CS 106X, Lecture 22
Graphs; BFS; DFS

reading:

Programming Abstractions in C++, Chapter 18

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Based on slides created by Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, Marty Stepp, Ashley Taylor and others.

Plan For Today

- **Recap**: Graphs
- **Practice**: Twitter Influence
- Depth-First Search (DFS)
- Announcements
- Breadth-First Search (BFS)

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Graphs

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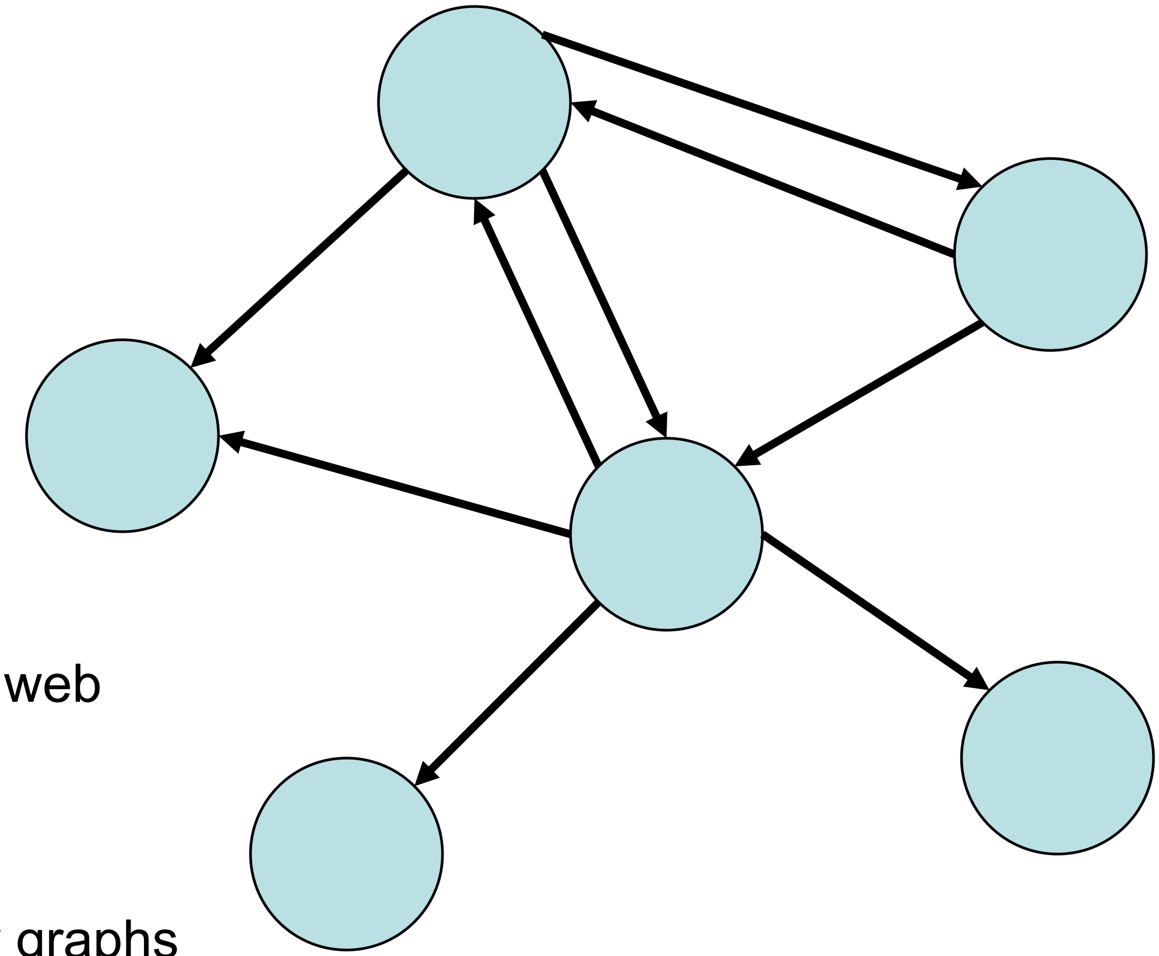

Graphs can model:

- Sites and links on the web
- Disease outbreaks
- Social networks
- Geographies
- Task and dependency graphs
- and more…

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Path: sequence of nodes/edges from one node to another
Path: node *x* is reachable from node *y* if a path exists from *y* to *x.*
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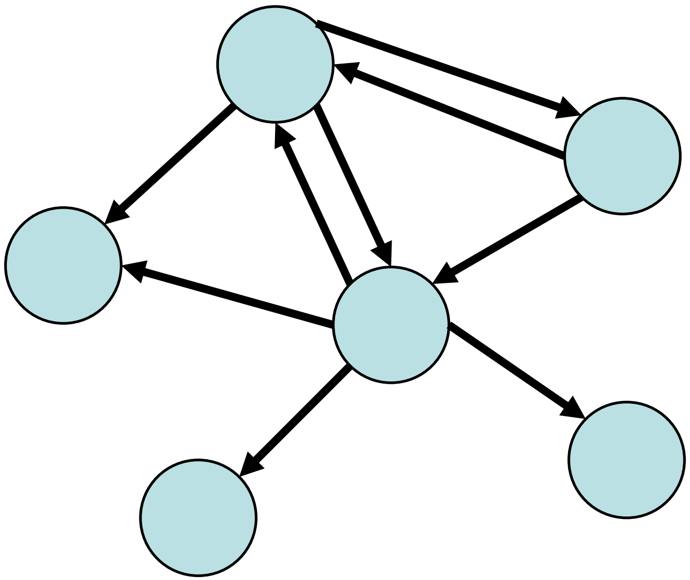

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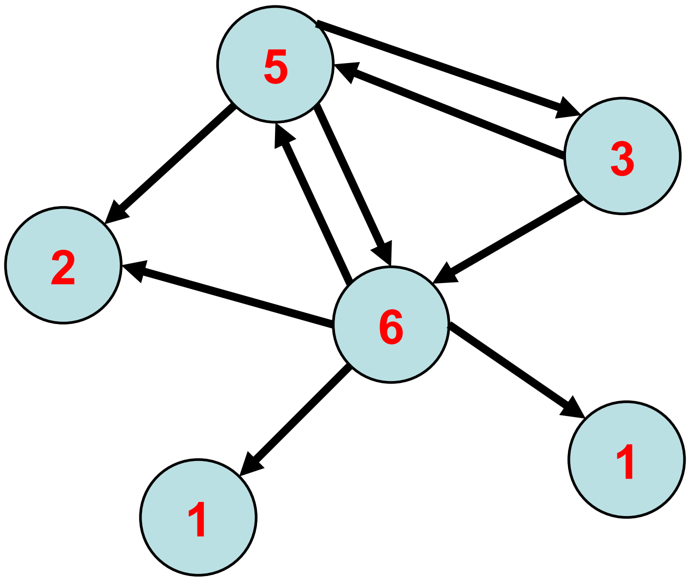

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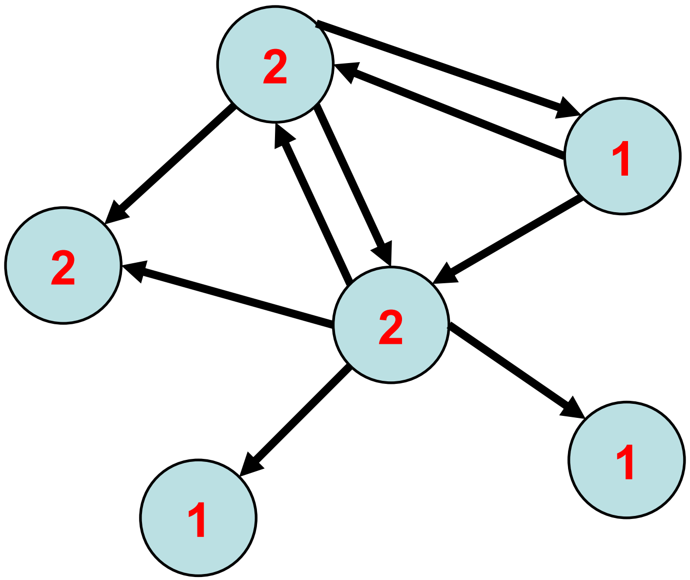

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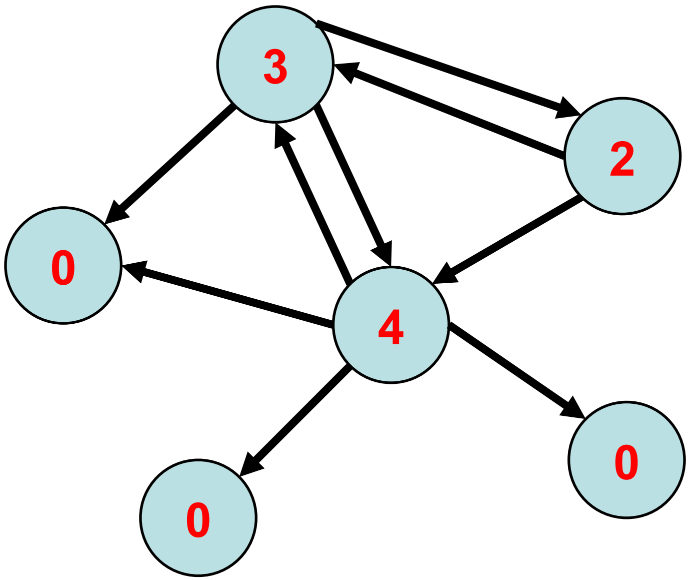

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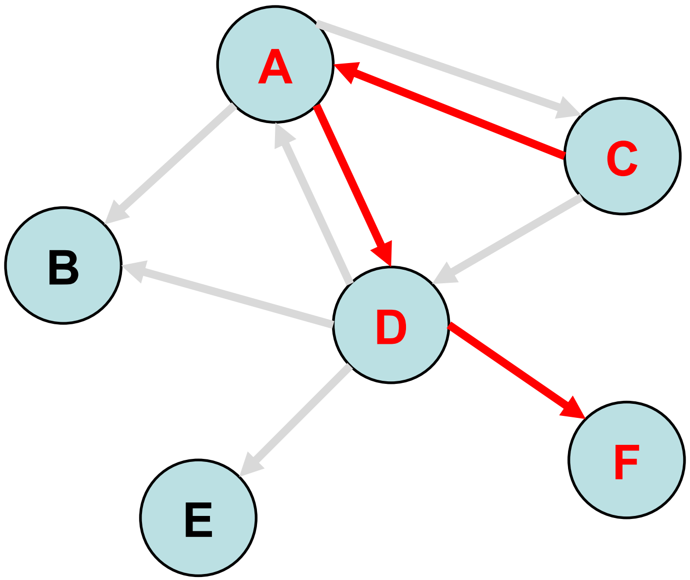

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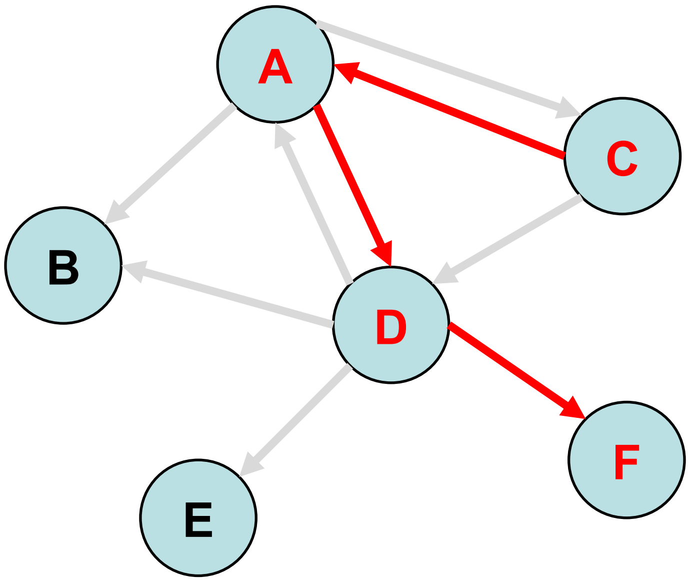

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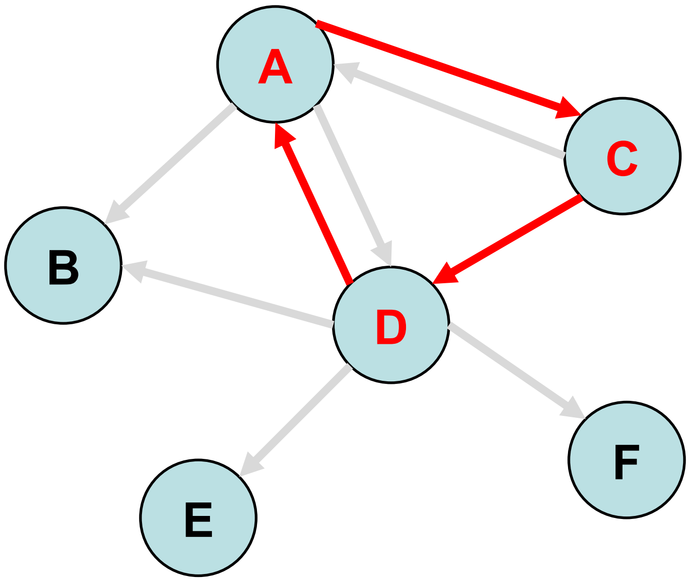

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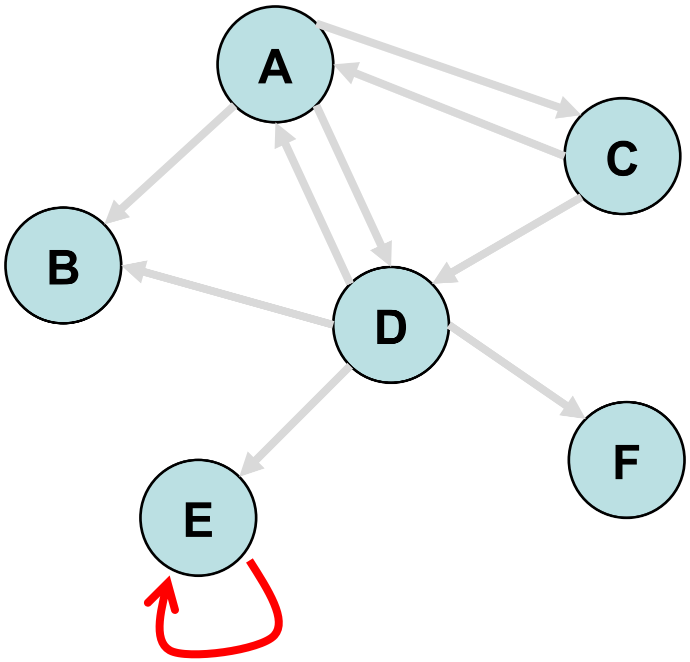
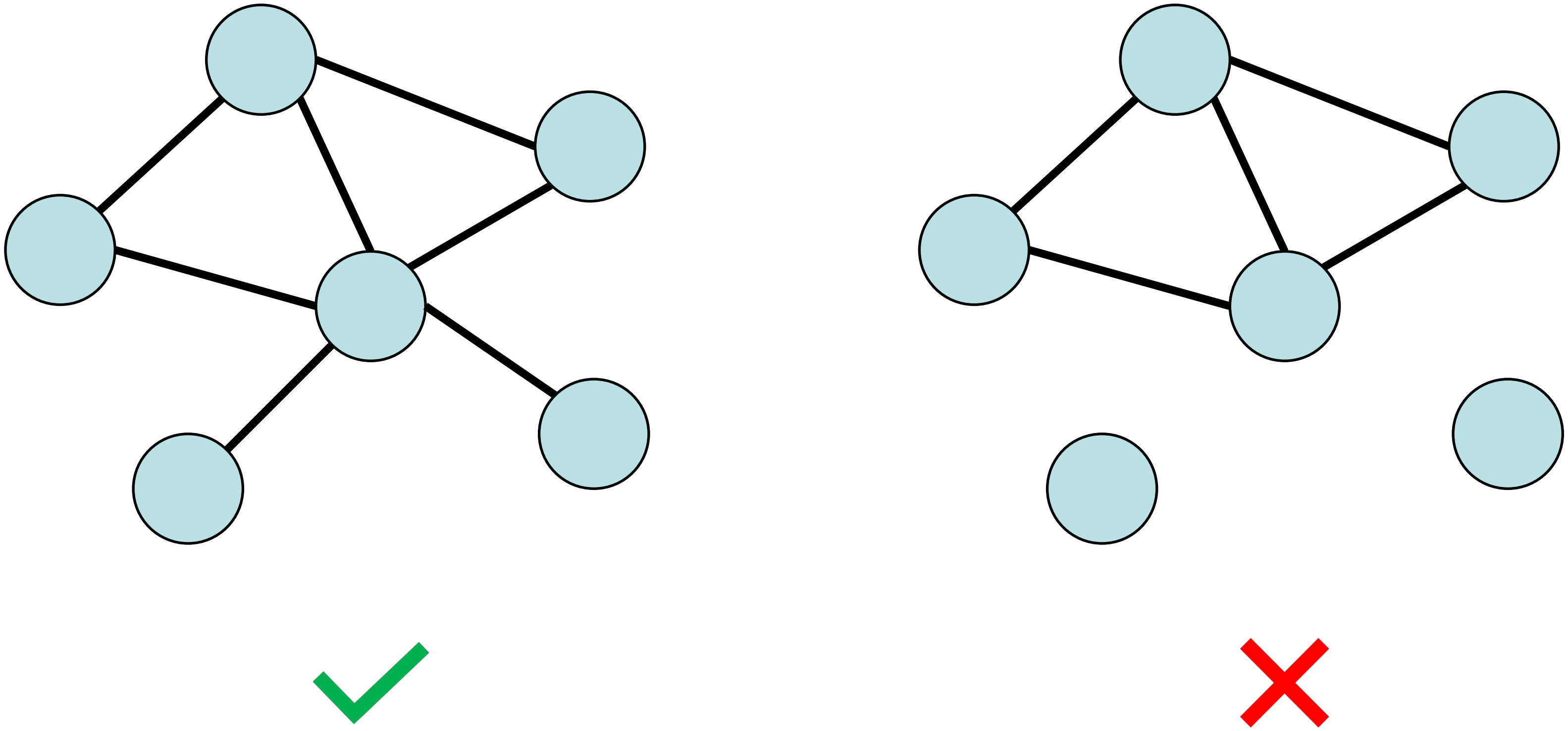


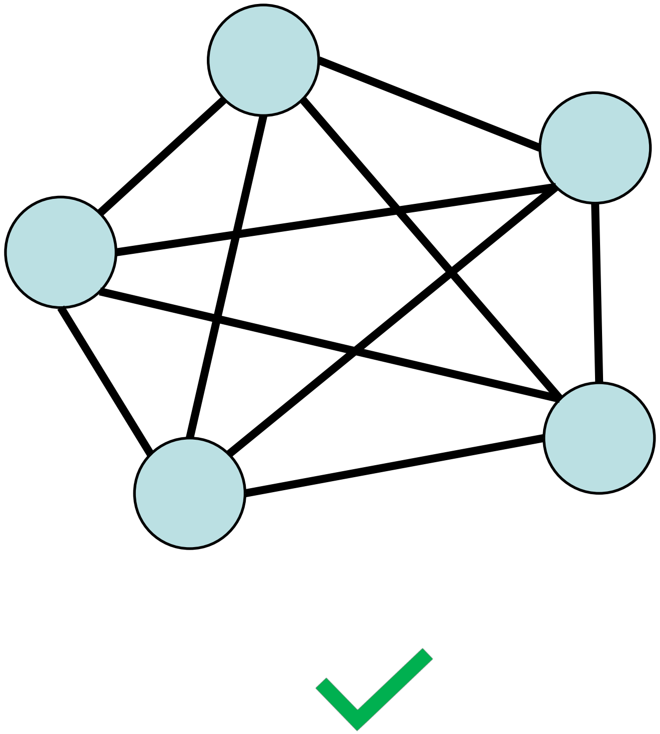
Graph Properties

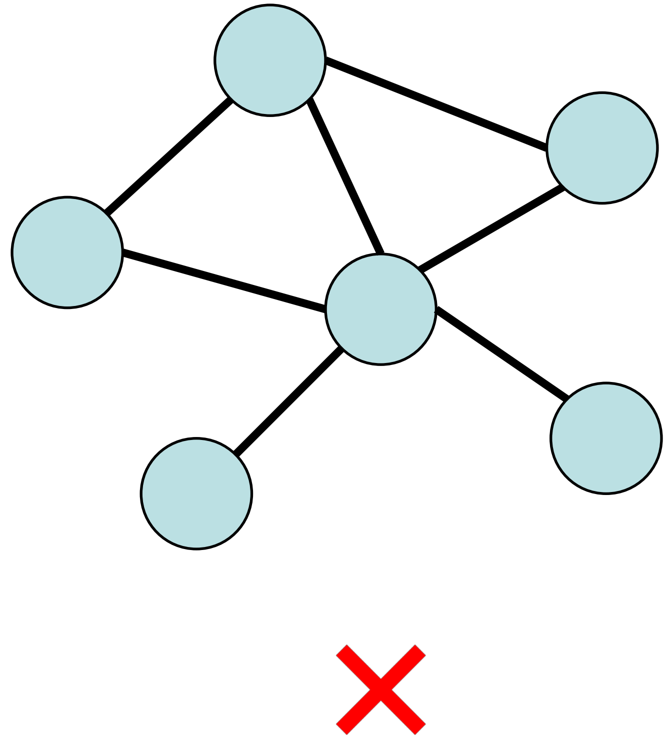
A graph is **connected** if every node is reachable from every other node.



Graph Properties

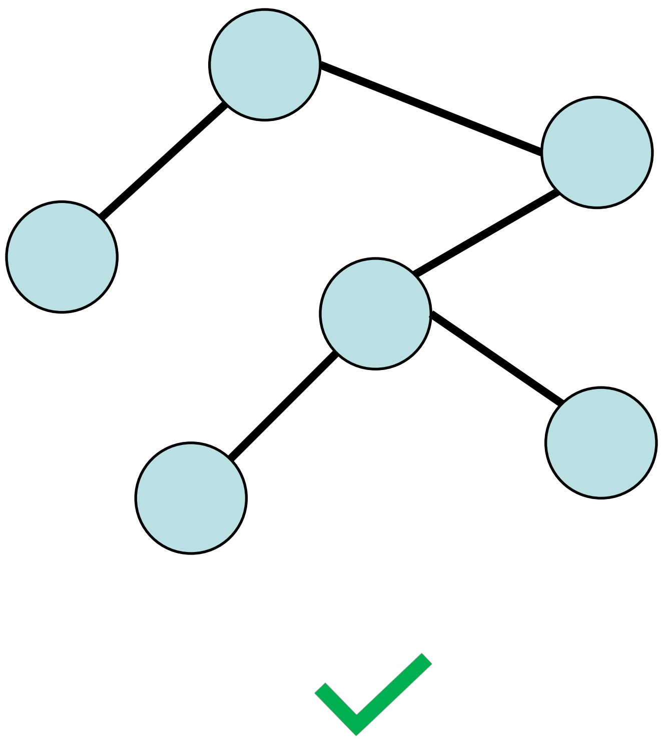
A graph is **complete** if every node has a direct edge to every other node.

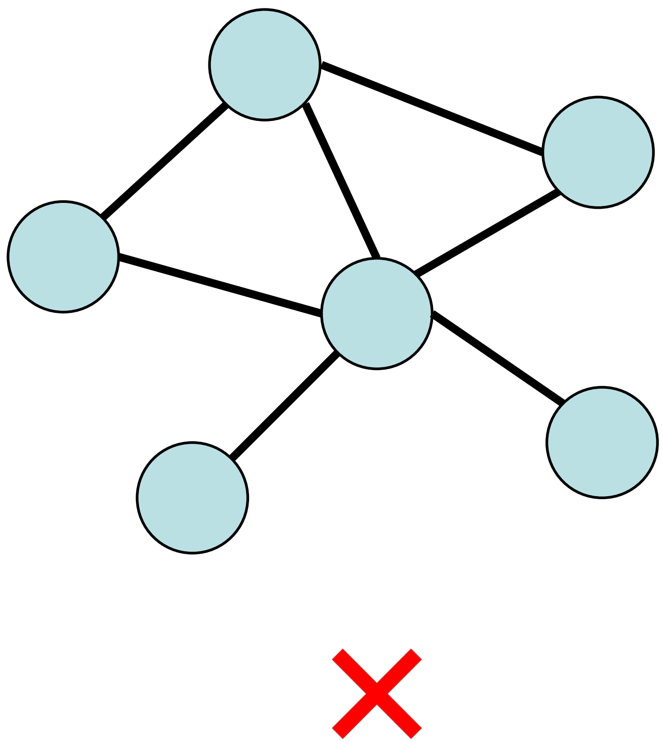




Graph Properties

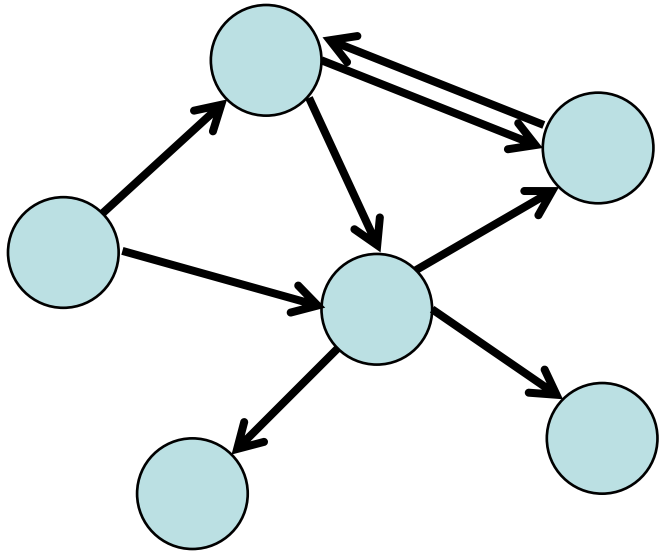
A graph is **acyclic** if it does not contain any cycles.



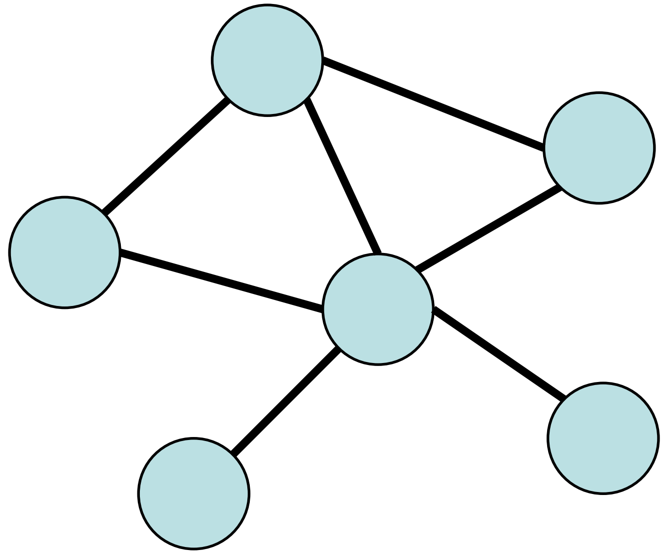


Graph Properties

A graph is **directed** if its edges have direction, or **undirected** if its edges do not have direction (aka are bidirectional).



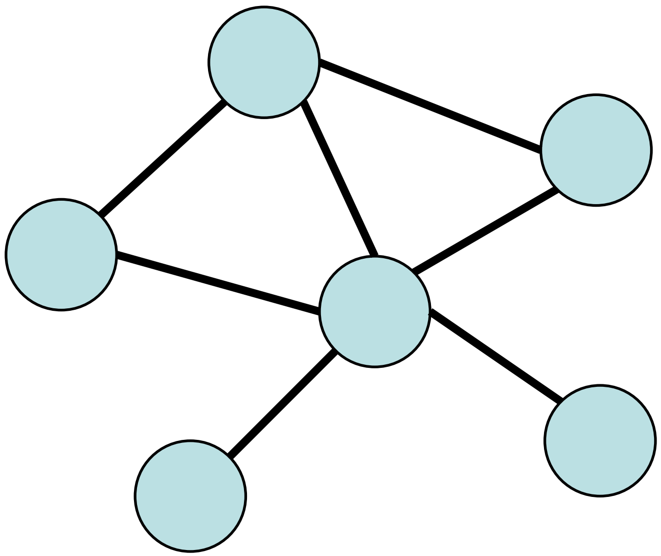
directed



undirected

Graph Properties

- Connected or unconnected
- Acyclic
- Directed or undirected
- Weighted or unweighted
- Complete



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Twitter Influence

- Twitter lets a user follow another user to see their posts.
- Following is directional (e.g. I can follow you but you don't have to follow me back ☹)
- Let's define being *influential* as having a high number of *followers-of-followers*.
  - Reasoning: doesn't just matter how many people follow you, but whether the people who follow you reach a large audience.

- Write a function **`mostInfluential`** that reads a file of Twitter relationships and outputs the most influential user.

https://about.twitter.com/en_us/company/brand-resources.html

BasicGraph members

```
#include "basicgraph.h"   // a directed, weighted graph
```

g`.addEdge(`***v1***`, `***v2***`);`	adds an edge between two vertexes
g`.addVertex(`***name***`);`	adds a vertex to the graph
g`.clear();`	removes all vertexes/edges from the graph
g`.getEdgeSet()` ***g***`.getEdgeSet(`***v***`)`	returns all edges, or all edges that start at ***v***, as a `Set` of pointers
g`.getNeighbors(`***v***`)`	returns a set of all vertices that ***v*** has an edge to
g`.getVertex(`***name***`)`	returns pointer to vertex with the given name
g`.getVertexSet()`	returns a set of all vertexes
g`.isNeighbor(`***v1***`, `***v2***`)`	returns `true` if there is an edge from vertex ***v1*** to ***v2***
g`.isEmpty()`	returns `true` if queue contains no vertexes/edges
g`.removeEdge(`***v1***`, `***v2***`);`	removes an edge from the graph
g`.removeVertex(`***name***`);`	removes a vertex from the graph
g`.size()`	returns the number of vertexes in the graph
g`.toString()`	returns a string such as `"{a, b, c, a -> b}"`

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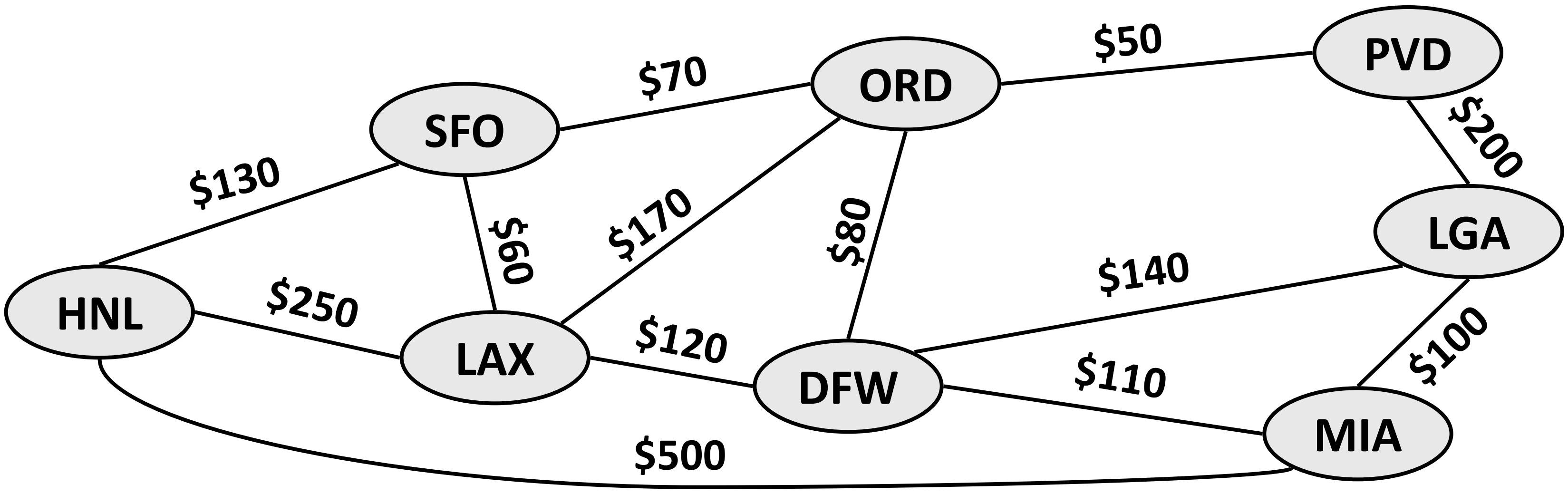
Searching for paths

- Searching for a path from one vertex to another:
  - Sometimes, we just want *any* path (or want to know there *is* a path).
  - Sometimes, we want to minimize path *length* (# of edges).
  - Sometimes, we want to minimize path *cost* (sum of edge weights).



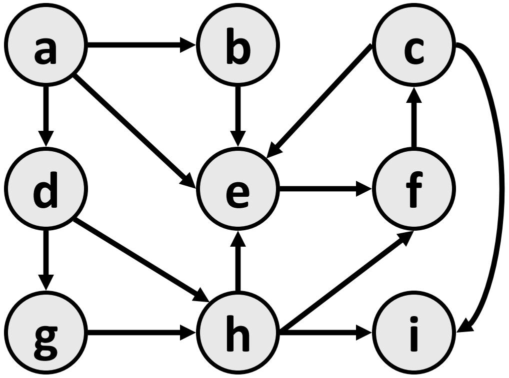
Finding Paths

- Easiest way: Depth-First Search (DFS)
  - Recursive backtracking!
- Finds a path between two nodes if it exists
  - Or can find all the nodes **reachable** from a node
    - Where can I travel to starting in San Francisco?
    - If all my friends (and their friends, and so on) share my post, how many will eventually see it?

Depth-first search (18.4)

- **depth-first search** (DFS): Finds a path between two vertices by exploring each possible path as far as possible before backtracking.
  - Often implemented recursively.
  - Many graph algorithms involve *visiting* or *marking* vertices.

- DFS from *a* to *h* (assuming A-Z order) visits:
  - **a**
    - **b**
      - **e**
        - **f**
          - **c**
            - **i**
    - **d**
      - **g**
        - **h**
  - path found: **{a, d, g, h}**



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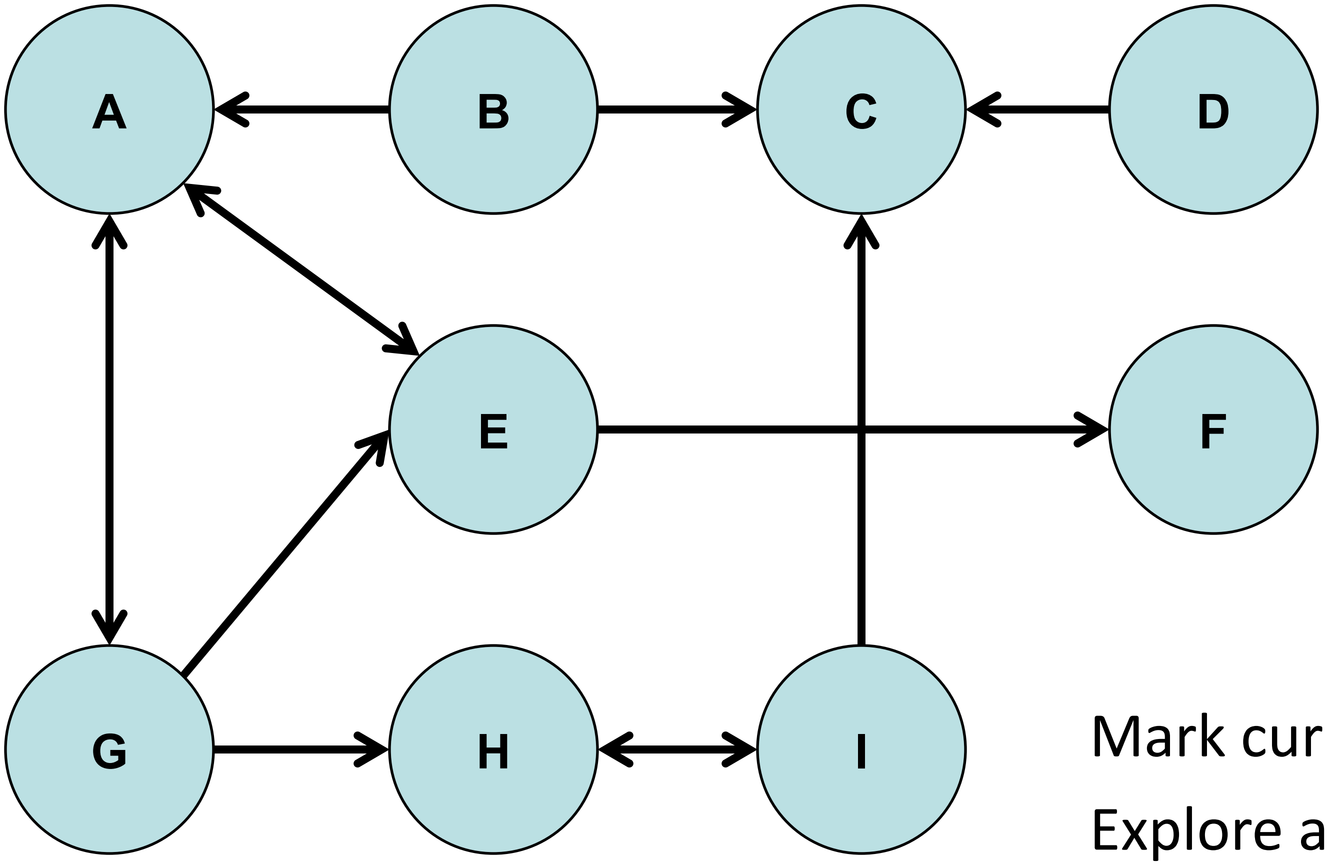
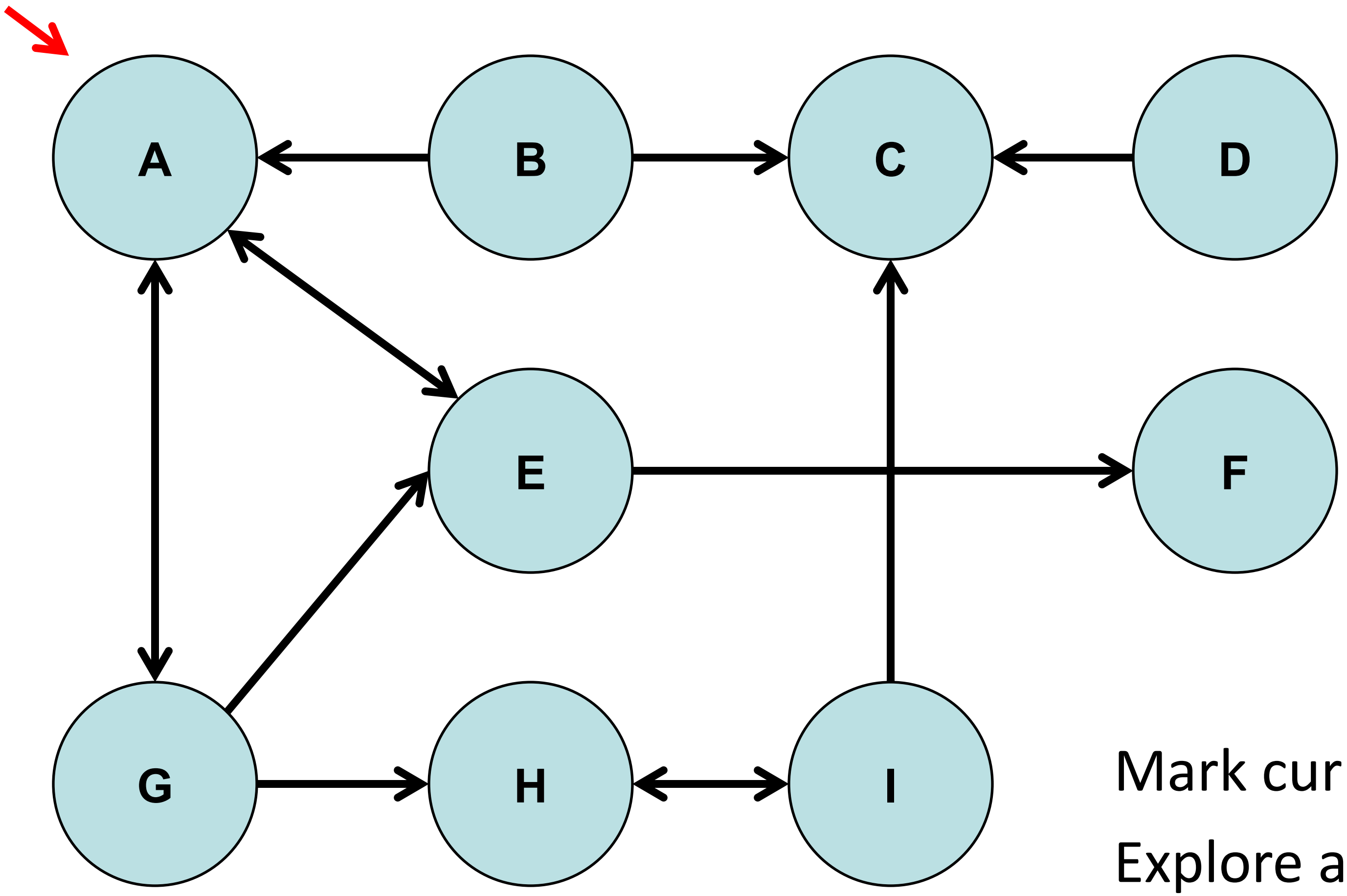


Mark current as visited

Explore all the unvisited nodes from this node

DFS



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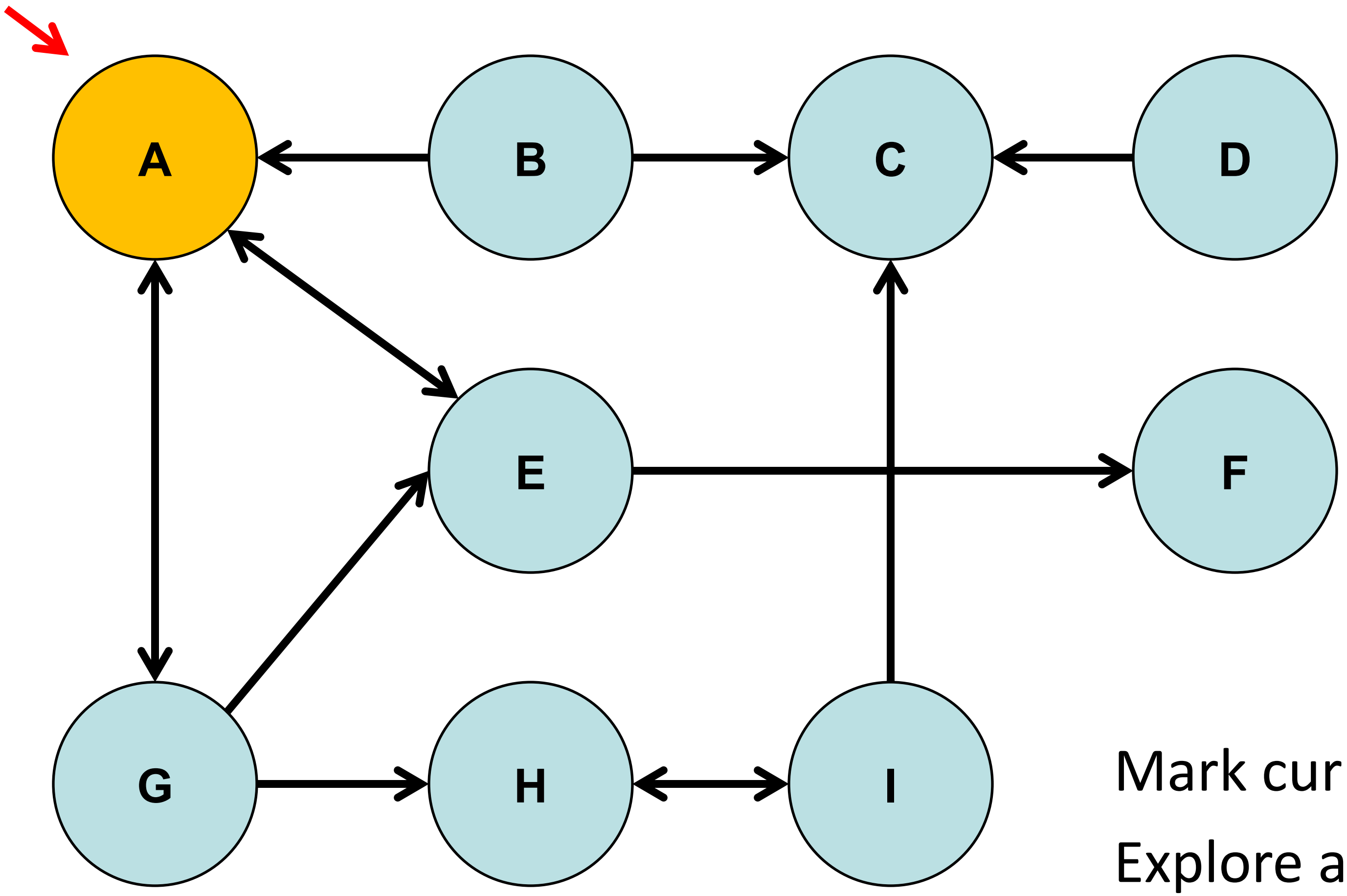

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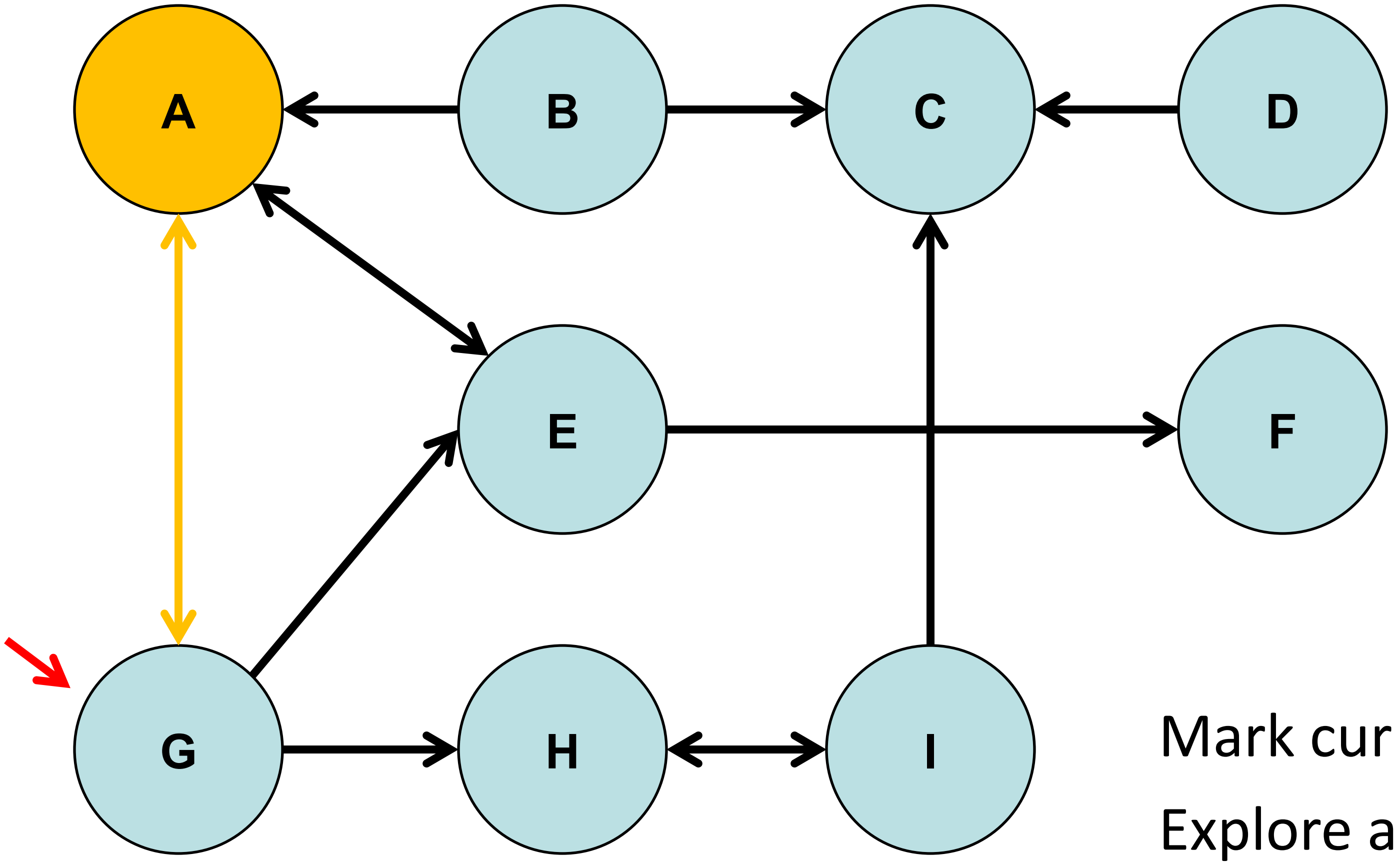

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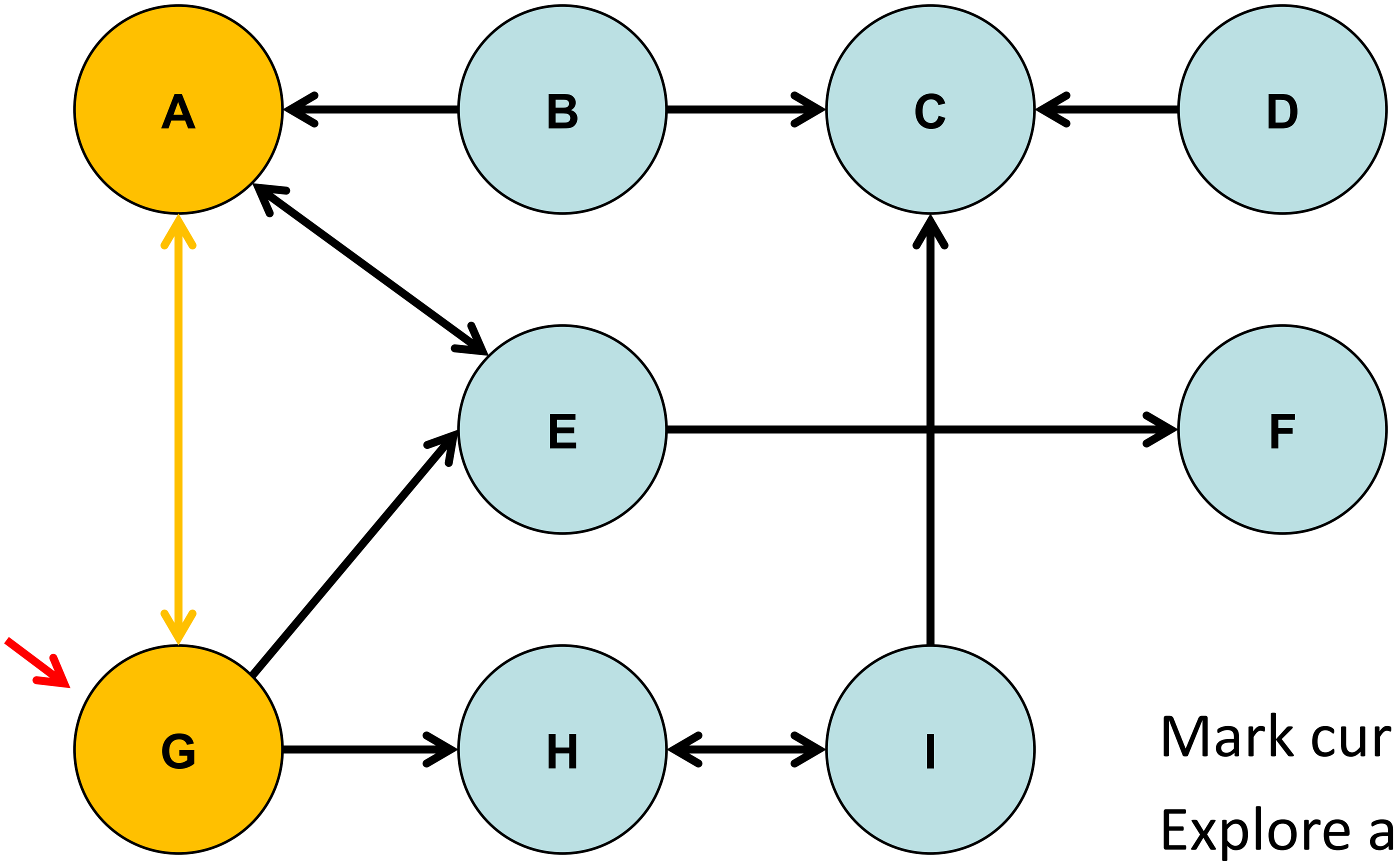

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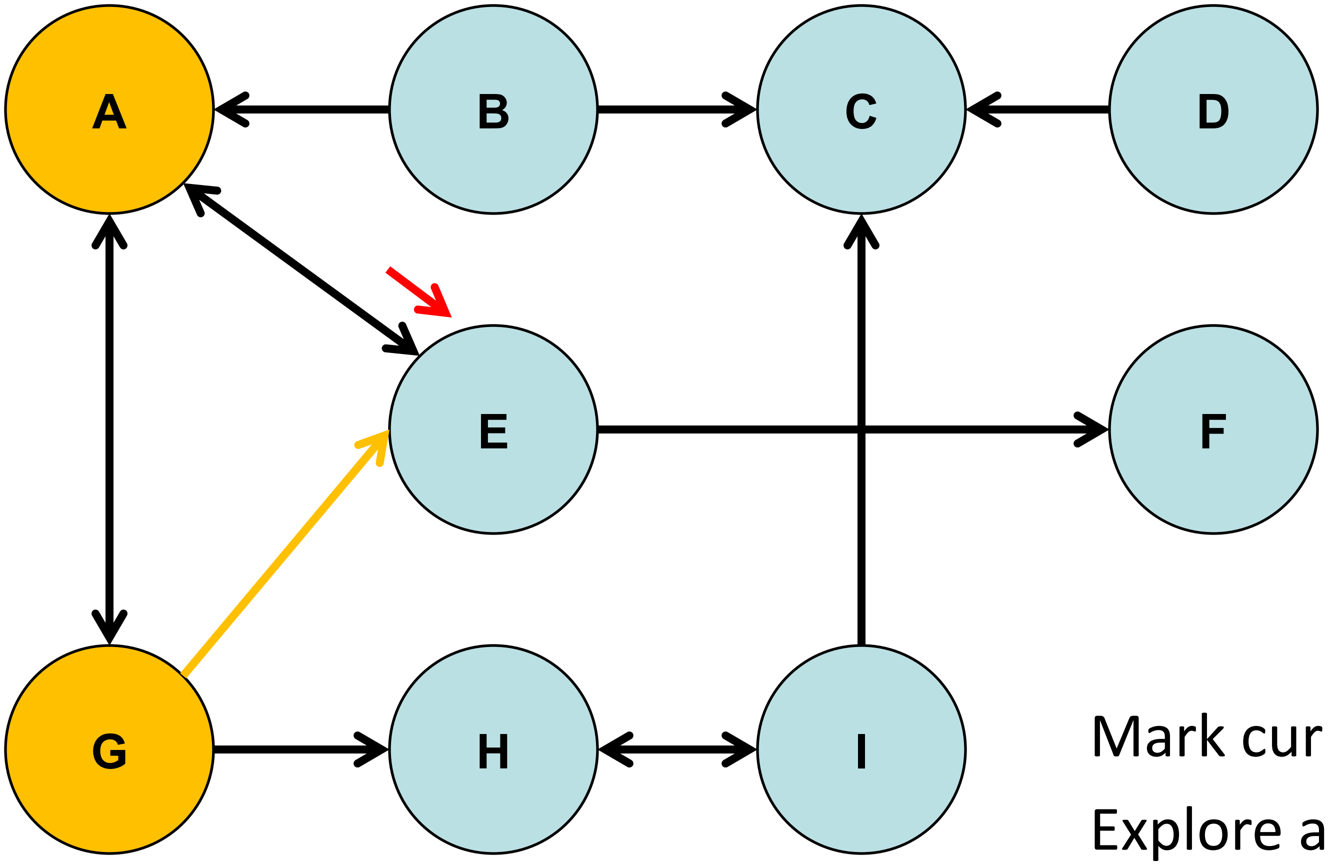

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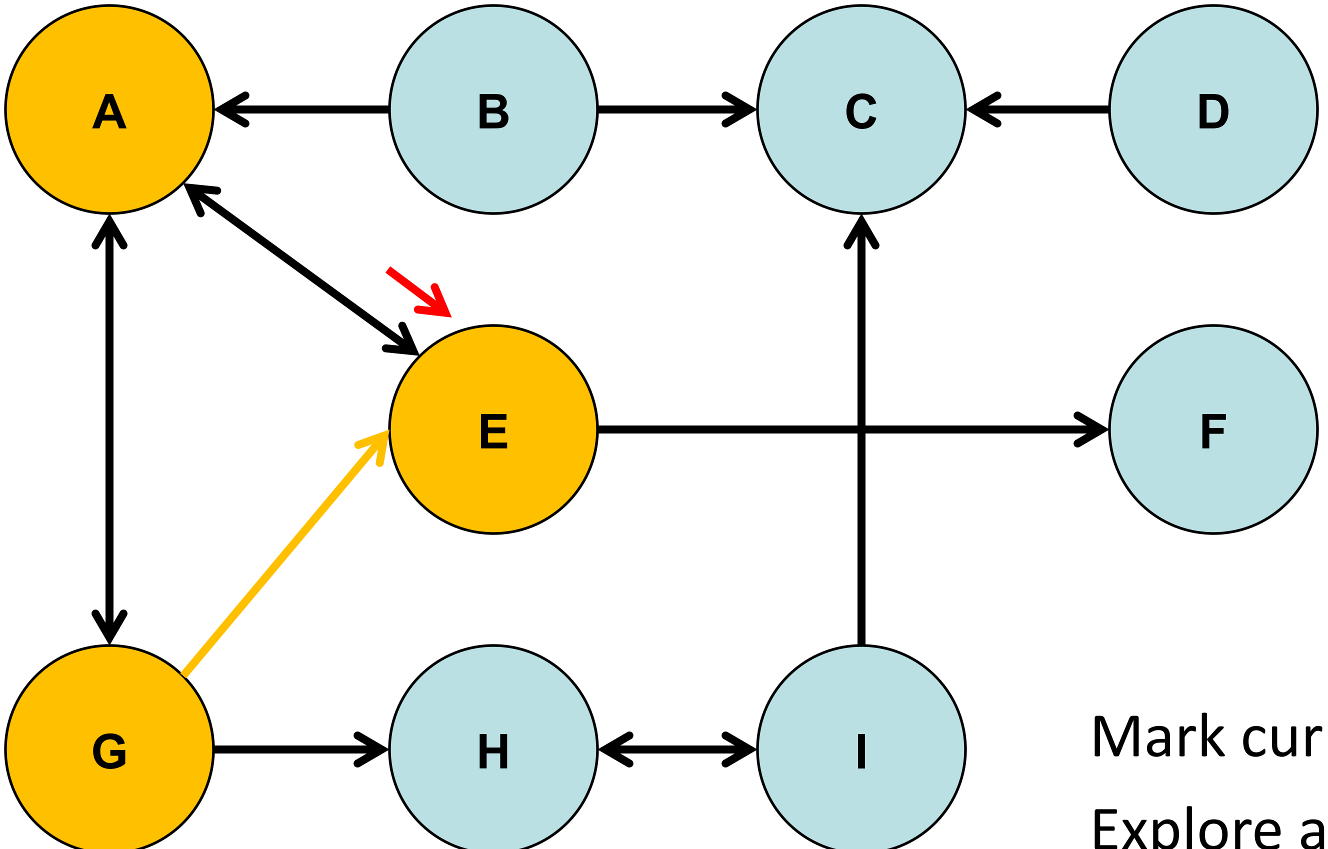

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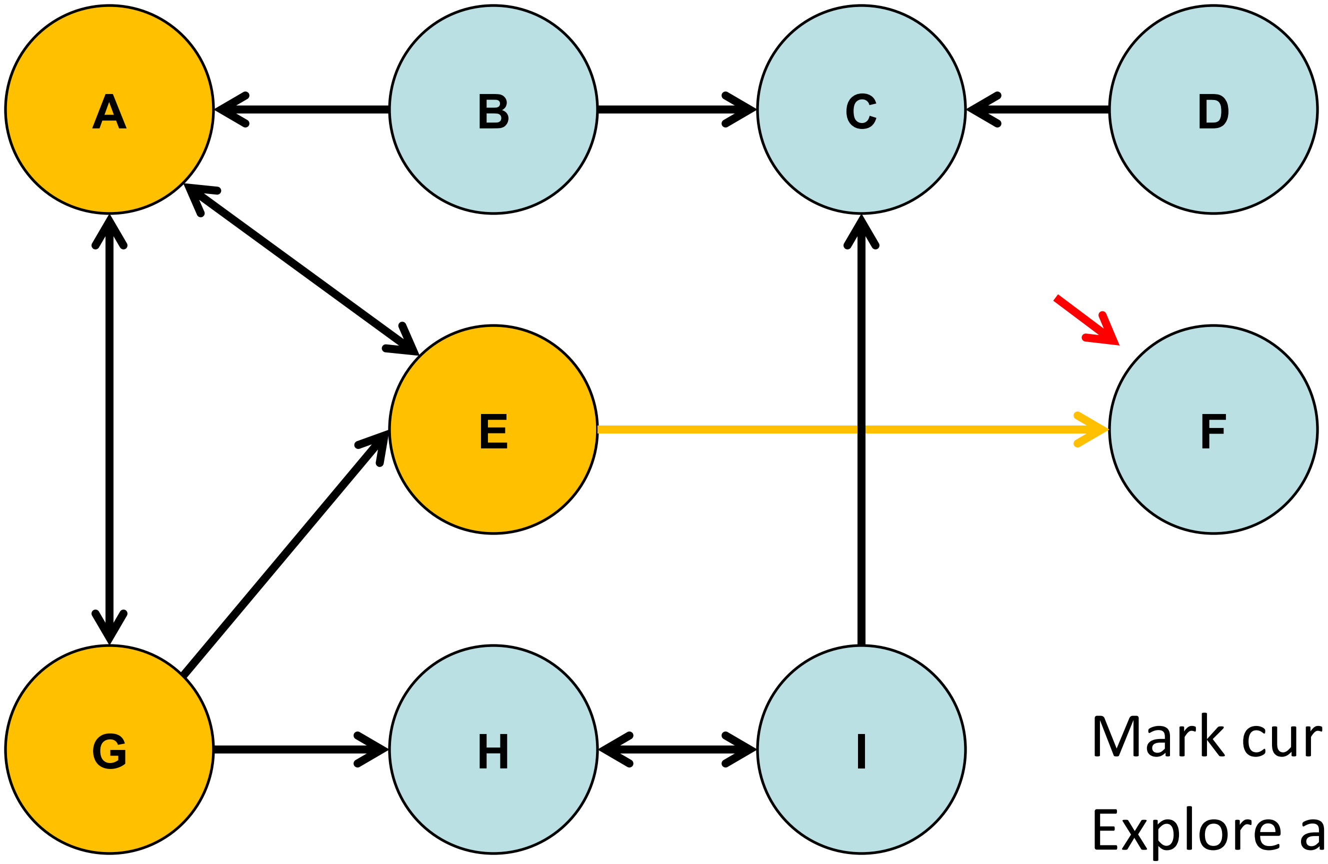

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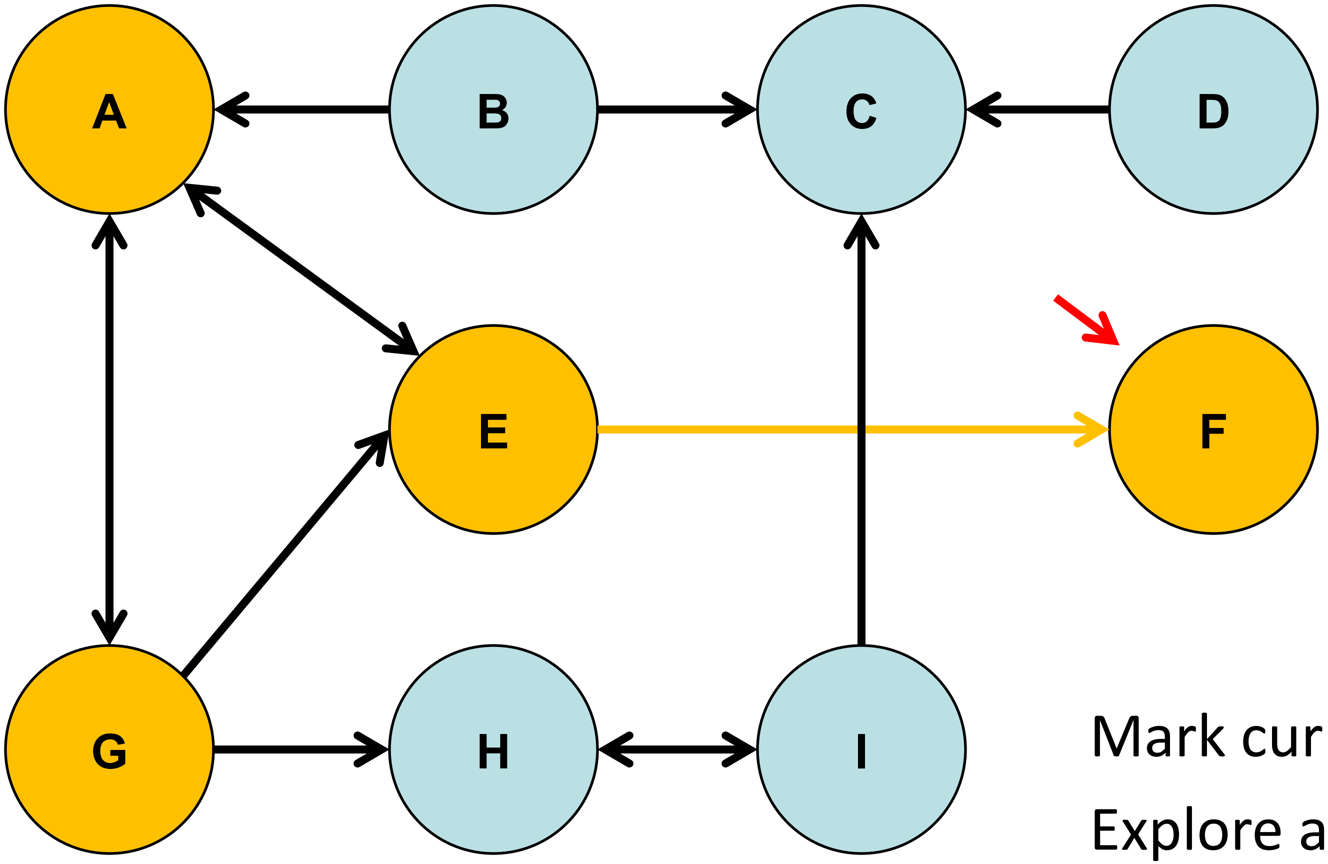

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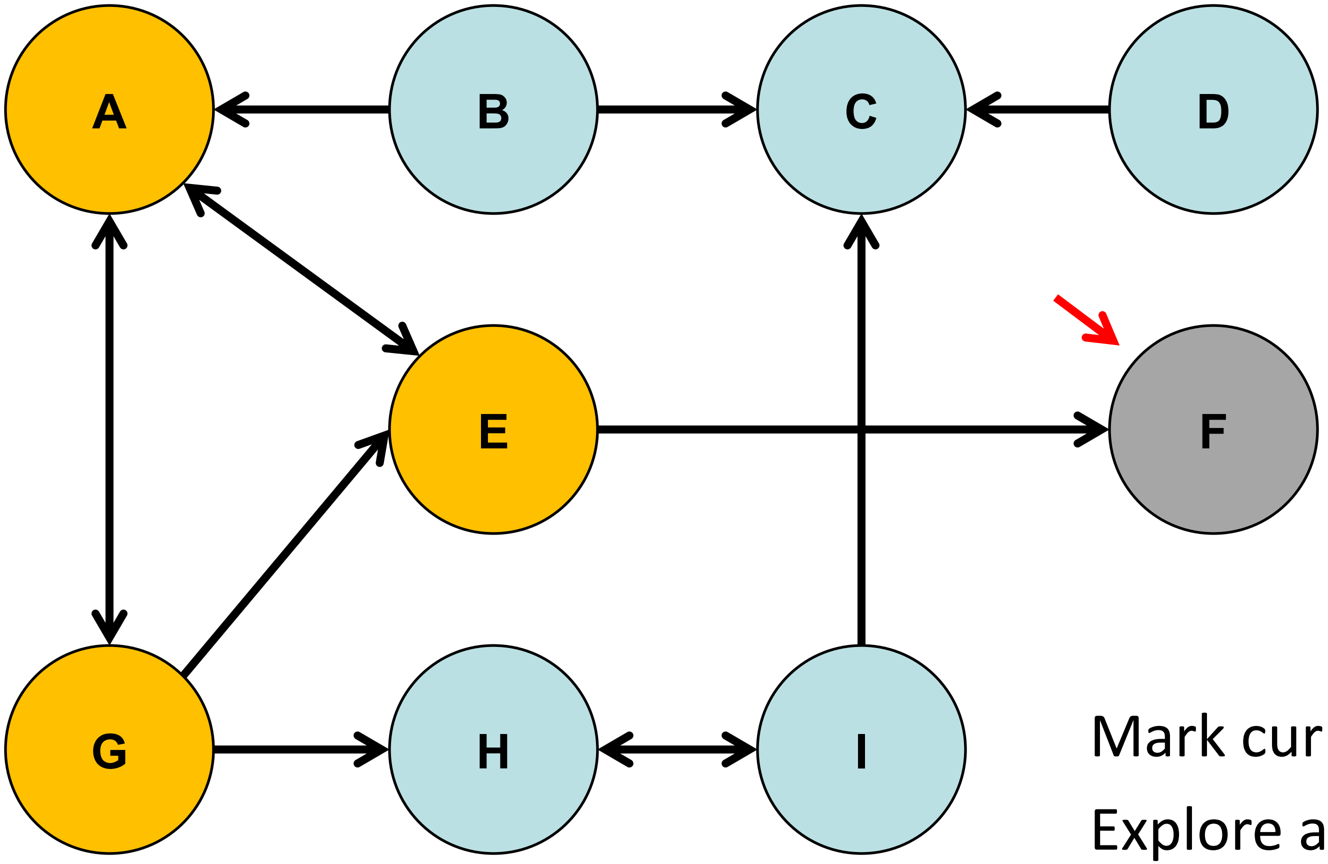

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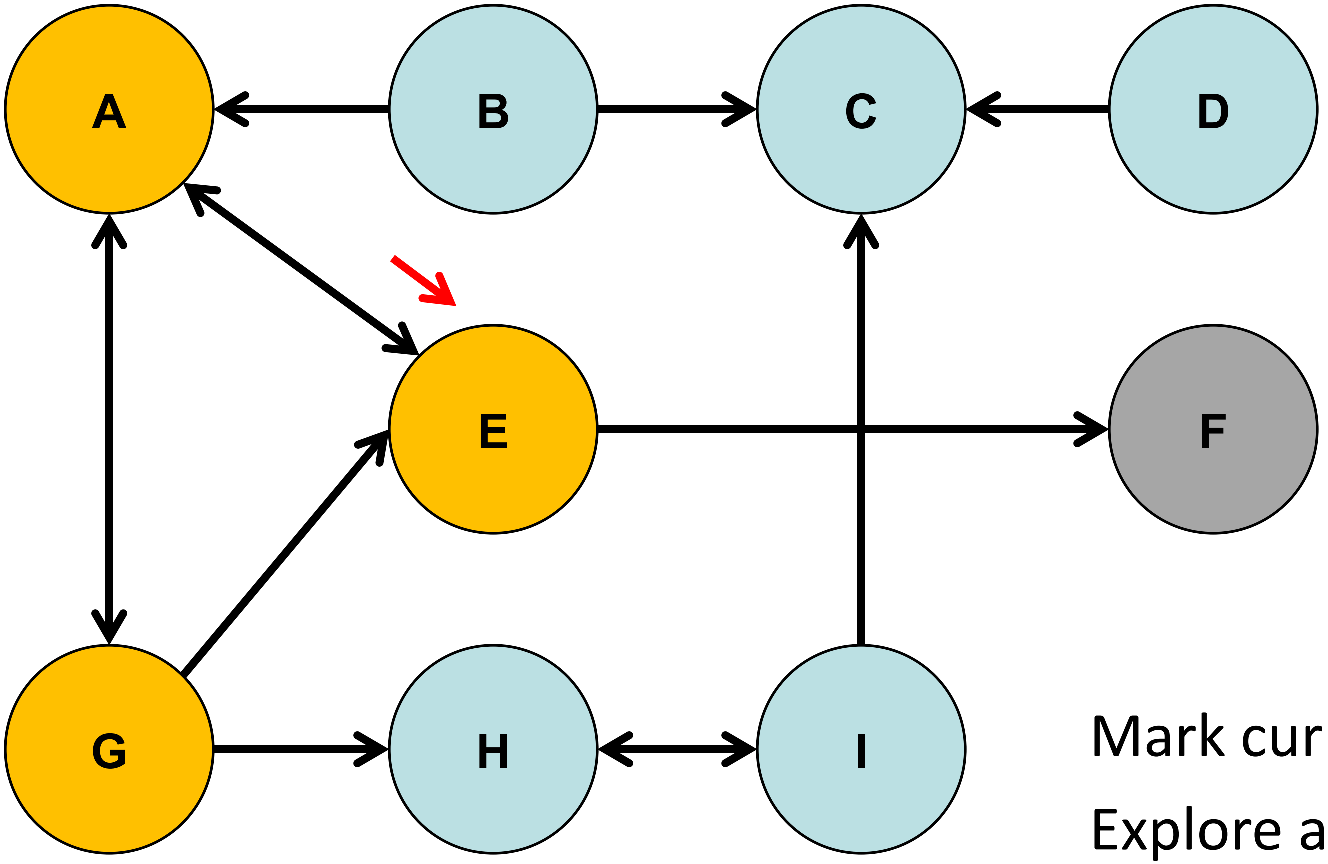

