Logic programming with Prolog

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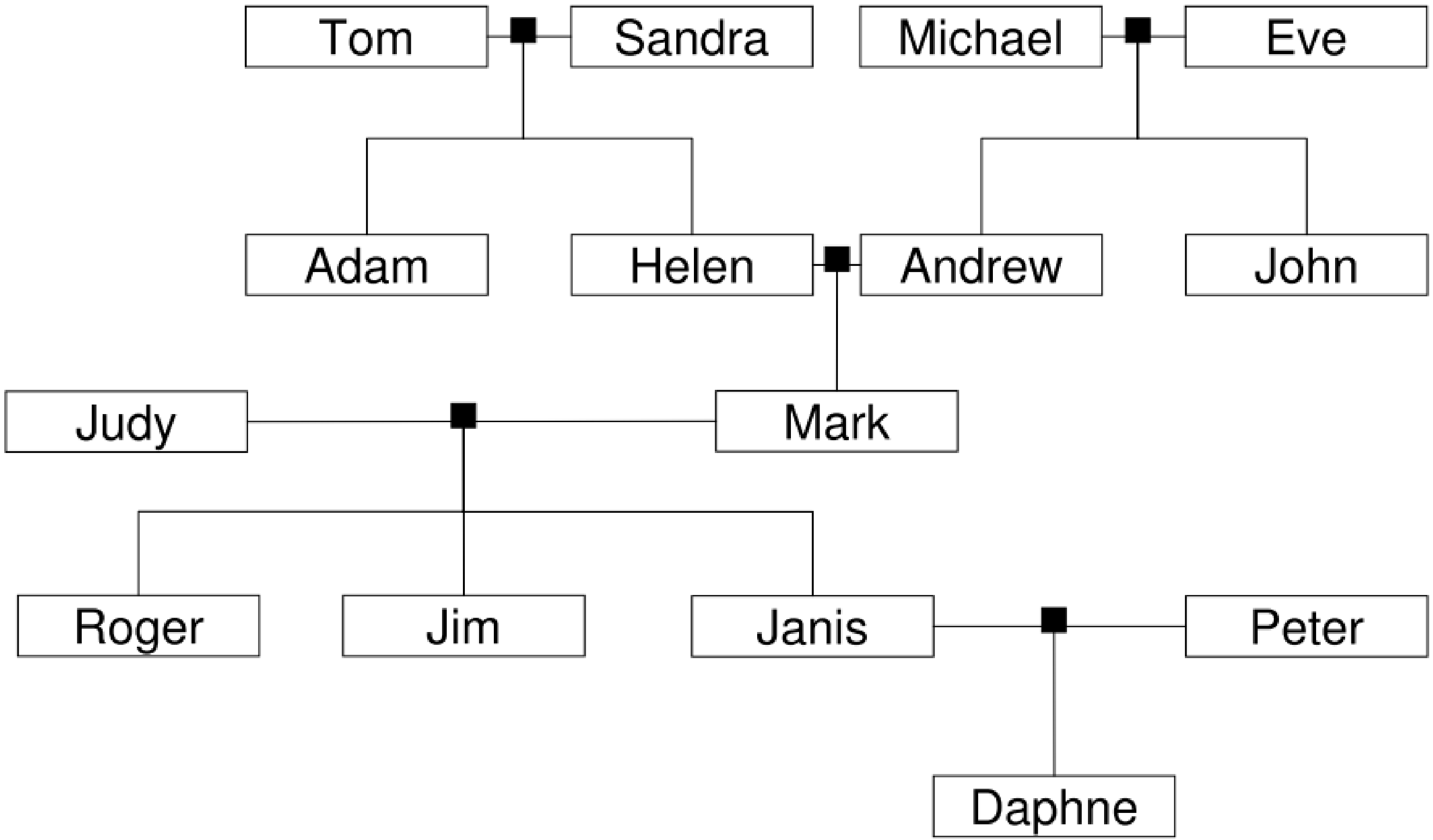
A running example: A family genealogy tree



Programs and statements

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Prolog programs consist of collections of statements (called assertions, or clauses).

Statements and procedures

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Prolog programs consist of collections of statements (called assertions, or clauses).

Statements are grouped into procedures. In the example, we have one procedure named parent, made up of several statements.

Procedures and arguments

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Prolog programs consist of collections of statements (called assertions, or clauses).

Statements are grouped into procedures.
In the example, we have one procedure named ‘parent’, made up of several statements.

