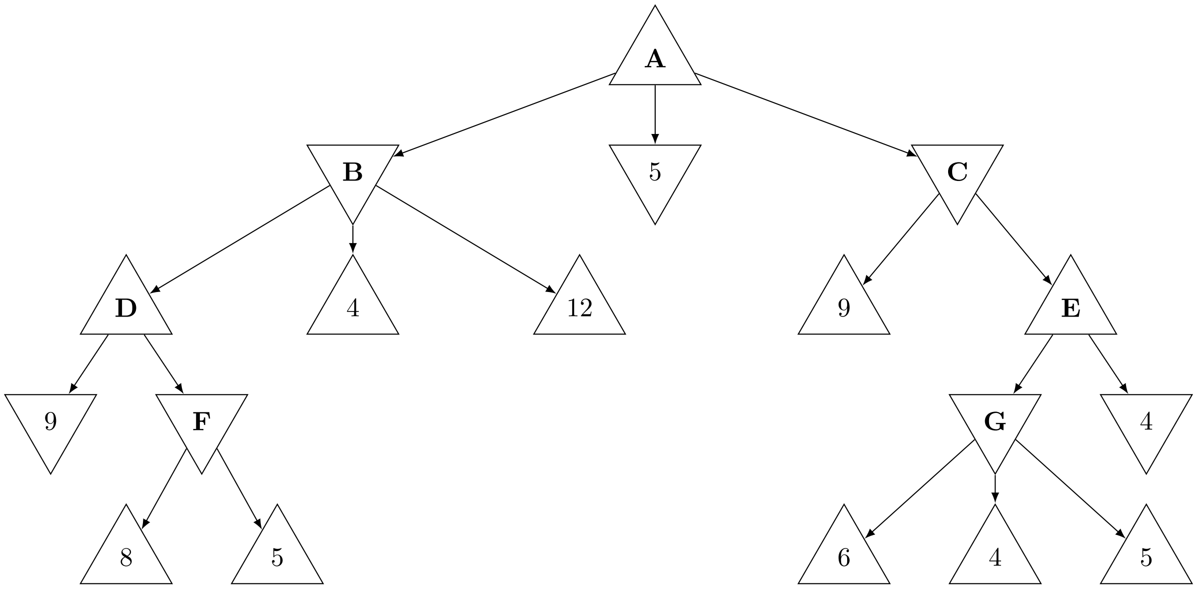
CS 540 Homework Assignment # 5

Zhenyu Zou

October 11, 2018

1 Question 1: Game Search Tree



1. (a) Game theoretical value at each node is following:
   level 4 = node F: min(8,5) = 5, node G: min(6,4,5) = 4
   level 3 = node D: max(9,5) = 9, node E: max(4,4) = 4
   level 2 = node B: min(9,4,12) = 4, node C: min(9,4) = 4
   level 1 = node A: max(4,4,5) = 5

   (b) If we start from the bottom and apply to the game theoretical value that we computed, it would follow the following procedure:
   node F: A→B→D→F→5
   node D: A→B→D→9
   node B: A→B→4
   node A: A→5
   node G: A→C→E→G→4

node E: A→C→E→4

node C: A→C→E

2. We denote node x = (α, β) to denote their alpha and beta value in order for potential pruning. At the final time of the pruning process, we derive the following alpha beta values for the nodes.
   Node F = $(\alpha = 9, \beta = 8)$, Node D = $(\alpha = 9, \beta = 8)$, Node B = $(\alpha = 9, \beta = 4)$
   Node A = $(\alpha = 5, \beta = 4)$, Node C = $(\alpha = 4, \beta = 4)$, Node E = $(\alpha = 4, \beta = 4)$
   Node G = $(\alpha = 4, \beta = 4)$
   The subtree that we have pruned is right child of Node F(5), child of Node B(12), children of Node G(5)

3. The final result for the root node will be th same, but we may have different path. Since the alpha-beta algorithm will take action greedily, it will always stops at the first node and stop. Then it may produce different solution path under the same graph. But, there will not exist a situation where applying the alpha-beta pruning will produce different answer. Since the logic behind is that. Say you have a level 1 max node trying to pick the maximum, and the level 2 min node is trying to pick the minimum node. Say you already have a pending max node that the value is 100. However, when you exploring for the min node, and you happen to see a node that is lower than 100. We can be sure that the min node must be lower than 100. Then there is no point keep exploring this min node, since no matter what this min node chooses. It will not affect the result of the level 1 max node.

2 Question 2: Probability

1. (a) Determine X = total of seven in two rolls. The whole space of rolling two fair die is $6 * 6 = 36$, and the probability of rolling any number is $P(n) = \frac{1}{36}$, and the total possible total of seven is $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$, therefore P(X) = $\frac{6}{36}$

   (b) Determine X = total of seven in three rolls. The whole space of rolling three fair die is $6*6*6 = 216$, and the probability of rolling any number is $P(n) = \frac{1}{36}$, and the total possible total of seven is $(1,1,5), (1,2,4), (1,3,3), (1,4,2), (1,5,1), (2,1,4), (2,2,3), (2,3,2), (2,4,1), (3,1,3), (3,2,2)$ $, (3,3,1), (4,1,2), (4,2,1), (5,1,1)$, therefore P(X) = $\frac{15}{216}$

   Assume x = number of rolls. Therefore, $P(x = 2) = \frac{1}{6} > P(x = 3) = \frac{15}{216}$

2. The probability of the first one being Heart Ace is $\frac{1}{50} * \frac{49}{49} = \frac{1}{50}$ and the second one being Heart Ace is $\frac{1}{49} * \frac{49}{50} = \frac{1}{50}$, therefore the total probability is $\frac{1}{50} + \frac{1}{50} = \frac{1}{25}$

3. The probability of picking a clover leaves that have four leaflets is P(Four) = $\frac{1}{10000}$,Then $((1-\frac{1}{10000}))^n$, is when you pick n times the outcome being not four leaflets. Thus, $1-(\frac{9999}{10000})^n = 0.9, (\frac{9999}{10000})^n = 0.1, n = \log_0 .99990.1 = 23024.6996$ times.

4. Assume R = picking a card that one side is red, B = picking a card that one side is black. Then, $P(R) = \frac{1}{2}, P(B) = \frac{1}{2}$. Since we have observed that one side is black, Then $P(R|B) = \frac{P(B|R)P(R)}{P(B)} = \frac{\frac{1}{3}\frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$

5. Since, $P(lottery|spam) = 0.42, P(lotter|non-spam) = 0.05, P(spam) = 0.5, P(non-spam) = 0.5$ And we are given that it contains lottery, thus

$$P(spam|lottery) = \frac{P(lottery|spam)P(spam)}{P(lottery)}$$

$$P(spam|lottery) = \frac{P(lottery|spam)P(spam)}{P(lottery, spam) + P(lottery, non-spam)}$$

$$P(spam|lottery) = \frac{P(lottery|spam)P(spam)}{P(lottery|spam)P(spam) + P(lottery|non-spam)P(non-spam)}$$

$$P(spam|lottery) = \frac{0.42 * 0.5}{0.42 * 0.5 + 0.05 * 0.5}$$

$$P(spam|lottery) \approx 0.893$$