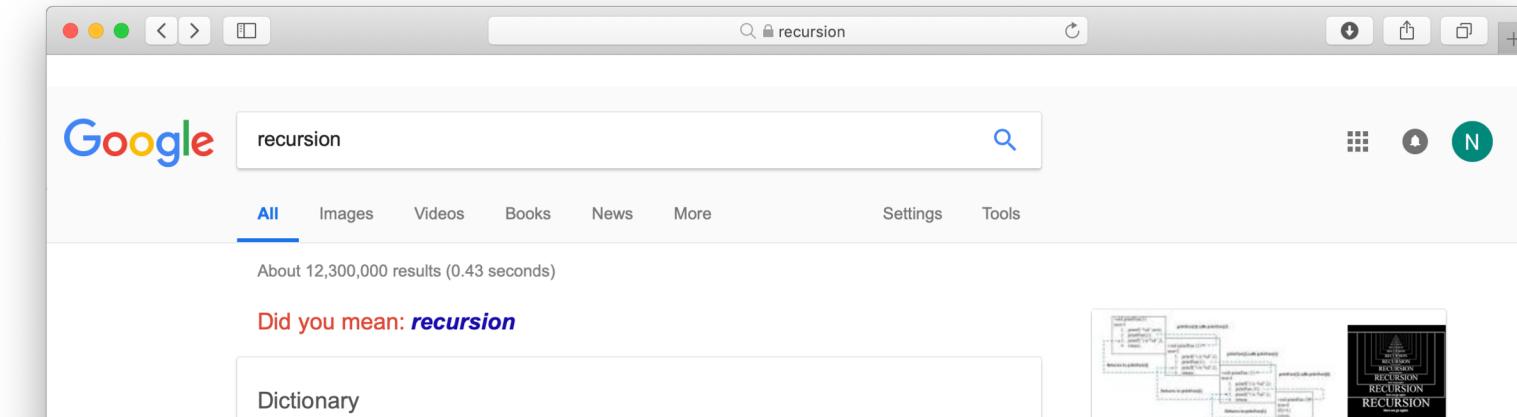


CS 106X, Lecture 8

More Recursion

reading:

Programming Abstractions in C++, Chapters 7-8



Plan For Today

- **Recap:** Recursion So Far
- More Practice: Reversing a File
- Recursion: Runtime
- Announcements
- Memoization
- Wrapper Functions

operator , operand Surrounded by ().

Plan For Today

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Recursive Thinking

- In code, recursion is when a function in your program calls itself as part of its execution.
- Conceptually, a recursive problem is one that is *self-similar*; it can be solved via smaller occurrences of the same problem.

The Recursion Checklist

- ❑ Find what information we need to keep track of.**
What inputs/outputs are needed to solve the problem at each step? Do we need a wrapper function?
- ❑ Find our base case(s).** What are the simplest (non-recursive) instance(s) of this problem?
- ❑ Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
- ❑ Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

Example 1: Factorial

$$n! = n * (n-1) * (n-2) * (n-3) * \dots * 1$$

- Write a function that computes and returns the factorial of a provided number, recursively (no loops).
 - e.g. **factorial(4)** should return **24**
 - You should be able to compute the value of any non-negative number. ($0! = 1$).

Factorial

```
// Returns n!, or 1 * 2 * 3 * 4 * ... * n.  
// Assumes n >= 0.  
int factorial(int n) {  
    if (n < 0) {                                // error handling  
        throw "illegal negative n";  
    if (n == 0) {                                // base case  
        return 1;  
    } else {  
        return n * factorial(n - 1); // recursive case  
    }  
}
```

Recursive Program Structure

```
recursiveFunc() {  
    if (test for simple case) { // base case  
        Compute the solution without recursion  
    } else { // recursive case  
        Break the problem into subproblems of the same form  
        Call recursiveFunc() on each self-similar subproblem  
        Reassamble the results of the subproblems  
    }  
}
```

Example 2: Fibonacci

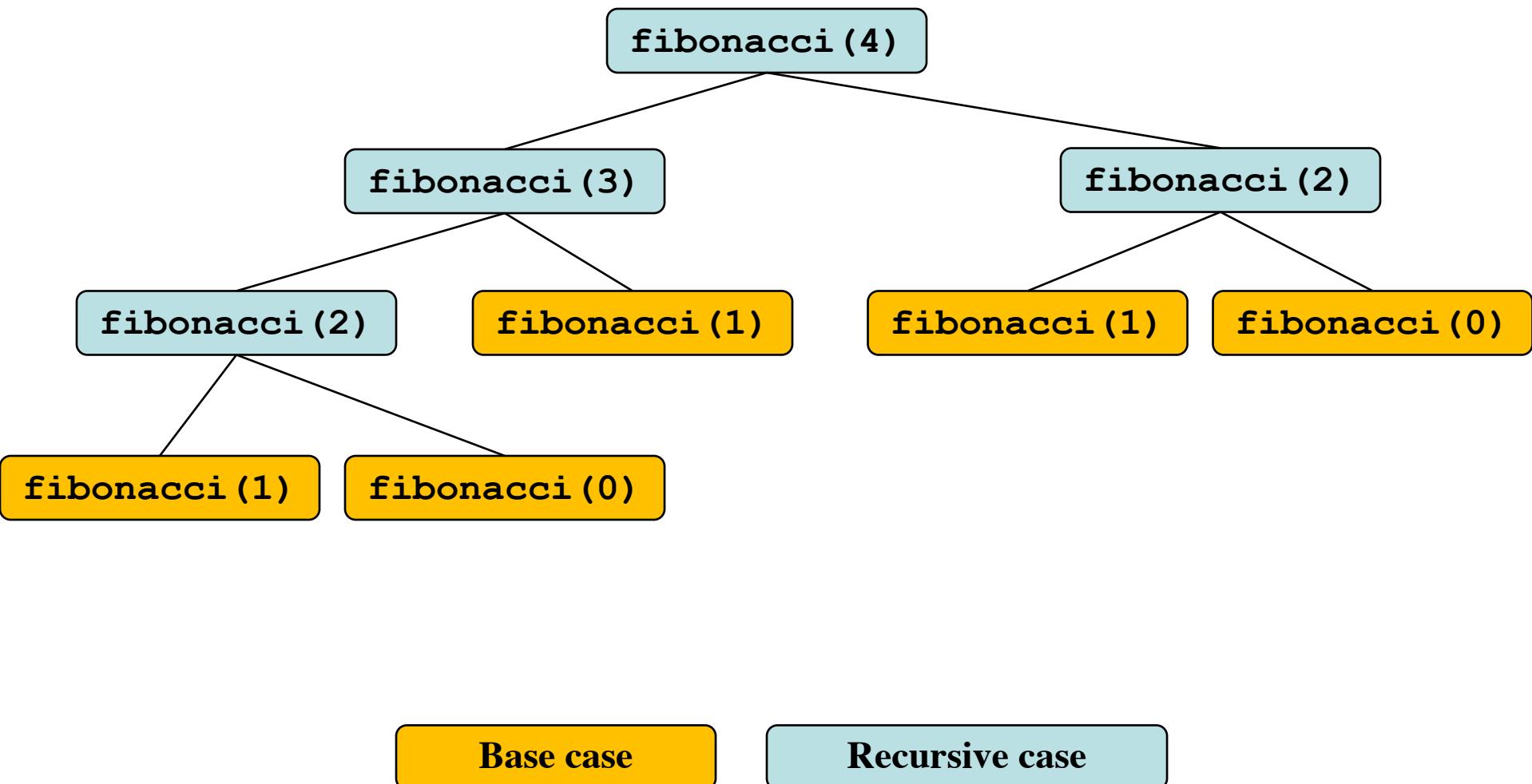
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

- The Fibonacci sequence starts with 0 and 1, and each subsequent number is the sum of the two previous numbers.
- Write a function that computes and returns the nth Fibonacci number, recursively (no loops).
 - e.g. **fibonacci (6)** should return 8

Fibonacci

```
// Returns the i'th Fibonacci number in the sequence
// (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...)
// Assumes i >= 0.
int fibonacci(int i) {
    if (i < 0) {                                // error handling
        throw "illegal negative index";
    } else if (i == 0) {                         // base case 1
        return 0;
    } else if (i == 1) {                         // base case 2
        return 1;
    } else {                                     // recursive case
        return fibonacci(i-1) + fibonacci(i-2);
    }
}
```

Recursive Tree



Base case

Recursive case



isPalindrome exercise

- Write a recursive function `isPalindrome` accepts a string and returns `true` if it reads the same forwards as backwards.