Each procedure defines a certain relationship between its arguments.

The programmer decides on how to interpret this relationship. Here, parent(tom, adam) will be interpreted as “*Tom is the parent of Adam*.”

Statements revisited: Facts and rules

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Prolog programs consist of collections of statements (called assertions, or clauses).

Statements are grouped into procedures.
In the example, we have one procedure named ‘parent’, made up of several statements.

Each procedure defines a certain relationship between its arguments.

The programmer decides on how to interpret this relationship. Here parent(tom, adam) will be interpreted as “Tom is the parent of Adam.”

There are two kinds of clauses: facts and rules.

Facts are propositions declared to be True.
In the example, the procedure named parent consists only by facts.

Questions and queries

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).

The collection of statements constitutes a (declarative) database.

We can pose queries on this database.

A query is the codification of a question.

There are only two types of queries:

1. *Is it indeed the case that a given statement is true?* (ground query)

2. *Under what conditions, if any, is a given statement true?* (non-ground query)

Ground queries

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Ground queries result in a Yes/No (or True/False) response:

For example, the question

“*Is it indeed the case that Peter is the parent of Daphne?*”

will be codified into a query and executed as

```
?- parent(peter, daphne).
```

Evaluation of ground queries

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

Prolog will take the query

?- parent(peter, daphne).

and will start searching the database from top to bottom, one statement at a time trying to match ("unify") it with a statement.

In trying the first statement, a match (unification) is *not* successful.

Evaluation of ground queries /cont.

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).

Prolog will then try to match the query

?- parent(peter, daphne).

against the next statement in the program.

Again, if not successful, it will try the next statement in the program…etc. until either a match is found or until the database is exhausted.

Evaluation of ground queries /cont.

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

In this example, Prolog will eventually succeed, having matched the query

```
?- parent(peter, daphne).
```

with the last statement.

Prolog will respond *Yes* (or *true*) to the query.

We have managed to prove that it is indeed the case that parent(peter, daphne) is true.

