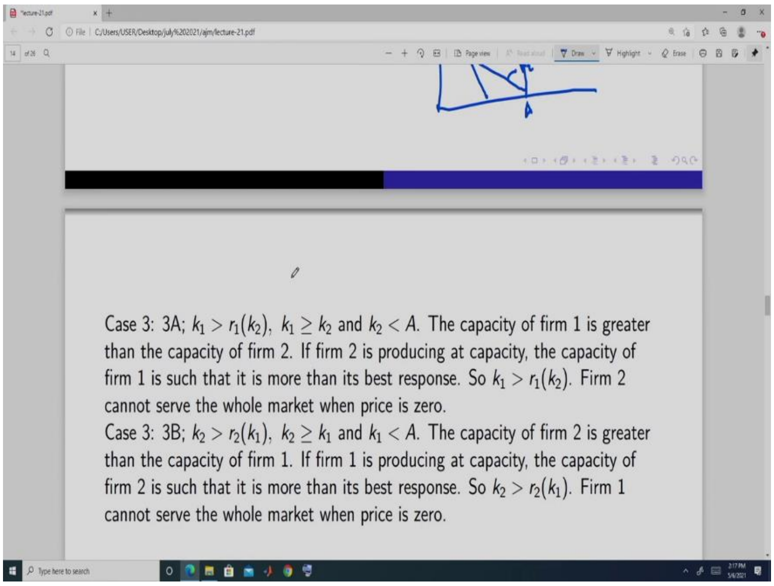

So, output of firm 1 here output of firm 2 our capacity of firm 1 capacity of firm 2 this we get as the reaction function of firm 1, this we get reaction function of firm 2. Now and in case 1 we have got this whole region and this whole region but when we combine these so, what is the case 1 saying? Case 1 says that r1, sorry, r1 k2 these reaction-$r_1(K_2) \geq K_1$ is greater than equal to k1 and this is k2- $r_2(K_1) \geq K_2$. So, if I fix this k1 then the reaction function from this we will get it should be here from this. So, k2 should be less than this.

So, here in this region and here if I fix suppose in here k2 then the reaction from this it should be in this line so, it should be less than this here. Now, if I specify k2 here then if we only have this then it should be here. But take this k1 k2 is should be less than this portion, so, that is why we will not include this portion and not include this portion only this portion is in case 1. So, suppose let us mark it with a different color this green color this region is actually the case 1 this, this green color, okay.

Now, in case 2 we know that case 2 is this k1, k1 is greater than equal to A and k2 is greater than equal to A. So, here this point is A and this point is A so, this region is actually giving us is giving us case 2. So, case 2 is this-$K_1 \geq A, K_2 \geq A$. So, the market size is sufficiently big so, that both the firm can satisfy the or meet the whole market, okay. And this portion is given by this case this k1. So, suppose the capacity of firm 1 is greater than the capacity of firm 2 and if firm 2 is producing at capacity that is k2 the capacity of firm 1 is such that it is more than its best response.

It is like this-$k_1 > r_1(k_2)$ and firm 2 cannot serve the whole market when price is 0. So, case 3 is so, this line we will get this this is the 45 degree line. Now, here case 3 is we have defined that there are two part case 3A and case 3B. So, 3A is r1 k2 this is less than k1-$k_1 > r_1(k_2)$, k1 is greater than equal to k2 and k 2 is less than A. So, this portion is obviously and this portion is also part of it because this r1 if you fix k here it has to be greater than.

So, these points is satisfy this points. So, we get this, okay. This is case 3A and 3B is given by this this region. This region is 3B and 3B is it is this I am writing 3B here. So, it is k1 less than equal to k 2, k1 is less than A and r2 k1 this is less than k2-$k_2 > r_2(k_1)$. So, this portion. So this portion also comes. So, these are the three cases and in case 3 we have two sub cases.

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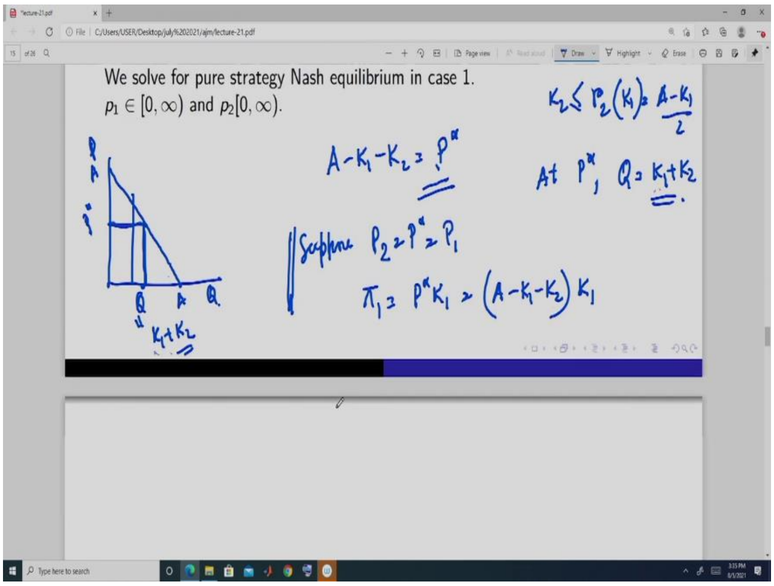

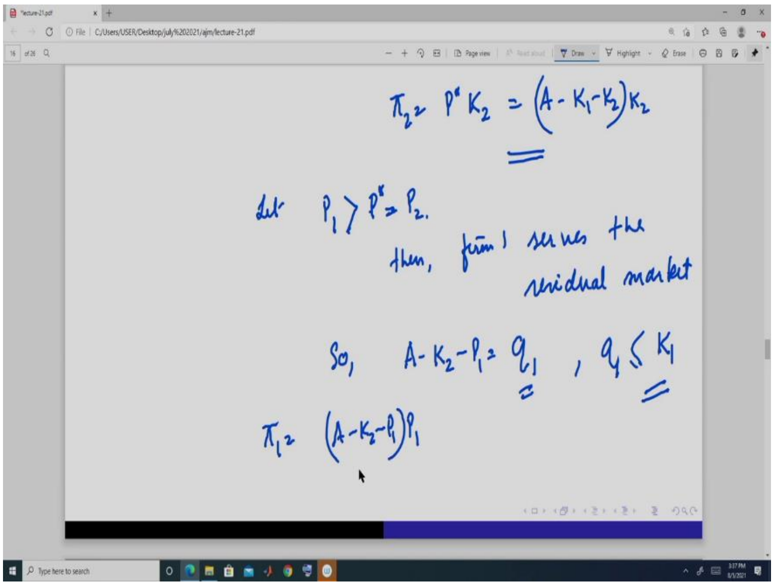

So, based on this we have got what we know the all the possible combinations of cases of k1 and k2 we have specified. Now, we have to find a pure strategy Nash equilibrium. So, that is what is going to be the p1 and what is going to be the p2 in this market p1 and p2. P1 the strategy set is this 0 to infinity, p2 strategy set is 0 to infinity. Suppose, we have this situation now, let us assume that there is a price and that is A minus this is P star we have a price like this-$A - K_1 - K_2 = P^*$. In this case we will definitely have a positive price like this.

Because in case 1 the capacity this is less than equal to this which is equal to this-$K_1 \leq P_1(K_2) = \frac{A-K_2}{2}$ and k2 is less than this-$K_2 \leq P_2(K_1) = \frac{A-K_1}{2}$. So, we will have a price like this. Now, we will show that this is the pure strategy Nash equilibrium in this case. Now, if suppose P2 is equal to P star, okay and suppose p1 is also equal to their price then the profit of firm 1 because it is selling k1 and price is this-$P_2 = P^* = P_1$. So, it is selling at this price and it is selling up to its capacity k1 and this is same as this, this-$\pi_1 = P^*K_1 = (A - K_1 - K_2)K_1$ and here mind this we will have a unique price here, why we will have a unique place?

Because our demand function is something like this it is A P Q it is A and this is A. So, it is a downward sloping straight name. So, if we fixed any fixed quantity here you will get a unique price if you fixed any quantity here we will get a unique price and at this price at P star demand Q is, equal to k1 plus k 2. So, if this is P star then this Q is equal to k 1 plus k2. So, it is such that both the firms can supply up to its capacity it is not less than that neither it is more than that, okay. And here we see that the profit of firm 1 is this- $\pi_1 = P^*K_1 = (A - K_1 - K_2)K_1$if it is.

And the profit of firm 2 profit of firm 2 is this and this is equal to this-$\pi_2 = P^*K_2 = (A - K_1 - K_2)K_2$. Now we have to show whether this we have to show that this is a pure strategy Nash equilibrium, okay. So, how do we proceed? Here let assume, lets suppose p1 is greater than P star and it is equal P2. So, if firm 2 sets a price P star then suppose firm 1, set a price higher than this, then this implies what? Then it will firm 1 serves the residual market so the demand curve of firm 1. So, the demand curve of firm 1 is- $A - K_2 - P_1 = q_1$, if $q_1 \leq K_1$ k2 q1 provided q1 is less than equal to, okay.

So, if this is the case then what is the profit of firm 1. Profit of firm1 is this-$\pi_1 = (A - K_2 - P_1)P_1$. Now, so, there can be many such prices which satisfy this we have to find that price which is the maximizing this.

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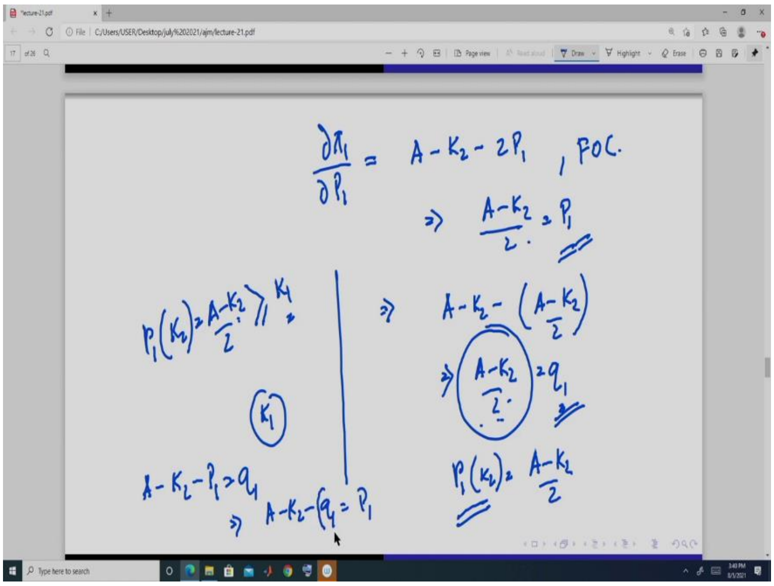

So, what we do let us we are taking partial because in this case there is a strategic interaction. So, price this profit is also a function of p2, okay although it is not appearing explicitly but it is appearing based on the condition, right? So, that is why because this is the condition, right? This-$P_1 > P^* = P_2$. So, that is why I am taking a partial. So, this is this first order condition will imply that it is this-$\frac{d\pi_1}{dP_1} = A - K_2 - 2P_1, \frac{A-K_2}{2} = P_1$. So, if you plug in this price in the demand function of firm 1 that is the residual demand function if this.

Now, this- $\frac{A-K_2}{2}$ is equal to q1 now, we know that this is r1 is given by this form. So, if firm 2 is supplying up to its capacity then what is the best response of firm 1 it is given by this-$r_1\ (K_2) = \frac{A-K_2}{2}$ and if firm 1 suppose charges a higher price and it is charges a higher price so, it is serving the residual market in that case what is the optimal price? it will set optimal price is this-$\frac{A-K_2}{2} = P_1$ an optimal quantity to sell is this much, right?

So, this is when it is acting as a monopolist in the residual market and this when it is given a capacity of firm 2, what is the optimal output of firm 1 it is given by this-$r_1\ (K_2) = \frac{A-K_2}{2}$. Now, in case 1 we know that this which is equal to this this is greater than or equal to k1-$r_1\ (K_2) = \frac{A-K_2}{2} \geq K_1$. So, in this case what is happening if our firm 1 sets a price think initially thinking that his price is going to be higher than the price set by firm 2 that is P star then it is acting as a monopolist in the residual market.

In that case, if it wants to maximize or it wants to set the monopoly price in the residual market, it sets a price this- $\frac{A-K_2}{2} = P_1$and the output itself is this-$\frac{A-K_2}{2}$. Now, this output is actually greater than equal to its capacity. So, it means what, even if it wants to sell this much amount of output, it cannot sell this much. So, it will sell k1. Now, remember this the demand function is of firm 2 firm 1 is this one- $A - K_2 - P_1$and this you can get this- $A - K_2 - q_1 = p_1$.

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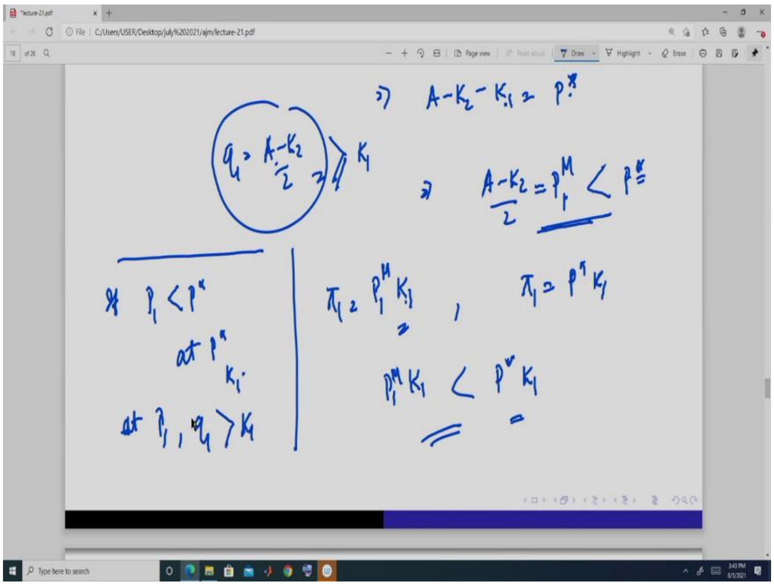

Now, here, when we plug in k1 here, when we plug in k1 we get the price P star. So, this means what and here the monopoly in the residual market if this is equal to this firm should be selling this much amount should be producing this. So, and this is actually greater than k1. Now if we plug in this here, we will get a price which is less than this. So, this means this means which is the this which is the you can I am writing this Pm.

This is supposed to P1m, okay this is less than star-$\frac{A-K_2}{2} = P_1^M < P^*$. Now, so, here profit because it will sell if it is acting as a monopolist in the residual market this, this is the profit of firm 1-$P_1^M K_1$ because it cannot sell this much. So, this is less than greater than equal to this here. So, it will be able to sell only this and if it sets P star its profit is this-$P^* K_1$ and we have shown that this is less than this. So, that is why we get this-$P_1^M K_1 < P^* K_1$.

So, this means that profit up firm 1 is higher if it sets a sets the price P star rather than trying to act as a monopolist in the residual market. Because if it wants to act as a monopolist in the residual market, it will end up charging a price which is less than this P star. So, that is why this is not optimal. So, this strategy of firm 1 is not optimal-$P_1 > P^* = P_2$. Now, if suppose firm 1 sets a price which is less than q if P1 is less than P star. So, at P star it sells k1. So, at P1 q 1 this will be greater than this will be greater than, k1.

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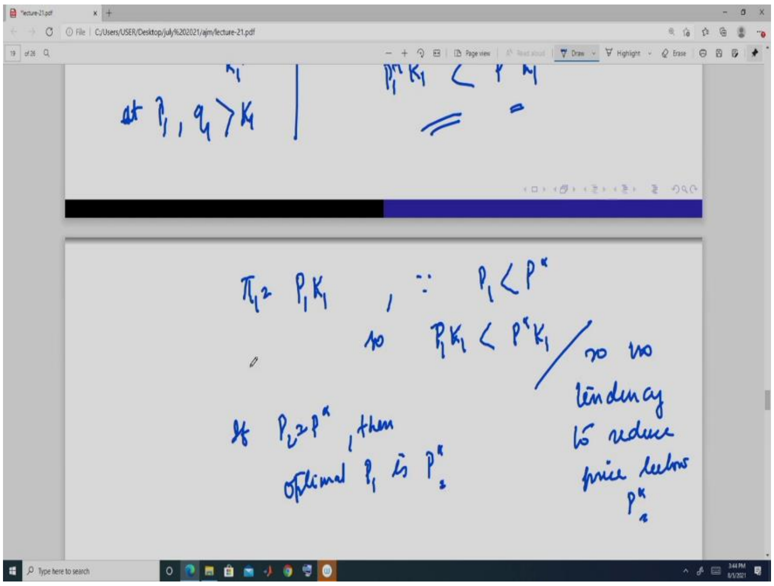

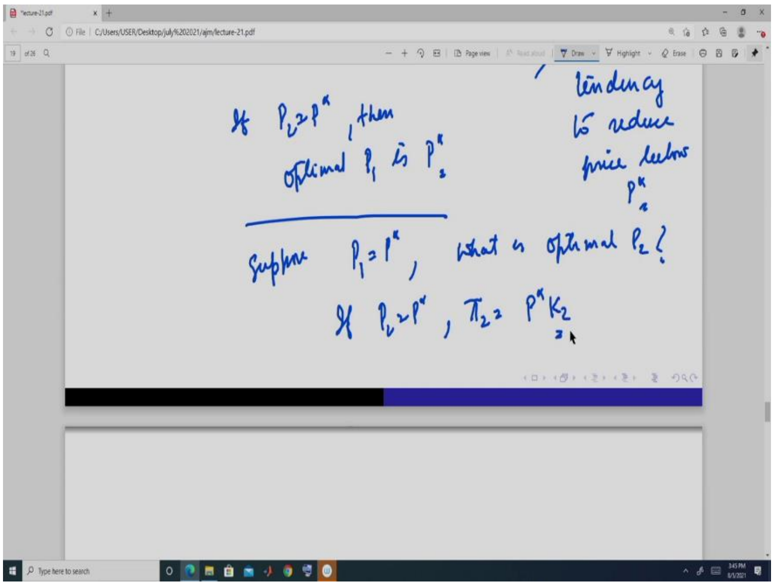

So, if this is the case then profit of firm 1 is going to be P1 into k1 and since P1 is less than P star. So, is less than P star k1. So, to know tendency to reduce price below P star. So, we get that if P2 is equal to P star then optimal P1 is P star, right? Now, we have to show the whether it is a Nash equilibrium or not. So, again suppose P1 is equal to P star, okay when P1 is equal to P star we have to find what is optimal P2. So, we know if P2 is also equal to P star then profit of firm 2 is P star into k2.

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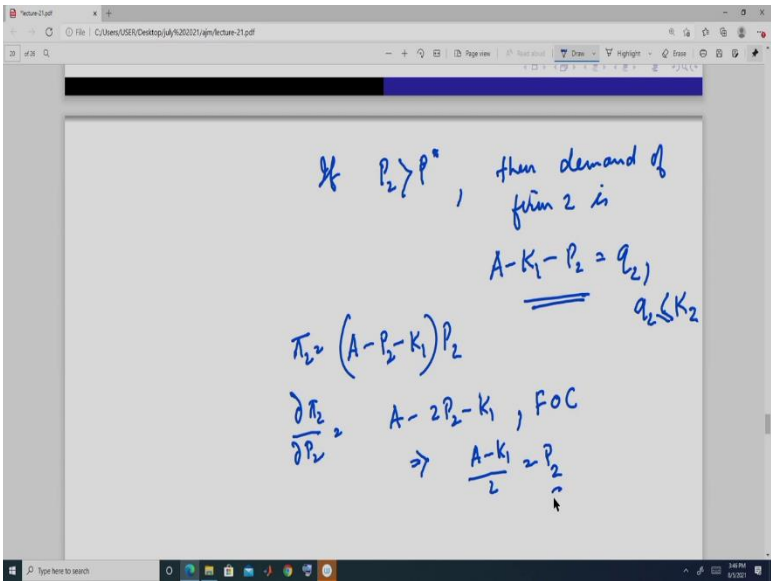

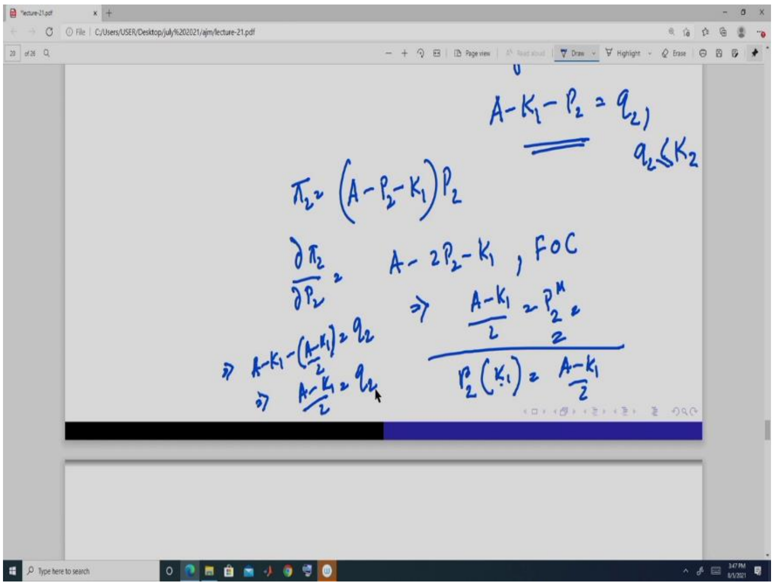

Now, if P2 is suppose greater than P star then of firm 2 is A minus k1 minus p2 this-$A - K_1 - P_2 = q_2$ it will start the residual market provided k2 is less than equal to k2. So, we have to find a price p2 such that it is greater than P star and also it is maximizing its profit. Because there can be many such p2s. So, P2 profit function of firm 2 now is this-$\pi_2 = (A - P_2 - K_1)P_2$. So, again optimizing with respect to P2, we get this first order condition gives this-$\frac{d\pi_2}{dP_2} = A - 2P_2 - K_1$.

So, this is what? So, this is again the Cournot reaction output. So, if r1 is producing up to its capacity that is k1 what is the optimal output of firm 2 it is given by this a which is equal to this-$P_2(K_1) = \frac{A-K_1}{2}$. So, let us this is M, so we get this. And from here we get so, we get this, it is this-$A - K_1 - \frac{A-K_1}{2} = q_2$.

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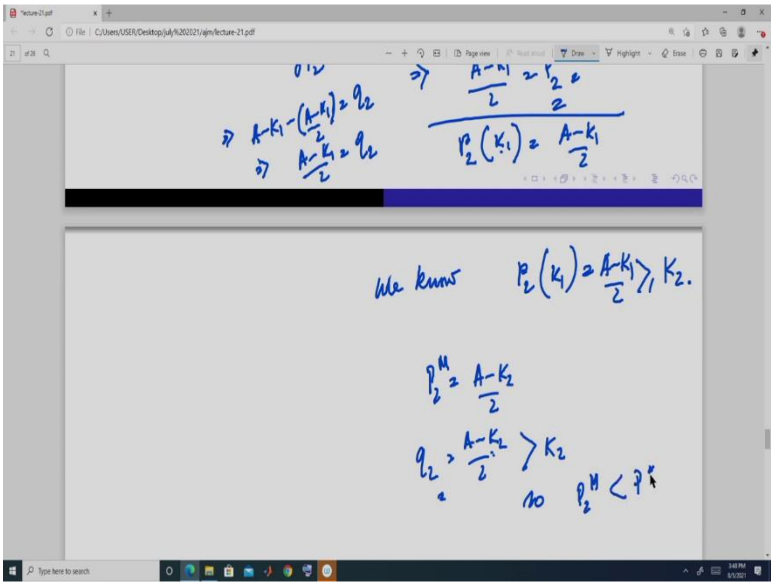

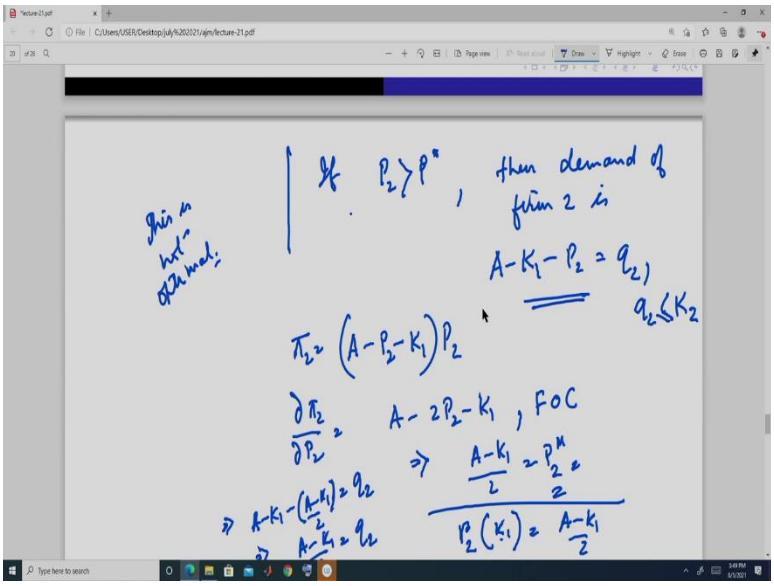

So, and we know that r2 k1 which is equal to A minus this is greater than equal to k2, i.e. $P_2\ (K_1) = \frac{A-K_1}{2} \geq K_2$. So, even if firm 2 wants to sell this it will not be able to sell it will only be able to sell k2. Now, if it wants to sell this it sets a price which is p2M. And p2 M is equal to $\frac{A-K_2}{2}$. So, at this price quantity is it sales is it can it wants to sell is this- $\frac{A-K_2}{2}$ since it is a downward sloping demand curve the residual demand curve is also downward sloping.

So, these since this is greater than k2, so, p2 M is actually less than P star here. So, if that is the case then profit of firm 2 when it sets P2 M and k2 this is going to be less than P star k2. So, this is not optimal $P_2 > P^*$.

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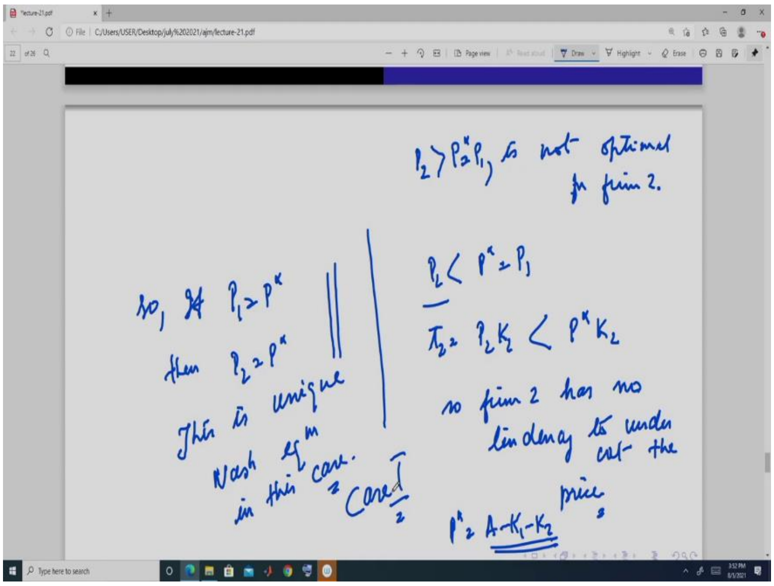

So, we get that P 2 greater p star than is not optimal for firm 2. Now, if p2 is less than P star which is equal to P1 say firm 1 sets that price in this case profit of firm 2 is P2 into k2 this is definitely less than p start k2- $P_2K_2 < P^*K_2$. So, firm 2 has no tendency to undercut the price. So, from here we get that. So, if p1 is equal to P star then P2 is also equal to P star. So, therefore, this is unique Nash equilibrium in this case and this is case 1.

So, in case 1 we get that there is going to be a unique Nash equilibrium and that Nash equilibrium is given by this case this here. So, it is such that the price is equal to prices such that it is the firms are producing at their capacity. So, P star is equal to this-$P^* = A - K_1 - K_2$. So, this is P star. So, when both the firms are going to set a common price that is P star and at that P star the demand is such that both the firms produce up to its capacity.

And this is the unique thing because we have seen that there is no tendency of any firm to set a price higher than P star neither there is a tendency to lower the price, okay. And for the other cases that is in case 2 and case 3 we will do later in the next class. Thank you.

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 30
Bertrand Competition with capacity constraint

Hello, everyone. Welcome to my course Introduction to Market Structures.

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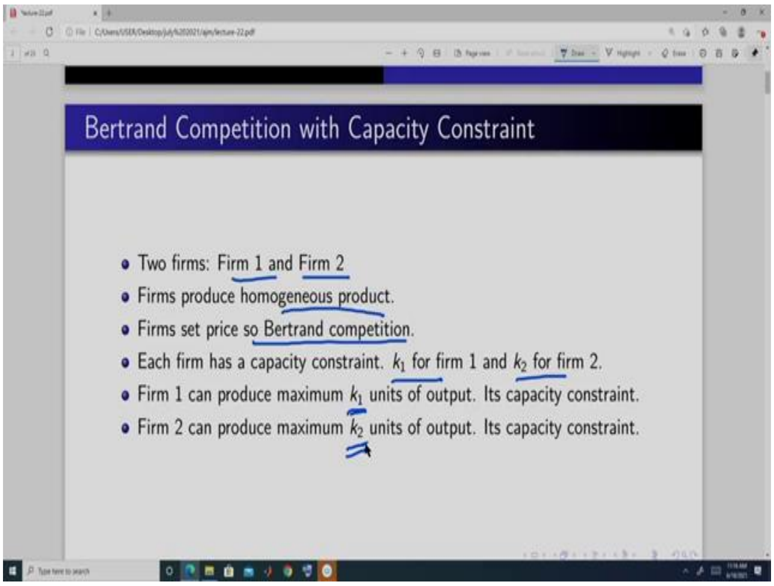

We were doing Bertrand competition with capacity constraints. So, and we have proved only find the pure strategy Nash equilibrium in case one and we will continue. So, our model is this we have two firms, firm 1 and firm 2, both produces homogeneous product that means they produce similar product, whether you buy from firm 1 or firm 2 it does not matter.

Again each firms sets price on their choice variable is price so it is a Bertrand competition. And what we do, we introduce capacity constraints. So, firm 1 capacity is k1 and k2 for firm 2. So, capacity here means that firm 1 can produce maximum k1 units of output and firm 2 can produce k2 units of output.

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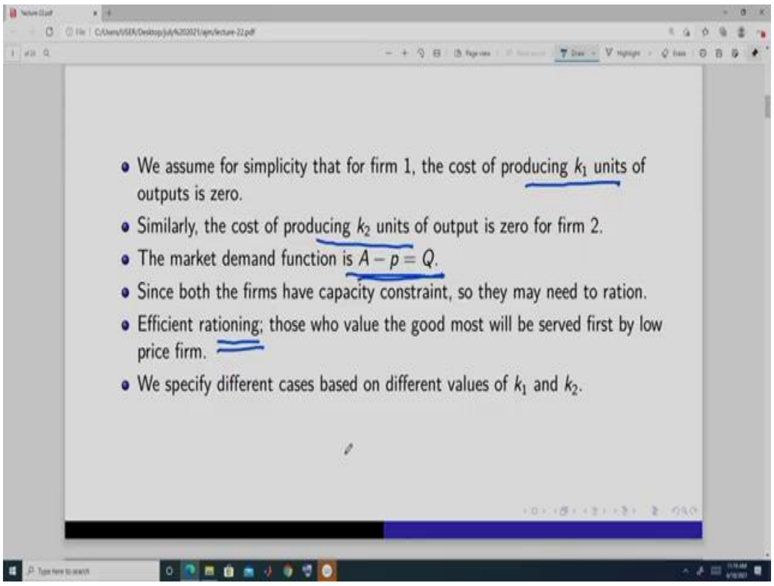

And for simplicity we assume that the cost of producing output is 0. So, till k1 there is no cost of production for firm 1 and it cannot produce more than k1. Similarly, for firm 2 there is no cost of production till k2 units of output and it cannot produce beyond k2, okay. So, this is the structure of the firm. And the market demand is this- A-p=Q. So, it is a linear straight line which is downward sloping, okay.

So, since both the firms have capacity constraint, so they may need to ration. What do we mean by ration? So, suppose the demand is 10 units and each firm can produce suppose firm 1 can produce 5 units, firm 2 can produce suppose 3 units. So, they can produce only 8 units, but the demand is 10. So, 2 person or 2 units cannot be met, cannot be supplied. Now, who is going to forego among the buyers or among the demanders, consumers, who is going to forgo this amount. So, that is the rationing. So, the firm use a method to ration.

Unlike suppose each individual demand 1 unit and suppose there are 10 individuals who demands 1 units and firm 1 can produce 5 units and firm 2 can produce suppose 3 units, so two person they cannot be supplied. So, who are going to be these two persons, right? So, here in this module we use efficient rationing. Efficient rationing means that one who value the goods most is going to get it.

So, it means that that if I am willing to pay more, I will get that good fast. And if my willingness to pay is less, I will get it later or I may not get it, okay. So, the firm which sets the lowest price they sells to the consumers whose willingness to pay is high, okay.

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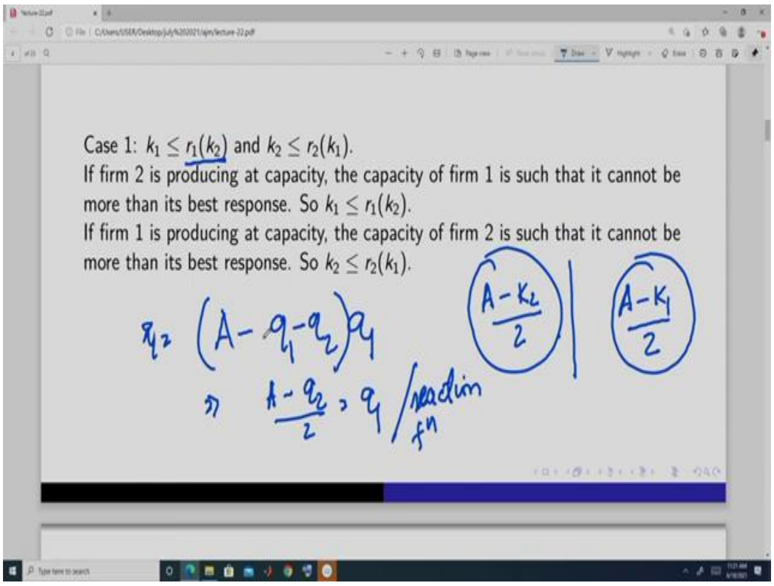

So, we specify different cases and we have done it like in case 1, it is suppose the capacity of firm 1, k1 is less than this amount-$r_1(k_2)$. What is this amount? This is the best response. Suppose firm 1, firm 2 produces k2 unit, what is the Cournot best response amount of output of firm 1 this is given by this. So, this is r1k. So, in this case, if we want to derive, so suppose our Cournot output of firm 1, output of firm 2 and, so this is the profit of firm 1-$\pi_1 = (A - q_1 - q_2)q_1$. Suppose q1 is less than equal to k1 and q2 is also less than equal to k2. So, here, in this case we get this to be, so this is the reaction function-$\frac{A-q_2}{2} = q_1$, Cournot reaction function.

Now, k2, q2 the maximum it can take is k2. So, suppose firm 2 produces at its capacity then what is the best response for firm 1, this amount-$\frac{A-k_2}{2}$. So, here it says that the capacity of firm 1 given a capacity of firm 2 is less than equal to this amount. Similarly, for firm 2 we will get a reaction function like this-$\frac{A-k_1}{2}$. So, k2, if k1 is fixed k2 is such that it is less than equal to this amount-$\frac{A-k_1}{2}$, okay and we have found pure strategy Nash equilibrium in this case in the previous lecture.

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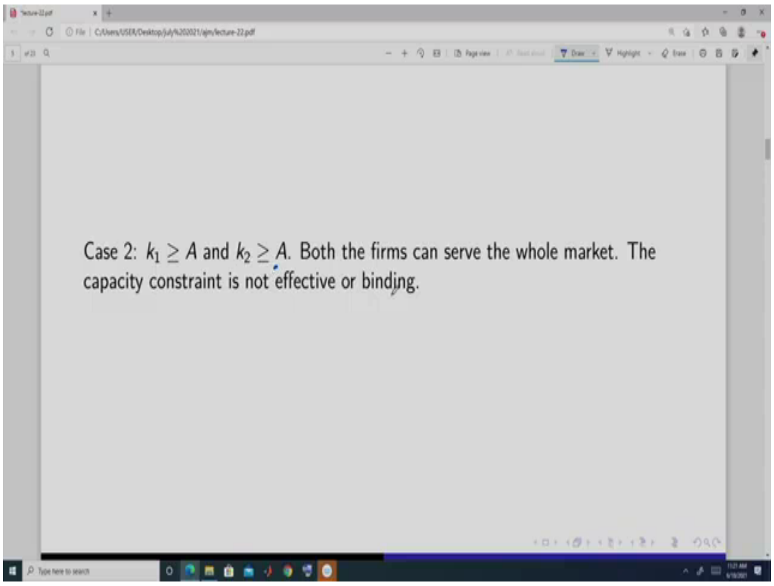

Next in case 2, we have suppose both the firms can produce more than its, more than the size of the market. Their capacity is more than the size of the market. Size of the market here is A, because if the price is 0 what is the maximum output demanded in the market and that is A. So, the maximum output that is going to be demanded in this market is A and both the firms can supply A units. So, actually the capacity constraint is not at all binding in this case, okay. It is more. So, both the firms can produce. So, we will also solve this today.

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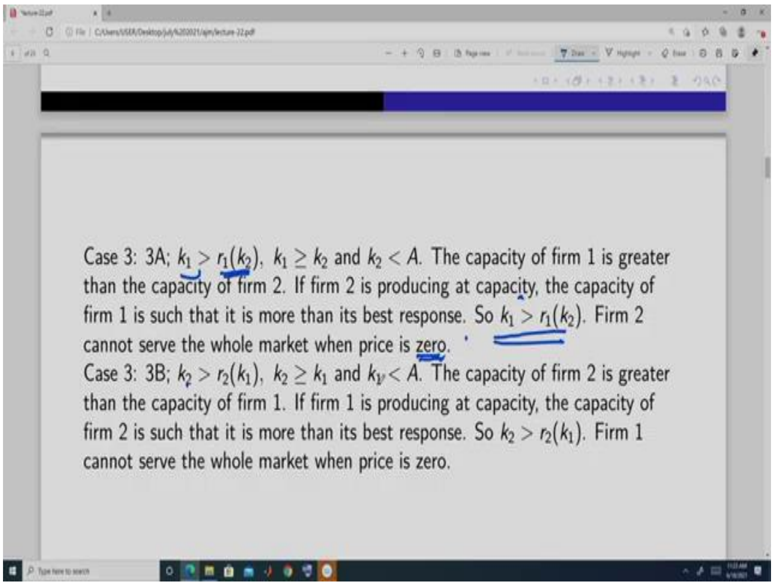

Next case is this, case 3A, where capacity of firm 1 is greater than this amount, (k_2), okay. In the case 1 it was less than this, but capacity of firm 1 is greater than the capacity of firm 2 and capacity of firm 2 is always less than A, but capacity of firm 1 can be greater than A. So, this is 3A. And 3B, so it means the capacity of firm 1 is greater than the capacity of firm 2. If firm 2 is producing at capacity, the capacity of firm 1 is such that it is more than its best response.

So, that is why it is this. Best response here it means the reaction function, Cournot reaction function that we have already done in the Cournot section that we dealt with. And also I have derived it explicitly in the last class. And firm 2 cannot serve the whole market when the price is 0, okay.

Similarly, this case 3, again 3B is just the opposite of this that is k2 is greater than equal to r2k1 that is if firm 1 is producing at its capacity k1 then the best response for firm 2 is given by this amount-$r_2(k_1)$ based on the reaction function. So, capacity of firm 2 is always greater than this. And this k2 is greater than equal to k1 and k1 is less than A, okay. So, when the price is 0, firm 1 cannot supply the whole market, okay.

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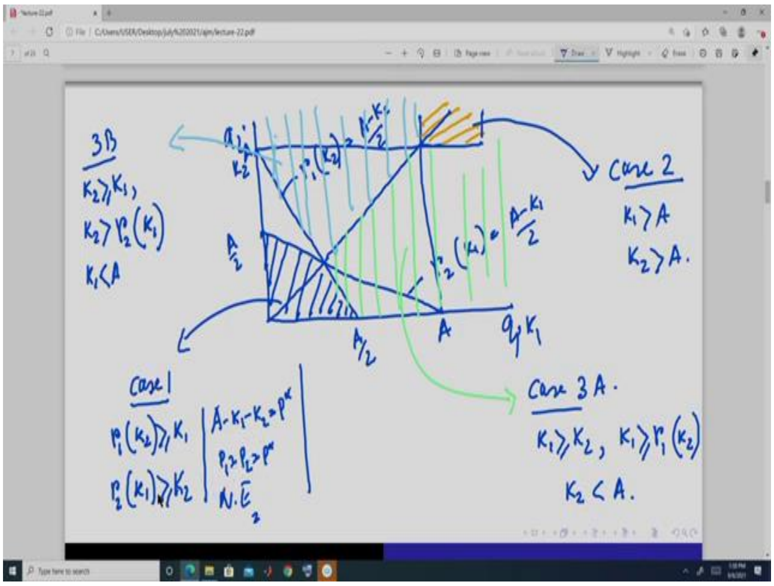

Okay, now, so in the diagram we can show it in this way. So, in this axis q1, q2 and capacity of firm 1, capacity of firm 2, so this is the reaction function of firm 1 given a capacity of firm 2. This is the reaction function of firm 2 given a capacity of firm 1 and this is we have shown already. And we have already proved the case 1 and the case 1 is this region. This is case 1 and here we have got the r1 this is greater than equal to k1, this-case $1 - r_1(K_2) \geq K_1, r_2(K_1) \geq K_2$. And here in this case we have shown the p star and p1 is equal to p2. So, this is the, this we have shown.

And this point is A, this is A by 2, this is A by 2, this point is A. And this region is case 2, case 2 where capacity of firm 1 is greater than A and capacity of firm 2 is greater than A, **okay** this whole region. And this region, this green region, this is case 3A, where we get, this is 3A where capacity of firm 1 is greater than equal to capacity of firm 2 and capacity of firm 1 is greater than equal to the optimal Cournot optimal output given a capacity of firm 2 and capacity of firm 2 is strictly less than A.

And this region, this light blue, this region is actually 3B. This is 3B. And here we get capacity of firm 2 is greater than equal to capacity of firm 1 and the capacity of firm 2 is greater than the Cournot optimal output of firm 2 given a capacity of firm 1 that is k1 and k1 is strictly less than A. So, these are the possible cases. And we have already shown, this case.

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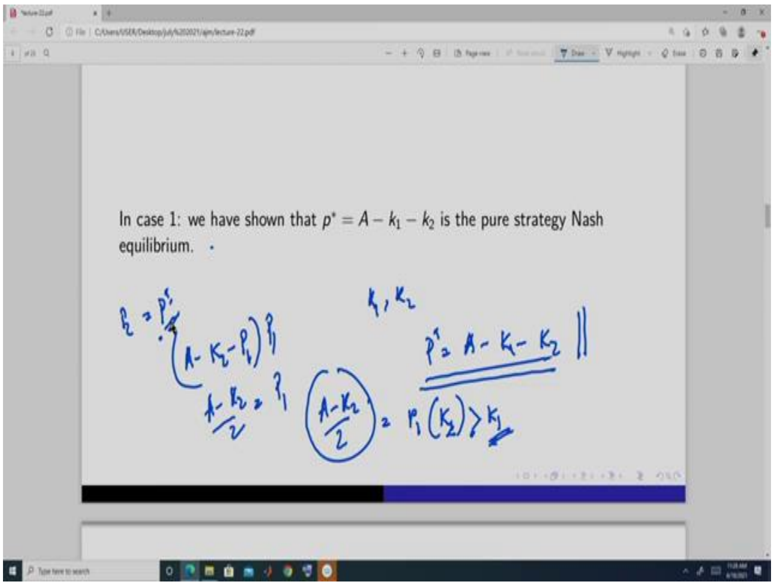

Now, in case 1 we have shown that firm 1 and firm 2 produces at capacity k1 and k2 and the price that we get if both of them produces that capacity is this p star and so this is the unique pure strategy Nash equilibrium in case 1 and we have shown this, okay. Next, we, how do we have got it, we have simply solved one problem that suppose firm 2 set this price p2 is equal to suppose this, okay.

Now, firm 1 suppose wants to be the monopolist in this. So, firm 1's demand curve will be, it will be this-$(A - K_2 - P_1)P_1$. So, it will be, price will be this-$\frac{A-K_2}{2} = P_1$. When we plug in this price quantity, it is this. This is what? This is actually this-$r_1(K_2)$. So, k1 is less than this-$r_1(K_2)$, right? So, it means what? It wants to produce this much, it wants to set a price p1 such that it produces this much amount, okay. But that will give it maximum, higher profit, but it cannot produce this much.

So, the nearest that it is possible to produce is k1, and when it produces k1 then the price it gets is the p star. So, that is why p star is this. And similarly, we can argue it for the case of firm 2 and we get this as the pure strategy Nash equilibrium, okay.

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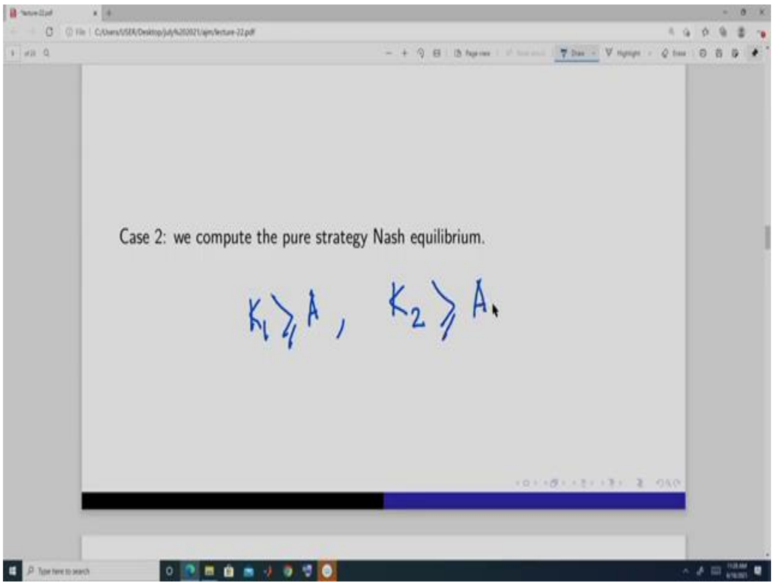

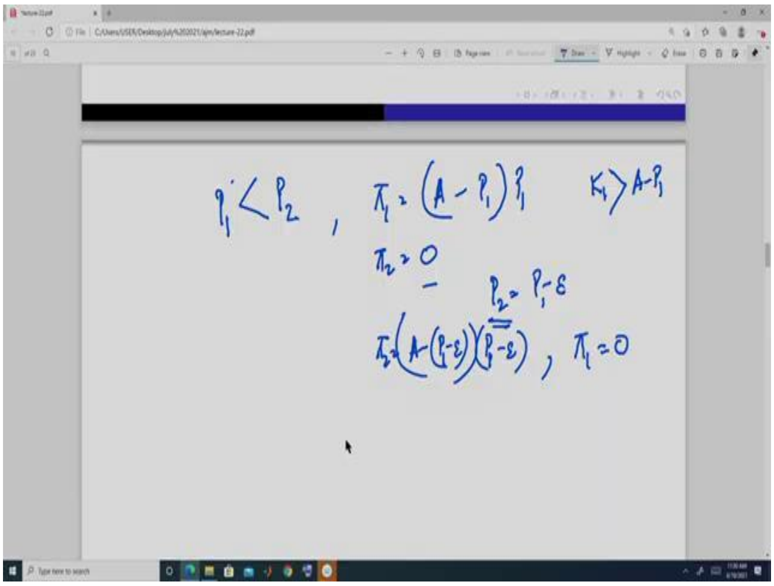

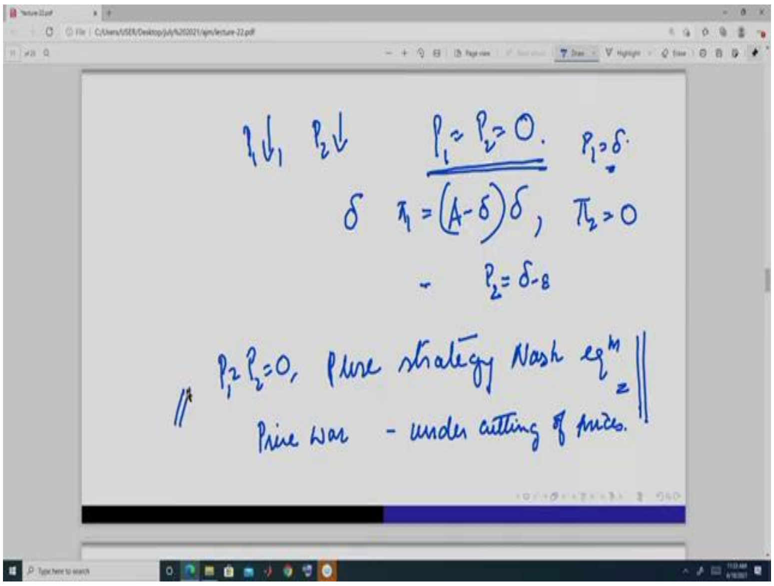

Next, here in this case, in case 2, capacity of firm 1 is greater than equal to A and capacity of firm 2 is greater than equal to A. Suppose firm 1 produces set of price which is p1 and suppose this is less than the price of firm 2. Firm 2 sets a price p2. Then profit of firm 1 is going to be this-$\pi_1 = (A - P_1)P_1$. Now, this is the demand. And we know k1 is always greater than this- $A - P_1$. So, firm 1 is not facing any constraint regarding the output that is the capacity constraint. Profit of firm 2 is 0, because firm 1 is supplying to all the buyers and so no one is going to buy from firm 2. So, firm 2 here is going to do what?

So, p2 best response is p1 small amount that is epsilon amount and this is this-$P_2 = P_1 - \varepsilon$. So, here when p2 is this then the profit of firm 2 is this-$\pi_2 = (A - (P_1 - \varepsilon))(P_1 - \varepsilon)$and profit of firm 1 is 0, because p1 is greater than p1 minus epsilon. So, what is happening? Firm 1, firm 2 is undercutting the price of firm 1. So, like this it will go on happening. So, price will go down. There is going to be continuous competition and each firm is going to undercut the price. So, finally, p1 is going to fall and similarly p2 is also going to fall.

So, what, it can go till what level. It can go till p1 is equal to p2 is equal to 0. So, any positive price suppose takes up some delta as a positive price, what is the demand at delta, this- $(A–\delta)\delta$. Delta is a very small positive amount suppose. Suppose its profit is this- $(A–\delta)\delta$ and p2 profit of this is this. So, what will happen? Again it will be price of, so p2 is delta minus epsilon amount. So, like this it will go on. So, any price greater than this p is equal to 0 is cannot be sustained. So, there is going to be someone who is going to undercut that price.

So, here pure strategy Nash equilibrium is p1 is this is the pure strategy Nash equilibrium. So, there is going to be continuous something called price war or undercutting of prices. So, finally, price goes down to 0 level, okay. So, in case 2 we get that the price is equal to 0. So, this is also obvious from the case that we have done in the first case when Bertrand competition between two firms and they have the same marginal cost and 0 fixed cost.

Now, there they have a positive marginal cost and that is the constant marginal cost. So, there the pure strategy Nash equilibrium is price of both the firms is equal to the constant marginal cost. So, same here, here the marginal cost is 0. So, the pure strategy Nash equilibrium is at that level of price that is price is equal to 0, okay. So, this proof and that proof is same. We get this.

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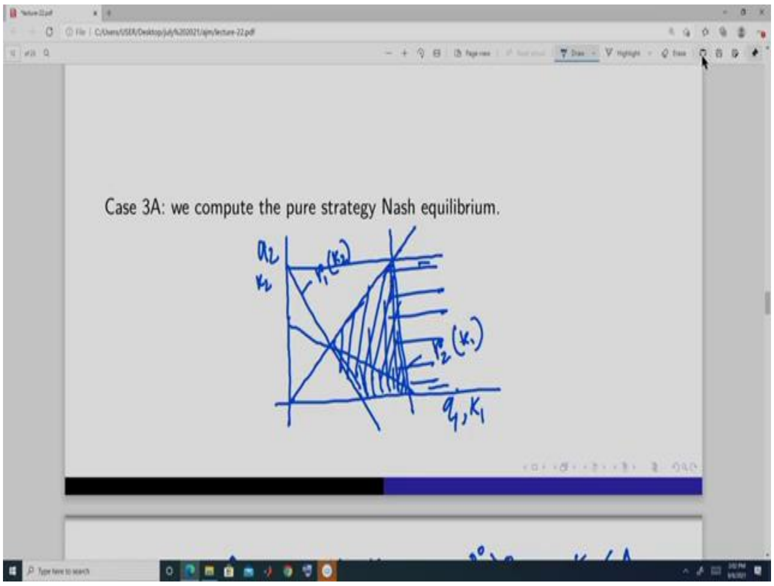

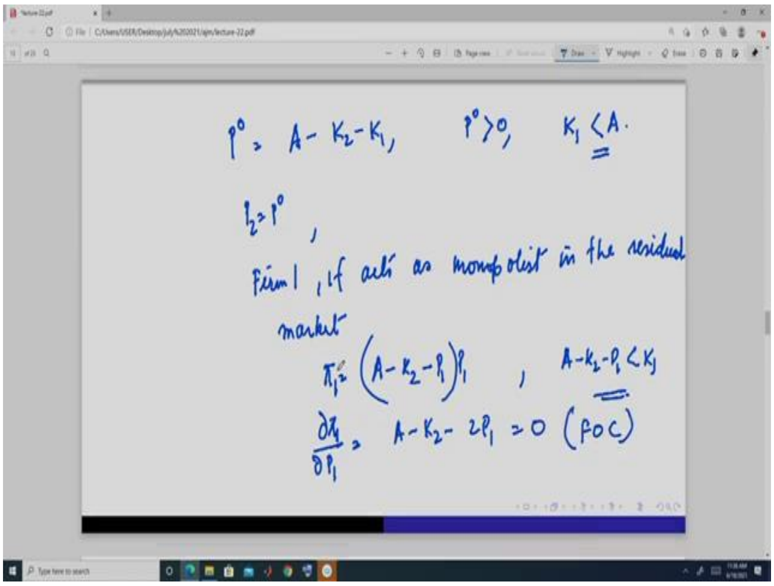

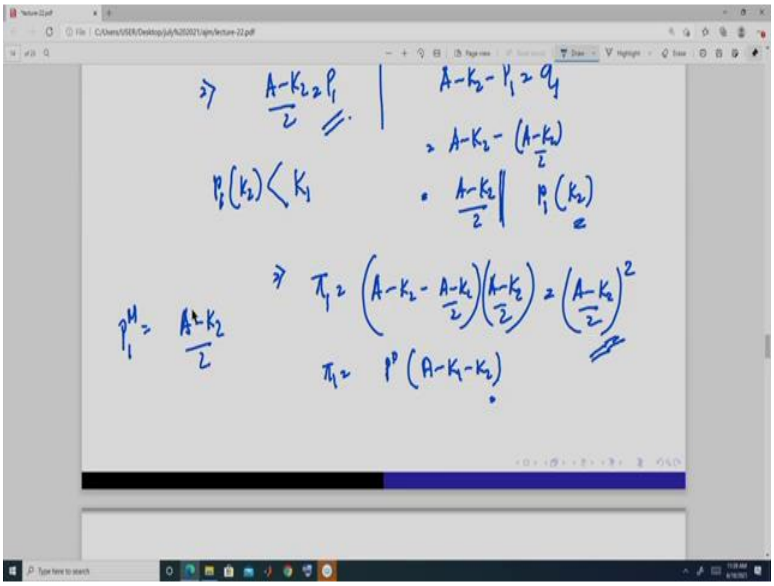

Next one is a slightly complicated one. And here we compute the pure strategy Nash equilibrium for case 3A. So, we have already seen that case 3A is this. Here and this is the reaction function of firm 1, reaction function of firm 2 this and so this region is, all these region is case 3A, but when A is, when k is less than A, so it is only this portion. And this is the whole all this region is 3CA and we will prove the pure strategy Nash equilibrium here.

So, now, suppose we have a price p naught and the price is this-$P^0 = A - K_2 - K_1$. So, we get p naught to be a positive when k1 is less than A, okay. It has to be less than A. So, when k1, the capacity of firm 1 is less than A, so it is only this, this portion. It is only this portion, this region. Although CA includes all this, but if we take A less than, k1 less than A it is only this portion, but for this also it will, we will not see any difference.

Now, in this situation suppose firm 2 set this price p1, firm 1, if acts as monopolist in the residual market, if it acts as a monopolist in the residual market what is going to happen? Suppose, so it maximizes its profit this with respect to p1, because firms are setting the price and this is equal to this which is equal to here first order condition- $A - K_2 - 2P_1 = 0$and this gives us this-$\frac{A-K_2}{2} = P_1$, right?

Now, this is the demand curve faced by the firm 1- $A - K_2 - P_1 = q_1$and since it acts as a monopolist in this and this price is this-$A - K_2 - \frac{A-K_2}{2}$. So, we get this. So, again we know this is what, this is the reaction function of firm 1. So, since this is the reaction function and we are given, this is greater, so we are given k1 is greater than this. So, firm 1 can supply this amount, right?

And so here profit of firm 1 is this-$\pi_2 = \left(A - K_2 - \frac{A-K_2}{2}\right) A - K_2$ if we plug it in here and this is equal to this-$\left(\frac{A-K_2}{2}\right)^2$. This when firm 2 is producing k2 the best response for firm 1 is to produce this much and it is doing it here. So, this, even if we are, this is actually the Cournot outcome, right? So, it is based on the Cournot reaction function. So, this profit is definitely greater than the profit that firm 1 gets if it sets the price p naught this-$P^0 = A - K_2 - K_1$. So, firm 1 will set this price when firm 2 sets p naught. And let us call this price p1 and because it acts as a monopolist in the residual market, so this-$P_1^M = \frac{A-K_2}{2}$.

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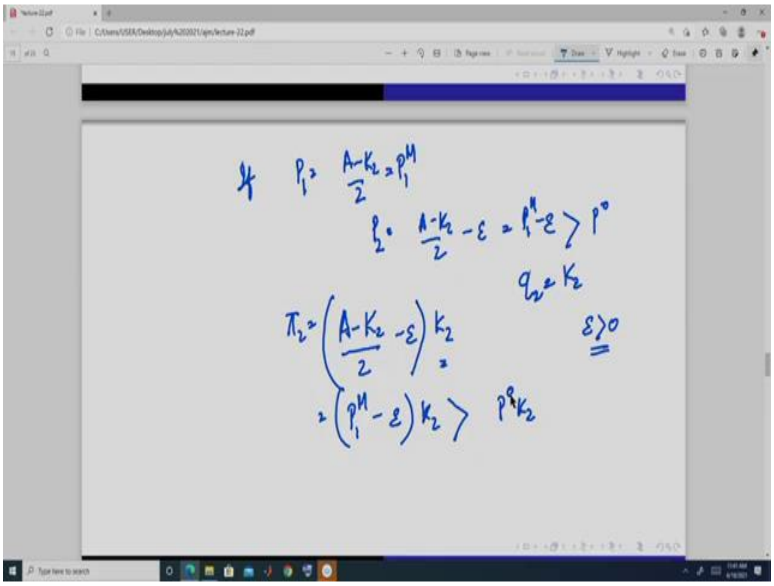

Now, if firm 1 sets this price if p1 is, so then it means it is acting as a residual, is acting as a monopolist in the residual market. So, then and price of p2 is this, firm 2 is this. So, firm 2 this is not a best response. What it will do? It will set a price p2 is equal to actually is equal to some epsilon amount which is greater than p naught and it is selling q2 is equal to k2, okay.

So, the profit of firm 2 is now-$\pi_2 = \left(\frac{A-K_2}{2} - \varepsilon\right) K_2 = (P_1^m - \varepsilon)K_2$, where epsilon, this is an epsilon amount which is positive and it is a very small number, okay so that they call sell k2 amount. So, this is definitely going to be greater than selling p naught k2 because this is greater than, this is less than this amount-$(P_1^m - \varepsilon)K_2 > P^0 K_2$, okay. So, firm 2 is best response is this.

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p_1^l, $\quad p_1^l K_1 = \left(\frac{A-K_2}{2}\right)^2$, $\quad K_1 < A$.

$$p_1^l = \left(\frac{A-K_2}{2}\right)^2 \frac{1}{K_1} \qquad K_1 \geq A$$

$$p_1^l = \left(\frac{A-K_2}{2}\right)^2 \frac{1}{q_1}$$

$$p_1^l < \left(\frac{A-K_2}{2}\right) = p_1^M, \quad p_1^l = \left(\frac{A-K_2}{2}\right)\left(\frac{A-K_2}{2}\right)\frac{1}{K_1}$$

$$\left(\frac{A-K_2}{2}\right)\frac{1}{K_1} < 1 \qquad \frac{A-K_2}{2} < K_1$$

$p_1^M, \; p_1^l$. $\quad p_1^M = p_1$, $\quad p_2 = \left(\frac{A-K_2}{2} - \varepsilon\right)$

K_2.

$$p_1 = \left(\frac{A-K_2}{2} - \varepsilon - \delta\right)$$

$- K_1$

$=$

$p_1^M, \; p_1^l$. $\quad p_1^M = p_1$, $\quad p_2 = \left(\frac{A-K_2}{2} - \varepsilon\right)$

K_2.

$$p_1 = \left(\frac{A-K_2}{2} - \varepsilon - \delta\right)$$

$- K_1$

$$\pi_1 = \left(\frac{A-K_2}{2} - \varepsilon - \delta\right) K_1 > \left(\frac{A-K_2}{2}\right)^2$$

At P_1^l, $P_1^l K_1 = \left(\frac{A-K_2}{2}\right)^2$

$$\frac{A-K_2}{2} > \left(\frac{A-K_2}{2} - \varepsilon - \delta\right) > P_1^l$$

$P_1 \downarrow$, $P_2 \downarrow$.

$P_1 = P_1^l$

, $P_1 = P_1^H$ it will fall till $P_1 = P_1^l$

$P_2 = P_1^l - \varepsilon$, K_2

Firm 1 again acts as monopolist in the residual market.

$P_1 < P_1^l$, $\pi_1 = P_1 K_1$

$\pi_1 = P_1 K_1 < \left(\frac{A-K_2}{2}\right)^2$

At P_1^l, $P_1^l K_1 = \left(\frac{A-K_2}{2}\right)^2$

$\pi_1 = \left(\frac{A-K_2}{2}\right)^2$

$P_2 = \left(\frac{A-K_2}{2} - \varepsilon\right)$.

Firm 1, $P_1 = \left(\frac{A-K_2}{2} - \varepsilon - \delta\right)$

P_1^H, P_1^l

$P_1^H \downarrow \to P_1^l$

$P^H \leftarrow$

Edgeworth cycle.

Price war so prices fall to P_1^l

Again price rises to P_1^H after $P_1 = P_1^l$

Now, let us find another price and this price is suppose p1l. And this price is such that p1l into k1 is this- $P_1^l K_1 = \left(\frac{A-K_2}{2}\right)^2$. So, suppose firm 1 is selling up to its capacity, so, okay this k1 has to be less than A and it is setting a price which is a positive price and the profit is equal to this- $\left(\frac{A-K_2}{2}\right)^2$ when it acts as a monopolist in the residual market. So, this price is actually we can write it in this form and it is this, okay. Now, if suppose k1 is very high and suppose k1 is greater than equal to A, then p1l you can think as some quantity q such that their product and this is equal to this-$P_1^l = \left(\frac{A-K_2}{2}\right)^2 \frac{1}{q_1}$, okay.

Now, here, the interesting thing here is this is actually less than this which is-$P_1^l < \left(\frac{A-K_2}{2}\right) = p_1^M$. Why, because p1l is equal to-$\left(\frac{A-K_2}{2}\right) \left(\frac{A-K_2}{2}\right) \frac{1}{k_1}$. Now, here this number- $\left(\frac{A-K_2}{2}\right) \frac{1}{k_1}$is less than 1, because of the condition that we get this, okay. So, that is why pl is less than. And in this case we will take this p1l and q in such a way that p1l is less than this price, okay. So, now we have two price p1m and p1l. Now, we know that when p1m is set by firm 1 then firm 2 set a price which is this- $\left(\frac{A-K_2}{2} - \varepsilon\right)$and it sells up to its capacity.

Now, firm 1 can also set a price p1 which is equal to minus epsilon minus suppose some small amount delta, okay and it sells an amount up to its capacity k1. So, here in this case profit of firm 1 is going to be this-$\left(\frac{A-K_2}{2} - \varepsilon - \delta\right) K_1$. Now, this is going to be greater than, because at p1l profit p1l into k1 is equal to this much amount, right? and this price A1 minus this, this is greater than p1l and this is less than this-$\frac{A-K_2}{2} > \left(\frac{A-K_2}{2} - \varepsilon - \delta\right) > P_1^l$. So, what happens, firm 1 is also going to undercut the price.

As firm 1 undercuts, so firm 2 also undercuts and so prices of p1 is going to fall, price of firm 2 is also going to fall and it is going to go falling till p1 is equal to p1l. So, the price of firm 1 starting from p1l it will fall till p1 is equal to p1l, okay. So, at p1 now suppose p2 is to sell it can slightly reduce its price which is small amount and sell k2 units of output.

Now, here firm 1 is not going to act like this. Here firm 1, because if it sets a price which is less than this, then profit of firm 1 is going to be this and this profit is going to be less than this amount- i.e. $\pi_1 = P_1 K_1 < \left(\frac{A-K_2}{2}\right)^2$, because at p1l we know p1l k1 is equal to A minus this-

$\left(\frac{A-K_2}{2}\right)^2$. So, firm 1 here in this case firm 1 again acts as monopolist in the residual market, okay. So, in this case profit of firm 1 is again going to be like this-$\left(\frac{A-K_2}{2}\right)^2$.

Now, firm 2 can gain if this is the price then firm 2 is again going to set a price like this-$\frac{A-K_2}{2} - \varepsilon$. Now, if firm 2 sets a price like this, then again firm 1, is going to set p1 which is equal to delta very small amount, so like this. And then there is going to be price war. So, prices fall to p1l then at this again price rises to p1m after p1 is equal to this. So, what is happening? So, there is going to be some kind of a cycle and this cycle is going to be between p1m and p1l. So, price will oscillate between this.

So, from here it will move to p1l by falling, so price will fall and it will reach this and then from here it will start move to this and then again, so it will be like this. So, this is called Edgeworth cycle, okay. Edgeworth is a great economist and he first discovered this cycle, okay. But he did not use any game theory to discover this, because at that time game theory was not there, but he talked about this price competition like, okay. So, and we get this kind of cycle in this situation in case 3A.

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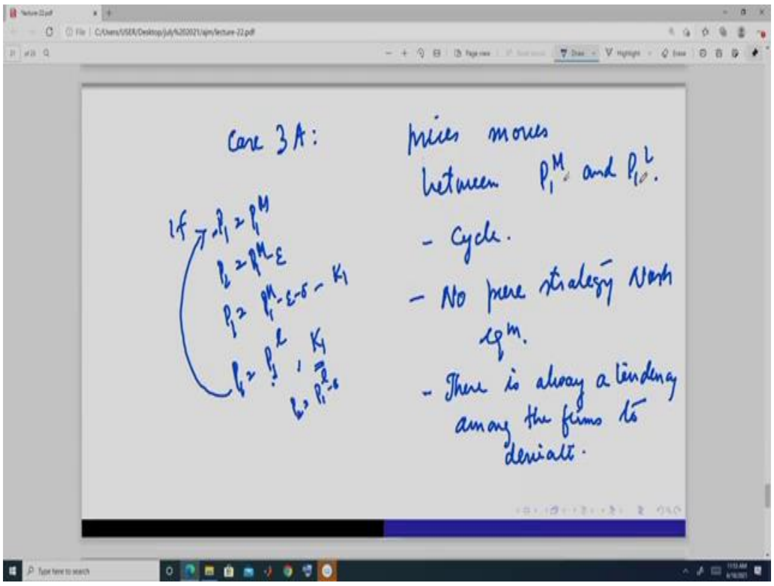

So, in case 3A what do we get, we get that the prices moves between p1m and p1l and there is a cycle, so no pure strategy Nash equilibrium, because there is always the tendency among the firms to deviate, right. So, if p1 is equal to p1m then p2 is equal to p1m epsilon, then p1 is a again a small amount delta, like this it will go on and it will fall till p1 is equal to p1l and when, after this it is again is going to be like this.

When firm 1 sets this price, it is selling k1 units. So, firm 2 it is going to slightly reduce the price, because at this price both the firms cannot sell up to its capacity. So, by slightly reducing the price, it can sell up to his capacity. So, p2 here is going to be this minus a very small amount and then it can sell up to its capacity, so then firm 1 is not going to go below this price. So, it will set this.

Now, when firm 1 set this price, firm 2 is never going to set this price, because p1l minus epsilon is less than p1m minus epsilon. So, firm 1, firm 2 will set this price. So, when firm 2 sets this price, firm 1 since we have this price, we know by slightly reducing it can sell up to its capacity. So, here again it will sell up to its capacity like this. It will go on cycling, okay. So, in case 3 we have no pure strategy and the price will oscillate or you can see cycle between p1m and p1l.

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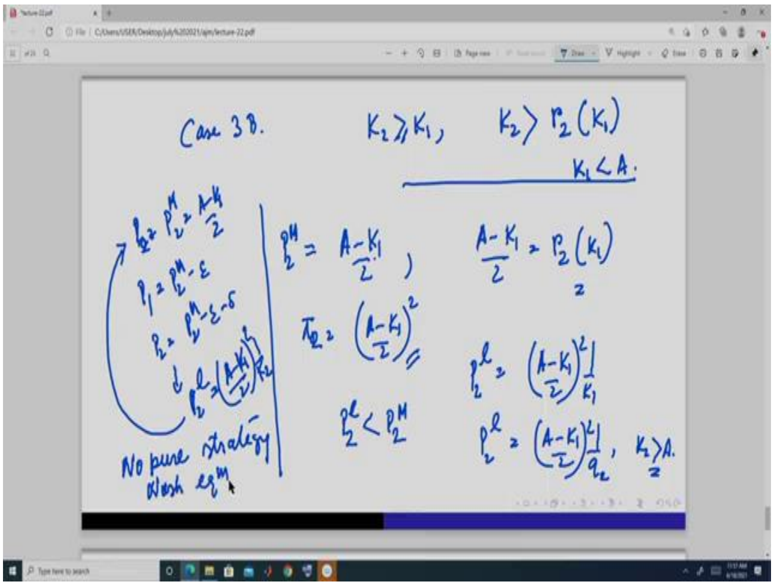

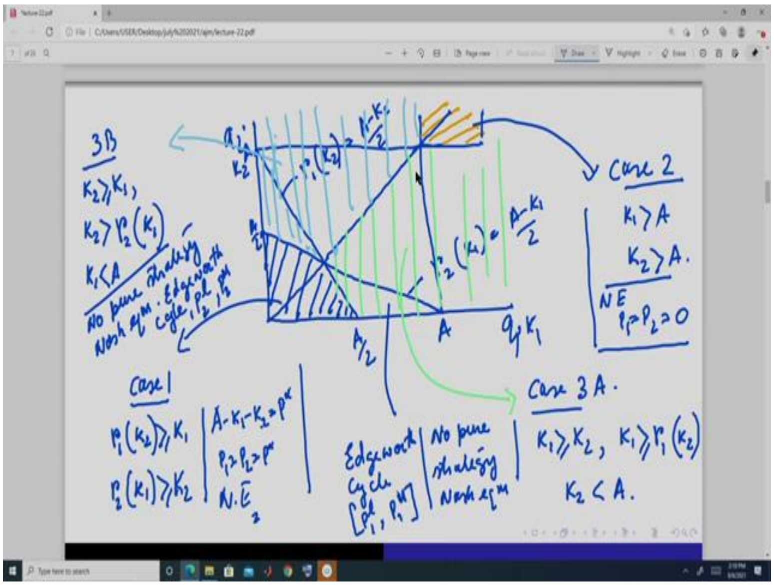

Now, we talk about the case 3, again, case 3B. So, here it is this-$K_2 \geq K_1$. Okay, Now, in this case, again, so capacity of firm 2 is greater than A. So, we will have p2m which is like this-$P_2^M = \frac{A-K_1}{2}$, because we can derive this. So, this is, so at this place because the reaction function of, is this and when it wants to produce this much or act as this much, so if firm 1 is already producing k1 units so the market demand is going to be this price only, right? as we have already got, right. So, this is, okay.

Now, similarly again, so here profit of firm 2 is this-$\left(\frac{A-K_1}{2}\right)^2$. Now, based on this we can derive one another price this- $P_2^l = \left(\frac{A-K_1}{2}\right)^2 \frac{1}{K_1}$ which is or and this p2l is less than m or we can find another price p2l which is like this if k2 is greater than A in this case. Now, here again we know the price is going to oscillate between these two price. So, firm 2 first set a price like this p2 is equal to, p2 is suppose pm2 and that is, then p1 is going to be p2m minus epsilon. Then firm 2's price is going to be epsilon minus some delta and like this price will fall and it will reach this, like this-$P_2^l = \left(\frac{\mathrm{A}-\mathrm{K}_1}{2}\right)^2 \frac{1}{K_1}$.

And after that it will again rise to this, because firm 2 is never going to reduce a price below p2l, because if it is better to act as a monopolist in the residual market because the profit is same here if price is p2l and the price is, if it acts a monopolist in the residual market. So, again when it acts as a monopolist in the residual market firm 1 is not going to reduce the price but it will set a price slightly less than p2m. And then again there is going to be price war, same argument. And based on that, we get that there is again no pure strategy Nash equilibrium.

So, what do we get? So, if you look at this diagram, so we got that in case of 3A we shown that in this region we have, so here we have no pure strategy Nash equilibrium. And instead what do we get, we get that there is, the price will, there is a something called Edgeworth cycle and price cycles between p1l and p1m in this range, okay. And in this 3AB we again got there is, it is same as this case, but only the thing is it is the capacity of firm 2 is greater. So, again here, there is no pure strategy Nash equilibrium. And what we get that the, there is Edgeworth cycle and price will oscillate or cycle between p2l and p2m, okay.

So, for all this reason we have no pure strategy Nash equilibrium, in this region also no pure strategy Nash. Here we have got there is a unique pure strategy Nash equilibrium and that is Nash equilibrium is p1 is equal to p2 it is equal to 0. So, in case 2, we get this outcome. We have shown that. And we have already shown that the in case 1 we have unique pure strategy Nash equilibrium and it is a positive pure strategy Nash equilibrium, price is positive. It takes a positive value and this is p star, where at p star the market demand is such that the firm 1 and firm 2 sales up to its capacity. Firm 1 sales k1 unit, firm 2 sales k2 unit.

But in this two region, in this portion and in this portion we found no pure strategy Nash equilibrium. There actually exists a mix strategy Nash equilibrium, but we are not going to do that, okay. So, now, here you may be this portion is also in the 3B, so I missed it. So, this whole portion is in 3B and this whole portion is in 3A.

So, in Bertrand competition with capacity constraint we see that when the capacity lies within certain range, then we find pure strategy Nash equilibrium. And if the capacity is sufficiently big that is each firm can supply or can meet the demand of the whole market that is the whole market, the maximum demand that can be there in this market is A units, so if each firm can meet this A units, then there is a pure strategy Nash equilibrium and the price is equal to 0.

But if the capacities are in the intermediate region between these two and suppose the capacity of firm 1 is greater than capacity of firm 2 and it lies within these two case then which is the case 3A then we find no pure strategy Nash equilibrium. And when the capacity of firm 2 is greater than the capacity of firm 1 and it lies within these range that is 3B then also we find no pure strategy Nash equilibrium.

And in this situation we find that there is a cycle and the cycle is in case 3A price will start from, if the price started from p1m that is the monopoly price in the residual market, it will move down till p1l. And when it reaches p1l it again it will move to p1m. So, like this there is going to be a price changes in the price by each firm. They will charging different prices.

Now, in 3B again we will have p2l and p2m and the price will cycle between these two. So, it will oscillate between these two. So, if the price starts from p2m then there is going to be a price war and it will come down to p2l. And then again price will rise and it will rise to p2m. So, like that it is going to cycle.

So, with this I end the portion on capacity constraint in Bertrand competition. So, with this I conclude the Bertrand with capacity constraint, okay. Now, actually, I had allotted three lectures on this Bertrand competition with capacity constraint, but I could complete it within two lectures. So, the next lecture I will allot it to one more topic in Bertrand competition and it is a very interesting. So, we will introduce decreasing returns to scale in Bertrand competition and we will see what kind of results we get. So, thank you very much.

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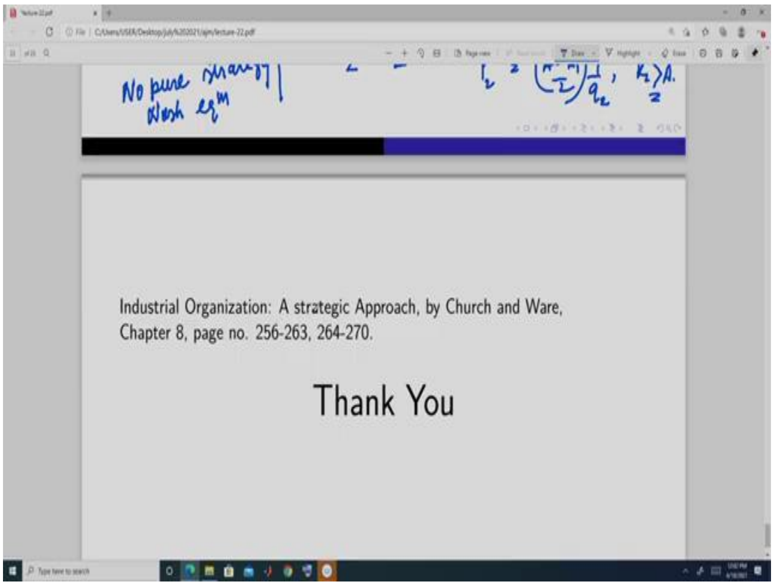

And for this reason you can go through this book Industrial Organization, A Strategic Approach by Church and Ware and these are the specific page numbers of the chapter 8, okay. Thank you.

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 31
Tutorial on Bertrand Competition

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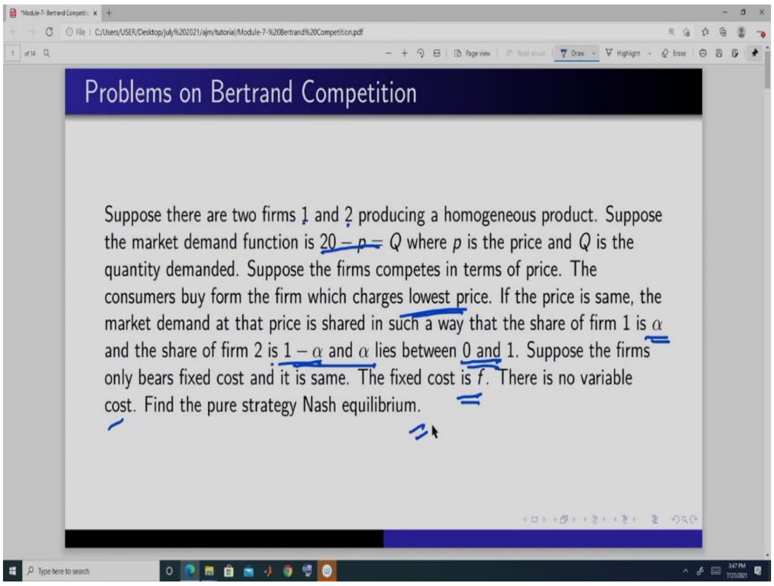

Let us do some problem on Bertrand competition. So, in the first problem, let us suppose assume that there are two firm, firm 1 and firm 2 producing homogeneous product, market demand function is this- 20–p=Q and they compete in terms of price and buyers or the consumers buy from their lowest price and if the market is, if the market price is such that the price is same set by firm 1 and firm 2, then the market demand is share in this ratio-1–α.

That is firm 1 will get alpha and firm 2 will get 1 minus alpha and alpha lies between 0 and 1. So, we have done that it is half, half, but not it is suppose alpha and 1 minus alpha. And suppose the firms only bears fixed cost and it is same and the fixed cost is f and there is no variable cost, find the pure strategy Nash equilibrium. We have solved one version of this in the class.

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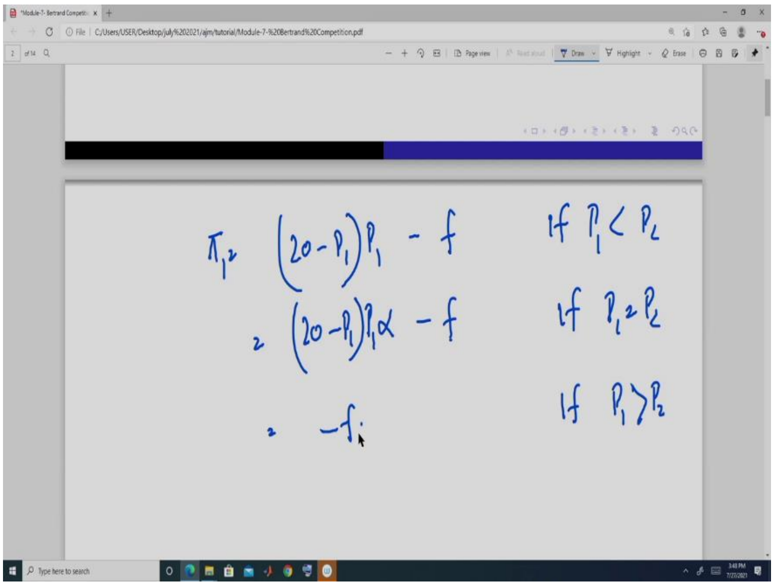

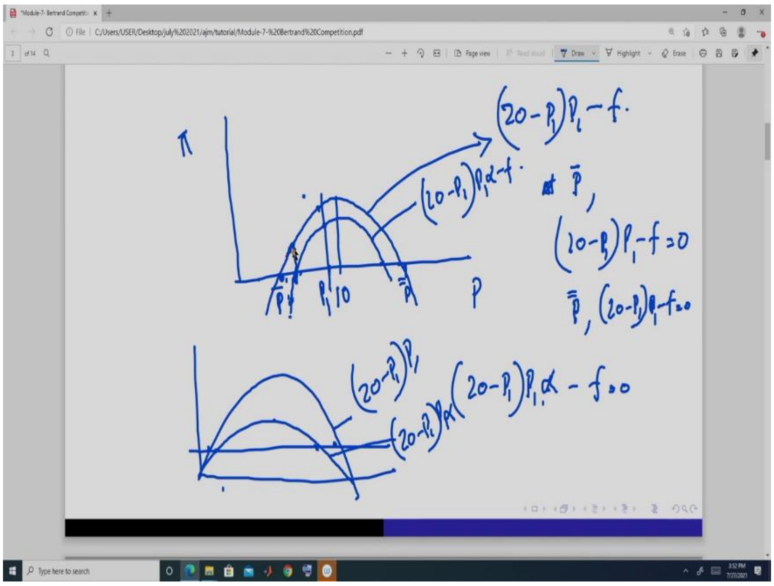

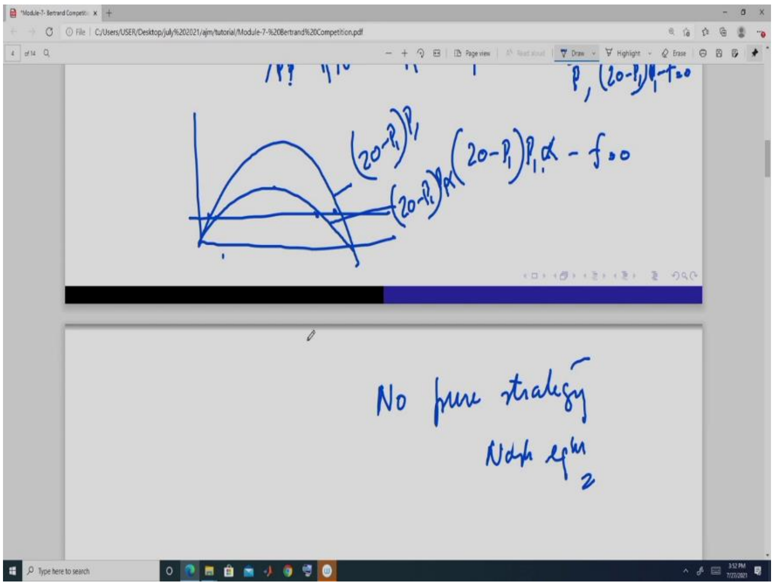

So, in the tutorial we will. So, the profit function of suppose firm 1 you can write it in, demand function is this, - $\pi_1 = (20 - P_1)P_1 - f$ if p1 is less than p2 or it is this-$(20 - P_1)P_1\alpha - f$ if p1 is equal to p2 and it is equal to 0 if p1 is less than. Now, here we can have two situation, either we can assume this. So, it means that we incur fixed cost only when we produce or since we want to produce but no one is buying from us, so we may have a situation like this also. We have already incurred fixed cost. So, it is something like this- –f, okay. So, we get this kind of things. So, you can do this.

So, I am assuming this or you can take 0 also. It will not make much difference. But it is better to take this, because we have assumed that the fixed cost is independent of the output. So, if we plot this function, what do we get? If we take profit here and let us first plot this function, this. Now, this we will have one p, this p, suppose this is p bar or, at p bar this is equal to, and again this is, again we have another p double bar at which, this is 0, so it is somewhere here, p double bar. And we will get the profit function here. This point is 10, right. This is the profit function here.

Now, if we plot this when they share it, this will be somewhere here when we equate it to 0, because this is now multiplied with this. This is lying between 0 and 1. So, this we can say f divided by now alpha, alpha lies between 0 and 1. So, this term is now more than f. So, this is going to be lying here, sorry this is going to lie here, because it has to be greater. And this point again this which is higher, when we have equate to 0, it will be lying here. So, it will be some curve like this. This is, it is like this. Or you can take like this, sorry. This is alpha, alpha is lying between 0 and 1. So, it is like this. f is something here. So, we get these points.

So, these points are these points here. So, similarly we will get this for firm 2 also, because everything is same. Now, if firm 1 suppose sets a price like this, then firm 2, if this is suppose firm, firm 2 if it sets this price, it will get this profit. But if it sets slightly less, it will get this profit. So, it will do this. So, they are going to compete like this and they will finally reach this price. At this price, what is happening? If this slightly reduce, it gets this.

But if it again matches, then it gets some price which is below this. So, again it will not do one thing. Firm 1 will not match. Then firm 2 it is not optimal to, it will go on increasing the price, then it will again reach this point, again this. So, we will get a cycle like this and we will have no pure strategy in this case, no pure strategy Nash equilibrium, okay. So, this is same thing. Only thing here I have only changed the sharing method.

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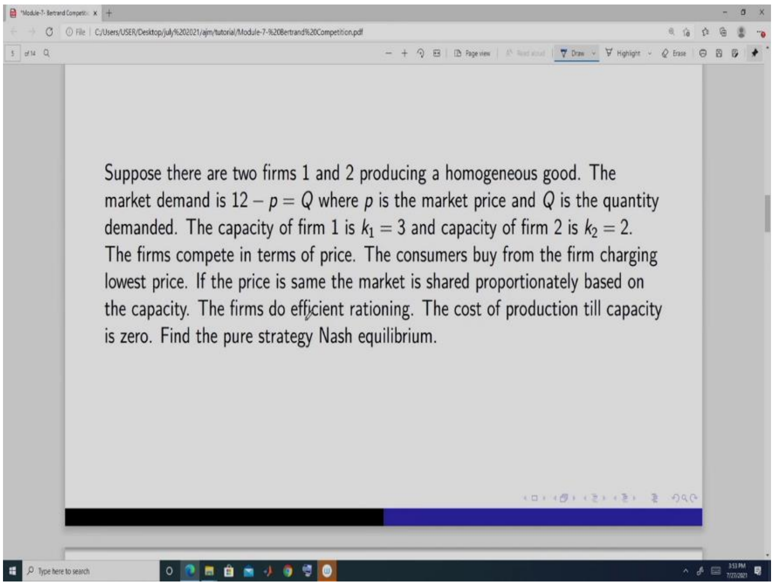

Now, let us do one another problem. Suppose there are two firms, firm 1 and firm 2 and they produces homogeneous product and the market demand function is this-12–p=Q. Capacity of firm 1 is k1, capacity of firm 2 is k2, k1 is 3, capacity of firm 2 is k2 and it is 2. And the usual Bertrand thing applies that is the consumers buys from the lowest firm price, firm that sets the lowest price. And if the price is same then the market is shared proportionally based on their capacity. Firms do efficient rationing. And the cost of production till capacity is 0. We have to find the pure strategy Nash equilibrium here.

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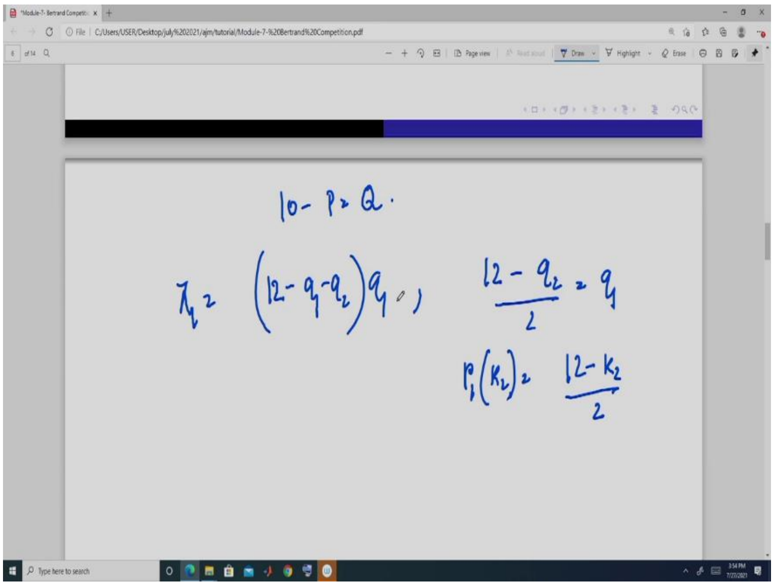

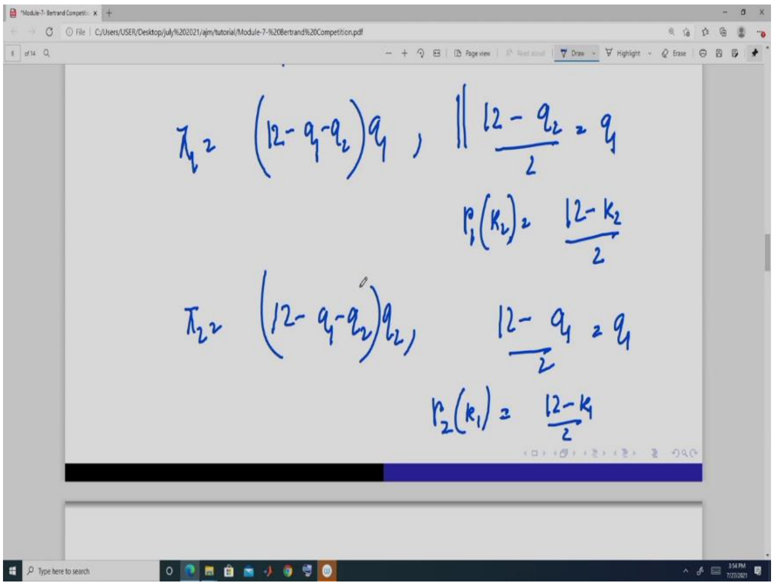

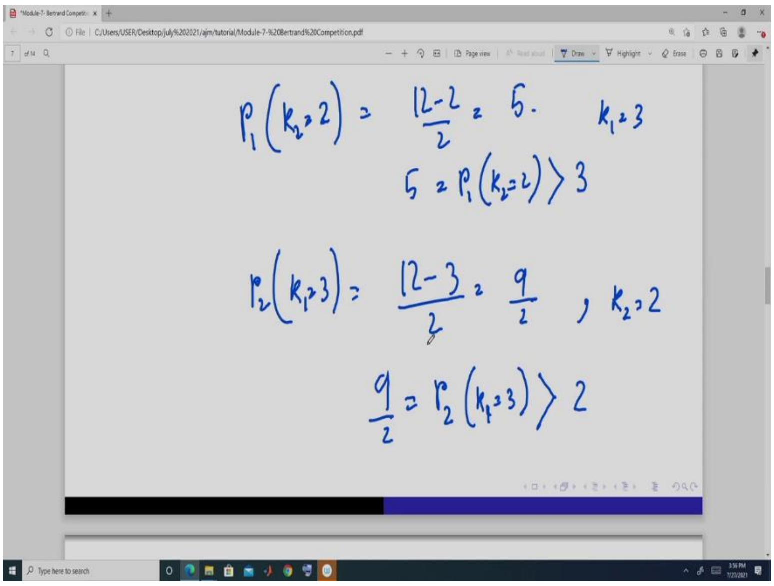

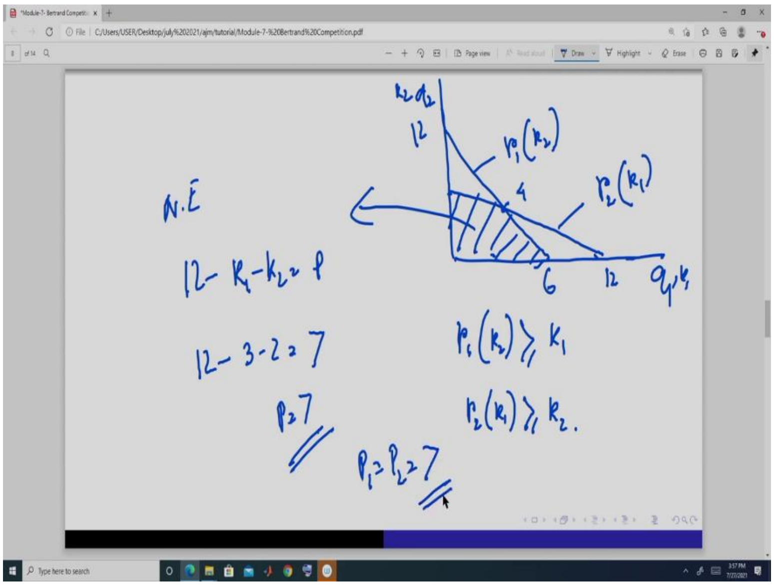

So, this demand function is this, right. So, if we try to find the Cournot this, Cournot reaction function it is I think 12, sorry. It is this-$\frac{12-q_2}{2} = q_1$. So, this we can write, this is the Cournot reaction function. You can write Cournot reaction function is, so again, now here plug in the, so if we take this, capacity of firm 2 is 2. So, this we get what, it is 5. And capacity of firm 1 is k1 is 3. So, this is this-$P_1(K_2 = 2) > 3$ which is equal to 5. Again, it is 3 and capacity of firm 2 is 2. So, in this situation we have got this.

So, the, if, reaction function is, if this is q1, q2, k2, k1, okay we have got this and point is actually 4. And for in this situation we know for this situation when the is Nash equilibrium, pure strategy Nash equilibrium is 12 minus k1 k2, this price. And here we have got this, because this is greater than k1 and we have got this. And this is, so price is equal to 7. Both the firm set the same price p1 is equal to p2 and it is equal to 7. So, this is the pure strategy Nash equilibrium-$P_1 = P_2 = 7$. We have shown this and we are simply using this, okay.

Introduction to Market Structure
Professor Amarjyoti Mahanta
Indian Institutes of Technology, Guwahati
Lecture – 32
Bertrand Competition with Decreasing Returns to Scale

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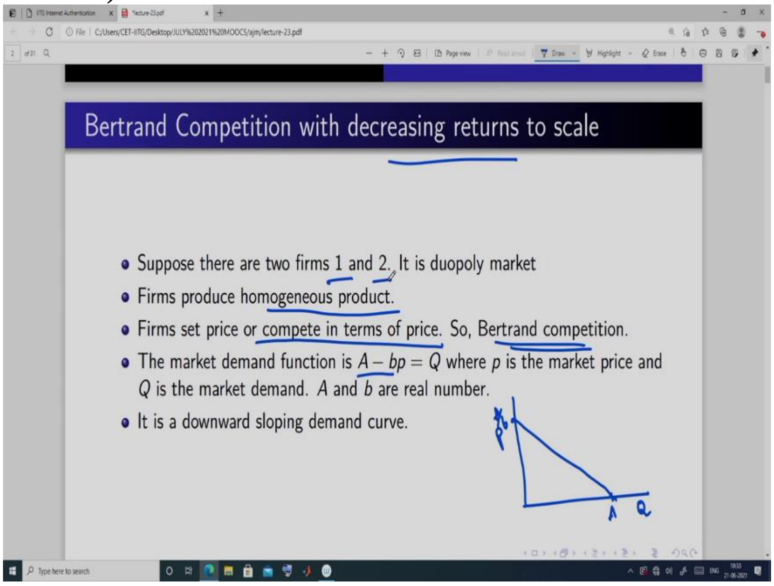

Hello, welcome to my course, Introduction to Market Structures. Today, we are going to do Bertrand Competition with Decreasing Returns to Scale. We are all familiar with decreasing returns to scale, so, we will do that. Suppose, we again assume that there is a duopoly market, that there are two firms, firm 1 and firm 2. So, since there are 2 firms so, it is a duopoly market.

And firms produce homogeneous product, homogeneous products means that the goods produced by firm 1 and firm 2 are completely substitutable. So, whether I buy from firm 1 or I buy from firm 2, it does not matter. And firms compete in terms of price. So, there is Bertrand competition. And the market demand is this, A minus bp equal to Q- A–bp=Q. Q is the here market demand and p is the market price, that is the lowest price in the market, okay.

And A and b are some positive real number. And so, since it is this, so our demand curve if we take p here and quantity here, it is A by p, and this is A. So, it is a downward sloping demand curve. And throughout this course, till now we have only done downward sloping demand curve, okay. So, we continue with that type of demand curve.

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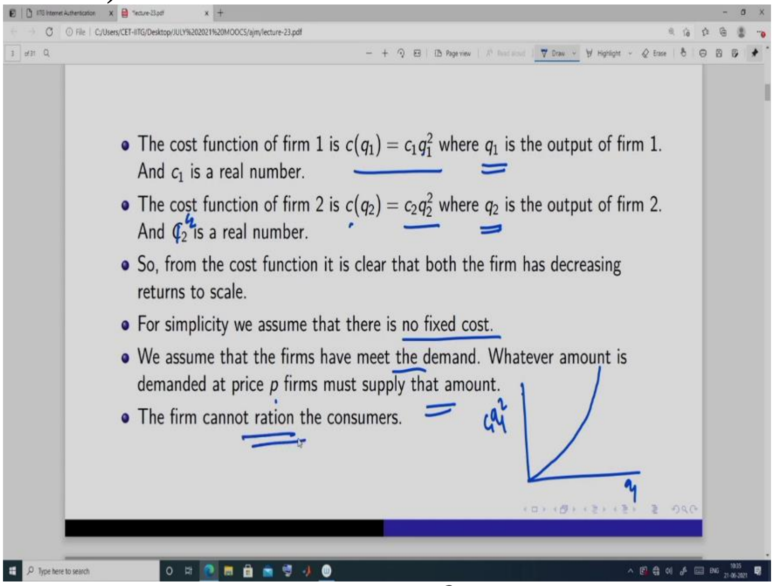

Now, the cost function of firm 1 is this-$c(q_1) = c_1q_1^2$. So, it is c1 q1 square. So, q1 is the output of firm 1, okay. So, it is square. So, this means the cost function is going to this kind of form, right. If I take q1 here and so, it is this, it is this, right?. So, and if we have this kind of cost function, then we know it is decreasing returns to scale. So, that is if we go on increasing output, so, the marginal cost is increasing, okay. And similarly, the cost function of firm 2 is c2 into q2 square, where q2 is the output of firm 2.

And here c1, this should have been small c. These two are positive real numbers, okay. So, it is from the cost function of both the firms, it is clear that they are facing decreasing returns to scale. And for simplicity in this case, we will assume that there is no fixed cost, okay. Again, further we assume that the firms meet the demand. It means that suppose, firm 1 is the lowest, is has set the lowest price.

So, there will be some quantity demanded at that price? So, firm 1 has to supply that whole amount, okay. So, whatever amount is demanded at price p, firms must supply that amount, okay. So, it mainly, so suppose there is a 100 units demand for the output, the firm cannot say that it is only going to supply 80, it has to supply the 100 units, okay. So, therefore the firms cannot ration the consumers, okay. So, it has to meet the demand.

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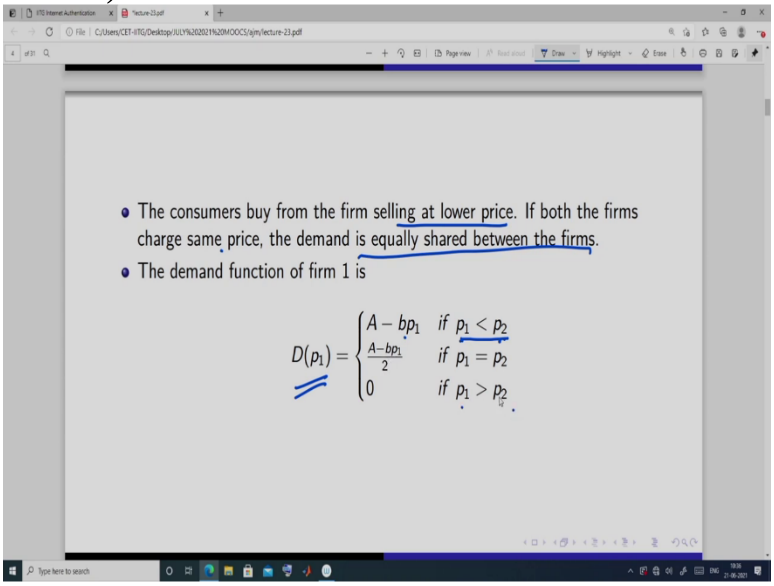

And further, we assume that the consumer, consumers buy from the firm selling at a lower price, okay. And if both the firms charge same price, demand is equally shared between the firms, okay. So, the prices if they set, same price, the demand is equally set. If a firm 1, suppose sets a price lower than firm 2, then everyone is going to buy from firm 1. So based on that, assumption, the demand curve faced by each firm, this is of this nature.

So, if price of firm 1 is less than price of firm 2, is p1 which is less than price of firm 2, p2 then demand is A every, the total demand or the aggregate, the market demand is this much-$A - bp_1$ and this is going to be supplied by firm 1. If the price of firm 1 and firm 2 is same, then at this price suppose this is the demand so, it is equally shared-$\frac{A-bp_1}{2}$. And if the price of firm 1 is greater than the price of firm 2, so, its demand is 0. So, no one is going to buy from this firm.

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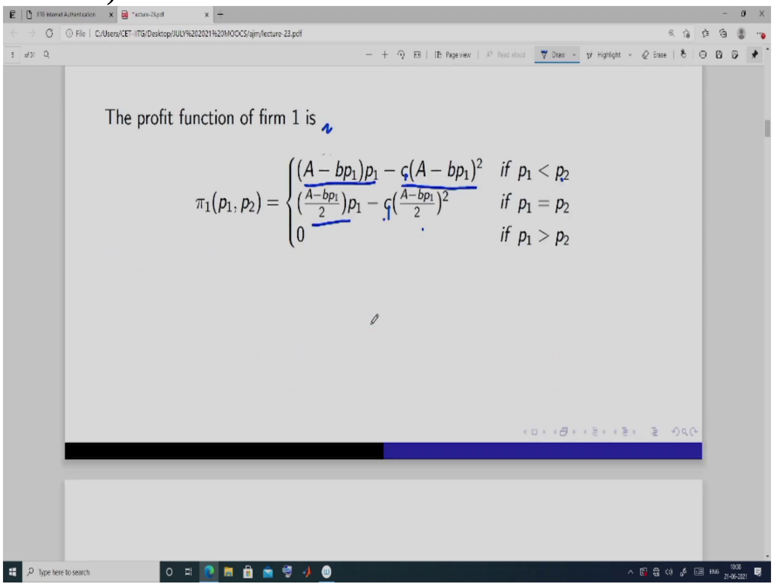

So, based on this demand curve now, we get the profit function of firm 1. So, profit function of firm 1 it is, it is this- $(A - bp_1)p_1 - c(A - bp_1)^2$. So, this is the total quantity supplied by firm 1 or produced by firm 1, since the price of firm 1 is lower than the price of firm 2 into price. So, this is the total revenue of firm 1. And this is the total cost, this is the output it produces or it sells and that is square of it. And here, what we do to keep the things simple, okay.

So, since it is A, we, there is c1 and, okay so, this c1. Again, if the prices are same firm 1 and firm 2 sets the same price, then the amount supplied or produced by firm 1 is this much,- $\frac{(A - bp_1)}{2}\, p_1 - c\frac{(A - bp_1)^2}{2}$ A minus bp 1 divided by 2. Because it is going to get half of the market demand into price. So, this is the total revenue. So, this is the output produced by firm 1. So, this is the total cost. So, this is the total profit of firm 2, if it shares the market, okay. So, this is c1 and this is c2, okay.

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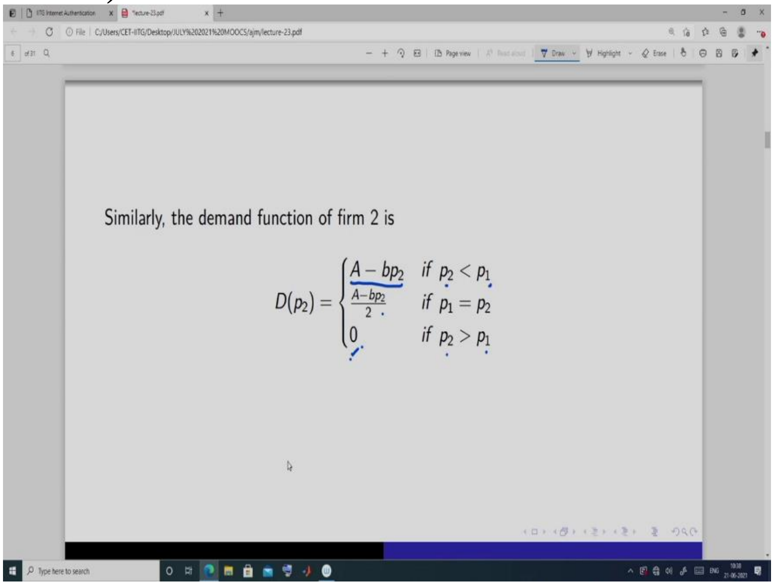

Similarly, the demand for firm 2 is this-$(A - bp_2)$. If the price of firm 2 is less than the price of firm 1, then the whole market is supplied by the firm 2. And so, this is the total demand that is faced by a firm 2. But if the price of firm 1 is same as the price of firm 2, then the total demand that firm 2 faces is half of this half-$\frac{(A-bp_2)}{2}$, So, that is why it is this, okay. And if the price of firm 2 is greater than the price of firm 1 so it gets 0 demand.