<code>isPalindrome("madam")</code>	→ true
<code>isPalindrome("racecar")</code>	→ true
<code>isPalindrome("step on no pets")</code>	→ true
<code>isPalindrome("able was I ere I saw elba")</code>	→ true
<code>isPalindrome("Q")</code>	→ true
<code>isPalindrome("Java")</code>	→ false
<code>isPalindrome("rotater")</code>	→ false
<code>isPalindrome("byebye")</code>	→ false
<code>isPalindrome("notion")</code>	→ false

isPalindrome solution

```
// Returns true if the given string reads the same
// forwards as backwards.
// By default, true for empty or 1-letter strings.
bool isPalindrome(string s) {
    if (s.length() < 2) {    // base case
        return true;
    } else {                  // recursive case
        if (s[0] != s[s.length() - 1]) {
            return false;
        }
        string middle = s.substr(1, s.length() - 2);
        return isPalindrome(middle);
    }
}
```

isPalindrome solution 2

```
// Returns true if the given string reads the same
// forwards as backwards.
// By default, true for empty or 1-letter strings.
// This version is also case-insensitive.
bool isPalindrome(string s) {
    if (s.length() < 2) {    // base case
        return true;
    } else {                  // recursive case
        return tolower(s[0]) == tolower(s[s.length() - 1])
            && isPalindrome(s.substr(1, s.length() - 2));
    }
}
```

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reverseLines exercise

Write a recursive function `reverseLines` that accepts a file input stream and prints the lines of that file in reverse order.

Example input file:

I am not a person who contributes
And I refuse to believe that
I will be useful

Expected console output:

I will be useful
And I refuse to believe that
I am not a person who contributes

Find Recursive Case: Start from
a little bit
harder case
from base case

I - - - -

Stack, reverse
function stack.

The Recursion Checklist

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Reversal pseudocode

Reversing the lines of a file:

- If the input stream is empty:
 - Stop
- If there are lines left:
 - Read the next line L
 - Print the rest of the lines in reverse order
 - Print the line L.

reverseLines solution

```
void reverseLines(ifstream& input) {  
    string line;  
    if (!getline(input, line)) {  
        // base case - stop  
    } else {  
        // recursive case  
        reverseLines(input);  
        cout << line << endl;  
    }  
}
```

```
if (x) {  
    do nothing  
} else {  
    do something  
}
```

implicit.

```
if (not x) {  
    do something  
}
```

reverseLines solution

```
void reverseLines(ifstream& input) {  
    string line;  
    if (getline(input, line)) {  
        // recursive case  
        reverseLines(input);  
        cout << line << endl;  
    }  
}
```

- Where is the base case?

Tracing our algorithm

- **call stack:** The function invocations running at any one time.

Stack

```
reverseLines(input);
```

```
void reverseLines(ifstream& input) { // call 1
    string line;
    if (!getline(input, line)) { // "I am not a person who contributes"
        void reverseLines(ifstream& input) { // call 2
            string line;
            if (!getline(input, line)) { // "And I refuse to believe that"
                void reverseLines(ifstream& input) { // call 3
                    string line;
                    if (!getline(input, line)) { // "I will be useful"
                        void reverseLines(ifstream& input) { // call 4
                            string line;
                            if (getline(input, line)) {
                                reverseLines(input); // fail
                                cout << line << endl;
                            }
                        }
                    }
                }
            }
        }
    }
input f: }
```

→ I am not a person who contributes
And I refuse to believe that
I will be useful

I will be useful
And I refuse to believe that
I am not a person who contributes

The “Recursive Leap of Faith”

- When writing a recursive function, pretend that someone has already written a function that can solve that problem for smaller inputs. How could you use it in your implementation?
- E.g. “what if we had a function that could print out a smaller file in reverse?”

```
void reverseLines(ifstream& input) {  
    string line;  
    if (getline(input, line)) {  
        reverseLines(input);  
        cout << line << endl;  
    }  
}
```

// "leap of faith"



The “Recursive Leap of Faith”



<https://www.youtube.com/watch?v=VgoflyRO-RI>

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Recursion & Big-O

```
void reverseLines(ifstream& input) {  
    string line;  
    if (getline(input, line)) {  
        reverseLines(input);  
        cout << line << endl;  
    }  
}
```

- What is the Big-O of the above function?
- (What is N?)

How many times is this
function called in total?

x

What is the runtime of each
individual function call?

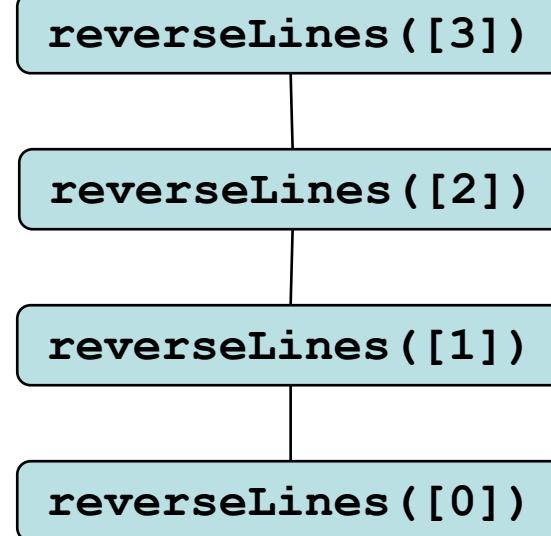
Recursion & Big-O

```
void reverseLines(ifstream& input) {  
    string line;  
    if (getline(input, line)) {  
        reverseLines(input);  
        cout << line << endl;  
    }  
}
```

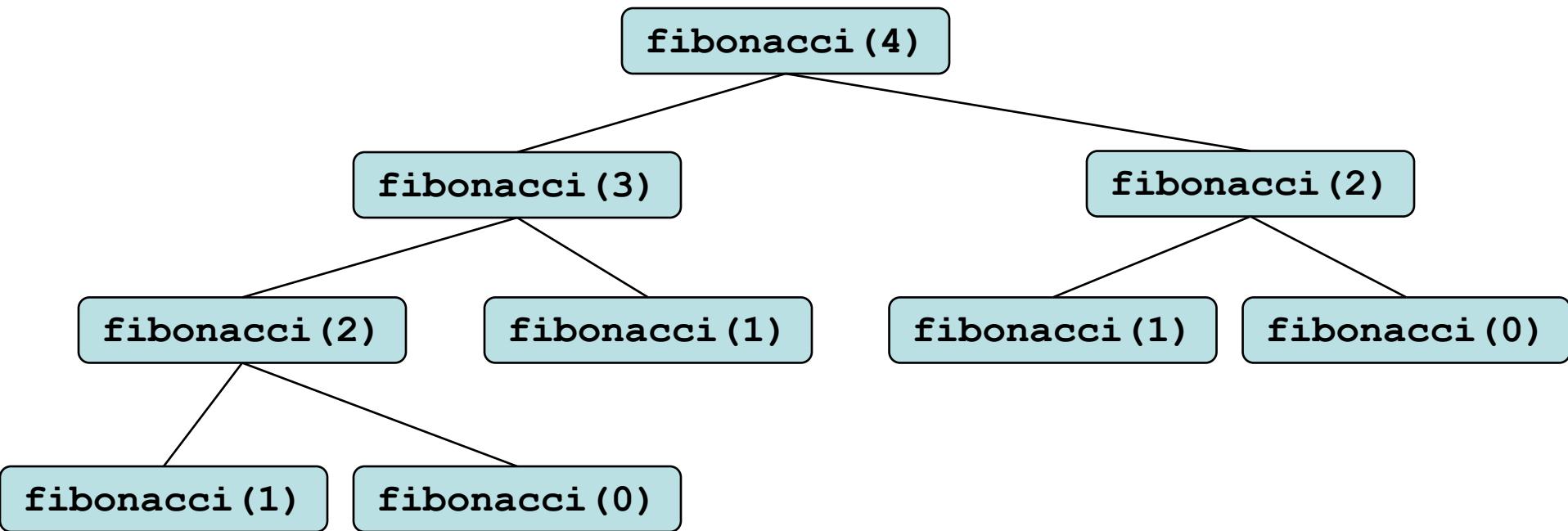
There is one recursive call for each line of the file.

