CS 106X, Lecture 25
Topological Sort

reading:

Programming Abstractions in C++, Chapter 18

Plan For This Week

- Graphs: Topological Sort (HW8)
- Classes: Inheritance and Polymorphism (HW8)
- Sorting Algorithms

Plan For Today

- Topological Sort
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- Announcements

- **Learning Goal:** understand how to implement topological sort, and why it and dependency graphs are useful.

Plan for Today

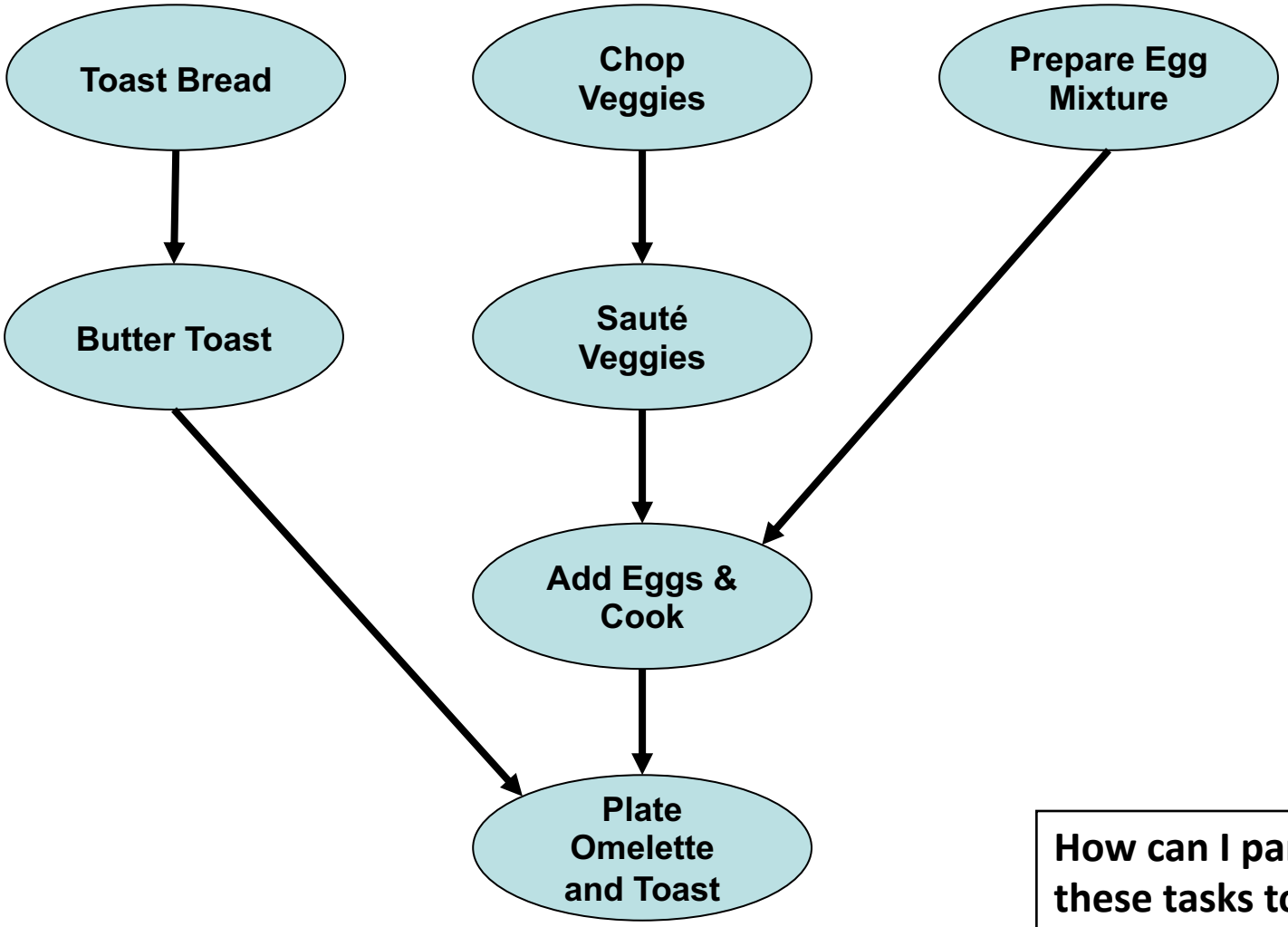
- **Topological Sort**
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- Announcements

Graphs So Far

- Graphs as Data Structures
  - Adjacency List
  - Adjacency Matrix
  - Edge List
- Searching
  - DFS
  - BFS
  - Dijkstra's Algorithm
  - A* Search

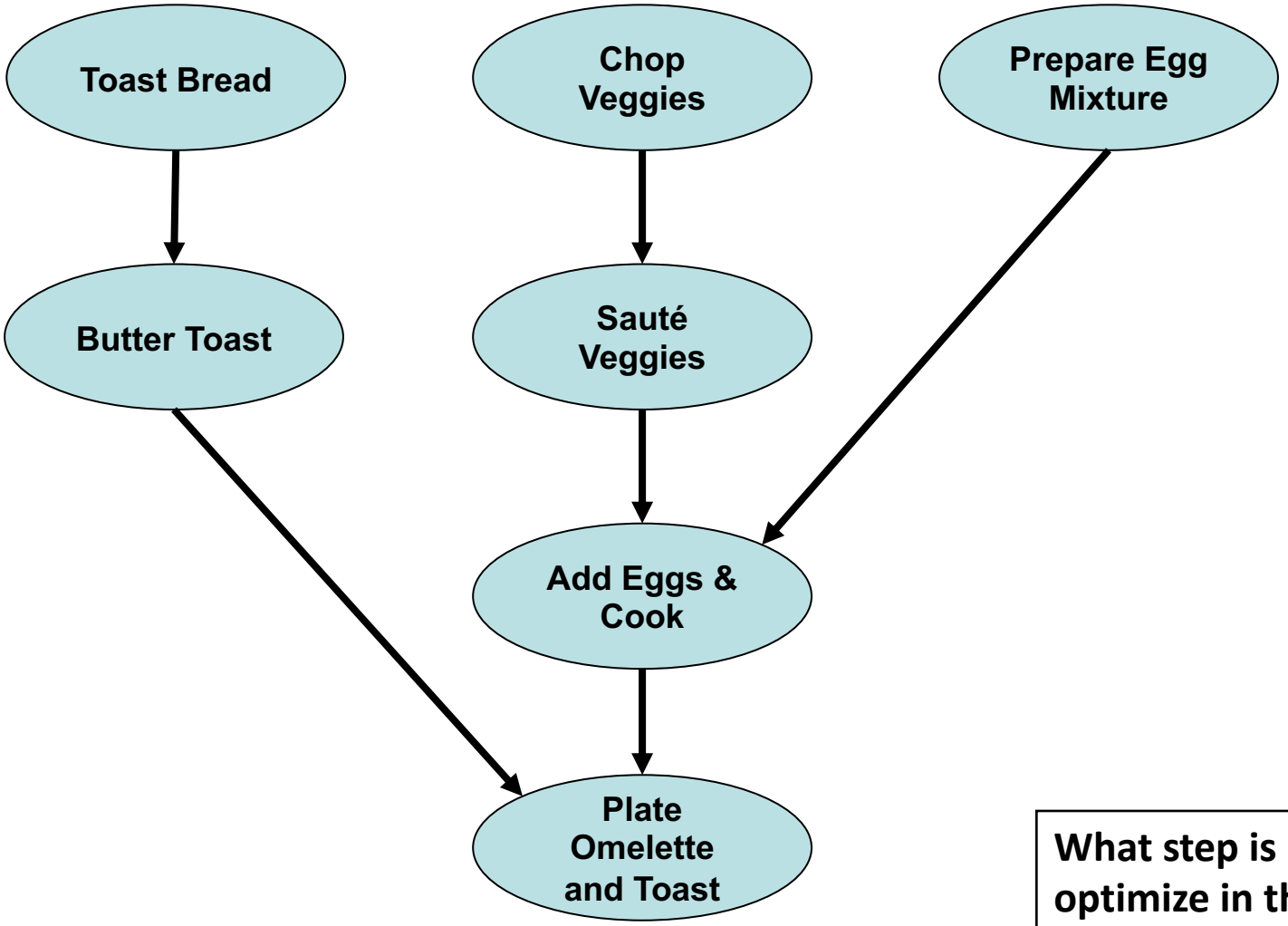
Dependency Graphs

Dependency Graphs: Tasks



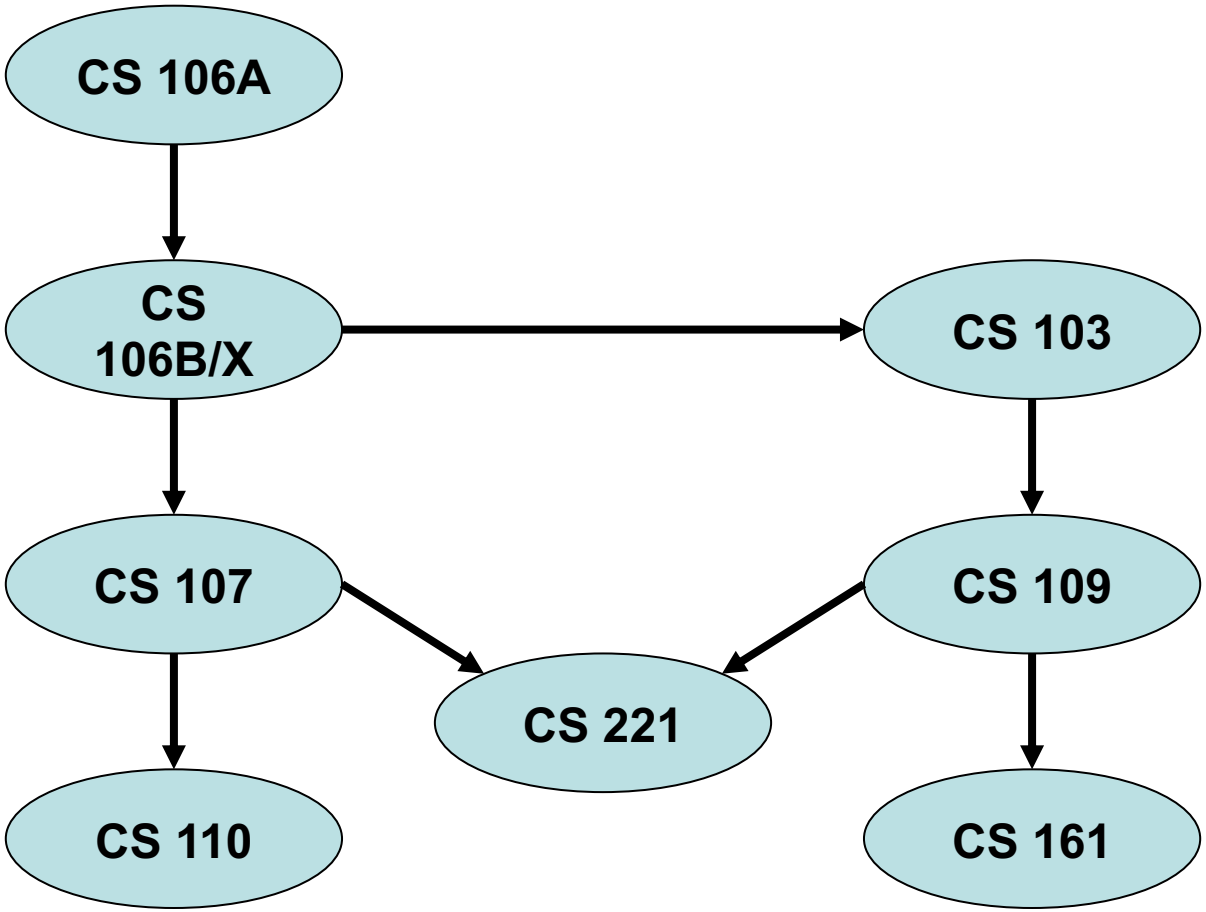
How can I parallelize these tasks to finish more quickly?

Dependency Graphs: Tasks



What step is best to optimize in this production process?

Dependency Graphs: Orderings

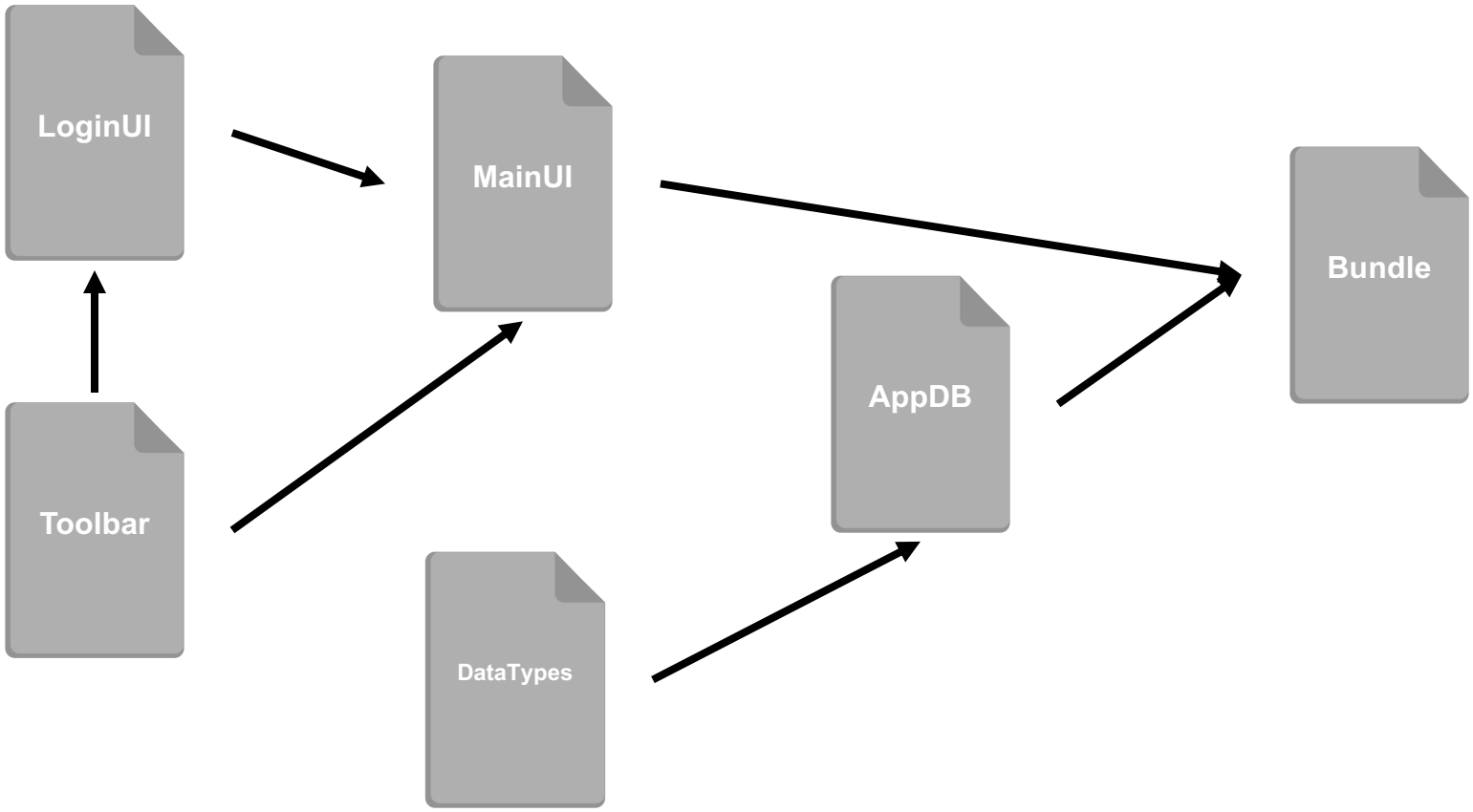


What are valid orderings of these tasks?

Dependency Graphs: Games

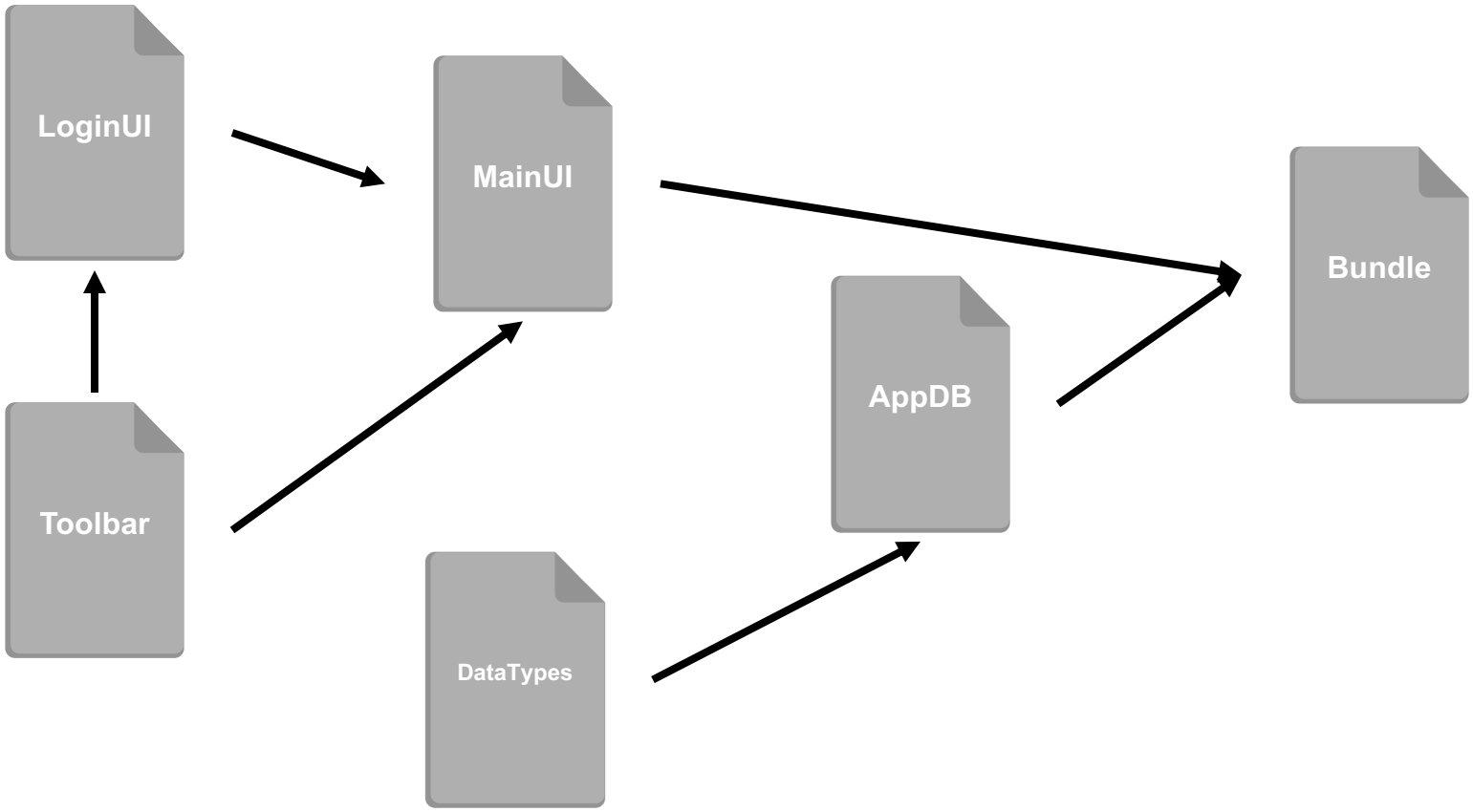
What is an effective AI strategy for resource management games (like Civilization)?

Dependency Graphs: Compilers



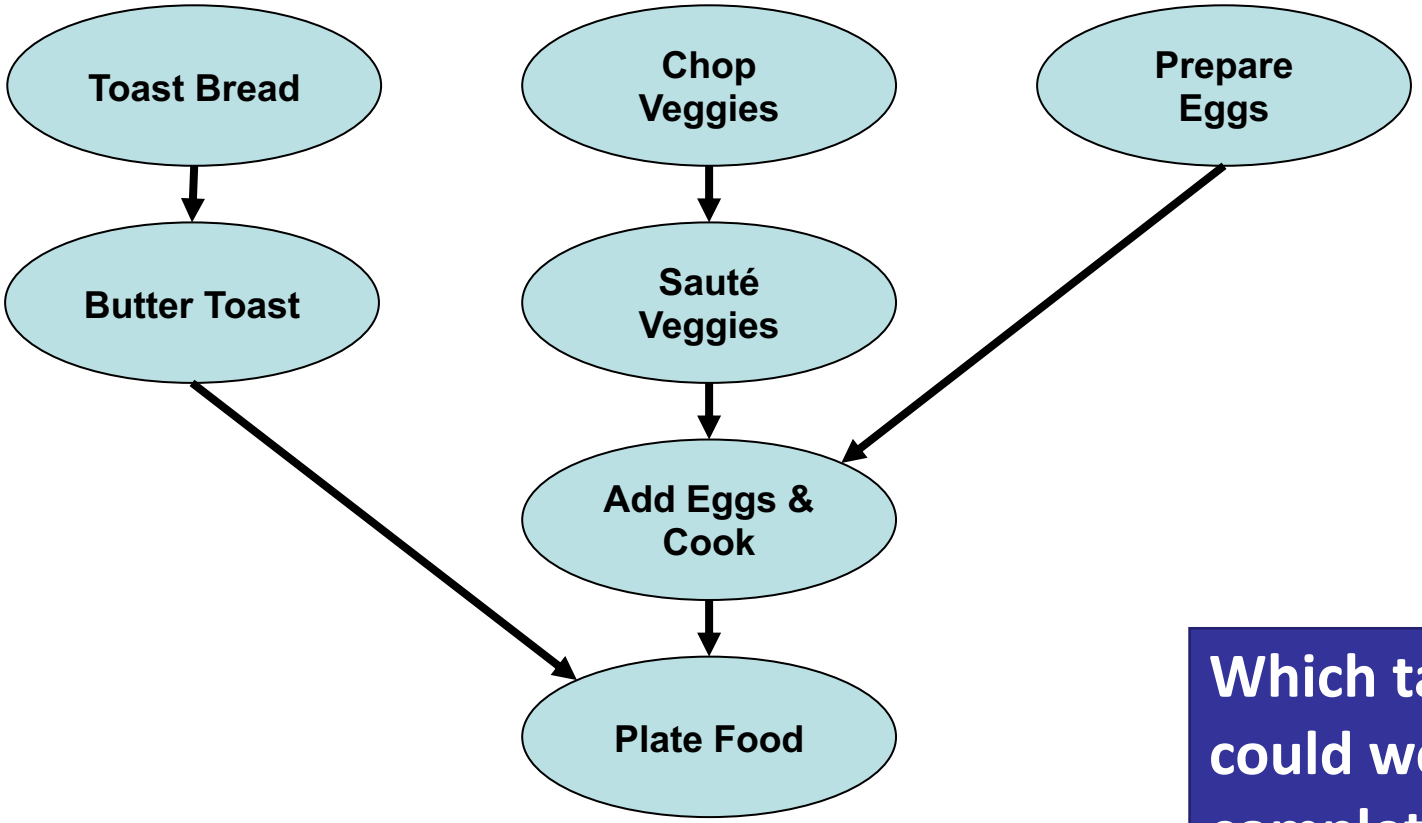
In what order should we recompile source files with dependencies?

Dependency Graphs: Web



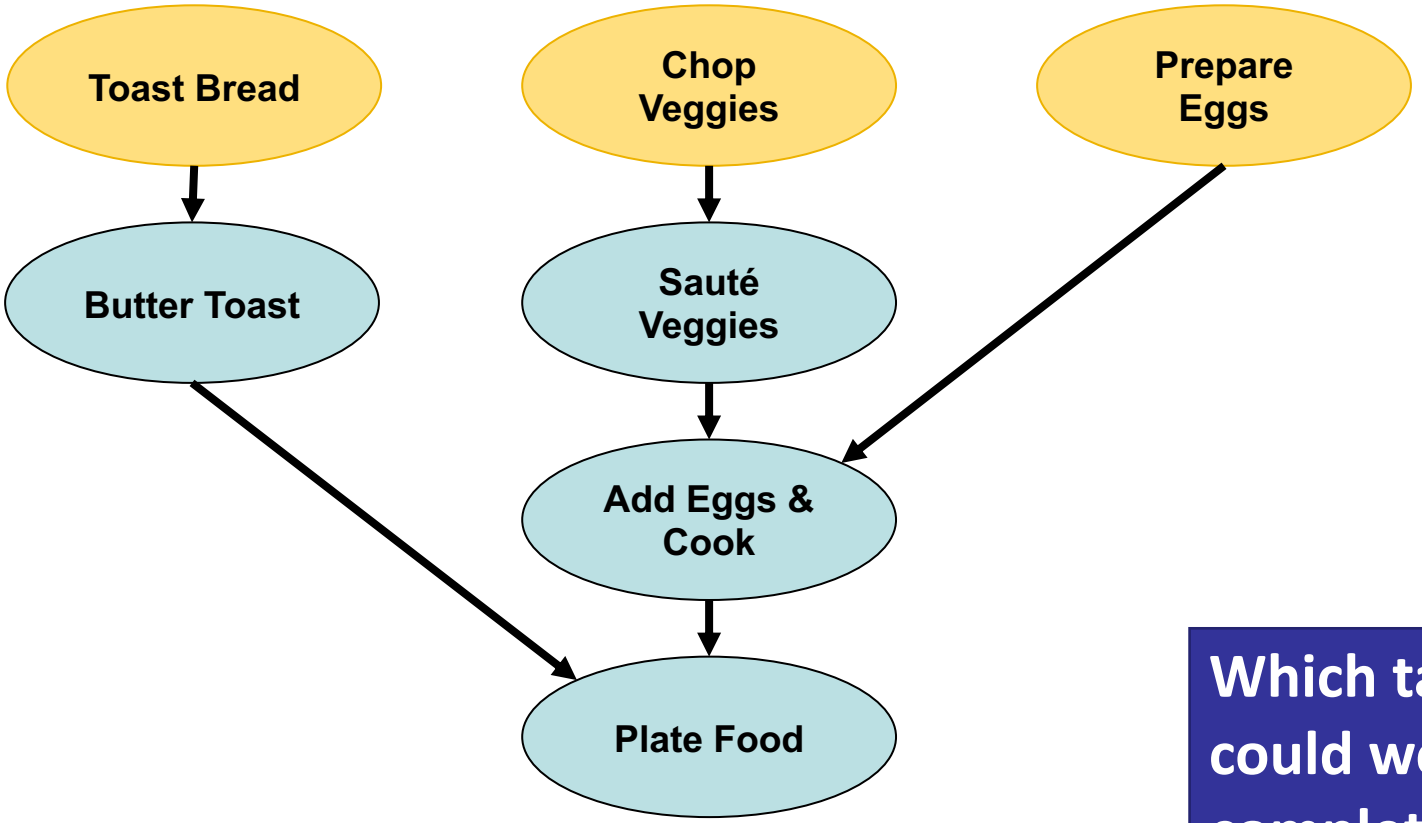
How can I efficiently package up and transmit web dependencies?

Topological Sort



Which tasks could we complete first?

Topological Sort



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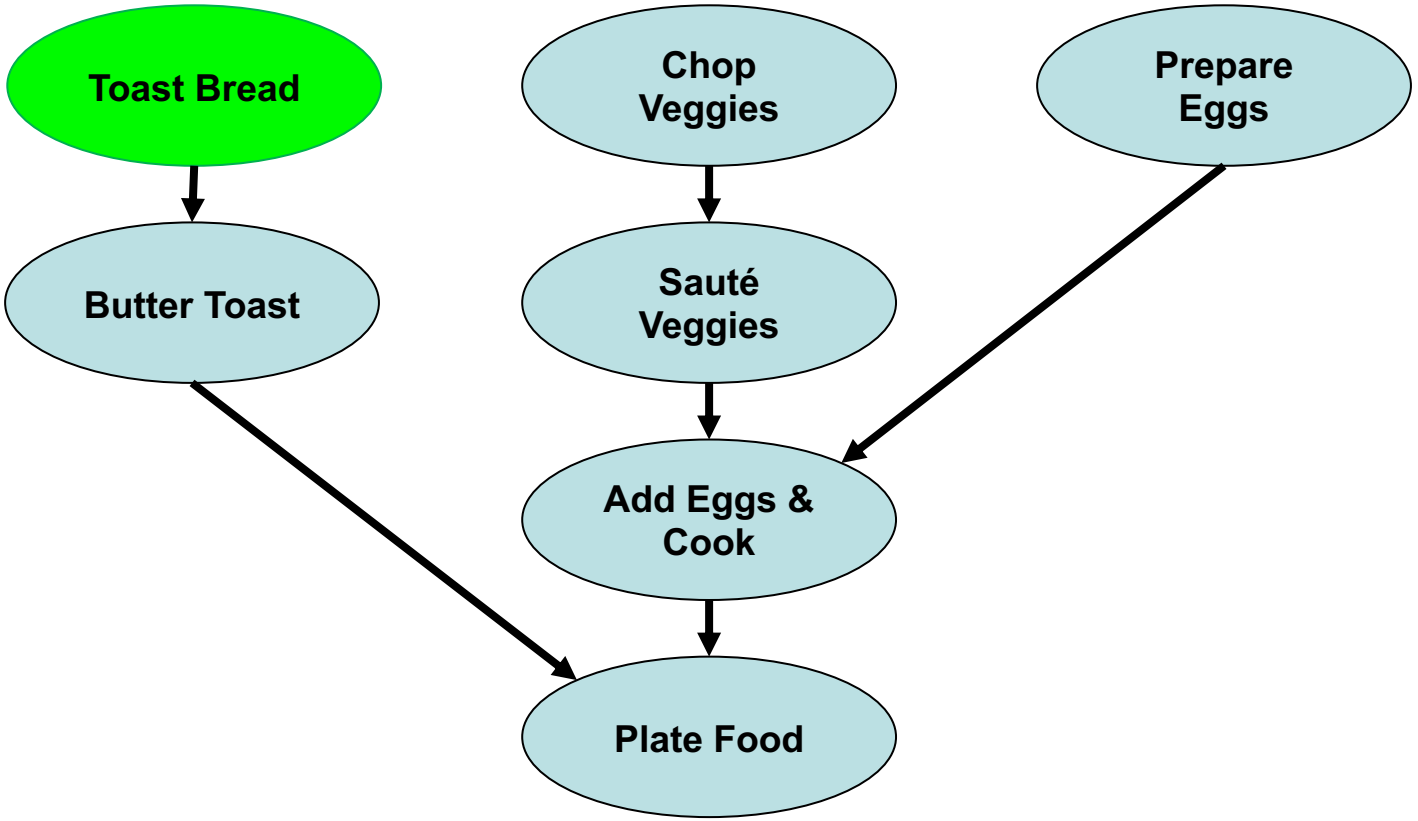

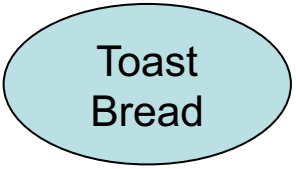

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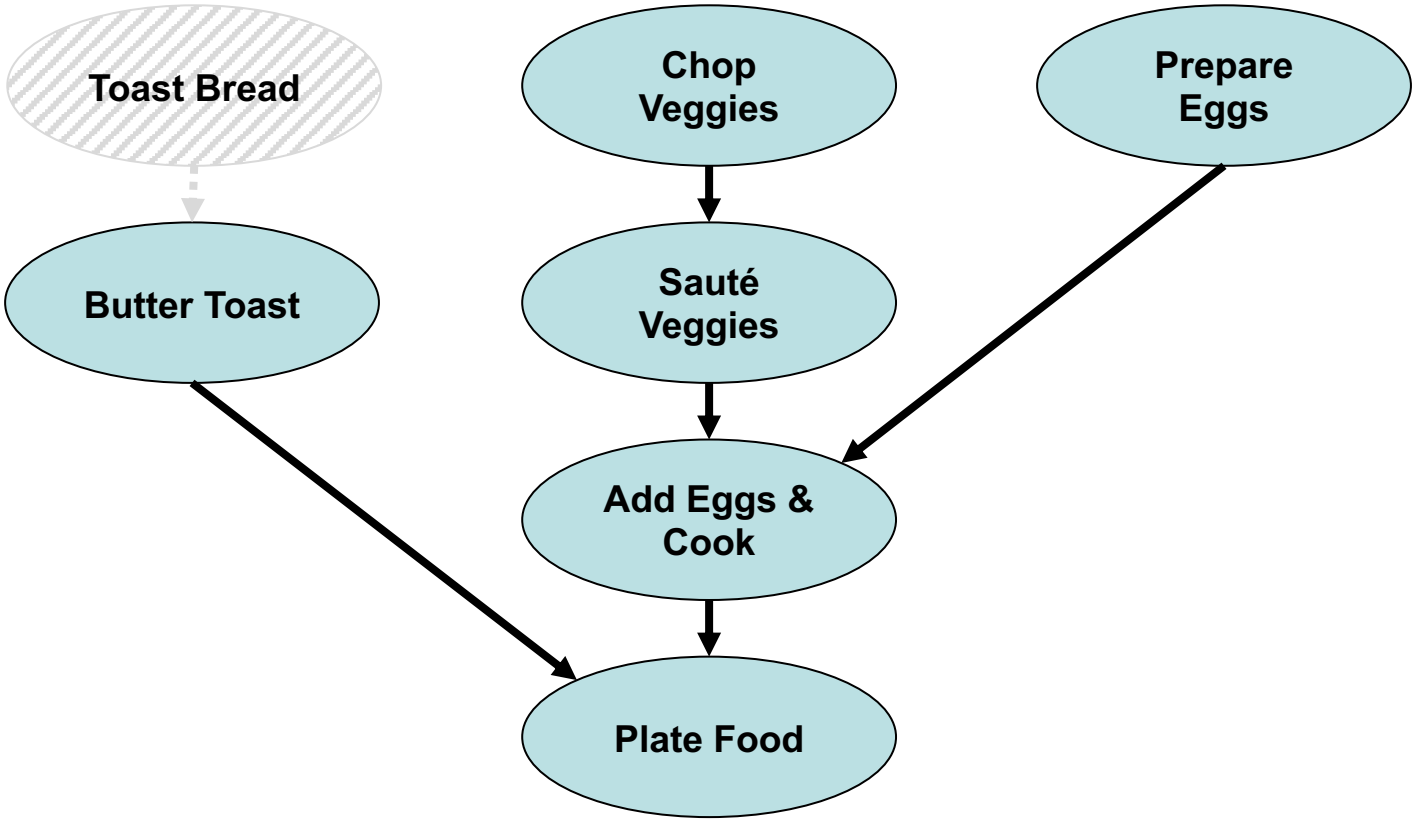

