

Problem 1: Bayes Decision Rule (30%)

DLCV #1

$$P(x|w_1) = \frac{1}{5} \text{ for } 0 \leq x \leq 5$$

$$P(x|w_2) = \frac{1}{3} \text{ for } 3 \leq x \leq 6$$

$$P(w_1) = \frac{3}{4}, P(w_2) = \frac{1}{4}$$

$$P(x) = \begin{cases} \frac{1}{5} \times \frac{3}{4} = \frac{3}{20} & \text{for } 0 \leq x < 3 \\ \frac{1}{5} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{4} = \frac{7}{30} & \text{for } 3 \leq x < 5 \\ \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} & \text{for } 5 \leq x \leq 6 \end{cases}$$

$$P_e = \min(P(w_1|x), P(w_2|x))$$

$$= \int_0^6 \min(P(w_1|x), P(w_2|x)) P(x) dx$$

$$= \int_0^3 P(w_2|x) P(x) dx + \int_3^5 P(w_2|x) P(x) dx + \int_5^6 P(w_1|x) P(x) dx$$

$$= \int_3^5 P(w_2|x) P(x) dx$$

$$= 2 \times \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{1}{5} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{4}} \times \frac{7}{30}$$

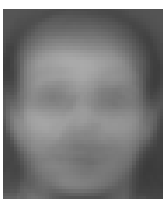
$$= \frac{1}{6}$$

decision region: $R_1: [0, 5]$ $5 < x \leq 6$
 $R_2: (5, 6]$ $P(w_1|x)=0$ $P(w_2|x)=1$

Problem 2: Principal Component Analysis and k-Nearest Neighbors Classification (70%)

(a) (10%) Perform PCA on the training set. Plot the mean face and the first three eigenfaces.

The mean face:



The first three eigen-faces:







(first,second, and third)

(b) (25%) Take person 1 image 1 , and project it onto the above PCA eigenspace. Reconstruct this image using the first $n = 3, 50, 100, 239$ eigenfaces. For each n , compute the mean square error (MSE) between the reconstructed face image and person 1 image 1 . Please plot these reconstructed images, with the corresponding MSE values.



The original image from 1_1.png

n	3	50	100	239
reconstructed image				
MSE	663.04	213.06	82.32	0.5093

(c) (35%) To apply the k-nearest neighbors classifier to recognize test set images, please determine the best k and n values by 3-fold cross-validation. For simplicity, the choices for such hyperparameters are $k = \{1, 3, 5\}$ and $n = \{3, 50, 159\}$. Please show the cross-validation results and explain your choice for (k, n) . Finally, use your hyperparameter choice to report the recognition rate on the test set.

$k \backslash n$	3	50	159
1	78.75% / 72.5% / 68.75%	95% / 88.75% / 96.25%	95% / 90% / 95%
3	63.75% / 62.5% / 52.5%	87.5% / 82.5% / 81.25%	87.5% / 82.5% / 81.25%
5	56.25% / 55% / 47.5%	81.25% / 76.25% / 75%	80% / 75% / 71.25%

The table shows the accuracy of cross-validation. From the table, $(50,1)$ and $(159,1)$ both result in highest accuracy in average. Thus, I chose both of them to report the recognition rate on the test set.

$(1/50)$: 96.25%

$(1/159)$: 94.375%

As shown, the choice of $(50,1)$ lead to higher accuracy