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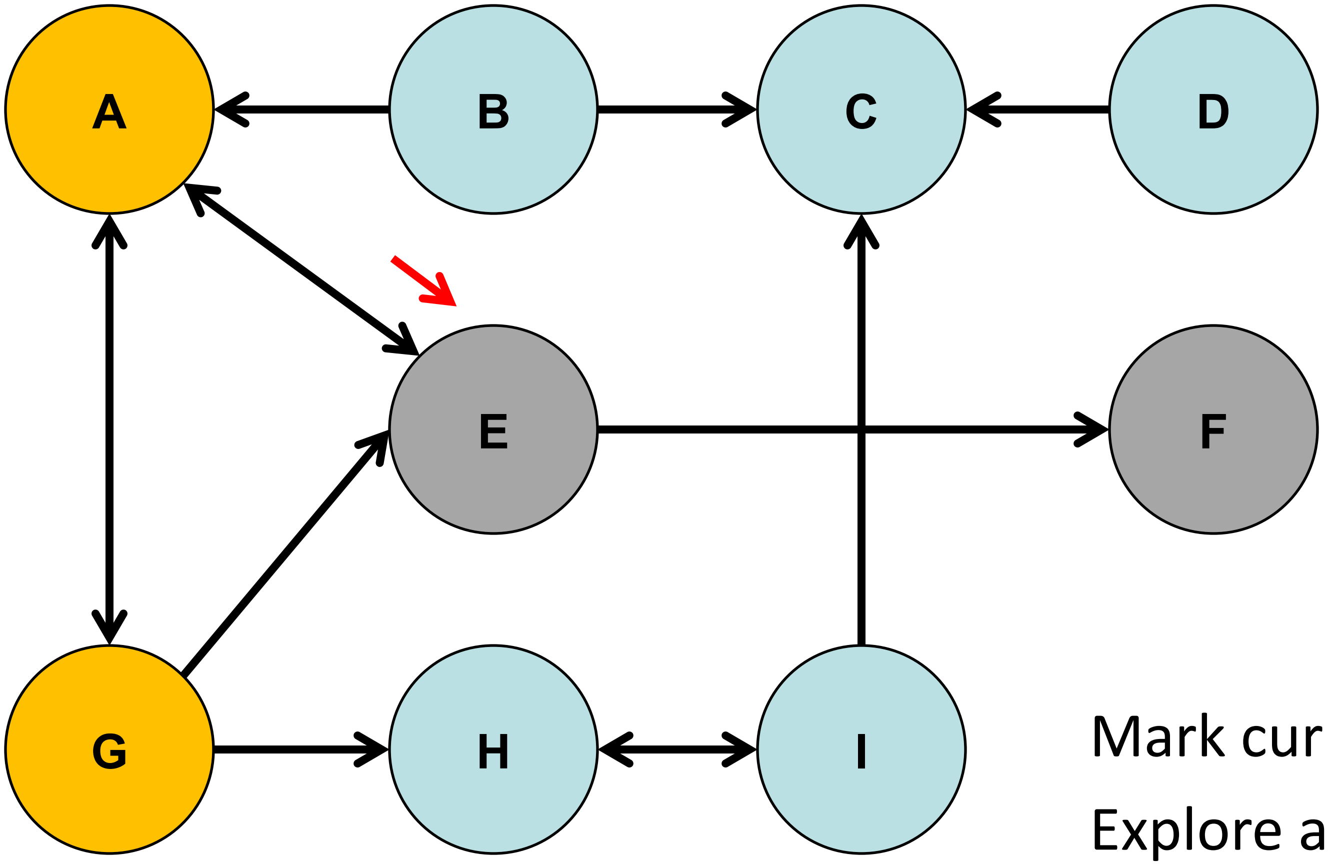

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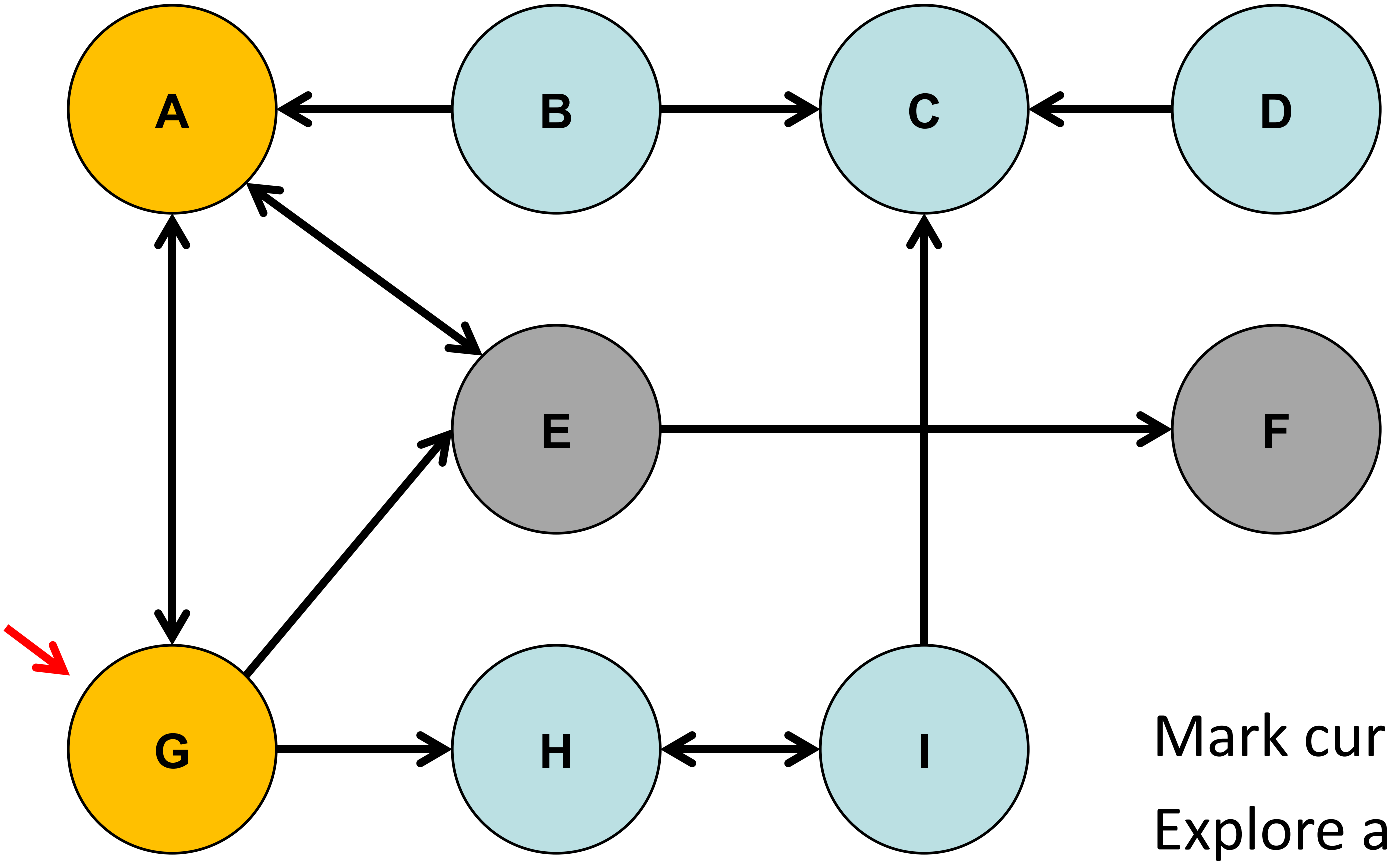

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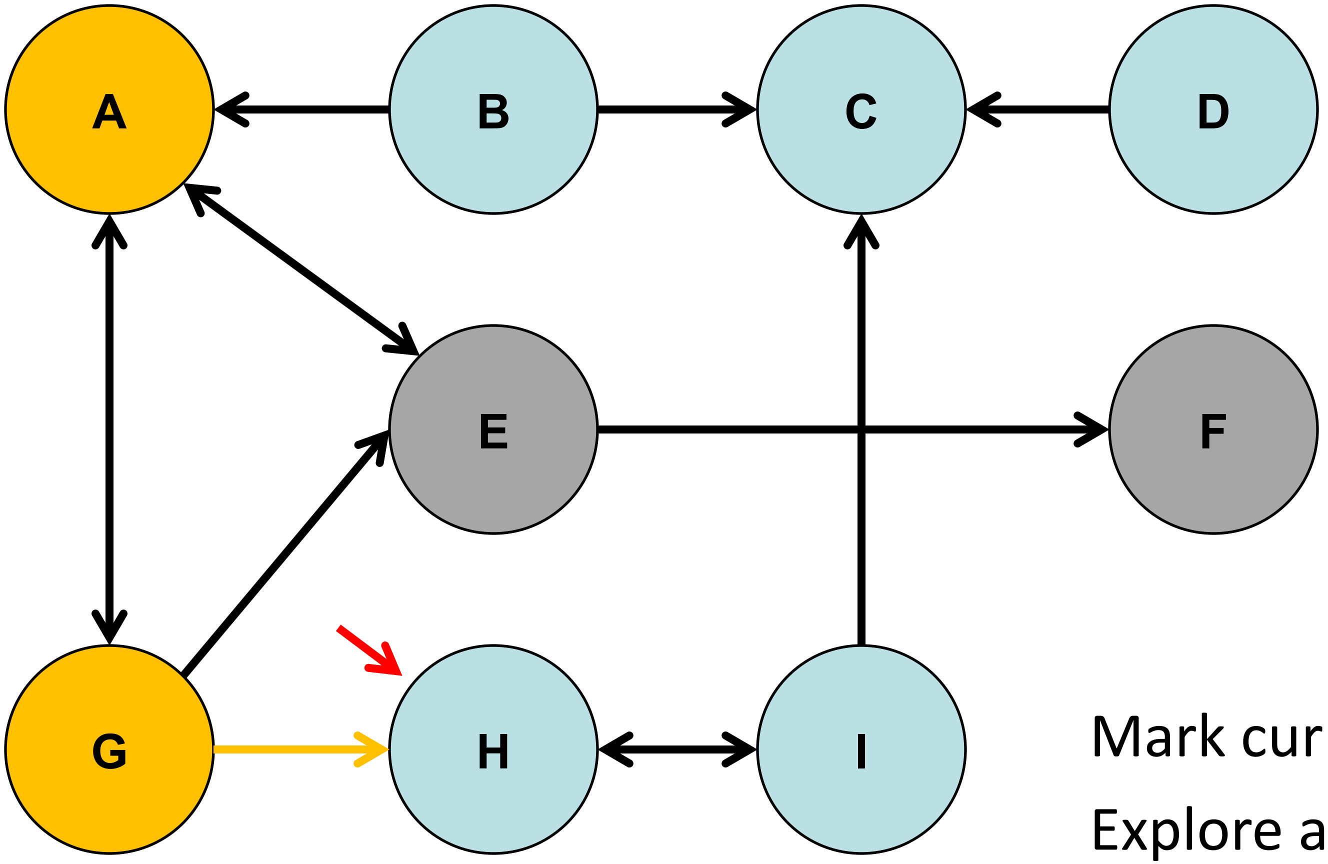

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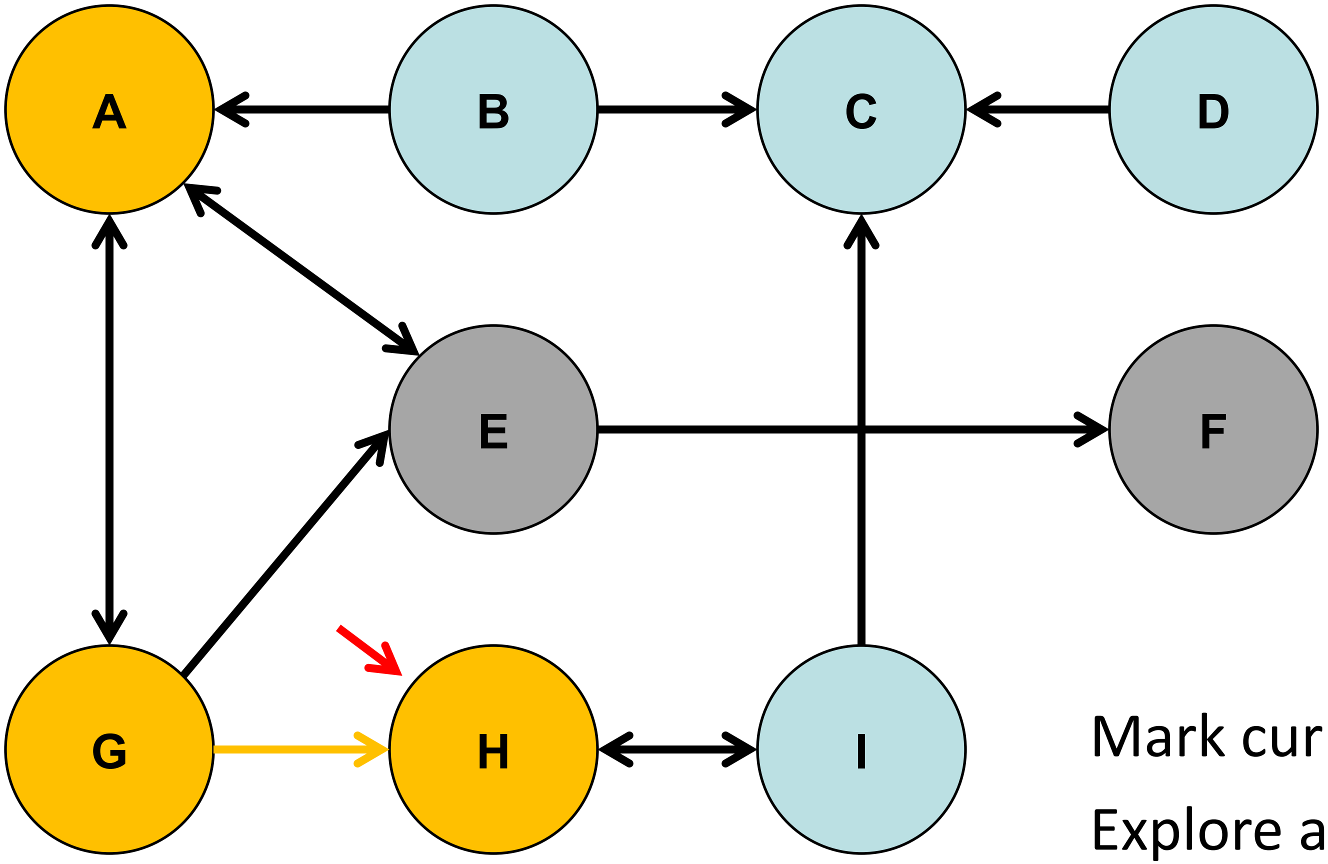

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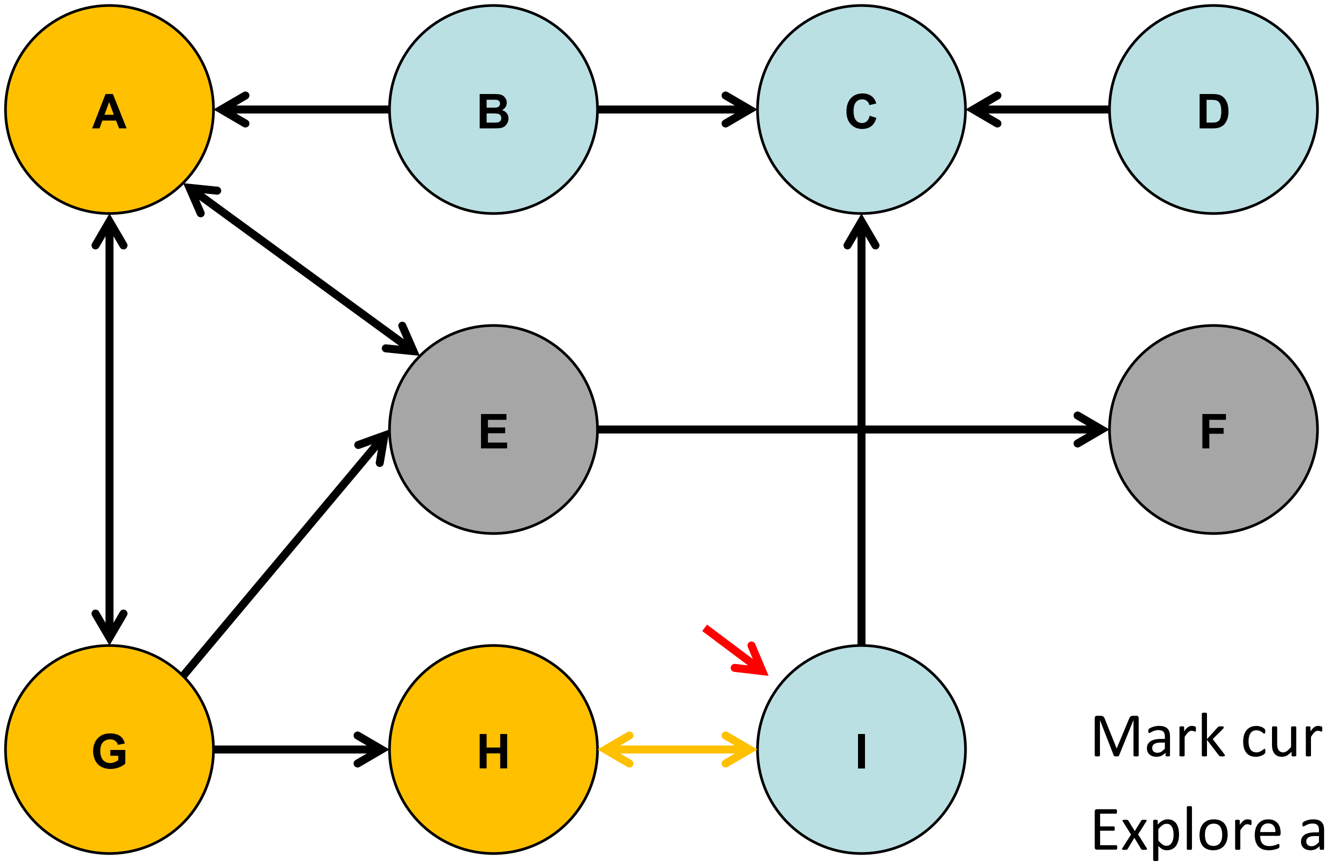

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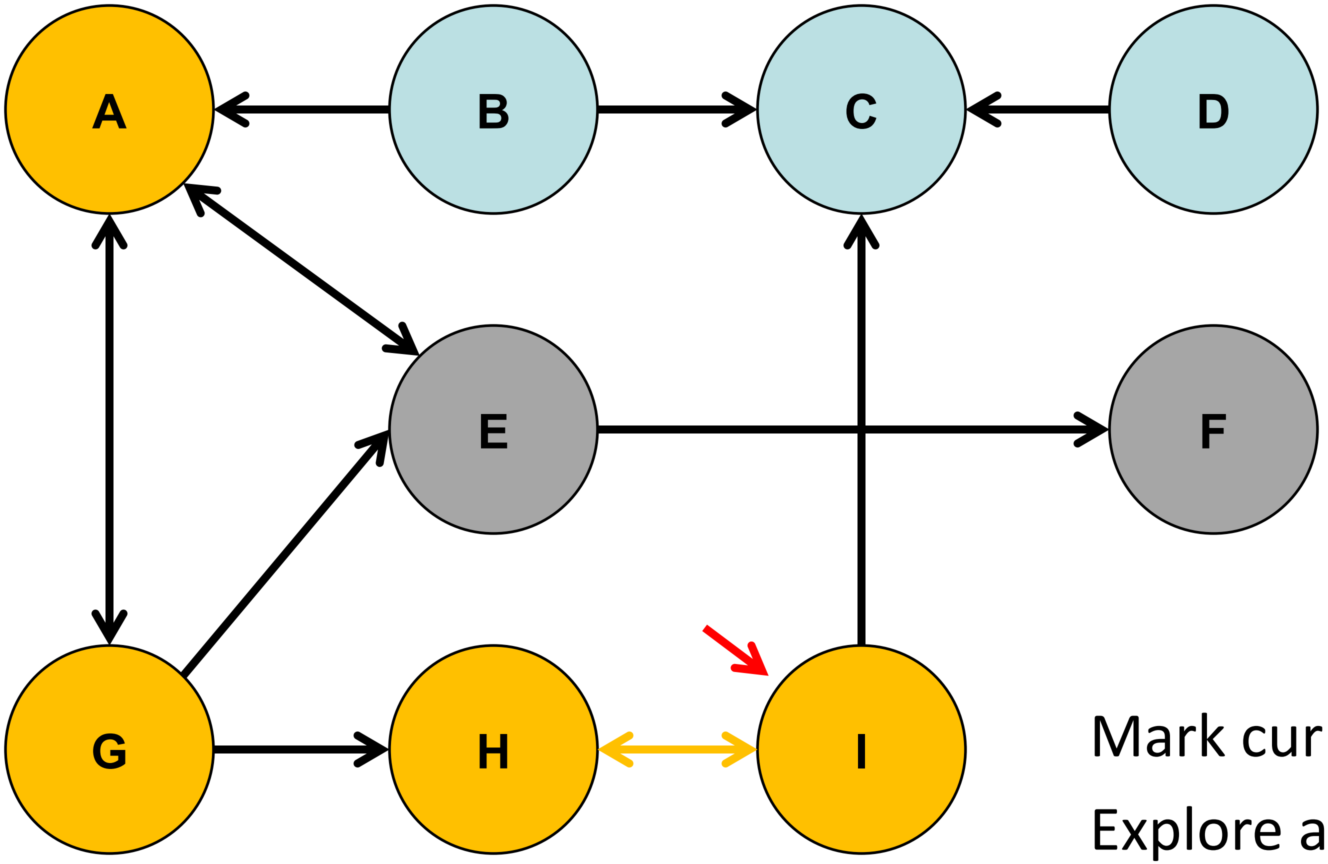