Non-ground queries

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
```

A non-ground query involves one or more variables.

The question *"Who is a parent of Daphne"* can be transformed into a non-ground query as

?- parent(X, daphne).

where X (note the capitalization) is a variable.

We are asking Prolog to seek instantiation(s) for variable X, provided any exist, that could make the query succeed.

Unification revisited

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).

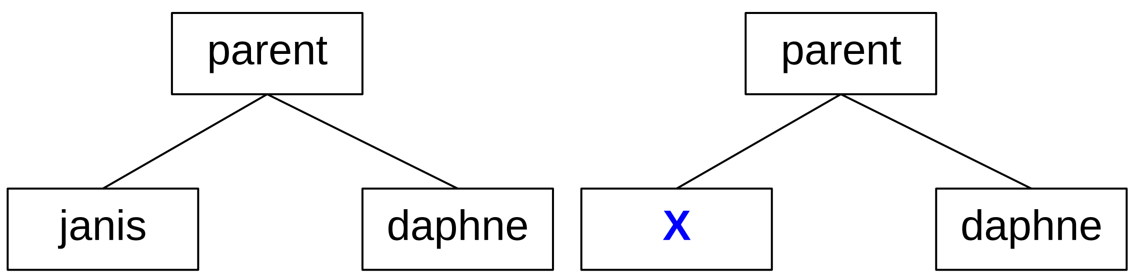
Unification (or matching) is a basic operation on terms.

A ground query can unify with a statement,e.g.

?- parent(tom, adam).

A non-ground query can unify with a statement only if substitution can be made for any variables so that the two terms can be made equal. In this example, we have

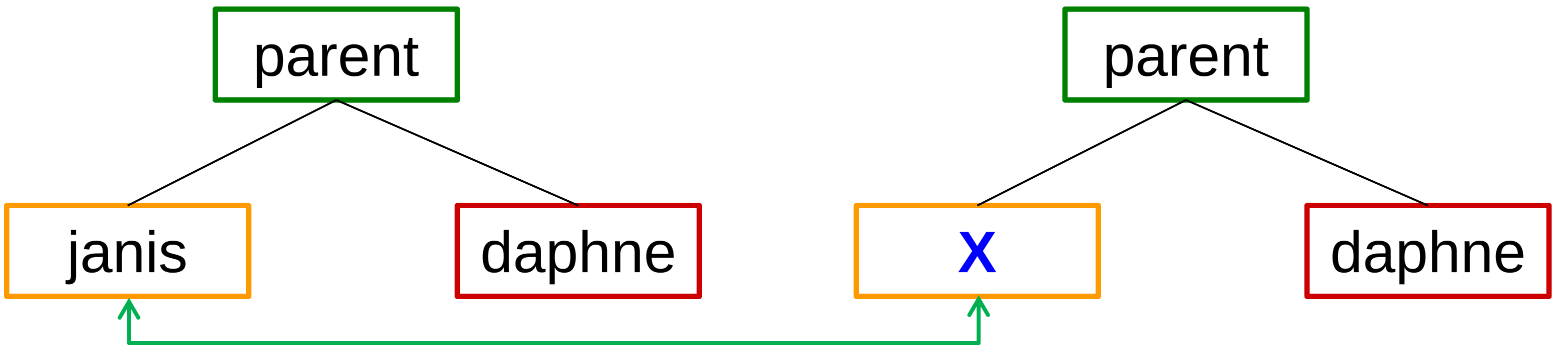
?- parent(X, daphne).



Tree representation of a statement.

Tree representation of a non-ground query.

Unification revisited /cont.



- The terms parent(janis, daphne) and parent(X, daphne) unify instantiating X to janis.

- There is in fact one more solution, because there are two possible choices to unify with parent(X, daphne).

Rules: Head and body

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).

A rule statement gives rules of implication between propositions. The general form is

head :- body.

(or conclusion :- condition.)

which reads

“The head (of the rule) is true, if the body is true.”,

or, alternatively:

“The head of the rule can succeed if the body of the rule can succeed.”

Formulae and rules

- Let us extend the database with a new procedure grandparent.

- Let p stand for isParentOf relation and let g stand for isGrandParentOf relation.

- We can define g in terms of p by the following formula: For persons x, y, z:

$$G = \forall\ x\ \forall\ y\ \forall\ z\ ((p(x,z) \wedge p(z,y)) \rightarrow g(x,y))$$

- We can represent the formula with the rule below:

 grandparent(X, Z) :- parent(X, Y), parent(Y, Z).

Rules and goals

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).

Consider the following rule statement:

$$\underbrace{\text{grandparent(X, Z)}}_{\text{head}} \text{ :- } \underbrace{\overbrace{\text{parent(X, Y)}}^{\text{goal}}\text{, }\overbrace{\text{parent(Y, Z)}}^{\text{goal}}\text{.}}_{\text{body}}$$

The body of rule grandparent contains two goals.

The goals are related by a conjunction (denoted by the comma symbol).

Extending the database

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
                     parent(Y, Z).
```

The rule is added to the database.

Evaluation of a ground query in the presence of rules

An example

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
parent(Y, Z).

Consider the query grandparent(judy, daphne).

Prolog will search its database from top to bottom and

a) **unify** the query with the head of the rule,

b) **instantiate** X to judy and Z to daphne,

grandparent(X, Z) :- parent(X, Y), parent(Y, Z).

?- grandparent(judy, daphne).

Evaluation of a ground query in the presence of rules

An example /cont.

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
                     parent(Y, Z).
```

c) **resolve** to two new queries (that correspond to the two goals of the rule):

parent(judy, Y), parent(Y, daphne).

Both queries must now be evaluated (in the order specified) and if both prove true, then the rule succeeds (and the answer to the query is Yes/True).

Evaluation of a ground query in the presence of rules

An example /cont.

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
                     parent(Y, Z).
```

The two goals

parent(judy, Y), parent(Y, daphne).

will be evaluated as follows:

The first goal parent(judy, Y), will be executed as any other query, unifying with parent(judy ,roger), and instantiating Y to roger.

Once the first goal succeeds, Prolog will try the next one on the right, for the same instantiation:

Can roger make the second goal succeed?

No. The query parent(roger, daphne) is not successful.

Evaluation of a ground query in the presence of rules

An example /cont.

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
parent(Y, Z).

Prolog will continue searching the database to find matches that can satisfy both goals

parent(judy, Y), parent(Y, daphne).

The first goal, parent(judy, Y), can unify with parent(judy, jim), instantiating Y to jim.

Can jim make the second goal succeed?

No. The query parent(jim, daphne) is not successful.

Evaluation of a ground query in the presence of rules

An example /cont.

parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
parent(Y, Z).

Prolog will continue searching the database to find matches that can satisfy both goals

parent(judy, Y), parent(Y, daphne).