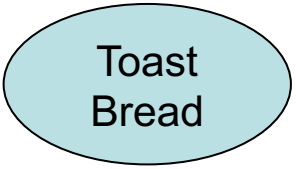

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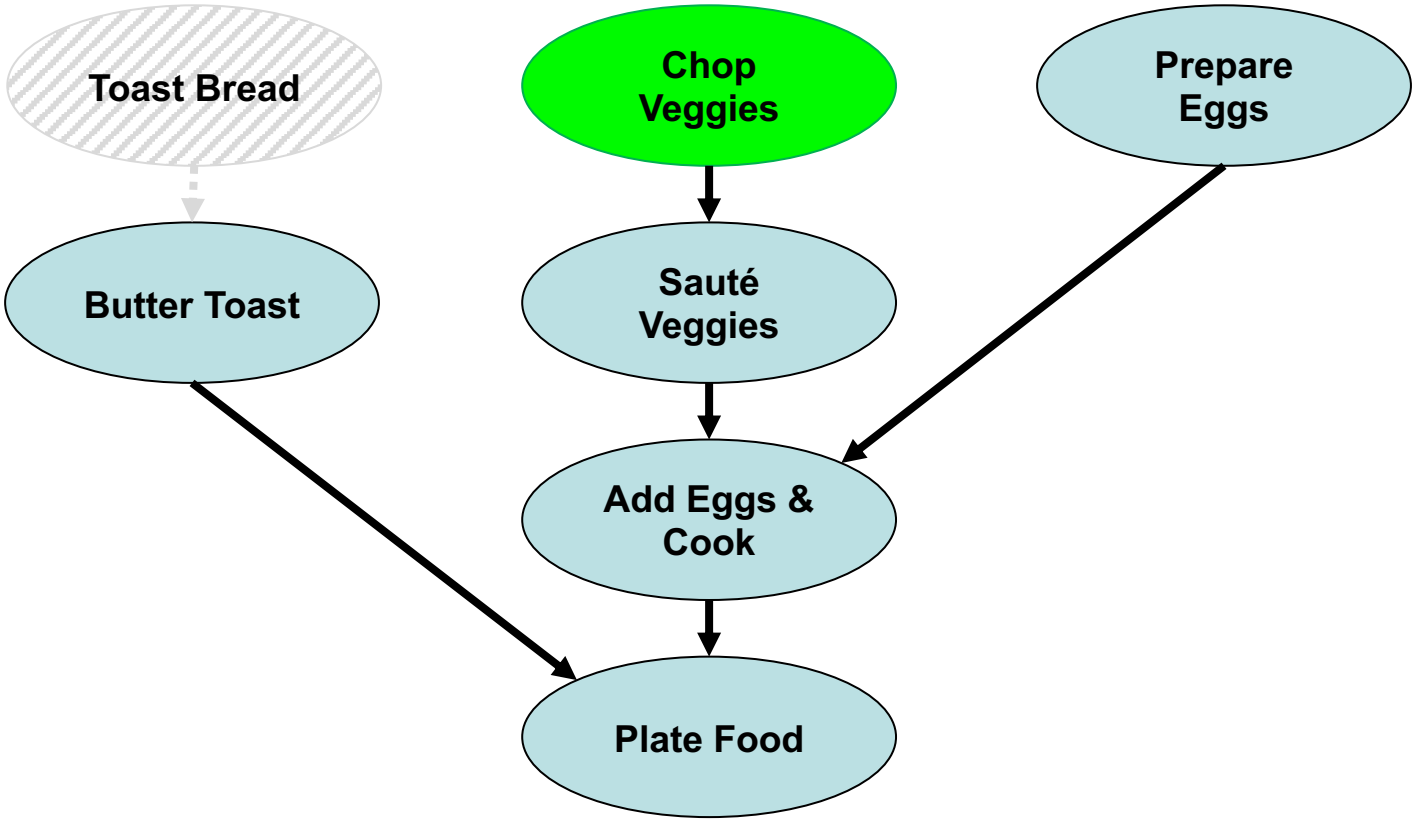

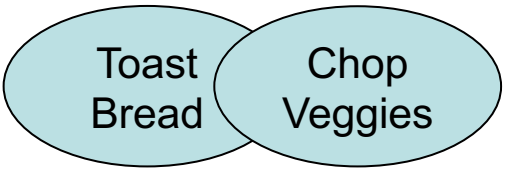

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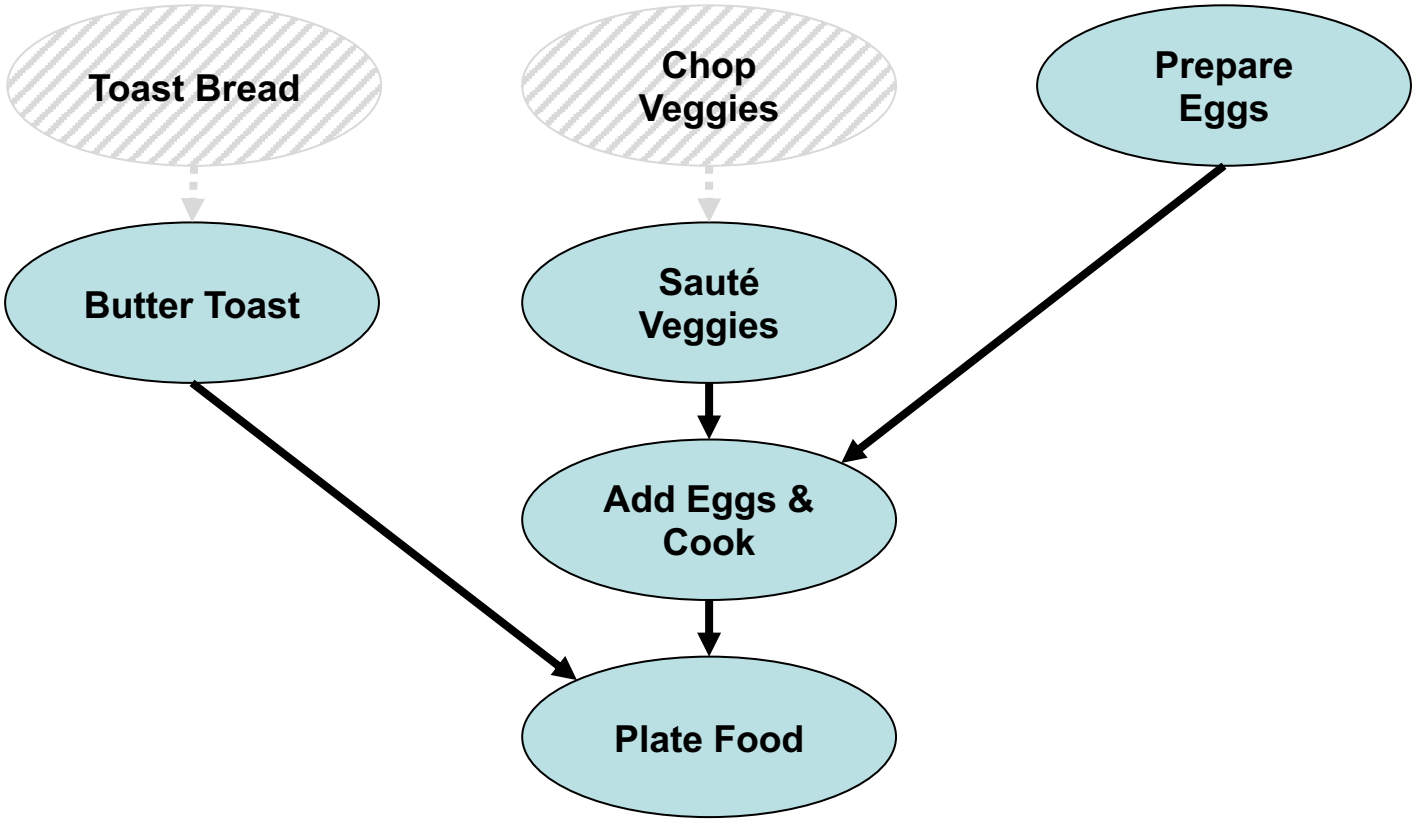

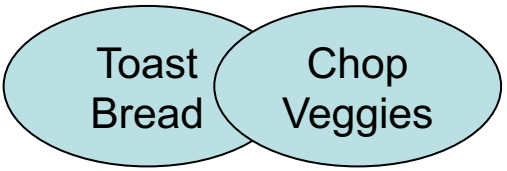

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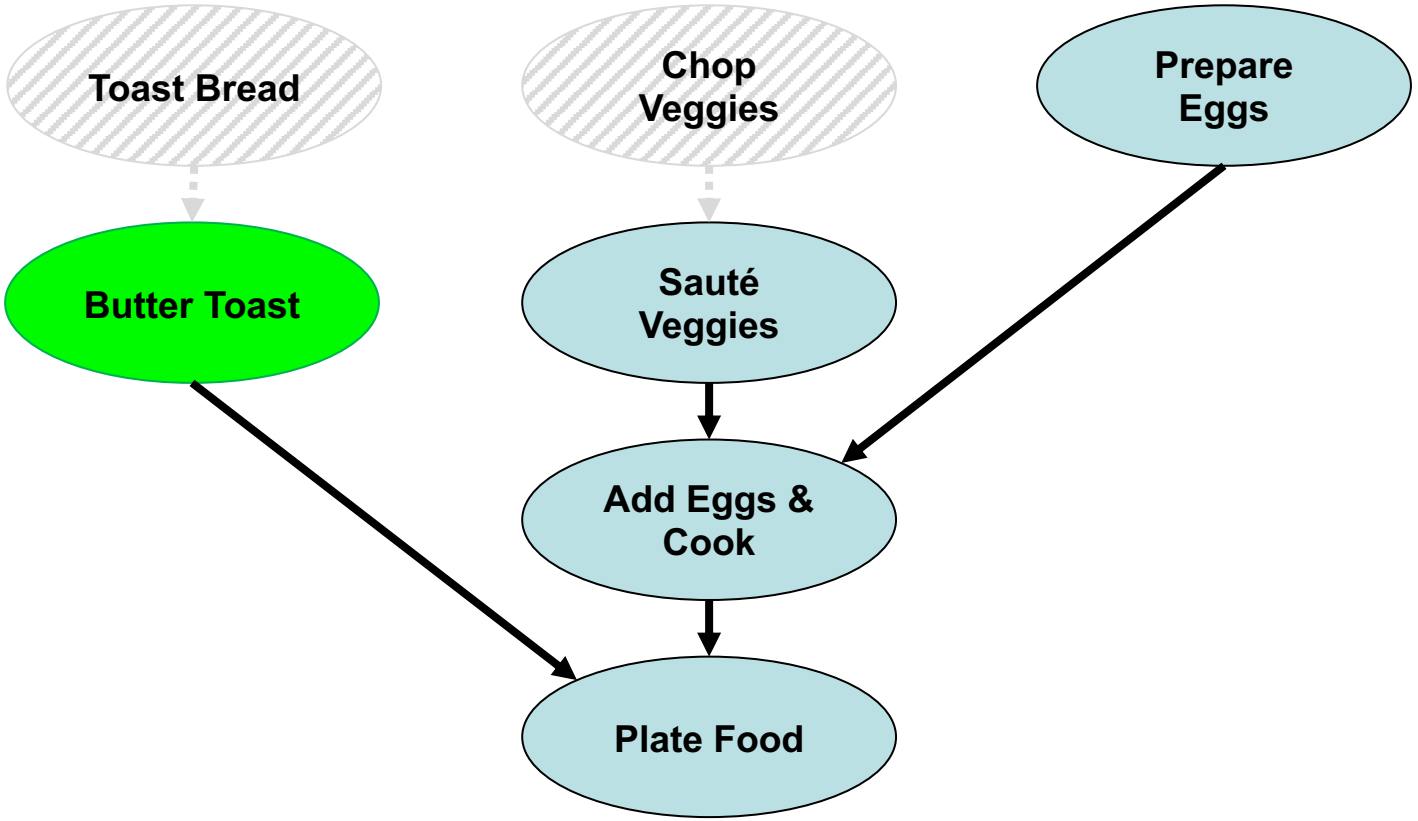

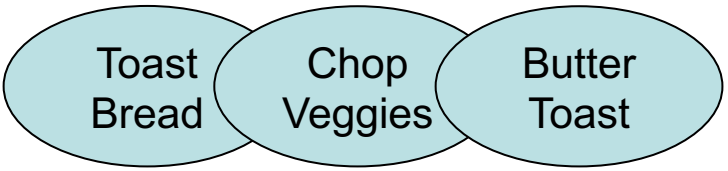

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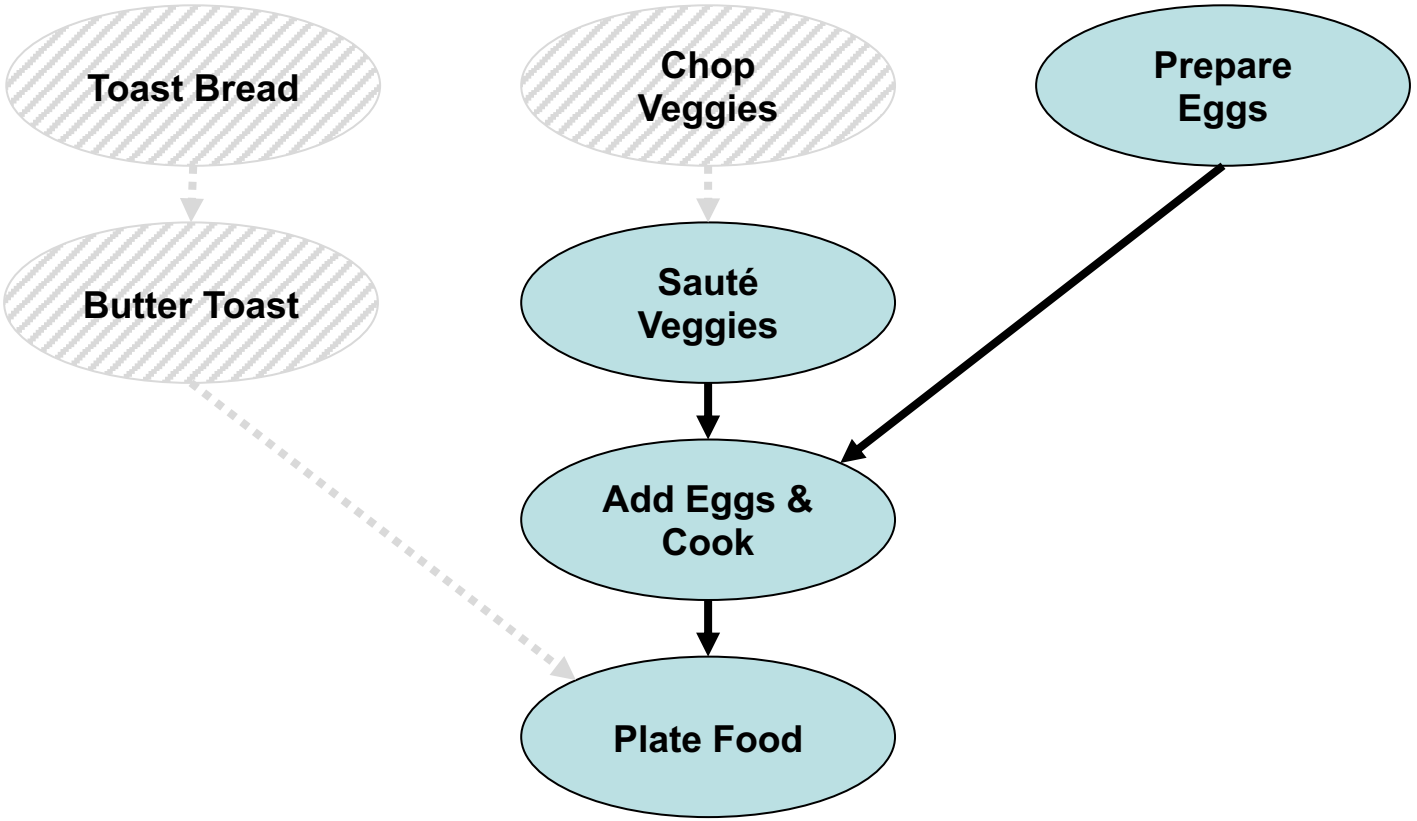

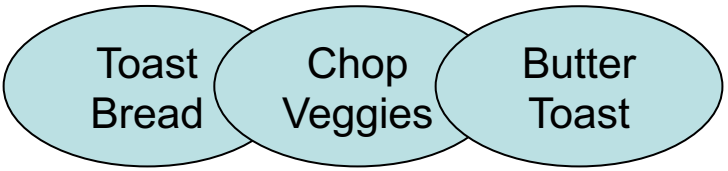

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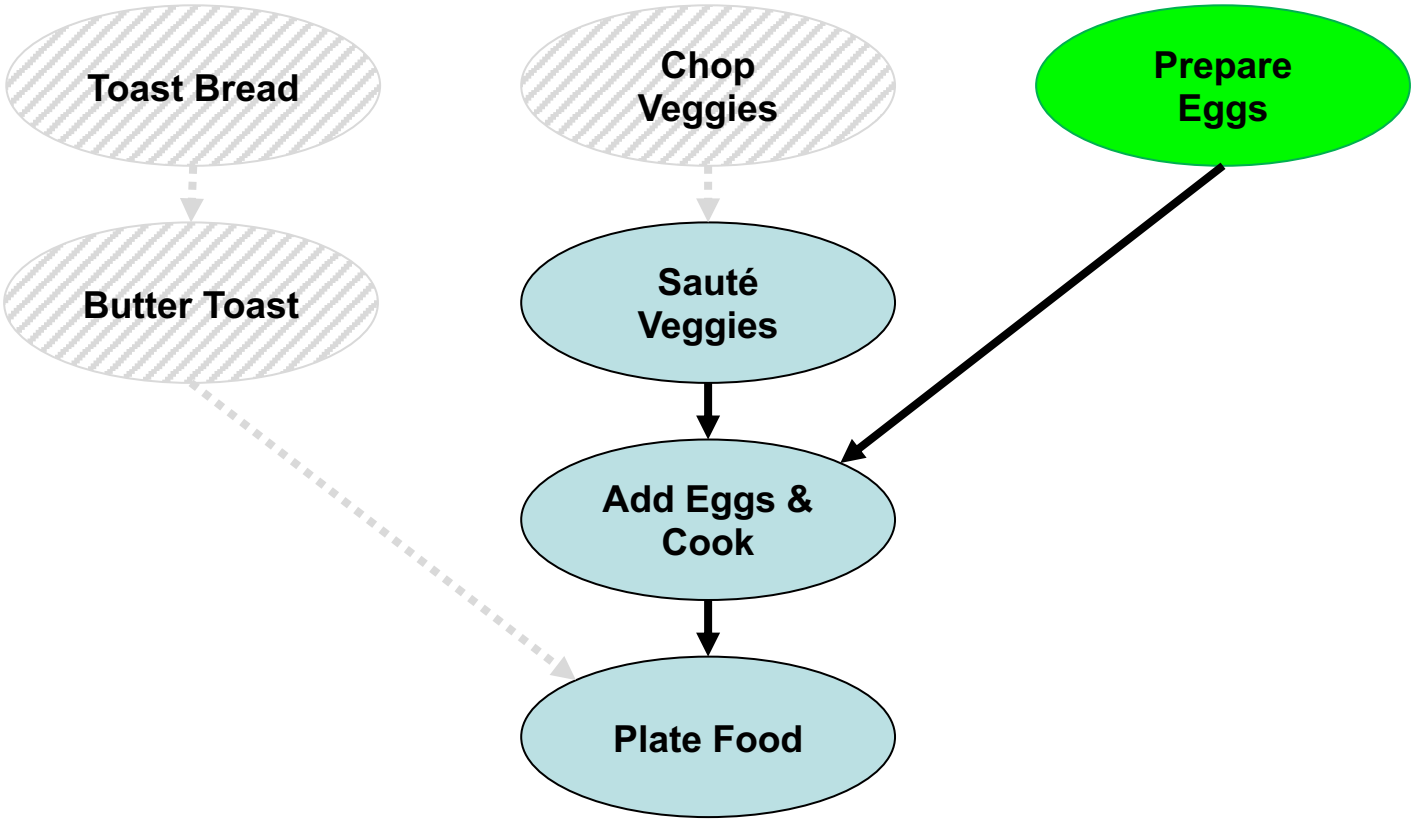

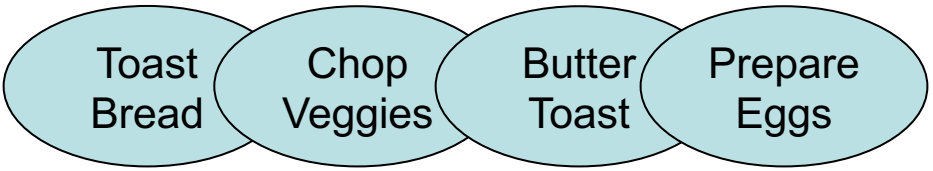

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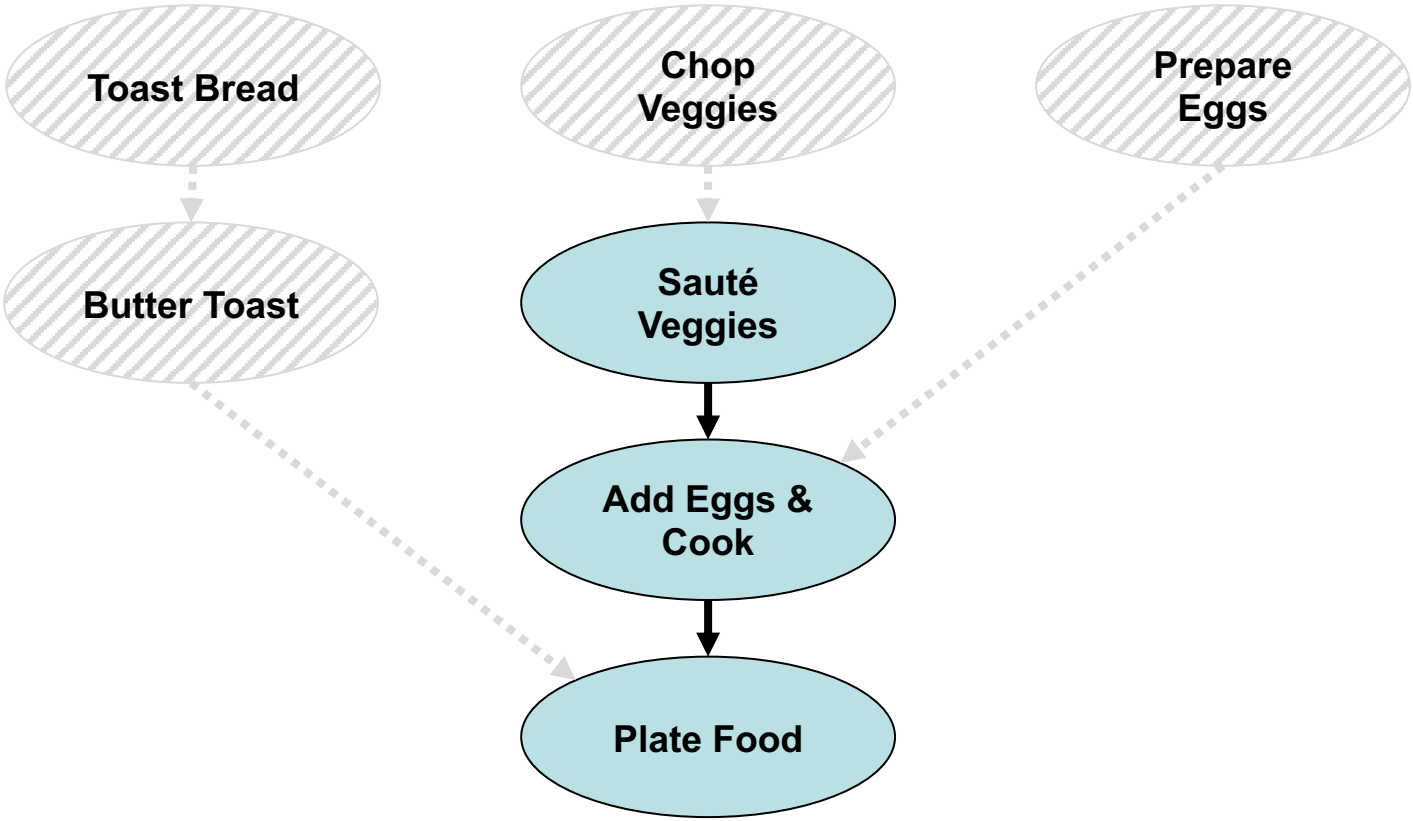

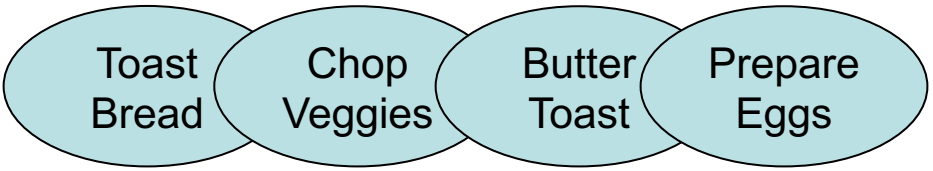

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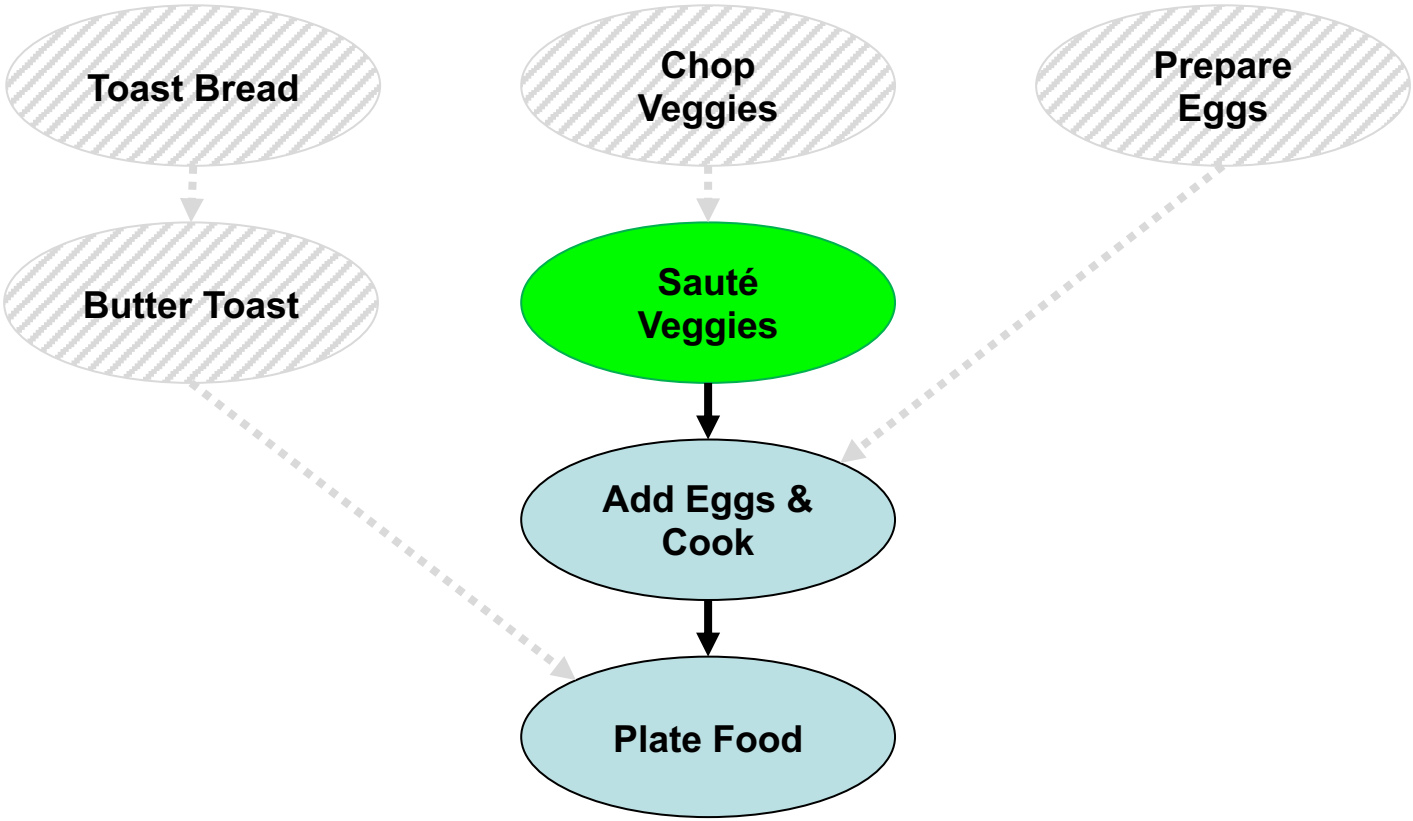

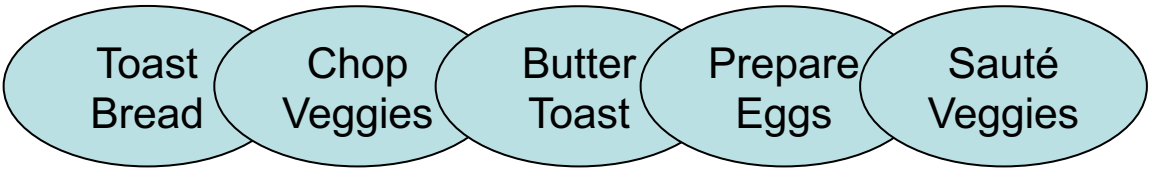

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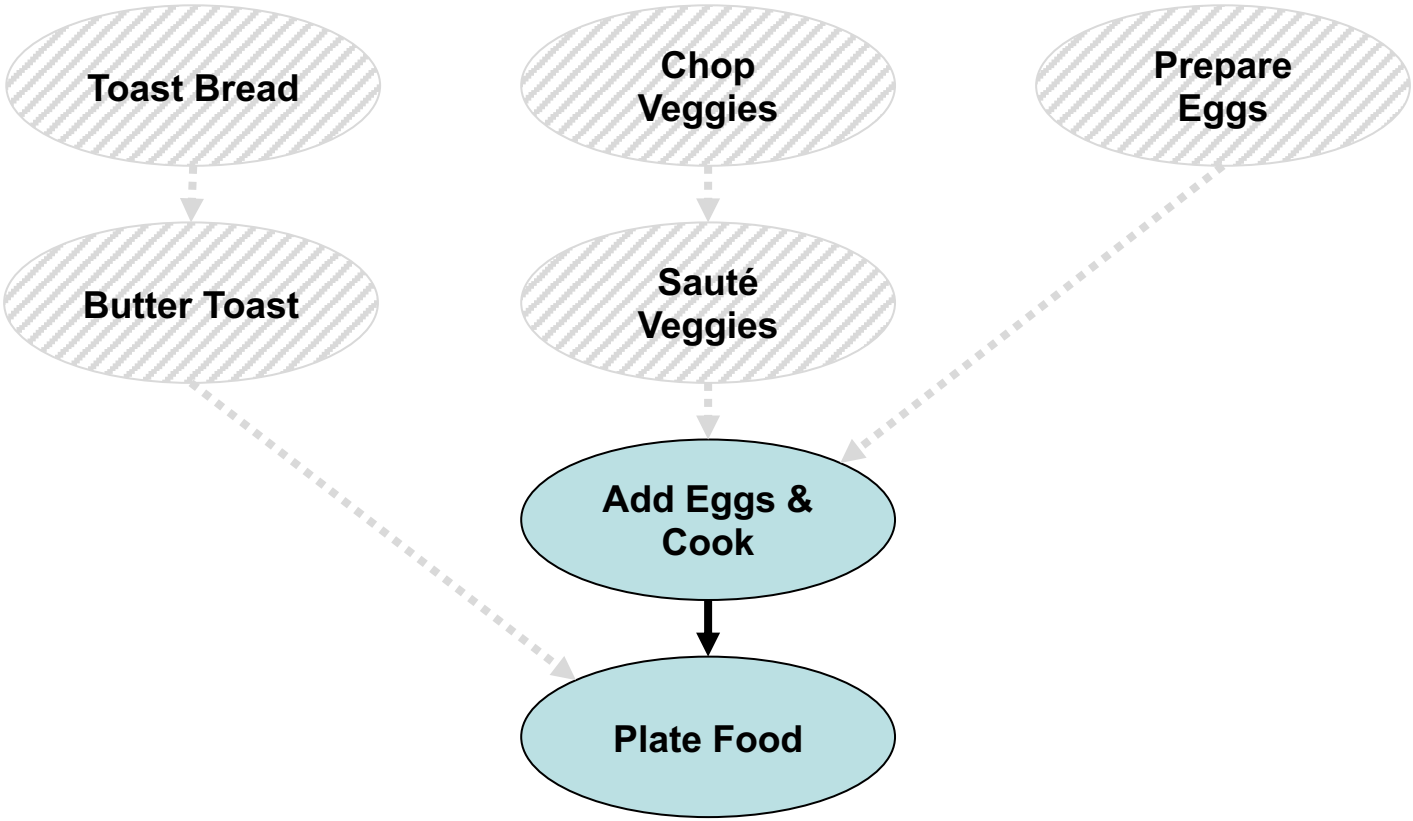

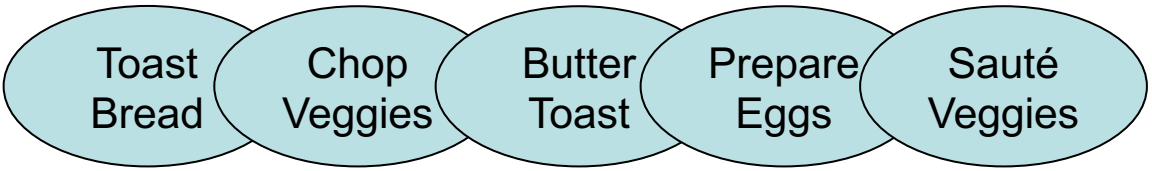

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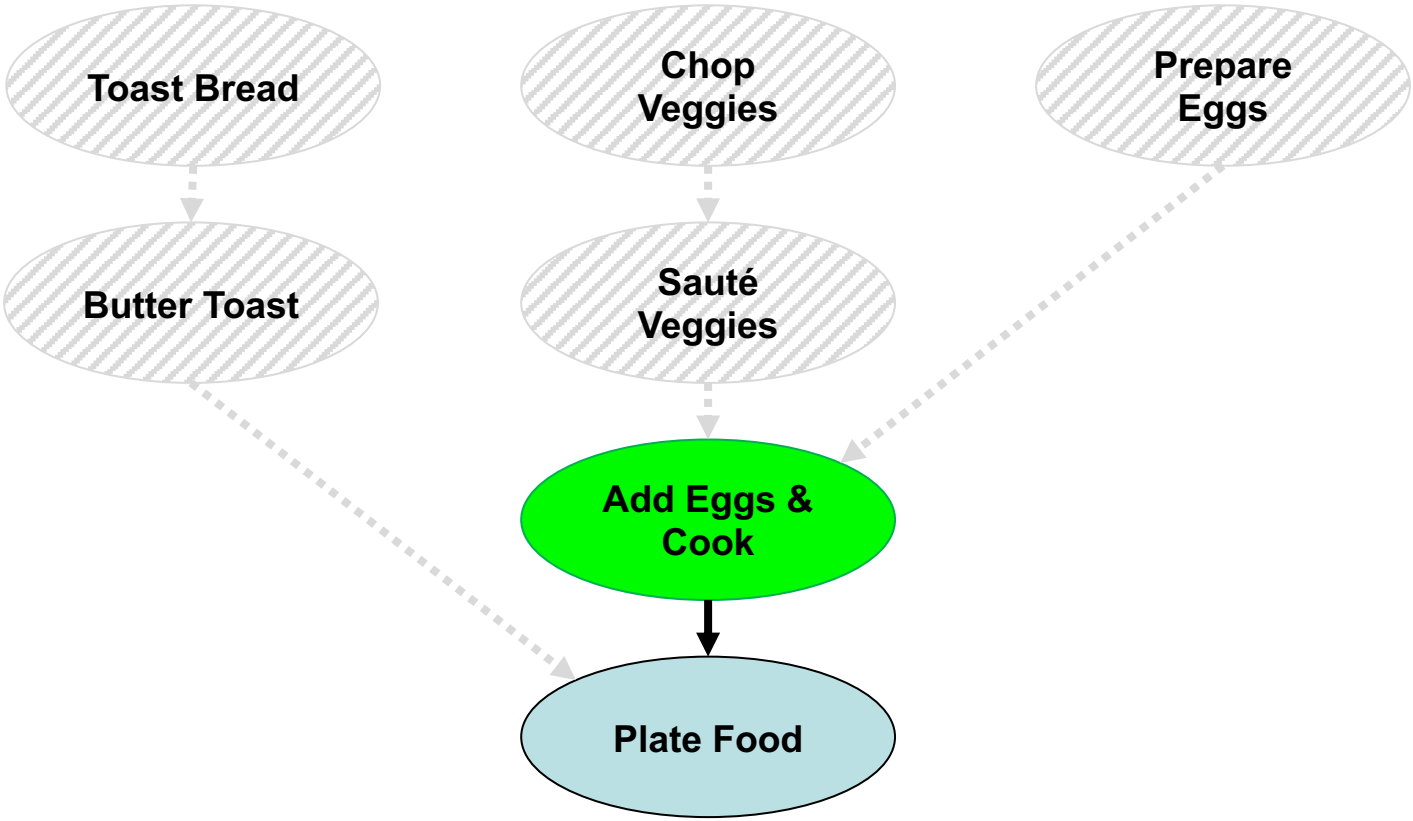

