TA section 2

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Review Exercises

#14:

Q: A, B 不獨立, 問: $\mathbb{P}(A|B) > \mathbb{P}(A)$?

A: No.

- Two events A and B are independent if and only if: $\mathbb{P}(A|B) = \mathbb{P}(A)$.
- If A,B are dependent, then ${\rm I\!P}(A|B) \neq {\rm I\!P}(A)$ that we can only identify.
- Example: $\Omega=\{1,2,3,4\}$, $\mathbb{P}(\{1\})=0.1, \mathbb{P}(\{2\})=0.2, \mathbb{P}(\{3\})=0.3, \mathbb{P}(\{4\})=0.4.$
- Let $A := \{3, 4\}, B := \{1, 2\}$. Then, $A \cap B = \phi$.
- $\mathbb{P}(A \cap B) = 0 \neq \mathbb{P}(A) \mathbb{P}(B) = 0.7 \times 0.3 = 0.21$. 即, A, B 不獨立, 但:
- $\mathbb{P}(A|B) = \mathbb{P}(A \cap B) / \mathbb{P}(B) = 0/0.3 = 0 < \mathbb{P}(A) = 0.7.$



3/9

- 10. 設A與B獨立, A與C獨立, 且 $B \cap C = \emptyset$ 。
 - (i) 試證A與B∪C獨立;
 - (ii) 試舉一例說明若 $B \cap C \neq \emptyset$, 則(i)之結論便不一定成立。
- (i). $(A \cap B) \cap (A \cap C) = A \cap (B \cap C) = A \cap \phi = \phi$,
- $\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}((A \cap B) \cup (A \cap C)) = \mathbb{P}((A \cap B) + \mathbb{P}((A \cap C)) = \mathbb{P}(A)\mathbb{P}(B) + \mathbb{P}(A)\mathbb{P}(C) = \mathbb{P}(A)(\mathbb{P}(B) + \mathbb{P}(C)) = \mathbb{P}(A)\mathbb{P}(B \cup C).$ So, $A \perp \!\!\! \perp (B \cup C).$

4/9

• (ii).
$$\Omega = \{1, 2, 3, 4\}$$
. $\mathbb{P}(\{w\}) = 1/4, \forall w \in \Omega$. Let $A = \{1, 2\}, B = \{1, 3\}, C = \{1, 4\}, \text{ then } B \cap C = \{1\} \neq \phi$.

- $\mathbb{P}(A) = \mathbb{P}(B) = \mathbb{P}(C) = 1/2$. $\mathbb{P}(B \cap C) = \mathbb{P}(\{1\}) = 1/4$.
- $B \cup C = \{1, 3, 4\}$, $\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}(\{1\}) = 1/4 \neq \mathbb{P}(A) \mathbb{P}(B \cup C) = 1/2 \times 3/4 = 3/8$.



13. 設
$$P(A)$$
, $P(B) > 0$, 且 $P(A|B) > P(A)$ 。試證 $P(B|A) > P(B)$ 。

• Given ${\rm l\!P}(A|B)>{\rm l\!P}(A)$, it implies that $P(A\cap B)>{\rm l\!P}(A)\,{\rm l\!P}(B)$. Then,

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} > \frac{\mathbb{P}(A)\mathbb{P}(B)}{\mathbb{P}(A)} = \mathbb{P}(B).$$



15. 試證

(i)
$$P(A^c|B) = 1 - P(A|B)$$
;

(ii)
$$P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C) \circ$$

- Hint:
 - (i) by $\mathbb{P}(B) = \mathbb{P}(B \cap A) + \mathbb{P}(B \cap A^c)$.
 - (ii) by $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$.

• (i).
$$\mathbb{P}(A^c|B) = \frac{\mathbb{P}(A^c \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A \cap B) + \mathbb{P}(A^c \cap B) - \mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B) - \mathbb{P}(A \cap B)}{\mathbb{P}(B)} = 1 - \mathbb{P}(A|B).$$

• (ii).
$$\mathbb{P}((A \cup B)|C) = \frac{\mathbb{P}((A \cup B) \cap C)}{\mathbb{P}(C)} = \frac{\mathbb{P}((A \cap C) \cup (B \cap C))}{\mathbb{P}(C)} = \frac{\mathbb{P}(A \cap C) + \mathbb{P}(B \cap C) - \mathbb{P}(A \cap B \cap C)}{\mathbb{P}(C)} = \mathbb{P}(A|C) + \mathbb{P}(B|C) - \mathbb{P}((A \cap B)|C).$$



4. 投擲一公正的骰子一次,令X表所得之點數除以4之餘數。試求X之值域 Ω_1 , 並對 $\forall x \in \Omega_1$, 給出 $P_X(X=x)$ 。

- $(1 \mod 4) = 1$; $(2 \mod 4) = 2$; $(3 \mod 4) = 3$; $(4 \mod 4) = 0$; $(5 \mod 4) = 1$; $(6 \mod 4) = 2$.
- 令投擲骰子之點數為 $Y, Y \in \{1, 2, 3, 4, 5, 6\}$, 點數除四餘數為 $X, X \in \{0, 1, 2, 3\}$;
- $\Omega_1 := \{0, 1, 2, 3\};$
- $\mathbb{P}(X=0) = \mathbb{P}(Y=4) = 1/6$, $\mathbb{P}(X=1) = \mathbb{P}(Y \in \{1,5\}) = 2/6$, $\mathbb{P}(X=2) = \mathbb{P}(Y \in \{2,6\}) = 2/6$, $\mathbb{P}(X=3) = \mathbb{P}(Y=3) = 1/6$;
- pmf: