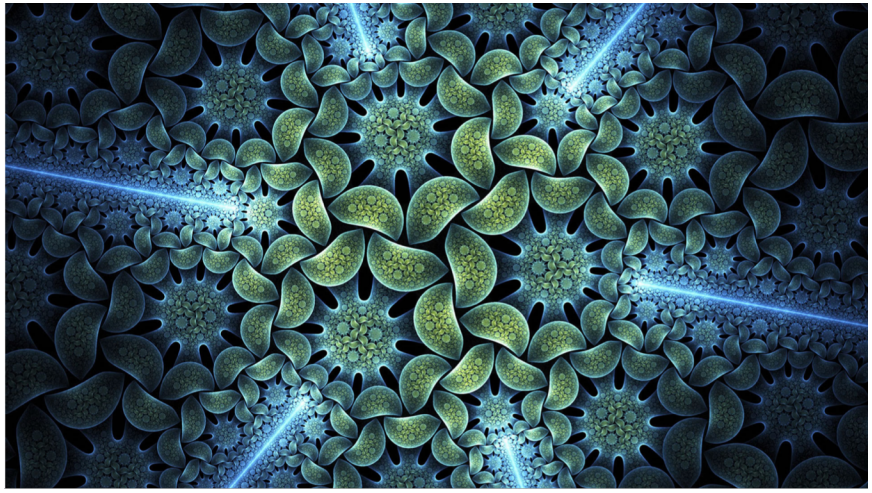
CS 106X, Lecture 9
Fractals

reading:

Programming Abstractions in C++, Chapter 8.4



Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Announcements

- HW3 – Recursion – going out at 3PM today
  - Fractals
  - Grammar Generator
  - Human Pyramid

Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Recursion & Big-O

```
void reverseLines(ifstream& input) {
    string line;
    if (getline(input, line)) {
        reverseLines(input);
        cout << line << endl;
    }
}
```

- What is the Big-O of the above function?
- (What is N?)

How many times is this function called in total? x **What is the runtime of each individual function call?**

Recursion & Big-O

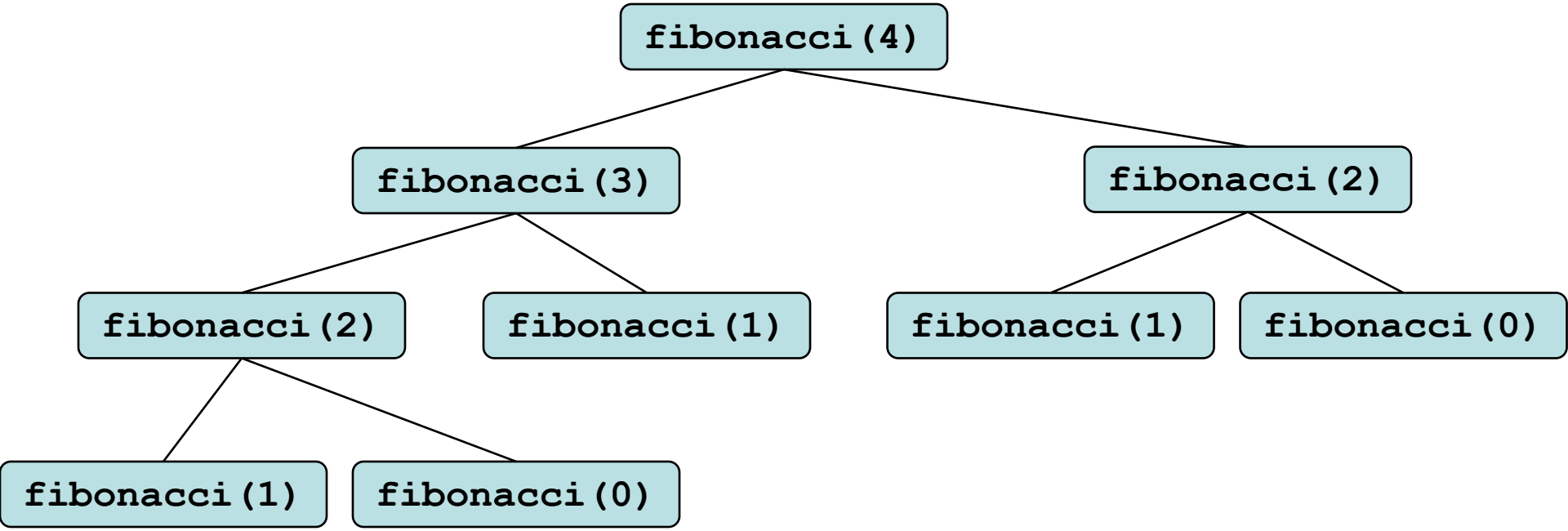
- The runtime of a recursive function is the number of function calls times the work done in each function call.
- The number of calls for a branching recursive function is usually

$$O(b^d)$$

where

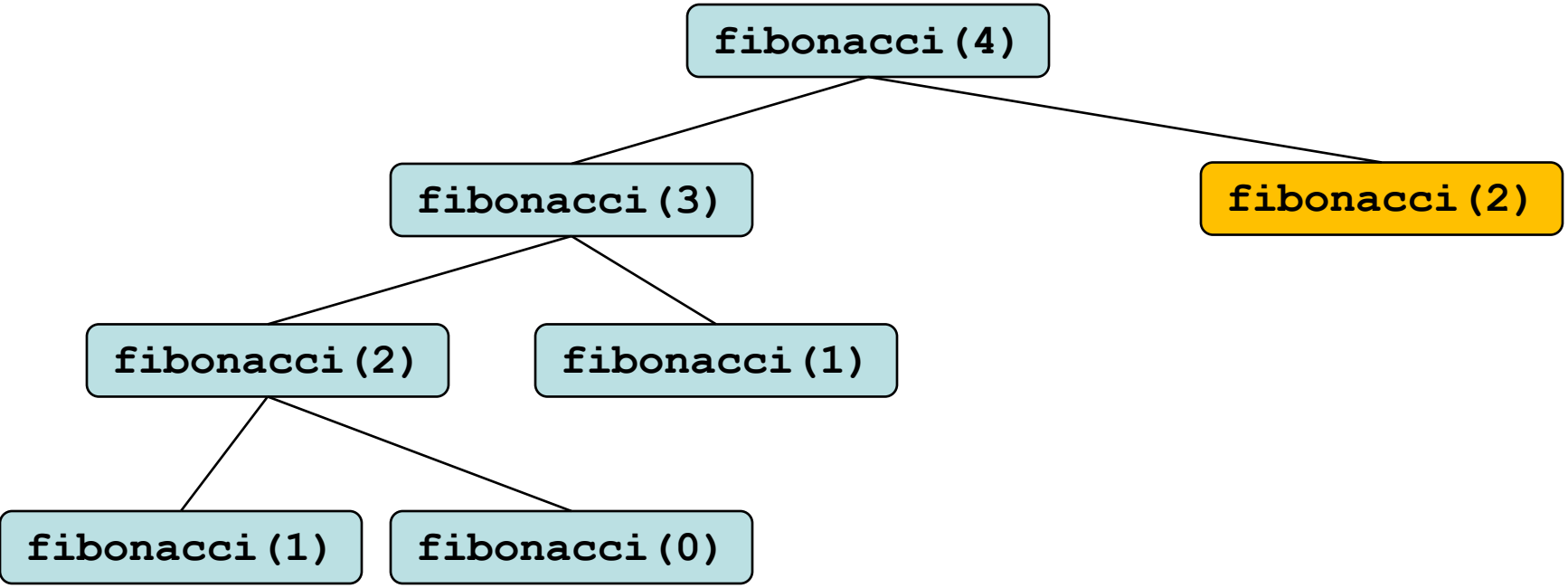
  - **b** is the worst-case branching factor (# recursive calls per function execution)
  - **d** is the worst-case depth of the recursion (the longest path from the top of the recursive call tree to a base case).

Fibonacci: Big-O



- Each recursive call makes 2 *additional* recursive calls.
- The worst-case depth of the recursion is the index of the Fibonacci number we are trying to calculate (N).
- Therefore, the number of total calls is $O(2^N)$.
- Each individual function call does O(1) work. Therefore, the total runtime is $O(2^N) * O(1) = O(2^N)$.

Recursive Tree



Is there a way to remember what we already computed?

Memoized Fibonacci

```
// Returns the nth Fibonacci number (no error handling).
// This version uses memoization.
int fibonacci(int i, Map<int, int>& cache) {
    if (i < 2) {
        return i;
    } else if (cache.containsKey(i)) {
        return cache[i];
    } else {
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);
        cache[i] = result;
        return result;
    }
}
```

Wrapper Functions

```
Map<int, int> cache;
int sixthFibonacci = fibonacci(5, cache);  // 5
```

- The above function signature isn't ideal; it requires the client to know to pass in an (empty) map.
- In general, the parameters we need for our recursion will not always match those the client will want to pass.
- Is there a way we can remove that requirement, while still memoizing?
- **YES!** A "wrapper" function is a function that "wraps" around the first call to a recursive function to abstract away any additional parameters needed to perform the recursion.

That's a Wrap(per)!

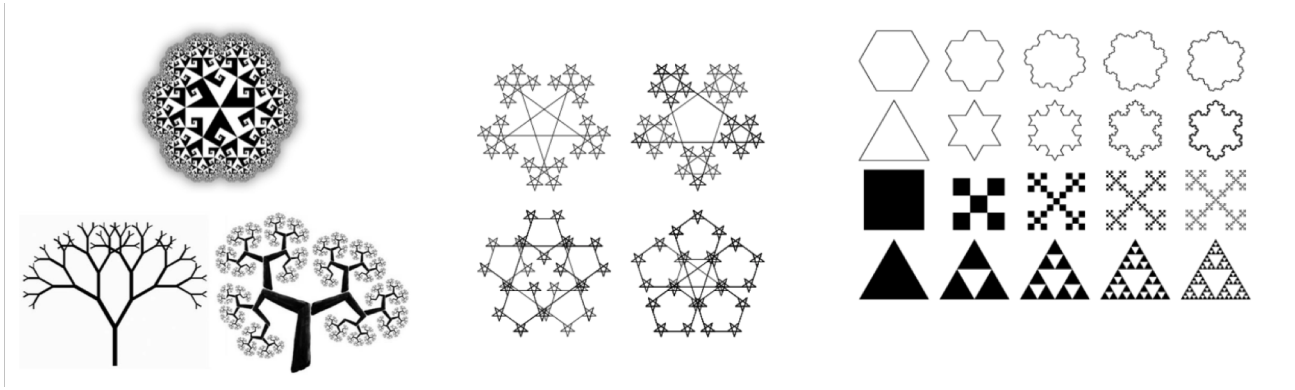
```
// "Wrapper" function that returns the nth Fibonacci number.
// This version calls the recursive version with an empty cache.
int fibonacci(int i) {
    Map<int, int> cache;
    return fibonacci(i, cache);
}

// Recursive function that returns the nth Fibonacci number.
// This version uses memoization.
int fibonacci(int i, Map<int, int>& cache) {
    if (i < 0) {
        throw "Illegal negative index";
    } else if (i < 2) {
        return i;
    } else if (cache.containsKey(i)) {
        return cache[i];
    } else {
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);
        cache[i] = result;
        return result;
    }
}
```

Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Fractals

A **fractal** is a recurring graphical pattern. Smaller instances of the same shape or pattern occur within the pattern itself.



Fractals in Nature

Many natural phenomena generate fractal patterns:

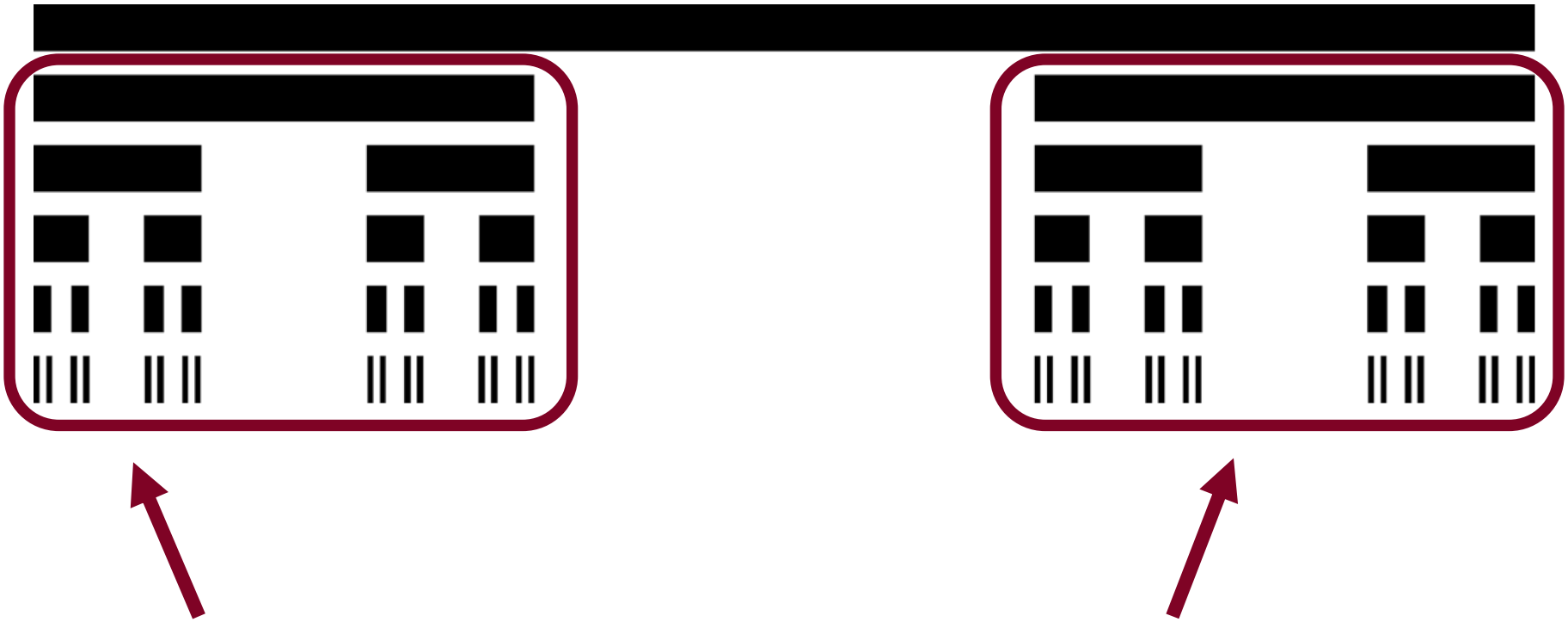
1. **earthquake fault lines**
2. **animal color patterns**
3. **clouds**
4. **mountain ranges**
5. **snowflakes**
6. **crystals**
7. **DNA**
8. **...**

Cantor Fractal

Cantor Fractal

Parts of a cantor set image ... are Cantor set images

Cantor Fractal



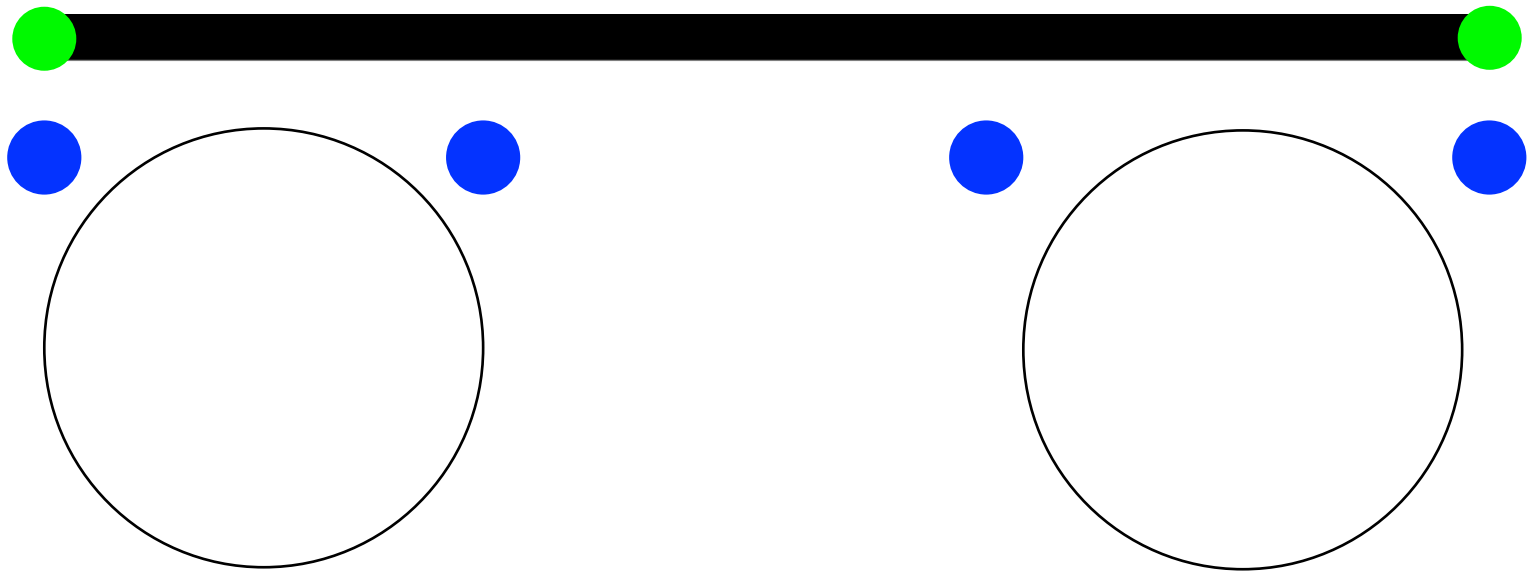
Another cantor set

Another cantor set

Level 1 Cantor Fractal

Level n Cantor

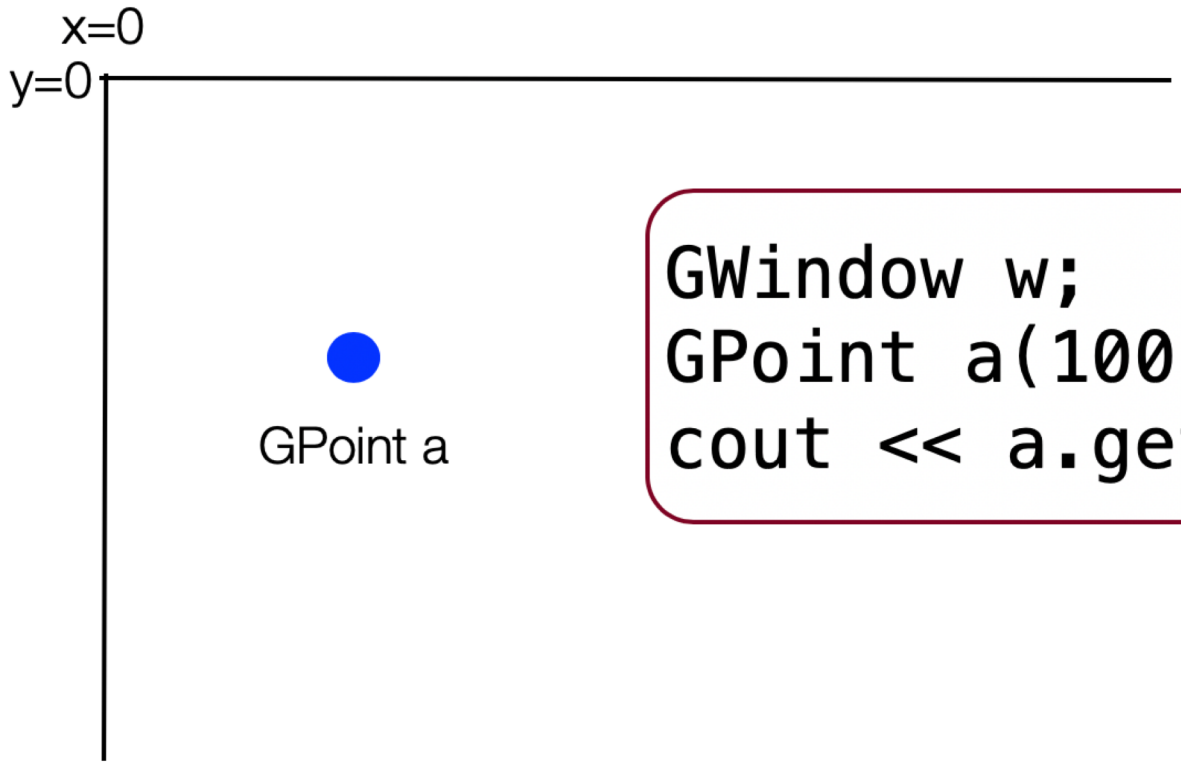
1. Draw a line from start to finish.



2. Draw a Cantor of size n-1

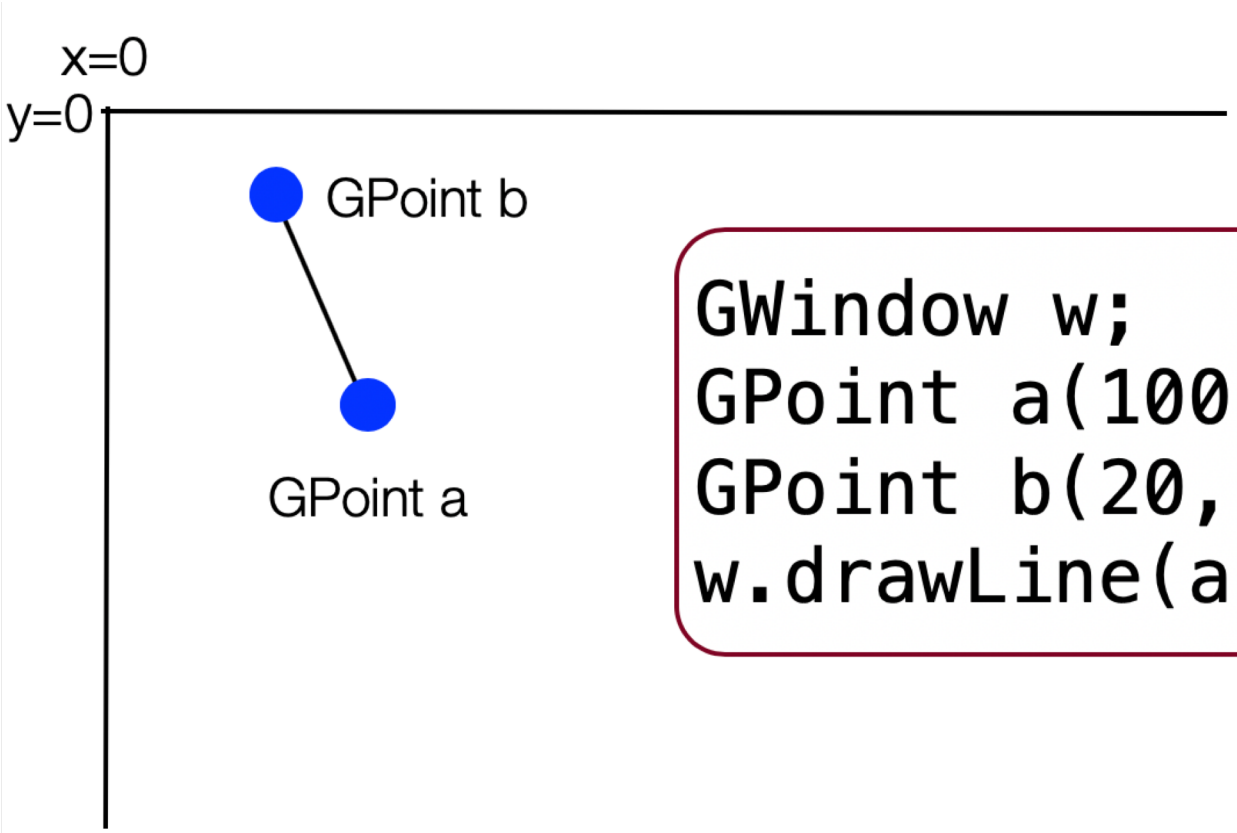
2. Draw a Cantor of size n-1

Stanford Graphics Libraries



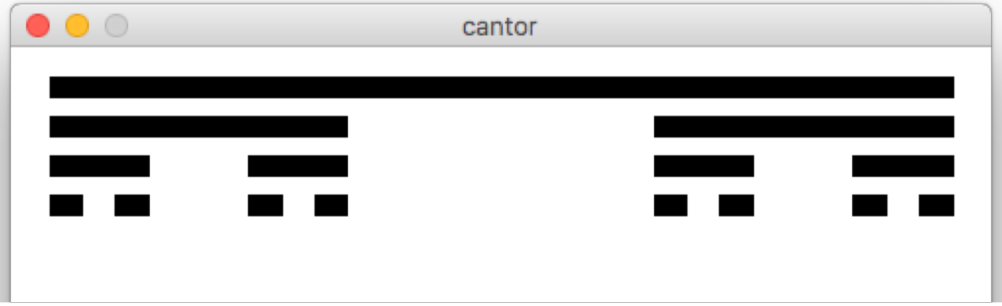

```
GWindow w;
GPoint a(100, 100);
cout << a.getX() << endl;
```

Stanford Graphics Libraries




```
GWindow w;
GPoint a(100, 100);
GPoint b(20, 20);
w.drawLine(a, b);
```

Cantor Fractal



Plan For Today

- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Coding a fractal

- Many fractals are implemented as a function that accepts x/y coordinates, size, and a *level* parameter.
  - The *level* is the number of recurrences of the pattern to draw.

- Example, Koch snowflake:

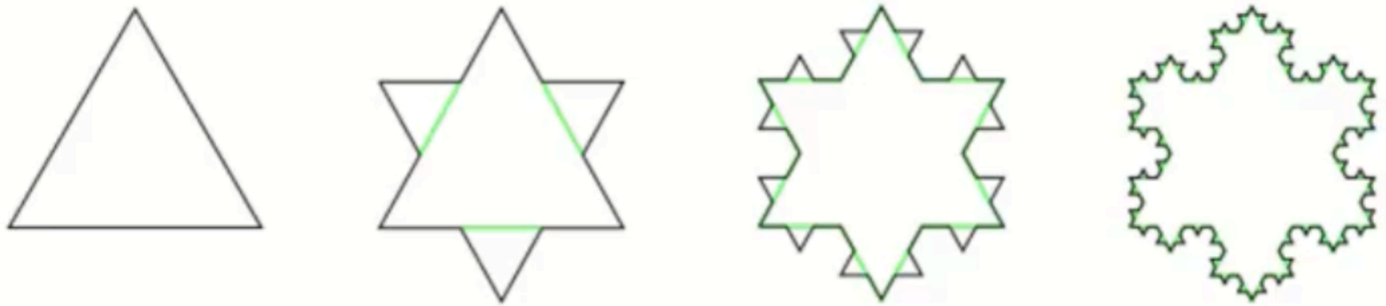
```
snowflake(window, x, y, size, 1);
```

```
snowflake(window, x, y, size, 2);
```

```
snowflake(window, x, y, size, 3);
```

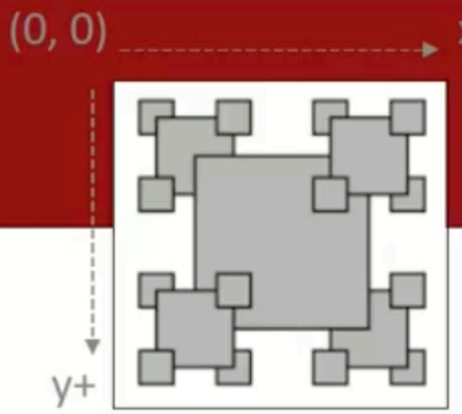
Koch snowflake

- **Koch snowflake**: A fractal formed by pulling a triangular "bend" out of each side of an existing triangle at each level.