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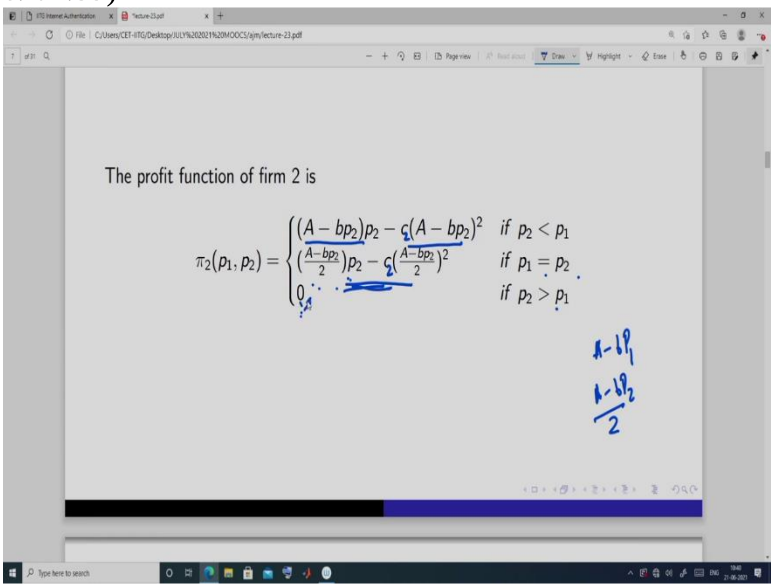

Based on this, we get the profit function of firm 2, this nature. So, here this is c2 and this is c2, okay. So, it is A minus b, this is the total demand into the price. So, this is the total revenue and this is the cost, okay. If firm 2 is only supplying everyone or it is meeting the demand. Because the price of firm 2 is less than price of firm 1. This is the profit when price of firm 1 and prize of firm 2 is same.

So, this is the, this is what? This is the total market demand at this price is this, or you can say this, p1 and is equal to p2. And half of this is this. So, this is the total supply, amount supplied by firm 2 into or produced by firm 2 into price. So, this is the total revenue. And this is the total cost. So, total revenue minus total cost gives you the profit-$\frac{(A-bp_2)}{2}\ p_1 - c\frac{(A-bp_2)^2}{2}$. And if the price of firm 2 is greater than the price of firm 1, then no one buys from firm 2, so price is 0, okay. So, this is the profit function so or the payoff of firm 2.

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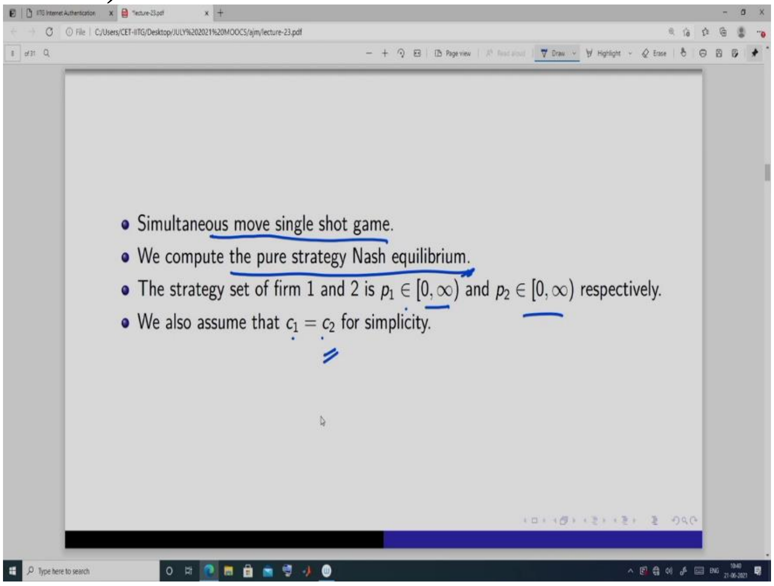

Now, this game it is a Bertrand competition. So, in a Bertrand competition we know the firm 1 and firm 2 both selects or chooses price at the same time. So, it is a simultaneous move single shot game. So, here we find the all compute the pure strategy Nash equilibrium. And the strategy set of firm 1 and firm 2 is this, for firm 1 and this for firm 2 respectively. We further assume, that c1 and c2 is same, okay. We will relax this assumption later on, okay. So, now we the find the pure strategy Nash equilibrium in this setup or in this market game, okay.

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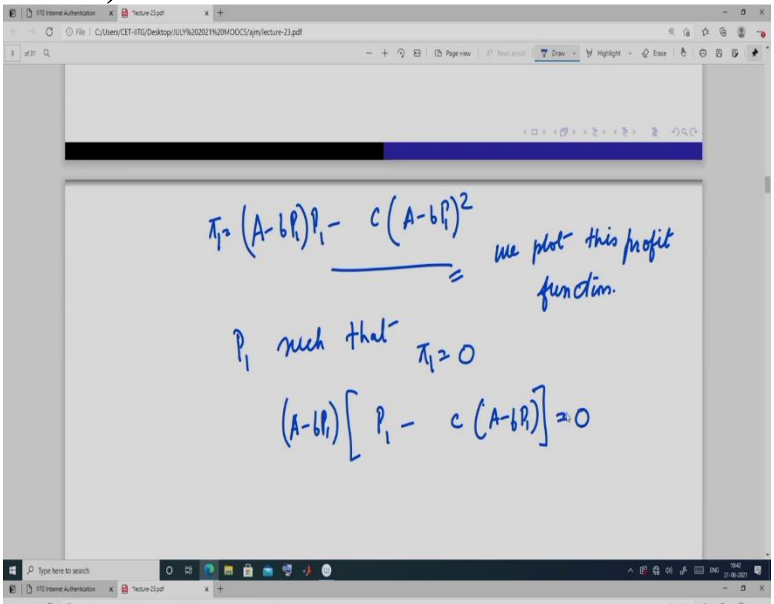

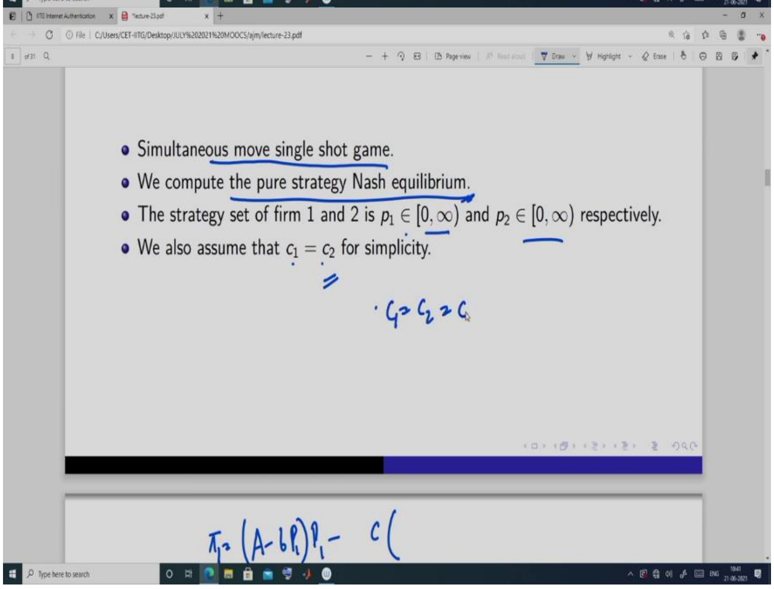

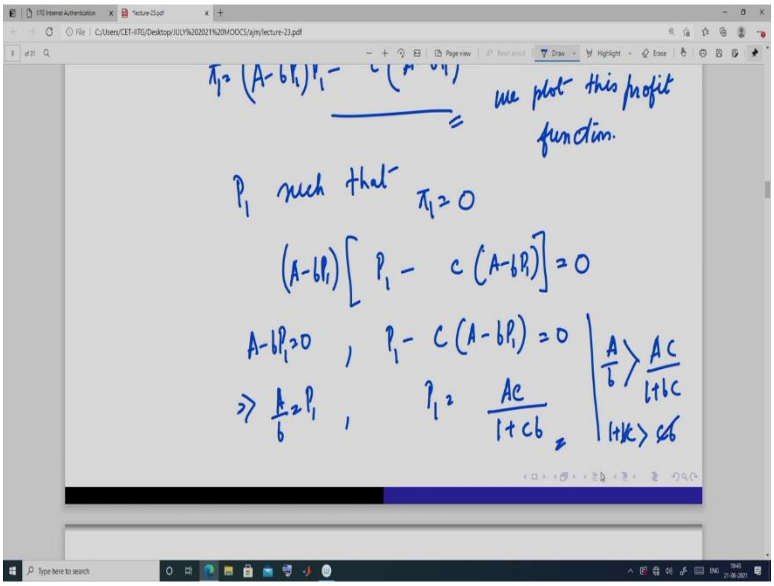

So first, we denote this, we do calculate it for profit of firm 1. So, it is this, right. Now c1 is equal to c2 and suppose that is, here we make it like this, c1 is equal to c2 is equal to sum c, okay. And so, it is this-$\pi_1 = (A - bP_1)P_1 - c(A - bP_1)^2$. Now, we plot this function, okay. What do we do here? So, we first find out the p's such that, this is equal to 0. So, let us write it in a better way.

So, p1 such that this is equal to 0. So, how do we find it? We simply take here in because of this simple functional form, we can, we get this-$(A - bP_1)[P_1 - c(A - bP_1)]$. So, from here we get. So, this gives me A by b and this gives me and from here, it is easy to see that A, this is greater than, this is greater than this. So, this we denote this.

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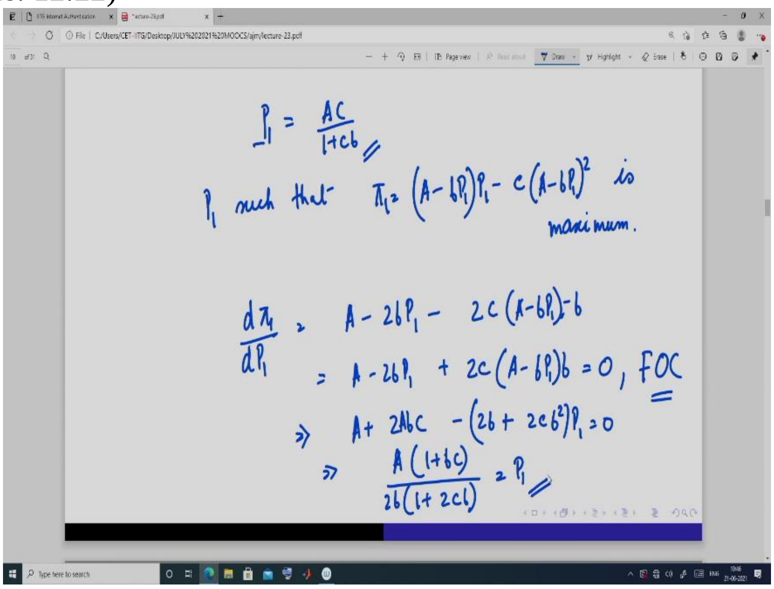

So, we denote that p1 under bar is equal to AC, remember we will use this price and we have 1 price like this- $\frac{AC}{1+cb}$, okay. Now, again let us find the a price such that, so p1 such that this- $\pi_1 = (A - bP_1)P_1 - c(A - bP_1)^2$is maximum, okay. How do we find out that price, that p? So, we find out that p by simply finding the monopoly price of this. And this is equal to, and this is equal to 0 is given by first order condition-$A - 2bP_1 + 2c(A - bP_1)b = 0$. So, first derivative is equal to 0. So, this gives what? This is 2Ab from here. So, we get this price, okay. So, this is one monopoly price-$P_1 = \frac{A(1+bc)}{2b(1+2cl)}$.

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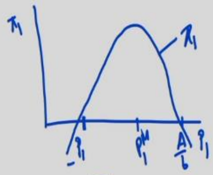
$$\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$$



P_1 such that $\hat{\pi}_1 = 0$

$$\Rightarrow \left(\frac{A-bP_1}{2}\right)\left[P_1 - c\left(\frac{A-bP_2}{2}\right)\right] = 0$$

$A-bP_1=0$, $\quad P_1 - c\left(\frac{A-bP_2}{2}\right) = 0$

$\Rightarrow \frac{A}{b} = P_1$, $\quad P(2+cb) = Ac$

$$P_1 = \frac{Ac}{2+cb}$$

maximum.

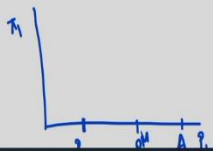
$$\frac{d\pi_1}{dP_1} = A - 2bP_1 - 2c(A-bP_1)\cdot -b$$

$$= A - 2bP_1 + 2c(A-bP_1)b = 0, \text{ FOC}$$

$$\Rightarrow A + 2Abc - (2b + 2cb^2)P_1 = 0$$

$$\Rightarrow \frac{A(1+bc)}{2b(1+2cb)} = P_1$$

$$\Rightarrow P_1^M = \frac{A(1+bc)}{2b(1+2cb)}$$



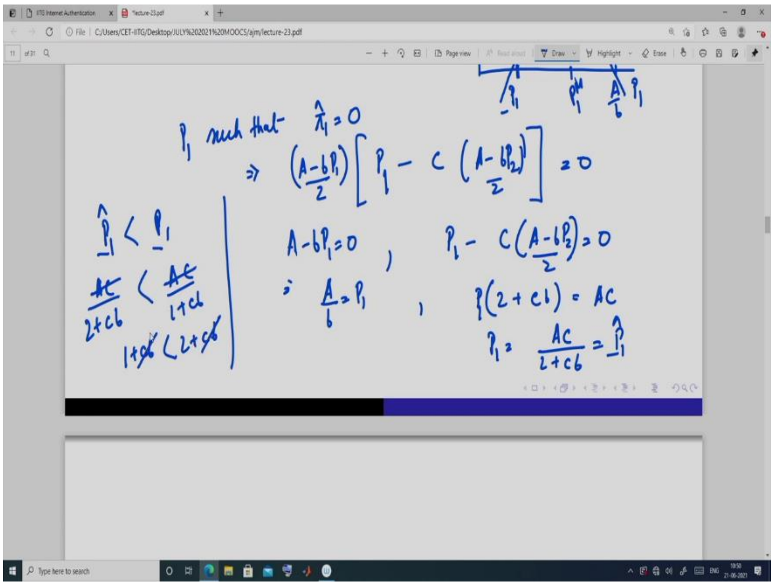
$$(A-bP_1)\left[P_1 - c(A-bP_1)\right] = 0$$

$A-bP_1=0$, $\quad P_1 - c(A-bP_1) = 0$ $\quad \Big| \; \frac{A}{b} > \frac{Ac}{1+bc}$

$\Rightarrow \frac{A}{b} = P_1$, $\quad P_1 = \frac{Ac}{1+cb}$ $\quad \Big| \; 1+bc > cb$

$$\underline{P}_1 = \frac{Ac}{1+cb}$$

P_1 such that $\pi_1 = (A-bP_1)P_1 - c(A-bP_1)^2$ is maximum.



So, based on this three price, what do we get? If we take here and profit of this, this is p1 lower bar. This is A by p and this is the monopoly price of firm 1. So, we denote this price as p1 M which is the monopoly price. And this is A, and this is, you will have something like this, this is the profit function.

Now, consider this, this function-$\pi_1 = \frac{(A-bP_1)}{2} P_1 - c\left(\frac{A-bP_1}{2}\right)^2$. And suppose, denote this by pi 1 hat, okay what do we get? So, this again, we want to find out p1, such that this is equal to 0. So, this is given by, so, this is equal to 0, 0. So, this gives me, or this gives me, okay. And this is same as the price we have got here and this. But this price, which we denote as p1 lower bar is not same as this p1. And we denote this as p1 hat lower bar, okay. And it is easy to see, that p1 hat lower bar is less than p1 bar. Because A minus C 2 plus cb is less than A-$\frac{AC}{2+cb} < \frac{Ac}{1+cb}$, okay.

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P_1 such that $\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$ is maximum.

$$\frac{d\hat{\pi}_1}{dP_1} = \frac{A-2bP_1}{2} - 2c\left(\frac{A-bP_1}{4}\right)\cdot(-b)$$

$$\Rightarrow \frac{A}{2} - bP_1 + \frac{Acb}{2} - \frac{cb^2P_1}{2} = 0, \ \underline{\underline{FOC}}$$

$$\hat{P}_1^M < P_1^M$$

$$\frac{A(1+cb)}{b(2+cb)}$$

$$\Rightarrow \hat{P}_1^M = \frac{A(1+cb)}{b(2+cb)}$$

$$\frac{d\pi_1}{dP_1} = A - 2bP_1 - 2c(A-bP_1)\cdot(-b)$$

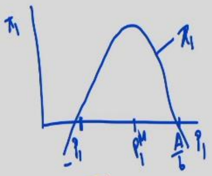
$$= A - 2bP_1 + 2c(A-bP_1)b = 0, \ \underline{\underline{FOC}}$$

$$\Rightarrow A + 2Abc - (2b + 2cb^2)P_1 = 0$$

$$\Rightarrow \frac{A(1+2bc)}{2b(1+cb)} = P_1$$

$$\Rightarrow P_1^M = \frac{A(1+2bc)}{2b(1+cb)}$$

$$\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$$



P_1 such that $\hat{\pi}_1 = 0$

$$\Rightarrow \left(\frac{A-bP_1}{2}\right)\left[P_1 - c\left(\frac{A-bP_1}{2}\right)\right] = 0$$

$$\frac{d\hat{\pi}_1}{dP_1} = \frac{A-2bP_1}{2} - 2c\left(\frac{A-bP_1}{4}\right)\cdot(-b)$$

$$\Rightarrow \frac{A}{2} - bP_1 + \frac{Acb}{2} - \frac{cb^2P_1}{2} = 0, \ \underline{\underline{FOC}}$$

$$\hat{P}_1^M < P_1^M$$

$$\frac{A(1+cb)}{b(2+cb)} < \frac{A(1+2cb)}{2b(1+cb)}$$

$$\Rightarrow \hat{P}_1^M = \frac{A(1+cb)}{b(2+cb)}$$

$$2(1+cb)(1+cb) < (2+cb)(1+2cb)$$

Now again, here we also find the p1 such that p1 hat, which is equal to A minus bp 1, that is when the market is equally shared between firm 1 and firm 2, such that this- $\pi_1 = \frac{(A-bP_1)}{2} P_1 - c\left(\frac{A-bP_1}{2}\right)^2$ is maximum. So, we get this, this is equal to 0, that is the first order condition- $\frac{A-2bp_1}{2} - 2c\left(\frac{A-2bp_1}{2}\right) - b \Rightarrow \frac{A}{2} - bp_1 + \frac{\frac{Acb}{2} \quad cb^2P_1}{2} = 0$. And we get from here, p1 is equal to A sorry, right? And we call this the, the monopoly price-$P_1^m = \frac{A(1+cb)}{b(2+cb)}$. if we have this. And it is easy to see, we will also mark it by hat, that p1 hat M is less than p1 M because it is A 1 plus cb divided by sorry, here I have made a mistake and it is, okay. So, we compare these two prices and we get.

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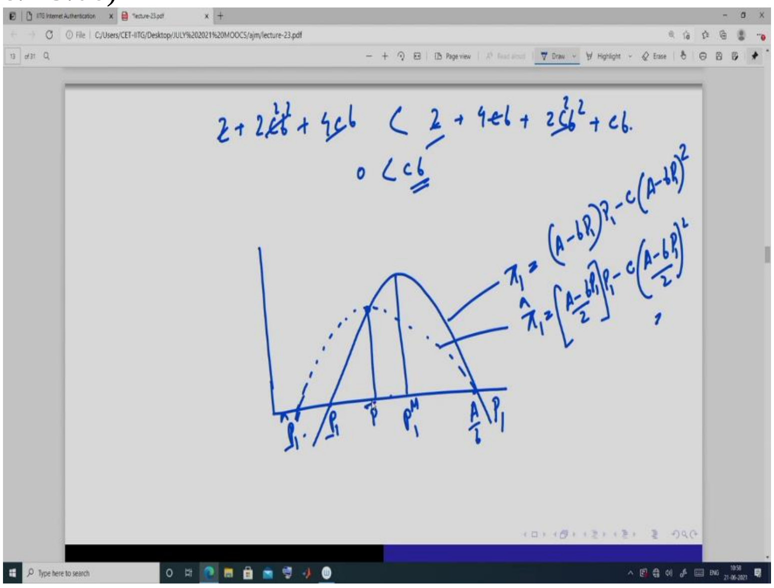

So, it is, just wait a minute let me, we get this and we get this to be this-$2 + 2c^2b^2 + 4cb < 2 + 4cb + 2c^2b^2 + cb$, and then will be, and so, this whole term get cancels and there is. And since both of them are positive so, we get this- 0<cb. So, this means that, okay let me draw this, because, everything will be now based on this diagram. This is A by b, this is suppose this is pM, this is p and see here this curve is, this is p1 lower bar hat. And p1 is somewhere here or it may be here. So, we will get something like this, this curve is this hat, where this is, and this is, this is this. So, we have a price like this, which we called p bar.

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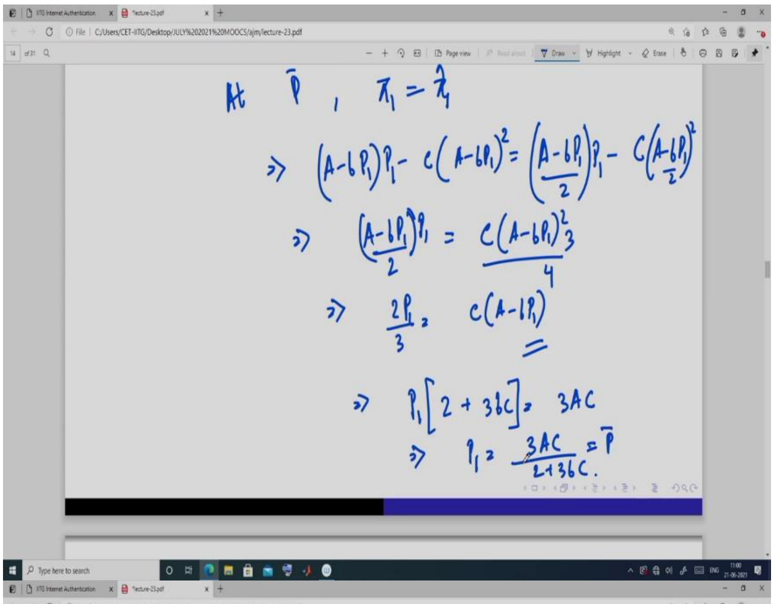

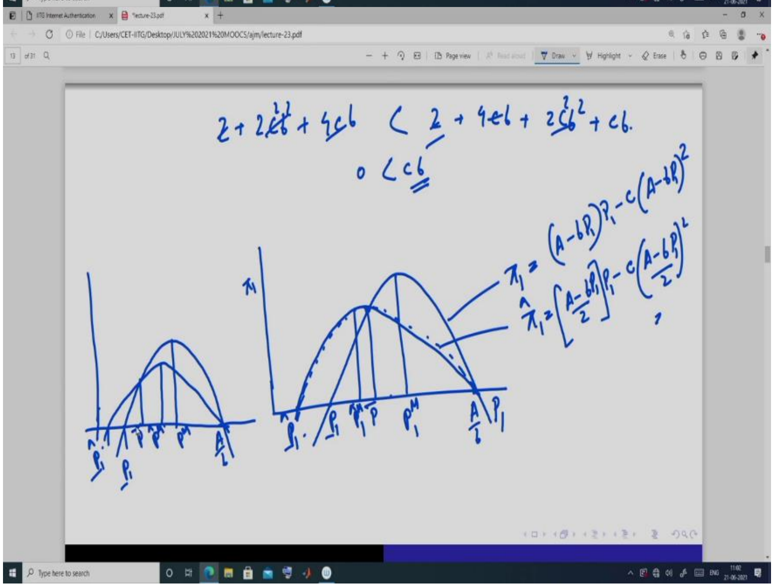

And this price is such that at p bar, we have p, this is equal to this-$\pi_1 = \pi\text{^}_1$. So, this means we have a situation where, whether the market is shared or whether the market is supplied solely by firm 1 profit is same, this-$(A - bP_1)P_1 - c(A - bP_1) = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$. So, if we this, we get this to be, we get this-$\frac{2P_1}{3} = c(A - bP_1)$. And from here, we get the price-$P_1 = \frac{3AC}{2+3bC}$. Now, this, so this is equal to p upper bar, this price.

And this price is this. And we can have a graph like this, where this is the pM bar monopoly price, okay. Or we may have a situation something like this, okay. And this is p upper bar and here this is the p upper bar, okay. So, here monopoly output when or the profit price that maximizes this profit, where market is shared is below this and here it is above this price, okay.

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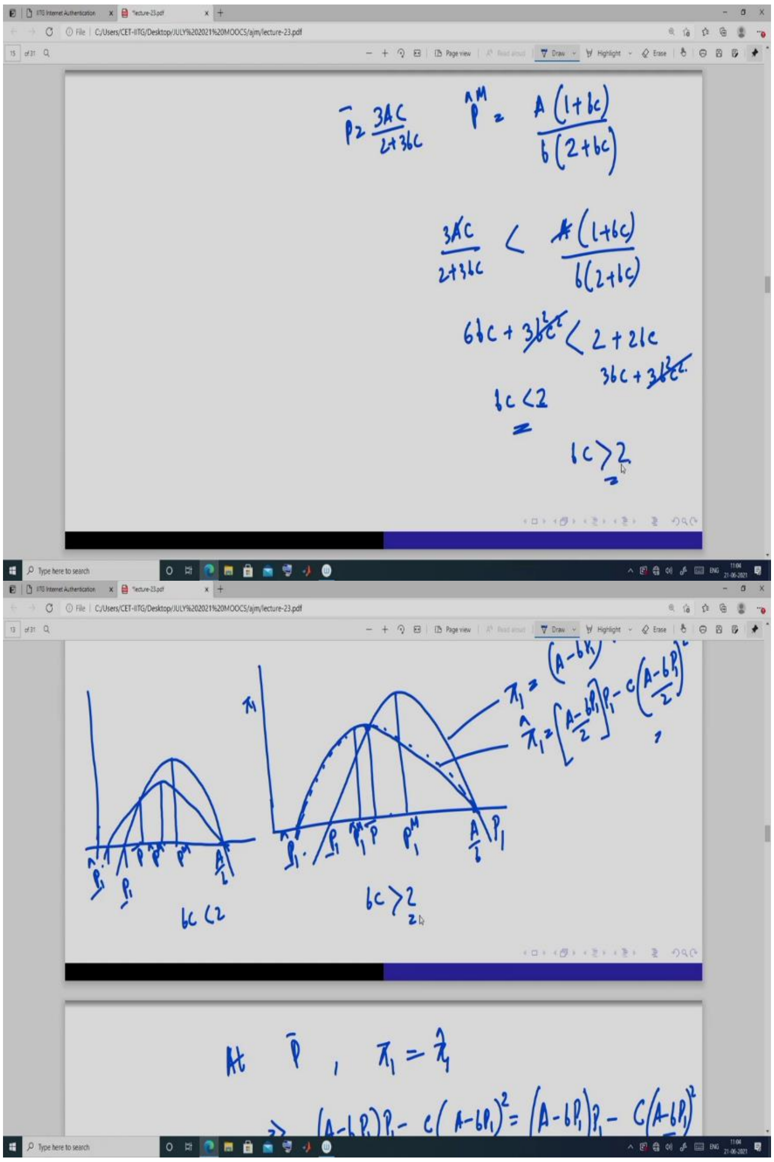

So, and we can see that under what condition we will get this. We will simply compare this, c this-$\frac{3AC}{2+3bC}$ and compare this price which is and which is this, this price-$\frac{A(1+bc)}{b(2+bc)}$. So, we get that if we compare 3AC, this is less than this. If we have this- bc<2, then this price is less than this or if this is greater than 2, i.e. bc>2, then this. So, here this, this case is b into c less than 2, here this is b into c is greater than 2, okay. So, we get this.

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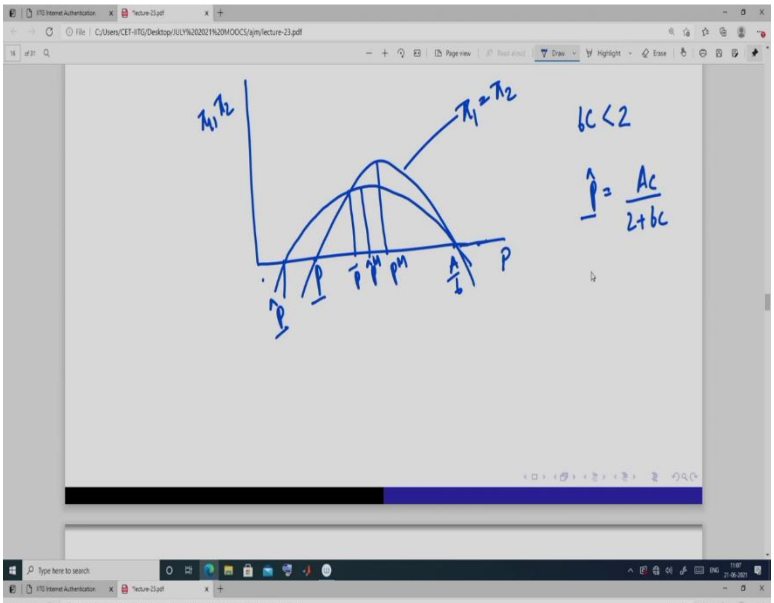

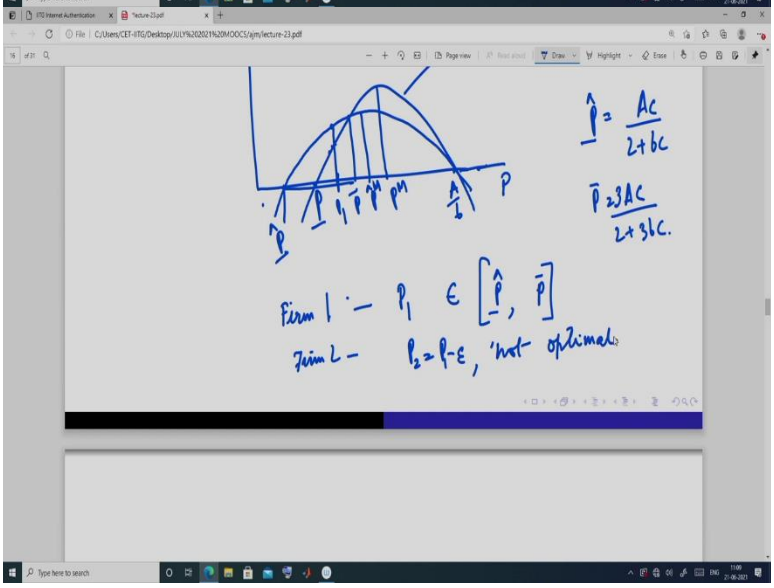

Now, here if you look at this profit function of firm 1 and firm 2, you will find that they are same. So, if we take this, this is suppose. So, we can write it like this, this is A by b, this is the monopoly price and this is p, where this is again you can write something like this. And this is, this is p bar and suppose this is p hat M, okay. Here we have profit of firm 1 and firm 2 here it is price, okay. We get this situation. Now, what we will do? We will try to find a pure strategy Nash equilibrium, in this case where bc is less than 2, okay.

Now see we have derived that here and that price is this, and another price is this. This-$\frac{Ac}{2+bc}$ p A is 3AC 2 plus 3bc, this-$\frac{3AC}{2+3bc}$, right? So, we will require these two price only. Now here, how to find the pure strategy Nash equilibrium? Now suppose, firm 1 sets a price p1, okay which lies between in this range, okay this range, anywhere. Then firm 2, if it sets a price p 2 which is p 1 minus epsilon amount, then it will get the whole market and suppose this is p1. And if it shares, they are going to get this profit if firm 1, firm 2 undercut its profit is going to be here. So, this is not optimal.

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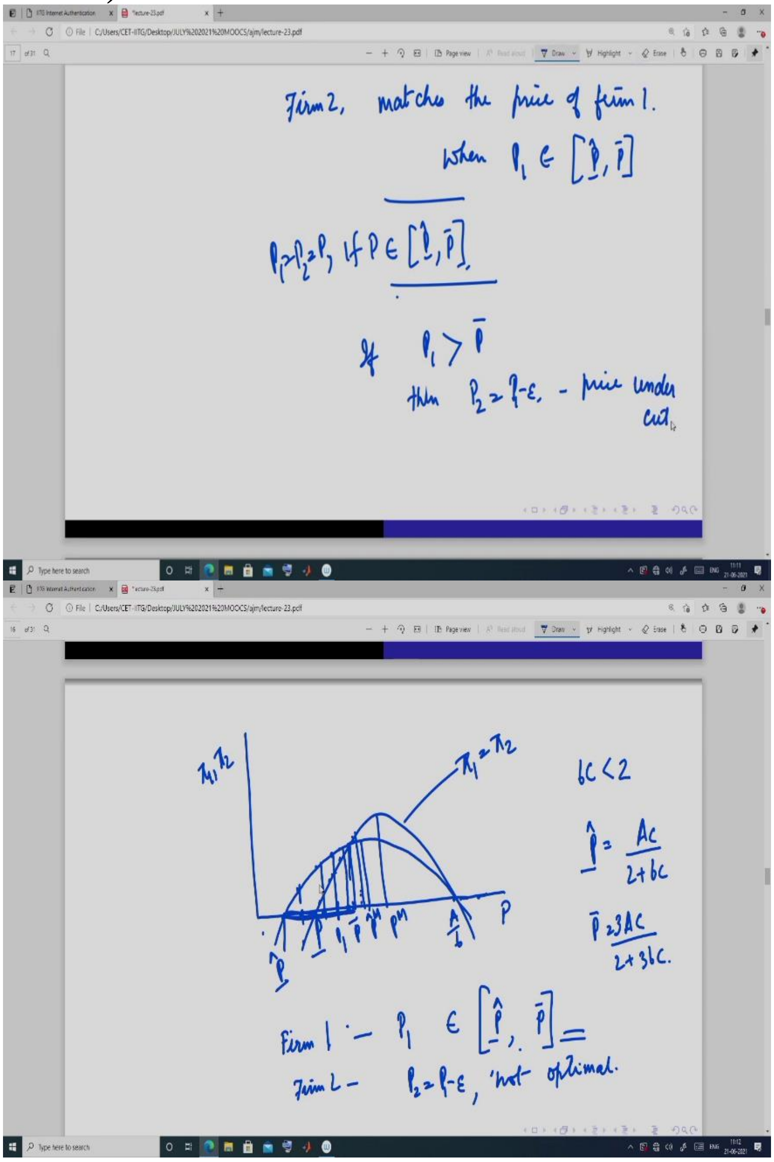

So, firm 2, what it does? Firm 2 matches the price of firm 1. Similarly, when p1 belongs to, because you take any price here, you take a price here. If the price is here, then if it undercuts its profit is going to be negative. But if it shares, market equally it is going to be positive. If it is price is here, then if it firm 1 is at this price, then firm 2 if it undercuts profit will be here, if it shares profit will be here.

Take this price, if firm 1 sets this price, if firm 2 undercuts profit will be here, but if it shares the price, it will remain here. So, at this price, both the profits are same. So, there is no tendency to undercut. So, in this range of price, that is in this range we see both the firm has a tendency to match the price. So, we get that p1 belonging to this, then if so, we get p1 is equal to p2 is equal to p, if p belongs in this range, okay.

Now, the same and if suppose p1 is here, here in this way. Then if it undercuts, firm 2 undercuts it is going to be at a higher curve, higher profit, then if it shares. So, if p1 is greater than p1 then p2 is p1 epsilon amount. So, there is a price, undercut. So, here any price there is going to be continuous price wars. But in this range, price are going to be matched.

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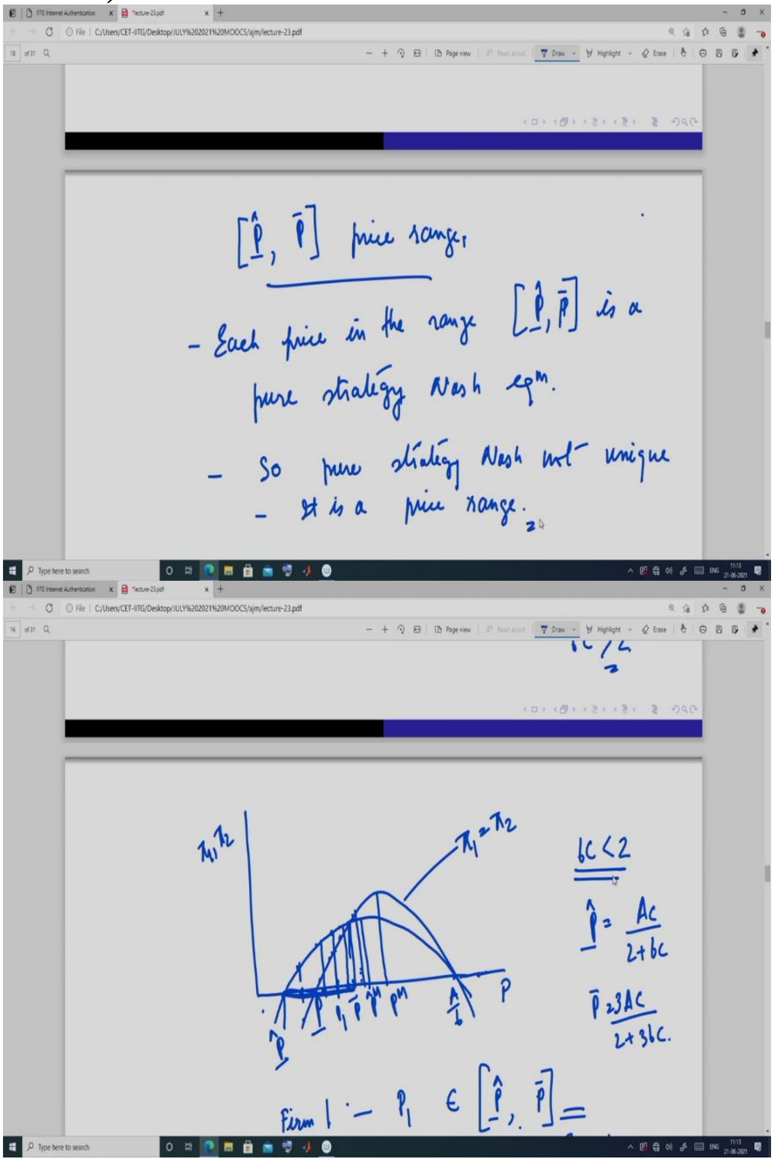

So, based on this a, we get that this range, this price range. So, so, we get this range. So, each price in the range. This is a pure strategy Nash equilibrium. So, so, pure Nash equilibrium is not unique. It is a price range, okay this whole range. So, what do we get? So, this is the case when we have done this and we get this situation.

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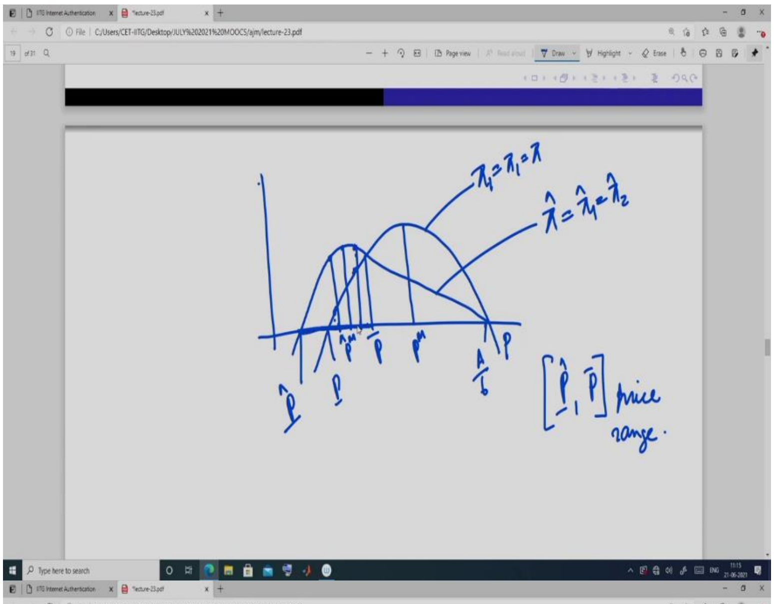

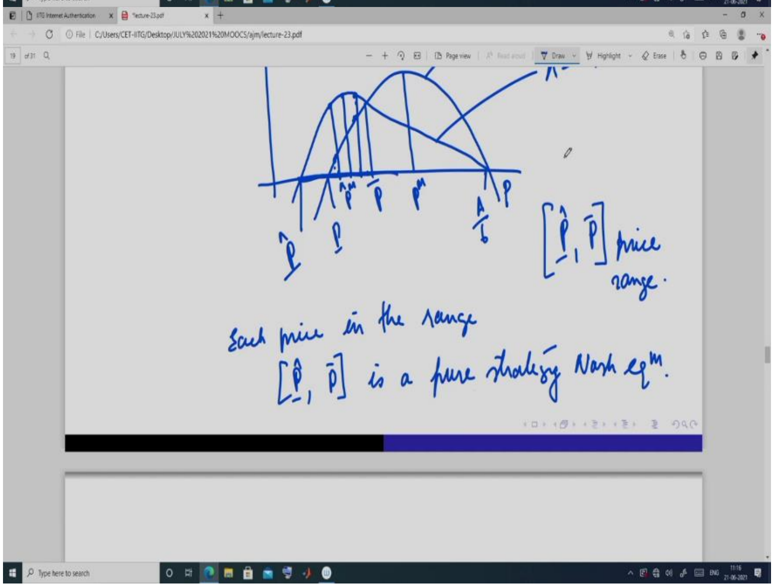

Now, if we simply take the other case, that is this one. This is hat, this is p. Here again, this whole range is the same thing. This same argument, this price range, each price in this range is a pure strategy Nash equilibrium. Same argument you take any price here, there is no tendency to undercut. Because if it undercuts, it will be in this curve and if it shares, it will be here. You take any price here, if it undercuts it will be here, here.

And if it shares it will be here. So, there is no tendency to undercut. So, in this whole region the tendency is to match the price of the other firm rather than to undercut it. So, but if price is greater than this, they will undercut. So, for this region we get that again, each price in the range p is a pure strategy Nash equilibrium, okay. And it is necessarily not unique, okay. So, this is the case we get, when we take the c1 and c2 to be same.

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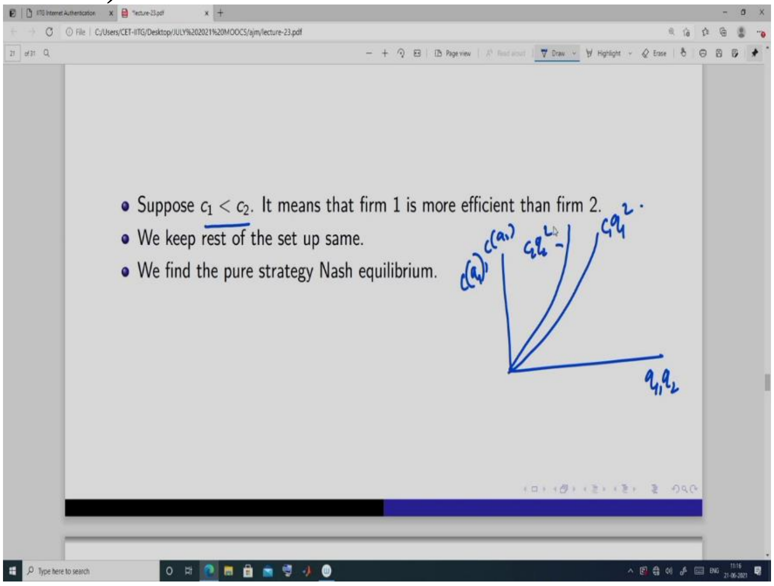

Now, we take suppose this, c1 and c2 are not same. And for simplicity, we take and for suppose one case, there is c1 is less than c2. It means, firm 1 is more efficient than firm 2. Because if we look at the cost curve, here c1 and c2 and here this if this, this is c2, because c2 is greater than c1, okay. We keep the rest of the setup same and we find the pure strategy Nash equilibrium in this case. What we will do?

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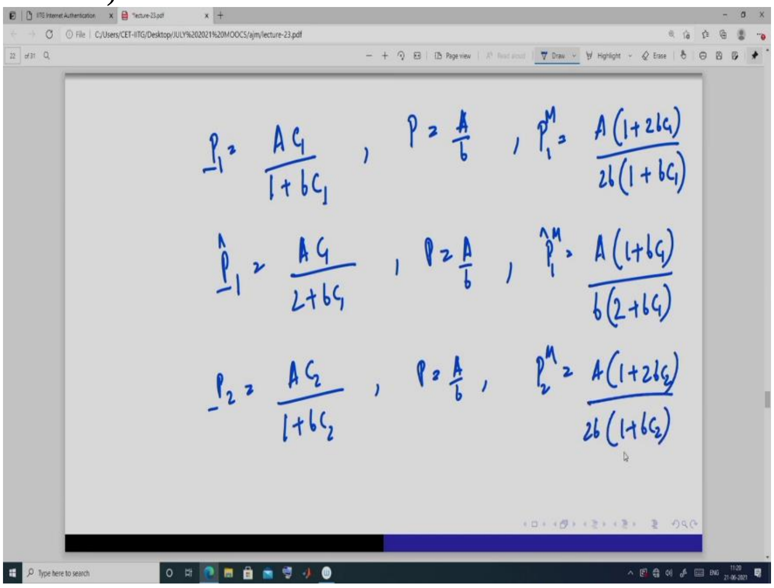

Here, we will simply find this is this price, sorry. So, first let us find this price. This price is going to be AC 1 plus this-$P_1 = \frac{AC_1}{1+bC_1}$. Similarly, we will have one price that is this-$P = \frac{A}{b}$ at which profit is 0. And this price is going to be, and this is going to be A C1 2 plus b c1, this I think it is this, it is this-$P_1^M = \frac{A(1+bc_1)}{b(2+bc_1)}$, okay. So, we have got this prices and similarly we will get this prices-$P_2 = \frac{AC_2}{1+bC_2}, P = \frac{A}{b}, P_2^M = \frac{A(1+2bc_2)}{2b(2+bc_2)}$. We will evaluate or we will find it in the same way. Only c1 will be replaced by c2 and c1 is less than c2.

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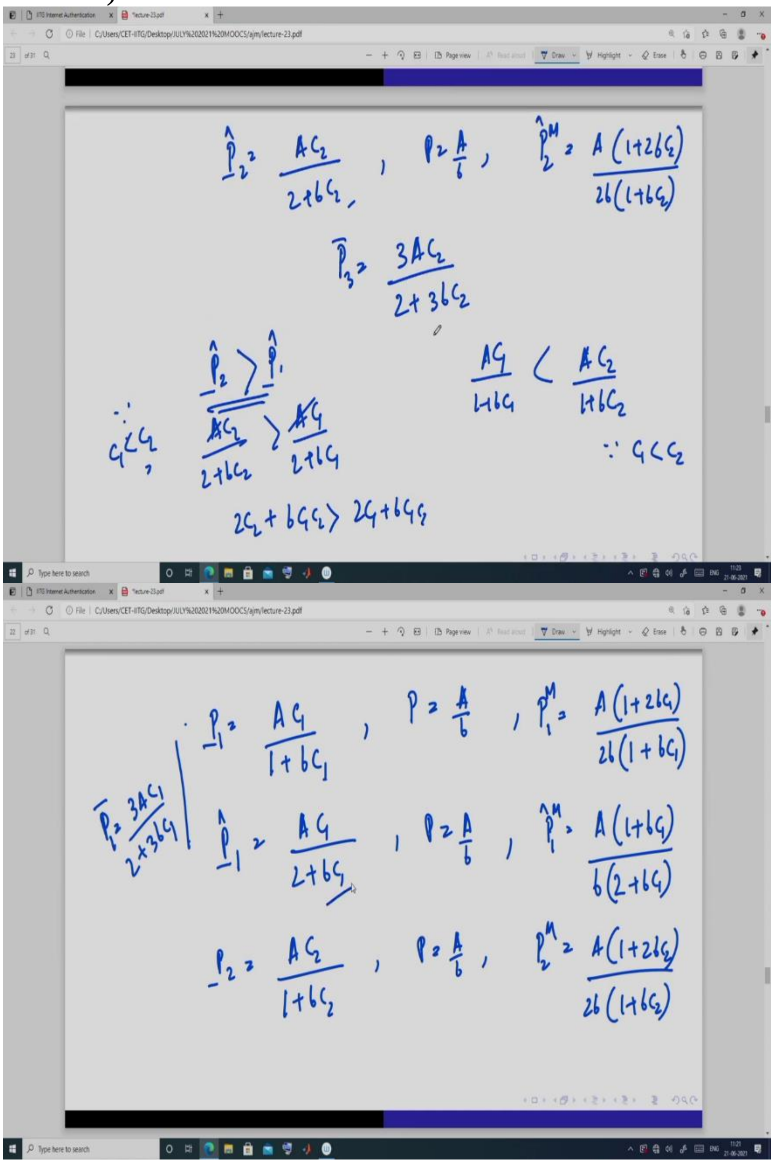

Now, here we will further, what we will have p bar 1 which is 3AC by 2 plus 3 bc1-$\bar{P}_1 = \frac{3AC_1}{2+3bC_1}$. And here again we will have this, which is 3AC 2 which is 2 plus 3 bc2, okay, this-$\frac{3AC_2}{2+3bC_2}$. Now we plot this, first do the comparison. Now here, if we compare this-$\frac{AC_1}{2+bC_1}$ and compare this-$\frac{AC_2}{2+bC_2}$, what do we get is that let see, p this is greater than this. Because c2 is greater than c1, AC 2, 2 bc 2. This is greater than A C1 2b c1, easy to see-$\frac{AC_2}{2+bC_2} > \frac{AC_1}{2+bC_1}$.

So, we cancel out, so, we get this-$2c_2 + bc_1c_2 > 2c_1 + bc_1c_2$. Now because c1 is, since c1 is less than c2. Now similarly we compare this, we also compare these two prices A. So, if this is the A C1 this is also going to be less than A C2 1 plus bc 2-$\frac{AC_1}{1+bC_1} < \frac{AC_2}{1+bC_2}$. Because you are simply, since c1 is less than c2, okay.

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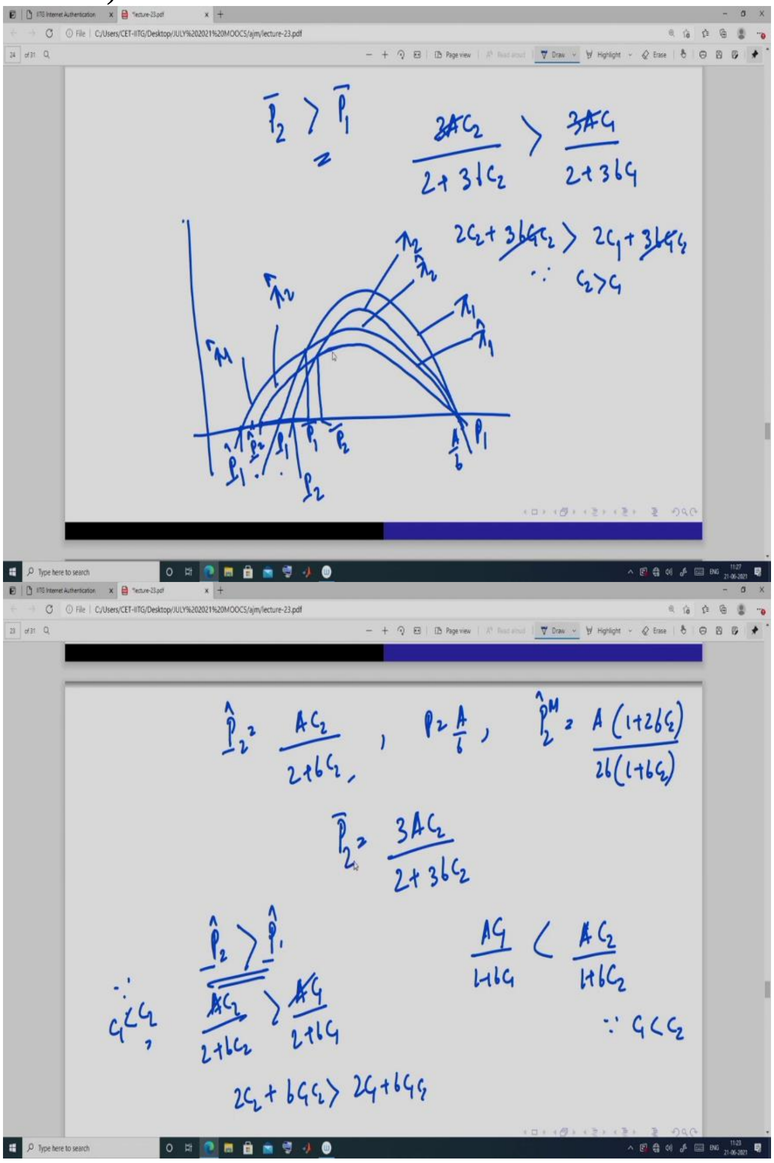

Again, we get that p1, p1 this is going to be greater than this-$P_2 > P_1$. How? Same argument, so since c2 is greater than c1, we get this. So, based on these and we can compare the monopoly prices also and we will get the same thing, okay. So, what we do? We get this, plot this, okay. This is for this and this is p1 bar, this is. So, in the last case, when these two prices of both the firms are seen, we got this as the pure strategy Nash equilibrium, okay.

But right now, we will get a slightly different. So, this is p1bar and we will have a price like this, this is and this price is p bar 2, we have a price like this. This curve is p2 hat when the firm 2 is sharing the market. And this price is p2 hat lower bar, okay and this price is p upper bar 2 and this price is, this price, these two curves are nothing. So, this is p1, okay this is p pi 2 hat, this is pi 1 hat, okay. This curve should always lie actually, because this, this point I have made a mistake.

And this price is this, okay. Now, look here firm 1 suppose, sets a price here, okay. Then if it is not matched by firm 2, then its profit is going to be here. Because this is, when it serves the firm, market alone and this is when it shares. But here if firm 2 shares the market, then here also its profit is going to be negative, because this is the profit curve of firm 2 when it shares.

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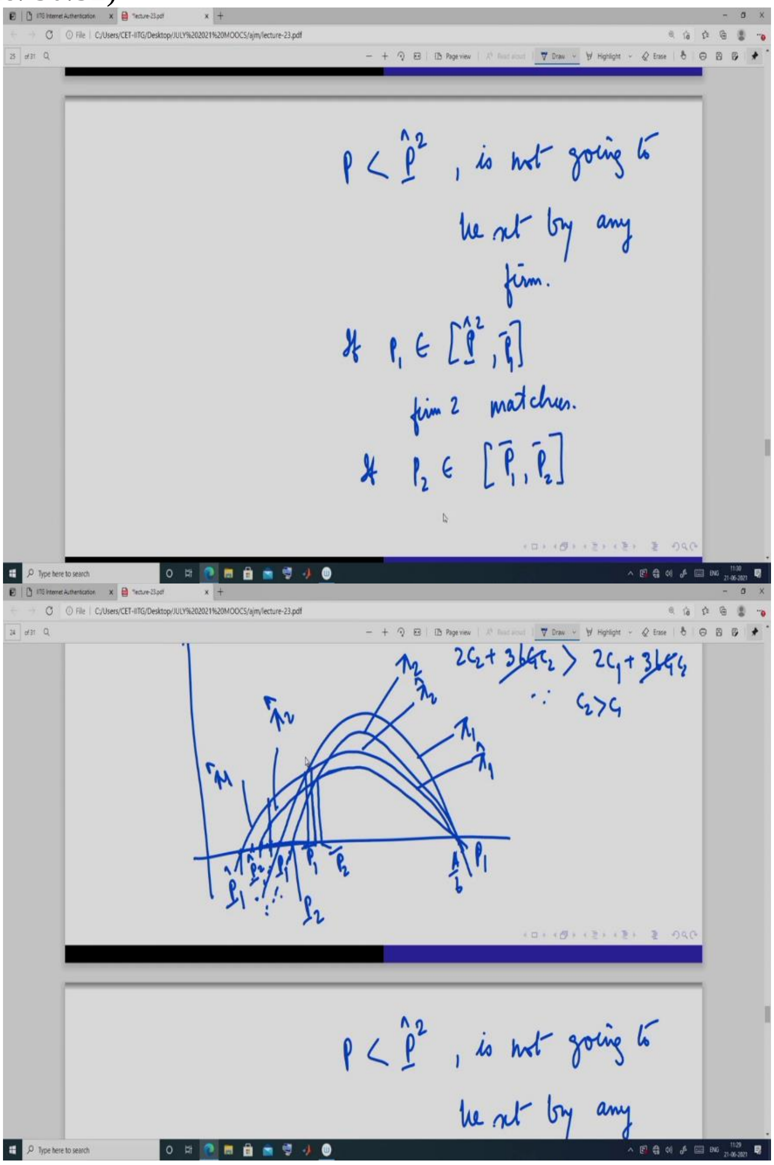

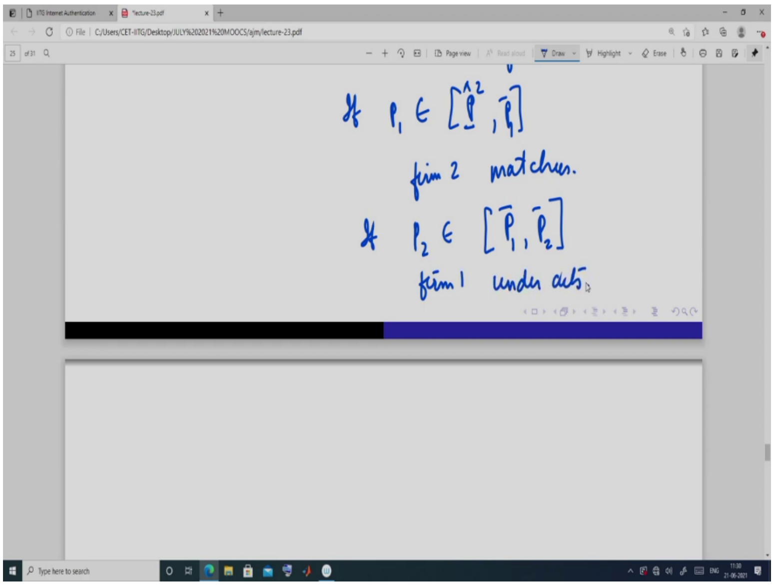

So, any price so, p below this is not going to be set by any firm, okay. This is now, so, we get this as the low. Here, if we take this price and if firm 1 sets this price, firm 2 if it undercuts, it is given by here, it will be negative. But if it shares, it will get this. And if firm 2 also undercuts, it will, since it will not undercut because it has same that price. And its profit is this, if it shares and profit of firm 2, if it shares is this. But if it undercuts, it will be here in this curve.

So, it will be negative so, that is why it will be if p1 belongs to this. And it is less than this, right? this is lower than this. So, firm 2 matches, we get this. Now, take a price, which is here in between these two, firm 1 sets this price. So, if firm 2 matches this price, profit of firm 1 is here, matches. And if firm 2 set, set same the same price it is here. But if firm 1 slightly reduces, its profit is at the higher level. So, if p2 suppose belongs to a price, which is firm 1 undercuts.

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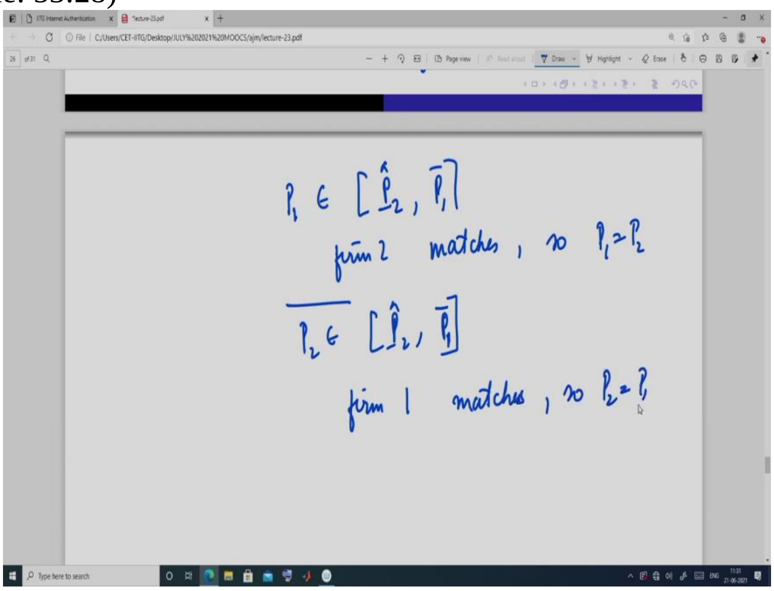
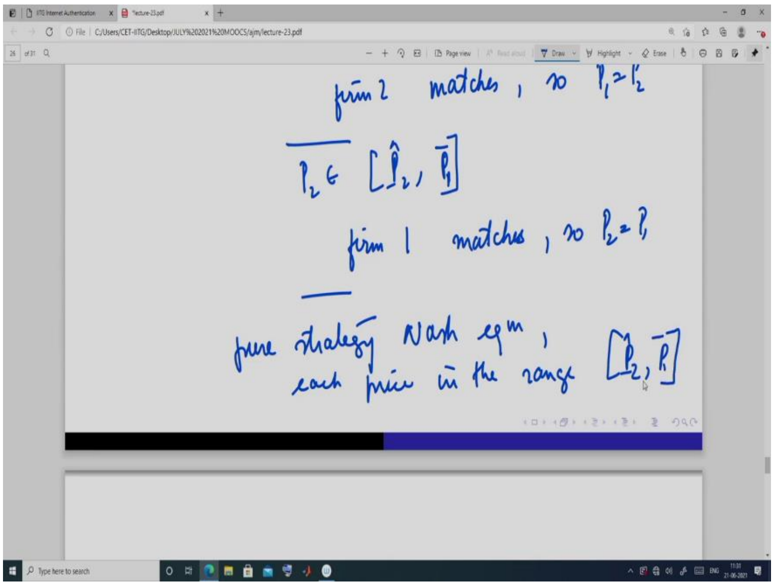
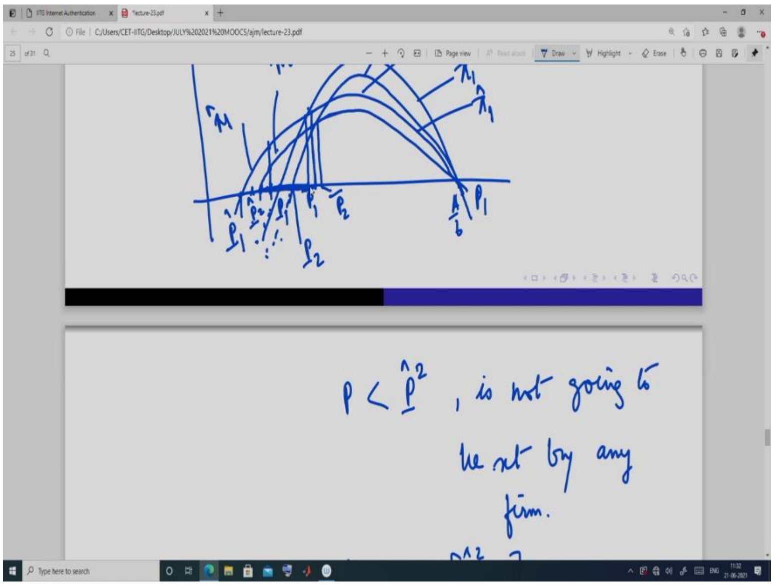
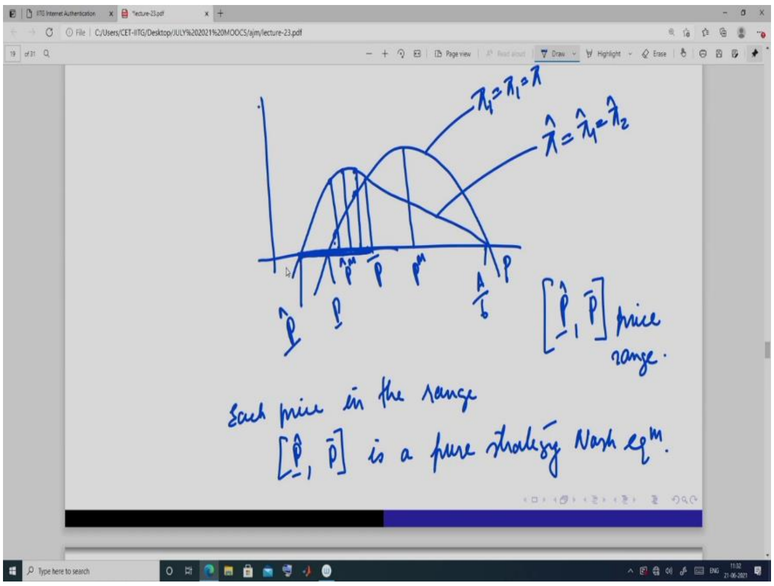



firm 2 matches, so $P_1 = P_2$
$P_2 \in [\hat{P}_2, \bar{P}_1]$
firm 1 matches, so $P_2 = P_1$
pure strategy Nash eqm, each price in the range $[\hat{P}_2, \bar{P}_1]$


π_1
$\hat{\pi}_1$
P_1
P_2
$\frac{A}{b}$
$P < \hat{P}^2$, is not going to be set by any firm.


$\pi_1 = \bar{\pi}_1 = \bar{\pi}$
$\hat{\pi} = \hat{\pi}_1 = \hat{\pi}_2$
$\hat{P}$
$\bar{P}$
P^m
$\frac{A}{b}$
P
$[\hat{P}, \bar{P}]$ price range.
Each price in the range $[\hat{P}, \bar{P}]$ is a pure strategy Nash eqm.

So, so when they are going to match? So, when p1 lies between this firm 2 matches. And so, p1 is equal to p2. Again, if p2 lies in this range, this range firm 1 matches. So, p2 is equal to p1. So, we get that the pure strategy, pure strategy Nash equilibrium, pure strategy each price in the range, this. So, each price in this range is a pure strategy. So, again here it is not necessarily unique, but it is a range.

So, it will be from this range, region to this. So, this is when we have asymmetry in cost function but there is a decreasing returns to scale, we get a range and that range is this to this. I will, but in when there is no asymmetry, when the cost function is same, then we do not get this, we do not get instead we get what? We get a this kind of, this whole range this, this range from p hat lower bar to p upper bar.

But here, we have two p hat lower bar and two p upper bar. And from that, we get the lower p upper bar and the higher p hat, okay. So, that is the and so, with this we conclude the Bertrand competition. So, Bertrand competition we have done in different forms. We have first assumed that when the cost function, marginal cost function is constant, it may be different. So, then we have found out the pure strategy Nash equilibrium.

In certain cases, we have got pure strategy Nash, Nash equilibrium in certain cases we have not got. Then we have introduced capacity constraint and in capacity constraint we have shown that under certain combination of capacities, we get pure strategy Nash equilibrium, under certain we do not get any pure strategy Nash equilibrium. And in this case, when we have decreasing returns to scale, we have shown that there is a range of pure strategy Nash equilibrium.

Or you can say it is a continuum of pure strategy Nash equilibrium. So, we get different varieties of Nash equilibrium in this Bertrand competition. So, that is why Bertrand competition is very interesting, okay. So, with this I end this portion, for this decreasing returns to scale, this class notes are sufficient. Because it is not there in any these, textbooks that I have mentioned, okay. Thank you.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Indian Institutes of Technology, Guwahati
Lecture – 33
Stackelberg Quantity Competition

Hello, welcome to my course, Introduction to Market Structures. Today, we are going to do Stackelberg Quantity Competition. So, till now we have done Cournot competition and a different versions of Bertrand competition. And today, we will do Stackelberg quantity competition. In Stackelberg quantity competition, we first assume that there are two firms, firm 1 and firm 2.

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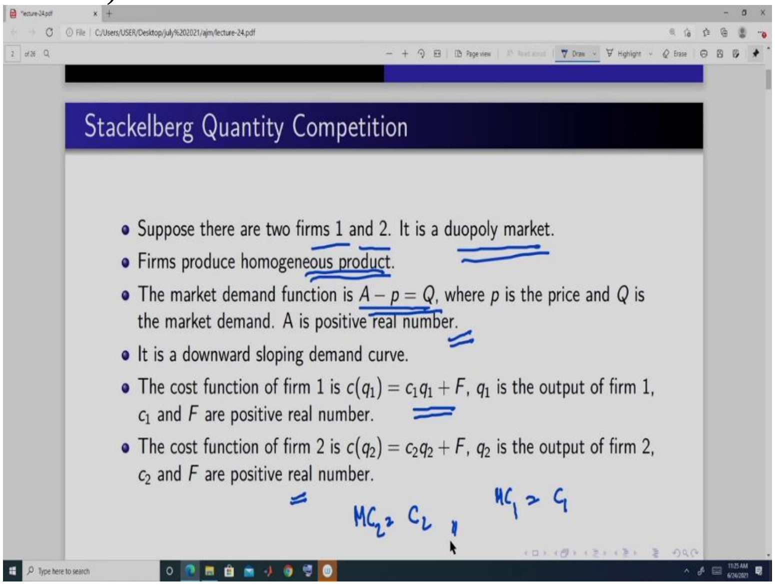

So, we consider a simple market and that is a duopoly market, okay. Now, firms produce homogeneous product. So, that means, whatever output are produced by firms 1 and firm 2, they are similar whether you buy from firm 1 or you buy from firm 2, it does not matter the products are same. And the market demand function is this- A–p=Q. So, it is again A minus p is equal to capital Q. So, p is the market price and big Q is the market demand and A is a positive real number.

So, this is our standard (market) downward sloping demand curve that we are using it for some time. Now, we specify the cost function on each firm. Cost function of firm 1 is this-$c(q_1) = c_1q_1 + F$. So, its marginal cost is constant. So, if we take this as a a cost function, and then the marginal cost of firms 1 is c1. So, this is the variable component and this is the fixed cost and we assume for a simplicity only this version of the cost function.

Again, the cost function of firm 2 is- $c(q_2) = c_2q_2 + F$ c2 into q2 plus F2, where q2 is the output of firm 2 and c2 and F are some positive real number. So, again marginal cost of firm 2 is constant and it is c2, okay. So, this is the specification of the market or specification of the agents who are operating in the market. So, the firms they have a cost function of this nature and the market demand is this. Now, we define the interaction.

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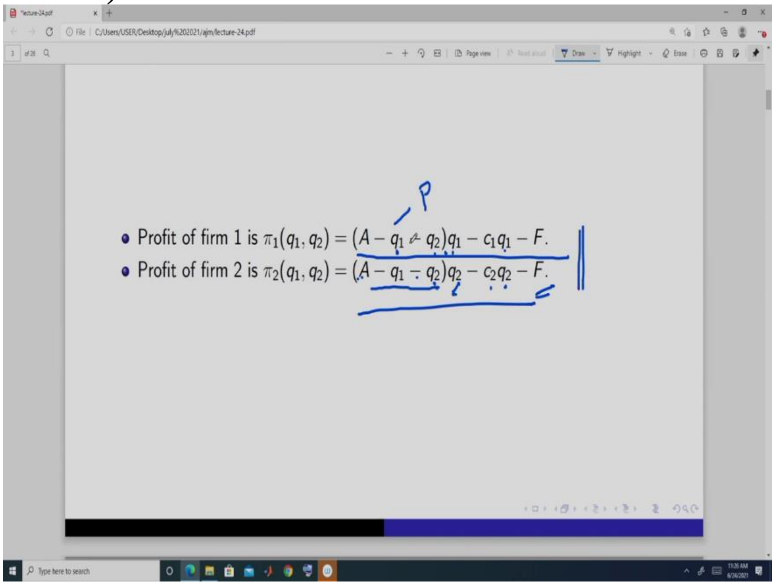

So, here based on this cost function and this market demand, we get the profit function of firm 1 is of this nature-$\pi_1\,(q_1, q_2) = (A - q_1 - q_2)q_1 - c_1q_1 - F$. This is Q1 output of firm 1, this is Q2 output of firm 2. So, this is the aggregate so, this is the market price and this is the output of firm 1. So, this is the total revenue minus total cost of firm 1, so, this is the profit.

And here again, this is A minus q1 minus q2. So, this is the price and into q2. So, this is the total revenue for firm 2 and c2 into q2, this is the variable cost, total variable cost minus F that is the fixed cost. So, this is the profit function of firm 2-$\pi_2\,(q_1, q_2) = (A - q_1 - q_2)q_2 - c_2q_2 - F$ So, we take this as the profit function, okay or you can say the payoff. And now we will define the game.

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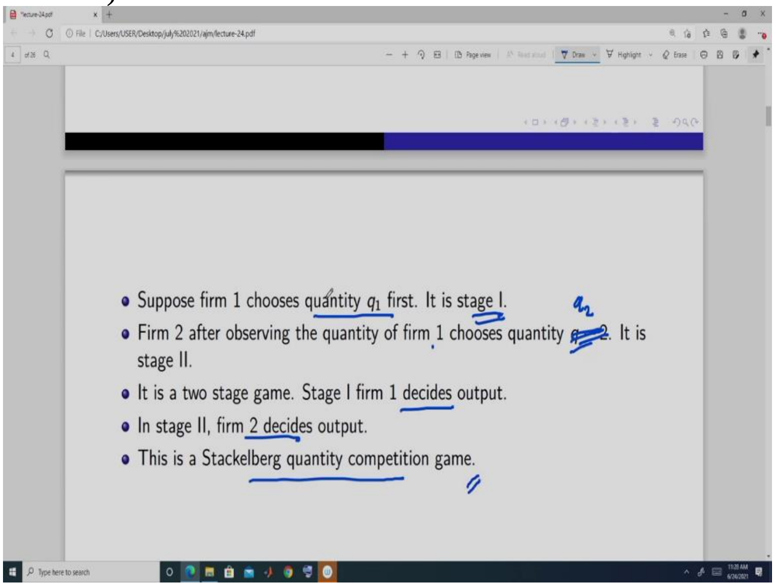

So, here firm 1 first chooses quantity q1, okay and it is called as stage 1. So, firm 1 first chooses this quantity. After observing the quantity of firm 1, firm 2 chooses this quantity and this is q2. So, this should be q2, this is not minus, a, q2. So, it is called the stage 2. So, we get it is an extensive form game or a dynamic game, which has two stages, stage 1, firm 1 decides output. And in stage 2, firms 2 decides output, okay.

So, this form of a market interaction is called a stackelberg quantity competition. Why compete, quantity competition? Because each firm chooses output. And why Stackelberg? Here, it is the decisions are taken sequentially. So, first firms 1 decides the output and then firm 2 after observing the output of firm 1, it chooses its output, that is q2, okay. So, this is the stackelberg. Now, we have already done dynamic games.

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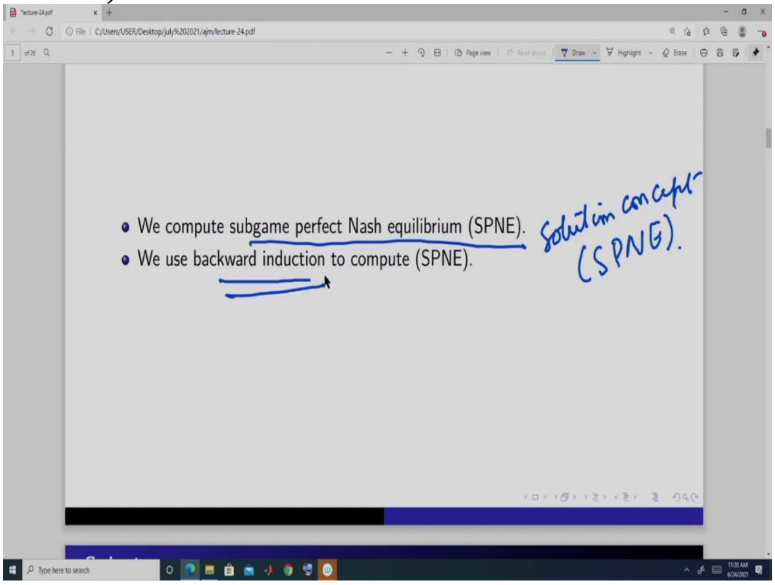

So, what do we do in dynamic games? We look for sub game perfect Nash equilibrium as the solution concept. So, solution concept we are going to use is SPNE, sub game perfect Nash equilibrium. And how do we find the sub game perfect Nash equilibrium? We use something called a backward induction, and we will use that in this market, okay. So, we have two firms, firm 1 decides its output in stage 1. First, I can say it in the first and after observing the output of firm 1, firms 2 decides the output it is going to produce, okay.

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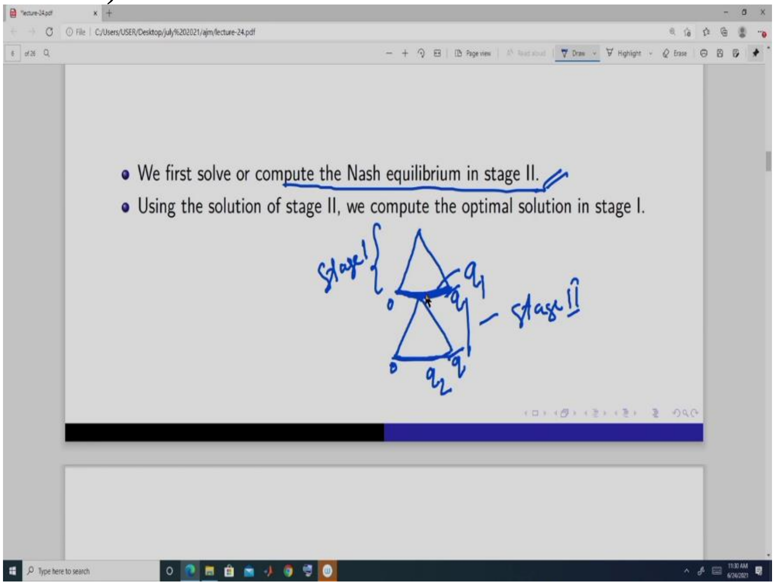

So, that is the stage 2. So, how do we solve this? So, when we are using backward induction, what we will do? We will first solve compute a Nash equilibrium in stage 2. So, it is going to be a game like this. If you look at the game structure, it is something like this where this is, you can say q1 and this is stage 1 and then this is stage 2. And this is q2, q2 again line because it is continuous.

So, I have drawn it line this so, it is from 0 and suppose some sufficiently big number q upper bar. And this is from 0 to sufficiently big number q upper bar, okay. So, this is stage 1 and this, this is stage 2 and this is stage 1. So, now how do we solve this? So, we use backward induction. So, then it means that we assume that suppose firm 1 has produced something, q1. And based on that, we find the optimal output of firm 2.

So, we first solve the stage 2, assuming some output, some outcome in stage 1. And then using that solution of stage 2, we compute the optimal solution in stage 1. So, this is the backward induction. So, first we take some, some value of q1 and then solve this. Because this is 1 sub game. And then we solve this whole game, using this outcome, okay.

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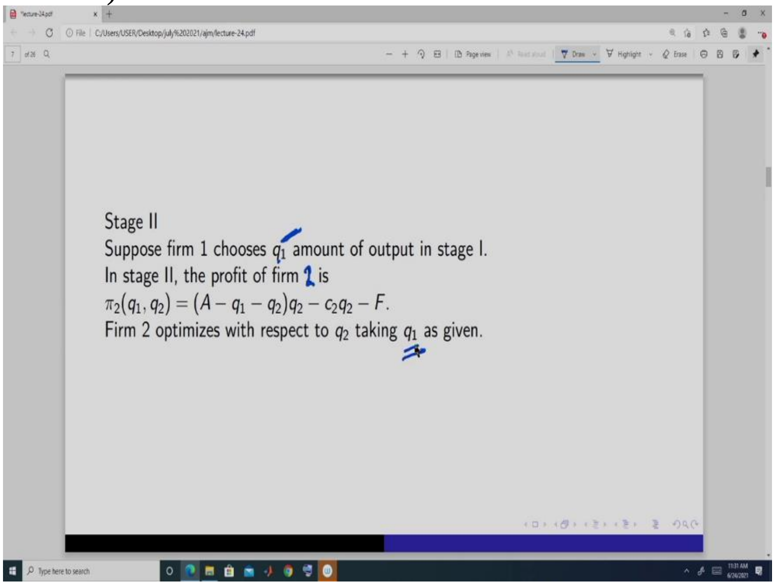

So, what we do? Suppose, the output of firm 1 is some amount q1, okay. So, in stage 2 profit of firm 2 is this, A minus q1 minus q2, q2 minus c2 q2 minus F. So, this is the total revenue and this is the total cost, okay. So, and firm 2, maximizes or or optimizes its profit with respect to q2. Taking q1 as given, because it cannot decide the output, it just simply observe, and it knows that okay firm 1 is already produced this much or it is committed, that it is met.

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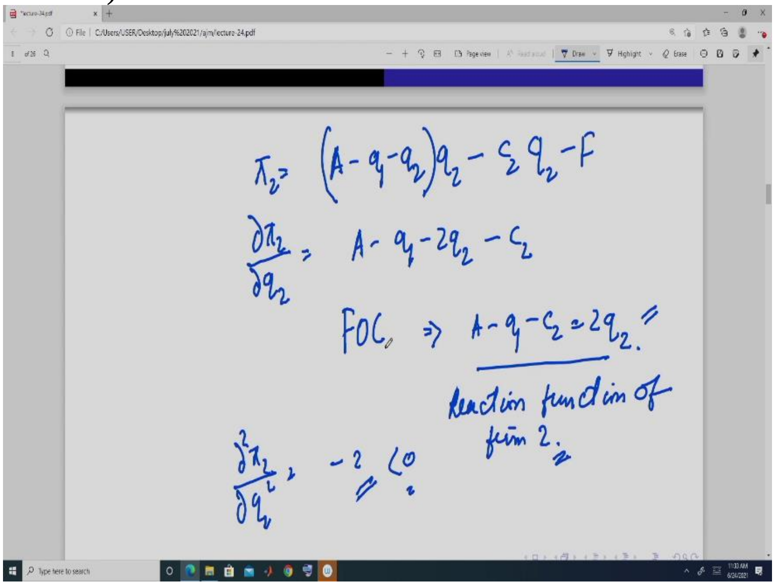

So, based on this we get, this is a profit- $\pi_2 \, (q_1, q_2) = (A - q_1 - q_2)q_2 - c_2 q_2 - F$ and when we are and this is differentiable. Because this function is differentiable actually. So, we get, we take the partial derivative with respect to q2 and that is again, first order condition for maximization gives me, this is the reaction function of firm 2- $A - q_1 - c - 2 = 2q_2$. We have derived such reaction function while doing Cournot competition, right? So, it is same as the cournot reaction function of firm 2, okay.

And if you take the second derivative, we will see that with respect to q2 it is negative. So, and it is always negative-$\frac{d^2\pi_2}{dq_2^2} = -2 < 0$. So, we do not have to bother whether it is a maximum point or not, the solution of this is a maximum point or not, is it going to maximize the profit or not, okay. So, we do not need to worry. So, based on this, we get this reaction.

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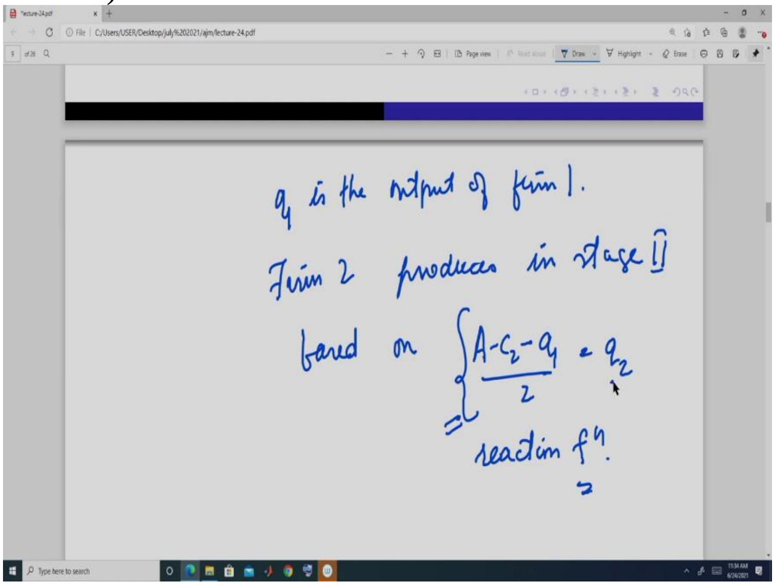

So, now what does this mean? That means, that whenever q1 is the output of firm 1, firm 2 produces in stage 2 based on this function, which is the reaction function-$\frac{A-c_2-q_1}{2} = q_2$. So, it produces based on this function, right? So, this is the outcome in stage 2. So, this is the optimal output. So, when firms 2 decides its output, it always decides based on this reaction function, okay.

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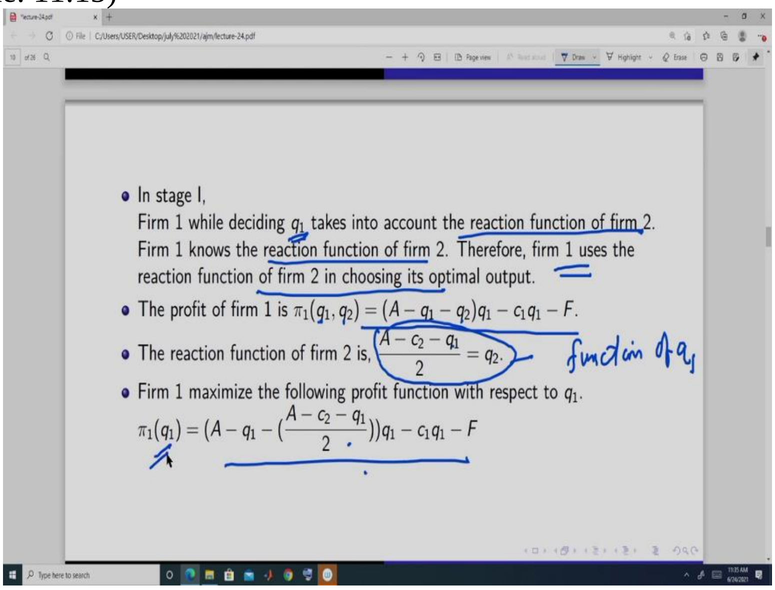

Now, we move to stage 1, what do. In stage 1, firm 2 is deciding q1, and it will take into account the reaction function of firm 2. Because it knows that if I produce some q1 then how firm 2 is going to react? It is going to react based on these reaction function. So, and firm 2 knows the reaction function of firm 2. So, firm 1 knows the reaction function of firm 2, how?

Because all these information's are a common knowledge cost function of firm 1 and firm 2, its common knowledge demand firms, demand mark, market demand is known to both the firms. So, so firm 1 can compute the reaction function of firm 2. So, firm 2 is going to use the reaction function of firm 1, firm 2 in choosing its optimal output, what it is going to do? So, firm 1 profit function is this, we have seen that.

Now, we know the reaction function and we have taken here profit function as a function of q1 and q2. But this q2, firm 1 knows is actually given by these reaction function. It is actually a function of q1. So, we plug in this here in place of this and we get this as the profit function. So, now the profit function of firm 1 is only a function of q1, its own output-$\pi_1(q_1) = \left(A - q_1 - \left(\frac{A - c_2 - q_1}{2}\right)\right) q_1 - c - 1\, q_1 - F$. Because it knows that how the firm 2 is going to react, based on its own output.

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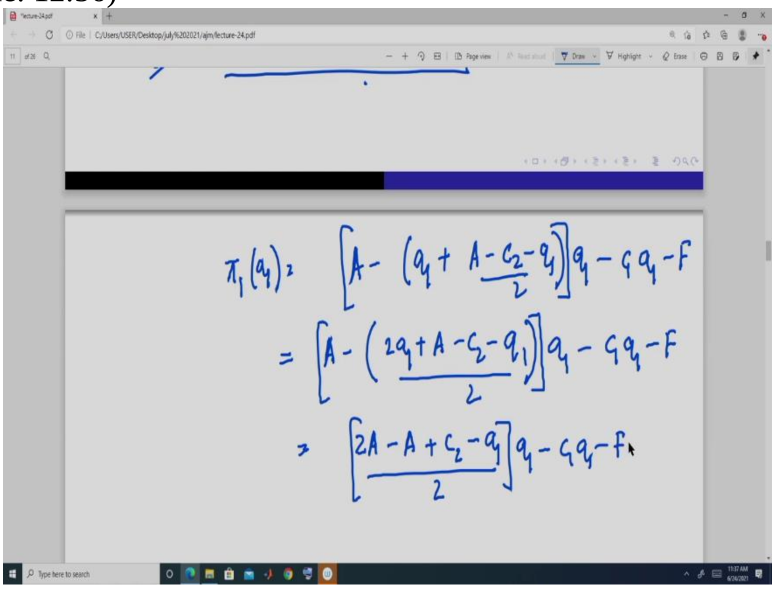


$$= \left[A - \left(\frac{2q_1 + A - c_2 - q_1}{2}\right)\right] q_1 - c_1 q_1 - F$$

$$= \left[\frac{2A - A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$\pi_1 = \left[\frac{A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$\frac{\partial \pi_1}{\partial q_1} = \frac{A + c_2}{2} - q_1 - c_1$$

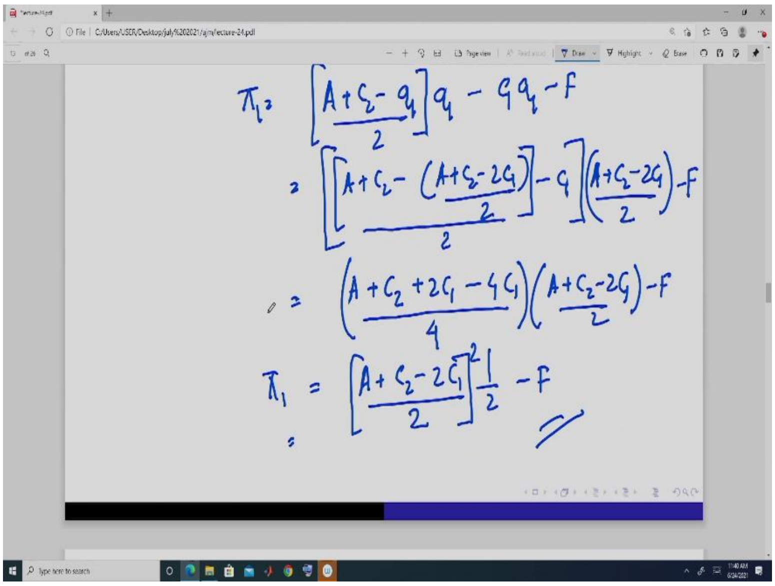
$$\text{FOC} \Rightarrow \frac{\partial \pi_1}{\partial q_1} = 0$$

$$\Rightarrow \frac{A + c_2 - 2c_1}{2} = q_1$$

Stage I, $q_1 = \frac{A + c_2 - 2c_1}{2}$

$$\pi_1 = \left[\frac{A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$= \left[\left[\frac{A + c_2 - \left(\frac{A + c_2 - 2c_1}{2}\right)}{2}\right] - c_1\right]\left(\frac{A + c_2 - 2c_1}{2}\right) - F$$



So, we now find the optimal q1. So, it is, it is this, this-$\pi_1(q_1) = \left[A - \left(q_1 + \frac{A-c_2-q_1}{2}\right)\right] q_1 - c_1 q_1 - F$. So, profit of firm 1 is actually, it is this-$\pi_1 = \left[\frac{A-c_2-q_1}{2}\right] q_1 - c_1 q - 1 - F$. Now we maximize this with respect to q1. So, we get, this first order condition gives me, so this equal to. So, this is the output of firm 1, in stage 1-$\frac{A-c_2-2c_1}{2}$. So, the stage 1 outcome, is q1 equal to this. So, the profit of firm 1 in stage 1, we have is this, it is this-$\left[[A + c_2 - \left(\frac{A-c_2-2c_1}{2}\right)\right] - c_1] \left(\frac{A-c_2-2c_1}{2}\right) - F$. Because there is this 2, so, this whole thing is going to be. So, this is the outcome in stage 1-$\left(\frac{A-c_2-2c_1}{2}\right)^2 \frac{1}{2} - F$. So, in stage 1 profit of firm 1 is this, and output of firm 1 is this. Now, based on this we can find out the outcome in stage 2.

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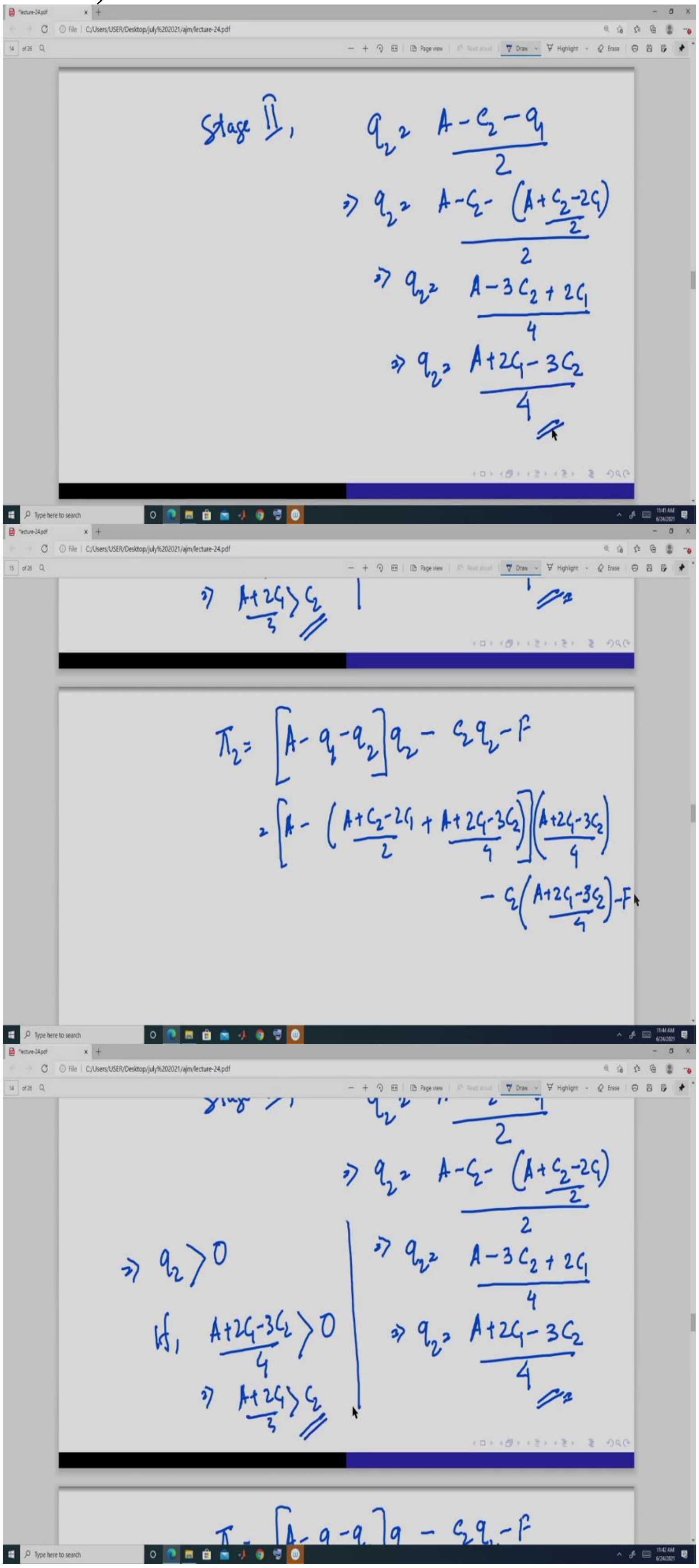

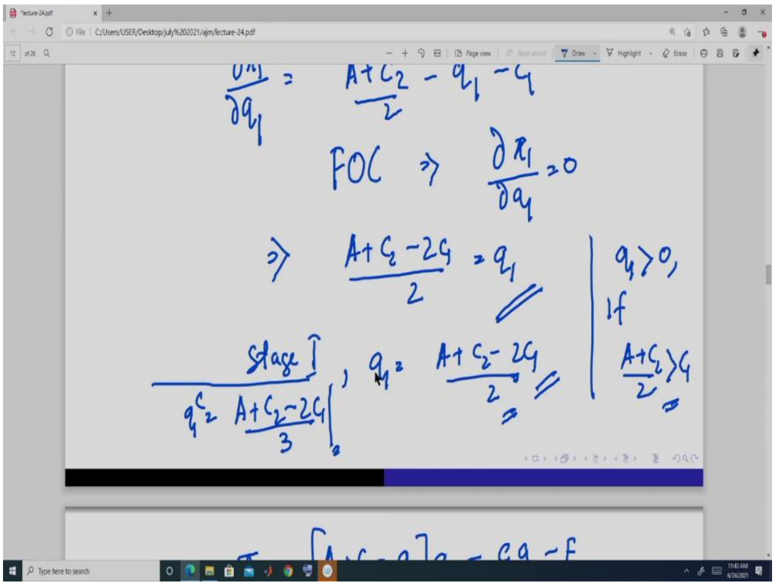

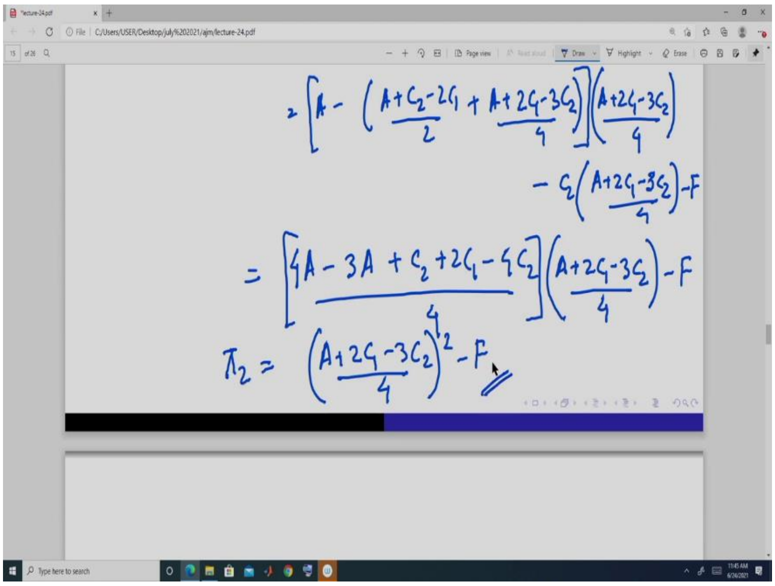

So, in stage 2, what happens? Q2 is A c2 minus q sorry, it is the Cournot reaction function-$q_2 = \frac{A-c_2-q_1}{2}$. So, we get this and so, q2 is, it is this-$q_2 = \frac{A-3c_2+2c_1}{4}$. So, we write q2 is equal to this-$\frac{A+2c_1-3c_2}{4}$. And so, now the profit of firm 2 in stage 2 is, is this-$(A-q_1-q_2)q_2 - c_2q_2 - F$. So, now here remember, this from here we have to put a condition that is q2 is positive, q2 is positive if, if this is. So, this implies c2 must be this-$\frac{A+2c_1}{3}$.

And here, this condition for this to be positive, q1 is positive if A plus c2. So, this is always going to be positive. Because, c, it is this output is actually if you look at this, this is greater than the Cournot output. Because in Cournot, you remember Cournot output is this-$q_1^c = \frac{A+c_2-2c_1}{3}$, right? We will show again this and we get this.

So, this is greater than Cournot and we know in Cournot it is positive. So, it is positive, but this is less than the Cournot output and this is so, we need this condition-$\frac{A+2c_1}{3} > c_2$. Now, let us compute the profit of firm 2. Profit of firm 2 now simply substitute the values, we get this. So, profit of firm 2 is this-$\pi_2 = \left(\frac{A+2c_1-3c_2}{2}\right) 62 - F$ and in stage 2.

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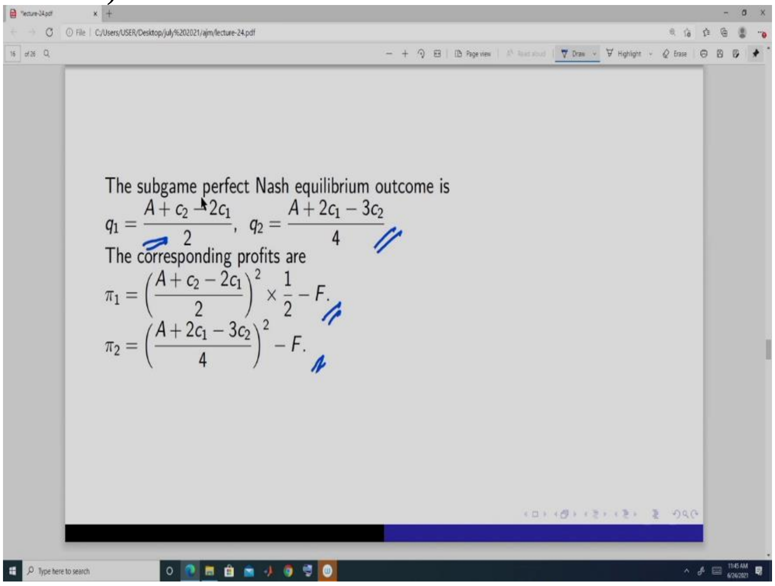

So, we get, so, here the sub game perfect Nash equilibrium outcome is this for, for firm 1, this is the output- $q_1 = \frac{A+c_2-2c_1}{2}$ and this is the output for firm 2-$q_2 = \frac{A+2c_1-3c_2}{4}$. And the profits are this. This will be decided in stage 1 and this will be decided in stage 2 and these enter profits, right? Now we can compare the profits.

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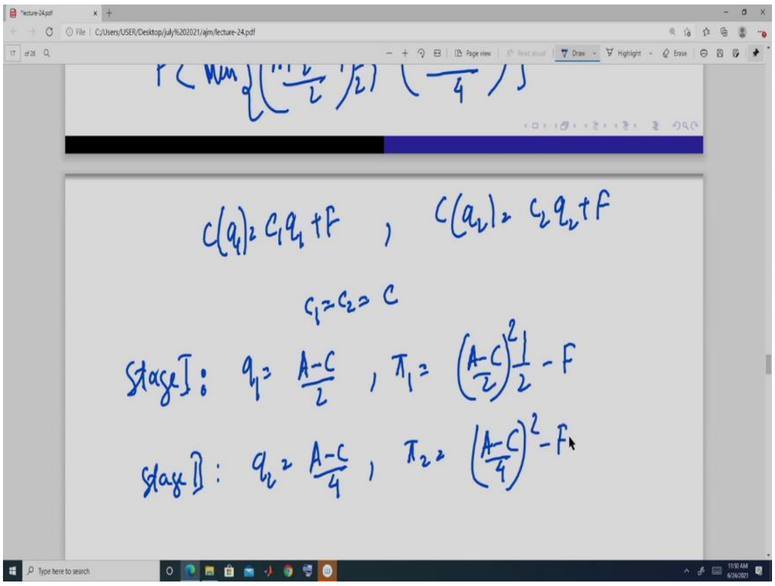

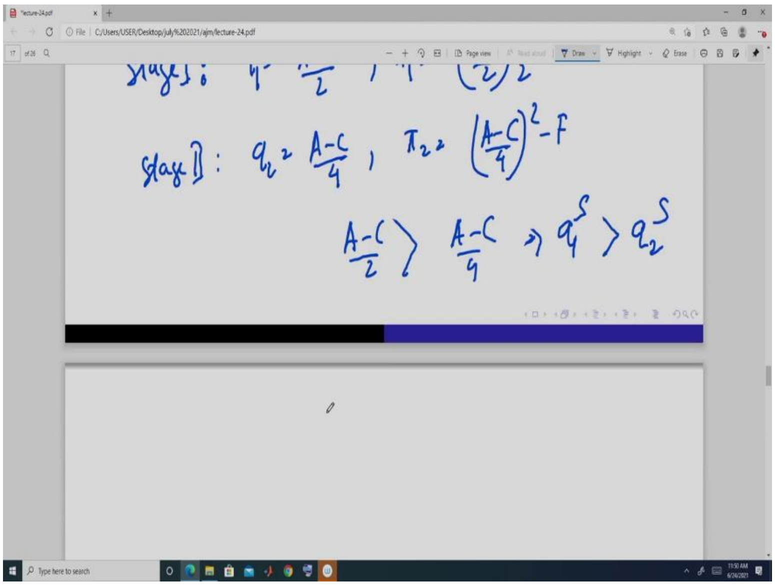

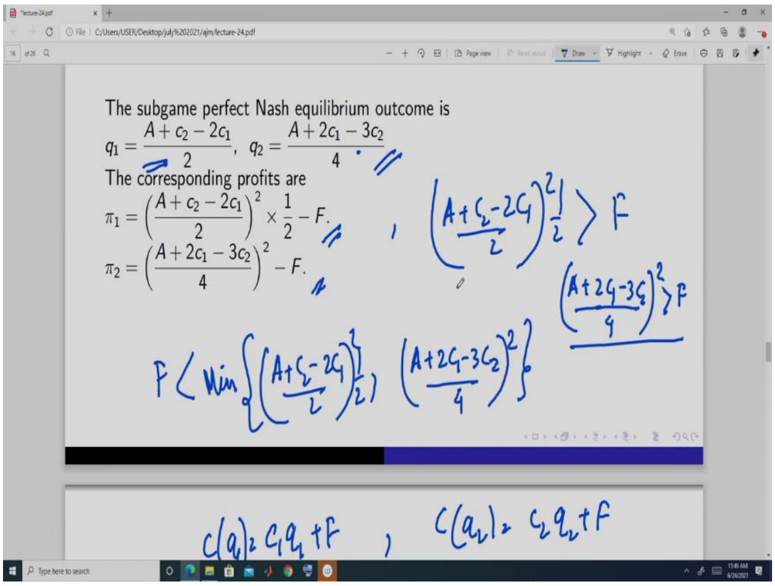

So, whether the profit of firm 1 is greater than profit of firm 2. But now here, there is a firm 1 and firm 2 are not similar. Firm 1's cost function is this-$c(q_1) = c_1q_1 + F$ and firms 2's cost function is this- $c(q_2) = c_2q_2 + F$. So, whether they are making higher profit or lower profit it also depends on this marginal cost, this, this marginal cost, see? So, it will be not possible to compute. So, we have to take that, okay if marginal cost is lower for firm 1 or it is greater for firm 1 like that, different conditions.

So, for simplicity suppose we do this assumption, c1 is equal to c2 which is equal to c, okay. But this, this kind of by taking this kind of output we will compare the profit and that we will do through diagram, not algebraically. Because algebraically it will be messy and it will not give us very clear picture, it will be a conditional statement. So, let us take this. So, then q1 is what? q1 is this-$\frac{A-c}{2}$ in stage 1, q1 is this and profit is this-$\pi_1 = \left[\frac{A+c_2-2c_1}{2}\right]^2 \frac{1}{2} - F$, okay.

So, here, here we have to make further an assumption. And that is the F should be less than this amount. So, F should be, this-$\left[\frac{A+c_2-2c_1}{2}\right]^2\frac{1}{2} > F$. And so, based on this, F should be less than minimum of this. So, f should be less than min of, we get this. And F should be minimum of this two- $\min\{\left[\frac{A+c_2-2c_1}{2}\right]^2\frac{1}{2}, \left[\frac{A+2c_1-3c_2}{4}\right]^2$. Otherwise, it will not be profitable to produce at all, okay. So, this, we should not overlook these kind of conditions.

So, now here profit of this. So, in stage 2, q2 is going to be here it is 2 c1 minus 3 c2. So, again c divided by 4, it is this. Now, if we compare these two outputs- Stage i: $q_1 = \frac{A-c}{2}, \pi_1 = \left(\frac{A-c}{2}\right)^2\frac{1}{2} - F$, Stage ii: $q_2 = \frac{A-c}{2}, \pi_2 = \left(\frac{A-c}{4}\right)^2 - F$, we get this is definitely greater than this. So, this implies q1 stackelberg is greater than q2 stackelberg where, firm 1 moves first and firm 2 moves second.