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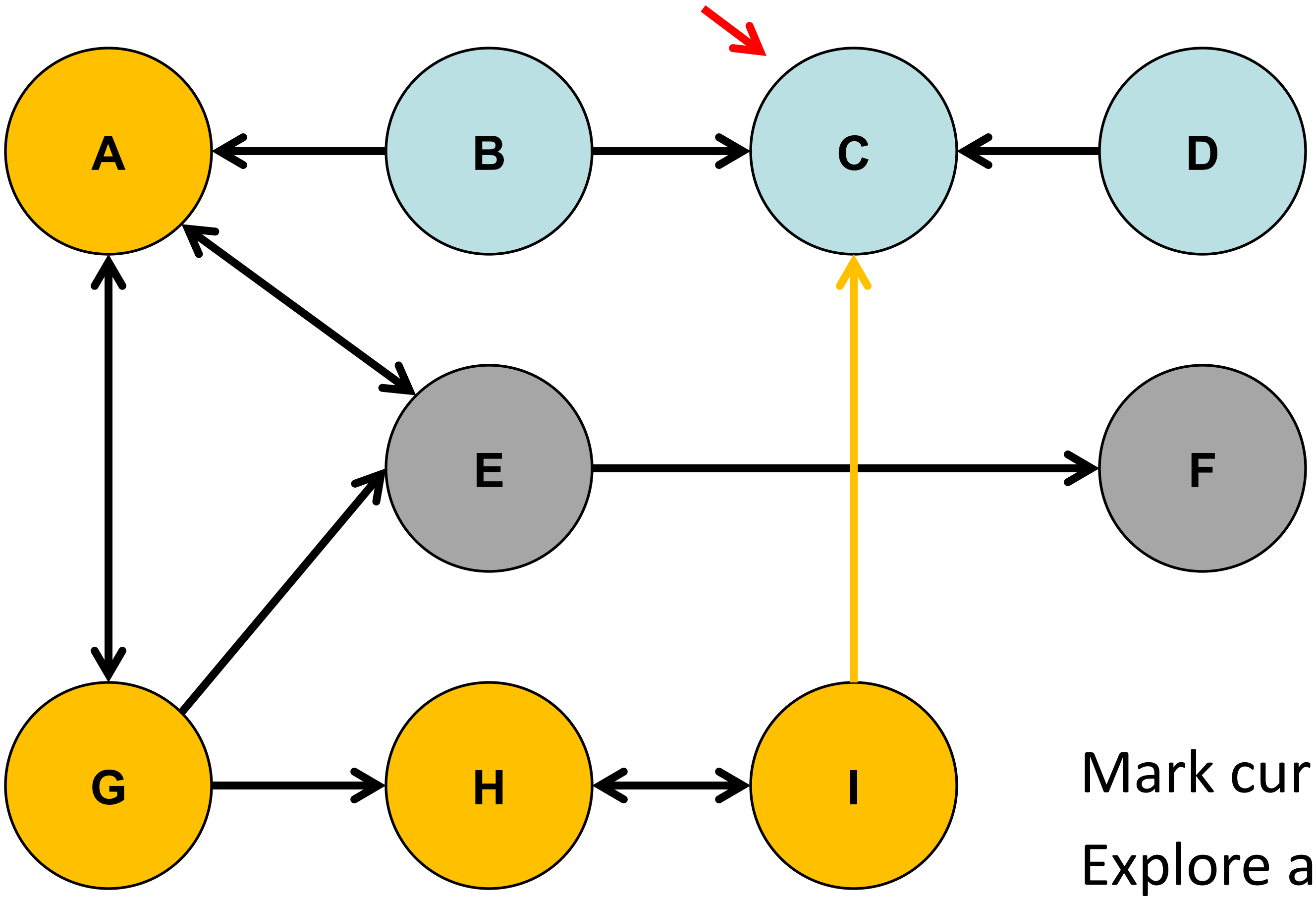

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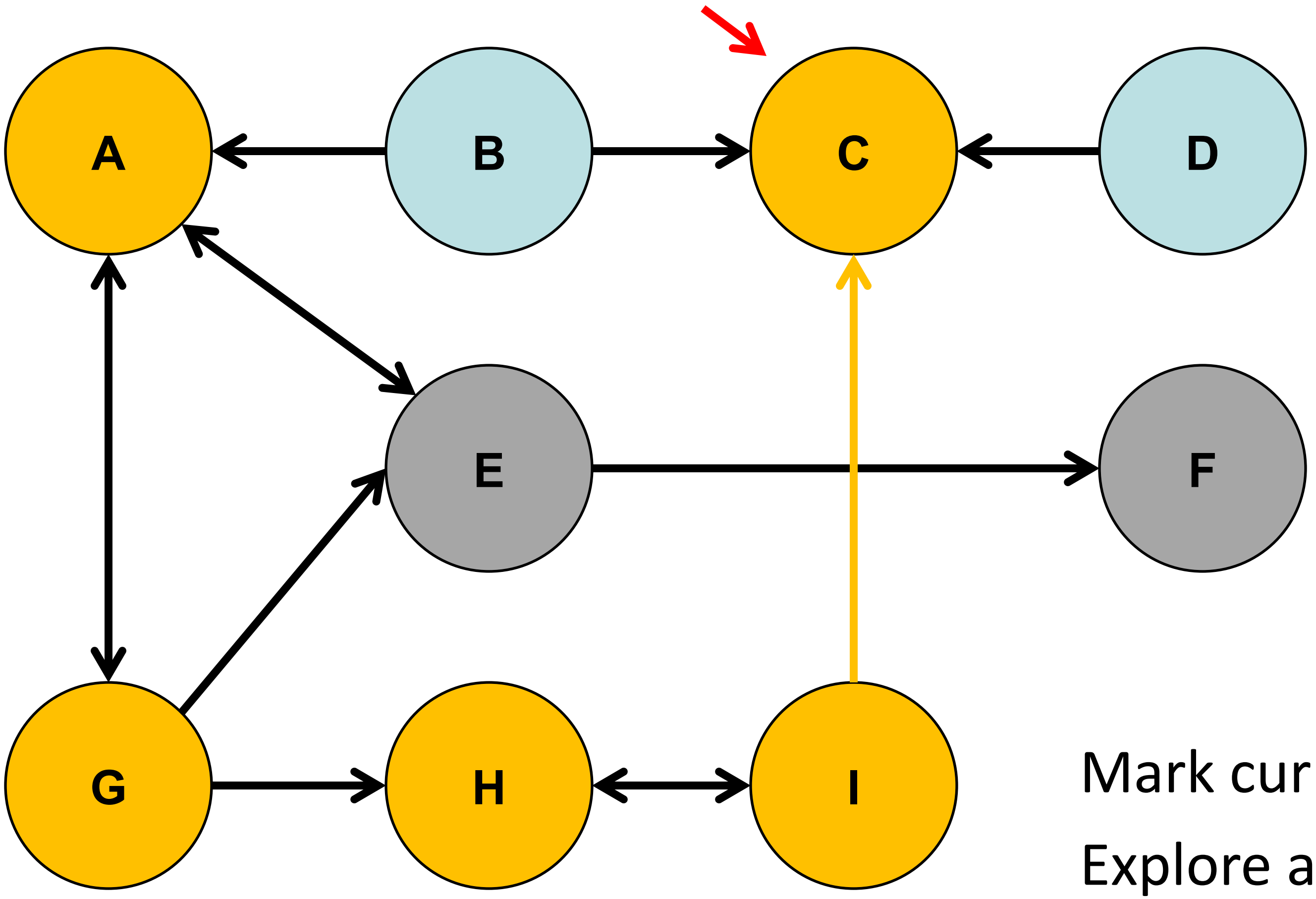

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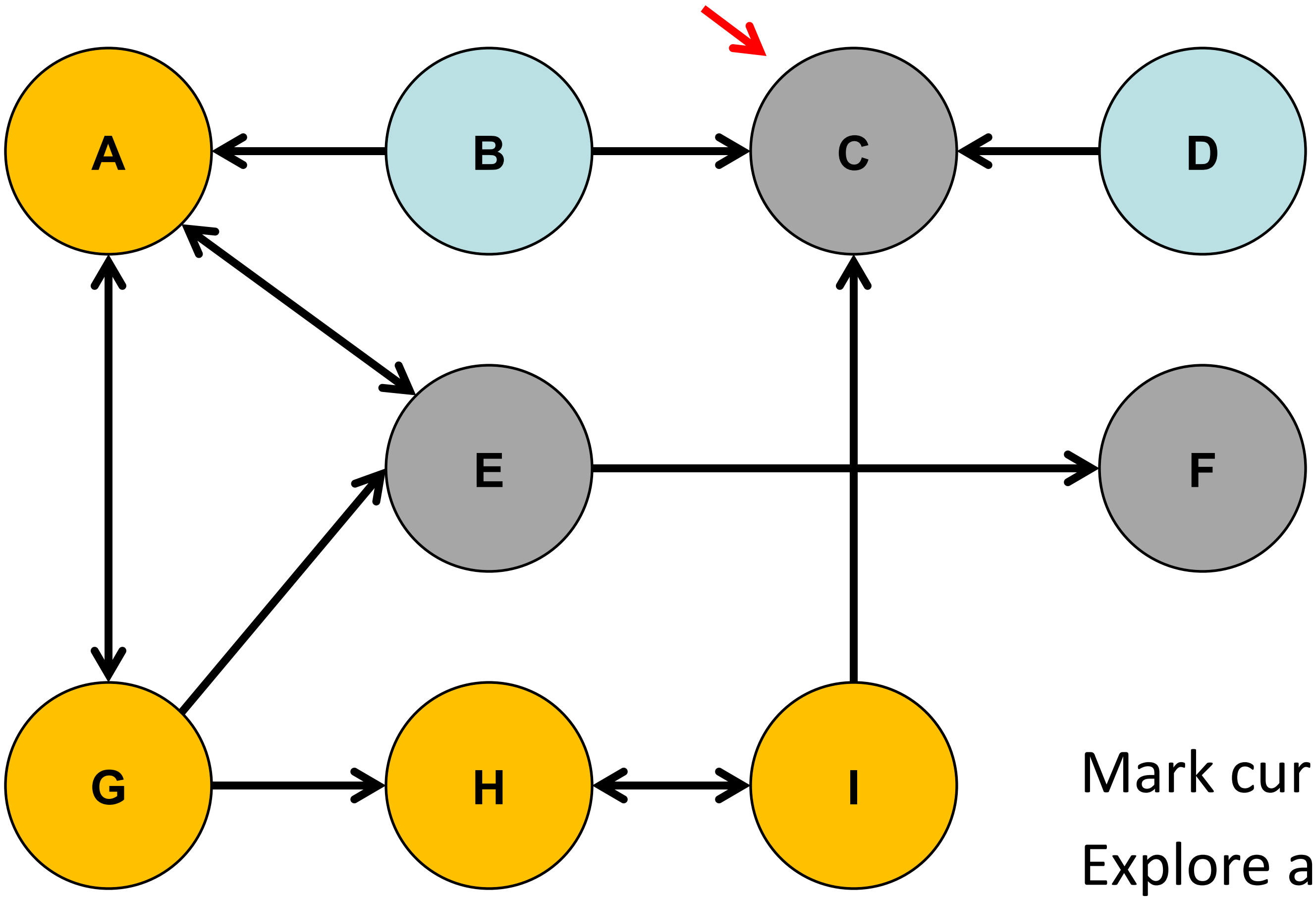

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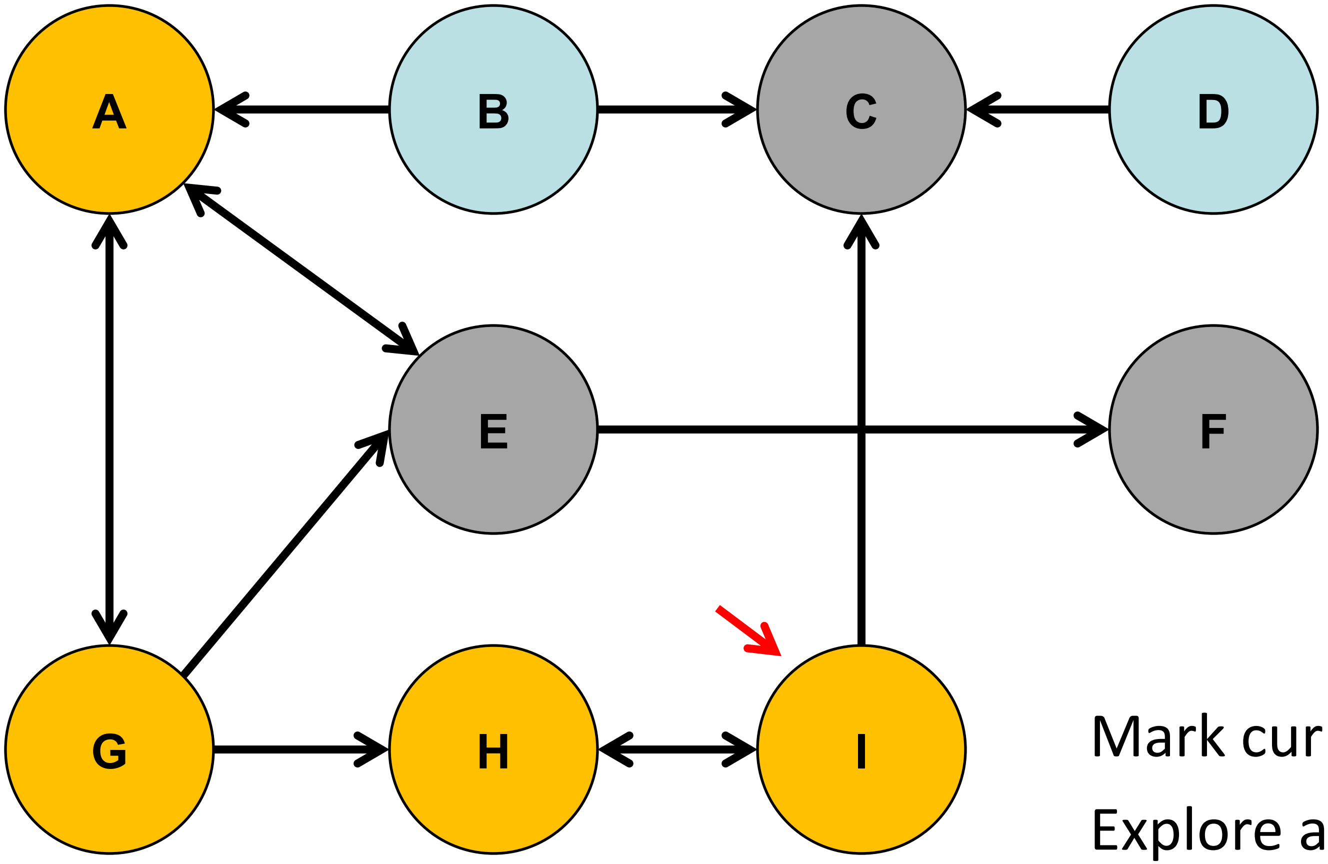

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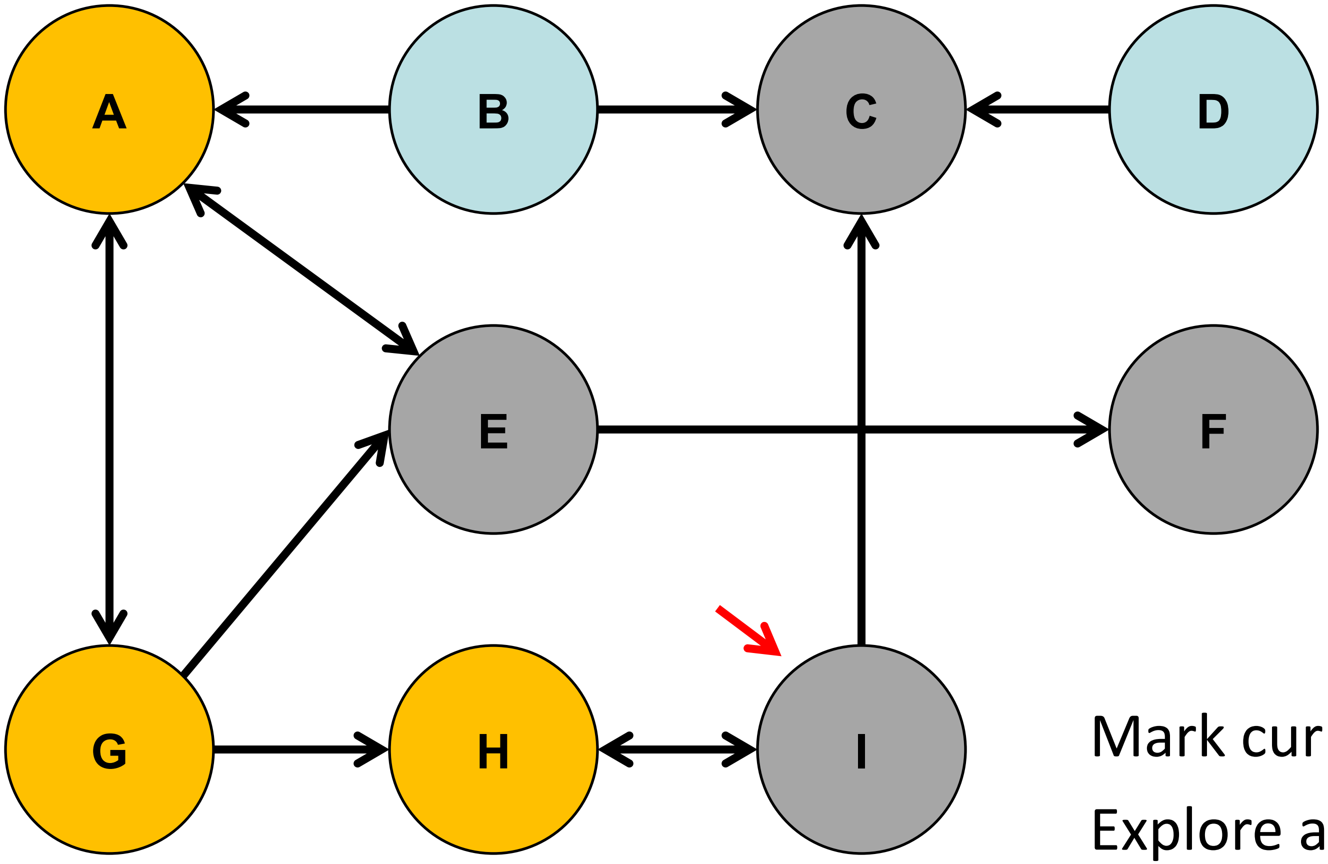

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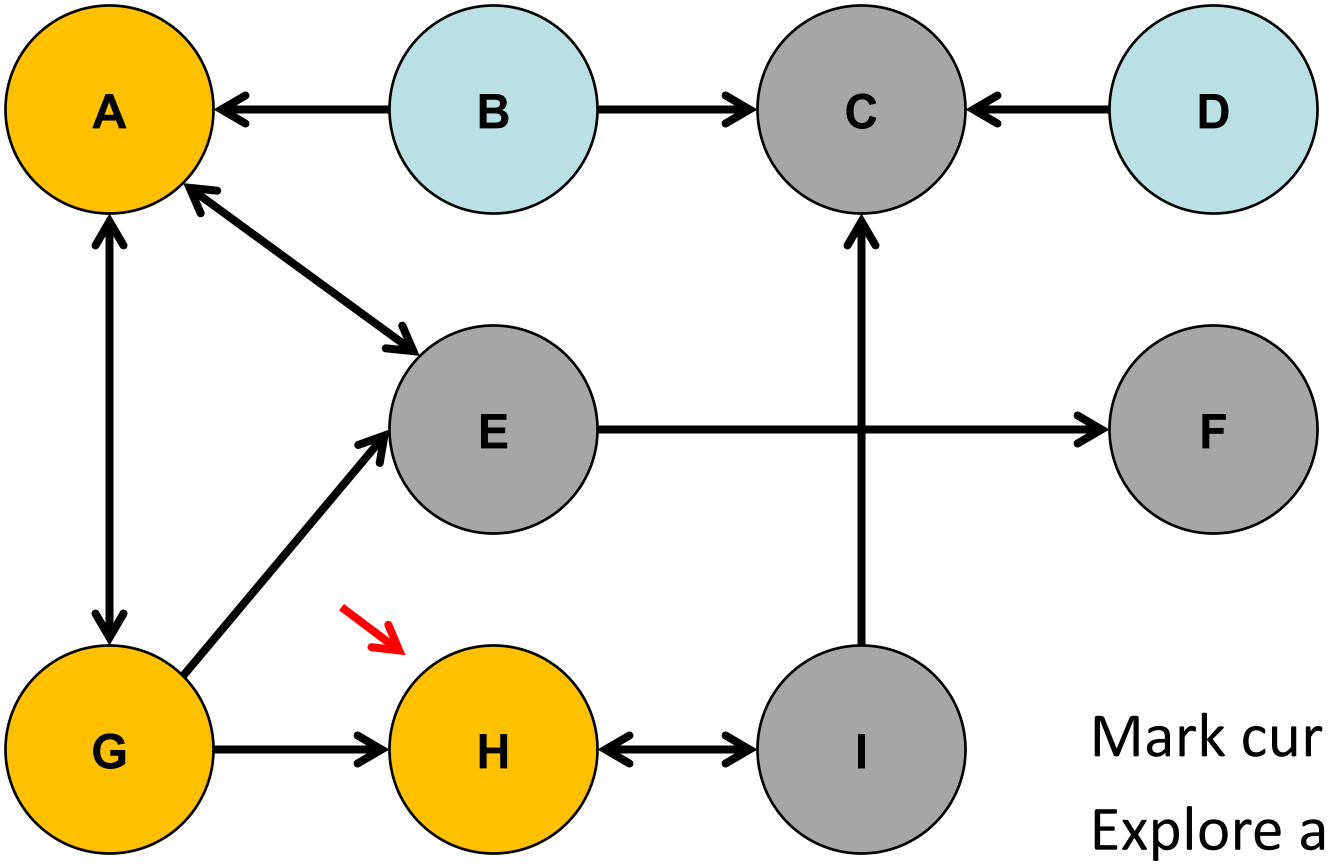

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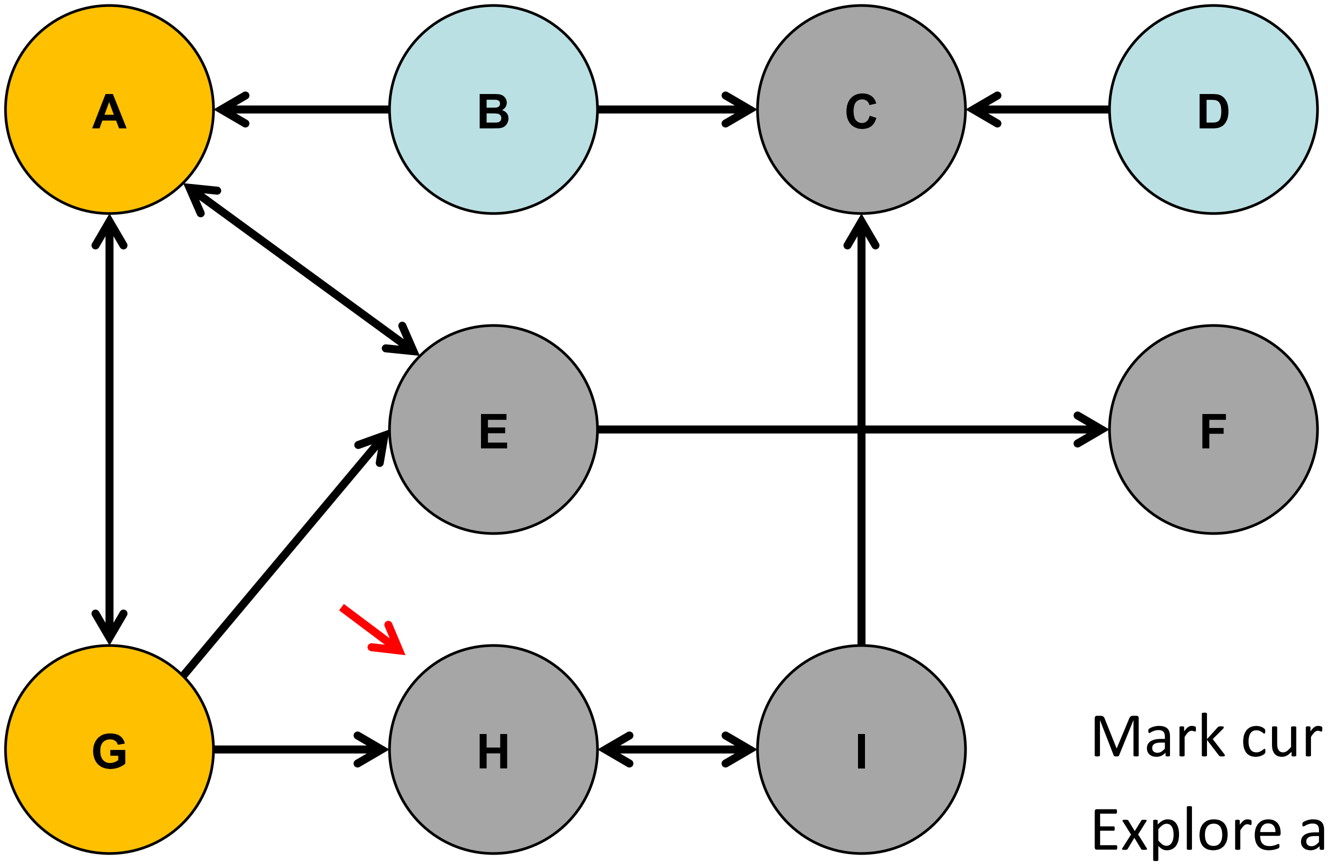