- Start with an equilateral triangle, then:
  - Divide each of its 3 line segments into 3 parts of equal length.
  - Draw an eq.triangle with middle segment as base, pointing outward.
  - Remove the middle line segment.

Boxy fractal



- Where should the following line be inserted in order to get the figure at right?

```
window.fillRect(x, y, size, size);  // draws a gray square

void fractal(GWindow& gw, int x, int y, int size, int order) {
    if (order >= 0) {
        // A) here
        fractal(gw, x-size/2, y-size/2, size/2, order-1);
        // B) here
        fractal(gw, x+size/2, y+size/2, size/2, order-1);
        // C) here
        fractal(gw, x+size/2, y-size/2, size/2, order-1);
        // D) here
        fractal(gw, x-size/2, y+size/2, size/2, order-1);
        // E) here
    }
}
```

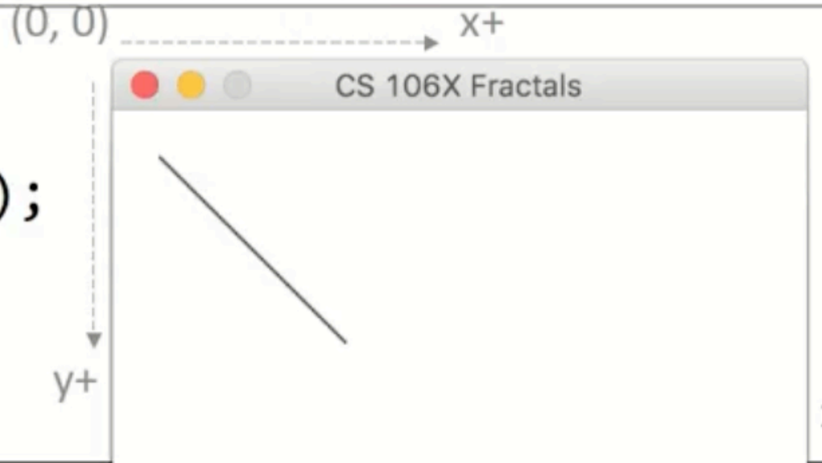
question adapted from Cynthia Lee's 106B/X slides

Stanford graphics lib

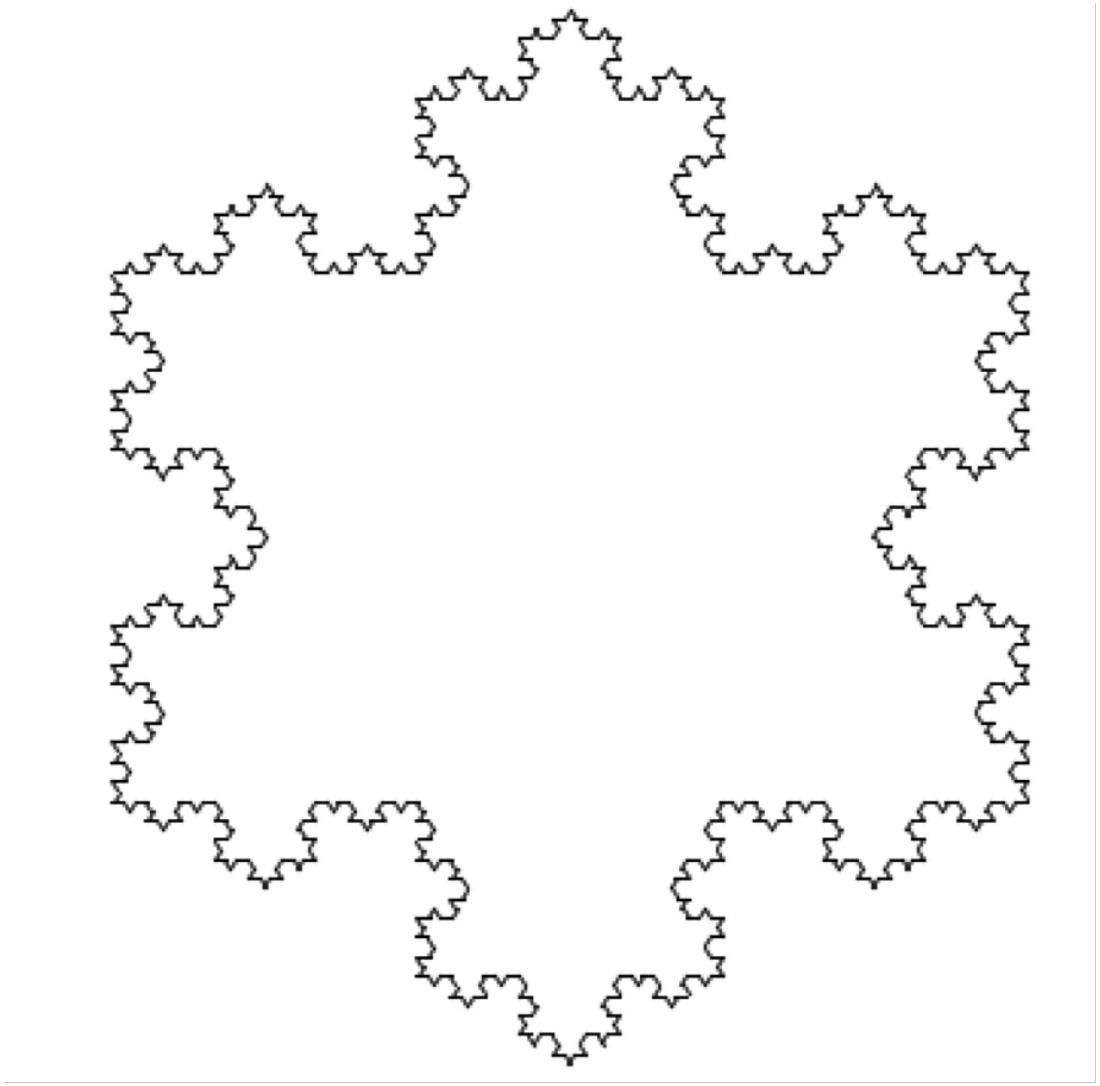
```
#include "gwindow.h"
```

`gw.drawLine(x1, y1, x2, y2);`	draws a line between the given two points
`gw.drawPolarLine(x, y, r, t);`	draws line from (*x*,*y*) at angle *t* of length *r*; returns the line's end point as a GPoint
`gw.getPixel(x, y)`	returns an RGB int for a single pixel
`gw.setColor("color");`	sets color with a color name string like "red", or #RRGGBB string like "#ff00cc", or RGB int
`gw.setPixel(x, y, rgb);`	sets a single RGB pixel on the window
`gw.drawOval(x, y, w, h);` `gw.fillRect(x, y, w, h); ...`	other shape and line drawing functions (see online docs for complete member list)

