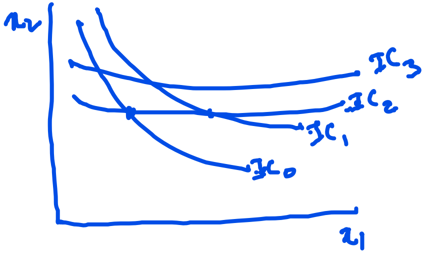
Problems on Consumer Behaviour and Demand Curve

1. Suppose the indifference curves are of the following nature. Name the properties they violate.



i) Monotonicity

ii) Strict convexity

i) Transitivity

2. Suppose the utility function of consumer is $U(x, y) = x^{\alpha} + y^{\alpha}, \; 0 < \alpha < 1$. The price of good x is 10 and price of good y is 20. Suppose the income of the consumer is 1000. Find the utility maximizing bundle of this consumer.

$$\mathcal{L} = x^{\alpha} + y^{\alpha} + \lambda\left[1000 - 10x - 20y\right]$$

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1} - \lambda 10$$

$$\frac{\partial \mathcal{L}}{\partial y} = \alpha y^{\alpha-1} - \lambda 20$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 1000 - 10x - 20y$$

FOC

$$\Rightarrow \alpha x^{\alpha-1} = \lambda 10$$

$$\Rightarrow \alpha y^{\alpha-1} = \lambda 20$$

$$\Rightarrow 1000 = 10x + 20y$$

$$\Rightarrow \frac{y}{x} = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow y = x\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$1000 = 10x + 20 \cdot x \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$= x\left[10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\frac{1000}{\left[10 + 20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]} = x$$

$$y = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\left[\frac{1000}{10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}\right]$$

3. Suppose the utility function of consumer is $U(x, y) = x^{\alpha} + y^{\alpha}$. The price of good x is p_x and price of good y is p_y. Suppose the income of the consumer is m. Derive the demand function of good x and good y of this consumer.

$$\mathcal{L} = x^{\alpha} + y^{\alpha} + \lambda \left[m - p_x x - p_y y \right]$$

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1} = \lambda P_x .$$

$$\frac{\partial \mathcal{L}}{\partial y} = \alpha y^{\alpha-1} = \lambda P_y$$

$$\Rightarrow \frac{y}{x} = \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = M = P_x x + P_y y$$

$$\Rightarrow M = P_x x + P_y \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}} x$$

$$\Rightarrow M = \left[P_x + \frac{(P_x)^{\frac{1}{1-\alpha}}}{P_y^{\frac{\alpha}{1-\alpha}}} \right] x$$

$$\Rightarrow \quad x = \frac{M \, P_y^{\frac{\alpha}{1-\alpha}}}{\left[P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}} \right]}$$

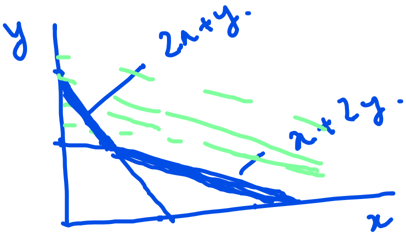
$$\Rightarrow \text{as } P_x \uparrow \quad x \downarrow$$

$$\Rightarrow \quad y = \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}} \cdot \frac{M \, P_y^{\frac{\alpha}{1-\alpha}}}{\left[P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}} \right]}$$

$$\Rightarrow \quad y = \frac{M \, P_x^{\frac{1}{1-\alpha}}}{P_y \left[P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}} \right]}$$

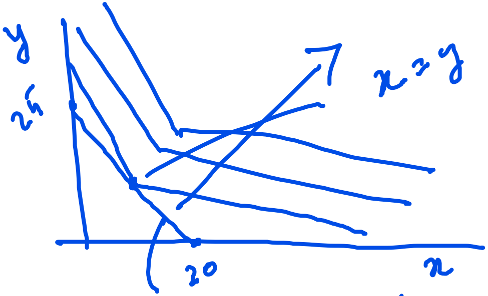
$$\text{as } P_y \uparrow \Rightarrow y \downarrow$$

4. Suppose the utility function of a consumer is $U(x, y) = \min\{2x + y,\ x + 2y\}$. The price of good x is 5 and price of good y is 4. Suppose the income of the consumer is 100. Find the utility maximizing consumption bundle.



$$2x + y = \bar{U} = x + 2y$$

$$U = \min\{2x + y,\ x + 2y\}$$



$$100 = 5x + 4y$$

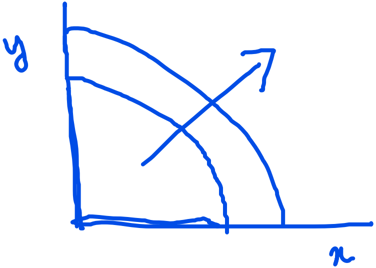
$$2x + y = x + 2y$$

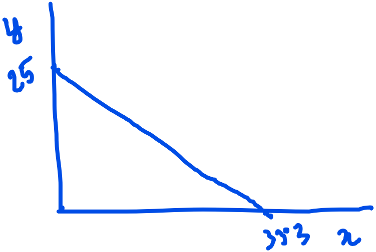
$$\Rightarrow \underline{\underline{x = y}}$$