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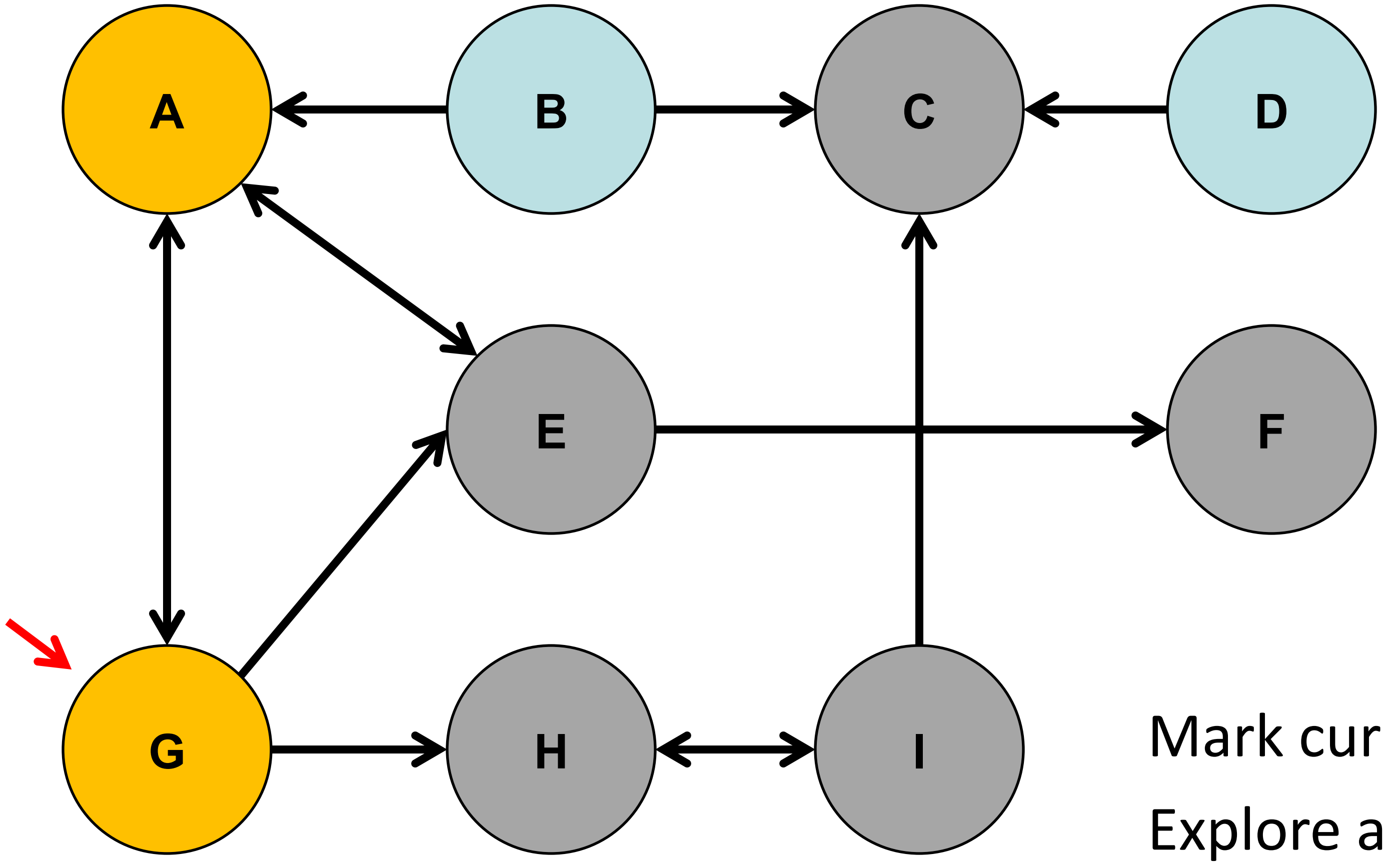

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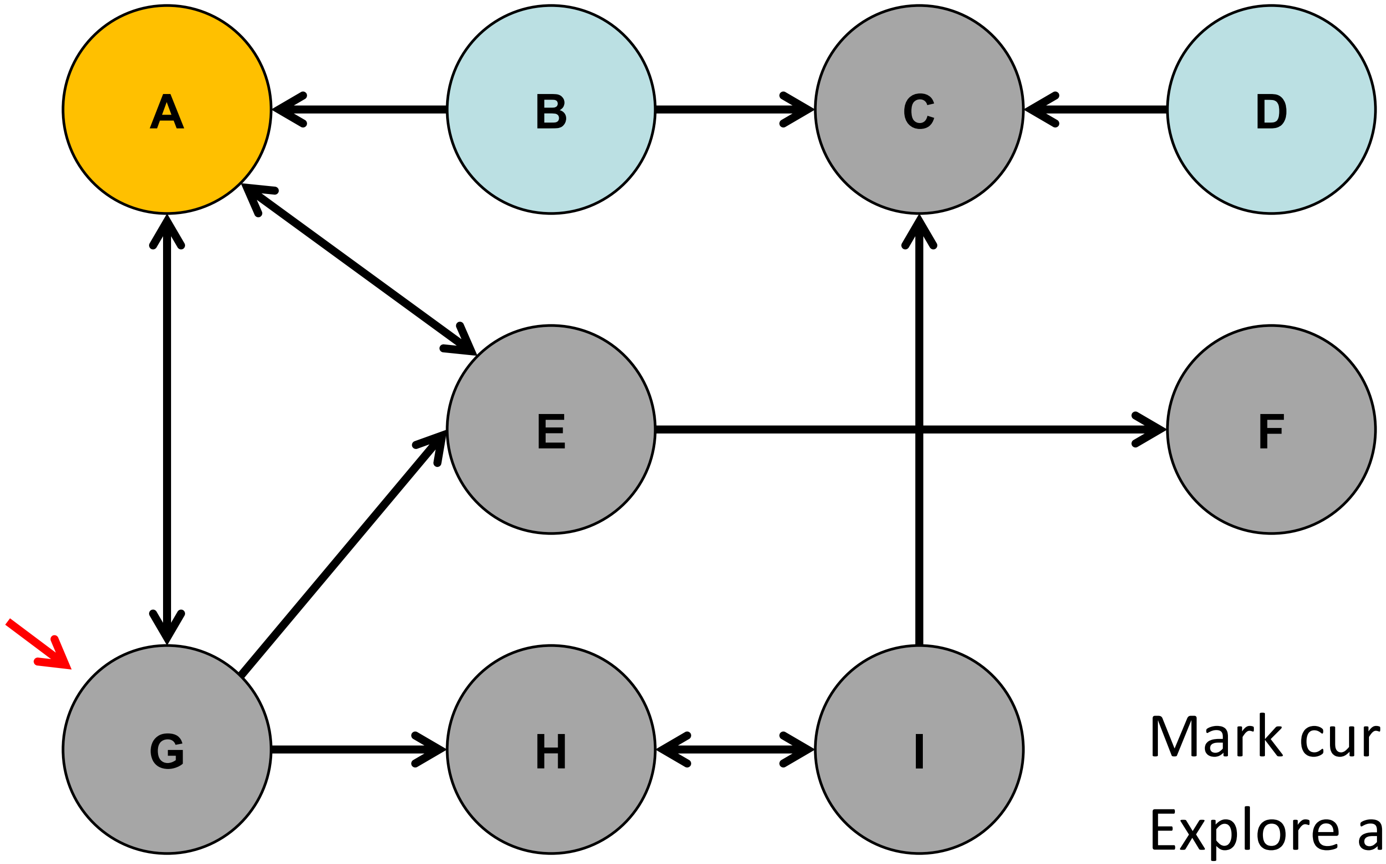

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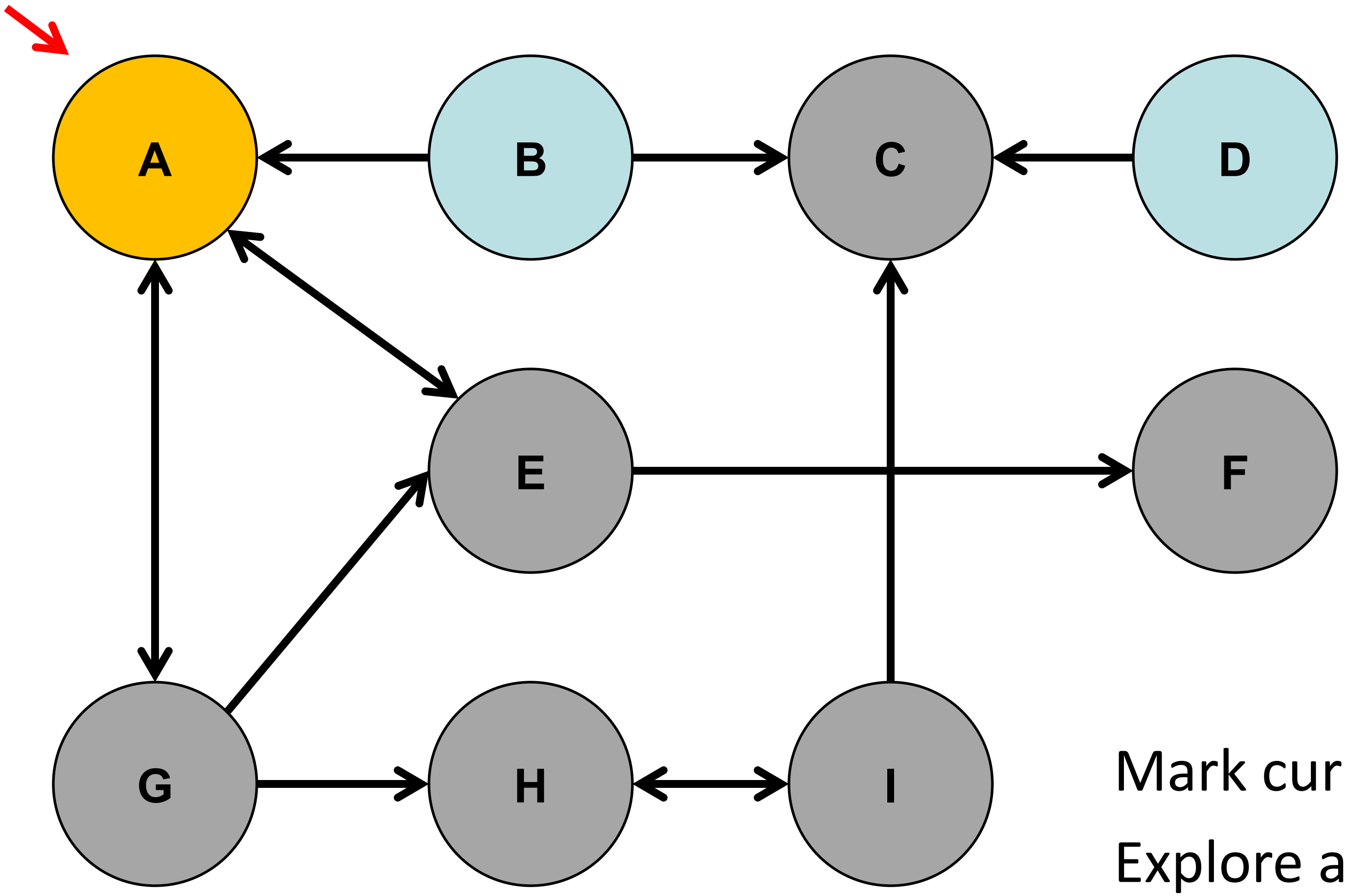

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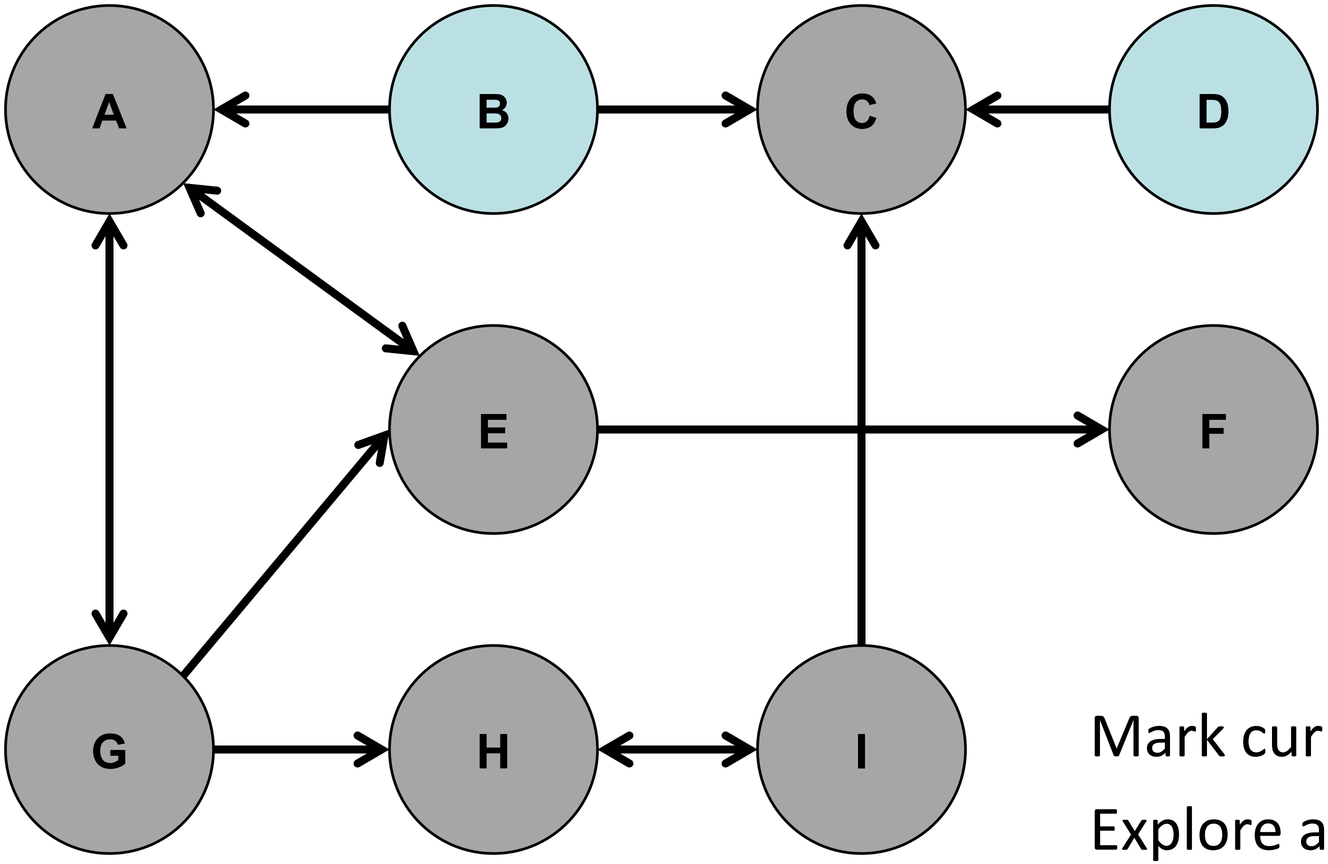

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DFS Details

- In an n-node, m-edge graph, takes $O(m + n)$ time with an adjacency list
  - Visit each edge once, visit each node at most once
- Pseudocode:

```
dfs from v1:
    mark v1 as seen.
    for each of v1's unvisited neighbors n:
      dfs(n)
```

- How could we modify the pseudocode to look for a specific path?

DFS that finds path

dfs from v_1 to v_2:

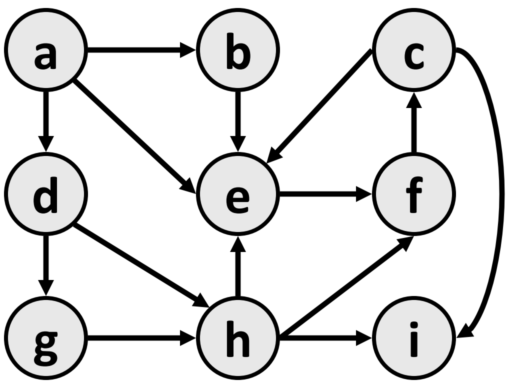
mark v_1 as **visited**, and **add to path**.

选对象

perform a **dfs** from each of v_1's unvisited neighbors n to v_2:

if **dfs**(n, v_2) succeeds: a path is found! yay!

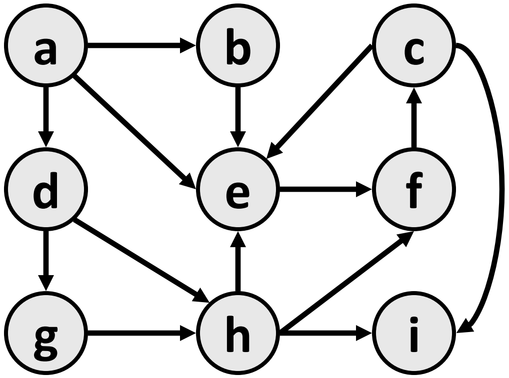
if all neighbors fail: **remove v_1 from path**.



- To retrieve the DFS path found, pass a collection parameter to each call and choose-explore-unchoose.

DFS observations

- *discovery*: DFS is guaranteed to find *a* path if one exists.



- *retrieval*: It is easy to retrieve exactly what the path is (the sequence of edges taken) if we find it
  - choose - explore - unchoose

- *optimality*: not optimal. DFS is guaranteed to find a path, not necessarily the best/shortest path
  - Example: dfs(a, i) returns {a, b, e, f, c, i} rather than {a, d, h, i}.

Plan For Today

- **Recap**: Graphs
- **Practice**: Twitter Influence
- Depth-First Search (DFS)
- Announcements
- Breadth-First Search (BFS)

Announcements

- Assignment 7 will go out this Friday, is due Wed. after break
  - Short graphs assignment (Google Maps!), implementing algorithms from this week
- Assignment 8 will go out the Wed. after break, is due the last day of class (Fri)
  - Graphs and inheritance assignment (Excel!)

Plan For Today

- **Recap**: Graphs
- **Practice**: Twitter Influence
- Depth-First Search (DFS)
- Announcements
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Finding *Shortest* Paths

- We can find paths between two nodes, but how can we find the **shortest** path?
  - Fewest number of steps to complete a task?
  - Least amount of edits between two words?
- When have we solved this problem before?

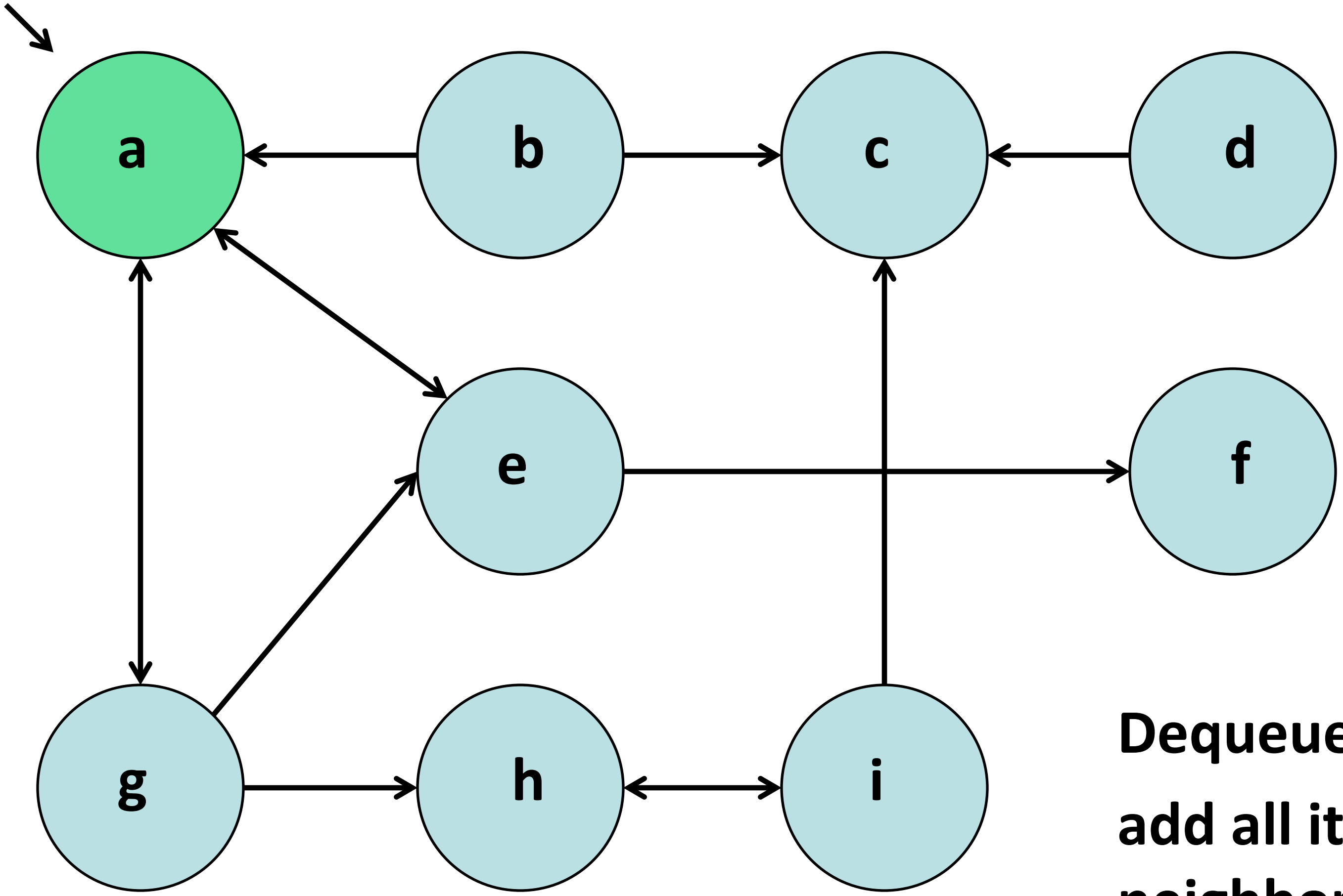
Breadth-First Search (BFS)

- Idea: processing a node involves knowing we need to visit all its neighbors (just like DFS)
- Need to keep a TODO list of nodes to process

Breadth-First Search (BFS)

- Keep a Queue of nodes as our TODO list
- Idea: dequeue a node, enqueue all its neighbors
- Still will return the same nodes as reachable, just might have shorter paths