The runtime of an individual function call is O(1).

Thus, the total runtime is $O(N)*O(1) = O(N)$.



Fibonacci: Big-O



- Each recursive call makes 2 *additional* recursive calls.
- The worst-case depth of the recursion is the index of the Fibonacci number we are trying to calculate (N).
- Therefore, the number of total calls is $O(2^N)$.
- Each individual function call does $O(1)$ work. Therefore, the total runtime is $O(2^N) * O(1) = O(2^N)$.

Recursion & Big-O

- The runtime of a recursive function is the number of function calls times the work done in each function call.
- The number of calls for a branching recursive function is usually $O(b^d)$

where

- **b** is the worst-case branching factor (# recursive calls per function execution)
- **d** is the worst-case depth of the recursion (the longest path from the top of the recursive call tree to a base case).

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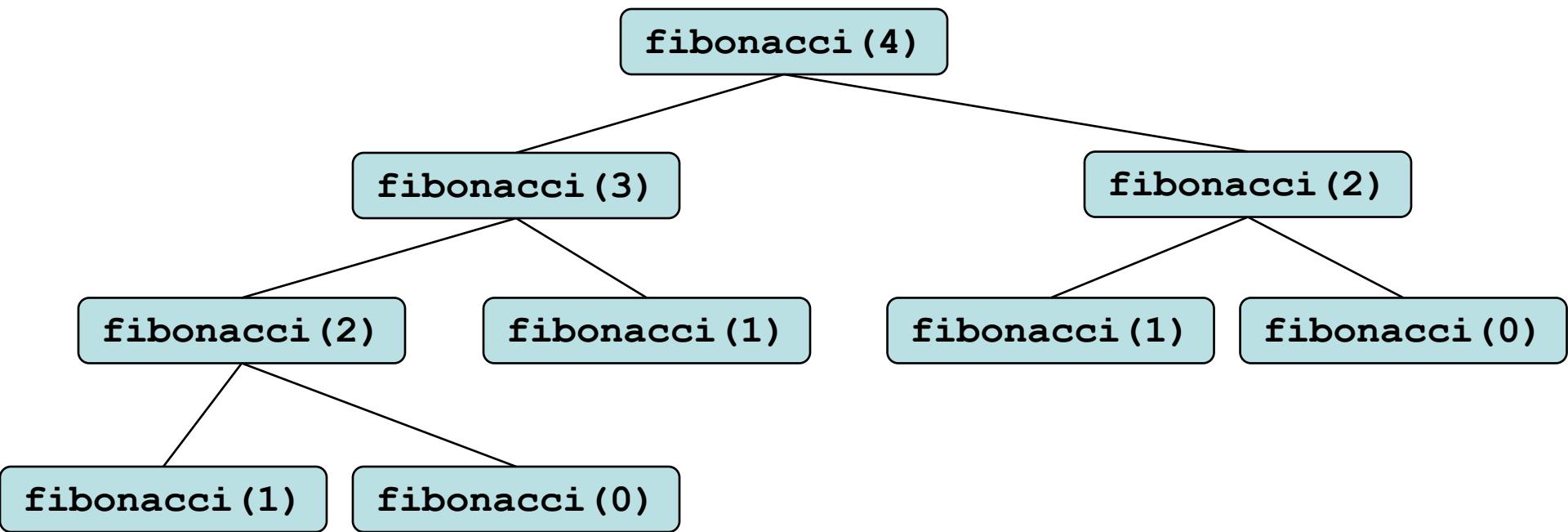
Announcements

- Computer Forum Career Fair **TODAY** until 4PM @ Packard Lawn
- Candy poll!
- HW1 IGs

Plan For Today

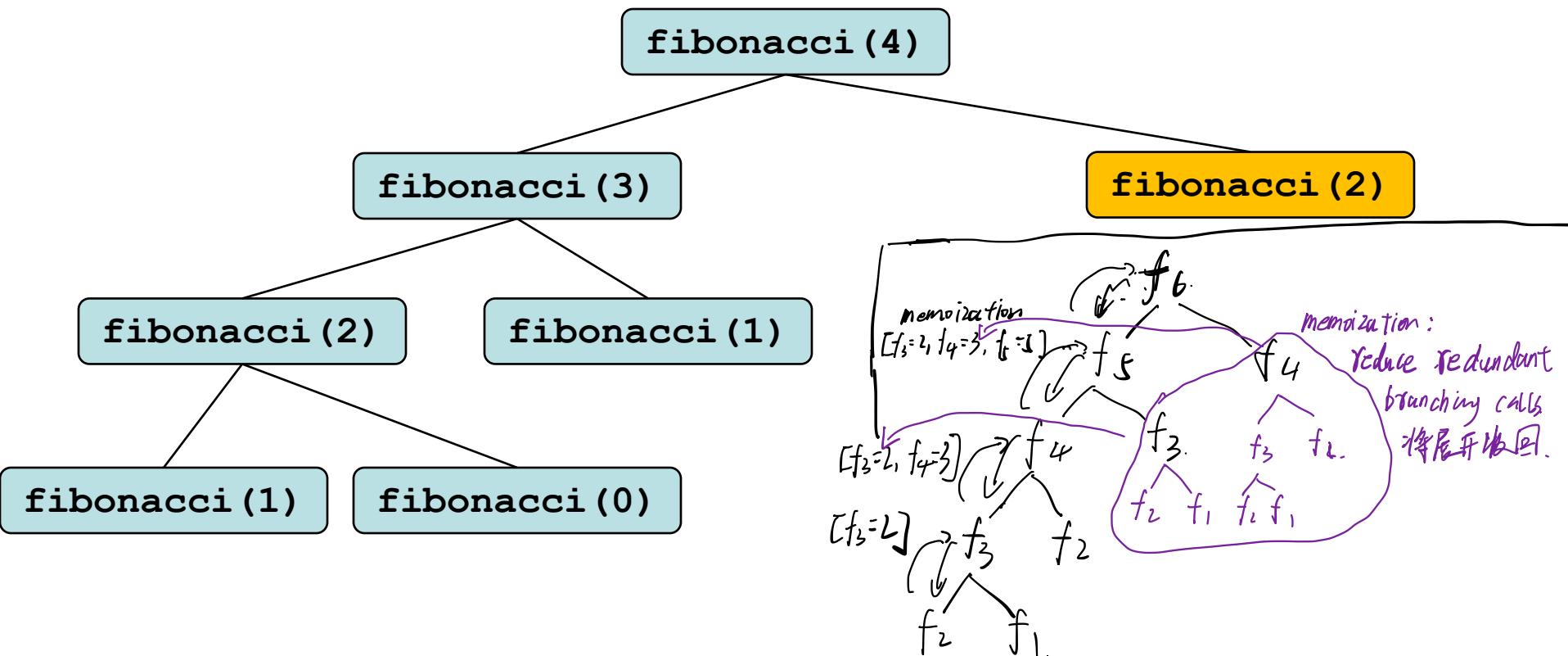
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Recursive Tree



As we recurse, we do extra work
recomputing some Fibonacci
numbers many times!

Recursive Tree



Is there a way to remember what we already computed?

Memoization

- **memoization:** Caching results of previous expensive function calls for speed so that they do not need to be re-computed.
 - Often implemented by storing call results in a collection.
- Pseudocode template:

```
cache = {}           // initially empty

function f(args):
    if I have computed f(args) before:
        Look up f(args) result in cache.
    else:
        Actually compute f(args) result.
        Store result in cache.
    Return result.
```

Memoized Fibonacci

```
// Returns the nth Fibonacci number (no error handling).
// This version uses memoization.
int fibonacci(int i, Map<int, int>& cache) {
    if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }
}
```

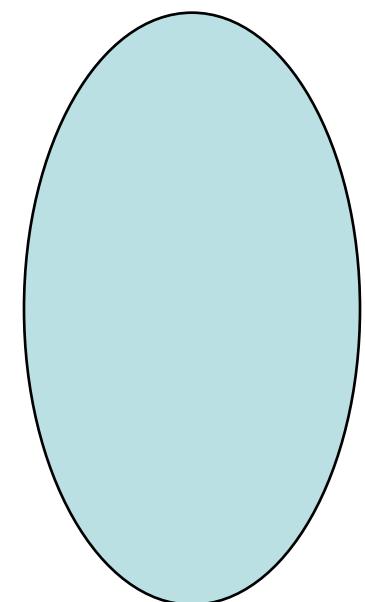
Static Cache. doesn't create new

Memoized Stack Trace

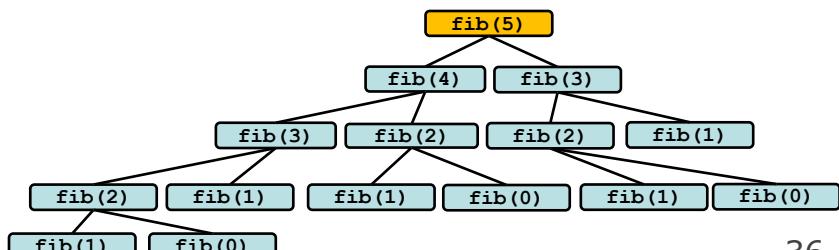
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

cache



call tree

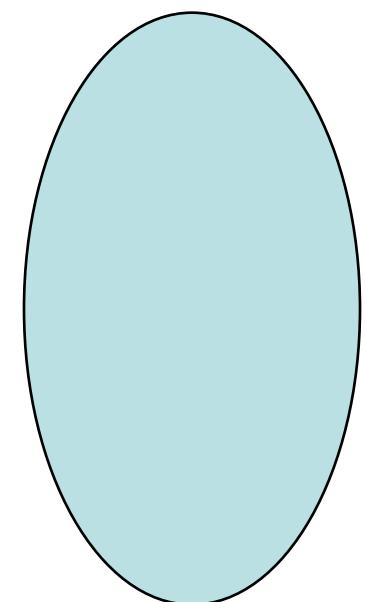


Memoized Stack Trace

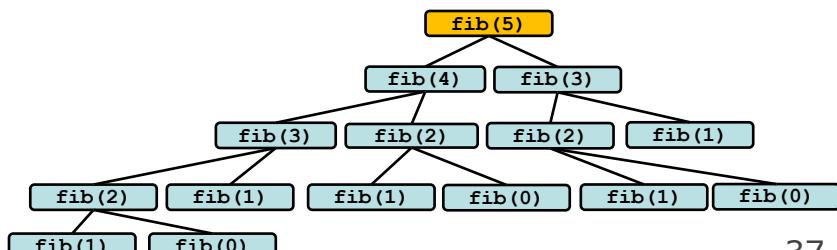
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        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

cache



call tree

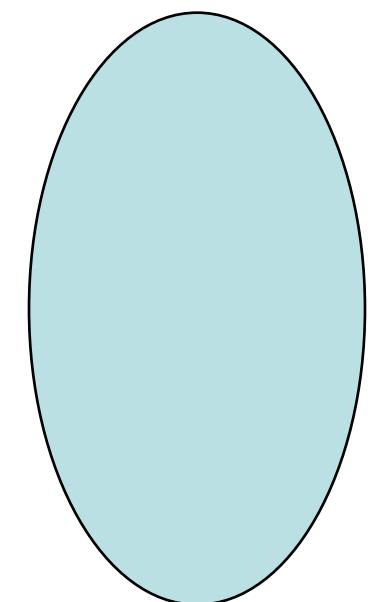


Memoized Stack Trace

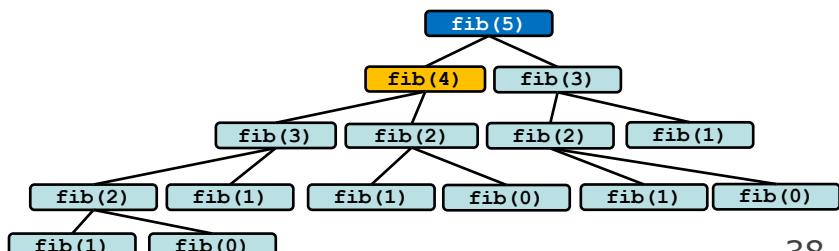
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int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache



call tree

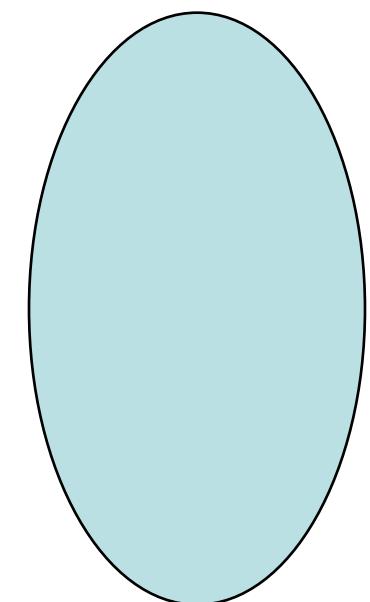


Memoized Stack Trace

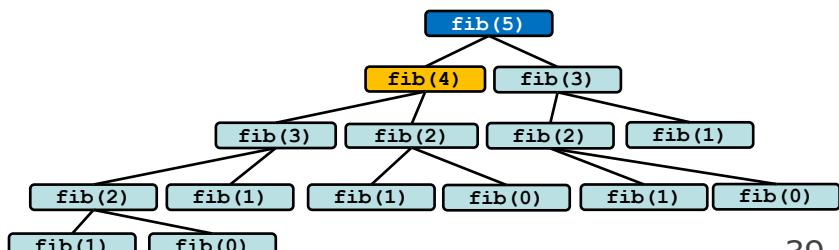
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        if (i < 2) {  
            return i;  
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            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache



call tree



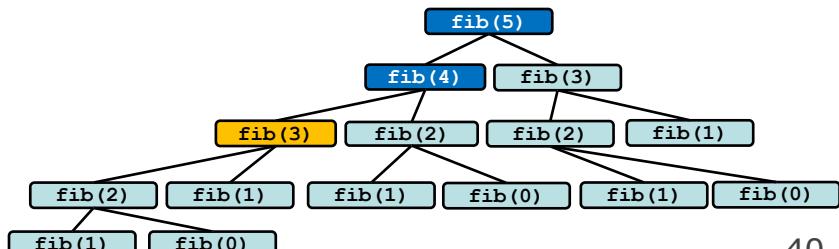
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            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
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                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

call tree



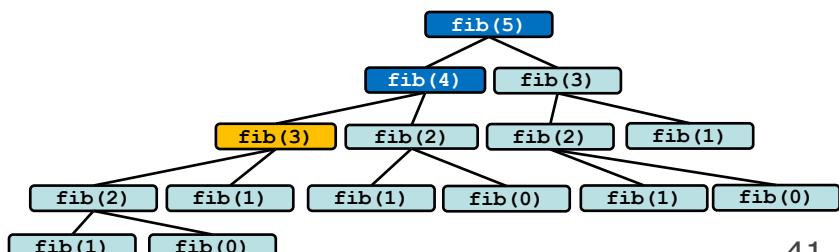
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                return i;  
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            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
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```

cache

call tree



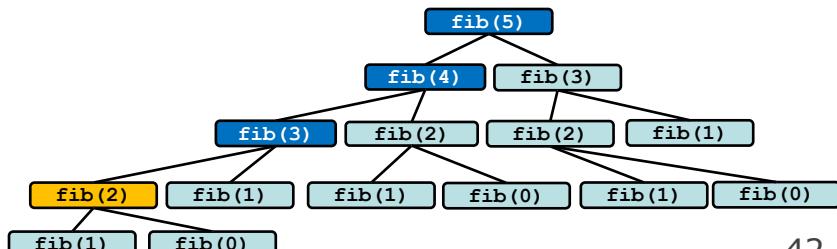
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        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
                    int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