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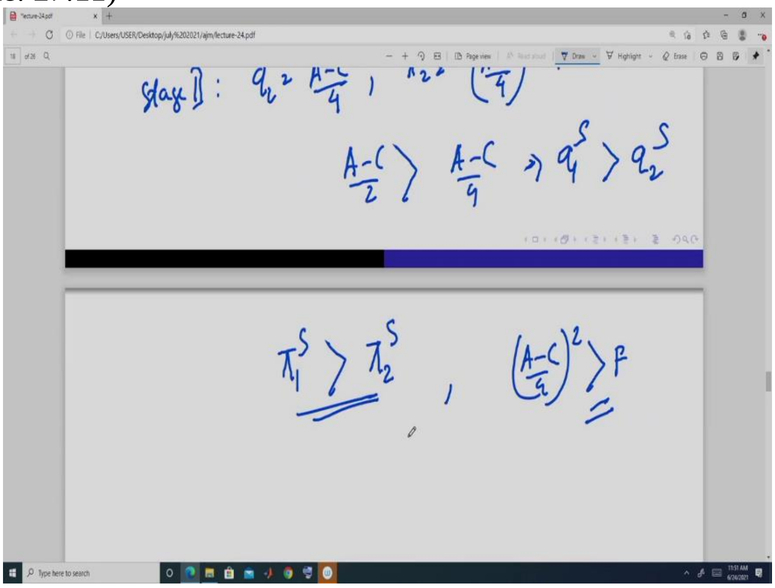

And profit, so, from this we get the profit also. Profit of firm 1 in stackelberg is greater than profit of firm 2 in stackelberg. Because, this is definitely less than this, obvious. And here, we have to make an assumption that this$\left(\frac{A-c}{4}\right)^2$ is greater than F, okay. So, we get this.

Now, the question is so, we get that if the firms are similar in terms of cost, then the firm which moves first or the firm which decides output first, it is going to on higher profit then the firms which is going to decide or choose output later or it is in second's stage, okay. So, this immediately gives that if I move first then I have an advantage. So, this is the first, here we can say that it has a first mover advantage.

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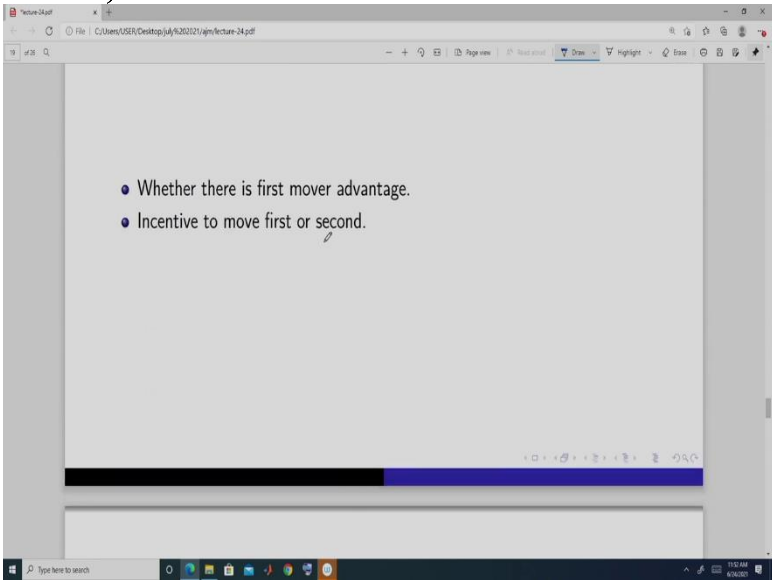

So, whether there is a first mover advantage? Yes, there is a first mover advantage, we have found it when the cost functions are similar. Now, we will see what is, what happens when costs are not similar in this. And we will do it diagrammatically and related to this topic we have a thing like whether there is an incentive to move first or move second, this we will consider later.

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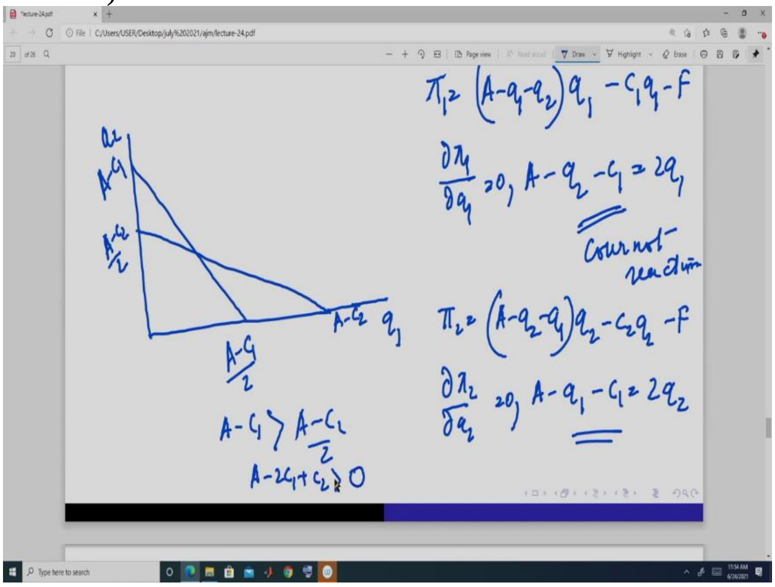

Now, see our profit function is of firm 1, is this-$(A - q_1 - q_2)q_1 - c_1 q_1 - F$. So, if we simply, this is the cournot reaction function-$A - q_2 - c_1 = 2q_1$. And for firm 2, this- $(A - q_2 - q_1)q_2 - c_2 q_2 - F$ we get this- $A - q_1 - c_1 = 2q_2$. So, this is again the firm 2's reaction function. If we plot them, we get this and this is A minus c1. So, here we assume there is an implicit assumption that this- $A - c_1$ is greater than A minus c2 half-$\frac{A-c_2}{2}$. So, this means that A minus twice c1 plus c2 is greater. Again, we have this less than this.

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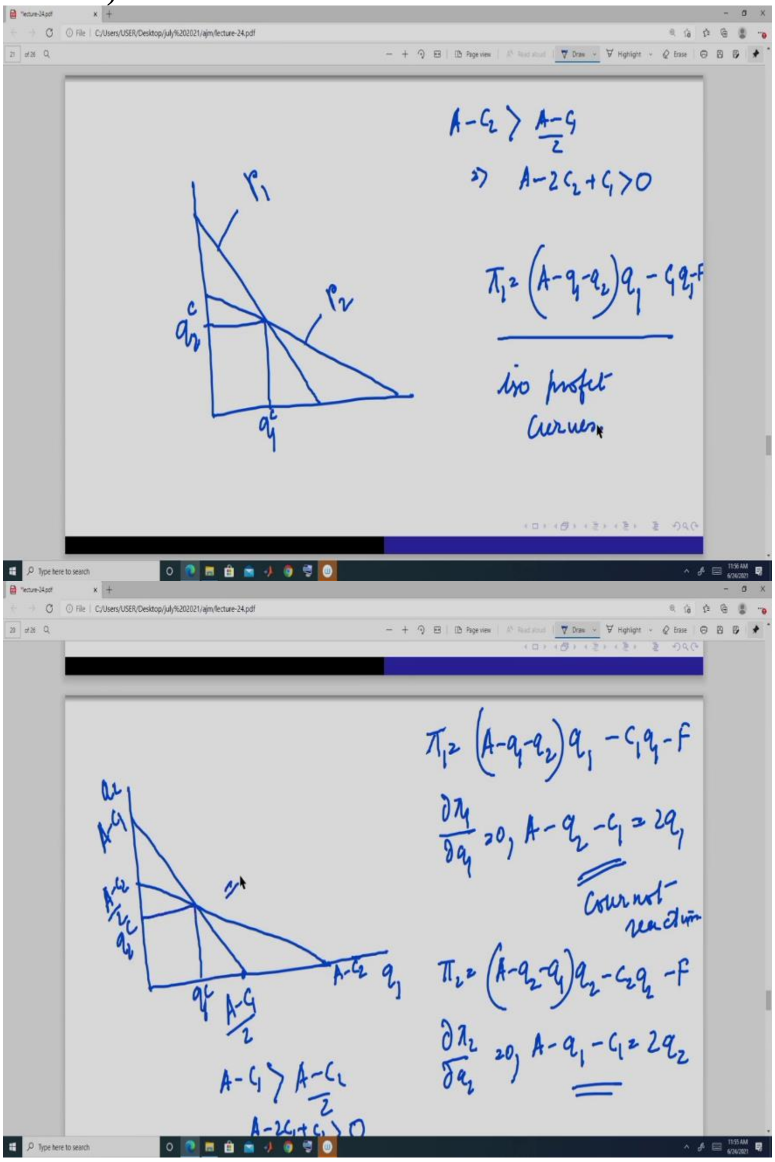

So, so, this implies, is greater than this and we know that we assumed this thing in the cournot thing. So, this is cournot output, this is cournot output, we have got this. Now, we will use this diagram, how? See, so, the if we take this, this is the reaction function of firm 1 and this is the reaction function of firm 2, this is the cournot output of firm 1 and this is a cournot output of firm 2, okay. And profit function of firm 1 which is a function of q1 and q2, this- $(A - q_1 - q_2)q_1 - c_1q_1 - F$. So, now we draw something called iso profit curves. What are iso profit curves? Iso profit curves are actually level curves.

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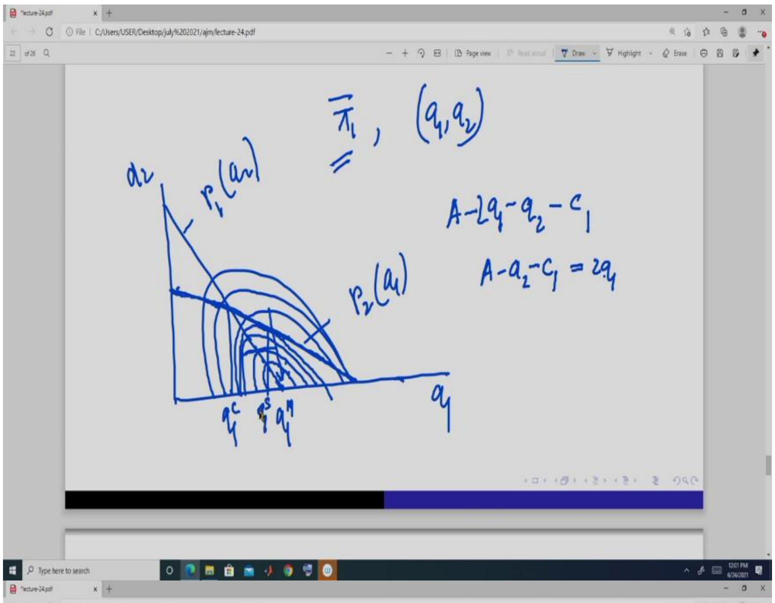

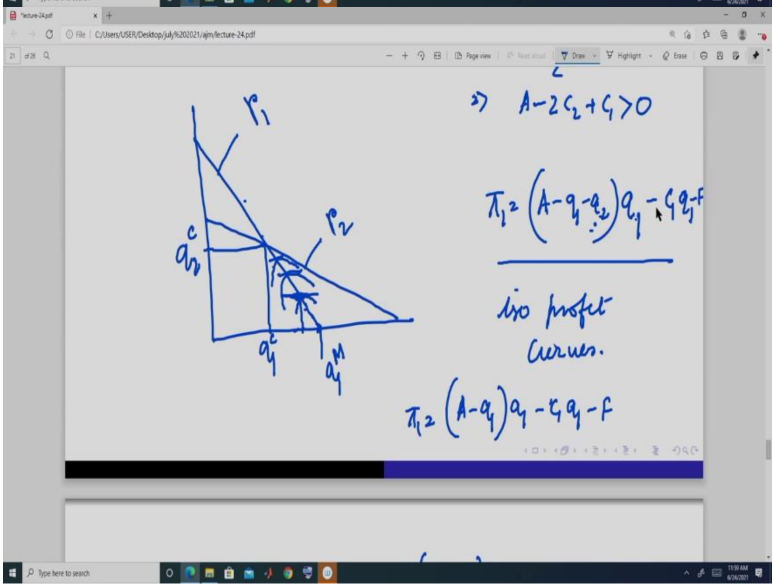

So, what we do? We fix the profit of firm 1 this and look at the combination of q1 and q2 such that it gives me the same level of profit. Now, here in this reaction function, if we fix the level of q2, some level and then find the optimal A, so, that is given by this reaction function. So, this function-$\pi_1 = (A - q_1 - q_2)q_1 - c_1q_1 - F$, if we plot this is going to be you can look at this. So, it is what? If we take the derivative, right?

Now, A this- $A - q_2 - c_1 = 2q_1$is equal to this line, but if this is q is greater than this, so, you fix the level. Suppose, I fixed the level here of q2. So, if q1 is less than this amount, then this is positive increasing. And if q is greater than this amount, then it is negative. So, we get a like this. So, this is like this, that is why it is a maximum point, given a fixed q2. Similarly, we will have a like, like this. Here it is increasing and then it is decreasing.

Again, it is like this. So, for each point we fix some q2 and we will get the, how it looks this. Then and when q2 is 0, this is what? It is this again, it is a, again it is what? It is a quadratic function-$(A - q - 1)q_1 - c_1q_1 - F$ or you can see it is also a function of the monopoly profit function, monopoly profit function, right? and this is maximum at this. So, this is the q1 when it is a monopoly. And when q2 increases, we get this.

We shift from this function to this function. And so, the, we know the profit is maximum and the monopolist. So, as we move here, so q2 is increasing, this is increasing. So, that means this portion is decreasing this, if we even if we keep constant what happens? Profit goes down. So, if we take these as the reaction functions of firm 1 and firm 2, cournot reaction function, so, this is the output when firm 1 is monopoly and this is the output when firm 2 is Cournot.

And here we get curves like this, where this is 10 means, derivative is 0 here of the iso profit curve. Iso profit curve, means this curve. So, the combination of q1 and q2 such that profit is same. So, it is this combination of q1 and q2 such this profit is constant. And if we go up like this, profit goes down. Because this is the monopoly profit and as q2 increases, you look at this function q2 increases, this portion is decreasing keeping q1 constant so, it is going profit is going down.

So, profit increases in this direction, so, this is the maximum point. So, now what firm 1 does? Firm 1 knows that firm 2 is going to be produced output based on this reaction 1. So, firm 1 in stage 1 will decide that point of this reaction function, which is going to give me the maximum profit. So, we will have this kind of reaction function, iso profit curve. And we will choose that one which is going to be the, giving me the maximum.

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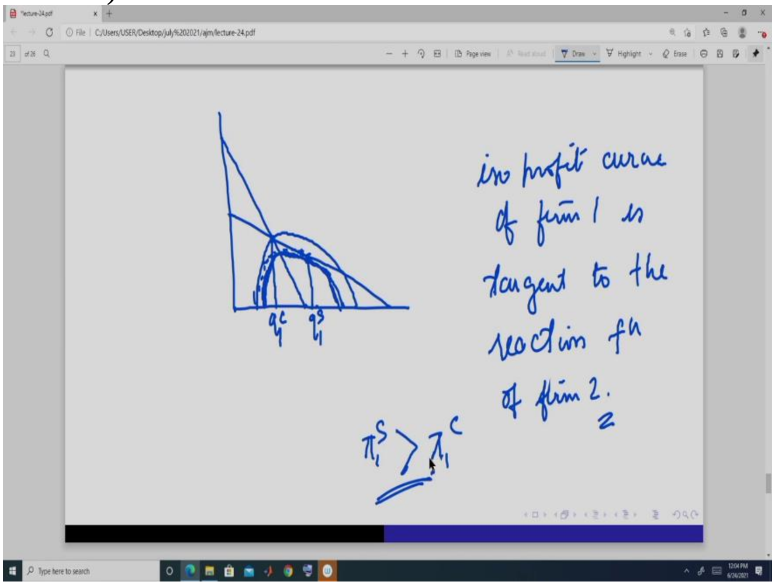

Now suppose, so, from here we get it has to be some tangent like this. So, it is suppose this. This is going to be the Stackelberg output, why it is going to be tangent? Because if a firm is suppose it is not tangent, so, let us draw again. Suppose, this line is tangent at this point. So, this is stackelberg output of firm 1, this is the Cournot, okay.

Now, suppose this is not the output, somewhere different. It has produced something less, then it is producing here, firm 2 is produced based on this. If it, if firm 1 produces here, firm 2 is going to produce based on this reaction function, so, it will be here. And that will be lying somewhere in a reaction function, iso profit curve, which lies below this. If it is more, here because this line is only tangent.

So, it will always all these iso profit curves are going to lie below this. So, profit of firm 1 is not maximized, if the iso profit curve lie above this iso profit. Because if this is tangent to this from below, it is like this. So, all the iso profit which lies above it are going to give less profit. So, any output here it means output of firm 2 is here. So, so there must be a (tan) iso profit curve which passes through this, but this will lie above this iso profit, which is tangent. So, that is why profit is going to be less.

So, that is why this is going to be the stackelberg, wherever it is tangent. So, iso profit curve of firm 1, is tangent to the reaction function of firm 2, right? And this is this Cournot output, right? So, this passes through this iso profit, and this is the stackelberg. So, what do we get? So, the profit in stackelberg is greater than profit in Cournot, whenever the iso reaction functions are like this, downward sloping we get the profit of firm 1 which moves first is higher in Stackelberg than the Cournot.

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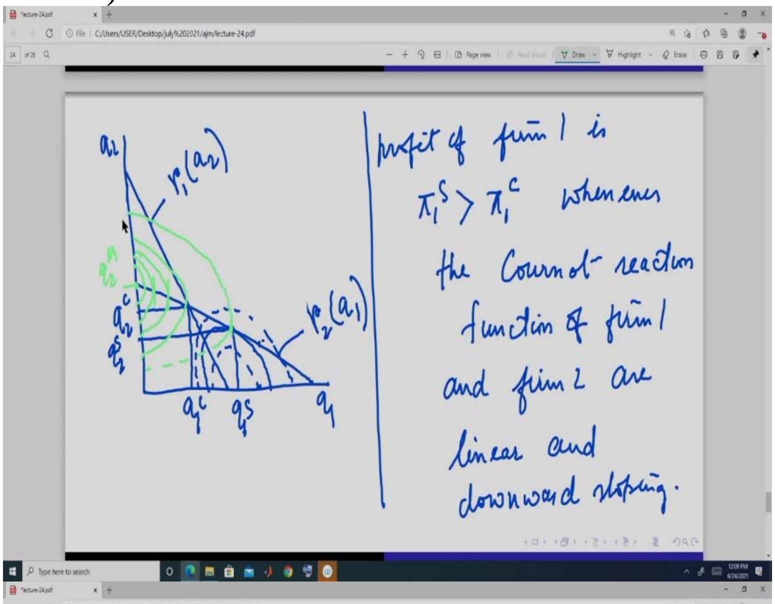

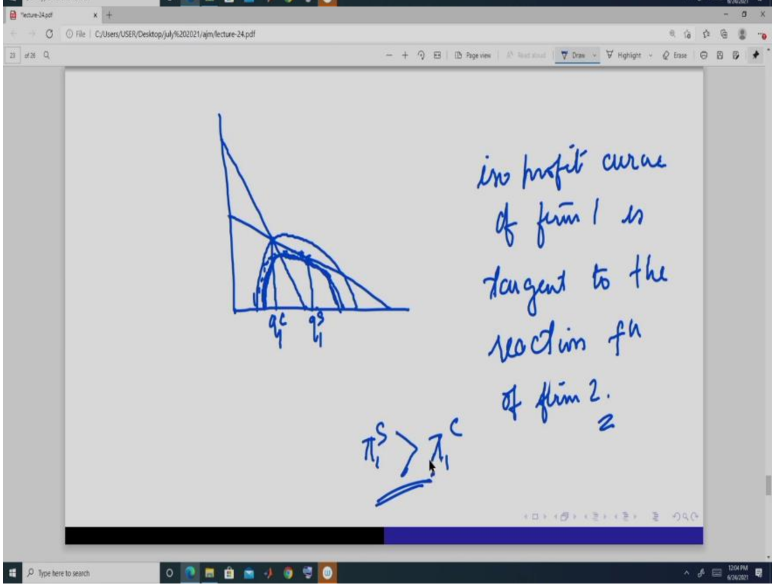

So, we write it in this form, that profit of firm 1 is this-$\pi_1^s > \pi_1^c$ whenever Cournot reaction function of firm 1 and firm 2 are, are you can say linear and downward sloping, or as given in this figure, right? Now, from here we again try to find out whether there is a first, so, from this we get to see what is happening. So, profit of firm is this. Now, let us look at from the context of firm 2, okay. This is the reaction function of, okay and suppose this is the reaction, okay.

This is q1 c cournot output, this is Cournot output c, right. We know whenever the cournot reaction function intersects, we get the cournot output. And we have seen this to be the stackelberg output of firm 1 and this is the stackelberg output of firm 2. Because in stackelberg competition what happens? Firm 1 takes the reaction function of firm 2 as given and based on that rather than the output. So, given this reaction function, firm 1 chooses that combination of that q1 which maximizes its profit, okay.

So, these are the iso profit curves of firm 1 and this is 1. Because this iso profit curves gives higher profit to firm 1 than this. But if you choose anything based on this suppose this output or this, if you choose this output firm 2 is going to produce here. If it produce here, so, the iso profit that passes through this point lies above this. So, profit is less.

Now, here let us draw the similar iso profit curves of firm 2. So, firm 2’s monopoly output is this, this is firm 2's monopoly output. Its reaction function is going to be some, something like this, somewhere like this where it is like this way and it will be like here. And this is the stackelberg, this is the cournot. So, from here, what do we get?

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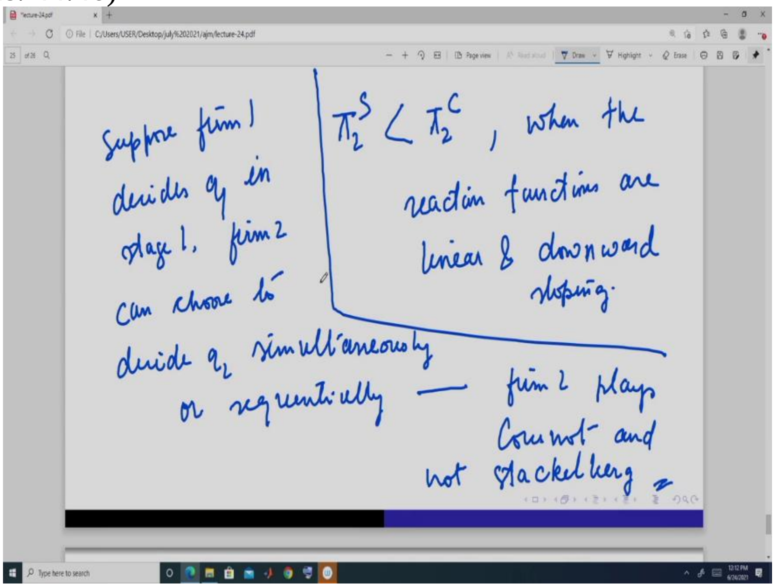

We get that the profit of firm 2 in stackelberg is always going to be less than the Cournot, right less than the cournot, when reaction functions are linear and downward sloping, we have got this. So, from here what do we get? We can conclude that if firm 1 is moving in stage 1, and if firm 2 is given a choice whether to move in stage 1 or in stage 2 like, a firm 2 has a choice to play cournot or to stackelberg firms 2 is always going to play Cournot from this.

So, that is why we see that firm 2 will never want to be the second mover in this case. Because then it has a first mover advantage. So, if we write the problem in this way that suppose firm 1 decides q1 in stage 1 and firm 2 can choose to decides q2 simultaneously or sequentially then we get in this scenario, firm 2 will play Cournot and not Stackelberg, okay.

So, in this context whether there is a first mover advantage or not we get that the firm 1 has a first mover advantage in this cournot in stackelberg quantity competition. And firm 2 has an incentive to move along with firm 1, rather than move later after observing the output of firm 1, okay. So, from this graphical analysis, we get this, okay.

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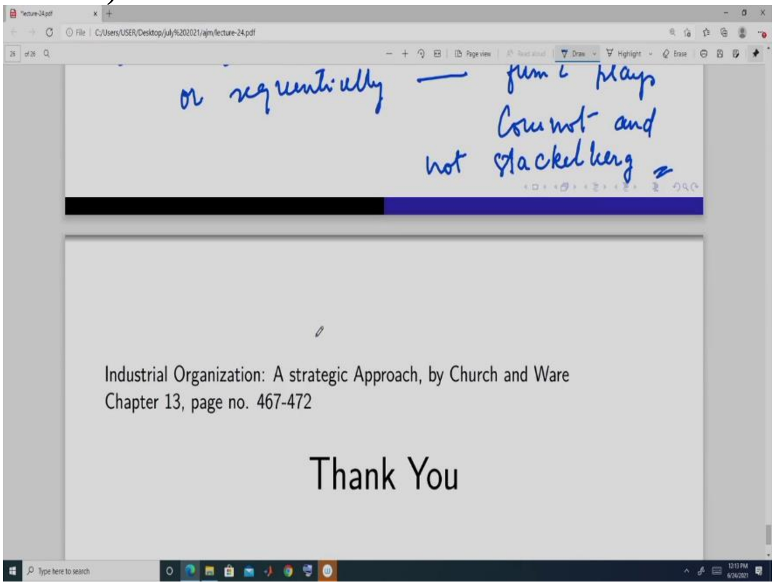

So, you can read this portion from this book by Church and Ware, A Strategic Approach, Industrial Organizational or strategies approach chapter 13-page number this, or these class notes are sufficient enough. And in the next class we will do the price competition in this kind of setup. Thank you.

Introduction to Market Structures
Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture 34
Tutorial on Bertrand Competition and Stackelberg Quantity Competition

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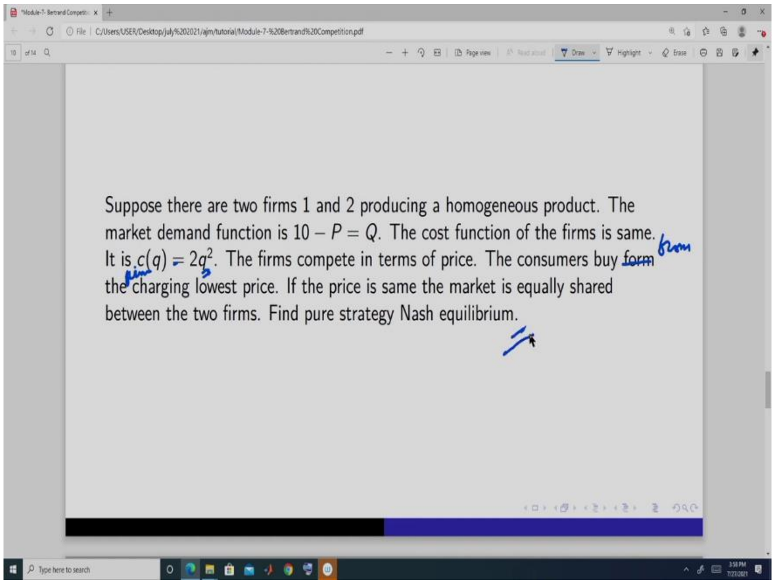

We will now use the decreasing returns scale. Suppose there are firms, firm 1 and firm 2, and the market demand function is this- 10–P=Q. Cost function of firm is same. It is this- $c(q) = 2q^2$, both the firm. And firm competes in terms of price, that is Bertrand competition and a consumer buys from the firm charging lowest price and if the price is same, the market is equally shared between the two firms. So, find a pure strategy Nash equilibrium. We have to solve the pure strategy.

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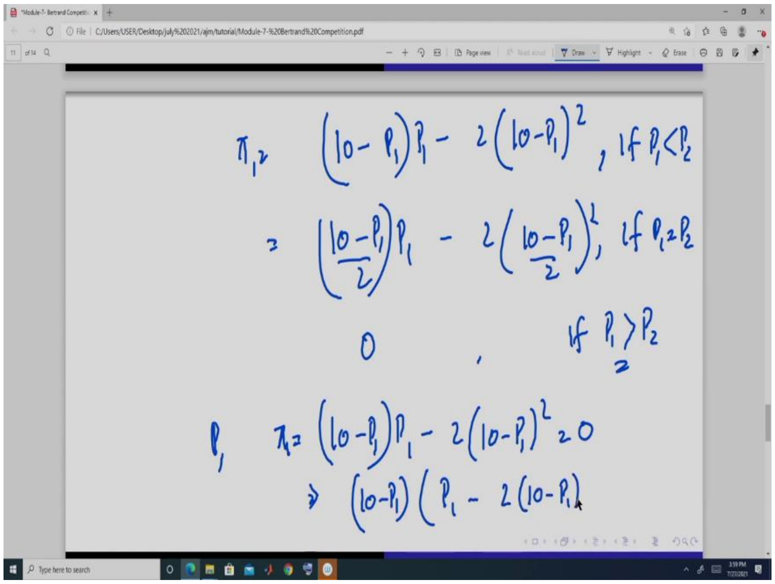

So, here again, we get the profit of firm 1 is this-$(10 - P_1)P_1 - 2(10 - P_1)^2$, if P 1 is less than P 2. And it is equal to this-$\frac{(10-P_1)}{2}P_1 - 2\left(\frac{10-P_1}{2}\right)^2$, if P 1 is equal to 2. And it is equal to 0 if P 1 is greater than P 2. We will get this. So, we find out P such that profit, this is-$(10 - P_1)P_1 - 2(10 - P_1)^2$ equal to 0. We get this at two price.

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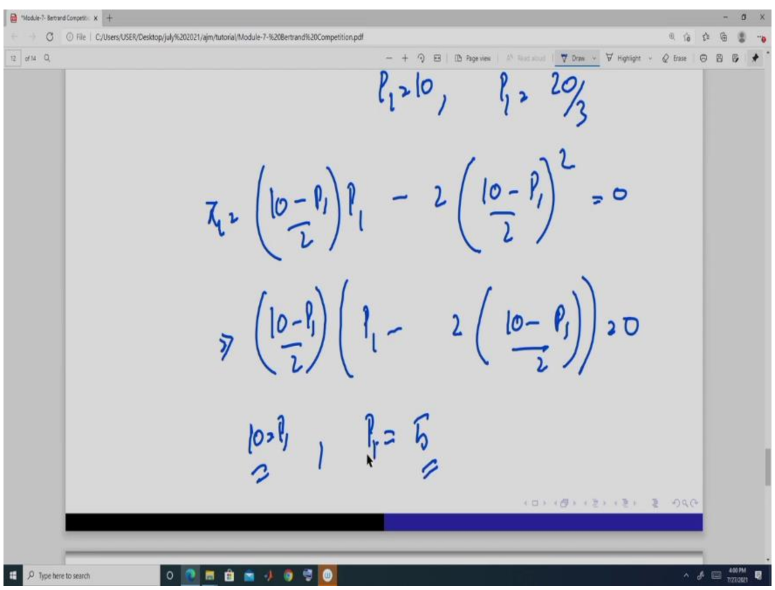

So, we get this is equal to 0. So, at P 1 is equal to 10 and P 1 is equal to 20 by 3, we get this. Again, we find out that this-$\frac{(10-P_1)}{2}P_1 - 2\left(\frac{10-P_1}{2}\right)^2$, and this is equal to 0. So, this is, this is equal to 0-($\frac{(10-P_1)}{2}$)$(P_1 - 2\left(\frac{(10-P_1)}{2}\right) = 0$. So, we get that P is equal to this, and P is equal to, P is equal to 5. We get this. P 1. So, these prices are going to be same for firm 1 and firm 2.

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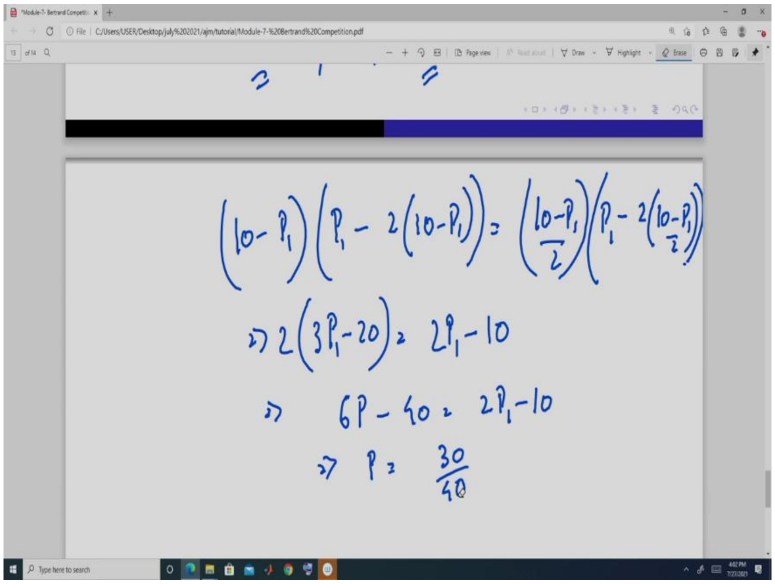

And we get another price when this-$(10 - P_1)(P_1 - 2(10 - P_1))$ is equal to this-($\frac{(10-P_1)}{2}$)$(P_1 - 2\left(\frac{(10-P_1)}{2}\right)$. So, this implies this-$2(3P_1 - 20) = 2P_1 - 10$. So, it is 30 by 4.

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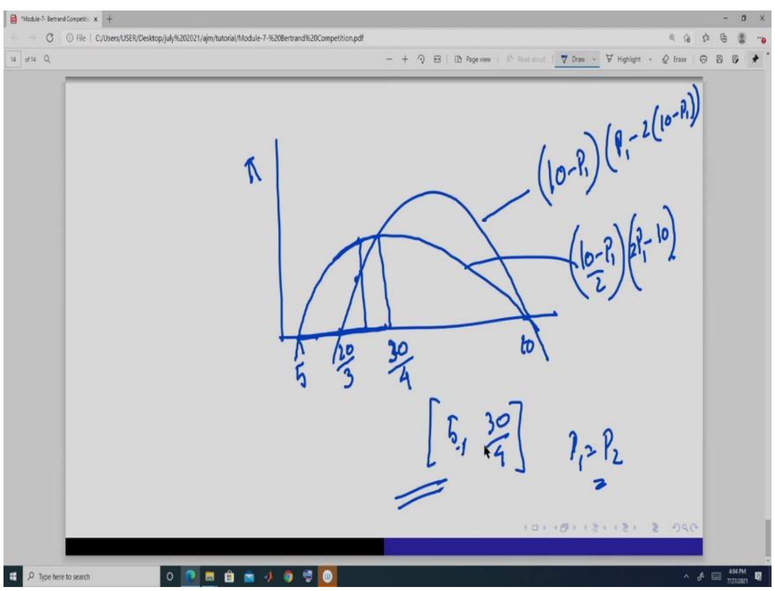

Now, if we plot this function, this function is this. This point is 10, this point is 20 by 3. And this function is, is this. This point is 5 and this is 10. This point is, one second, 30 by 4, okay. So, and here will be profit. So, we will get the same prices for firm 1 and firm 2. And here we know that this range, because if firm 1 sets a price, suppose here, firm 2, if it undercuts, it will get this, but if it sets the same price, it is going to get this. So, if we look at this whole range of prices, the Nash equilibrium lies in this range-[5, 30/4]. So, each price in this range, price in this range constitute a Nash equilibrium. And P 1 is always equal to P 2 and they lie within this range, okay.

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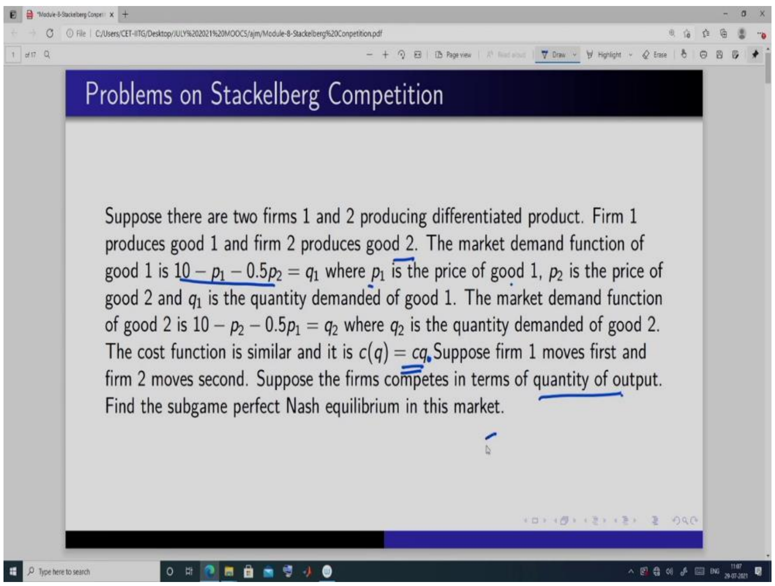

Let us solve some problem on Stackelberg competition. So, in this problem, suppose there are two firms 1 and 2, producing differentiating products. Firm 1 produces good 1, and firm 2 produces good 2. And our market demand function is this-$10 - p_1 - 0.5p_2 = q_1$, okay where p 1 is the price of good 1, p 2 is the price of good 2, and q 1 is the quantity demanded of good 1, which I produced by firm 1.

And market demand function for good 2 is this-$10 - p_2 - 0.5p_1 = q_2$. 10 p 2 minus point, 0.5 p 1 equal to, so here q 2 is the quantity of good 2, which is produced by firm 2. And the cost function for both the firms are same and it is given by c q, okay. Suppose firm 1 moves first and firm 2 moves second. So, it is a sequential gain and that is why it is a Stackelberg. And firms competes in terms of quantity. So, this is again Stackelberg quantity competition with differentiated product. And we have to find the subgame perfect Nash equilibrium.

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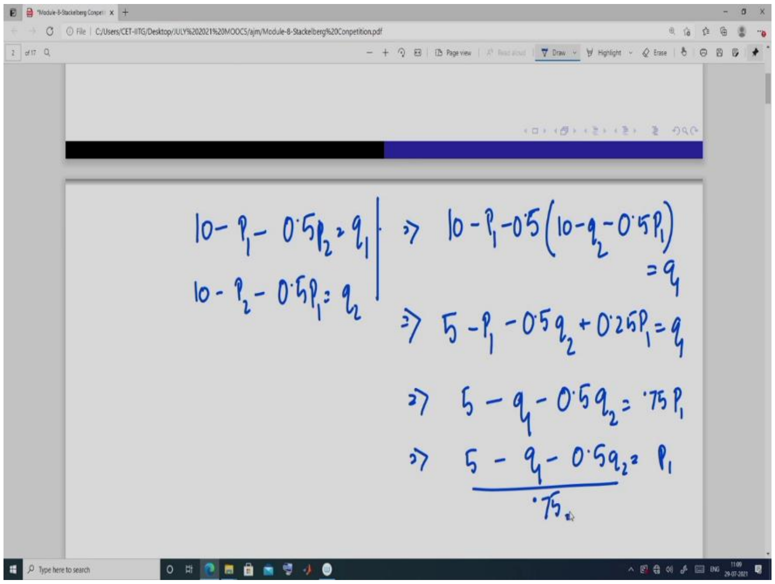

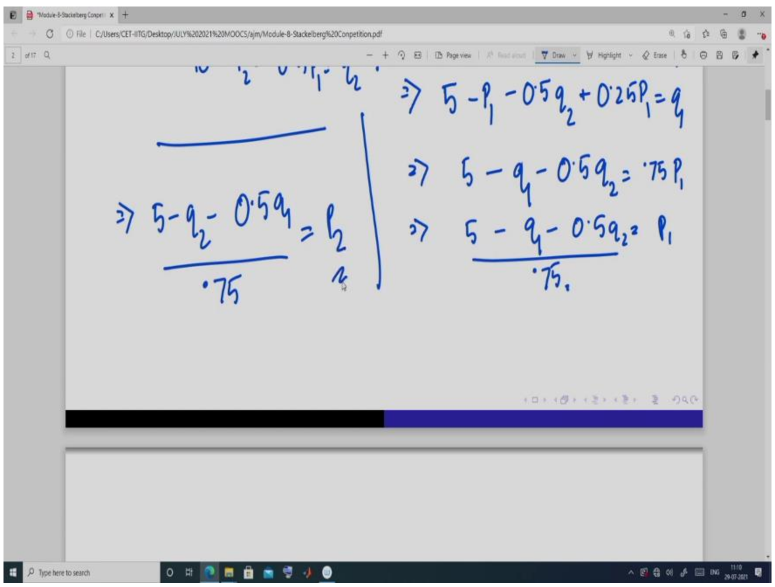

So, from this, first what we will do, we have to convert this-$10 - p_1 - 0.5p_2 = q_1$, this demand equation. So, this, you can write simply because the demand for good 2 is this-$10 - p_2 - 0.5p_1 = q_2$, right? So, from here, you can, you can write this-$10 - p_1 - 0.5(10 - p_2 - 0.5p_1)$. Substitute here, this, you take this here, we will get q 2. And this you can write as q 1. So, this is 5, q 2, which we can write in this. So, this is the inverse demand function of good 1, this -$5 - q_1 - 0.5q_2 = P_1$. Similarly, since the demand functions are more or less same, inverse demand function for good 2, we can write q 1, 0.75 equal to q 1, equal to p 2. This is the price of good 2-$\frac{5-q_2-0.5q_1}{0.75} = p_2$.

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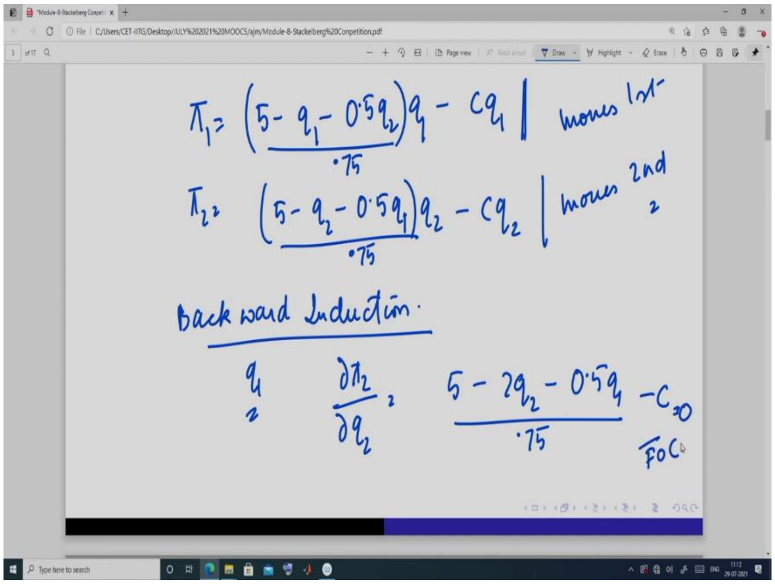

So, from this, we can write the profit function of firm 1. Profit function of firm 1 is 5 minus q 1, 0.5 q 2, q 1, cost is c, marginal cost is constant, c, and it is q 1 divided by. This is the profit function of firm 1-$\pi_1 = \left(\frac{5 - q_1 - 0.5q_2}{.75}\right) q_1 - cq_1$. And profit function of firm 2 is, it is this-$\pi_2 = \left(\frac{5 - q_2 - 0.5q_1}{.75}\right) q_2 - cq_2$.. Now here, this moves first and this moves second, okay. So, we will use backward induction to solve this problem., right? So, suppose q 1 is the output in stage 1. Then, we will find the reaction function of firm 2. It is, it is this minus c, which is equal to 0, which is the first order condition-$\frac{5 - 2q_2 - 0.5q_1}{.75} - c = 0$.

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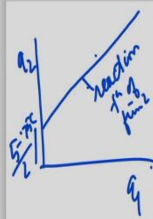


$$\Rightarrow 5 - .75c + 0.5q_1 = 2q_2.$$

$$\Rightarrow \frac{5 - .75c + 0.5q_1}{2} = q_2 \quad \text{reaction f}^{n} \text{ of firm 2}$$

$$\pi_1 = \left[5 - q_1 + 0.5\left(\frac{5 - .75c + 0.5q_1}{2}\right)\right]q_1 - cq_1$$

$$\pi_1 = \left(\frac{10 - 2q_1 + 2.5 - .37c + .25q_1}{2}\right)q_1 - cq_1$$

$$10 - p_1 - 0.5p_2 = q_1$$

$$10 - p_2 - 0.5p_1 = q_2$$

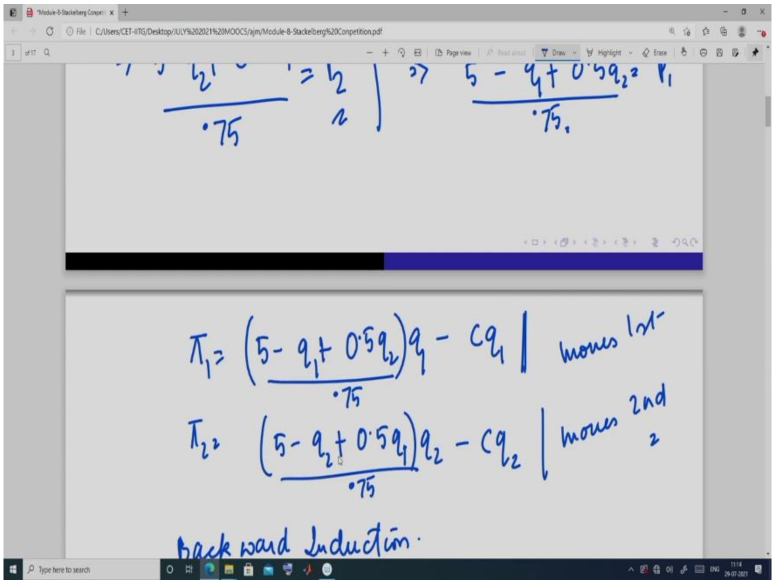
$$\Rightarrow 10 - p_1 - 0.5(10 - q_2 - 0.5p_1) = q_1$$

$$\Rightarrow 5 - p_1 + 0.5q_2 + 0.25p_1 = q_1$$

$$\Rightarrow 5 - q_1 + 0.5q_2 = .75p_1$$

$$\Rightarrow \frac{5 - q_1 + 0.5q_2}{.75} = p_1$$

$$\Rightarrow \frac{5 - q_2 + 0.5q_1}{.75} = p_2$$



So, from this, the reaction function of firm 2 is 5 minus, and this is the reaction function of firm 2- $5 - .75 + 0.5q_1 = 2q_2$. So, given any output of firm 1 in stage 1, how much output firm 2 is going to produce? It is based on this, this is the reaction function of firm 2- $\frac{5-.75+0.5q_1}{2} = q_2$. So, now what do we do? We plug in this in the profit function. Just wait a minute.

This is, okay, this should be plus. I have made a mistake, see. This is plus. So, this is plus. So, it will be plus here, it will be plus here. So, here, it will be plus here, it will be plus here, plus here. So, this reaction function is actually q 1 q 2, this is, if q 1 is equal to 0, it is taking a positive value. This q 2 is taking a positive value. That is something like this. This point is, you can say, this, okay. Now, let us substitute this here, and this is going to be, so we are substituting here, so it is this, 0.5. It is this-$\pi_1 = \left(\frac{10-2q_1+2.5-.37c+.25q_1}{2}\right) q_1 - cq_1$.

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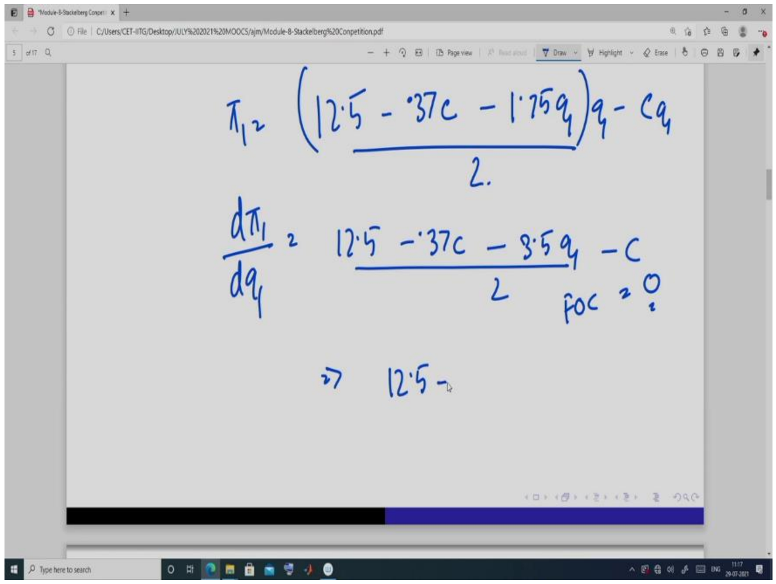

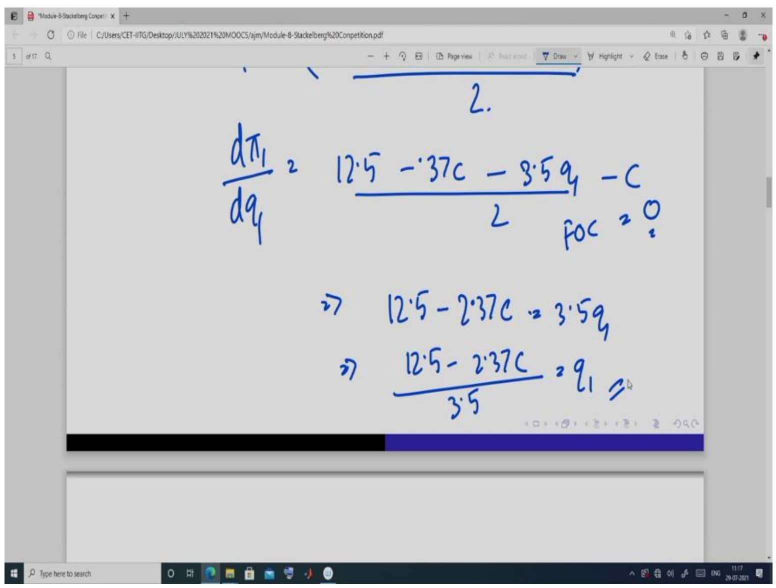

Which, we can write as 12.5. This is 1.75 q 1. We are getting this-$\pi_1 = \left(\frac{12.5-.37c-1.75q_1}{2}\right) q_1 - cq_1$.. Now, we will take this with respect to q 1, and we will get. So, first order condition, we will make it 0, so we get the output of firm, wait, this must be, okay so it is, it will be 2, so it is 2.37 c. So, it is. So, q 1 is this-$\frac{12.5-2.37c}{3.5} = q_1$.

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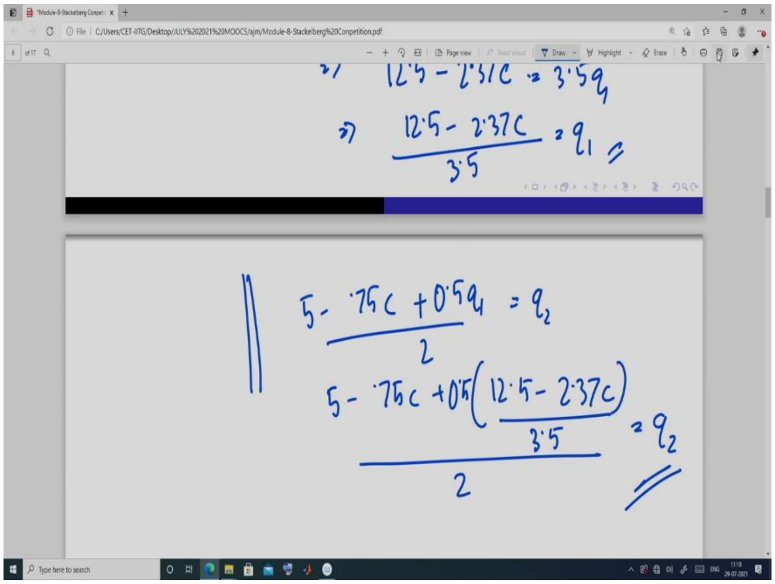

And we plug in this q 1 in the reaction function. Reaction function **is** 5 minus this, 0.75 c plus point, point q 1. This is q 2. So, plug in this. This is going to be, so we have, this is the subgame perfect Nash equilibrium in this case, okay. So, q 1 is this and q 2 is this, right. By plugging in the reaction function, the value of q 1, we will get the q 2, okay.

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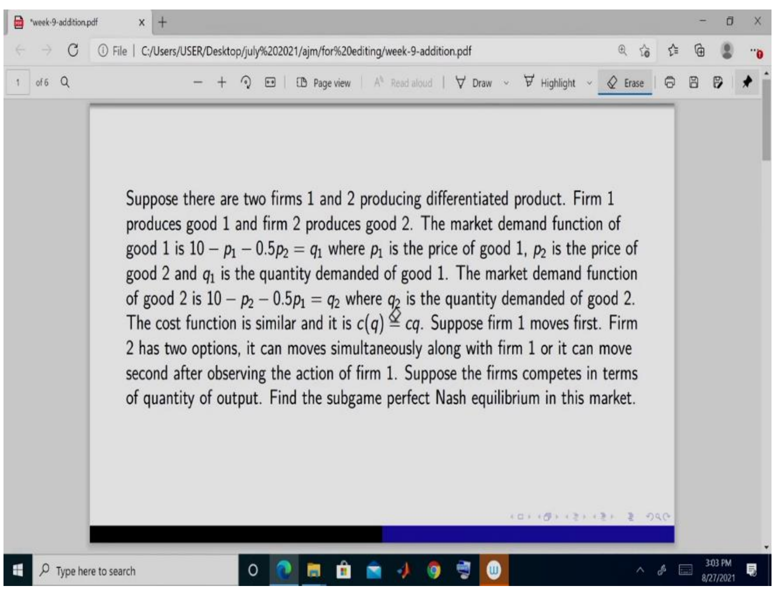

So, we are going to solve this second problem. And this is actually an extension of the first problem.

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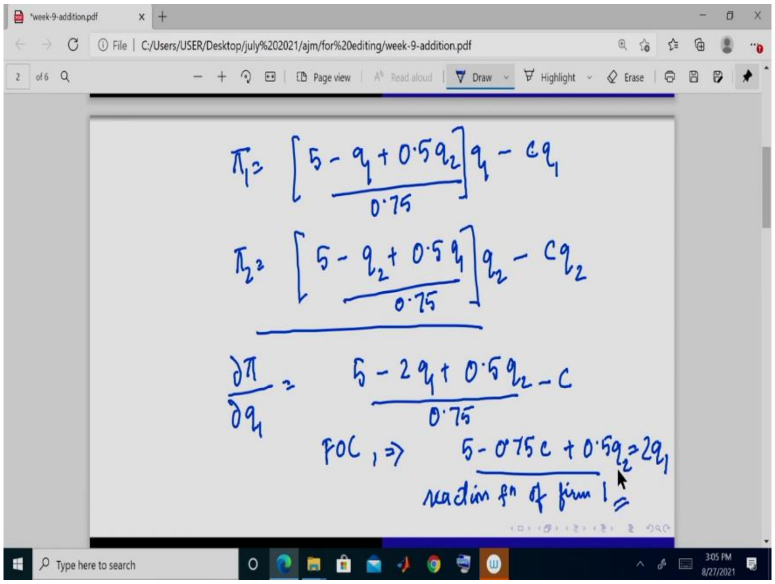

So, in the first problem, we have got the profit function of firm 1, of this nature. This$-\pi_1 = \left[\frac{5-q_1+0.5q_2}{0.75}\right] q_1 - cq_1$. And the profit function of firm 2, this-$\pi_2 = \left[\frac{5-q_2+0.5q_1}{0.75}\right] q_2 - cq_2$. So, here we will find the Cournot reaction function from this profit function. So, if we take this, we will get, then first order condition is going to give us. So, this is the reaction function of firm 1. Given output of firm 2, q 2, what is the optimal output of firm 1? This-$5 - .75 + 0.5q_2 = 2q_1$.

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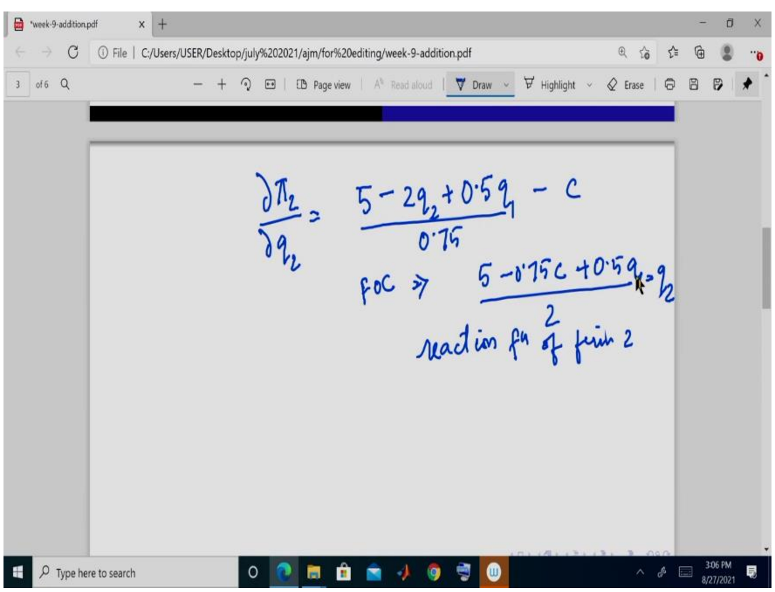

Now, we find out the reaction function of firm 2. And first order condition is going to imply this, so this is the reaction function of firm 2-$\frac{5-.75c+0.5q_1}{2} = q_2$.. So, given any output of firm 1, q 1, what is the optimal output of firm 2 q 2, we have got this. So, if we plot them, this is q 1, this is q 2. This function, if suppose q 2 is 0, then this is going to take a positive value. So, it is something like this. And this point is, this. And, this, if q 1, takes a value 0, it will be somewhere here. And, slope is going to be less than this slope. So, this point is 50 minus, and this is the Cournot outcome. Now, we will study the Stackelberg outcome based on these reaction functions.

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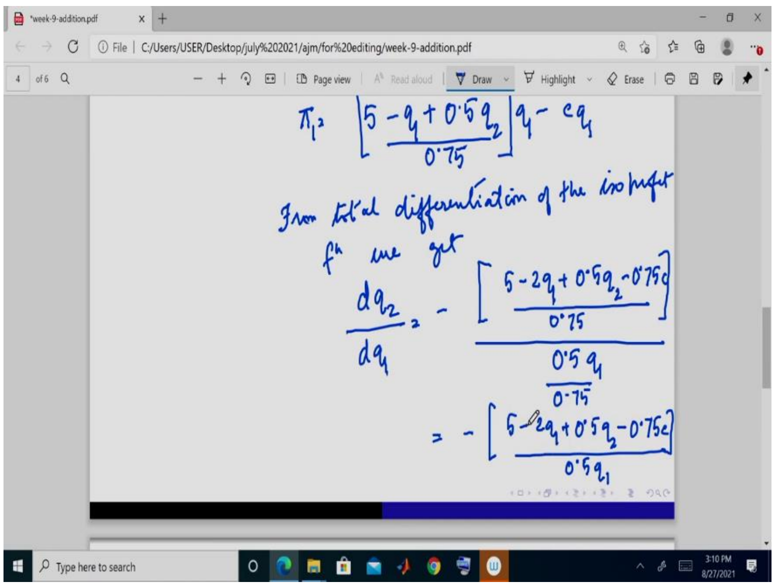

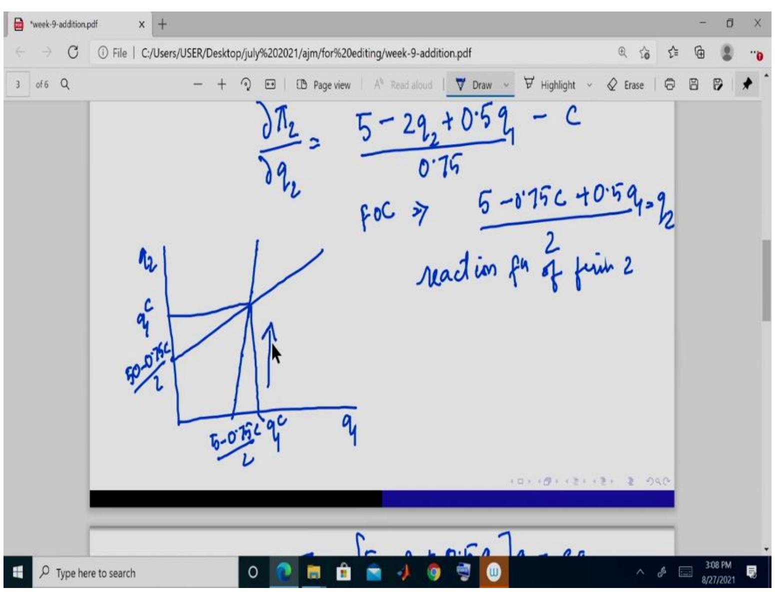

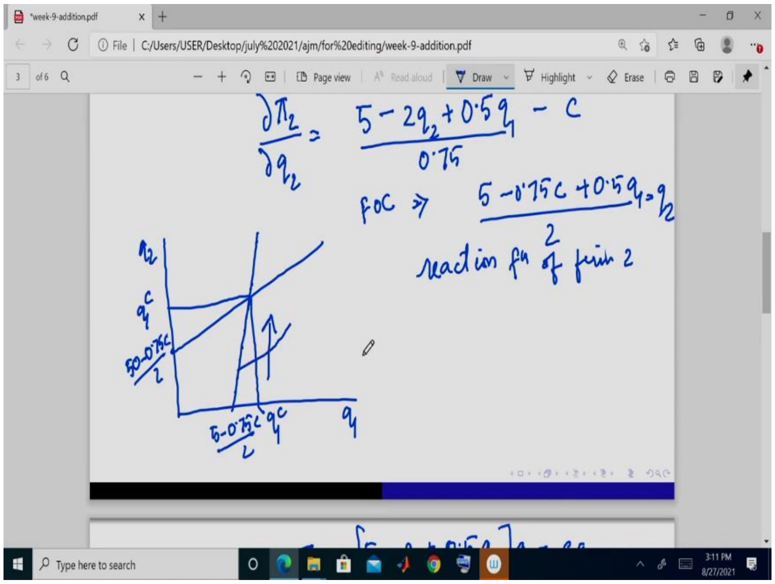

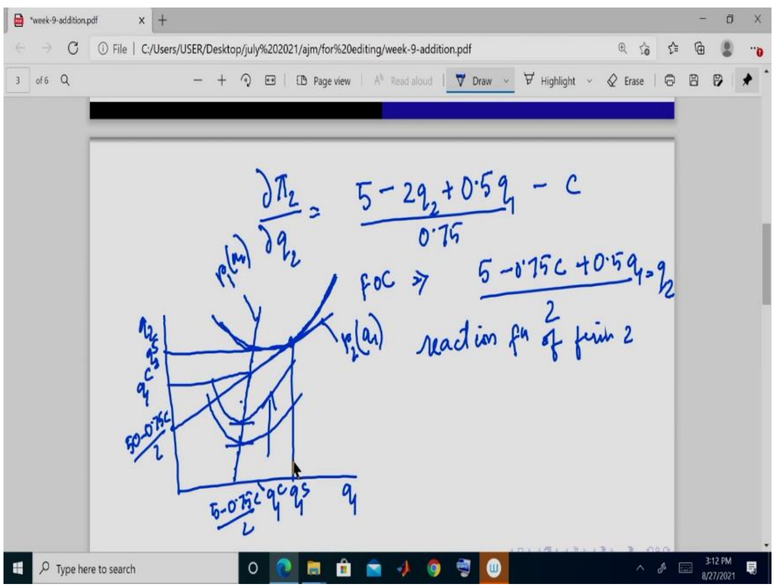

So, we know, the profit function of firm 1 is, it is this. So, we will look at the iso-profit curve of this, generated from this. Now, if you look at this, here, consider this reaction function. So, here if we move in this way, output of firm 2 is increasing, q 2 is increasing. Now, here, if we keep on increasing q 2, keeping q 1 fixed, see the profit is increasing. So, here if we fix this, profit is increasing in this direction. So, profit increases in case of the firm 1. And if we take the, from total differentiation of the iso-profit function, we get this- $\frac{dq_2}{dq_1} = -\frac{\left[\frac{5-2q_1+0.5q_2-0.75c}{0.75}\right]}{\frac{0.5q_1}{0.75}}$. And here, this portion will be simply, it will be this. So, this is going to be this.

Now here, look this portion is actually the reaction function of firm 1. So, we fix q 2, and if we keep on increasing q 1, here, if we fix q 2 and keep on increasing q 1, this is going to take a negative value. It takes a 0 value, when it is in this reaction. So, it will be something like this.

And, if we keep on decreasing q 1, keeping q 2 fixed, then what is happening? This is going to take a negative value, sorry, this is going to take a positive value and this is negative. So, it is going to be a negative. So, it will be something like this. It will be like this. So, it is 0 at this point. So, these are the iso-profit of firm 1.

And the Stackelberg outcome here, it is going to be some point here and it is given by this point, q 1 s, q 2 s because Stackelberg outcome is such that the iso-profit curve of firm 1 is going to be tangent to the reaction function of firm 2. This is the reaction function of firm 2, and this is the reaction function of firm 1. This. So, this is the Stackelberg. Now, we have to find out whether firm 2 is going to choose along with firm 1, in stage 1, or it is going to choose after firm 1 has chosen is output and that is in stage 2.

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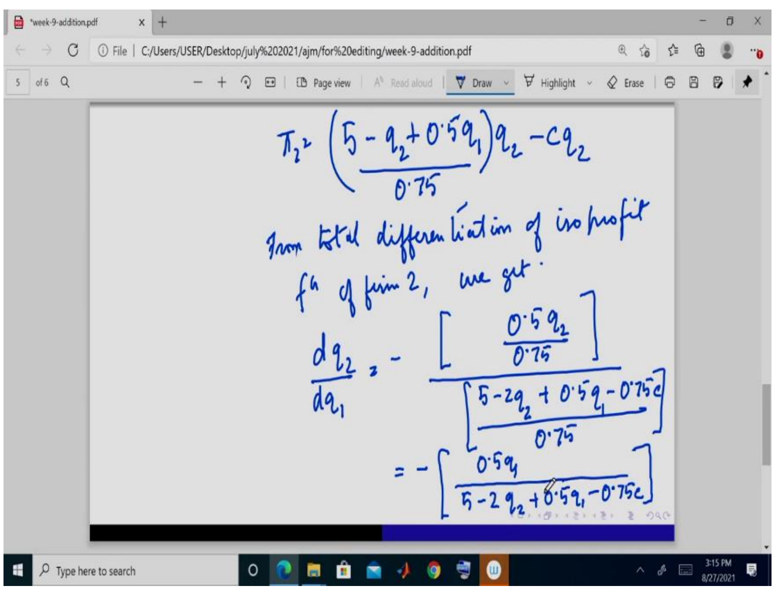

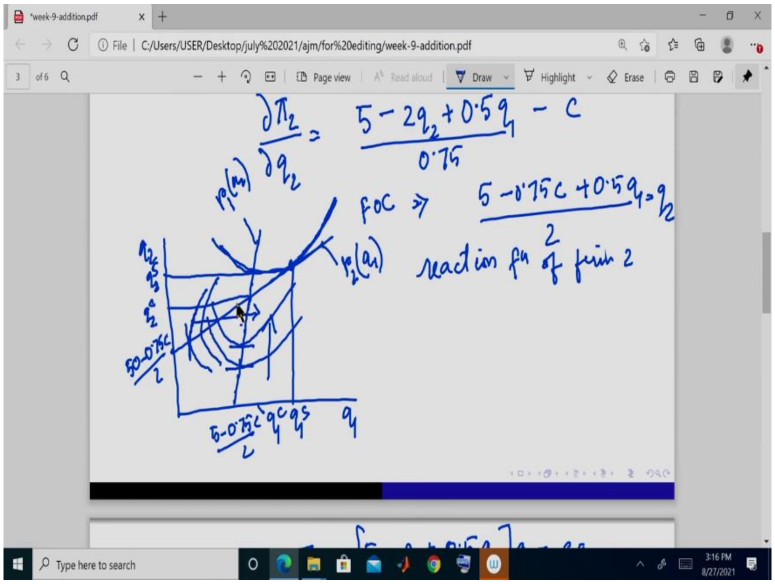

So, we again find the iso-profit curve of firm 2. So, the profit function of firm 2 is this. And then here, if we fix q 2, and keep on increasing q 1, if we fix q 2 here, q 2 here, and keep on increasing the output of firm 1, q 1, then the profit of firm 2 increases. And we got it from this here.

So, it is same in this case also, in this direction profit increases. So, we get from total differentiation of iso-profit function of firm 2, we get this, which is equal to, which is equal to this. So, this is the reaction function of firm 2-$\frac{dq_2}{dq_1} = -\left[\frac{0.5q_1}{5-2q_2+0.5q_1-0.75c}\right]$. So, in this, when we are at the reaction function of firm 2, this takes 0. So, this is a 90 degree.

So, iso-profit curves are like this, this because if we keep fixed q 2, and keep on increasing, q 1 here it will be, this will be negative. And if we keep fixed q 1, and keep on increasing q 2, this will be negative. And this is negative, so it will be positive. So, if we keep this fixed at the reaction function and then keep on (in) decreasing this, then this is going to be positive. So, this is negative. So, that is why this portion is negative and this portion is positive. So, these are the iso-profit curves of firm 2.

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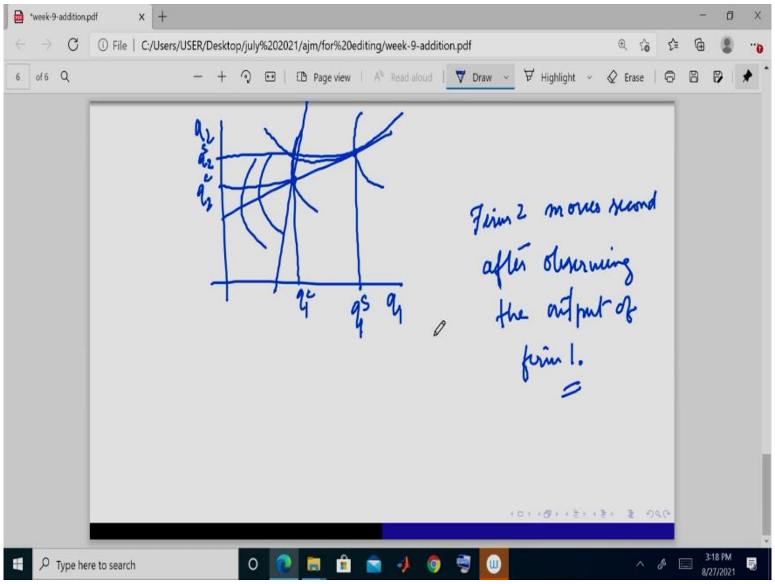

Now, we derive the result. This is q 1, q 2. This is the reaction function of firm 1, this is the, yes, this is Cournot thing, and suppose this is the Stackelberg. And these are the iso-profit of firm 2. So, if it moves simultaneously, along with firm 1, then this is the outcome, Cournot outcome. But if it moves second, then this is the outcome. So, profit here, is more than the profit this, at this point.

So, that is why firm 2 moves second after observing the output of firm 1. So, but in the homogenous good case, we have found that firm 1, firm 2 will always try to move along with, simultaneously along with firm 1. But here, in this case we have found that firm 2 is preferring to move second, that is, in stage 2, after observing the output of firm 1. So, here, we have, there is a second mover advantage.

Introduction to Market Structure
Professor Amarjyoti Mahanta
Department Humanities and Social Sciences
Indian Institutes of Technology, Guwahati
Lecture 35
Stackelberg Price Competition

Hello, welcome to my course introduction to market structures.

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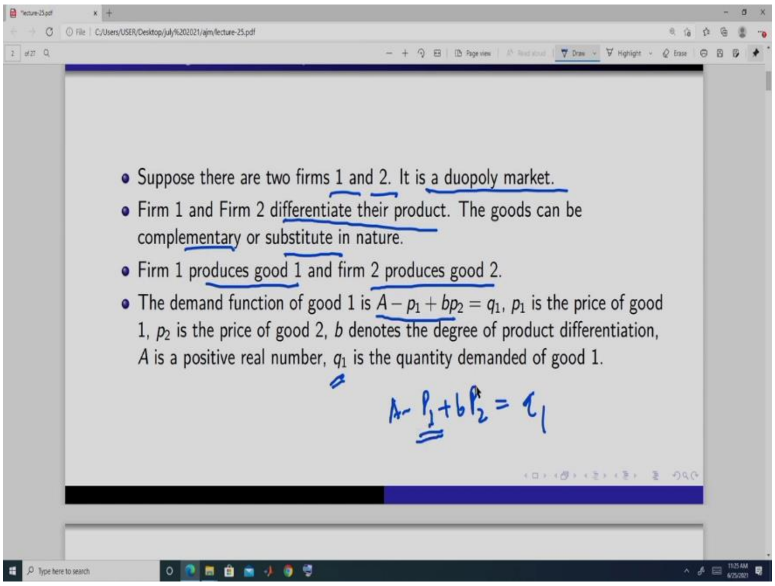

So, in the last class we have done stackelberg quantity competition and today we are going to do stackelberg price competition. In stackelberg quantity competition, we have seen that if the reaction function are linear and downward sloping, then there is always a first towards first mover's advantage. So, it means that a firm which chooses output first that it has a, it makes higher profit then the firm which chooses output secondly, or in stage is two.

So, here today instead of price, instead of quantity firms are going to choose prices. So, again we assume more or less similar setup. So, there are two firms, firm 1 and, firm 1 and firm 2. And it is a duopoly market. Now, here what we have? What is different in this setup is that the products are differentiated, okay. So, differentiated product means they are not perfectly substitutable. So, the goods can be either complimentary or they are substitute in nature.

So, what does it mean? So, when I say substitute, it is not a very close substitute. It is something like, the brands the different brands, they substitute their product in terms of quality. So, in that sense we make this assumption, that they are substitutable. And complementarity, here it means that goods are completely complimentary. That means, if I want this good 1, then I will also demand good 2.

But if it is substitute, then if I demand good 1 then I will not demand good 2, okay. Like, if we think of goods having differences say in terms of some attributes or in terms of some qualities, like this, okay. But that is not explicitly modeled here, we only model it in a very simplest form. So, firm 1 produces good 1 and firm 2 produces good 2. And the demand function of good 1 is of this nature-$\mathrm{A} - \mathrm{p}_1 + \mathrm{bp}_2 = \mathrm{q}_1$.

So, the demand function is A minus p1 plus b, p2 is equal to q1. So, p is the price of good 1, p2 is the price of good 2 and b denotes the degree of product differentiation, okay. And here A is a positive real number and q is the output. So, if you look at this demand curve, it is again downward sloping in this p1. This is the price of good 1. So, it is again downward sloping, we will come to this slightly later.

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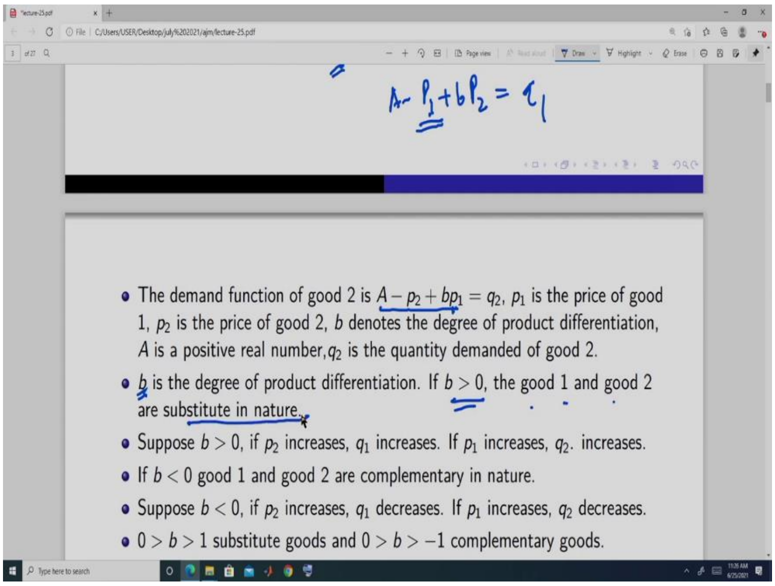

Next the demand for good 2 is again like this- $\mathrm{A} - \mathrm{p}_2 + \mathrm{bp}_1 = \mathrm{q}_2$, a minus p2 plus bp 1 is equal to q2. Here p1 is the price of good 1, p2 is the price of good 2, b denotes the degree of product differentiation. So, it is same as the b earlier for, in the, that we have, so, used in the demand for a good 1 and q is the quantity demanded of good 2. See, b is the degree of product differentiation. If b is positive, then good 1 and good 2 are substitute in nature.

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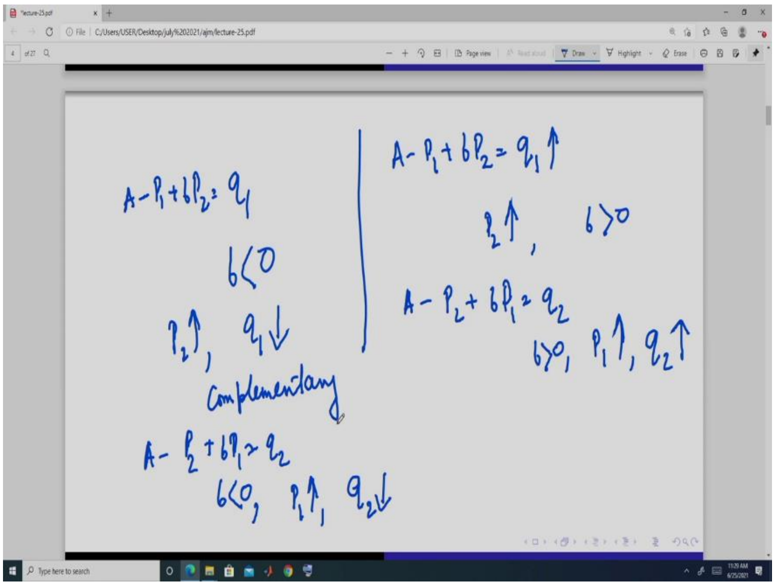

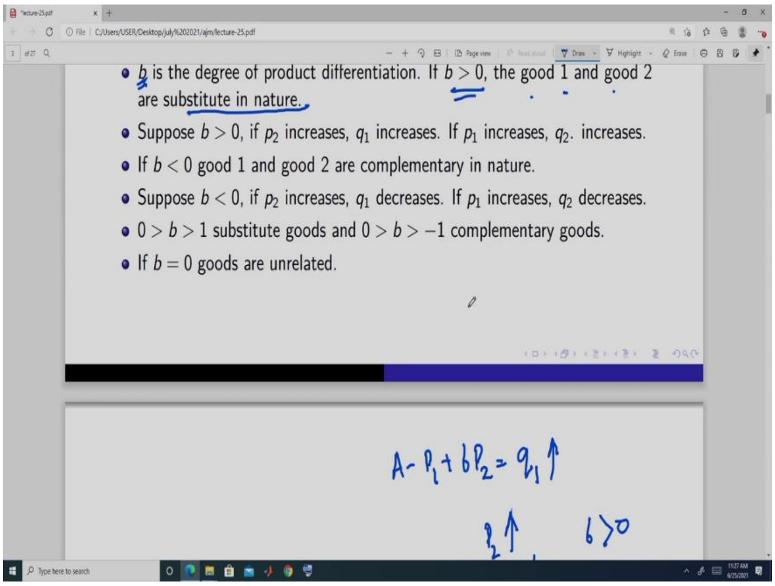

So, it, it is something like this. So, demand if, it is like this. Now here, if b is positive that means what? If p2 suppose increases, that is if p2 increases, q1 increases, demand for q1 increases, that is good 1 increases. So, good 2, when the price of good 2 increases, demand for good 1 increases. So, that means they are substitutes in nature, because the moment the price of a good increases, we reduce its demand.

So, that is why they are substitutes in this case, when b is positive. Similarly here, also we will get same thing. If b is positive and if price of good 1 increases, then quantity demanded of good 2, that is going to increase, okay. So, people will substitute, buy more of good 2 then good 1 when the price of good 1 increases, okay. So, this is the case. Now, if in the same demand function, if you take this and now suppose b is negative. So, that means what is happening?

If p2 increases, price of good 2 is increasing, then this portion is increasing so, this and it is negative, so, q1 falls. So, here when the price of, good 2 increases, demand for good 1 is also going, is going down. So, goods are complimentary here, okay. Similarly, if we take this good and suppose b is negative, then when price of good 1 increases q2 falls. Because the complementarity goods are, they are, they are going to be consumed together.

They required in certain ratio or in some here like pen and refill, or desktop and mouse, something like those, those kind of things. So, so since it is complimentary, so, if the price of one good increases, so, that is the demand for that good is going to go down so, the demand for other goods is also going to go down. So, in this way, there is product differentiation between firm 1 and firm 2. And how they are doing this product differentiation? We are not going to do that, okay. So, we will come to that later on when we do product differentiation.

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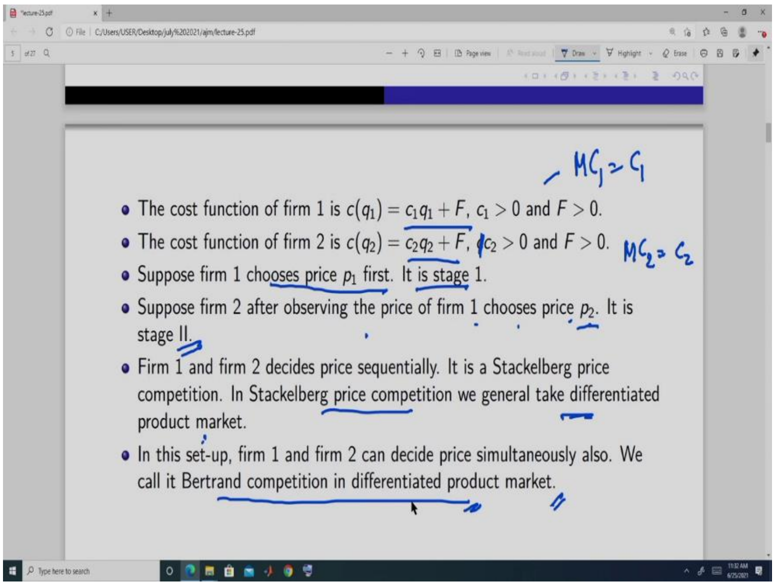

Now we specify the cost function. Cost function of firm 1 is c1 into q1 plus F-$c(q_1) = c_1q_1 + F$. So, c1 is some positive number. So, if we have a cost function like this, then we know the marginal cost of firm 1 is a constant and it is c and it is c as positive number. Similarly, the cost function of firm 2 is of this nature, c2 into q2 plus F- $c(q_2) = c_2q_2 + F$, c2 is a positive real number sorry, is a positive real number. So the marginal cost of firm 2 is c2.

So, it is a constant marginal cost, okay. Now we define the game, how? What actually the firms chooses? So, firm 1 chooses pricing first. So, it is stage 1, and firm 2 after observing the price of firm 1 chooses price p2, so it is stage 2. So, here firm 1 and firm 2, decides price sequentially.

So, firm 1 chooses price, price first and then firm 2 chooses price, okay. And so that is why it is a stackelberg price competition.

Because we are choosing price, so price and since it is, decisions are taken sequentially so, that is why it is stackelberg. Now, generally when we do stackelberg price competition, we take, take differentiated product market, why? Because we have seen, that if we take homogeneous good market and if there is a price competition, then we also have seen something called a bertrand paradox. So only bertrand paradox when it, when we do not see any bertrand paradox.

When either there is a capacity constraint or there is decreasing returns to scale. So, that is why we bring in product differentiation to move out of bertrand paradox, okay. Now, in this setup, what we have defined now, like the cost function and the demand function, the way the products are differentiated, that can be studied as a bertrand competition in where? Prices are taken, are taken simultaneously or the firms choose their price simultaneously.

So, then it becomes a bertrand competition in differentiated product market. But we are not going to do that. And it is same, almost same there is no difference. So, it is simply more or less same as the Cournot competition, only here it is price and in Cournot it is quantity, okay. But in this stackelberg price competition, why we are going to do it? We will see that, the end that we get a very interesting result, okay. So, we are going to do stackelberg price competition.

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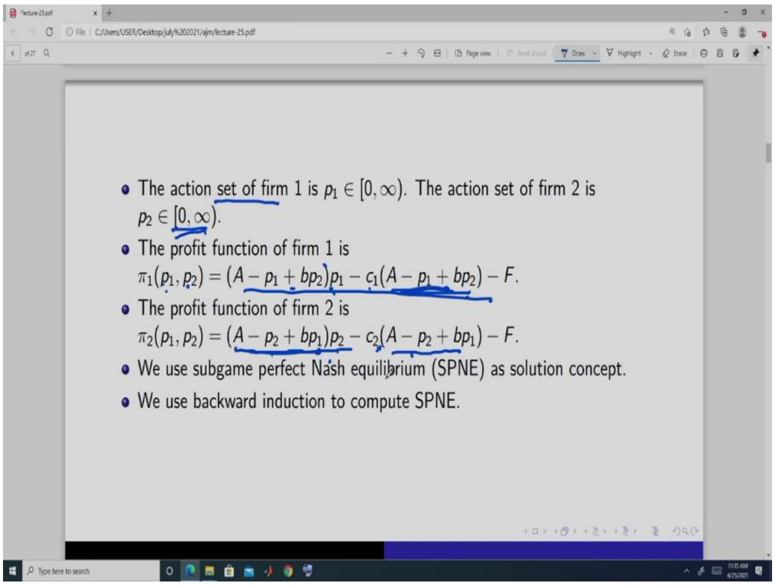

So, it is prices are decided sequentially. So, that is why, since it is an extensive game, so, we are defining the action set. Now the action set is not same as the strategy set. Your strategies are mainly in a dynamic game, we define a strategy for an act, strategy is a set of complete set of action in all contingencies. So, here since it is a continuous state we cannot define contingency in that way.

Because each price in a range are going to be, because p1 can take many possible values. And then we have to define p2 for each such contingency. So, that is it is not possible or we can do it using something called the reaction function and we will do that actually, okay. So, the action set of firm 2 is this, okay. So, firm 1, now the profit of firm 1 it is, it is going to be function of p1 p2 and it is, this-$(A - p_1 + bp_2)p_1 - c_1(A - p_1 + bp_2) - F$. And profit of firm 2 it is this, this is what? This is the demand function.

So, it is quantity q1 into p1, so, this is the total revenue minus this is the total cost. Because this is the q1, quantity of good 1, c1 into this portion, this is the quantity. Now, so, total revenue minus total cost so, gives me the profit. Here, this portion is the quantity demanded of good 2 into price. So, total revenue, this is c2 marginal cost into the total output or the total quantity demanded or that is being supplied by firm 2.

So, it is a minus p2 plus bp1, this is the demand function of firm 2 or good 2. So, this is a total cost, this portion is the total cost. So, total revenue minus total cost gives me the profit-$\pi_2 = (A - p_2 + bp_1)p_2 - c_2(A - p_2 + bp_1) - F$. Now, so, these profit functions are the p of functions of firms. So, we will use sub game perfect Nash equilibrium as a solution concept in this case and we will find out the SPNE through backward induction.

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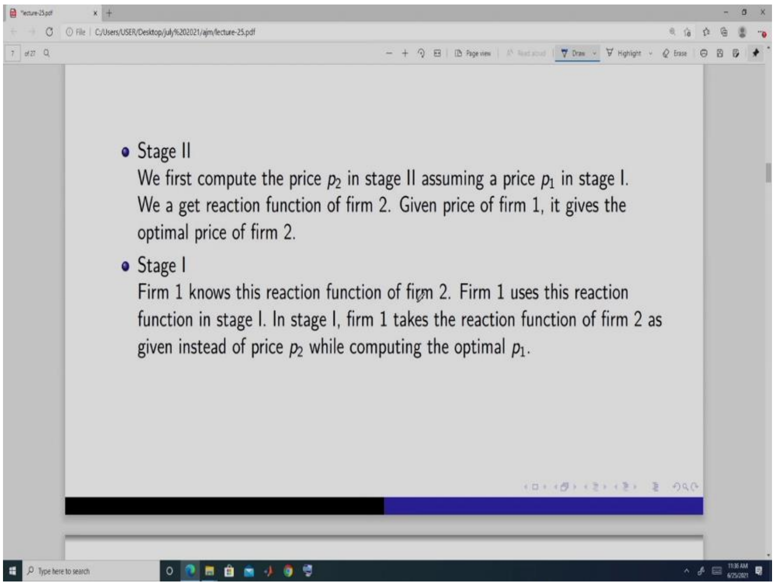

So, what we do? So, first we will solve the stage 2, which is going to be, going to be the last stage. Assuming that there is a p1 in stage 1. So, we will get a reaction function of firm 2? In that when we optimize p2? So, we will know that if p1 is some, takes a specific value we will know what is the optimal value of p2 based on that reaction function. Now firm 1 will know this reaction function.

A firm 1 can compute this reaction function and firm 2 instead of taking p2 as given, what we will do? It will take that reaction function as given. And based on that, it is going to decide its optimal p1.

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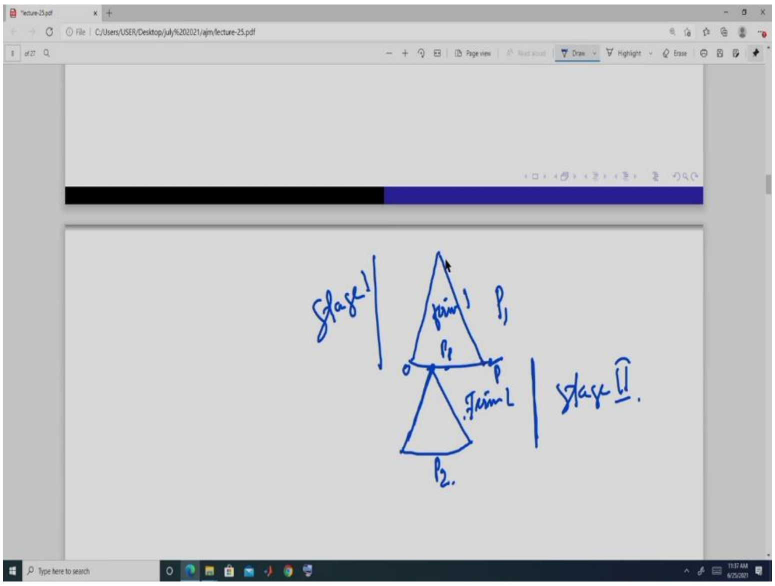

So, it is something like this, that in stage 1, in this stage, stage 1 p1 is decided, okay. So, p1 takes some value here to come very, some positive number like this any point. And suppose this is and then after observing this, firm 2 is going to decide. So, this is firm 1 and this is firm 2. And this is stage 2.

So, here p2 is going to be decided, here p1 has been decided. So, this is stage 2. So, we solved first this and taking the optimal solution in this stage, we solved this, this is the way we use the way we solve the dynamic game, okay.

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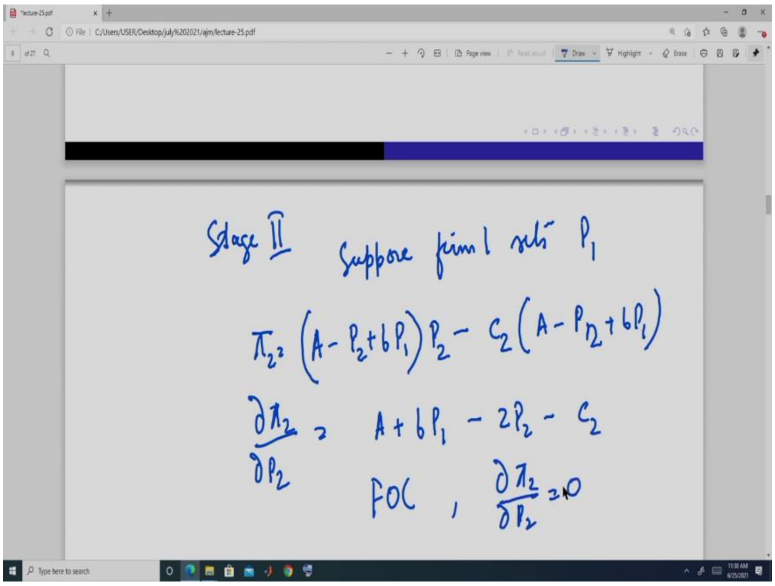

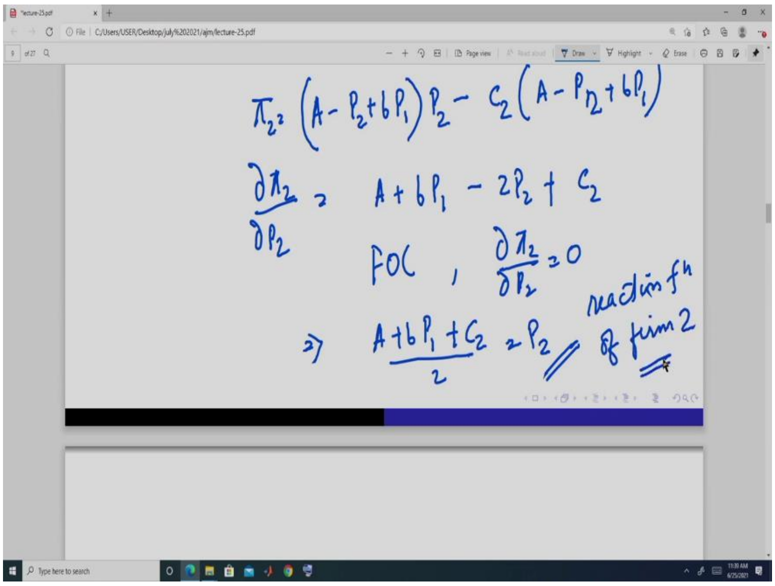

So, so let us solve this stage 2, p1. Since, this is a differentiable function so, we can take the partial with respect to p2 and we will get, this is firm 2, so, it will be p2. Now, first order condition gives me, that is equal to 0. so, I get so, this is the reaction function of firm 2, we have got this-$\frac{A+bP_1+c_2}{2} = P_2$. Now, based on this, we will so, this is the stage 2. In stage 2, we know that how firm 2 is going to react.

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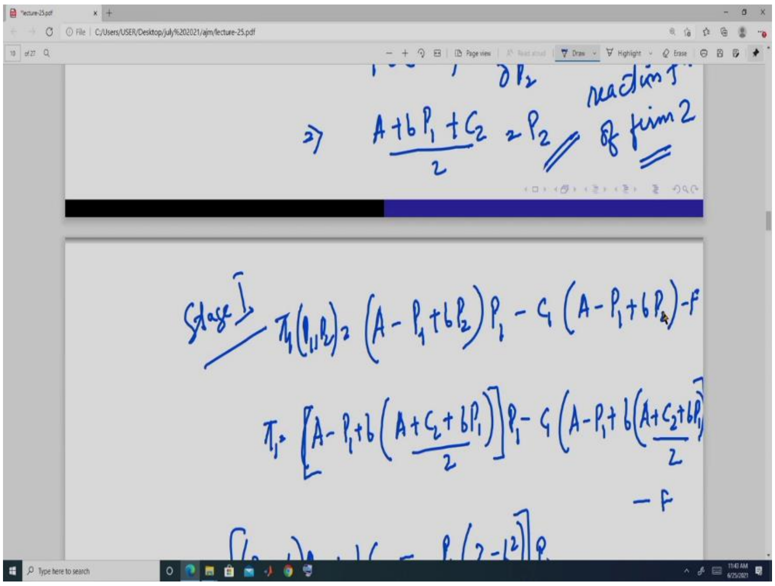

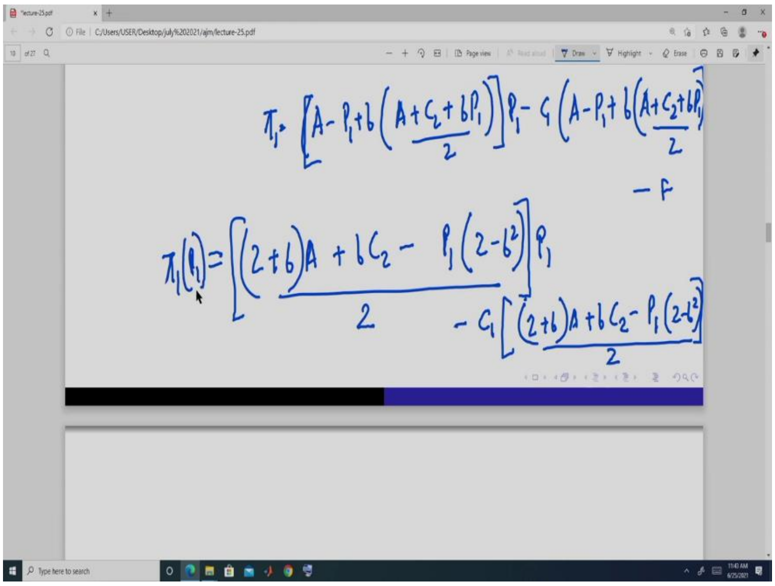

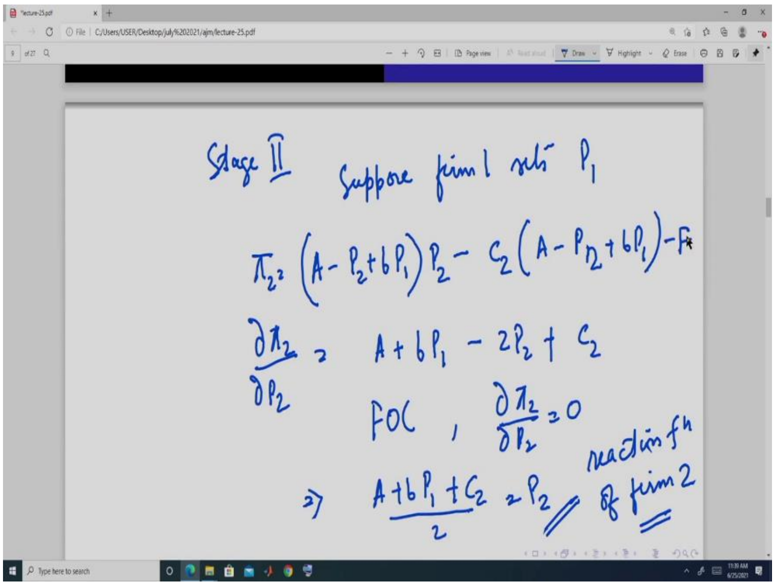

So, stage 1, firm 1 is going to use this. Firm 1, so profit function is this- $(A - p_1 + bp_2)p_1 - c_1(A - p_1 + bp_2) - F$. Fixed cost I have forgotten to put the fixed cost, okay this. Now firm 1 knows how p2 is going to be decided. If it chooses p1 what is going to be the p2, it knows from this reaction function. So, firm 1 will plug in that value here, that function here instead. It will be this, then this portion will remain same bc 2 this is going to be plus b square and there it is going to be minus.

So, it is going to be minus 2, p1 sorry. And is going to be this, minus c1. So, again here it is going to be the same thing. So, a plus b sorry, 2 A plus bc 2 minus this, okay profit of firm 1 is this- $\left[\frac{(2+b)A+bC_2-P_1(2-b^2)}{2}\right] P_1 - c_1 \left[\frac{(2+b)A+bC_2-P_1(2-b^2)}{2}\right]$. So, here it was this function of p1 and p2. If you look, it is a function of p1 and p2, p 1 and p2, p. Now, p2 it knows it is given by this reaction function, so, this becomes only a function of p1.

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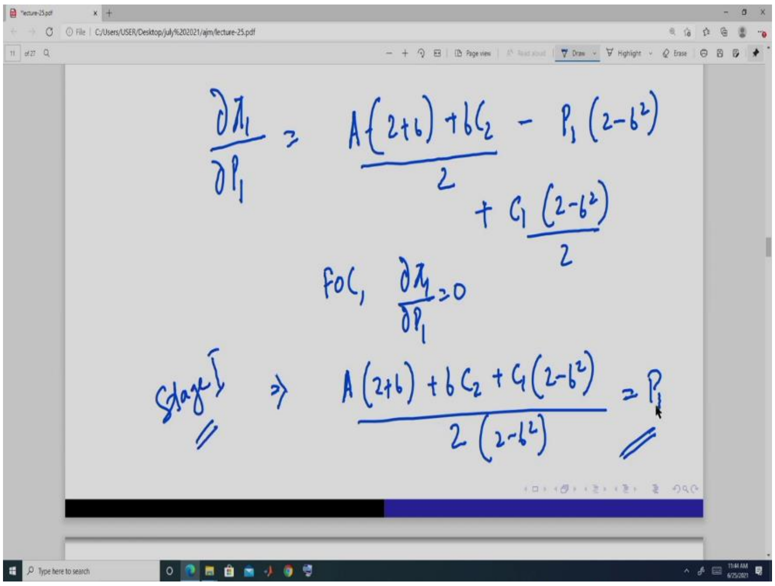

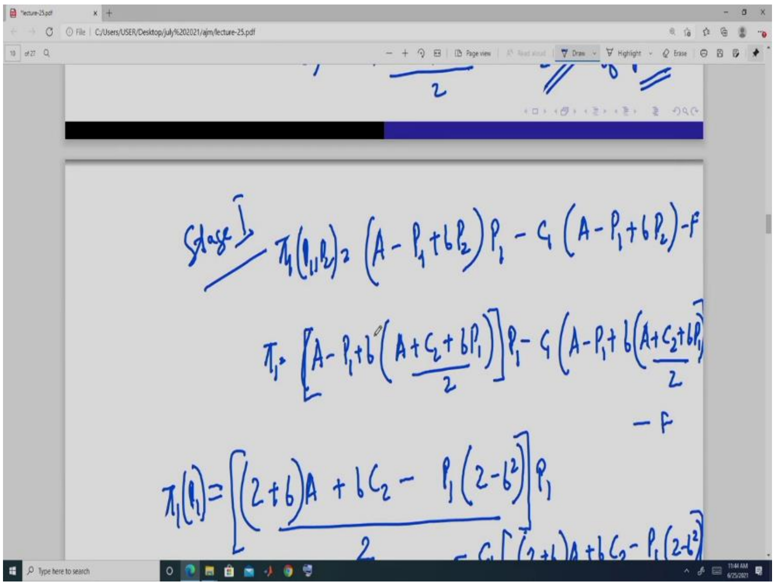

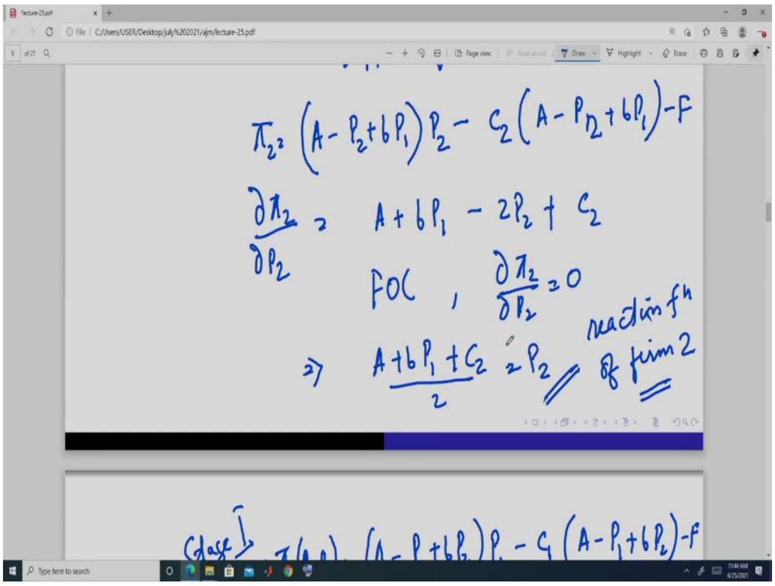

Now, we know again this is a differentiable function. So, this gives me, this-$\frac{d\pi_1}{dP_1} = \frac{A(2+b)+bC_2}{2} - P_1\,(2-b^2) + \frac{c_1(2-b^2)}{2}$. And first order condition gives me, this is equal to 0. So, we get is equal to p1 this, this is stage 1-$\frac{A(2+b)+bC_2+c_1(2-b^2)}{2(2-b^2)} = P_1$. In stage 1, we get this p1 and in stage 2 we get, this.

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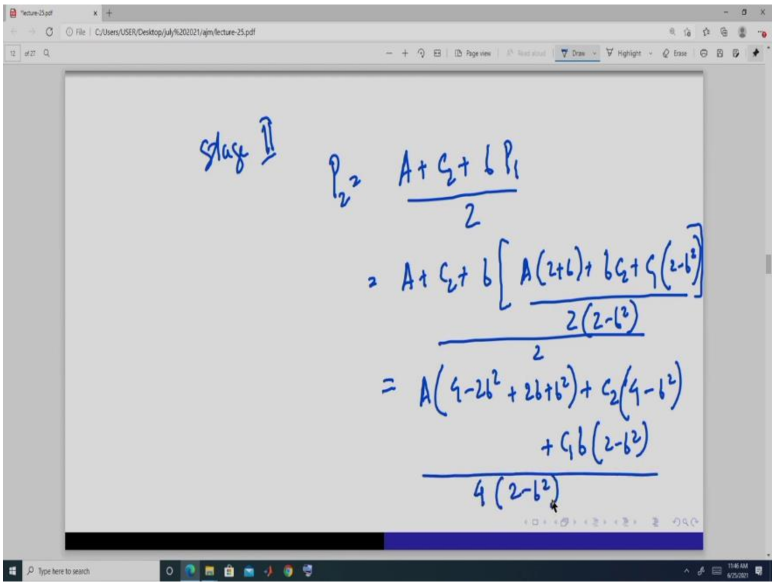

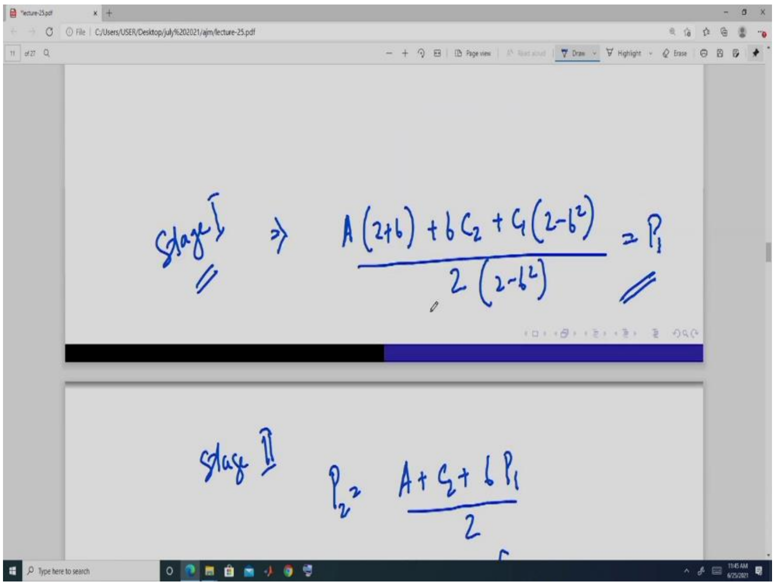

So, in stage 2 now, we will get p2 as so, this is the reaction function of firm 1, firm 2. So, plug in the value of optimal value of p1. So, it is, it is this. So, it will be, is divided by 4, this.