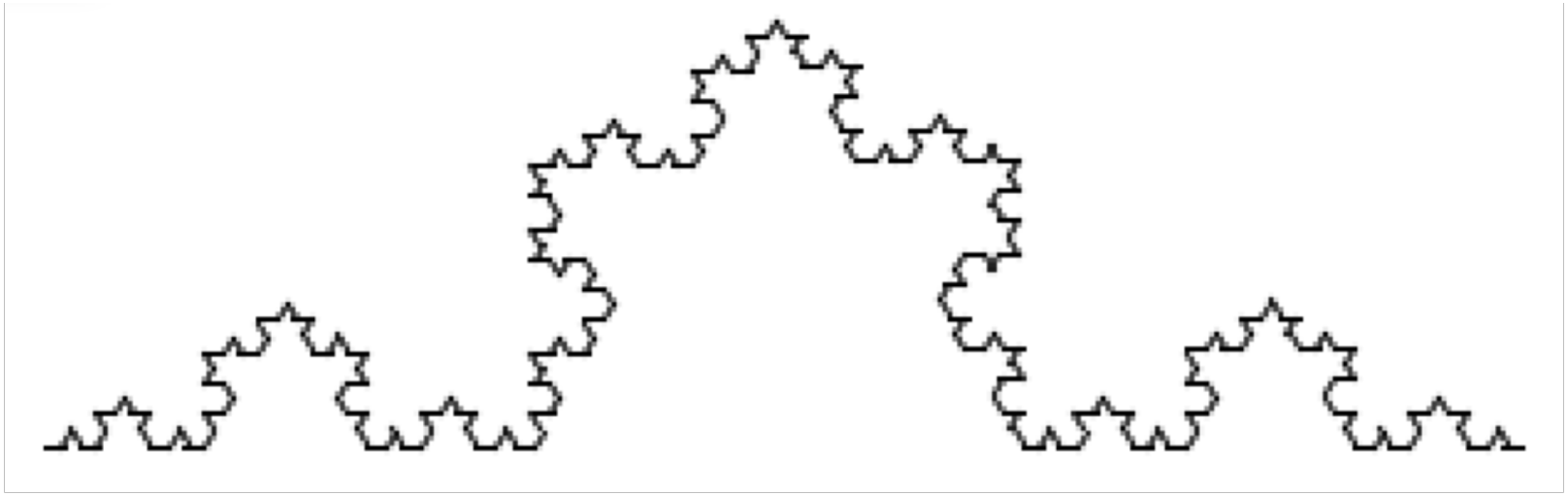
```
GWindow gw(300, 200);
gw.setTitle("CS 106X Fractals");
gw.drawLine(20, 20, 100, 100);
```