The first goal, parent(judy, Y), can unify with parent(judy,janis), instantiating Y to janis.

Can janis make the second goal succeed?

Yes. The query parent(janis, daphne) is successful.

Evaluation of a non-ground query in the presence of rules: An example

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
                     parent(Y, Z).
```

The question

“Who are the grandparents of Daphne?”

is codified into the query

?- grandparent(G, daphne).

This will unify with the head of rule grandparent, instantiating variable Z to daphne and resolving into two goals:

parent (G, Y), parent(Y, daphne).

Evaluation of a non-ground query in the presence of rules: An example

```
parent(tom, adam).
parent(tom, helen).
parent(sandra, adam).
parent(sandra, helen).
parent(michael, andrew).
parent(michael, john).
parent(eve, andrew).
parent(eve, john).
parent(helen, mark).
parent(andrew, mark).
parent(judy, roger).
parent(judy, jim).
parent(judy, janis).
parent(mark, roger).
parent(mark, jim).
parent(mark, janis).
parent(janis, daphne).
parent(peter, daphne).
grandparent(X, Z) :- parent(X, Y),
                     parent(Y, Z).
```

```
?- grandparent(X, daphne).
X = judy ;
X = mark ;
No
```

Upon finding a match, Prolog will stop here and wait for instructions. The semicolon symbol (;) inquires whether more matches can be found.

A period symbol (.) would indicate our intention to stop the search.

Multi-line rules: Disjunction

- We can now further extend the database with a rule to define ancestor relation:

```
ancestor(X, Y) :- parent(X, Y).
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
```

disjunction

conjunction

- A rule can be placed in more than one lines.

- In this case, there is a disjunction between the two lines, and there is a conjunction between the two goals of the body of the second rule.

Further extending the database

```
man(tom).
man(michael).
man(adam).
man(andrew).
man(john).
man(mark).
man(roger).
man(jim).
man(peter).
woman(sandra).
woman(eve).
woman(helen).
woman(judy).
woman(janis).
woman(daphne).
```

```
father(X, Y) :- man(X),
                parent(X, Y).
mother(X, Y) :- woman(X),
                parent(X, Y).
```

We extend the database by introducing four procedures:

man, woman: made up of facts, and
father, mother: rules.

```
father(X, Y)   :- man(X),
                  parent(X, Y).

mother(X, Y) :- woman(X),
                parent(X, Y).
```

Anonymous variables in rules

- If any parameter of a relation is not important, we can replace it with an anonymous variable denoted by the underscore character (_) as follows:

```
is_father(X) :- father(X, _).
is_mother(X) :- mother(X, _).
```

- We can now pose more queries such as *"Is Tom a father?"*
- To answer this type of question, it does not matter who Tom is the father of, as long as Tom is found as the first term in a father statement. The query is as follows:

```
?- is_father(tom).
true
```

Anonymous variables in queries

- Alternatively we can use anonymous variables in queries, such as

```
?- father(tom, _).
true
```

Lists

- Lists are represented in square brackets […].
- The empty list is represented by [].
- Every non-empty list can be represented in two parts:
  - The head, which is the first element.
  - The tail, which is the list containing the remaining elements.

  - The head of [john, eve, paul] is john.
  - The tail of [john, eve, paul] is [eve, paul].

List representations

- The symbol | in [H|T] represents a list whose head is H and whose tail is T.
- We can represent the above example as [john | [eve, paul]]
- Since [eve, paul] is also a list with head eve and tail [paul], we can write the above list as [john | [eve | [paul]]]
- Any one-element list can be written as that element joined to the empty list. Thus, [paul] is the same as [paul | []]
- We can now write the full list as [john | [eve | [paul | [ ]]]]

Example: Checking for list membership

- We want to define a procedure member(X,L) which succeeds if X is an element of a list L.
- We can define list membership recursively as follows:
- X is a member of L if

  X is the head of L (regardless of what the tail is), or

  member(X, [X|_]).

  X is a member of the tail of L (regardless of what the head is).

  member(X, [_|T]) :- member(X, T).

Example: Checking for list membership /cont.

```
?- member(a, [a, b, c]).
true

?- member(a, []).
false.