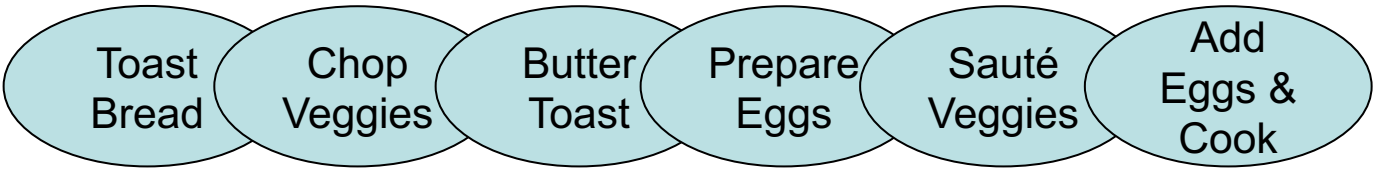

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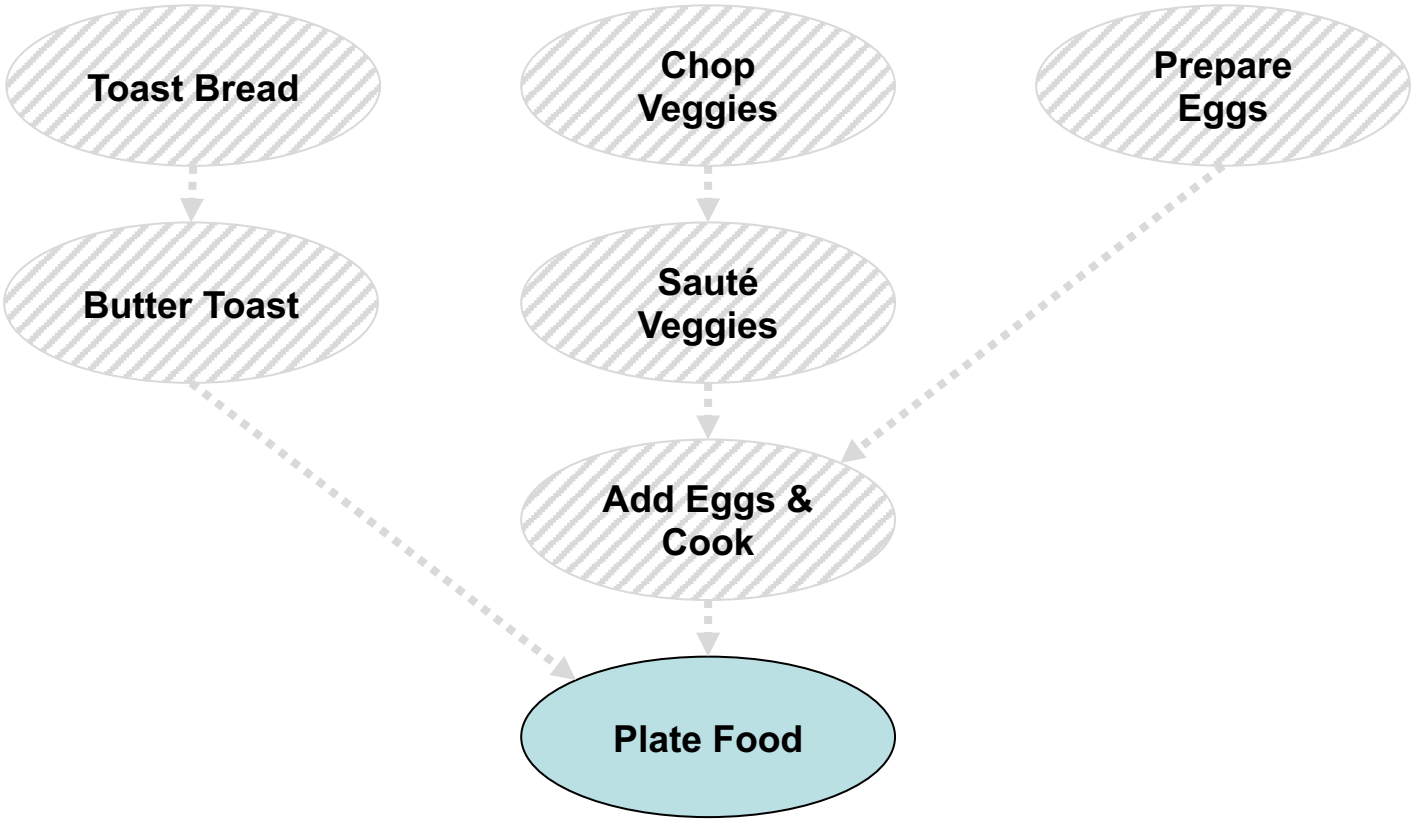

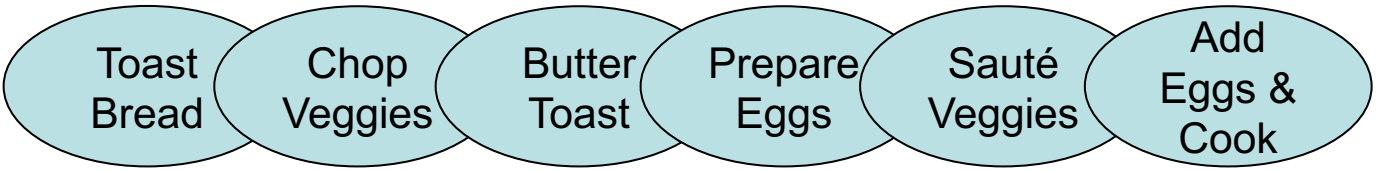

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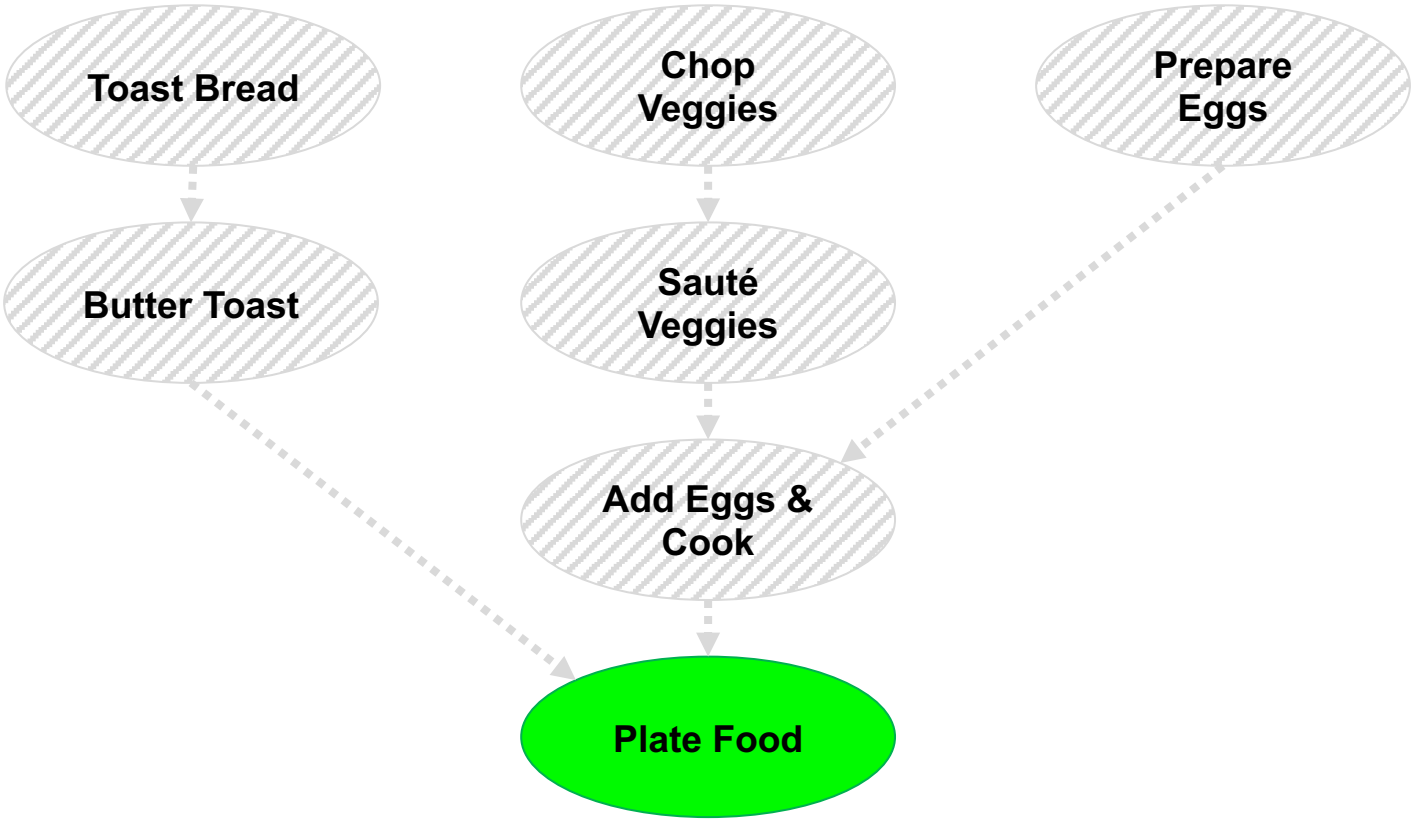
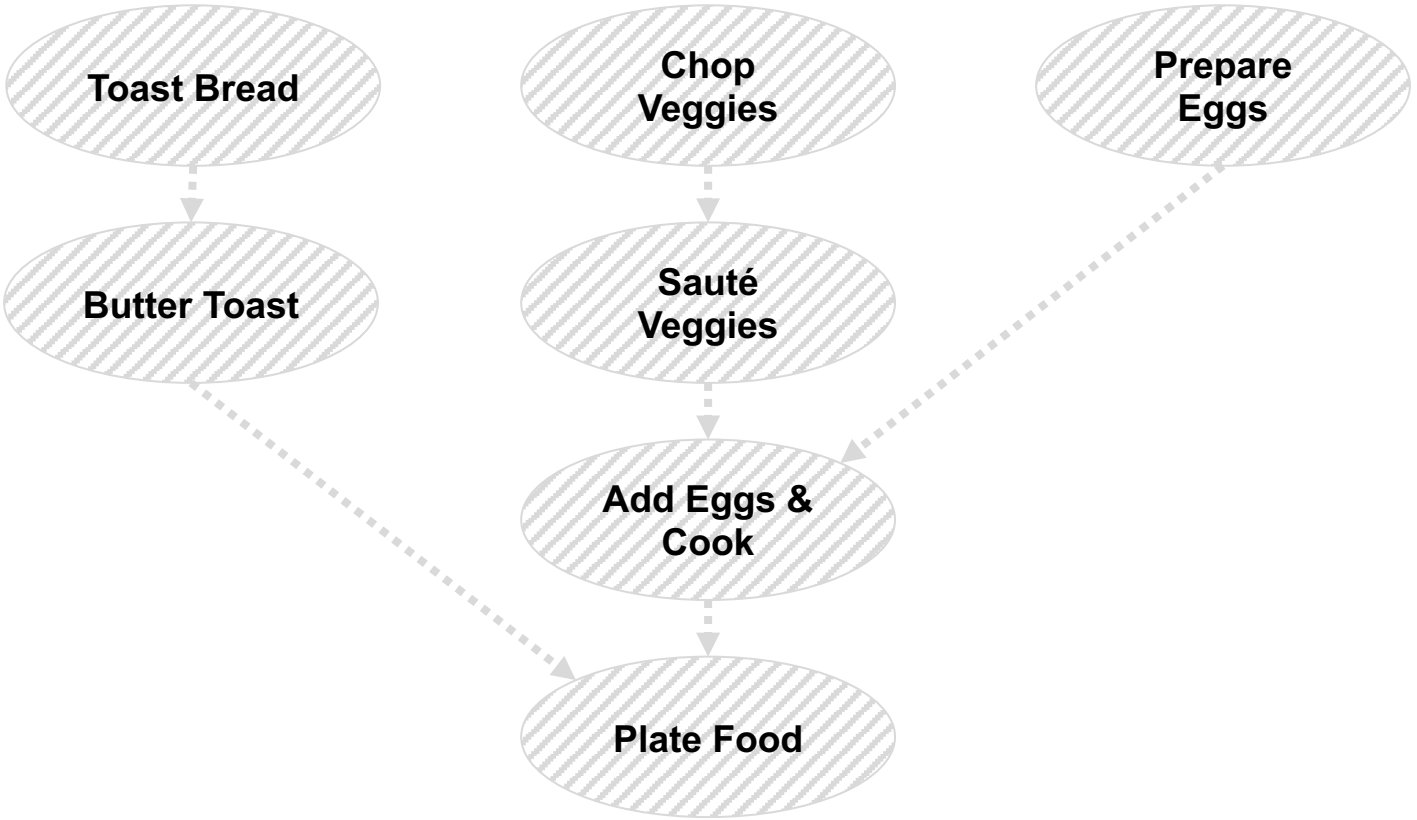
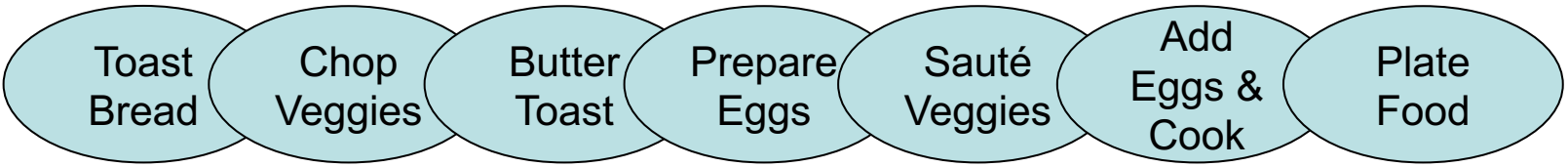


Toast Bread, Chop Veggies, Butter Toast, Prepare Eggs, Sauté Veggies, Add Eggs & Cook, Plate Food

Topological Sort





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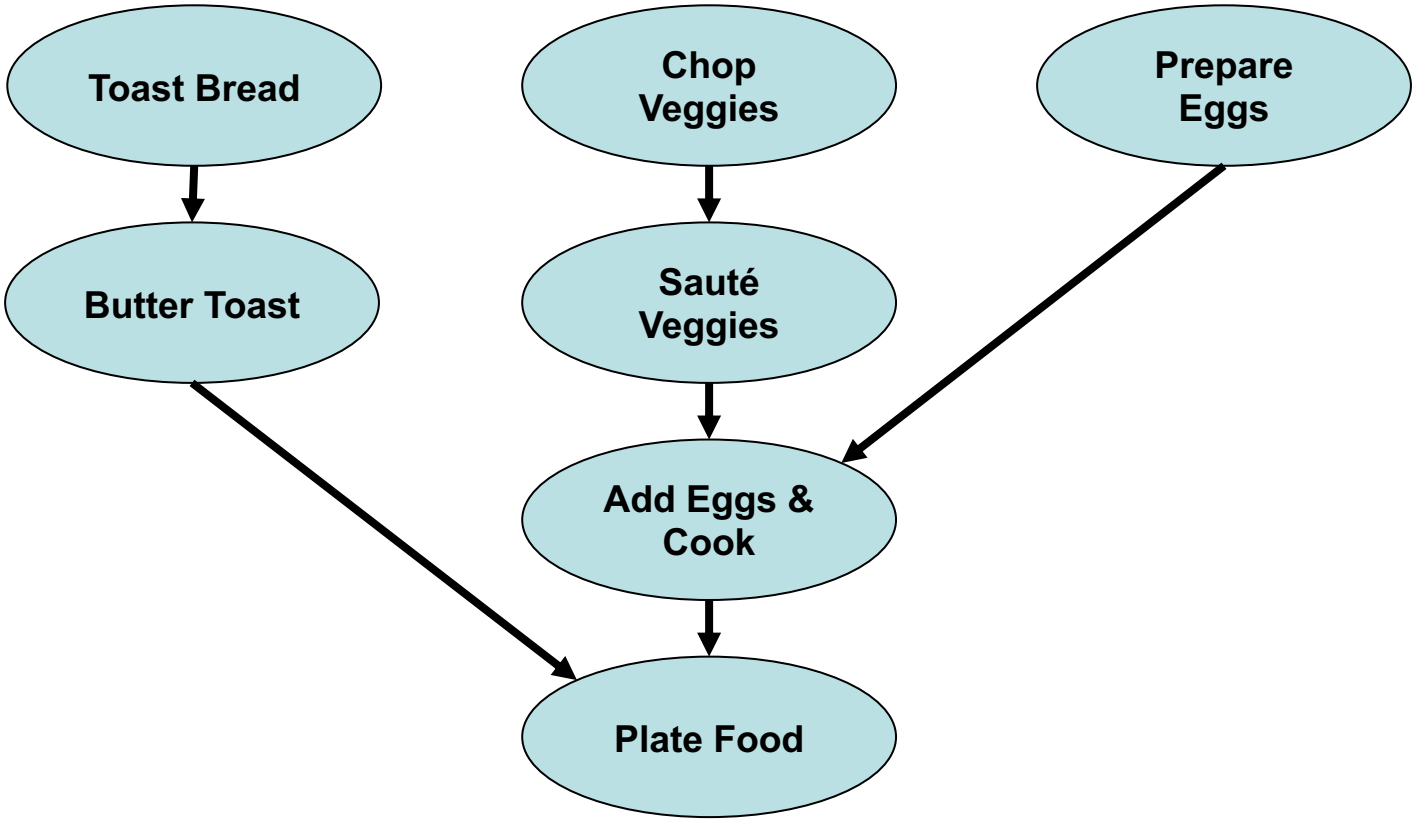
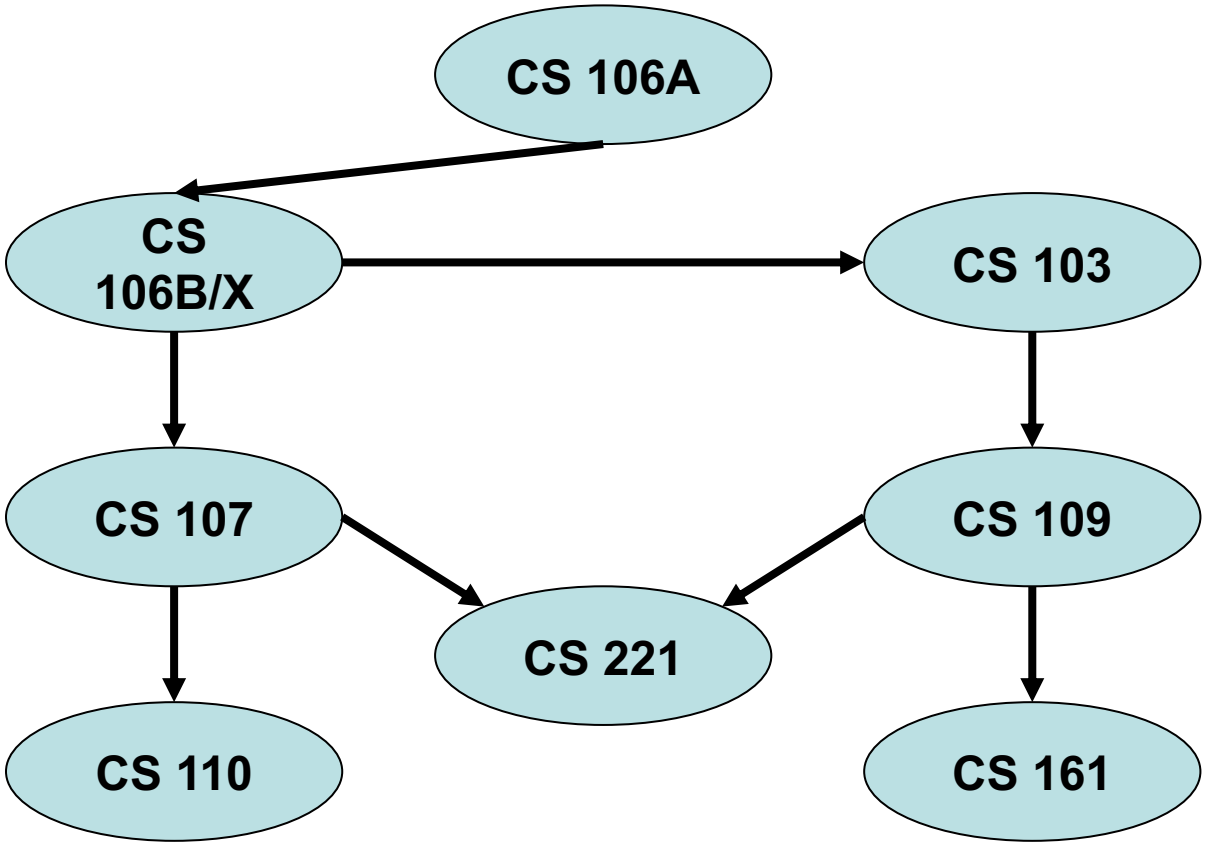


Toast Bread, Chop Veggies, Butter Toast, Prepare Eggs, Sauté Veggies, Add Eggs & Cook, Plate Food

Topological Sort

Given a directed (acyclic!) graph $G = (V, E)$, a topological sort is a total ordering of G's vertices such that for every edge (v, w) in E, vertex v precedes w in the ordering.



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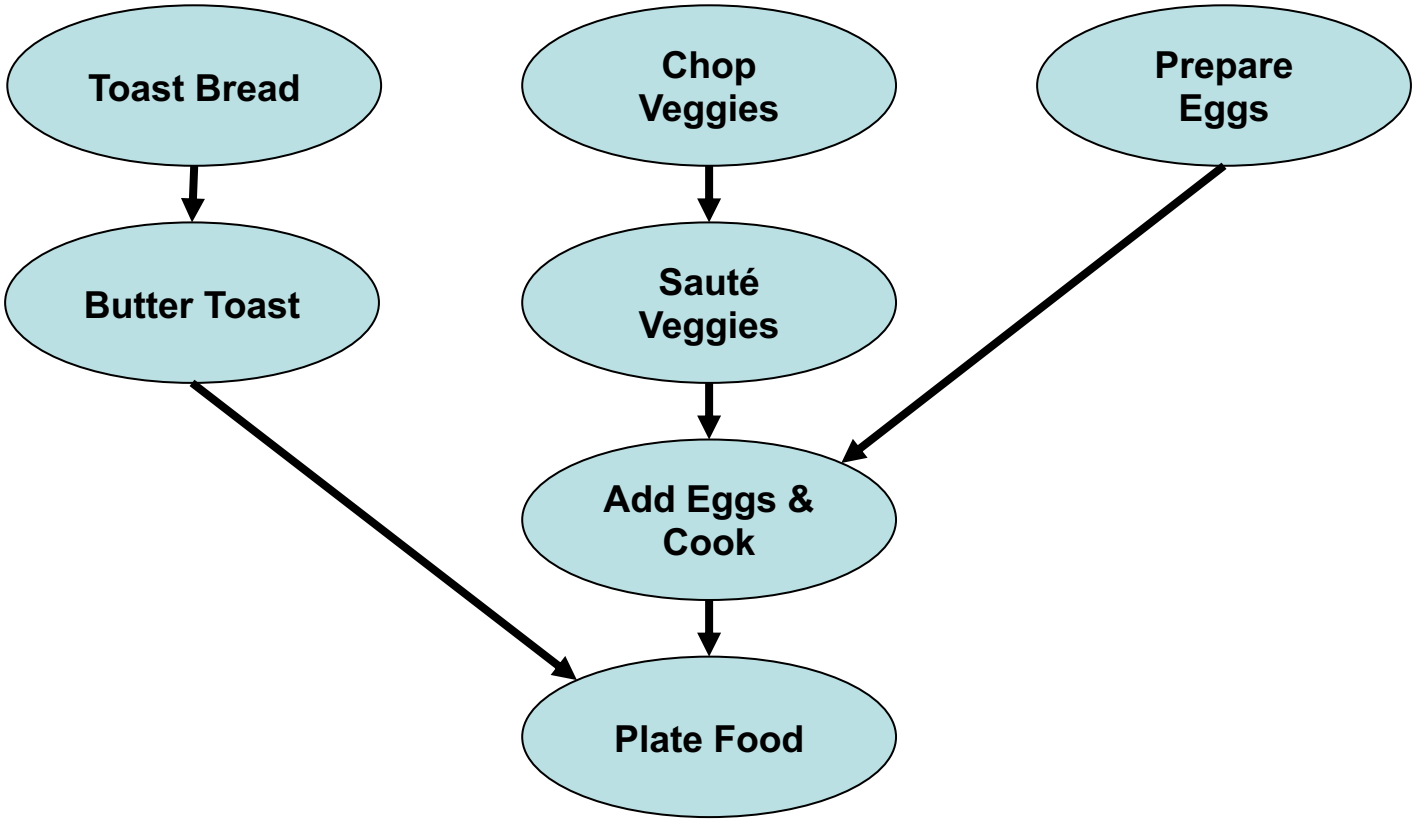

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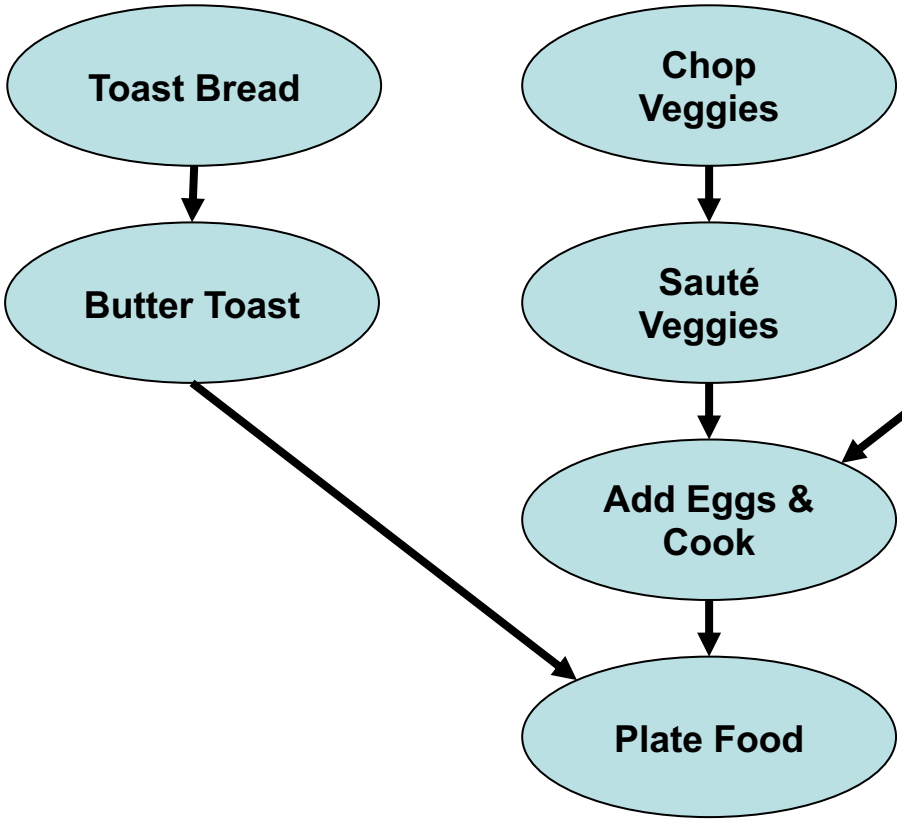

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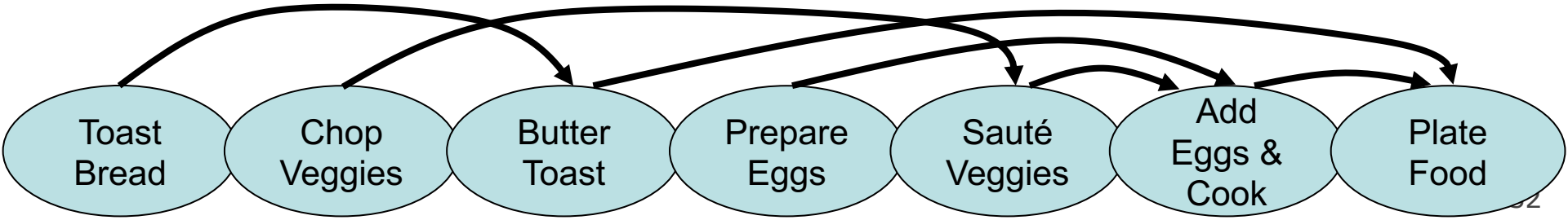

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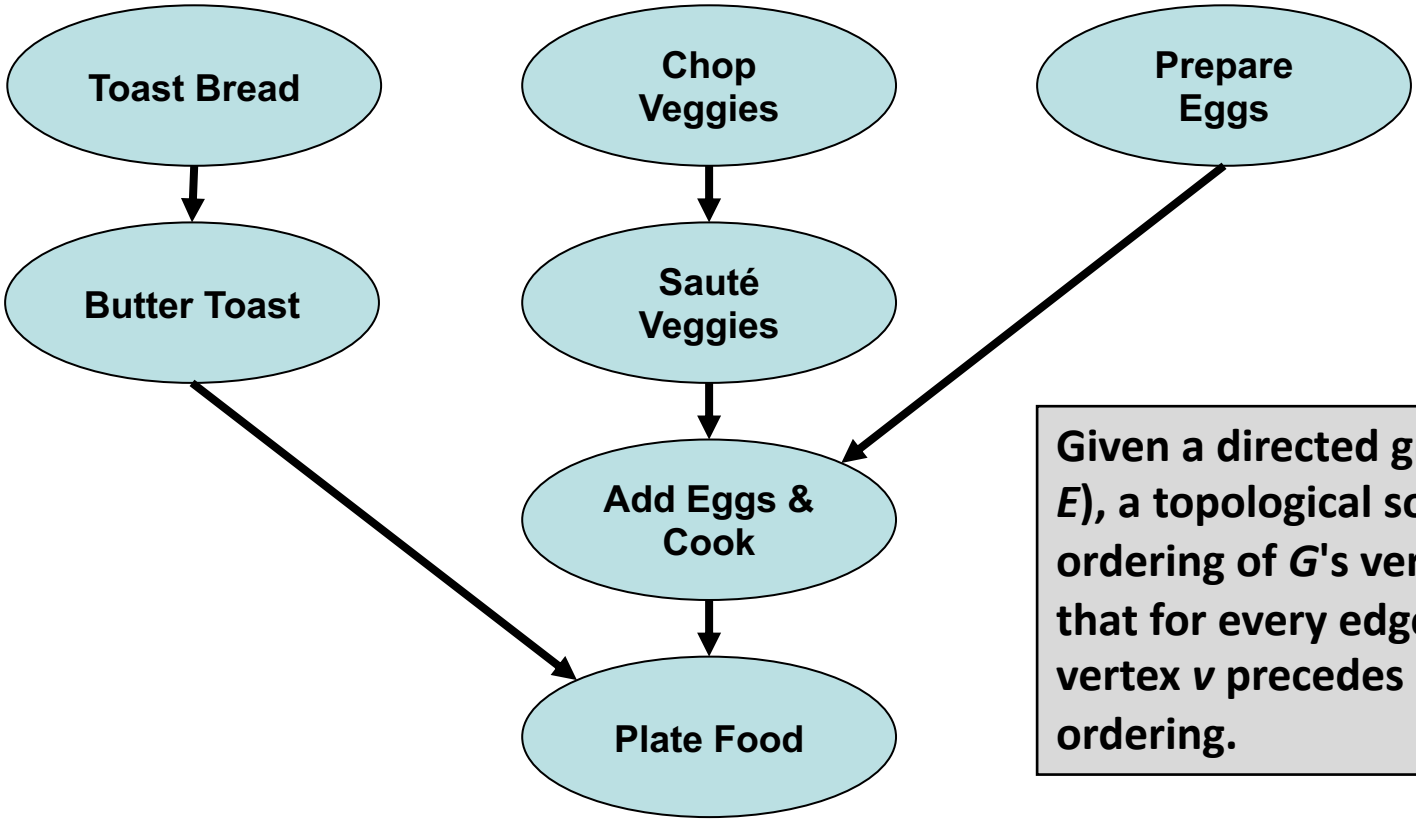

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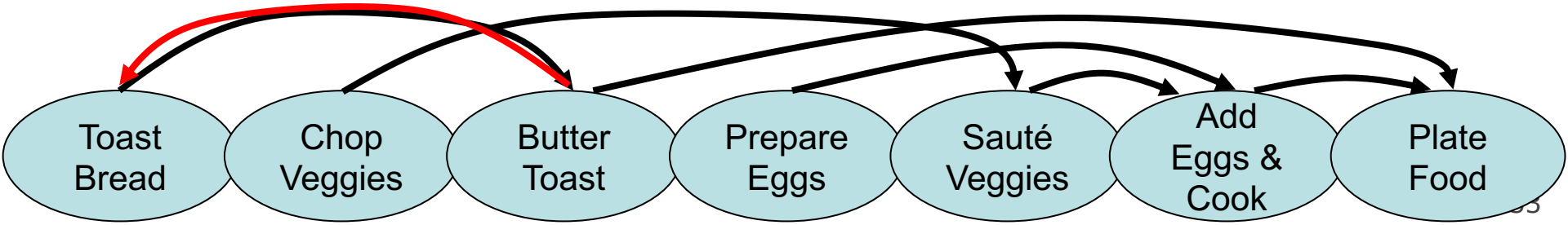

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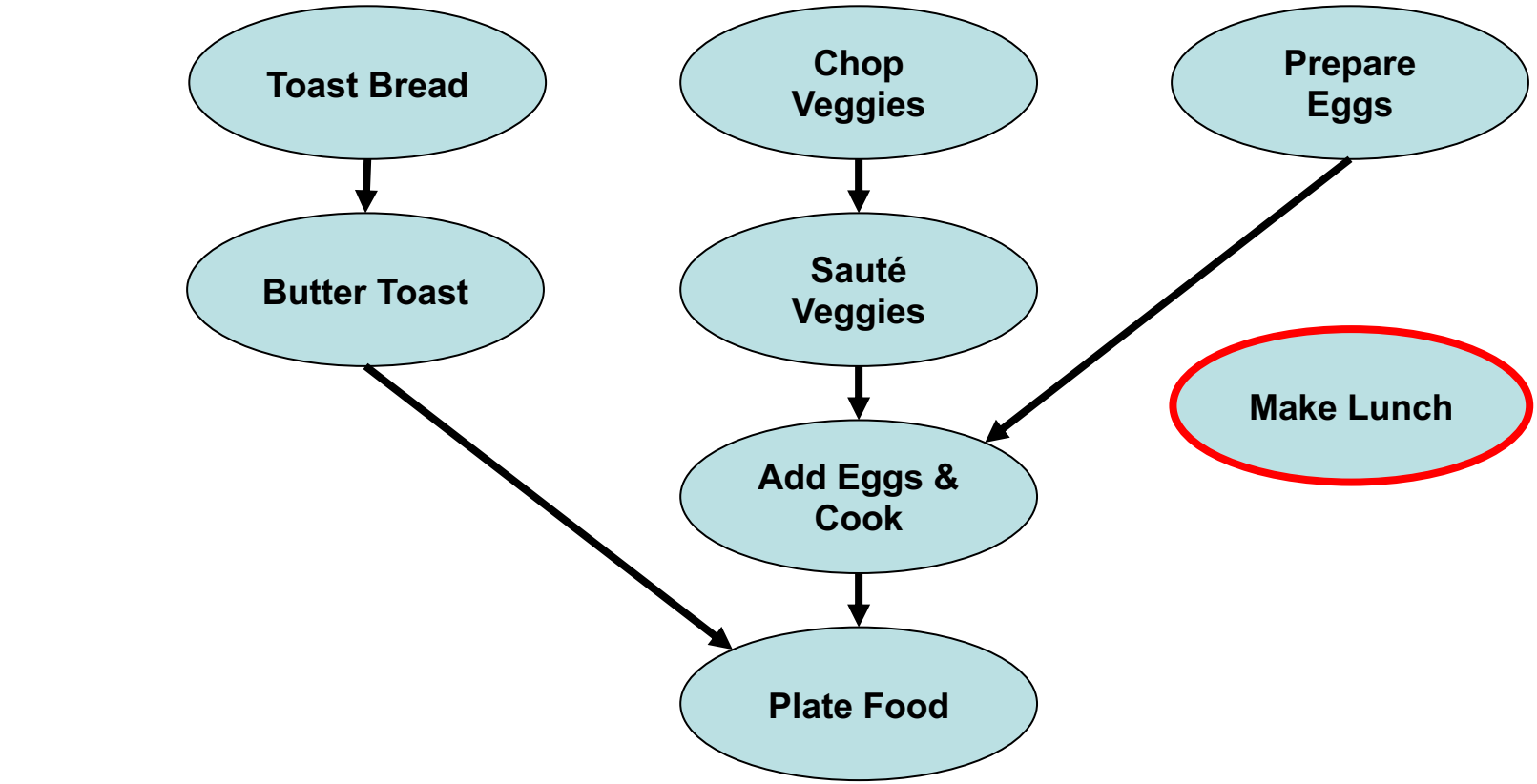

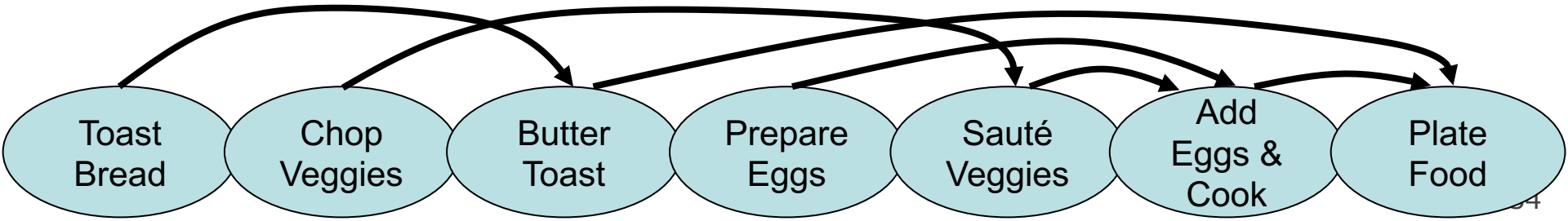

Topological Sort: Idea

1. Find node with in-degree of 0
2. Add it at the end of our topological ordering so far, and remove it and its edges from the graph
3. Repeat until no nodes left

Topological Sort Algorithm

```
ordering := { }.     // (empty list)
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
   Delete v and its outgoing edges from graph
   ordering += v
```

- This algorithm modifies the passed-in graph ☹
- Have we handled all edge cases?
- What is the runtime of this algorithm?