BFS

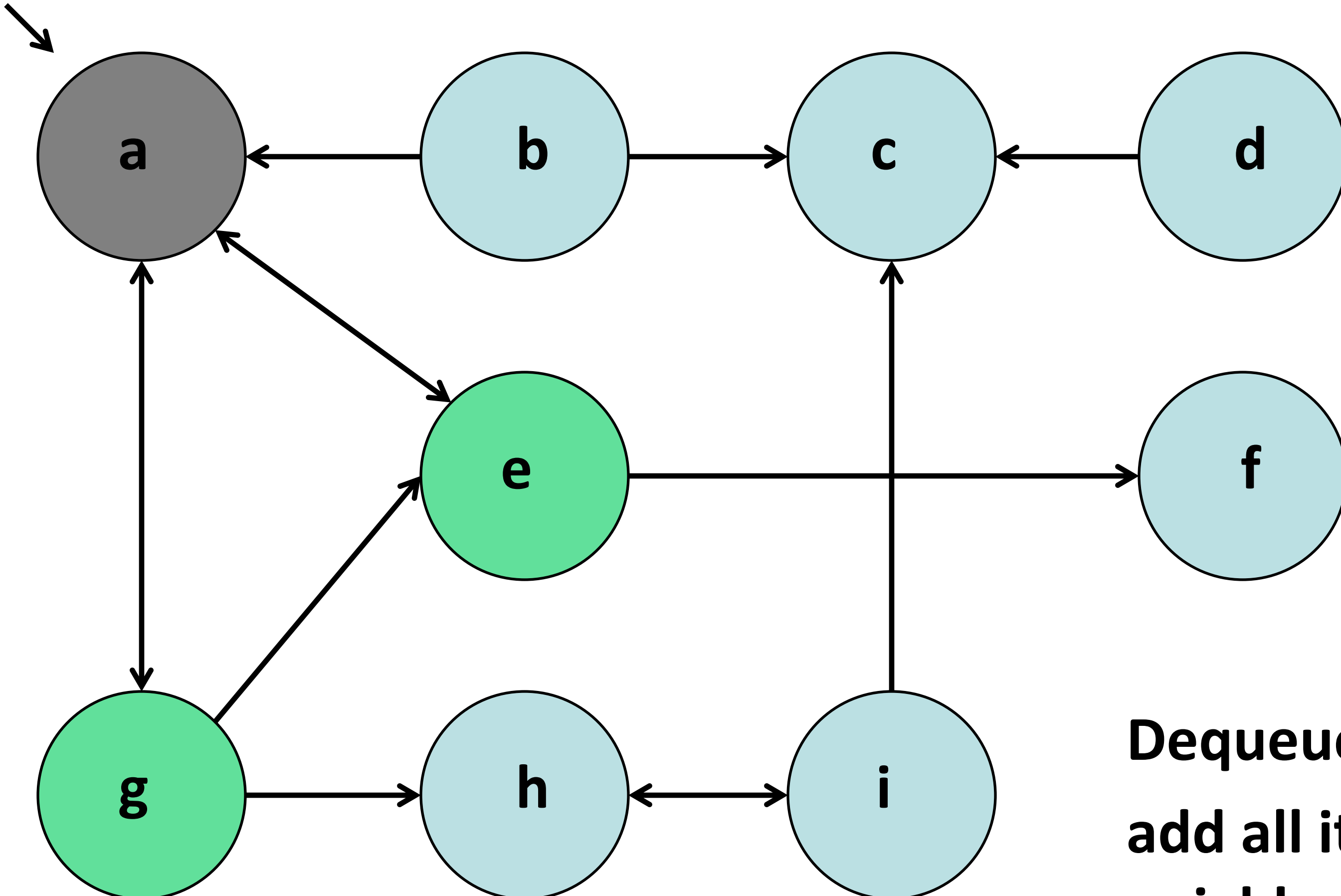


Dequeue a node

add all its unseen neighbors to the queue

queue: a

BFS

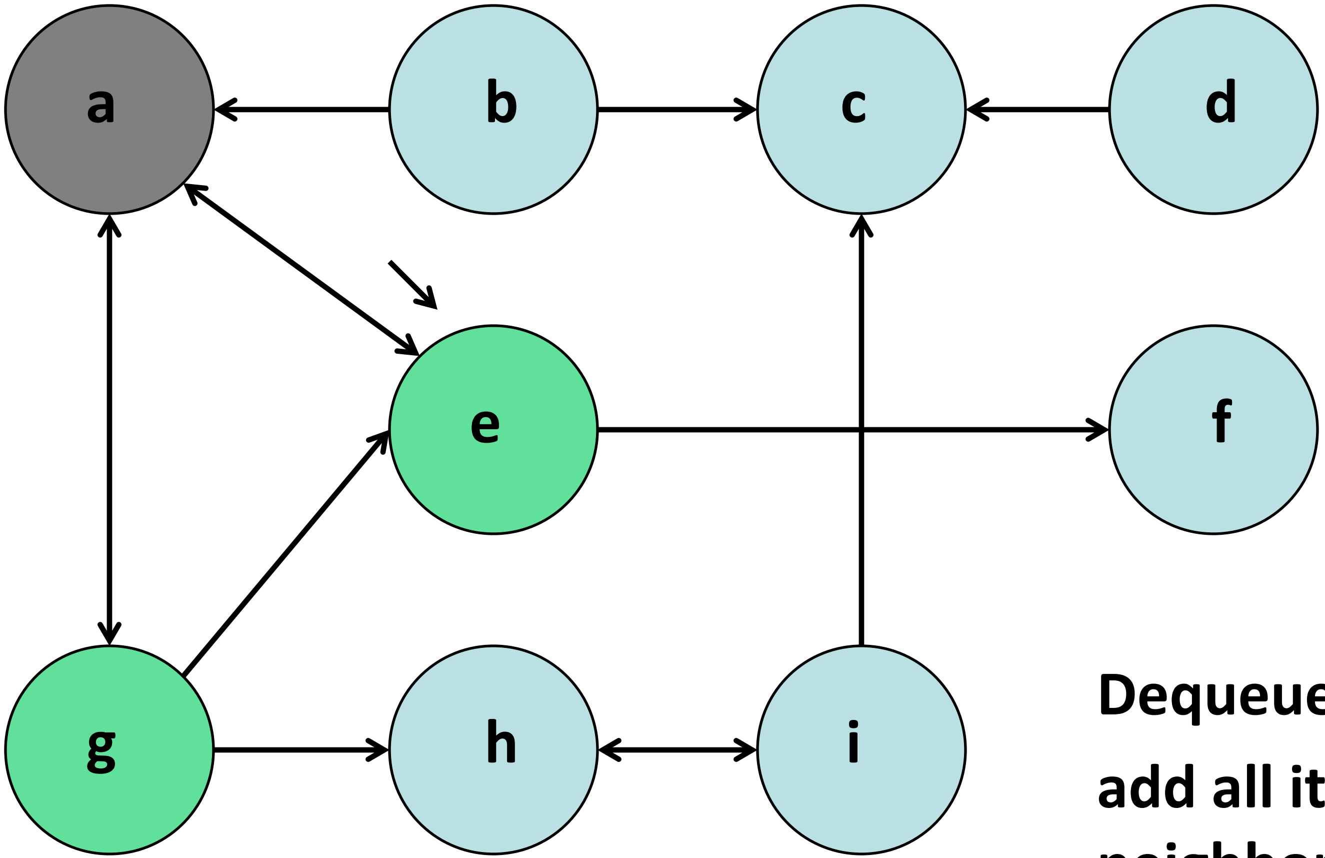


Dequeue a node

add all its unseen neighbors to the queue

queue: e, g

BFS

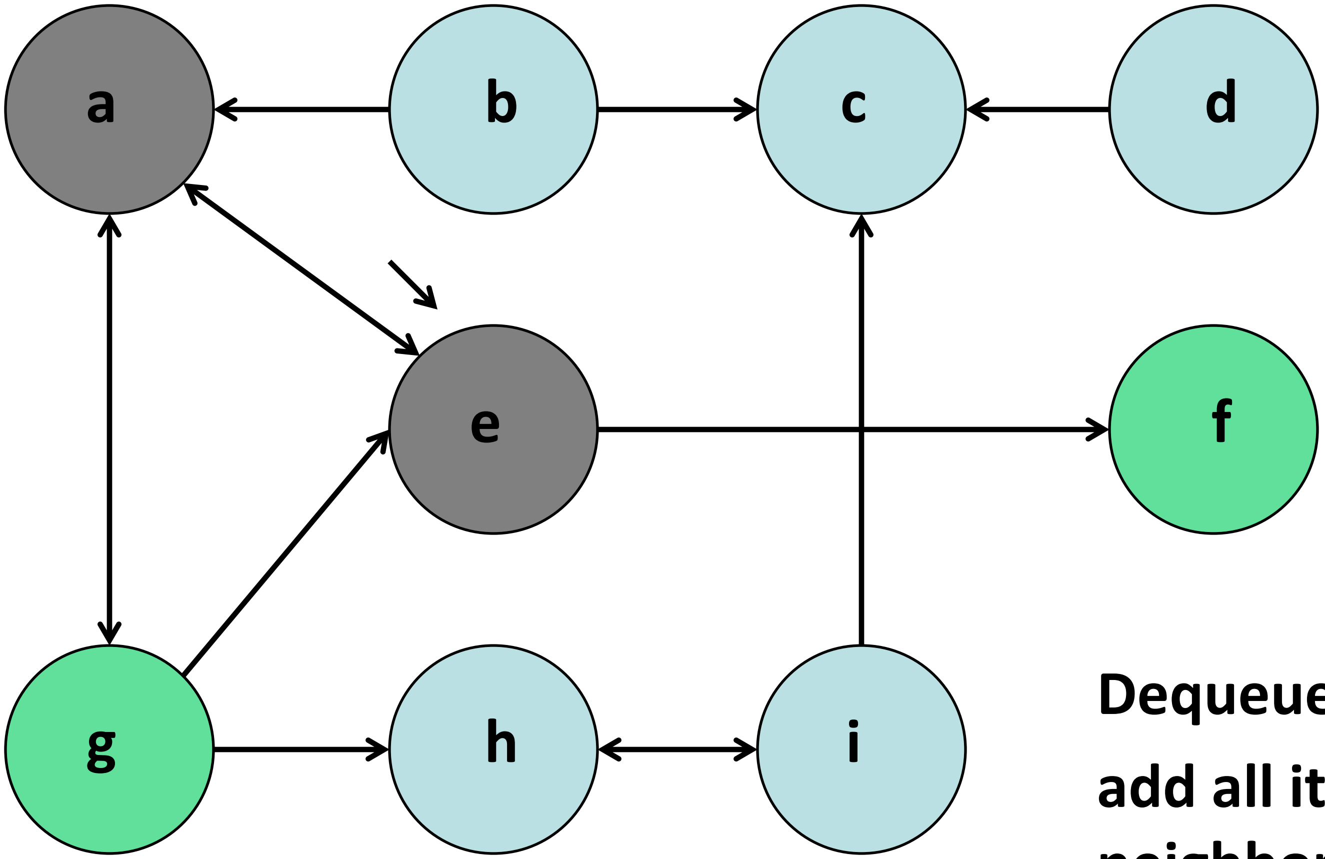


Dequeue a node

add all its unseen neighbors to the queue

queue: e, g

BFS

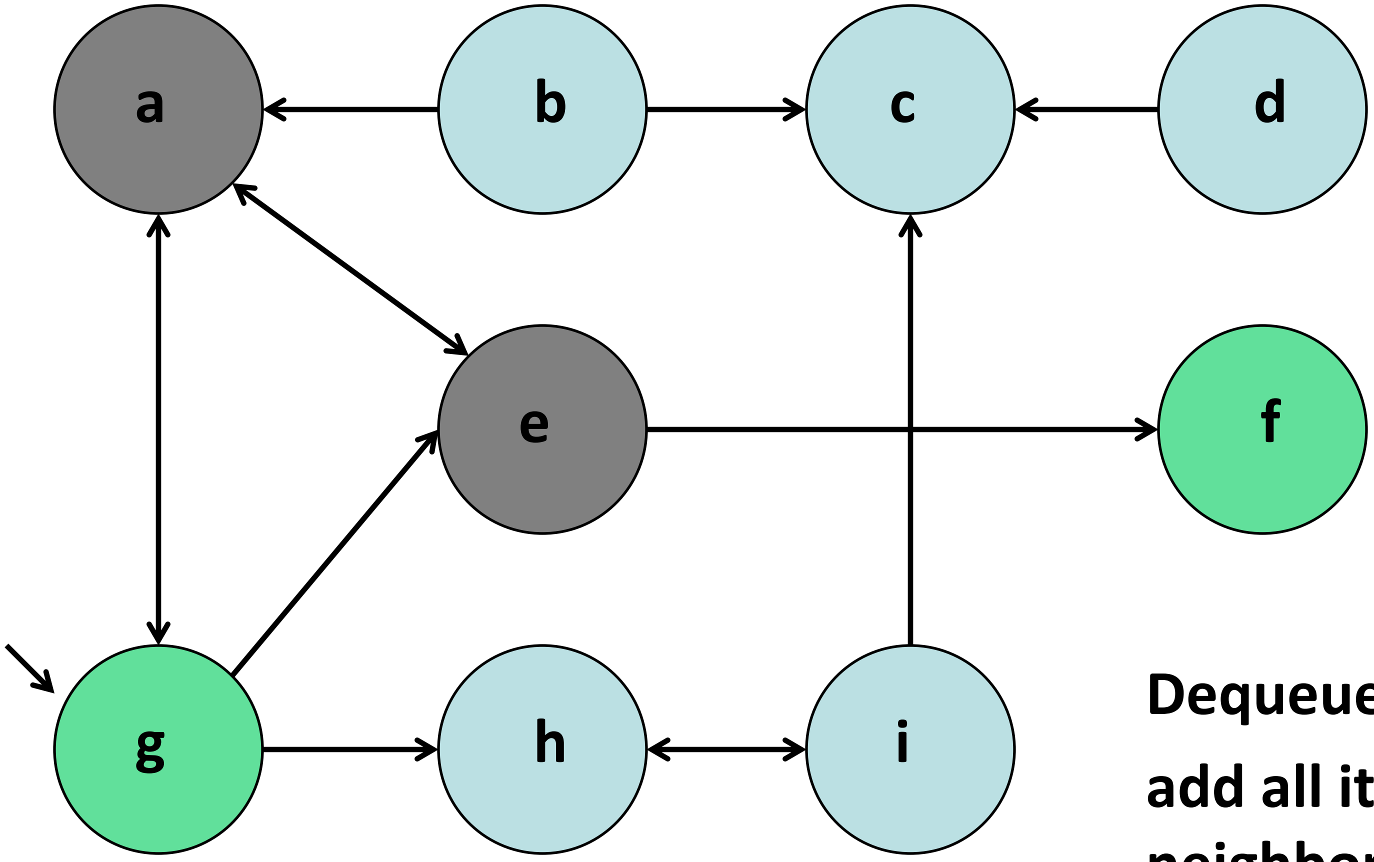


Dequeue a node

add all its unseen neighbors to the queue

queue: g, f

BFS

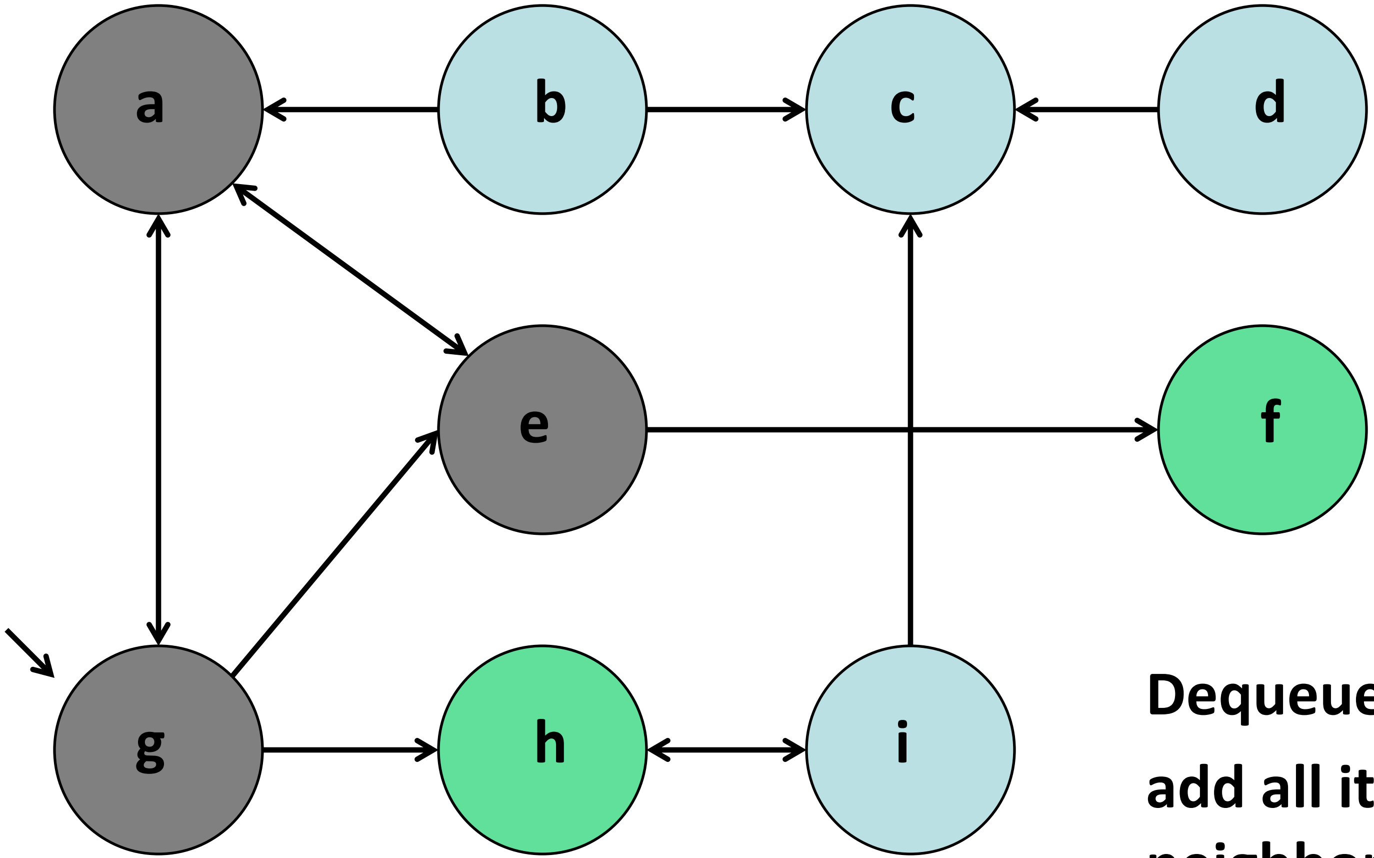


Dequeue a node

add all its unseen neighbors to the queue

queue: g, f

BFS

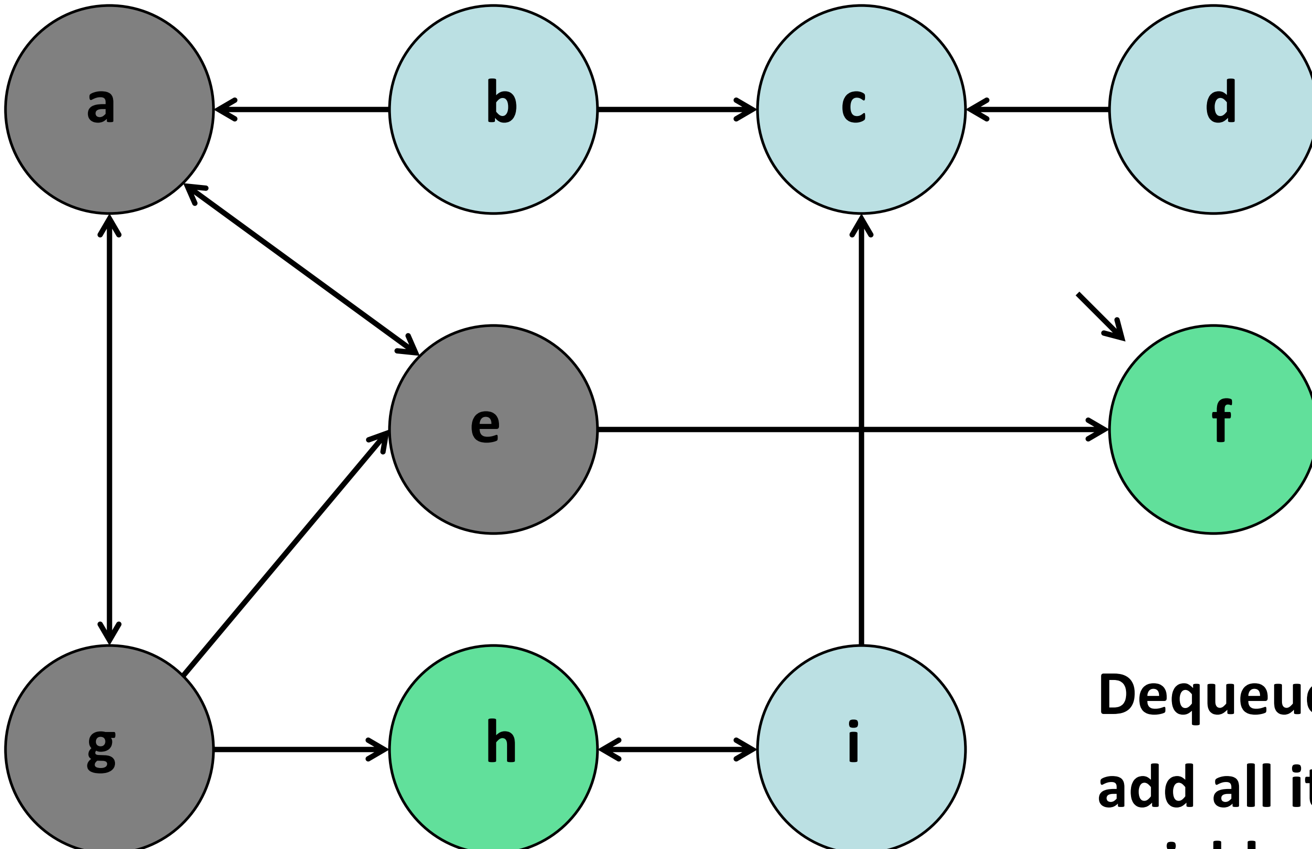


Dequeue a node

add all its unseen neighbors to the queue

queue: f, h

BFS

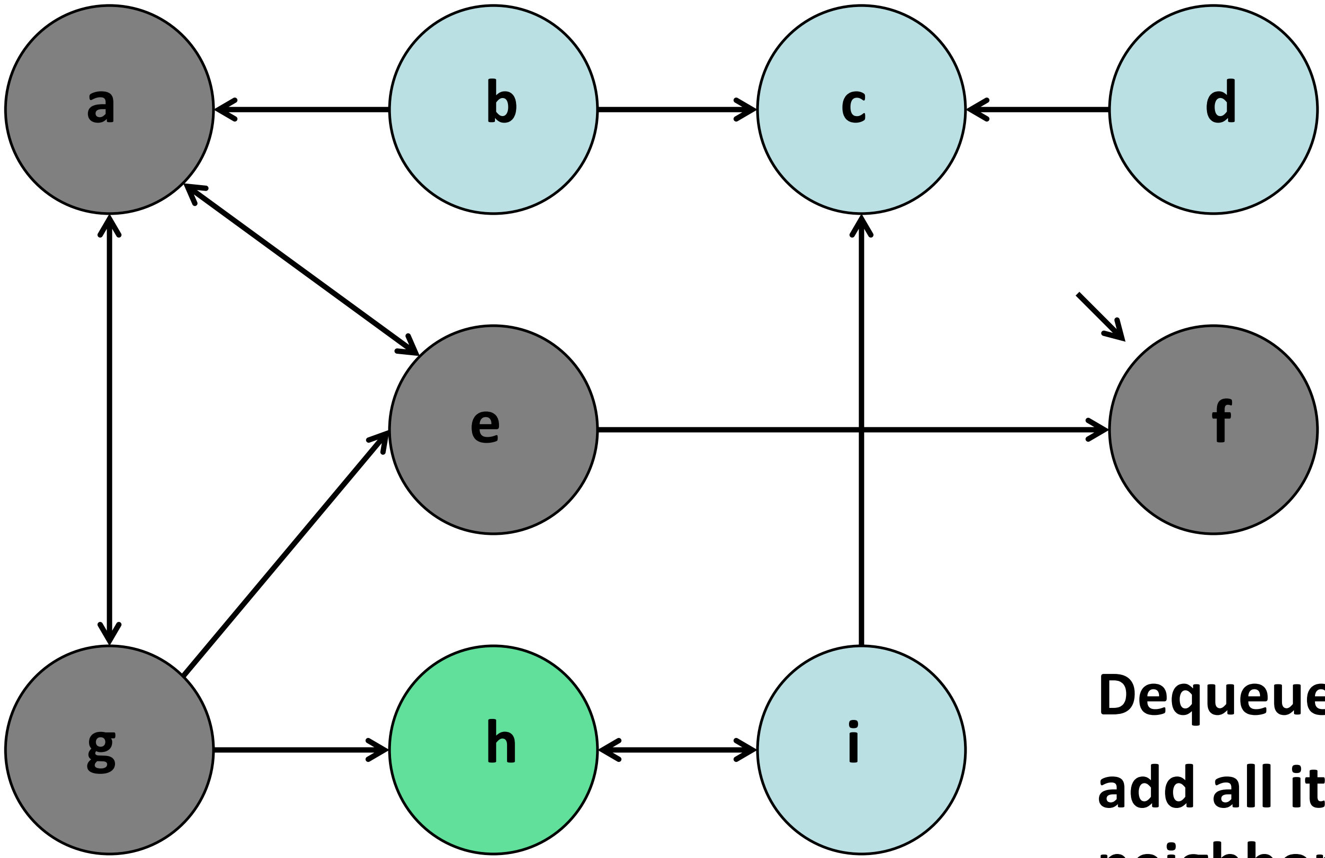


Dequeue a node

add all its unseen neighbors to the queue

queue: f, h

BFS

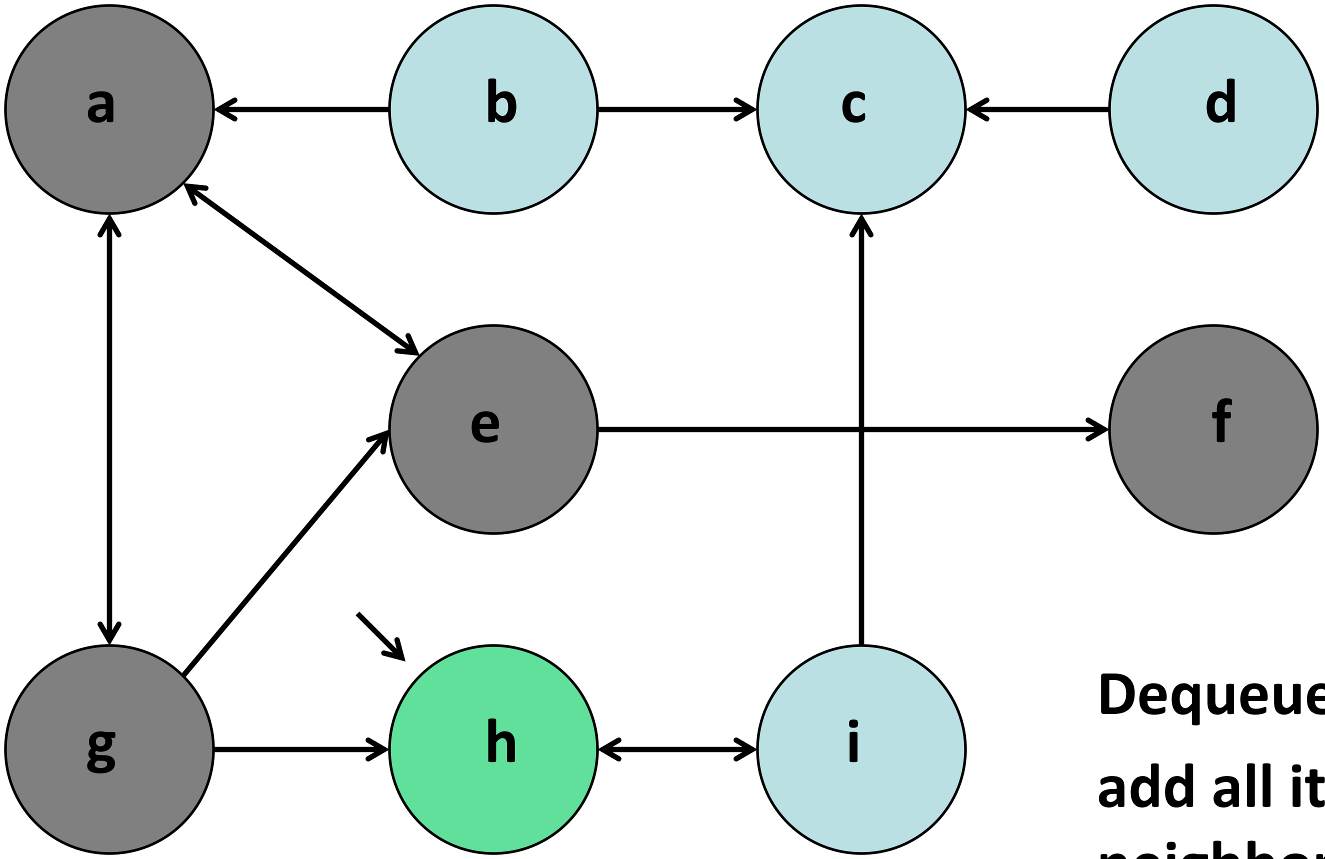


Dequeue a node

add all its unseen neighbors to the queue

queue: h

BFS

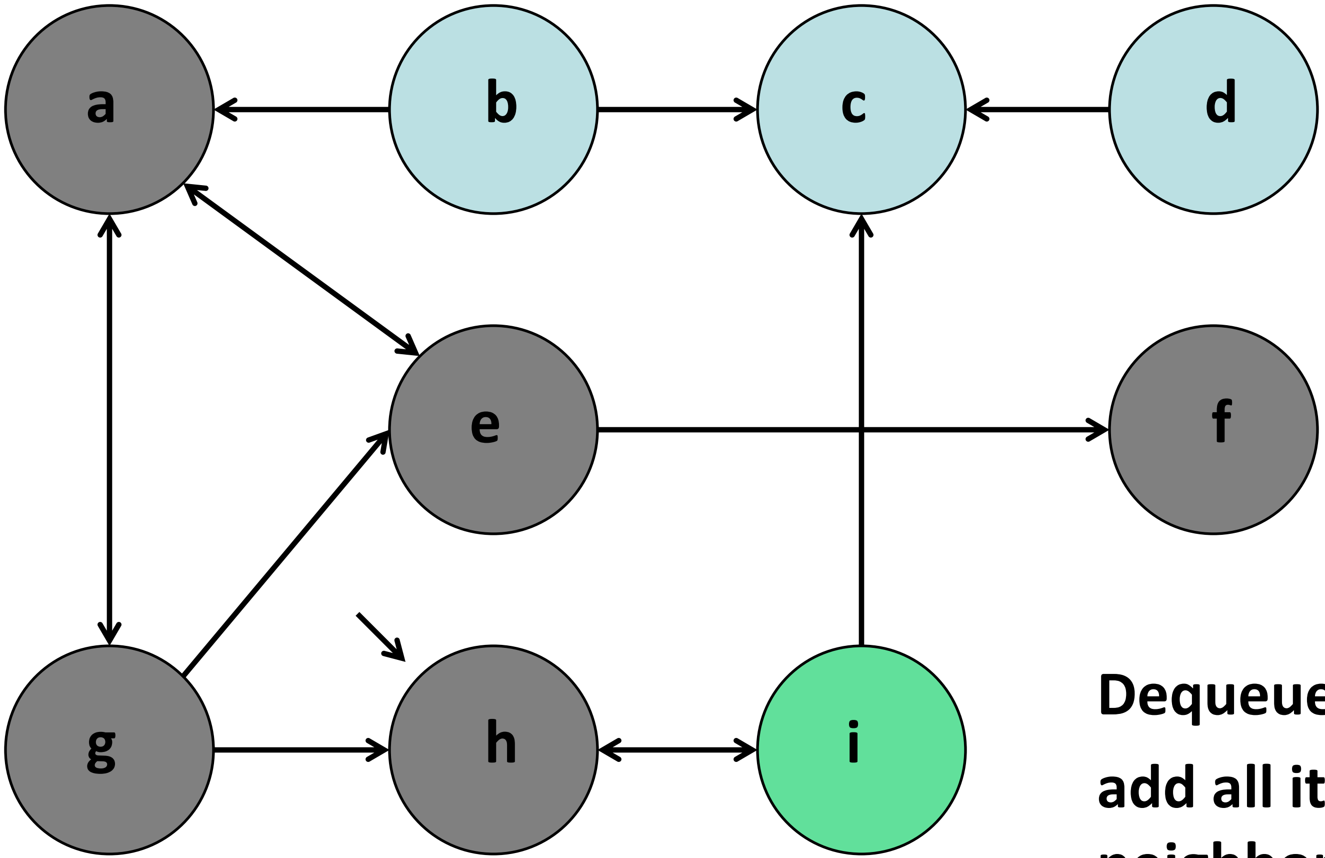


Dequeue a node

add all its unseen neighbors to the queue

queue: h

BFS

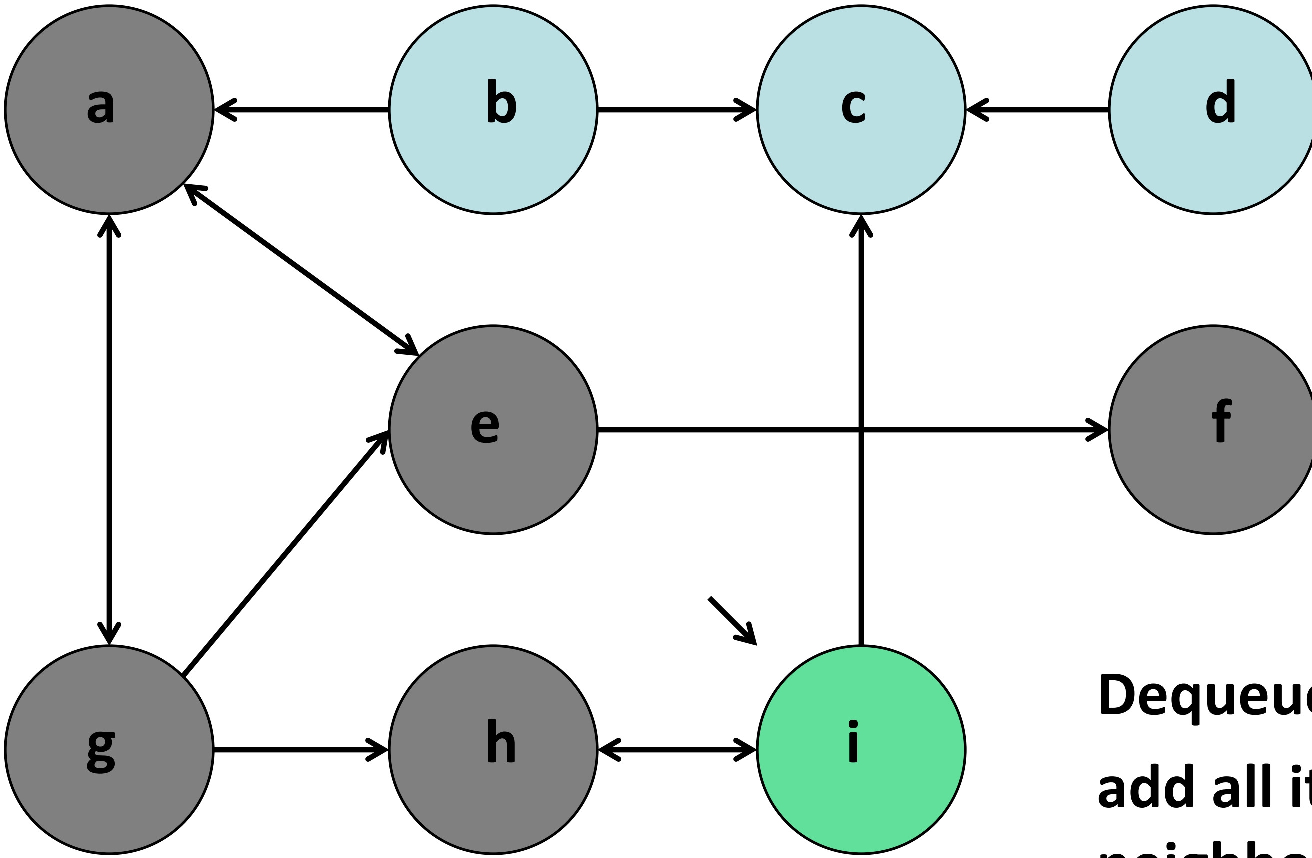


Dequeue a node

add all its unseen neighbors to the queue

queue: i

BFS

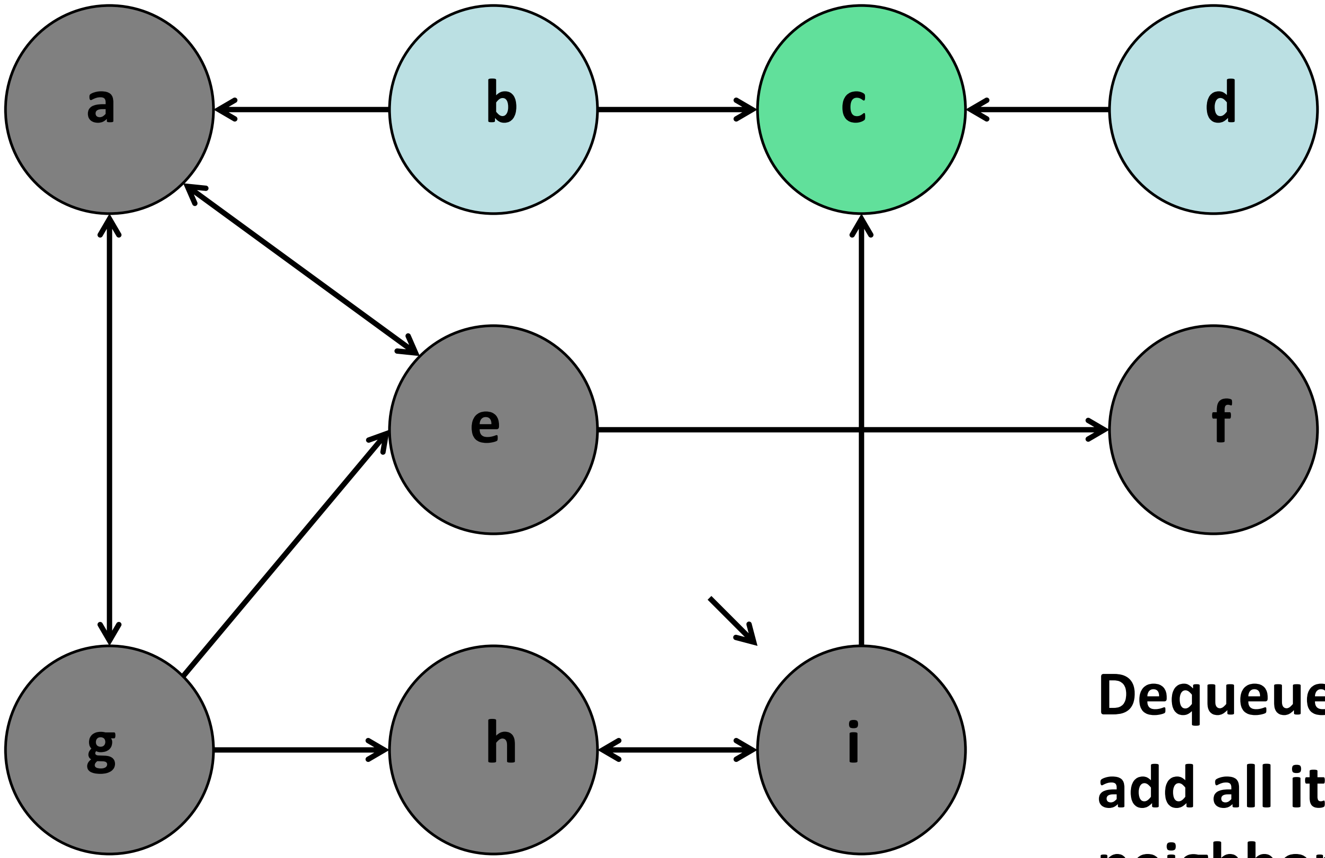


Dequeue a node

add all its unseen neighbors to the queue

queue: i

BFS

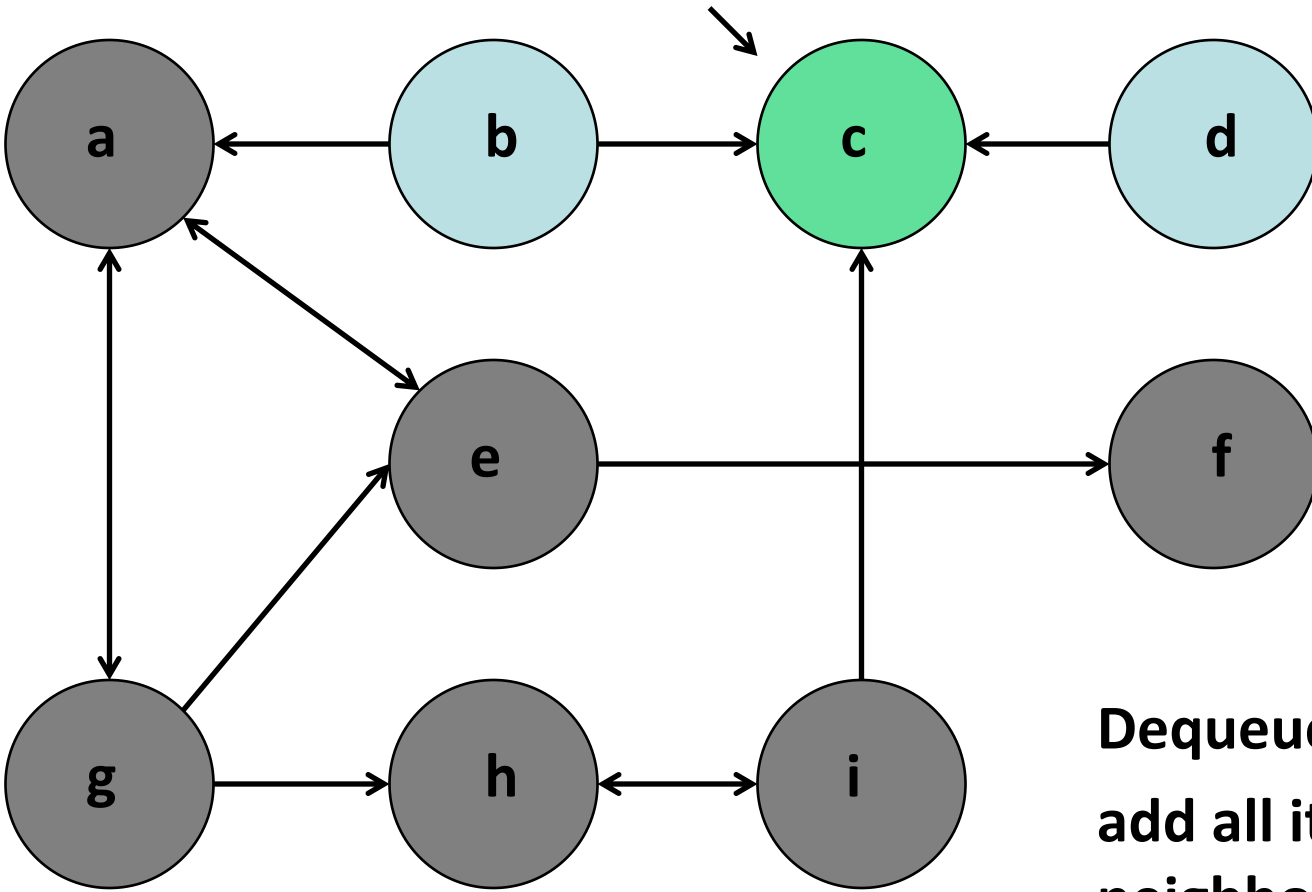


Dequeue a node

add all its unseen neighbors to the queue

queue: c

BFS

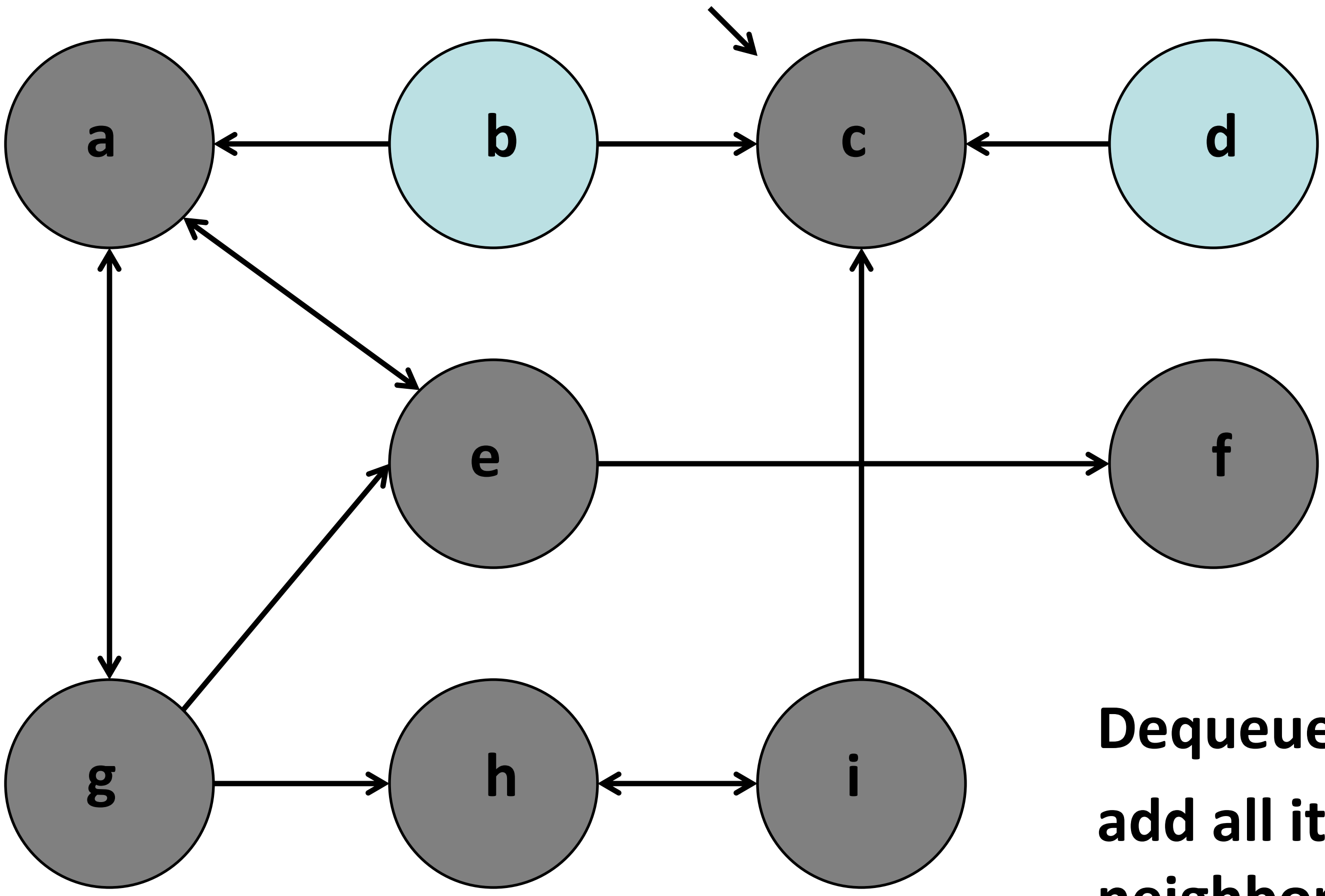


Dequeue a node

add all its unseen neighbors to the queue

queue: c

BFS



Dequeue a node

add all its unseen neighbors to the queue

queue: c

BFS Details

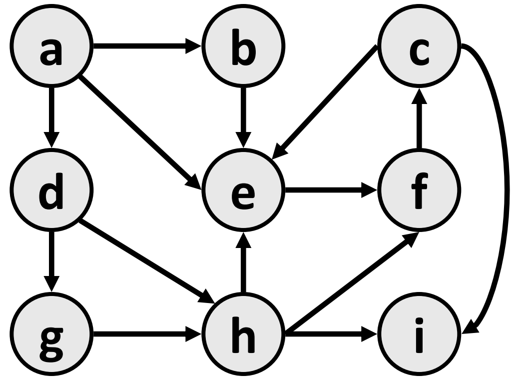
- In an n-node, m-edge graph, takes $O(m + n)$ time with an adjacency list