call tree



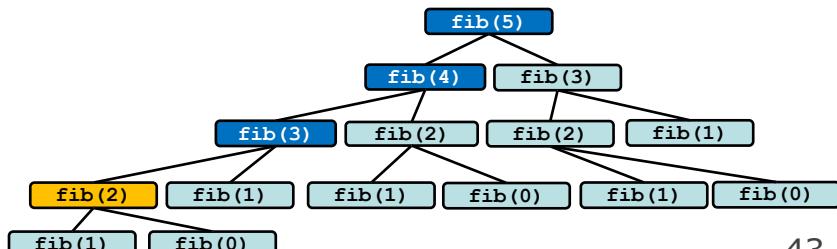
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        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
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                }  
            }  
        }  
    }  
}
```

cache

call tree



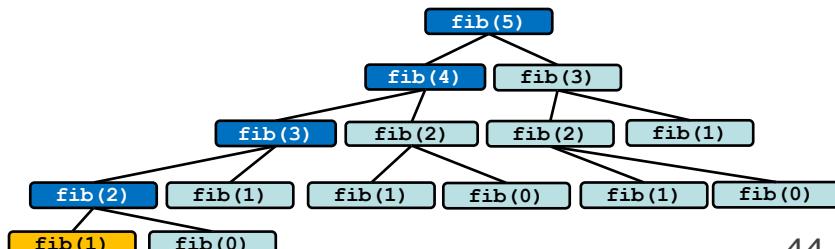
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            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                int fibonacci(int i, Map<int, int>& cache) { // i=1  
                    if (i < 2) {  
                        return i;  
                    } else if (cache.containsKey(i)) {  
                        return cache[i];  
                    } else {  
                        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
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                        return result;  
                    }  
                }  
            }  
        }  
    }  
}
```

cache

call tree



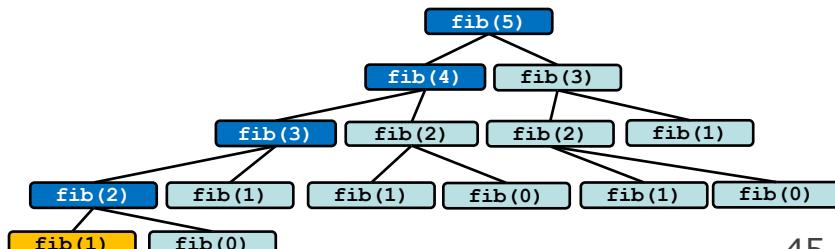
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        int fibonacci(int i, Map<int, int>& cache) { // i=3  
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                int fibonacci(int i, Map<int, int>& cache) { // i=1  
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                        return result;  
                    }  
                }  
            }  
        }  
    }  
}
```

cache

call tree



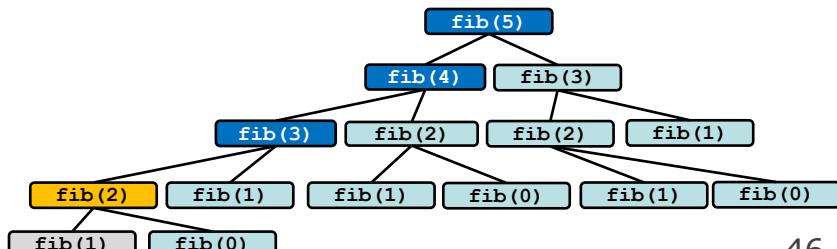
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                    return i;  
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                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

call tree



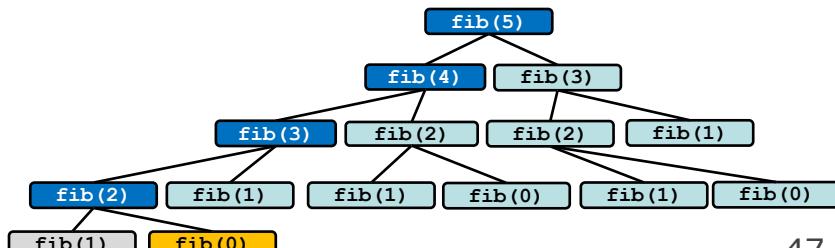
Memoized Stack Trace

```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                int fibonacci(int i, Map<int, int>& cache) { // i=0  
                    if (i < 2) {  
                        return i;  
                    } else if (cache.containsKey(i)) {  
                        return cache[i];  
                    } else {  
                        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                        cache[i] = result;  
                        return result;  
                    }  
                }  
            }  
        }  
    }  
}
```

cache

call tree



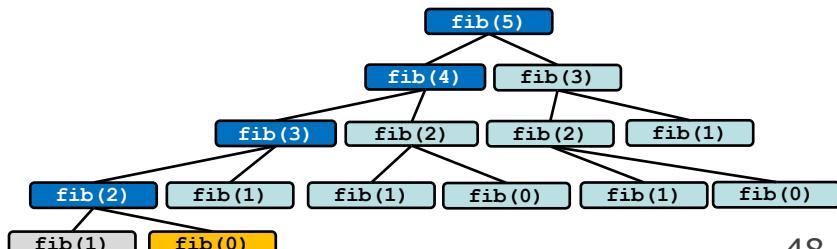
Memoized Stack Trace

```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                int fibonacci(int i, Map<int, int>& cache) { // i=0  
                    if (i < 2) {  
                        return i;  
                    } else if (cache.containsKey(i)) {  
                        return cache[i];  
                    } else {  
                        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                        cache[i] = result;  
                        return result;  
                    }  
                }  
            }  
        }  
    }  
}
```

cache

call tree



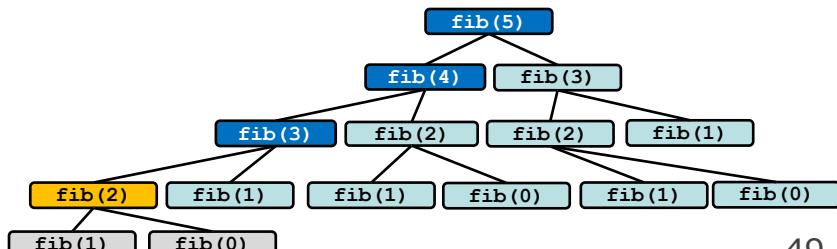
Memoized Stack Trace

```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
                    int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

call tree



Memoized Stack Trace

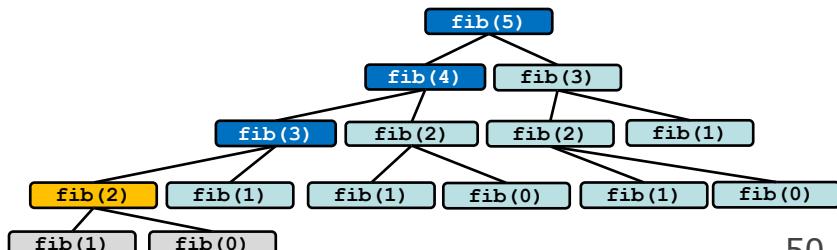
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=2  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
                    int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

2 -> 1

call tree



Memoized Stack Trace

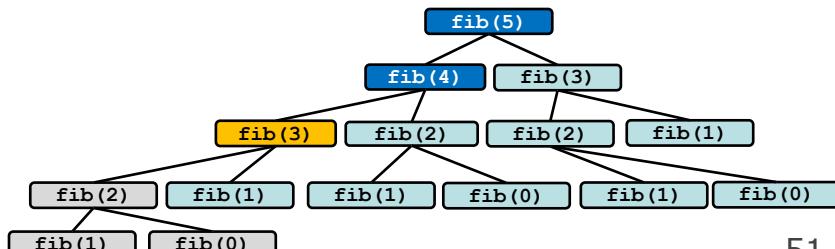
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

2 -> 1

call tree



Memoized Stack Trace

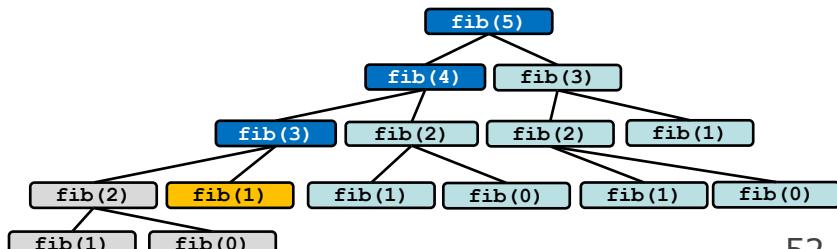
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=1  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
                    int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