?- member([b, c], [[a], [b, c]]).
true

?- member(X, [a, b]).
X = a ;
X = b ;
false.
```

Example: The last element in a list

- In this example, we want to define a rule which succeeds if an element is found to be in the last position of a non-empty list.
- We can identify two cases for this:
- The list has one element.
- The list has more than one element.

Example: The last element in a list /cont.

- Case 1: The list has only one element.
- In this case, the last element is the only existing element of the list.
- The following rule,

last(L, [L]).

reads *“Rule last succeeds if element L is found to be the only element of a given list.”*

Example: The last element in a list /cont.

- Case 2: The list has more than one element.
- In this case, we need to reduce the problem to the one that can be handled by case 1.
- In other words, the clause will succeed once it chops off all elements, one by one, until it ends up with one element.

Example: The last element in a list /cont.

- The following rule,

  last(L, [H|T]) :- last(L, T).

  reads *“Rule last can succeed for a list whose head is H and whose tail is T, if it can succeed for a new list which is the tail T of the original list.”*

Example: The last element in a list /cont.

- In other words, let us get rid of the first element and see if we end up with only one element in which case the rule of case 1 will determine that this remaining element is indeed the last element.

- However, if after getting rid of the first element we end up with a list with more than one elements, we must repeat this chopping off the head of the list, until we end up with a list which has only one element and subsequently handled by the first rule (of case 1).

Example: The last element in a list /cont. Evaluation of a ground query [1 of 3].

Given the rule,

```
last(L, [L]).
last(L, [H|T]) :- last(L, T).
```

Consider the query

```
?-  last(c, [a, b, c]).
```

Prolog will (search its database from top to bottom and)

unify the query with the head of the second statement of the rule,
instantiate variable L to c and variable [H|T] to [a | b, c],
resolve to a new query (that corresponds to the body of the rule):

```
last(c, [b, c]).
```

This new query must now be evaluated.

Example: The last element in a list /cont. Evaluation of a ground query [2 of 3].

Given the rule,

last(L, [L]).

last(L, [H|T]) :- last(L, T).

The query
?- last(c, [b, c]).

Will cause Prolog to (perform a new search and)

unify the query with the head of the second statement of the rule,
instantiate variable L to c and variable [H|T] to [b | c],
resolve to a new query (that corresponds to the body of the rule):

last(c, [c]).

This new query must now be evaluated.

Example: The last element in a list /cont. Evaluation of a ground query [3 of 3].

Given the rule,

```
last(L, [L]).
last(L, [H|T]) :- last(L, T).
```

The query

```
?-  last(c, [c]).
```

Will cause Prolog to (perform a new search and)

unify the query with the head of the first statement of the rule, and yield a success, indicating that it is indeed the case that c is found in the last position of the list.

Example: Calculating the size of a list

- Consider a rule size/2 to read in a list and calculate its length.

```
size([],0).
size([H|T],N) :- size(T,N1), N is N1+1.
```

- We can execute queries as follows:

```
?- size([],N).
N = 0.
?- size([a,b,c],N).
N = 3.
?- size([[a,b],c],N).
N = 2.
?- size([[a,b,c]],N).
N = 1.
```

Built-in utility functions

- The built-in function findall(X, P, L) returns a list L with all values for X that satisfy predicate P.
- To eliminate redundancies in a list, we can use the built-in function list_to_set(List, Set) that converts the list (with possibly repeated elements) into a set.
- The built-in function length(List, L) returns the length L of a given list.

Example with **findall** and **list_to_set** in a query

- Let us obtain a set of all fathers:

```
?- findall(F, father(F, _), Lst).
Lst = [tom, tom, michael, michael, andrew, mark, mark, mark, peter].

?- findall(F, father(F, _), Lst), list_to_set(Lst, Set).
Lst = [tom, tom, michael, michael, andrew, mark, mark, mark, peter],
Set = [tom, michael, andrew, mark, peter].
```

Example with **findall** and **list_to_set** in a rule

- The query findall(F, father(F, _), Lst), list_to_set(Lst, Set) is rather long and complex.

- We can encapsulate its size and complexity in a rule:

```
get_all_fathers(Set) :- findall(F, father(F, _), Lst),
                        list_to_set(Lst, Set).
```

```
?- get_all_fathers(Set).
Set = [tom, michael, andrew, mark, peter].
```

Example with **findall** and **length** in a rule

- Let us construct a rule qualifies_for_benefits(P) that succeeds if P is a mother of at least three children.

```
qualifies_for_benefits(P) :-
                        woman(P),
                        findall(P, parent(P, _), L),
                        length(L, N),
                        N >= 3.

?- qualifies_for_benefits(Name).
Name = judy ;
false.
```