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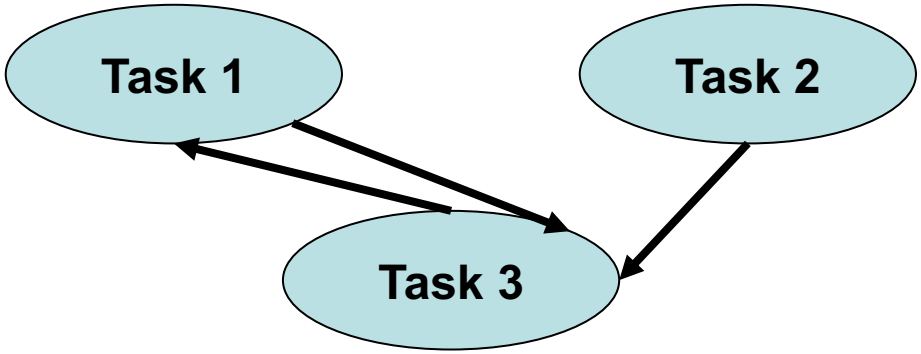

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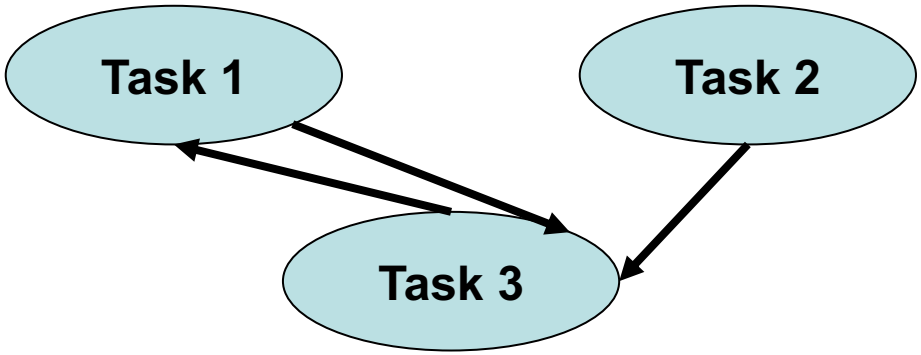

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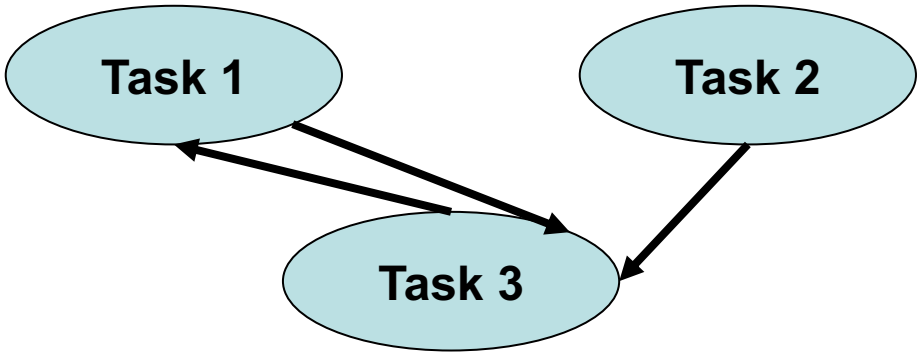

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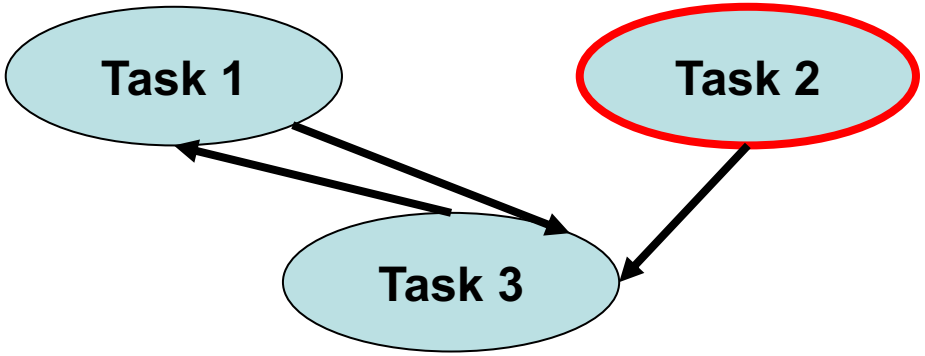
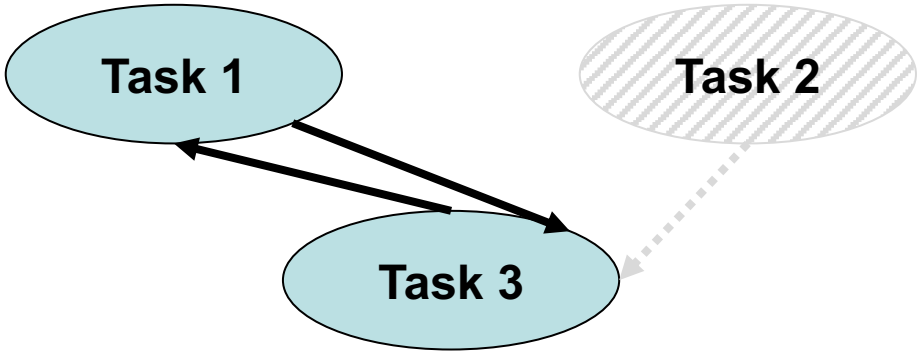


Ordering: { }

Topological Sort Algorithm: Cycles

```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
   Delete v and its outgoing edges from graph
   ordering += v
```



Ordering: { }

Topological Sort Algorithm: Cycles

```
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Repeat until graph is empty:
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   ordering += v
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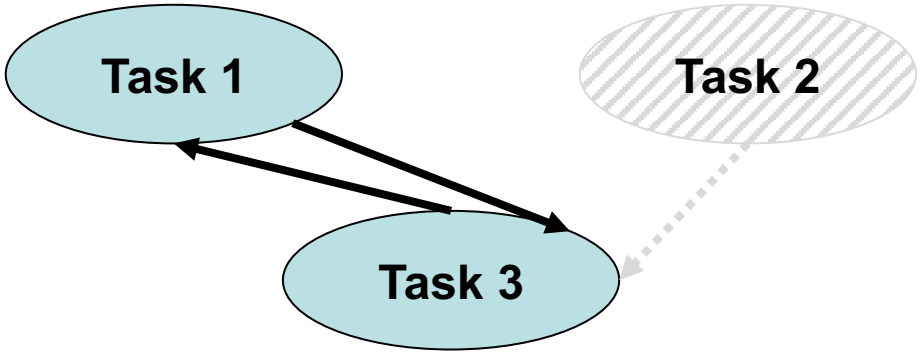
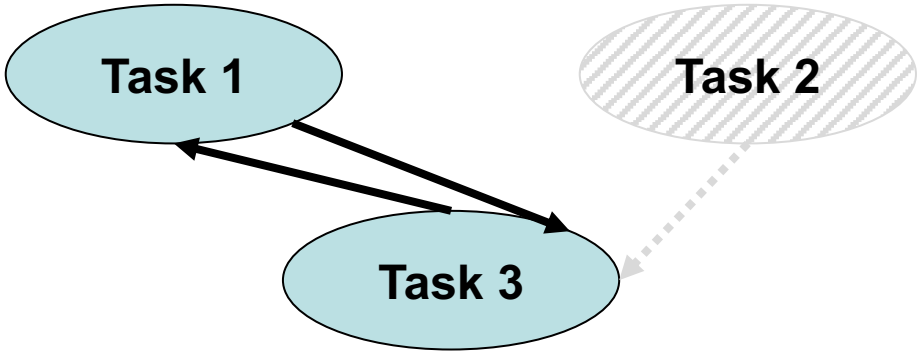


Ordering: { “Task 2” }

Topological Sort Algorithm: Cycles

```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
   Delete v and its outgoing edges from graph
   ordering += v
```



Ordering: { “Task 2” }

Topological Sort Algorithm: Cycles

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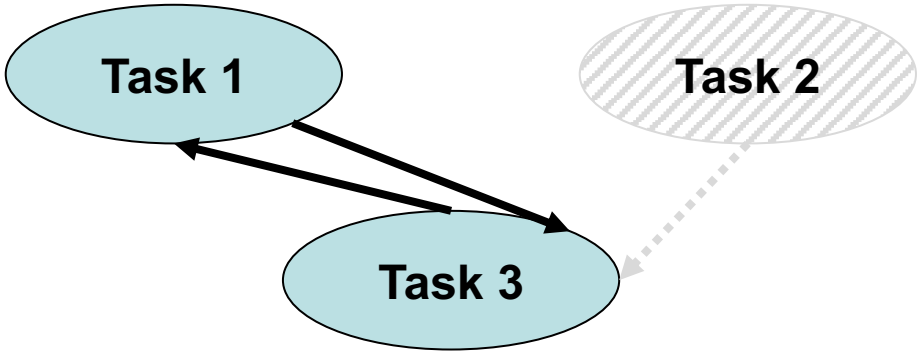
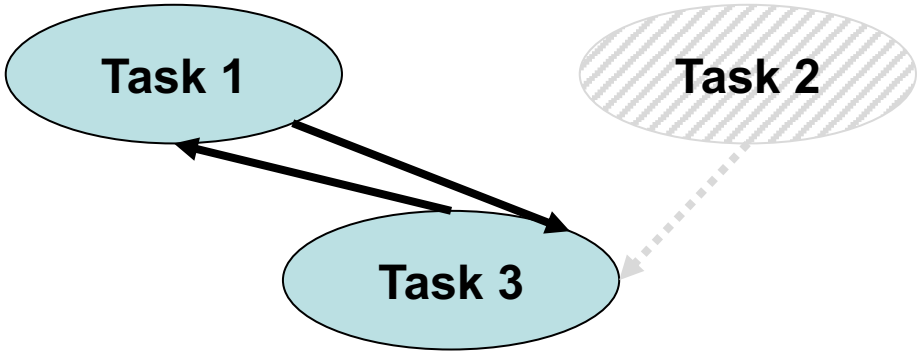


Ordering: { “Task 2” }

Topological Sort Algorithm: Cycles

```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
      - if none, no valid ordering possible
   Delete v and its outgoing edges from graph
   ordering += v
```



Ordering: { “Task 2” }

Topological Sort Algorithm: Cycles

```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
      - if none, no valid ordering possible
   Delete v and its outgoing edges from graph
   ordering += v
```

- This algorithm modifies the passed-in graph ☹
- Have we handled all edge cases?
- What is the runtime of this algorithm?

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
      - if none, no valid ordering possible
   Delete v and its outgoing edges from graph
   ordering += v
```

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

O(1)

```
ordering := { }.
```

O(V)

```
Repeat until graph is empty:
```

O(V)

```
    Find a vertex v with in-degree of 0
        - if none, no valid ordering possible
```

O(E)

```
    Delete v and its outgoing edges from graph
```

O(1)

```
    ordering += v
```

O(V(V+E))

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

O(1)
```
ordering := { }.
```

O(V)
```
Repeat until graph is empty:
```

O(V)
```
Find a vertex v with in-degree of 0
   - if none, no valid ordering possible
```

O(E)
```
Delete v and its outgoing edges from graph
```

Is the worst case here really O(E) *every time?* For example, if one node has *all* the outgoing edges, then we'll have an O(E) operation that time, but a no-op every other time.

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

O(1)
```
ordering := { }.
```

O(V)
```
Repeat until graph is empty:
```

O(V)
```
Find a vertex v with in-degree of 0
```
```
   - if none, no valid ordering possible
```

O(E)

```
Delete v and its outgoing edges from graph
```

Key Idea: every edge can be deleted *at most once.* Therefore, the total runtime of this operation is O(E), not O(V*E).

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

O(1)

```
ordering := { }.
```

O(V)

```
Repeat until graph is empty:
```

O(V)

```
    Find a vertex v with in-degree of 0
        - if none, no valid ordering possible
```

O(E) *total*

```
    Delete v and its outgoing edges from graph
```

O(1)

```
    ordering += v
```

Tighter bound: $O(V^2 + E)$

Topological Sort Algorithm: Runtime

For graph with V vertexes and E edges:

O(1)
ordering := { }.

O(V)
Repeat until graph is empty:

O(V)
Find a vertex *v* with in-degree of 0

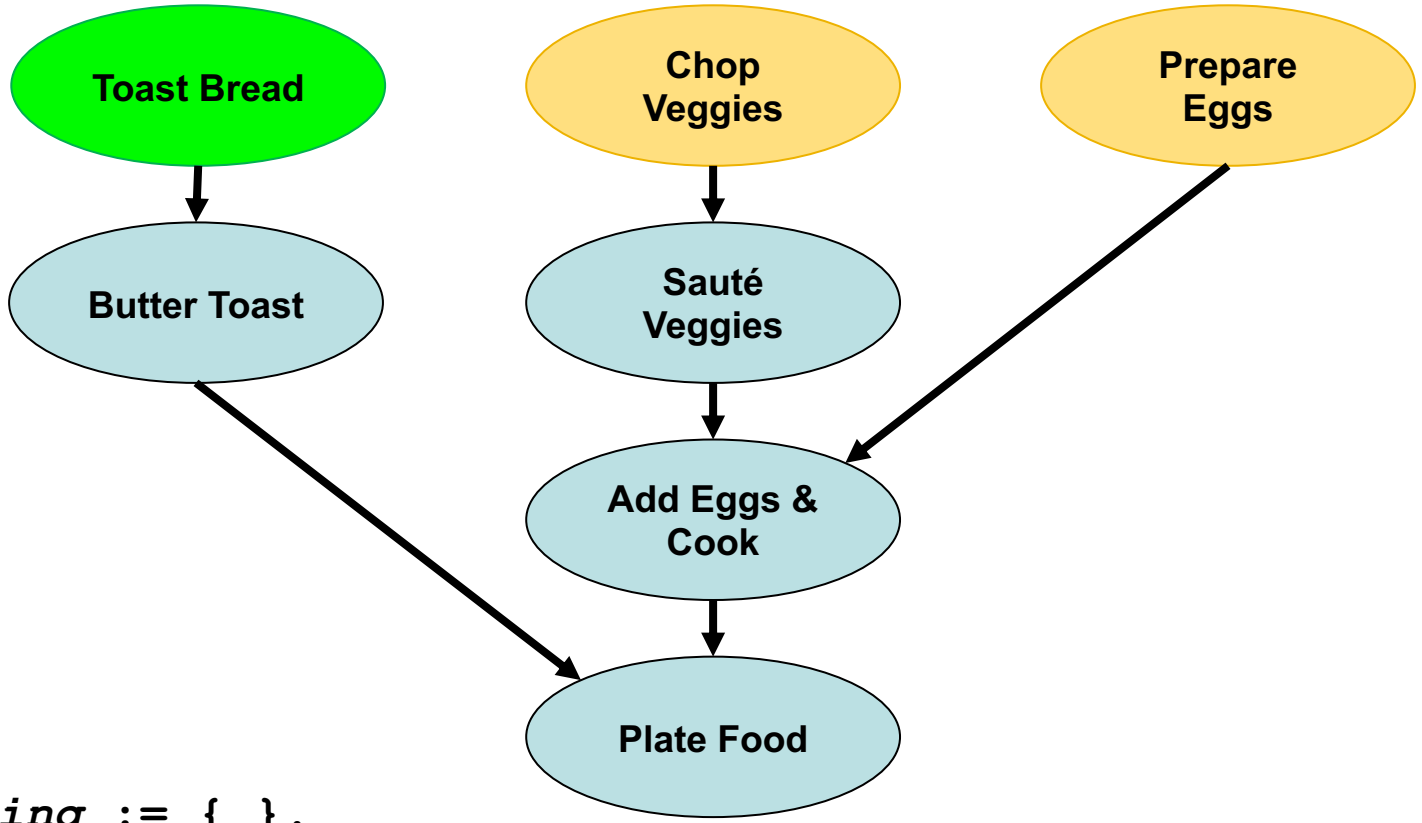
- if n… possible

Can we make this more efficient?

O(E) *total*
Delete *v* and its outgoing edges from graph

O(1)
ordering += v

Topological Sort: Take 2

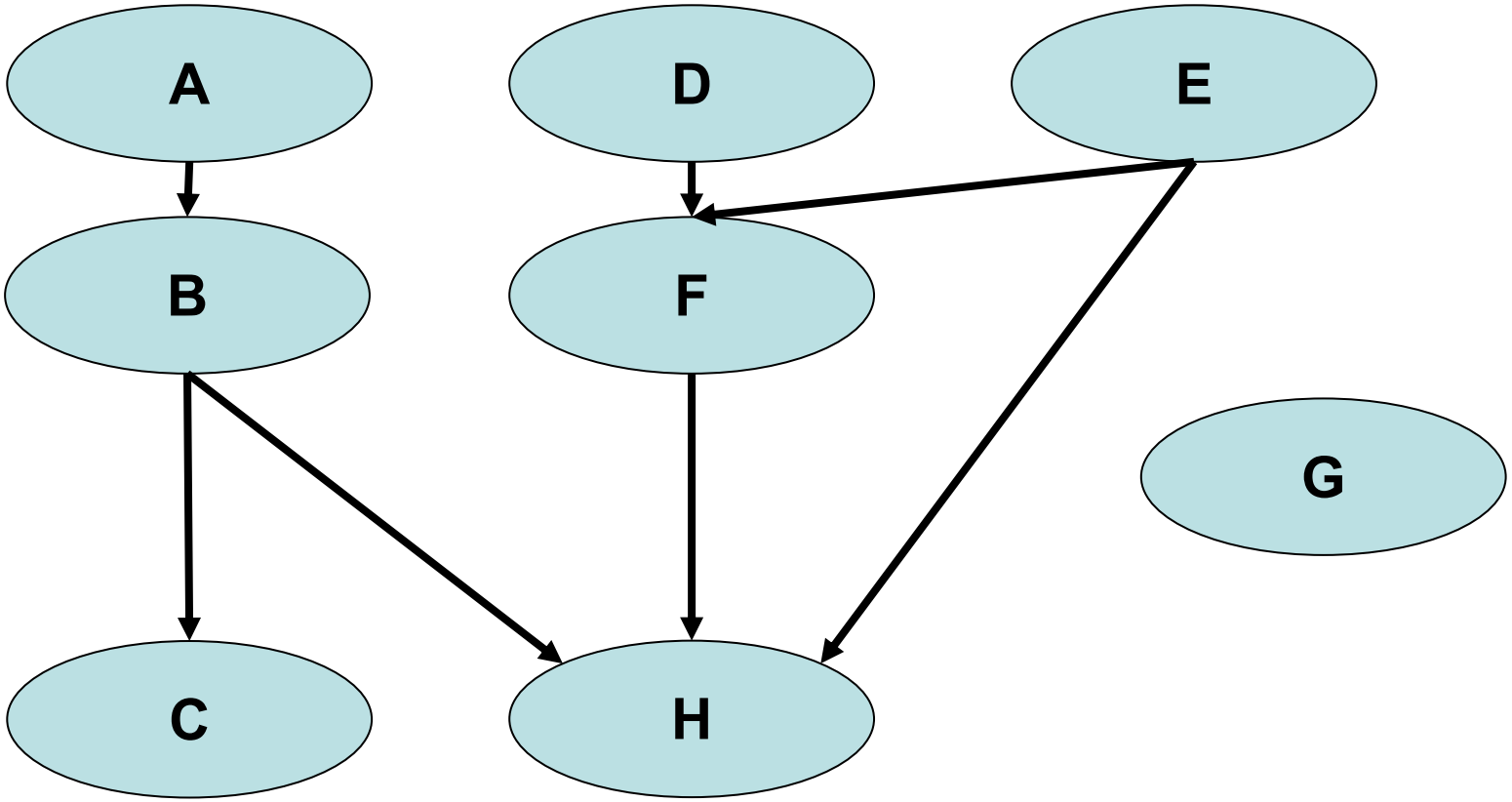



```
ordering := { }.
Repeat until graph is empty:
   Find a vertex v with in-degree of 0
       - if none, no valid ordering possible
   Delete v and its outgoing edges from graph
   ordering += v
```

3 Key Ideas for Improvement

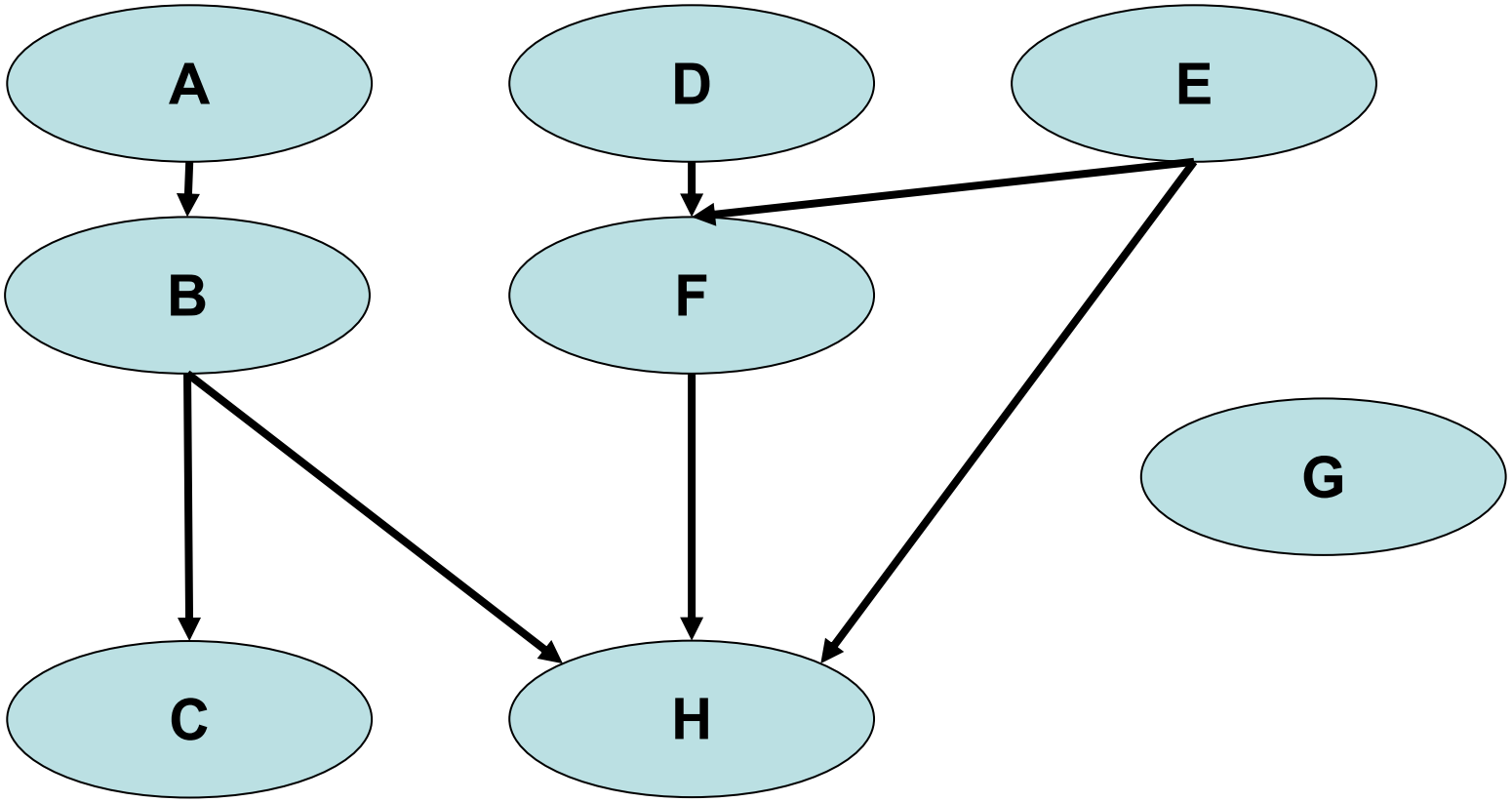
- Keep a *queue of nodes with in-degree 0* so we don't have to search for nodes multiple times.
- A node's in-degree only changes when one of its prerequisites is completed. Therefore, when completing a task, check if any of its neighbors now has in-degree 0.
- Keep a *map of nodes' in-degrees* so we don't need to modify the graph.

Topological Sort: Take 2



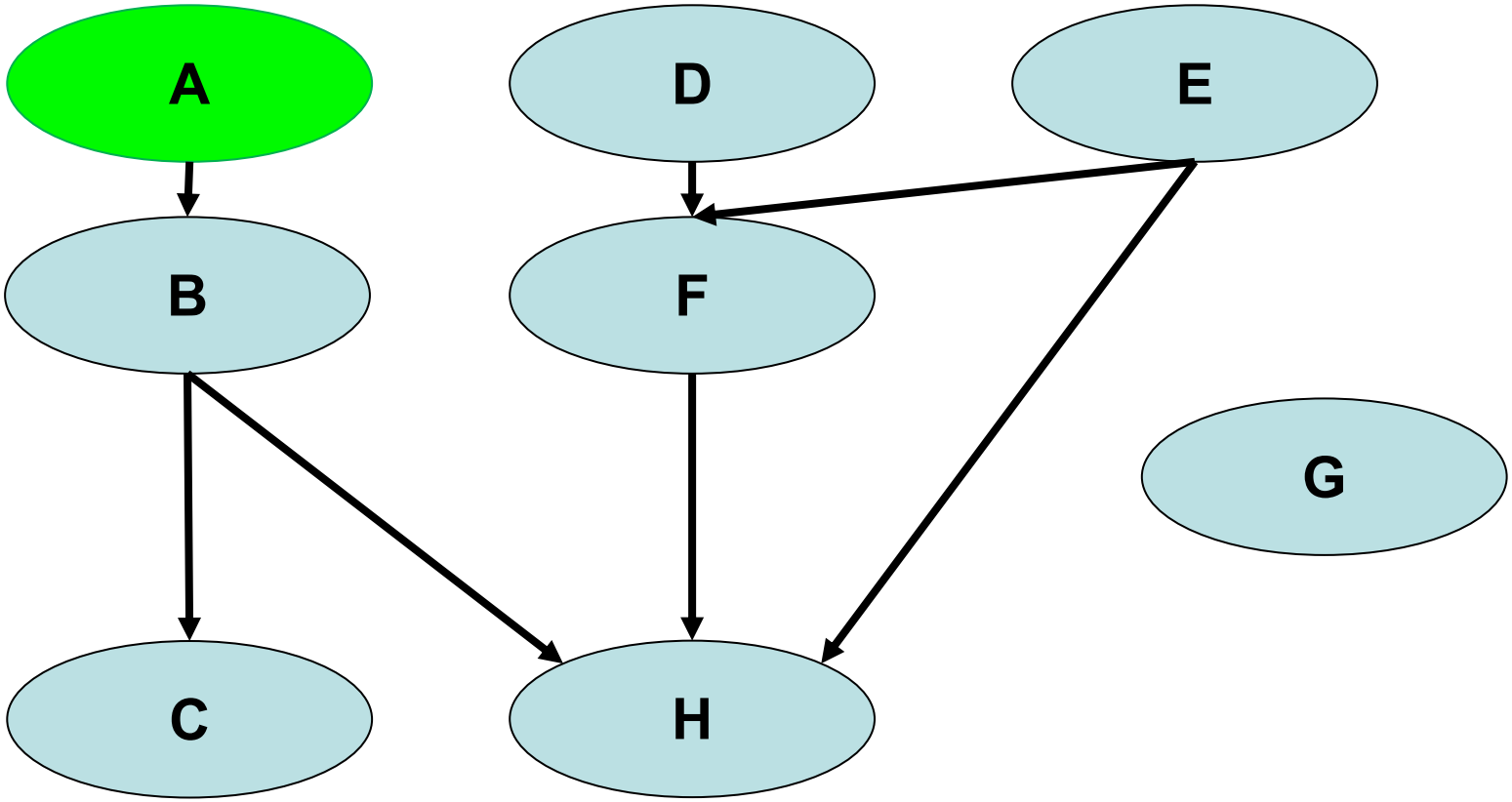
0-In-Degree Queue: { }
In-Degree Map: { }

Topological Sort: Take 2

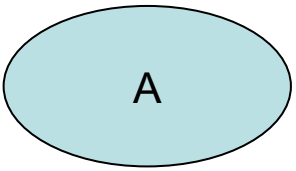


0-In-Degree Queue: { “A”, “D”, “E”, “G” }
In-Degree Map: { “A”:0, “B”:1, “C”:1, “D”:0, “E”: 0, “F”: 2, “G”:0, “H”:3 }

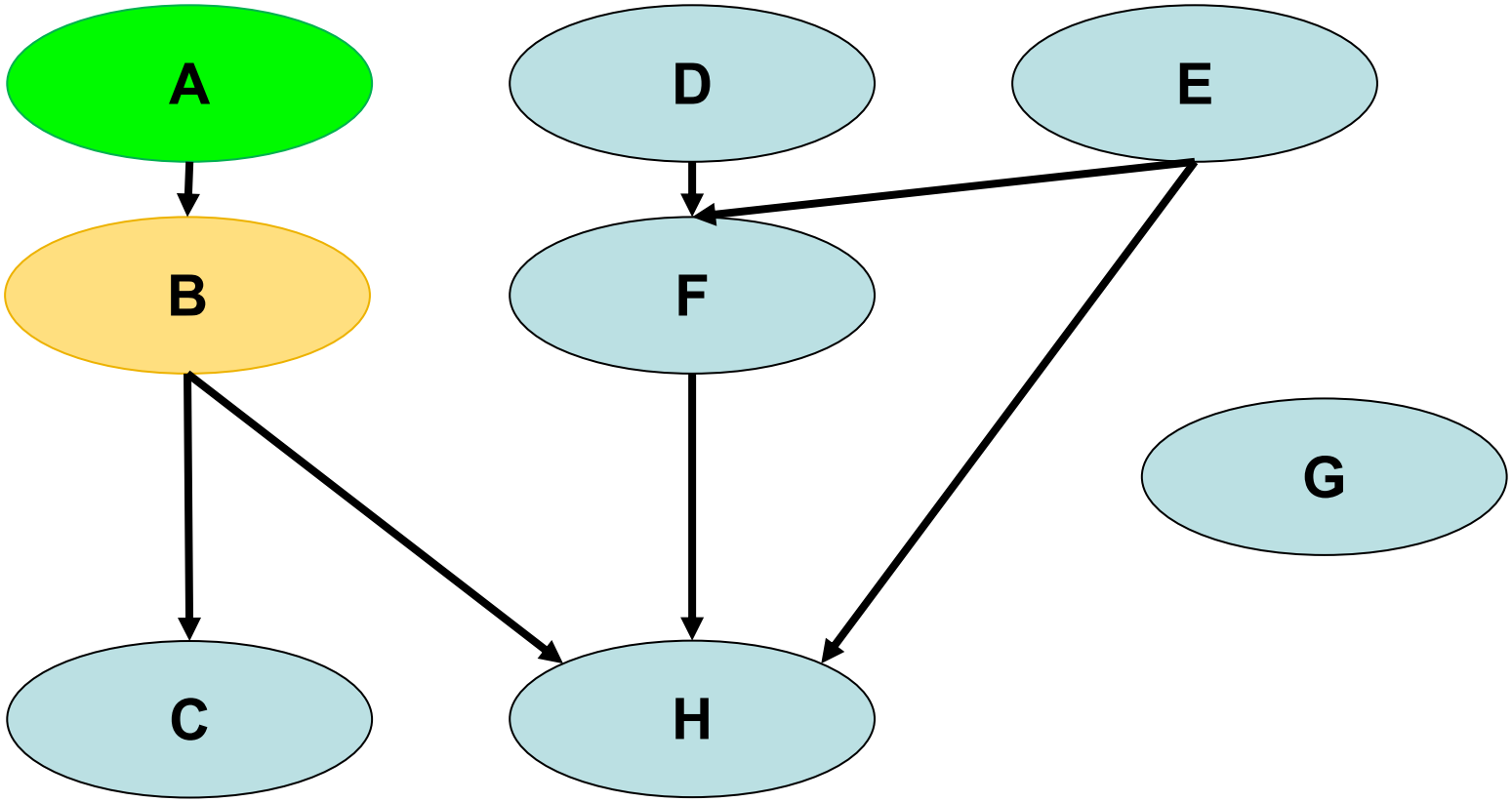
Topological Sort: Take 2



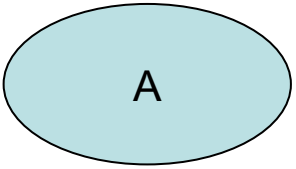
0-In-Degree Queue: { ~~"A"~~, "D", "E", "G" }
In-Degree Map: { "A":0, "B":1, "C":1, "D":0, "E": 0, "F": 2, "G":0, "H":3 }



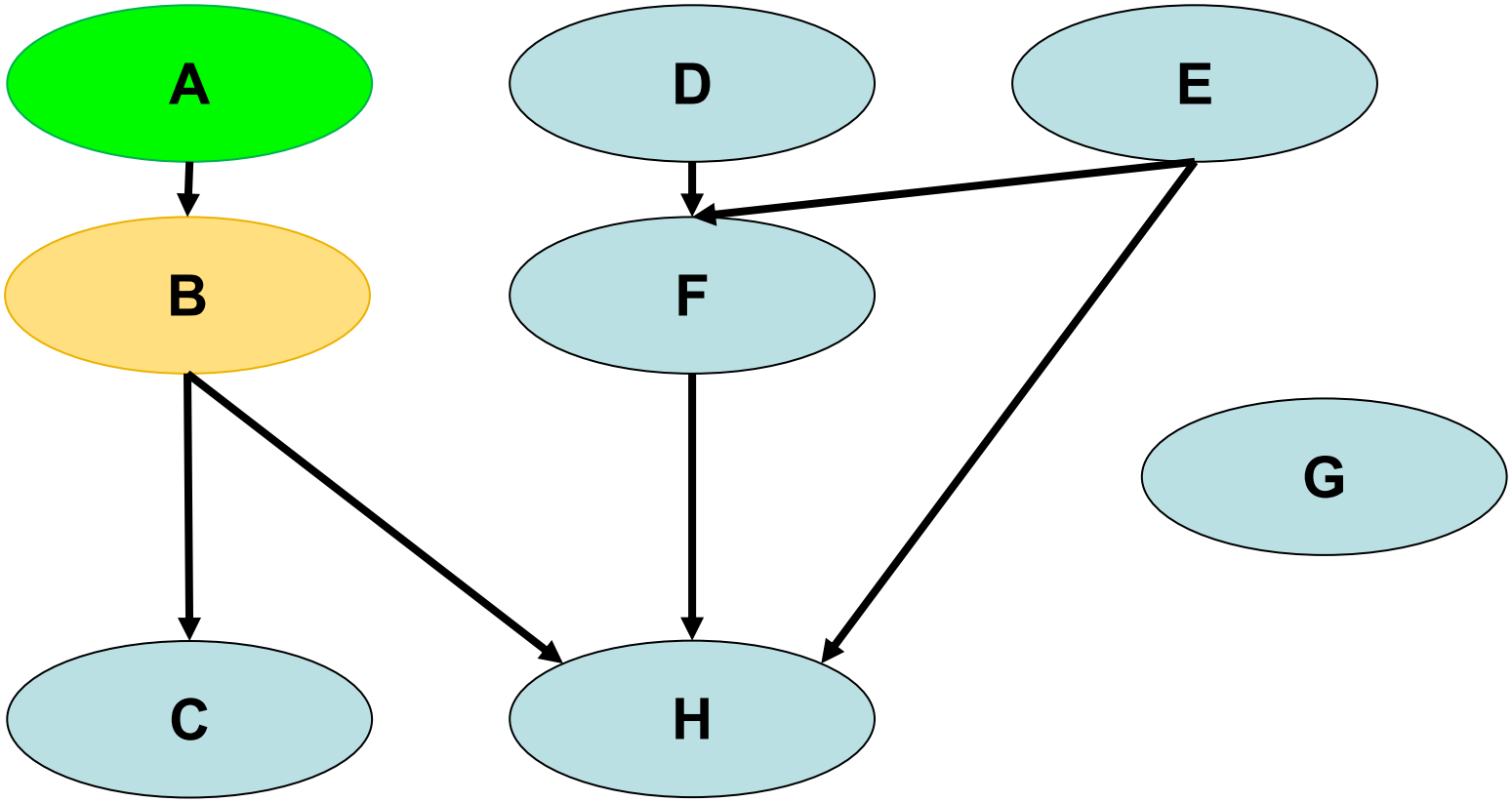
Topological Sort: Take 2



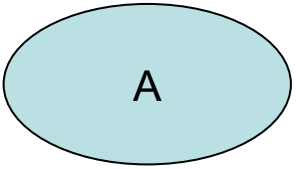
0-In-Degree Queue: { “D”, “E”, “G” }
In-Degree Map: { “A”:0, “B”:1, “C”:1, “D”:0, “E”: 0, “F”: 2, “G”:0, “H”:3 }