2 -> 1

call tree



Memoized Stack Trace

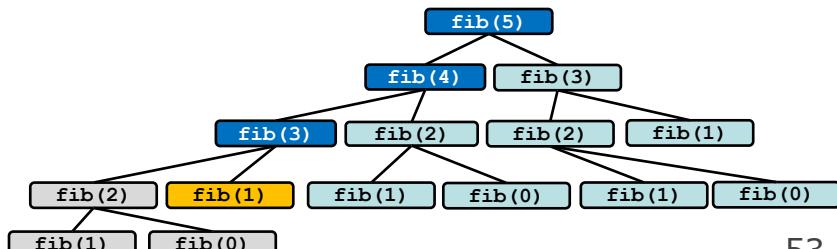
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            int fibonacci(int i, Map<int, int>& cache) { // i=1  
                if (i < 2) {  
                    return i;  
                } else if (cache.containsKey(i)) {  
                    return cache[i];  
                } else {  
                    int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                    cache[i] = result;  
                    return result;  
                }  
            }  
        }  
    }  
}
```

cache

2 -> 1

call tree



Memoized Stack Trace

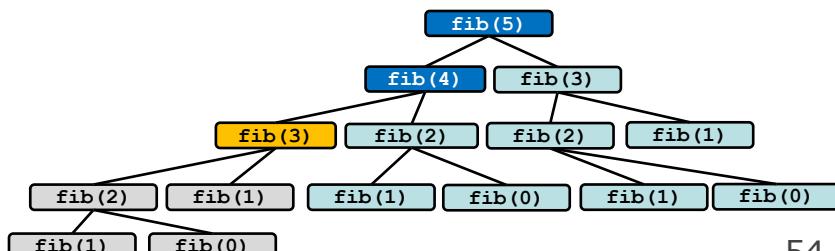
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

2 -> 1

call tree



Memoized Stack Trace

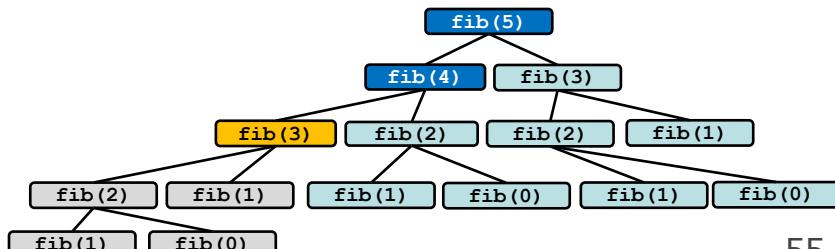
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=3  
            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

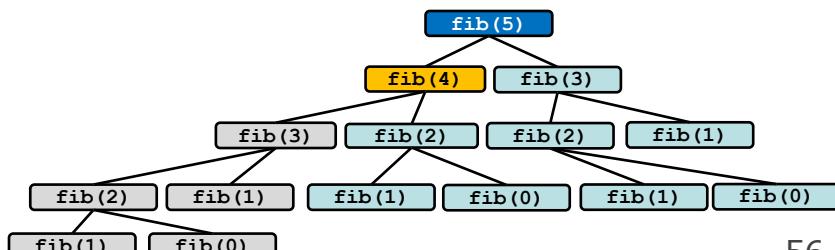
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

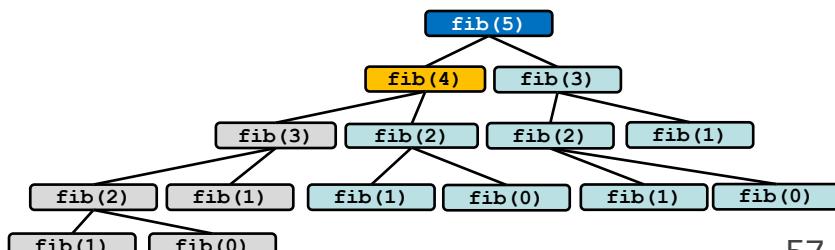
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

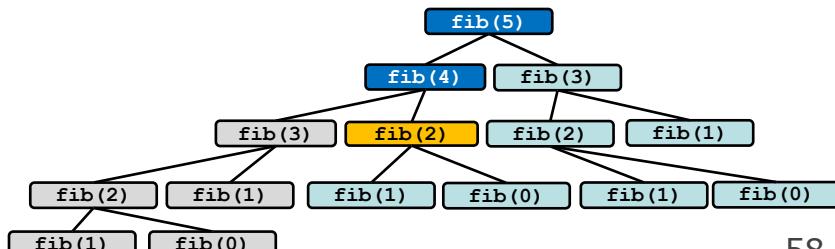
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=2  
            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

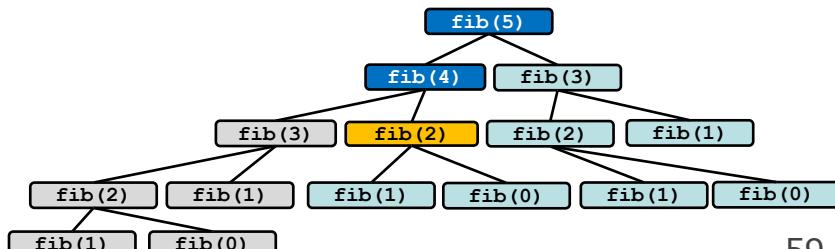
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        int fibonacci(int i, Map<int, int>& cache) { // i=2  
            if (i < 2) {  
                return i;  
            } else if (cache.containsKey(i)) {  
                return cache[i];  
            } else {  
                int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
                cache[i] = result;  
                return result;  
            }  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

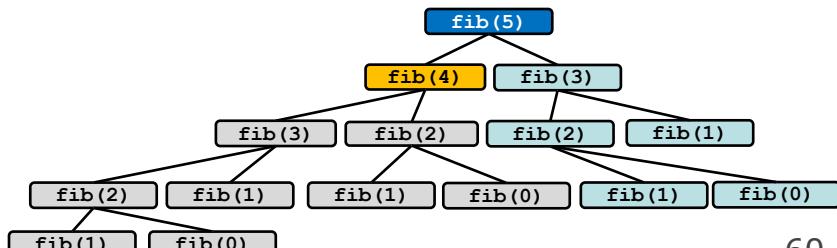
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Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
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        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2

call tree



Memoized Stack Trace

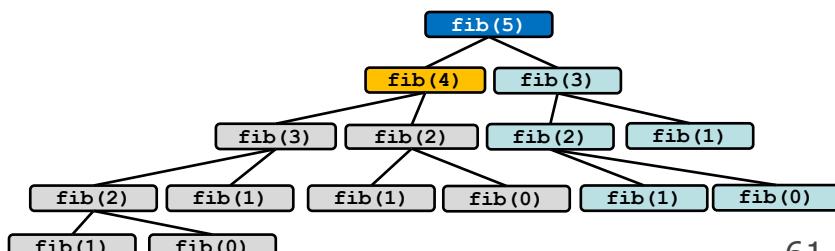
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=4  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2
4 -> 3

call tree



Memoized Stack Trace

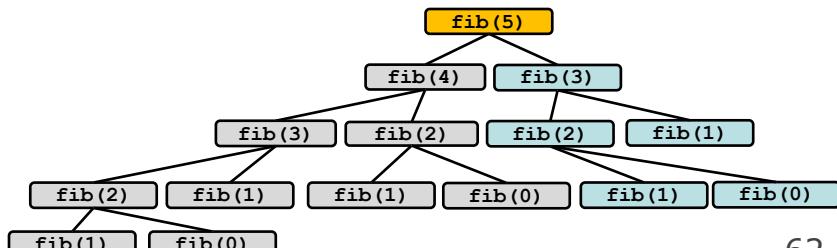
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

cache

2 -> 1
3 -> 2
4 -> 3

call tree



Memoized Stack Trace

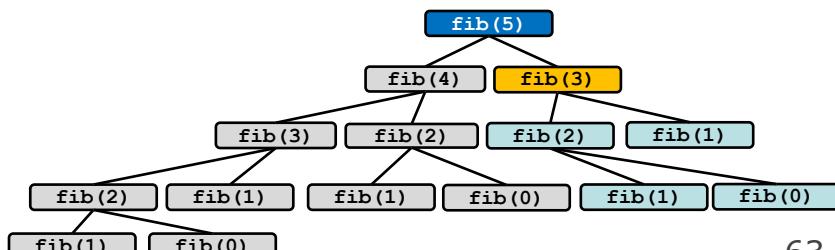
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=3  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2
4 -> 3

call tree



Memoized Stack Trace

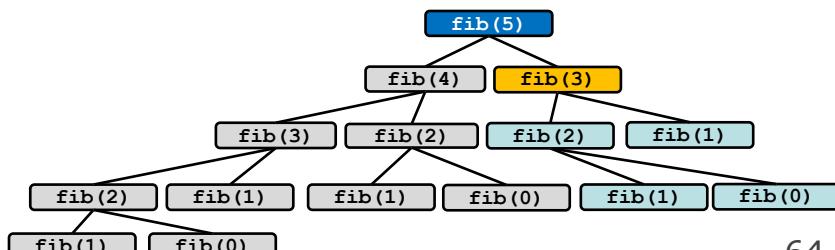
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    int fibonacci(int i, Map<int, int>& cache) { // i=3  
        if (i < 2) {  
            return i;  
        } else if (cache.containsKey(i)) {  
            return cache[i];  
        } else {  
            int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
            cache[i] = result;  
            return result;  
        }  
    }  
}
```