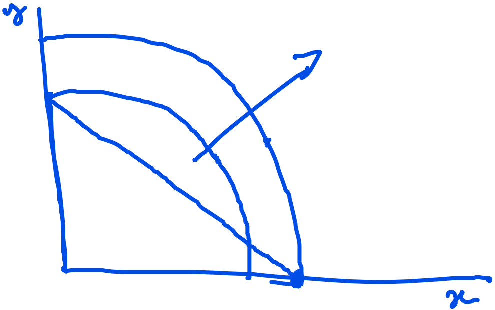
$$100 = 5x + 4y.$$

$$\frac{100}{9} = x, \quad \frac{100}{9} = y$$

5. Suppose the utility function of consumer is $U(x, y) = x^2 + y^2$. Suppose the price of good x is 3 and price of good y is 4. The income of the consumer is 100. Find the utility maximising bundle of this consumer.



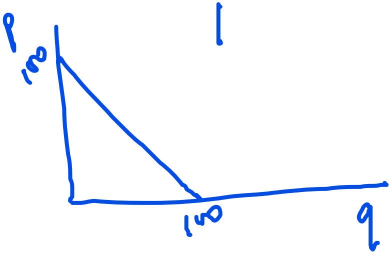


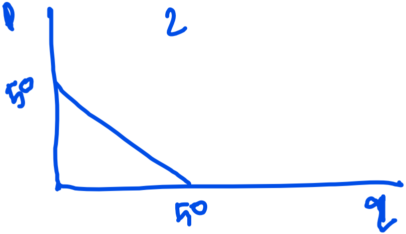


$$x = \frac{1}{3}$$

$$y = 0$$

6. Suppose the demand function of consumer 1 of good a is $100 - p = q$ and the demand function of consumer 2 of good a is $50 - p = q$. If there are only two consumers what is the market demand function of good a?





For $P \geq 50$

$100 - P = Q$

For $P < 50$

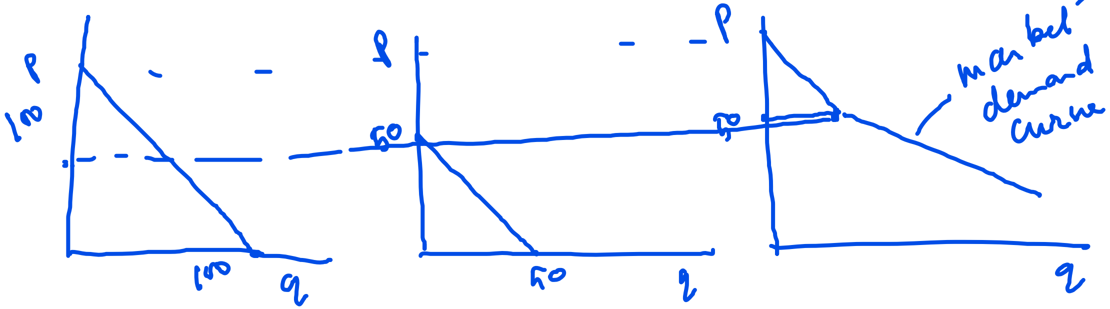
$100 - P + 50 - P$

$\Rightarrow 150 - 2P = Q$

At $P = 50$

1 $\quad q = 50$

2: $\quad q = 0$



7. Suppose demand function of good 1 is $15 - 3p = q$ and suppose demand function of good 2 is $16 - 4p = q$.
What is price elasticity of demand of good 1 at price $p = 3$?
What is the price elasticity of demand of good 2 at price $p = 3$?
Which has more elastic demand curve?

$$|\varepsilon_{d_1}| = \left| \frac{\partial Q_1}{\partial P} \cdot \frac{P}{Q_1} \right| = \left| \frac{\frac{\partial Q}{Q_1}}{\frac{\partial P}{\partial P}} \right|$$

$15 - 3p = q$, At $p = 3$

$q = 6$

$$\frac{\partial q}{\partial p} = -3$$

$$= \left| -3 \cdot \frac{3}{6} \right| = \underline{\underline{1 \cdot 5}}$$

$$|\varepsilon_{d_1}| = 1 \cdot 5 .$$

$$\left|\varepsilon_{d_2}\right| = \left|\frac{\partial q_2}{\partial p} \cdot \frac{p}{q_2}\right|$$

At $p = 3$

$$\frac{\partial q_2}{\partial p} = -4$$

$$16 - 4p = q$$

$$\Rightarrow q = 4.$$

$$\left|\varepsilon_{d_2}\right| = \left|-\frac{4}{4} \cdot 3\right| = 3$$

$$|\varepsilon_{d_2}| > |\varepsilon_{d_1}| \quad \text{at } p = \frac{3}{2}$$

$$|\varepsilon_{d_1}| = \left| -\frac{3}{15-3p} \cdot p \right|$$

$$|\varepsilon_{d_2}| = \left| -\frac{4}{16-4p} \cdot p \right|$$

$$\Rightarrow \frac{3}{15-3p} < \frac{4}{16-4p}$$

$$\Rightarrow 48 - \cancel{12p} < 60 - \cancel{12p}$$

$$|\varepsilon_{d_1}| < |\varepsilon_{d_2}|$$