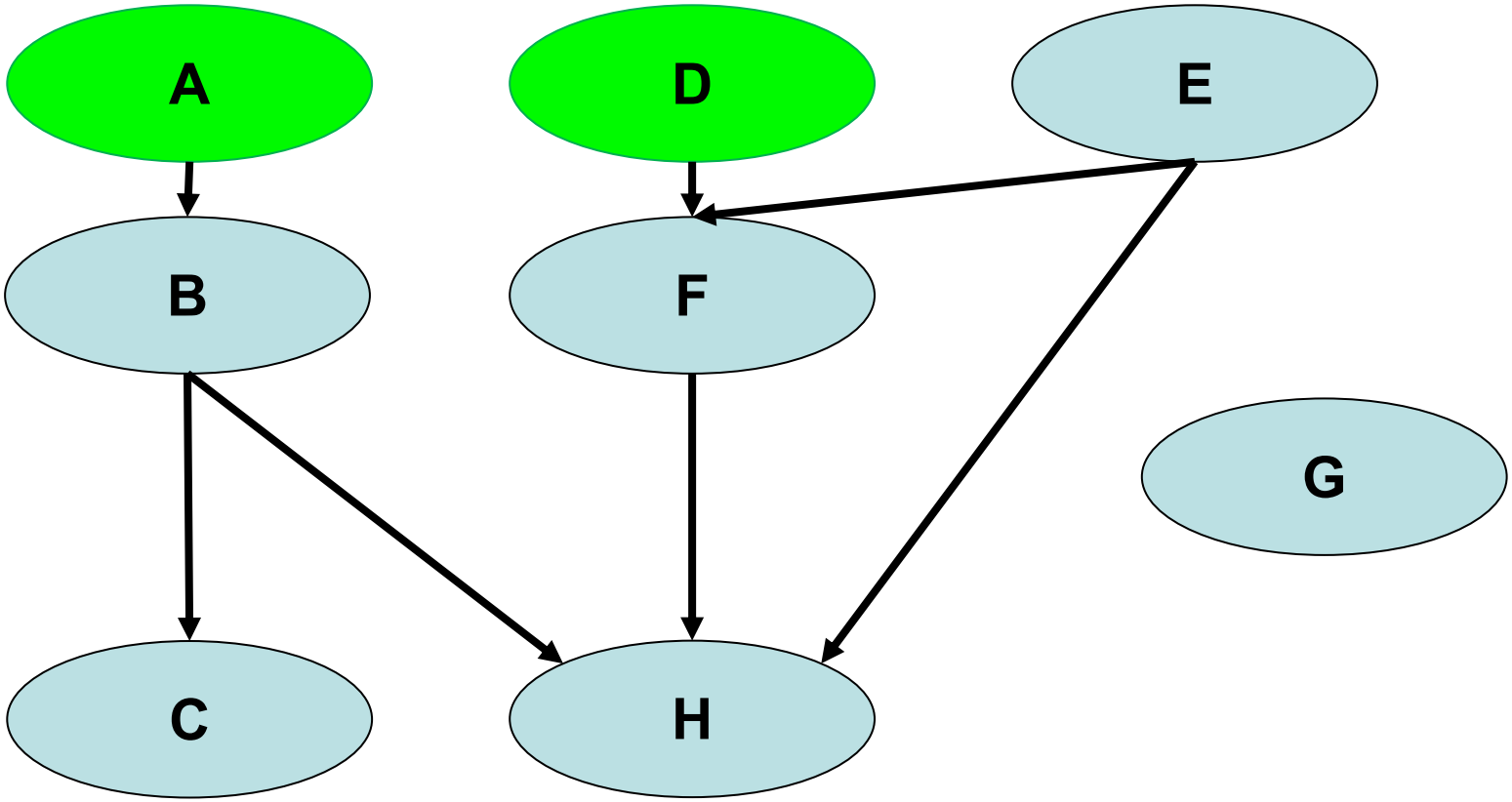
Topological Sort: Take 2



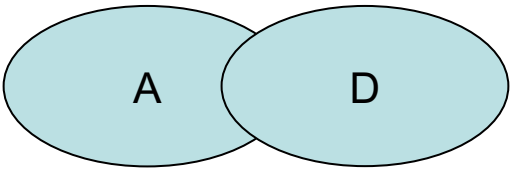
0-In-Degree Queue: { “D”, “E”, “G”, “B” }
In-Degree Map: { “A”:0, “B”:0, “C”:1, “D”:0, “E”: 0, “F”: 2, “G”:0, “H”:3 }



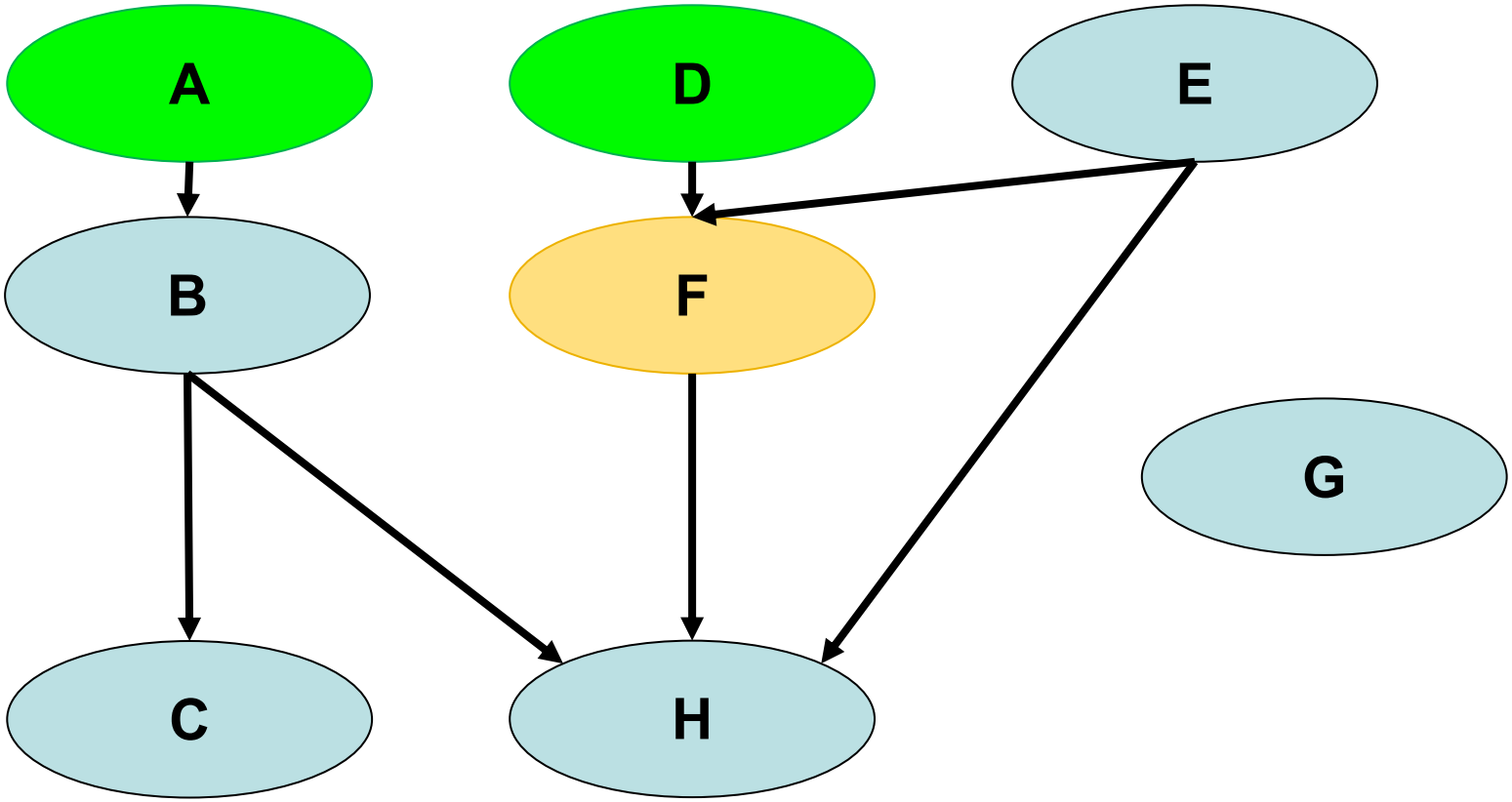
Topological Sort: Take 2



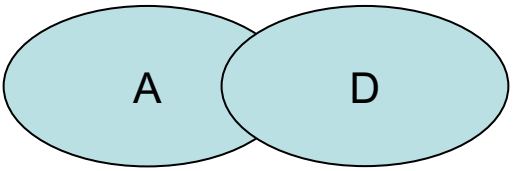
0-In-Degree Queue: { ~~"D"~~, "E", "G", "B" }
In-Degree Map: { "A":0, "B":0, "C":1, "D":0, "E": 0, "F": 2, "G":0, "H":3 }



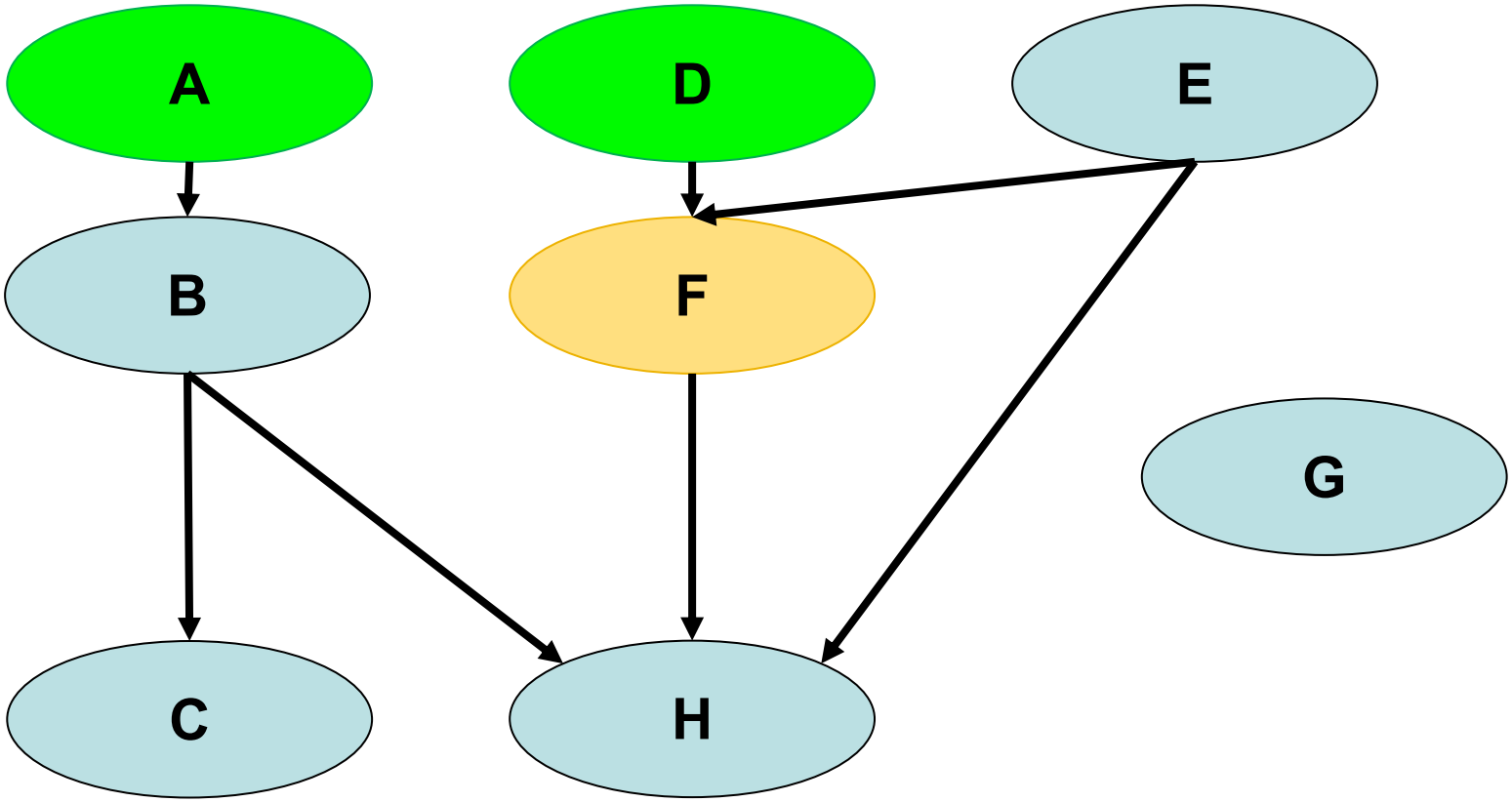
Topological Sort: Take 2



0-In-Degree Queue: { “E”, “G”, “B” }
In-Degree Map: { “A”:0, “B”:0, “C”:1, “D”:0, “E”: 0, “F”: 2, “G”:0, “H”:3 }



Topological Sort: Take 2



0-In-Degree Queue: { “E”, “G”, “B” }
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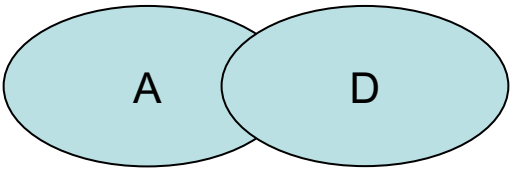
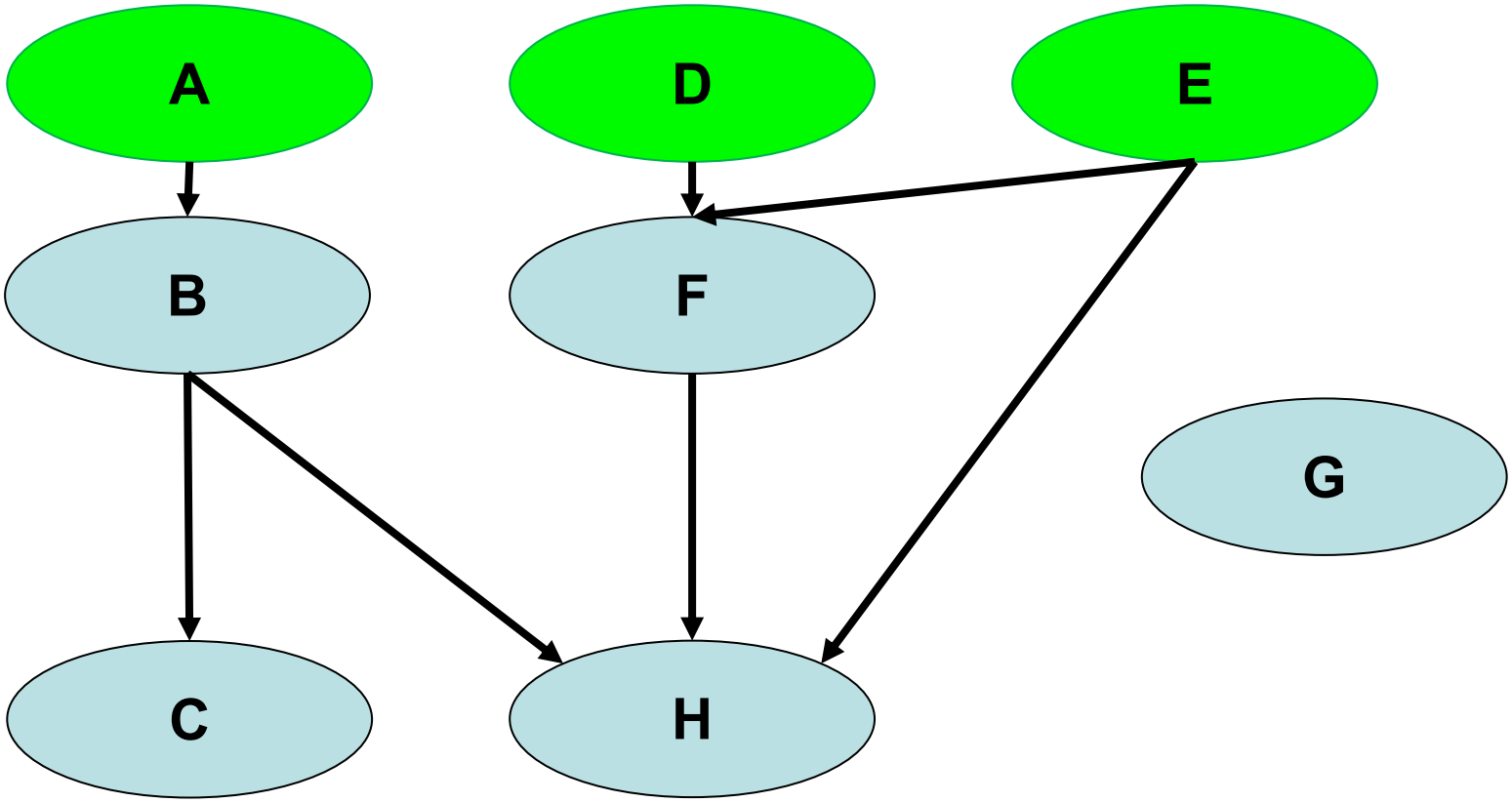


Topological Sort: Take 2



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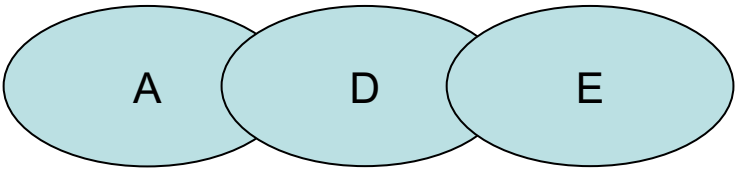
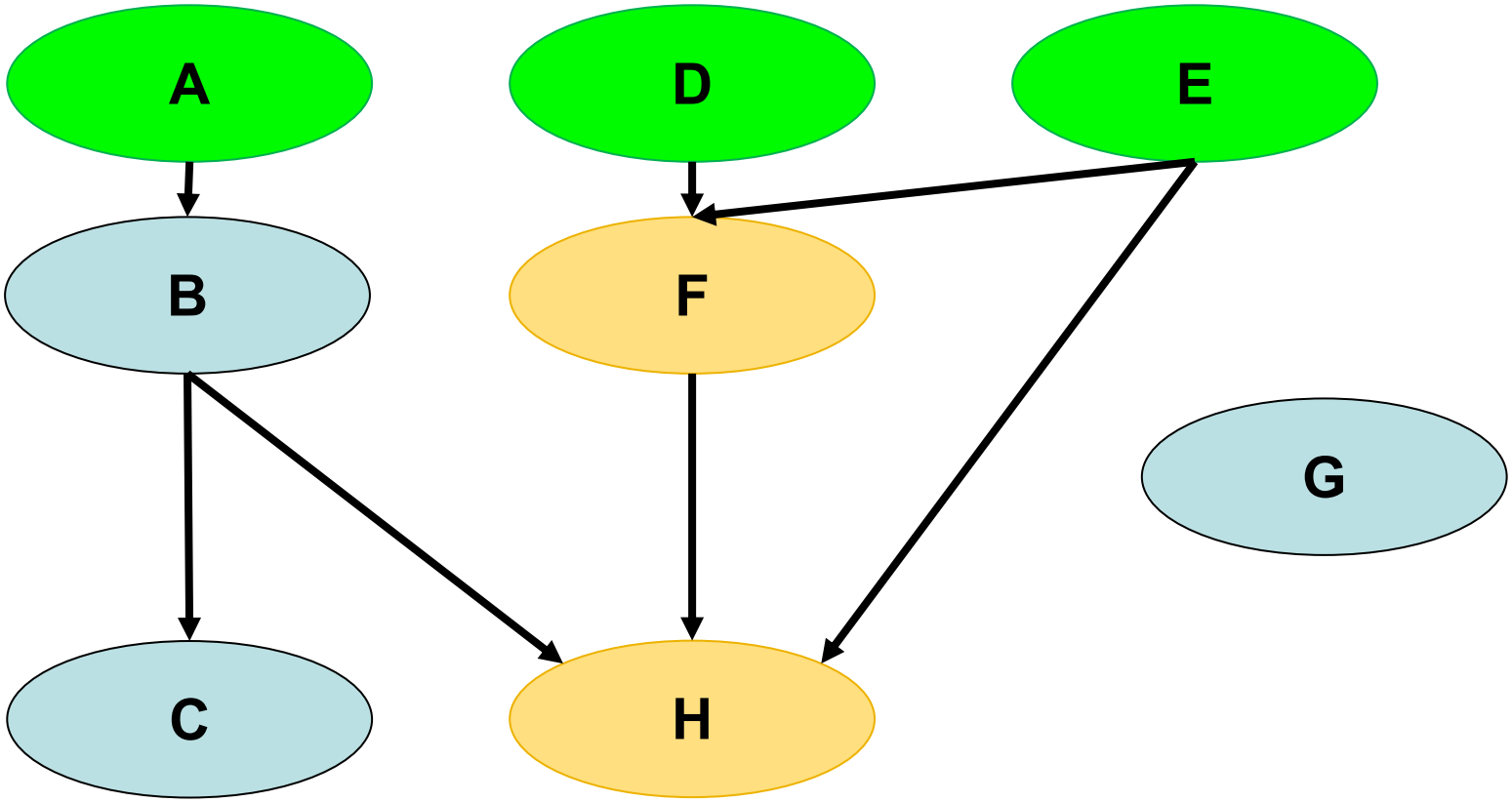
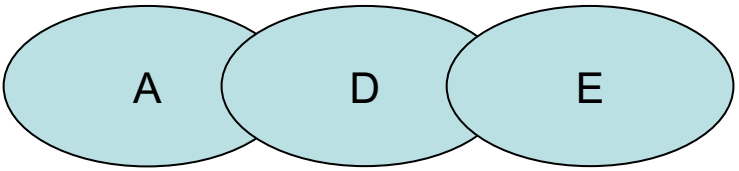


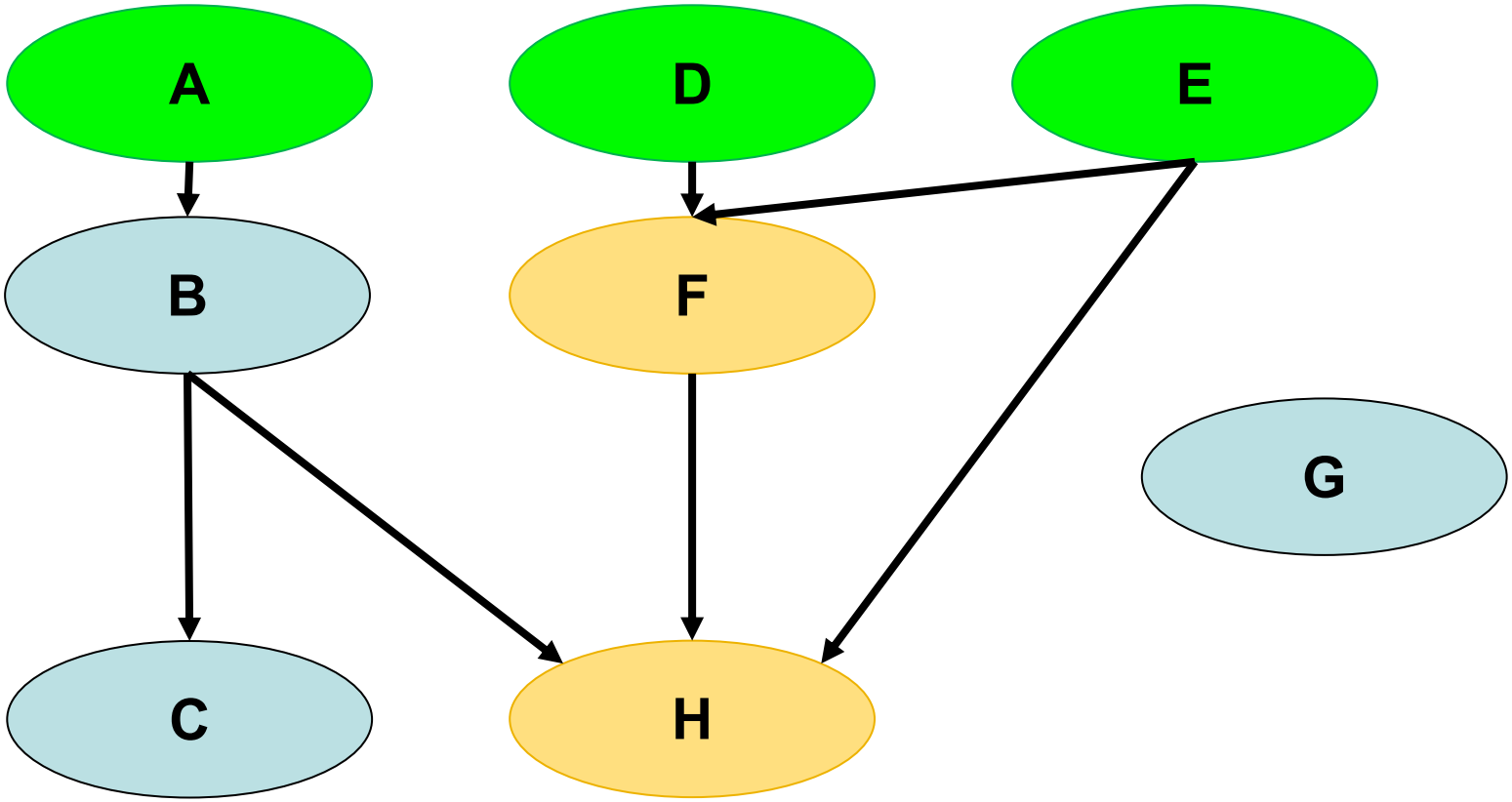
Topological Sort: Take 2



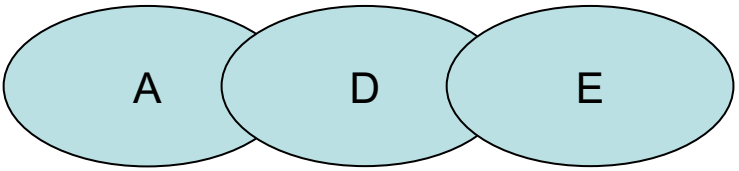
0-In-Degree Queue: { “G”, “B” }
In-Degree Map: { “A”:0, “B”:0, “C”:1, “D”:0, “E”: 0, “F”: 1, “G”:0, “H”:3 }



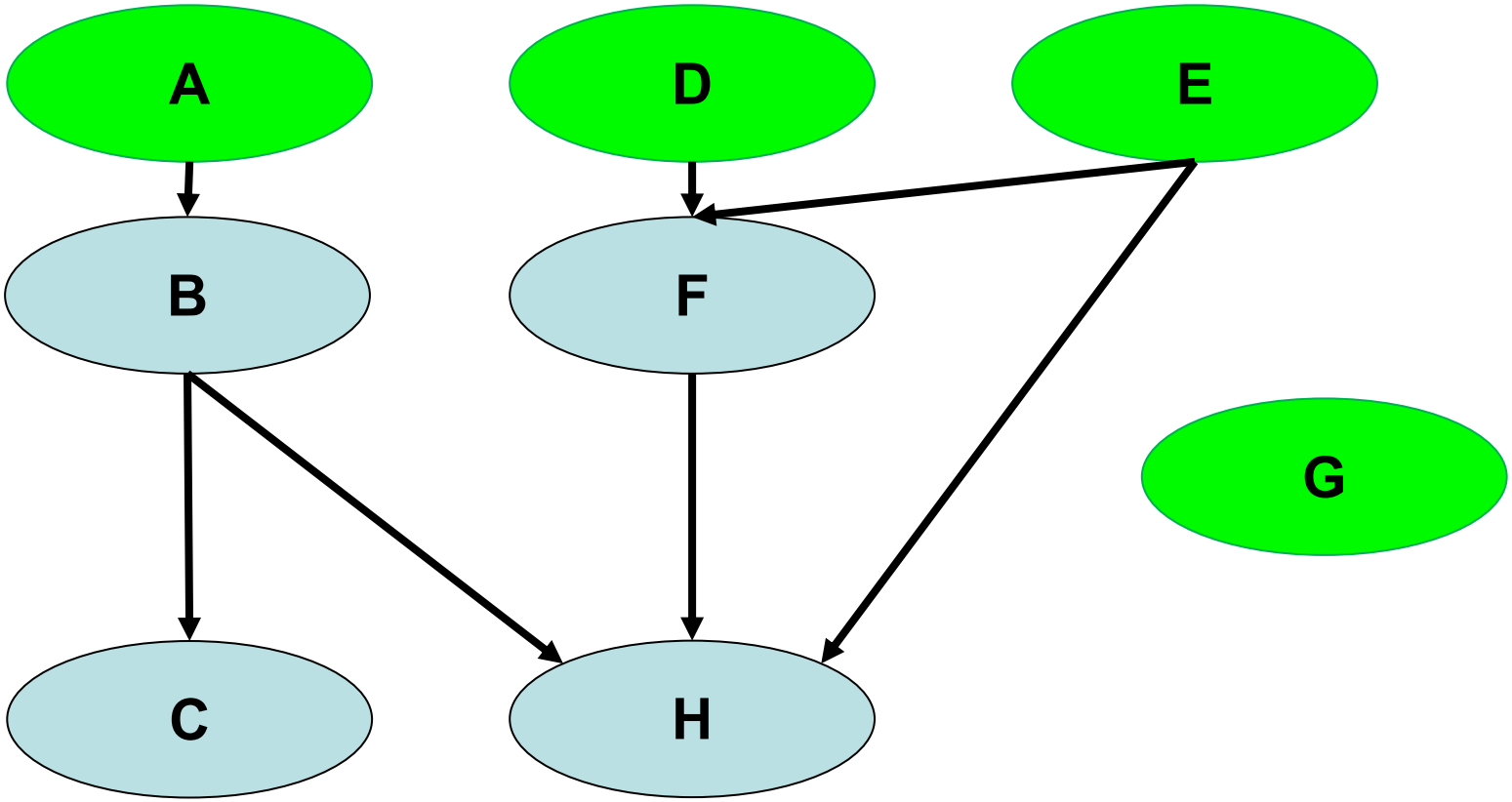
Topological Sort: Take 2



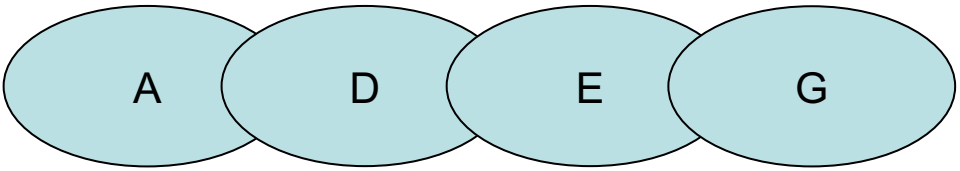
0-In-Degree Queue: { "G", "B", "F" }
In-Degree Map: { "A":0, "B":0, "C":1, "D":0, "E": 0, "F": 0, "G":0, "H":2 }



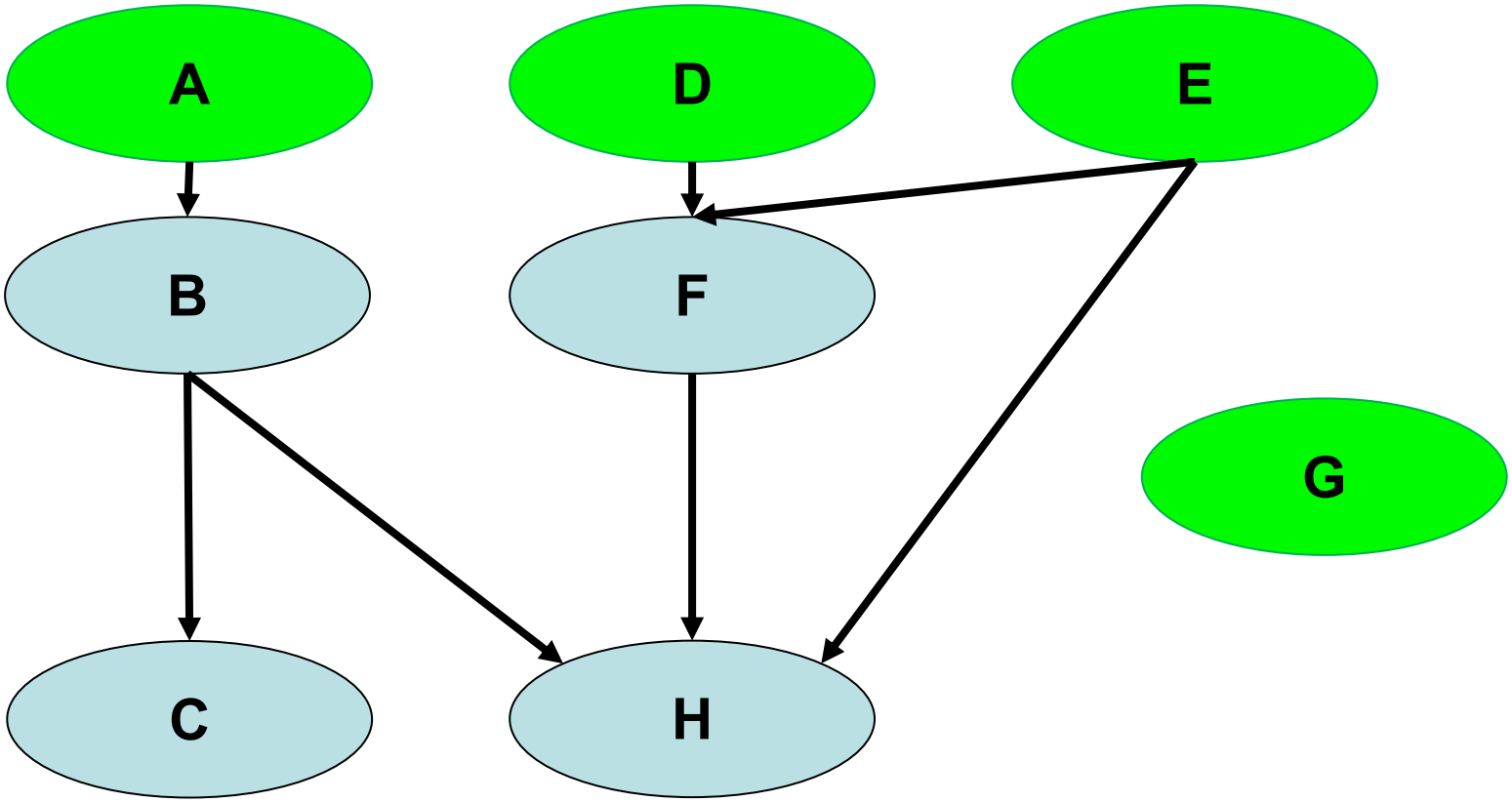
Topological Sort: Take 2



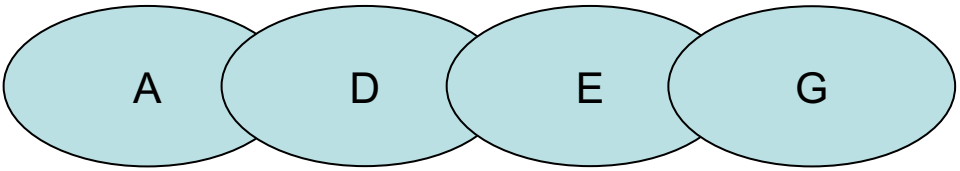
0-In-Degree Queue: { ~~"G",~~ "B", "F" }
In-Degree Map: { "A":0, "B":0, "C":1, "D":0, "E": 0, "F": 0, "G":0, "H":2 }