cache

2 -> 1
3 -> 2
4 -> 3

call tree



Memoized Stack Trace

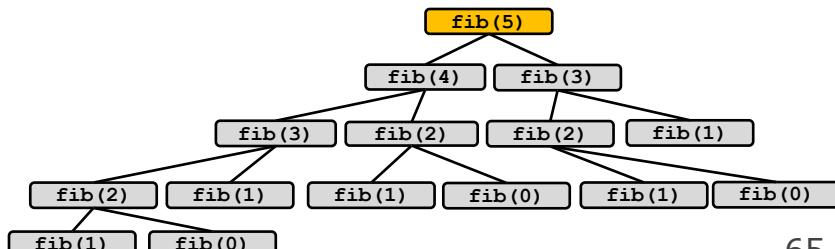
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

cache

2 -> 1
3 -> 2
4 -> 3

call tree



Memoized Stack Trace

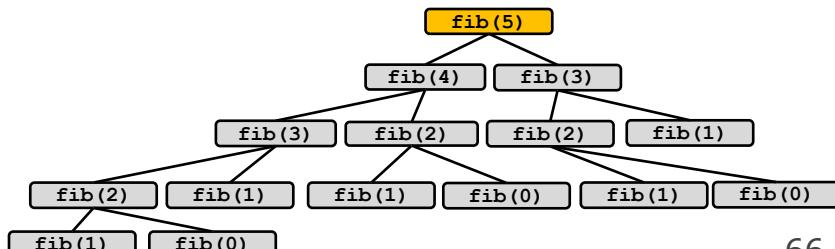
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache);
```

```
int fibonacci(int i, Map<int, int>& cache) { // i=5  
    if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

cache

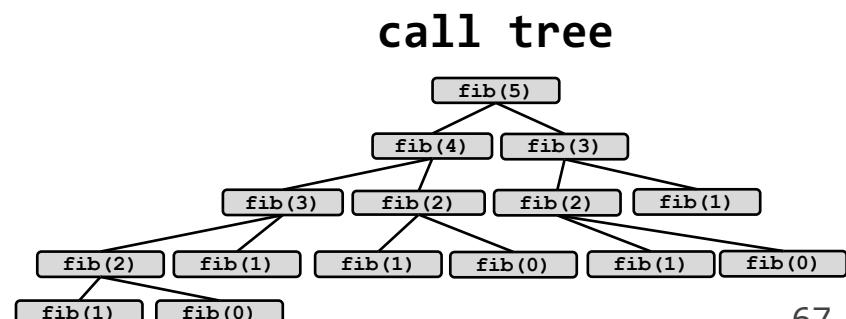
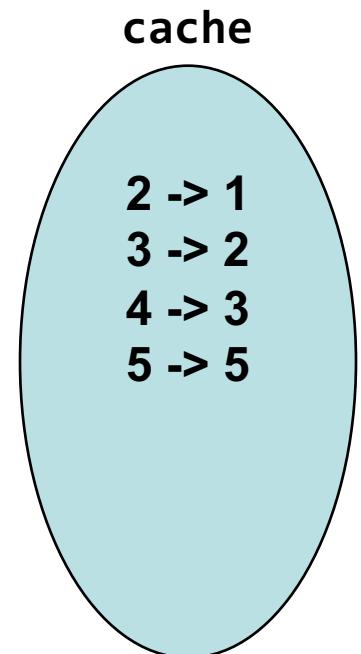
2 -> 1
3 -> 2
4 -> 3
5 -> 5

