CS 540 HW 2

Yien Xu

September 17, 2017

Question 2: State Space

An (m, n, k)-puzzle is a sliding puzzle with m columns, n rows, and k empty squares, where $1 \leq k < mn$. As usual, one move is to move a single tile to an adjacent (up, down, left, right) empty square, if available.

1. (10 points) How many tiles are there in a (m, n, k)-puzzle in general? For example, there are 8 tiles in a $(3, 3, 1)$-puzzle.

   The puzzle has $m \times n$ squares in total but k empty squares. So there are $mn - k$ tiles.

2. (20 points) Let each tile have a distinct name. How many distinct states are there in the state space? Show your derivation.

   If we have $m \times n$ squares in total and k empty squares, then we can choose k squares to be empty from mn square. Since every tile has a distinct name (this is actually a permutation), we have to re-arrange them by multiplying $(mn - k)!$.

$$\binom{mn}{k} \cdot (mn - k)! = \frac{(mn)!}{k! \cdot (mn - k)!} \cdot (mn - k)! = \frac{(mn)!}{k!}$$

3. (20 points) Draw a graph that corresponds to the state space of a $(2, 2, 1)$-puzzle, and **briefly describe your graph**. This is not an art project: You may represent the states in ways easy for you to type, and you do not necessarily need drawing programs – you may even use plaintext. You can also hand draw the graph, but please clearly show the nodes and edges.

   Let a square box depict a state. Every box contains four numbers, which correspond to the name of a tile, where 0 represents an empty square. All edges are double ended with cost 1, which means puzzles can go back and forth easily. According to the derivation above, there are

$$\frac{4!}{1!} = 4 \times 3 \times 2 = 24$$

   states possible. The state graph is split into two halves. For example, if the initial state is $\{0, 1, 2, 3\}$, then only the left half of the graph can be reached; if the initial state is $\{0, 2, 1, 3\}$, then only the other half can be reached.