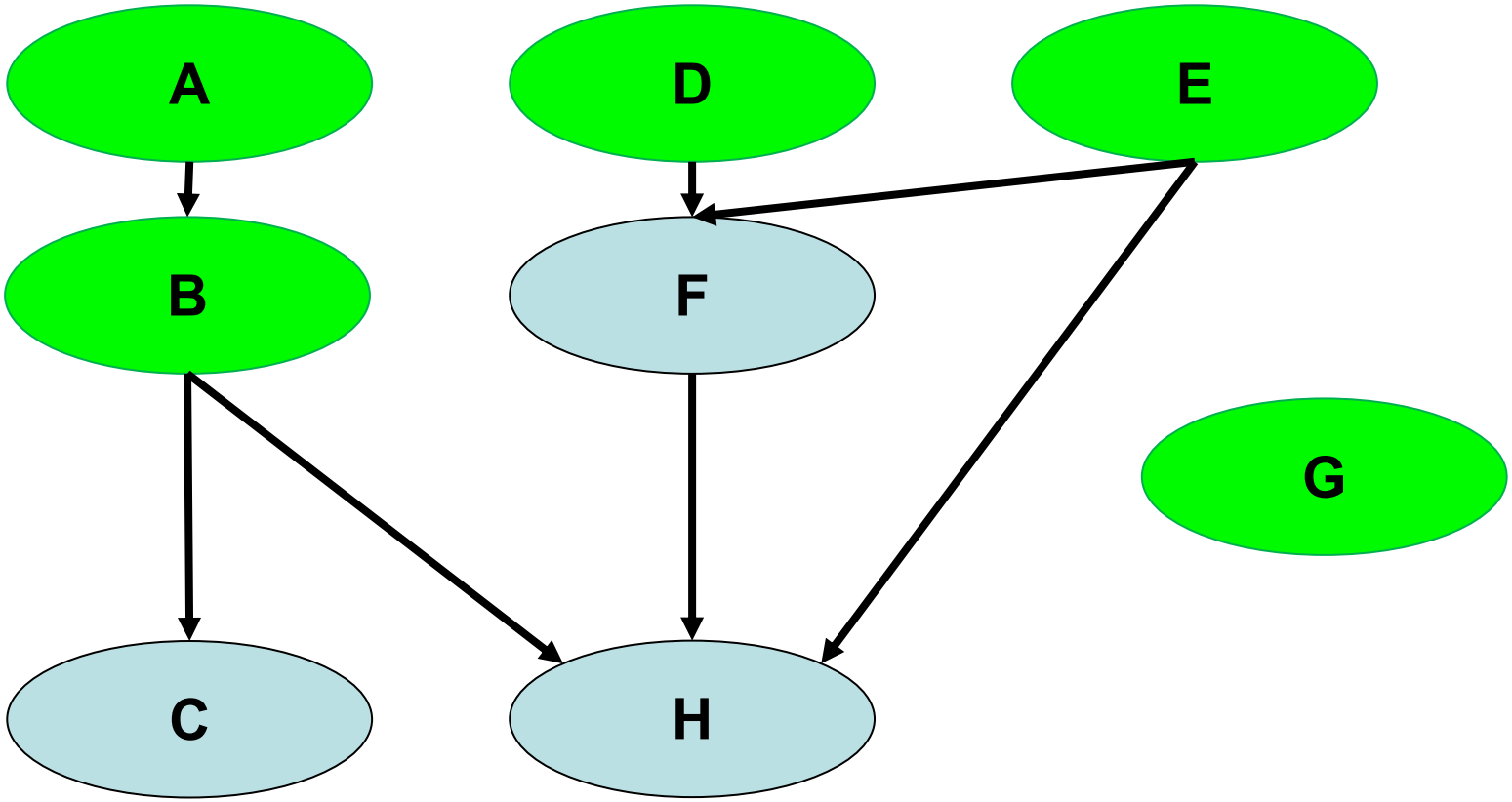
Topological Sort: Take 2



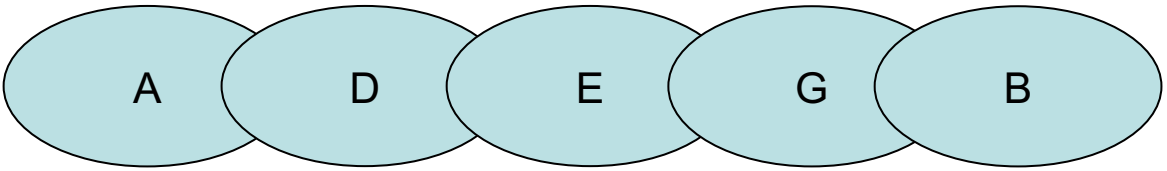
0-In-Degree Queue: { “B”, “F” }
In-Degree Map: { “A”:0, “B”:0, “C”:1, “D”:0, “E”: 0, “F”: 0, “G”:0, “H”:2 }



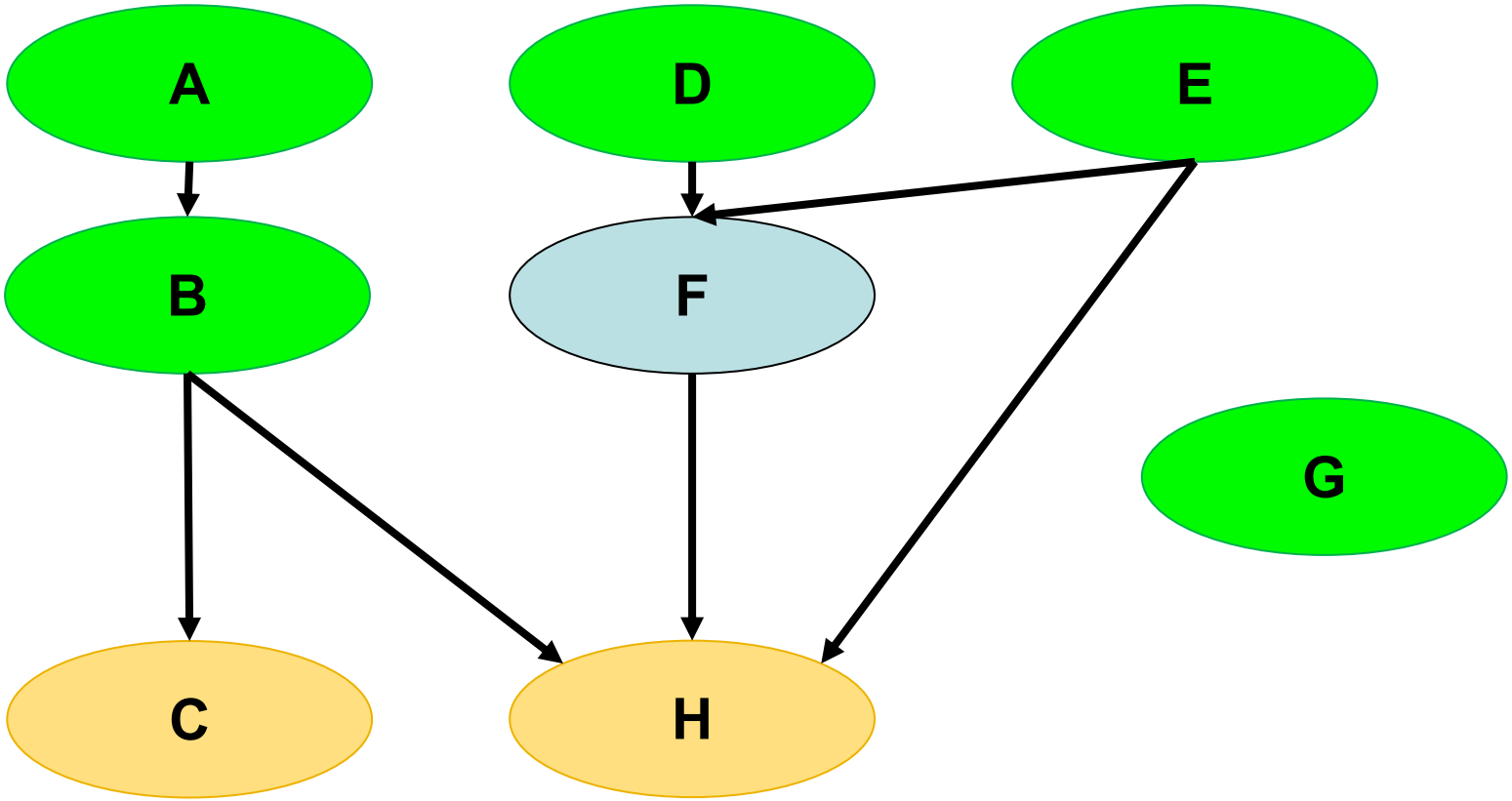
Topological Sort: Take 2



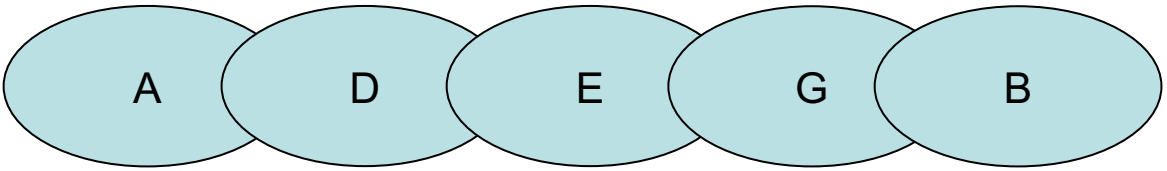
0-In-Degree Queue: { ~~"B"~~, "F" }
In-Degree Map: { "A":0, "B":0, "C":1, "D":0, "E": 0, "F": 0, "G":0, "H":2 }



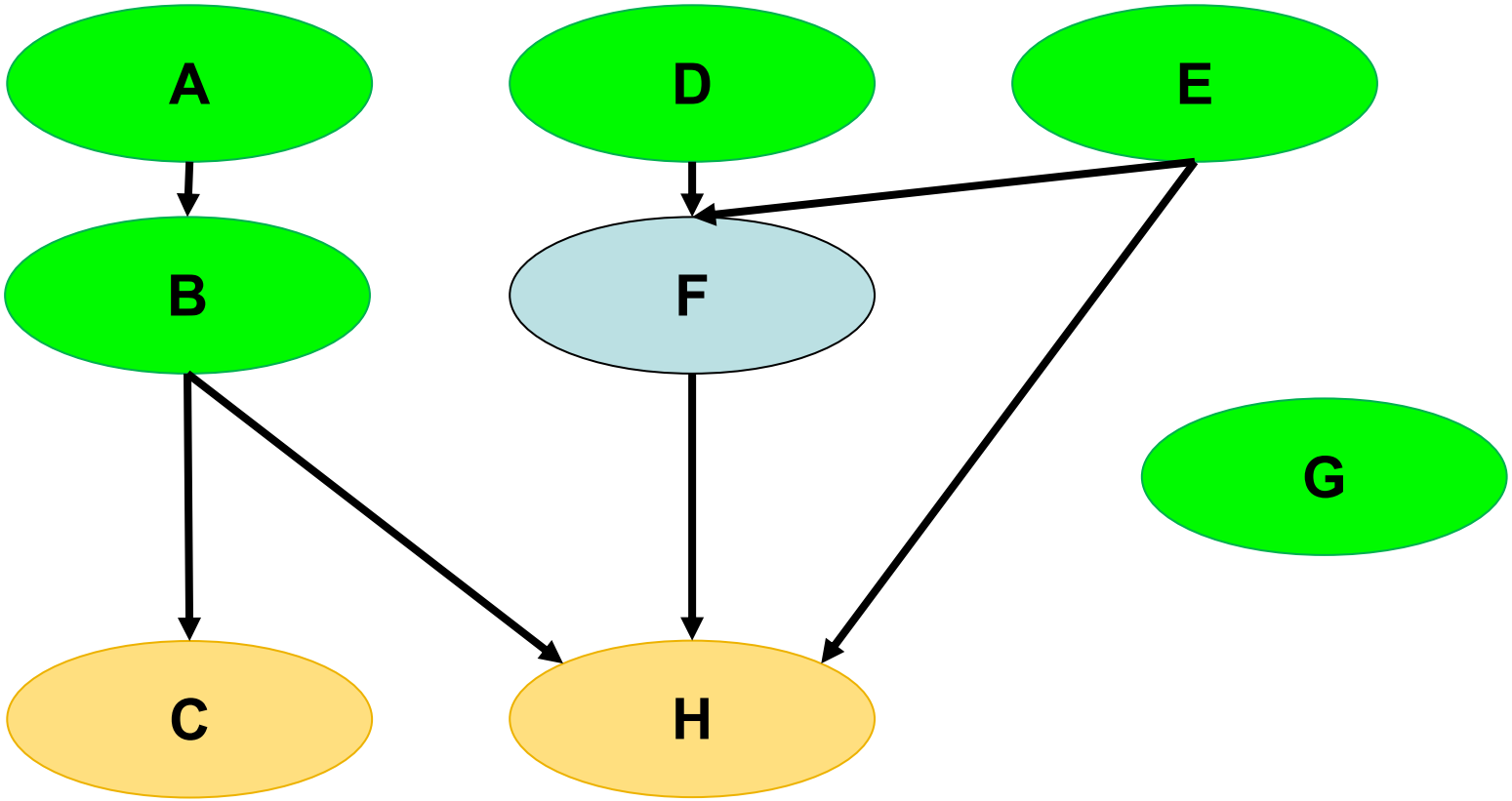
Topological Sort: Take 2



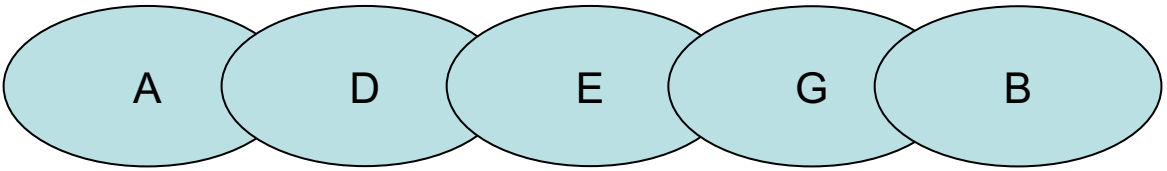
0-In-Degree Queue: { “F” }
In-Degree Map: { “A”:0, “B”:0, “C”:1, “D”:0, “E”: 0, “F”: 0, “G”:0, “H”:2 }



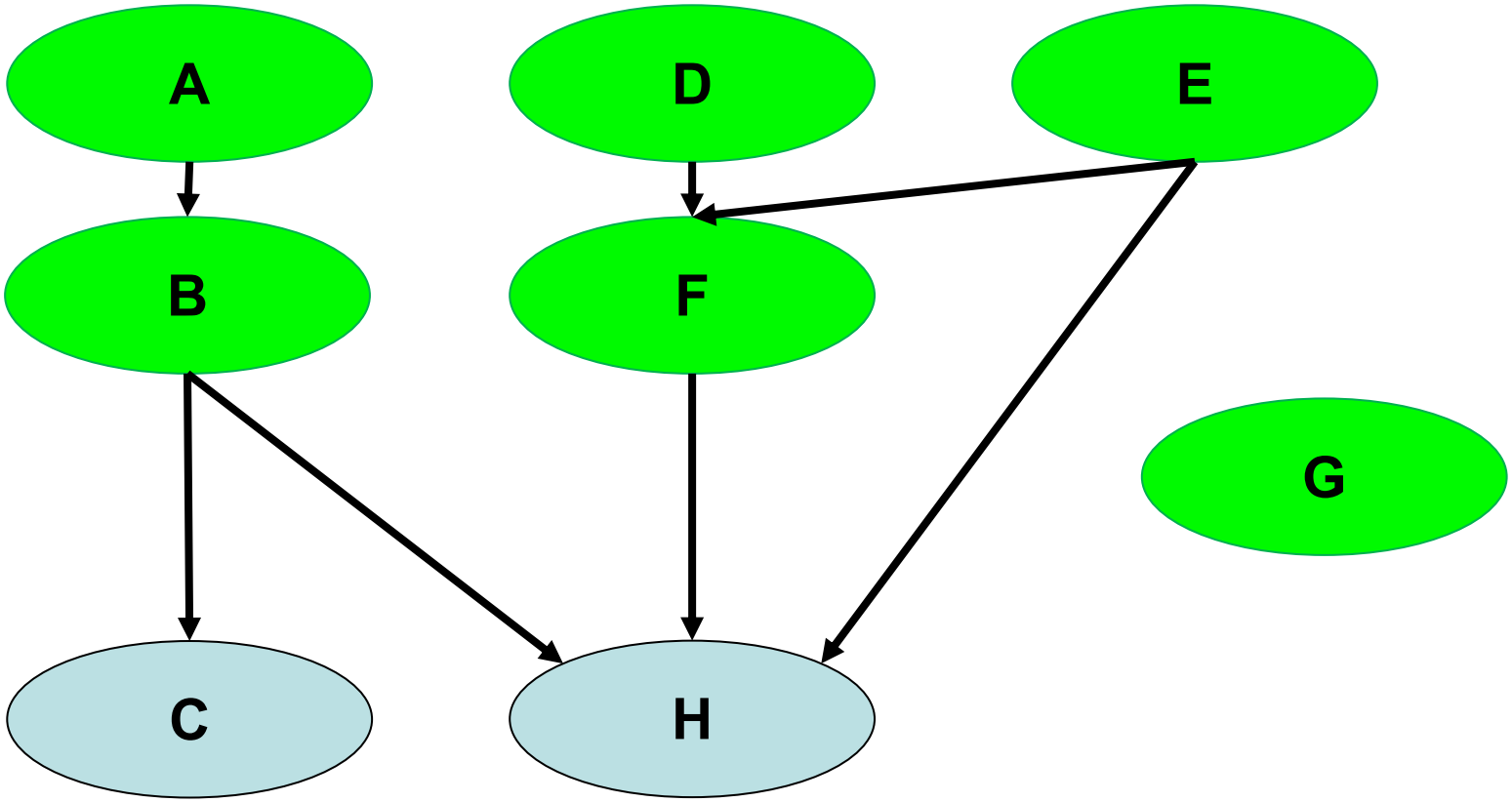
Topological Sort: Take 2



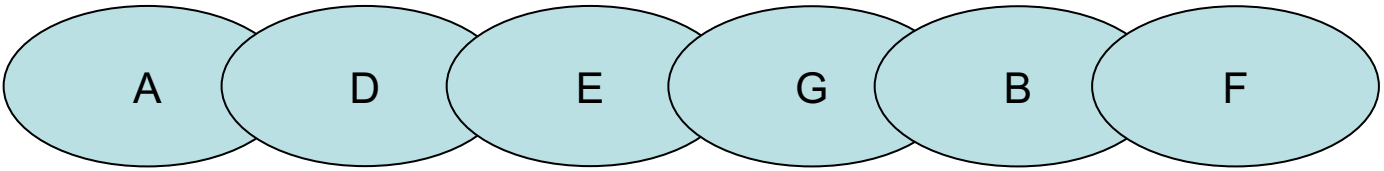
0-In-Degree Queue: { "F", "C" }
In-Degree Map: { "A":0, "B":0, "C":0, "D":0, "E": 0, "F": 0, "G":0, "H":1 }



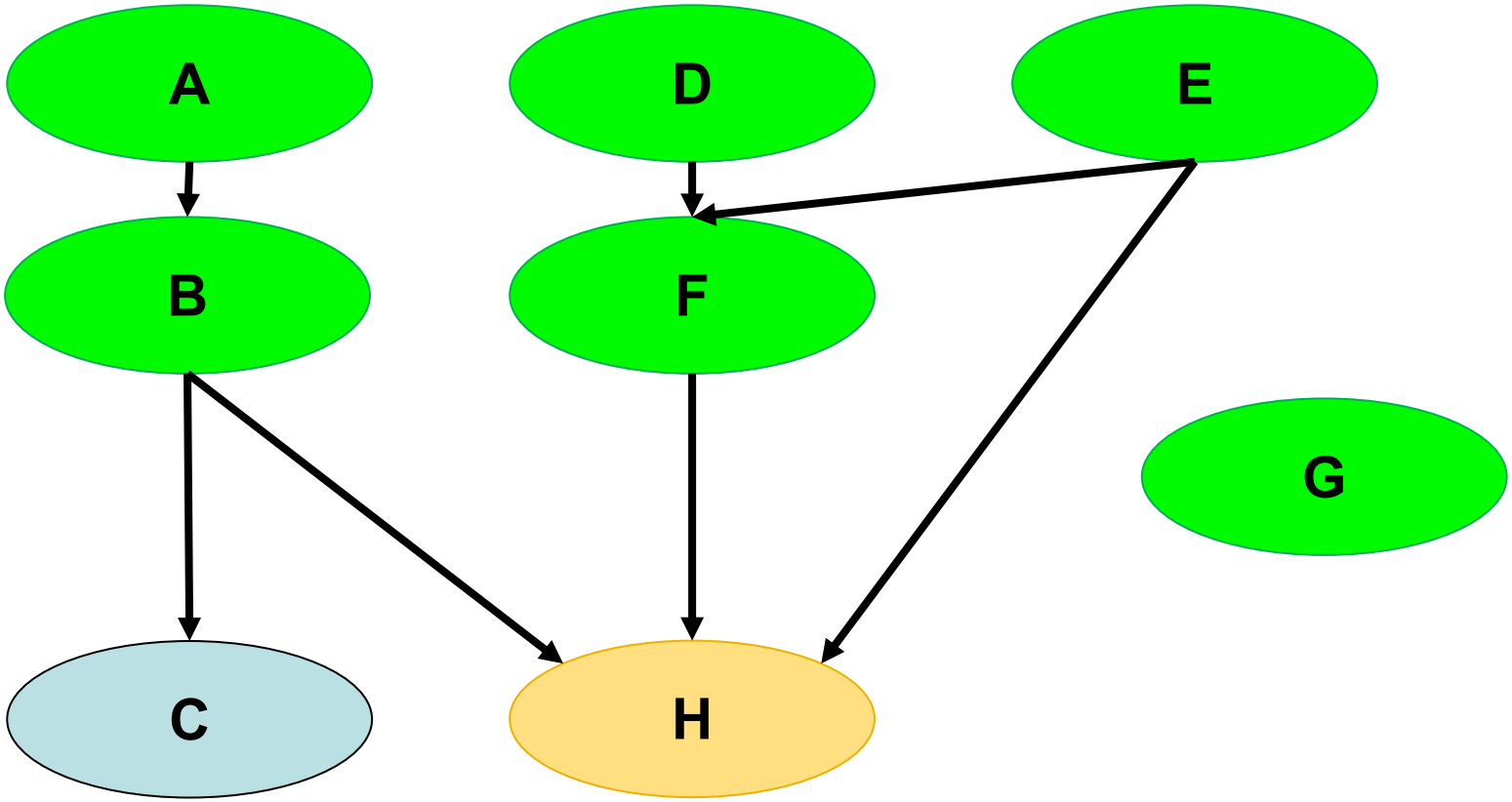
Topological Sort: Take 2



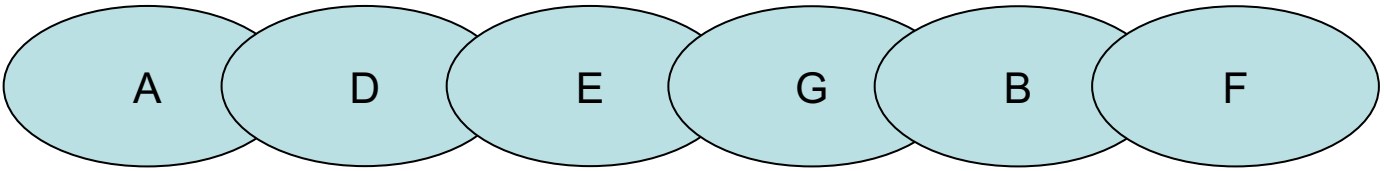
0-In-Degree Queue: { ~~"F"~~, "C" }
In-Degree Map: { "A":0, "B":0, "C":0, "D":0, "E": 0, "F": 0, "G":0, "H":1 }



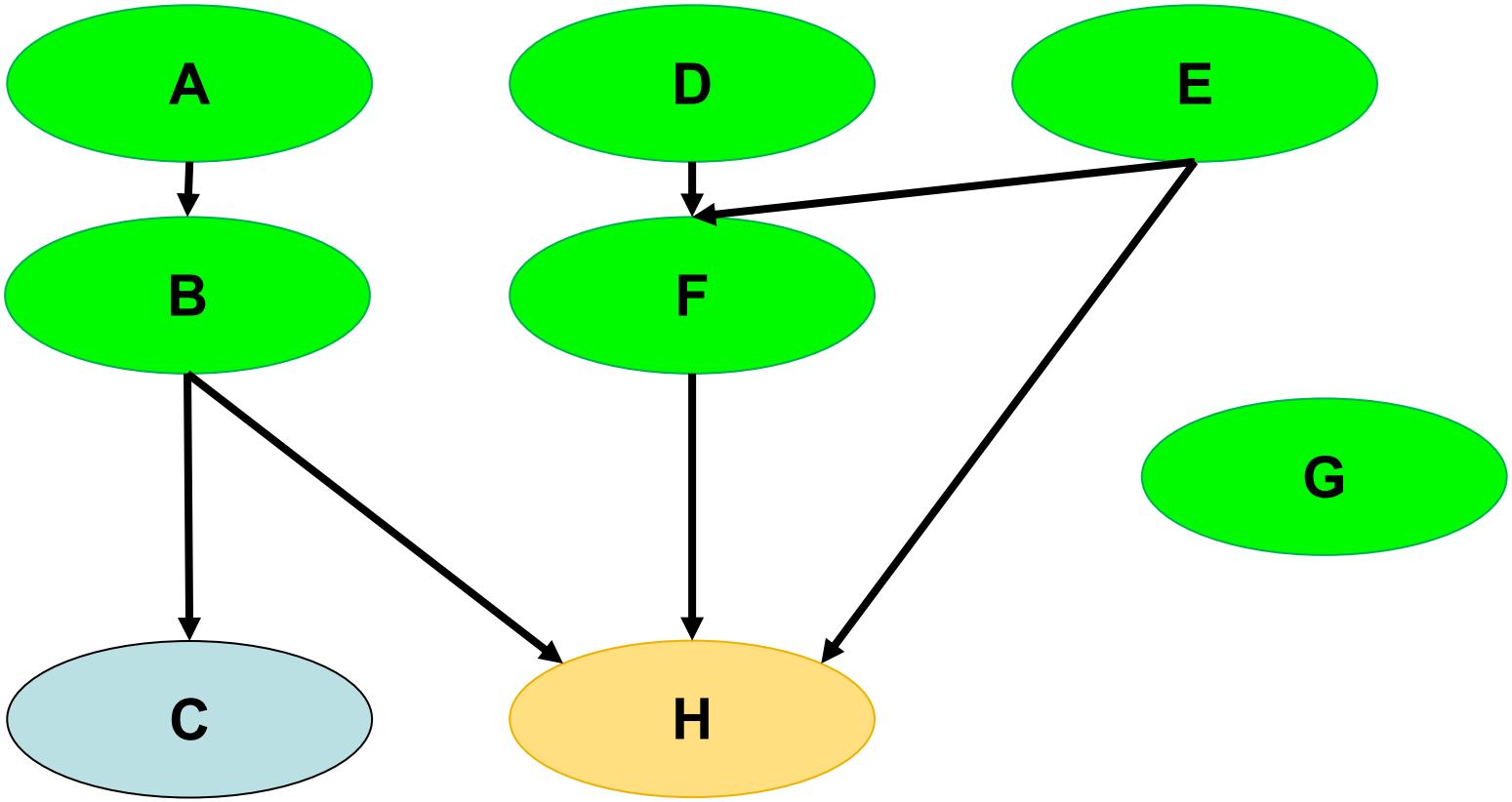
Topological Sort: Take 2



0-In-Degree Queue: { “C” }
In-Degree Map: { “A”:0, “B”:0, “C”:0, “D”:0, “E”: 0, “F”: 0, “G”:0, “H”:1 }



Topological Sort: Take 2



0-In-Degree Queue: { "C", "H" }
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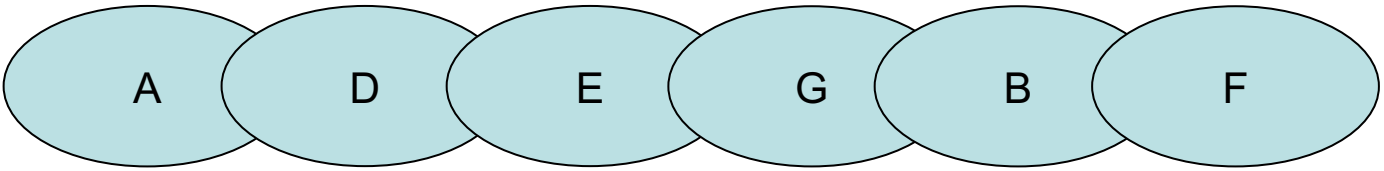

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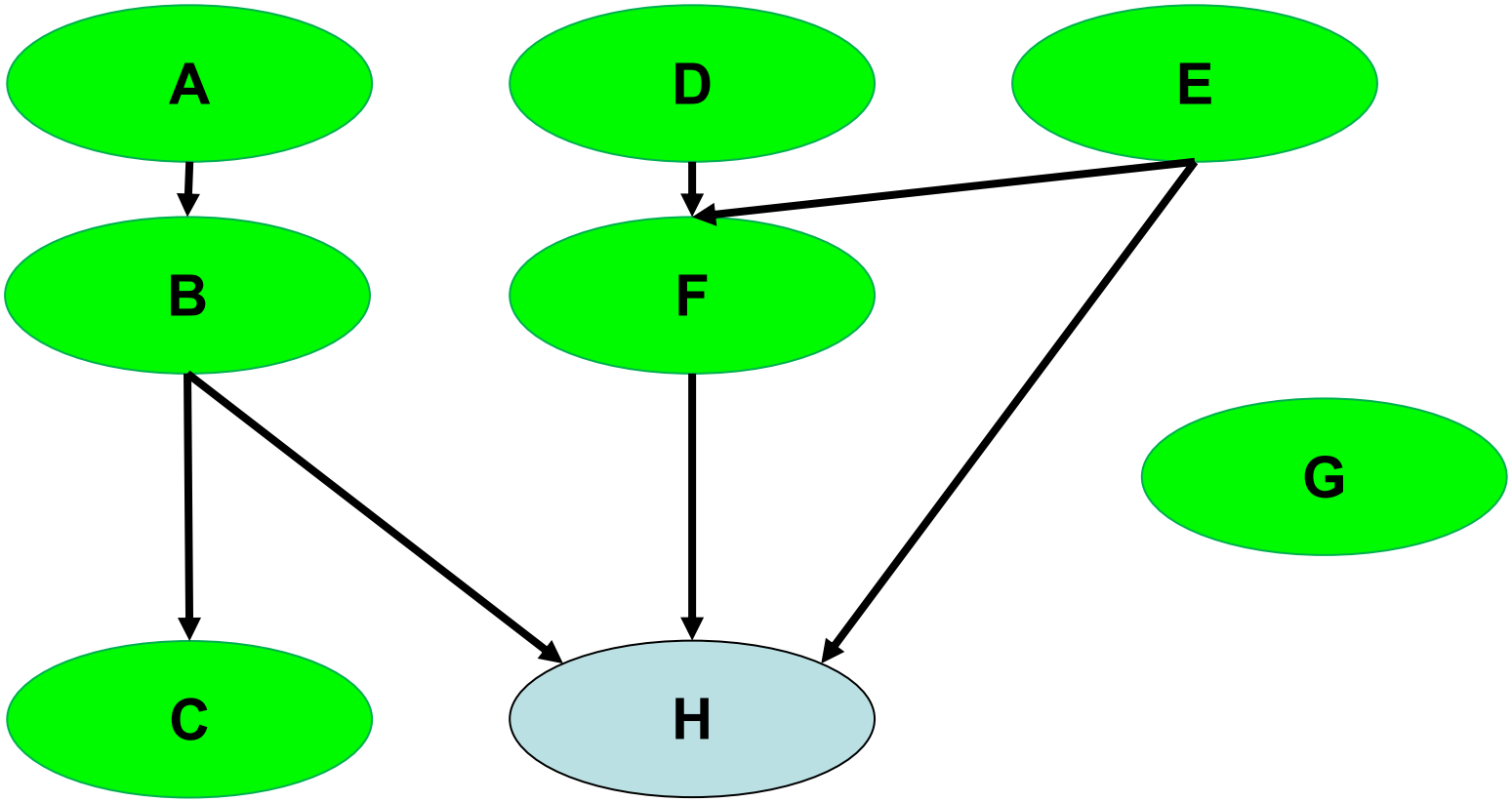

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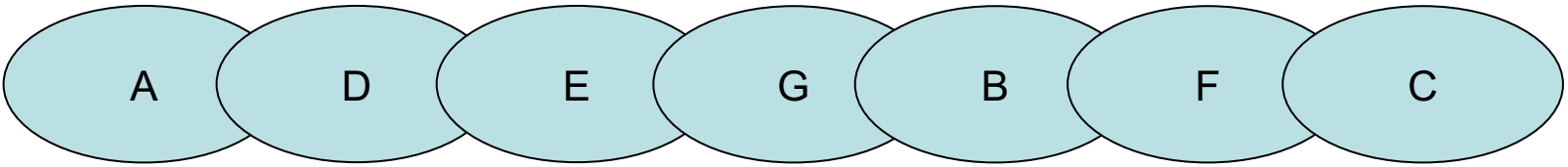