  - Visit each edge once, visit each node at most once

bfs from v_1 to v_2:
    create a *queue* of vertexes to visit,
        initially storing just v_1.
    mark v_1 as **visited**.

    while *queue* is not empty and v_2 is not seen:
        dequeue a vertex v from it,
        mark that vertex v as **visited**,
        and add each unvisited neighbor ***n*** of v to the *queue*.

- How could we modify the pseudocode to look for a specific path?

BFS observations



- *optimality*:
  - always finds the shortest path (fewest edges).
  - in unweighted graphs, finds optimal cost path.
  - In weighted graphs, *not* always optimal cost.
- *retrieval*: harder to reconstruct the actual sequence of vertices or edges in the path once you find it
  - conceptually, BFS is exploring many possible paths in parallel, so it's not easy to store a path array/list in progress
  - solution: We can keep track of the path by storing predecessors for each vertex (each vertex can store a reference to a *previous* vertex).
- DFS uses less memory than BFS, easier to reconstruct the path once found; but DFS does not always find shortest path. BFS does.

Recap

- **Recap**: Graphs
- **Practice**: Twitter Influence
- Depth-First Search (DFS)
- Announcements
- Breadth-First Search (BFS)

Next time: more graph searching algorithms

Overflow

BFS that finds path

bfs from v_1 to v_2:
create a *queue* of vertexes to visit,
initially storing just v_1.
mark v_1 as **visited**.

while *queue* is not empty and v_2 is not seen:
dequeue a vertex *v* from it,
mark that vertex *v* as **visited**,
and add each unvisited neighbor ***n*** of *v* to the *queue,*
while setting *n*'s previous to *v*.