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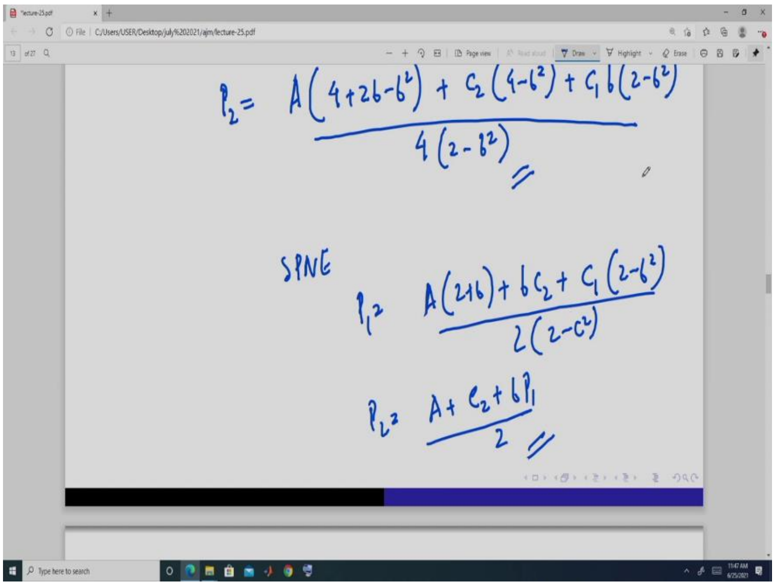

So, p2 is actually equal to, is this-$P_2 = \frac{A(4+2b-b^2)+c_2(4-b^2)+c_1 b(2-b^2)}{4(2-b^2)}$. And here, when we state SPNE, we say that p1 is equal to A this-$\frac{A(2+b)+bC_2+c_1(2-b^2)}{2(2-b^2)} = P_1$, p2 is given by this reaction function-$P_2 = \frac{A+c_2+bP_1}{2}$, SPNE. And the actual outcome is actually this, p2 is this and p1 is this. Because when you plug in this here, we get this.

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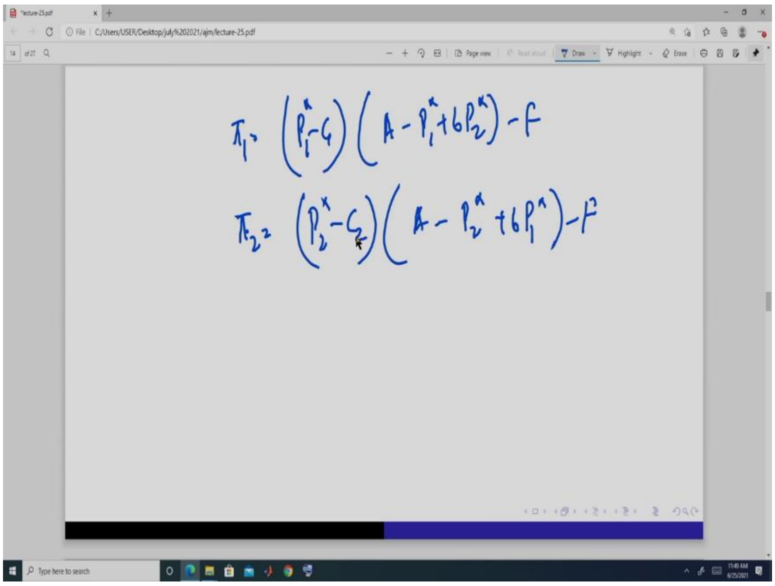

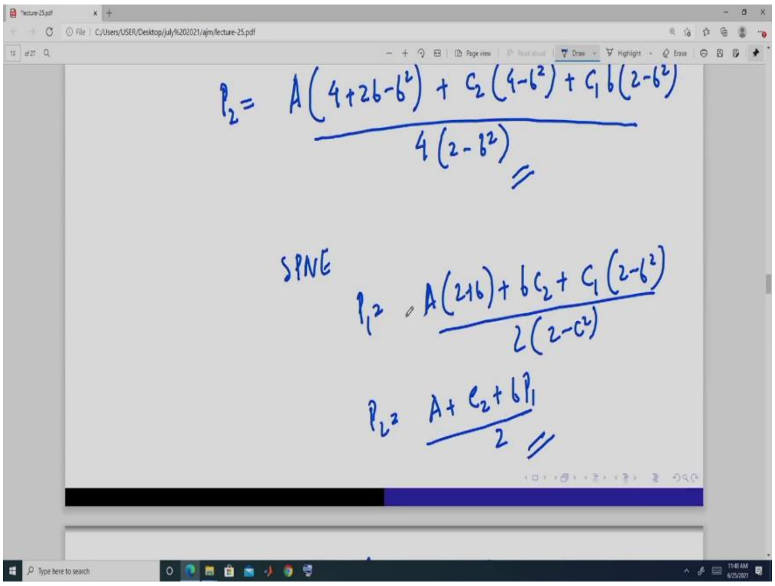

Now we can compute the profit function, but profit function is going to be very messy thing here. It will look a very messy, because profit function now it is you can write it in this form. So, plug in the optimal values, sub game perfect Nash equilibrium. And this is going to be c2, this, where p1 is, p1 takes this value, p2 takes this value. So, I am not putting these value because it will look very messy.

But you can do it is, but how do we compare this? We are not going to compare profit of firm 1 and firm 2, what we are going to do? In fact you can also do it for but it will be better if you take the p, c1 and c2 as same, then it will make sense otherwise it is, again it will be very conditional statement. But we will see whether there is a first mover advantage or a second mover advantage. How do we do that?

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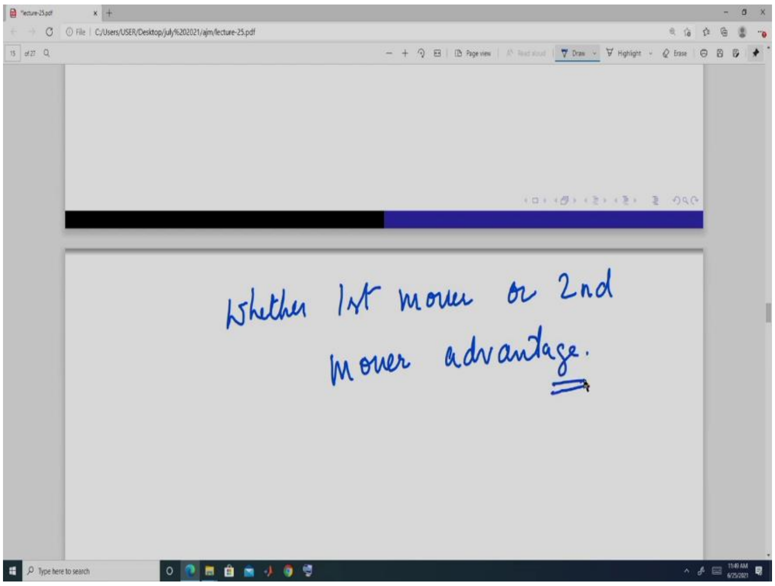

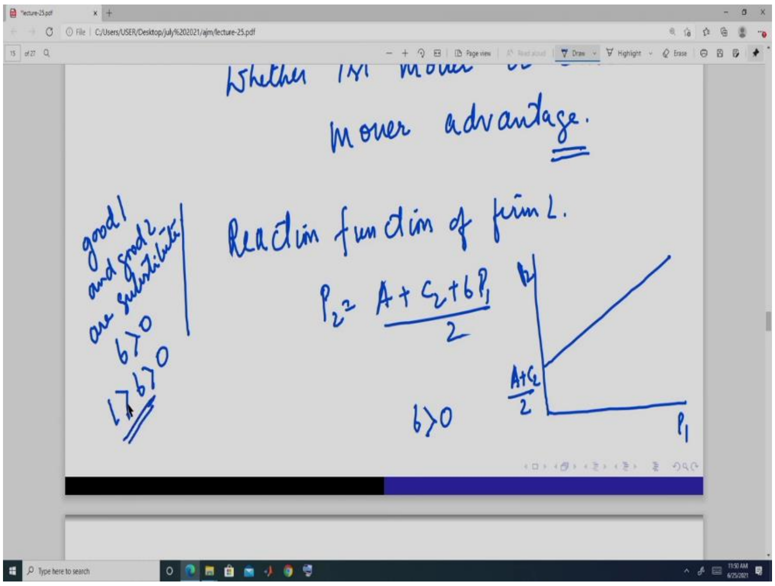

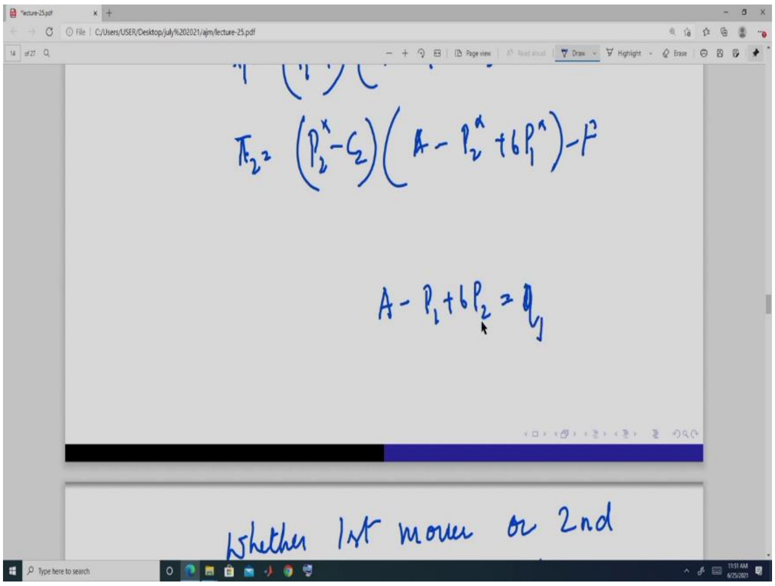

So, we now check whether first mover or second mover advantage, okay we do that. How do we do it? We, see we have already know the reaction function of firm 2. We have derived it, we what do we have got? Reaction function of firm 2 is, is this. So, if we plot this, this is p1 and this is p2, we get a curve like this where this point is this, when? When b is positive. So, first let us take the case when good 1 and good 2 are substitute.

So, b is positive and further b, why it is less than 1? Because if it is greater than 1, then see in, in this demand function, if it is greater than 1 then the effect of p2 will be more on good 1 then p2, p1, effect of p2. So, that is why this is always taken as this as less than 1, okay. And when it is 0, there is completely unrelated they are not. So, we take this here and we get this reaction function which we have already done.

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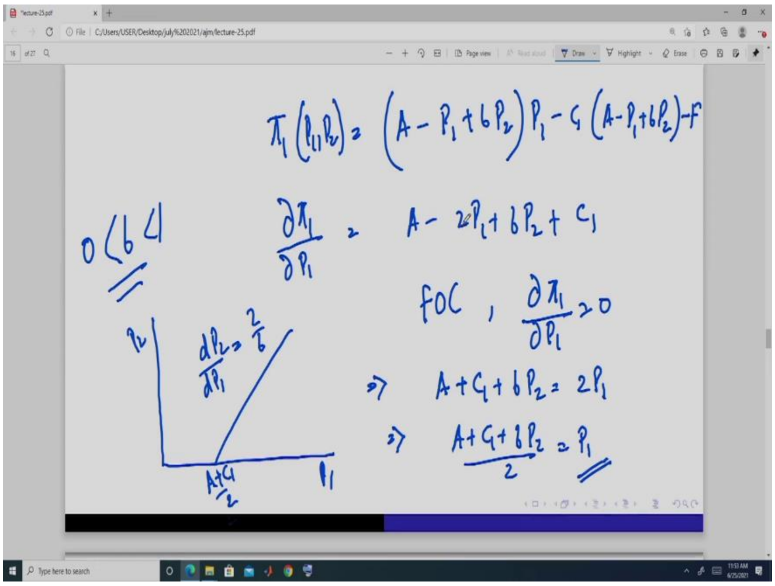

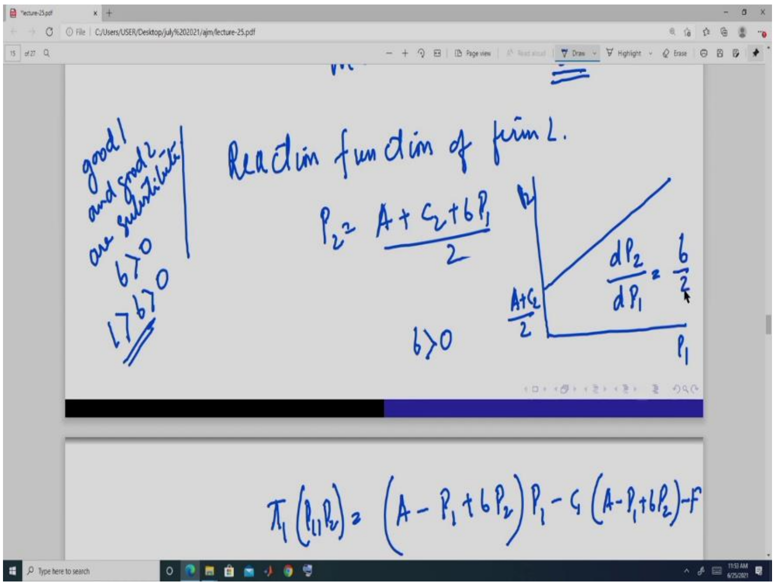

Again we know the profit of firm 1, when firm 1 is this, this. Taking p2 as given, we get this. Again, the first order condition, first order condition gives me that this is equal to 0. So, this gives me, it is this-$\frac{A+c_1+bP_2}{2} = P_1$. So, this is the reaction function of firm 1. And we have assumed that b is this. So, goods are substitute, so this is going to be, this is going to look like, where this.

And in this case the slope of this is going to be what? 2 by b, it is positive. And in this case slope is going to be dp2 by dp1. It is going to be b by 2, right? So, it is positive again because b is positive, it is this, But slope of this is less than this.

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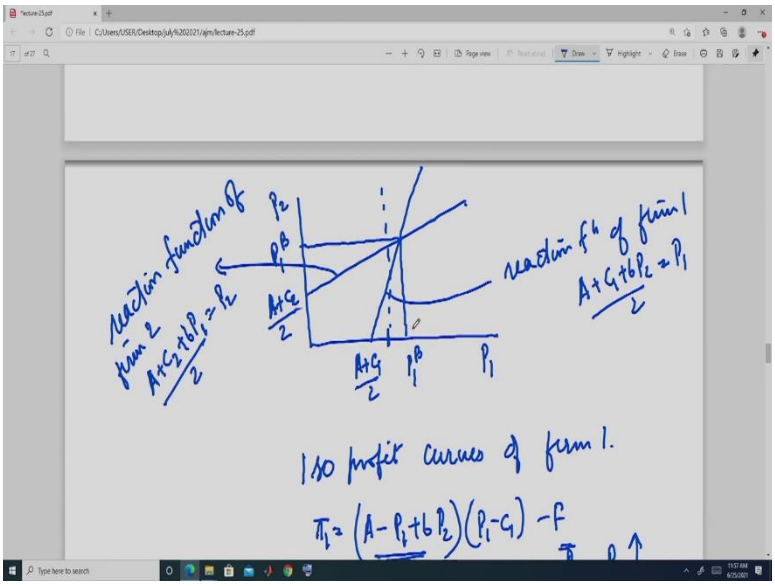

So, we get that this and this is, this, this is the outcome when these two reaction functions intersect. So, we say this to be, when both the price are decided simultaneously, so, this is the bertrand competition when there is product, when there is differentiated product market, okay this. But, what we are dealing here? We are dealing with stackelberg. So, stackelberg case is when the firm 1 decides its output first and firm to decides its output second.

But you can reverse the sequence also it does not matter, okay. Now, but we will compare, we were trying to find out whether there is first mover advantage or second mover advantage. In the last class also, when we did the quantity competition, we did that analyze based, analysis based on the Cournot reaction function. So, here we will do the analysis based on the bertrand reaction function. Now, in this case the iso profit curves of firm 1.

Because firm 1, knows the reaction function of firm 2, firm 2 also knows this reaction function of firm 1. So, this you can say this is the reaction function of firm 1, which is A plus c1 bp 2 this, and this is the reaction function of firm 1, where it is sorry, okay this. Now, iso profit curve, function is this, profit when we fix the level of profit and then look at the combination of p1 and p2. So, this you can, this-$\pi_1 = (A - P_1 + bP_2)(P_1 - c_1) - F$.

Now, if we fix p1, fix p1 and if you keep on increasing p2. Since b is positive so, this portion, this is fixed, this portion is increasing, this portion. So, that means, if I fix p p1 suppose here and then keep on increasing p2 like this, what is going to happen? Profit is increasing, right? So, profit increases in this horizon, in this vertical direction, okay. Because goods are substitute, right? because when the price of good 2, keeping the price of good 1 fixed the demand for good 1 is going to increase.

Because people will shift from good 2 to good 1, because price is increasing. So, that is why its profit is going to be increased. Because more, more quantity will be demanded.

Now, so we know this, this diagram.

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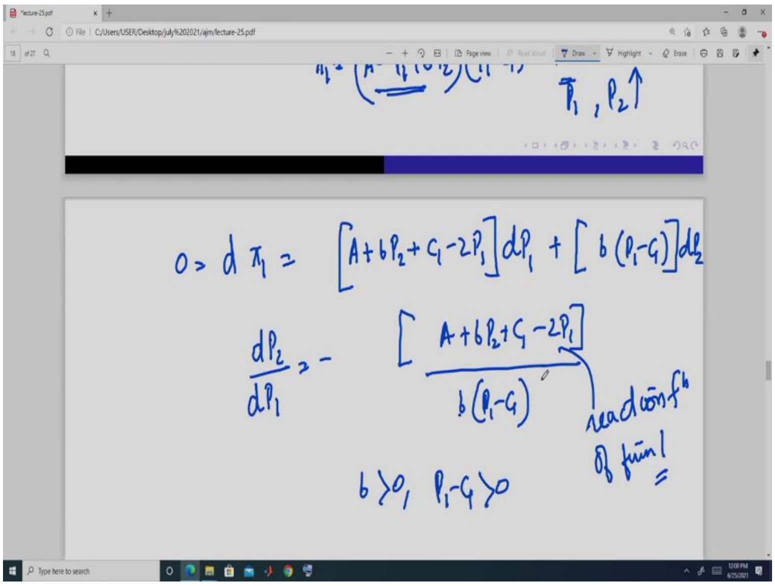

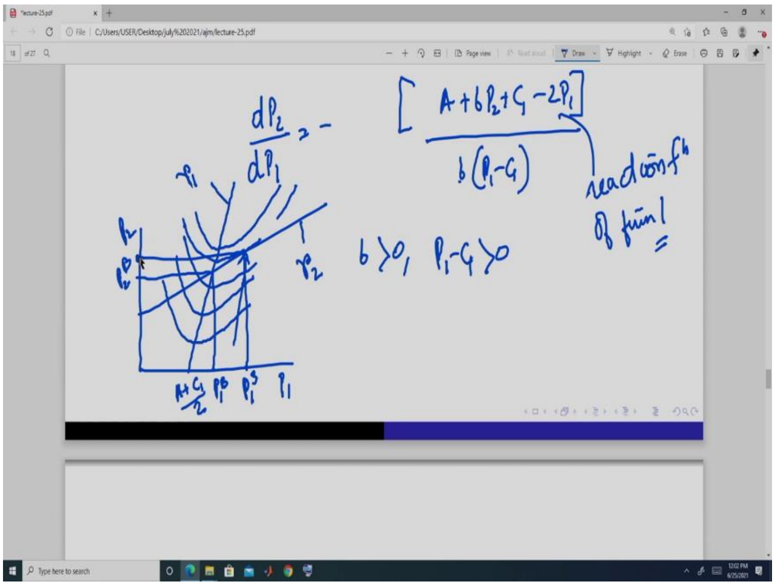

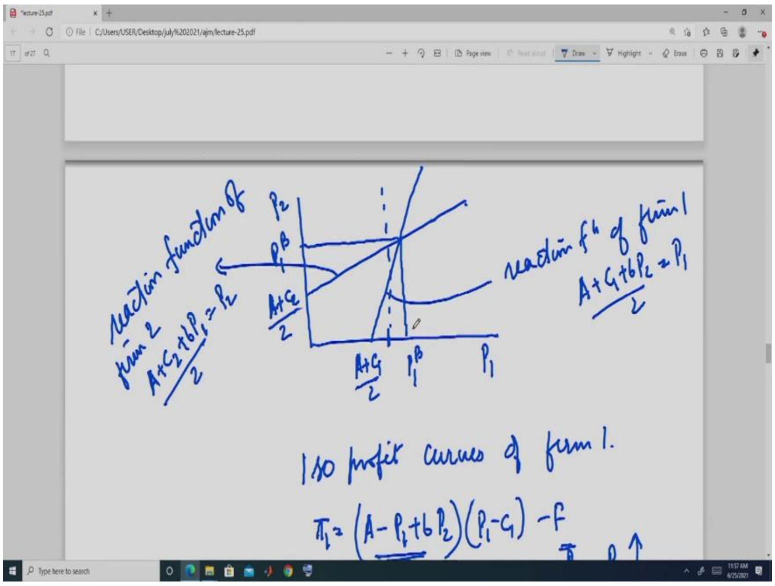

Now, how, this function how it looks? So, if we would find the total differentiation of this, we will get this what? This and when we are moving along an iso profit curve, this is equal to 0. So, the level, profit level is fixed. So, we get this- $\frac{dP_2}{dP_1} = -\frac{A+bP_2+c_1-2P_1}{b(P_1-c_1)}$. Now, here if you look at this, this is what? This is the reaction function of firm 1, of firm 1. And this portion, since p1 is always going to be greater than c1 because c1 is a marginal cost.

Otherwise 2 we will get the bertrand paradox. So, this portion is always positive because, because b is positive and c minus, p minus c1 is positive so, this portion is always positive. So, the sign of this how p1 as p1 varies, how the curves look like this iso profit curve this depends on the sign of this reaction function. And we know this reaction function in this portion, it is negative.

Because p1 is, at this point if we fix the p2, at this point p1 is then that reaction function is this function is. Because this is equal to this. So, if we fix p2, p1 at this level it is going to be such that this takes a value 0. But if it is less, it is going to be what? positive. And since this is positive and this, there is a negative sign so, it is going to be negative. But if p, here it is this is greater than this.

So, this is going to this minus this portion, this portion is going to take a negative value. So, here it is a negative sign so, it is going to be positive. So, we get, so, suppose this is sorry, suppose this is the reaction function of firm 1, firm 1 and this is the reaction of firm 2. So, the iso profit curves are going to be like this, and they are increasing.

So, profits are increasing in this direction. This is the reaction function of firm 1 and this is the reaction function of firm 2. This is the bertrand outcome. And the stackelberg outcome is such that it is going to be tangent. So, it is suppose this point. So, stackelberg is going to be this.

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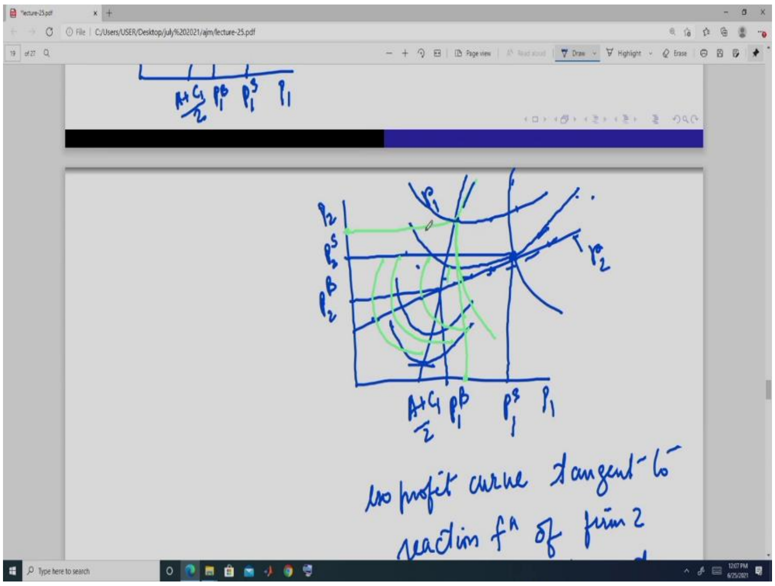

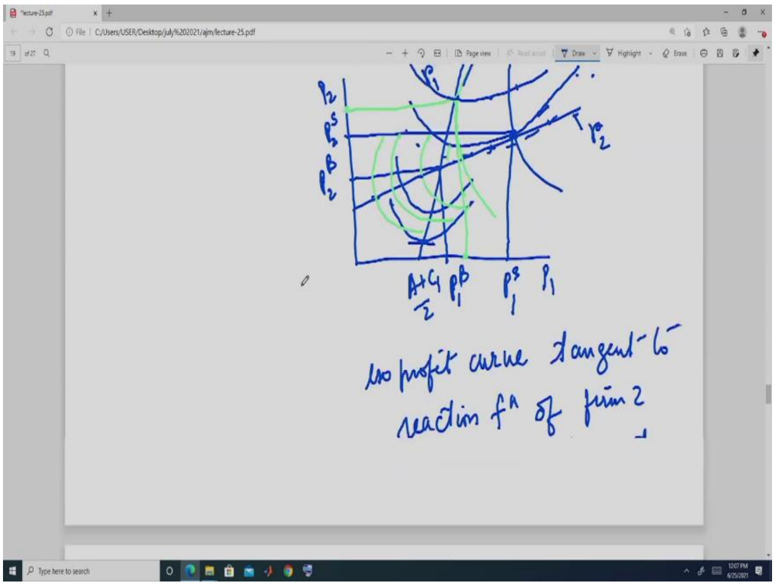

So, what do we get? I will draw this again, this is the reaction function of firm2. This is the reaction function for firm 1, and iso profit curves are like this, so it is here. And we will get a curve like this and this is suppose, this is the bertrand outcome, when both the firms decides the price simultaneously, this is stackelberg. When firm 1 moves first and firm 2 moves second, like this.

So, now, if it chooses any other point here, because firm 1 knows firm 2 is going to choose based on this reaction function. So, it will choose that point in this reaction function which will give it the maximum profit and profits are increasing in this direction, right. So, if you take here, so, it will be below, if you take here it will again below this. And as this is the, iso profit curves which is tangent.

So, any other point it will be such that, that iso profit curve passing through that point will be lying below this. So, the profit is going to be less. So, that is why the iso profit curve tangent to reaction function of firm 2 gives the, stackelberg outcome, okay. So, the stackelberg outcome that we have computed, we have got it in this way. When, when b is positive, that is goods are substitutes. Now, look in this way.

So, we have got this. Similarly, the iso profit curves of firm 2 is going to be something like this. Yeah, it will be like this, it will be like this and it will be somewhere sorry, it will be something like this. Then this is the stackelberg outcome when so, this green one, this gives me the stackelberg outcome when firm 2 moves first and firm 1 moves second, okay. Now, if you look at this see what is happening. It will, it will be such that it will, it is going to be tangent, right. The moment it is tangent to this, what is happening?

It will lie above this, because there is a highly unlikely chance that it is going to be tangent. So, if it is like this, then it means what? The iso profit curve of firm 1 is here, but when it moves first it is here. So, this is above this. So, what happens? When firm 2 moves second and, firm 1 move second and firm 2 moves first, then actually firm 1 gets a higher profit. So, now it is here, right? and if you look at these here are increasing.

Here it is this, but if it moves first, firm 1, firm 2 moves first, then it is A will be somewhere here. Because it will be based on this reaction function. So, that is why firm 2 get higher profit when it moves second, and when firm 2, firm 1 moves first.

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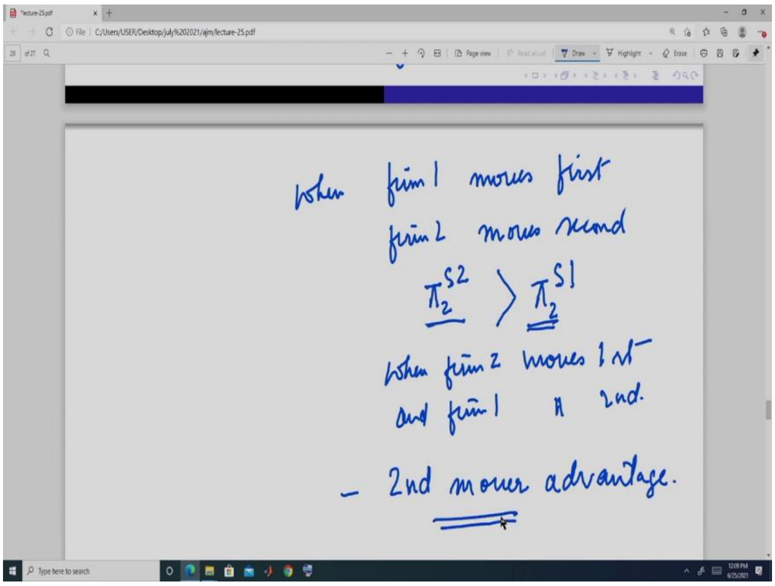

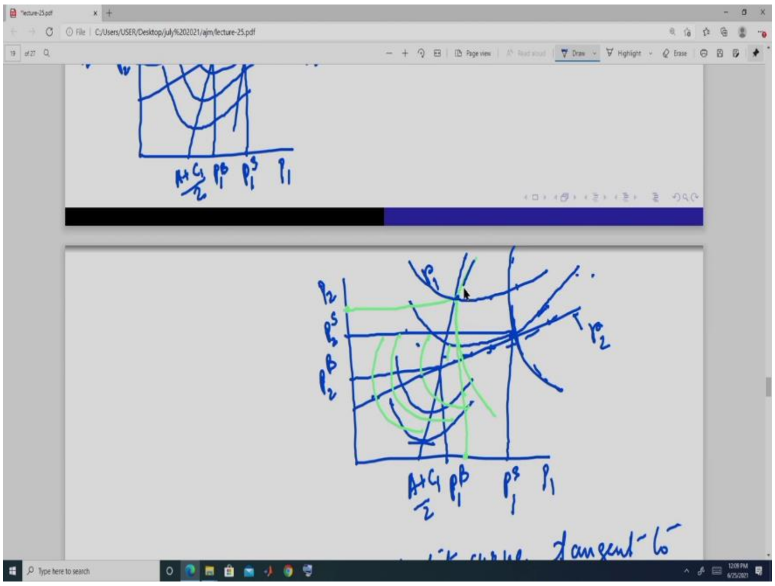

So, what do we get? When firm 1 moves first and firm 2 moves second, so, profit of firm 2 here when it moves second, this is going to be greater of firm 2, when it moves first when, firm 2 moves first and firm 1 move second. So, this is the case, we have got it from this. So, what is happening? This is the iso profit of firm 2. But if it moves first, its iso profit is going to be somewhere here.

It will be based on this reaction function, right. Because it will choose that point in this reaction function of firm 1, which will give you the maximum a . So, it will definitely going to be lie below this. So, because this a is never going to b, because this is tangent that this a. So, that is why, firm we will we get there is always a second mover advantage. So, what we, do we get?

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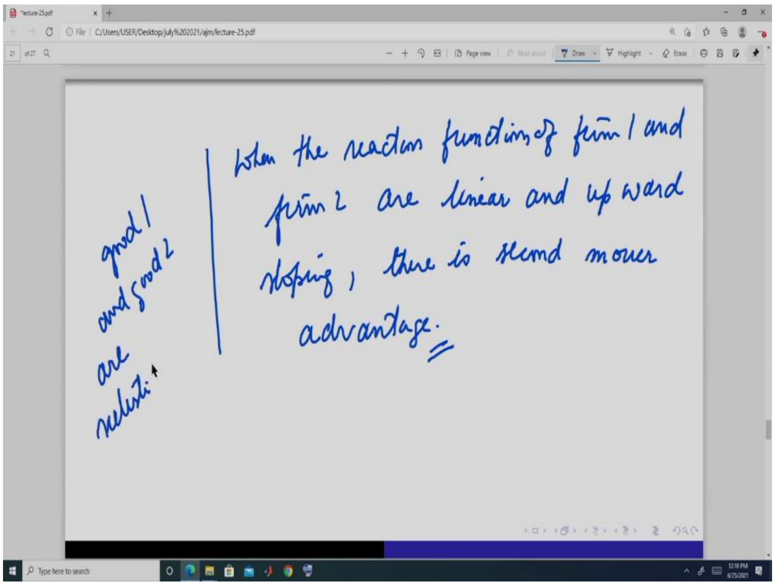

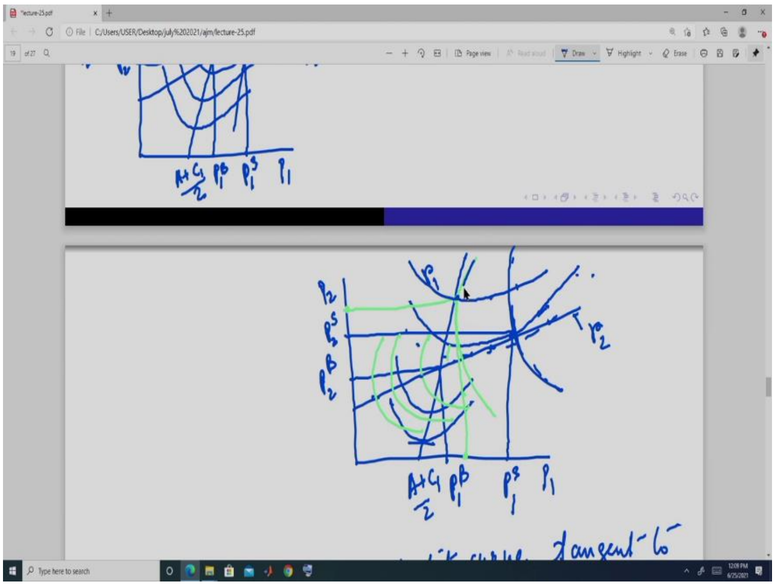

We get that whenever the reaction functions, so, this result we write it in this form. When the reaction function of firm 1 and firm 2 are linear and upward sloping, there is a second mover advantages. So, this means when they are upward sloping, this means good 1 and good 2 are substitute, okay. If it is not substitute, then the reaction functions would not have been upward sloping, we have got it from this.

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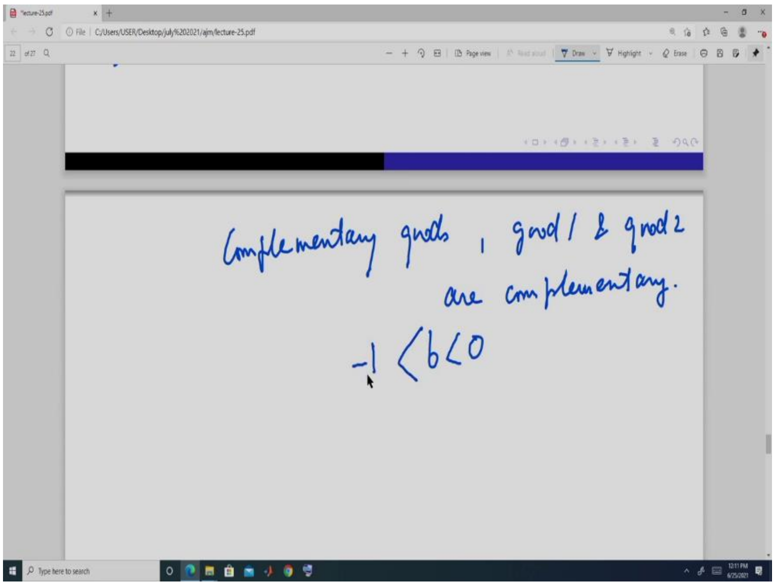

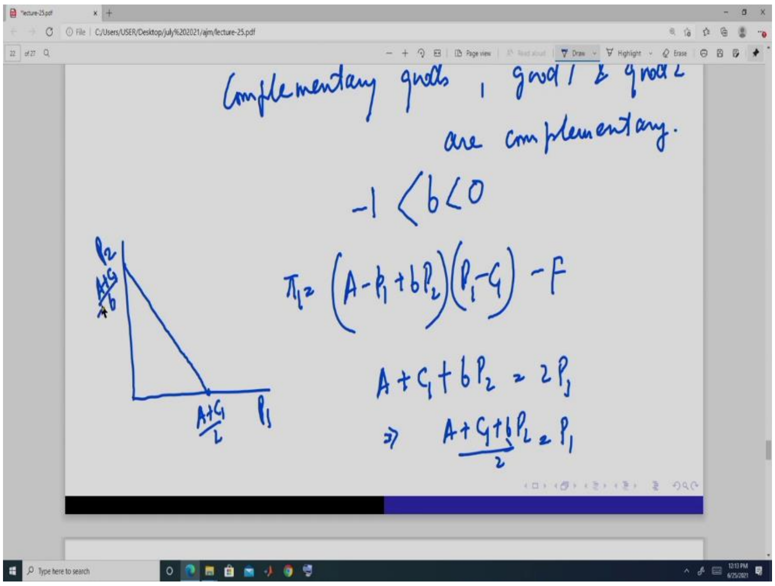

Now, in case of complimentary goods we know, in case of complimentary goods, that is good 1 and good 2 are complimentary. So, that means b is negative and b is greater than minus 1, okay. So, this so, reaction function is profit of firm 1 is this. We know the reaction function of firm 1. This is going to be a negative portion, part. So, going to be like this. So, this reaction function, if we take price of good 1 here, price of good 2, our price of firm here.

So, this this is negative, this portion because b is negative. So what it, what it will be? So, if this is 0, it is going to be A plus. And if this is 0, this is going to be divided by, is going to be divided by minus b. So, it is positive number, we will get like this.

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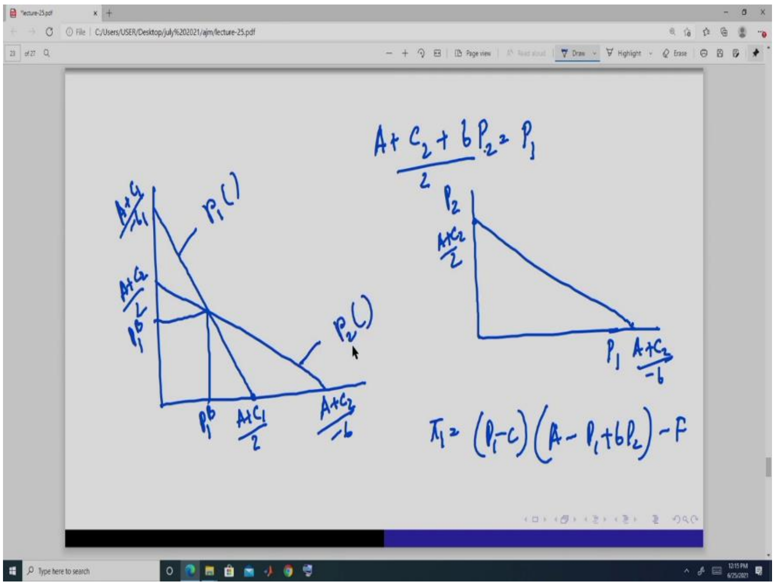

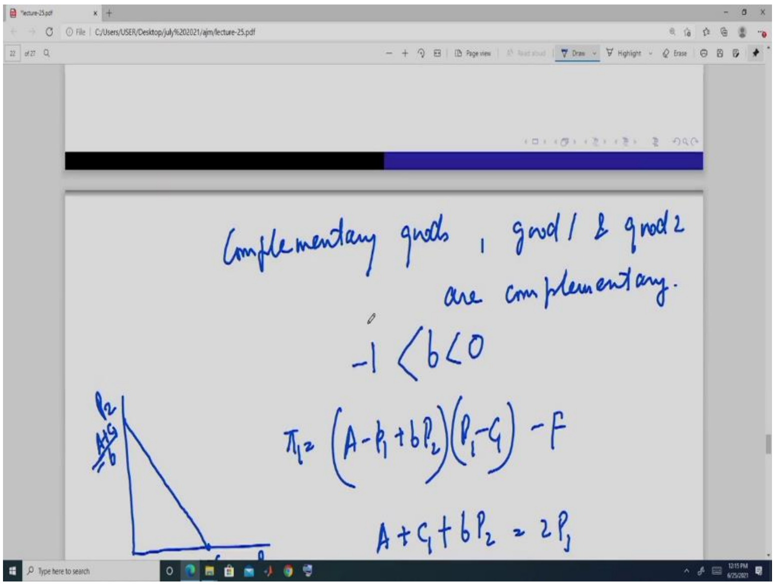

Similarly, the reaction function of firm 2, it is going to be A plus c2, it is going to be like this- $\frac{A+c_2+bP_2}{2} = P_1$, sorry, this. And since this is going to be negative so, and this is going to be like this. And this is so, this is positive like this. So, it is going to be like this. This is going to be the price when they are choosing the price simultaneously. So, this is bertrand a. Now in this case, what we have done?

We have assumed that the goods are complimentary. And firm 1 chooses price first, firm to chooses price second. And the profit function, the iso profit curve is this-$\pi_1 = (P_1 - c)(A - P_1 + bP_2) - F$. Now, when p2 increases, since b is negative this is going to go down. So, this is so, profit in this direction it is going to go down. Reaction function of firm 1, reaction function of firm 2.

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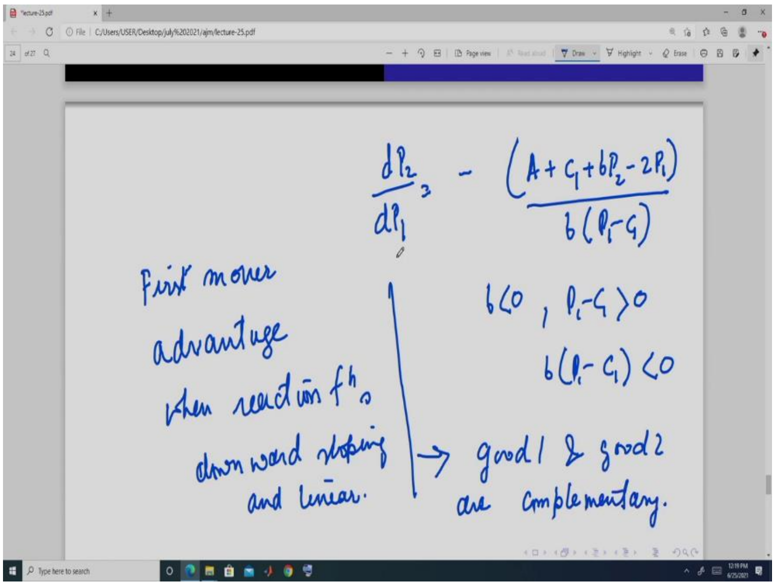

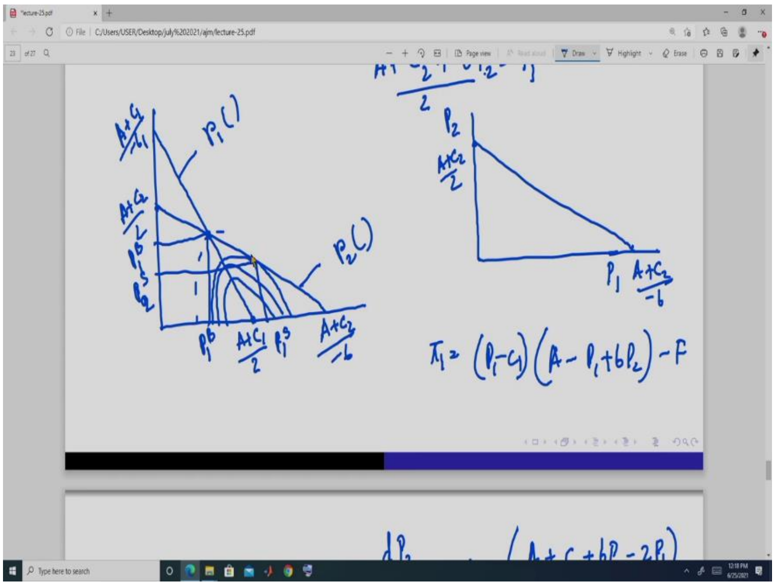

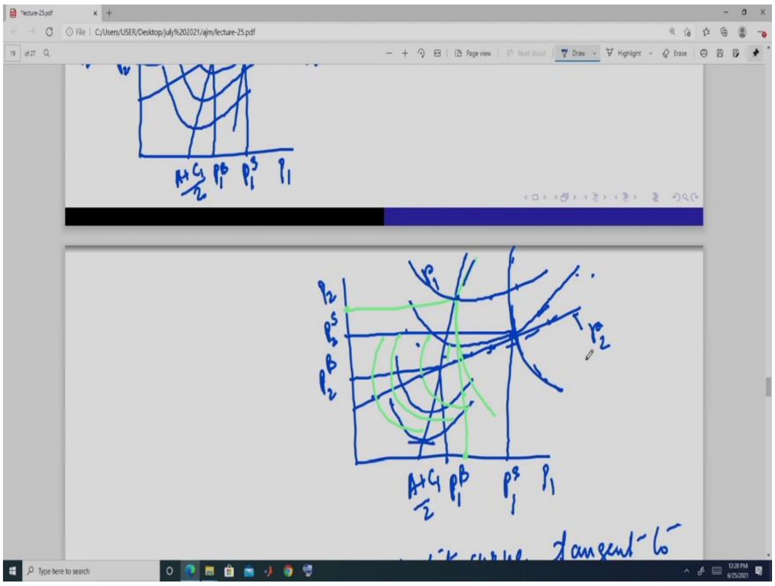

So, again by simply looking at this, we get this to be- $\frac{dP_2}{dP_1} = -\frac{A+c_1+bP_2-2P_1}{b(P_1-c_1)}$. Now, b is negative so, since b is negative so, since b is negative, p1 minus c1 is positive. So, this is positive so, this whole term- $b(P_1 - c_1)$ is negative. So, this is, whole term is positive. So, this when prices are here, this portion reaction function this takes negative value. And when it, positive value and when it is here it is taking negative value. So, the iso profit curves are going to be like this, okay. And profit is going to be higher here.

So, again it will be this point is going to be some stackelberg price. And this is going to be stackelberg price, right? Now, here if you look at this here so this is when they move. Now, in firm 1, if you look at firm 2, it will be at, so the of firm 2, this is the reaction function of firm 2. So, it will choose that it is tangent to this one. So, this is the stackelberg. Now, whether there is a first mover or second mover advantage.

Now, in this case you will see that this point is should give firm 2 the maximum profit and this is when they move simultaneously. And this is when firm 2 moves second and firm 1 moves first. So, here this is going to give it a higher profit, than this. So, firm 2 has a disincentive to move, second. So, we get that again there is a first mover, first mover advantage when reaction functions are downward sloping and linear.

So, this is the case when good 1 and good 2 are complimentary in nature. And they are complimentary then we get the result is same as the Cournot, as the quantity competition and when there is no homogeneous product. But when the goods are substitute, we get that there is a second mover advantage and we show that second mover advantage based on this, that if firm 2 move second, its will be in this iso profit curve.

And the iso profit curve of firm 1 is this because it is tangent to this. But if it and this is always going to lie above any iso profit curve that is tangent to this, because this is the reaction function of firm 1. So, that is why firm 2 here is going to gain by moving second rather than moving first or moving simultaneously along with firm 1, okay. So, with this, we end the stackelberg competition.

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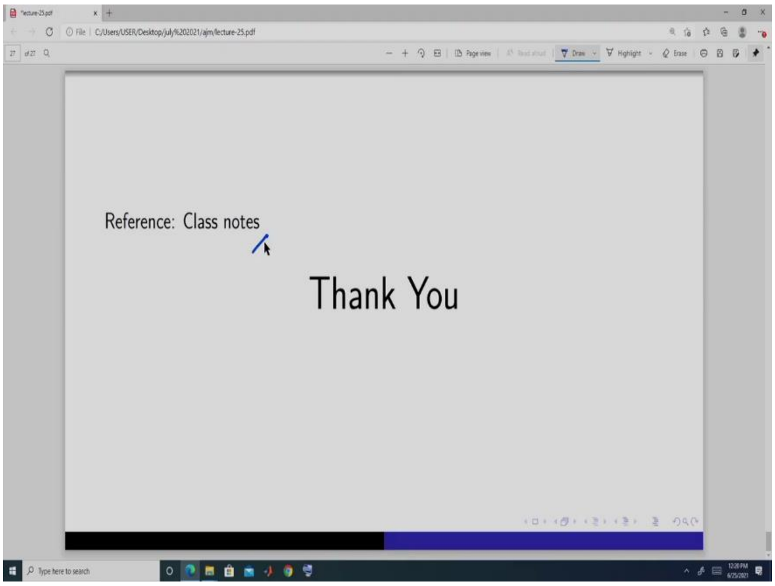

And stack it for this portion price competition this class note is sufficient. And it is more or less straightforward, it is not very difficult. Only problem is when you have to find out the, whether there is a first mover advantage or the second mover advantage and it is better if you do it based on the diagrams, rather than comparing those algebraic expressions. Because these algebraic expressions are going to be very messy in this situation.

Because we have got the and because this is the price of firm 1 and, and when we plug in this, in this reaction function we get the price of firm 2 in this form. And we have to plug in this, in this function. So, this is going to be a very messy thing. But, we can find out whether, we can find out a way to compare the situations whether there is first mover advantage or second mover advantage based on the diagrammatic analysis or based on the analysis of the graphs, okay. So, this portion you can look at from these notes is sufficient.

And if you compare it with the quantity competition, you will see if in the case of complimentary goods it is same as the quantity competition. Even if we are doing product differentiation, okay. So, with this I end this portion. So, thank you.

Introduction to Market Structures
Department of Humanities and Social Sciences
Indian Institute of Technology Guwahati
Professor Amarjyoti Mahanta
Lecture 36
Simultaneous move Hotelling Model

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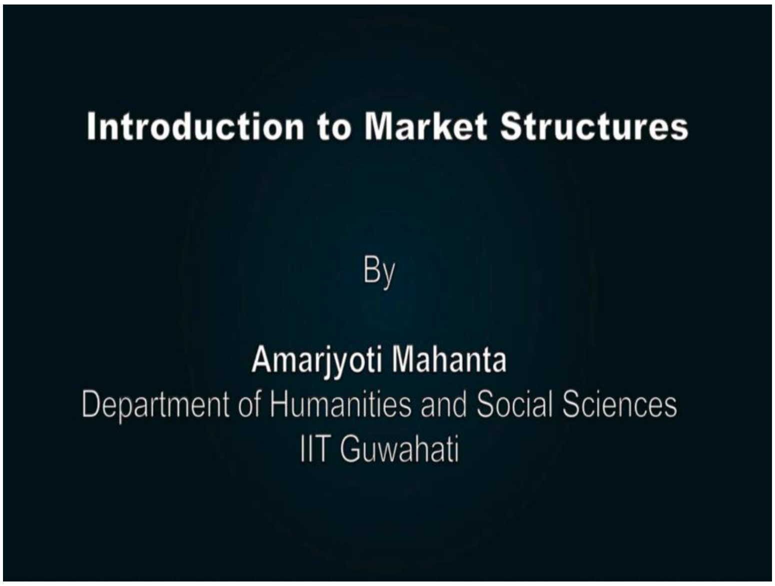

Hello, welcome to my course Introduction to Market Structures. Today, we are going to do product differentiation.

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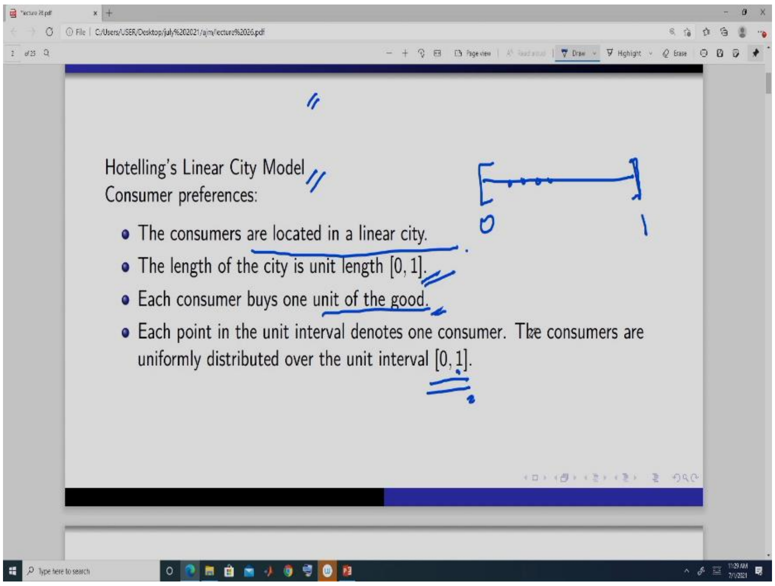

And we have already done a version of product differentiation while doing stackelberg price competition. In the stackelberg price competition, what we have done, we have assumed that the firm 1 and firm 2 can differentiate their product and that lead to either production of some complimentary goods or some substitute goods, it depends.

Now, we are going to do it completely in a different way and the model that we are going to do is a very famous model and that is Hotelling's Linear City Model. So, Hotelling's Linear City Model is mainly about a location, like think of stretch of the length of suppose unit length and there are some ice cream parlours. Now, the ice cream parlours they sell the similar kind of ice cream, but they can vary based on the location in that stretch, okay of one kilometre you can take.

So, depending on their different location what happens, people will go to that ice cream parlour which is nearer to that person. So, that is also a way of differentiating product or you can say that you have different attributes and these attributes can be mapped in a distance of 0 to 1. So, it is like it takes a value which lies between 0 and 1 okay.

So, let us do the specific model. So, what we do, we do that who what do we assume that the first consumers are located in a linear city, okay. So, it is a straight line and the length of the city is unit length. So, it can be any but for simplicity, we assume that it is unit length and each buyer buys only one unit of the good. So, this is you can say, it is an example of discrete good. So, here a consumer buys only one unit, okay and each point in the interval this unit interval, so, it is something like this.

So, this is suppose unit interval, here each point denotes one consumer, okay or you can say that the consumers are uniformly distributed over the unit interval [0, 1]. So, the distribution of consumers where each consumer buys one unit is uniformly distributed over [0,1], okay it is same thing, okay. But this is more technically correct, the consumers are uniformly distributed over the unit length, okay.

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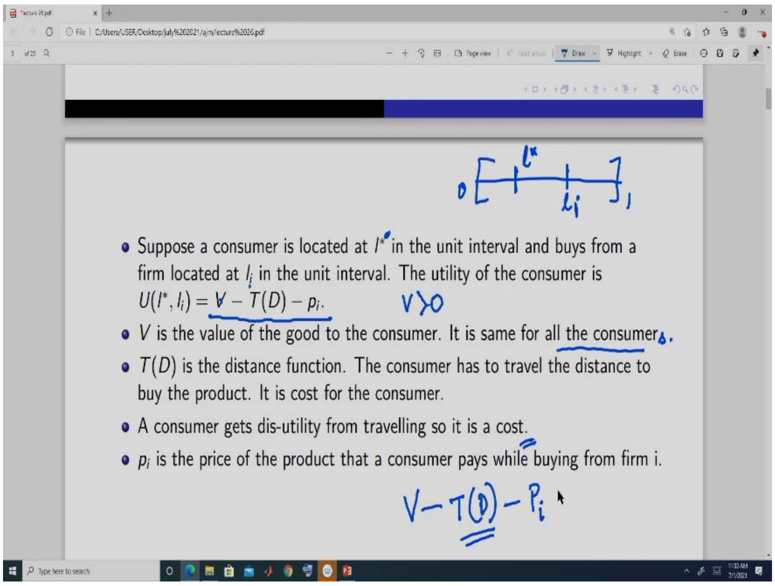

Now, suppose a consumer is located at this. So, here it is unit length. This is 1 and 1 and suppose the consumer is located here this is L star, this point, okay and buys from a firm located at Li and suppose this is another firm which is firm I and its location is Li. So, the utility of the consumer is given by this function- $U(l^*, l_i) = V - T(D) - p_i$, okay. So, what is this function? This function is, this V is a value that other consumer gets from the consumption of this good. So, each value is something like this that you have a utility function and in that utility function, you plug in the optimal value and you get the value function.

Now, here, when we say value function it means because the each consumer is buying only one unit, so this is you can say is the maximum amount a consumer is willing to pay to buy that. So, that is what we mean by the value, okay. So, this is V and we assume that V is a positive number, okay and again for simplicity, we assume that it is same for all the consumers, okay. Now, so, this portion is clear when we define the utility, we first find that there is a portion that is the value, okay. Now value means the maximum a consumer is willing to pay for that and this portion is the distance function. So, this is a kind of negative thing so, it is a kind of a cost or you are getting this utility from it.

So, the distance that a consumer has to travel from its own location and from the location where the firm is located so, that distance a consumer has to travel to buy that good. So, that is creating a kind of its disutility and so, it is taken as a cost, okay. So, or you can say it is the travel, you have to travel this portion and then while traveling you incur some cost and that is this cost, okay because this model has been used in many ways. If you study suppose R1 economics, how the city grows, how a city grow over time, then also you will find a Hotelling Model and this kind of the basic is this model, okay.

And since the consumer gets disutility from traveling so, it is taken as a cost and so, this portion is V that what I get from consuming goods or the maximum amount that I am willing to pay minus this. So, this D is the distance that one person has to travel or a consumer has to travel and this is given by a function. So, this is the cost that is the traveling cost minus the price of the goods set by firm I and that is Pi, okay. So, this is the utility. Why do we need this utility function? You will see why we need.

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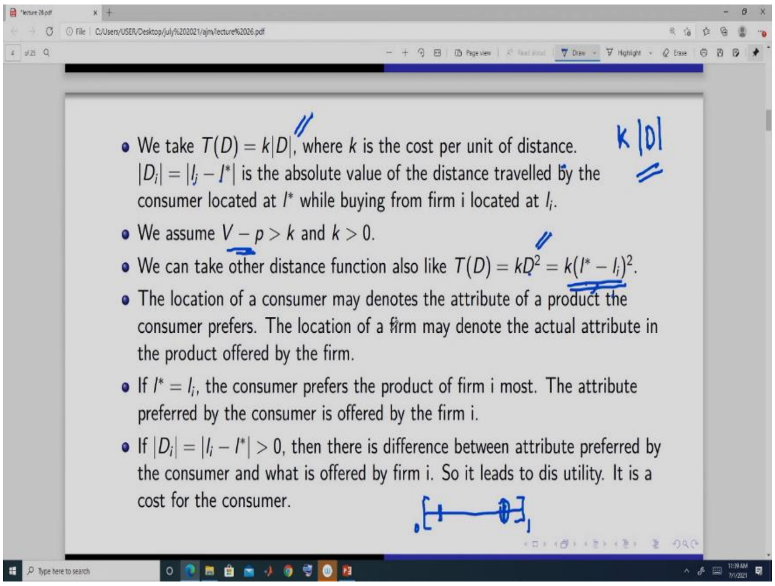

Now, let us define this distance function-$T(D) = k|D|$. So, this distance function is defined in this way k absolute value of D. Now here, k is the cost per unit of distance, okay and suppose the distance is D then it is k into D and this distance is defined in this way-$|D_i| = |l_i - l^*|$. So, it means that suppose take this 0, 1 suppose this is the location of a consumer that is L star and this is the location of a firm Li. So, then the distance is you can say D is absolute value of the distance is Li minus L star.

This distance this one. So, this into k gives you the total amount of cost incurred on traveling or the disutility that you get from traveling you can say that. Because this consumer is suppose buying from this, suppose then you get this much this small k into this amount of disutility or you can say it is a cost and then we specify further, you will see that this k we assumed in such a way that this is V minus p is this k is less than this amount-V–p>k and k>0, okay.

You will see why we need to assume this. So, this is one form of distance function and since this distance is always a positive value so, that is why we take it in terms of absolute value or you can take another form of distance function. Suppose you take the square of this. So, if you take the square of this, it is something like this-$T(D) = kD^2 = k(l^* - l_i)^2$, okay. So, the further away so, it is now this is no more linear in this. So, if you are further away then, your utilities this utility is much more compared to when you were nearer it, okay. Now this you can think of this linear city model in terms of like this.

Suppose you want to buy a product and that product has some attributes and that attribute you can give value to it and it like you can give value from 0 to 1 and suppose your preferred your location denotes the attribute that you want and the location of the firm denotes the attribute that is offered by the firm. Now, if there is a distance then it means what, that the exact kind of attribute that you want a product to have that is not being offered, okay. So, there is a mismatch. So, you get some amount of disutility from it but if suppose it is matched so, if your location and the location of the firm is same, then it means that the attribute that you want in that product, that same attribute is being offered by the firm.

So, it is something like this. So, you want suppose the speed of your mobile phone and you want a specific number to that, okay you want a specific speed, but the mobile phone companies are not offering that but it is offering something else. So, the difference between these two we can take that as a distance and that distance is giving you some form of a disutility because what you want you are not getting it, but if the distance is not there, then you prefer that product most. So, from here what do we get?

So, you will buy from that firm which is nearest to you because that means that firm is closest to you and so, that firm is giving you that attribute which is closer to the attribute you want, right? But here, if you think in terms of attribute as if you take attributes see there is again another problem here. So, we have to be very careful. So, like this so if we are talking about this point as one attribute and this point and another attribute, we do not want to imply that the quality here is better than the quality here.

It is not something like that. So, it simply it is, we are silent on the quality. What we are saying simply is that these are different nature or different kinds and the quality of these each kind is same or it is good, okay or simply you can think that when you are choosing from suppose, you are walking in sea beach and you want to buy an ice cream. So, you will look for that parlour, ice cream parlour which is nearest to you from where you are standing. So that idea, okay. So, consumers are going to buy from that firm which is nearest, okay. So that is why this distance function plays an important role. So, the way we define the distance function we can vary the results, okay.

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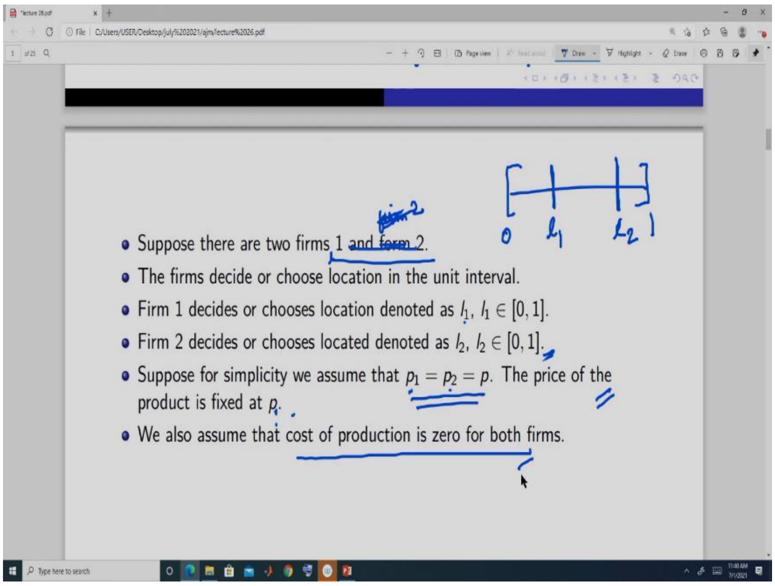

Now, we come to the firm. So, we first take the simplest firm and that is we have two firm. Firm 1 sorry two firms, 1 and 2, okay. And the firms decide or choose location in the unit. So here, in this unit location there is one location that is chosen by firm 1 and another is chosen by firm 2, okay. So, firm 1's location is denoted by L1 and the location of firm 2 is denoted by L2, okay. And for simplicity, we assume that the prices are same, p1 is equal to p2 they are same, okay and it remains fixed.

So, the firms in our model in the actual the original model hotelling and that which is further developed in a game theoretic form that includes this price. So, price is endogenously determined in that model but that is slightly complicated model. So, since this course is not that advanced course so, we will not take that second stage of the game in this model. We will only consider only this location thing. So, this price we will assume that it is fixed and it is same for each firm, okay and further to simplify the things, we assume that the cost of production is 0 for both the firms, okay.

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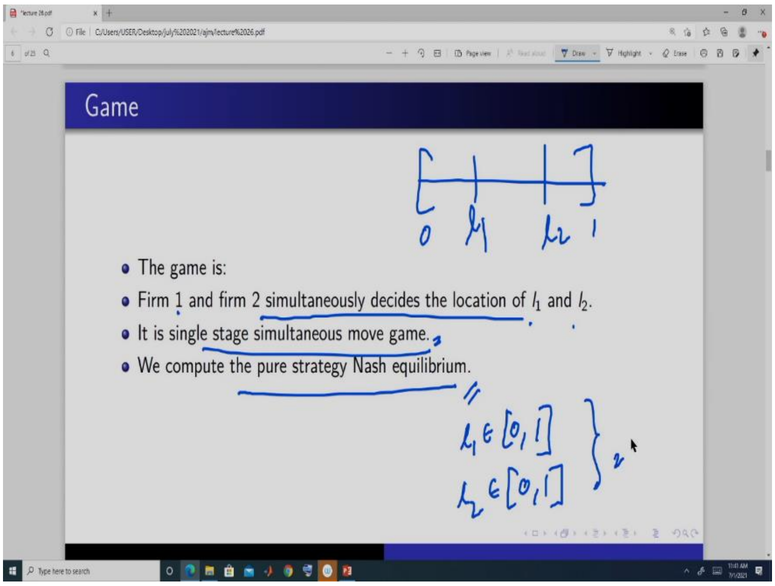

Now, what is the game here? So, the game is firm 1 and firm 2 simultaneously decides the location l1 and l2. So, in this unit interval 0 and 1, in this interval firm 1 chooses the location l1 and firm 2 chooses the location l2 and these decisions are taken simultaneously. So, this is a single stage simultaneous move game. So, in this game what do we do, we compute the pure strategy Nash equilibrium. Now here the strategy said you can say l1 is any location between 0 and l2 and strategy set of firm 2 is at any point in this unit interval 0 and 1, okay. So, this is the strategy. Now we move to the solution, okay. So, we find the pure strategy.

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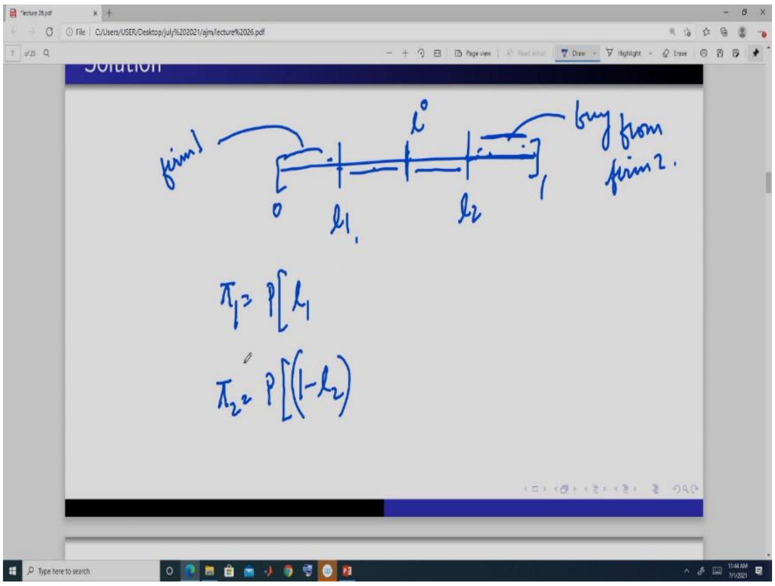

Now here, let us first define the payoff, okay. So, profit of firm 1, you can say suppose, okay before doing that, let us take one. Suppose, this is the unit interval and suppose this is the l1 and this is l2, this is the location of firm 1 and this is the location of firm 2. So, the profit of firm 1, so, these people are going to buy from firm 1, these people are definitely going to buy from firm 2 because if they move from here to here, this distance is quite bigger than simply this distance. So, this firm people are going to buy from this. So, price is fixed, cost there is no cost. So, it is going to get l1.

This is form and profit of firm 2 price is fixed, it is going to get 1 minus l2 because this distance any point here this whole distance is this is 1 from 0 to 1 and this is l1. So, this is left so it is 1 minus l1 this distance. So, this much it is going to get and it is going to firm 1 is going to get some portion of the people who are located in this region. So, we will get a person like this suppose L star or l dot or l not such that this person is going to be indifferent from buying from this and this because prices are same, value is same and if the distance is same, so, this distance must be equal to this distance for this person to be indifferent between buying from firm 1 and firm 2. So, how do we locate this l not. So, we have to complete this part. So, we have not yet done this.

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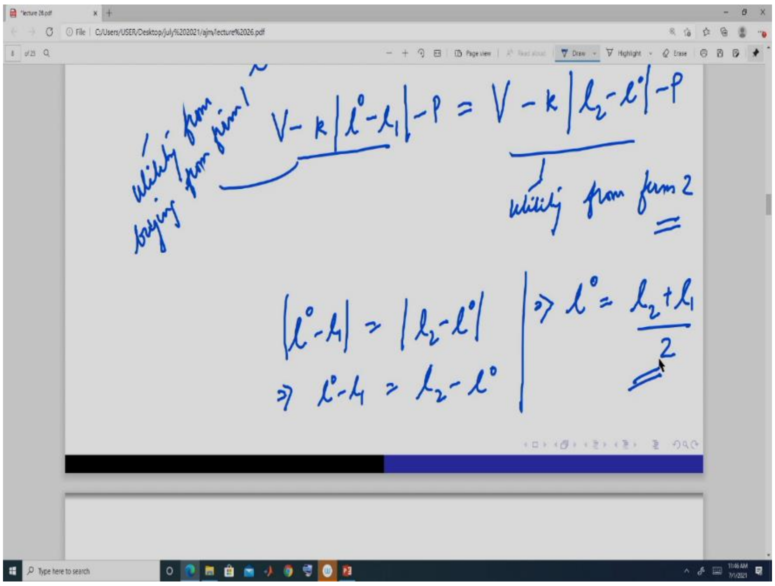

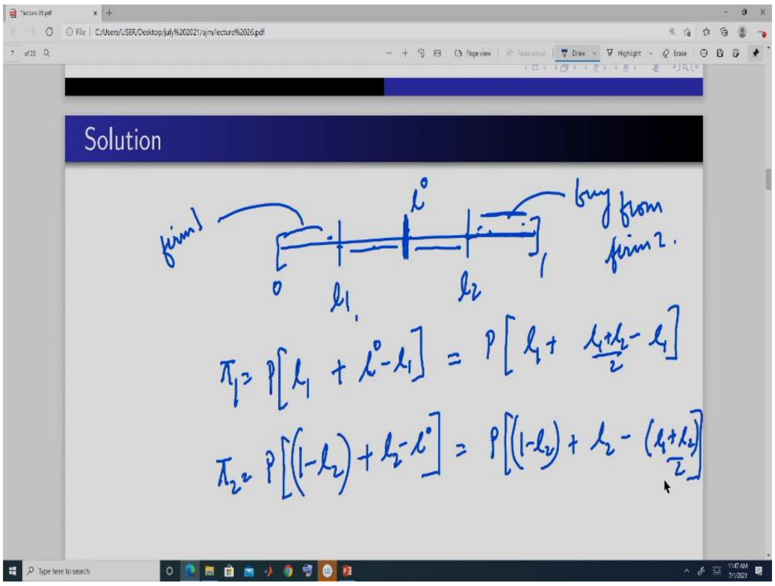

So, l not is l not is such that V minus k l **not** minus l1 should be equal to k l2 minus l dot minus P, i.e. $V - k|l^o - l_1| - P = V - k|l_2 - l^o| - P$ this is the utility from firm 1 and this is the utility from firm 2, okay. So, from this we have to find out this l not. So, this is you can say if we do the simple manipulations, we will get this is equal to this-$|l^o - l_1| = |l_2 - l^o|$ and since the absolute values are or this modulus are always positive, so, we can say l1 is l dot is going to lie somewhere here.

So, it is this point is greater than l1 and this point is less than l2. So, we take it this in this form, this distance is this-$l^o - l_1 = l_2 - l^o$. So, from here, what do we get? We get this-$l^o = \frac{l_2+l_1}{2}$. So, this l dot here, so, this person is who is indifferent and we have find out the location of that person. Location of that person is this, half of this distance between this. So, the profit of firm 1 here it should have been like this plus l2-$\pi_2 = P[(1 - l_2) + l_2 - l^o]$. So, this is equal to what is this? This is l1 plus l2 minus and profit of firm 2 is this- $P[(1 - l_2) + l_2 - \frac{l_1+l_2}{2}]$.

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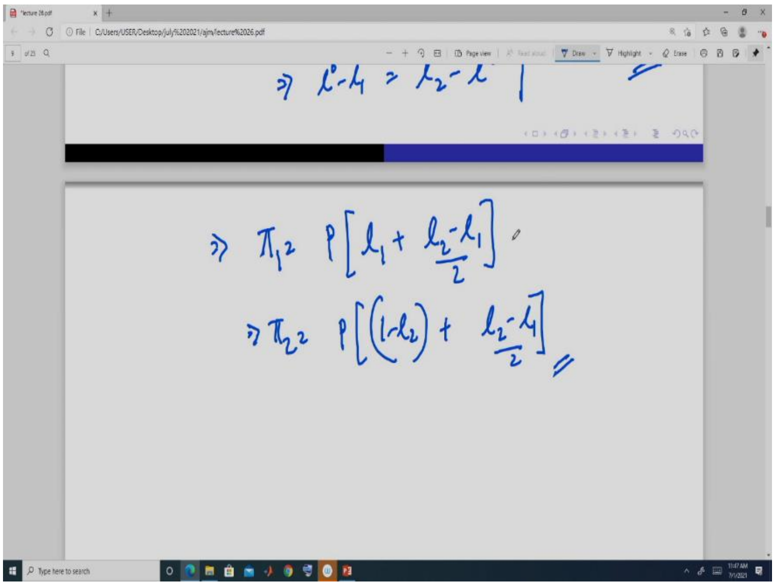

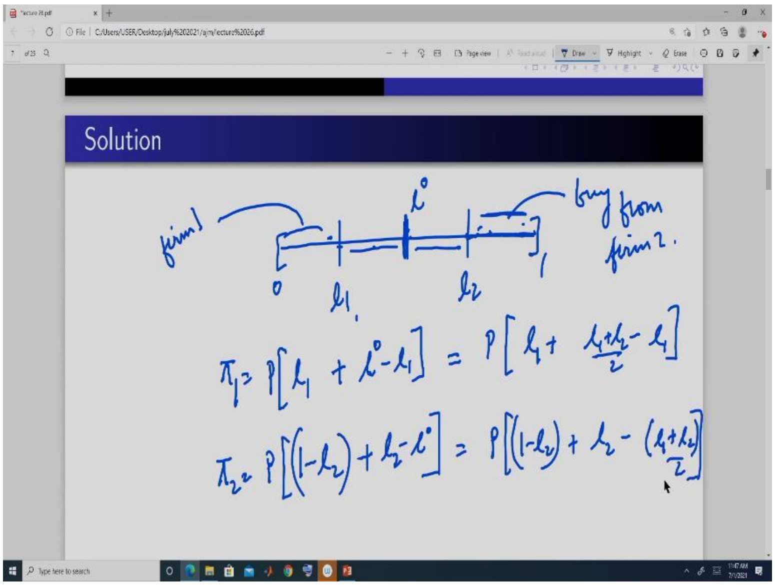

So, we can say that the profit of firm 1 is- $\mathrm{P}\left[\mathrm{l}_1+\frac{\mathrm{l}_2-\mathrm{l}_1}{2}\right]$ and profit of firm 2 is this- $\mathrm{P}\left[(1-\mathrm{l}_2)+\frac{(\mathrm{l}_2-\mathrm{l}_1)}{2}\right]$. Now the thing is we have to decide the optimal or the Nash pure strategy Nash equilibrium l1 and l2. So, how do we do that. So, we have on this this as the given some l1, l2, okay.

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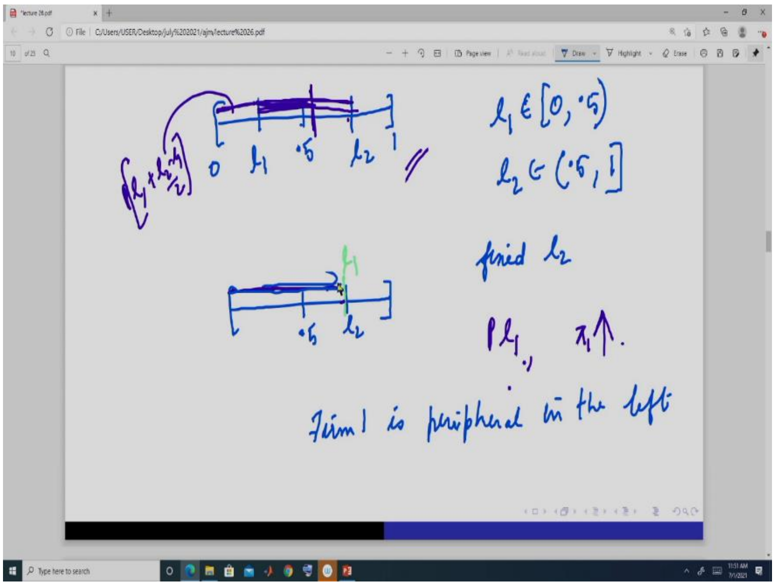

So, let us look at that. Suppose take this example, this case sorry not example, this case. This point is the half, okay suppose l1 is here and l2 is here, okay is this any point here lying between 0 and 0.5, any point here lying between 0.5 and 1. Is it optimal is any point? So, l1 lying between this point is suppose not included. Now, if we take this case now think of this equation. Here, fix l2, l2 is somewhere here because we have taken this. If you switch l1 here and suppose the l1 is in this same position, same position here and this is 11, okay.

What is the profit of firm 1? Now earlier profit of in this case, profit of firm 1 was this P into l1 plus half of this distance. This half of this, this-P $\left[l_1 + \frac{l_2 - l_1}{2}\right]$ now if it switches here in this position its profit is so, this whole is getting it is getting now earlier only this and half of this suppose this is the half. So, it was getting this much profit, now it is getting whole amount. So, what is happening? This gives the profit of firm 1 increases here.

So that is why this is not a Nash equilibrium, pure strategy Nash equilibrium. If firm 2 chooses this point, firm 1 choose point adjacent to it. Why? Because see if you look it from this side firm 1 is the peripheral firm, right? So, firm 1 is peripheral in the left, okay? Okay, So, there is no firm where it is going to lie here. So, if it lies adjacent, this whole portion is going to be its own market share. So that is why firm 1 is going to choose adjacent to firm 2. So, firm 1 locates to firm 2, okay.

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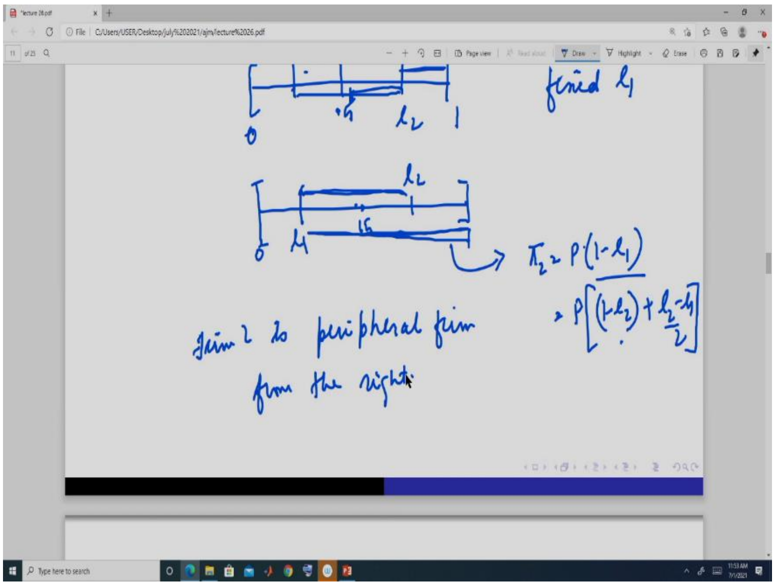

Now take this case, so, this is l1, this is l2. So, fix l1, okay. Now based on the same argument, you will see that if firm because now what is the profit of firm 2? Firm 2 is this length plus half of this length, half of this length which is suppose given by this amount so, it is this. Now, instead of that suppose it moves to this position. If we fixed l1, if it moves here this whole portion is the profit of firm 2. So, its profit now it is P into this distance, this whole distance.

So, this whole distance you can say is 1 minus l1 which is greater than P 1 minus l2 plus l2 minus l1, when l2 is this l1 is this fixing. So, this-$\pi_2 = P(1-l_1)$ is greater than this-$P\left[(1-l_2)+\frac{(l_2-l_1)}{2}\right]$. So, that is why again firm 2 here from this argument, you will see firm 2 is peripheral firm from the right, okay.

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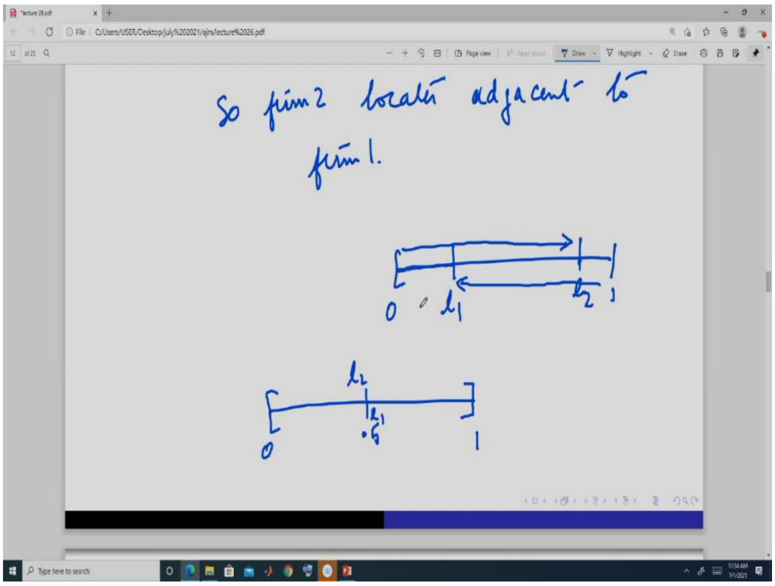

So, firm 2 locates adjacent to firm 1. So, from this argument we see that the peripheral firms will always located adjacent to the firm because this if you look at this position l1, l2 so, this is peripheral from left but this is not peripheral from left. This is peripheral from right. So, this firm will tendency will be to move here. This firm which is peripheral in the right will have a tendency to move here, okay. So, they will move. Now in this way what do we see that 0.5 which is the middle, centre of this unit length. So, l1 will locate here and l2 will also locate here.

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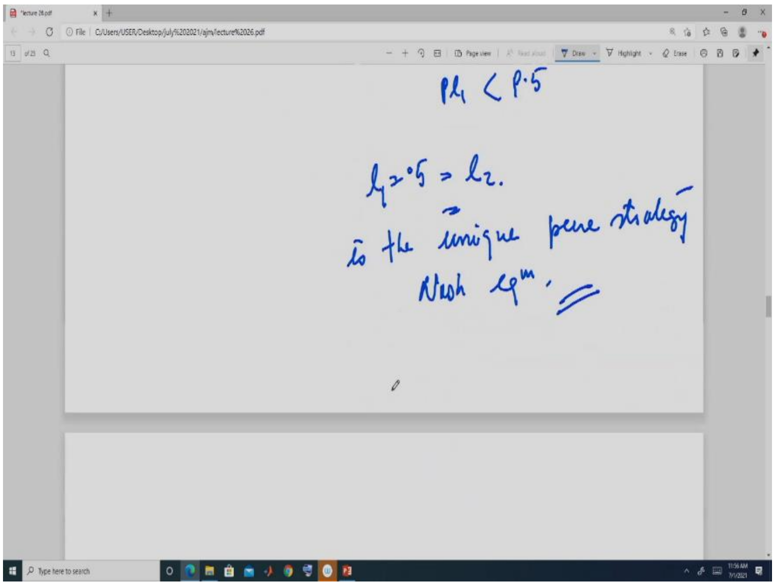

And so, in this situation what do we get? Because they will always locate at the adjacent position, right? Now, if you take any point other than this. Like take l1 here. So, if l1 is here then from that argument we know l2 is going to be here. So firm 1's profit is only this much which is less than half. So, profit is this- $pl_1 < P.5$. So, firm 1 if it moves here, it knows l2 is also going to because this l2 is greater than this here. So, profit of firm 2, this is greater than.

So, firm 1 if it locates here, this point firm 2 is also going to locate here, so it is better to locate here. So that is why l1 is equal to this. This- $l_1 = 0.5 = l_2$ is the unique pure strategy Nash equilibrium in this case when we have two firms and they decides the location simultaneously because any point other than this, there is a tendency to deviate, okay.

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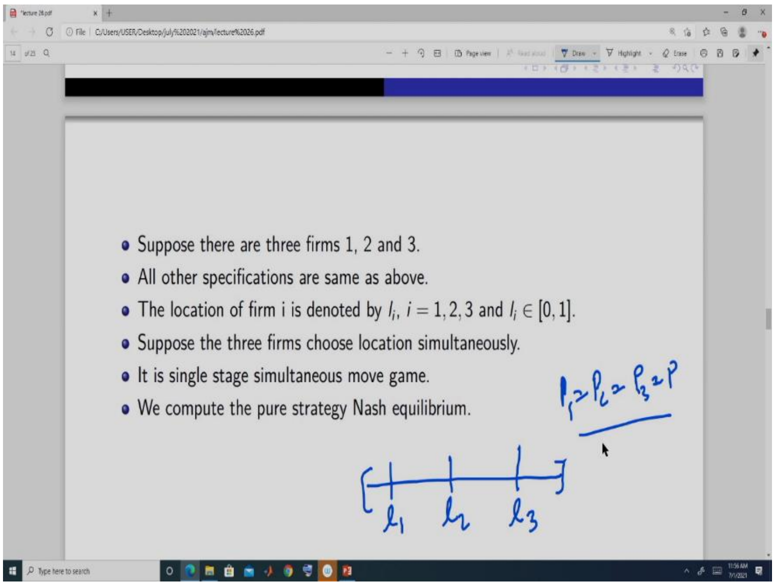

Now, let us introduce one more firm. So, suppose instead of two firms, we have now three firms and it is something like this. Firm 1 chooses the location l1. Firm 2 chooses the location l2 and firm 3chooses the location l3, okay. And rest of the specifications are same that the price is same that is p1 is equal to p2 is equal to p3 is equal to P. There is no cost of production and we have one consumer in each point. So, that is consumers are uniformly distributed in the in unit interval, okay.

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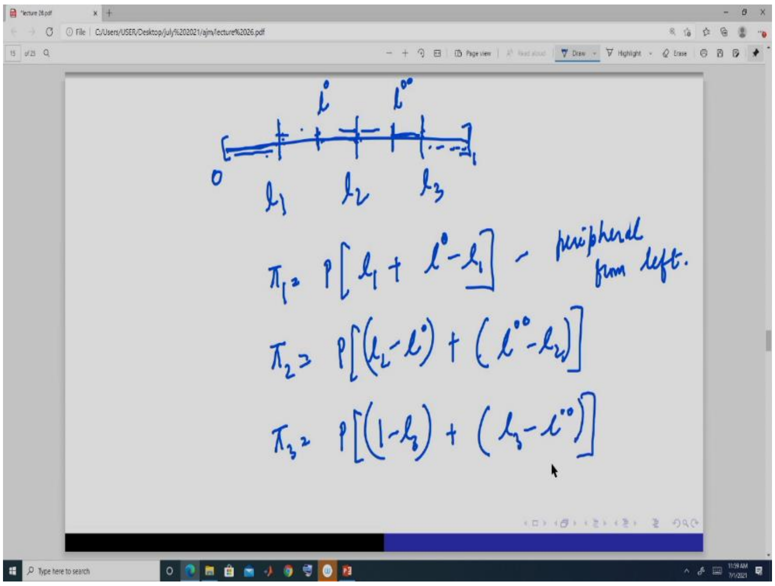

Now in this setup, see if we have a situation like this suppose this is 0 this is 1. So, this is the unit interval, suppose location of firm 1 is this, location of firm 2 is this, location of firm 3 is this. Now how do we define the payoff? we will have a person who is going to be indifferent between buying from l1, firm 1 and firm 2. Similarly, we will have a person here who is indifferent between buying from firm 2 and firm 3.

So, this location is suppose, l dot and this l not and this is suppose l double dot, okay. Now, how do we define the profit? Firm 1, firm 1's profit is this people are only going to buy from this firm only they because if they travel to any other firm their cost is going to be higher. So, profit of firm 1 this many people they are going to buy from firm 1 and so, these people are going to buy from firm 1. So, that is l not. This is the profit of firm 1-$\pi_1 = P[l_1 + l^o - l_1]$ if the locations are of this nature and then profit of firm 2 is it is in the middle, it is not a peripheral firm anymore because this is peripheral from left and profit of firm 1 is going to get half of this and sorry half of this and half of this right or these people.

So, its location is l2 minus plus this plus l double not minus l2- $\pi_2 = p[(l_2 - l^o) + (l^{oo} - l_2)]$ and profit of firm 3, firm 3 which is peripheral from right its profit is going to be this much- $\pi_3 = P[(1 - l_3) + (l_3 - l^{oo})]$. It is always going to get no one is going to buy from any other firms when firm 3 is located here. So, everyone is going to buy from firm 3 who are lying in this region and it is going to get this length because this person is indifferent between so it is this.

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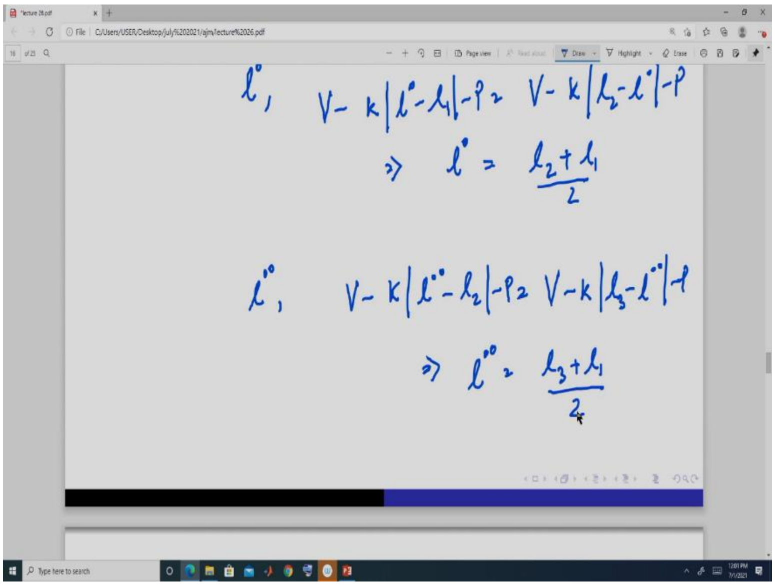

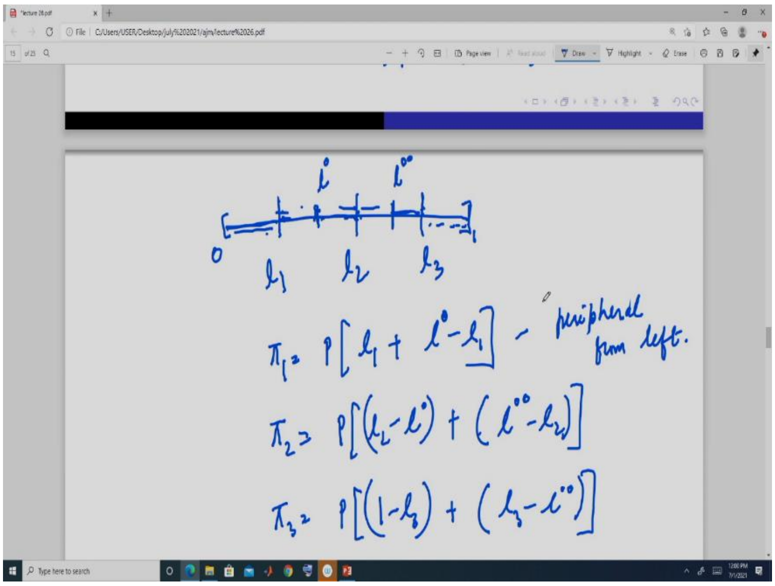

Now we have to find out these two l dot and l not and l double not. So, l double not is such that V is equal to this- $V - k|l^{o} - l_1| - P = V - k|l_2 - l^{o}| - P$. So, from here we get l this- $l^{o} = \frac{l_2 + l_1}{2}$ again. So, it is going to be centre of this length, okay this. So, exact this much length is going to be how much? So, we will do that. Now, let us first find this l double not. l double not is such that l double not minus l2 is equal to l3. So, it is indifferent from buying from firm 2 and firm 3, it is this- $V - k|l^{oo} - l_2| - P = V - k|l_3 - l^{oo}| - P$. So, we get l this, this is the location of l double not- $l^{oo} = \frac{l_3 + l_1}{2}$ and this is the location of l not.

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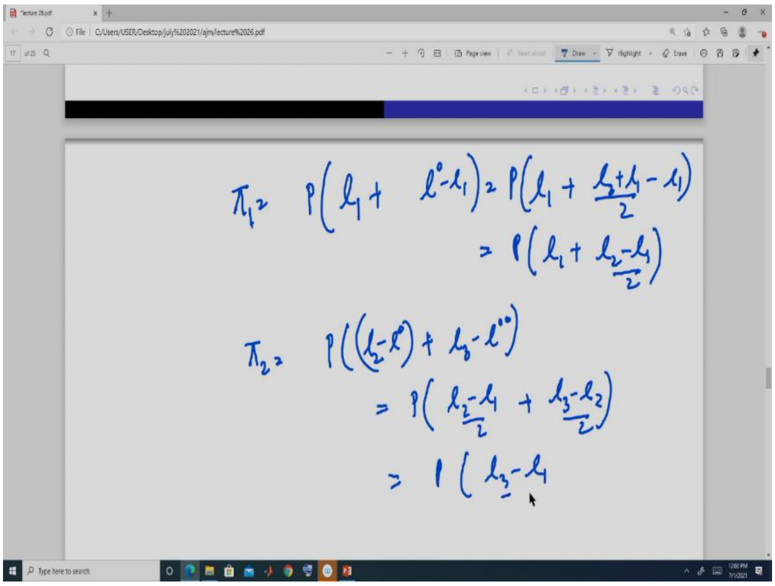

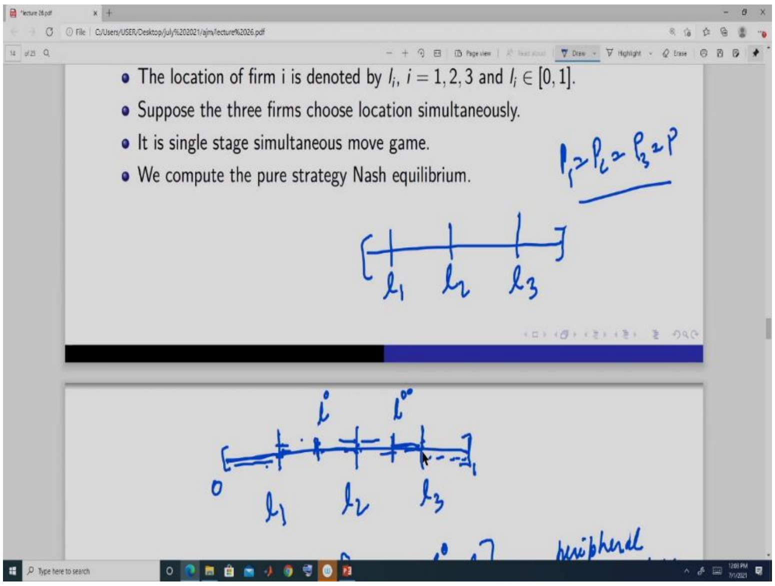

So, from here we get the profit of firm 1 is l1 plus. So, it is l dot minus l1, right? So, this is P into which is equal to-$\pi_1 = P(l_1 + l^o - l_1) = P\left(l_1 + \frac{l_2+l_1}{2} - l_1\right) = P(l_1 + \frac{l_2-l_1}{2})$ and this profit of firm 2 is so, this is equal to this portion is this only. So, this is plus here this is l3 minus l2. So, this is actually here this is farm two's profit is half of length is market share is half of this distance, okay, this- $P(\frac{l_3-l_1}{2})$.

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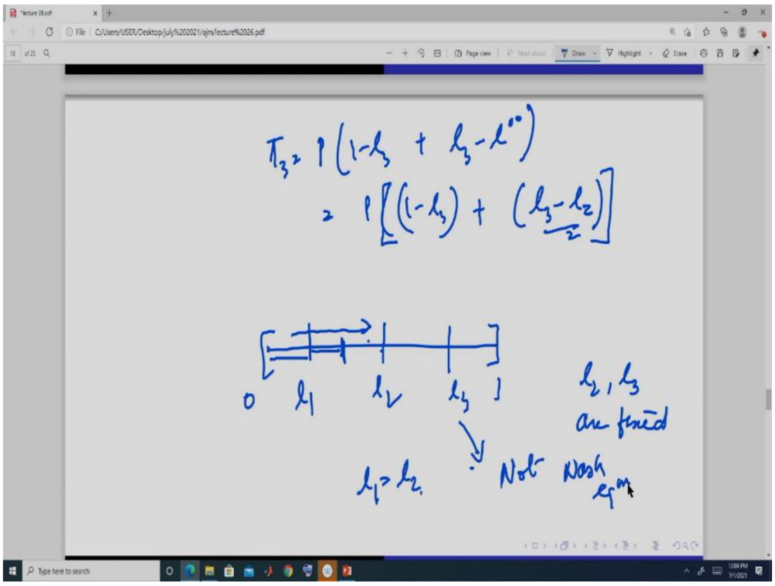

And profit of firm 3 is which is now equal to 1 minus l3 plus l3 minus l2 this- $\pi_3 = P[(1 - l_3) + (\frac{l_3-l_2}{2})$. Now, we have to find the exact location of l1, l2, l3. Now, if you take this and suppose this is l1 this is l2 and this is l3. Now if firm 1 fix this two suppose l2 and l3 are fixed. So, if it moves here what is happening? Earlier it was profit is this much plus half of this. So, it is this. Now, if you move what is happening?

Your V is getting profit is going higher because you are getting more length. So, your market share is increasing. So, it is best for firm 1to locate adjacent to firm 2. So, l1 is equal to l2 in this situation. So, that is why this is, this is not Nash equilibrium, right?

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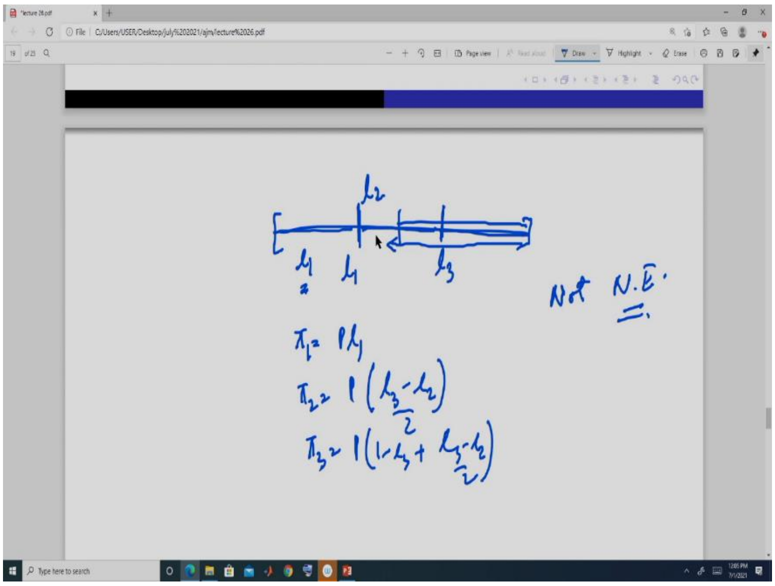

Similarly, if you look from the perspective of firm, so, now suppose l1 and l2 this is somewhere. Let us look at l3. It is here right. Now l3, so, what it is? This is suppose is got by l1 so, profit of is l1, profit of firm 2 is half of this because this side it is firm 1 and this side it is firm 2. So, it is this l3 minus l2 half of this and profit of firm 3 here if you look it is this 1 minus l3 plus half of this distance this. So, if so, it is getting some this much now, if it moves like this, then it is profit is going to increase. So, firm 3 so that is why (locate) locating here it is not Nash equilibrium if given this two a firm 3 moves like this its profit increases. So that is why it is not a Nash equilibrium.

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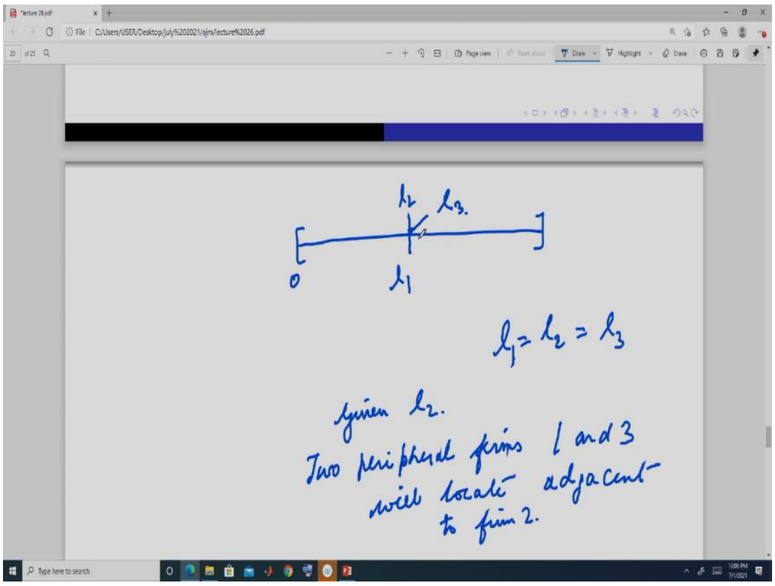

So, from here what do we get? We get that firm so l1 is going to be l2 from the first argument and from the second argument, we get that this it should also be adjacent. So, this should also be l3 should also be here. So, what that it means so given l2, two peripheral firms 1 and 3 will locate adjacent to firm 2 from 1 will from move from left and another will move from right.

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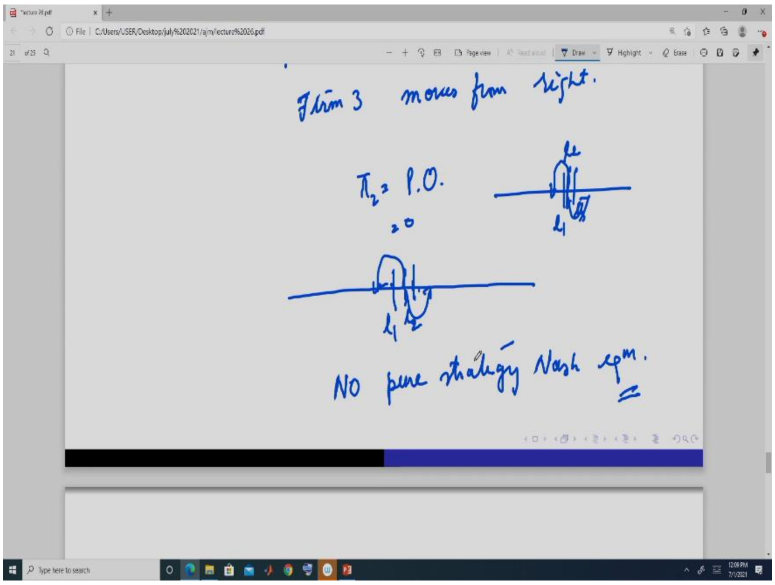

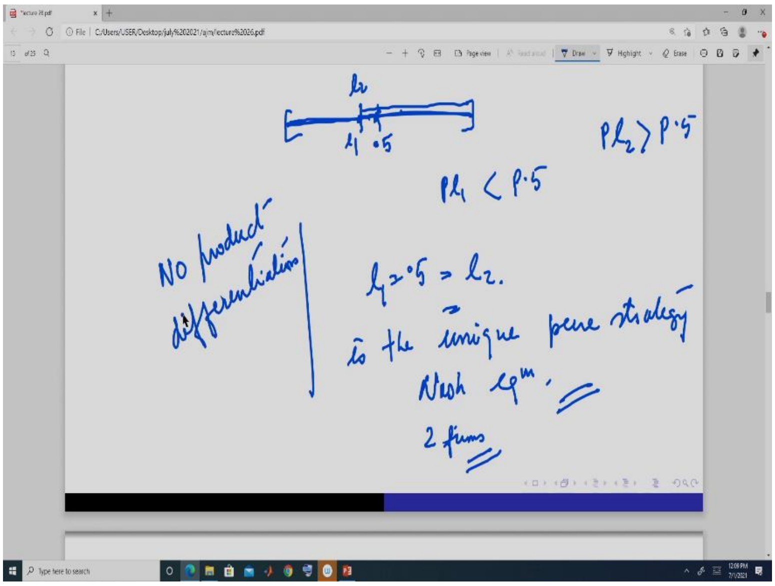

So, firm 1 moves from left and firm 3 moves from right and here we do not get any because then what is the profit of firm 2, profit of firm 2 is 0 because it is sandwiched between firm 1 and firm. So, it is here. So, it is sandwiched between firm 1 and firm 3. So, firm 2 will lie here or it will shift to this. So, if it is l1 is this and l two is and suppose this is adjacent so, firm 2 will think of moving here or it moving here. If it moves then firm 1 will be the centre. In this case, firm 3 will be the centre. So, then again it will try to move.

So, what do we get when we have three firms and they choose the location simultaneously then and price is fixed. So, then no pure strategy Nash equilibrium, okay. So, this is our when we have two firm we get that it is going to be at the centre of the this and so, it results into no product differentiation, right? So, it is a unique and so, it is since it is going to be the at the centre. So, in this situation we have no product differentiation, right? or you can say minimum product differentiation. And here we do not know what is going to happen because we do not have any pure strategy Nash equilibrium.

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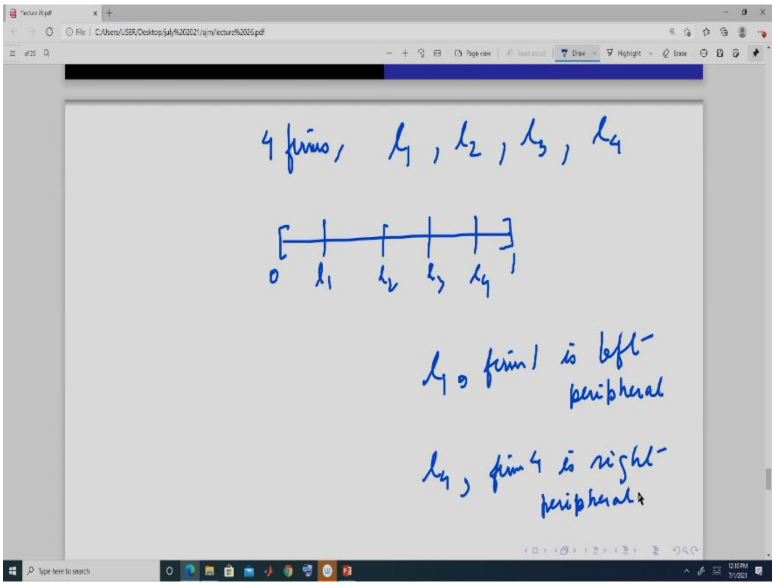

Now suppose we have four firms. Keep everything same and suppose the location of firm 1 is l1, location of firm 2 is l2, location of firm 3 is l3, the location of firm 4 is l4 and they choose from this and suppose this is l1, this is l2, this is l3, this is l4. Now from the previous argument, we know here l1 that is firm 1 is left peripheral firm and l4, firm 4 is right peripheral.