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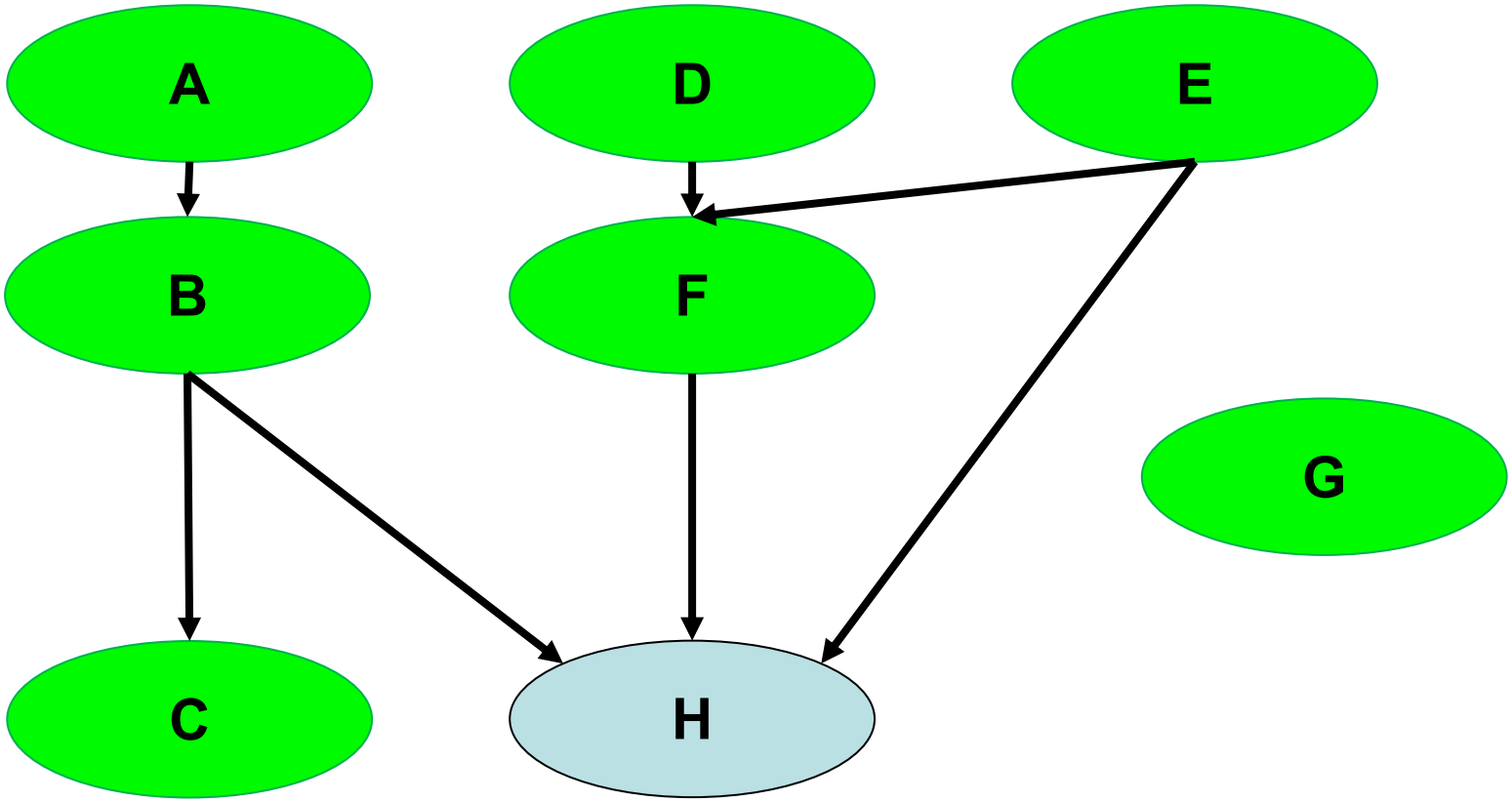
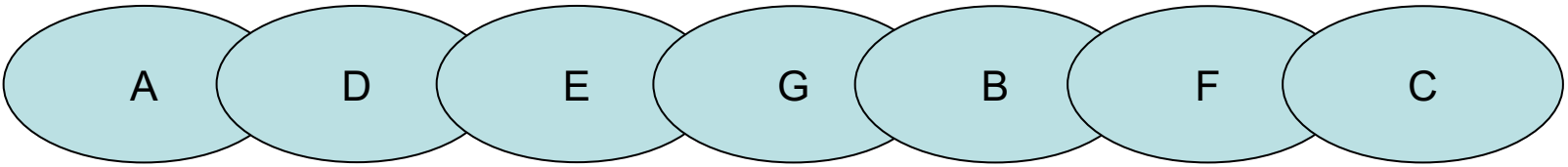


0-In-Degree Queue: { “H” }
In-Degree Map: { “A”:0, “B”:0, “C”:0, “D”:0, “E”: 0, “F”: 0, “G”:0, “H”:0 }



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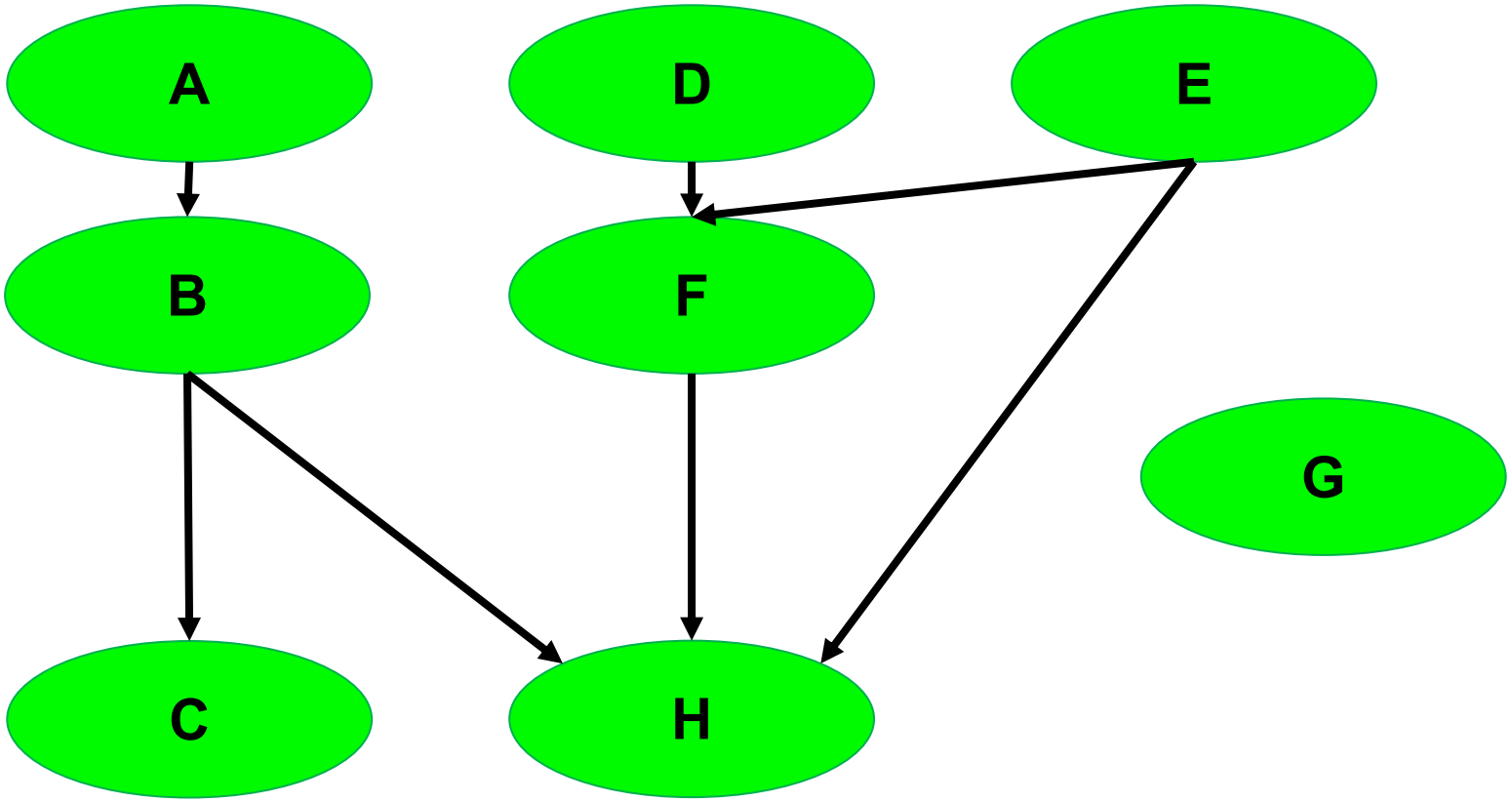

0-In-Degree Queue: { ~~"H"~~ }

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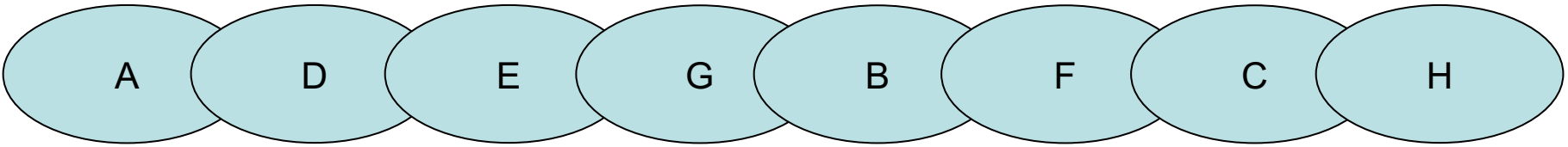

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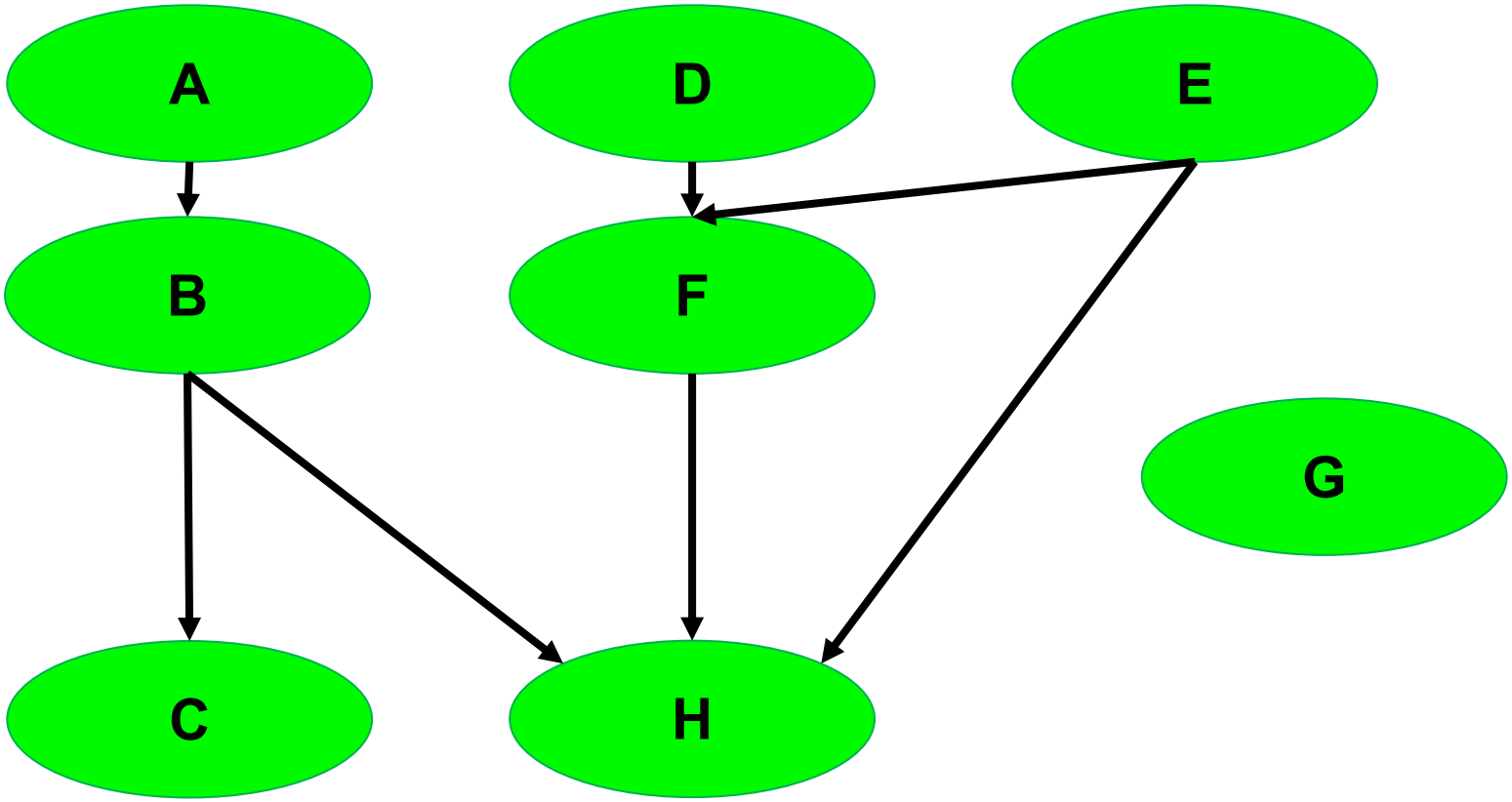

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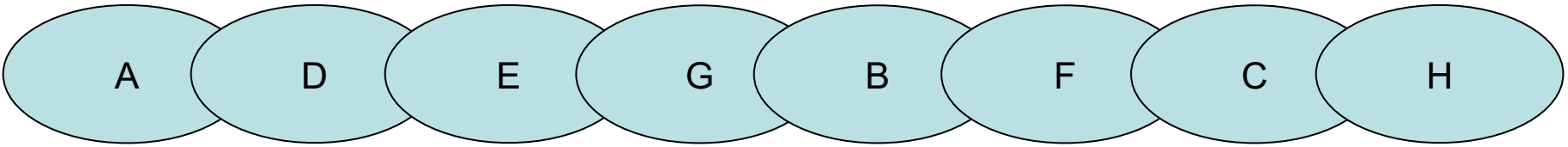

Plan for Today

- **Topological Sort**
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- Announcements

Kahn's Algorithm

map := {each vertex → its in-degree}.

queue := {all vertices with in-degree = 0}.

ordering := { }.

Repeat until queue is empty:

Dequeue the first vertex *v* from the queue.

ordering += *v*.

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queue += {any neighbors whose in-degree is now 0}.

- **This algorithm doesn't modify the passed-in graph! ☺**
- **Have we handled all edge cases?**
- **What is the runtime of this algorithm?**

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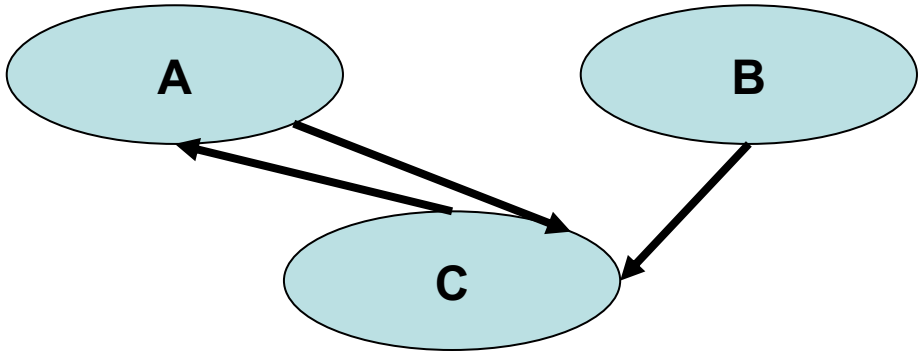

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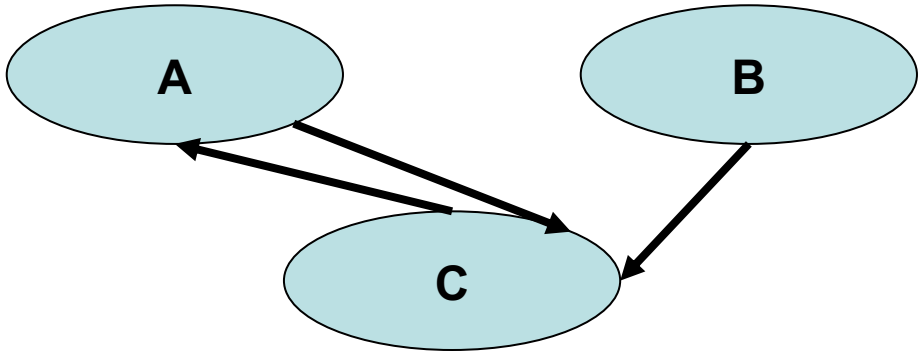
0-In-Degree Queue: { "B" }
In-Degree Map: { A":1, "B":0, "C": 2 }

Ordering: { }



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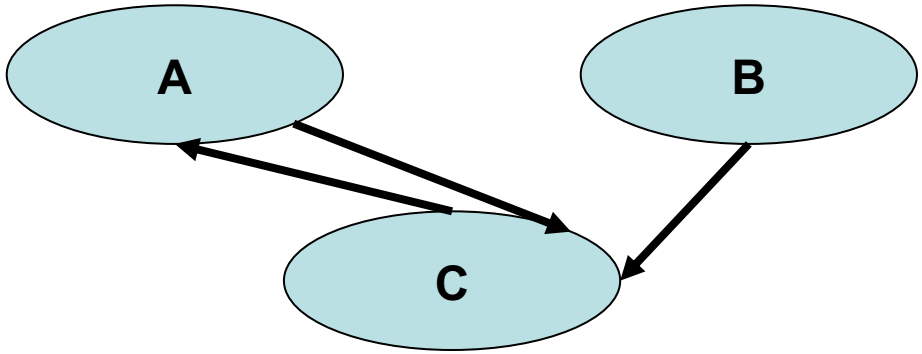

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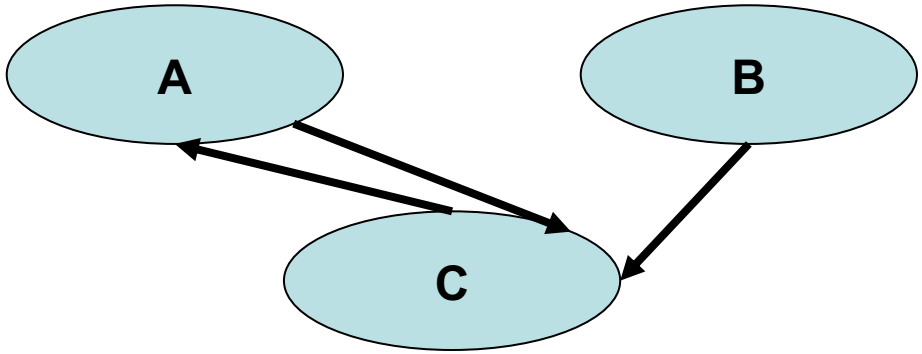

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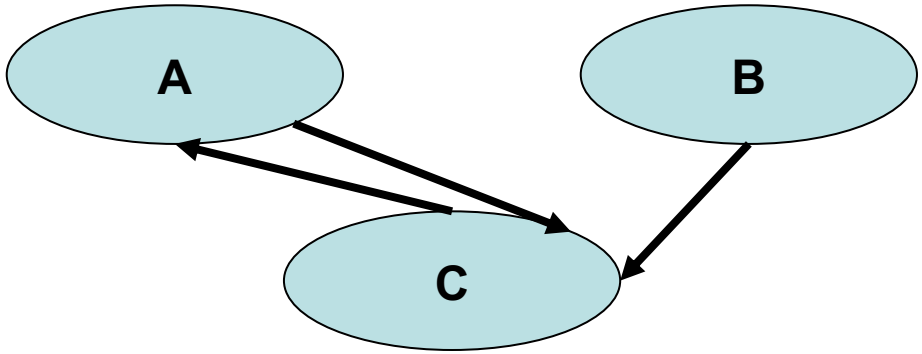

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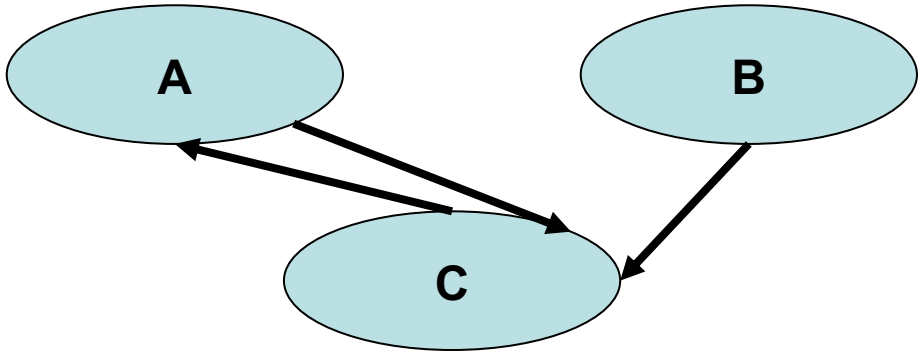

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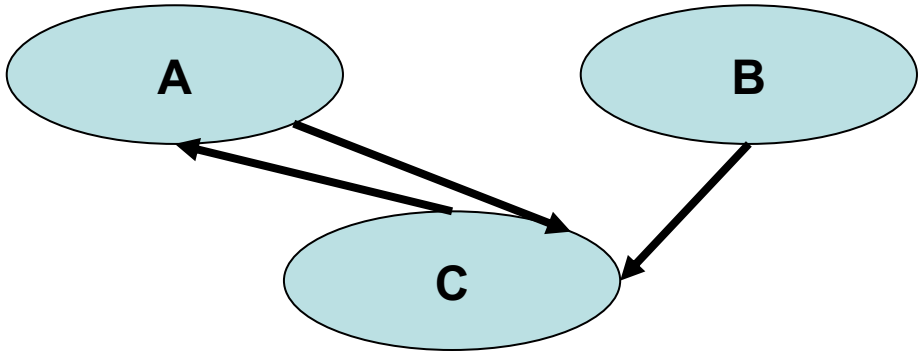

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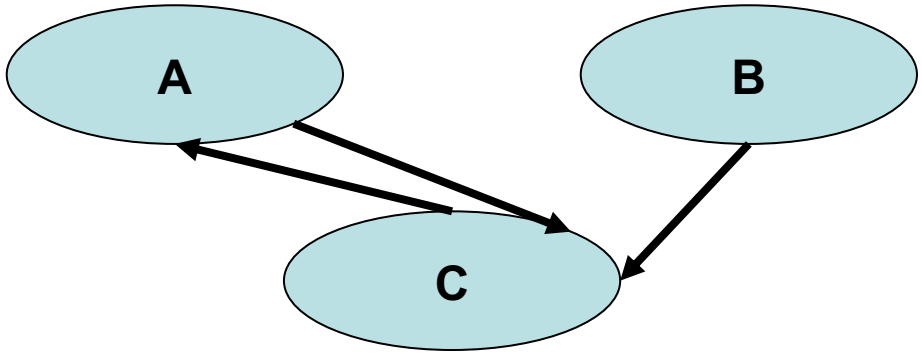

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Dequeue the first vertex *v* from the queue.

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Decrease the in-degree of all *v*'s neighbors by 1 in the *map*.

queue += {any neighbors whose in-degree is now 0}.

If all vertices processed, success! Otherwise, there is a cycle.

- **This algorithm doesn't modify the passed-in graph! ☺**
- **Have we handled all edge cases?**
- **What is the runtime of this algorithm?**

Kahn's Algorithm: Runtime

For graph with V vertexes and E edges

map := {each vertex → its in-degree}.

queue := {all vertices with in-degree = 0}.

ordering := { }.

Repeat until queue is empty:

- Dequeue the first vertex *v* from the queue.
- *ordering* += *v*.
- Decrease the in-degree of all *v*'s neighbors by 1 in the *map*.
- *queue* += {any neighbors whose in-degree is now 0}.

Kahn's Algorithm: Runtime

For graph with V vertexes and E edges

O(V)

map := {each vertex → its in-degree}.

queue := {all vertices with in-degree = 0}.

ordering := { }.

O(V)

Repeat until queue is empty:

O(1)

Dequeue the first vertex *v* from the queue.

ordering += *v*.

O(E)

Decrease the in-degree of all *v*'s neighbors by 1 in the *map*.

queue += {any neighbors whose in-degree is now 0}.

O(VE)

Kahn's Algorithm: Runtime

For graph with V vertexes and E edges

O(V)

map := {each vertex → its in-degree}.

queue := {all vertices with in-degree = 0}.

ordering := { }.

O(V)

Repeat until queue is empty:

O(1)

Dequeue the first vertex *v* from the queue.

ordering += *v*.

O(E) *total*

Decrease the in-degree of all *v*'s neighbors by 1 in the *map*.

queue += {any neighbors whose in-degree is now 0}.

O(V + E)

Plan for Today

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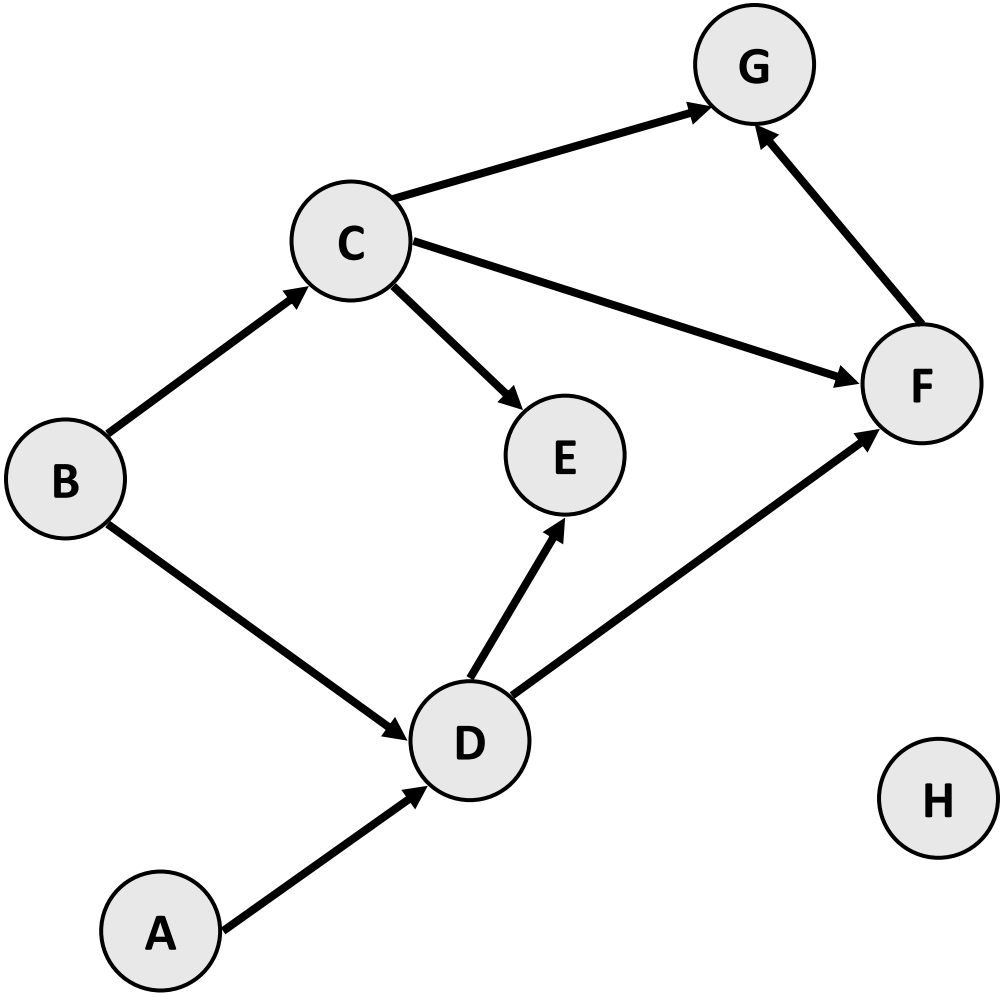
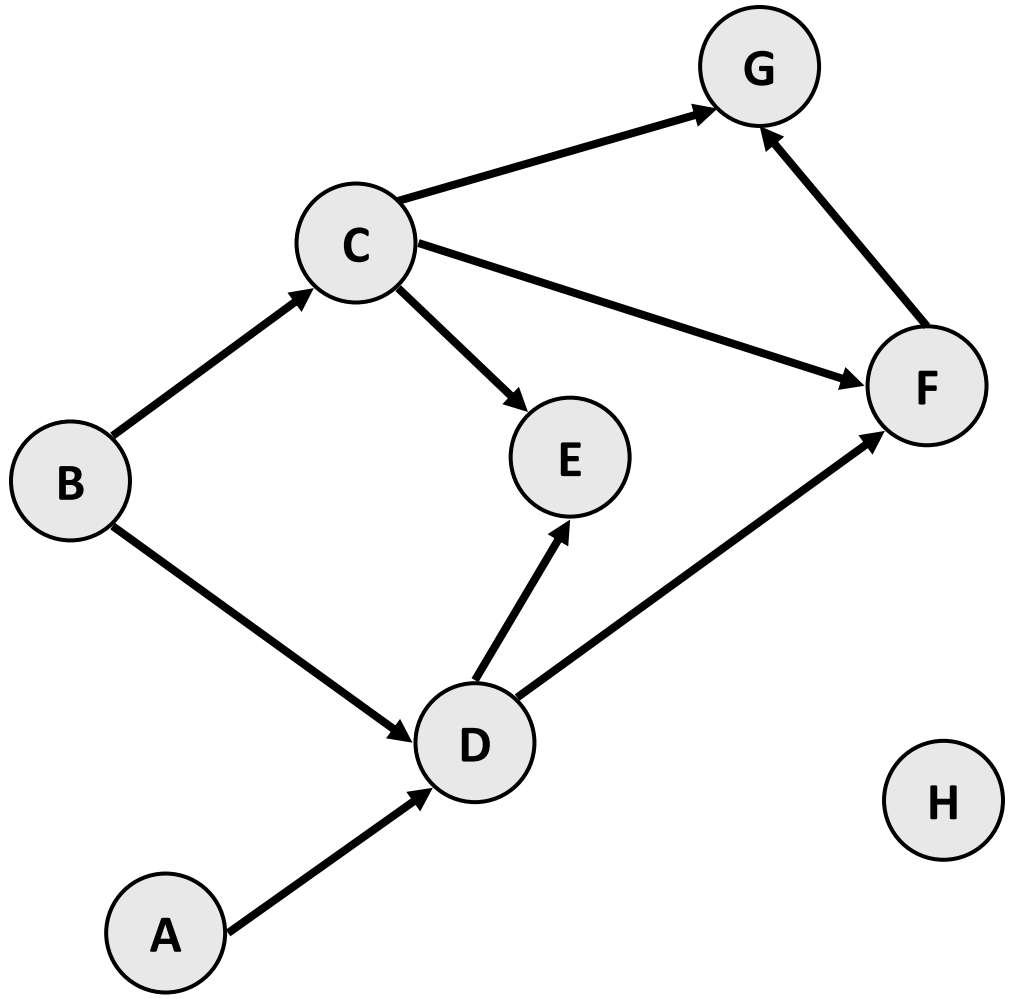
Revisiting Recursive DFS

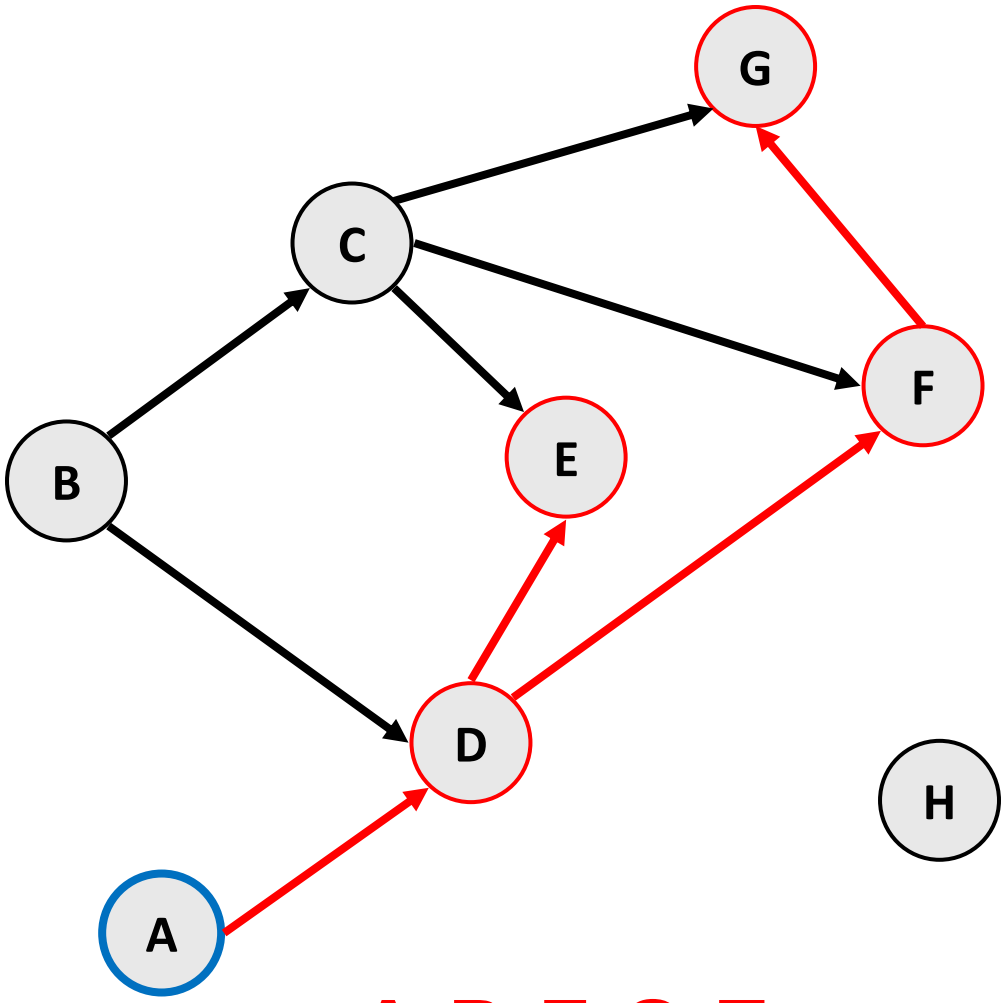


Revisiting Recursive DFS



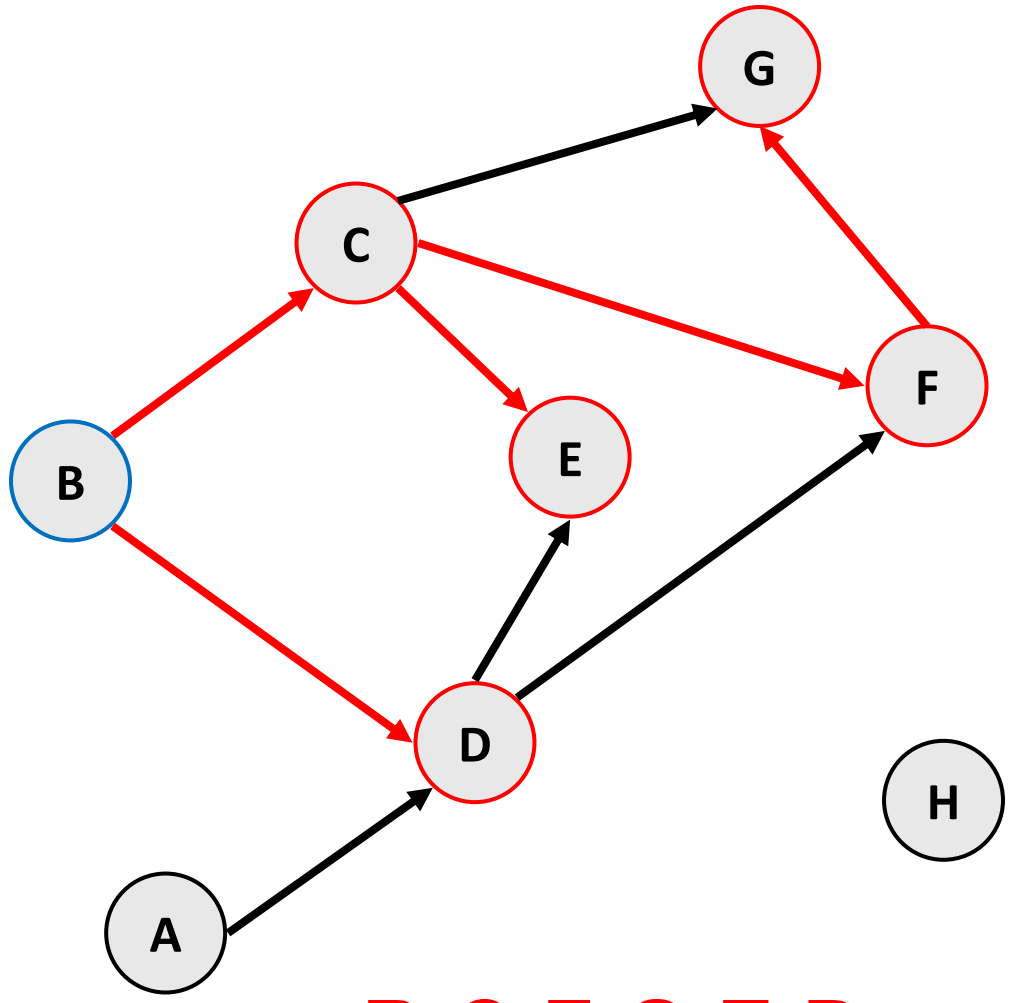
Revisiting Recursive DFS

- **Key idea:** visiting node *a* means we must have visited all nodes that have *a* as a prerequisite.



Revisiting Recursive DFS

- **Key idea:** visiting node *a* means we must have visited all nodes that have *a* as a prerequisite.
- But different starting points may yield different subsets. Is there a way we can combine them into one complete topological ordering?