call tree

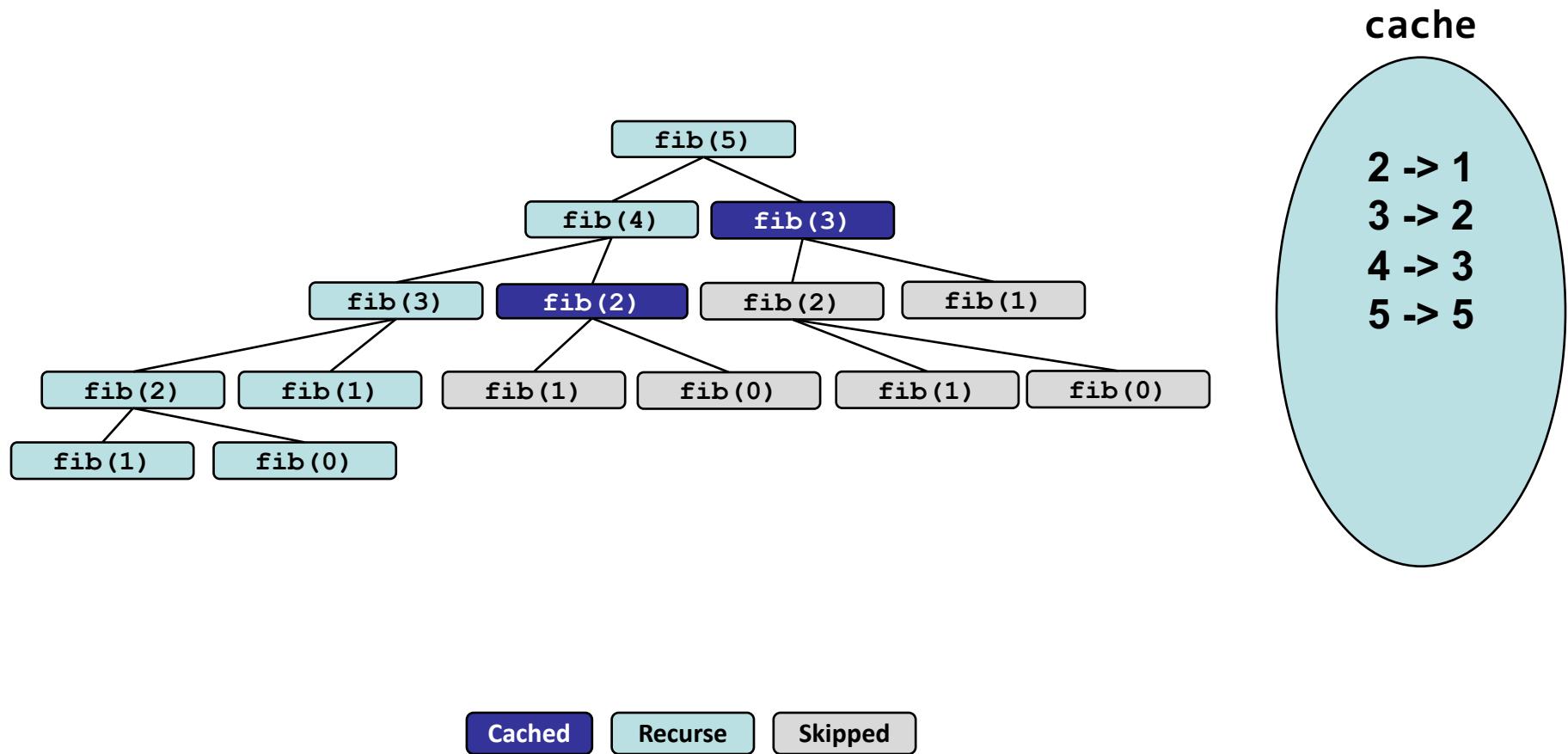


Memoized Stack Trace

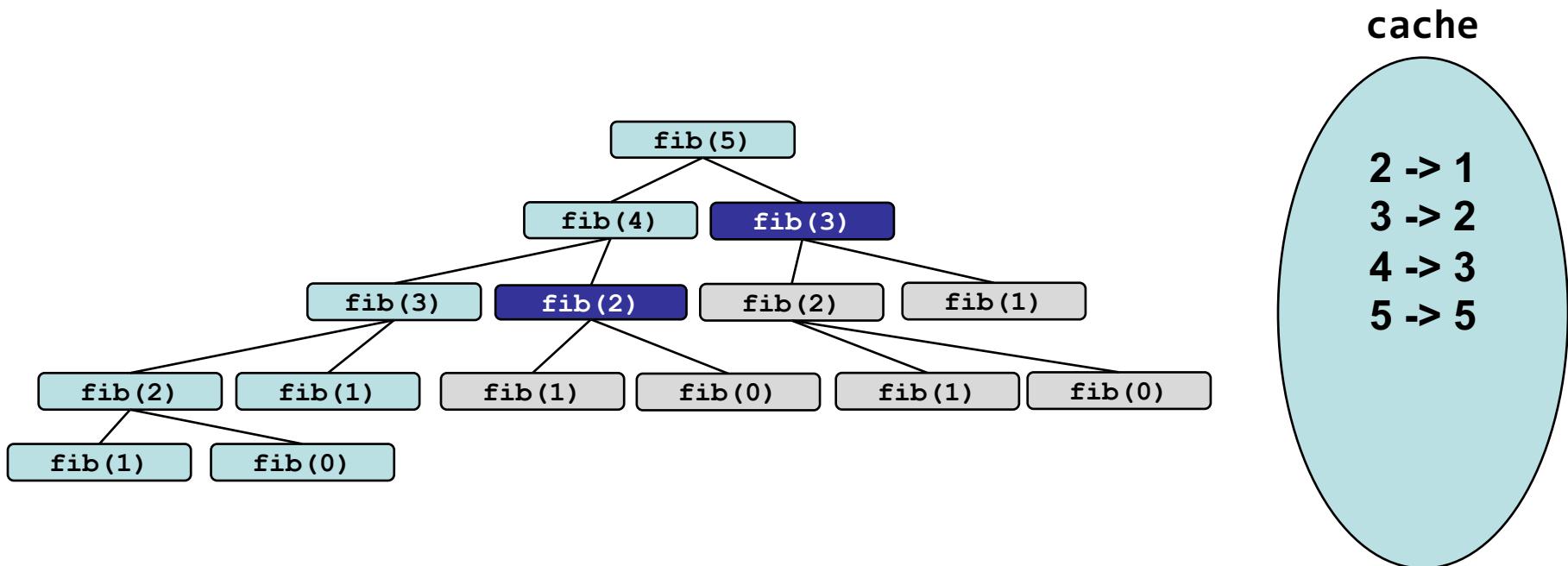
```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache); // 5
```



Memoized Fibonacci



Memoized Fibonacci



What is the runtime of this function?

$O(2^N)$ is technically correct for worst-case, but we can be more precise.

The branch factor for the memoized version becomes 1, because once we calculate $\text{fib}(n-1)$, $\text{fib}(n-2)$ will always be cached. Thus, the runtime is $O(n)!$

Plan For Today

- **Recap:** Recursion So Far
- More Practice: Reversing a File
- Recursion: Runtime
- Announcements
- Memoization
- Wrapper Functions

Wrapper Functions

```
Map<int, int> cache;  
int sixthFibonacci = fibonacci(5, cache); // 5
```

- The above function signature isn't ideal; it requires the client to know to pass in an (empty) map.
- In general, the parameters we need for our recursion will not always match those the client will want to pass.
- Is there a way we can remove that requirement, while still memoizing?
- **YES!** A “wrapper” function is a function that “wraps” around the first call to a recursive function to abstract away any additional parameters needed to perform the recursion.

That's a Wrap(per)!

```
// "Wrapper" function that returns the nth Fibonacci number.  
// This version calls the recursive version with an empty cache.  
int fibonacci(int i){  
    HashMap<int, int> cache;  
    return fibonacci(i, cache);  
}  
  
// Recursive function that returns the nth Fibonacci number.  
// This version uses memoization.  
int fibonacci(int i, HashMap<int, int>& cache) {  
    if (i < 0) {  
        throw "Illegal negative index";  
    } else if (i < 2) {  
        return i;  
    } else if (cache.containsKey(i)) {  
        return cache[i];  
    } else {  
        int result = fibonacci(i-1, cache) + fibonacci(i-2, cache);  
        cache[i] = result;  
        return result;  
    }  
}
```

包装住内部 recursive function
提供一个“全局”环境

全局在递归中用 pass by reference

The Recursion Checklist

- Find what information we need to keep track of.**
What inputs/outputs are needed to solve the problem at each step? Do we need a wrapper function?
- Find our base case(s).** What are the simplest (non-recursive) instance(s) of this problem?
- Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
- Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?



crawl exercise

- Write a function **crawl** that accepts a file name as a parameter and prints information about that file.
 - If the name represents a **normal file**, just print its name.
 - If the name represents a **directory**, print its name and information about every file/directory inside it, indented.

```
course
  handouts
    syllabus.doc
    lecture-schedule.xls
homework
  1-gameoflife
    life.cpp
    life.h
    GameOfLife.pro
```

Input: 'str' →
is File

- **recursive data:** A directory can contain other directories.

Stanford C++ files

```
#include "filelib.h"
```

Function	Description
createDirectory(<i>name</i>)	creates a new directory with given path name
deleteFile(<i>name</i>)	removes file from disk
fileExists(<i>name</i>)	whether this file exists on the disk
getCurrentDirectory()	returns directory the current C++ program runs in
getExtension(<i>name</i>)	returns file's extension, e.g. "foo.cpp" → ".cpp"
getHead(<i>name</i>), getTail(<i>name</i>)	separate a file path into the directory and file part; for "a/b/c/d.txt", head is "a/b/c", tail is "d.txt"
isDirectory(<i>name</i>)	returns whether this file name represents a directory
isFile(<i>name</i>)	returns whether this file name represents a regular file
listDirectory(<i>name</i> , <i>v</i>)	fills the given Vector<string> with the names of all files contained in the given directory
readEntireFile(<i>name</i> , <i>v</i>)	reads lines of the given file into a vector of strings
renameFile(<i>old</i> , <i>new</i>)	changes a file's name

The Recursion Checklist

- ❑ Find what information we need to keep track of.**
What inputs/outputs are needed to solve the problem at each step? Do we need a wrapper function?
- ❑ Find our base case(s).** What are the simplest (non-recursive) instance(s) of this problem?
- ❑ Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
- ❑ Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

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Wrapper Functions

We cannot vary the indentation without an extra parameter:

```
void crawl(string filename, string indent) {
```

This doesn't match the function signature we must implement, so we need to use a wrapper function.

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crawl solution

```
// Wrapper to print out this file and everything in it.
void crawl(const string& filename) {
    crawl(filename, "");
}

// Prints information about this file recursively,
// and (if it is a directory) any files inside it.
void crawl(const string& filename, const string& indent) {
    cout << indent << getTail(filename) << endl;
    if (isDirectory(filename)) {
        // recursive case; print contained files/dirs
        Vector<string> filelist;
        listDirectory(filename, filelist);
        for (string subfile : filelist) {
            crawl(filename + "/" + subfile,
                  indent + "    ");
        }
    }
}
```

Default Parameters

We could also use a default parameter *in this case*:

```
void crawl(const string& filename, const string& indent = "") {  
    cout << indent << getTail(filename) << endl;  
    if (isDirectory(filename)) {  
        // recursive case; print contained files/dirs  
        Vector<string> filelist;  
        listDirectory(filename, filelist);  
        for (string subfile : filelist) {  
            crawl(filename + "/" + subfile,  
                  indent + "    ");  
        }  
    }  
}
```

But for the purposes of this example we'll stick with a wrapper function. Sometimes default parameters can do the trick, but not always – e.g. if we need to pass by reference, or initialize a data structure dynamically).

Plan For Today

- **Recap:** Recursion So Far
- More Practice: Reversing a File
- Recursion: Runtime
- Announcements
- Memoization
- Wrapper Functions

Next time: fractals (Recursion you can see!)