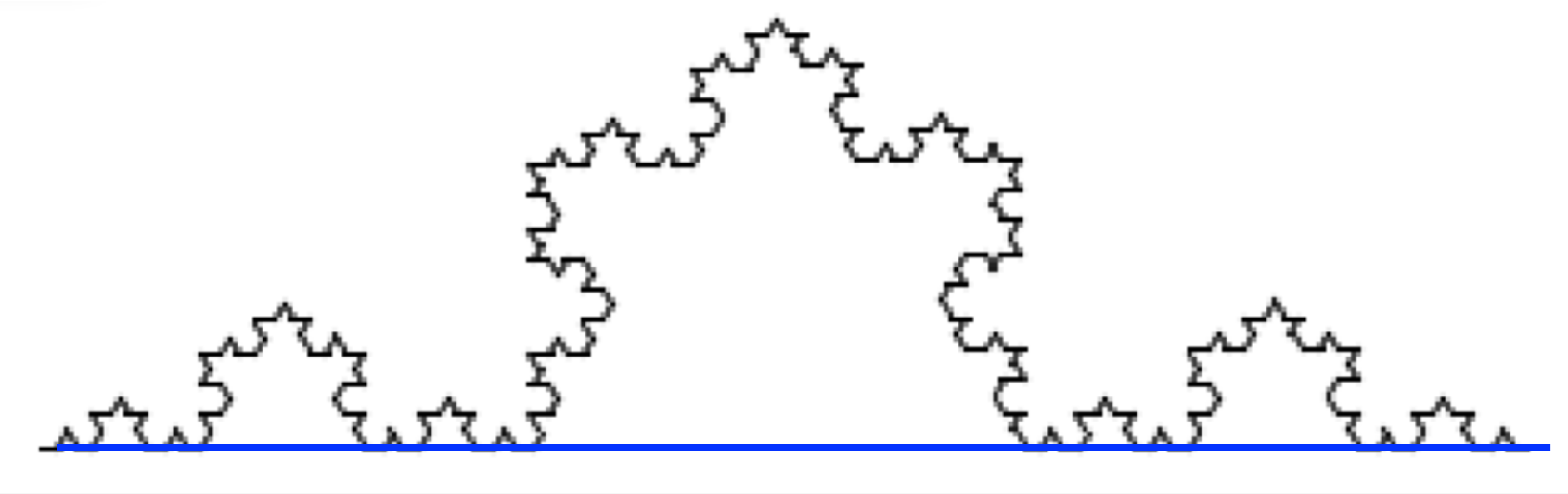
Snowflake



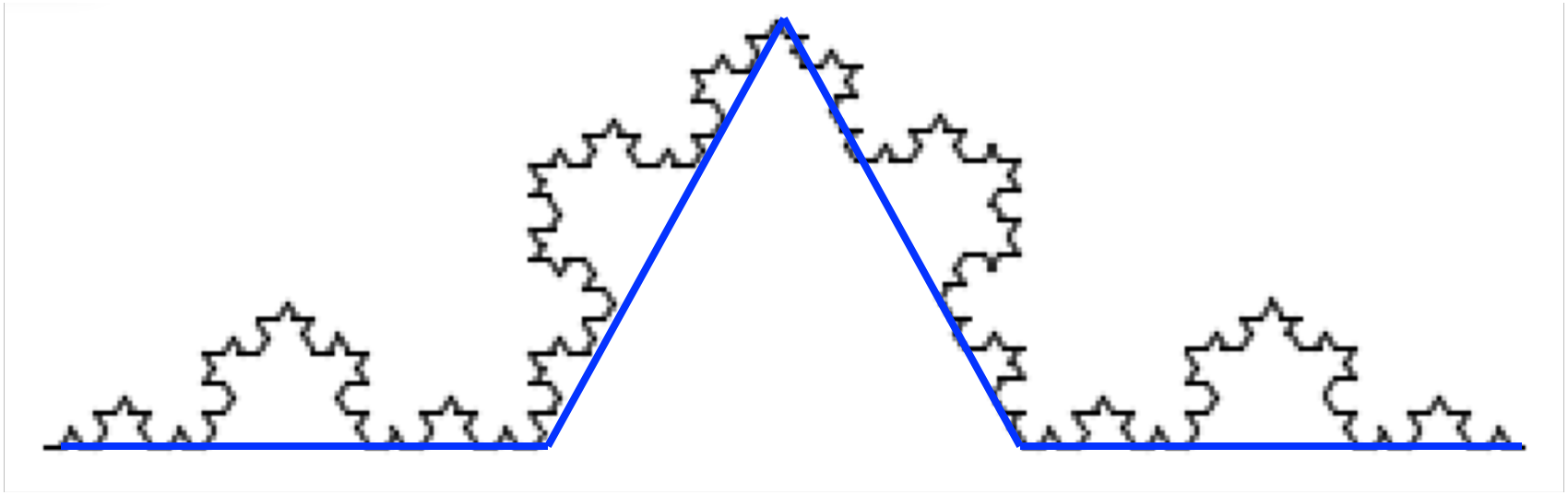
Snowflake



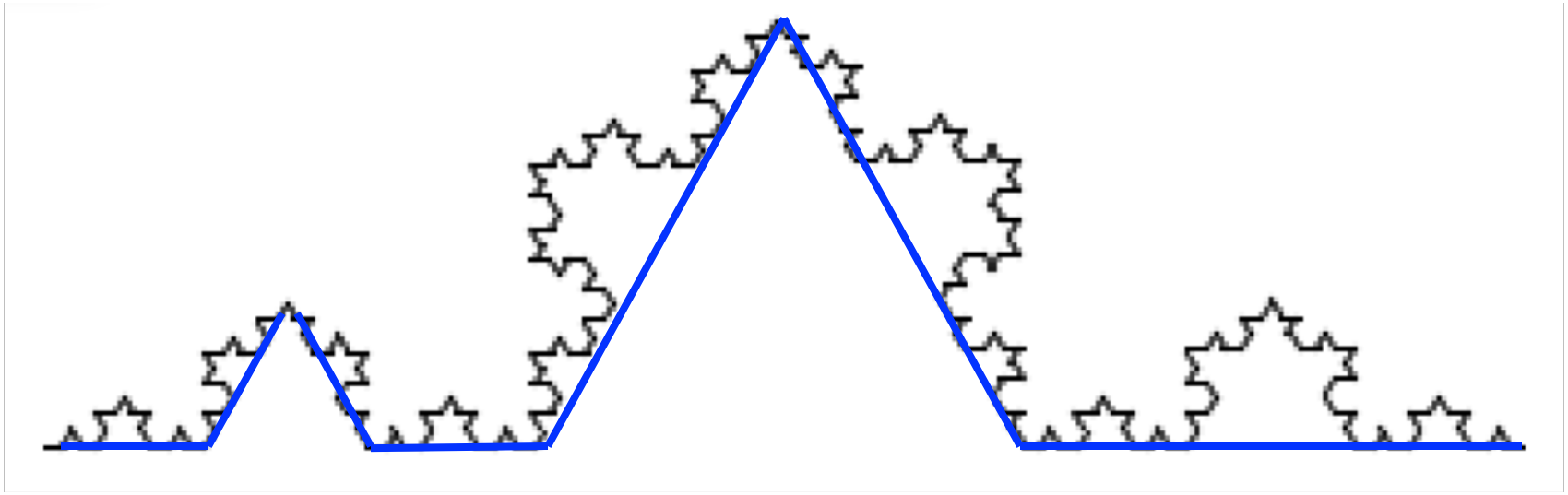
Snowflake



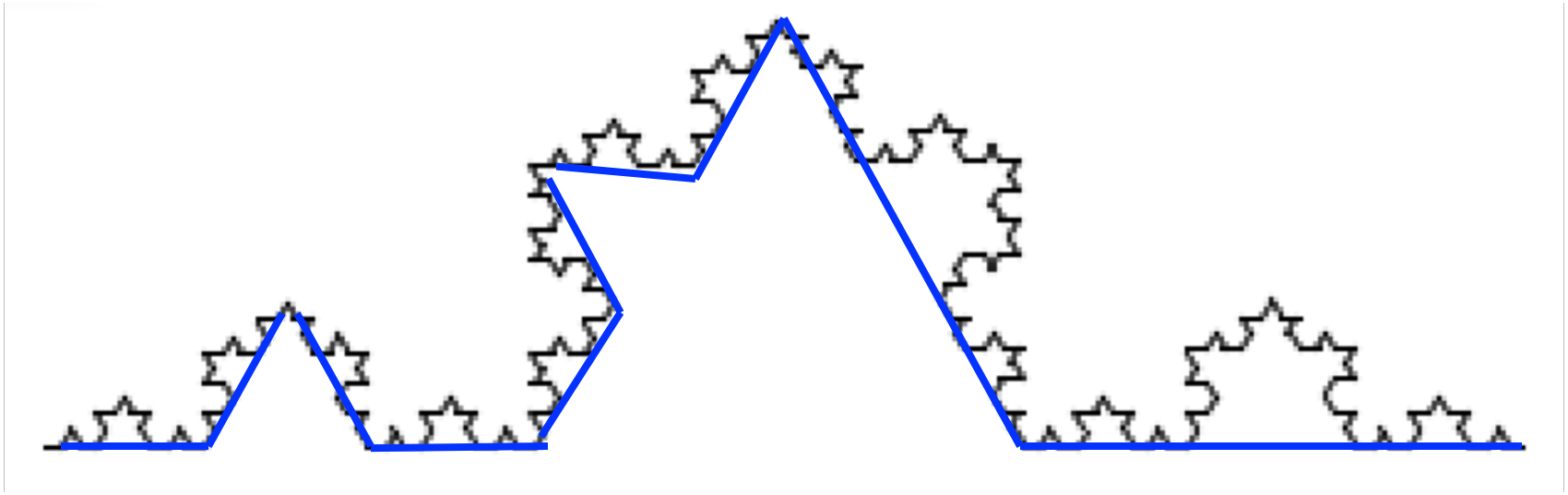
Snowflake



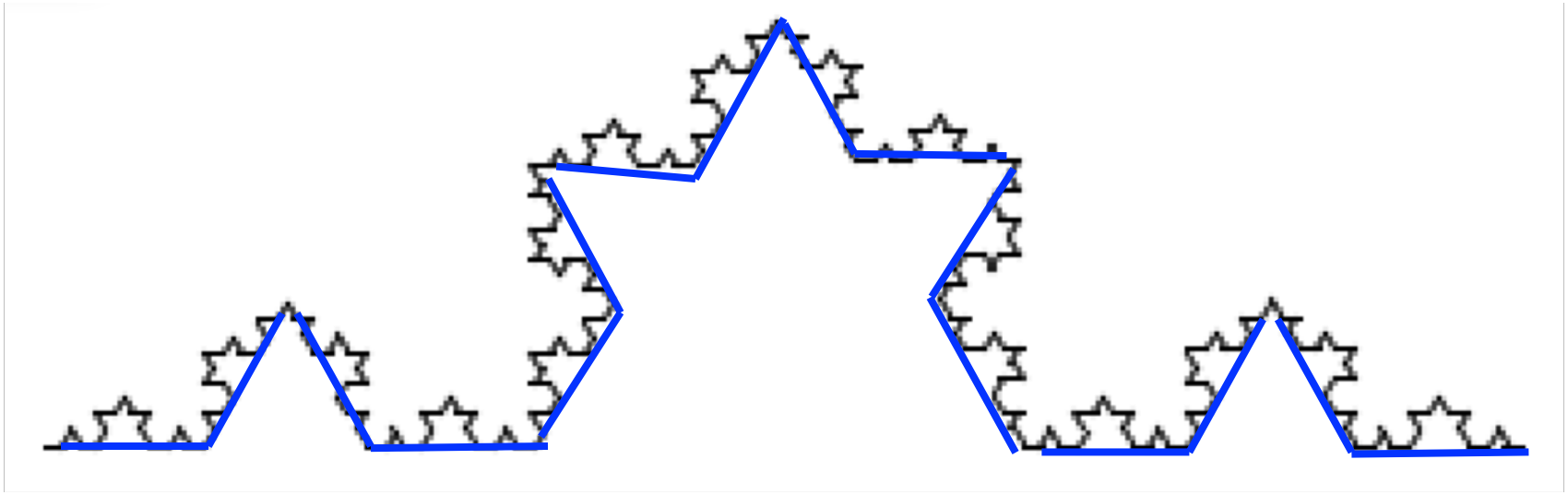
Snowflake



Snowflake



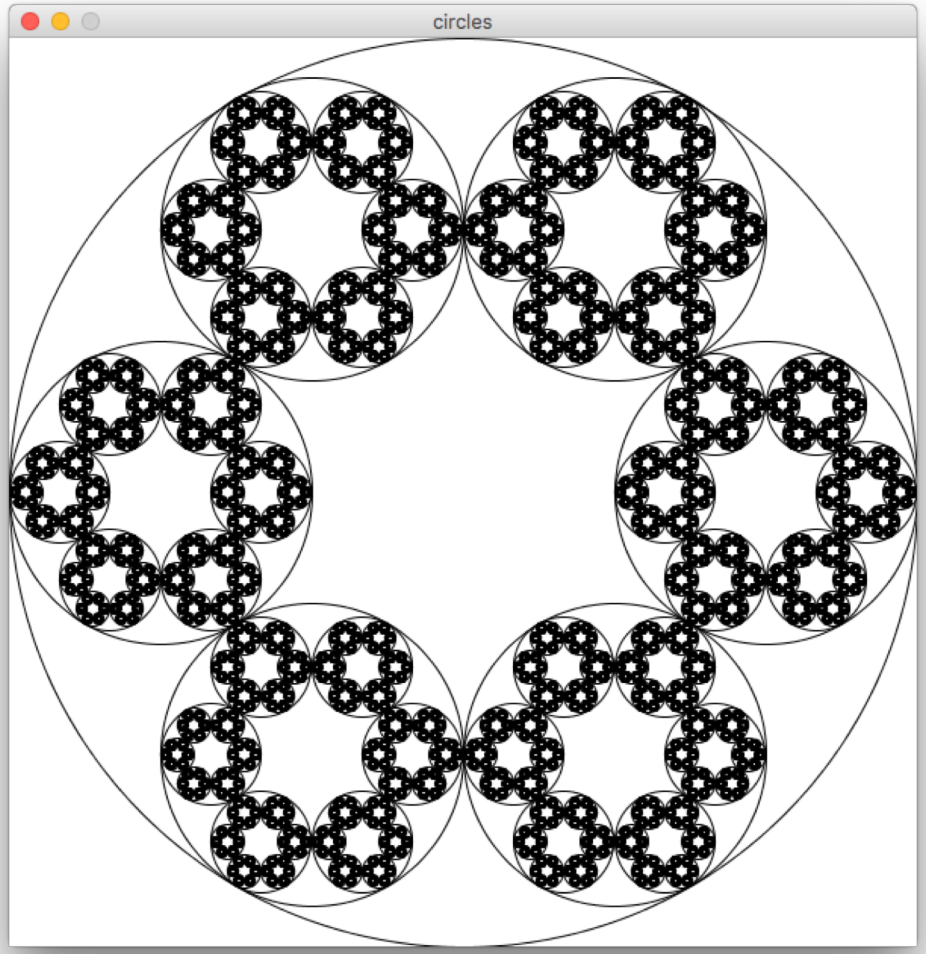
Snowflake



Plan For Today

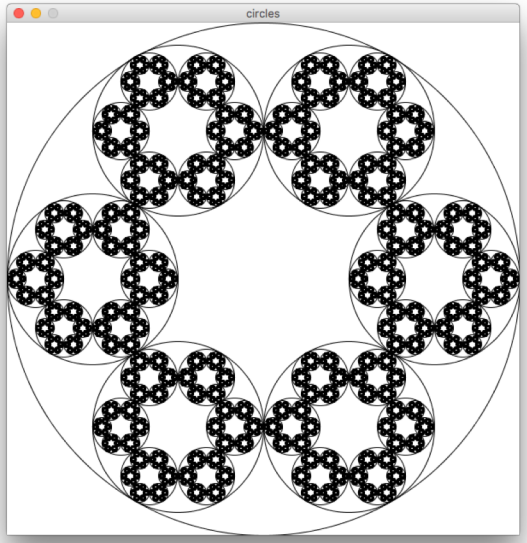
- Announcements
- **Recap:** Runtime and Memoization
- Fractals
  - Cantor fractal
  - Snowflake fractal
  - Emblem fractal

Emblem Fractal



Emblem Fractal

- We want to draw this figure at a given center and radius on-screen.
- An order-0 emblem is nothing
- An order-1 emblem is a circle of the specified size
- An order-n emblem is a circle of the specified size, containing 6 order n-1 emblems at increments of 60 degrees around the circle 2/3 away from the center, with 1/3 the radius.



Recap

- **Fractals**
  - Fractals are self-referential, and that makes for nice recursion problems!
  - Break the problem into a smaller, self-similar part, and don't forget your base case!

References and Advanced Reading

- References:
  - http://www.cs.utah.edu/~germain/PPS/Topics/recursion.html
  - Why is iteration generally better than recursion? http://stackoverflow.com/a/3093/561677

- Advanced Reading:

  - Tail recursion: http://stackoverflow.com/questions/33923/what-is-tail-recursion

  - Interesting story on the history of recursion in programming languages: http://goo.gl/P6Einb