B-C-F-G-E-D

Revisiting Recursive DFS

A D F G E

B C F G E D

C F G E

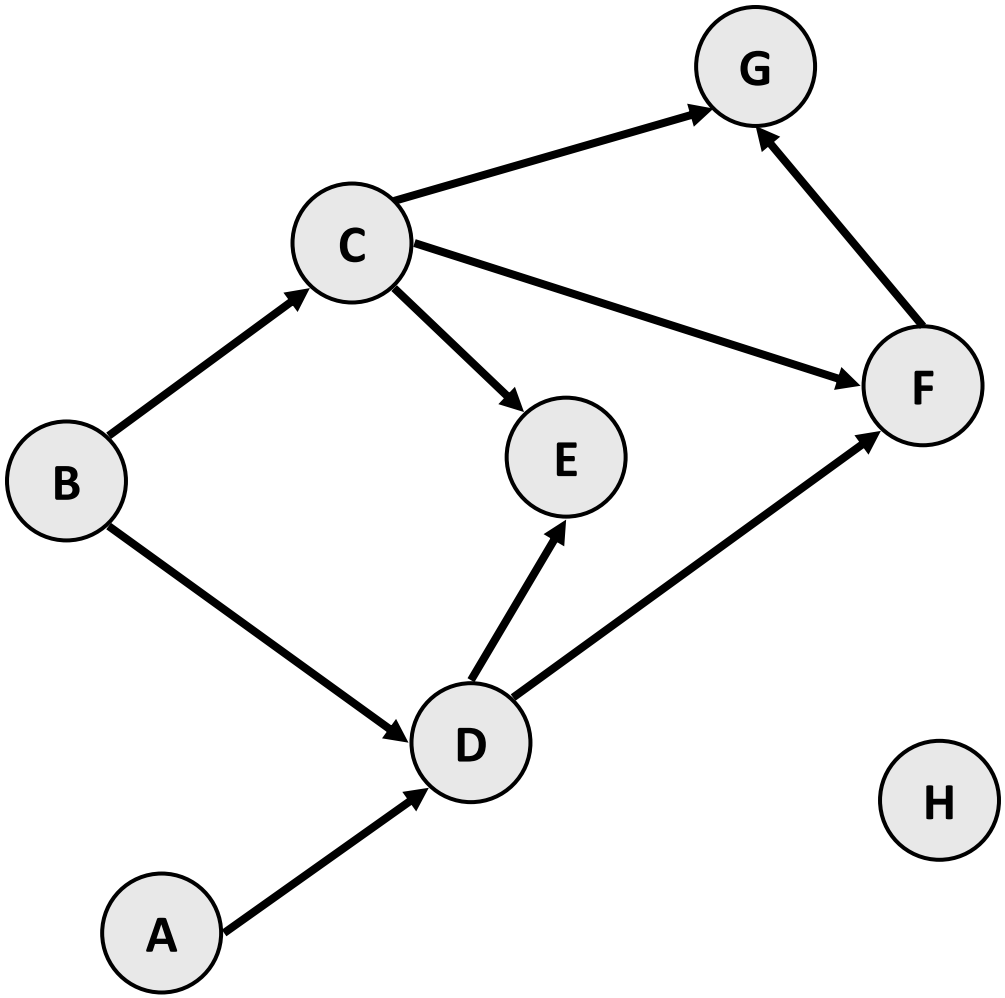
D F G E

E

F G

G

H



Revisiting Recursive DFS

A D F G E

B C

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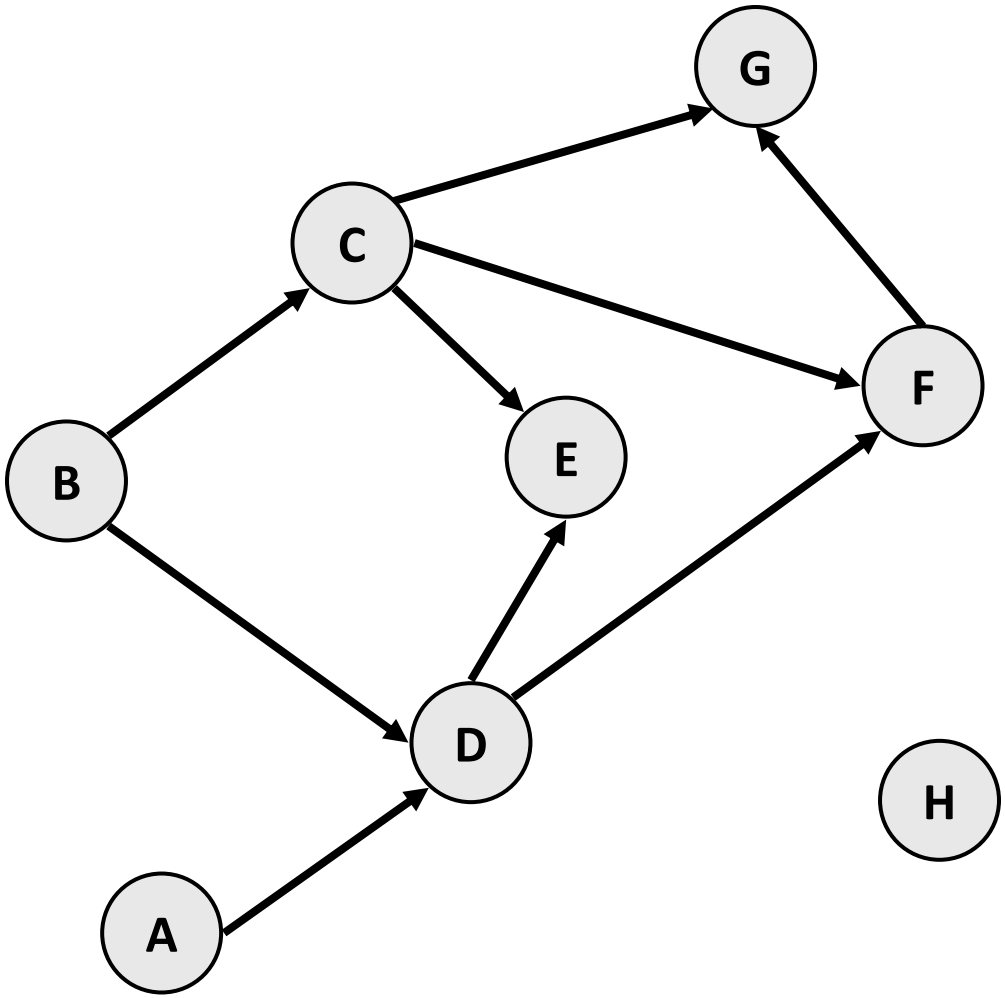
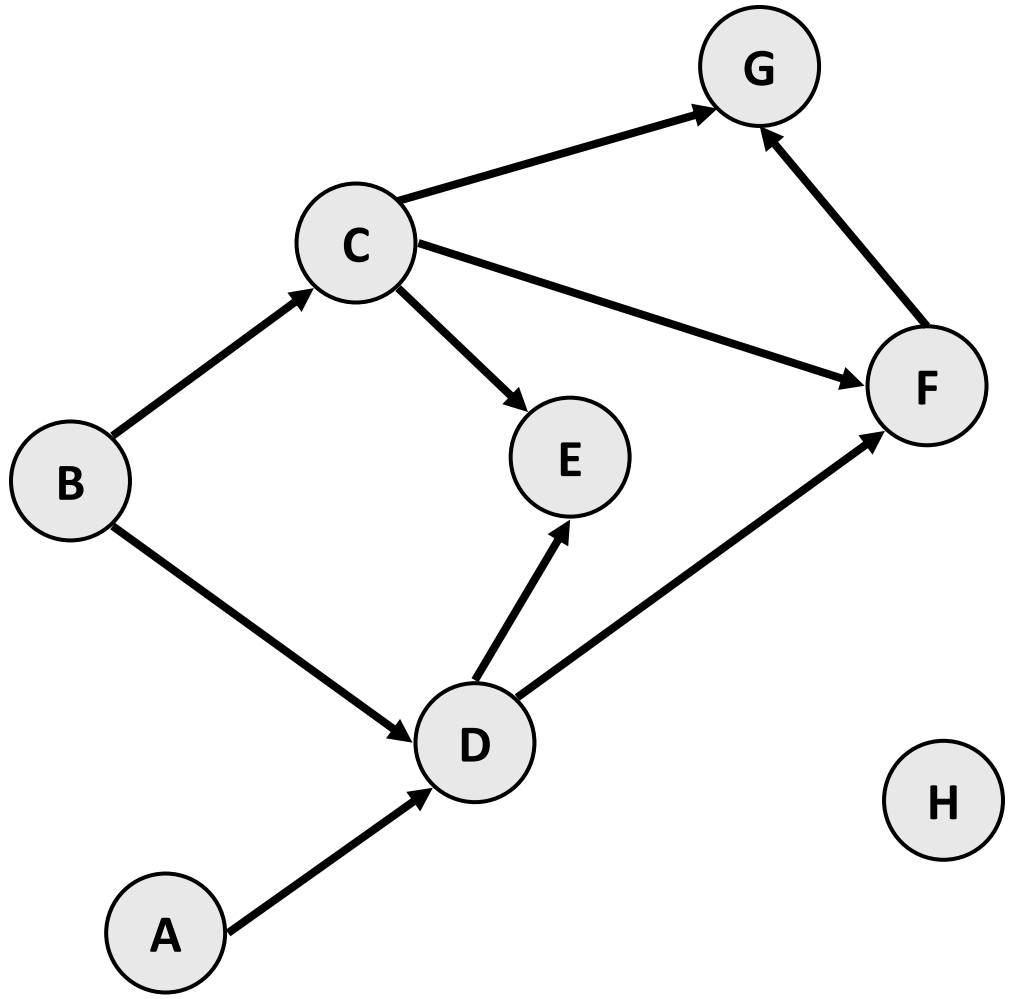


Revisiting Recursive DFS

H B C A D F G E



Revisiting Recursive DFS

H

G

F G

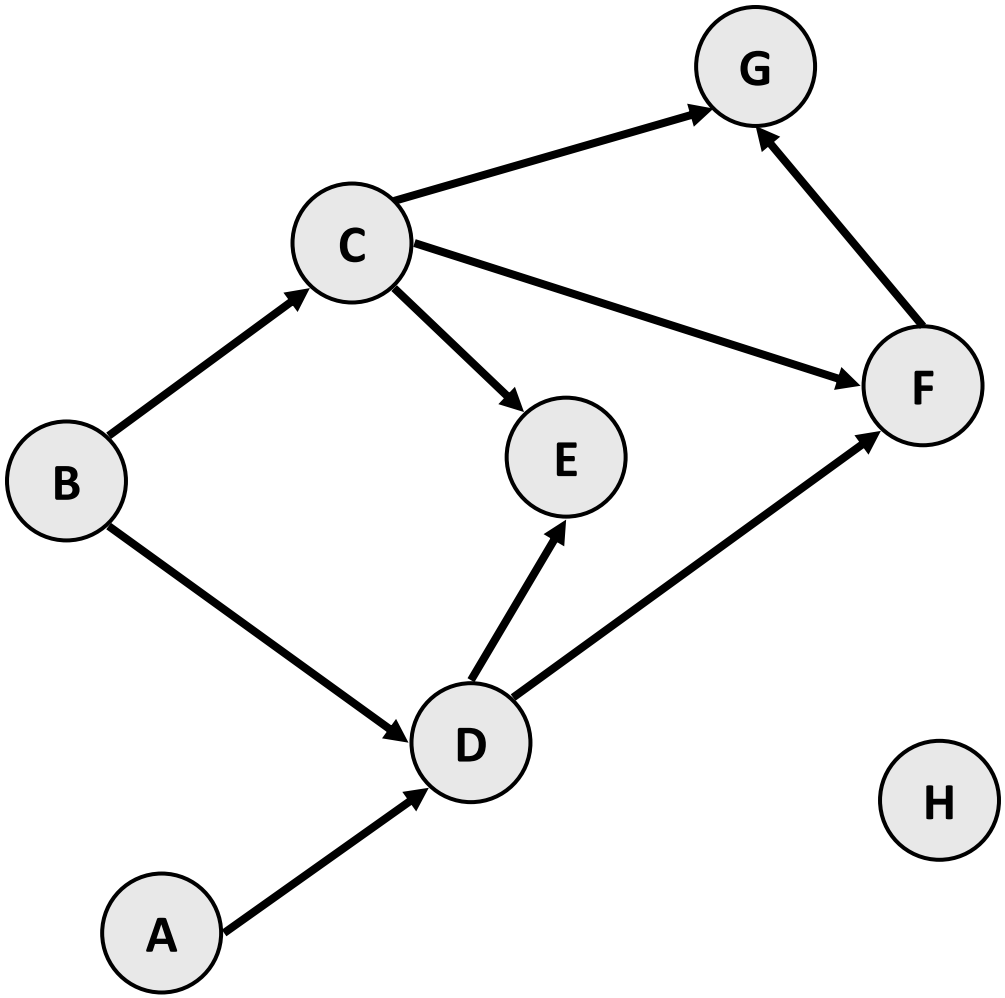
E

D F G E

C F G E

B C F G E D

A D F G E



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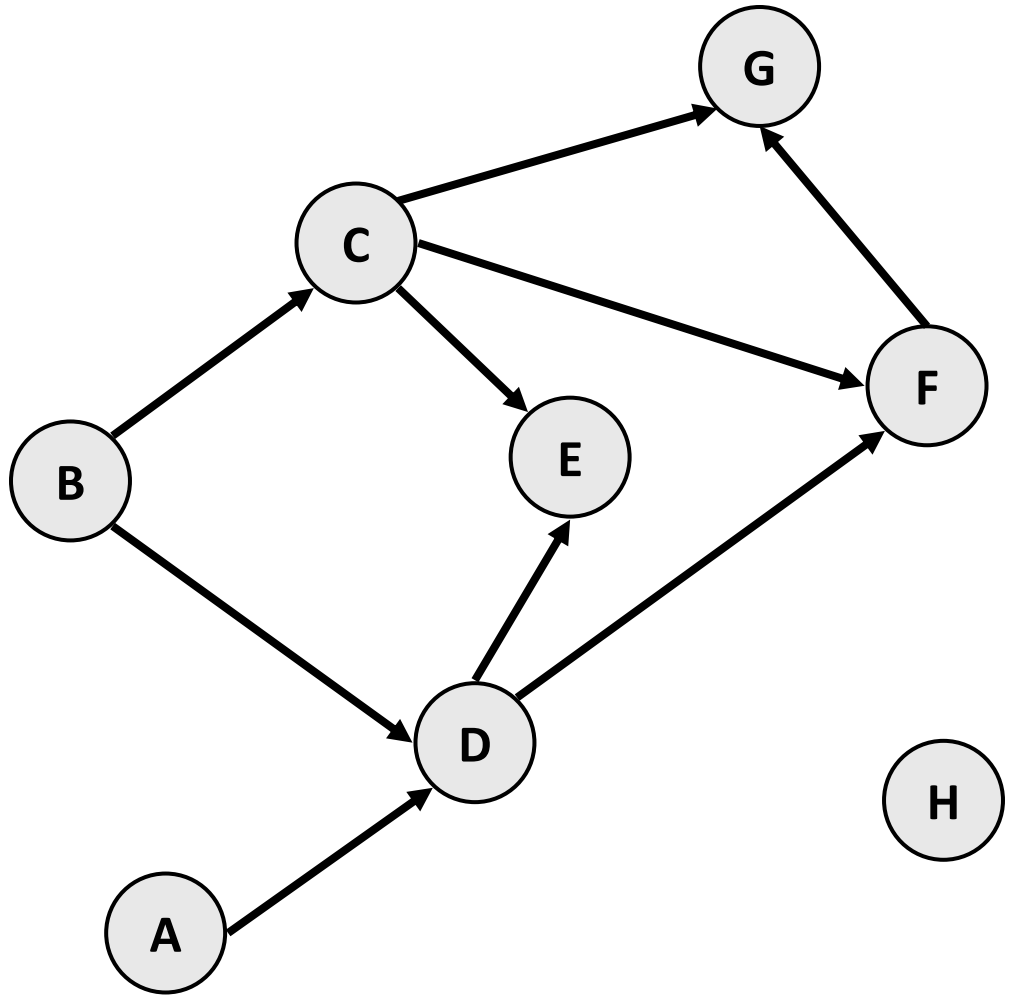
E

D

C

B

A



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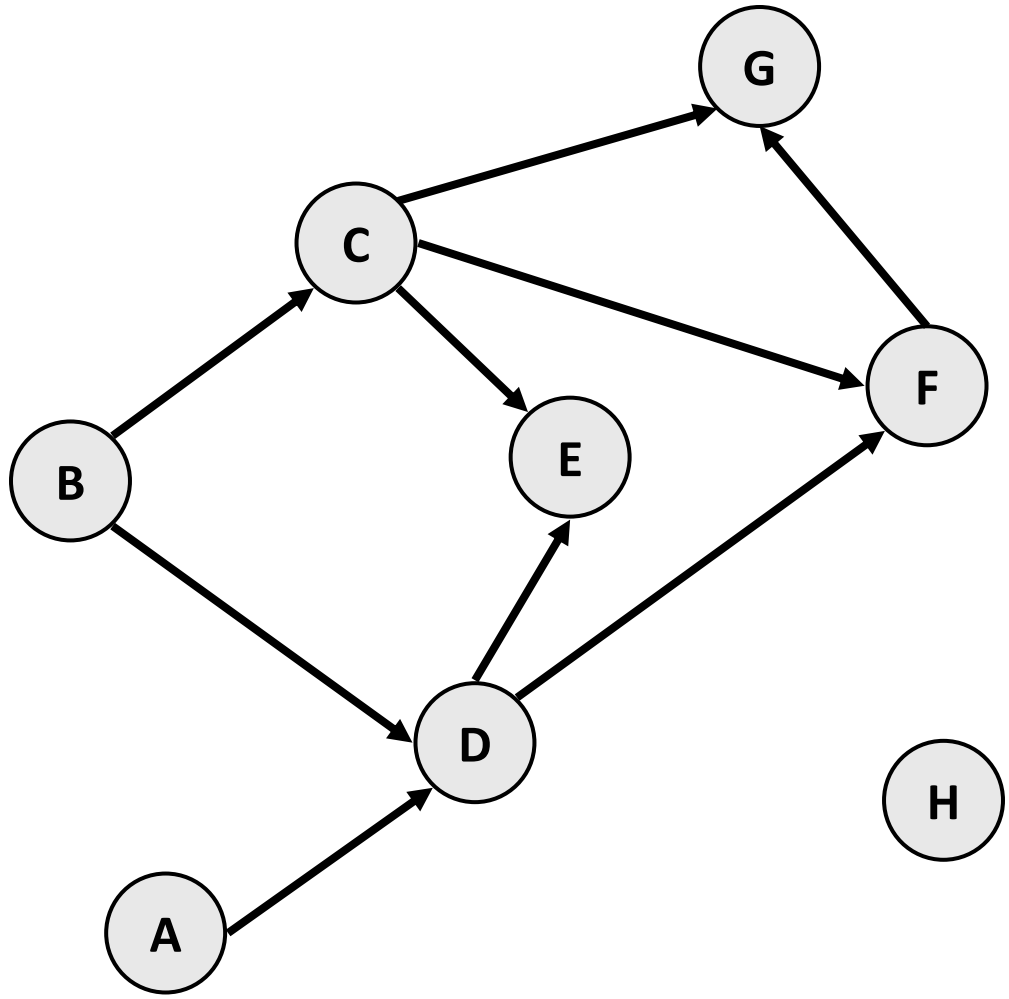
A B C D E F G H



Topological Sort: DFS

- function **topologicalSortDFS**():

  L = { }. // to store the sorted vertexes

  while graph contains any unmarked vertexes:

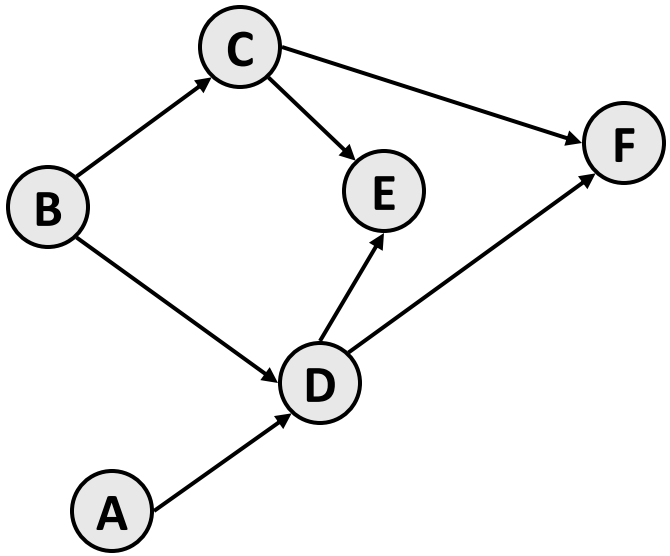
  - select any unmarked vertex *v*.
  - **visit**(*L*, *v*).

- function **visit**(*L*, v):

  if *v* is unmarked:

  - for each neighbor *n* of *v:*

    visit(*L*, *n*).
  - mark *v*.
  - add *v* to front of L.



Topological Sort: DFS

- function **topologicalSortDFS**():

  L = { }. // to store the sorted vertexes

  while graph contains any unmarked vertexes:

  - select any unmarked vertex *v*.
  - **visit**(*L*, *v*).

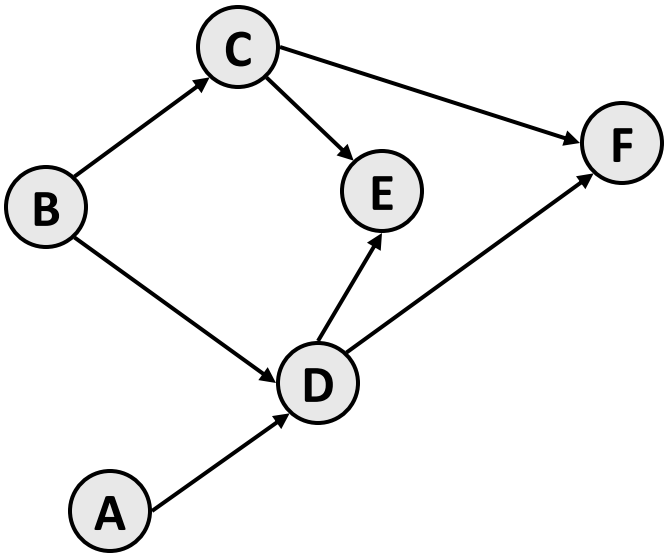
- function **visit**(*L*, v):

  if *v* has a temp.mark, error.

  if *v* is unmarked:

  - temp.mark *v.*
  - for each neighbor *n* of *v:*

    visit(*L*, *n*).
  - mark *v*.
  - add *v* to front of L.



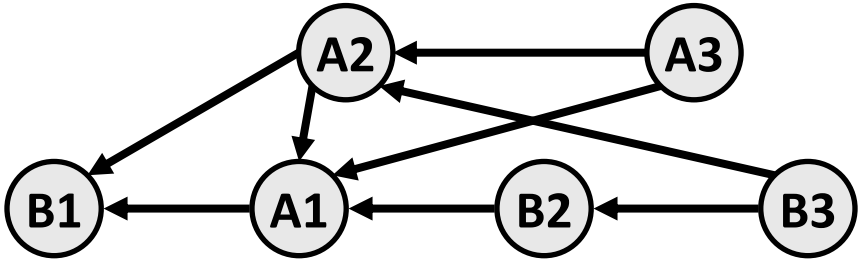
Plan for Today

- **Topological Sort**
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- Announcements

Topological Sort: Spreadsheets

Cells with formulas referencing other cells have **dependencies**.

	A	B
1	**20** =B1*2	**10**
2	**30** =A1+B1	**25** =A1+5
3	**50** =SUM(A1:A2)	**0.833** =B2/A2



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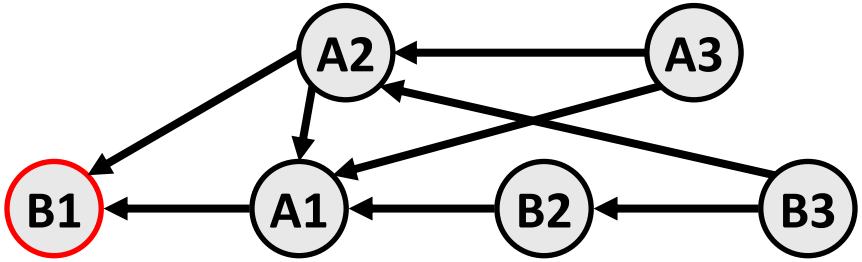

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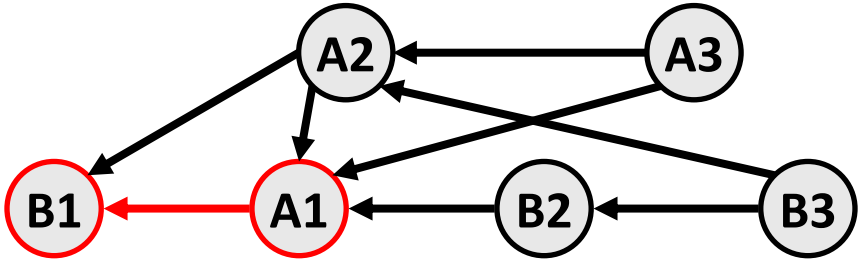

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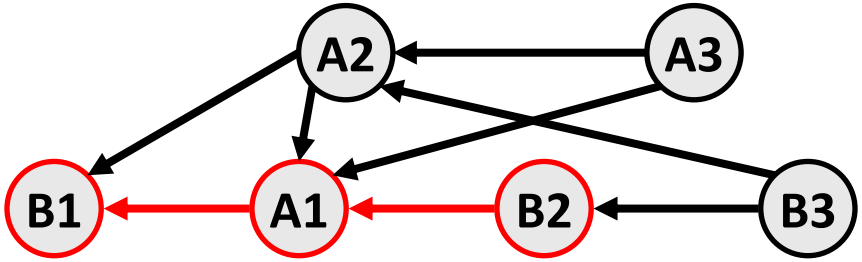

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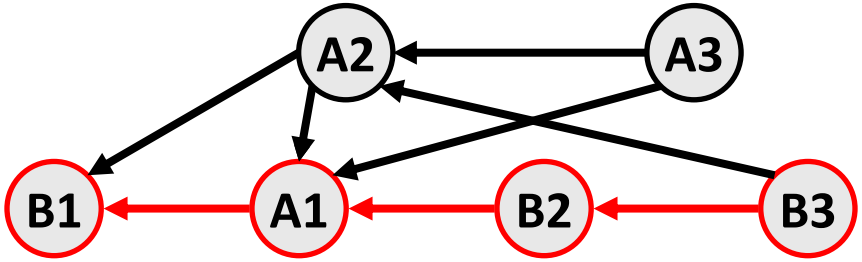

Topological Sort: Spreadsheets

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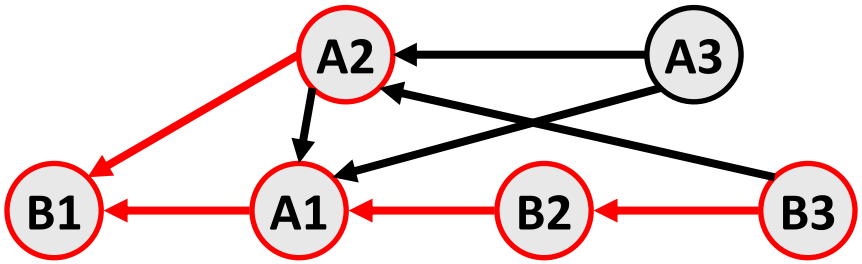

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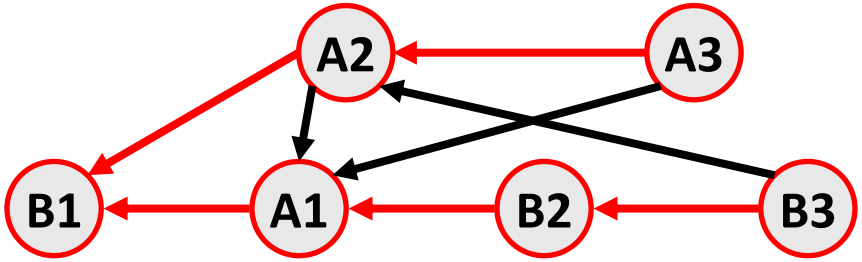

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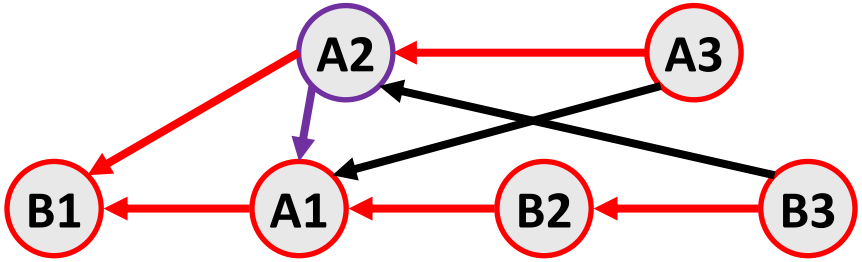

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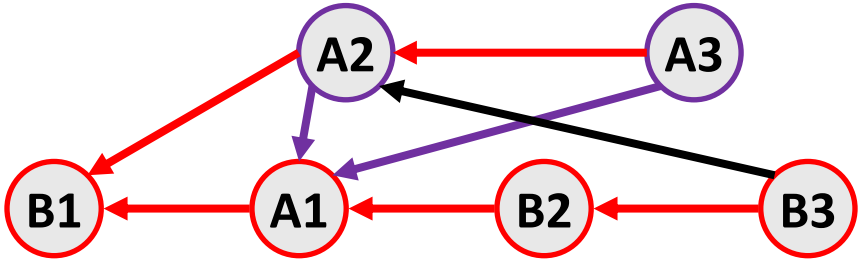

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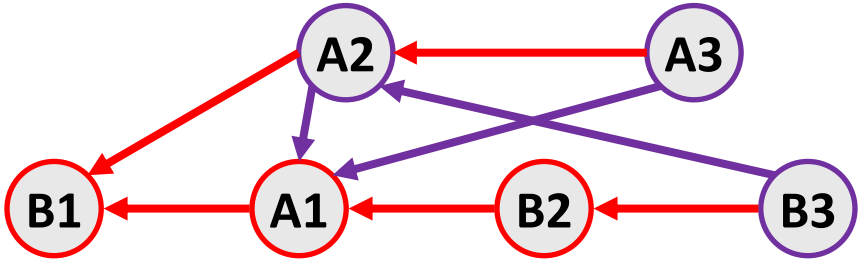

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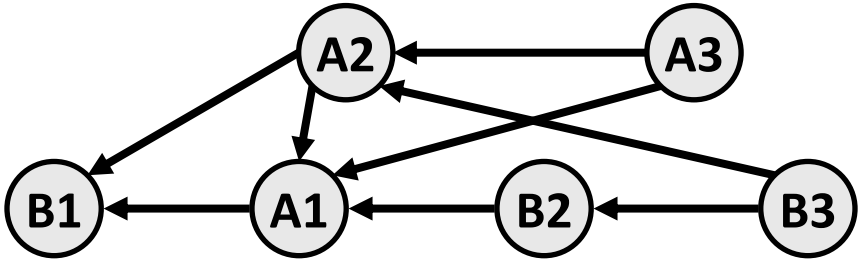

Topological Sort: Spreadsheets

Cells with formulas referencing other cells have **dependencies**. We can use topological sort to efficiently update cells!

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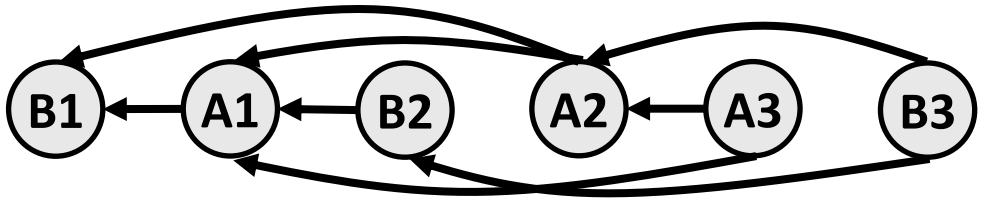

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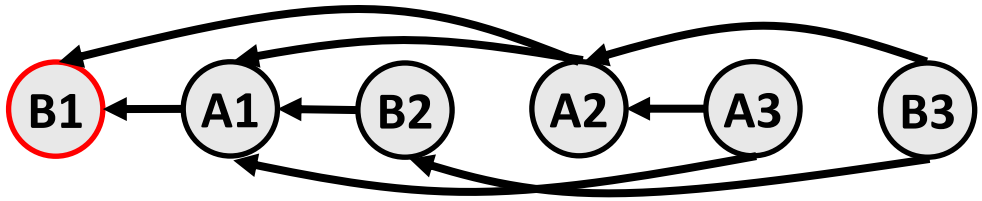

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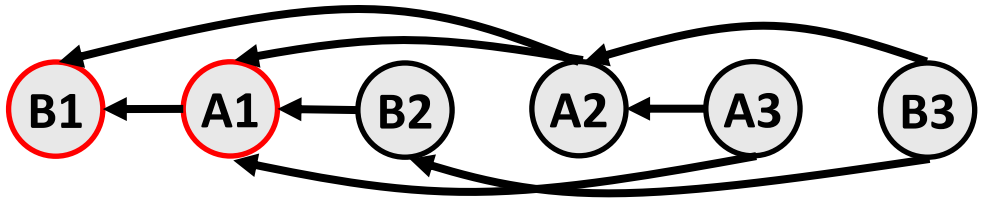

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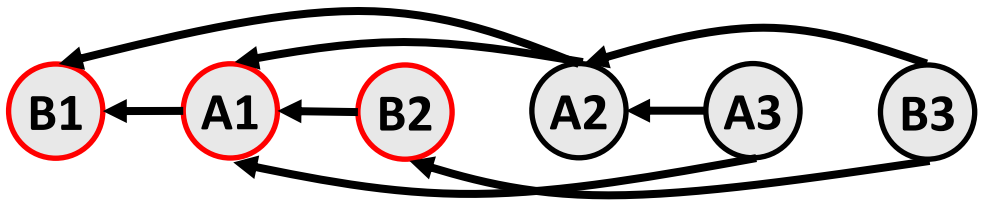

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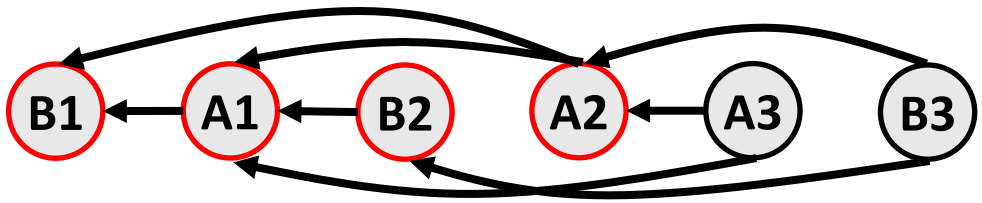

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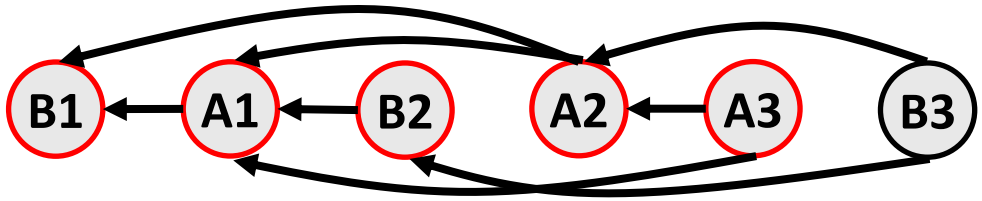

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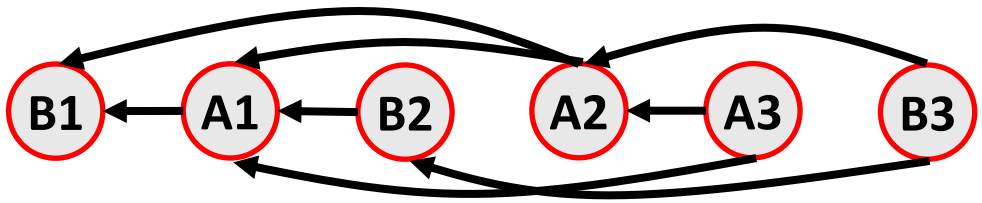

Plan for Today

- Topological Sort
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- **Announcements**

Announcements

- Homework 8 (Excel) goes out on Wednesday 11/28, is due Friday 12/7. **No late submissions will be accepted**.
- Guest lecture on Hashing next Monday: **Zach!**

Recap

- Topological Sort
  - Kahn's Algorithm
  - Recursive DFS
  - Spreadsheets
- Announcements

Next time: Classes – Inheritance and Polymorphism