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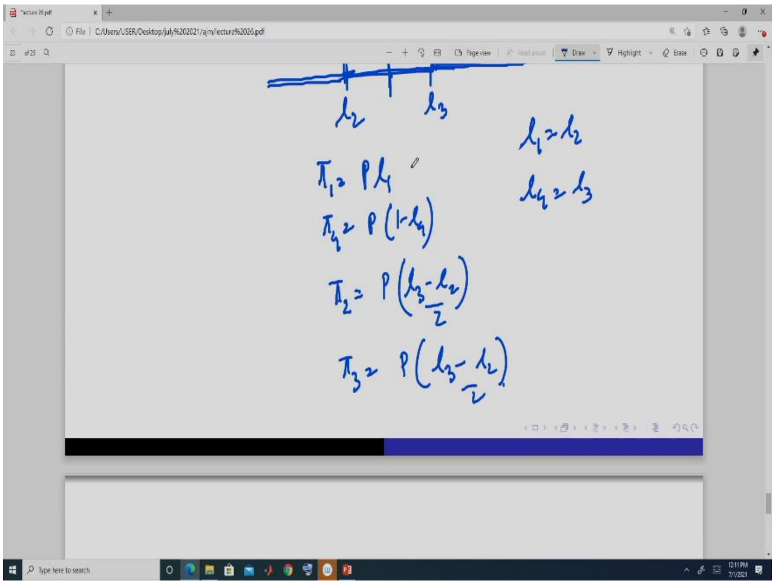

So, from the previous argument what do we get if l2 and l3 are fixed, here if l2 and l3 are fixed, so firm 1 will locate here adjacent and firm 3 firm 4 will locate here adjacent. So, in this case what is going to happen? Profit of firm 1 is going to be P into l1 this whole distance. Profit of firm 4 is going to be this length and we further we have l1 is equal to l2 and l4 is equal to l3. Now here if you look at what is going to be the profit of firm 2?

Profit of firm 2 is going to be so half of this distance. So, this is right because the we will get a person who is going to be indifferent between buying from l2 and l3, firm 2 or firm 3 and we can get this at what point this is the half of this distance. So, this is going to be this- $\pi_2 = P\left(\frac{l_3 - l_2}{2}\right)$ and for firm 3, it is going to be same, this. Now we can find out what is going to be the solution of this by solving this here. Is it possible to get a solution?

Now see, if this distance is less than this distance, then firm 2 will not lie here but it will lie right of firm 1 and we will take this. So that is why this distance must be equal to this distance. Similarly, if this distance is greater than this distance firm 3 will not lie (ri) left of firm 4 but will shift to the right of firm 4.

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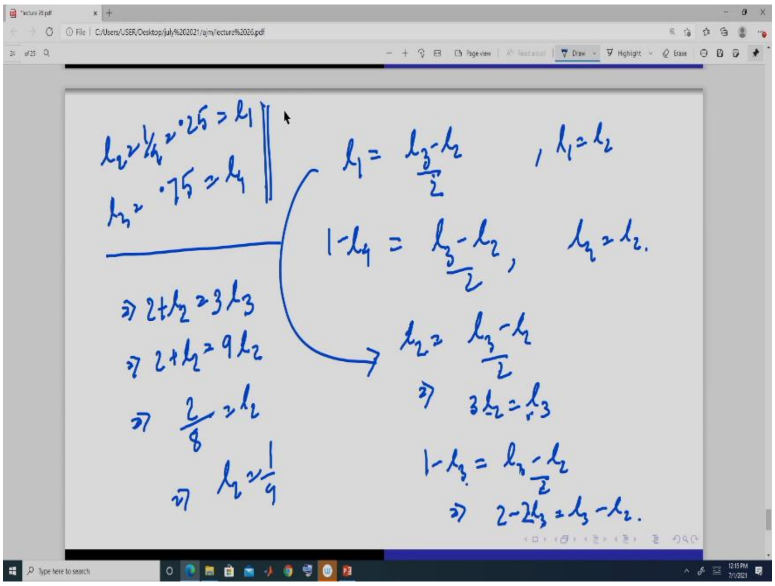


Firm 1 moves from left

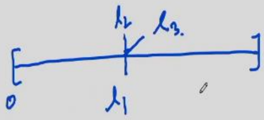
Firm 3 moves from right.

$\pi_2 = $ P.O.

$= 0$

No pure strategy Nash eqm.

$\pi_3 = 1\left(1 - l_3 + \frac{l_3 - l_2}{2}\right)$



$l_1 = l_2 = l_3$

given l_2.

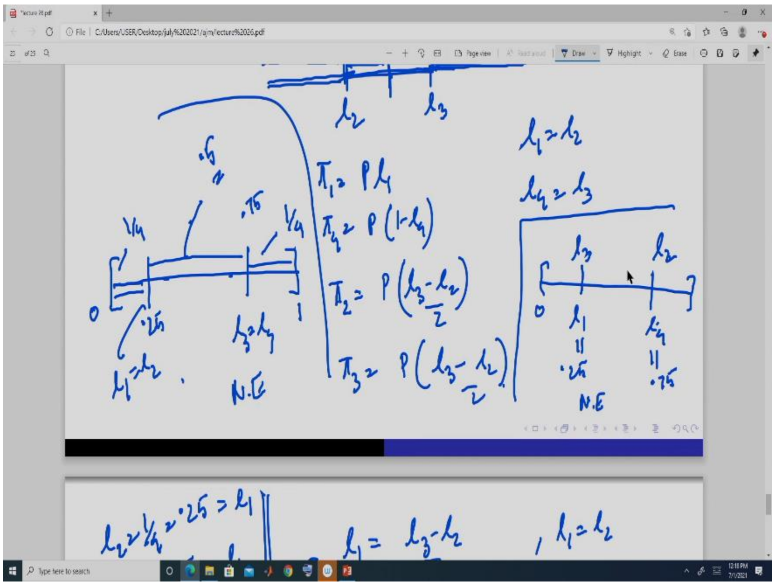
... firms 1 and 3

---

4 firms, l_1, l_2, l_3, l_4

l_1, firm 1 is left-peripheral

l_4, firm 4 is right-peripheral.



So, from this argument what do we get? We get that l1 must be equal to this-$\frac{l_3-l_2}{2}$, when l1 is equal to l2 and further, we get 1 minus l4 is equal to l3 minus l2 when l4 is equal to l2, right. So, from here, what do we get? So, l1 is equal to l2 from this. So, l2 should be equal to this from this here. So, this gives me this- $3l_2 = l_3$ and from here l3 and l4 are same. We get this so, from this what do we get? 2 minus 2 plus l2 should be equal to 4-2 $+ l_2 = 4l$. We have this, so it will go bring here should be 3 l this and from this, we have already this here. So, we get l2 in 3.

So, place here. So, it will be 9 l2. So, this is equal to 2 by 8 l2. So, l2 is 1 by 4. So, what do we get? We get here, we get that l**2** is 1 by 4 or what is 0.25. So, immediately l3 is 0.75 it is three times into this and this is equal to l1 and this is equal to l4. So, this is one pure strategy Nash equilibrium. You can take a different also combination of this. So, this is l2, l3 l1 is same and l2 and l4 is same. So, there you can get different combinations. So, this is one pure strategy- $l_2 = \frac{1}{2} = 0.25 = l_1$. So here, if you look at this diagram, it will be like this.

This distance is so this point is 0.25, so this distance is one-fourth and this is the location of firm 1 and firm 2, this is the location of firm 3 and this distance is again, fourth and this distance is and this distance is, you can say 0.5 distance so they are getting half-half, okay. So, this is a one pure strategy Nash equilibrium. So, in this case, when we have four firms, we get that there exists a pure strategy Nash equilibrium, but when we have three firms, we see that we do not have any pure strategy.

And again, when we have two firms, we see that there is a pure strategy and there is no product differentiation and when three, we do not know what is going to happen. When we have four, we get that the product differentiation is of this nature. Two firms who are going to be similar and another two firms are going to be similar but these two firms are different from these two firms, okay or another combination can be this that is l1, l3, this is equal to 0.25 and l2, l4 is equal to 0.75. This can be one another pure strategy Nash equilibrium. So, this is again a Nash equilibrium.

This is also this is also a Nash equilibrium. So, in this case we get this firm 1 and firm 3 they are similar, but and firm 2 and firm 4 are similar, but, these two are different. Here firm 1 and firm 2 are similar, firm 3 and firm 4 are similar, but this group is different from this group. So, this is a kind of product differentiation. So, we get that there are some firms which are similar but they are going to be different from the other set of the firms. So, we can go on trying this if we have suppose, five firms if we have six firms if we have seven firms and then we will see get a pattern like this that the peripheral firms are always going to lie adjacent to their nearest neighbour, okay.

And this we will not go further more than four firms because the calculations are slightly complicated, even five-six firms It is easy, but if you go beyond that, then you will have to solve many equations. So, you can try those on your own. And that is it.

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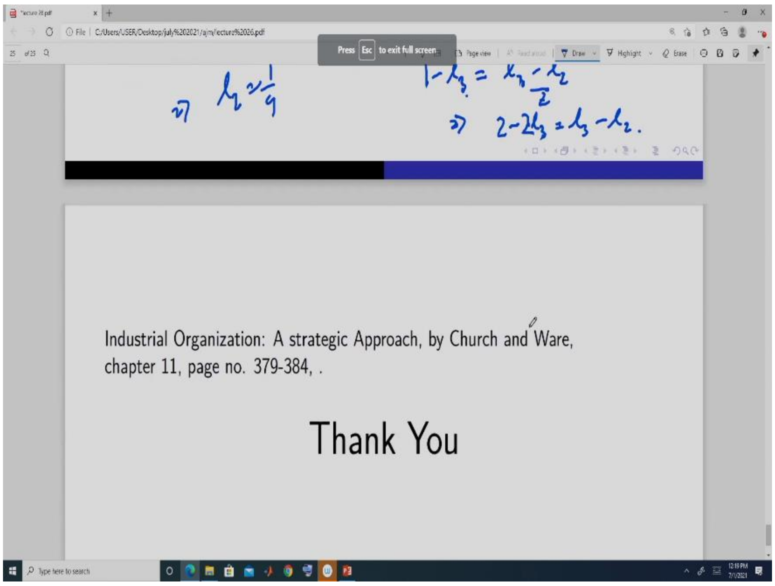

So, if you want to read from where this portion I have done, it is from this book Industrial Organization, a strategic approach by Church and Ware. These are the specific page numbers 379 to 384, chapter 11. Thank you.

Introduction to Market Structures
Department of Humanities and Social Sciences
Indian Institute of Technology Guwahati
Professor Amarjyoti Mahanta
Lecture 37
Tutorial on Stackelberg Price Competition

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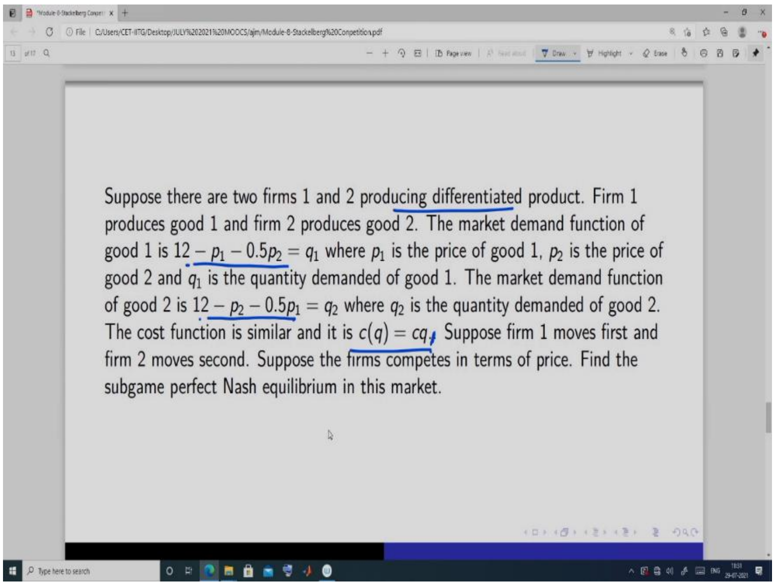

Suppose the firm 1 and firm 2 produces differentiated good and the demand function of good 1 is this- $12 - p_1 - 0.5p_2 = q_1$ which is produced by firm 1, demand function of good 2 is this-$12 - p_2 - 0.5p_1 = q_2$ which is produced by firm 2 and the cost functions are same. It is like this- $c(q) = cq$ suppose firm 1moves first and firm 2 move second and find the subgame perfect Nash.

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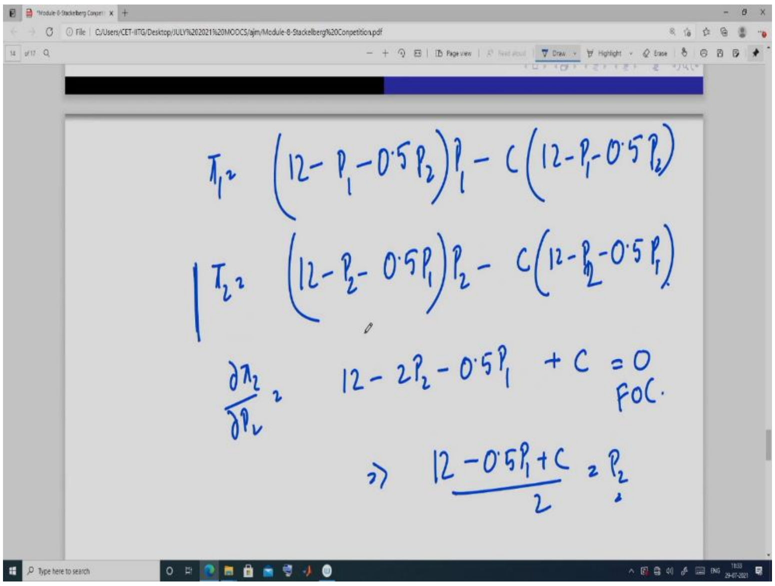

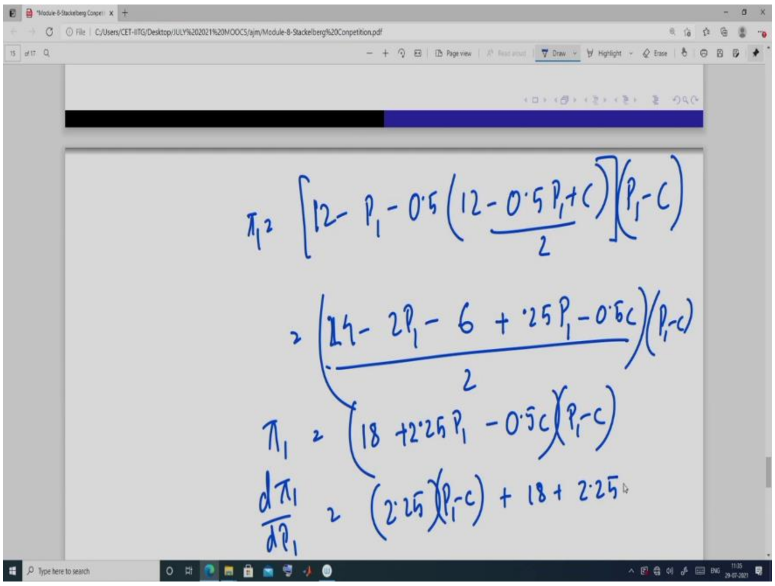

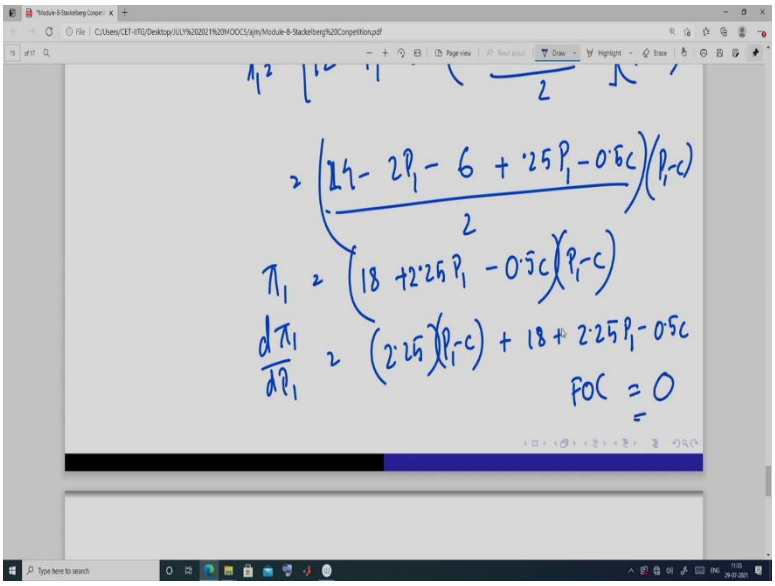

So, this is a standard problem and you can solve this. Profit of firm 1 is 12 P1, this is 0.5 P2, P1 C **is** this-$\pi_1 = (12 - P_1 - 0.5P_2)P_1 - C(12 - P_1 - 0.5P_2)$ and profit of firm 2, so, there it is a negative sign. So, it means the products are if the price increases, then demand goes down or if P2 increases demand for good 1 goes down. So, it means goods are complimentary in nature, is this- $\pi_2 = (12 - P_2 - 0.5P_1)P_2 - C(12 - P_2 - 0.5P_1)$. So, we will use backward index and first solve this, respect to P2 and we will get equal to zero first order condition.

So, the reaction function of firm 2 is it is this- $\frac{12-0.5P_1+C}{2} = P_2$. Now, you plug in this in the profit function of firm 1, take this. So, this is like 24 minus 2P1it is 6, it is this- $\pi_1 = (18 + 2.25P_1 - 0.5c)(P_1 - c)$ and you will differentiate this with respect to P1, you will get what. So, if you take the derivative of this part first, we will get this and first order condition will give that this is equal to zero- $(2.25)(P_1 - c) + 18 + 2.25P_1 - 0.5c = 0$.

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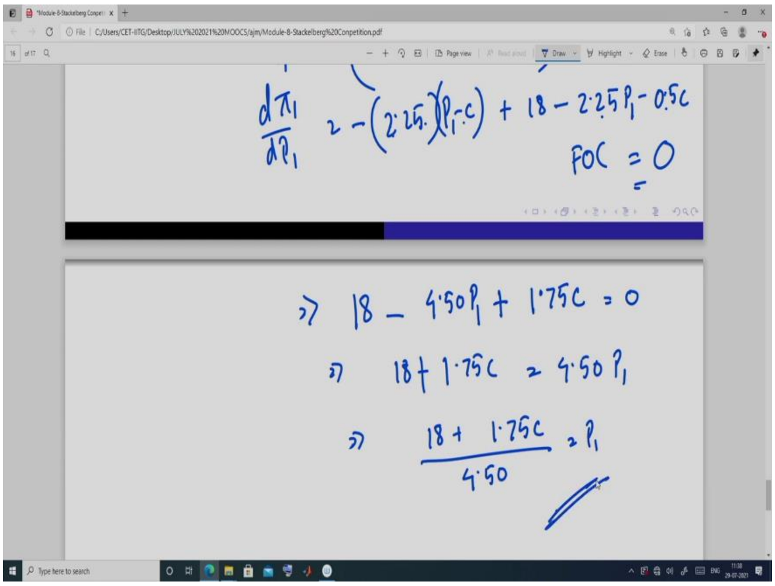

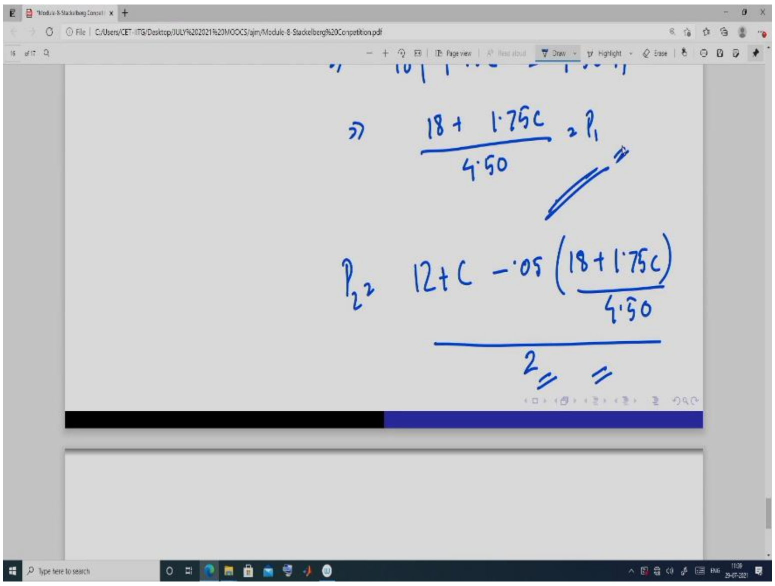

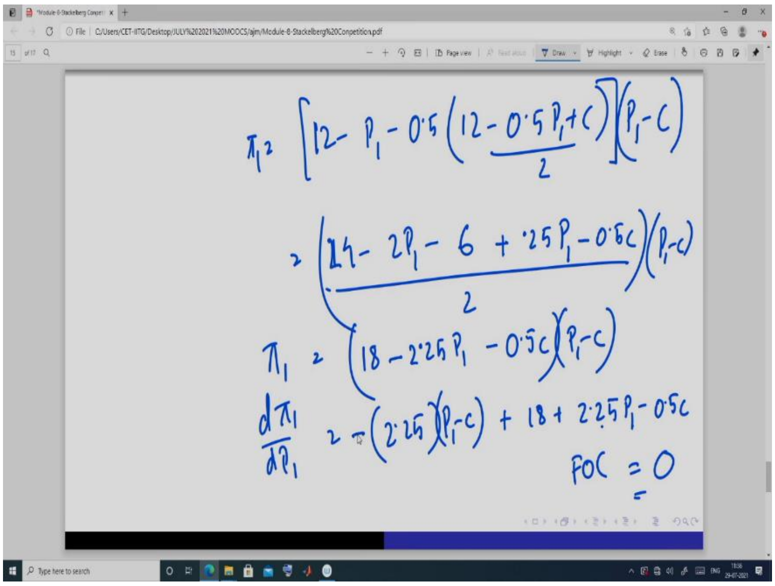

So, we get this part is equal to 18 plus it is to 4.5 P1. Cost is 2.75 to be negative. So, it will be negative and positive. So, this term is actually negative, okay. So, this is equal to zero. So, 18 minus I think there is some problem here. This term is going to be plus and this is, so it is plus 2.7. So, it will be this part is 2.5 plus 2.25 and there is minus 0.5.

So, it will be 1.75 C with this and this will be so, this is the price and you plug in this price in this reaction function and you will get P2 is equal to 12 plus C minus 0.5 this, 18 plus 1.75 C- $P_2 = \frac{12+c-0.05\left(\frac{18+1.75c}{4.50}\right)}{2}$ okay. So, we get this in the stage 2 and this in the stage 1. So, these are the subgame perfect Nash equilibrium prices in this price competition, okay.

Introduction to Market Structures
Professor Amarjyoti Mahanta
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Module 11: Product Differentiation and Entry Deterrence
Lecture 38
Sequential Move Hotelling Model

Hello, welcome to my course, Introduction to Market Structures. So, we were doing product differentiation.

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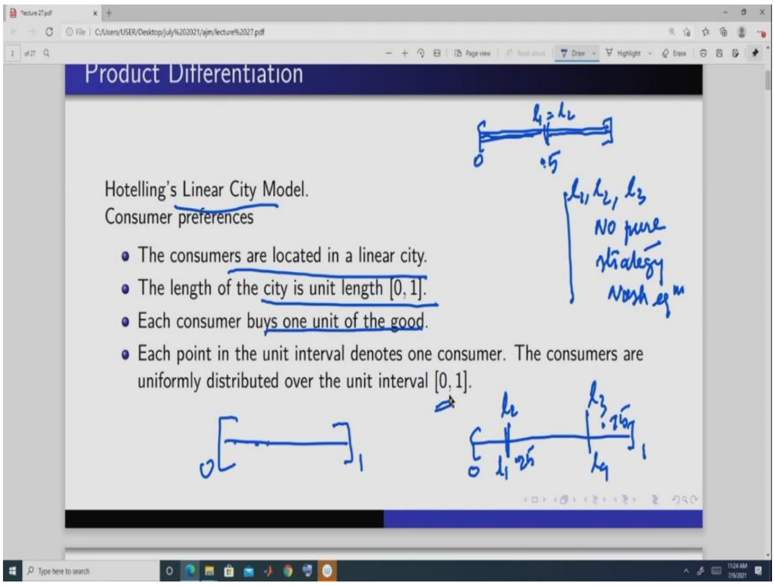

And we have done Hotelling's Linear City Model. And in the Hotelling's linear city model in the last class, what we have done? We have done that the firms enter, or firms chooses the location simultaneously. So, we have done 3 cases. When we have 2 firms, what we have got? When we have 2 firms, we have got that the firms choose the same location and that is in the unit interval. They will choose 0.5, 0.5. So, l1 is 0.5 and also l2 is 0.5.

So, firm 1 will share this, will get this market share and firm 2 will get this market share. And in case 3, when we have 3 firms, we have got that l1, l2, l3, we have got that there is no pure strategy Nash equilibrium, when we have 3 firms. And but when we have 4 firms, we have shown that when we have 4 firms, we get that firm 1 and firm 2 are going to choose the same location and that is 0.25 and firm 3 and firm 4, they are going to choose the same location and that is 0.75, in this 0 1-unit interval, okay.

So, this is one of the Nash equilibrium but there can be different combinations and you can get it. Different Nash equilibrium. So, this much we have done in the Hotelling model. Today we are going to do Hotelling model, but the firms are going to enter sequentially. So, we keep the specification same. So, we say that the consumers are located in a linear city and the length of the city is 0 1. Each consumer buys 1 unit of the good and each point in the interval, so this interval 0 1 interval, each point denotes 1 consumer. So, it means that the consumers are uniformly distributed within these 0 1 interval, okay. So, this is the specification, is same as the last one.

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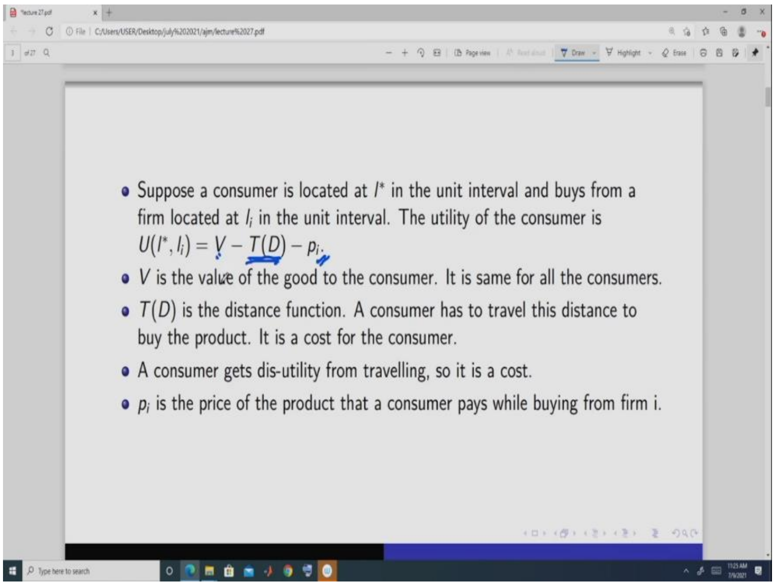

And we also keep the distance function same so the utility function, that the utility that a consumer gets from consuming this good is V. And this is the dis-utility from travelling a distance if that consumer has to travel some distance to buy from a firm. And this is the price. And for simplicity, we will see that the price is fixed and that is p. And V is the value of good to the consumer from the consumption of that good.

And T D is the distance function. And since travelling is a cost, so they get, it acts as a dis-utility for the consumer. And p is the price, so since it is, you are paying, so that is why it is a negative thing. So, you are, by consumption of that good, you get a value of V and then rest you are taking as a dis-utility, because you are paying, so it is going out from you.

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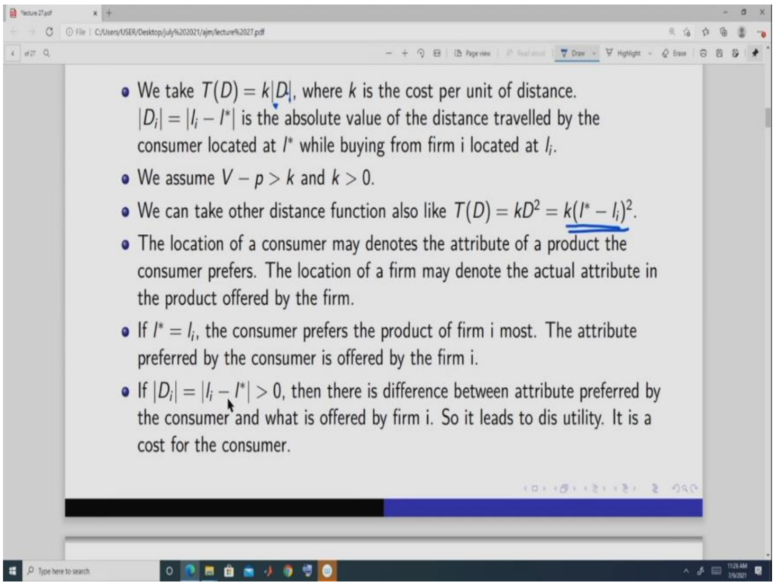

And the distance function is this-$T(D) = k|D|$, where k is the cost per unit of distance, and this is the modulus, absolute value, of the length of the, length between the location of the firm and the location of the consumer, okay. We can take any other form of distance function and it can be this quadratic form. But we will stick to this, as we have done in the last class.

So, these locations you can understand, locations in terms of the attributes, also different degree of attributes or different nature of attributes. And if a person, a consumer is located at the same place at the location of the firm, that means, you can say that the attributes are same. Whatever attribute a consumer wants, that attributes is being offered by the firm.

But if there is a difference, then it means the attribute in a product, that a consumer wants, that is not same as the attribute offered by the firm, and which is closest to the attributes of needs or attribute demanded by a consumer, okay. So, there is some difference, if this is, this distance is positive, okay. So, this is, this is same as what we have done earlier.

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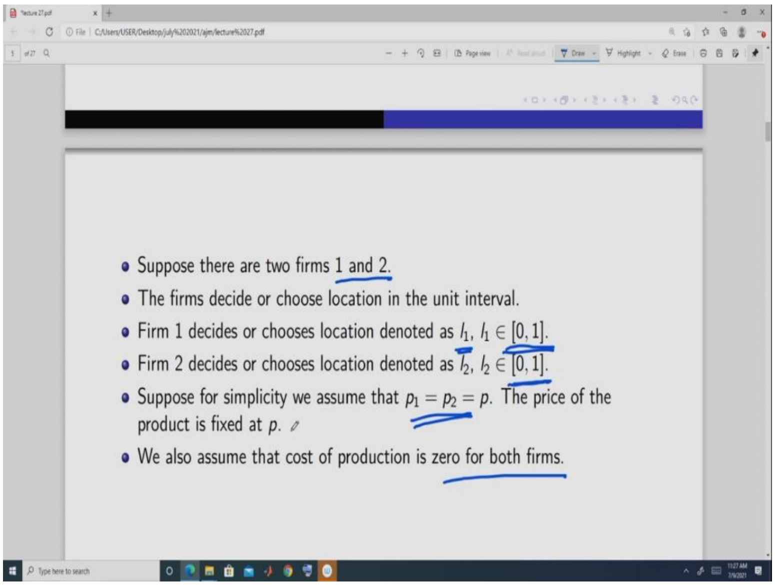

And for, at the beginning, we take 2 firms, firm 1 and firm 2 and firms choose the location in the unit interval. So, firm 1 choose l1 and l1 lies in this 0- and 1-unit interval. Firm 2 decides location and we denote it as l2 and it lies between 0 and 1. And for simplicity, we assume that there is price is fixed and it is same, okay. And we assumed also that there is 0 cost of production for both the firms. So, there is no cost of production, okay. So, this we have till now, it is same.

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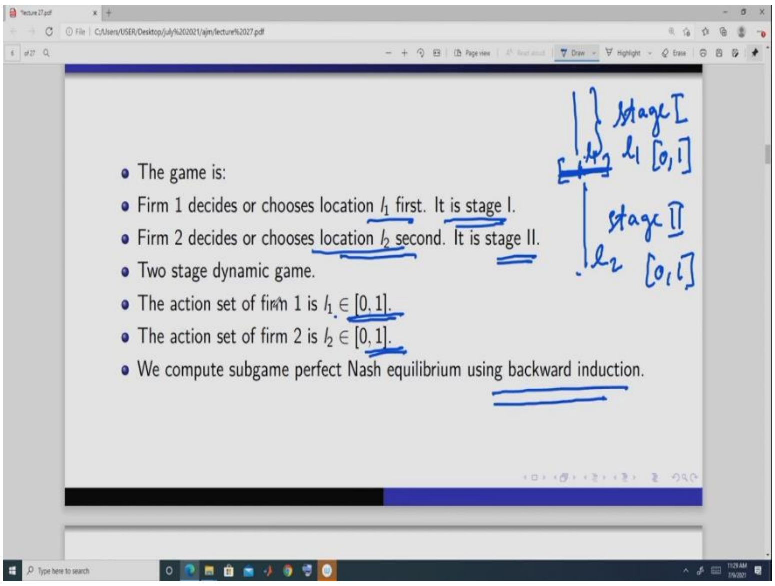

What is new? It is this. The game is new. What happens in this game? Firm 1 decides location l1 in first, so this is stage 1. Then after observing the location of firm 1, firm 2 decides its location. This as l2 and this is stage 2. So, firm 2 locates after observing the location of firm 1, okay. So, this is a dynamic game or you can say this is a 2 stage dynamic game and action set because here action set and the since it is a dynamic game, so action set and the strategy sets are not same.

So, the action set of firm 1 is to choose a location in between this. And the action set of firm 2 is to choose a location between. So, if you look at this game then in stage 1, l1 is chosen from 0 and 1. So, this is stage 1. Then in stage 2, so here, you can say, it chooses a location from here, suppose somewhere l1 is here, then after observing this, firm 2 chooses and that is stage 2, location l2, lying between 0 and 1, okay. So, this is observed, l1 is observed, while l2 is being chosen, okay. So, we will compute the subgame perfect Nash equilibrium and we will use backward induction, okay.

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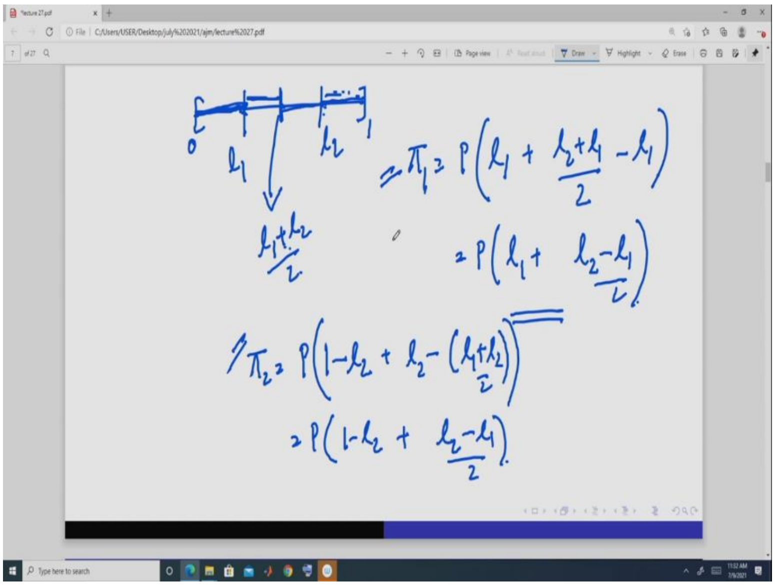

Now, how to payoffs? Let us define the payoff function. So, suppose this is the unit interval and suppose this is the location of firm 1 in stage 1. And this is the location of firm 2 in stage 2. So, the payoff of firm 1, we know, it will get this much 0 l1, because all these consumers nearest firm is firm 1. So, profit of firm 1 p l1. And we will get a consumer, who is going to be indifferent between buying from firm 1 and buying from firm 2. And this point we have derived it in the last class and this point is l1 plus l2 divided by 2.

So, this much distance again firm 2 is going to get. So, what is this distance? So, this distance is, so this point is this, l2 plus l1 divided by 2. And this distance, this point is l1 minus l1. So, if this is the l1, so this holds also that this-this, because it is starting from 0 so this distance is l1. So, we get this. So, this is, we get this. So, this is the payoff of firm 1-$\pi_1 = P(l_1 + \frac{l_2 - l_1}{2})$.

And payoff of firm 2? Price is fixed, this. And this much, it is going to get, because all for these, all these consumers, firm 2 is the nearest one, because firm 1 is lying here, left of firm 2. So, this distance is 1 minus l2, this, and this distance we know, this location is, this point is this, so this distance is l2 minus, we get this. So, this is, this much it is going to get. And it is this- $\pi_2 = P(1 - l_2 + \frac{l_2 - l_1}{2})$. So, these two are the payoff functions. And we will have to find the location such that these payoffs are maximum. And mainly the subgame perfect Nash equilibrium also gives that.

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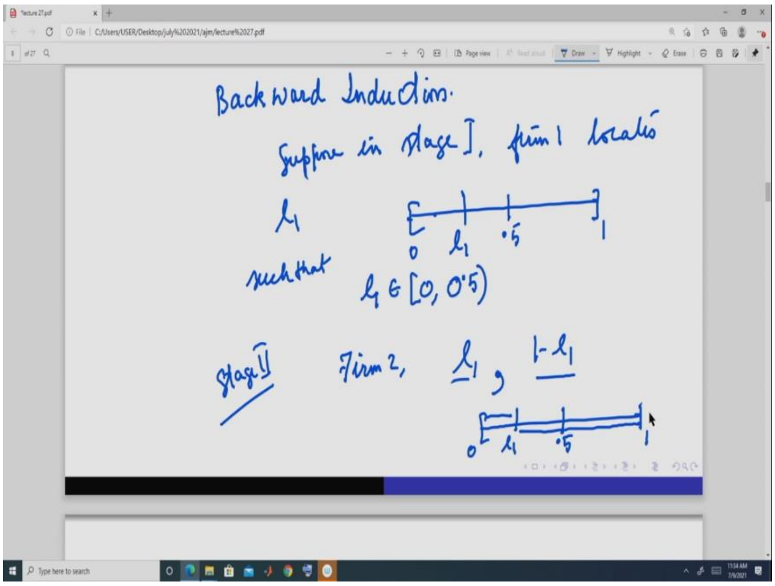

So, we will solve this. How we are going to solve it? We are going to solve using backward induction, okay. So, here, so we assume, suppose, in stage 1, firm 1 locates l1 as here. This is 0.5. This is 0. This is 1. And suppose l1 is. So, l1 such that l1 is this. So, this point is not included anywhere between 0 and 0.5, but 0.5 not included, okay anywhere. It can be this point, this point, this point, anywhere, okay.

Now, here if this is the case, so now, let us move to stage 2. So, this we assume suppose, this is in the stage 1. So, stage 2, what we will do? So, stage 2, firm 2 compares l1 and compares 1 minus l1, these 2 distances, it will compare first. Now, when it compares these 2 distances that is, this is l1, so it compares this distance and it compares this distance, okay. So, in this case, this distance is greater.

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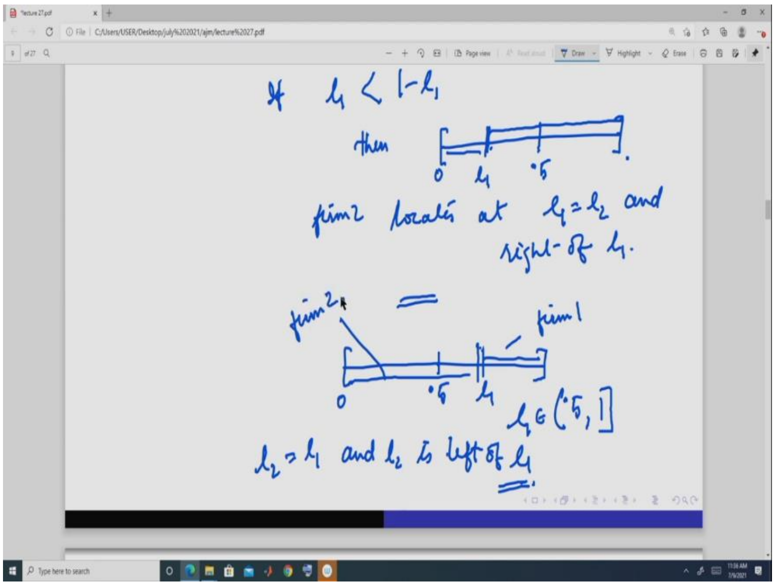

So, if l1 is less than 1 minus l1, then firm 2 will locate here, then firm 2 locates at, l1 is equal to l2 and right of l1, here. So, firm 2 gets this whole market and firm 1 gets this whole market, right? So, here this is the outcome in stage 2. So, now, so we know, how given a location of firm 1 that is l1, how firm 2 is going to behave now, in this case.

Now, again suppose, this is l1. So, this l1 belongs to this, so 0.5, it is not included. Then in this case also the argument is same as before, okay. So, here also, we will find that firm 2 is going to locate l2 is equal to l1 and l2 is left of l1. It is here. So, firm 1 will get this. This is firm 1 and this much is for firm 2, right?

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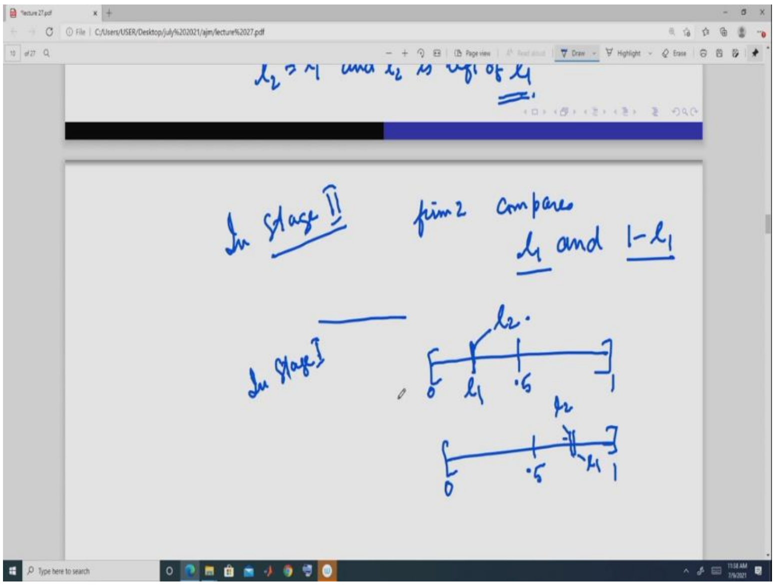

Now, so this is the way we know, how firm 2 is going to behave, given any location of firm 1, in stage 2. So, in stage 2, we have got that it compares, firm 2 compares l1 and 1 minus l1. Based on this it chooses, whether it will lie in the leftward of l1 or it is in the rightward of l1, okay. So, in stage 1, firm 1 knows this that, if so firm 1 knows that if it locates here firm 2 is going to locate here. If firm 1 here, then firm 2 in stage 2 is going to locate here. Again, firm 1 if it locates here, it knows firm 2 is going to locate here. So, it gets such that it gets this.

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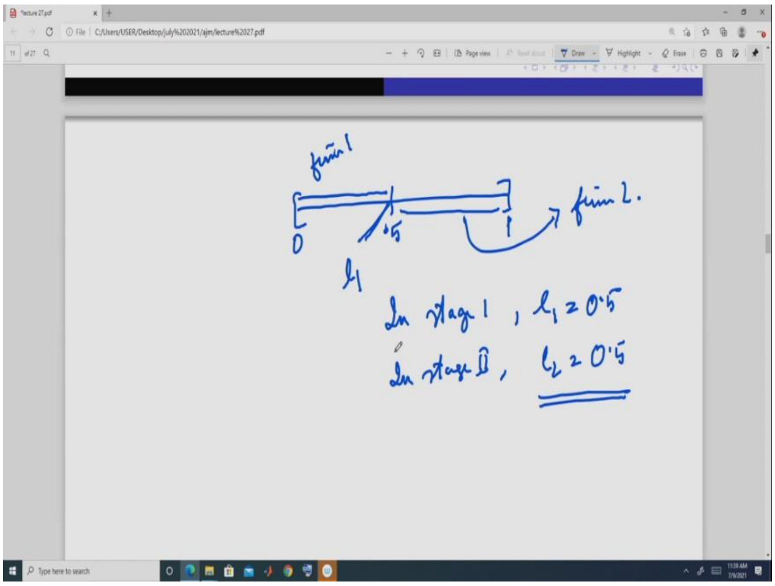

So, that is why in stage 1 the optimal location of firm 1 is to locate at 0.5. So, firm 1 gets this. And in stage 2, so in stage 1, l1 is equal to this- 0.5, and in stage 2, l2 is equal to 0.5. So, this much is for firm 1 and this much is for firm 2 or you can switch then it is same. So, we get that if we have 2 firms then the location is this.

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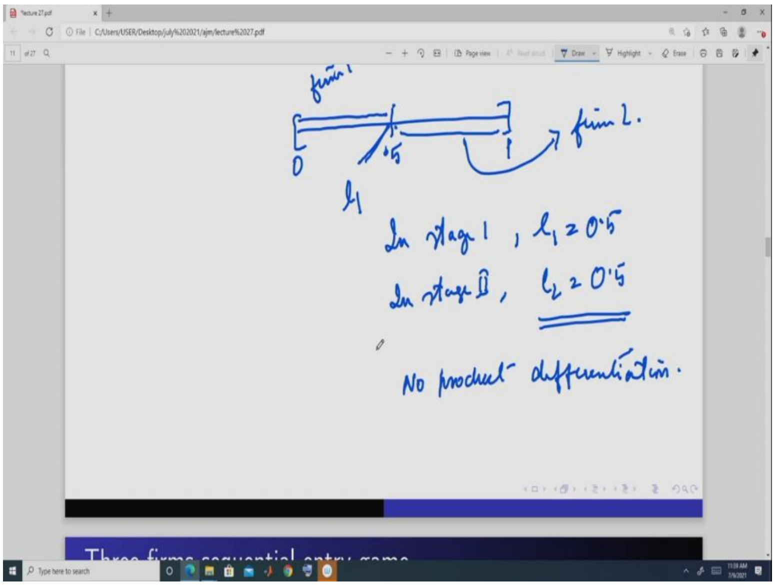

So, what is the implication of this? So, this means that there is no product differentiation, okay. Both of them are going to have same location. So, they are not going to differentiate. Or you can say that if you think it, in terms of attributes, so this much amount of attribute is going. So, it is going to be same for both the firms, okay.

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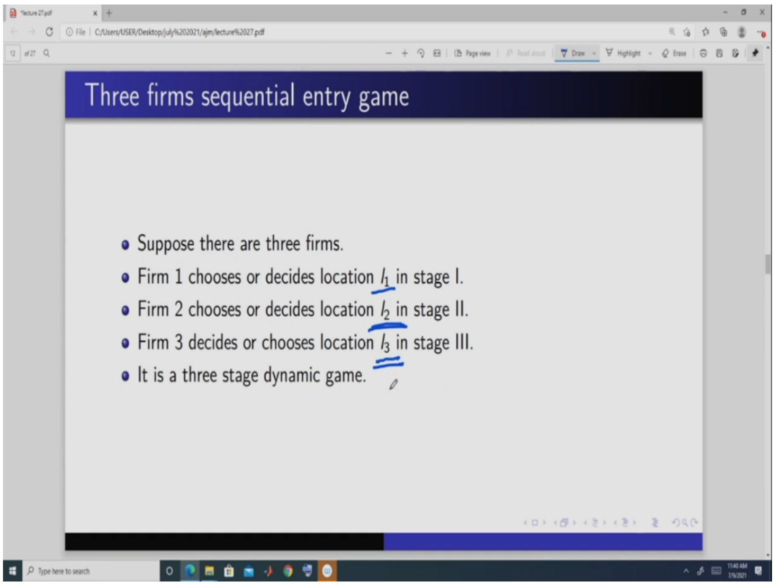

Now, let us move to 3 firms. So, we keep everything same, except we now increase 1 more firm. So, in stage 1, l1 is decided or it is chosen. In stage 2, l2 the location of firm 2 is chosen, and in stage 3 location of firm 3 is chosen. So, it is a 3-stage game now. Everything is same as earlier, okay.

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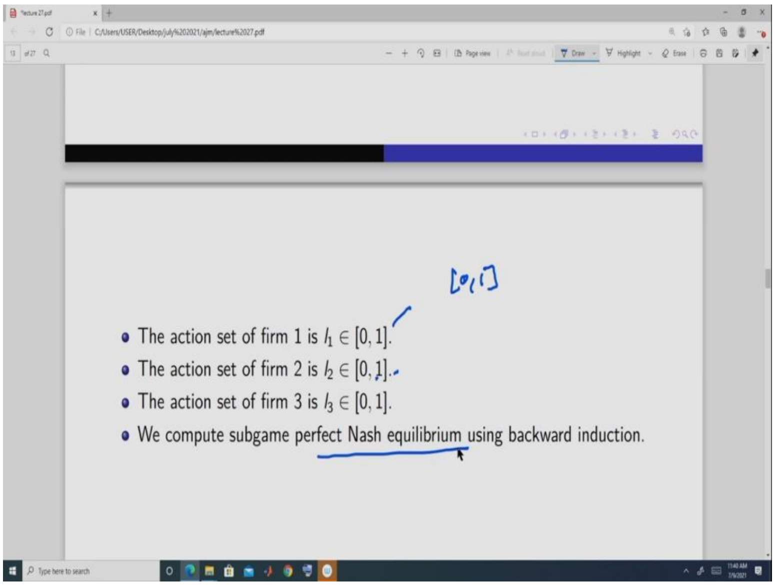

So, action set of firm 1 is l1, which lies in this, 0 and 1. Action set of firm 2 is again l2 and it lies in this, range 0 to 1-unit interval. And action set of firm 3 is again l3 and it lies between 0 and 1. And since it is a 3 stage game, so we use subgame perfect Nash equilibrium. And so we compute subgame perfect Nash equilibrium. And we use backward induction to solve it, okay. So, now let us first define the payoffs, okay.

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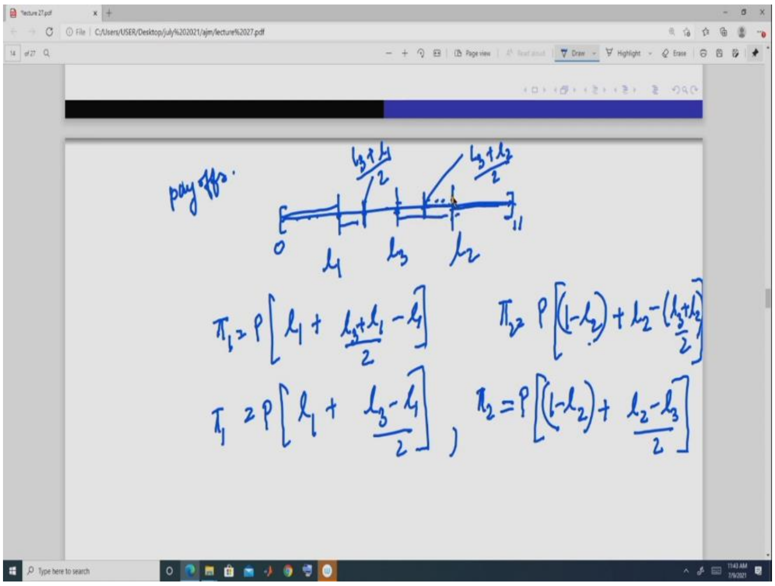

Suppose this is the unit interval, okay. This is the location of firm 1, this is the location of firm 2 suppose, and this is the location of firm 3, okay. Now, here, payoff of firm 1, price is fixed, it is p. Since it is here, so all these consumers, firm 1 is the nearest to them. So, they are going to buy from firm 1. So, l1 is there.

Then here, in between this, so there is a point like this, and this point is, l3 plus l1 divided by 2. So, this person is indifferent from, between, buying from this firm and this firm. So, all these persons lying here are going to buy from firm 1 because firm 1 is nearest for them, nearest to them so it is going to be. So, the payoff function for firm 1 is this-P $\left[l_1 + \frac{l_3 - l_1}{2}\right]$. This much and half of this distance and this is this. That is this much.

Now, payoff of firm 2. Payoff of firm 2 is, again price is same, so p and this much, all these consumers are going to buy from firm 2 because they are nearest to the firm 2. So, 1 minus l2, this distance, this, this is 1, so l2 is this point, so this distance is, all the consumers lying here, are going to buy from firm 2. So, this is, plus there, we are going to get a point like this, who is indifferent between or this distance is same, so middle of this, middle of these 2 points, this point.

So, this point is again given by l3 plus l2 divided by 2. So, this many people are going to buy from firm 2. So, it is going to be l2 minus l3, this- $\pi_2 = P\left[\,(1 - l_2) + l_2 - \frac{l_3 + l_2}{2}\right]$. So, the payoff of firm 2 is going to be 1 minus l2, this much, plus l2 minus l3 divided by 2- P $\left[\,(1 - l_2) + \frac{l_2 - l_3}{2}\right]$. This distance. This distance is half of this distance. This distance is half of this distance. So, it is this.

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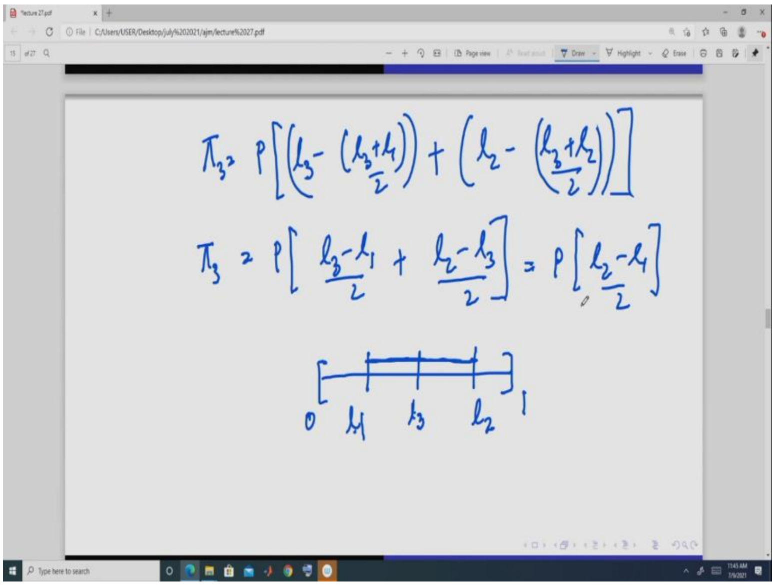

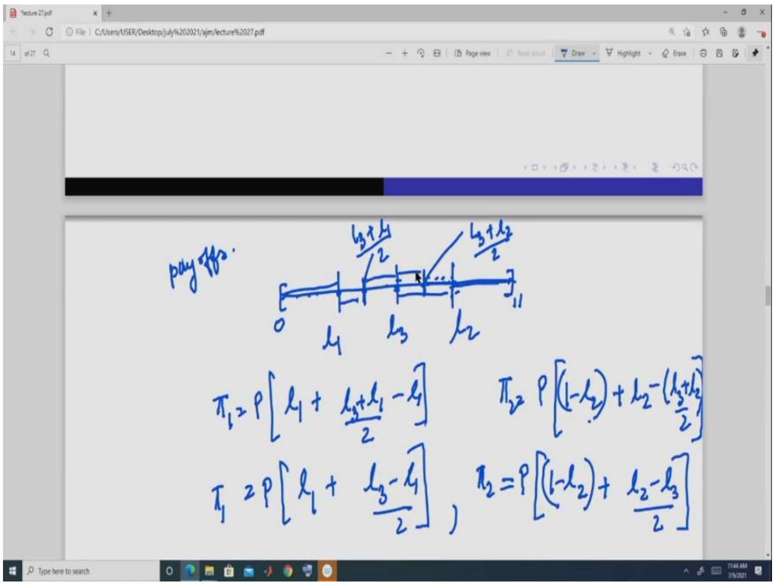

Now, the payoff of firm 3, price is same, this. Now, firm 3 gets this much and this much. So, this much is what? This much is l3 minus, l3 plus, this, plus l2 minus, l3 plus l2. So, this is the payoff of firm 3- P $\left[\frac{l_3 - l_1}{2} + \frac{l_2 - l_3}{2}\right]$. So, actually this is, you can see, half of this distance, this distance. Or here if you look at this position, this is 0, this is 1, location of firm 1, location of firm 2 and suppose, firm 3 here. So, firm 3's market share is half of this distance, okay l2 minus l1 divided by 2, this, ok. Remember this, we will use it. Now, let us find the solution. So, these are the payoff functions and firms are going to choose the location so that those are maximized. So, we will use the backward induction.

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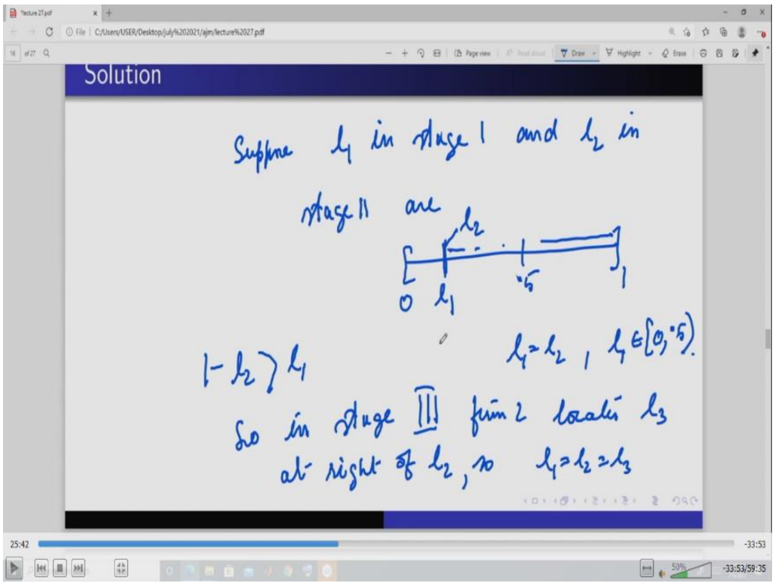

So, suppose, l1 in stage 1 and l2 in stage 2, are of this nature. This is 0.5. This is l1 and this is l2, okay of this nature. So, l1 is equal to l2 and l1 anywhere like this. Then if this is the case, then it is said, so this distance is so, so here, see 1 minus l2 is greater than l1. So, in stage 3, firm 3 locates l3 at right of l2 and so l1 is equal to l2 is equal to l3. So, firm 2 is now sandwiched. So, it will get 0 consumers. So, that is why firm 2, if given firm 1's location here, firm 2 will never look at the same position as firm 1's.

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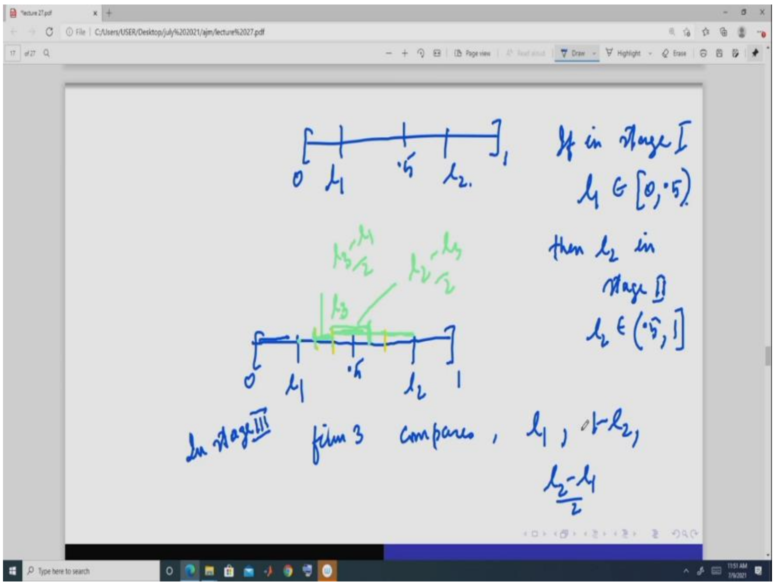

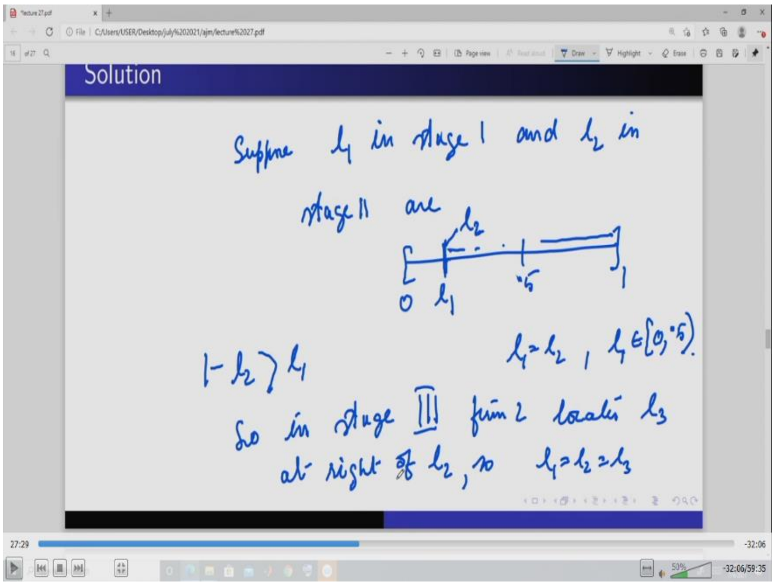

It will locate in some way like this. So, if l1 is here, somewhere here, then firm 2 will lie somewhere here, here in this region, here because otherwise firm 2, firm 3 in stage 3, we know, it is going to locate here, so l3 is here and it is going to be at right of l2, so it will get the whole market and l2 will not get anything. So, now so this is the case.

Another thing here, again in stage 3, so we get that it will be of this nature. So, if this is l1, this is l2, then this is the case in stage 3. Firm 3 compares, what, l1, this distance, it compares 1 minus l2, this distance. And if it lies anywhere here, here or here, anywhere in position like here, here or here, it is going to get, suppose it lies here, then its market share is, suppose this is l3, its market share is half of this distance, so this and it is going to get half of this distance, this.

So, we know, this half of this is, l3 minus l1 divided by, so this distance is l3 minus l1 divided by 2. This half of this is suppose this, so this distance is l2 minus l3, so, if we take this, so we get again what? So, this firm 2's a is l2 minus l1. So, it is going to compare this-l_1, this-$1 - l_2$and it is going to compare this length-$\frac{l_2 - l_1}{2}$, these three.

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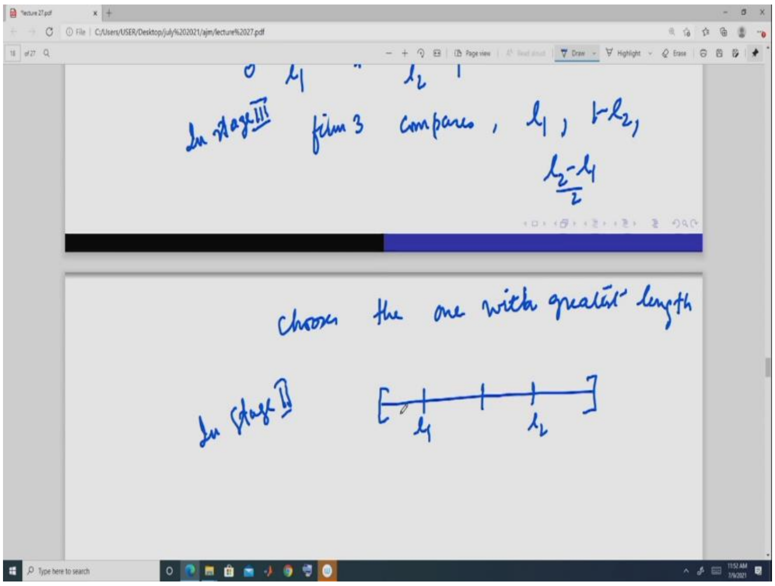

So, firm 3 compares this, it compares these 3 distances. And chooses the one with greatest length, whichever is the as greater, it will choose that, the length of which one is greater. So, in stage 2, firm 2 knows that firm 3 is going to compare these 3 things.

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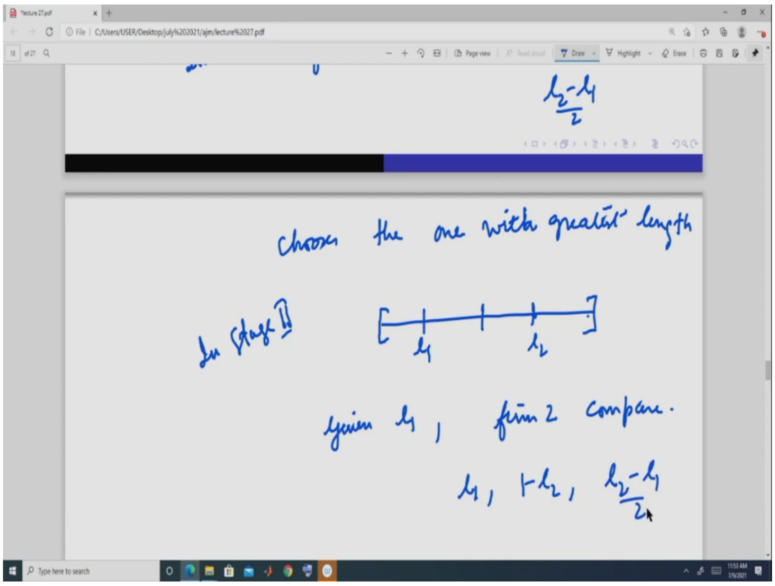

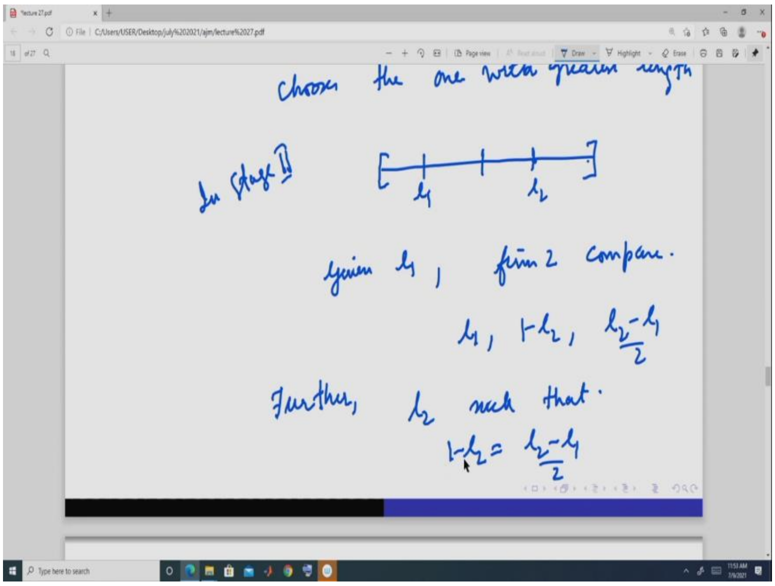

So, that is why given l1, firm 2 compares this. This, it can compare this location. So, firm 3 again can compare l1, 1 minus l2. And further firm 3 firm 2 will choose l2 such that 1 minus l2 is equal to-$\frac{l_2-l_1}{2}$. Because, because this much firm 2 can ensure, because given l1, it knows this.

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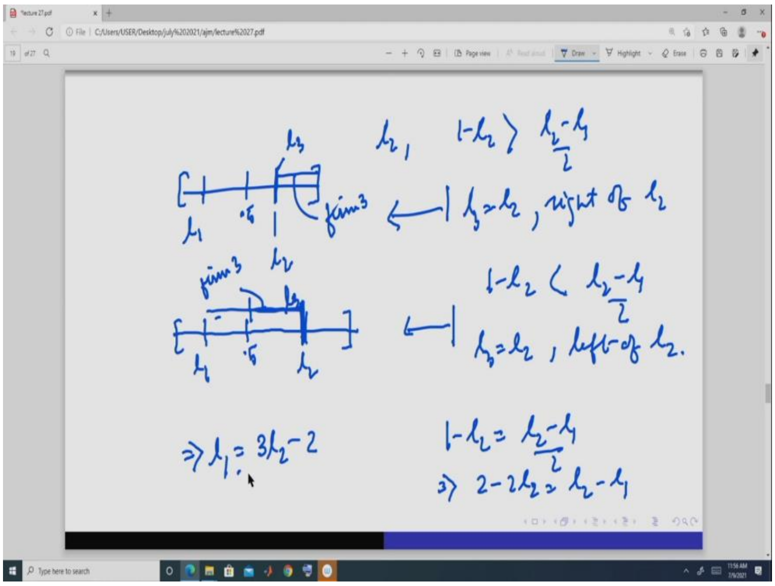

It can ensure this, because, if this distance, if suppose, it has chosen l2 and suppose 1 minus l2 is greater than this-$\frac{l_2-l_1}{2}$. Then firm 2 is going to choose l3 is going to be equal to l2 and right of l2. So, firm 2 will get less this much only. Now, if this is the case, then l3 is again equal to l2 and it is left of l2. So, here in this case it will be like this.

Suppose, this is l1 and suppose this is l2, then this is going to be l3 and this is going to be the firm 3's market share. In this situation, so firm 3 will be here and it will get the half of this distance. This is going to be firm 3, okay. So, that is why, for this reason, firm 2 will always ensure this-$1-l_2=\frac{l_2-l_1}{2}$, okay. Now, while ensuring this, we get the value of l1. Because see, from here we get, so if we get this condition that 1 minus l2 should be equal to this-$\frac{l_2-l_1}{2}$. So, this implies this-$2-2l_2=l_2-l_1$. So, then this implies what? l1 should be equal to 3 l2 minus 2, okay. So, now from here, here again, firm 1, firm 2 can do what?

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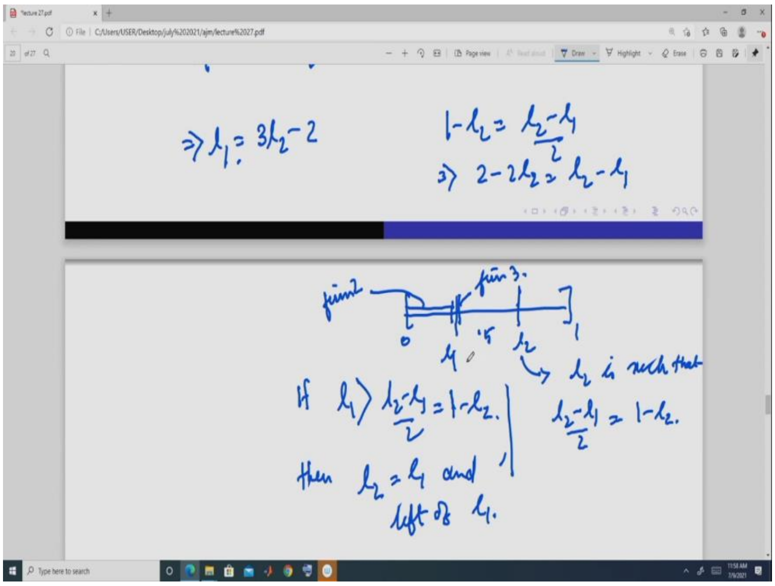

So, it will do this. Now, if suppose, this is 0.5, so suppose this is l1 and if suppose this is l2, so this l2 is such that l2 minus l1 half is equal to 1 minus l2, okay. Now, if l1 is greater than this-$l_1 > \frac{l_2 - l_1}{2} = 1 - l_2$, so firm 2 will locate here. So, if this, then l2, is equal to l1 and left of l1. So, this much is going to get by firm 2 and firm 3 will gets locate here, in stage 3, right? So, firm 2 will do this calculation in stage 2.

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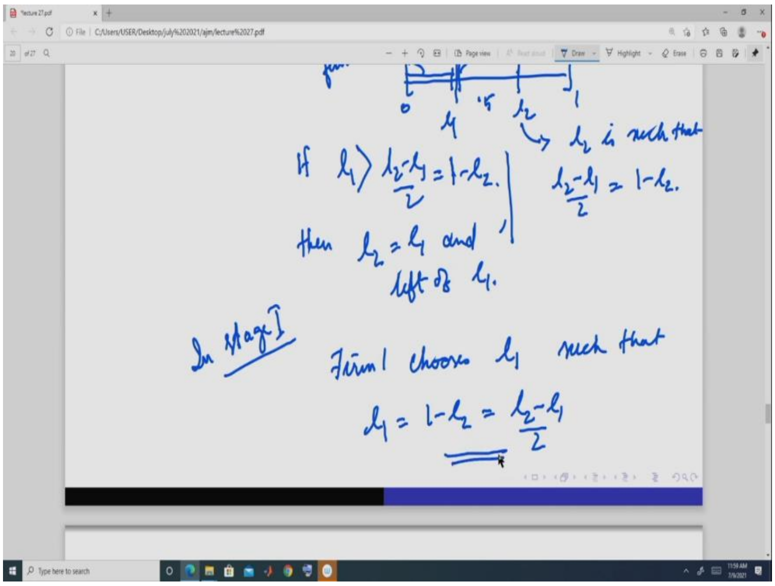

So, that is why, in stage 1, firm 1 will, firm 1 chooses l1 such that l1 is equal to 1 minus l2, which is equal to, given firm 2 is always going to ensure this-$l_1 = 1 - l_2 = \frac{l_2 - l_1}{2}$. So, if we get this, then we solve this.

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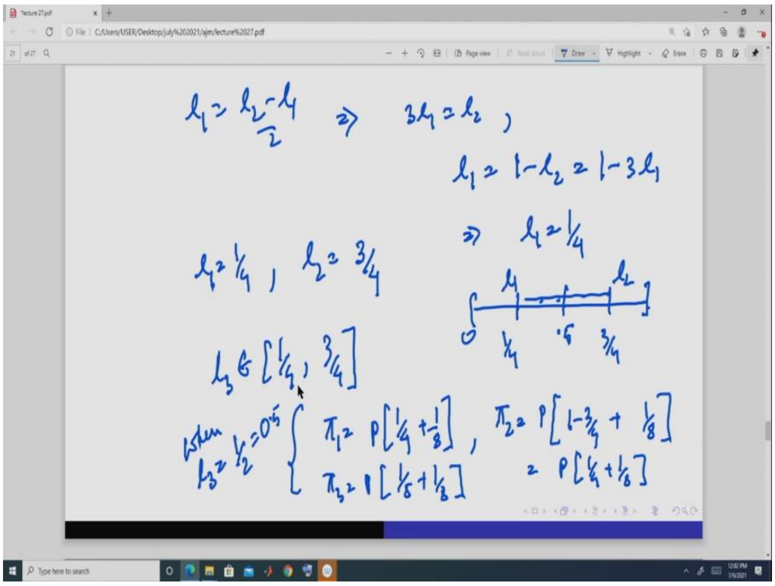

So, from here, we know that, if we solve this, we will get, so 3 l1 is equal to l2, from equating this to l1 is equal to l2 minus. This implies this, i.e.- $l_1 = \frac{l_2 - l_1}{2} => 3l_1 = l_2$. And further, when you plug in that here and l1 is equal to 1 minus l2 which is equal to 1 minus 3 l1 then we get that l1 is equal to 1 by 4. So, l1 is equal to 1 by 4, l2 is equal to 3 by 4 and then l3, see what is happening, this is 1 by 4, l1, this is 3 by 4, l2.

What is this distance? 3 by 4 minus 1 by 4, so this distance is, half and firm 2, wherever it locates, it gets half of this, so that is 1 by 4. So, firm 3 l3 is going to be indifferent lying between this and anywhere here-[1/4, ¾], anywhere. So, the payoff of firm 1, if firm 1, if it is going to be p, this plus suppose it is at the center, suppose it is 0.5, then it will be half minus 1 by 4, so it is again 1 by 8- $\pi_1 = P\left[\frac{1}{4} + \frac{1}{3}\right]$. Payoff of firm 2. It is this-$\pi_2 = P\left[1 - \frac{3}{4} + \frac{1}{8}\right]$. 1 minus 3 by 4 plus this, half of this, so this is 1 by 8. So, it is 1 by-$\pi_3 = P\left[\frac{1}{5} + \frac{1}{8}\right]$. And it is 1 by 8 plus 1 by 8. So, this is the case, when l3 is equal to half or it is 0.5. It can choose any point here. So, depending on that we will get different payoffs, okay.

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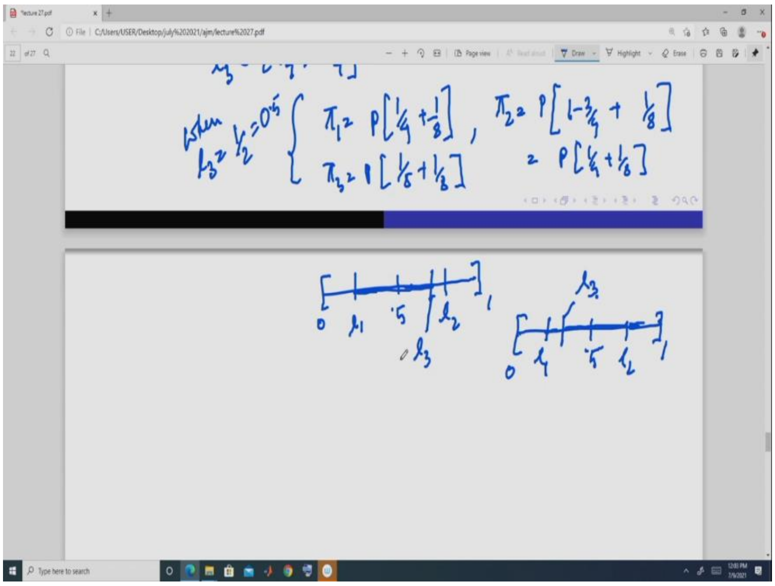

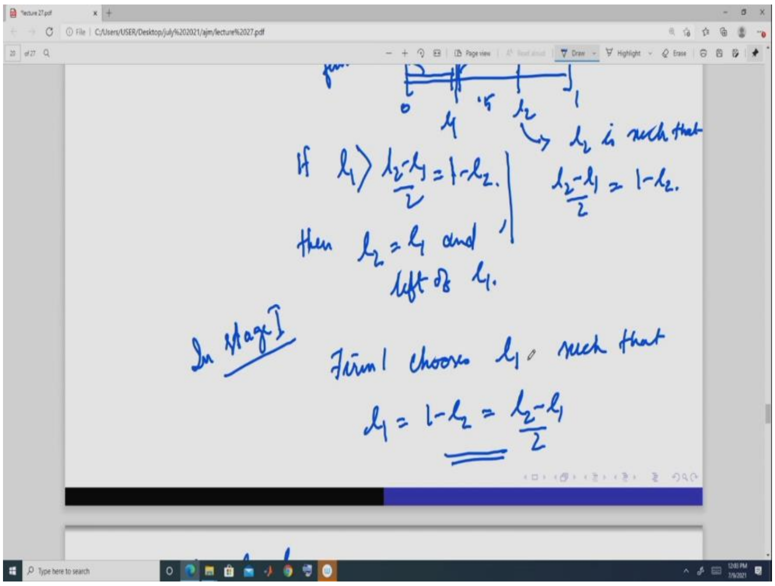

So, if it is here, now if it is here, then we will, firm 1 get, is going to get half of this. And if this is l3, if it is here, if l3 is here, then it is going to get half of this, and half of this is going to get by the firm 2. But firm 3 is always going to get 1 by 4 only. If it lies here also, it will get this much, 1 by 4. If it is here it will also get this, 1 by 4. So, firm 3's payoff is 1 by 4 only. But payoff of firm 1 and firm 2 can be different depending on the exact location of firm 2, firm 3, okay.

So, we get that actually there exist a subgame perfect Nash equilibrium, pure strategy subgame perfect Nash equilibrium or simply subsume perfect Nash equilibrium, when we have 3 firms, in this case. But when we had 3 firms and the firms are choosing the location simultaneously, we found that there is no pure strategy Nash equilibrium, right. So, here we have found and it is not unique, okay.

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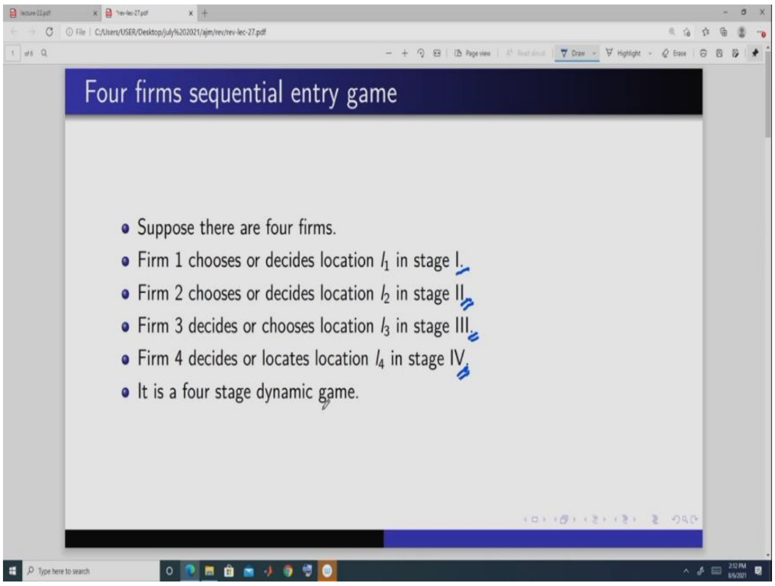

Now, suppose let us take the 4 firms sequential entry game. So, here we have 4 firms and firm 1 decides or locate in stage 1, firm 2 in stage 2, firm 3 in stage 3 and firm 4 in stage 4. So, this is a four-stage dynamic game, okay. And the payoffs are going to be similar as in the case of 2 stage or in the 3 stage that we have done just before this.

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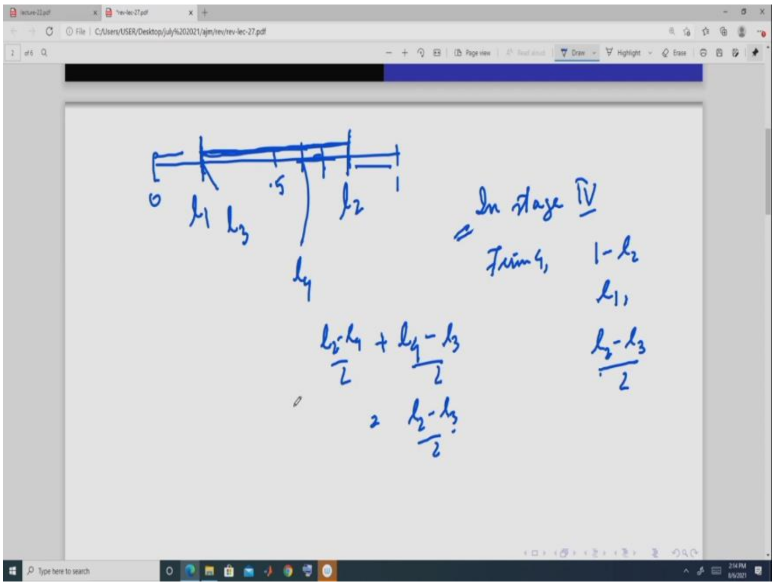

So, we will again use backward induction to solve this. So, suppose, this is 1 0 and suppose this is 0.5, this is suppose l1, this is suppose l2 and suppose this is l3, okay. Then in stage 4 firm 4 compares 1 minus l2 with l1 this distance-$1 - l_2$ this distance-l_1 and half of this distance-$\frac{l_2 - l_3}{2}$, will compare this, which is, because here, if it locates anywhere here, suppose, this is suppose l4, then what is the payoff of firm 4? So, what is the distance that the firm 4 is getting? What is the length it is getting?

So, it is getting half of this, so 2 minus this much, and again half of this, so plus l4 minus l3 by 2. So, this is-$\frac{l_2 - l_4}{2} + \frac{l_4 - l_3}{2} = (l_2 - l_3)/2$. So, this is the same way, whether it locates here, here any point, it will get only half of this whole this region, in this region, if this is l2 and this is l1 and this side it is, right of l1 is, l3. So, it will get this much. So, firm 4 going to compare this three, in stage 4.

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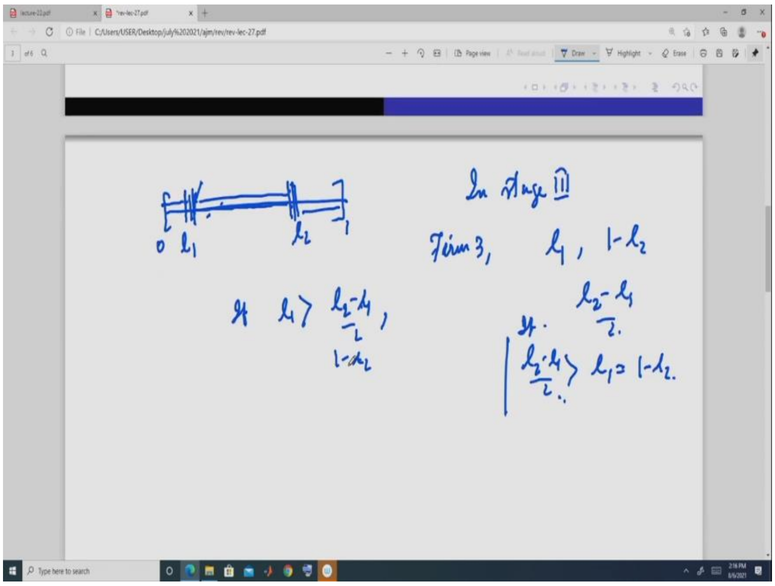

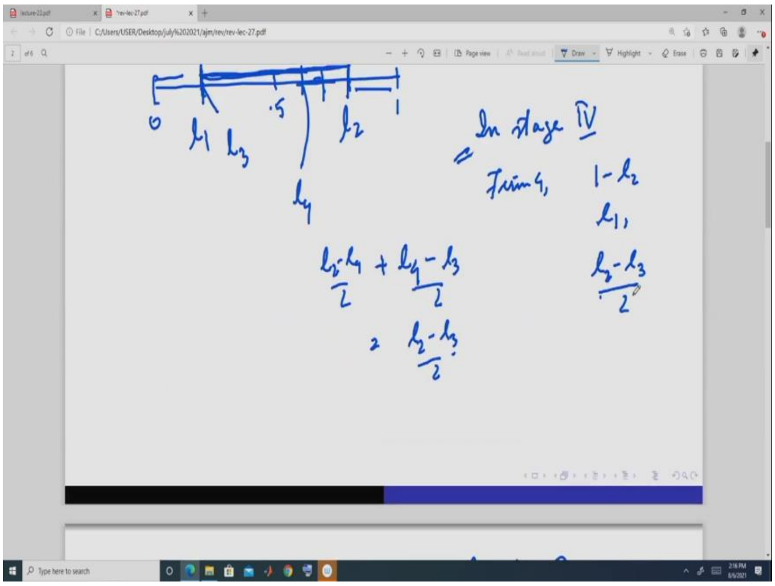

Now, in stage 3, firm 3 is given this situation that there is firm 3 is going to again compare this, going to compare this and is going to compare this, this distance. So, these 3 distance and it knows that if suppose, this is greater. Suppose, this is greater than-$\frac{l_2 - l_1}{2} > l_1 = 1 - l_2$ and this is suppose equal to. So, if this is greater, then if it locates here any point, then firm 4 is also going to locate here and it will get the half of this a or will locate here, it will get the half of this a, anywhere firm 4.

We have already seen, it knows from, so and if so this is the a. If this is the case, then firm 3 will either locate here or it will locate here, if this is the case. So, and if suppose, based on this comparison, and if l1 is suppose greater than l2 minus l1 by 2. And it is also greater than 1 minus. Then it will locate here, but there is a chance that if it is greater than this, then firm 4 also locates here.

So, it will better compare between these two, these two in this situation. And whichever is greater, it will locate, try to, it will get, locate in such a way that it gets maximum of, from these two length, okay. So, from this, we get that the firm 3 is going to make this comparison, okay.

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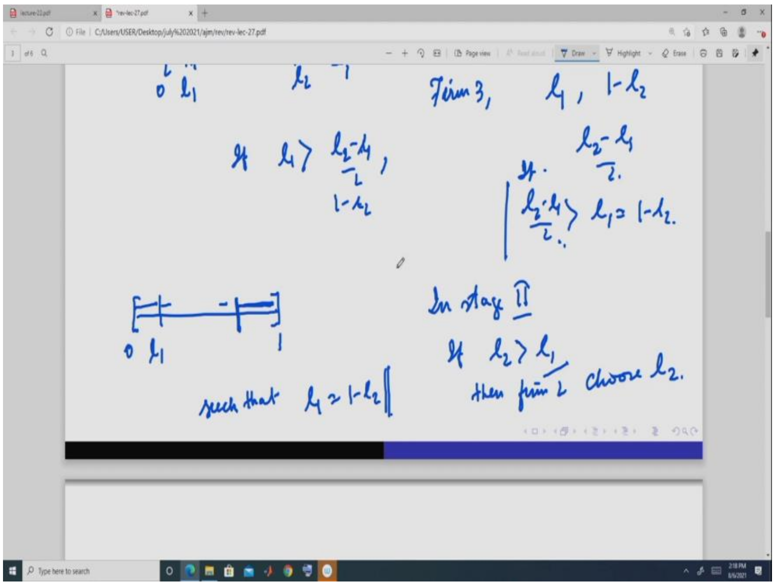

Now, in stage 2, firm 2 will know this that firm 3 is going to make this comparison, firm 4 is going to make this comparison. So, if this is l1 and if suppose l2 is greater than l1, or l2 is locating somewhere, somewhere here. Then it will at least ensure such that this distance is equal to this distance, okay. Or because at least it will, it can ensure this much-$l_2 > l_1$. So, firm

2, if this, then firm 2 will choose l2 such that this happens- $l_1 = 1 - l_2$, okay. Now, so firm we will do this. Because if suppose this is, suppose less than this.

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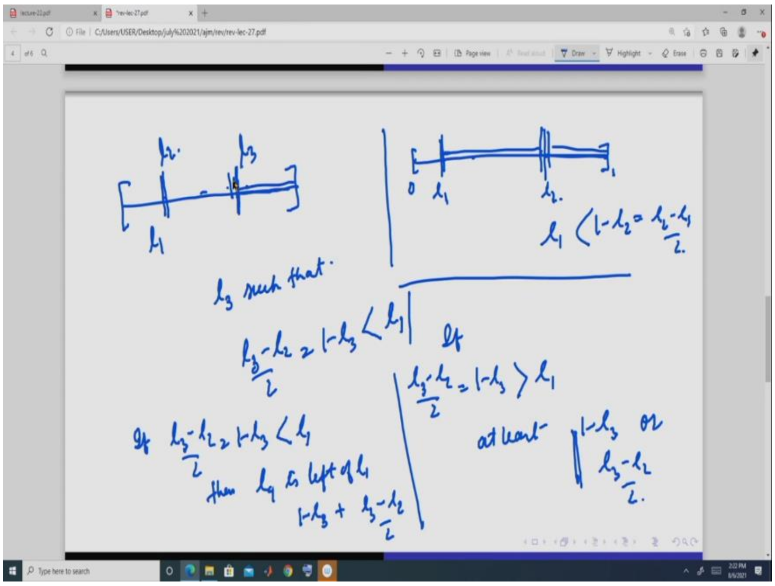

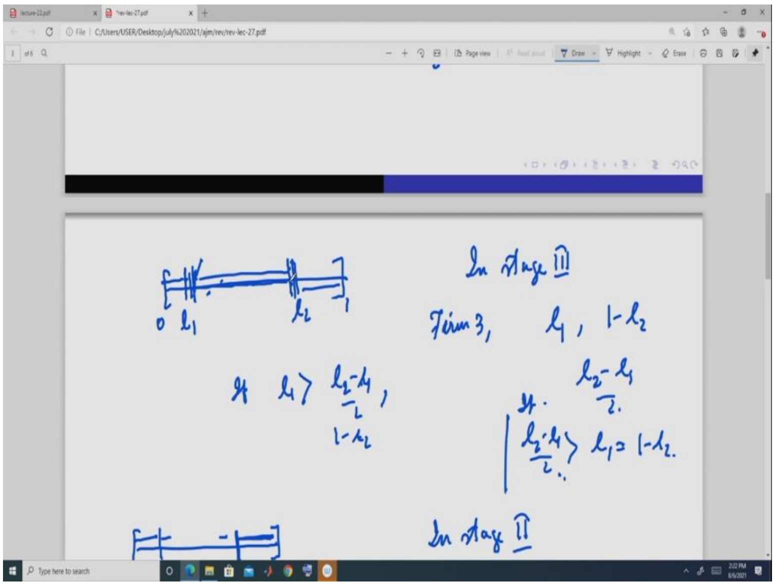

Now, it may happen that the situation is something like this that this distance is, okay. Now, if firm 2 chooses half of this, so it is chooses in such a way that this distance is, this is l2, so it is 1 minus l2 and it gets half of this-$1 - l_2 = \frac{l_2 - l_1}{2}$, okay. And this is greater than, suppose l1, okay. It can do this, firm 2 can do this.

Now, if in this case, what will happen? Firm 3 can locate here, it will get this much for sure, and firm 4 may locate here or may locate anywhere here and including this. So, firm 2 here, is not sure that it is going to get a none, it is going to get a positive length. So, that is why, at least, if it ensures this much then it will always get this. So, if it does chooses a position which is greater than this, then it will ensure that this distance is equal to this distance, instead of looking at it in this way, okay.

But firm 2 has another possibility, see because there are now 4 firms, after firm 2 enters, there is, 2 more firms are going to enter. Firm 3 in stage 3 and firm 4. So, if this is l1 then if firm 2 chooses this l2, then what is going to happen? Then firm 3 is not going to choose this then, because firm 3 will also have a threat that the firm 4 is going to enter and then it will get this.

So, firm 3 will choose l3 such that in this situation, at least this is equal to-$\frac{l_3-l_2}{2} = 1 - l_3$. So, it ensures this, ok. Now, it may happen that this distance is less than l1-$\frac{l_3-l_2}{2} = 1 - l_3 < l_1$. If this happens then firm 4 is going to locate here, okay. So, firm 3 is going to get this and half of this. If this is the case then, then l4 is left of l1 and so firm a will get, the length of a is, firm, it will get this plus, so this is going to ensure, this much- $1 - l_3 + \frac{l_3-l_2}{2}$. So, that is why it is going to ensure.

Now, if is greater than l1 then, if this is the case, then firm 4 will either locate here and get this or it will locate here or anywhere here it will get. So, at least 1 minus l3 or this much-$\frac{l_3-l_2}{2}$ is ensured. So, that is why firm 3 will locate here, in this case. And firm 4 will locate either here or here. Now, firm 2 knows this. So, it can either choose in here or it can choose something like this. So, firm 2 has two possibilities here, okay given l1.

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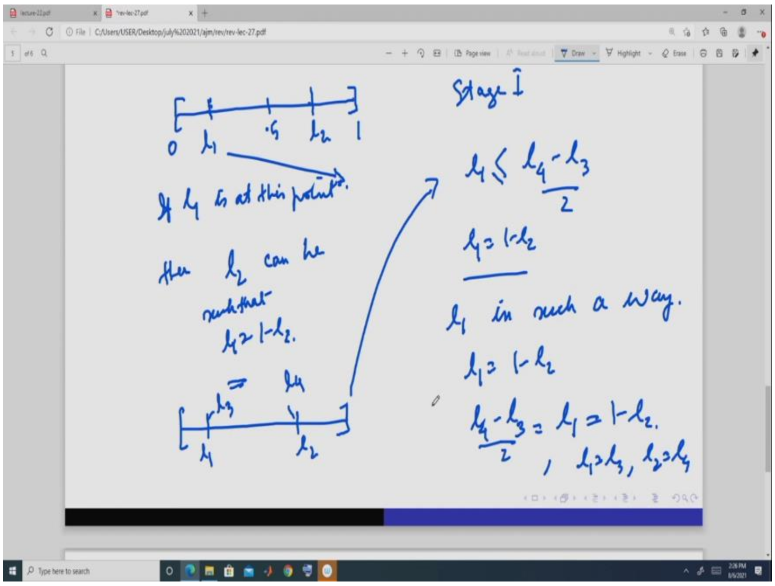

Now, firm 1 lets come to stage 1. So, firm 1, it knows that if it chooses a position like this then there is a possibility that firm is going to choose here, such that this is equal to this. So, if l1 is this, is suppose less than do not write, 0.5. But so, if l1 is this point, okay. Then l2 can be such that l1 is equal to 1 minus l2 or, okay so if it is this case, then what do we know? If this is the case, then firm 3 is going to either, firm 3 is going to choose here or it will choose here or it will here, depending on the sign, okay.

Now, what is going to happen here? So, this is going to be one situation, right. Another, then if firm 3 is here, then firm 4 is here. So, if this is the case, then we get, if l1 is here, l2 is here, then this position is l3 or this position is l4. We get this. When do we get this? We get this when l1 is less than equal to l2, this- $\frac{l_4 - l_3}{2}$. And we know that l1 is equal to. Because this is going to be ensured, in this case. So, firm 1 is going to do this calculation. It will choose l1, so l1 in such a way that it is going to solve these things. This- $l_1 = 1 - l_2$. And given this is equal to, which is equal. So, we get this- $\frac{l_3 - l_4}{2} = l_1 = 1 - l_2$. Now, from this, so here and further l1 is equal to l3, l2 is equal to l4.

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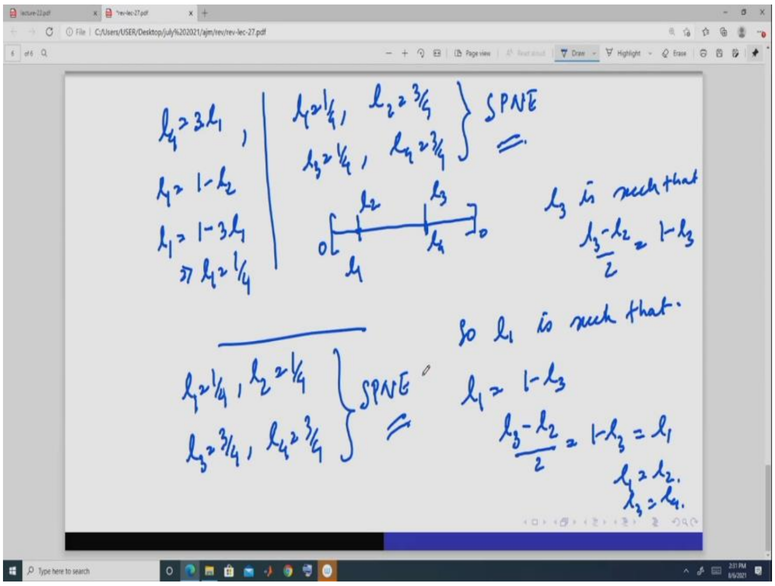

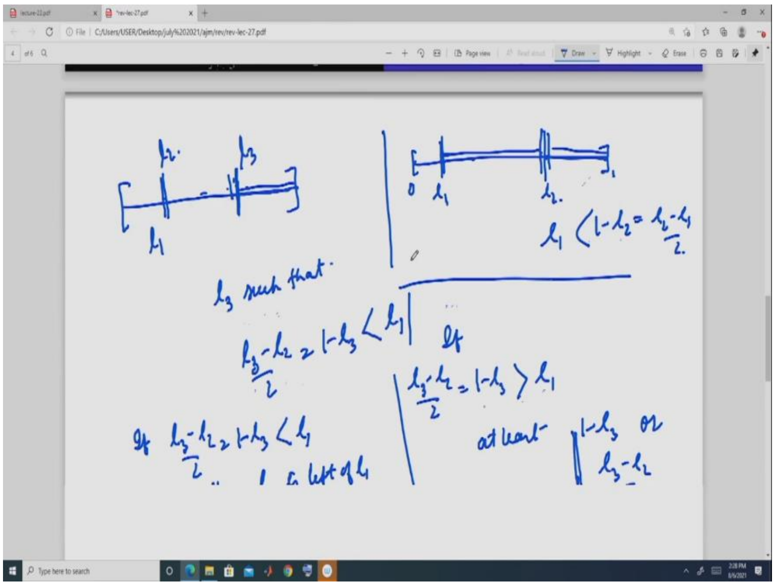

Now, from l4, so from this, we get that l1 is equal to 3 times l4, this. And l1 is equal to 1 minus l2, use this. This here a is 3 times, so from this we get l4 is 3 times l1. And we have this. So, plug in here, l1 is equal to 1, so here, it is l4 is a, so 3 l1. So, this will give me l1 is equal to 1 by 4.

Now, if l1 is equal to 1 by 4 then l2 is here, it is same as here, so it is 3 by 4. And l3 is 1 by 4, l4 is $\frac{3}{4}$. So, this is one subgame S P N E. Subgame Perfect Nash Equilibrium. Now, another thing that we have seen is, this possibility that this is l1 and firm 2 locates here, then l3 is this. So, l3 will be such as, such that l3 minus l2 is equal to 1 minus l3.

Now, firm 1, if this is greater, then we have seen, firm 4 is going to locate here, then this. So, l1 is again such that l1 is equal to 1 minus l3. l3 minus l2 is equal to 1 minus l3 is equal to l1 and l1 is equal to l2 and l3 is equal to l4. We can have that outcome. So, these equations are, if you look at them are same as this. So, again in this situation, we get that l1 is again 1 by 4, l2 is 1 by 4, l3 is 3 by 4 and l4 is 3 by 4. This is another Subgame Perfect Nash Equilibrium.

So, from this, we get that there are two this kind of this, but there are many, if you look at these things, then here l1 can be this position and then we can change that also, in this here. So, there are many possibilities. But the locations are going to be of this nature. So, we see that in this kind of product differentiation or that is the product differentiation through decision of or through the choice of location, then we see that there are two firms which are going to produce similar product and another two firms which are going to produce similar product but they are different from this.

So, we will see variety in the market but we will not see that much variety. So, there will be some firms producing similar homogenous product and there will be also some firms which and there will be difference between these two types of firms, okay. So, in when we have 4 firms they are entering sequentially, we find this as one of them.

These as the two Nash equilibriums, subgame perfect Nash equilibrium, where we find that there will be some firms producing homogenous product and some firms which are differentiating from those two set of firms, okay. So, this is the outcome in this situation, okay. So, thank you very much. And for this portion you can read class notes.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Indian Institute of Technology, Guwahati
Department of Humanities and Social Sciences
Module 11: Product Differentiation and Entry Deterrence
Lecture 39
Dixit's Model of Entry Deterrence

Hello. Welcome to my course Introduction to Market Structures.

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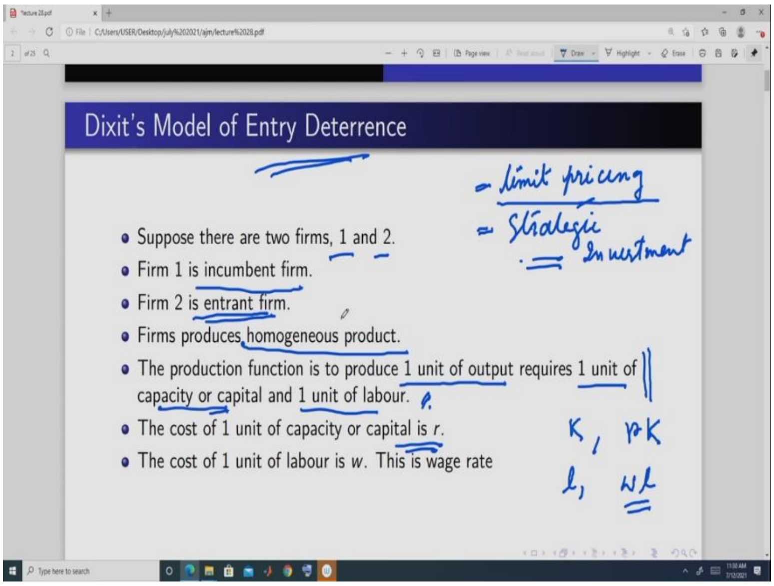

So, today we are going to start a new topic and that is Dixit's Model of Entry Deterrence. Entry deterrence means that suppose there already exist a firm in the market, and now one new firm may want to enter that market. Then this firm, which is already existing in the market, it may act in such a way that its action that gives a signal and that signal actually deter the entry of firm 2 or the firm which wants to enter this market. So, this is the entry deterrence.

And now the firms may follow many strategies to deter the entry. And if it is successful in deterring the entry of firms, then that firm which is already there, it can get the monopoly profit or it will be the sole producer in the market, so it will over time, it may earn lot of profit. So, this the strategies or the policies so that a firm deter the entry, this is important to study. Why?

Because it mean, because the firm, those can be considered as part of the monopoly practices. Because you deter the entry of new entrant or new firm so you get a monopoly rent out of it. And we have seen that if a firm is monopoly then it is not social welfare maximizing or it is not Pareto optimal also, right? So, because of this reason the regulatory bodies, they always keep and watch that whether the firms are deterring the entry of the new firms or not, right? So, that is why we study this.

Now, in the entry deterrence this can be done broadly in two ways. By setting the price very low, so that is called the limit pricing. Or you have so much capacity that you can produce a huge amount of output, and suppose you incur some economies of scale because of that so your average cost is low. So, you get some your cost of production is low, so you can sell at lower price or you can sell more so the other firms may not enter.

So, mainly these two are the strategies that the firm follows. And we are not going to do limit pricing, because if we do the, because the limit pricing models, it requires asymmetric information also and we have not done asymmetric information. So, that is why we are not going to do this. But we are going to do strategic investment. And that is this model. And this model was first proposed by Avinash Dixit, so that is why it is Dixit's Model of Entry Deterrence, okay. So, now let us move to the model.

So, suppose, there are 2 firms, firm 1 and firm 2. Firm 1 is an incumbent firm. Incumbent firm means that firm 1 is already existing in the market, okay. And firm 2 is an entrant firm, it is going to enter, okay. And for simplicity, we are assuming that both the firms produces homogeneous product. So, we know what is the meaning of homogeneous product. It means that the output produced by firm 1 and firm 2 are perfectly substitutable. So, whether you buy from firm 1 or from firm 2, it does not matter.

And for simplicity further we assumed a very specific form of production function and it is, production function is, to produce 1 unit of output requires 1 unit of capacity or we call it capital and 1 unit of labor, so you will require both capital and labor and 1 unit of capital and 1 unit of labor is going to give you 1 unit of output.

So, this is to keep all the calculation simple, we assume this thing and this assumption is actually straight from the Dixit's Model and in the Dixit Model also this assumption was made, okay. Now, we have to specify the cost of these inputs. So, the cost of capital or capacity is r per unit. So, if you are employing suppose k units of capital then your cost is on capital is r into k. And the cost of 1 unit of labor is w, so it is the wage rate. So, if you are employing l amount of labor, so there wage cost is this, w into l, okay.

Now, here capacity or you can think this capacity as the machine and that is creating a kind of capacity in the production. So, it is, you can think of same as a, it is same as capital, okay. That we have been assuming till now in this course, okay. So, here capacity and capital is same thing in this model, okay.

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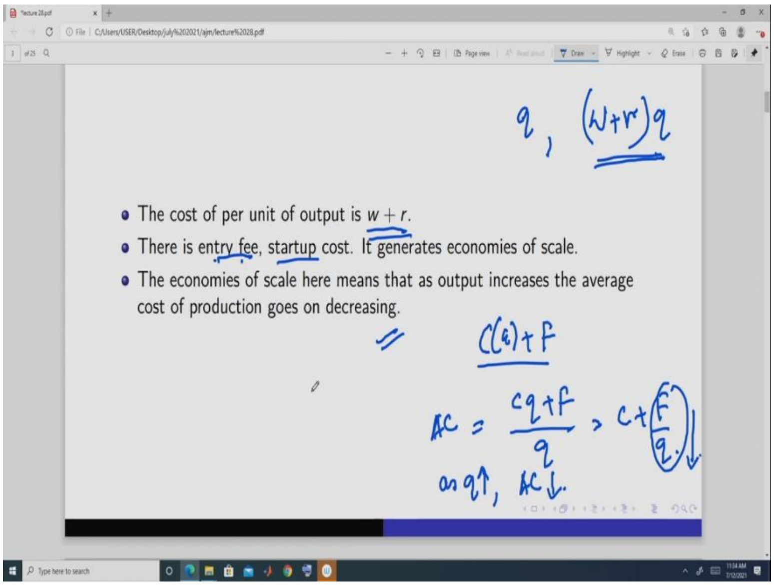

Now, so because of these two costs, so cost per unit of output is this- w+r. So, if you want to produce q units of output then your cost is, because you will require q units of labor and q units of capital and the price of capital is r and the price of labor is w, so this is the- (w+r)q, okay. Now, here we introduce one more cost and that is the startup cost or entry fee. Now, how do you motivate this cost? Entry fee or startup cost is something like this. Suppose a firm wants to enter this market.

Now, first it will have to know the demand in that market. Now, it will not know that demand automatically, it will have to spend some time. It will and that and it will have to go through the market, through, do some survey and then find out survey of the existing market and survey by doing some kind of market research like how much, what is the possibility of demand of this good in this market?

So, those will cost some amount and those are fixed. Like if you know that the demand in this market is this much. Suppose hundred units is that the maximum a firm can sell. Then it means that the amount of expenditure you have incurred to know that information that is gone, you will not and it is fixed. If you produce, if you vary your output that cost is not going to change that you have already incurred. So, that is why it is a fixed cost, you can say. And it is we call it an entry cost, because you do a little bit of market research.

And also, you want to study the technology that is available, what kind of labors you require, what kind of machines you require. So, for those kind of things you will spend some time and you will use some resources, to know, to get those information. So, that is going to generate some cost. And that is actually constitutes this entry fee or you can say startup cost, okay.

And see, this startup cost is a fixed cost. Now, fixed cost, it will generate economies of scale. Now, what do we mean by economies of scale? Economies of scale here, it means suppose, your cost function is of this nature- c(e) +f, right. Now, this is suppose very specific this- cq+F. If you take this, this is what AC, Average Cost-$\frac{cq+F}{q}$. Now, this you can write this as-c $+\frac{F}{q}$. Now, what is happening? As output increases, your AC is going down. Because this portion $\frac{F}{q}$ is going down, as q increases.

So, this is what economies of scale means in this context that as the output increases, average cost goes on decreasing, okay. So, if you incur some this entry cost or startup cost, you will always try to produce as much output as possible, because then it will reduce your cost per unit of output, okay. So, this is the specification that we have, specification of the Dixit's Model.

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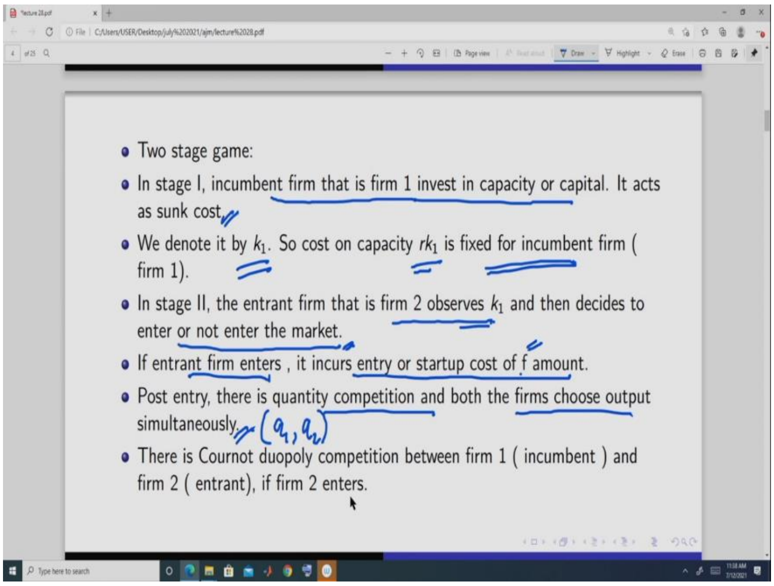

Now, let us specify the game. So, it is a two-stage game. In stage 1 incumbent firm that is firm 1 invest in capacity or capital, okay. So, it will fix, buy some machine or it will buy some fix set up some capacity in that firm. And that expenditure incurred on buying those capital or setting up those capacity that is a sunk cost.

So, that is sunk cost means, you will not here, sunk cost in this in the sense, it means, definitely since it is a capacity if you want to resell it, you will get some money for it. So, in that sense it is not sunk. But it is sunk in the sense that if you are suppose, your capacity is to produce 100 units and you are producing only 50 units, so this 50 which is lying idle, you are incurring some cost on that. So, in that sense, it is sunk. So, you have a capacity to produce 100 units and you are producing only 50 units. So, for 50 units you are getting some amount of revenue, because you are selling, producing 50 units and you are selling 50 units.

But since your capacity is to produce 100, so this 50 which is, which, that amount of machines that you are not using or the capacity that you are not using, which is lying idle, that is that, but you have already incurred the cost, while setting up that much capacity or while buying that much amount of machine, so that is not giving you that much amount of capacity or machines are not giving you any return, so in that sense it is a sunk cost. And we denote the capacity or capital of a firm by k1, okay. So, cost on capacity is, if it decides to have a capacity of k1 is rk1 and it is fixed for incumbent firm. So, if you have decided k1 units of capacity then r into k1 is cost, you have incurred, okay.

Now, in stage 2, the entrant firm that is firm 2, observes k1. So, firm 2, which is deciding to enter, it knows, what is the capacity of firm 1, okay. And that is this. And then decides to enter or not enter the market. So, it takes the decision to enter or not into the market after observing the capacity of firm 1 or the observing the capacity of the incumbent firm.

If the entrant firm enters, that is, if firm 2 enters it, incurs entry cost or startup cost of f amount, okay. So, it will be the amount is some f. The post entry, once the firm 2 enters, firm 1 already exists, there is quantity competition and both the firms choose output simultaneously. So, we have actually, we have separated these decisions that is firm 1 now make a decision whether to, make a decision on the amount of capacity in stage 1.

Firm 2 observes this capacity in stage 2 and decides whether to enter or not. Suppose it wants to enter. As it enters then there is firm 1 and firm 2 simultaneously decides the output of each of their output, q1 and q2. Their output, here q1 of firm 1 and q2 of firm 2. So, you can say that the game is something like this. In stage 1 capacity that is k1 is decided, in stage 2, firm 2, after observing k1, decides whether to enter or not. And then there is Cournot quantity competition between firm 1 and firm 2. Now, this Cournot competition, you can also think as a part of stage 2, okay.

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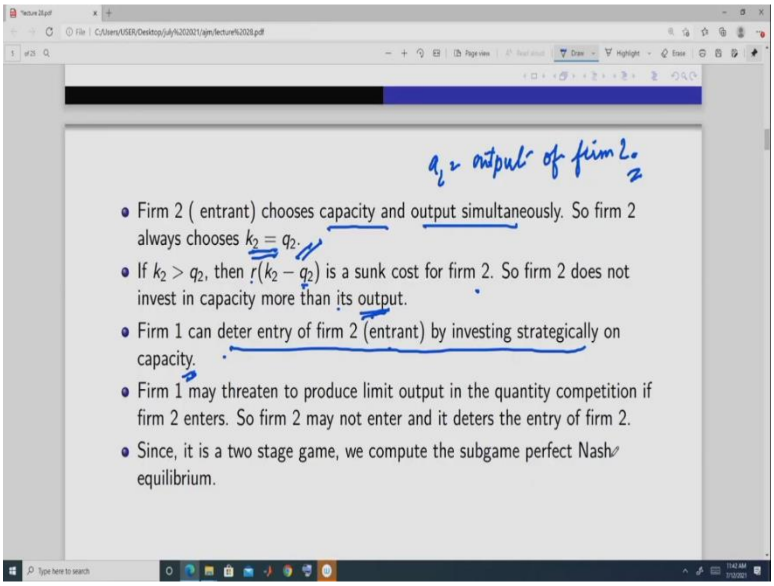

Now, here firm 2, it is deciding its capacity and output simultaneously. It is not like firm 1 deciding its output in stage 2 after the firm 2 has entered or not, after the decision of the firm 2 to enter or not to enter. But its decision on the capacity is done in stage 1. But for firm 2 these two decisions that is how much amount of capacity to have, or how much amount of machines to have, and the output it is going to produce that is taken simultaneously. So, firm 2 always chooses k2 is equal to q2. Where q2 is the output of firm 2, okay.

Now, this is always going to be true. k2 is equal to q2. Why? Because if k2 is greater than q2 then r, this much amount of capital or capacity, i.e. $r(k_2 - q_2)$ is sunk for firm 2. So, it is not getting any return, so firm 2 does not invest in capacity more than is invest, more than is output. If it have more capacity or more machine then the output it needs to produce, then that amount, which is incurred in setting up that capacity or the cost that it incurs, is going to be a sunk cost and it will not generate any income. So, that is why it is, firm 2 will always choose k2 is equal to q2.

Now, firm 1 in stage 1, can deter entry of firm 2, by investing strategically on capacity. So, it is something like this. So, firm 1 may threaten that I have a huge amount of capacity, if you enter, I will produce something called limit output. And so firm 1 says that if you produce, if you enter, then I will produce limit output. And if firm 1 produces limit output then firm 2 will not enter. And so that is why it is it deters the entry.

So, based on this capacity that capacity itself is going to give you, give firm 2, a kind of a signal that whether firm 1 is actually going to produce limit output or not. Now, since this is, see, this is something like this. So, firm 1 decides the capacity. Now, if it have a sufficiently big amount of capacity, then what, firm 2 may know that may think it. It like this, if I enter firm 1 is going to produce this much amount of output till its capacity, okay.

So, then what is going to happen? So, that is why firm 2 is not going to enter. The moment firm 2 does not enter then firm 1, what it can do? It can produce the monopoly output. So, it will not actually produce the limit output. But because it has a capacity, a huge amount of capacity that may deter the entry. Because firm 2 may think that if I enter, firm 2 will firm 1 is going to produce still its capacity and that is going to be my limit output.

Now, what is limit output? We will discuss later. But limit output, think it is something that deters the entry of firm 2. Now, since it is a 2 stage game, so we are going to find the, compute the subgame perfect Nash equilibrium. And how do we do? We use backward induction to do.

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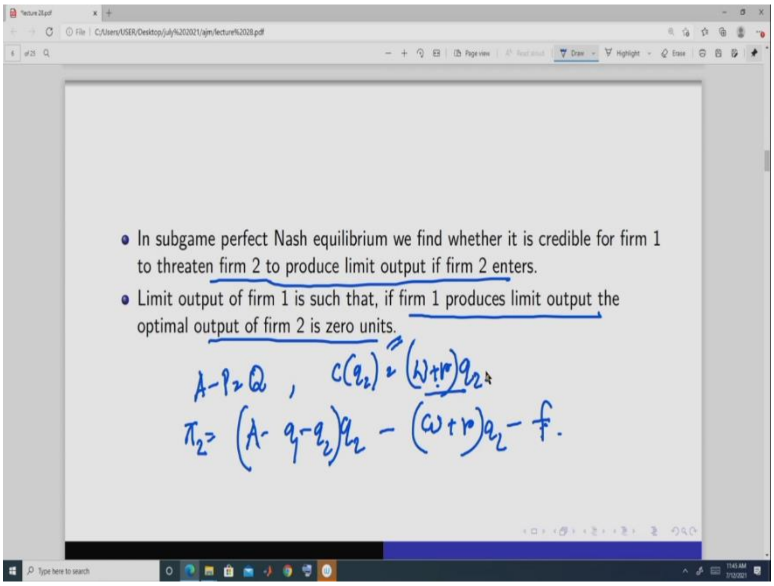

Now, again here in this subgame perfect Nash equilibrium, we find whether it is credible for firm 1 to threaten firm 2 to produce limit output, if firm 2 enters. Now, see, if we have seen in Cournot outcome that if generally the Cournot, in Cournot outcome, if firm produces the Cournot outputs that is profit maximizing, right?

But if it wants to produce more than the Cournot output, then it is going to give it. That firm less k. So, there is a possibility that, that if I threaten the firm 2, that if you enter I will produce till my capacity and since I have a huge capacity, so you will not be able to, you will make a loss. But then it may happen that itself may be creating a loss for me, so that is why, that may not be, that threat may not be credible enough. So, that is why I may not threaten it. So, in subgame, we will have to find whether this threat is actually credible or not, okay.

Now, we will define, what is limit output. Limit output of firm 1 that is the incumbent firm is such that if firm 1 produces limit output the optimal output of firm 2 is 0 units. So, if firm 1 produces limit output firm 2 does not produce. Now, here, how do we get it? Suppose we will do it.

I will show it using 1 example. Suppose the market demand is this. So, the profit of firm 2 and suppose the firm 2's cost function is, c or let us take in this case only, so that it will be very specific. Because to produce 1 unit of output, it requires 1 unit of labor and 1 unit of capital. If it were produces q2 units of output so its cost is w plus r into q2. This-$c(q_2) = (w + r)q_2$.

So, the profit of firm 2, if q1 is the output of firm 1. So, this is going to be the market price, this is going to be the total revenue-$\pi_2 = (A - q_1 - q_2)q_2 - (w + r)q_2 - f$, this the sunk cost f. So, cost function is this, to produce the output q units. Its cost is this. This it has already incurred. So, we have put an additional thing. It is not part of the total cost function, okay.

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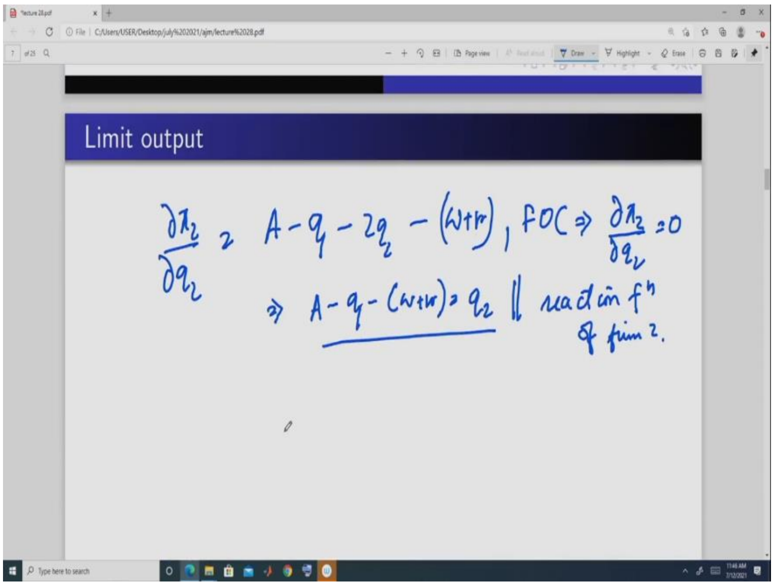

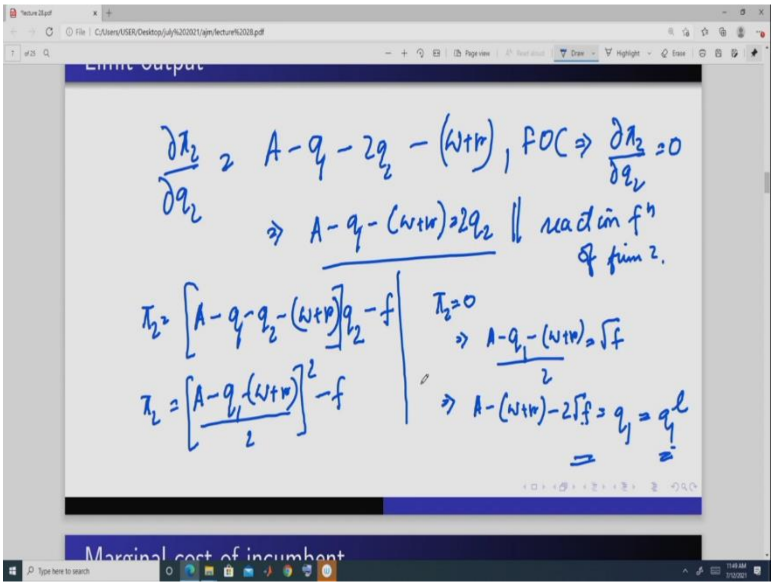

So, if we, how to find the limit output? If we optimize this, with respect to q2, what do we get? And then first order condition gives me equal to 0-$A - q_1 - 2q_2 - (w + r) = 0$. So, then this implies what? This implies. So, this is the reaction function of firm 2-$A - q_1 - (w + r) = q_2$. So, plug in the output of firm 1, we get the output of firm 2. Now, here if we plug in that output of firm 1 then the optimal output is 0. That is what the limit output is.

Now, you will see, how to derive the limit output of this. Now, this profit function of firm 2, you can write it in this form-$\pi_2 = [A - q_1 - q_2 - (w + r)]q_2 - f$. This form. Here taking q2 common, oh sorry, this. Now, we know q2 can be written as a function of q1 from the reaction function. So, this becomes and you plug in that here, so it will be square, this-$\pi_2 = \left[\frac{A-q_1-(w+r)}{2}\right]^2 - f$. Now, equate this equal to 0.

So, what do we get? We get, this implies this. So, from here, we get and we denote this as limit output of firm 1-$A - (w + r) - 2\sqrt{f} = q_1 = q_1^l$. If firm 1 produces this much amount of output, see, if firm 1 produces. Here it will be 2. So, firm 2 is not going to produce anything because its a is going to be 0. Because plug in this here, you will get that this cancels out, what is left is this. And square of that you will get f. So, profit is 0. So, firm 2 does not produce, does not want to enter at this here. So, this is the limit output. So, we will use this concept in this model, okay.

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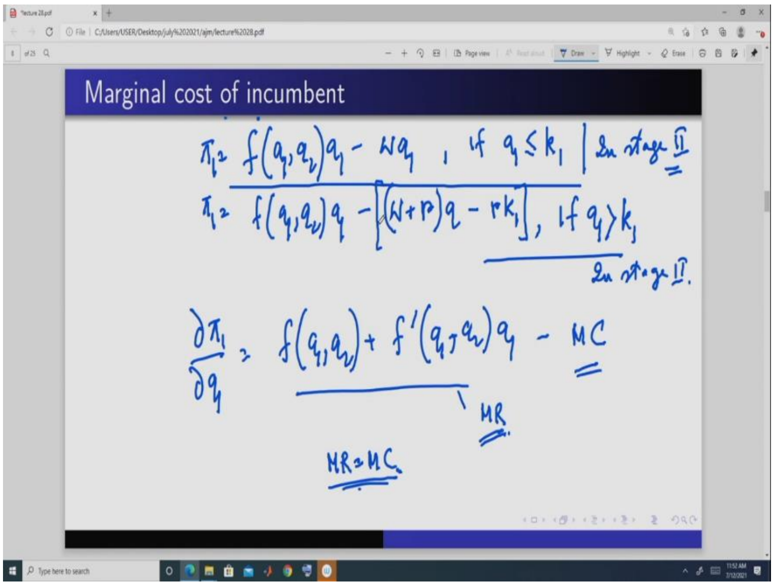

Next, we define the marginal cost of the incumbent. Because suppose, we require the marginal cost. Why? Because the profit function of firm 1, you can say if we take a general demand function like this, so output of firm 1 is q1 output of firm 2 is q2. So, this is the price into this, it is this, is the profit of firm 1-$\pi_1 = f(q_1, q_2)q_1 - wq_1$. If q1 is less than k1, less than equal to, and the profit of firm 1 is going to be what? This is going to be cost in stage 2, right.

Now, here this now this plus r this. So, this is the output, minus this is going to be the cost. If q1 is greater than, output is greater than its capacity or its capital, this in stage 2, so this cost portion is changing according to the output and given capacity in stage 2. So, this is the general demand.

So, now we know, when we take this, what do we get? We get, see, this is not a continuous thing, equal to, you can say marginal cost. Because this here. So, this portion, you can think as, something as, marginal revenue, okay. And this is marginal cost. So, the reaction function of the Cournot reaction function is such that marginal revenue is actually equal to marginal cost, okay. And we have done this while looking at the iso-profit curves of the firms, while doing Stackelberg, right? So, now, let us get this marginal cost first. Define the marginal cost based on this cost function.

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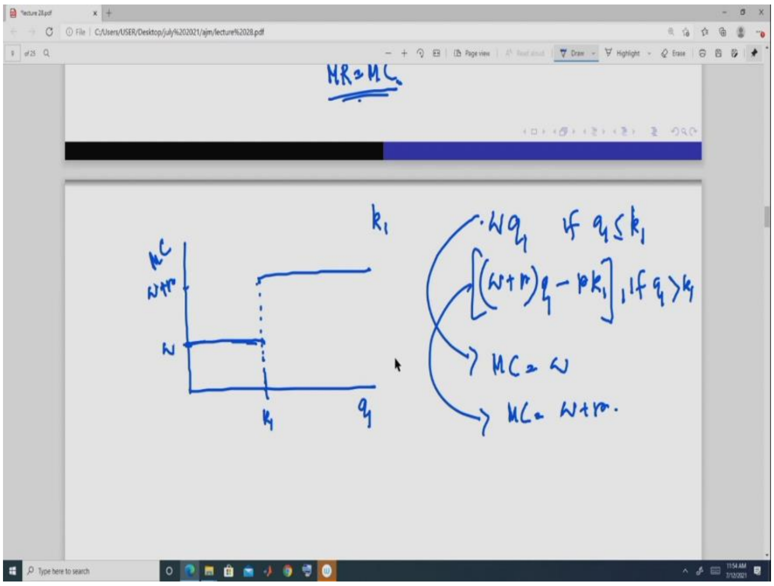

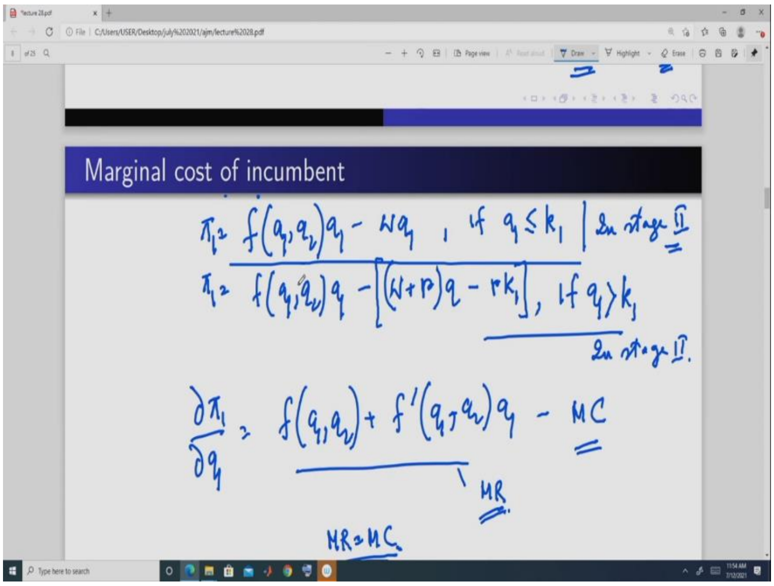

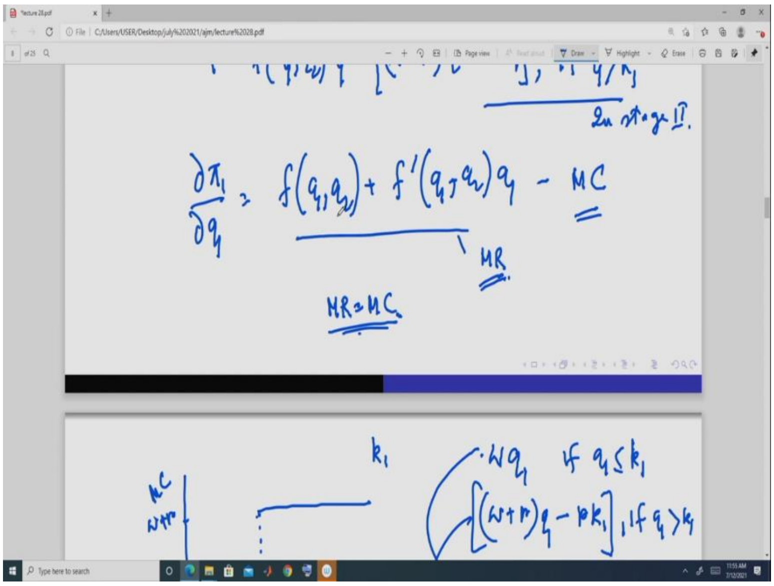

So, suppose k1 is the capacity and k1 is this. In this axis we take the output of firm 1, in this axis marginal cost. So, till capacity its cost is this. So, suppose this is w, so till this much, its cost is this. Now, if q1 is less than equal to, small k1, but it is this-$[(w + r)q_1 - rk_1]$. If q1 is greater than k1. Or the here, in this, these are the total cost so M C is w. There M C is w plus r.

So, suppose this is w plus r. So, from here, so the marginal cost is a horizontal curve, but it is stepwise. So, it is this much, still this much output. And from here it is this, okay. This is the marginal cost. Now, what is the objective of firm 1? Firm 1, while choosing its output, because we are going to find the subgame perfectness equilibrium, now to find the subgame perfect Nash equilibrium, we use backward induction. So, that is, we first take the last stage.

So, here in this game, the last stage is which game? Last stage is the Cournot competition, provided firm 2 has entered, right. Here, so in Cournot competition, firm 1 will try to maximize this profit, which is given by this here, either it will maximize this or it will maximize this, depending on its capacity k1.

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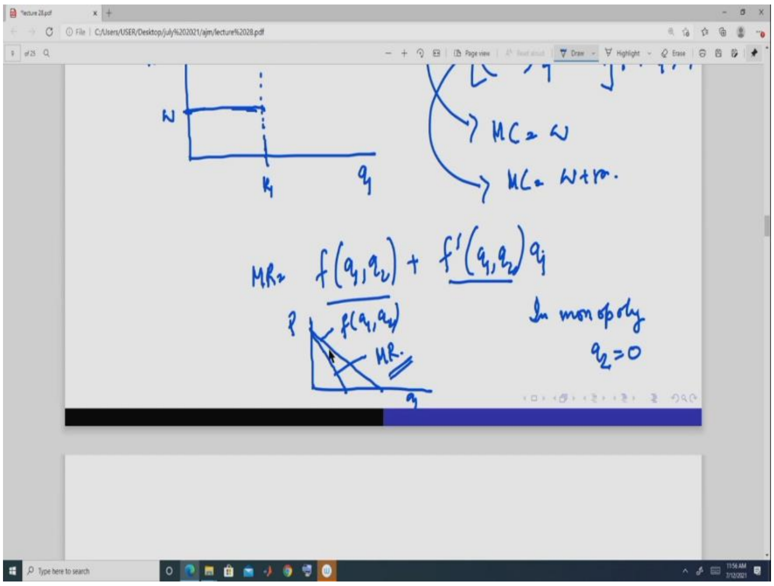

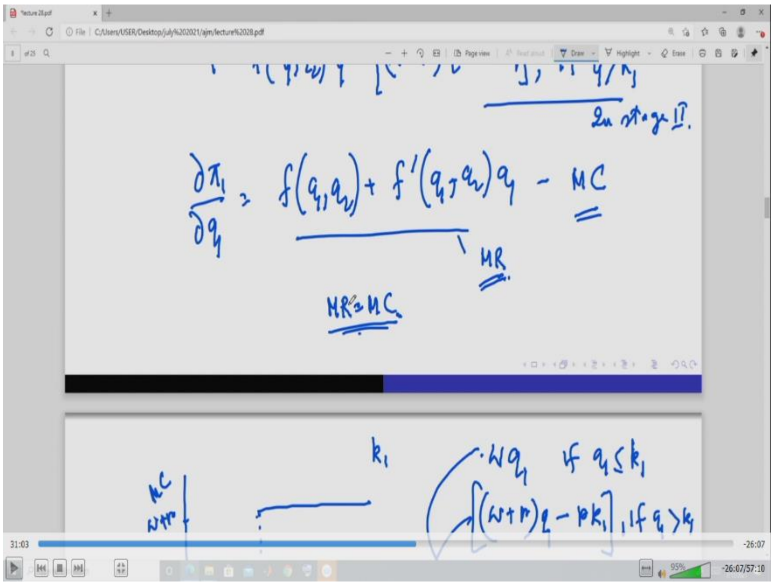

So, these marginal revenue curves, these, these marginal revenue, see, they are, we know marginal revenue curve is this, for a general demand function, downward sloping demand function. This as-$f(q_1, q_2) + f'(q_1, q_2)q_1$, is a downward sloping, right? if we take the output here and if we take this, it is downward slope.

Since if we take the price here, this is the demand function, this portion is going to be the negative into a, so into the output, so it will be somewhere here, this is the marginal revenue. We have done this while doing the monopoly thing. But in monopoly, what we have done? We have taken this is equal to 0. But here, here it will take some positive amount. Now, see if the q2 takes a higher value, this is going to be.

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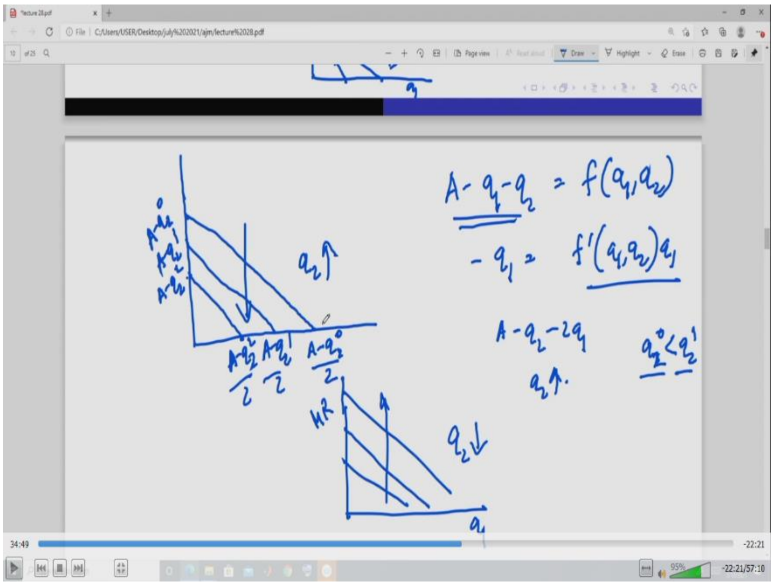

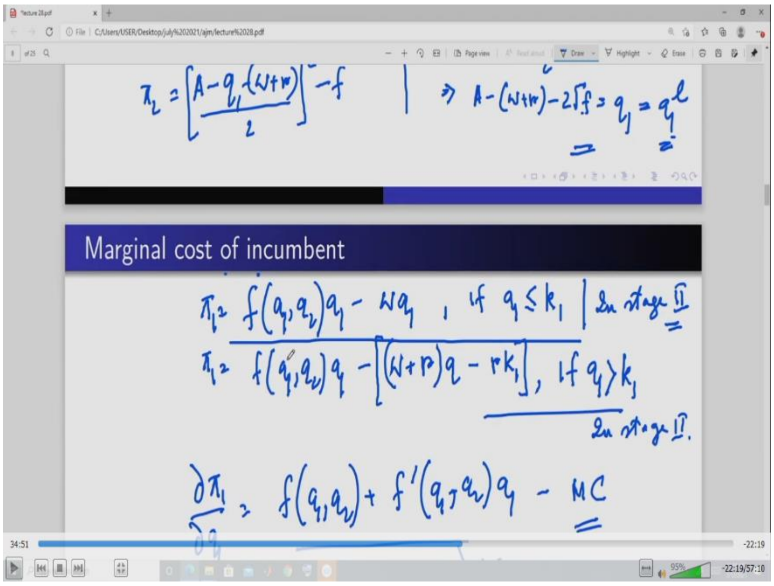

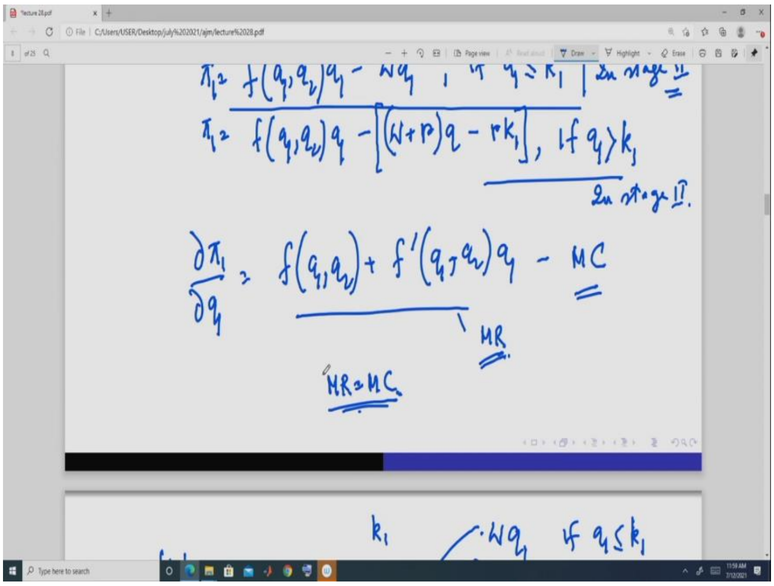

Let us take an example. And it will be clear. So, this is the f. This is the inverse demand function you can think-A $- q_1 - q_2$. And if we take the derivative of this with respect to q1, it is minus 1 and into this, so it is this. Now, if you look at this, this curve, now fix one q2 and this point is going to be this and this is going to be, it is this.

Now, if you increase the q2 suppose q2 is increased, then what will happen? This is going to go down, suppose instead of bar fix q2 at q not, then this is suppose q2 1 this is q2 1, and q21 is greater than q not 2. So, we will get like this. Suppose take this. This is q2 2 and this is suppose a q2 2. So, as q2 increases, we move in this direction and we go this way, when q2 falls. This is marginal revenue, right?

And from this a and also from profit maximization, what we have done in Cournot? We know at the optimal output there this marginal revenue should be equal to marginal cost. Because the given output of firm 2, the reaction function, output of firm 1 is given by the reaction function of firm 1, okay.

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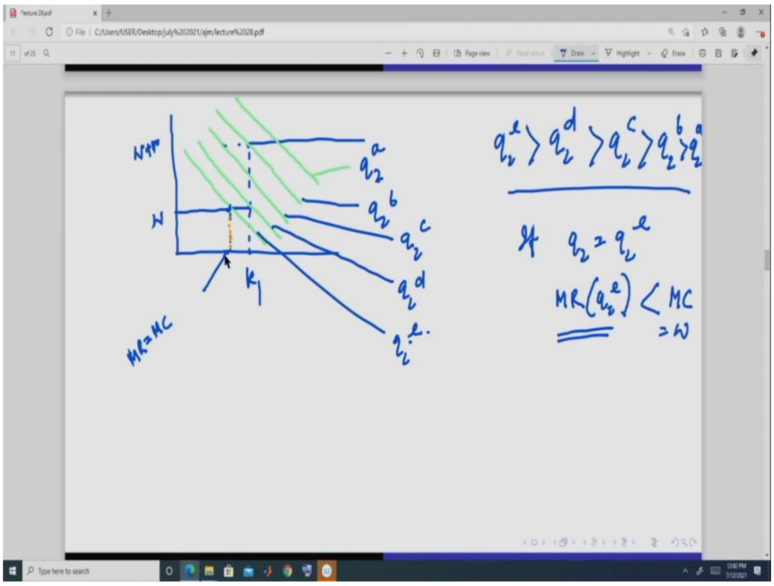

So, it is going to be something like this. Now, suppose this is one marginal revenue. It is like this. This is for suppose q2 is a, this is suppose q2 b, this is suppose q2 c, this is suppose q2 d, and this is suppose q2 e. Now, from here, if we look at this marginal revenue curves, we get that q2 e should be greater than q2 d, c, b, a, right? We get this ranking of the output of firm 2- $q_2^e > q_2^d > q_2^c > q_2^b > q_2^a$.

Now, here what is happening? At this point, what is happening? At this output, M R which is given here, is equal to M C, which is this. Capacity is this. But if I produce this much in this, so what do we get? We get that if the output of firm 2 is this, then we should, my output is given by this. Because marginal revenue equal to here. Because here, if I look at my capacity here, okay what is happening? Marginal revenue is this much. Marginal cost is this.

So, at this capacity, if q2 is q2 e, marginal revenue this, this is less than marginal cost and what is the marginal cost here? w. So, definitely it is less than w plus r. So, in this situation what is going to? Optimal output is this. From that a Cournot thing. So, the reaction function is going to be, it is going to produce less than its capacity.

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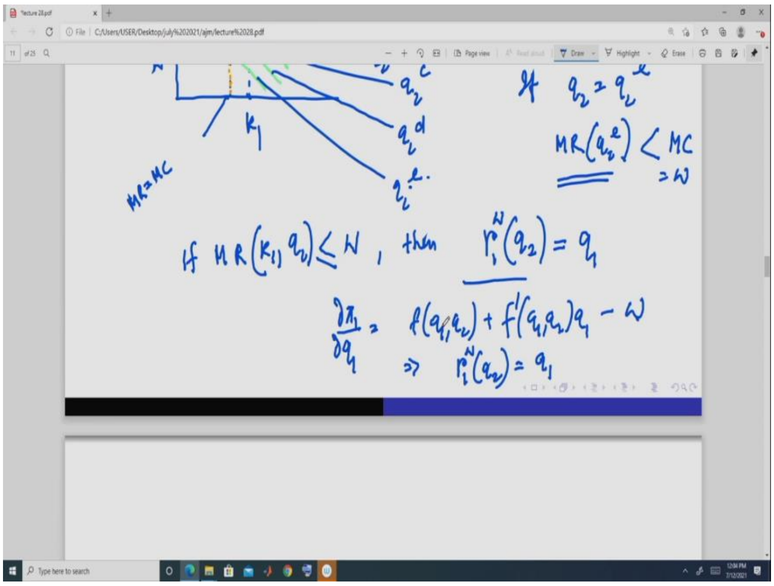

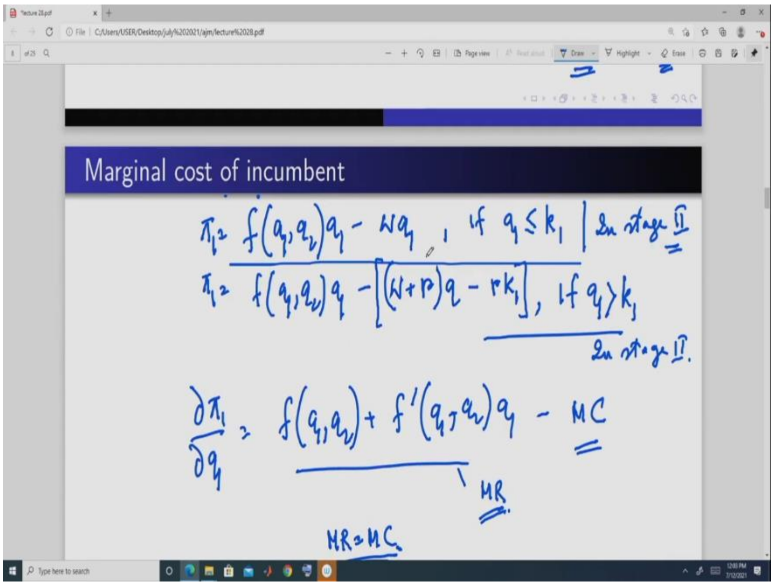

So, that is why, what do we get? So, we write the reaction function in this form. So, if marginal revenue at this, suppose, firm 1 is producing its capacity, and firm 2 is producing some amount, some amount it can be of any of this point, if this is less than w, less than equal to w, then reaction function of firm 1 given some a, is going to be, what? It will going to be based on only w.

So, it is going to be, from here and we get this. If it this, we get this. Why? Because, if we take the derivative with respect of the first equation, then it is, we will do a specific example, so that will be more clearer to you. It is going to be this only-$f(q_1, q_2) + f'(q_1, q_2)q_1 - w$. Till this much, if the q2 is this and suppose, firm 1 produces this, then here, marginal revenue is equal to 1, right?

So, this is going to be the optimal Cournot thing. Marginal revenue is equal to marginal cost, right? So, it is going to be given by this. And so this reaction function is, since the marginal cost is this, we write it in this form. We denote it w only, is equal to this-$p_1^w(q_2) = q_1$, okay. Now, suppose, q2 is here, here take this. In this what is happening?

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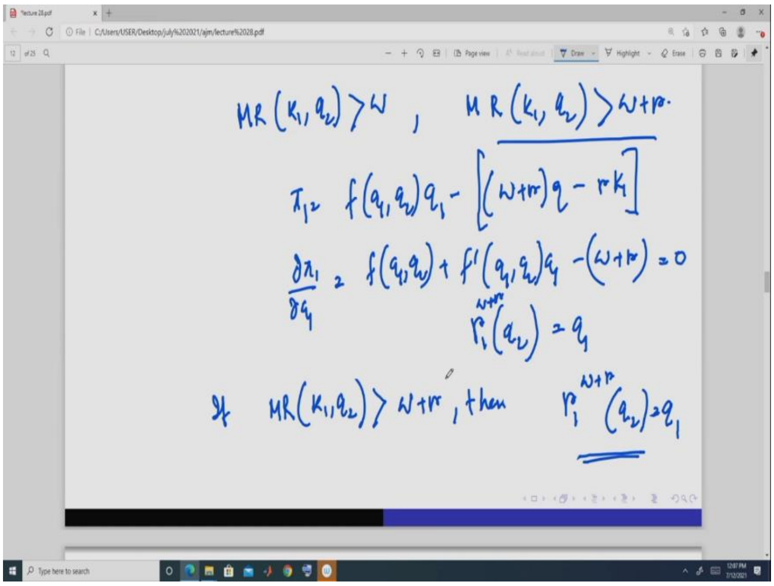

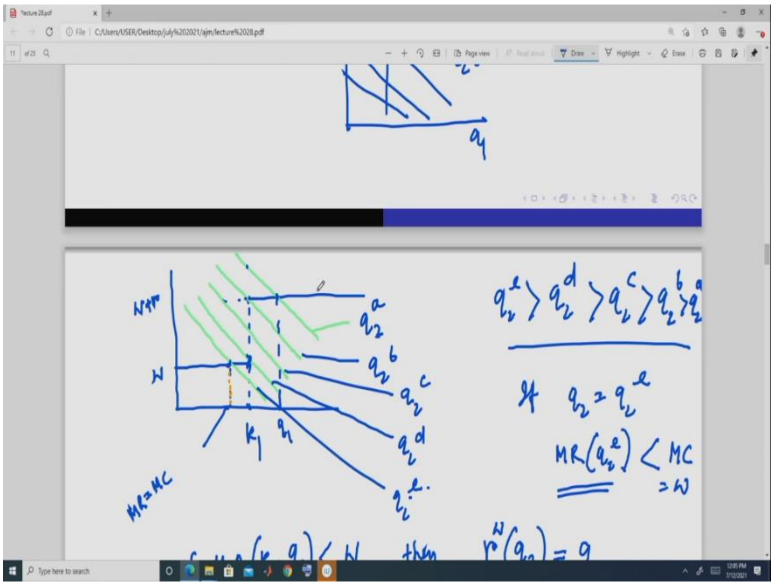

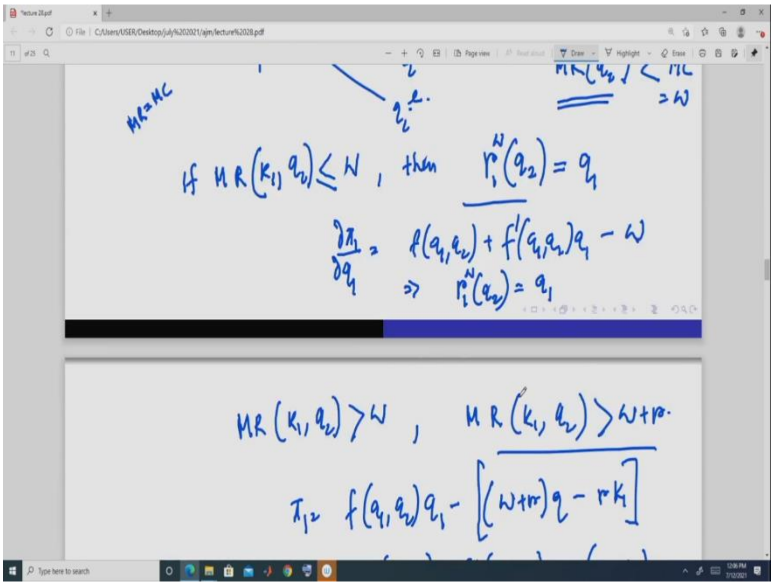

Marginal revenue is greater than w. It is greater than w and it is greater than w plus also. Further, is greater than. So, since it is greater, marginal revenue is greater and marginal cost is here, so it is better to produce some more output, so that you get more additional revenue is more than the additional cost. So, you will produce up to this much amount of output, right?

So, here your cost function is your profit function in this Cournot is going to be this-$f(q_1, q_2)q_1 - [(w+r)q - rk_1]$. This so, your reaction function. So, this portion remains same as this. But here it is this, w plus r and it should be equal to 0. So, this reaction function, we denote as w plus r is equal to. So, if this is the case then if marginal revenue is greater than, here also, this, it should be always greater than actually, greater than-$MR(k_1, q_2) > w + r$. Then w plus r then the reaction function is this-$r_1^{w+r}(q_2) = q_1$.

Now, if it is here, from this point to this point, anything, marginal revenues, here marginal cost is same as marginal revenue, here marginal revenue is more, but if it produces more, so it will, marginal cost here, but marginal revenue here, so it will not produce. Here if it produces, marginal revenue will be here, marginal cost is going to be here. So, it will not produce.

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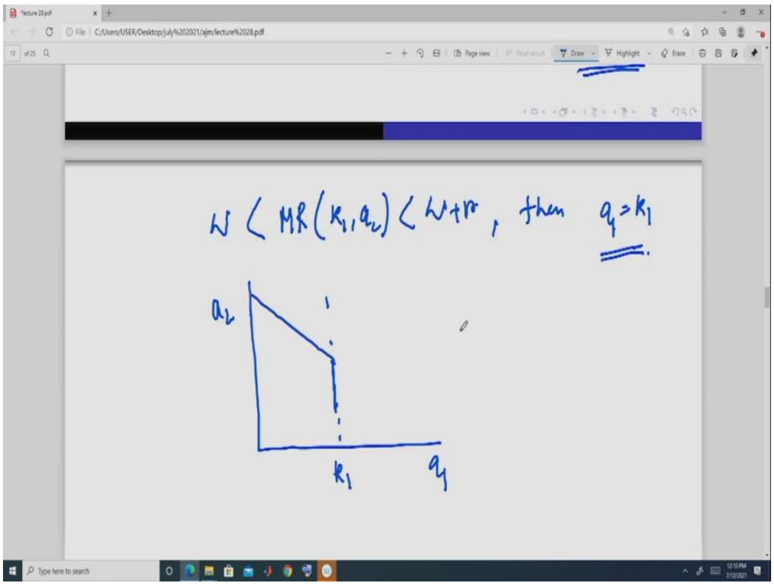

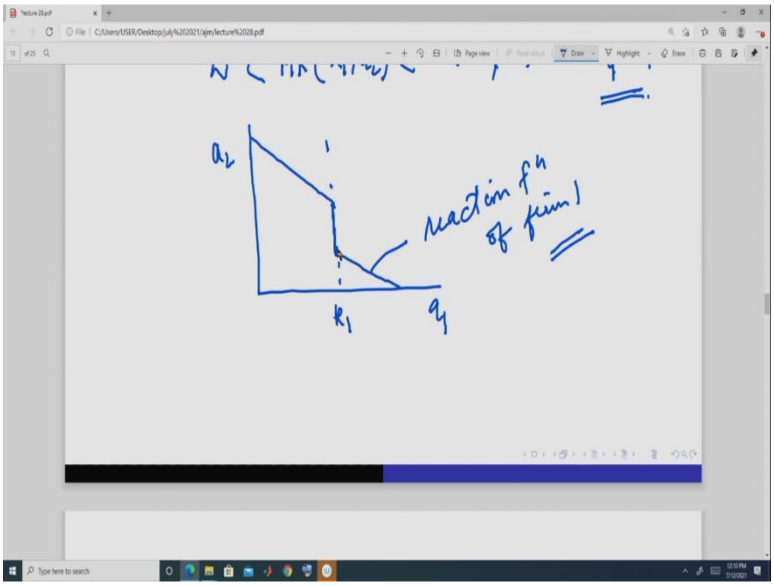

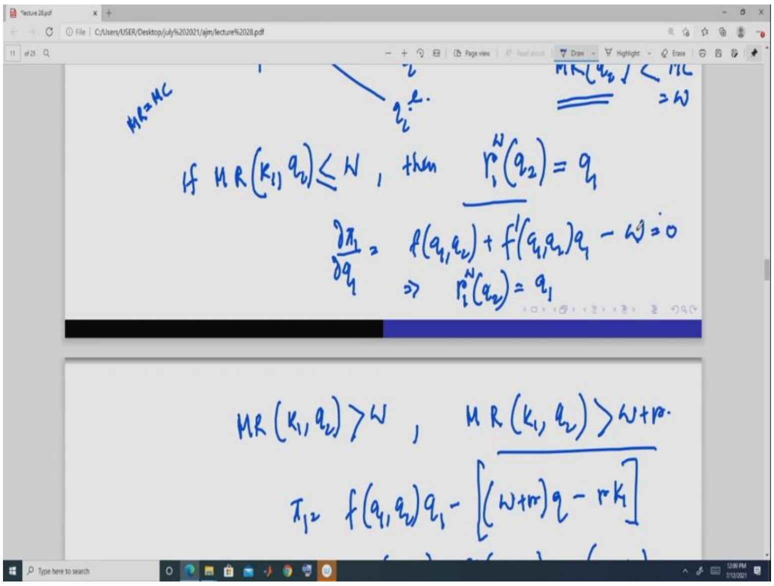

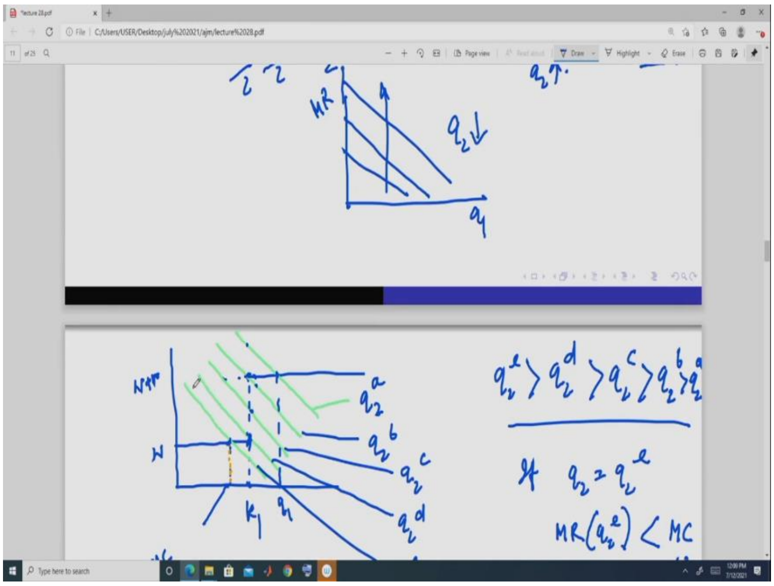

So, in this range, if the output is such, output of firm two is such that the marginal revenues are of this nature, that is, if the marginal revenue of, if it lies between this and this-$w < MR(k_1, q_2) < w + r$, then it will produce same, up to its capacity. I got this. Then this means what? So, this means, see, we can now draw the reaction function of firm 1. This is output of firm 1, this is output. Suppose capacity is this, k1.

Now, from here, what do we have got? This is the reaction function, when this and when the marginal revenue is this, when this is the marginal revenue, when the output of firm 2 is very high. So, that means, when output is here. So, if it is here, then in this case, so marginal reaction function is given by this a-$p_1^w(q_2) = q_1$. And this reaction function so it is given by this, equal to 0. So, it is suppose something like this.

Now, at from this, this region, this region, so some intermediate amount of q2. So, here it will be producing only q1. So, for some this, it will be some amount like this. Then again if it was like this, for these curves, or when the marginal revenues are like this, so here it is increasing, right. So, output of firm 2 is less and less. So, it will be here. Here in this position. So, then reaction function is given by this region, this portion. So, it will be like this. So, this is going to be the reaction function of firm 1, okay. So, it is going to be kink at the capacity. This so, let us do 1 example and it will be clear to you.

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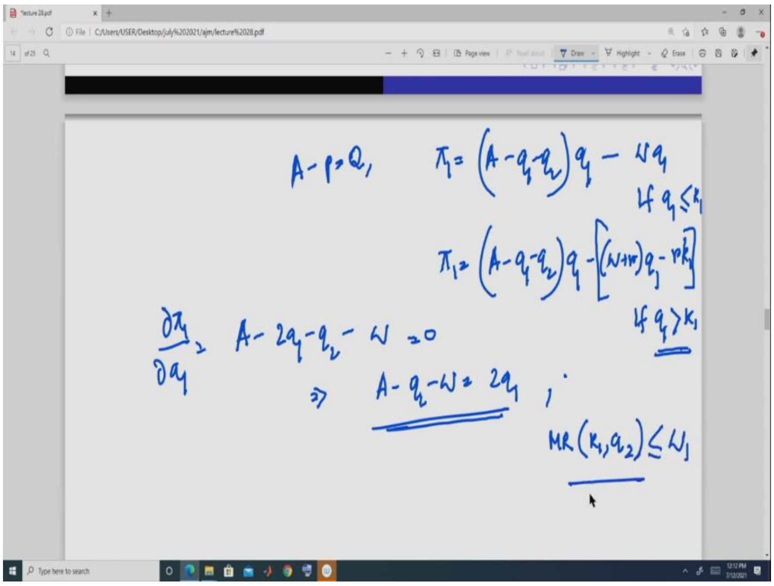

Suppose the market demand function is A p is equal to q- A-p=Q. So, the profit function of firm 1 is-$\pi_1 = (A - q_1 - q_2)q_1 - wq_1$. If this is less than equal to k1 and it is going to be this, okay-$\pi_1 = (A - q_1 - q_2)q_1 - [(w + r)q_1 - rk_1]$. Now, here if we take the derivative of this, it is this-$A - 2q_1 - q_2 - w = 0$. So, this is the reaction function-$A - q_2 - w = 2q_1$. So, this is the reaction function, when. Again.

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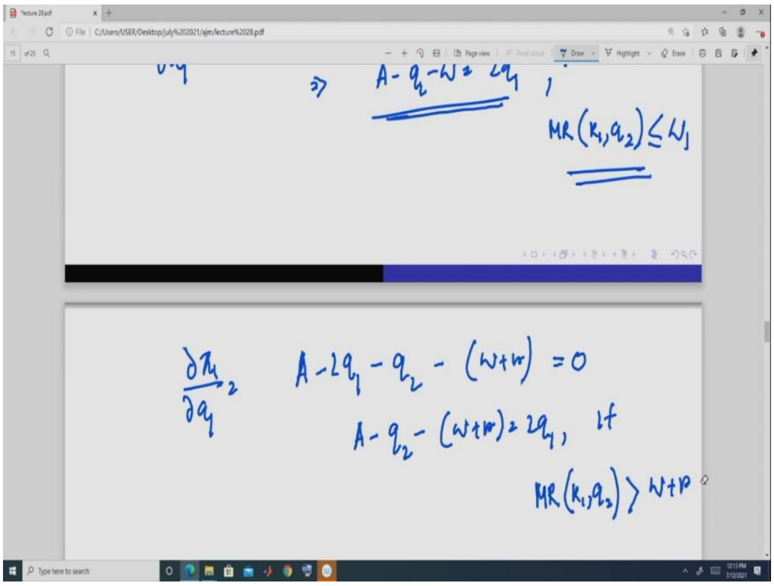

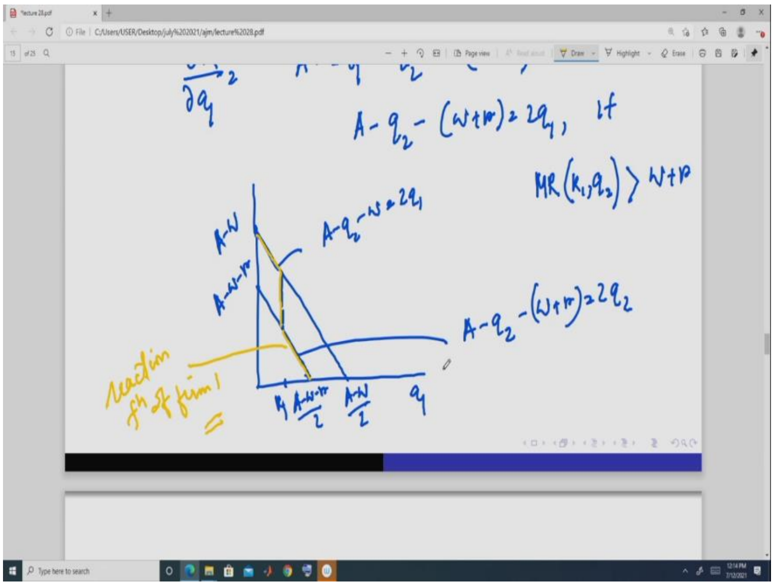

So, this is equal to 0, is reaction function, if it is w plus r-$A - q_2 - (w + r) = 2q_1$. So, but, if we try to plot these two-reaction function, it is going to be this. It is this. Now, this function is this one, so this is and this is. Suppose k is here, so reaction function of it is going to be this, this, this. So, this is the reaction function of firm 1, this yellow line. So, it is kink like this, if k is this.

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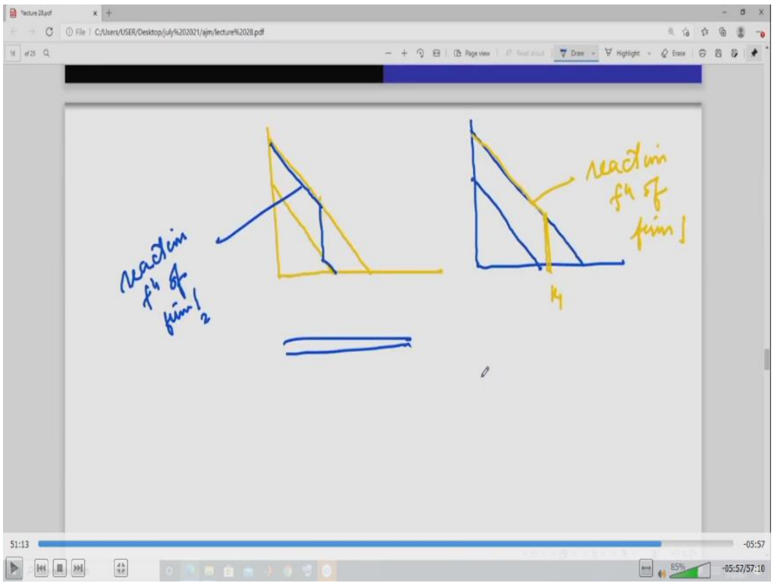

Now, if k is here, then the reaction function is going to be like this, this blue line. Now, we may have a situation like this. This is k1. Reaction function is this. This is the reaction function of firm 1. Here this blue line is the reaction function of firm 1. So, we have understood the reaction function of firm 1. It is going to be of this nature, right? We have solved 1 example also.

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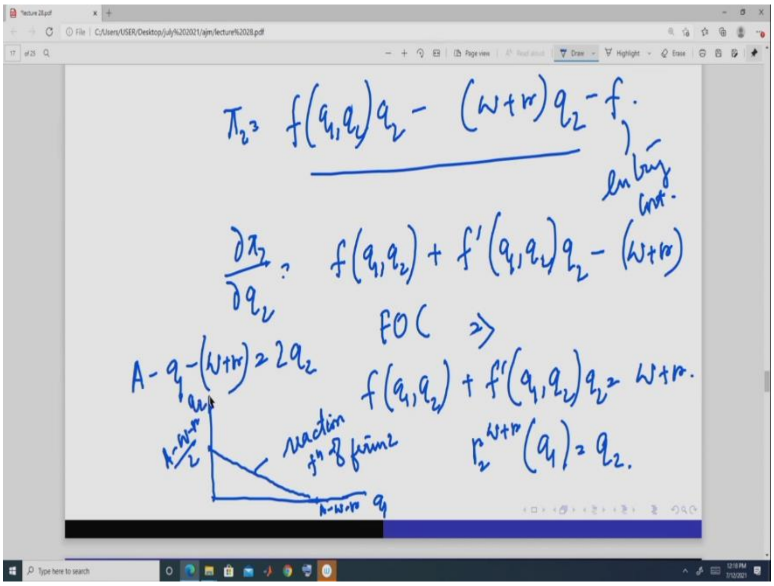

Now, what is going to be the reaction function of firm 2? Firm 2, its reaction function, if we take the general demand function, profit function is this, f-$\pi_2 = f(q_1, q_2)q_2 - (w + r)q_2 - f$. This is the entry cost. So, it is going to be-$f(q_1, q_2) + f'(q_1, q_2)q_2 - (w + r)$. So, first order condition implies what? This, so, we will get the reaction function. Reaction function of firm 2 is always going to be this-$f(q_1, q_2) + f'(q_1, q_2)q_2 = (w + r)$.

And if we plug in here, then it will be going to be-$A - q_1 - (w + r) = 2q_2$. So, two q2. So, it is, this is going to be a minus by 2, and this is suppose, so this is the reaction function of firm 2. In this axis we have taken q1, in this axis we have taken q2, like this we will get. So, we know the reaction functions of firm 1 and reaction functions of firm 2, okay.

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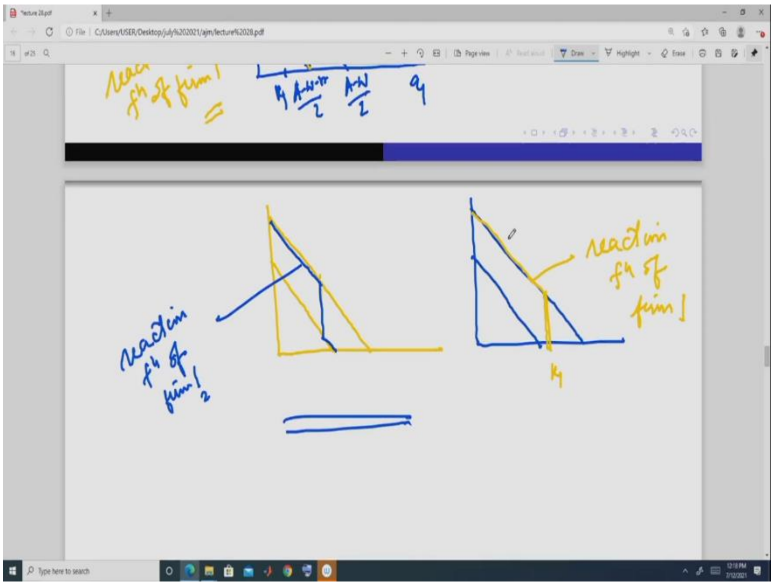
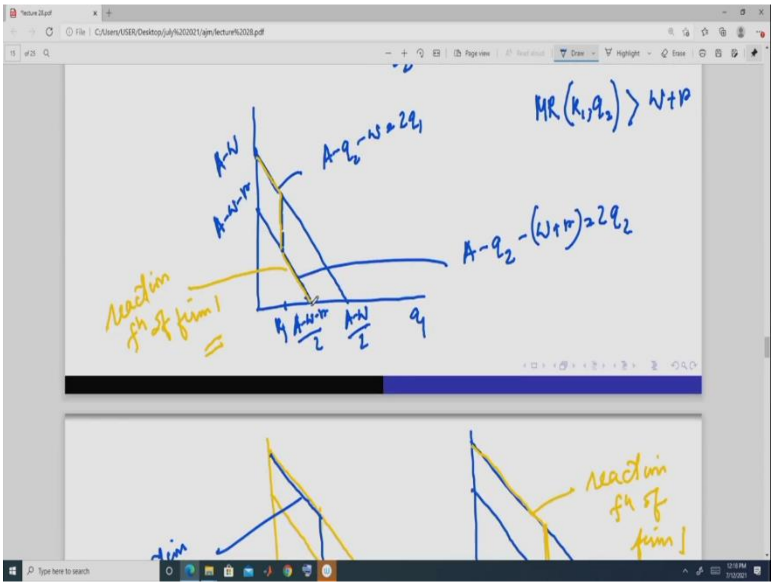
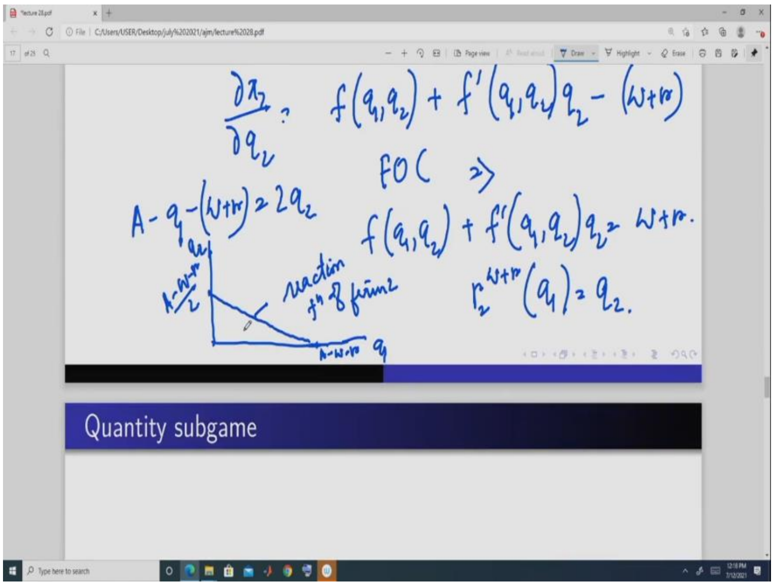

reaction fn of firm 1
reaction fn of firm 2
q1


MR(k1,q2) > w+r
A-q2-w = 2q1
A-q2-(w+r) = 2q2
A-w
reaction fn of firm 1
q1


∂π2/∂q2 = f(q1,q2) + f'(q1,q2)q2 - (w+r)
FOC ⇒
f(q1,q2) + f'(q1,q2)q2 = w+r.
A-q1-(w+r) = 2q2
reaction fn of firm 2
q1
Quantity subgame

So, firm 1's reaction function is kinked. It can be of this nature, it can be of this nature or it can be of this nature, right. And firm 1’s reaction function is this.

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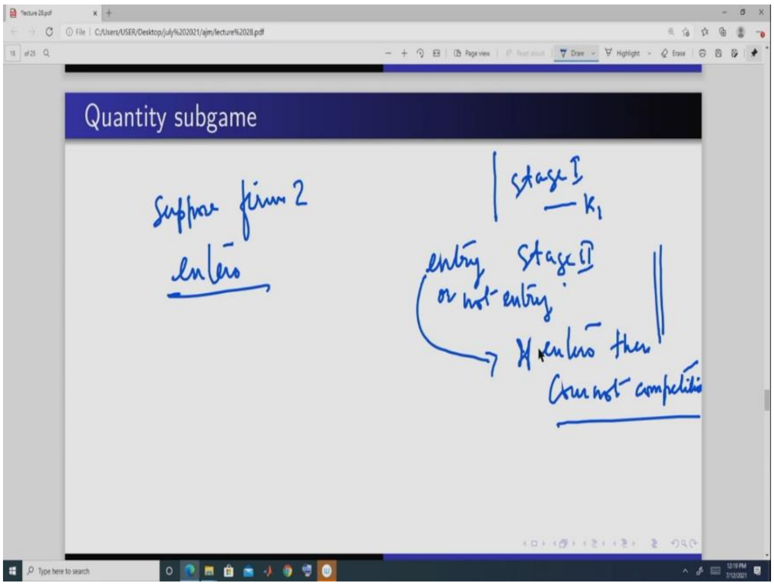

Now, we try to solve this game. So, this game, it is like this. In stage 1, this is decided, small k1. In stage 2, entry or not entry. If enters, then Cournot competition, right? So, this is the a. So, we suppose, firm 2 enters. So, in this stage, this is going to be the last stage, quantity competition, that is the Cournot competition. So, we will do that first, okay. And then we will move backwards.

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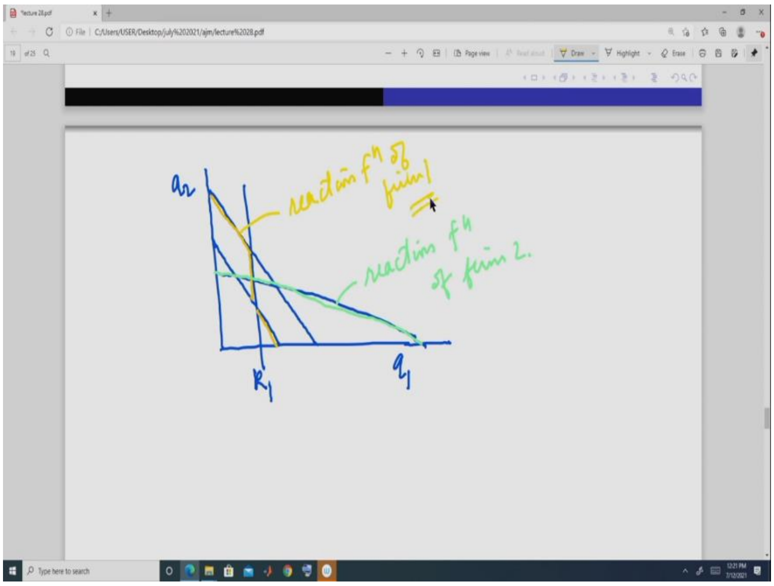

So, one possibility can be of this nature. Suppose this is q1 and this is q2, and this is suppose k1, okay. And this is the reaction function of. So, this yellow line is the reaction function of firm 1, okay. And this green line is the reaction function of firm 2. Firm 1 given capacity k. We get this, because the capacity is this.

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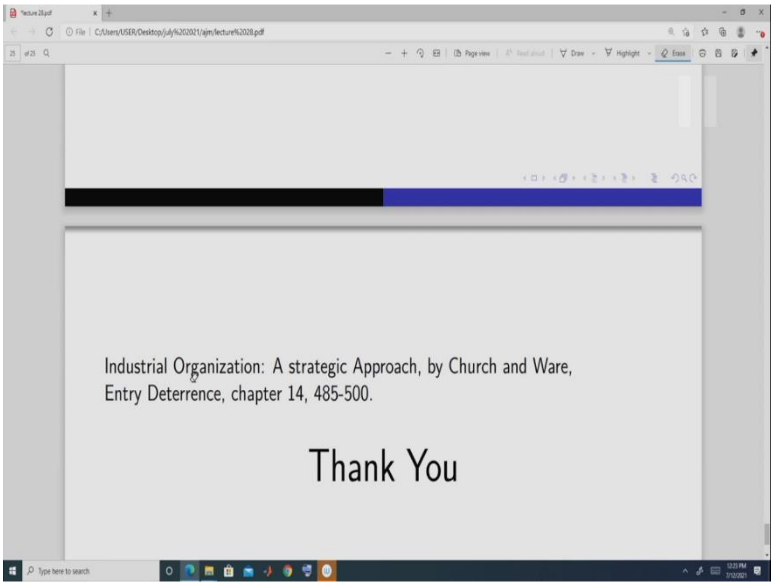

You can read this portion from this chapter 14 of this book by Industrial Organization: A Strategic Approach, by Church and Ware, Entry Deterrence. This chapter is entry deterrence and these are the specific page numbers. So, thank you now. So, we will continue this in the next class.

Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 11
Lecture 40
Tutorial

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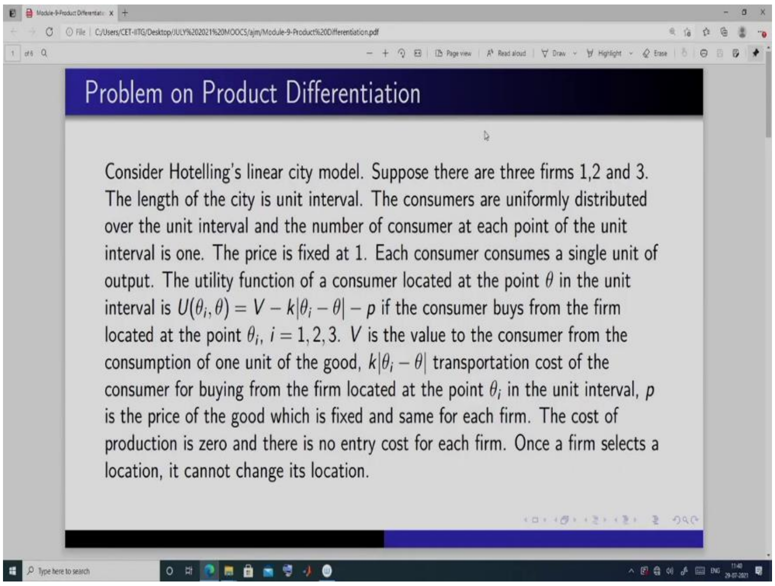

So, let us solve 1 problem on product differentiation and in this problem what I have done? I have combined both the sequential and the simultaneous decision. Because in the case of product differentiation we have done separately 1 section on where firm decides the location simultaneously. And in another section we have done firms decides the location sequentially, okay. So, now here in this question is these 2 things are combined.

So, let us look at the problem. So, consider Hotelling linear city model. Suppose there are 3 firms, firm 1, 2 and 3. The length of the city is unit interval. The consumers are uniformly distributed over the unit interval and the number of consumers at each point of the unit interval is, okay 1. So, it is specification is same till now. The price is fixed at 1 and each consumes a single unit of the output.

Utility function of a consumer located at the point theta in the unit interval is this-$U(\Theta_i, \Theta) = V - k|\Theta_i - \Theta| - p$ if this V that is the value the consumer gets from the consumption of that good and this is the, disutility that it derives from by travelling some or making some distance travelling some distance to buy that product.

And this is the price, price is 1, so it is equal to 1. If consumer buys from the firm located at point theta i when whereas the consumer is located at the point theta, okay. And theta I will take depending on the location from which firm it is buying it will give the utility. And V is the value to the consumer from the consumption of 1 unit of the good. And this is the transportation cost of the consumer for buying from the firm located at point theta i. Interval and p is the price 1 which is fixed and same for each firm, okay.

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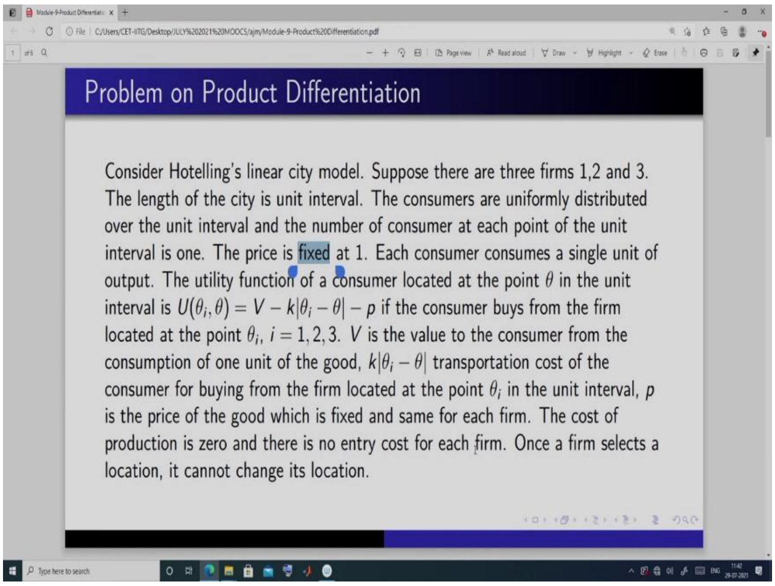

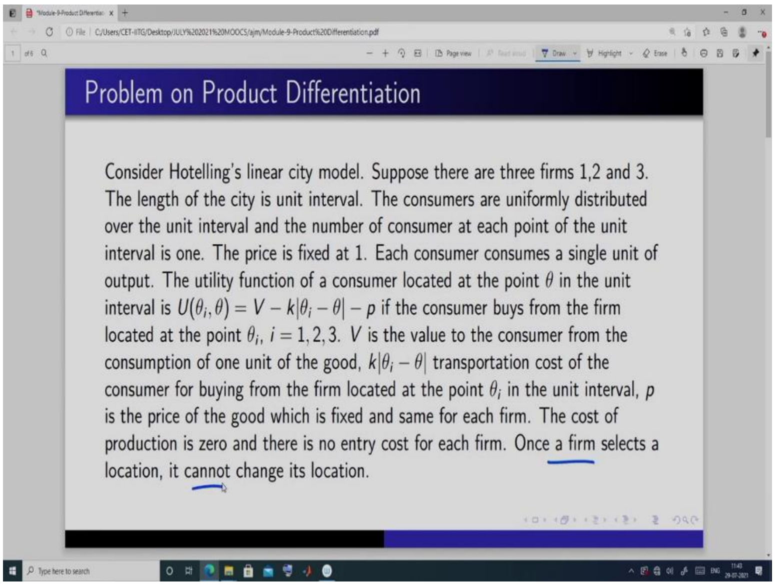

It is fixed at this and the cost of production is 0 and there is no entry cost because here the firms are also in entering sequentially 1 after another, so no entry cost. And once firm select a location it cannot change his location. So, once its chooses the location it fixed like firm 1 when it moves fast, if it chooses the location and then when firm 2 and 3 they enters it cannot change his long it will be there.

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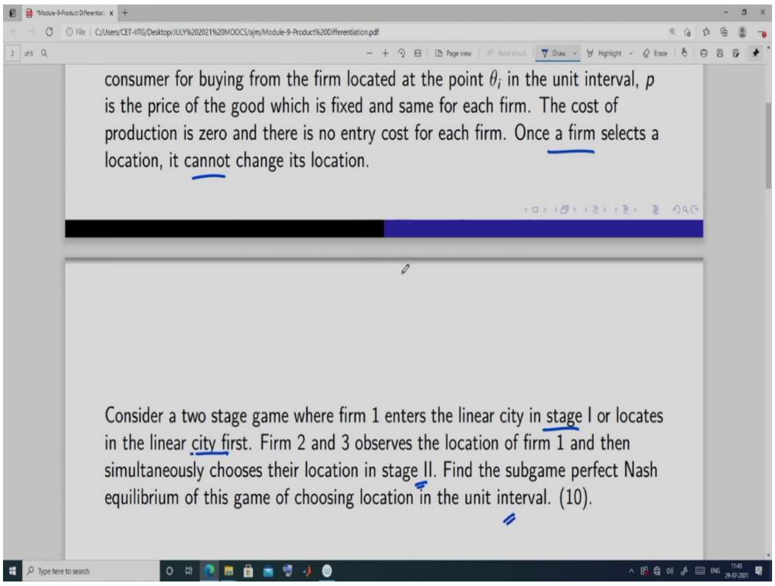

Now, consider a 2-stage game where firm 1 enters the linear city in stage 1 or locates in the linear city first firm 2 and 3 observes the location of firm 1 and then simultaneously choose their location in stage 2. So, firm 2 and 3 they are moving simultaneously. Find the subgame perfect Nash equilibrium of this game of choosing location in the unit interval, okay.

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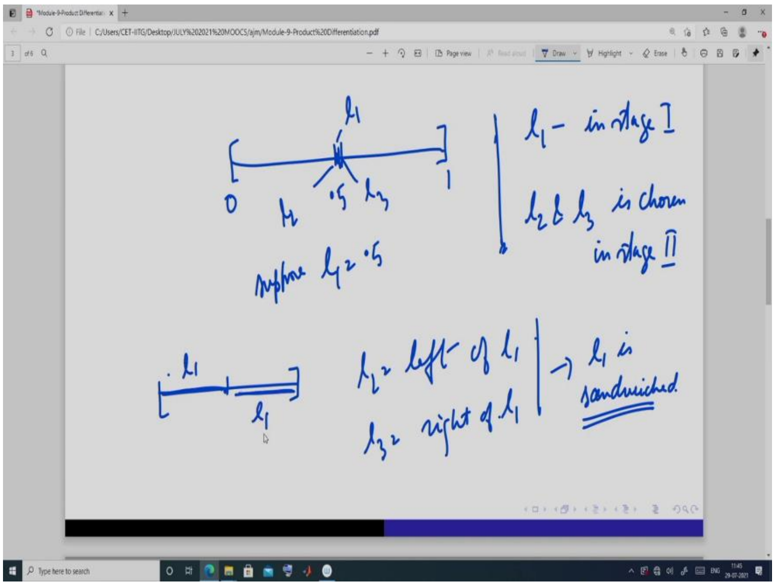

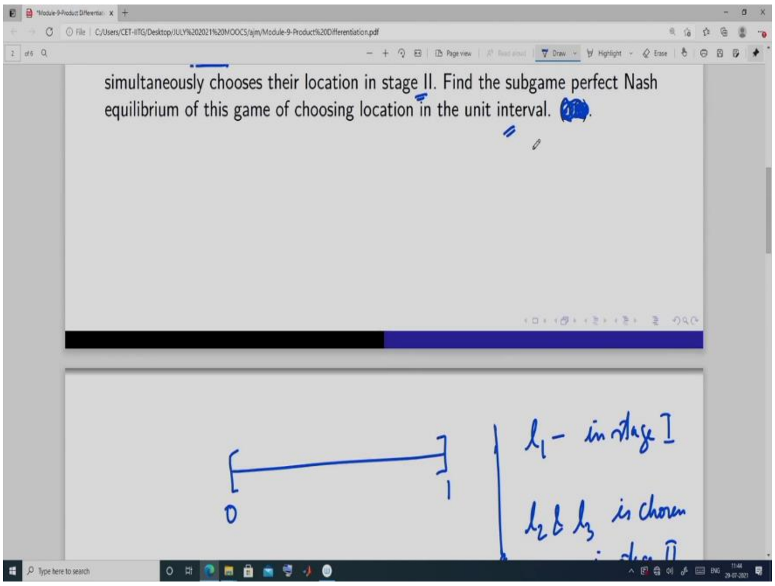

Now, let us solve this. So, this is the unit interval, okay. Now, what happens l1 is chosen in stage 1, l 2 and l 3 is in stage 2? okay So, we will solve this using backward induction and we have to find the subgame perfect Nash equilibrium, okay. So, what we are going to do? So, first suppose l 1 is this or let us solve this. Suppose this is suppose l 1 is at this point, 0.5 this is the middle and l 1 is here. Then in stage 2 firm 2 can choose this and firm 3 can choose this position.

So, this is l 2 and this is l 3. So, l 2 is to left of l 1 and l 3 is right of l 1. So, l 1 is sandwich, sandwich between these 2. So, when there it is sandwich, so it means what? It is not getting any consumers. So, its profit is 0, right? So, it will not select this. So, from here we get that it will be either in this or it will be in this, okay. It does not matter.

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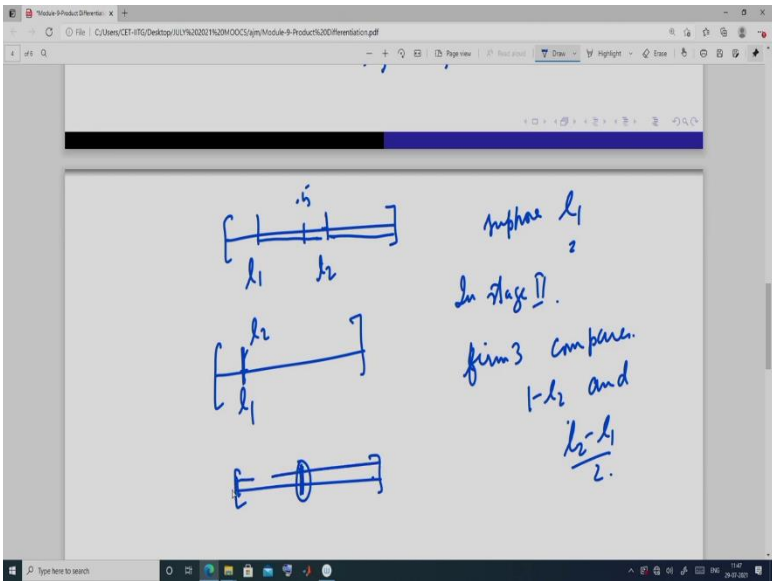

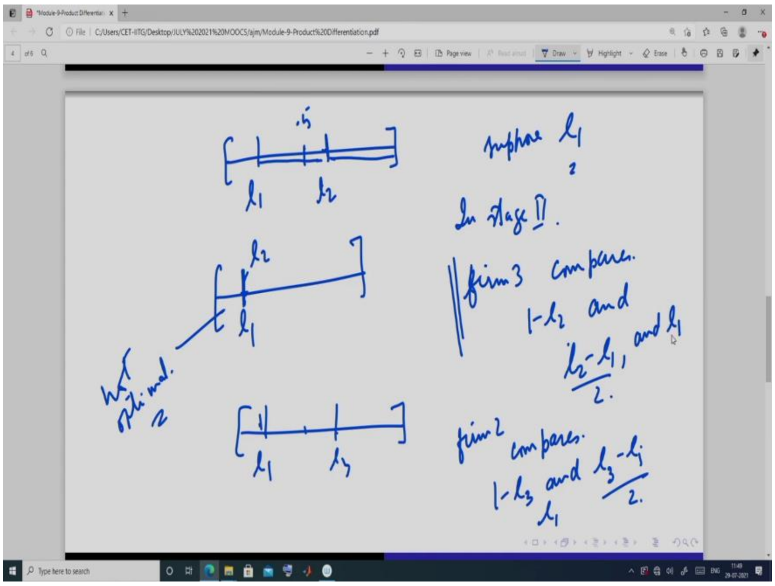

Now, suppose l 1 is here. Now, in stage 2 what firm what is happening firm 2 and 3 are entering simultaneously. Now, if suppose firm 2 selects this, okay firm 2 select this. And firm 3 while deciding since its output because it is deciding simultaneously along with it. So, it will select if its it will compare this half of this and it will compare this. So, it will firm 3 compares 1 minus l 2 and l 2 minus l 1 half of this because if firm 2 its locate suppose here, l 2 is here right of this. Then the best response of firm 3 is to locate here, okay.

Because you can simply use the argument that you have got from when there are 2 firms and suppose unit interval we have got this you get at the middle. Because if it locates here the other firm also locate here and it gets the whole. So, that is why this is not i. So, you can argue in that way. So, in this line here it is this is not optimal. So, if this is not optimal, then it will look at somewhere here. So, it will it has chosen suppose this l 2 is this. If l 2 is this, then firm 3 will compare this- $1 - l_2$ and $\frac{l_2 - l_1}{2}$.

Now, because l 2 and l 3 is being chosen simultaneously. So, if this is l 1 and suppose this is l 3 because l 3 is not going to be here again same using the same argument. Then firm 2 compares 1 minus l 3 this distance and it compares this half of this- $\frac{l_3 - l_1}{2}$ and it will also compare this because suppose it will compare this this and it will compare l 1. If l 1 is greater than it will locate here and if here also it will compare l 1. So, out of these 3 lengths it will choose the 1 which is the maximum.

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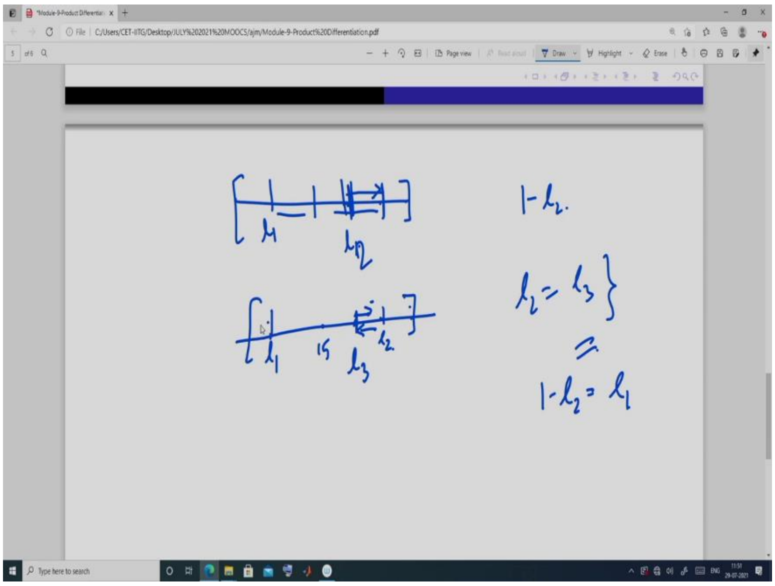

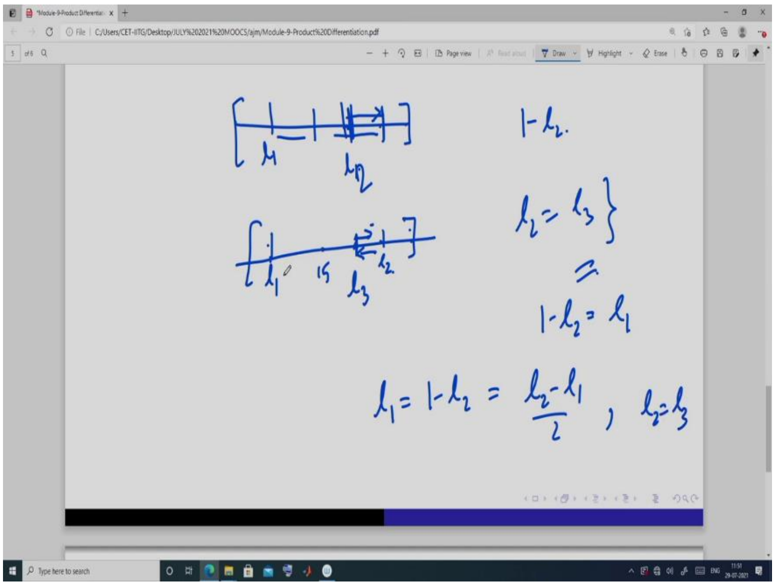

So, from this argument we get that firm 1, so given in location l 1. Firm 2 and 3 they will always try to see. If this is l 1 and see this is l 1, this is l 2 then firm 1, 2 will either look at here or it will look at here. Because if it locates here, then best response of firm 2 is not to here but to shift till this point, because it will get a more a. And so, l 1 is this, this is 0.5 and if this is l 3 then l 2 will locate here along with this l 3. So, this is and I hope it is clear.

Because if it is l 2 is here then firm 1, 3 will move this or if l 3 is there, firm 2 will move will gain by moving in this direction. So, that is why l 2 is equal to l 3, this is true. Now, when this is true? If we from this diagram because this length is very small, so we get that this should be true. Now, this would be true when? When 1 minus l2 this is equal to l 1 this distance is equal to this distance-$1 - l_2 = l_1$. So, from this we get a condition that l 1 must be equal to 1 minus l 2 and l 2 minus l 1 and l 2 is equal to l 3.

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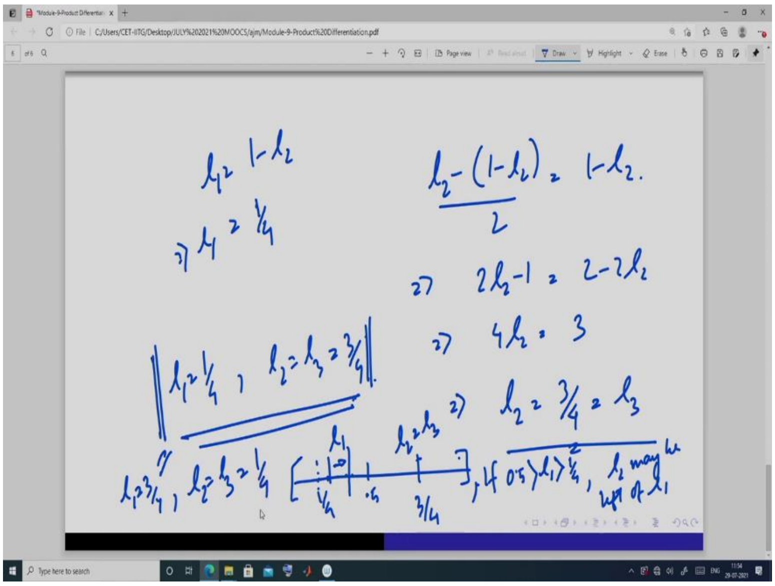

So, and when we solve this what do we get? We get this-$\frac{l_2-(1-l_2)}{2} = 1 - l_2$, so we get 3 by 4 and this is equal to l 3 and l 1 is equal to 1 minus l 2. So, l 1 is equal to 1 by 4. So, l 1 is equal to 1 by 4, l 2 is equal to equal to l 3 it is equal to, so this is another-$l_1 = \frac{1}{4}, l_2 = l_3 = \frac{3}{4}$. So, this is a subgame perfect Nash equilibrium. So, what do we get? We get this 1 by 4 is going to be l 1 is 3 by four is going to be equal to l 3. Because if you look at any other point you will see that there is it is not an Nash equilibrium, why?

Because if it moves here slightly less than this. So, then what is happening? l1 is reducing, so there is should not move here. But if it moves slightly here in this, so if it is here suppose l 1 then what is happening? This length is greater than this length, so this firm will shift here and there firm will shift here. So, it will be like this. So, if l 1 is greater than 1 by 4 and it is less than 0.5 then again what is going to happen? This l 2 will be either, so l 2 may be left of l 1.

So, again firm 1 may get sandwiched, so that is why it should not move here. So, that is why this is 1 subgame perfect Nash equilibrium. And we may have another subgame perfect Nash equilibrium which is l 1 is equal to 3 by four and l 2 is equal to l 3 and it is equal to 1 by 4. So, we will have only these 2. Now, here you will see what is happening? So, this model is about product differentiation and this is about horizontal product differentiation. So, here we will see that 1 firm is differentiating and the other 2 firms are producing homogeneous product. They are not differentiating, but they are differentiating with respect to the product of firm 1, okay. Thank you.

Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 12: Entry Deterrence, Bundling and Tying
Lecture 41
Dixit is Model of Entry Deterrence

Okay, Hello everyone! Welcome to my course, Introduction to Market Structures.

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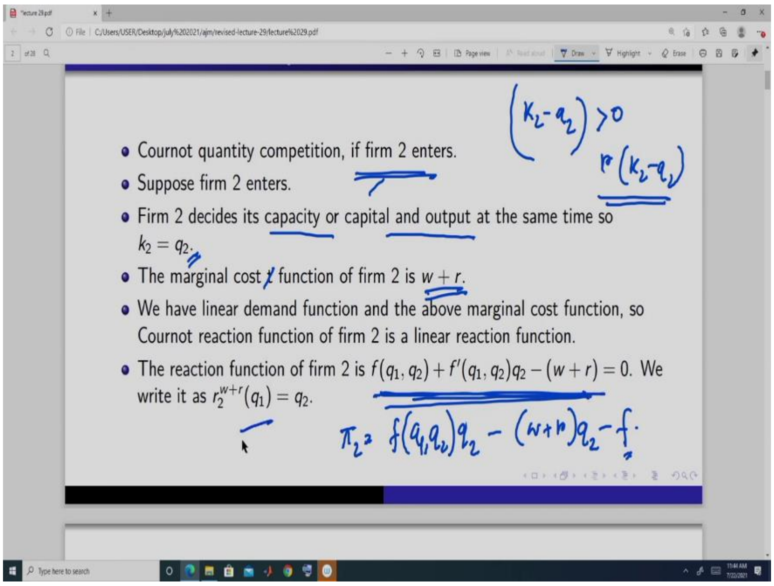

So, we were doing entry deterrence model and specifically we were doing the Dixit model. So, in the last class we have derived a reaction function of firm 1 and the reaction function of firm 2 given that firm 2 enters. Now, firm 1 here is the incumbent firm. So, it can invest in its capacity which is same as capital and firm 2 is an entrant firm and it enters if it is profitable for that firm to enter. If its profit is 0 if it enters, then that firm does not enter, okay.

So, today we are going to solve this problem and we are going to solve this assuming a general downward sloping demand curve, okay. Suppose we assumed at the firm 2 suppose enters, okay. So firm 2 enters, okay. So, when it enters we know there is Cournot competition, okay. So, based on that we will get that there are Cournot reaction functions, but the reaction function of firm 1 is going to be slightly different than the usual Cournot reaction function that we have done a while doing the Cournot oligopoly or Cournot duopoly but for firm 2, it is going to be same as the Cournot reaction, okay.

Now further firm 2 decides its capacity and capital at the same time, okay. So, it is this, so it means that when it is deciding his output at the same time it is deciding the amount of capacity it wants to have. So, it will never going to have capacity more than its output. So, if it is more so, if suppose capacity is this-$k_2 - q_2$ and it produces some amount this and this amount is suppose positive, then the cost on this excess capacity is this much-$r(k_2 - q_2)$.

So, it is not going getting any return on this excess capacity. So, that is why it is not optimal strategy for firm 2 to have a capacity higher than its output. So, that is why k2 is always going to be equal to q2 that is output of firm2. And marginal cost, function of firm 2 is this- w+r because it decides its capacity and the labor at the same time.

So, and since suppose we assume that the demand curve is linear, then we get the reaction function of firm 2 in this form-$f(q_1, q_2) + f'(q_1, q_2)q_2 - (w + r) = 0$. Why do we get this? Because we take the profit function of firm 2 can be written in this form this is suppose the demand curve or demand function output of firm 2, so this is the total revenue, this is the total cost for on production and the cost for entry is this. So this is the profit- $\pi_2 = f(q_1, q_2)q_2 - (w + r)q_2 - f$ and if we take the derivative with respect to q2 we get this reaction function, okay and we denote it in this way which is a function of q1, output of firm 1, okay.

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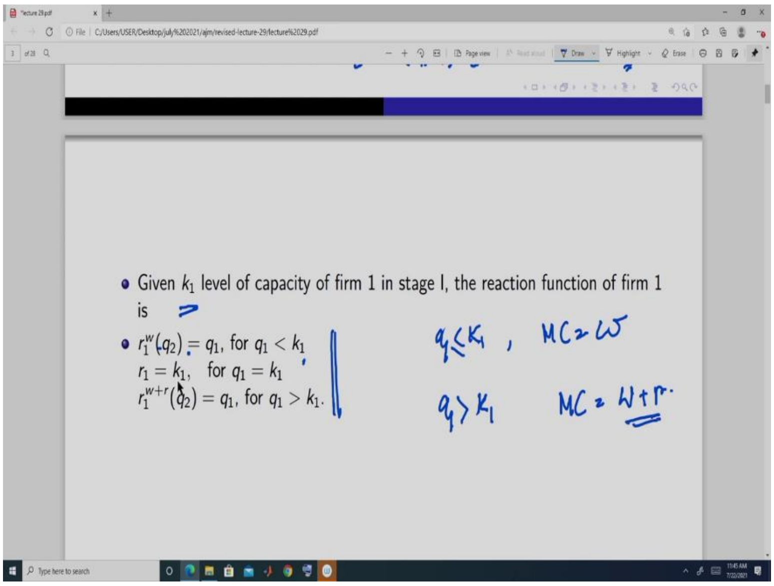

And reaction function of firm 1 suppose there is a k1 capacity which is an arbitrary number, okay in stage 1. Then the reaction function of firm 1 in stage 2 it is given like this, this is the case- $r_1^w(q_2) = q_1$ when q1 is less than k1. So, when the it is producing less than its capacity, then it requires marginal cost is only this. So, when q1 is less than this less than equal to this, marginal cost is only w. And when q1 is greater than this k_1, then the marginal cost is it is this-w+r, because it has to now expand its capacity. So, that is why from there we get this and we have done it in detail in the last class, okay.

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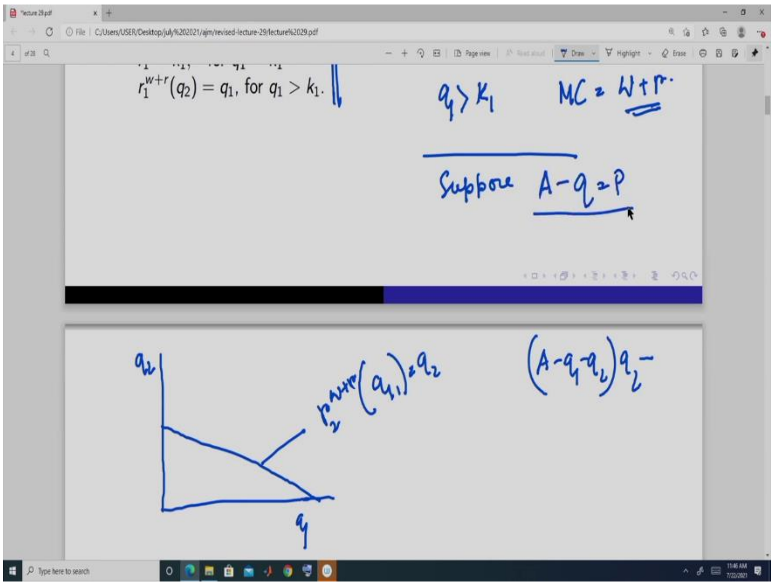

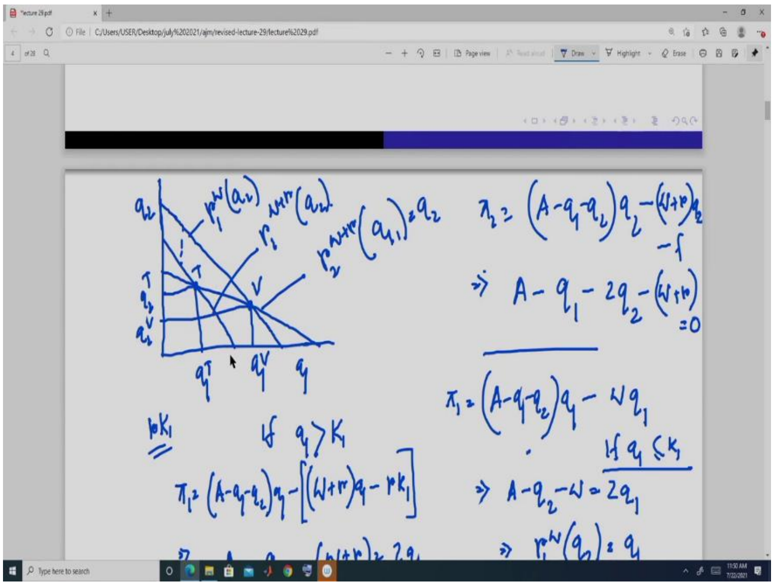

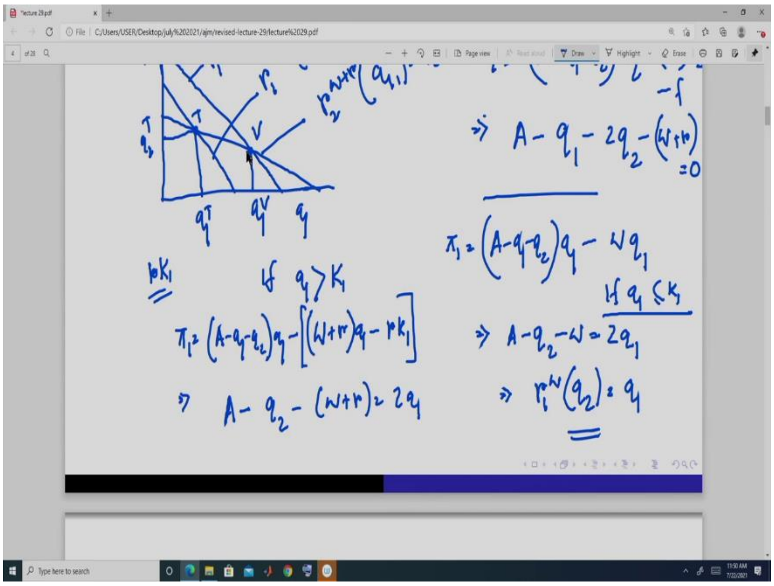

Now, we will denote some specific outputs and our analysis will be based on those outputs suppose this is the output of firm 1 output of firm 2 and this is the reaction function of firm 2, okay this is, now here you can look at suppose the demand curve is this. Demand curve, so let us write here, suppose demand curve is this-A-q=P, okay then we get this, this is the profit function- $\pi_2 = (A - q_1 - q_2)q_2 - (w + r)q_2 - f$ and reaction function of firm 2 it is this- $A - q_1 - q_2 - (w + r) = 0$. So, it is this thing and the reaction function of firm 1 is like this-$\pi_1 = (A - q_1 - q_2)q_1 - wq_1$.

So, this is what, this it is this, this is reaction function- $r_1^w(q_2) = q_1$ if it is this suppose some k, k is big enough. So, we get this and the reaction function if output is greater than is this, then so it will be because it will be producing till it is k1, based on this capacity it will be this, in the A additional amount will be based on the, **so**, the marginal cost is this, not the total cost.

So, total cost for A because it has already incurred this much cost this in stage 1, right. So, it will be this one and so, the reaction function of firm 1 is… So, it is something that was this and this reaction function is this of firm 1. Now, this is the symmetric output, because this is the reaction function and this is the reaction function. So, we take this point to be the point T and this is this asymmetric thing, this point is V and this is V.

This is the Cournot outcome when we have reaction function is this, this-$A - q_2 - w = 2q_1$ and if from here, now, if we can have k here and the reaction function of firm 1 is this. If k is here then the reaction function is like this, okay we have done that.

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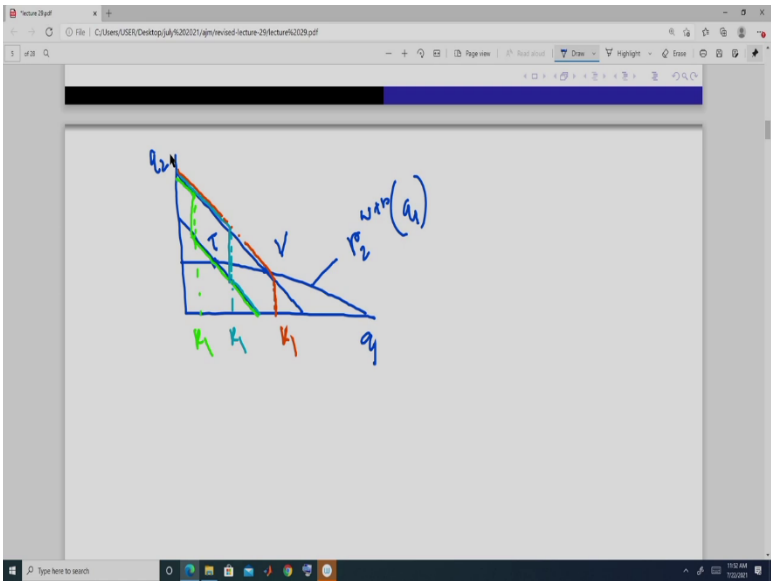

Now, what do we can see here this is the point T this is point V. So, we can have the reaction function of firm 1, if this is k1 then the reaction function is this green line we may have a situation like this if suppose this is k1, then this light blue line is the reaction function of firm 1 and if this is k1, then th**e** red line is the reaction function of firm 1 and we will do the…

So, the reaction functions are of this nature and we will now see that what is the outcome in stage 2. Why we are looking at stage 2? Because it is a two stage game and in a two stage game we look at subgame perfect Nash equilibrium and in subgame perfect Nash equilibrium we solved it using backward induction.

So, we will first solve the stage 2 assuming that suppose firm 2 enters, because moment firm 2 enters there is Cournot competition and we now we have defined the reaction functions of firm 1 and firm 2 here. Now first in the stage 2 we may have a situation where firm 1 needs to expand its capacity. When do we see that?

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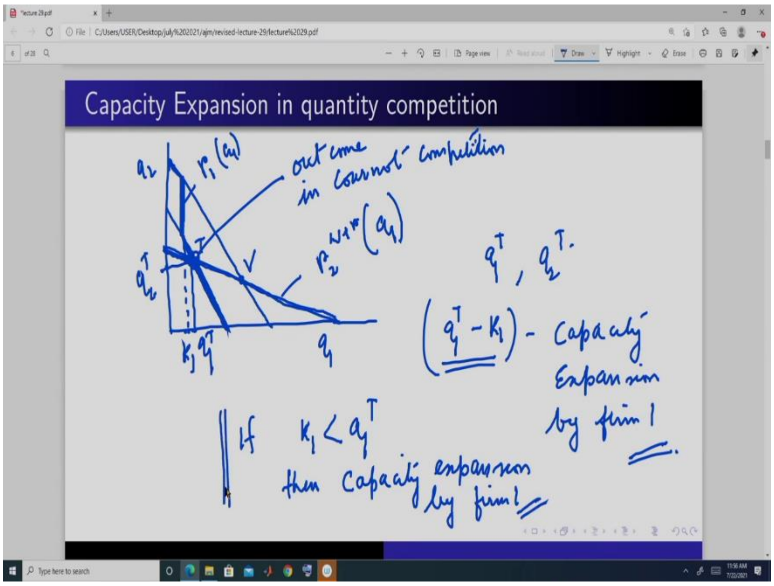

Suppose this is the outcome, this is q1, q2, this is the reaction function of firm 2, okay. This point is V this point is T and suppose K is here. So, the reaction function of firm 1 is this thick blue line, okay. Now, if this is the reaction function of firm 1 and the reaction function of firm 2 is this, then they intersect at this point. So, the outcome in stage 2 is this point, so this is the outcome in Cournot competition, right?

So, if this is the outcome in Cournot competition because this is the reaction function of firm 1, okay this line, this kink line. So, they intersect here, so, what is the output of firm 1? Output of firm 1 is this, output of firm 2 is this capacity of firm 1 is k1. So, this much amount-$q_1^T - K_1$ it has to expand. So, this much is amount of capacity expansion by firm 1, okay in this situation. Because this is the reaction function of firm 1 and this is the reaction function of firm 2 they intersect here.

So, this point is the Cournot outcome, Cournot quantity competition outcome. So, the output of firm 1 is this, output of firm 2 is this and here this much, this is greater than the capacity of firm 1. So, this so, it will lead to an capacity expansion by firm 1, okay. So, in this what happens if k1 is less than this output q1 T, then we see that there is capacity expansion by firm 1, this is the outcome in this kind of scenario when the capacity of firm 1 is not sufficient.

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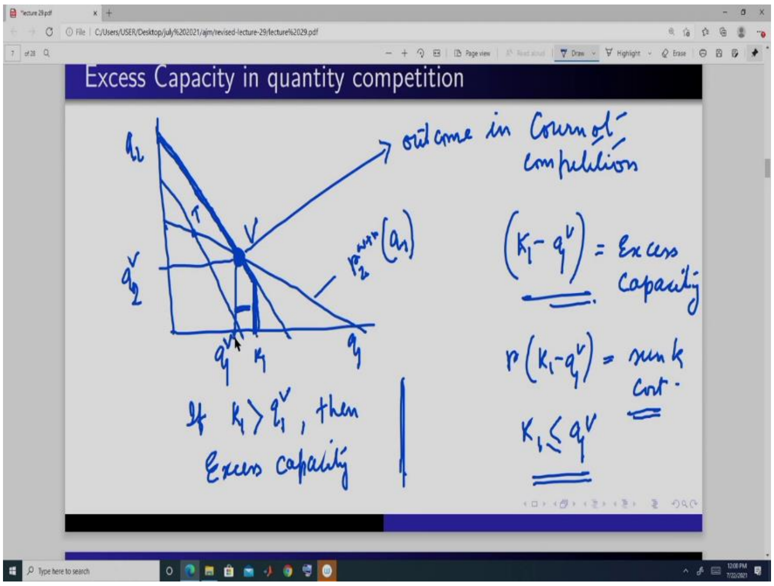

Now let us when we have excess capacity, firm 1 may have a situation where it has excess capacity. So, this is suppose the reaction function of firm 2. So, this point is p and this point is v we know these points, okay. Now suppose the capacity of firm 1 is here k1. What is the reaction function of firm 1? Reaction function of firm 1 is this thick blue line, kink at this point and reaction function of firm 2 is this, this point is the outcome in Cournot competition.

Now, if this is the outcome in Cournot competition, what is the output? This is the output of firm 2, this is the output of firm 1; okay capacity is this much, so this much sis the excess capacity k1, this should be small k, okay q1 V. So, this is the amount- $K_1 - q_1^V$ on excess capacity. Now, what is the cost in this?

This is you can say, this is equal to sunk cost, because firm 1 is not going to get any return on this. So, firm 1 will never want to have this unnecessary cost because it is not getting any return to this. So, that is why here we get that the k1 should always be less than equal to q1 V, this is the outcome- $K_1 \leq q_1^V$. And here this is the excess capacity and when do we get excess capacity? When k1 is greater than q1 V.

So, if k1 is greater than this output of, this much level of output of firm 1, then there is excess capacity, right? we get this. Now, when this is one outcome and firm 1 we will never want to have excess capacity here because if it has then it is a sunk cost for it, this much amount of cost is it is not getting any return on it, it is unnecessary. Because if it has this then also the outcome is this, okay if capacity is here is equal to this outcome is same. So, this must cost is unnecessary.

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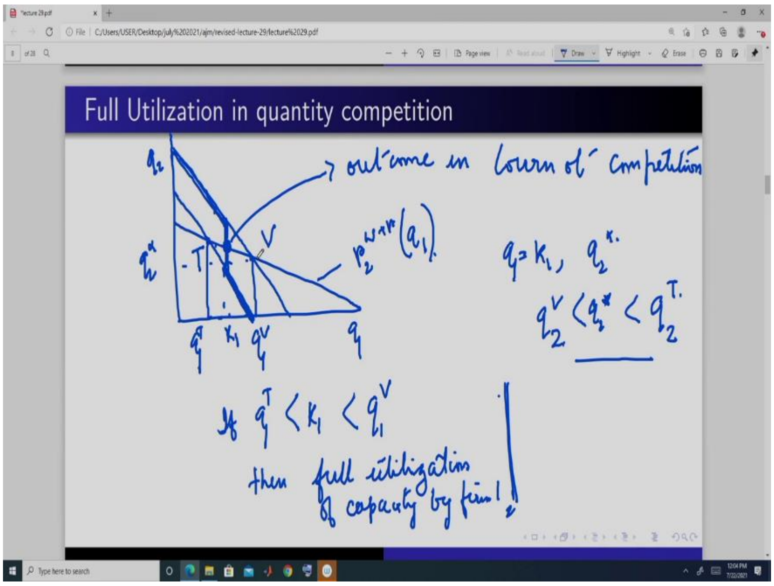

Next, let us look at when there is full utilization of capacity. Suppose this is the reaction function of firm 2, This is T and this is V, okay as we have already defined, okay. Now suppose capacity is here, so the reaction function of firm 2 1 is going to be this blue line this and where do they intersect, they intersect at this point. So, this is the outcome in Cournot competition.

If this is the outcome, then what is the output of firm 2? Firm 2 produces this much and firm 1 produces this k1. So, kl is equal to this-q1 and this is q star, where, q star is greater than this much amount, q1 V, sorry, and it is less than this in this situation- $q_2^V < q_2^* < q_2^T$, okay and output of this. So, when, so here you can take any if you look at this any point here, K here, it will be like this only, here it will be like this. So, if k lies within this range then full utilization of capacity by firm 1, okay.

So, we have seen that in stage 2 suppose firm 2 enters, then what are the possible outcomes depending on the capacity of firm 1? So, if capacity is very low like this, then the firm 1 needs to expand this capacity and the outcome is at this point T point, if capacity is too much, then the outcome is at when firm 2 enters is at V and the firm 1 has some excess capacity.

If the capacity of firm 2 lies within this range, then the outcome is at this point which lies between in between T and V and there is full utilization of capacity, okay. So, these are the three possible outcomes when firm 2 enters in stage 2, okay.

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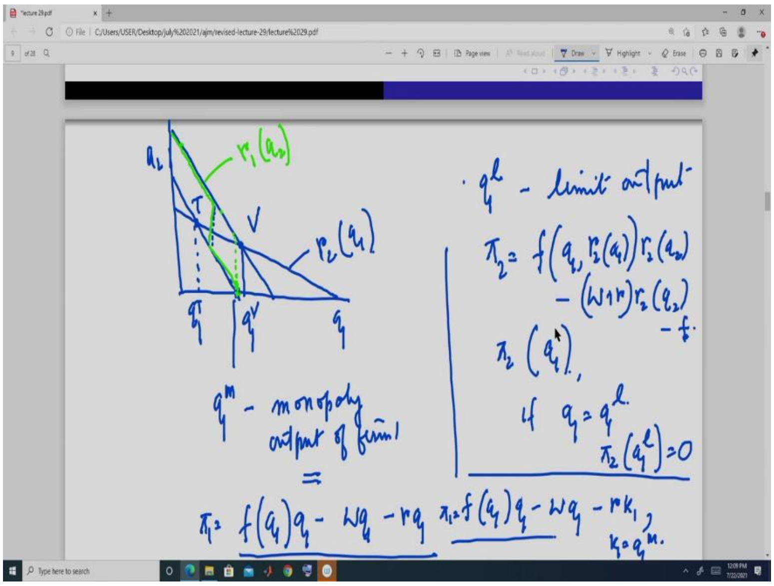

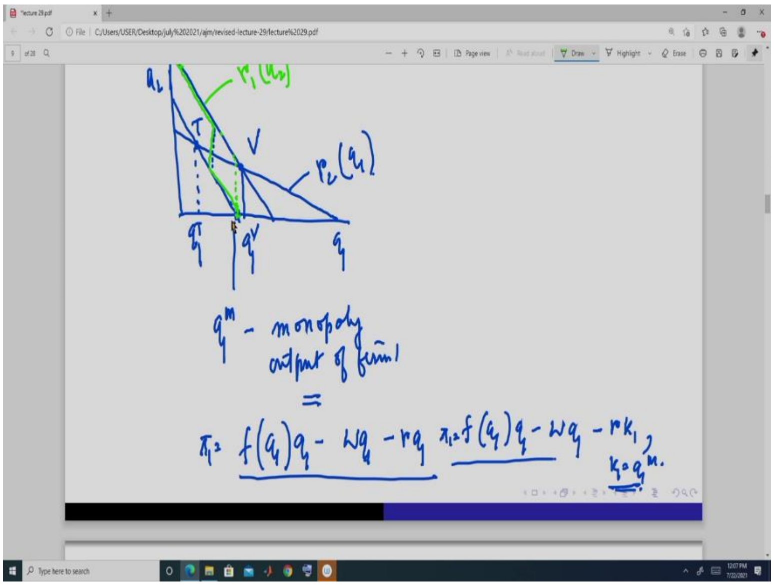

Now we will move to stage 1, but before moving stage 1 what we will do? We will define certain outputs of firm 1, because based on those outputs we are going to decide the optimal amount of capacity chosen by firm 1. So, let us look at this; this is the reaction function of firm 2, this is the point T and this is the point V, okay.

Now, this is the output of firm 2 which is q1 T, this is q1 V. Suppose capacity is somewhere here and we get a, suppose let us draw the reaction function by this green line, this is the green is the reaction function of firm 2 and this is reaction function of firm 1, okay. Now here this output is q1 m, this is the monopoly output of firm 1, okay. And in this monopoly outcome what do we have? The profit of firm 1 you can write this, here this much is going to be the output so, we will require this q1, this much amount of capacity also.

So, we can write it in this way and this is the monopoly profit-$\pi_1 = f(q_1)q_1 - wq_1 - rq_1$, okay. Now here this thing given or instead of this we can write it in such a way that to keep this same, where k1 is equal to this-$K_1 = q_1^m$, okay. This will also give me this point, okay because then the reaction function is going to be this point, this point okay and so this is going to be the monopoly outcome.

Now, we are going to also have another that is output that is this q1 l and this is the limit output, we have already defined what do we mean by limit output. Because we can write this profit of firm 2 can be written as a function of because we can write this… so, this function-$\pi_2 = f\big(q_1, r_2(q_1)\big)r_2(q_2) - (w + r)r_2(q_2)$ this is actually a function of q1 we can write it if we plug in the reaction function we get this.

Now, if this q1 is equal to output which is q1 l the limit output then it is this-$\pi_2\big(q_1^l\big) = 0$. So, that is why this is called the limit output of firm 1. If firm 1 produces this output, profit of firm 2 is going to be 0 we have to incorporate the entry cost also and the entry cost is F here, okay.

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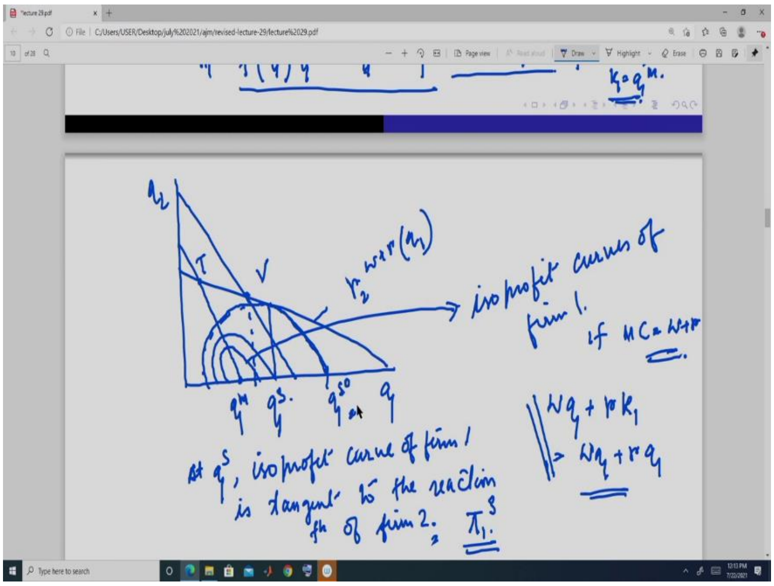

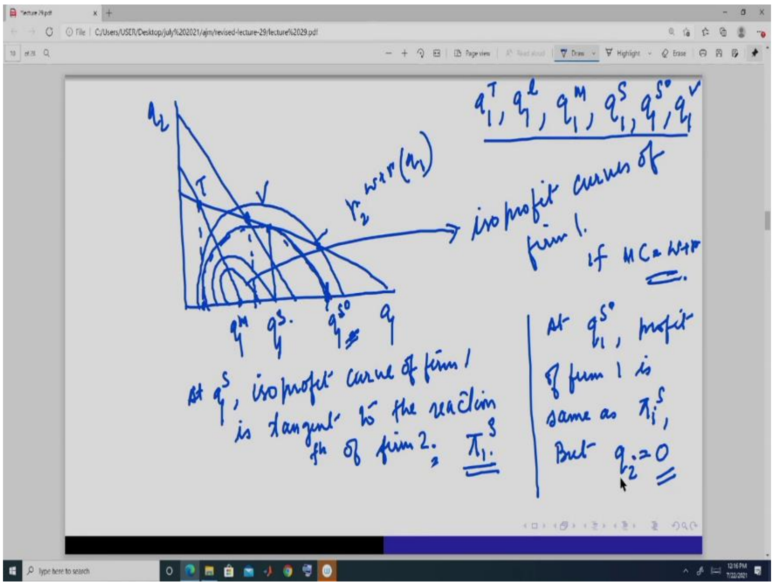

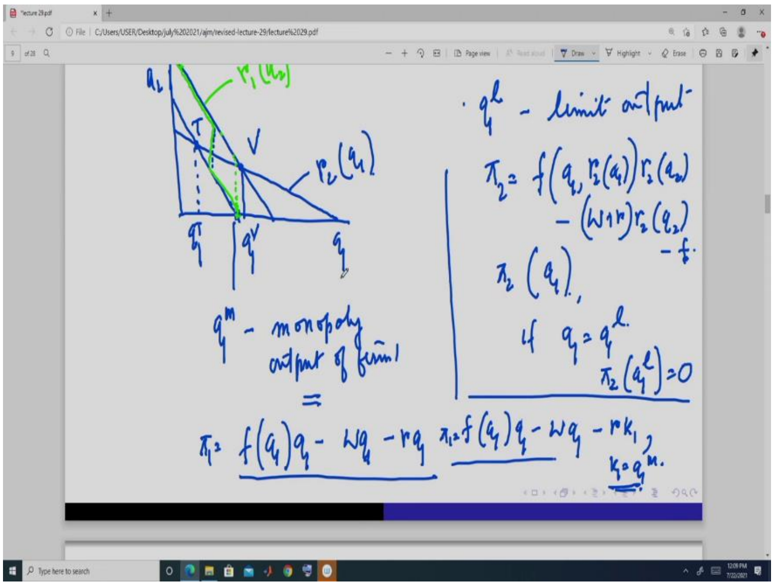

Next we define another output and that is the Stackelberg output of firm 1. Now, these outputs q1 m, q1 l, q1 T, q1 V all these can be of different values and we can rank them also and depending on their ranking we will get the outcome in stage 1, okay. So, this is output of firm 1, output a firm 2 this is the reaction function of firm 2.

Let us draw it like this, this is point T, this is point V, this is q1 m and we know these are the isoprofit function, oh sorry. Here when we are drawing it, it is isoprofit curves of firm 1 if its reaction function, if its marginal cost is, if MC is equal to w plus r, right? and you will see it is in fact MC because otherwise the there is no point in having excess capacity we have seen that in capacity greater than this output, this output, right?

So, we may have a situation oh sorry, it is not we may have; we will have a situation where, so it will, the capacity will be such that it is going to produce that much amount, it will never have excess capacity, okay. So, if it does not have excess capacity, then it means, then this portion, this at the optimal point this is equal to this-$wq_1 + rk_1 = wq_1 + rq_1$, if we are always going to have full utilization, okay and we will see that we will always have full utilization, okay. So, that is why we are taking this as the isoprofit curves rather than taking from here.

Now, again we will have this and we may have Stackelberg outcome like this. This is the Stackelberg outcome of firm 1 because at q1 s isoprofit curve of firm 1 is tangent to the reaction function of firm 2, we have this. So, we will get some profit and suppose this profit of firm is this Stackelberg represented in this way. Then each point in this isoprofit curve will give me this because isoprofit curves means all the combination of output of firm 1 and output of firm 2, so that the profit, level of profit is fixed.

So, we will have output here also. So, at q1 s naught, profit of firm 1 is same as the Stackelberg output, but q2 is equal to 0, okay this. We will also require this output because see this profit at this point is same as profit at this point. Now, here you can see that if we take isoprofit curve like this and suppose we look at this point, then this point lies above this isoprofit curve.

So, the profit here is going to be less than this profit, okay if that is the case, then we would prefer this Stackelberg rather than this, right and here we will use this to in certain situation to determine whether when what kind of optimal capacity the firm 1 should have, okay that is why we required this output and this Stackelberg thing, okay. So, I hope it is clear so, we have defined these outputs q1 T, q1 l, q1 m, q1 s, q1 s naught and q1 V okay and all our analysis, further analysis will be based on these outputs, okay. So, I am, I hope it is now clear the definition of these outputs of firm 1 okay.

So, this point is giving me q1 T, this point is giving me q1 V, this point is giving me q1 l m, q1 s is given by this point where isoprofit curve firm 1 is tangent to the reaction function of firm 2 and q1 s naught is given by this point where the output of firm 2 is 0, where the profit of firm 1 is same as the Stackelberg output or where these two, this profit level on this k, it can, it could have been this point also not but we have taken this, because this is also a point where profit is same as this level, but the output of firm 2 is 0 but we have taken the higher one; this one. So, remember this, okay out of these two profit, these two outputs we have taken this and not this, okay.

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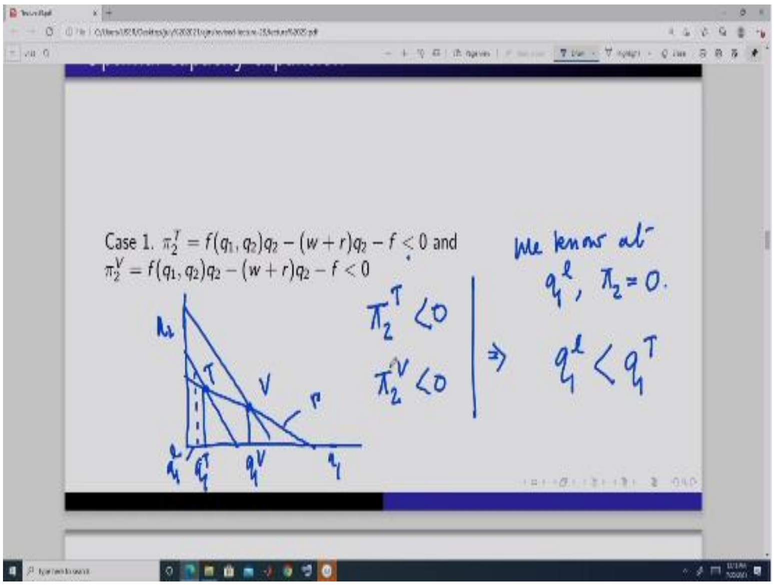

Now, let us look at stage 1. So in stage 1, let us look at this situation where profit of firm 2 in at point T is 0 is less-$\pi_2^T = f(q_1, q_2)q_2 - (w + r)q_2 - f < 0$, is negative and a profit of firm 2 at point V is also negative. So, if this is the reaction function of firm 2 and this is point V and this is point T, then profit of firm 2 at T is negative, profit of firm 2 at V is negative. And we know at q1 l profit of firm 2 is equal to 0, so this implies, q1, this is what q1 T this is q1 V.

So, q1 T is implies that q1 l is actually less than q1 T, so q1 l is somewhere here, then only the profit at this is going to be negative of firm 2. So, we will see what is the best thing to do here.

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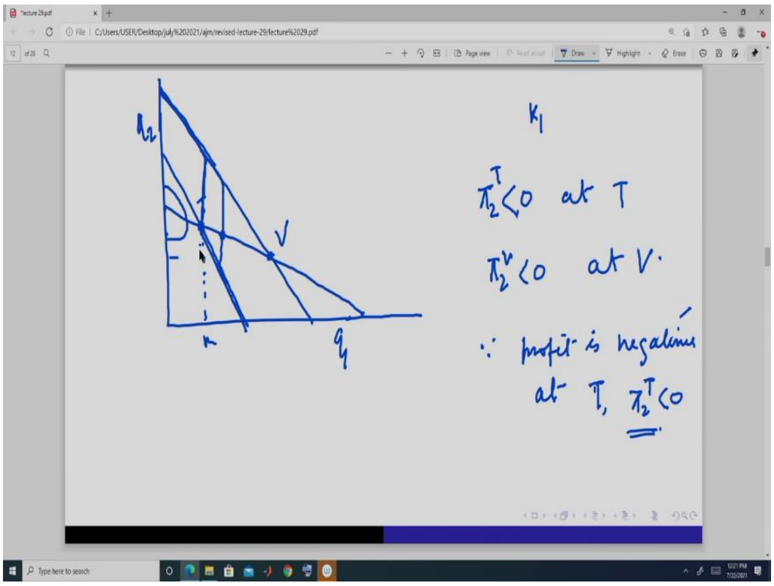

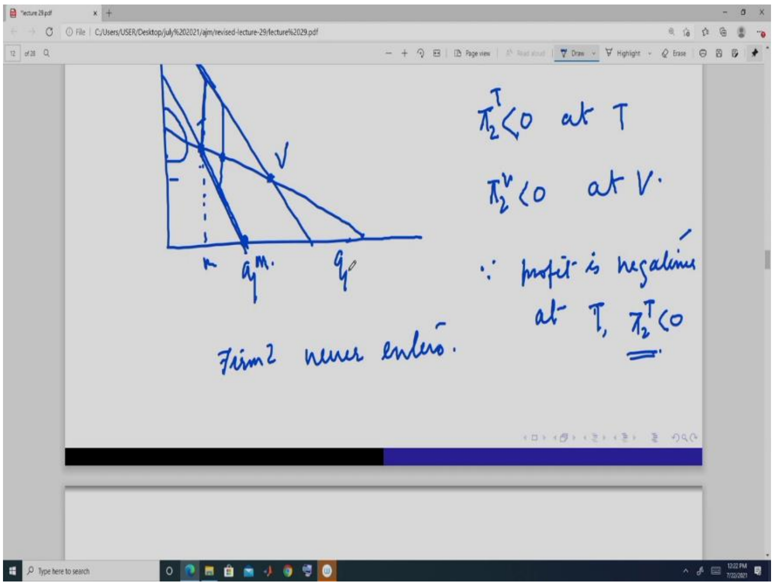

So, in this case, when case 1 we have got, okay this is suppose the reaction function have firm 2. So, firm 2 firm 1 while doing this calculation in stage 1, while deciding k1, it will see that profit of firm 2 here is 0 at T, again this is also 0 at V, right? So, now suppose the capacity is here, so the reaction function is this and if firm 2 enters, the outcome is this, right? if firm 2 enters but here profit is negative.

So, firm 2 will not enter, so since profit is negative at T, that is this, So, firm 2 never enters here, whatever, because if the capacity is here, then reaction outcome should be this, but since profit is negative here, it will be negative here also, it will be negative and since it is negative here also, okay. So, what do we get in this situation?

We, because see, if we look at this reaction function of firm 2, isoprofit curves are somewhere here So, this is the monopoly, right? so profits are decreasing like this. If it is negative here, then it will be negative here also, right? So, that is why it is even if capacity is here reaction function is this firm 2 to enters output is this in stage 2 it will earn negative profit, so it is better not to enter and earn 0.

So, here in this situation, output the firm 1 is going to be this one, this monopoly output because firm 2, never enters, so firm 1 will not be threatened by the entry of firm 1, so because even if it gives a signal that I am going to enter, it is not going to threaten firm 1 because firm 1 knows that firm 2 is not going to enter.

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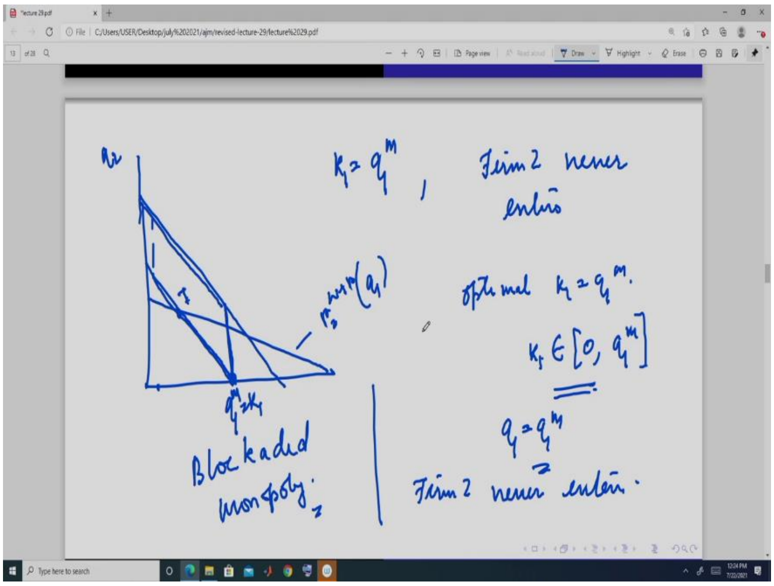

So, in this situation we get a thing that k1 is always going to be of this much amount-$K_1 = q_1^M$, monopoly amount and firm 2 never enters. So in, if this is the reaction function of firm two, is going to it will and this is going to be the outcome, okay. So, optimal k1 is this or even if it is anything, it will never go into, because if suppose the k is this, the reaction function is this, so this point is T, so it is profit is 0 negative. So, it is not profitable to enter.

So, you can write k1 can lie between 0 and this $[0, q_1^M]$ and the outcome is this-q_1^M and firm 2 never enters and so, this case is called something called blockaded monopoly. Firm 1 produces monopoly output and still firm 2 does not enter. So, it gets the monopoly profit, why? Because the entry cost is you can think in such that if it enters firm 2 earns a negative profit, okay. So, that is the reason.

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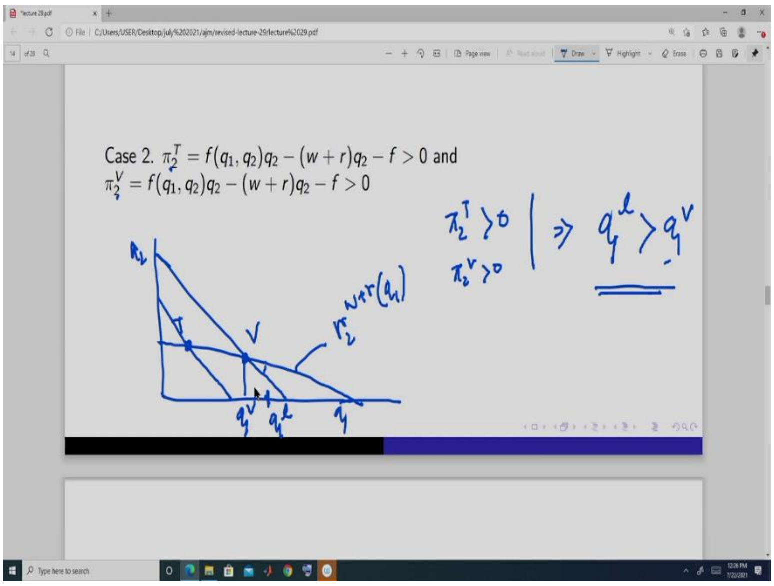

Next look at the case when firm 2 is profit at point T is positive and also at point V it is positive. So, it is this is output of firm 2, so this is, this curve is the reaction function of… so this point is T and this point is V. So, here also profit of positive at this point, positive at this point. So, this means, so this implies that q1 l is greater than this output. So, q1 l is somewhere here. So, if this is the thing, then what should the firm 1 do in stage 1?

Because while in stage 1, firm 1 while deciding k1 it will do this calculation, it will see. So, it will know that in this point also firm 1 is, firm 2 is making quality profit in these two, so it is making positive profit, okay. Because the q1 l is somewhere here, limit output is greater than q1 l and we know that if suppose the capacity is here, so, we will do that thing.

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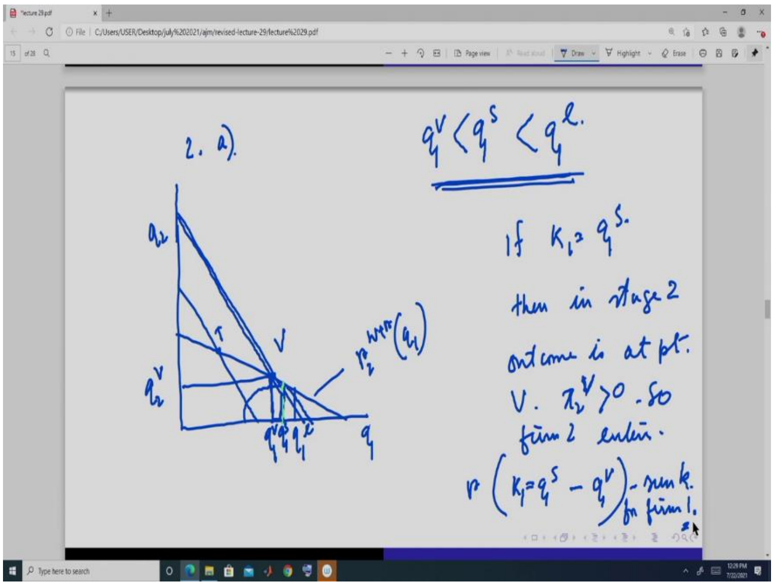

So, we will have to situations now. 2a; suppose the case is this q1 S is greater than q1 V, have this q1 q2, okay we have this and suppose this is q1 l, this is q1 V and suppose this is q1 S. So, it is somewhere here, this is the Stackelberg thing, okay we have this. Now, if k1 is equal to q1 S, so capacity is this. So, let us this green line gives me the capacity, then what is going to be the reaction function of firm 1?

It is going to be this and this. So, if this, then in stage 2 because it knows the outcome is this, profit is, stage 2 outcome is at point V and profit of firm 2 at V is positive. So, firm 2 enters and the output of firm 1 is q1 V, output of firm 2 is q2 V. Now, this much it is going to get as a sunk cost. So, if this is the situation, then what is the… So, this here minus is the sunk cost for firm 1.

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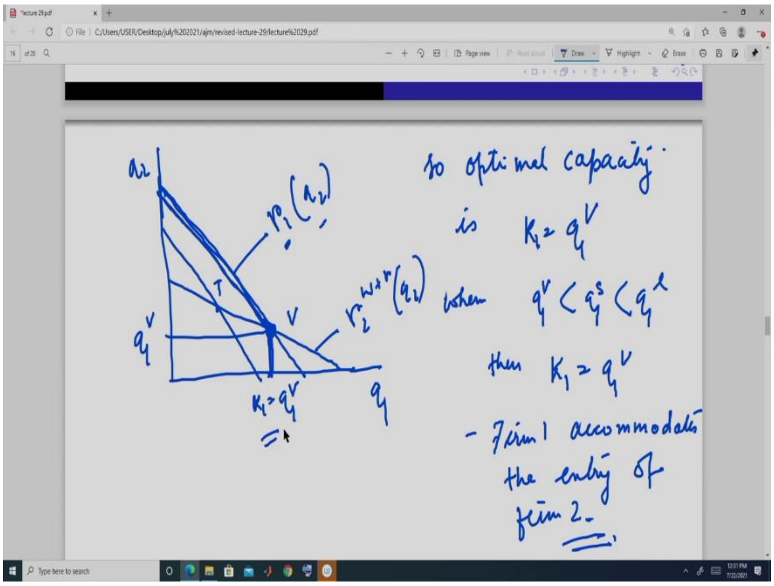

So, here optimal capacity is, so optimal capacity is this-$K_1 = q_1^V$. So, when k1 is this and firm 1 accommodates the entry of firm 2 and the outcome that we get is… And this is the outcome, okay so in this case, firm 1 accommodates the entry.

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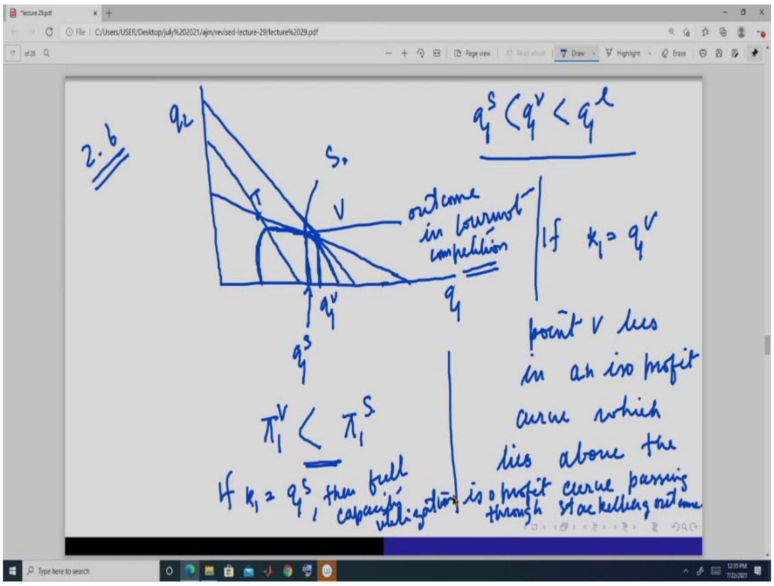

Now, let us look at this. Situation is this is always going to be the case-$q_1^s < q_1^v < q_1^l$ and this case is going to be that. This is q1 S, now here if k1 is equal to q1 V following from the previous case, so this is 2b, okay. Then if this is, so this is the outcome, right? we get here and since this is the Stackelberg output, so, that it is somewhere isoprofit is tangent here. So, this point, point V lies in a, in an isoprofit curve which lies above the isoprofit curve passing through Stackelberg outcome.

This point in this, so, profit of firm 1 V is less than profit of firm 1 at this point, suppose this is S. So, if now because of this, if k1 is equal to q1 S this capacity is this. So, outcome is here this is the outcome in Cournot competition, full capacity utilization, now this is the third case, if this then full capacity utilization. So, profit will be at a higher rate this if k1 is this.

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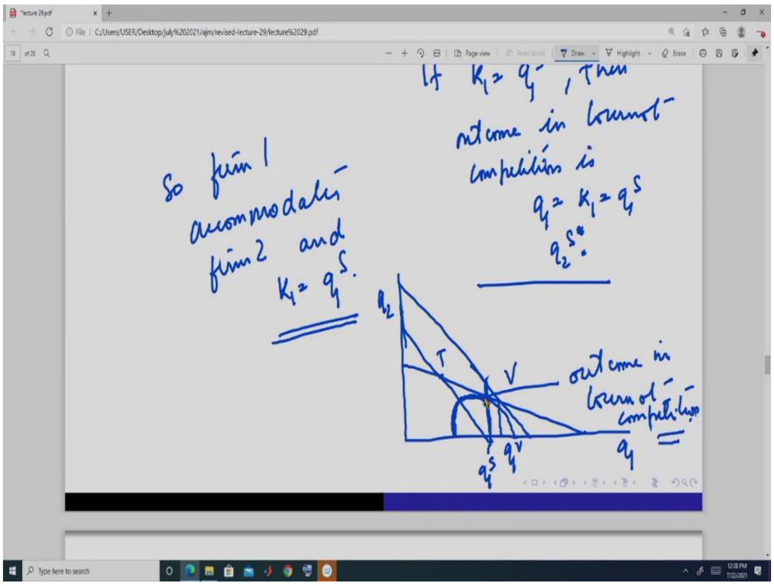

So, if k1 is equal to this then the outcome in Cournot competition is q1 is equal to k1 which is equal to q1 S and q2 is this Stackelberg output, this q1 star. So, here profit will be higher than this point. So, but if it produces less, if its capacity is less here, so the reaction function will be this. So, again it will be at an isoprofit which is higher than this because it is at this point it is not tangent.

So, that is why it is optimal for, so, firm 1 again accommodates firm 2 and k1 is equal to the Stackelberg output this-$K_1 = q_1^s$ and the outcome that we get is something like this, okay. So, the reaction function is going to be this, so they intercept, this is the outcome in Cournot competition, okay. So, in case 2, we see the firm 2, firm 1 always accommodates, it cannot deter the entry of firm 2.

In case 1 what we have seen that the firm 2 never enters blockaded monopoly, in case 2 when the profit at this point and this point both are positive, that is profit at T and profit at V are positive, that means the limit output is more than q1 V that means which is greater than q1 V then we have firm 1 always accommodates the entry of firm 2.

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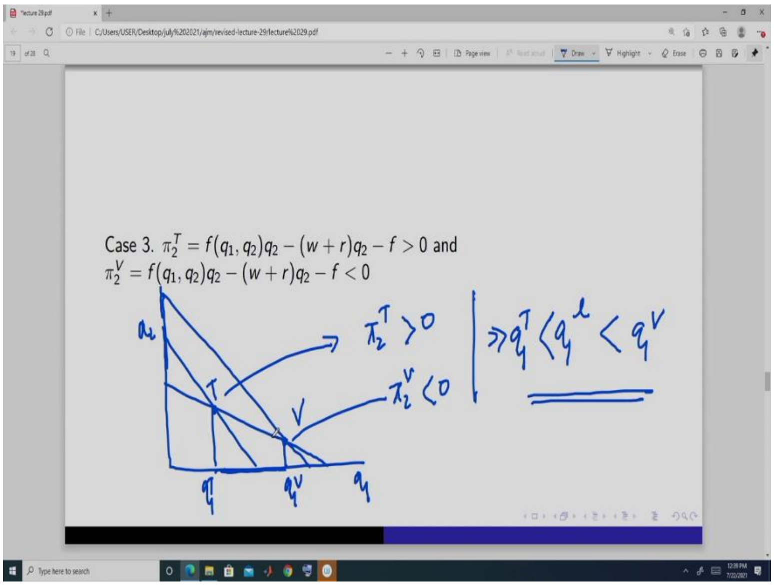

Now let us look at case 3. In case 3, we have a situation when we get this outcome that suppose this is the reaction function of firm 2, this is point T, this is point V, okay. Now profit here of firm 2, this is positive, profit here of firm 2 it is negative. So, this implies that q1 l is less than q1 V this and it is greater than q1 T. So, q1 l limit output lies in this range, okay. Now what are the possible outcome in this situation? So, let us look at the first output.

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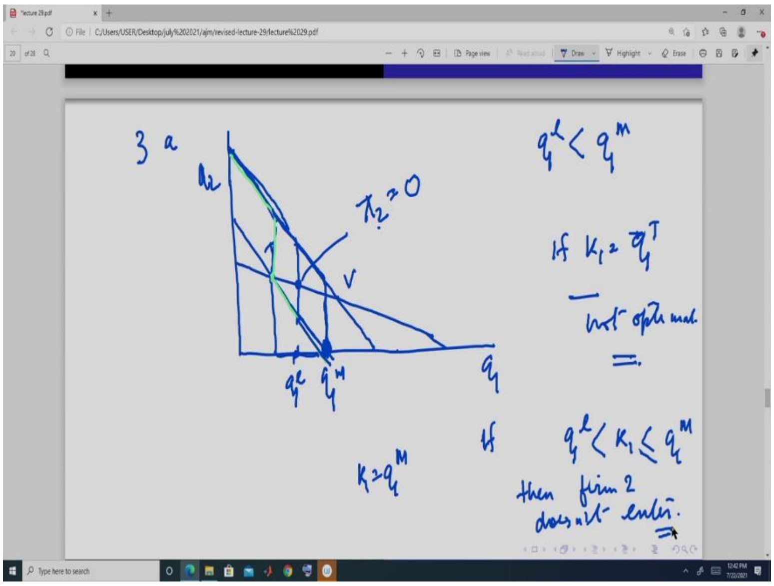

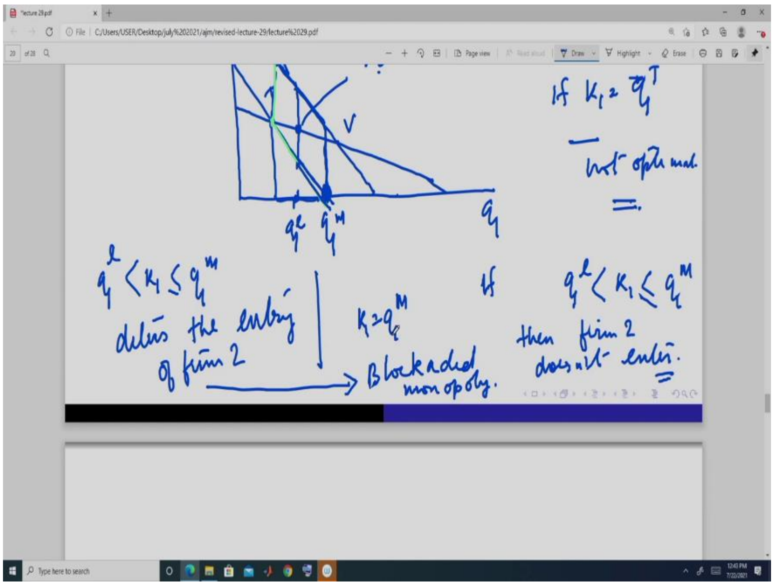

This is supposed 3a; here if this is less than monopoly output of firm 1, q1 q2, okay. This is point T, this is point V and this is q1 m. Now here q1 l lies here, somewhere here, suppose it is at this point, okay. So, here this is suppose q1 l, this point is. Now, if here K1 is equal to suppose T is this, so then the reaction function is something like this, given by this green line is going to be like this and at this point a so it will, it means firm 2 will enter. Because at this profit is positive for firm 2. So, this is so this- $K_1 = q_1^T$ is not optimal and does not deter.

But, here if q1 k1 is this, is suppose this much outcome, if firm 2 enters then outcome will be this firm 1's output is this. So, here profit of firm 2 is equal to 0 at this here. So, then this is will deter the entry of firm 2, firm 2 will not enter because it is earning 0 profit. So, the actual outcome is of firm 1 is going to be this.

So, in this situation and if it produces k1 is suppose is equal to and this much reaction function will be this, actually the output is this because if capacity is more than this, if capacity is more than this, then firm 1 monopoly output, then it will always, then firm 2 does not enter. If it does not enter, then what is going to happen? It means firm 1 has deterred the entry.

So, k1 lying in any point here in this range is, k1 lying in this range it deters the entry of firm 2 and this is also a situation of something like blockaded monopoly because firm 1 has producing what? Firm 1 is producing monopoly output and but still it is blocking the entry it is deterring the entry of firm 2, right?

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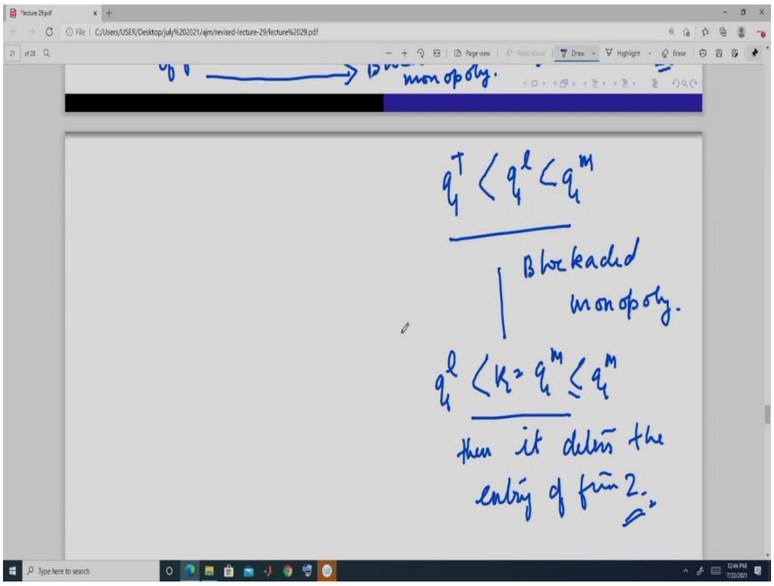

So, whenever, whenever this limit output is less than this- $q_1^l < q_1^m$ and this is greater than this- $q_1^T < q_1^l < q_1^m$, then we have blockaded, again we have blockaded monopoly and k1 is equal to q1 M or it is anywhere here k1 m less than equal to q1. If this is the situation, then it deters the entry of firm 1, sorry deters the entry of firm 2, okay. I hope this is clear.

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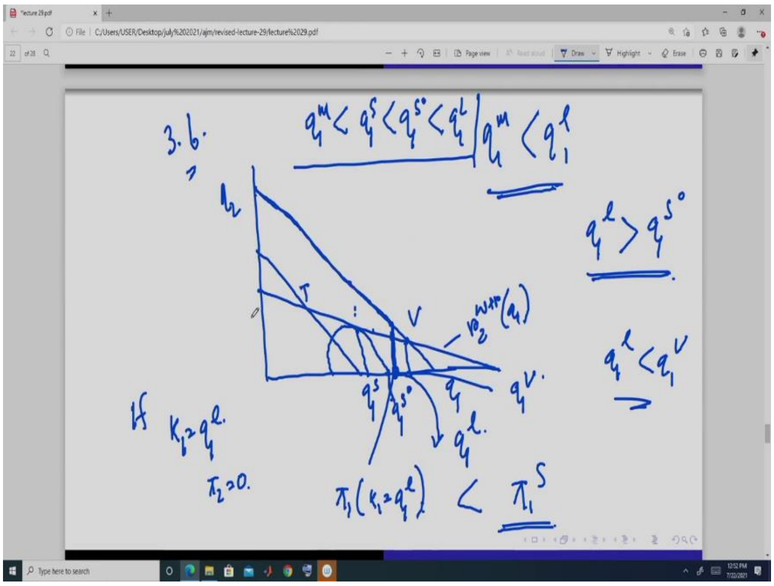

Next, let us do another situation. This is suppose case 3. So, in case 3b; we will… this is the reaction function of firm 2 and this is point T and this is point V and this is q1 S. So, here q1 l is greater than q1 m because we have already taken when it is less in the previous case and this is q1 S, okay naught this point. And q1 l is suppose greater than q1 S naught, okay. So, it is this point is q1 l.

So, this in case b what we have taken this is greater than q1 S greater than q1 S naught and this is greater than q1 l, okay this is-$q_1^m < q_1^s < q_1^{s^o} < q_1^l$. Now here if suppose it blocks the entry. How it can block? Let us do one, let us draw these curves slightly like this, so that this point is V, okay. So, this is q1 l. In earlier one q, so q1 l is less than q1 V this has to be satisfied this point is q1 V. So, here if this is the situation then what do we get?

It can block because this is A. So, if k1 is equal to q1 l, then it is going to do what? We will get this reaction function and then profit of firm 2 is 0. So, outcome is, this is the reaction function of firm 1, right. So, firm 1, firm 2 does not enters, firm 2 does not enter and reaction function is this, so outcome is this.

So, this is the profit point of firm 1, this we need produces k1 is equal to k1 this. But suppose and this is the profit, if output of firm 2 is 0 and firm 1 is getting the Stackelberg profit, then the profit is here this is the monopoly profit. So, it is closer. So, this profit is less than the profit of Stackelberg.

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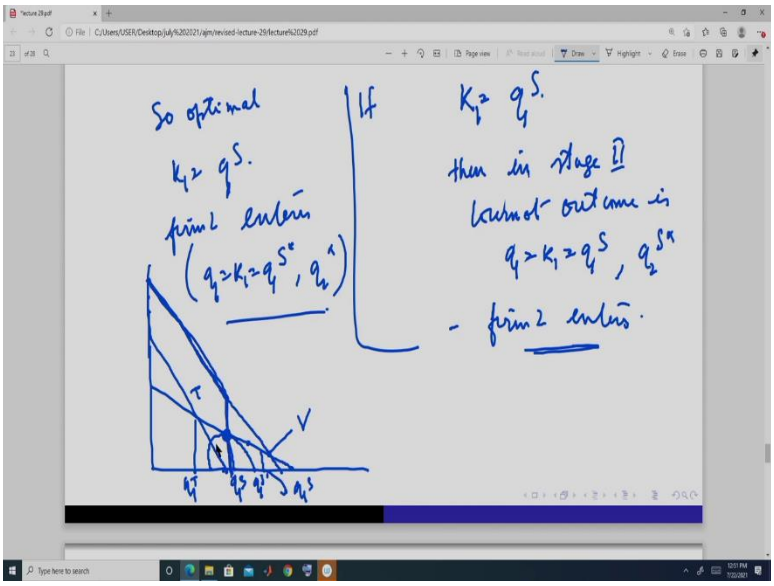

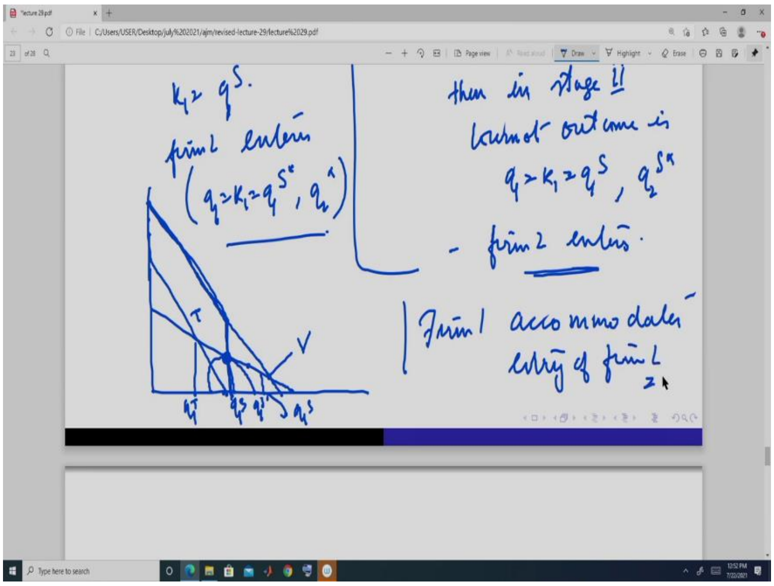

So, in this case if k1 is equal to output of this, this Stackelberg thing then in stage 2 Cournot outcome is q1 is equal to k1 is equal to this and q2 is this, this because firm 2 is going to, if this is the case firm 2 is going to earn a positive profit because q1 l is this, this point is q1 limit output of firm 1. So, firm 2 is enter.

So, in this case, firm 2 enters, but if firm 2 enters, it gets more profit then by deterring the entry of firm 1, firm 2. By deterring the entry of firm 2, it gets less profits, then by allowing the firm 2 to enter and having capacity this much. So in this case, so, the optimal k1 is this, this- q_1^S and firm 2 enters and the outcome, Cournot outcome is k1 is equal to this- $q_1 = K_1 = q_1^S$ and the profit this much-$(q_1 = K_1 = q_1^S, q_2^*)$.

And you can see and the reaction function of firm 1 is going to be this much, this one and this is going to be the outcome, okay. So, in case 3b; we see that firm 1 accommodates the entry of firm 2. So, here firm 1 accommodates entry of firm 2, okay.

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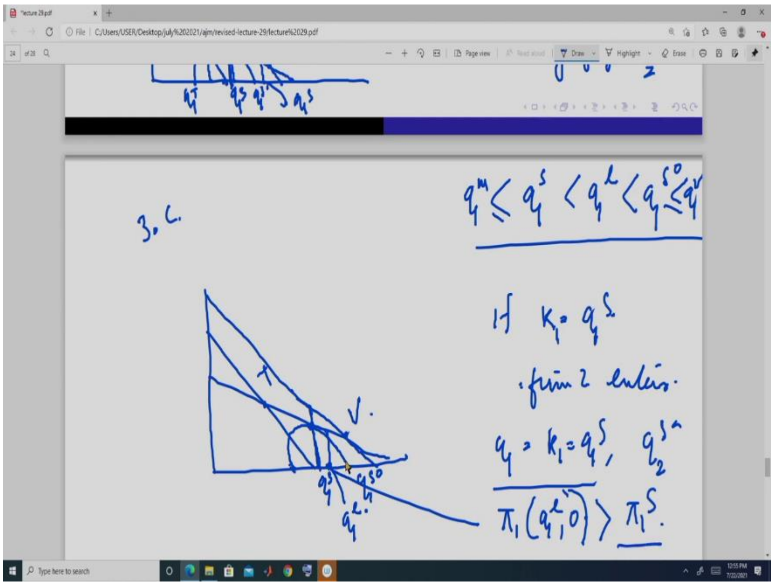

Next we will look at this 3c; when we get a situation like this, then this is we do not know. So, suppose this is the outcome, this is q1 S, this is q1 S dot and the limit output is somewhere in between okay and this is suppose V, okay. Now here if k1 is equal to q1 S, okay limit output is here, right? then what is going to happen?

Its profit is this and here limit output is this much so, in this A reaction function limit output is at this point, so it is here Stackelberg. So, firm 2 this is going to be the A, so firm 2 enters and if firm 2 enters what is going to be an outcome? It is going to be k1 is we have seen this and this is output a firm 1 and q2 is this Stackelberg, profit is here. So, at this point and this point, since they are in the same isoprofit, so the profit is same.

But if you compare this limit output and this you will see this is greater than this. So, if firm 2 suppose produces 0 output and firm 1 produces this. So, this the profit it gets is more than the Stackelberg thing. So, at this point this profit of firm 1 when it produces this and output of firm 2 is 0, this is greater than the Stackelberg, so we get it from, why it is greater?

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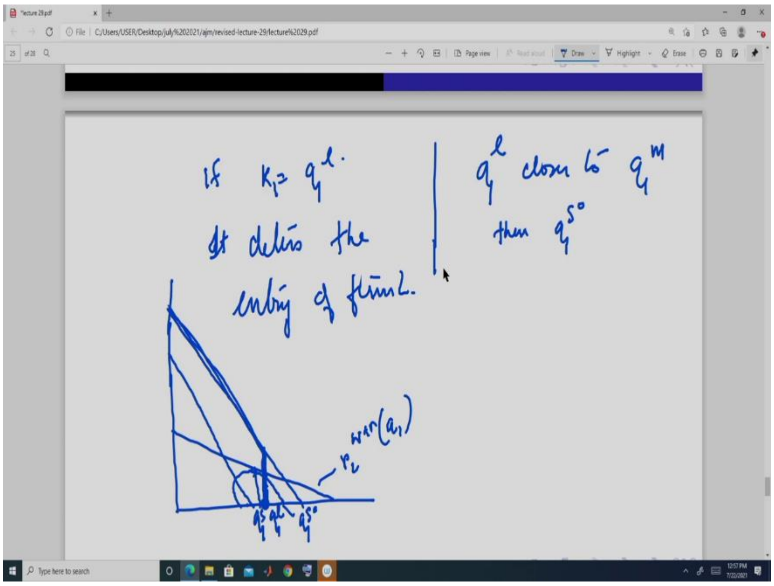

Because q1 l this is closer to q1 m then q1 S naught, so this is closer to q1 m and this is here q1 S naught and q1 A is this. So, in this situation, in this situation if k1 is equal to q1 l, then it deters the entry of firm 2 and the output is, in this situation we will get the reaction function suppose this is of firm 2, here this is this, and the this thick line is going to be reaction function and it is going to produce here.

Not here, it is going to produce here, outcome is going to be here in this A. Because k1 is this and this is since it is left of this A. So, profit here is higher than the Stackelberg profit this point, so it deters the entry of firm 2.

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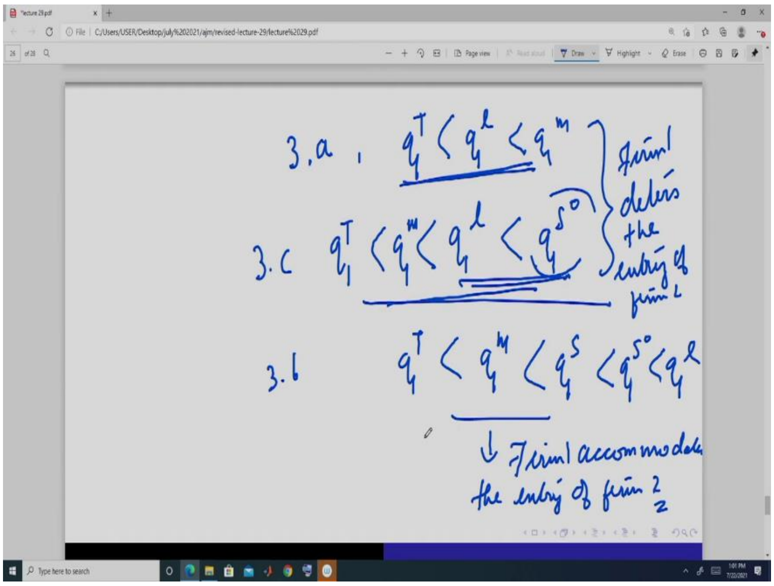

So, what do we get we get that firm 1 can deter the entry of firm 2 in this situation, when in case 3a where q1 l is less than q1 m and in case 3c when q1 l is less than q1 S naught and this is… so, this is the important thing. And in case 3b when we have in this it deters, firm 1 deters the entry of firm 2, here firm 1 accommodates the entry of firm 2, okay.

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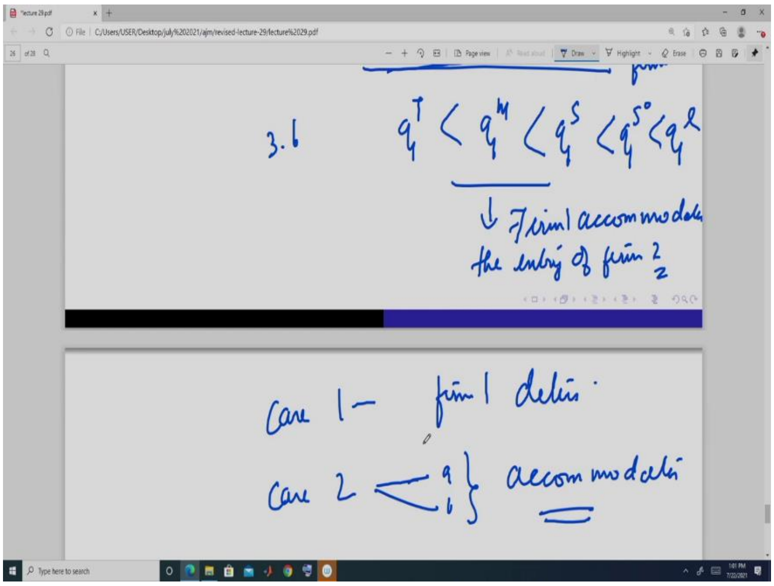

And we have seen in case 1, firm 1 deters and in case 2 in all the cases we have seen, we a and b; it accommodates. So, these are the subgame perfect Nash equilibrium. In case 1, firm 1 deters is it is a blockaded monopoly, in case 2 it always accommodates because profit of firm 2 is positive both at point V and point T. In case 3, we get mixed outcome in case 3a where we have this situation firm 1 deters, in case 3c when we have this, that is limit output is less than this outcome, this output of firm 1, then firm 1 deters

But if limit output is greater than this, then firm 1 accommodates the entry of firm 2 and there is this Stackelberg outcome in Cournot competition, okay. So these are the outcomes that we get in this Dixit's entry deterrence model. So we see that firm 1 will not always deter the entry of firm 2, it sometimes it will accommodate. But in certain cases we see that it will deter the entry. So, it is not a uniform thing to say that firm 1 will always want to deter the entry of firm 2, okay.

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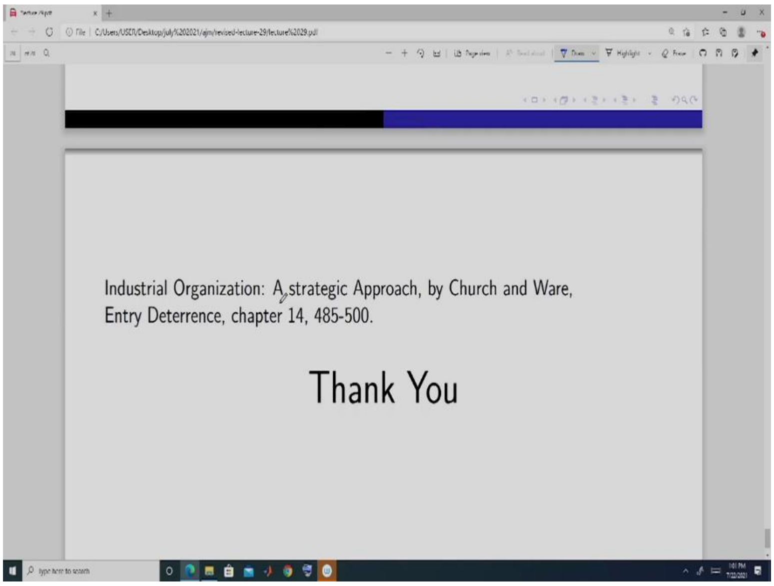

So you can read this portion from chapter 14 of this book, Industrial Organization: A Strategic Approach by Church and Ware. So these are the specific page numbers. Thank you.

Introduction to Market Structures

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Module 12: Entry Deterrence, Bundling and Tying
Lecture 42
Bundling and Tying

Hello and welcome to my course introduction to market structures. Today we are going to do bundling and tying, this is the last module of this course.

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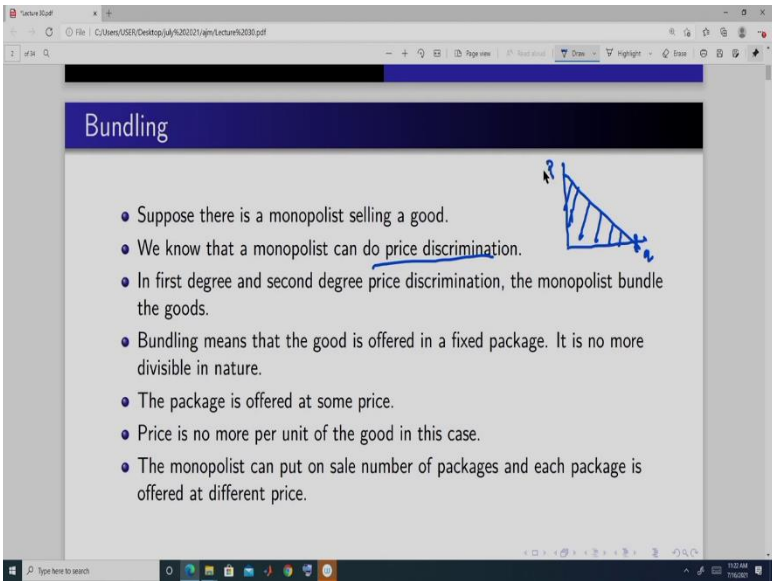

Bundling; one form of bundling we have already done, we have done the monopoly and in the monopoly we have seen that the monopolist can do a price discrimination and what do we mean by price discrimination? That means that they charge different prices to different quantities and that is in first degree price discrimination or they can charge different prices in different markets if the consumers cannot move around from market 1 to market 2.

Or they will package it in different way, packaging and that is a bundling like in our case, till now we have assumed that the goods are continuously divisible. So, you can get any quantity you want. But if it is packaged, then it is only available in fixed quantities, you cannot change that quantity, so that divisibility property that is gone.

So, monopolist can also do that, we have seen that in first degree price discrimination. So and we have derived in first degree price discrimination that the whole bundle, so suppose if we have a demand curve like this, then this whole amount and suppose there is no cost no margin, marginal cost is 0, then this whole amount, this amount will be packaged at a price which is given by this area below this curve demand curve when this is the quantity and this is the price. So, this is we have seen in first degree price discrimination, we have also done graphically the second degree price discrimination.

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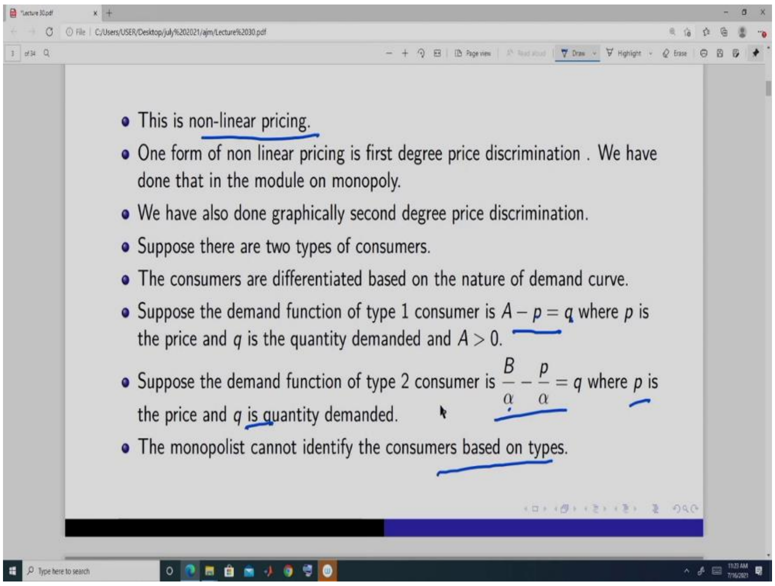

But we will do it algebraically today and we will consider only the monopoly because if we take the duopoly or oligopoly its calculations are slightly more involved. So, we are not doing it, okay. So and since if monopolist is doing bundling, so, this is a form of nonlinear pricing. And for simplicity, we assumed that there are two types of consumers. So, it can have more than two consumers; types of consumers.

And the demand function of type 1 consumer is this-A-p=q where p is the price and q is the quantity and demand function of consumer 2 is this- $\frac{B}{\alpha} - \frac{p}{\alpha} = q$, B by alpha minus p by alpha, where p is the price and q is the quantity demanded.

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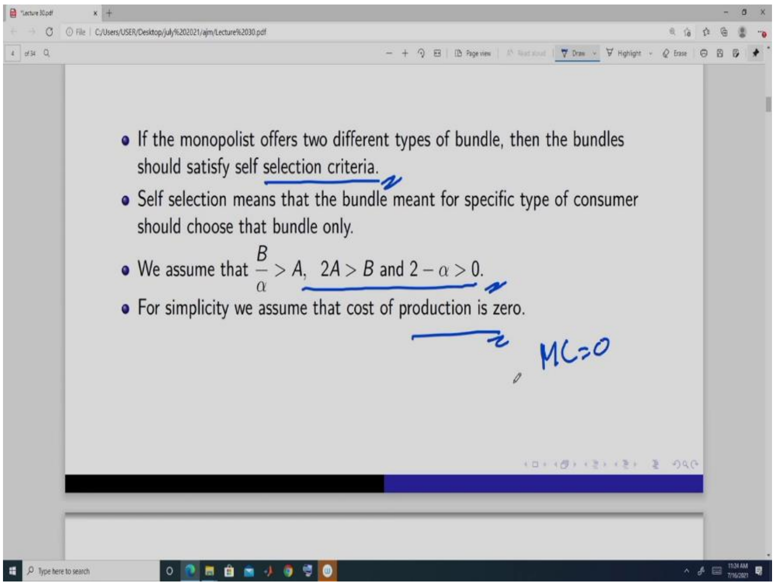

Now, the monopolist cannot identify these buyers, it knows that there are two types of buyers, but it does not know who is type 1 and who is type 2. So, it will choose or it will decide or it will design the package in such a way so that there is something called self-selection that is if I meant this bundle for type 1 consumer, then this type 1 consumer should choose that bundle only. And if I choose, if I design a bundle for type 2 consumers, then the type 2 consumers should select that only.

So, type 2 consumers should not select the bundle of type 1 consumer and type 1 consumers should not select the bundle of type 2 consumers, okay. So, this is the criteria of self-selection and we have to insure that. And we assume further these technical conditions and we will see why we need them, okay. And for simplicity we assume that there is 0 cost of production so that means marginal cost is 0 and there is no fixed cost, okay.

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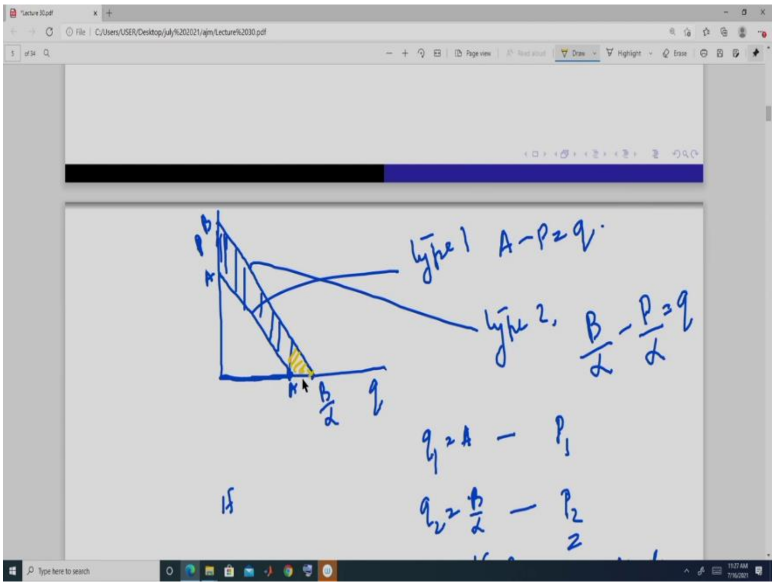

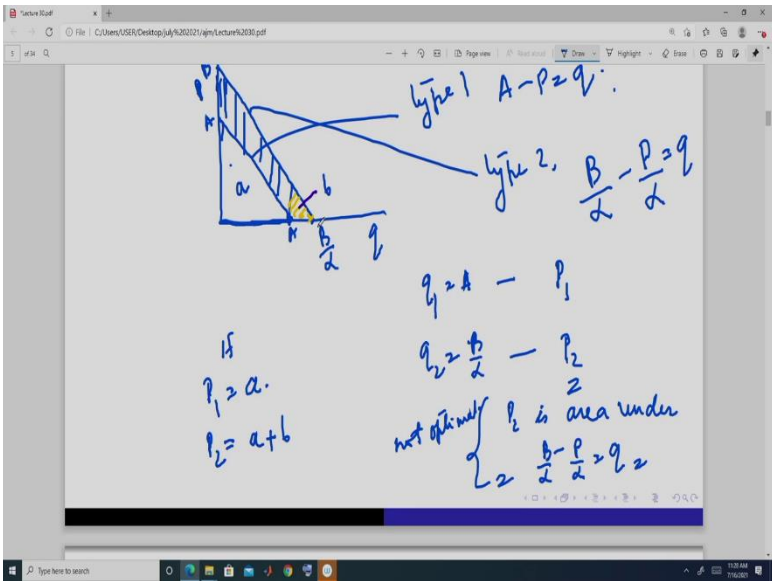

So now if we plot this, so this is quantity and this is price. This is suppose, this is the type 1 where A minus p. So, this is A and this is A and suppose this is type 2 B by alpha minus p by alpha is equal to q, sorry this is quantity, okay. So, this point is B and this point is B by alpha, okay. Now, the monopolist can sell this point is this, so, sell one bundle that is for type 1, type 1 it is bundle this at some price p1 and then for bundle B it can sell the amount this at some price p2, okay.

So, we for one bundle it has to be something like here. Now see if we see touch this and if p2 is this whole area, if p2 is this whole area, area under this demand curve if p2 is area under this demand curve if p2 is area under this demand curve-$\frac{B}{\alpha} - \frac{p}{\alpha} = q$ then p2 consumer 2, if it buys this bundle, then it gets this much amount of surplus. So, this is not an optimal strategy because it violates self selection, but here it can charge this much.

So, p2 and this area is suppose B and this area is suppose sorry let us denote it in different ways otherwise it will be confusing. This is suppose this and this is suppose small a. So, p1 can be equal to a, small a, this whole region and p2 can be a plus b, then the consumer 2, type 2 is indifferent between buying this bundle a and this bundle this and this and so, and we assume that the consumer, type 2 consumer buys this bundle when p2 is equal to this-a+b. Now, the question is whether this is the optimal bundle or not. So, how do we do this?

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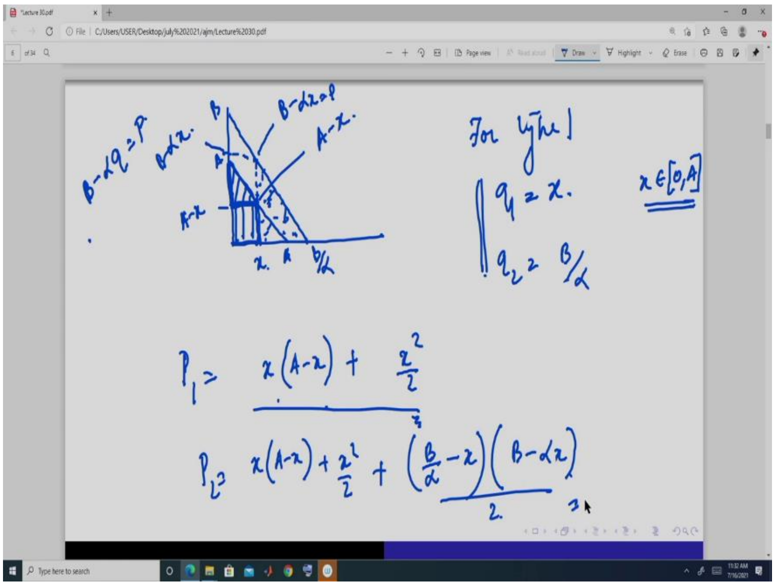

So, what do we do, we do, we solve it in this form. Suppose this is x, suppose for type 1 q1 is x and for type 2 it is b by alpha, okay, this, this is the package, okay. And here x would lie within this range, okay it can be A also. Now, within this, if this is the A, consumer 1 for type 1 it can charge this whole region for paying this. This region, this point is a minus x. So, this point is a minus x. So, this region is this rectangle into this triangle plus this triangle.

So, this area is x A minus this plus this distance is A minus this, so it is x, this distance is x so x squared, so this is the area of this triangle- $x(A - x) + \frac{x^2}{2}$, okay. And if A is equal to; x is equal to A then this is 0 and we have only this part, okay. And so this is p1, okay and p2 it can charge this much additional amount. So, this point is what? This point is B by alpha, sorry, this when we plug in this quantity and here this is we can write this as inverse demand curve and x is the A.

So, this point is B minus alpha is equal to P, this point is B minus alpha x. So, this triangle, so this triangle, area of that triangle, this dotted triangle is what? This height into base divided by 2. So, this is B by alpha minus x because this is x so this distance, this is the base and the height is this much. This is the height divided by 2- $(\frac{B}{\alpha} - x)(B - \alpha x)$. So, now p2 is this plus it can charge this triangle. Sorry, this area, not this triangle this whole area so that is the p1, it will get this- $P_2 = x(A - x) + \frac{x^2}{2} + (\frac{B}{\alpha} - x)(B - \alpha x)$.

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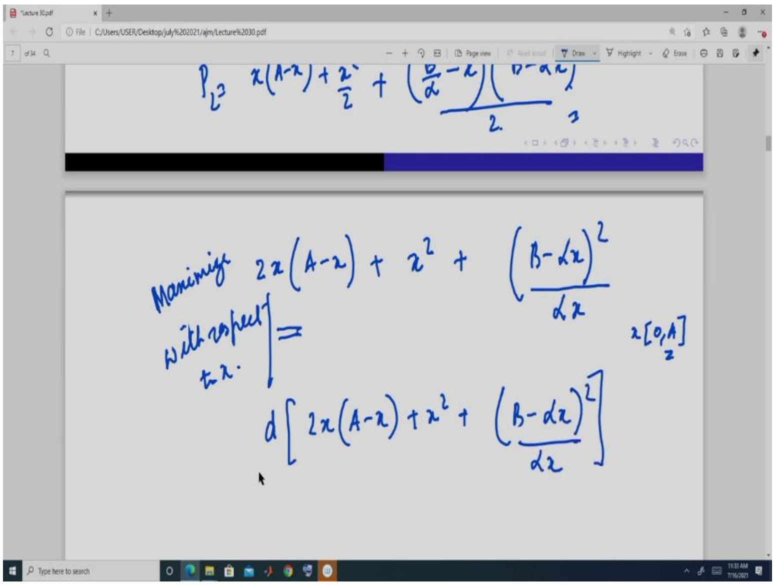

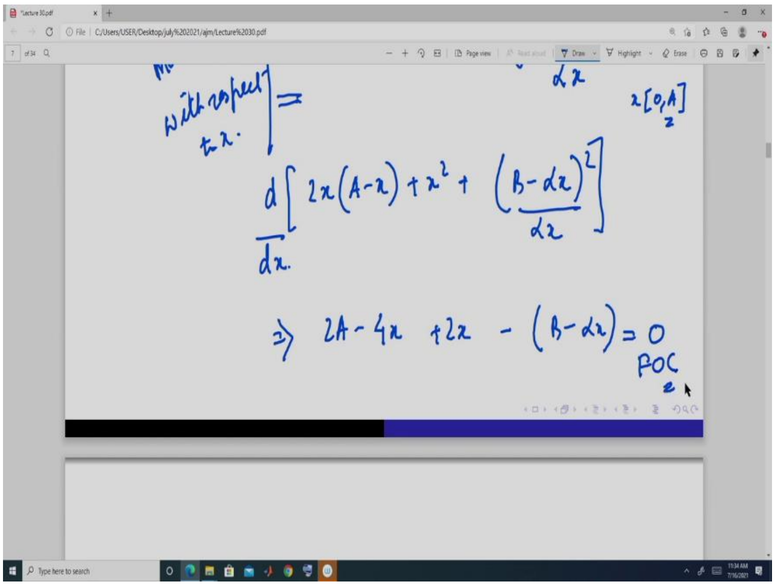

So, monopolist is going to choose x such that this is maximum. So, it will maximize this- $2x(A - x) + x^2 + \frac{B-\alpha x}{\alpha x}$ with respect to x and since this is differentiable function in x and x lies between 0 and A. So, this point is we differentiate this and equate it to 0 the first order condition and so what do we get? And this is equal to this, this is the first order condition-$2A - 4x + 2x - (B - \alpha x) = 0$. So, we solve this what do we get?

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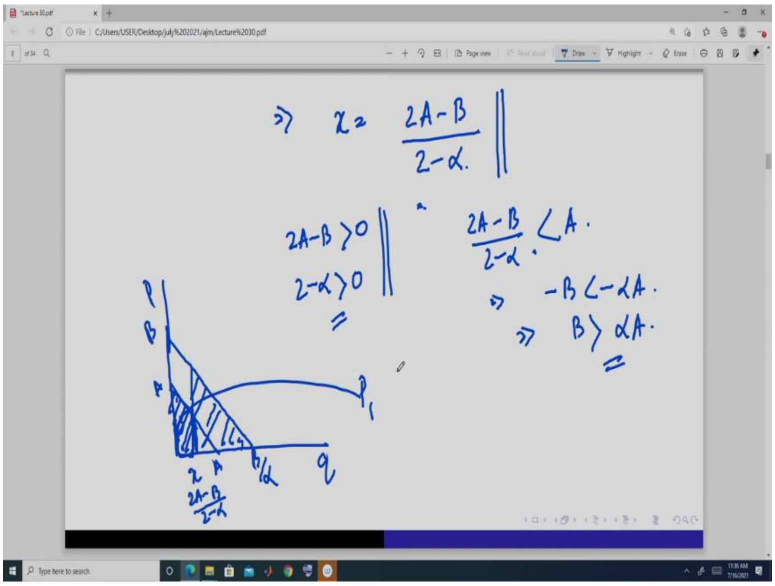

We get x is equal to 2A minus B divided by 2 minus alpha- $x = \frac{2A-B}{2-\alpha}$. So, that is why we assume this conditions, we require these conditions so that this takes a value like. Now, if you look at this, this point is actually less than A. So, this less than A means, so this means B should be greater than alpha A and this should be positive. So, this is positive-$2A - B > 0$ and this is positive-$2 - \alpha > 0$ both or both can be negative.

But, for simplicity assume both are positive. So, this ensures that we get this. Now here, this is the condition ensures that this point is somewhere here. This point is somewhere here x. So, this is the price p1 is equal to this region which is this and this region is p1 which is actually equal to since this x is equal to 2A minus B divided by 2 minus alpha. So, this is if we plug in that here in this portion, in this portion we will get it. So, and p2 is this region plus this region.

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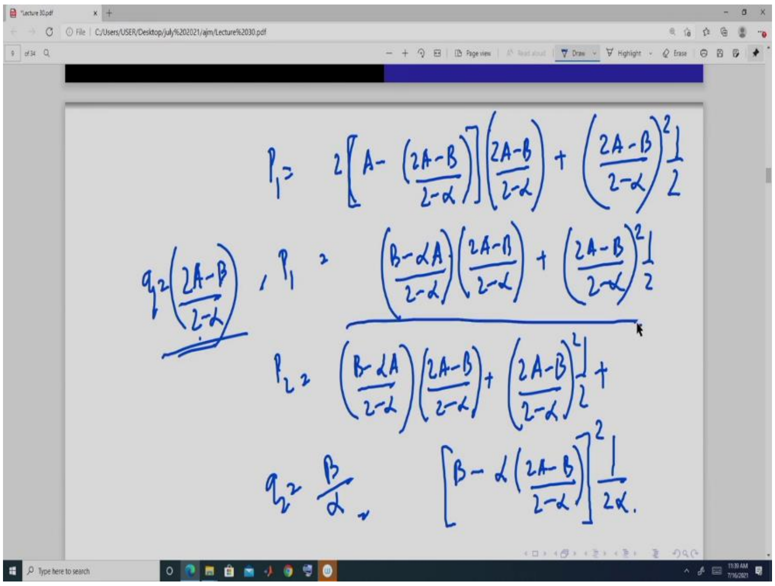

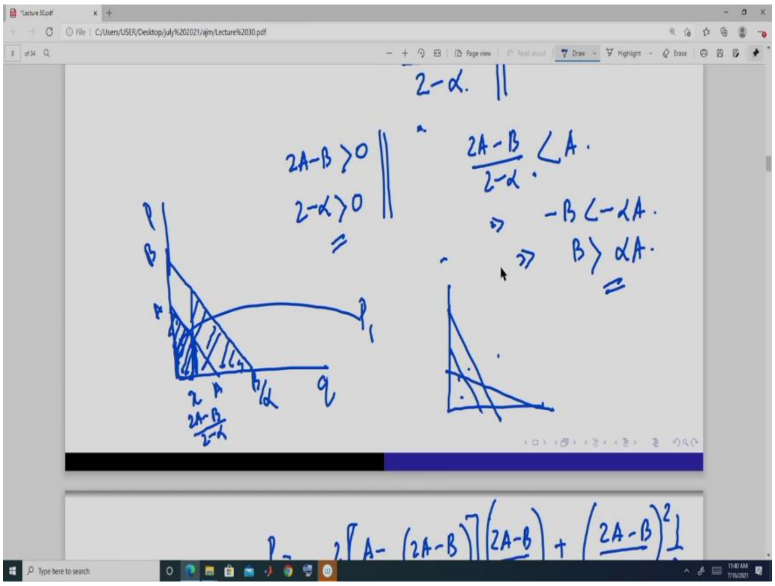

So, here we get that p1 is equal to 2 this-$P_1 = 2\left[A - \left(\frac{2A-B}{2-\alpha}\right)\right]\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 . \frac{1}{2}$, this and this is equal to, it is this-$\left(\frac{B-\alpha A}{2-\alpha}\right)\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 . \frac{1}{2}$ and p2 is this, this is at q1 is okay, this is the bundle-$\left(\frac{2A-B}{2-\alpha}\right) = q_1$. This is the package at this price, p2 is this whole thing plus one more additional term and that is, it is this-$P_2 = \left(\frac{B-\alpha A}{2-\alpha}\right)\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 . \frac{1}{2} + \left[B - \alpha\left(\frac{2A-B}{2-\alpha}\right)\right]$. So, q2 its bundle is B by alpha So, B by alpha at this price and q1 this at this price, this is the way the monopolist can package the product and it is like this.

So, this much amount at this price whole and this much amount at this area plus this area, okay. So, this is one way of bundling, okay. So, but if we have a, suppose if we have a demand curve of, suppose one demand curve is this, another demand curve is this, another demand curve is this, then also we can look whether, so there are three types of buyers, we can see whether we can bundle it or not.

So, here one possibility is that there may be three types of bundles or packages or there may be only two, okay . But here this will be much more complicated than this here, because there are three types of demand curves and we will have to look whether it is because we have to satisfy the self selection criteria. So, consumer 1 should not buy the package meant for consumer 2 or for consumer 3 and neither consumer 2 should buy the package of consumer 1 and consumer 3, either consumer 3 should by the practice of consumer 1 and consumer 2.

So, all these possibilities have checked. So, that is why if we simply increase one more type of consumers the complications increases by huge amount, but with two things it is still manageable but the idea is I think it is clear. So, the main idea is that if we can differentiate the demand curves, then it is possible to bundle the product, okay.

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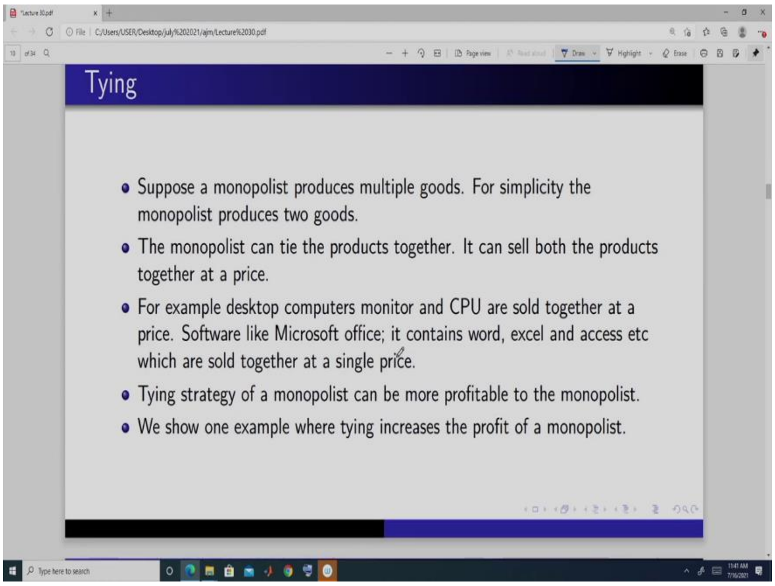

Now we switch to another topic and that is tying. What do we mean by tying? So, tying here again we take a monopolist, now monopolist can suppose a monopolist selling computer, desktop computer. So, desktop computer come as a two thing, one is the monitor and other is the CPU. So, this comes together. So, this is you can think of as a kind of a tying.

So, monitor is tied with the CPU or CPU is tied into a monitor or you can simply assemble this thing and you can buy them separately. So, this is or if you look at MS Office, so like MS Word, MS Excel, MS Access all these things are tied together. So you can buy only MS Office and all these three things are available, but each of them can be considered as separate product also, right?

So, this is a tying strategy and the monopolists can follow up tying strategy, okay. Now it can be shown that the tying is sometimes profitable and that is why the monopolists tie. Now here we are assuming that the monopolists are not a single good producer monopolist. So, they are producing multiple products, okay.

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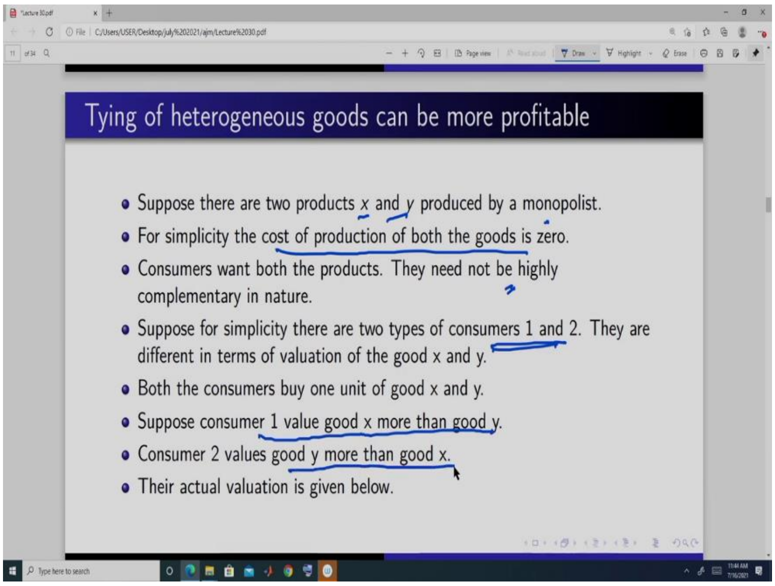

So, suppose there are two goods x and y and these two goods are produced by a monopolist, okay. And for simplicity, we again take the cost of production to be equal to 0. So, that is the marginal cost is 0 and there is no fixed cost, okay. And consumer, assume for simplicity that they want both the product, so we have some consumers and they want both the product.

And these products may not be strictly complementary in nature like in case of monitor and CPU they are complementary, you cannot have, the monitor is useless if you do not have a CPU and CPU is useless if you do not have a monitor, but in case of like MS Word, MS Excel, Access then they are not strictly complementary, although they have some degree of complementarity, but they are not highly complementary.

Like you can, if you are only working, if you do not work with any data or numbers then if you do not, you are not engaged in any form of calculation, only you require it for typing then MS Word is sufficient and you can do it that is okay. But if you are engaged in both calculations and writing, then you require both Excel and Word. But if you are only engaged in calculation and you do not require and need to do any writing, then Excel is sufficient for you, okay.

So, we assume that there is, both the products are required by the consumer but they are not highly complementary in nature. And for simplicity suppose we assume there are two types of consumers and the types of consumers are differentiated based on their valuation for this product and for simplicity what we have done we assumed that suppose consumer 1 value good x more than good y and consumer 2 values good y more than good x, okay. And their actual valuation is and is given like this.

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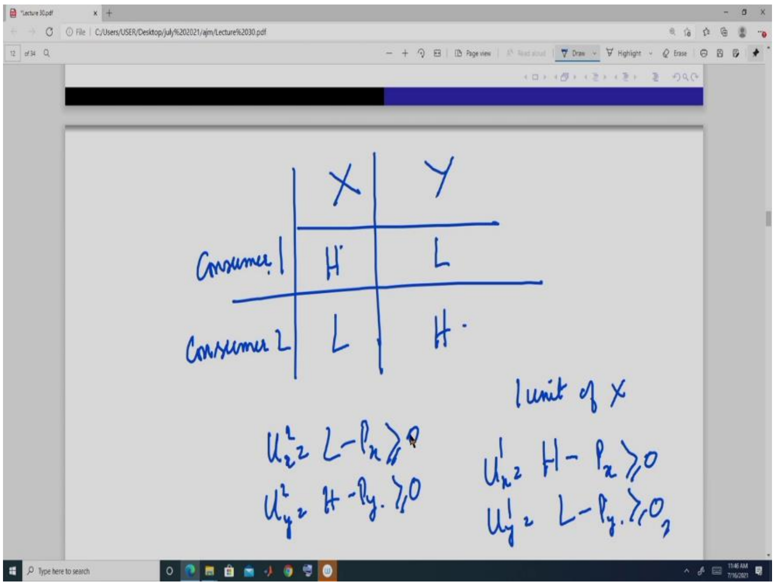

Suppose we have two goods X and Y okay and we have consumer 1 whose valuation is suppose high and this is low and consumer 2 is valuation is low and high and suppose these are same for to keep the things simple, otherwise it will create a lot of, we will have to do a lot of calculations and here my objective is to give you the idea not do the actual calculation.

So, we, so these are valuation. So, that means, if consumer 1 buy one unit of X, the utility it gets is H, net utility is this H minus Px and the utility it gets from y is H minus p, like this, okay. These are the price and both we assume that both of them consumes only one unit of good. So, consumer 1 each consumes one unit of X and one unit of Y if both of them are affordable.

Consumer 2 buys affordable or if they give them non negative, if this is they will consume for this consumer 1. And for consumer 2, utility is this- $U_x^2 = 2 - P_x, U_y^2 = H - P_y$. And he will buy both the product one unit of each if both of them are positive, okay. Now, how do the monopolist price this good? It has many strategies.

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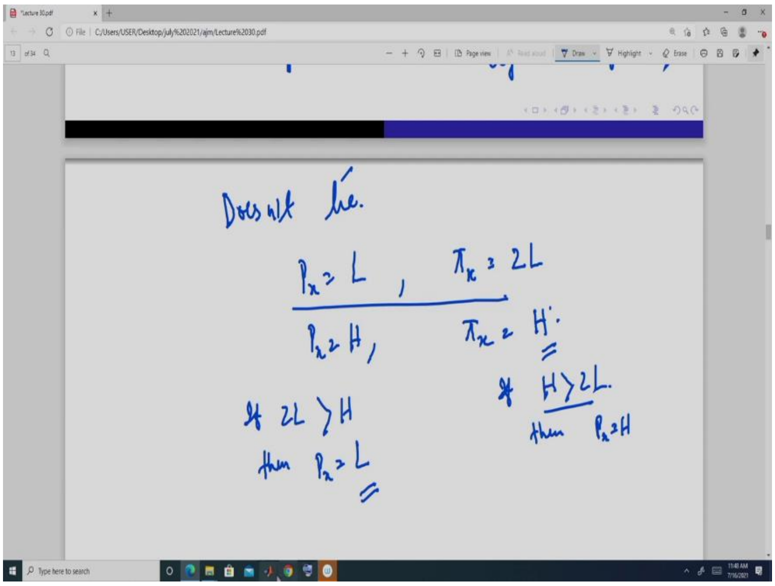

Suppose it does not tie, does not tie the product. So then it means what so Px it is, it can charge L. If it charges L, then profit from because it is a monopolist, so both the consumer can buy it, so it will have 2L. This is one possibility or it can charge this H and profit from x is only consumer 1 buys. So, this strategy is profitable when H is greater than 2L, right. Otherwise, so if this is the case, then Px is equal to H or if 2L is greater than this H then Px is equal to L, this.

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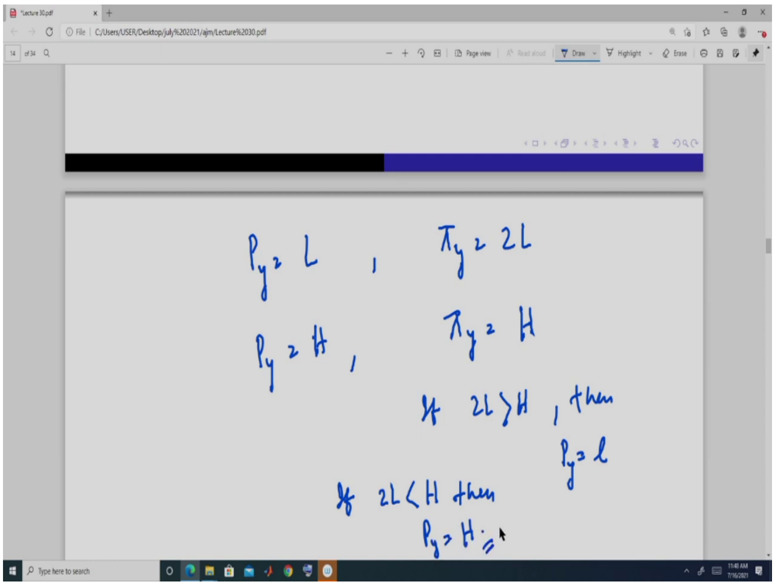

Similarly, for good y also. So, y is equal to L then profit of y is 2L if Py is equal to H, then consumer 2 buys, so, it is H only. So, if 2L is greater than H then price is, then is l and if 2L is less than H then Py is equal to h. So, these are the pricing strategies if it does not tie, okay if tying is not there.

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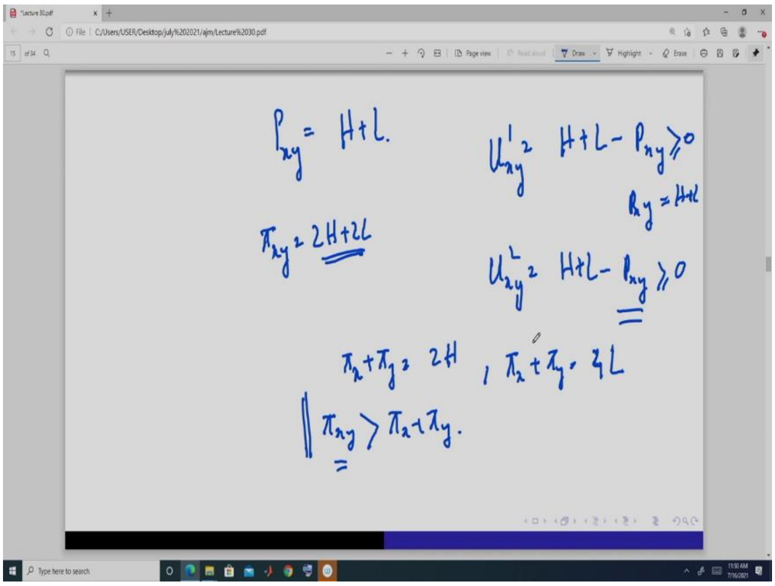

But if it ties it can charge a price, it can charge a price that is Pxy it is a tied product, but you are have to buy both the product and you have to pay only one price and that can be H plus L. Now here if this is the A consumer 1 utility of consumer 1 is H plus L is this-$U^1_{xy} = H + L - P_{xy}$, so it is going to be positive if this is, it is going to be non-negative if Pxy is equal to H plus L.

And this is again if we have this, so, profit sif it ties is 2H plus 2L-$U^2_{xy} = H + L - P_{xy}$. Now here in this situation, if we have what are the possibilities? Possibilities are here, either it gets, if it does not tie, if its profit is x is 2H or it can be 4L, if you compare with this, you will get that Pxy is always greater than this-$\pi_{xy} > \pi_x + \pi_y$.

So, because of this reason, we get that the tying is a more profitable strategy then not tying. So in this situation, the monopolist is always going to tie this product x and y. So that is why we get certain products which are bundled together, which are tied together. We do not see, we do not get them as an individual product, but we get them as a composite product, where many things are together, okay. Even if the goods are not highly complementary, okay.

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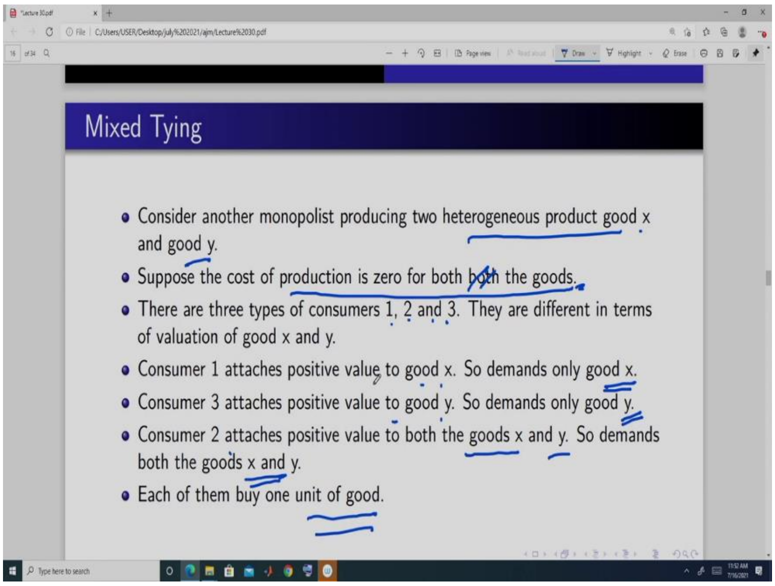

Next we can see that whether only tying is here or there can be other possible way of tying? So, we see something called a mix tying and when do we see mix tying? We will now do that. Again consider a monopolist which produces two heterogeneous product good X and good Y, okay. And suppose for simplicity cost of production is zero for both the product and there are three types of consumers 1, 2 and 3.

Now, we increase that type of consumer and they are different because their valuations are different. Consumer 1 attaches positive value to good X and so demands only good X. Consumer 1 does not demand good Y. Consumer 3 attaches positive value to good Y and so demands only good Y and consumer 2 attaches positive value to good X and positive value to good Y, so demands both the goods if it gets positive utility from or non-negative utility from them. And for simplicity each of them buys only one unit of good, okay. This is the setup.

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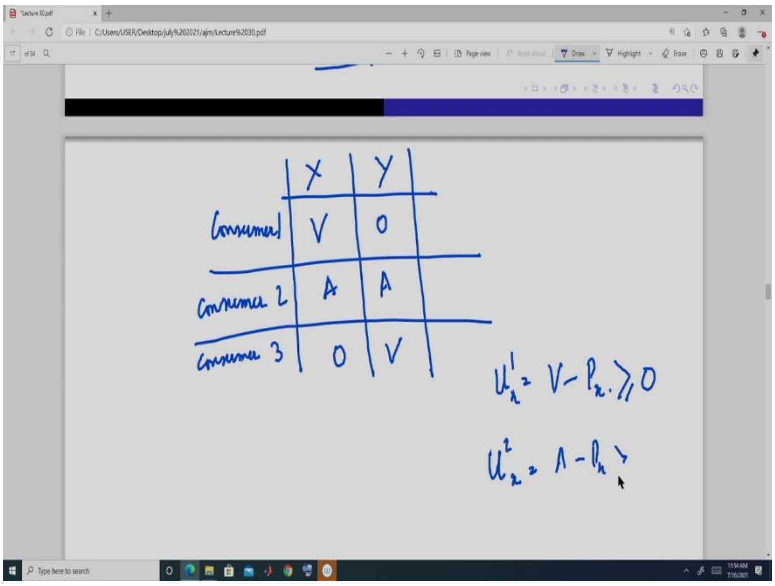

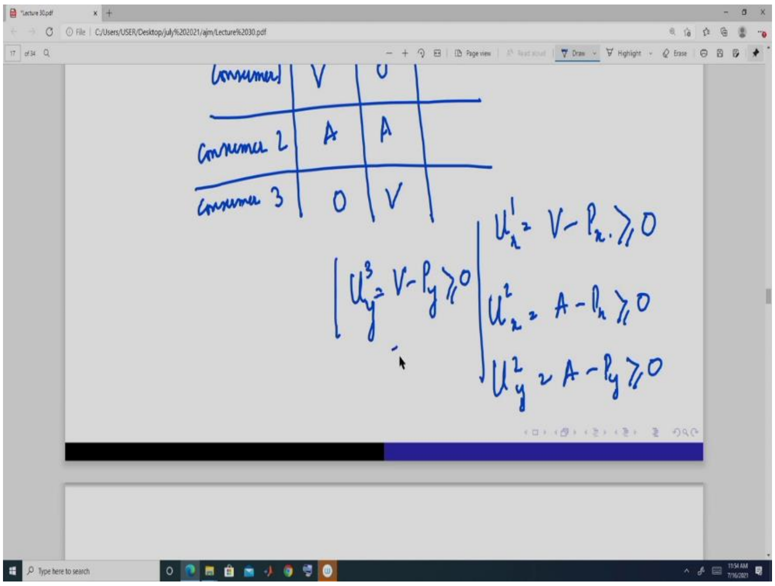

Now, we have to see how the, what is the strategy of the monopolist and before that let us look at the valuation. So, this is suppose X this is suppose Y. Consumer 1, consumer 2, consumer 3, suppose it values this and this is 0, and this is 0 and suppose this is A and this is A same. This is one simplest configuration that we can have, not even complicate this, only then the calculations will be more complicated, okay.

So, consumer 1 buys good X if V is this-$U_x^1 = V - P_x \geq 0$. Consumer 2 buy X if A minus Px-$U_x^2 = A - P_x \geq 0$, again consumer 2 buy good 2, if A minus Py gives this much-$U_y^2 = A - P_y \geq 0$ and consumer 3 buy only good A if V minus Py is greater 0-$U_y^3 = V - P_y \geq 0$. So, these are the possible A. Now, we have to see how the monopolist is going to price this and this information are there to the monopolist, okay.

And here even monopolist we do not need to specify whether monopolist can identify this types of consumers or not, even if the monopolist cannot identify, it will package the price itself will give **a** you, will lead to a self selection kind of thing, okay. So that in specification is not required.

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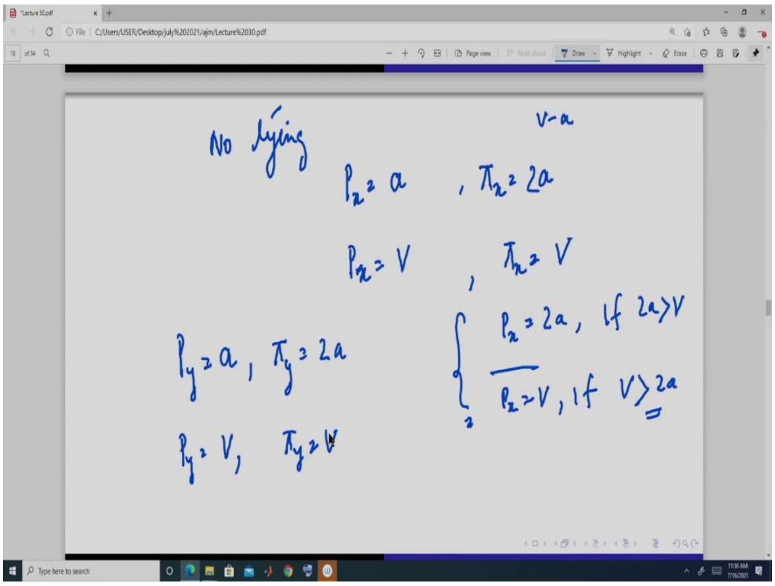

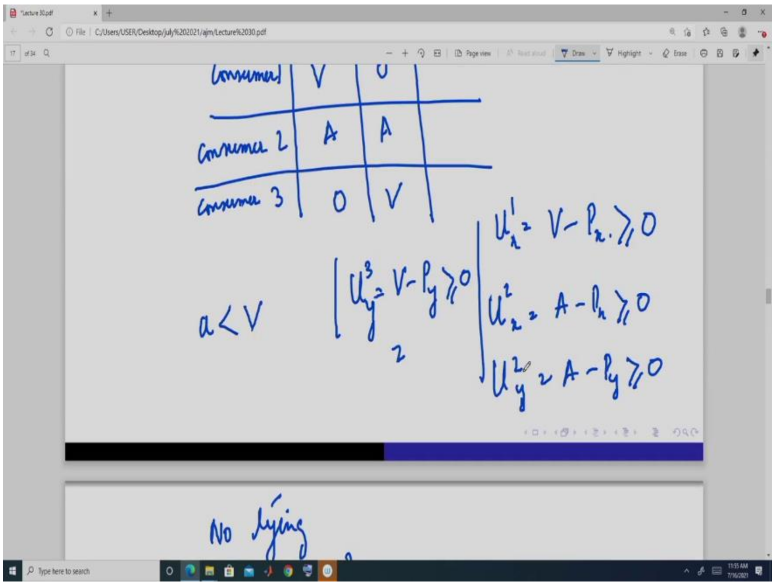

Now here, suppose no tying, okay. So then Px can be suppose a and further here we assume that suppose a is less than V, okay. So, if P is this α then profit from x is consumer 1 is going to buy because its utility is a minus V now, it is positive. So, it will get 1 and consumer 2 is also going to buy good a, so it is profit is this. Consumer 3 is not going to buy this and it can set this is equal to V also. Now then profit is this V.

So, the price here is going to be 2a, if 2a is greater than V or it is going to be this V if V is greater than 2a. So, it is same as the outcome we have got earlier and for price of good y, it can set the price a. Then consumer 2 is going to buy it, profit of firm monopolist is consumer 2 is going to buy and consumer 3 is going to buy. So, it is 2a if it sets the price this, only consumer 3 is going to buy, so it is going to this.

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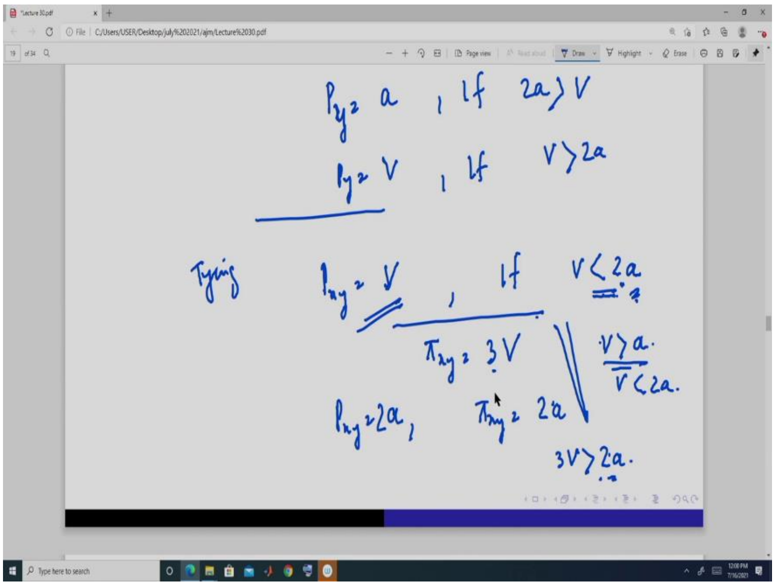

So, following this thing, we get that the Py is equal to a if 2a is greater than V and Py is equal to V if V is greater than 2a. If it is same, then they are indifferent you can charge anything, okay. Now, we have got this if there is no tying and if there is tying, suppose it does tie, then Pxy it can be V, if it is V then what it is? Then the profit and if it is V and if suppose V is less than 2a.

If this is the case, then profit from tying is, consumer 1 is going to get buy because it will get x, it is y is going to be useless for him but still he gets x and his valuation is non negative. So, his net utility is not negative if price is V. So, one consumer 1 will buy, consumer 2, since it is this he is going to get some positive utility if price is V of x y. So, this person is going to buy, so it is again V.

Third person, consumer 3 is also going to buy why because his utility from good V, good y is V and price is V. So, he is getting x but he has no value from x but it is still gets. So, the profit here is 3V, okay and if its price is suppose 2a, then this thing is going to only, if this is the case, then only consumer 2 is going to buy. So, profit here it is going to be 2a.

Now, if this is the case plus 2V and if we have suppose V is greater than a but V is less than V, right? then we will definitely have a situation where 3V is greater than 2a, okay. Because from this, from this condition- V>a, we get this, even if this is true we will get this, right. So, if they tie, then the price is always going to be this if this is true, okay and given this- V<2a. If this is not true, then it is not going to get this-3V. So, it will be only 2V, in that case also it is going to get 2V. So, it is going to be greater than this 2a.

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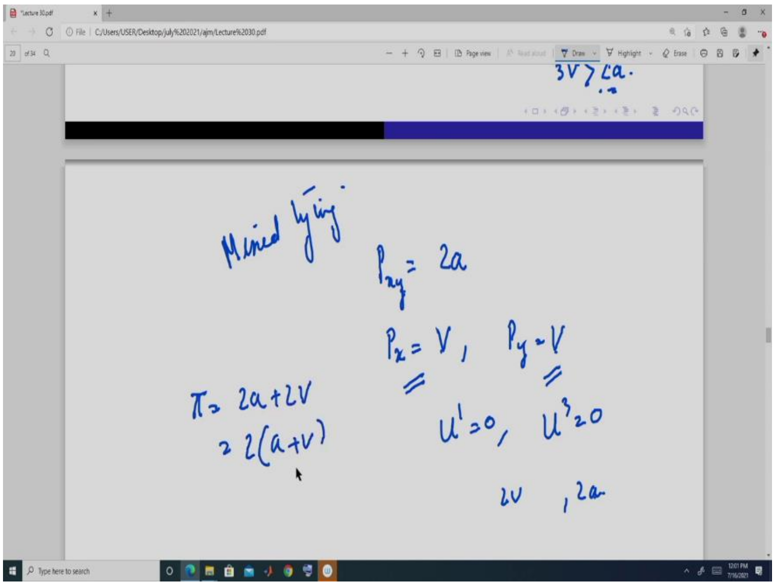

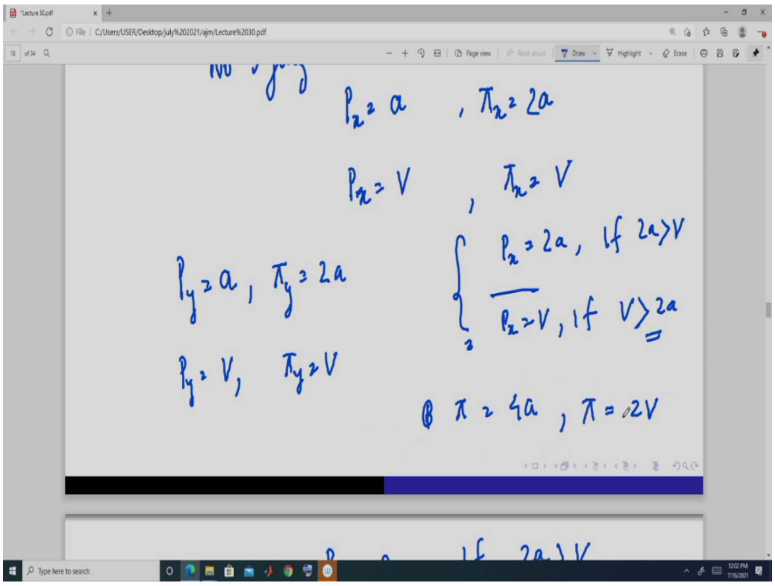

Now, so this is thing but monopolists can follow another strategy and that is the mixed tying. What it can do in mixed tying? It can set one price where it is getting both x and y at a price this-$P_{xy} = 2a$ and it can set a price V this. Now here in this equation-$P_x = V$ see if so what is going to happen? If consumer 1 will buy this, consumer 3 is going to buy this product, so at this, so their utility is going to be 0. It is going to be 0 and for this person for consumer 2, if it buys this and the separately its price is 2V, but if it buys together it is 2a, okay 2a is less than 2V.

So that is why it is going to buy that, so the profit in this situation to the monopolist is 2a plus 2V, which is 2 a plus V, this-2(a+V) is greater than both, this 3V here . And it is in this case profit is how much? Profit is this or this or it is this, so it is profit, here it can be 4a or it can be 2V, right. And these are going to be less than this. So, that is why we get that mixed tying can also be one possible. So, the main idea is to extract as much consumer surplus as possible and so the monopolist can use different strategies to extract surplus.

So, one is the if it is possible, it will only do bundling to extract more, if it is not producing more than if it is only producing one good then it can do bundling. If it is producing more than one good, then again it can do bundling and it can do tying it can do mixed tying and again do can bundle the good also. So, there are so many possibilities it can, the idea is that the monopolist will always try to extract as much consumer surplus as possible.

Next topic we going to do is tying as product differentiations. Now here what do we mean by product so, product differentiation, we see that the products are different to a firm producing another type of, one type of product, another firm producing another type of product and they are different based on some characteristics or some attributes of this good.

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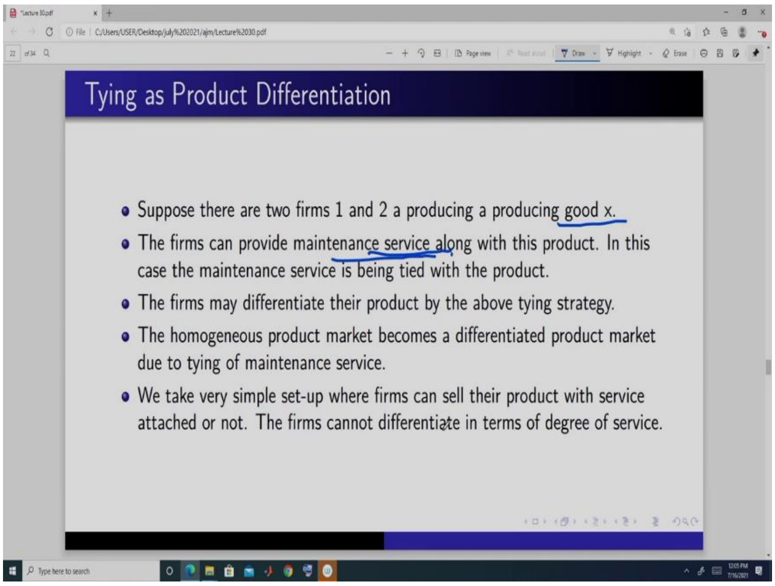

Now here we can see that suppose that both the firms produce a homogeneous product and that is suppose this x, but you require some kind of a maintenance service, maintenance service to for this good. So, by providing maintenance service and by not providing such maintenance service, you can differentiate this product. So, as a product itself it is homogeneous, but by additional this service, maintenance service, you are differentiating this.

So, for simplicity we take that the firm can either provide the service, maintenance service or it may not provide the maintenance service. So, these is a way of differentiating. So, you can have one product which you get the product and also along with it you get the maintenance service, another is you only get the product, okay.

So, in this case, we see that the products are now different; one comes along it maintenance services tied along with that product. In another it was only the product and if you want some service you will have to pay it separately. Now here we are assuming that the service is fixed, it is either you give service or you do not give service. So it is you cannot differentiate it in terms of degree of service that whether you will give 1-year service, 2-year service that is not possible, actually to keep the things simple.

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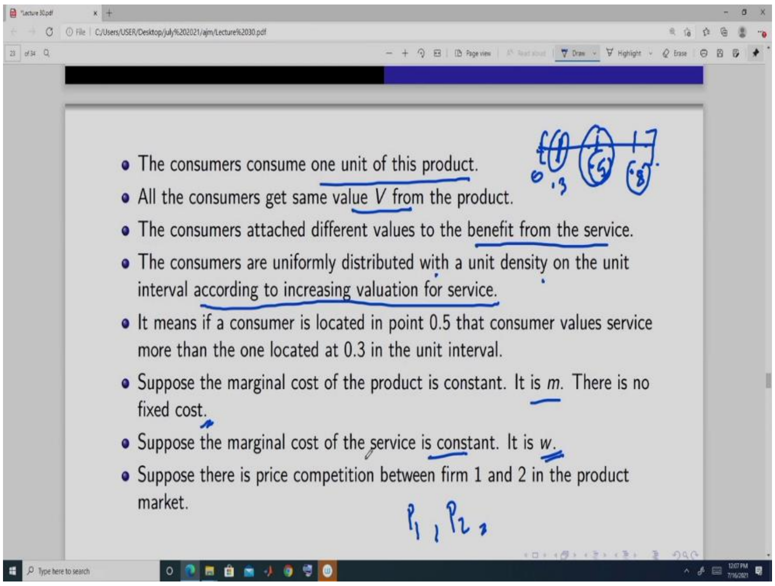

And consumers consume only one unit of good, they get the same value and that is this. Now but the consumers attach different values to the benefit from the service and how do we model it? We take this something similar to the hoteling model. And here we assume that the consumers are uniformly distributed with unit density on the unit interval according to increasing valuation of services.

So, if we take this 0 and this here, suppose this is consumer, so this consumer located here is going to value the service more than consumer this which is suppose located at this point, but this which is located at this point is going to value the service more than this, okay. So, this is how the consumers are located and there is you can say that the continuum of consumers located in this region or in this unit interval, each point or each dot represents one consumer, okay.

Now, suppose the marginal cost of this product is this-m and there is no fixed cost and suppose the marginal cost of the service, it is also constant and it is this w, okay. And in the product market for while selling this product that is there is price competition that means firm 1 decides a price p1 and firm 2 decides a price p2, okay.

So, you can say that there is a bertrand competition and so here we may get three possible outcomes and see and it is, you can say that it is a two stage game. What happens? okay We will discuss the game later.

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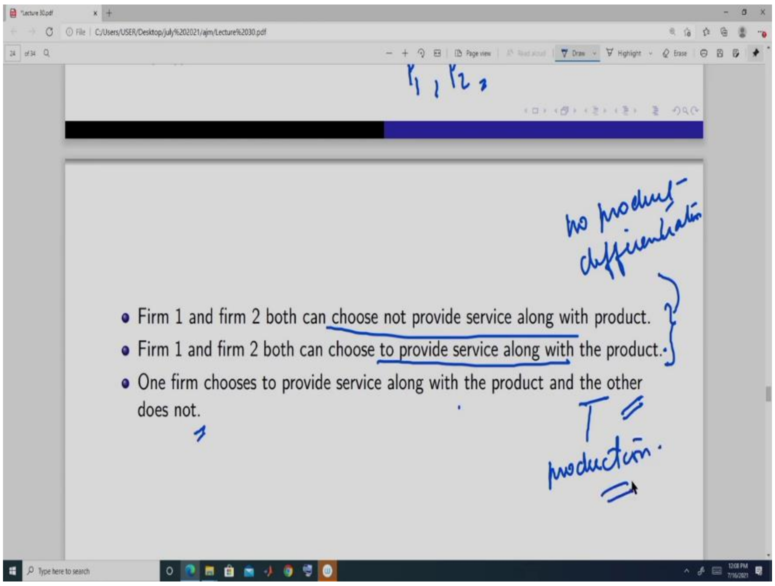

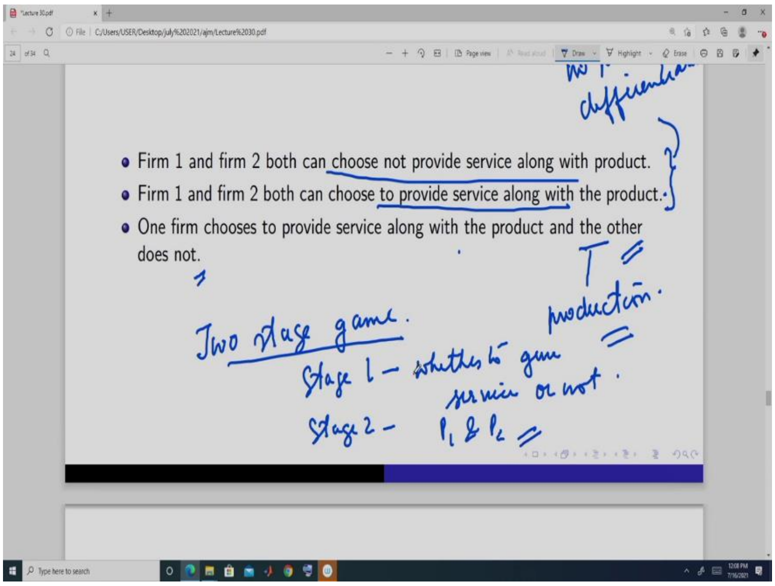

But first, what are the possible thing; one the both the firms choose, does not choose service along with the product, they give only the product that is one outcome. Another outcome is both the firms choose to give the or provide the service that is another outcome. Or another outcome is one firm chooses to give service and other firm chooses not to give the service, okay.

So, here we get the product differentiation. In this two case, there is no product differentiation and here we see that there is product differentiation, okay. And product differentiation is through tying a service and the game is a two stage game. Stage 1, it decides whether to give whether to give service or not and in stage 2 it decides p1 and p2, okay. So, we will consider it using subgame perfect Nash equilibrium.

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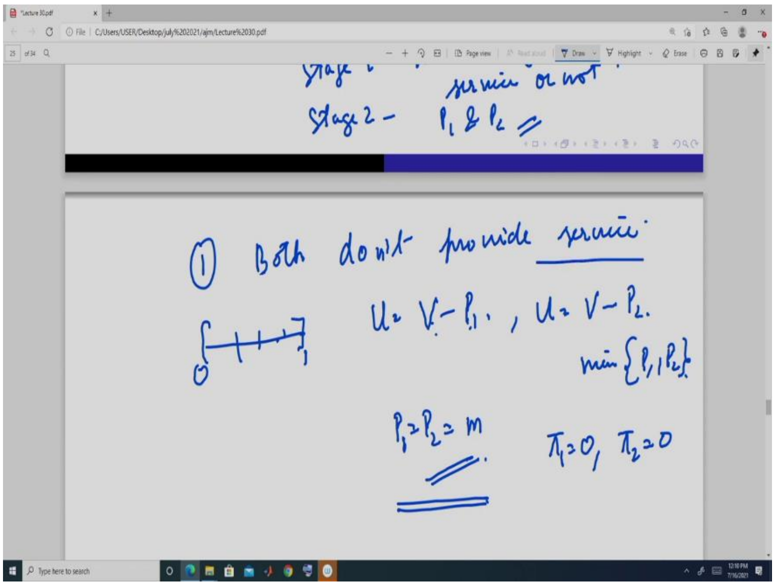

Suppose the first case, suppose both case 1; both provides, both do not provide the service. So, the utility that the consumer gets is V minus p1 if it buys from the A and p2, okay. V is the utility and it does not get any services, so it is same. So, here what is going to happen if this person is given any location, here or here any point it will choose that where p1 is which is lowest.

So, it is minimum of p1 and p2, right. So, we know here if it charges anything greater it is not going to buy. So, ultimately here p1 is equal to p2 and it is equal to m, this is the optimal strategy and profit of firm 1 is 0, profit of firm 2 is 0 in this case, when both do not provide service, okay.

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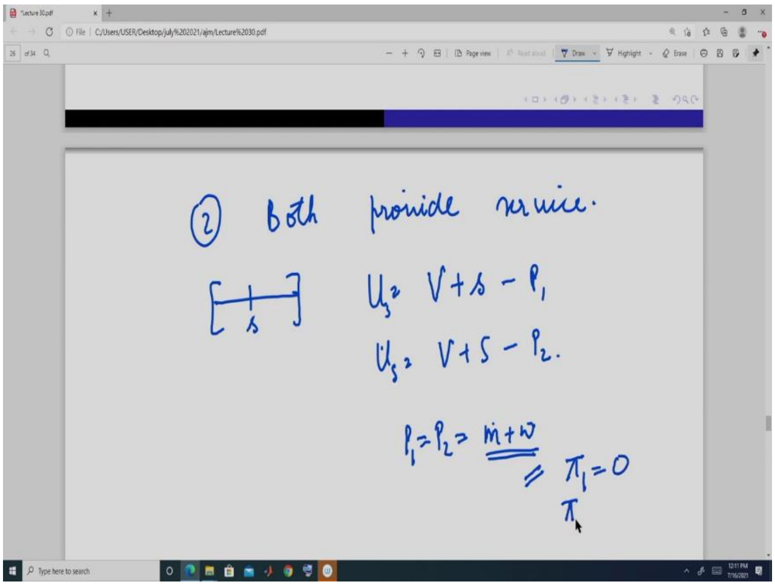

Now, suppose both provide service. When both provide service then what is going to happen? Utility of any consumer is going to be this plus this if this is the location of that consumer- U = V + s, consumer s minus p1 if it buys from here and it is going to be this if it is buys from firm 2. So, this remain same, so there this consumer is going to buy from s, from that firm whose price is less.

So, again here if you charge any higher price none is going to buy from that. So, that is why there is only one possibility that p1 is equal to p2 and that is m plus w because this is the marginal cost, sum of the marginal cost. This is the marginal cost of the product and this is the marginal cost of the service, okay. So, here again in this case profit of firm 1 is 0, profit of firm 2 is 0. So, bertrand paradox is fully operational here and here in this two case, in first case.

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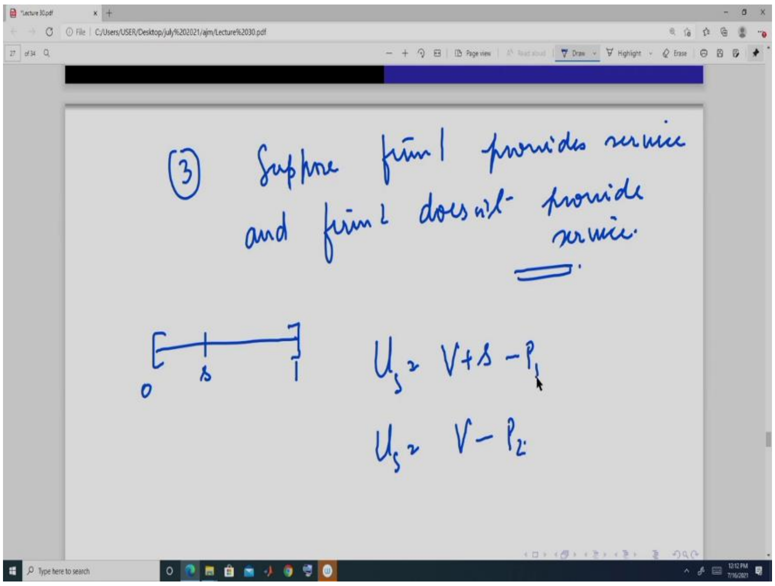

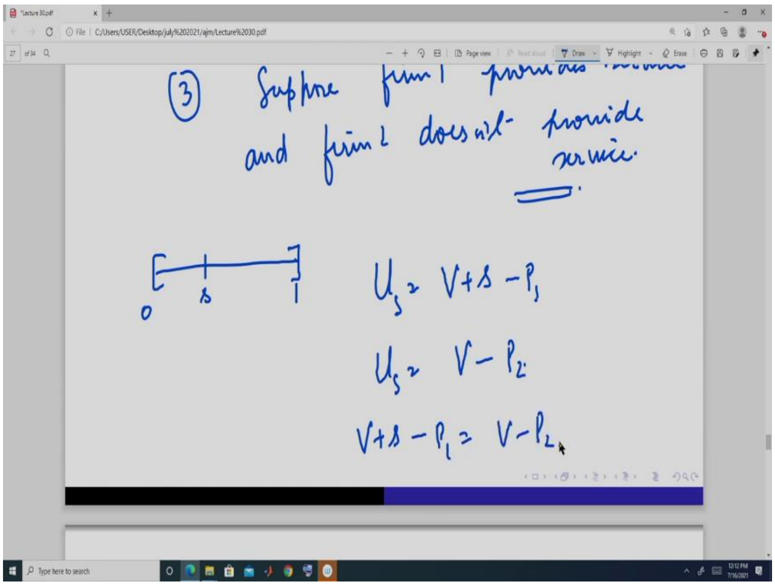

But in the case 3, suppose firm 1 provides service and firm 2 does not provide service. This is one possibility, okay. Now, let us look at these consumers. This is suppose consumer x, sorry consumer s, utility of consumer s is V if it buys from firm 1- $U = V + s - P_1$. And utility is this minus this, if it buys from firm 2 because it does not get the service, here it gets service. So, from these, these two valuations are not same. So, we get that now the price of firm 1 and firm 2 can be different. How can it be different?

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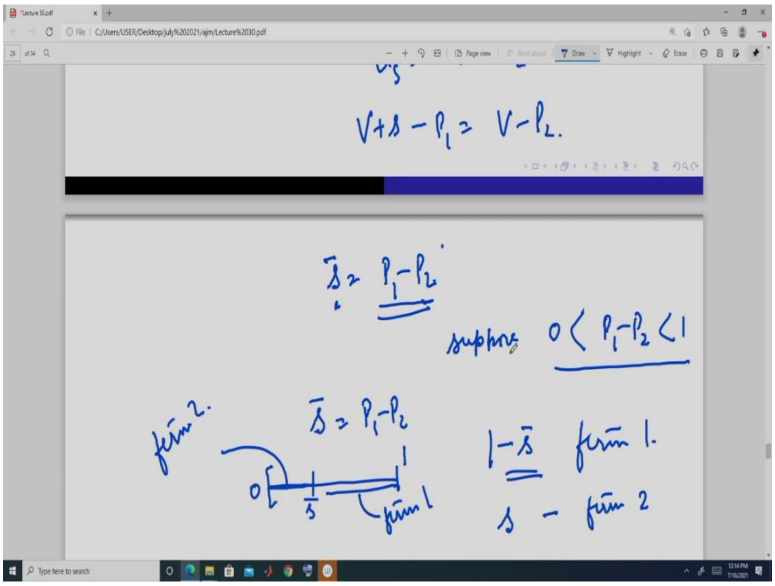

See we can find a value of s such that it is equal to P- s $= P_1 - P_2$ and here see all these person's s is more than this, valuation is more than the s, all this, all these person's valuation is less than s. So, in this case if this suppose this is s interior, so we assume this-$0 < P_1 - P_2 < 1$, okay. So, if this is the case then what do we get? We get that we have s bar where p1, given p1 and p2 and this person gets same utility from buying from firm 1 or firm 2.

But they are getting different product and in product of firm 1 is that the same product with service; product of firm 2 is only the product no service. So, the market of firm 1 is 1 minus this s bar, this-1-s, okay. And market of firm 2 is all these consumers are going to buy from firm 1 and all this, so, if this is s bar, then this region is for firm 2 and this region is for firm 1. So, this is the firm 2, okay now we define the profit function.

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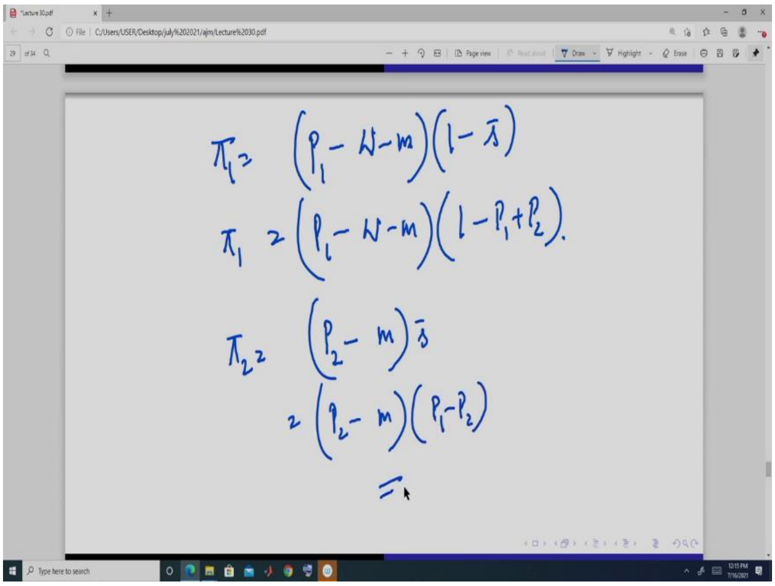

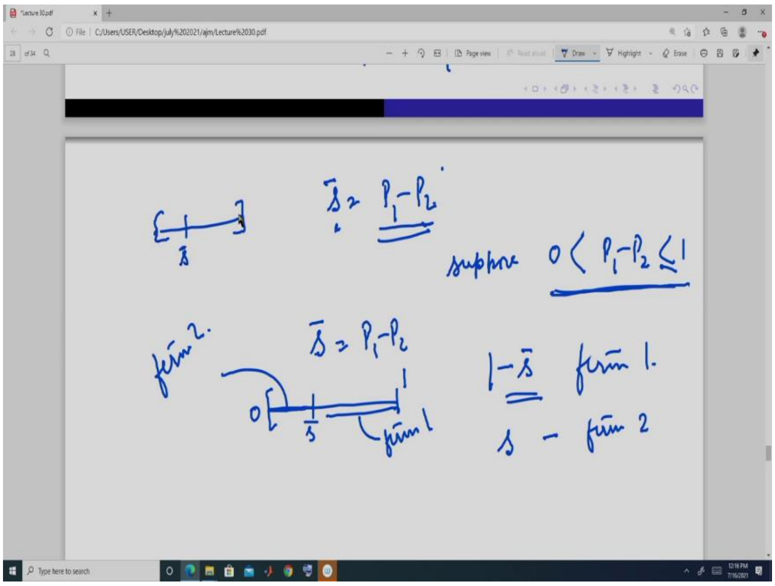

Profit function of firm 1 is it will charge the price p1 and it will marginal cost from service marginal cost from product into it will be serving this many people which is equal to this-$\pi_1 = (P_1 - w - m)(1 - P_1 + P_2)$ and profit of firm 2 is this-$\pi_2 = (P_2 - m)(P_1 - P_2)$. Here while doing this we have assumed this thing, now we can take this, okay. Why because if so that we get an interior thing as lies within, not here, neither here.

If it is at the boundary, then it is means either firm 1 is serving or firm 2 is serving. So, it is so that means what either only one type of product is there, we do not see any product differentiation in this case. So that is why we will assume this only, consider this thing.

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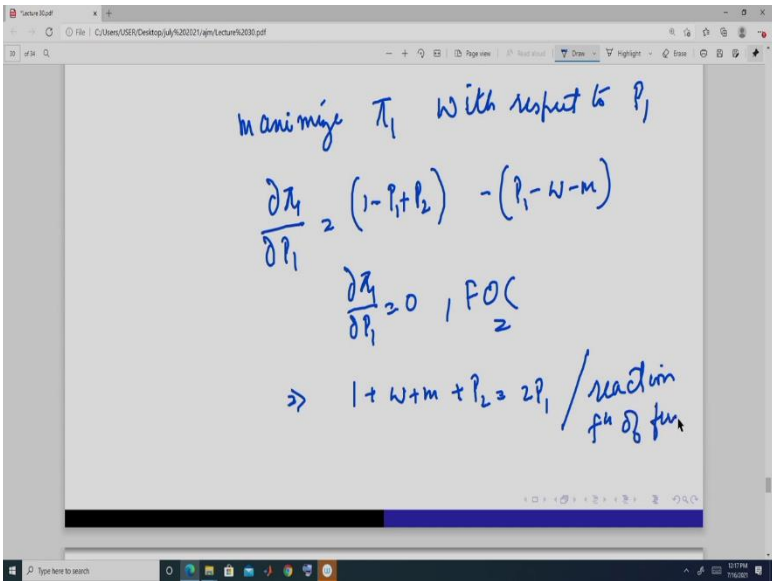

Now here we maximize profit, maximize p1 with respect to p1. So, we get what? This will give you this-$\frac{d\pi_1}{dP_1} = (1 - P_1 + P_2(+(P_1 - w - m)$ and first order condition gives this equal to 0 first order condition. So, we get the reaction function of firm 1 in this form. This is the reaction function of firm 1-$1 + w + m + P_2 = 2P_1$, okay.

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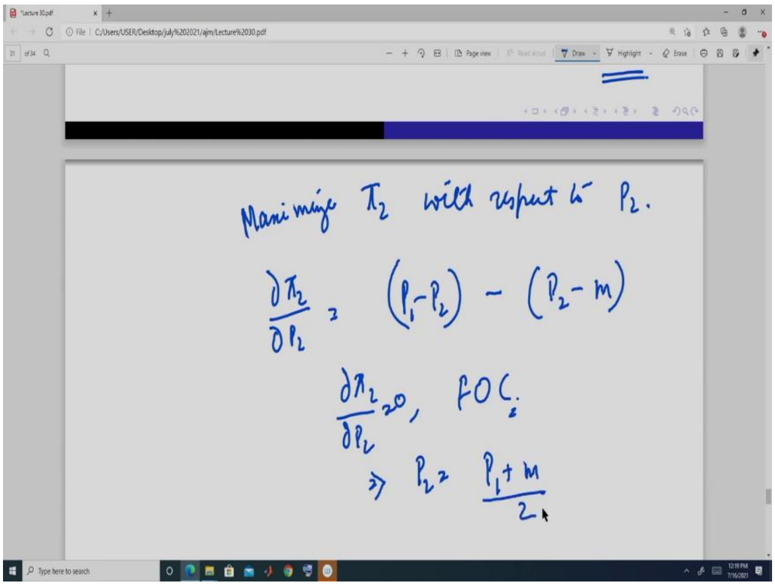

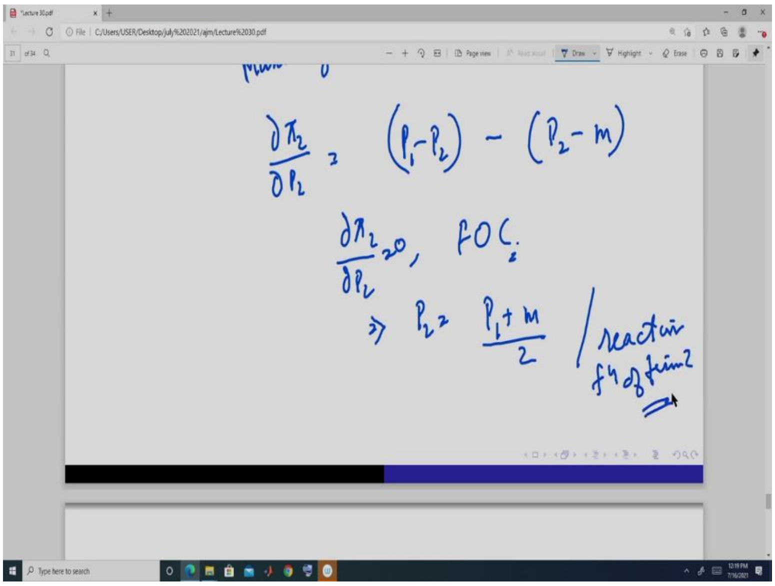

Similarly, firm 2 what it does maximize p2 with respect to p2. Now, here what is going to happen so, we have we know the a it is going to be this and this is going to this-$\frac{d\pi_2}{dP_2} = (P_1 - P_2) - (P_2 - m)$ first order condition gives me this is equal to 0 first order condition, so we get p2 is equal to p1 plus m divided by 2. This is the reaction function of firm 2-$P_2 = \frac{P_1+m}{2}$.

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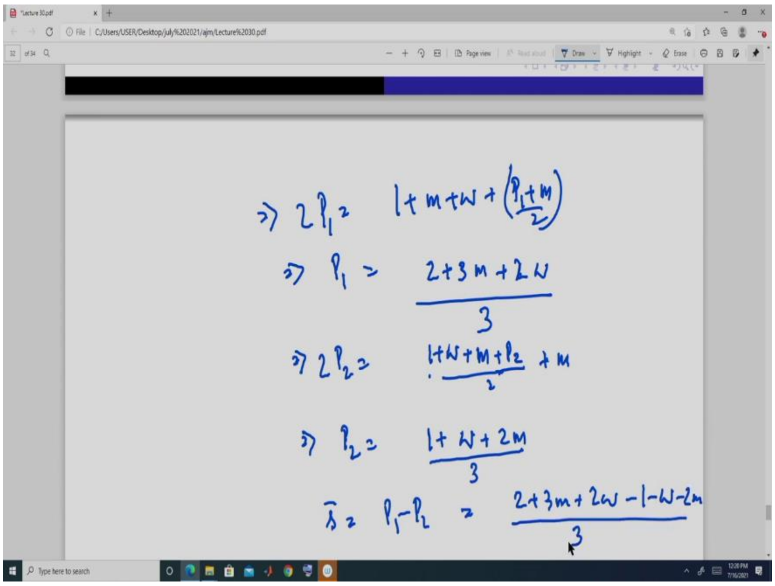

Now, we solve these two reaction functions what do we get p1 is equal to 1 plus m plus w by… So, this implies p1 is equal to 1 plus; 2 plus 2m plus not here it is also m. So, it will be 3m; 3m plus 2w divided by 3-$P_1 = \frac{2+3m+2w}{3}$ and p2 is what? So, 2p is 1 plus w plus twice m divided by this-$P_2 = \frac{1+w+2m}{3}$. Now, what is s bar? s bar is p1 minus p2 this is 2 plus 3m plus 2w minus 1 minus w minus 2m divided by 3.

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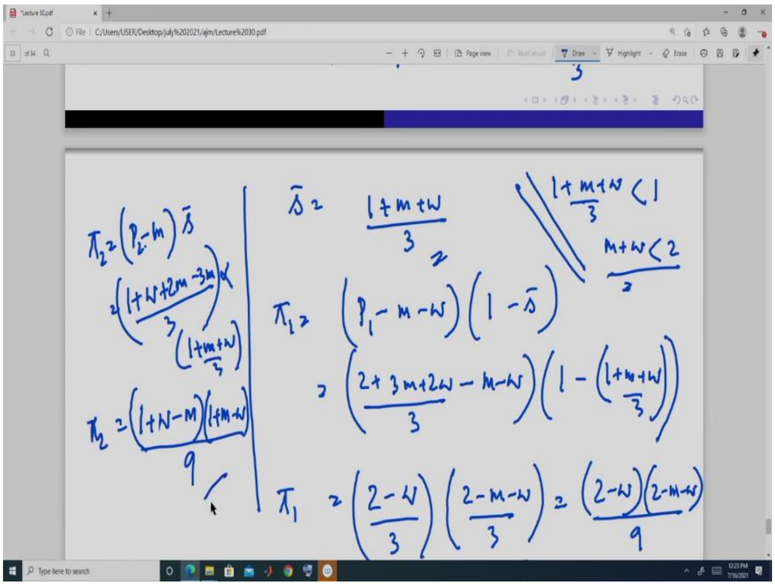

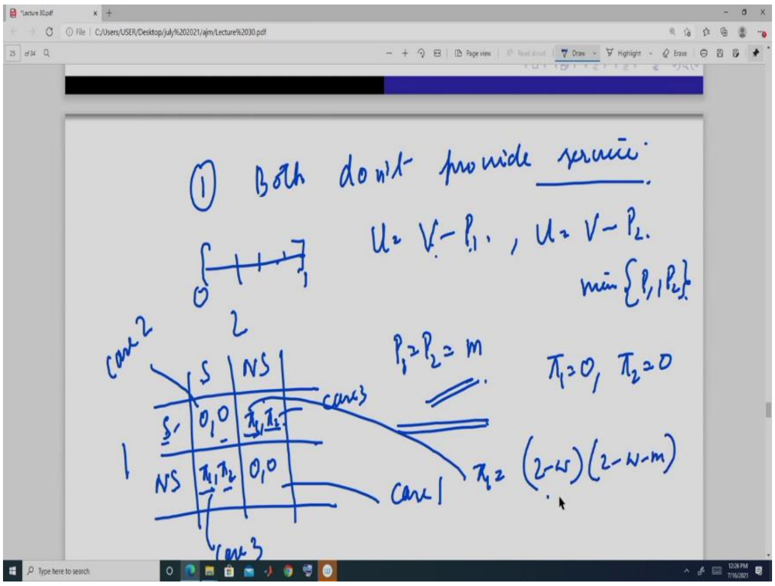

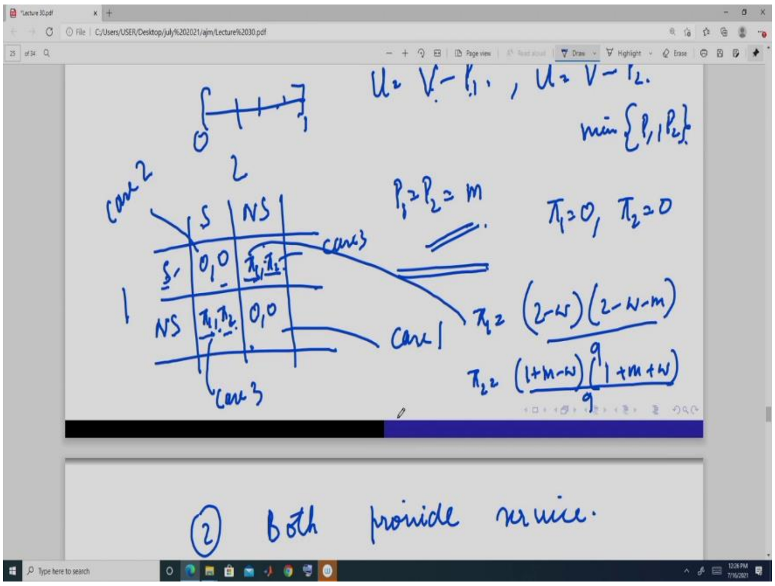

So, s bar is equal to 1 plus m plus w divided by 3-$\frac{1+m+w}{3}$. So, profit of firm 1 in this case is p1 minus w minus m minus w 1 minus s bar. So, this is 2 plus 3m plus 2w minus…-$\pi_1 = \left(\frac{2+3m+2w}{3} - m - w\right)\left(1 - \frac{1+m+w}{3}\right)$, right? Now, here we have to assume that this is definitely greater than 0 and this is less than 1. So, we have to assume that this is less than 2- m+w<2, okay for this condition to satisfy this-$0 < P_1 - P_2 \leq 1$. So, for simplicity assume this okay, this condition needs to be satisfied.

Now, solving this we get 2 is this, this is the profit of firm 1-$\left(\frac{2-w}{3}\right)\left(\frac{2-m-w}{3}\right)$. So, this is 2 minus w 2 minus m minus w divided by 9. And profit of firm 2, sorry profit of firm 2 is p2 minus m s bar, p2 is this 1 plus w plus 2m minus 3m into this, this so this is 1 plus w minus m 1 plus n minus w divided by 9, this is the profit of firm **2**-$\pi_2 = \frac{(1+w-m)(1+m-w)}{9}$ and this is positive and this is also positive. But, in this situation what do we get 0 and 0.

Now, here we can do something. So, strategy, in stage 1 strategy of firm 1 and strategy of firm 2. So, it provides service, it does not provide service, it provides service, is no service firm 1, firm 2. If both provide service it is 0 0, if both does not provide service it is 0 0, these two outcome 1, 2, so this is case 2 and this is case 1, this is case 3 and this is case 3.

Here we know profit a firm 1 and firm 2 both are positive. So, if firm 1, we have computed this firm 1 provide a service firm 2 and we will get the symmetrically opposite here. So, we get two Nash equilibrium under this and this. So, this we have got, this is what, this profit now, this profit, so it is 2 minus w, so it is 2 minus w and then 2 minus w minus m divided by 9. And profit of 2 is 1 plus w 1 plus m divided by 9.

So, in this is and it will be opposite here. So, this will be firm 1s and this will be firm 2s. And here this since both of them are positive so either this is a Nash equilibrium or this is a Nash equilibrium. So, here what do we get, so the subgame perfect Nash equilibrium outcome is that either firm 1 does provide the product differentiated product and firm 2 does not provide the service or firm 1 does not provide the service and firm 2 provide the service.

So, this is our form of tying where we see that both the firms are not doing tying, only one of them is tying and the tying is allowing them to differentiate the product and that is a subgame perfect Nash equilibrium outcome we have got, okay. And so, tying can be helpful in this sense and that can be a strategy which can be used in not only by the monopolist only also in an duopoly to differentiate the product.

So, from a homogeneous product market, you can switch to a differentiated product market by tying these kind of services. So, this is a very important strategy that a firm may utilize, okay. So, this was our last class. So, we I end this here.

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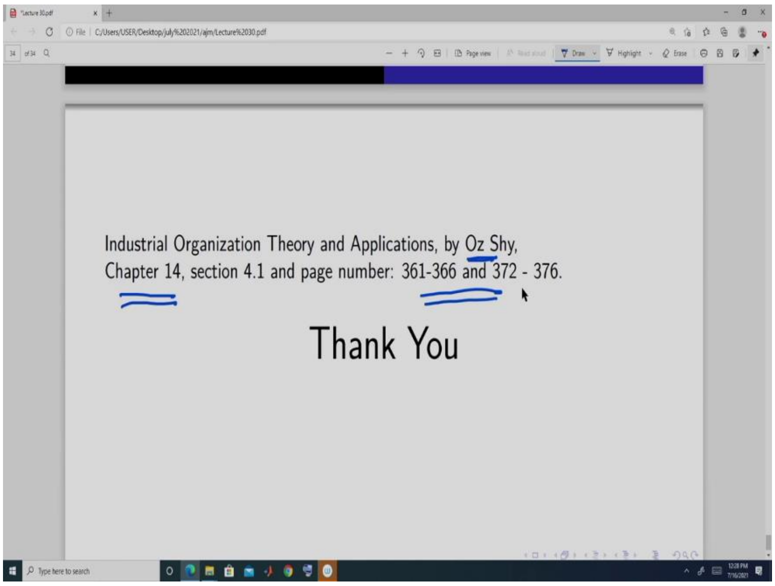

And for this portion tying and bundling, you can read this chapter 14 of this book by Industrial Organization Theory and Application by Oz Shy and these are the specific page numbers. So, this is a very long chapter and we have done a very small portion of it. And so that is why I have specified the pages.

So, here what we have done basically, in this A, we have done first we have mainly our objective was to look at how the interactions takes place in a market between the buyers and the sellers. So, from the buyer side we get the demand. So, that since they are buying, so we have derived the demand.

Firms they produce. So, they inquire incur some cost. So, we have done the cost minimization and from there we have derived the cost function. Now, then we have specified different setups under which these buyers and the firms interact. First is the competitive market where both the firms and the buyers takes the price as given and based on that, we get the market equilibrium we have shown that the perfectly competitive market is a Pareto optimal and also social welfare maximizing outcome.

Next, we have looked at monopoly; what is the monopoly price, what is the monopoly outcome? In the monopoly there is only one firm and we used the conventional downward sloping demand curve. And we have looked at the different price discrimination strategies of the monopolist also.

Then, we have introduced strategic interaction and to do that we have introduced game theory and we have done two types of, two forms of games; that is complete information static game, one shot simultaneous move. So, we know how to find a pure strategy and mix strategy of such games and then we have done dynamic games and we know we have used the solution concept that a subgame perfect Nash equilibrium.

And using these two we move to these two tools, we have done the analysis of the other forms of markets; the first Cournot duopoly where we have shown that it is not a pareto, it is something like a prisoner's dilemma kind of outcome, the Nash equilibrium outcome is and then we have looked at the Cournot oligopoly thing and we have also looked at the limit outcome of that.

Then we have introduced price competition that is the bertrand competition and we have discussed the bertrand paradox and their different setups. And then we have introduced capacity constraint and then we have seen that under certain condition the bertrand paradox may not be there, when capacities are not sufficiently big.

And then we have done in decreasing returns to scale in the production function. And then if we have price competition and decreasing returns to scale, then what is the outcome we see that there is a whole range of price that exist, okay. It is so, the Nash equilibrium, pure strategy Nash equilibrium is not unique.

Then we have done Stackelberg; both quantity competition and price competition and here we have androgenize, we have also looked not androgenize, we have look whether there is a first mover advantage or there is a second advantage. And we have got that in quantity competition there is first mover advantage and in a price competition if the goods are substitute theory the second mover advantage, but if the goods are complement then we see that there is a second mover advantage.

After that we have introduced a hoteling kind of model where we have done the product differentiation and we have done to under 2 setup that is, when the price is, when the firms choose the location simultaneously and when firms choose their locations sequentially. And we have found the pure strategy Nash equilibrium in the first case and the subgame perfect Nash equilibrium in the second case.

Next we have introduced entry deterrence; entry deterrence to see whether a firm is deterring because that may lead to a monopoly power in the firm. So, mainly it is important from the perspective of the regulatory authorities. Regulatory authorities always see whether the existing incumbent firm is engaged in some form of entry deterring strategies or not. And strategic investment can be a possible entry deterring strategy we have seen that, but it is not always, not in all the cases, only in certain cases.

Then, today we have done the bundling and the tying and there also we have shown the pure strategy Nash equilibrium, only in case of tying when there is a, we have introduced two firms, but in case of bundling we have only done it for monopoly thing, okay. So with this, I hope that you have enjoyed this course. Thank you.

Introduction to Market Structure
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 12
Lecture 43
Tutorial on Dixit's Model of Entry Deterrence

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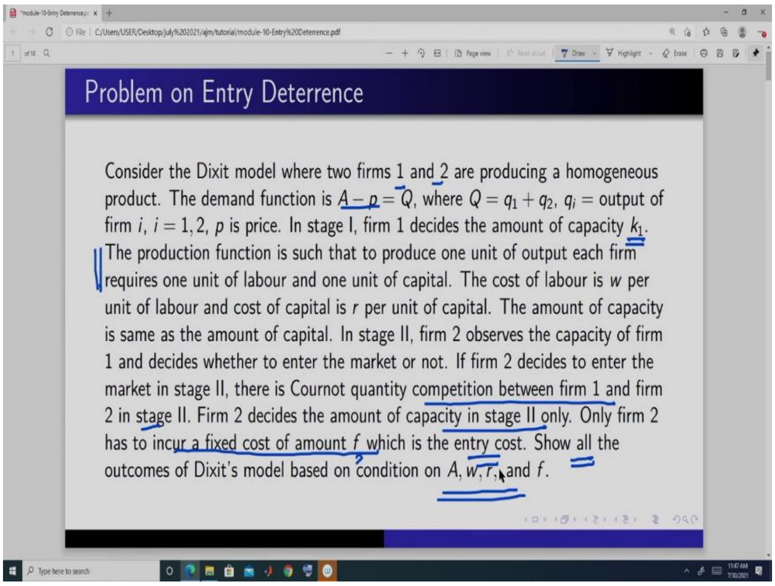

Let us solve one problem on Entry Deterrence or you can take this as an example. Now suppose there are two firms, firm 1 and firm 1 and firm 2, market demand function is this- A-p=Q and q1 and q2 are the outputs firm 1 and firm 2, p is the market price here in stage one firm decides the amount of capacity that is k1, the production function is such that to produce one unit of output each firm requires one unit of labor and one unit of capital.

This is exactly same as the Dixit model, the cost of labor is w per unit of labor and the cost of capital is r per unit of capital, amount of capacity is same as the amount of capital, okay. And in stage two firm 2 observe the capacity of firm 1 and decides whether to enter the market or not to enter, if firm 2 decides to enter the market in stage two, there is cournot competition between firm 1 and firm 2 in stage two.

So, in stage two, there are two things that is being decided. So, first firm two decides whether to enter or not. And once it enters, then there is cournot competition between firm 1 and firm and firm 2 decides the amount of capacity in stage two only. So, only firm 2 has to incur a fixed cost of amount f which is that entry cost firm 1 has no entry cost because it is an incumbent firm and firm 2 has an entry cost because it is an entrant firm.

And now we will show all the possible outcomes in the Dixit model based on providing conditions on A w because these are the parameters in this model. So, we will find them.

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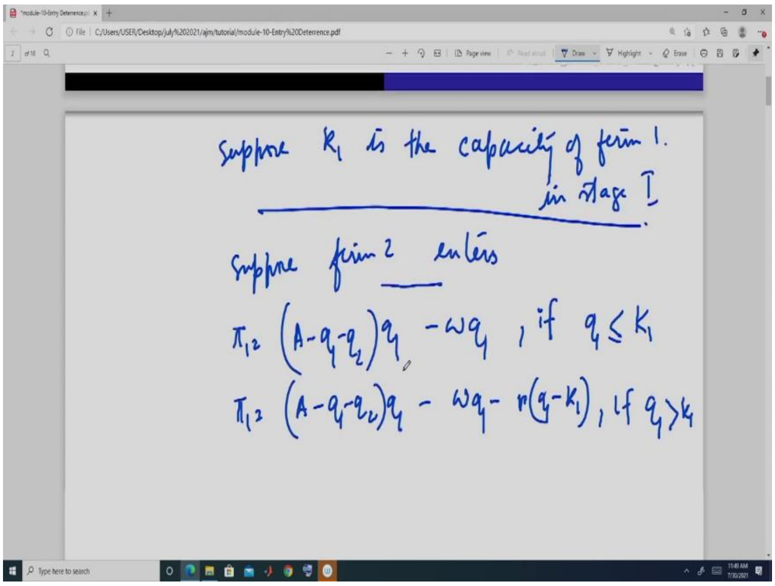

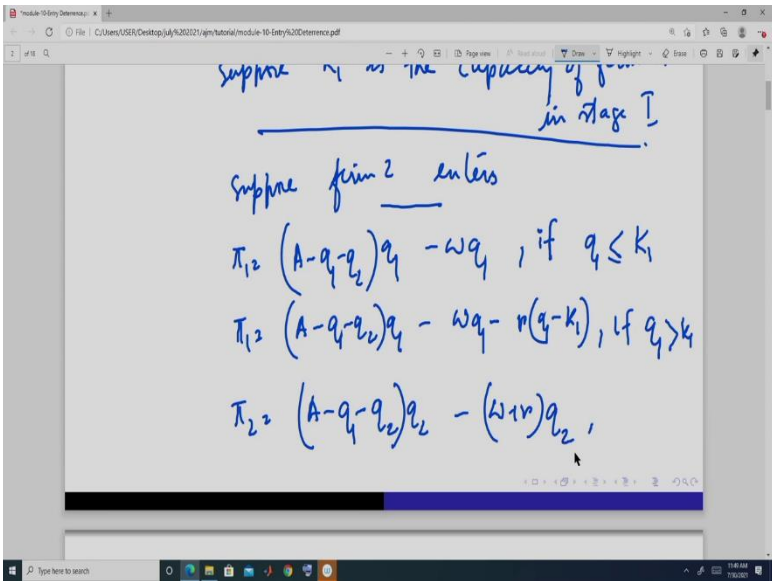

So, first suppose k1 is the capacity of firm 1 in stage 1, so this is k1 okay. Now let us move and suppose firm 2 enters, okay then profit of firm 1 is A minus q1 q2 this-$\pi_1 = (A - q_1 - q_2)q_1 - wq_1$ if q1 is less than or equal to k1 and otherwise, it is this-$\pi_1 = (A - q_1 - q_2)q_1 - wq_1 - r(q_1 - k_1)$ if $q_1 > k_1$ because the expenditure on the capacity is already borne in stage 1, okay so we get this and profit of firm 2 is and this is the total revenue this-$\pi_2 = (A - q_1 - q_2)q_2 - (w + r)q_2$.

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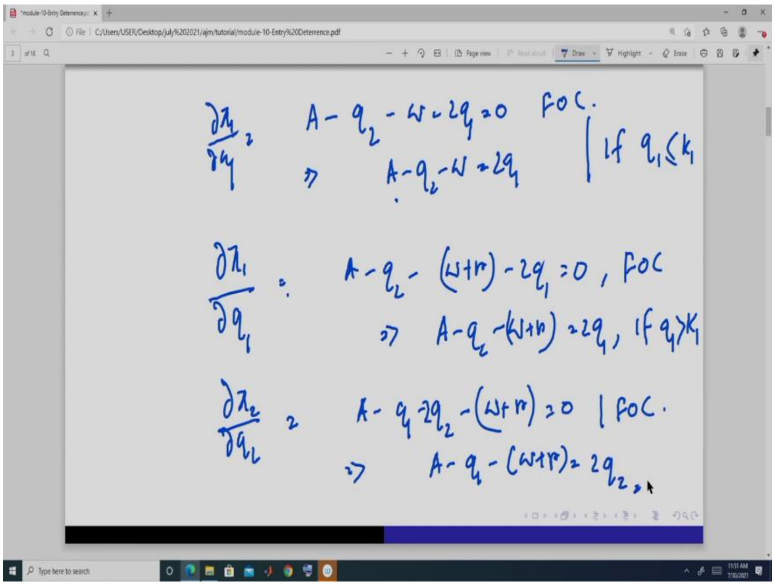

Now from here we can derive the reaction function of firm 1 and firm 2. So from, so we get and this gives me, this is the first order condition-$A - q_2 - (w + r) - 2q_1 = 0$ implying and, so, these are the reaction functions-$A - q_2 - w - r = 2q_1, A - q_1 - (w + r) = 2q_2$.

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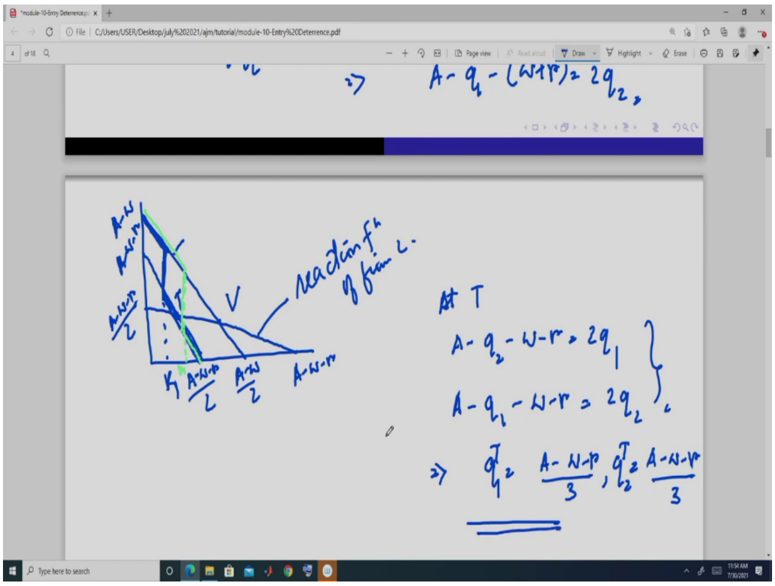

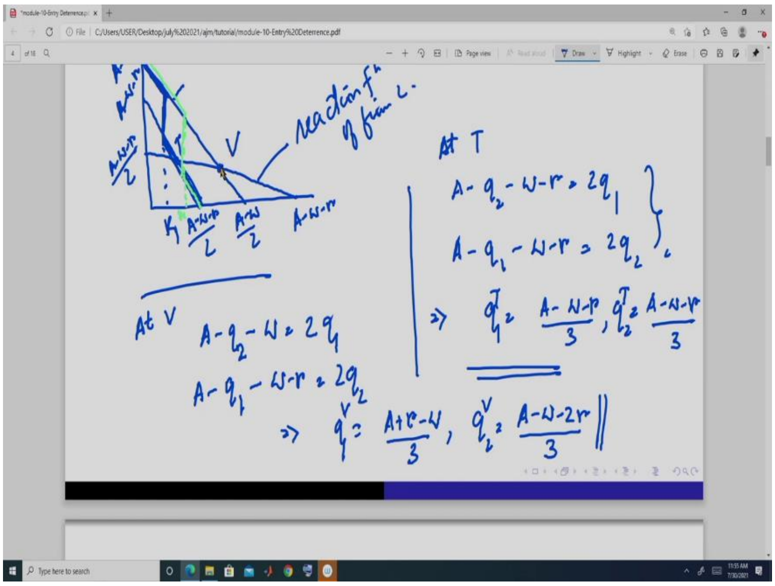

And if we plot them, we will get something like this, this, this is the reaction function of firm 2 this is point T, this is the point V that we have used and suppose this if this is the capacity, then the reaction function of firm 1 is this thick line, if this is the capacity, if this is k is the capacity then the reaction function of firm 1 is this green line this. So, we get the reaction function of this nature based on the exact value of capacity, okay.

Now, we will find out the output in this point T and V because our analysis is dependent on these two outputs. So, this is when this reaction function intersects with this reaction function. So, T point at T these two intersects- $A - q_2 - w - r = 2q_1, A - q_1 - (w + r) = 2q_2$. So, from this we get that k1 is, we get this so, this is the point at T-$q_1^T = \frac{A-w-r}{3}, q_2^T = \frac{A-w-r}{3}$, okay.

At V we get the reaction function of firm 1 is this $A - q_2 - w - r = 2q_1$ and reaction function of firm 2 is this- $A - q_1 - (w + r) = 2q_2$.. So, solving these two we get, so this is at the point V-$q_1^V = \frac{A+r-w}{3}, q_2^V = \frac{A-w-2r}{3}$, okay. So, we get that this is less than this and this is greater than this so this point we have got.

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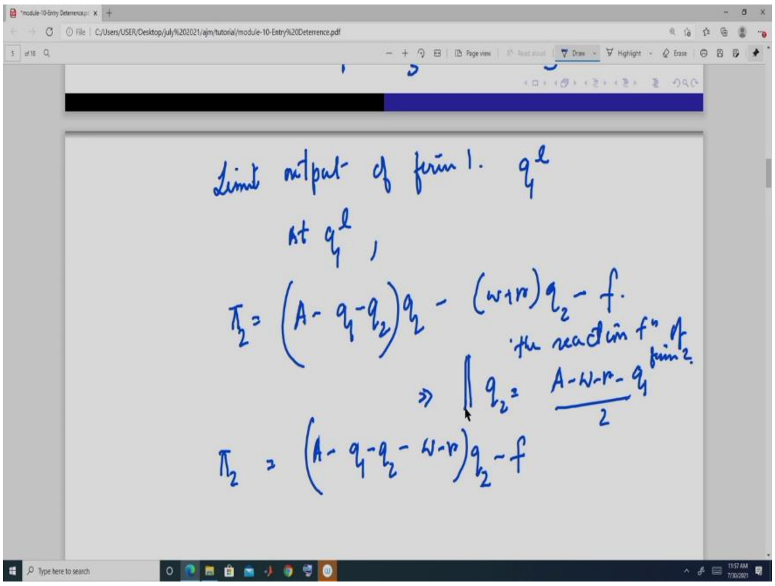

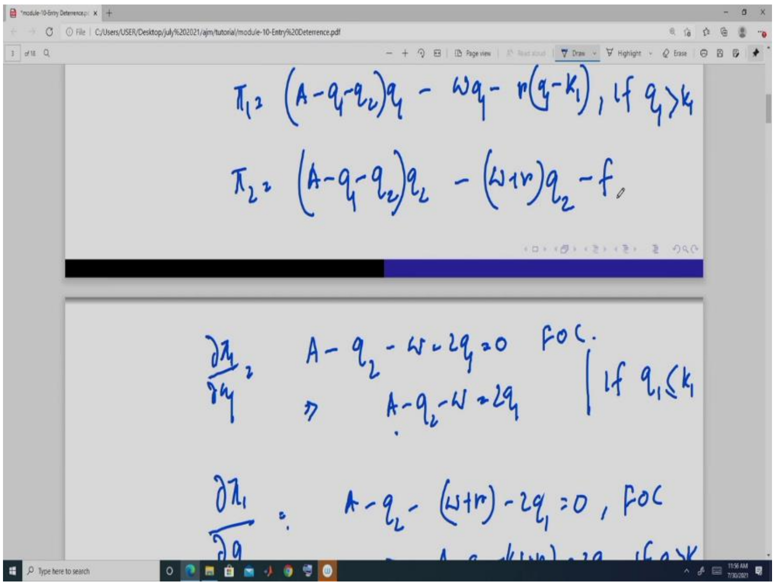

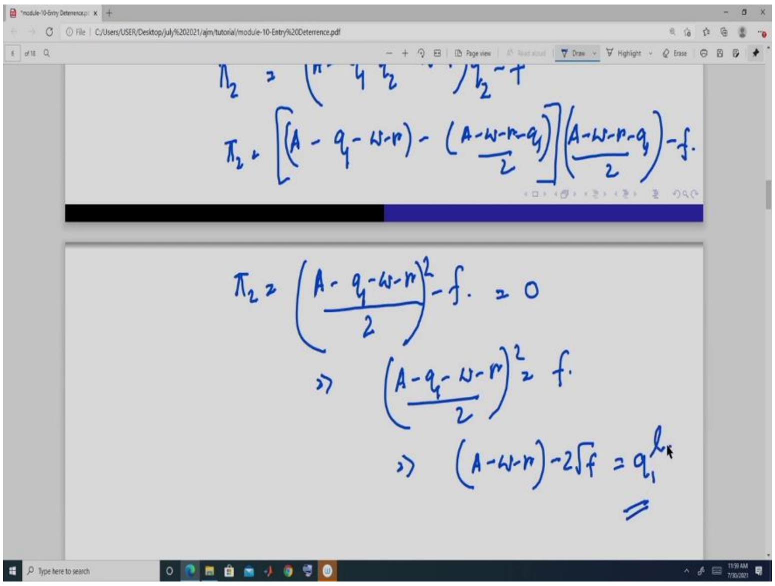

Now we will find the limit output of firm 1, limit output of firm 1 it such that it is q 1 l. So, at q 1 l I have made a mistake here. So, while specifying the profit function of firm 2 here I have to specify the entry cost f also, okay but this is a fixed cost so, it is not going to appear in the reaction function of firm 2, okay now into this and we know from the reaction function q2 is it is this-$q_2 = \frac{A-w-r-q_1}{2}$ so, there is this, this is the reaction function of firm 2 and we have got this.

So, plug in reaction function here we get this-$\pi_2 = \left[(A - q_1 - w - r) - \left(\frac{A-w-r-q_1}{2}\right)\right]\left(\frac{A-w-r-q_1}{2}\right) - f$ and if we solve this we get, get this-$\left(\frac{A-w-r-q_1}{2}\right)^2 - f$. Now, we have to find q1 such that this is equal to 0. So, we get equating this equal to 0 we get, so this is the limit output-$(A - w - r) - 2\sqrt{f} = q_1^l$.

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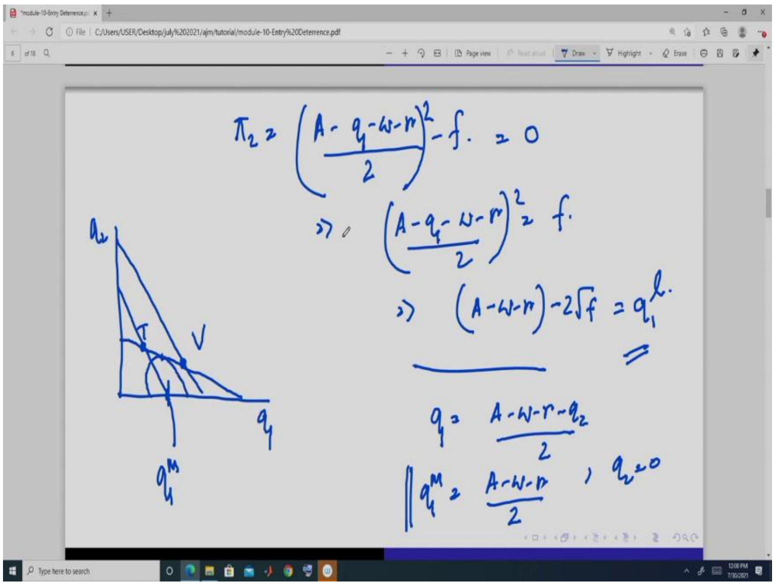

Now, let us again draw the reaction curves. So, this is the point T this is point V and we know this is the monopoly outcome of a firm. So, this is it is in this reaction function. So, q1 is, it is this here, now, q2 is equal to 0 so, q1 M is is this-$q_1^m = \frac{A-w-r}{2}$. So, this is the monopoly output. This is the limit output. Now, we have to find out the stackelberg thing here, so stackelberg thing it is something like this. It is something like this so this point.

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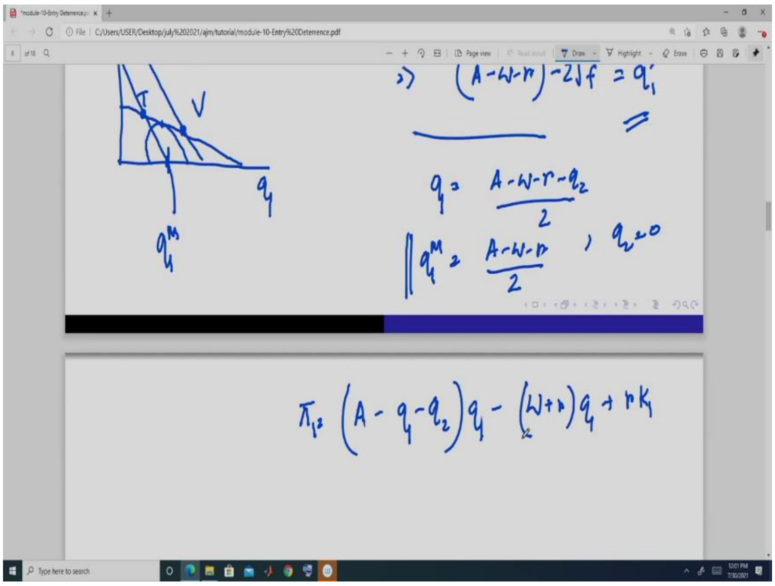

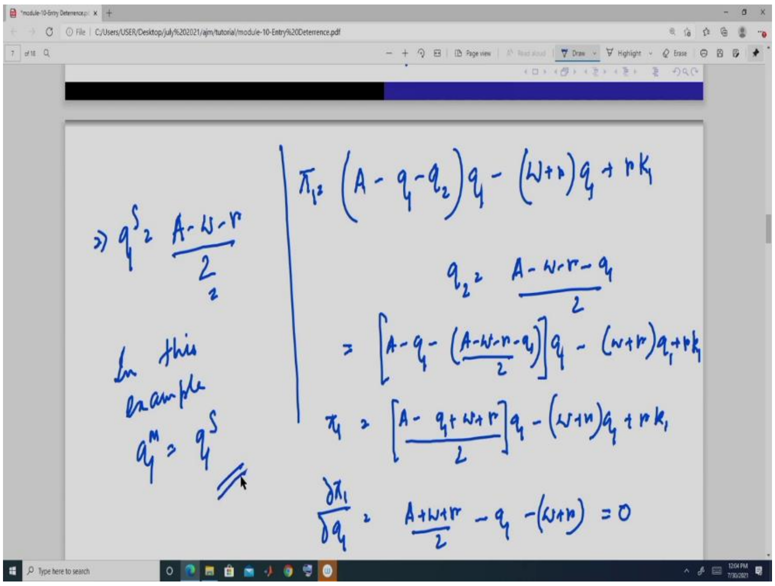

So, how do we find the stackelberg thing. So, this is the profit function of firm 1 you can write it in this form, if we take this reaction function because this reaction function means that its capacity is less than what is the amount of output it is producing, right. So, now here we know the reaction function of firm 2, it is this-$q_2 = \frac{A-w-r-q_1}{2}$, plug in this here, we will get, now optimize this, this is done in the stackelberg we get equal to 0-$\frac{d\pi_1}{dq_1} = \frac{A+w+r}{2} - q_1 - (w+r) = 0$. So, from this we get so, in this example we get that q1 M is equal to q1 S, okay and we have got this.

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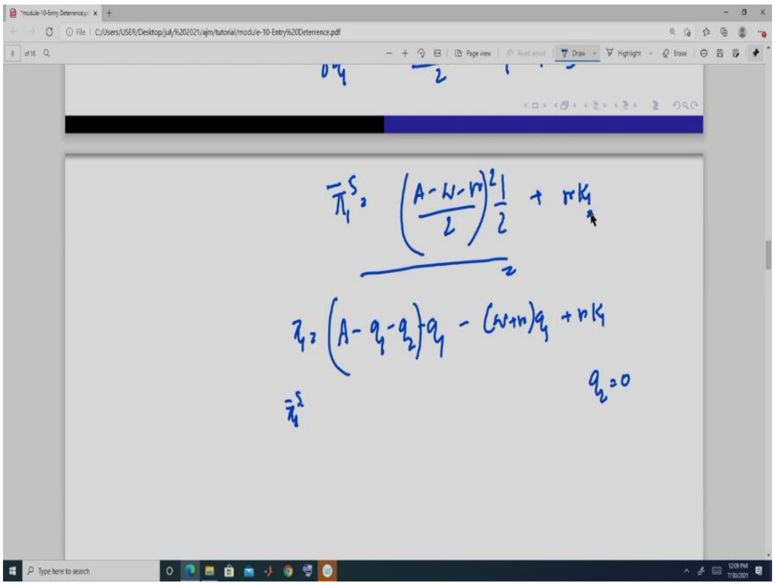

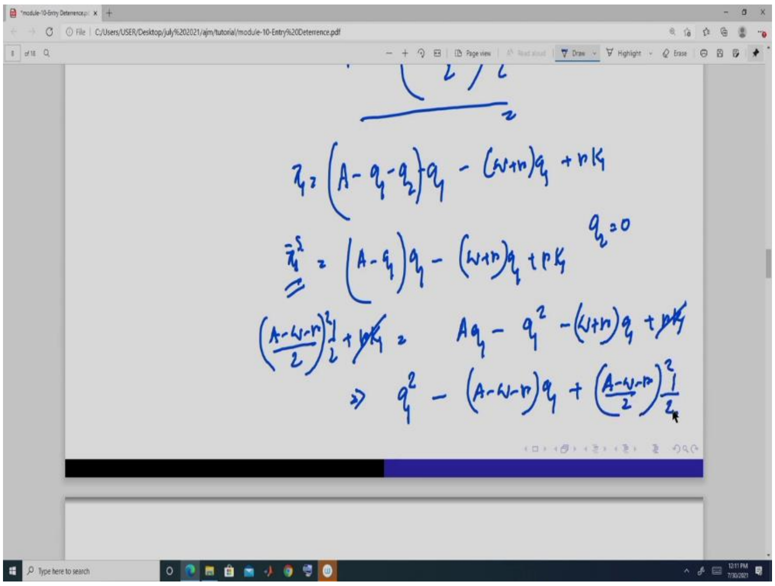

Now, we have to find out this, this stackelberg we have got this to the same, we have to find out this part, this is S naught q1 S naught so, how do we find q1 s naught, so stackelberg profit if you substitute that here stackelberg profit is so, k here, if you look at this here k will not be more than, should not have excess capacity, because it is going to end soon. So, profit of firm 1 in this case you can, output is this.

So, this stackelberg profit can write this A plus, A minus because if you take this common you get this minus, this plus so, you can, you will get this only because actually if you look at this term what you will get, you will get that this k1 is actually equal to the stackelberg output and so, that cost is borne in stage 1 you will not get any additional cost here in terms of the A but if you remove it and add it together you will get the profit in this term only.

So, that in this term only because what you can do is you can write it in this term. If you simply take this position or simply hit this, okay stackelberg thing which we have got from this here, this point. Now how to find out this thing, this we can find out because here I have when I have got this stackelberg I have simply taken the stackelberg profit when we are not taking this capacity thing anymore consideration, okay.

Actually the capacity will be same as the output and so, we will get the same thing if we combine the cost of the stage 1 and the profit in stage two together, okay. So, this is going to be the total profit when we have the outcome as the stackelberg thing okay or you can simply take this and then go and this A is, this is the, now when q2 is 0 and fix this profit at the stackelberg thing, okay.

So, here I can write this in terms of this also if we strictly follow this A then we will get this, we will have, this is equal to 0. So, we will have, this term is cancels out, we get this-$q_1^2 - (A - w - r)q_1 + \left(\frac{A-w-r}{2}\right)^2 \cdot \frac{1}{2}$.

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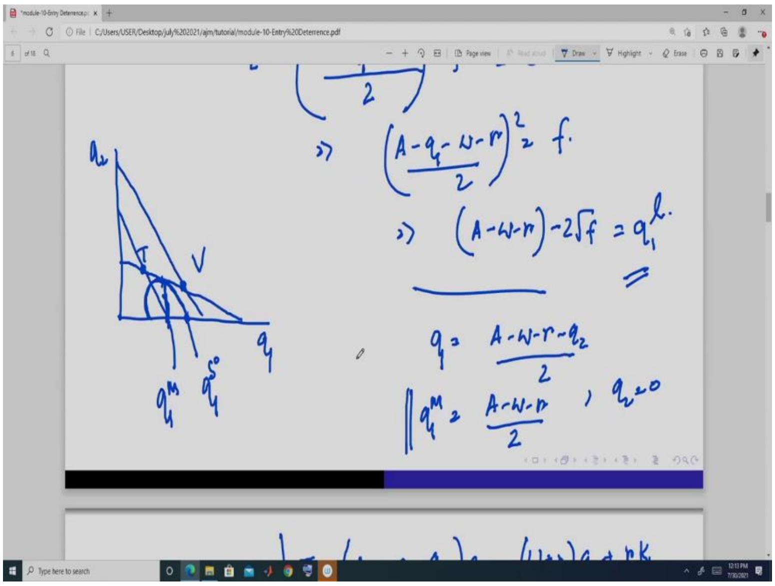

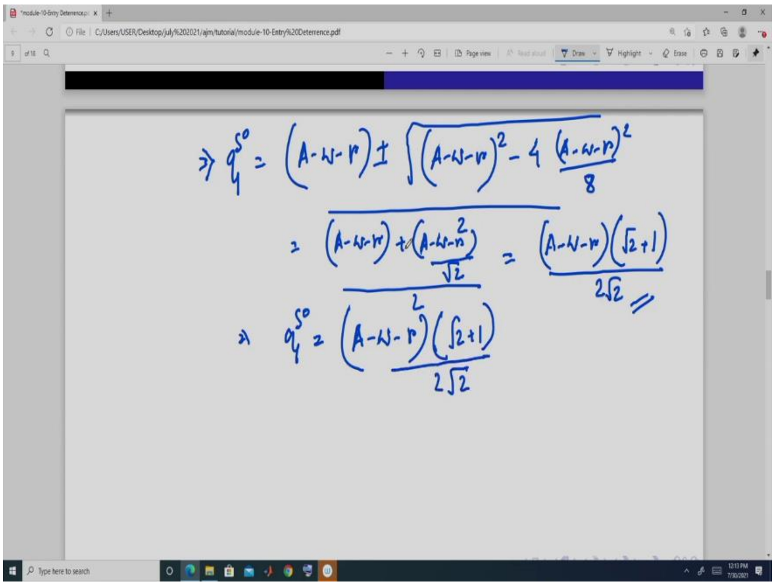

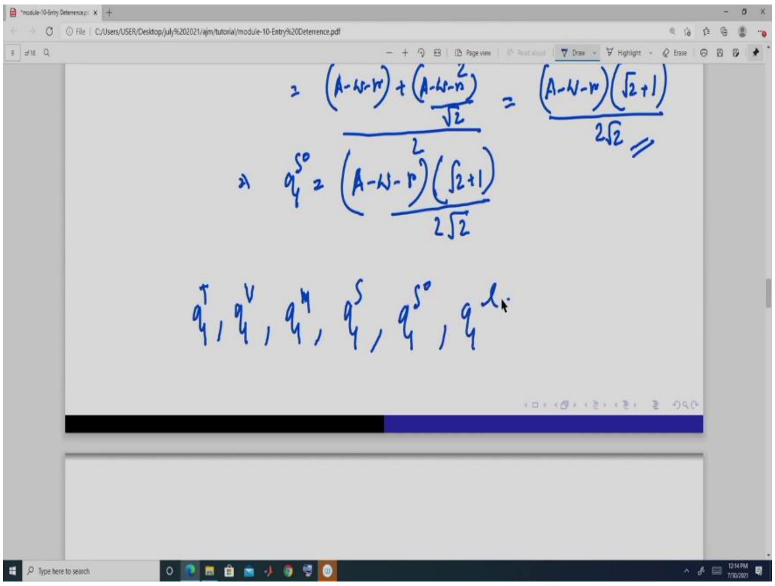

So, from this, let me get this divided by this which is this if we this is what, this is the, this q1 S naught, this, this output is this, this. Now, this minus A will be if we take it will be this one so, we do not bother we required this, okay. Now we we have derived all the outputs. So, what outputs we have? We have q1 T we have q1 V we have q1 M we have q1 S we have q1 S naught we have q1 l.

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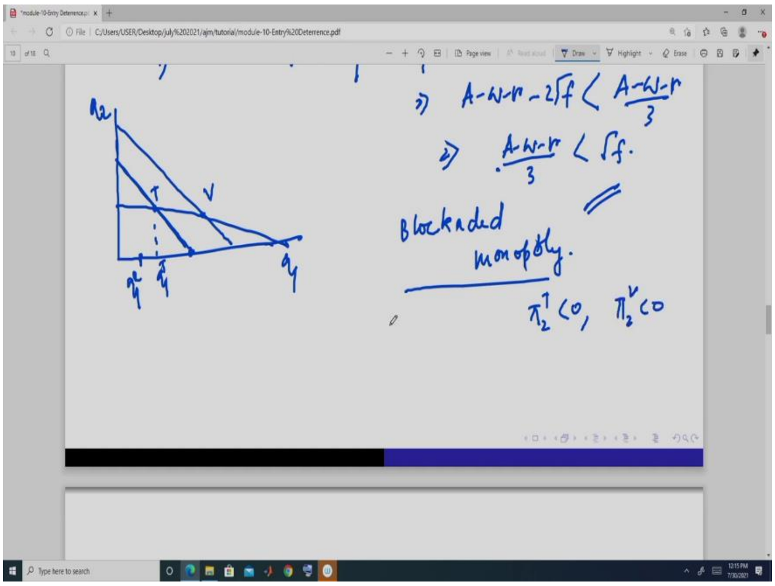

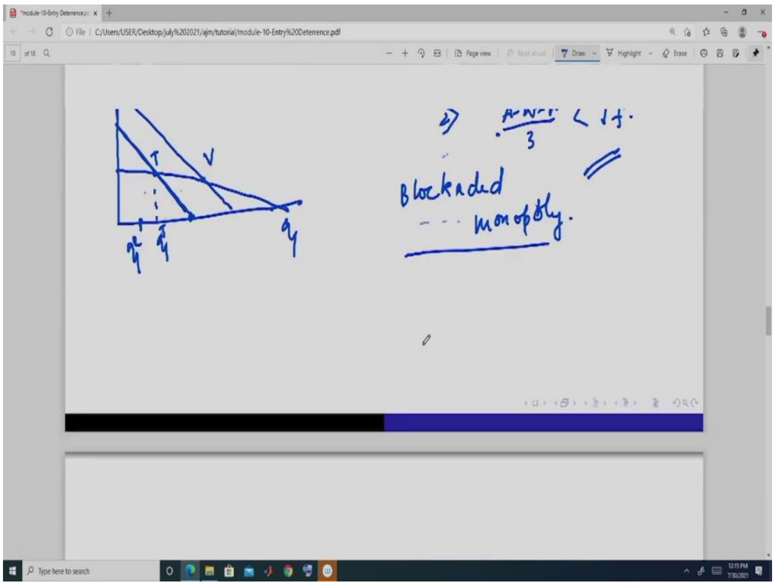

Now let us do the case 1, if q1 S is less than q1 T. So, this implies A minus W, this is less than this- $A - w - r - 2\sqrt{f} < \frac{A-w-r}{3}$. So, this means this is the case in this situation there is, if this is the case then what we have got, we have got that q1 this q1 is T, so firm 1 will so this is the case of blockaded monopoly. Next case 2 so, in this case what we had profit upon 2 at T is.

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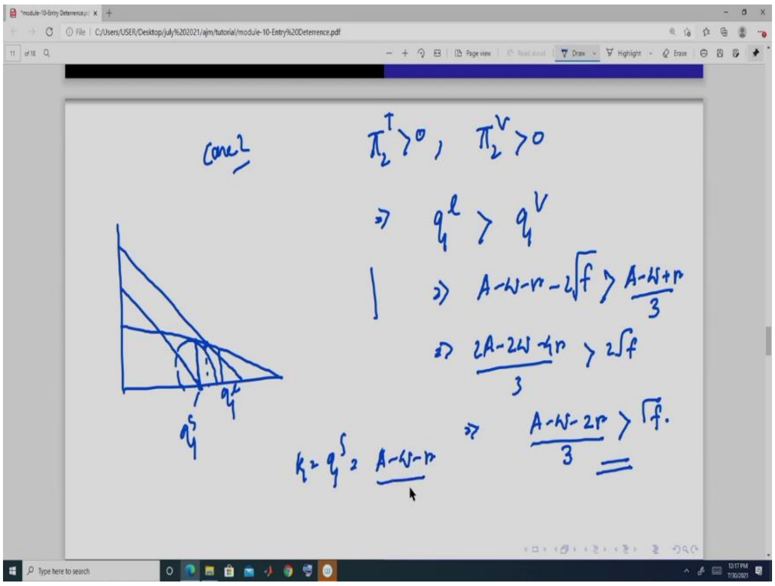

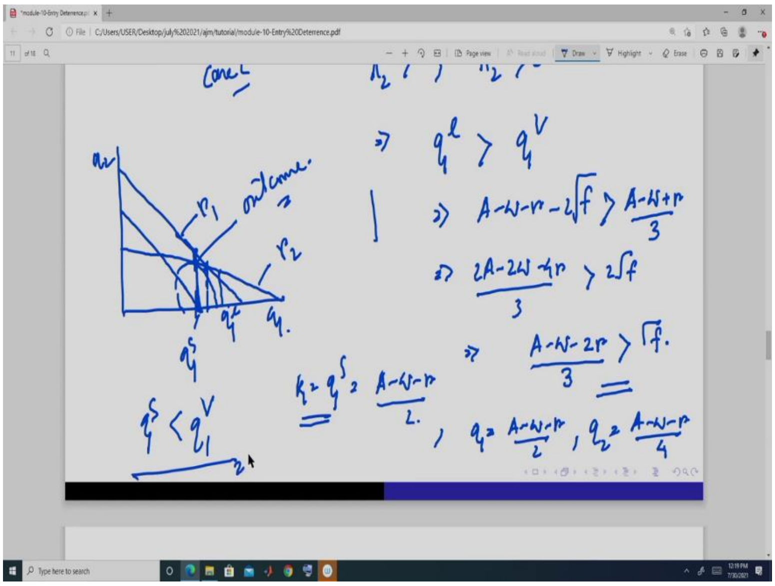

Next case 2 profit upon 2 positive, this is also positive. So, this implies q1 l is greater than q1 V. So, this implies, it is this-$A - w - r - 2\sqrt{f} > \frac{A-w+r}{3}$ this so, if this is the case-$\frac{A-w-2r}{3} > \sqrt{f}$ then what do we see? Here q l is somewhere here it is this, if this is the case and we know stackelberg is this. So, here in this case q1 is going to be q1 S and it is going to be and here q1 is going to be the stackelberg outcome this- $\frac{A-w-r}{2}$and q2 is going to be this- $\frac{A-w-r}{4}$because here it is like this.

So, if it tries to it will not be able to deter firm 1 will not be able to deter the entry of firm 2 because if it has this much amount of capacity then output is going to be this, now, if it produces capacity like stackelberg it is here, reaction function is this. So, it is reaction function of firm 2 is this, this is r2 and this is r1.

So, intersect and this is the outcome. So, it is the stackelberg outcome and firm 1 can ensure stackelberg outcome by having the capacity as the stackelberg outcome, capacity of the stackelberg outcome. So, firm 1 will do that, it will accommodate the entry of firm 2 and it will have a capacity of the stackelberg and outcome is going to be the stackelberg.

Now here we had two, we had, I had earlier while discussing this problem, not problem the in the main discuss and we had taken two cases where this point is greater than this or this point is less than but we will not require this because the stackelberg thing is less than the q this because we have seen that here. This is always less than this-$q_1^s < q_1^v$ because of this we do not require any further here, we will only get 1 case in 1 outcome in case 2.

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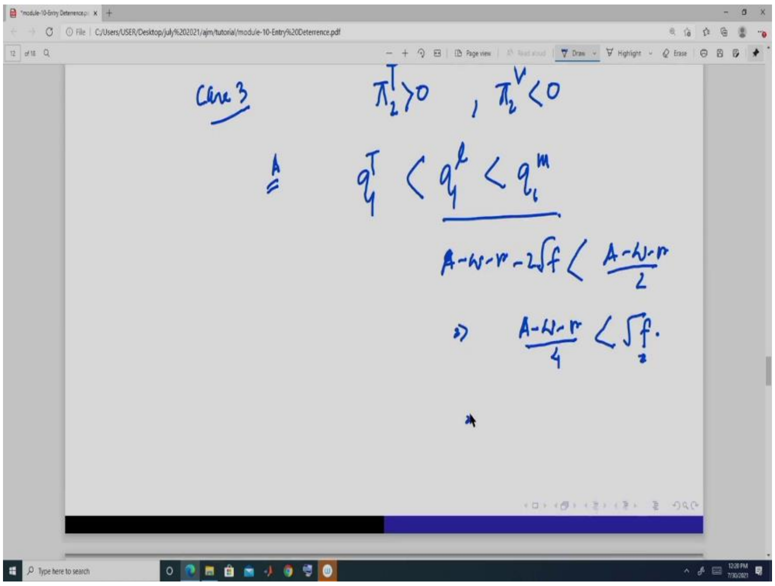

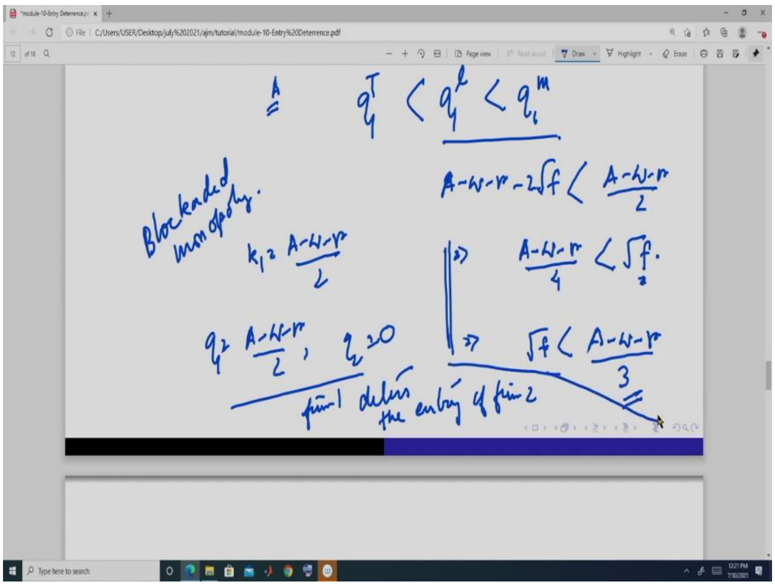

Now move to case 3, in case 3 we have profit of firm 2 is positive and T and it is negative and this so, in this case the first case A what we have is, so, this is, so, from this-$q_1^T < q_1^l < q_1^m$ we get this-$A - w - r - 2\sqrt{f} < \frac{A-w-r}{2}$ and from this we know then it is this outcome-$\frac{A-w-r}{4} < \sqrt{f}$, just opposite of that it is this- $\frac{A-w-r}{3} > \sqrt{f}$, so, absolute lie in this range to get this so, here again we have the blockaded monopoly and k1 is equal to a monopoly output and this is equal to and q2 is equal to 0 so firm 1 deters the entry of firm 2 okay, so this is 1.

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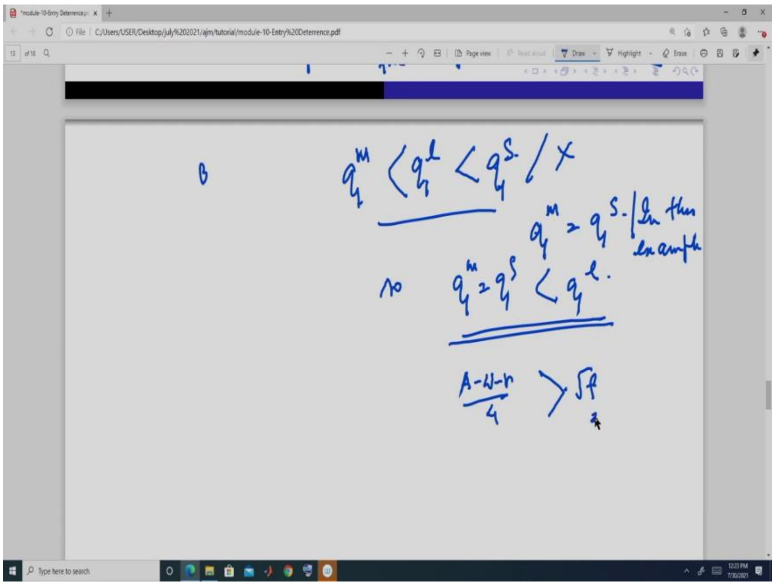

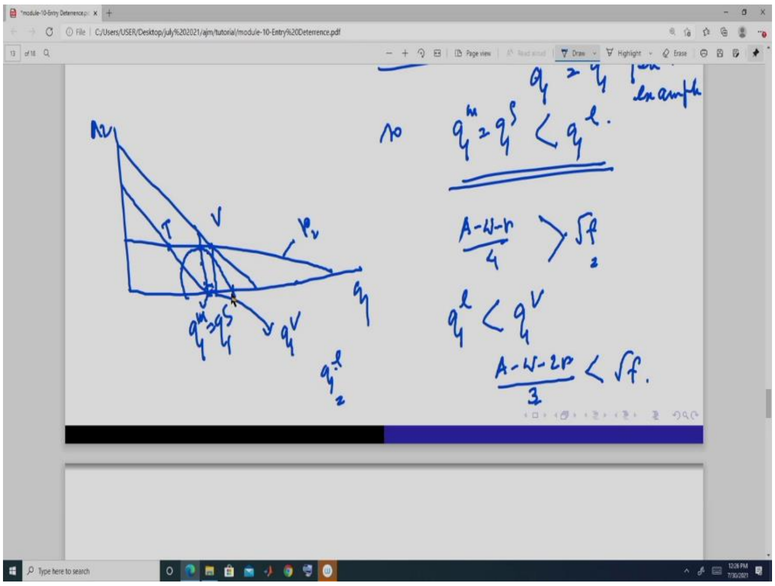

Next let us **3** B here q1 l is greater than M, okay. Now, this has we had taken one more case it is like this, but we know in our case is equal to - $q_1^m = q_1^s$so, we will have only this case-$q_1^m = q_1^s < q_1^l$ because of this we will not have this case, okay. Because in this example we do not get anything because this is equal to this. So, we get this, now here this implies that should be here, but we have 1 more condition that is q1 l is less than q1 V.

So, from here we get this is less than this, right. So, here we get this will give me this condition this is the case **so** we get this. So, is just the sign is opposite, okay and you can see that this is less than this-$\frac{A-w-2r}{3} < \sqrt{f}$. Now, here what we are getting, we are getting this is the reaction function of firm 2 and this is point T, this is point V, this is q1 M is equal to q1 S so ql lie here, this is point is q1 V, this is this point.

So, in this range q1 l lie, okay. Now if it lie here, this is the stackelberg thing, right. Now, if it produces the stackelberg thing, then it is going to be like this, right? and it will accommodate the entry. Now, if it gives the capacity as this then it is going to deter now, the A is whether it is going to ensure this or not, deter or it is going to accommodate because if it keeps k1 capacity as the A stackelberg thing then it is a commodity. So, that we can compare looking at this point.

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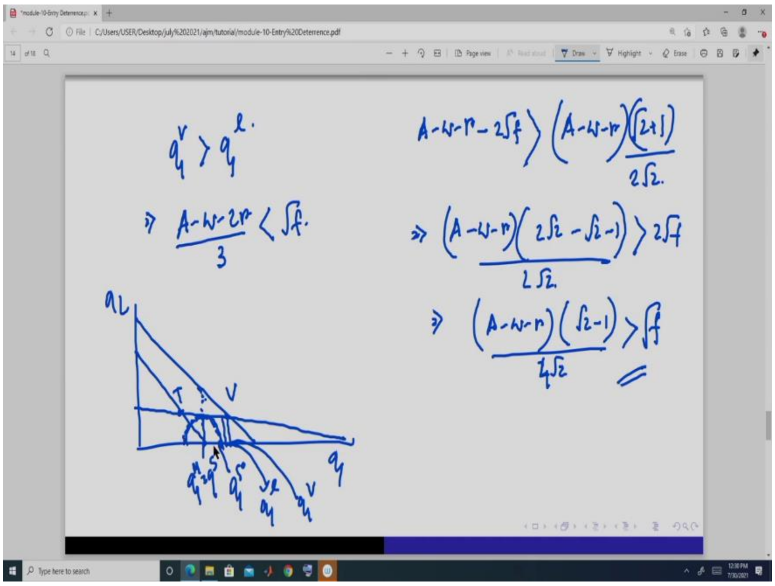

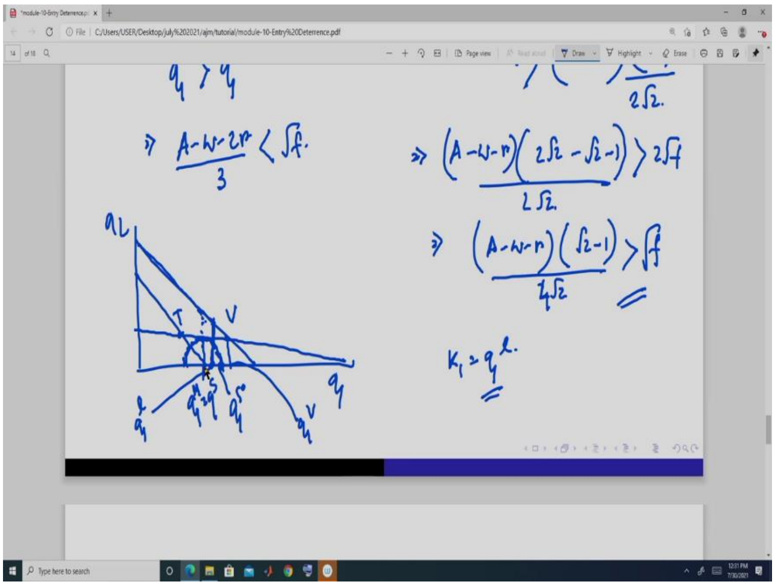

Now here we can compare first this whether this output $A - w - r - 2\sqrt{f}$ can be greater than the this output-$(A - w - r).\frac{\sqrt{2}+1}{2\sqrt{2}}$. Now, if we take this here, if we take this, this, it is this and we get this thing-$\frac{(A-w-r)(\sqrt{2}-1)}{4\sqrt{2}} > \sqrt{f}$ and at the same time we require q1 V to be greater than q1 l. So, we this, this we have got from this. So, we so, you have to calculate that whether this is less than this because f, sorry whether this is greater than this because f has to be less than this amount and f has to be greater than the amount, okay.

So, this is so, you compare this now, if this is greater suppose if this is true, then what it means, so, then this means that, this point is T this point is V q1 q2 this is qM which is equal to qS so this is qS this is q1 S naught suppose this point is q1 l and this point is q1 V. Now, if it accommodates, then what is going to happen? If it accommodates it is going to get a profit of. It can do the stackelberg and it is going to get this profit curve right, now, it will be at this here.

Here we have this is less than okay I have made this mistake. So, this point is q1 okay. Now, if so, this is the iso profit curve of the A firm 1 when it is acting when it is getting this stackelberg outcome and if firm 2 produce 0 its profit is this, this point, but if it deters the entry by having capacity as this much k1 equal to q1l, then the reaction function of firm 1 is this, outcome is this, this is closer to this monopoly than this.

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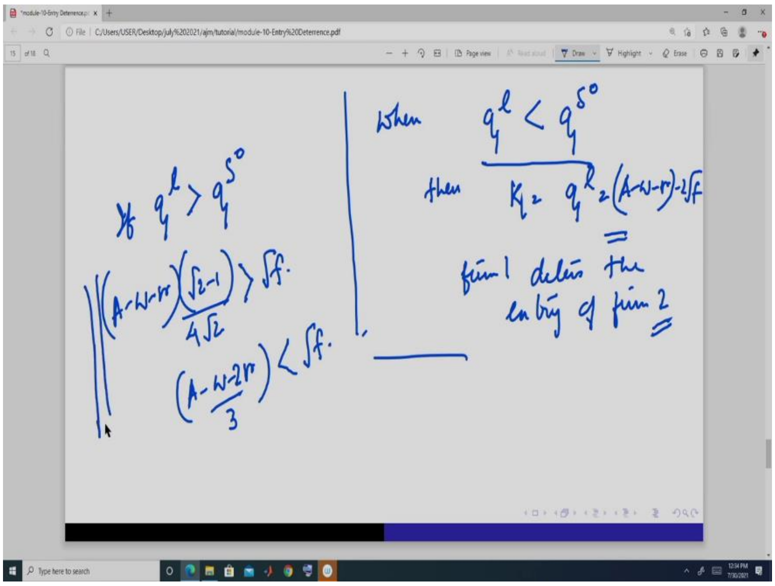

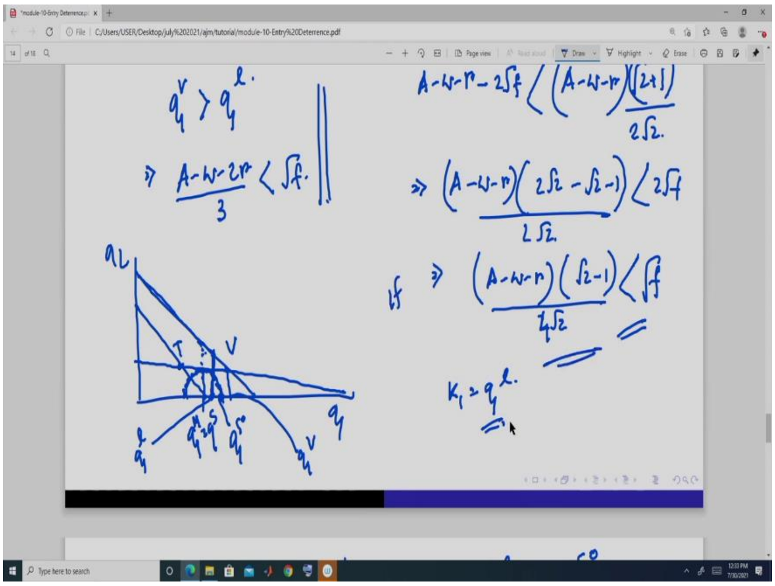

So, here in this situation when q1 l is less than q1 S naught then q1 then k1 is actually equal to ql which is equal to this and firm 1 deters the entry of firm 2, but in this situation if we have, okay this we have this and we have this, this would be opposite side, okay. It should be this and so, here this is not binding and this is binding whichever you have to check that if this is true, if this is true, then it will choose this and it will deter the entry of this.

But if the case is opposite that is if q1 this is greater than q1 S naught that means it will just change the sign of this that is 2 minus 1 4 root 2 should be this here. If this is the case and it has to be, okay I get this.

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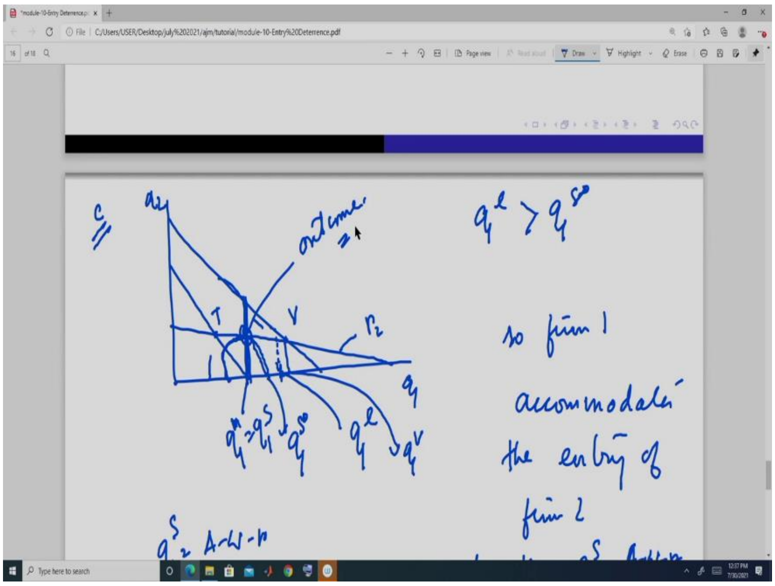

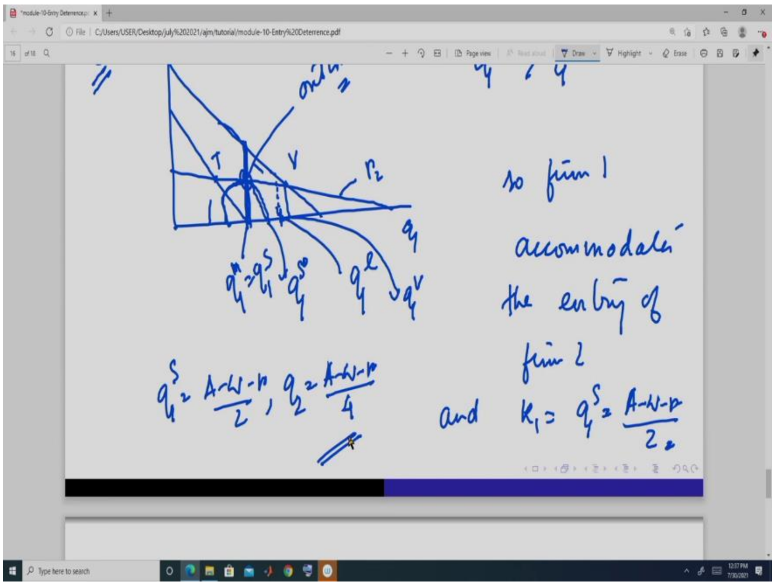

So, when both these condition is satisfied-$(A - w - r).\frac{\sqrt{2}-1}{4\sqrt{2}} > \sqrt{f}, \frac{A-w-2r}{3} < \sqrt{f}$, then we get a situation something like this, this is T this is V q1 is equal to q1 S and so, this point is q1 S naught and this is suppose q1 l this is q1 V. So, here q1 l is greater than q1. Now here q1 l is greater than q1 S naught this, in this situation which is suppose C, in this situation what do we get. So, this gives us this condition, these two conditions f should lie within this range then this gives us what.

So, it tells us that if firm 1 wants to deter it will have to need to have this much capacity then the outcome is this. So, it is farther away from the monopoly and this is the iso profit curve of the stackelberg and this is the iso profit curve with f. So, firm 1 here accommodates the entry of firm 2 and K1 is equal to q1 S and it is equal to A this, okay.

So, when this is the A, final outcome there is in the Cournot competition is going to be like this and this is going to be the outcome right? then to get this and q. So, these are all the possible outcomes that we get in this example of entry deterrence, here one, we do not get one possibility because we have this situation.

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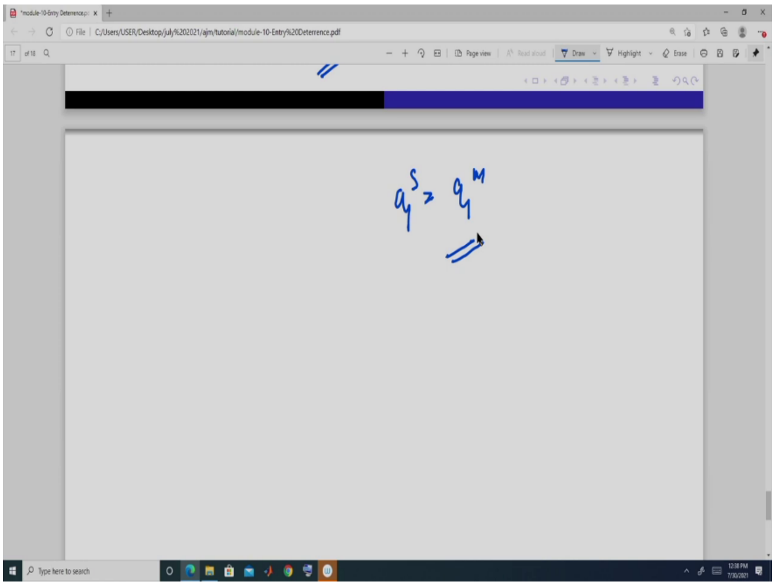

Because of q1 S is equal to q1 M. So, in case 2 we are having only one case so, we do not have many possibilities in case 2, okay. So thank you.

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